Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.3-General-binomial/1.1.3.4/54-1.1.3.4-a

Nasser M. Abbasi

May 18, 2024 Compiled on May 18, 2024 at 9:59pm

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**896**]. This is test number [54].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.78 (894)	0.22 (2)
Mathematica	99.78 (894)	0.22 (2)
Maple	84.82 (760)	15.18 (136)
Fricas	74.33 (666)	25.67 (230)
Mupad	49.89 (447)	50.11 (449)
Giac	49.00 (439)	51.00 (457)
Sympy	46.76 (419)	53.24 (477)
Maxima	37.17 (333)	62.83 (563)
Reduce	27.12 (243)	72.88 (653)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

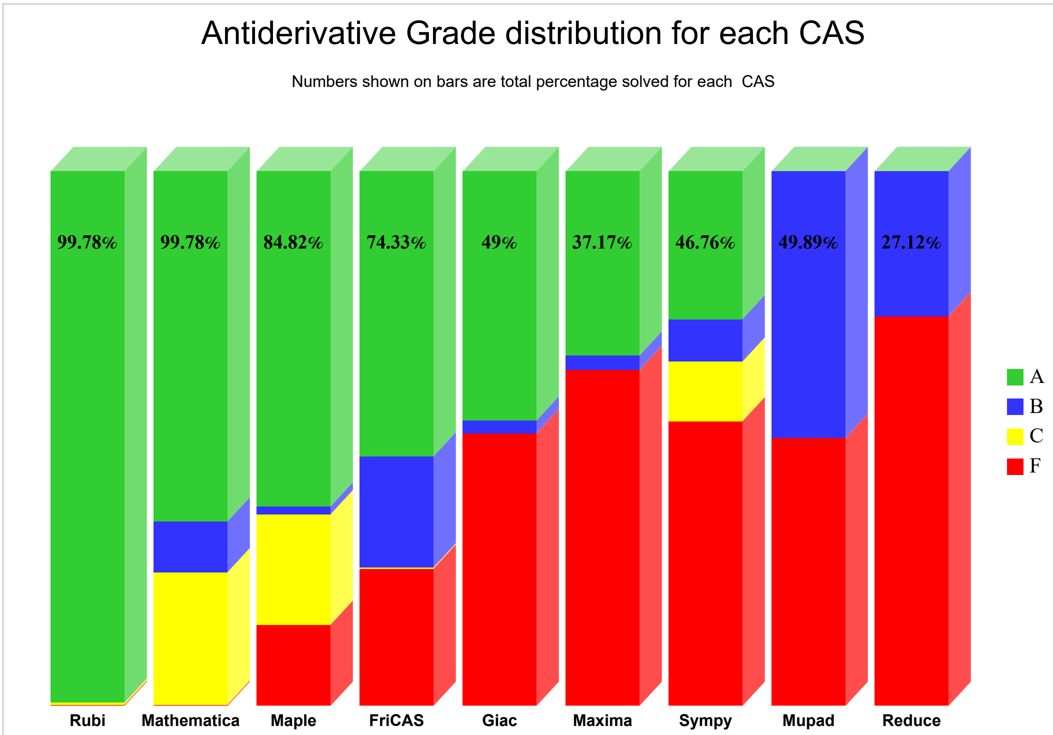
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

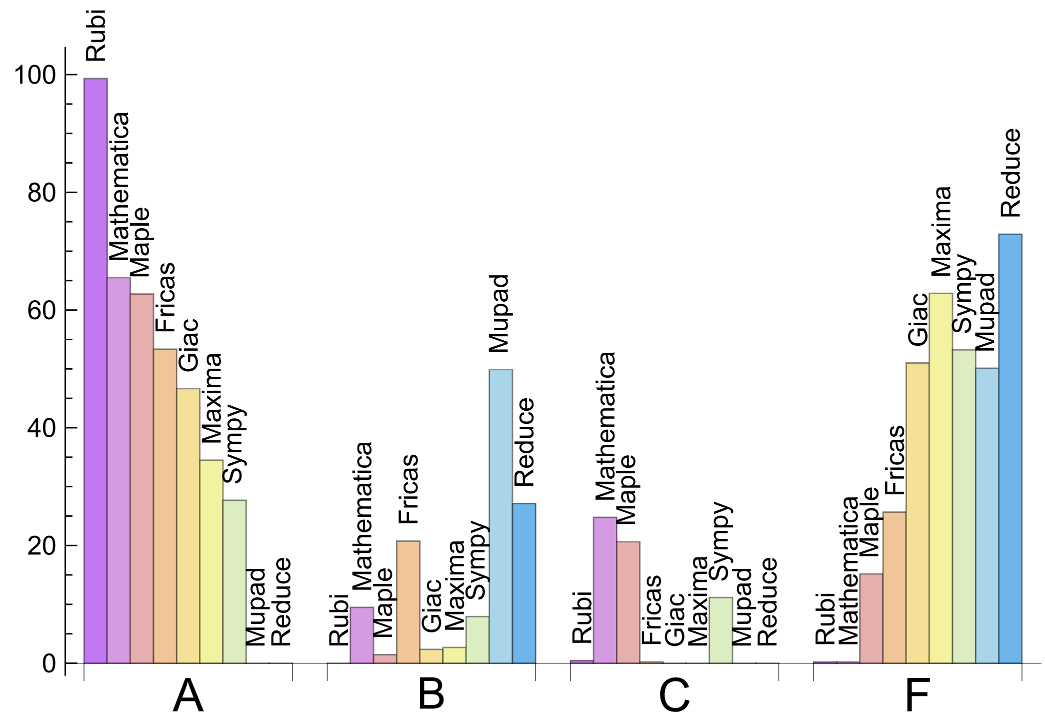
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.330	0.000	0.446	0.223
Mathematica	65.513	9.487	24.777	0.223
Maple	62.723	1.451	20.647	15.179
Fricas	53.348	20.759	0.223	25.670
Giac	46.652	2.344	0.000	51.004
Maxima	34.487	2.679	0.000	62.835
Sympy	27.679	7.924	11.161	53.237
Mupad	0.000	49.888	0.000	50.112
Reduce	0.000	27.121	0.000	72.879

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	2	100.00	0.00	0.00
Mathematica	2	100.00	0.00	0.00
Maple	136	100.00	0.00	0.00
Fricas	230	49.13	50.00	0.87
Mupad	449	0.00	100.00	0.00
Giac	457	97.37	0.00	2.63
Sympy	477	81.13	18.87	0.00
Maxima	563	90.59	0.00	9.41
Reduce	653	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.08
Giac	0.17
Reduce	0.23
Rubi	0.59
Fricas	0.82
Maple	2.21
Mupad	2.55
Mathematica	3.52
Sympy	14.27

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Reduce	86.36	0.76	70.00	0.68
Maxima	126.07	1.10	109.00	1.02
Mathematica	144.72	1.13	111.00	0.93
Giac	152.19	1.15	118.00	1.03
Sympy	188.75	1.63	97.00	1.05
Rubi	203.62	1.02	122.00	1.00
Maple	303.58	1.94	152.00	0.97
Mupad	352.38	1.98	119.00	1.12
Fricas	651.17	3.48	212.50	1.77

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

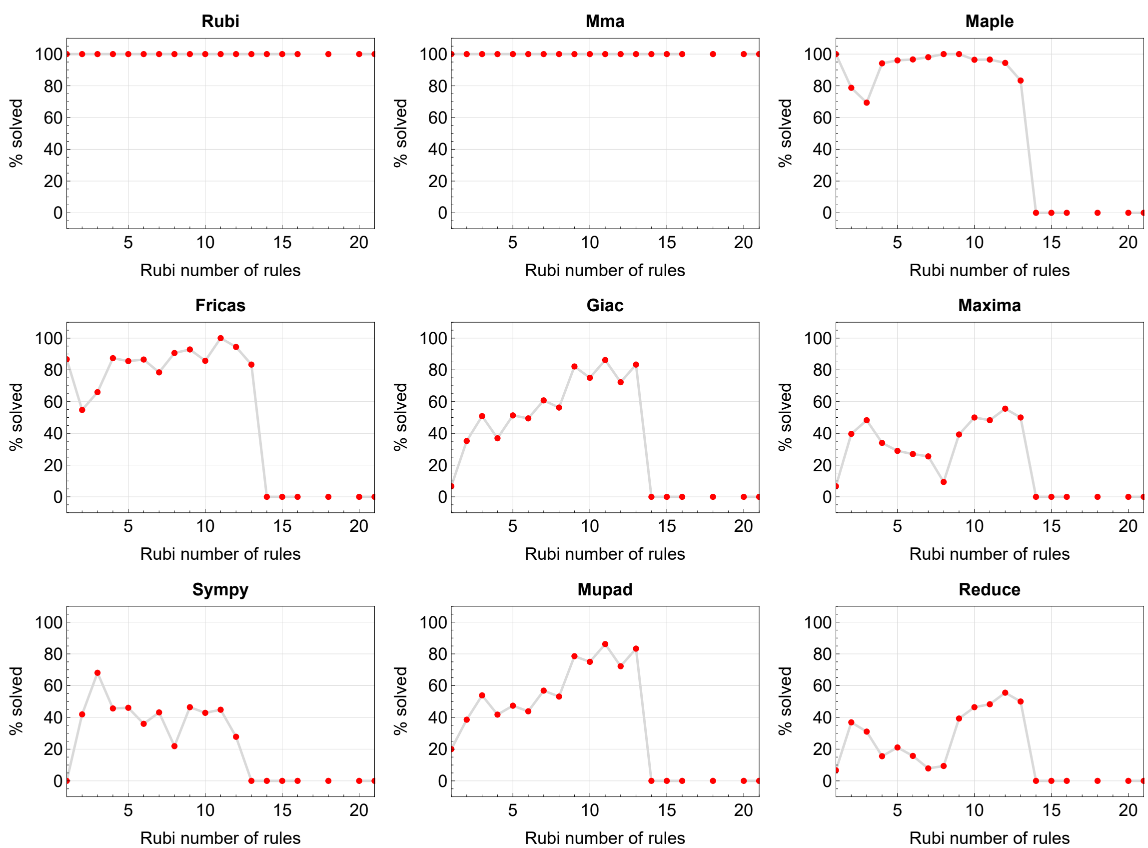


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

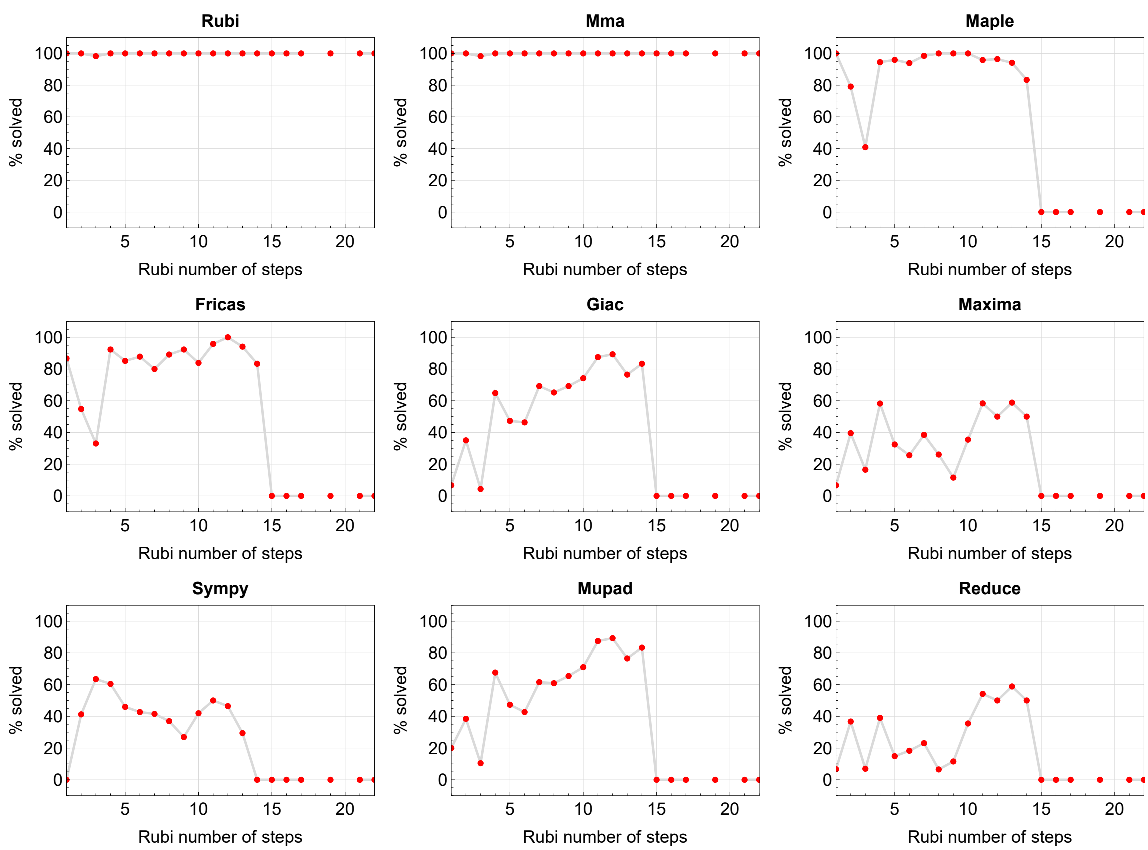


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

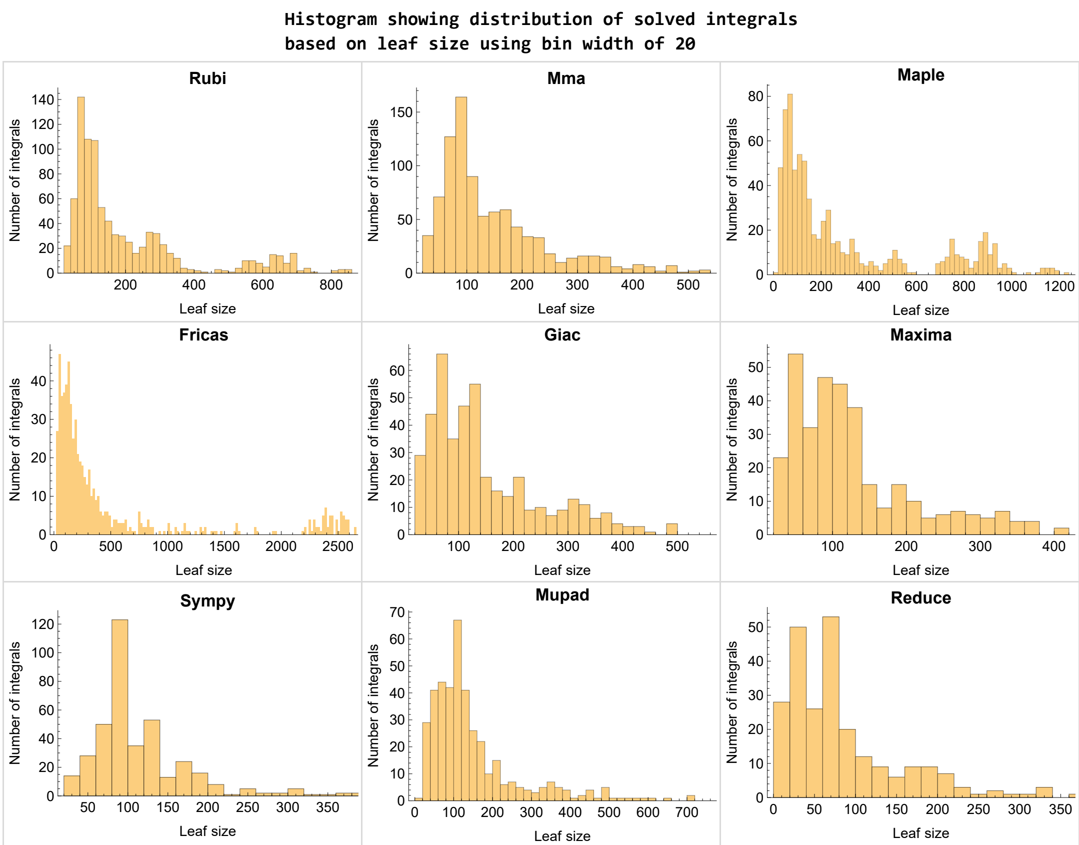


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

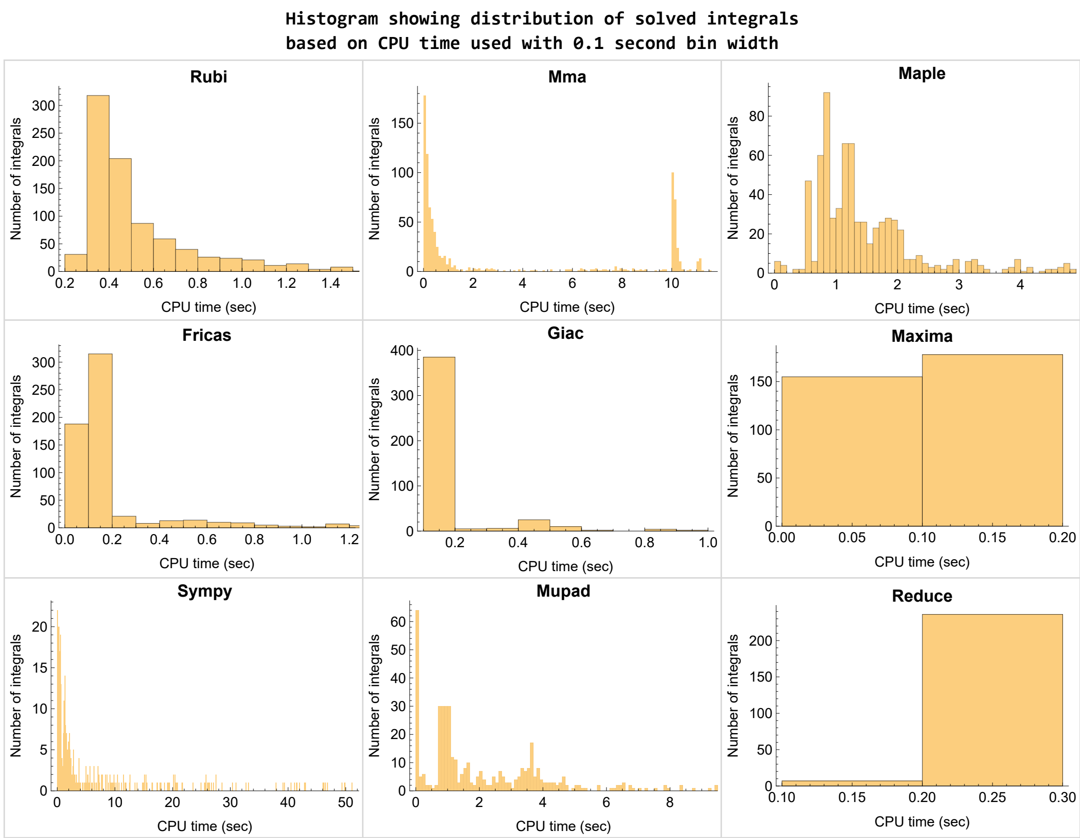


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

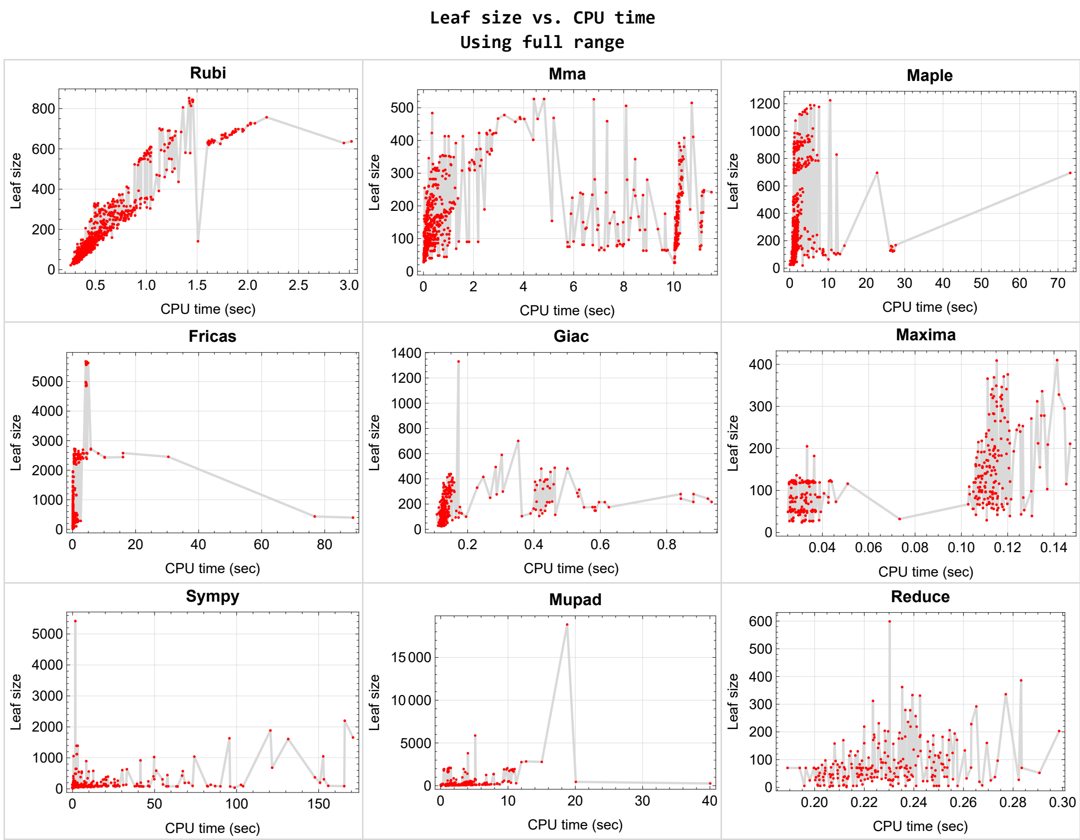


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {169, 170, 171, 172, 173, 174, 186, 187, 188, 189, 190, 191, 202, 203, 204, 205, 206, 218, 219, 220, 221, 222, 233, 234, 235, 236, 237, 238, 241, 244, 247, 249, 252, 255, 257, 260, 263, 265, 268, 271, 273, 276, 281, 440, 450, 466, 491, 515, 516, 517, 518, 519, 520, 521, 522, 594, 628, 882, 888, 894}

Mathematica {439, 441, 442, 443, 444, 452, 454, 455, 464, 465, 467, 468, 469, 477, 478, 479, 480, 481, 482, 490, 493, 494, 495, 497, 498, 506, 507, 508, 509, 510, 511, 512, 513, 514, 520, 522, 536, 537, 538, 539, 540, 546, 547, 548, 549, 550, 556, 558, 560, 566, 567, 568, 569, 570, 578, 579, 580, 582, 583, 591, 592, 593, 594, 595, 596, 604, 605, 606, 608, 609, 610, 611, 612, 613, 614, 622, 623, 624, 625, 626, 628, 629, 630, 631, 632, 638, 640, 641, 642, 648, 649, 650, 651, 652, 658, 659, 660, 661, 662, 668, 669, 670, 671, 672, 680, 682, 686, 687, 688, 689, 690, 691, 699, 703, 704, 705, 706, 707, 709, 710, 717, 719, 721, 722, 723, 724, 725, 726, 727, 728, 742, 745, 746, 758, 760, 761, 776, 777, 778, 779, 780, 795, 796, 798, 799, 814, 817, 818, 836, 848, 850, 851, 866, 867, 868, 870, 871, 879, 881, 888, 894}

Maple {439, 440, 441, 442, 443, 444, 450, 451, 452, 453, 454, 455, 464, 465, 466, 467, 468, 469, 477, 478, 479, 480, 481, 482, 490, 491, 492, 493, 494, 495, 496, 497, 498, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 536, 537, 538, 539, 540, 546, 547, 548, 549, 550, 556, 557, 558, 559, 560, 566, 567, 568, 569, 570, 578, 579, 580, 581, 582, 583, 591, 592, 593, 595, 596, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 622, 623, 624, 625, 626, 627, 629, 630, 631, 632, 638, 639, 640, 641, 642, 648, 649, 650, 651, 652, 658, 659, 660, 661, 662, 668, 669, 670, 671, 672, 848, 851}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
```

```
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
```

```
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

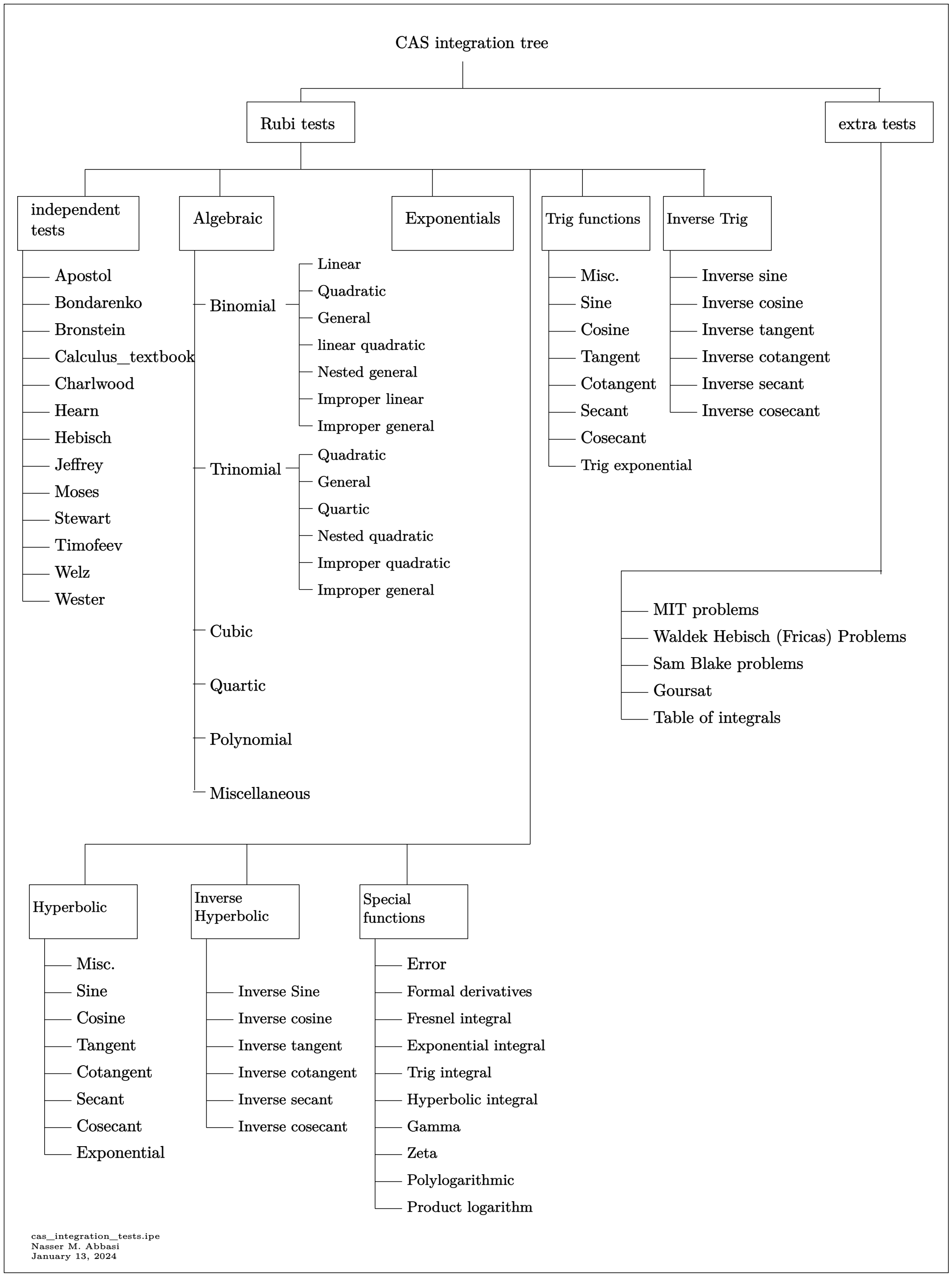
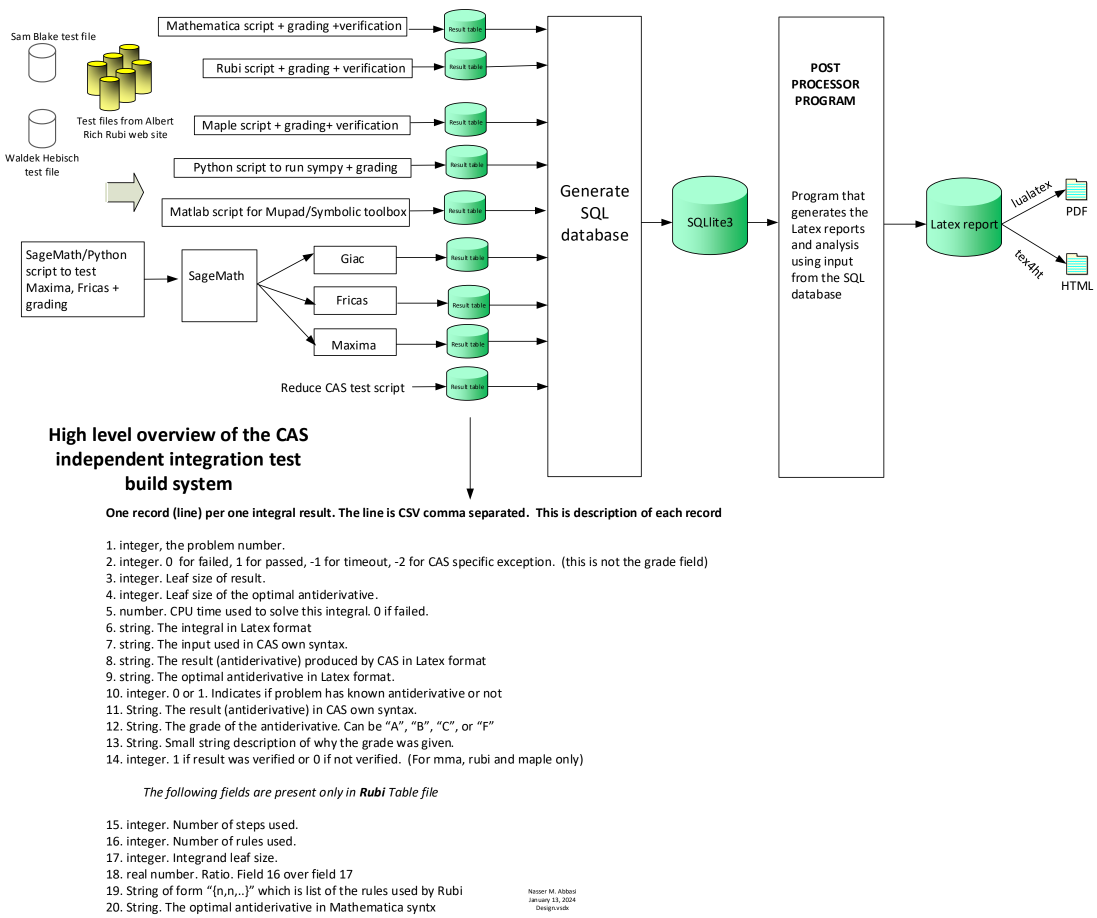


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 888, 891, 894, 895, 896 }

B grade { }

C grade { 628, 887, 889, 890 }

F normal fail { 892, 893 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 175, 176, 177, 178, 179, 180, 192, 193, 194, 195, 196, 197, 207, 208, 209, 210, 211, 212, 223, 224, 225, 226, 227, 238, 241, 244, 247, 249, 252, 255, 257, 260, 263, 265, 268, 271, 273, 276, 279, 281, 284, 287, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 445, 446, 447, 448, 449, 453, 457, 458, 459, 460, 461, 462, 463, 470, 471, 472, 473, 474, 475, 476, 483, 484, 485, 486, 487, 488, 489, 496, 499, 500, 501, 502, 503, 504, 505, 531, 532, 533, 534, 535, 537, 541, 542, 543, 544, 545, 551, 552, 553, 554, 555, 556, 557, 561, 562, 563, 564, 565, 571, 572, 573, 574, 575, 576, 577, 584, 585, 586, 587, 588, 589, 590, 597, 598, 599, 600, 601, 602, 603, 615, 616, 617, 618, 619, 620, 621, 633, 634, 635, 636, 637, 643, 644, 645, 646, 647, 653, 654, 655, 656, 657, 663, 664, 665, 666, 667, 673, 674, 675, 676, 677, 678, 679, 692, 693, 694, 695, 696, 697, 698, 708, 711, 712, 713, 714, 715, 716, 729, 730, 731, 732, 733, 734, 735, 743, 744, 747, 748, 749, 750, 751, 752, 759, 762, 763, 764, 765, 766, 767, 768, 778, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 797, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 834, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 872, 873, 874, 875, 876, 877, 878, 883, 884, 885, 886, 888, 891, 894, 895, 896 }

B grade { 25, 33, 442, 443, 444, 454, 455, 497, 498, 512, 513, 514, 536, 538, 539, 540, 546, 547, 548, 549, 550, 558, 559, 560, 566, 567, 568, 569, 570, 610, 611, 612, 613, 614, 629, 630, 631, 632, 638, 639, 640, 641, 642, 648, 649, 650, 651, 652, 658, 659, 660, 661, 662, 668, 669, 670, 671, 672, 687, 688, 689, 690, 691, 706, 707, 709, 710, 724, 725, 726, 727, 728, 742, 745, 746, 758, 760, 761, 776, 777, 779, 780, 879, 880, 881 }

C grade { 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 198, 199, 200, 201, 202, 203, 204, 205, 206, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 242, 243, 245, 246, 248, 250, 251, 253, 254, 256, 258, 259, 261, 262, 264, 266, 267, 269, 270, 272, 274, 275, 277, 278, 280, 282, 283, 285, 286, 288, 415, 439, 440, 441, 450, 451, 452, 456, 464, 465, 466, 467, 468, 469, 477, 478, 479, 480, 481, 482, 490, 491, 492, 493, 494, 495, 506, 507, 508, 509, 510, 511, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 578, 579, 580, 581, 582, 583, 591, 592, 593, 594, 595, 596, 604, 605, 606, 607, 608, 609, 622, 623, 624, 625, 626, 627, 628, 680, 681, 682, 683, 684, 685, 686, 699, 700, 701, 702, 703, 704, 705, 717, 718, 719, 720, 721, 722, 723, 736, 737, 738, 739, 740, 741, 753, 754, 755, 756, 757, 769, 770, 771, 772, 773, 774, 775, 795, 796, 798, 799, 814, 815, 816, 817, 818, 832, 833, 835, 836, 848, 849, 850, 851, 866, 867, 868, 869, 870, 871, 882, 887, 889, 890 }

F normal fail { 892, 893 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 241, 244, 247, 249, 252, 255, 257, 260, 263, 265, 268, 271, 273, 276, 279, 281, 284, 287, 289, 290, 291, 292, 293, 294, 295, 297, 298, 299, 300, 301, 307, 308, 309, 310, 311, 312, 313, 315, 316, 317, 318, 319, 326, 327, 328, 329, 330, 331, 334, 335, 336, 337, 343, 344, 345, 346, 347, 348, 351, 352, 353, 354, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 379, 396, 397, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 445, 446, 447, 448, 449, 457, 458, 459, 460, 461, 462, 463, 470, 471, 472, 473, 474, 475, 476, 483, 484, 485, 486, 487, 488, 489, 499, 500, 501, 502, 503, 504, 505, 531, 532, 533, 534, 535, 541, 542, 543, 544, 545, 551, 552, 553, 554, 555, 561, 562, 563, 564, 565, 571, 572, 573, 574, 575, 576, 577, 584, 585, 586, 587, 588, 589, 590, 594, 597, 598, 599, 600, 601, 602, 603, 615, 616, 617, 618, 619, 620, 621, 628, 633, 634, 635, 636, 637, 643, 644, 645, 646, 647, 653, 654, 655, 656, 657, 663, 664, 665, 666, 667, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 852, 853, 854, 855, 856, 857, 860, 861, 862, 863, 864, 865 }

B grade { 25, 33, 296, 314, 332, 333, 349, 350, 377, 378, 395, 858, 859 }

C grade { 64, 65, 66, 67, 68, 80, 81, 82, 83, 84, 85, 86, 98, 99, 100, 101, 102, 103, 104, 105, 239, 240, 242, 243, 245, 246, 248, 250, 251, 253, 254, 256, 258, 259, 261, 262, 264, 266, 267, 269, 270, 272, 274, 275, 277, 278, 280, 282, 283, 285, 286, 288, 439, 440, 441, 442, 443, 444, 450, 451, 452, 453, 454, 455, 456, 464, 465, 466, 467, 468, 469, 477, 478, 479, 480, 481, 482, 490, 491, 492, 493, 494, 495, 496, 497, 498, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 536, 537, 538, 539, 540, 546, 547, 548, 549, 550, 556, 557, 558, 559, 560, 566, 567, 568, 569, 570, 578, 579, 580, 581, 582, 583, 591, 592, 593, 595, 596, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 622, 623, 624, 625, 626, 627, 629, 630, 631, 632, 638, 639, 640, 641, 642, 648, 649, 650, 651, 652, 658, 659, 660, 661, 662, 668, 669, 670, 671, 672, 848, 851 }

F normal fail { 302, 303, 304, 305, 306, 320, 321, 322, 323, 324, 325, 338, 339, 340, 341, 342, 355, 356, 357, 358, 359, 371, 372, 373, 374, 375, 376, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 687, 688, 689, 690, 691, 706, 707, 708, 709, 710, 724, 725, 726, 727, 728, 742, 743, 744, 745, 746, 758, 759, 760, 761, 776, 777, 778, 779, 780, 795, 796, 797, 798, 799, 814, 815, 816, 817, 818, 832, 833, 834, 835, 836, 849, 850, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 98, 99, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 140, 141, 142, 143, 149, 150, 151, 152, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 241, 244, 247, 248, 249, 252, 255, 257, 260, 263, 265, 268, 271, 272, 273, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 299, 300, 301, 307, 308, 309, 310, 311, 312, 313, 314, 317, 318, 319, 326, 327, 328, 329, 330, 331, 332, 335, 336, 337, 343, 344, 345, 346, 347, 348, 349, 352, 353, 354, 360, 361, 362, 363, 368, 369, 370, 396, 397, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 445, 446, 447, 448, 449, 457, 458, 459, 460, 461, 462, 463, 470, 471, 472, 473, 474, 475, 476, 483, 484, 485, 486, 487, 488, 489, 499, 500, 501, 502, 503, 504, 505, 531, 532, 533, 534, 535, 541, 542, 543, 544, 545, 551, 552, 553, 554, 555, 563, 564, 565, 571, 572, 573, 574, 575, 576, 577, 584, 585, 586, 587, 588, 589, 590, 594, 597, 598, 599, 600, 601, 602, 603, 615, 616, 617, 618, 619, 620, 621, 628, 634, 635, 637, 643, 644, 645, 646, 647, 654, 656, 657, 673, 674, 675, 676, 677, 678, 679, 680, 682, 697, 698, 711, 712, 713, 714, 715, 716, 717, 718, 729, 730, 734, 735, 736, 737, 766, 781, 782, 783, 784, 786, 787, 788, 789, 790, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 819, 820, 821, 822, 823, 825, 826, 834, 837, 838, 839, 840, 841, 842, 843, 844, 848, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 872, 875, 882 }

B grade { 25, 33, 88, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107, 109, 135, 136, 137, 138, 139, 144, 145, 146, 147, 148, 153, 154, 155, 156, 157, 296, 333, 350, 364, 365, 366, 367, 377, 378, 379, 395, 439, 440, 441, 442, 443, 444, 450, 451, 452, 453, 454, 455, 456, 464, 465, 466, 467, 468, 469, 477, 478, 479, 481, 482, 490, 491, 492, 493, 494, 495, 496, 497, 498, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 561, 562, 578, 579, 580, 581, 582, 583, 591, 592, 593, 595, 596, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 622, 623, 624, 625, 626, 627, 629, 630, 631, 632, 633, 636, 653, 655, 663, 664, 665, 666, 667, 681, 692, 693, 694, 695, 696, 699, 700, 701, 731, 732, 733, 747, 748, 749, 750, 751, 752, 753, 754, 762, 763, 764, 765, 767, 768, 769, 770, 785, 791, 810, 828, 829, 830, 831, 845, 846, 847, 861, 862, 863, 864, 865, 873, 874 }

C grade { 824, 827 }

F normal fail { 239, 240, 242, 243, 245, 246, 250, 251, 253, 254, 256, 258, 259, 261, 262, 264, 266, 267, 269, 270, 274, 275, 302, 303, 304, 305, 306, 320, 321, 322, 323, 324, 325, 338, 339, 340, 341, 342, 355, 356, 357, 358, 359, 371, 372, 373, 374, 375, 376, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 832, 833, 835, 836, 849, 850, 866, 867, 868, 869, 870, 871, 876, 877, 878, 879, 880, 881, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896 }

F(-1) timedout fail { 297, 298, 315, 316, 334, 351, 480, 536, 537, 538, 546, 547, 548, 549, 550, 556, 557, 558, 559, 560, 566, 567, 568, 569, 570, 638, 639, 640, 641, 642, 648, 649, 650, 651, 652, 658, 659, 660, 661, 662, 668, 669, 670, 671, 672, 683, 684, 685, 686, 687, 688, 689, 690, 691, 702, 703, 704, 705, 706, 707, 708, 709, 710, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 738, 739, 740, 741, 742, 743, 744, 745, 746, 755, 756, 757, 758, 759, 760, 761, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 792, 793, 794, 795, 796, 797, 798, 799, 811, 812, 813, 814, 815, 816, 817, 818 }

F(-2) exception fail { 539, 540 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 175, 176, 177, 178, 179, 192, 193, 194, 195, 207, 208, 209, 210, 223, 224, 225, 226, 227, 247, 289, 290, 291, 292, 293, 298, 299, 300, 301, 307, 308, 309, 310, 311, 316, 317, 318, 319, 326, 327, 328, 329, 330, 334, 335, 336, 337, 343, 344, 345, 346, 347, 351, 352, 353, 354, 360, 361, 362, 363, 364, 367, 368, 369, 370, 377, 378, 379, 395, 396, 397, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 445, 446, 447, 457, 458, 459, 460, 470, 471, 472, 473, 483, 484, 485, 486, 499, 500, 501, 502, 571, 572, 573, 574, 584, 585, 586, 587, 597, 598, 599, 600, 615, 616, 617, 618, 781, 782, 783, 784, 800, 801, 802, 803, 819, 820, 821, 822, 823, 837, 838, 839, 840, 852, 853, 854, 855, 856 }

B grade { 25, 33, 163, 180, 196, 197, 211, 212, 294, 295, 296, 297, 312, 313, 314, 315, 331, 332, 333, 348, 349, 350, 365, 366 }

C grade { }

F normal fail { 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 198, 199, 200, 201, 202, 203, 204, 205, 206, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 302, 303, 304, 305, 306, 320, 321, 322, 323, 324, 325, 338, 339, 340, 341, 342, 355, 356, 357, 358, 359, 371, 372, 373, 374, 375, 376, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 437, 438, 439, 440, 441, 442, 443, 444, 448, 449, 450, 451, 452, 453, 454, 455, 456, 461, 462, 463, 464, 465, 466, 467, 468, 469, 474, 475, 476, 477, 478, 479, 480, 481, 482, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 544, 545, 546, 547, 548, 549, 550, 554, 555, 556, 557, 558, 559, 560, 564, 565, 566, 567, 568, 569, 570, 575, 576, 577, 578, 579, 580, 581, 582, 583, 588, 589, 590, 591, 592, 593, 594, 595, 596, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 636, 637, 638, 639, 640, 641, 642, 646, 647, 648, 649, 650, 651, 652, 656, 657, 658, 659, 660, 661, 662, 666, 667, 668, 669, 670, 671, 672, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896 }

F(-1) timedout fail { }

F(-2) exception fail { 531, 532, 533, 541, 542, 543, 551, 552, 553, 561, 562, 563, 633, 634, 635, 643, 644, 645, 653, 654, 655, 663, 664, 665, 673, 674, 675, 676, 692, 693, 694, 695, 711, 712, 713, 729, 730, 731, 732, 733, 747, 748, 749, 750, 762, 763, 764, 765, 766, 872, 873, 874, 875 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 175, 176, 177, 178, 179, 180, 192, 193, 194, 195, 196, 197, 207, 208, 209, 210, 211, 212, 223, 224, 225, 226, 227, 241, 247, 265, 268, 271, 273, 276, 281, 284, 289, 290, 291, 292, 293, 294, 307, 308, 309, 310, 311, 312, 326, 327, 328, 329, 330, 331, 343, 344, 345, 346, 347, 348, 360, 361, 362, 363, 364, 397, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 445, 446, 447, 448, 449, 457, 458, 459, 460, 461, 462, 463, 470, 471, 472, 473, 474, 475, 476, 483, 484, 485, 486, 487, 488, 489, 499, 500, 501, 502, 503, 504, 505, 531, 532, 533, 534, 535, 541, 542, 543, 544, 545, 551, 552, 553, 554, 555, 561, 562, 563, 564, 565, 571, 572, 573, 574, 575, 576, 577, 584, 585, 586, 587, 588, 589, 590, 597, 598, 599, 600, 601, 602, 603, 615, 616, 617, 618, 619, 620, 621, 633, 634, 635, 636, 637, 643, 644, 645, 646, 647, 653, 654, 655, 656, 657, 663, 664, 665, 666, 667, 673, 674, 675, 676, 677, 678, 679, 692, 693, 696, 697, 698, 711, 713, 714, 715, 716, 729, 730, 731, 732, 733, 734, 735, 747, 748, 749, 750, 751, 752, 762, 763, 764, 767, 768, 781, 782, 783, 784, 785, 786, 787, 800, 801, 802, 803, 804, 805, 806, 819, 820, 821, 822, 823, 824, 825, 837, 838, 839, 840, 841, 842, 852, 853, 854, 855, 856, 857, 858, 872, 873 }

B grade { 24, 25, 33, 238, 249, 252, 257, 260, 279, 377, 378, 379, 395, 396, 694, 695, 712, 765, 766, 874, 875 }

C grade { }

F normal fail { 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 198, 199, 200, 201, 202, 203, 204, 205, 206, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 242, 243, 245, 246, 248, 250, 251, 253, 254, 256, 258, 259, 261, 262, 264, 266, 267, 269, 270, 272, 274, 275, 277, 278, 280, 282, 283, 285, 286, 288, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 439, 440, 441, 442, 443, 444, 450, 451, 452, 453, 454, 455, 456, 464, 465, 466, 467, 468, 469, 477, 478, 479, 480, 481, 482, 490, 491, 492, 493, 494, 495, 496, 497, 498, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 536, 537, 538, 539, 540, 546, 547, 548, 549, 550, 556, 557, 558, 559, 560, 566, 567, 568, 569, 570, 578, 579, 580, 581, 582, 583, 591, 592, 593, 594, 595, 596, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 638, 639, 640, 641, 642, 648, 649, 650, 651, 652, 658, 659, 660, 661, 662, 668, 669, 670, 671, 672, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 753, 754, 755, 756, 757, 758, 759, 760, 761, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 843, 844, 845, 846, 847, 848, 849, 850, 851, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896 }

F(-1) timedout fail { }

F(-2) exception fail { 244, 255, 263, 287, 523, 524, 525, 526, 527, 528, 529, 530 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 175, 176, 177, 178, 179, 180, 192, 193, 194, 195, 196, 197, 207, 208, 209, 210, 211, 212, 223, 224, 225, 226, 227, 279, 284, 287, 289, 290, 291, 292, 293, 294, 298, 299, 300, 301, 307, 308, 309, 310, 311, 312, 317, 318, 319, 326, 327, 328, 329, 330, 331, 334, 335, 336, 337, 343, 344, 345, 346, 347, 348, 352, 353, 354, 360, 361, 362, 363, 364, 368, 369, 370, 377, 378, 379, 395, 396, 397, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 445, 446, 447, 448, 449, 451, 456, 457, 458, 459, 460, 461, 462, 463, 470, 471, 472, 473, 474, 475, 476, 483, 484, 485, 486, 487, 488, 489, 492, 499, 500, 501, 502, 503, 504, 505, 531, 532, 533, 534, 535, 541, 542, 543, 544, 545, 551, 552, 553, 554, 555, 561, 562, 563, 564, 565, 571, 572, 573, 574, 575, 576, 577, 584, 585, 586, 587, 588, 589, 590, 597, 598, 599, 600, 601, 602, 603, 615, 616, 617, 618, 619, 620, 621, 633, 634, 635, 636, 637, 643, 644, 645, 646, 647, 653, 654, 655, 656, 657, 663, 664, 665, 666, 667, 673, 674, 675, 676, 677, 678, 679, 692, 693, 694, 695, 696, 697, 698, 711, 712, 713, 714, 715, 716, 729, 730, 731, 732, 733, 734, 735, 747, 748, 749, 750, 751, 752, 762, 763, 764, 765, 766, 767, 768, 781, 782, 783, 784, 785, 786, 787, 800, 801, 802, 803, 804, 805, 806, 819, 820, 821, 822, 823, 824, 825, 837, 838, 839, 840, 841, 842, 852, 853, 854, 855, 856, 857, 858, 872, 873, 874, 875 }

C grade { }

F normal fail { }

F(-1) timedout fail { 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 198, 199, 200, 201, 202, 203, 204, 205, 206, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 283, 285, 286, 288, 295, 296, 297, 302, 303, 304, 305, 306, 313, 314, 315, 316, 320, 321, 322, 323, 324, 325, 332, 333, 338, 339, 340, 341, 342, 349, 350, 351, 355, 356, 357, 358, 359, 365, 366, 367, 371, 372, 373, 374, 375, 376, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 439, 440, 441, 442, 443, 444, 450, 452, 453, 454, 455, 464, 465, 466, 467, 468, 469, 477, 478, 479, 480, 481, 482, 490, 491, 493, 494, 495, 496, 497, 498, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 536, 537, 538, 539, 540, 546, 547, 548, 549, 550, 556, 557, 558, 559, 560, 566, 567, 568, 569, 570, 578, 579, 580, 581, 582, 583, 591, 592, 593, 594, 595, 596, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 638, 639, 640, 641, 642, 648, 649, 650, 651, 652, 658, 659, 660, 661, 662, 668, 669, 670, 671, 672, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 753, 754, 755, 756, 757, 758, 759, 760, 761, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 843, 844, 845, 846, 847, 848, 849, 850, 851, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 161, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 226, 228, 229, 230, 231, 233, 234, 235, 236, 237, 244, 247, 265, 271, 276, 279, 292, 310, 326, 327, 328, 329, 343, 344, 345, 346, 360, 362, 363, 398, 426, 427, 428, 434, 435, 436, 437, 445, 446, 447, 448, 457, 458, 459, 460, 461, 470, 471, 472, 473, 474, 483, 484, 485, 486, 487, 499, 500, 501, 502, 503, 531, 532, 533, 541, 542, 543, 553, 554, 563, 564, 882 }

B grade { 24, 25, 131, 132, 133, 135, 136, 137, 138, 139, 142, 144, 145, 146, 147, 158, 159, 160, 162, 163, 175, 176, 177, 180, 197, 211, 223, 224, 225, 227, 238, 241, 249, 252, 255, 257, 260, 263, 268, 289, 290, 291, 299, 300, 301, 307, 308, 309, 317, 318, 319, 335, 336, 337, 352, 353, 354, 361, 368, 369, 370, 377, 378, 379, 397, 417, 418, 419, 425, 534, 544 }

C grade { 239, 240, 242, 243, 245, 246, 248, 250, 251, 253, 254, 256, 258, 259, 261, 262, 264, 266, 267, 269, 270, 272, 275, 277, 278, 280, 293, 294, 295, 296, 297, 298, 302, 303, 304, 305, 306, 311, 312, 313, 314, 315, 316, 320, 321, 322, 323, 324, 325, 330, 331, 332, 333, 334, 338, 339, 340, 341, 342, 347, 348, 349, 350, 351, 355, 356, 357, 358, 359, 364, 365, 366, 367, 371, 372, 373, 374, 375, 376, 380, 381, 383, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 399, 400, 401, 402, 403, 404, 405, 886 }

F normal fail { 438, 439, 440, 441, 442, 443, 444, 449, 450, 451, 452, 453, 454, 455, 456, 462, 463, 464, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 480, 481, 482, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 504, 505, 508, 509, 510, 511, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 535, 536, 537, 538, 539, 540, 545, 546, 547, 548, 549, 550, 551, 552, 555, 556, 557, 558, 559, 560, 561, 562, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 578, 579, 580, 581, 582, 586, 587, 588, 592, 593, 594, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 645, 646, 648, 649, 650, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881 }

F(-1) timedout fail { 33, 34, 35, 55, 56, 57, 134, 140, 141, 143, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 232, 273, 274, 281, 282, 283, 284, 285, 286, 287, 288, 382, 388, 395, 396, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 420, 421, 422, 423, 424, 429, 430, 431, 432, 433, 506, 507, 512, 577, 583, 584, 585, 589, 590, 591, 595, 596, 643, 644, 647, 651, 652, 663, 722, 723, 813, 883, 884, 885, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 175, 176, 177, 178, 179, 180, 192, 193, 194, 195, 196, 197, 207, 208, 209, 210, 211, 212, 223, 224, 225, 226, 227, 238, 241, 244, 247, 249, 252, 255, 257, 260, 263, 265, 268, 271, 273, 276, 279, 281, 284, 287, 289, 290, 291, 299, 300, 301, 307, 308, 309, 317, 318, 319, 377, 378, 379, 380, 395, 396, 397, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433 }

C grade { }

F normal fail { 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 198, 199, 200, 201, 202, 203, 204, 205, 206, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 242, 243, 245, 246, 248, 250, 251, 253, 254, 256, 258, 259, 261, 262, 264, 266, 267, 269, 270, 272, 274, 275, 277, 278, 280, 282, 283, 285, 286, 288, 292, 293, 294, 295, 296, 297, 298, 302, 303, 304, 305, 306, 310, 311, 312, 313, 314, 315, 316, 320, 321, 322, 323, 324, 325, 326, 327, 328,

329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347,
348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366,
367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 381, 382, 383, 384, 385, 386, 387, 388, 389,
390, 391, 392, 393, 394, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411,
412, 413, 414, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449,
450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468,
469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487,
488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506,
507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525,
526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544,
545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563,
564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582,
583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601,
602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620,
621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639,
640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658,
659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677,
678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696,
697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715,
716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734,
735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753,
754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772,
773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791,
792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810,
811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829,
830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848,
849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867,
868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886,
887, 888, 889, 890, 891, 892, 893, 894, 895, 896 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	34	33	28	27	27	29	29	25	28
N.S.	1	1.03	1.00	0.85	0.82	0.82	0.88	0.88	0.76	0.85
time (sec)	N/A	0.291	0.010	0.398	0.036	0.072	0.018	0.118	0.204	0.044

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	32	29	27	28	25	27	28	22	26
N.S.	1	1.10	1.00	0.93	0.97	0.86	0.93	0.97	0.76	0.90
time (sec)	N/A	0.286	0.012	0.334	0.027	0.075	0.054	0.117	0.245	0.038

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	30	29	26	28	30	26	40	27	25
N.S.	1	1.03	1.00	0.90	0.97	1.03	0.90	1.38	0.93	0.86
time (sec)	N/A	0.294	0.013	0.096	0.032	0.088	0.112	0.122	0.202	0.756

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	33	31	26	30	31	29	37	28	29
N.S.	1	1.14	1.07	0.90	1.03	1.07	1.00	1.28	0.97	1.00
time (sec)	N/A	0.300	0.020	0.072	0.033	0.087	0.285	0.116	0.230	0.722

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	26	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.79	0.85
time (sec)	N/A	0.290	0.007	0.424	0.026	0.093	0.021	0.120	0.248	0.771

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	24	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.86	0.89
time (sec)	N/A	0.286	0.006	0.418	0.031	0.071	0.019	0.123	0.231	0.038

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	27	29	26	29	25	28
N.S.	1	1.00	1.00	0.97	0.87	0.94	0.84	0.94	0.81	0.90
time (sec)	N/A	0.293	0.012	0.093	0.032	0.106	0.052	0.121	0.211	0.049

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	24	24	28	24	23	25	24
N.S.	1	1.00	1.00	0.86	0.86	1.00	0.86	0.82	0.89	0.86
time (sec)	N/A	0.290	0.011	0.095	0.031	0.066	0.057	0.121	0.238	0.806

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	32	28	29	29	31	31	26	29
N.S.	1	1.00	1.03	0.90	0.94	0.94	1.00	1.00	0.84	0.94
time (sec)	N/A	0.293	0.012	0.096	0.039	0.092	0.147	0.122	0.231	0.040

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	25	27	29	29	29	26	28
N.S.	1	1.00	1.07	0.89	0.96	1.04	1.04	1.04	0.93	1.00
time (sec)	N/A	0.286	0.014	0.096	0.027	0.074	0.151	0.121	0.230	0.039

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	46	51	52	51	51	54	53	36	51
N.S.	1	1.10	1.21	1.24	1.21	1.21	1.29	1.26	0.86	1.21
time (sec)	N/A	0.345	0.018	0.753	0.031	0.090	0.024	0.122	0.224	0.062

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	51	50	52	49	53	52	32	49
N.S.	1	1.09	1.11	1.09	1.13	1.07	1.15	1.13	0.70	1.07
time (sec)	N/A	0.308	0.018	0.556	0.027	0.094	0.069	0.120	0.213	0.044

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	52	49	49	52	54	51	69	38	49
N.S.	1	1.02	0.96	0.96	1.02	1.06	1.00	1.35	0.75	0.96
time (sec)	N/A	0.327	0.028	0.763	0.031	0.065	0.144	0.118	0.252	0.050

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	52	51	46	54	55	51	70	39	52
N.S.	1	1.02	1.00	0.90	1.06	1.08	1.00	1.37	0.76	1.02
time (sec)	N/A	0.334	0.023	0.558	0.026	0.078	0.418	0.116	0.212	0.065

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	54	53	37	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	0.98	0.96	0.67	0.93
time (sec)	N/A	0.325	0.009	0.685	0.035	0.090	0.026	0.124	0.234	0.051

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	51	50	35	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.02	1.00	0.70	0.96
time (sec)	N/A	0.322	0.008	0.687	0.029	0.081	0.025	0.129	0.223	0.049

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	52	51	53	49	52	37	50
N.S.	1	1.00	1.00	0.98	0.96	1.00	0.92	0.98	0.70	0.94
time (sec)	N/A	0.325	0.021	0.558	0.030	0.076	0.066	0.125	0.247	0.053

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	53	49	48	37	48
N.S.	1	1.00	1.00	0.98	0.96	1.06	0.98	0.96	0.74	0.96
time (sec)	N/A	0.316	0.019	0.555	0.028	0.096	0.078	0.126	0.237	0.051

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	50	53	53	53	54	37	52
N.S.	1	1.00	0.96	0.94	1.00	1.00	1.00	1.02	0.70	0.98
time (sec)	N/A	0.333	0.020	0.557	0.037	0.072	0.170	0.123	0.218	0.055

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	46	51	53	53	51	37	50
N.S.	1	1.00	1.00	0.92	1.02	1.06	1.06	1.02	0.74	1.00
time (sec)	N/A	0.322	0.022	0.556	0.033	0.083	0.194	0.123	0.212	0.054

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	48	54	53	58	56	37	53
N.S.	1	1.00	1.02	0.91	1.02	1.00	1.09	1.06	0.70	1.00
time (sec)	N/A	0.329	0.019	0.559	0.036	0.093	0.487	0.119	0.223	0.055

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	51	53	54	53	37	50
N.S.	1	1.00	1.00	0.90	1.02	1.06	1.08	1.06	0.74	1.00
time (sec)	N/A	0.323	0.026	0.554	0.035	0.088	0.526	0.113	0.273	0.782

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	107	121	119	119	136	125	70	107
N.S.	1	1.04	1.13	1.27	1.25	1.25	1.43	1.32	0.74	1.13
time (sec)	N/A	0.471	0.033	0.700	0.034	0.093	0.032	0.118	0.234	0.697

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	71	107	121	119	119	138	125	70	107
N.S.	1	1.06	1.60	1.81	1.78	1.78	2.06	1.87	1.04	1.60
time (sec)	N/A	0.396	0.028	0.675	0.032	0.077	0.033	0.123	0.239	0.043

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	46	107	121	119	119	136	125	69	107
N.S.	1	1.10	2.55	2.88	2.83	2.83	3.24	2.98	1.64	2.55
time (sec)	N/A	0.336	0.039	0.677	0.027	0.097	0.032	0.119	0.219	0.044

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	87	113	119	120	117	134	124	66	105
N.S.	1	0.99	1.28	1.35	1.36	1.33	1.52	1.41	0.75	1.19
time (sec)	N/A	0.368	0.036	0.555	0.029	0.081	0.111	0.128	0.201	0.049

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	115	121	120	123	133	143	72	105
N.S.	1	1.00	1.02	1.07	1.06	1.09	1.18	1.27	0.64	0.93
time (sec)	N/A	0.466	0.051	0.563	0.032	0.093	0.193	0.126	0.207	0.054

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	116	106	117	122	123	131	148	71	113
N.S.	1	1.02	0.93	1.03	1.07	1.08	1.15	1.30	0.62	0.99
time (sec)	N/A	0.468	0.055	0.566	0.036	0.086	0.546	0.122	0.263	0.053

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	116	106	111	123	123	129	150	71	118
N.S.	1	1.02	0.93	0.97	1.08	1.08	1.13	1.32	0.62	1.04
time (sec)	N/A	0.461	0.064	0.562	0.033	0.088	1.281	0.121	0.253	0.908

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	115	118	106	123	123	129	149	72	122
N.S.	1	1.01	1.04	0.93	1.08	1.08	1.13	1.31	0.63	1.07
time (sec)	N/A	0.463	0.046	0.770	0.034	0.084	3.673	0.119	0.234	0.072

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	112	116	102	123	123	129	145	72	121
N.S.	1	0.99	1.03	0.90	1.09	1.09	1.14	1.28	0.64	1.07
time (sec)	N/A	0.491	0.161	0.566	0.028	0.078	19.233	0.123	0.212	0.739

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	90	121	102	123	123	133	136	72	121
N.S.	1	0.99	1.33	1.12	1.35	1.35	1.46	1.49	0.79	1.33
time (sec)	N/A	0.388	0.060	0.546	0.033	0.095	84.217	0.116	0.252	0.104

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	52	118	104	121	121	0	127	70	122
N.S.	1	1.08	2.46	2.17	2.52	2.52	0.00	2.65	1.46	2.54
time (sec)	N/A	0.295	0.034	0.550	0.028	0.077	0.000	0.127	0.227	0.073

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	80	121	104	121	121	0	127	70	122
N.S.	1	1.05	1.59	1.37	1.59	1.59	0.00	1.67	0.92	1.61
time (sec)	N/A	0.332	0.033	0.555	0.044	0.073	0.000	0.123	0.223	0.699

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	119	121	104	121	121	0	127	70	122
N.S.	1	1.02	1.03	0.89	1.03	1.03	0.00	1.09	0.60	1.04
time (sec)	N/A	0.447	0.034	0.572	0.033	0.077	0.000	0.122	0.236	0.072

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	121	119	119	136	125	70	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.16	1.07	0.60	0.91
time (sec)	N/A	0.474	0.023	0.720	0.031	0.095	0.042	0.124	0.245	0.045

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	121	119	119	134	125	70	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.15	1.07	0.60	0.91
time (sec)	N/A	0.451	0.021	0.688	0.028	0.074	0.031	0.131	0.221	0.044

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	121	119	119	136	125	70	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.16	1.07	0.60	0.91
time (sec)	N/A	0.433	0.021	0.710	0.034	0.065	0.036	0.120	0.194	0.045

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	121	119	119	136	125	70	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.16	1.07	0.60	0.91
time (sec)	N/A	0.447	0.018	0.719	0.033	0.084	0.035	0.126	0.219	0.043

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	120	119	119	133	124	70	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.14	1.06	0.60	0.91
time (sec)	N/A	0.440	0.019	0.722	0.026	0.105	0.031	0.120	0.218	0.043

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	120	119	119	134	124	70	106
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.15	1.06	0.60	0.91
time (sec)	N/A	0.443	0.020	0.710	0.037	0.082	0.036	0.120	0.233	0.044

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	109	116	115	115	128	120	68	103
N.S.	1	1.00	1.00	1.06	1.06	1.06	1.17	1.10	0.62	0.94
time (sec)	N/A	0.434	0.017	0.711	0.026	0.095	0.043	0.125	0.218	0.044

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	121	118	121	129	124	70	106
N.S.	1	1.00	1.00	1.08	1.05	1.08	1.15	1.11	0.62	0.95
time (sec)	N/A	0.423	0.040	0.580	0.026	0.087	0.116	0.121	0.283	0.049

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	120	116	121	128	119	70	104
N.S.	1	1.00	1.00	1.07	1.04	1.08	1.14	1.06	0.62	0.93
time (sec)	N/A	0.427	0.036	0.585	0.028	0.116	0.118	0.123	0.217	0.046

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	115	122	121	121	133	127	70	109
N.S.	1	1.00	1.02	1.08	1.07	1.07	1.18	1.12	0.62	0.96
time (sec)	N/A	0.432	0.041	0.569	0.026	0.091	0.220	0.122	0.215	0.048

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	113	119	120	121	133	124	70	108
N.S.	1	1.00	1.00	1.05	1.06	1.07	1.18	1.10	0.62	0.96
time (sec)	N/A	0.427	0.039	0.571	0.035	0.074	0.243	0.117	0.200	0.046

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	117	121	121	129	127	70	113
N.S.	1	1.00	1.00	1.06	1.10	1.10	1.17	1.15	0.64	1.03
time (sec)	N/A	0.434	0.043	0.569	0.033	0.082	0.622	0.112	0.223	0.051

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	113	114	120	121	133	124	70	111
N.S.	1	1.00	1.00	1.01	1.06	1.07	1.18	1.10	0.62	0.98
time (sec)	N/A	0.420	0.039	0.568	0.026	0.075	0.638	0.123	0.217	0.047

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	118	111	122	121	131	127	70	118
N.S.	1	1.00	1.03	0.97	1.06	1.05	1.14	1.10	0.61	1.03
time (sec)	N/A	0.439	0.026	0.572	0.028	0.070	1.820	0.123	0.218	0.077

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	109	108	120	121	131	124	70	116
N.S.	1	1.00	1.00	0.99	1.10	1.11	1.20	1.14	0.64	1.06
time (sec)	N/A	0.439	0.044	0.570	0.027	0.079	2.155	0.121	0.230	0.074

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	117	107	122	121	134	128	70	123
N.S.	1	1.00	1.02	0.93	1.06	1.05	1.17	1.11	0.61	1.07
time (sec)	N/A	0.438	0.038	0.571	0.028	0.099	7.069	0.114	0.235	0.764

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	102	119	121	129	123	70	120
N.S.	1	1.00	1.00	0.93	1.08	1.10	1.17	1.12	0.64	1.09
time (sec)	N/A	0.450	0.044	0.569	0.039	0.079	15.061	0.124	0.196	0.080

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	118	104	122	121	134	128	70	121
N.S.	1	1.00	1.03	0.90	1.06	1.05	1.17	1.11	0.61	1.05
time (sec)	N/A	0.437	0.034	0.570	0.043	0.091	32.742	0.118	0.201	0.084

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	101	119	121	131	125	70	119
N.S.	1	1.00	1.00	0.92	1.08	1.10	1.19	1.14	0.64	1.08
time (sec)	N/A	0.420	0.048	0.554	0.029	0.098	102.683	0.118	0.247	0.728

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	119	104	121	121	0	127	70	119
N.S.	1	1.00	1.05	0.92	1.07	1.07	0.00	1.12	0.62	1.05
time (sec)	N/A	0.438	0.034	0.599	0.036	0.094	0.000	0.122	0.200	0.717

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	121	121	0	127	70	121
N.S.	1	1.00	1.03	0.89	1.03	1.03	0.00	1.09	0.60	1.03
time (sec)	N/A	0.433	0.036	0.553	0.035	0.088	0.000	0.121	0.189	0.727

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	104	121	121	0	127	70	121
N.S.	1	1.00	1.00	0.89	1.03	1.03	0.00	1.09	0.60	1.03
time (sec)	N/A	0.427	0.051	0.539	0.035	0.117	0.000	0.114	0.204	0.070

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	74	71	68	74	75	70	77	5	76
N.S.	1	0.99	0.95	0.91	0.99	1.00	0.93	1.03	0.07	1.01
time (sec)	N/A	0.410	0.036	0.728	0.027	0.109	0.335	0.119	0.237	0.072

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	53	47	49	50	51	46	52	5	52
N.S.	1	0.98	0.87	0.91	0.93	0.94	0.85	0.96	0.09	0.96
time (sec)	N/A	0.348	0.023	0.931	0.036	0.101	0.254	0.126	0.207	0.728

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	33	31	32	31	30	27	32	5	31
N.S.	1	0.94	0.89	0.91	0.89	0.86	0.77	0.91	0.14	0.89
time (sec)	N/A	0.318	0.015	0.721	0.027	0.125	0.264	0.124	0.254	0.710

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	34	33	35	32	26	34	2	36
N.S.	1	1.12	1.00	0.97	1.03	0.94	0.76	1.00	0.06	1.06
time (sec)	N/A	0.335	0.016	0.710	0.027	0.109	0.606	0.128	0.206	0.830

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	51	49	46	48	47	41	69	5	46
N.S.	1	1.02	0.98	0.92	0.96	0.94	0.82	1.38	0.10	0.92
time (sec)	N/A	0.365	0.025	0.720	0.032	0.111	0.614	0.131	0.249	0.771

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	70	70	64	70	73	61	99	5	70
N.S.	1	1.01	1.01	0.93	1.01	1.06	0.88	1.43	0.07	1.01
time (sec)	N/A	0.384	0.032	0.724	0.033	0.108	0.709	0.124	0.213	0.763

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	156	171	84	182	167	114	217	5	164
N.S.	1	0.85	0.93	0.46	0.99	0.91	0.62	1.19	0.03	0.90
time (sec)	N/A	0.521	0.113	0.815	0.114	0.111	0.327	0.117	0.210	0.292

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	157	154	65	157	162	114	207	5	144
N.S.	1	0.94	0.92	0.39	0.94	0.97	0.68	1.24	0.03	0.86
time (sec)	N/A	0.564	0.116	0.812	0.109	0.109	0.258	0.125	0.217	1.073

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	147	152	60	154	145	87	186	5	162
N.S.	1	0.91	0.94	0.37	0.95	0.90	0.54	1.15	0.03	1.00
time (sec)	N/A	0.532	0.095	0.816	0.109	0.108	0.277	0.131	0.241	1.070

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	140	152	45	131	382	92	161	5	126
N.S.	1	0.93	1.01	0.30	0.87	2.55	0.61	1.07	0.03	0.84
time (sec)	N/A	0.490	0.052	0.805	0.107	0.103	0.230	0.134	0.250	0.960

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	130	129	42	128	369	71	133	1	123
N.S.	1	0.90	0.89	0.29	0.88	2.54	0.49	0.92	0.01	0.85
time (sec)	N/A	0.469	0.067	0.933	0.109	0.112	0.259	0.127	0.213	0.288

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	139	134	114	140	372	90	155	5	126
N.S.	1	0.95	0.91	0.78	0.95	2.53	0.61	1.05	0.03	0.86
time (sec)	N/A	0.509	0.094	0.806	0.111	0.162	0.245	0.128	0.248	0.910

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	136	135	113	140	411	73	161	5	126
N.S.	1	0.91	0.91	0.76	0.94	2.76	0.49	1.08	0.03	0.85
time (sec)	N/A	0.496	0.104	0.815	0.110	0.118	0.313	0.129	0.216	0.274

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	156	154	130	147	158	112	197	5	178
N.S.	1	0.95	0.93	0.79	0.89	0.96	0.68	1.19	0.03	1.08
time (sec)	N/A	0.559	0.120	0.813	0.113	0.091	0.328	0.131	0.202	0.996

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	153	154	130	148	176	99	176	5	145
N.S.	1	0.91	0.92	0.77	0.88	1.05	0.59	1.05	0.03	0.86
time (sec)	N/A	0.542	0.130	0.799	0.114	0.093	0.355	0.123	0.196	0.941

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	173	173	150	178	180	139	216	5	161
N.S.	1	0.94	0.94	0.82	0.97	0.98	0.76	1.17	0.03	0.88
time (sec)	N/A	0.585	0.141	0.815	0.107	0.100	0.357	0.135	0.258	0.922

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	79	72	76	82	121	82	106	61	86
N.S.	1	0.96	0.88	0.93	1.00	1.48	1.00	1.29	0.74	1.05
time (sec)	N/A	0.403	0.077	0.848	0.031	0.096	0.630	0.129	0.249	0.882

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	55	50	57	60	81	56	91	48	62
N.S.	1	0.92	0.83	0.95	1.00	1.35	0.93	1.52	0.80	1.03
time (sec)	N/A	0.351	0.043	0.729	0.026	0.099	0.515	0.126	0.219	0.085

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	40	41	38	40	44	36	65	37	37
N.S.	1	0.98	1.00	0.93	0.98	1.07	0.88	1.59	0.90	0.90
time (sec)	N/A	0.322	0.017	0.707	0.033	0.109	0.323	0.122	0.206	0.060

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	52	46	48	51	70	46	61	45	47
N.S.	1	1.02	0.90	0.94	1.00	1.37	0.90	1.20	0.88	0.92
time (sec)	N/A	0.346	0.034	0.715	0.036	0.089	0.317	0.129	0.239	0.152

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	75	64	76	76	118	70	80	61	78
N.S.	1	0.99	0.84	1.00	1.00	1.55	0.92	1.05	0.80	1.03
time (sec)	N/A	0.404	0.052	0.731	0.032	0.090	0.811	0.127	0.225	0.777

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	95	85	96	106	154	100	149	78	100
N.S.	1	0.98	0.88	0.99	1.09	1.59	1.03	1.54	0.80	1.03
time (sec)	N/A	0.447	0.103	0.732	0.027	0.088	0.975	0.128	0.237	0.824

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	193	203	114	218	271	156	244	99	209
N.S.	1	0.89	0.94	0.53	1.00	1.25	0.72	1.12	0.46	0.96
time (sec)	N/A	0.556	0.149	0.804	0.109	0.105	0.589	0.128	0.232	1.027

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	186	185	94	192	257	151	236	98	179
N.S.	1	0.93	0.92	0.47	0.96	1.28	0.75	1.17	0.49	0.89
time (sec)	N/A	0.548	0.138	0.809	0.107	0.145	0.670	0.124	0.254	0.962

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	182	181	87	187	240	126	211	88	193
N.S.	1	0.94	0.93	0.45	0.96	1.24	0.65	1.09	0.45	0.99
time (sec)	N/A	0.533	0.130	0.824	0.119	0.117	0.543	0.127	0.211	1.038

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	183	165	71	162	578	126	189	81	158
N.S.	1	1.01	0.91	0.39	0.89	3.18	0.69	1.04	0.45	0.87
time (sec)	N/A	0.602	0.124	0.835	0.106	0.154	0.587	0.125	0.203	0.304

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	160	65	157	573	102	166	78	150
N.S.	1	1.00	0.93	0.38	0.91	3.33	0.59	0.97	0.45	0.87
time (sec)	N/A	0.590	0.135	0.822	0.111	0.106	0.472	0.124	0.223	0.933

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	165	146	67	160	548	117	186	65	145
N.S.	1	0.96	0.85	0.39	0.94	3.20	0.68	1.09	0.38	0.85
time (sec)	N/A	0.548	0.096	0.799	0.108	0.153	0.405	0.131	0.212	0.934

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	158	145	65	158	537	97	160	67	143
N.S.	1	0.93	0.86	0.38	0.93	3.18	0.57	0.95	0.40	0.85
time (sec)	N/A	0.578	0.095	0.803	0.112	0.159	0.388	0.128	0.222	0.295

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	181	164	139	166	570	122	180	84	156
N.S.	1	1.01	0.91	0.77	0.92	3.17	0.68	1.00	0.47	0.87
time (sec)	N/A	0.602	0.136	0.830	0.110	0.161	0.415	0.133	0.207	1.004

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	178	163	138	172	618	109	188	96	159
N.S.	1	0.99	0.91	0.77	0.96	3.43	0.61	1.04	0.53	0.88
time (sec)	N/A	0.595	0.244	0.822	0.111	0.115	0.435	0.132	0.274	0.285

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	198	185	155	186	259	153	231	107	209
N.S.	1	0.99	0.93	0.78	0.93	1.30	0.77	1.16	0.54	1.05
time (sec)	N/A	0.646	0.153	0.814	0.112	0.102	0.477	0.125	0.222	1.004

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	195	183	154	186	277	138	206	114	176
N.S.	1	0.98	0.92	0.77	0.93	1.38	0.69	1.03	0.57	0.88
time (sec)	N/A	0.649	0.158	0.832	0.115	0.093	0.485	0.130	0.214	1.060

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	106	94	100	115	179	112	131	130	117
N.S.	1	0.99	0.88	0.93	1.07	1.67	1.05	1.22	1.21	1.09
time (sec)	N/A	0.491	0.073	0.855	0.029	0.111	1.381	0.134	0.217	0.134

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	83	92	76	94	142	94	93	115	94
N.S.	1	0.94	1.05	0.86	1.07	1.61	1.07	1.06	1.31	1.07
time (sec)	N/A	0.421	0.043	0.749	0.033	0.113	1.450	0.138	0.249	0.118

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	65	64	57	72	89	70	61	97	70
N.S.	1	0.98	0.97	0.86	1.09	1.35	1.06	0.92	1.47	1.06
time (sec)	N/A	0.371	0.029	0.721	0.026	0.131	1.222	0.136	0.204	0.741

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	29	42	42	42	28	17	44
N.S.	1	1.00	0.94	0.91	1.31	1.31	1.31	0.88	0.53	1.38
time (sec)	N/A	0.271	0.017	0.714	0.030	0.088	0.633	0.129	0.207	0.035

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	69	59	61	77	119	75	74	114	71
N.S.	1	1.01	0.87	0.90	1.13	1.75	1.10	1.09	1.68	1.04
time (sec)	N/A	0.374	0.051	0.721	0.033	0.114	0.450	0.133	0.250	0.176

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	100	87	98	109	197	107	136	142	107
N.S.	1	0.99	0.86	0.97	1.08	1.95	1.06	1.35	1.41	1.06
time (sec)	N/A	0.459	0.063	0.733	0.034	0.109	0.885	0.135	0.226	0.806

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	108	123	136	229	133	131	157	130
N.S.	1	1.00	0.89	1.01	1.11	1.88	1.09	1.07	1.29	1.07
time (sec)	N/A	0.520	0.089	0.744	0.029	0.090	0.985	0.136	0.232	0.825

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	213	216	116	228	364	192	259	197	213
N.S.	1	0.92	0.93	0.50	0.98	1.57	0.83	1.12	0.85	0.92
time (sec)	N/A	0.593	0.203	0.971	0.111	0.098	1.716	0.135	0.234	0.959

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	209	210	109	223	347	163	234	187	227
N.S.	1	0.94	0.94	0.49	1.00	1.56	0.73	1.05	0.84	1.02
time (sec)	N/A	0.591	0.167	0.829	0.117	0.114	1.169	0.125	0.248	0.348

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	209	194	90	196	792	162	210	180	187
N.S.	1	1.00	0.93	0.43	0.94	3.81	0.78	1.01	0.87	0.90
time (sec)	N/A	0.676	0.175	0.864	0.112	0.128	1.488	0.135	0.224	0.976

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	188	85	191	789	141	187	176	183
N.S.	1	1.00	0.94	0.43	0.96	3.96	0.71	0.94	0.88	0.92
time (sec)	N/A	0.644	0.170	0.866	0.109	0.141	0.985	0.131	0.235	0.939

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	192	181	85	195	756	155	206	158	175
N.S.	1	0.97	0.91	0.43	0.98	3.82	0.78	1.04	0.80	0.88
time (sec)	N/A	0.620	0.177	0.793	0.114	0.138	1.181	0.138	0.208	0.917

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	185	178	83	193	743	136	187	171	173
N.S.	1	0.95	0.92	0.43	0.99	3.83	0.70	0.96	0.88	0.89
time (sec)	N/A	0.616	0.165	0.808	0.123	0.149	0.628	0.132	0.257	0.918

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	192	178	86	195	752	153	207	158	175
N.S.	1	0.96	0.89	0.43	0.97	3.74	0.76	1.03	0.79	0.87
time (sec)	N/A	0.624	0.143	0.833	0.116	0.135	0.571	0.139	0.226	0.938

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	183	175	84	192	743	133	180	171	173
N.S.	1	0.93	0.89	0.43	0.97	3.77	0.68	0.91	0.87	0.88
time (sec)	N/A	0.608	0.136	0.813	0.110	0.104	0.425	0.134	0.224	0.276

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	193	159	199	776	162	204	183	185
N.S.	1	1.00	0.93	0.76	0.96	3.73	0.78	0.98	0.88	0.89
time (sec)	N/A	0.675	0.174	0.839	0.114	0.171	0.527	0.133	0.237	1.209

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	205	189	158	201	812	143	209	202	188
N.S.	1	1.00	0.92	0.77	0.98	3.94	0.69	1.01	0.98	0.91
time (sec)	N/A	0.714	0.171	0.845	0.114	0.121	0.508	0.136	0.230	0.979

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	225	214	175	221	366	189	254	206	240
N.S.	1	0.99	0.94	0.77	0.97	1.61	0.83	1.11	0.90	1.05
time (sec)	N/A	0.725	0.203	0.848	0.113	0.145	0.598	0.137	0.254	1.017

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	222	210	174	221	384	173	229	219	207
N.S.	1	0.98	0.93	0.77	0.97	1.69	0.76	1.01	0.96	0.91
time (sec)	N/A	0.714	0.212	0.848	0.120	0.102	0.670	0.131	0.220	0.949

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	28	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.72	0.79
time (sec)	N/A	0.286	0.035	0.547	0.031	0.074	0.631	0.115	0.232	0.700

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	28	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.72	0.79
time (sec)	N/A	0.292	0.030	0.508	0.032	0.083	0.443	0.111	0.220	0.047

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	30	46	29	26	31
N.S.	1	1.00	0.90	0.72	0.69	0.77	1.18	0.74	0.67	0.79
time (sec)	N/A	0.292	0.034	0.513	0.036	0.093	0.613	0.125	0.200	0.707

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	28	27	29	44	29	25	31
N.S.	1	1.00	0.95	0.76	0.73	0.78	1.19	0.78	0.68	0.84
time (sec)	N/A	0.293	0.032	0.512	0.027	0.132	0.276	0.127	0.198	0.045

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	30	27	29	44	29	27	31
N.S.	1	1.00	0.95	0.81	0.73	0.78	1.19	0.78	0.73	0.84
time (sec)	N/A	0.290	0.038	0.157	0.034	0.097	0.345	0.117	0.238	0.050

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	34	30	27	28	46	29	29	31
N.S.	1	1.00	0.87	0.77	0.69	0.72	1.18	0.74	0.74	0.79
time (sec)	N/A	0.282	0.039	0.153	0.036	0.085	0.380	0.120	0.256	0.785

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	30	27	29	42	29	30	30
N.S.	1	1.00	0.95	0.81	0.73	0.78	1.14	0.78	0.81	0.81
time (sec)	N/A	0.284	0.040	0.157	0.035	0.086	0.441	0.121	0.227	0.045

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	39	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.62	0.81
time (sec)	N/A	0.327	0.062	0.705	0.031	0.110	1.045	0.126	0.240	0.064

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	39	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.62	0.81
time (sec)	N/A	0.328	0.069	0.593	0.026	0.093	0.822	0.124	0.214	0.049

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	52	51	54	80	53	37	51
N.S.	1	1.00	0.94	0.83	0.81	0.86	1.27	0.84	0.59	0.81
time (sec)	N/A	0.326	0.064	0.592	0.027	0.095	0.830	0.117	0.203	0.053

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	52	51	53	78	53	36	51
N.S.	1	1.00	0.97	0.85	0.84	0.87	1.28	0.87	0.59	0.84
time (sec)	N/A	0.327	0.061	0.593	0.037	0.079	0.513	0.119	0.201	0.050

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	54	51	53	78	53	38	51
N.S.	1	1.00	0.97	0.89	0.84	0.87	1.28	0.87	0.62	0.84
time (sec)	N/A	0.327	0.076	0.580	0.032	0.089	0.596	0.126	0.253	0.052

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	54	51	53	80	53	40	51
N.S.	1	1.00	0.94	0.86	0.81	0.84	1.27	0.84	0.63	0.81
time (sec)	N/A	0.330	0.064	0.582	0.031	0.116	0.705	0.127	0.249	0.052

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	54	51	53	76	53	41	51
N.S.	1	1.00	0.97	0.89	0.84	0.87	1.25	0.87	0.67	0.84
time (sec)	N/A	0.325	0.070	0.718	0.030	0.091	1.099	0.127	0.202	0.053

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	91	76	73	78	114	77	50	69
N.S.	1	1.00	1.07	0.89	0.86	0.92	1.34	0.91	0.59	0.81
time (sec)	N/A	0.370	0.084	0.874	0.035	0.081	1.951	0.120	0.204	0.739

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	76	73	78	114	77	50	69
N.S.	1	1.00	0.95	0.89	0.86	0.92	1.34	0.91	0.59	0.81
time (sec)	N/A	0.366	0.086	0.584	0.027	0.086	1.353	0.120	0.232	0.034

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	76	73	76	114	77	48	69
N.S.	1	1.00	0.94	0.89	0.86	0.89	1.34	0.91	0.56	0.81
time (sec)	N/A	0.365	0.074	0.598	0.027	0.075	1.501	0.124	0.204	0.034

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	80	76	73	75	112	77	47	69
N.S.	1	1.00	0.96	0.92	0.88	0.90	1.35	0.93	0.57	0.83
time (sec)	N/A	0.362	0.075	0.586	0.046	0.083	1.108	0.117	0.206	0.033

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	78	73	75	112	77	49	69
N.S.	1	1.00	1.00	0.94	0.88	0.90	1.35	0.93	0.59	0.83
time (sec)	N/A	0.348	0.098	0.572	0.033	0.111	1.292	0.120	0.223	0.037

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	78	73	75	112	77	52	69
N.S.	1	1.00	0.91	0.92	0.86	0.88	1.32	0.91	0.61	0.81
time (sec)	N/A	0.355	0.070	0.572	0.027	0.104	1.184	0.121	0.291	0.035

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	78	78	73	75	110	77	52	69
N.S.	1	1.00	0.94	0.94	0.88	0.90	1.33	0.93	0.63	0.83
time (sec)	N/A	0.360	0.097	0.569	0.028	0.089	1.422	0.126	0.202	0.037

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	70	67	55	58	143	428	64	7	111
N.S.	1	0.96	0.92	0.75	0.79	1.96	5.86	0.88	0.10	1.52
time (sec)	N/A	0.381	0.110	0.777	0.113	0.122	66.774	0.128	0.204	0.785

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	40	39	108	381	39	5	93
N.S.	1	1.00	1.00	0.75	0.74	2.04	7.19	0.74	0.09	1.75
time (sec)	N/A	0.324	0.085	0.773	0.130	0.129	4.922	0.130	0.206	0.802

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	40	39	120	371	39	9	102
N.S.	1	1.00	1.00	0.75	0.74	2.26	7.00	0.74	0.17	1.92
time (sec)	N/A	0.329	0.082	0.764	0.121	0.125	147.707	0.126	0.231	0.791

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	71	70	57	58	149	0	59	9	69
N.S.	1	0.97	0.96	0.78	0.79	2.04	0.00	0.81	0.12	0.95
time (sec)	N/A	0.361	0.137	0.761	0.116	0.104	0.000	0.122	0.212	0.774

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	269	172	203	295	1289	605	289	7	1933
N.S.	1	1.20	0.76	0.90	1.31	5.73	2.69	1.28	0.03	8.59
time (sec)	N/A	0.890	0.310	0.863	0.145	0.127	30.322	0.132	0.212	1.085

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	263	153	190	212	1603	581	260	7	1640
N.S.	1	1.27	0.74	0.92	1.02	7.74	2.81	1.26	0.03	7.92
time (sec)	N/A	0.815	0.312	0.840	0.121	0.119	12.617	0.534	0.268	1.010

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	251	152	187	278	1245	558	280	4	1915
N.S.	1	1.23	0.75	0.92	1.36	6.10	2.74	1.37	0.02	9.39
time (sec)	N/A	0.827	0.318	0.832	0.134	0.142	5.609	0.125	0.232	0.435

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	261	156	191	212	1636	561	250	6	1700
N.S.	1	1.28	0.76	0.94	1.04	8.02	2.75	1.23	0.03	8.33
time (sec)	N/A	0.788	0.379	0.947	0.133	0.127	9.882	0.268	0.201	1.019

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	253	158	188	278	1290	586	280	9	2023
N.S.	1	1.22	0.76	0.91	1.34	6.23	2.83	1.35	0.04	9.77
time (sec)	N/A	0.799	0.312	0.842	0.136	0.111	51.007	0.131	0.201	1.088

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	115	100	82	91	255	0	98	116	144
N.S.	1	1.11	0.96	0.79	0.88	2.45	0.00	0.94	1.12	1.38
time (sec)	N/A	0.457	0.220	0.838	0.113	0.113	0.000	0.124	0.253	0.818

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	95	77	65	68	222	0	68	101	116
N.S.	1	1.16	0.94	0.79	0.83	2.71	0.00	0.83	1.23	1.41
time (sec)	N/A	0.404	0.158	0.826	0.120	0.135	0.000	0.121	0.227	0.793

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	61	61	190	1042	63	73	115
N.S.	1	1.00	1.00	0.86	0.86	2.68	14.68	0.89	1.03	1.62
time (sec)	N/A	0.366	0.146	0.727	0.118	0.133	74.233	0.129	0.235	0.816

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	96	79	66	67	232	0	66	115	139
N.S.	1	1.16	0.95	0.80	0.81	2.80	0.00	0.80	1.39	1.67
time (sec)	N/A	0.411	0.163	0.781	0.107	0.104	0.000	0.124	0.216	0.908

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	294	181	216	311	1426	1658	313	159	1884
N.S.	1	1.25	0.77	0.92	1.32	6.04	7.03	1.33	0.67	7.98
time (sec)	N/A	0.867	0.961	0.895	0.115	0.150	170.942	0.137	0.220	1.065

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	287	169	210	235	1773	1885	288	125	1578
N.S.	1	1.26	0.74	0.92	1.03	7.78	8.27	1.26	0.55	6.92
time (sec)	N/A	0.862	0.883	0.744	0.111	0.150	120.594	0.530	0.251	1.072

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	277	168	213	301	1417	1632	302	125	1922
N.S.	1	1.21	0.74	0.93	1.32	6.21	7.16	1.32	0.55	8.43
time (sec)	N/A	0.852	0.919	0.741	0.117	0.125	95.627	0.143	0.228	1.098

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	305	184	213	240	1788	2200	277	171	1757
N.S.	1	1.27	0.77	0.89	1.00	7.45	9.17	1.15	0.71	7.32
time (sec)	N/A	0.876	0.894	0.856	0.125	0.122	165.903	0.288	0.254	1.075

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	297	188	214	312	1463	0	313	194	2080
N.S.	1	1.23	0.78	0.88	1.29	6.05	0.00	1.29	0.80	8.60
time (sec)	N/A	0.885	0.837	0.852	0.133	0.101	0.000	0.150	0.256	1.503

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	124	99	85	101	344	0	89	219	145
N.S.	1	1.11	0.88	0.76	0.90	3.07	0.00	0.79	1.96	1.29
time (sec)	N/A	0.457	0.251	0.989	0.108	0.150	0.000	0.127	0.239	0.945

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	104	92	81	96	314	0	84	204	133
N.S.	1	1.03	0.91	0.80	0.95	3.11	0.00	0.83	2.02	1.32
time (sec)	N/A	0.424	0.243	0.937	0.113	0.134	0.000	0.129	0.228	0.863

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	92	82	96	313	0	84	203	136
N.S.	1	1.00	0.88	0.79	0.92	3.01	0.00	0.81	1.95	1.31
time (sec)	N/A	0.421	0.231	0.741	0.110	0.104	0.000	0.123	0.299	0.862

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	125	102	86	100	347	0	88	236	163
N.S.	1	1.11	0.90	0.76	0.88	3.07	0.00	0.78	2.09	1.44
time (sec)	N/A	0.470	0.240	0.785	0.109	0.103	0.000	0.122	0.238	0.892

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	307	192	234	341	1618	0	328	331	1944
N.S.	1	1.18	0.74	0.90	1.31	6.20	0.00	1.26	1.27	7.45
time (sec)	N/A	0.904	1.185	0.941	0.119	0.098	0.000	0.137	0.242	0.516

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	317	193	235	271	1959	0	314	336	1672
N.S.	1	1.20	0.73	0.89	1.02	7.39	0.00	1.18	1.27	6.31
time (sec)	N/A	0.902	1.112	0.744	0.130	0.133	0.000	0.536	0.277	1.176

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	306	189	230	336	1588	0	322	333	1952
N.S.	1	1.18	0.73	0.88	1.29	6.11	0.00	1.24	1.28	7.51
time (sec)	N/A	0.939	1.061	0.743	0.135	0.128	0.000	0.148	0.239	1.164

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	334	208	236	273	1934	0	299	362	1786
N.S.	1	1.24	0.77	0.87	1.01	7.16	0.00	1.11	1.34	6.61
time (sec)	N/A	0.985	1.228	0.852	0.119	0.131	0.000	0.306	0.235	0.453

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	326	209	237	346	1608	0	334	386	2109
N.S.	1	1.20	0.77	0.87	1.27	5.91	0.00	1.23	1.42	7.75
time (sec)	N/A	0.943	1.139	0.861	0.117	0.112	0.000	0.145	0.283	1.295

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	107	75	67	118	99	219	104	56	154
N.S.	1	1.04	0.73	0.65	1.15	0.96	2.13	1.01	0.54	1.50
time (sec)	N/A	0.413	0.091	0.823	0.027	0.110	0.424	0.127	0.238	0.791

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	77	56	49	84	75	168	73	44	114
N.S.	1	1.05	0.77	0.67	1.15	1.03	2.30	1.00	0.60	1.56
time (sec)	N/A	0.374	0.066	0.795	0.031	0.093	0.300	0.130	0.218	0.775

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	34	31	49	50	117	44	31	44
N.S.	1	1.09	0.74	0.67	1.07	1.09	2.54	0.96	0.67	0.96
time (sec)	N/A	0.324	0.044	0.796	0.032	0.105	0.234	0.122	0.213	0.763

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	65	61	50	67	122	87	61	62	80
N.S.	1	1.02	0.95	0.78	1.05	1.91	1.36	0.95	0.97	1.25
time (sec)	N/A	0.320	0.114	0.802	0.103	0.125	5.611	0.124	0.252	1.005

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	79	65	56	107	140	134	70	73	76
N.S.	1	1.11	0.92	0.79	1.51	1.97	1.89	0.99	1.03	1.07
time (sec)	N/A	0.340	0.171	0.865	0.114	0.121	14.754	0.121	0.210	1.118

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	78	65	158	169	160	120	83	93
N.S.	1	1.00	0.92	0.76	1.86	1.99	1.88	1.41	0.98	1.09
time (sec)	N/A	0.339	0.206	0.872	0.106	0.113	39.332	0.127	0.218	1.390

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	293	89	349	0	91	83	0	76	0
N.S.	1	0.97	0.29	1.15	0.00	0.30	0.27	0.00	0.25	0.00
time (sec)	N/A	0.595	5.758	1.272	0.000	0.088	1.216	0.000	0.253	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	264	75	325	0	67	82	0	52	0
N.S.	1	0.99	0.28	1.21	0.00	0.25	0.31	0.00	0.19	0.00
time (sec)	N/A	0.493	5.757	0.868	0.000	0.092	1.116	0.000	0.287	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	266	81	317	0	57	85	0	63	0
N.S.	1	0.99	0.30	1.18	0.00	0.21	0.32	0.00	0.23	0.00
time (sec)	N/A	0.498	6.403	0.897	0.000	0.109	1.374	0.000	0.232	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	266	80	328	0	64	94	0	63	0
N.S.	1	0.98	0.29	1.21	0.00	0.24	0.35	0.00	0.23	0.00
time (sec)	N/A	0.506	10.094	1.389	0.000	0.139	1.431	0.000	0.250	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	297	80	352	0	89	97	0	63	0
N.S.	1	0.97	0.26	1.15	0.00	0.29	0.32	0.00	0.21	0.00
time (sec)	N/A	0.546	10.103	2.064	0.000	0.098	1.638	0.000	0.300	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	577	91	503	0	102	83	0	79	0
N.S.	1	0.99	0.16	0.87	0.00	0.18	0.14	0.00	0.14	0.00
time (sec)	N/A	1.004	5.997	1.490	0.000	0.138	1.263	0.000	0.247	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	547	75	479	0	76	83	0	55	0
N.S.	1	1.00	0.14	0.87	0.00	0.14	0.15	0.00	0.10	0.00
time (sec)	N/A	0.948	5.832	0.996	0.000	0.090	1.338	0.000	0.289	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	81	469	0	64	85	0	61	0
N.S.	1	1.00	0.15	0.86	0.00	0.12	0.16	0.00	0.11	0.00
time (sec)	N/A	0.931	6.353	1.082	0.000	0.120	1.285	0.000	0.260	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	544	80	480	0	70	92	0	63	0
N.S.	1	1.00	0.15	0.88	0.00	0.13	0.17	0.00	0.12	0.00
time (sec)	N/A	0.937	10.094	1.186	0.000	0.102	1.344	0.000	0.251	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	575	80	504	0	97	97	0	63	0
N.S.	1	0.99	0.14	0.87	0.00	0.17	0.17	0.00	0.11	0.00
time (sec)	N/A	1.025	10.095	1.879	0.000	0.143	1.663	0.000	0.275	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	605	80	530	0	123	97	0	63	0
N.S.	1	0.99	0.13	0.86	0.00	0.20	0.16	0.00	0.10	0.00
time (sec)	N/A	1.037	10.108	2.809	0.000	0.113	1.710	0.000	0.320	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	107	80	68	118	124	267	104	67	206
N.S.	1	1.04	0.78	0.66	1.15	1.20	2.59	1.01	0.65	2.00
time (sec)	N/A	0.410	0.105	0.834	0.033	0.085	0.677	0.125	0.237	0.936

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	77	56	49	84	99	216	73	55	211
N.S.	1	1.05	0.77	0.67	1.15	1.36	2.96	1.00	0.75	2.89
time (sec)	N/A	0.358	0.073	0.803	0.039	0.105	0.446	0.130	0.225	0.845

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	34	31	49	73	165	44	42	150
N.S.	1	1.09	0.74	0.67	1.07	1.59	3.59	0.96	0.91	3.26
time (sec)	N/A	0.322	0.053	0.941	0.039	0.084	0.317	0.124	0.202	0.795

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	83	85	66	80	169	109	80	85	131
N.S.	1	1.02	1.05	0.81	0.99	2.09	1.35	0.99	1.05	1.62
time (sec)	N/A	0.332	0.157	0.849	0.114	0.107	11.931	0.125	0.251	1.204

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	98	81	79	134	166	223	112	94	111
N.S.	1	1.03	0.85	0.83	1.41	1.75	2.35	1.18	0.99	1.17
time (sec)	N/A	0.353	0.168	0.898	0.107	0.086	17.337	0.124	0.235	1.340

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	103	81	78	171	188	243	131	96	110
N.S.	1	1.03	0.81	0.78	1.71	1.88	2.43	1.31	0.96	1.10
time (sec)	N/A	0.360	0.271	0.881	0.112	0.124	46.202	0.128	0.219	1.577

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	317	93	373	0	115	172	0	95	0
N.S.	1	0.94	0.28	1.11	0.00	0.34	0.51	0.00	0.28	0.00
time (sec)	N/A	0.589	7.798	1.266	0.000	0.090	2.044	0.000	0.292	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	286	77	349	0	91	170	0	71	0
N.S.	1	0.96	0.26	1.17	0.00	0.30	0.57	0.00	0.24	0.00
time (sec)	N/A	0.541	7.730	0.872	0.000	0.111	1.809	0.000	0.249	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	288	83	334	0	80	172	0	82	0
N.S.	1	0.98	0.28	1.13	0.00	0.27	0.58	0.00	0.28	0.00
time (sec)	N/A	0.545	8.003	0.872	0.000	0.122	2.244	0.000	0.252	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	289	82	329	0	67	184	0	82	0
N.S.	1	0.98	0.28	1.12	0.00	0.23	0.63	0.00	0.28	0.00
time (sec)	N/A	0.537	10.083	1.418	0.000	0.160	2.482	0.000	0.277	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	290	82	351	0	89	196	0	82	0
N.S.	1	0.97	0.27	1.17	0.00	0.30	0.66	0.00	0.27	0.00
time (sec)	N/A	0.540	10.097	2.063	0.000	0.089	2.996	0.000	0.321	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	601	96	527	0	126	172	0	98	0
N.S.	1	0.98	0.16	0.86	0.00	0.21	0.28	0.00	0.16	0.00
time (sec)	N/A	1.031	8.199	1.461	0.000	0.123	2.113	0.000	0.263	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	571	78	503	0	100	172	0	74	0
N.S.	1	0.98	0.13	0.87	0.00	0.17	0.30	0.00	0.13	0.00
time (sec)	N/A	0.995	7.723	0.999	0.000	0.153	1.907	0.000	0.245	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	573	569	83	486	0	87	173	0	80	0
N.S.	1	0.99	0.14	0.85	0.00	0.15	0.30	0.00	0.14	0.00
time (sec)	N/A	0.993	7.853	0.907	0.000	0.160	2.194	0.000	0.311	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	575	569	85	482	0	77	182	0	82	0
N.S.	1	0.99	0.15	0.84	0.00	0.13	0.32	0.00	0.14	0.00
time (sec)	N/A	1.000	10.086	1.227	0.000	0.146	2.292	0.000	0.269	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	573	568	82	503	0	95	194	0	82	0
N.S.	1	0.99	0.14	0.88	0.00	0.17	0.34	0.00	0.14	0.00
time (sec)	N/A	0.993	10.091	1.929	0.000	0.153	2.827	0.000	0.305	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	605	598	82	529	0	123	199	0	82	0
N.S.	1	0.99	0.14	0.87	0.00	0.20	0.33	0.00	0.14	0.00
time (sec)	N/A	1.031	10.087	2.813	0.000	0.093	2.857	0.000	0.347	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	105	80	68	118	76	175	101	45	104
N.S.	1	1.02	0.78	0.66	1.15	0.74	1.70	0.98	0.44	1.01
time (sec)	N/A	0.423	0.090	0.806	0.030	0.096	0.387	0.122	0.219	0.882

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	75	56	49	83	52	124	70	33	52
N.S.	1	1.03	0.77	0.67	1.14	0.71	1.70	0.96	0.45	0.71
time (sec)	N/A	0.354	0.074	0.803	0.026	0.123	0.352	0.124	0.209	0.808

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	48	33	30	48	29	75	38	20	29
N.S.	1	1.04	0.72	0.65	1.04	0.63	1.63	0.83	0.43	0.63
time (sec)	N/A	0.319	0.051	0.795	0.029	0.106	0.233	0.125	0.266	0.812

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	37	54	102	71	40	45	57
N.S.	1	1.00	1.00	0.77	1.12	2.12	1.48	0.83	0.94	1.19
time (sec)	N/A	0.295	0.087	0.810	0.118	0.137	1.982	0.124	0.229	0.992

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	57	58	47	109	123	80	62	61	67
N.S.	1	0.98	1.00	0.81	1.88	2.12	1.38	1.07	1.05	1.16
time (sec)	N/A	0.325	0.129	0.865	0.103	0.140	7.857	0.131	0.219	1.129

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	88	78	67	178	170	163	121	82	95
N.S.	1	0.98	0.87	0.74	1.98	1.89	1.81	1.34	0.91	1.06
time (sec)	N/A	0.358	0.345	0.869	0.115	0.115	19.107	0.125	0.209	1.249

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	269	89	325	0	67	80	0	57	0
N.S.	1	1.00	0.33	1.20	0.00	0.25	0.30	0.00	0.21	0.00
time (sec)	N/A	0.517	10.093	1.257	0.000	0.128	1.392	0.000	0.265	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	74	309	0	42	78	0	35	0
N.S.	1	1.00	0.31	1.29	0.00	0.18	0.33	0.00	0.15	0.00
time (sec)	N/A	0.444	10.042	0.859	0.000	0.106	1.079	0.000	0.218	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	78	311	0	50	82	0	45	0
N.S.	1	1.00	0.32	1.28	0.00	0.21	0.34	0.00	0.19	0.00
time (sec)	N/A	0.459	10.041	0.874	0.000	0.104	1.094	0.000	0.221	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	273	78	329	0	62	90	0	46	0
N.S.	1	1.00	0.28	1.20	0.00	0.23	0.33	0.00	0.17	0.00
time (sec)	N/A	0.518	10.046	1.385	0.000	0.100	1.306	0.000	0.240	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	553	91	479	0	78	80	0	60	0
N.S.	1	1.01	0.17	0.87	0.00	0.14	0.15	0.00	0.11	0.00
time (sec)	N/A	0.950	10.097	1.497	0.000	0.106	1.460	0.000	0.258	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	523	75	463	0	51	80	0	38	0
N.S.	1	1.01	0.15	0.90	0.00	0.10	0.15	0.00	0.07	0.00
time (sec)	N/A	0.885	10.074	0.972	0.000	0.107	1.185	0.000	0.224	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	509	520	77	463	0	54	82	0	43	0
N.S.	1	1.02	0.15	0.91	0.00	0.11	0.16	0.00	0.08	0.00
time (sec)	N/A	0.918	10.044	0.915	0.000	0.142	1.016	0.000	0.232	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	551	78	481	0	70	88	0	46	0
N.S.	1	1.00	0.14	0.87	0.00	0.13	0.16	0.00	0.08	0.00
time (sec)	N/A	0.984	10.043	1.167	0.000	0.090	1.202	0.000	0.265	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	78	504	0	97	94	0	46	0
N.S.	1	1.00	0.13	0.87	0.00	0.17	0.16	0.00	0.08	0.00
time (sec)	N/A	1.046	10.042	2.132	0.000	0.115	1.361	0.000	0.246	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	77	68	116	88	175	114	34	152
N.S.	1	1.00	0.75	0.66	1.13	0.85	1.70	1.11	0.33	1.48
time (sec)	N/A	0.423	0.097	0.824	0.034	0.101	0.458	0.123	0.229	1.027

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	55	49	81	63	124	77	22	60
N.S.	1	1.00	0.75	0.67	1.11	0.86	1.70	1.05	0.30	0.82
time (sec)	N/A	0.367	0.076	0.818	0.027	0.088	0.357	0.120	0.223	0.929

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	33	30	47	41	75	38	13	33
N.S.	1	1.00	0.72	0.65	1.02	0.89	1.63	0.83	0.28	0.72
time (sec)	N/A	0.335	0.060	0.826	0.033	0.108	0.254	0.128	0.234	0.791

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	49	70	167	78	53	36	65
N.S.	1	1.00	1.00	0.84	1.21	2.88	1.34	0.91	0.62	1.12
time (sec)	N/A	0.323	0.122	0.821	0.109	0.127	5.324	0.121	0.225	1.053

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	83	77	68	144	230	264	99	61	131
N.S.	1	0.98	0.91	0.80	1.69	2.71	3.11	1.16	0.72	1.54
time (sec)	N/A	0.350	0.237	0.878	0.111	0.105	27.583	0.123	0.209	1.258

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	113	100	97	215	286	192	137	83	167
N.S.	1	0.97	0.85	0.83	1.84	2.44	1.64	1.17	0.71	1.43
time (sec)	N/A	0.374	0.265	0.904	0.107	0.136	68.396	0.132	0.252	1.469

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	103	406	0	122	80	0	57	0
N.S.	1	1.00	0.35	1.36	0.00	0.41	0.27	0.00	0.19	0.00
time (sec)	N/A	0.574	10.108	2.542	0.000	0.154	9.557	0.000	0.243	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	268	78	348	0	95	80	0	39	0
N.S.	1	1.01	0.29	1.31	0.00	0.36	0.30	0.00	0.15	0.00
time (sec)	N/A	0.503	10.087	1.649	0.000	0.122	4.303	0.000	0.252	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	73	336	0	83	78	0	20	0
N.S.	1	1.00	0.29	1.34	0.00	0.33	0.31	0.00	0.08	0.00
time (sec)	N/A	0.457	10.041	0.829	0.000	0.107	2.860	0.000	0.272	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	271	86	350	0	102	82	0	24	0
N.S.	1	1.00	0.32	1.29	0.00	0.38	0.30	0.00	0.09	0.00
time (sec)	N/A	0.509	10.042	1.356	0.000	0.088	9.280	0.000	0.268	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	302	72	386	0	119	90	0	24	0
N.S.	1	0.99	0.24	1.27	0.00	0.39	0.30	0.00	0.08	0.00
time (sec)	N/A	0.561	10.044	1.887	0.000	0.134	26.227	0.000	0.241	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	552	79	504	0	104	80	0	42	0
N.S.	1	1.01	0.14	0.92	0.00	0.19	0.15	0.00	0.08	0.00
time (sec)	N/A	0.939	10.087	2.060	0.000	0.090	5.251	0.000	0.279	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	524	535	71	490	0	94	80	0	21	0
N.S.	1	1.02	0.14	0.94	0.00	0.18	0.15	0.00	0.04	0.00
time (sec)	N/A	0.927	10.069	0.962	0.000	0.107	3.376	0.000	0.268	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	550	72	504	0	106	82	0	24	0
N.S.	1	1.00	0.13	0.92	0.00	0.19	0.15	0.00	0.04	0.00
time (sec)	N/A	0.969	10.037	1.502	0.000	0.101	7.245	0.000	0.260	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	580	580	72	540	0	127	88	0	24	0
N.S.	1	1.00	0.12	0.93	0.00	0.22	0.15	0.00	0.04	0.00
time (sec)	N/A	1.010	10.043	1.990	0.000	0.131	17.228	0.000	0.214	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	611	610	72	573	0	156	94	0	24	0
N.S.	1	1.00	0.12	0.94	0.00	0.26	0.15	0.00	0.04	0.00
time (sec)	N/A	1.040	10.042	2.680	0.000	0.106	46.083	0.000	0.207	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	73	70	116	98	338	104	42	145
N.S.	1	1.00	0.71	0.68	1.13	0.95	3.28	1.01	0.41	1.41
time (sec)	N/A	0.418	0.097	0.850	0.051	0.095	0.590	0.129	0.236	1.025

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	56	49	84	75	240	63	31	60
N.S.	1	1.00	0.77	0.67	1.15	1.03	3.29	0.86	0.42	0.82
time (sec)	N/A	0.374	0.078	0.975	0.043	0.087	0.536	0.126	0.221	1.065

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	48	33	30	49	52	144	32	22	33
N.S.	1	1.04	0.72	0.65	1.07	1.13	3.13	0.70	0.48	0.72
time (sec)	N/A	0.335	0.059	0.804	0.025	0.094	0.348	0.131	0.244	0.875

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	81	70	73	81	240	100	67	102	80
N.S.	1	1.05	0.91	0.95	1.05	3.12	1.30	0.87	1.32	1.04
time (sec)	N/A	0.347	0.166	0.885	0.111	0.125	8.624	0.126	0.236	1.011

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	106	99	94	170	348	1608	101	136	198
N.S.	1	0.95	0.88	0.84	1.52	3.11	14.36	0.90	1.21	1.77
time (sec)	N/A	0.381	0.225	0.987	0.115	0.153	131.439	0.124	0.240	1.196

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	297	108	397	0	153	80	0	116	0
N.S.	1	1.01	0.37	1.35	0.00	0.52	0.27	0.00	0.39	0.00
time (sec)	N/A	0.568	10.132	2.947	0.000	0.130	63.318	0.000	0.236	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	280	99	372	0	141	80	0	98	0
N.S.	1	1.00	0.35	1.33	0.00	0.51	0.29	0.00	0.35	0.00
time (sec)	N/A	0.541	10.108	1.283	0.000	0.096	41.133	0.000	0.221	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	282	103	364	0	145	78	0	31	0
N.S.	1	1.00	0.36	1.29	0.00	0.51	0.28	0.00	0.11	0.00
time (sec)	N/A	0.497	10.060	0.822	0.000	0.101	26.011	0.000	0.243	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	298	116	389	0	163	82	0	35	0
N.S.	1	0.99	0.39	1.30	0.00	0.54	0.27	0.00	0.12	0.00
time (sec)	N/A	0.539	10.071	2.263	0.000	0.121	90.131	0.000	0.221	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	331	83	425	0	178	0	0	35	0
N.S.	1	0.99	0.25	1.27	0.00	0.53	0.00	0.00	0.10	0.00
time (sec)	N/A	0.623	10.048	2.853	0.000	0.117	0.000	0.000	0.214	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	576	581	109	555	0	162	80	0	120	0
N.S.	1	1.01	0.19	0.96	0.00	0.28	0.14	0.00	0.21	0.00
time (sec)	N/A	1.018	10.115	3.457	0.000	0.090	82.424	0.000	0.261	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	564	92	528	0	154	80	0	103	0
N.S.	1	1.01	0.17	0.95	0.00	0.28	0.14	0.00	0.18	0.00
time (sec)	N/A	1.039	10.103	1.473	0.000	0.088	41.918	0.000	0.242	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	565	81	520	0	154	80	0	32	0
N.S.	1	1.00	0.14	0.92	0.00	0.27	0.14	0.00	0.06	0.00
time (sec)	N/A	0.978	10.085	0.966	0.000	0.123	26.304	0.000	0.217	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	578	579	86	545	0	167	82	0	35	0
N.S.	1	1.00	0.15	0.94	0.00	0.29	0.14	0.00	0.06	0.00
time (sec)	N/A	0.983	10.044	2.038	0.000	0.117	57.203	0.000	0.222	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	609	83	581	0	186	88	0	35	0
N.S.	1	1.00	0.14	0.95	0.00	0.30	0.14	0.00	0.06	0.00
time (sec)	N/A	1.048	10.050	3.234	0.000	0.111	165.610	0.000	0.276	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	152	123	138	0	295	298	330	112	0
N.S.	1	0.94	0.76	0.86	0.00	1.83	1.85	2.05	0.70	0.00
time (sec)	N/A	0.516	1.056	3.158	0.000	0.638	18.551	0.229	0.230	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	343	112	777	0	0	97	0	90	0
N.S.	1	1.06	0.35	2.40	0.00	0.00	0.30	0.00	0.28	0.00
time (sec)	N/A	0.714	10.183	4.148	0.000	0.000	21.703	0.000	0.358	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	625	94	1140	0	0	97	0	66	0
N.S.	1	1.08	0.16	1.96	0.00	0.00	0.17	0.00	0.11	0.00
time (sec)	N/A	1.224	10.112	3.401	0.000	0.000	7.069	0.000	0.393	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	118	96	109	0	221	253	177	88	0
N.S.	1	0.98	0.79	0.90	0.00	1.83	2.09	1.46	0.73	0.00
time (sec)	N/A	0.449	0.401	1.201	0.000	0.491	1.619	0.177	0.234	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	308	93	744	0	0	97	0	66	0
N.S.	1	1.08	0.33	2.60	0.00	0.00	0.34	0.00	0.23	0.00
time (sec)	N/A	0.633	10.080	2.374	0.000	0.000	2.936	0.000	0.363	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	580	622	98	1123	0	0	100	0	70	0
N.S.	1	1.07	0.17	1.94	0.00	0.00	0.17	0.00	0.12	0.00
time (sec)	N/A	1.180	10.066	2.693	0.000	0.000	3.775	0.000	0.418	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	117	82	100	0	207	160	0	89	0
N.S.	1	0.99	0.69	0.85	0.00	1.75	1.36	0.00	0.75	0.00
time (sec)	N/A	0.468	0.391	1.261	0.000	0.694	7.085	0.000	0.228	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	311	97	743	0	0	100	0	76	0
N.S.	1	1.10	0.34	2.63	0.00	0.00	0.35	0.00	0.27	0.00
time (sec)	N/A	0.637	10.067	3.258	0.000	0.000	13.808	0.000	0.448	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	564	536	81	1127	0	0	97	0	71	0
N.S.	1	0.95	0.14	2.00	0.00	0.00	0.17	0.00	0.13	0.00
time (sec)	N/A	1.040	10.097	4.523	0.000	0.000	10.180	0.000	0.479	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	78	75	84	81	180	131	109	88	0
N.S.	1	0.99	0.95	1.06	1.03	2.28	1.66	1.38	1.11	0.00
time (sec)	N/A	0.379	0.459	1.566	0.119	0.297	27.570	0.137	0.229	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	265	80	745	0	76	97	0	71	0
N.S.	1	0.99	0.30	2.77	0.00	0.28	0.36	0.00	0.26	0.00
time (sec)	N/A	0.549	10.113	4.342	0.000	0.150	73.181	0.000	0.567	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	183	144	162	0	355	634	494	133	0
N.S.	1	0.91	0.72	0.81	0.00	1.77	3.15	2.46	0.66	0.00
time (sec)	N/A	0.591	0.616	2.898	0.000	0.445	33.108	0.285	0.261	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	374	116	801	0	0	199	0	111	0
N.S.	1	1.03	0.32	2.20	0.00	0.00	0.55	0.00	0.30	0.00
time (sec)	N/A	0.765	10.197	5.642	0.000	0.000	70.828	0.000	0.380	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	621	656	96	1164	0	0	199	0	87	0
N.S.	1	1.06	0.15	1.87	0.00	0.00	0.32	0.00	0.14	0.00
time (sec)	N/A	1.259	10.149	4.412	0.000	0.000	23.507	0.000	0.358	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	149	121	134	0	273	546	415	109	0
N.S.	1	0.93	0.75	0.83	0.00	1.70	3.39	2.58	0.68	0.00
time (sec)	N/A	0.508	0.537	1.617	0.000	0.416	3.813	0.247	0.246	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	339	96	768	0	0	199	0	87	0
N.S.	1	1.05	0.30	2.37	0.00	0.00	0.61	0.00	0.27	0.00
time (sec)	N/A	0.665	10.087	3.980	0.000	0.000	8.341	0.000	0.386	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	653	84	1140	0	0	202	0	89	0
N.S.	1	1.06	0.14	1.86	0.00	0.00	0.33	0.00	0.14	0.00
time (sec)	N/A	1.237	10.081	3.982	0.000	0.000	11.718	0.000	0.430	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	148	101	117	0	255	289	0	112	0
N.S.	1	0.97	0.66	0.77	0.00	1.68	1.90	0.00	0.74	0.00
time (sec)	N/A	0.520	0.576	1.413	0.000	0.450	20.528	0.000	0.242	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	338	85	759	0	0	202	0	95	0
N.S.	1	1.08	0.27	2.42	0.00	0.00	0.64	0.00	0.30	0.00
time (sec)	N/A	0.688	10.084	3.709	0.000	0.000	25.773	0.000	0.447	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	214	165	186	0	409	1028	701	154	0
N.S.	1	0.89	0.68	0.77	0.00	1.70	4.27	2.91	0.64	0.00
time (sec)	N/A	0.659	0.768	6.415	0.000	0.509	49.733	0.353	0.248	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	405	116	825	0	0	308	0	132	0
N.S.	1	1.00	0.29	2.04	0.00	0.00	0.76	0.00	0.33	0.00
time (sec)	N/A	0.823	10.245	7.138	0.000	0.000	153.202	0.000	0.429	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	661	687	99	1188	0	0	308	0	108	0
N.S.	1	1.04	0.15	1.80	0.00	0.00	0.47	0.00	0.16	0.00
time (sec)	N/A	1.289	10.169	6.378	0.000	0.000	54.815	0.000	0.407	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	180	141	158	0	323	896	590	130	0
N.S.	1	0.90	0.70	0.79	0.00	1.61	4.46	2.94	0.65	0.00
time (sec)	N/A	0.575	0.668	2.371	0.000	0.458	8.368	0.303	0.247	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	370	84	792	0	0	308	0	108	0
N.S.	1	1.02	0.23	2.18	0.00	0.00	0.85	0.00	0.30	0.00
time (sec)	N/A	0.738	10.098	5.948	0.000	0.000	19.463	0.000	0.430	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	650	684	87	1166	0	0	311	0	108	0
N.S.	1	1.05	0.13	1.79	0.00	0.00	0.48	0.00	0.17	0.00
time (sec)	N/A	1.343	10.060	5.569	0.000	0.000	25.524	0.000	0.464	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	179	127	143	0	309	403	0	131	0
N.S.	1	0.95	0.68	0.76	0.00	1.64	2.14	0.00	0.70	0.00
time (sec)	N/A	0.574	0.826	1.917	0.000	0.509	49.341	0.000	0.243	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	373	88	786	0	0	311	0	114	0
N.S.	1	1.06	0.25	2.23	0.00	0.00	0.88	0.00	0.32	0.00
time (sec)	N/A	0.733	10.052	4.817	0.000	0.000	49.846	0.000	0.500	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	100	114	0	245	194	146	90	0
N.S.	1	1.00	0.83	0.94	0.00	2.02	1.60	1.21	0.74	0.00
time (sec)	N/A	0.478	0.576	1.414	0.000	0.417	14.917	0.166	0.227	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	312	98	753	0	0	94	0	69	0
N.S.	1	1.09	0.34	2.63	0.00	0.00	0.33	0.00	0.24	0.00
time (sec)	N/A	0.663	10.147	2.735	0.000	0.000	17.962	0.000	0.330	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	594	80	1124	0	0	94	0	47	0
N.S.	1	1.09	0.15	2.07	0.00	0.00	0.17	0.00	0.09	0.00
time (sec)	N/A	1.145	10.108	2.546	0.000	0.000	6.398	0.000	0.335	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	84	79	94	0	184	151	94	64	0
N.S.	1	1.01	0.95	1.13	0.00	2.22	1.82	1.13	0.77	0.00
time (sec)	N/A	0.404	0.966	1.194	0.000	0.445	2.247	0.164	0.221	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	278	80	728	0	0	94	0	47	0
N.S.	1	1.12	0.32	2.92	0.00	0.00	0.38	0.00	0.19	0.00
time (sec)	N/A	0.582	10.074	2.931	0.000	0.000	2.194	0.000	0.330	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	589	83	1119	0	0	97	0	53	0
N.S.	1	1.09	0.15	2.06	0.00	0.00	0.18	0.00	0.10	0.00
time (sec)	N/A	1.178	10.048	3.144	0.000	0.000	2.722	0.000	0.391	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	76	66	87	0	183	60	108	71	0
N.S.	1	1.01	0.88	1.16	0.00	2.44	0.80	1.44	0.95	0.00
time (sec)	N/A	0.377	0.441	1.315	0.000	0.244	6.843	0.146	0.217	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	275	82	740	0	58	97	0	59	0
N.S.	1	1.12	0.33	3.01	0.00	0.24	0.39	0.00	0.24	0.00
time (sec)	N/A	0.587	10.054	4.166	0.000	0.100	20.136	0.000	0.447	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	120	93	129	0	307	0	132	67	0
N.S.	1	1.01	0.78	1.08	0.00	2.58	0.00	1.11	0.56	0.00
time (sec)	N/A	0.477	0.848	1.349	0.000	0.388	0.000	0.175	0.230	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	312	87	787	0	0	0	0	50	0
N.S.	1	1.09	0.31	2.76	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.649	10.127	5.740	0.000	0.000	0.000	0.000	0.317	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	607	77	1154	0	0	94	0	27	0
N.S.	1	1.10	0.14	2.09	0.00	0.00	0.17	0.00	0.05	0.00
time (sec)	N/A	1.212	10.143	3.980	0.000	0.000	82.058	0.000	0.272	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	86	81	92	0	234	95	93	45	0
N.S.	1	1.01	0.95	1.08	0.00	2.75	1.12	1.09	0.53	0.00
time (sec)	N/A	0.418	0.757	0.875	0.000	0.251	24.916	0.173	0.207	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	287	79	754	0	92	94	0	30	0
N.S.	1	1.11	0.31	2.92	0.00	0.36	0.36	0.00	0.12	0.00
time (sec)	N/A	0.606	10.078	4.733	0.000	0.108	27.071	0.000	0.281	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	585	629	77	1177	0	108	97	0	32	0
N.S.	1	1.08	0.13	2.01	0.00	0.18	0.17	0.00	0.05	0.00
time (sec)	N/A	1.230	10.051	7.634	0.000	0.102	46.477	0.000	0.320	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	44	39	0	57	90	100	25	70
N.S.	1	1.00	0.66	0.58	0.00	0.85	1.34	1.49	0.37	1.04
time (sec)	N/A	0.332	0.922	1.167	0.000	0.099	86.921	0.195	0.271	1.196

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	314	95	784	0	117	97	0	32	0
N.S.	1	1.11	0.34	2.77	0.00	0.41	0.34	0.00	0.11	0.00
time (sec)	N/A	0.635	10.065	7.436	0.000	0.108	155.848	0.000	0.364	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	119	100	157	0	345	0	126	124	0
N.S.	1	0.98	0.83	1.30	0.00	2.85	0.00	1.04	1.02	0.00
time (sec)	N/A	0.481	1.026	0.927	0.000	0.320	0.000	0.182	0.256	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	325	108	809	0	170	0	0	112	0
N.S.	1	1.10	0.37	2.74	0.00	0.58	0.00	0.00	0.38	0.00
time (sec)	N/A	0.672	10.174	5.514	0.000	0.172	0.000	0.000	0.573	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	643	86	1190	0	169	0	0	38	0
N.S.	1	1.08	0.14	2.00	0.00	0.28	0.00	0.00	0.06	0.00
time (sec)	N/A	1.257	10.138	5.358	0.000	0.116	0.000	0.000	0.423	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	44	39	0	59	0	72	27	73
N.S.	1	1.00	0.56	0.49	0.00	0.75	0.00	0.91	0.34	0.92
time (sec)	N/A	0.359	0.828	0.795	0.000	0.109	0.000	0.154	0.282	1.035

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	323	107	785	0	150	0	0	41	0
N.S.	1	1.09	0.36	2.64	0.00	0.51	0.00	0.00	0.14	0.00
time (sec)	N/A	0.646	10.113	6.722	0.000	0.126	0.000	0.000	0.386	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	624	665	85	1225	0	167	0	0	43	0
N.S.	1	1.07	0.14	1.96	0.00	0.27	0.00	0.00	0.07	0.00
time (sec)	N/A	1.281	10.066	10.543	0.000	0.137	0.000	0.000	0.464	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	67	62	0	93	0	0	44	115
N.S.	1	1.00	0.64	0.60	0.00	0.89	0.00	0.00	0.42	1.11
time (sec)	N/A	0.384	0.782	1.237	0.000	0.124	0.000	0.000	0.248	1.248

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	350	121	829	0	178	0	0	43	0
N.S.	1	1.09	0.38	2.59	0.00	0.56	0.00	0.00	0.13	0.00
time (sec)	N/A	0.675	10.103	12.211	0.000	0.127	0.000	0.000	0.546	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	107	80	68	118	99	212	104	57	96
N.S.	1	1.04	0.78	0.66	1.15	0.96	2.06	1.01	0.55	0.93
time (sec)	N/A	0.418	0.093	0.760	0.035	0.109	0.509	0.127	0.226	0.853

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	77	56	49	84	75	162	73	45	76
N.S.	1	1.05	0.77	0.67	1.15	1.03	2.22	1.00	0.62	1.04
time (sec)	N/A	0.369	0.068	0.790	0.038	0.098	0.337	0.121	0.221	0.812

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	34	31	49	50	110	44	32	53
N.S.	1	1.09	0.74	0.67	1.07	1.09	2.39	0.96	0.70	1.15
time (sec)	N/A	0.332	0.046	0.808	0.036	0.092	0.253	0.123	0.208	0.790

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	124	146	115	115	127	68	125	55	140
N.S.	1	1.04	1.23	0.97	0.97	1.07	0.57	1.05	0.46	1.18
time (sec)	N/A	0.422	0.265	0.880	0.145	0.108	12.538	0.390	0.229	1.427

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	138	163	119	205	187	87	155	63	264
N.S.	1	0.97	1.15	0.84	1.44	1.32	0.61	1.09	0.44	1.86
time (sec)	N/A	0.431	0.468	1.180	0.114	0.150	17.856	0.405	0.245	1.417

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	146	168	134	263	235	85	214	64	342
N.S.	1	0.91	1.04	0.83	1.63	1.46	0.53	1.33	0.40	2.12
time (sec)	N/A	0.440	0.603	1.196	0.120	0.096	40.567	0.447	0.252	2.087

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	165	212	166	409	245	83	0	74	0
N.S.	1	0.91	1.17	0.92	2.26	1.35	0.46	0.00	0.41	0.00
time (sec)	N/A	0.442	1.203	1.276	0.115	0.124	5.288	0.000	0.242	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	135	181	273	320	243	83	0	49	0
N.S.	1	0.92	1.24	1.87	2.19	1.66	0.57	0.00	0.34	0.00
time (sec)	N/A	0.394	0.828	1.177	0.114	0.138	2.128	0.000	0.298	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	129	174	132	253	0	85	0	50	0
N.S.	1	0.93	1.25	0.95	1.82	0.00	0.61	0.00	0.36	0.00
time (sec)	N/A	0.394	0.711	1.236	0.127	0.000	1.545	0.000	0.248	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	116	167	143	135	0	116	0	54	60
N.S.	1	1.03	1.48	1.27	1.19	0.00	1.03	0.00	0.48	0.53
time (sec)	N/A	0.361	0.462	1.208	0.109	0.000	1.375	0.000	0.237	1.566

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	40	36	55	55	187	0	38	97
N.S.	1	1.00	0.75	0.68	1.04	1.04	3.53	0.00	0.72	1.83
time (sec)	N/A	0.301	0.239	0.853	0.028	0.136	1.302	0.000	0.233	1.176

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	62	55	90	81	646	0	49	132
N.S.	1	0.99	0.74	0.65	1.07	0.96	7.69	0.00	0.58	1.57
time (sec)	N/A	0.344	0.285	0.874	0.037	0.161	1.959	0.000	0.217	1.487

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	113	86	74	124	105	1392	0	60	174
N.S.	1	0.97	0.74	0.63	1.06	0.90	11.90	0.00	0.51	1.49
time (sec)	N/A	0.402	0.314	1.068	0.027	0.116	2.381	0.000	0.224	1.904

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	0	0	0	83	0	70	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.97	0.00	0.81	0.00
time (sec)	N/A	0.363	6.762	0.000	0.000	0.000	1.189	0.000	0.306	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	72	0	0	0	82	0	45	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	1.00	0.00	0.55	0.00
time (sec)	N/A	0.328	7.102	0.000	0.000	0.000	1.277	0.000	0.253	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	77	0	0	0	85	0	65	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	1.08	0.00	0.82	0.00
time (sec)	N/A	0.319	7.365	0.000	0.000	0.000	1.284	0.000	0.233	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	0	90	0	65	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	1.06	0.00	0.76	0.00
time (sec)	N/A	0.341	10.102	0.000	0.000	0.000	1.349	0.000	0.244	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	78	0	0	0	94	0	66	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	1.11	0.00	0.78	0.00
time (sec)	N/A	0.357	10.104	0.000	0.000	0.000	1.589	0.000	0.248	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	107	80	68	118	99	212	104	57	96
N.S.	1	1.04	0.78	0.66	1.15	0.96	2.06	1.01	0.55	0.93
time (sec)	N/A	0.426	0.099	0.764	0.030	0.103	0.519	0.126	0.231	0.962

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	77	56	49	84	76	162	73	46	76
N.S.	1	1.05	0.77	0.67	1.15	1.04	2.22	1.00	0.63	1.04
time (sec)	N/A	0.350	0.073	0.764	0.029	0.103	0.563	0.136	0.207	0.868

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	34	31	49	51	110	44	32	54
N.S.	1	1.09	0.74	0.67	1.07	1.11	2.39	0.96	0.70	1.17
time (sec)	N/A	0.309	0.049	0.769	0.031	0.102	0.293	0.116	0.261	0.799

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	125	148	115	116	149	70	126	55	160
N.S.	1	1.03	1.22	0.95	0.96	1.23	0.58	1.04	0.45	1.32
time (sec)	N/A	0.383	0.274	0.854	0.116	0.092	11.455	0.425	0.261	1.038

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	140	166	119	207	364	87	164	63	307
N.S.	1	0.95	1.12	0.80	1.40	2.46	0.59	1.11	0.43	2.07
time (sec)	N/A	0.430	0.543	1.173	0.112	0.167	16.464	0.399	0.248	1.414

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	145	169	140	265	458	83	219	64	375
N.S.	1	0.90	1.05	0.87	1.65	2.84	0.52	1.36	0.40	2.33
time (sec)	N/A	0.435	0.708	1.122	0.117	0.160	37.939	0.408	0.263	2.052

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	161	211	180	410	482	83	0	70	0
N.S.	1	0.90	1.19	1.01	2.30	2.71	0.47	0.00	0.39	0.00
time (sec)	N/A	0.439	1.055	1.406	0.141	0.128	5.619	0.000	0.256	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	129	180	269	322	470	82	0	45	0
N.S.	1	0.91	1.28	1.91	2.28	3.33	0.58	0.00	0.32	0.00
time (sec)	N/A	0.355	0.877	1.205	0.117	0.139	2.048	0.000	0.254	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	131	180	133	255	0	85	0	51	0
N.S.	1	0.93	1.28	0.94	1.81	0.00	0.60	0.00	0.36	0.00
time (sec)	N/A	0.375	0.827	1.260	0.125	0.000	1.770	0.000	0.242	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	116	170	141	136	0	117	0	54	0
N.S.	1	1.03	1.50	1.25	1.20	0.00	1.04	0.00	0.48	0.00
time (sec)	N/A	0.368	0.618	1.227	0.113	0.000	1.685	0.000	0.299	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	40	36	55	56	189	0	38	98
N.S.	1	1.00	0.75	0.68	1.04	1.06	3.57	0.00	0.72	1.85
time (sec)	N/A	0.299	0.303	0.864	0.027	0.101	1.686	0.000	0.217	1.284

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	62	55	90	82	648	0	49	132
N.S.	1	0.99	0.74	0.65	1.07	0.98	7.71	0.00	0.58	1.57
time (sec)	N/A	0.343	0.407	0.881	0.035	0.112	2.348	0.000	0.209	1.552

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	113	86	74	124	105	1392	0	60	174
N.S.	1	0.97	0.74	0.63	1.06	0.90	11.90	0.00	0.51	1.49
time (sec)	N/A	0.405	0.495	0.938	0.028	0.136	3.134	0.000	0.204	2.041

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	0	0	0	83	0	74	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.97	0.00	0.86	0.00
time (sec)	N/A	0.364	7.957	0.000	0.000	0.000	1.370	0.000	0.279	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	0	0	0	83	0	49	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.97	0.00	0.57	0.00
time (sec)	N/A	0.357	7.739	0.000	0.000	0.000	1.313	0.000	0.237	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	77	0	0	0	85	0	63	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	1.02	0.00	0.76	0.00
time (sec)	N/A	0.356	7.965	0.000	0.000	0.000	1.715	0.000	0.227	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	78	0	0	0	92	0	66	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	1.08	0.00	0.78	0.00
time (sec)	N/A	0.354	10.102	0.000	0.000	0.000	2.125	0.000	0.257	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	78	0	0	0	94	0	65	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	1.09	0.00	0.76	0.00
time (sec)	N/A	0.350	10.132	0.000	0.000	0.000	1.827	0.000	0.241	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	78	0	0	0	94	0	66	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	1.11	0.00	0.78	0.00
time (sec)	N/A	0.352	10.112	0.000	0.000	0.000	2.394	0.000	0.258	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	107	80	68	118	76	172	110	35	80
N.S.	1	1.04	0.78	0.66	1.15	0.74	1.67	1.07	0.34	0.78
time (sec)	N/A	0.399	0.091	0.783	0.028	0.113	0.459	0.124	0.260	0.928

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	77	56	49	84	52	121	79	35	57
N.S.	1	1.05	0.77	0.67	1.15	0.71	1.66	1.08	0.48	0.78
time (sec)	N/A	0.347	0.069	0.779	0.038	0.108	0.357	0.124	0.259	0.840

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	34	31	49	30	71	47	35	34
N.S.	1	1.09	0.74	0.67	1.07	0.65	1.54	1.02	0.76	0.74
time (sec)	N/A	0.310	0.046	0.769	0.037	0.110	0.232	0.121	0.271	0.823

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	107	131	107	105	280	63	105	35	135
N.S.	1	1.02	1.25	1.02	1.00	2.67	0.60	1.00	0.33	1.29
time (sec)	N/A	0.369	0.201	0.843	0.118	0.113	6.477	0.416	0.248	1.173

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	121	148	112	211	398	80	152	35	277
N.S.	1	0.92	1.12	0.85	1.60	3.02	0.61	1.15	0.27	2.10
time (sec)	N/A	0.399	0.438	1.199	0.147	0.104	8.627	0.421	0.302	1.711

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	152	172	140	283	406	82	220	35	367
N.S.	1	0.90	1.02	0.83	1.68	2.42	0.49	1.31	0.21	2.18
time (sec)	N/A	0.435	0.577	1.219	0.114	0.109	16.543	0.455	0.259	2.198

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	137	182	269	328	426	80	0	35	0
N.S.	1	0.96	1.27	1.88	2.29	2.98	0.56	0.00	0.24	0.00
time (sec)	N/A	0.398	1.090	1.270	0.142	0.128	2.654	0.000	0.248	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	107	163	225	244	411	78	0	31	0
N.S.	1	0.96	1.47	2.03	2.20	3.70	0.70	0.00	0.28	0.00
time (sec)	N/A	0.334	0.648	1.147	0.124	0.094	1.598	0.000	0.268	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	95	152	133	122	0	71	0	31	57
N.S.	1	1.01	1.62	1.41	1.30	0.00	0.76	0.00	0.33	0.61
time (sec)	N/A	0.316	0.430	1.174	0.115	0.000	1.171	0.000	0.248	0.979

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	40	36	55	35	107	0	35	36
N.S.	1	1.00	0.75	0.68	1.04	0.66	2.02	0.00	0.66	0.68
time (sec)	N/A	0.300	0.273	0.858	0.031	0.119	1.060	0.000	0.278	1.090

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	62	55	90	58	490	0	35	58
N.S.	1	0.99	0.74	0.65	1.07	0.69	5.83	0.00	0.42	0.69
time (sec)	N/A	0.348	0.324	0.864	0.031	0.091	1.489	0.000	0.264	0.983

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	113	86	74	124	82	1120	0	35	105
N.S.	1	0.97	0.74	0.63	1.06	0.70	9.57	0.00	0.30	0.90
time (sec)	N/A	0.386	0.392	1.081	0.043	0.177	2.722	0.000	0.263	1.080

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	0	0	0	80	0	35	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.93	0.00	0.41	0.00
time (sec)	N/A	0.350	10.054	0.000	0.000	0.000	1.542	0.000	0.253	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	0	0	0	80	0	33	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.93	0.00	0.38	0.00
time (sec)	N/A	0.348	10.045	0.000	0.000	0.000	1.547	0.000	0.220	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	76	0	0	0	82	0	33	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	1.00	0.00	0.40	0.00
time (sec)	N/A	0.344	10.044	0.000	0.000	0.000	1.047	0.000	0.257	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	78	0	0	0	88	0	35	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	1.04	0.00	0.41	0.00
time (sec)	N/A	0.365	10.042	0.000	0.000	0.000	1.562	0.000	0.271	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	78	0	0	0	94	0	35	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	1.09	0.00	0.41	0.00
time (sec)	N/A	0.381	10.044	0.000	0.000	0.000	1.498	0.000	0.252	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	105	80	68	118	76	172	101	35	80
N.S.	1	1.05	0.80	0.68	1.18	0.76	1.72	1.01	0.35	0.80
time (sec)	N/A	0.403	0.093	0.885	0.036	0.135	0.533	0.123	0.216	0.873

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	75	56	49	83	52	121	69	35	57
N.S.	1	1.06	0.79	0.69	1.17	0.73	1.70	0.97	0.49	0.80
time (sec)	N/A	0.347	0.072	0.772	0.033	0.108	0.363	0.121	0.235	0.830

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	48	33	30	47	29	70	38	35	34
N.S.	1	1.12	0.77	0.70	1.09	0.67	1.63	0.88	0.81	0.79
time (sec)	N/A	0.313	0.049	0.789	0.030	0.094	0.250	0.121	0.211	0.847

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	106	131	106	103	146	63	104	27	125
N.S.	1	1.03	1.27	1.03	1.00	1.42	0.61	1.01	0.26	1.21
time (sec)	N/A	0.371	0.199	0.872	0.137	0.088	6.793	0.364	0.211	1.204

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	123	151	112	209	206	82	146	35	254
N.S.	1	0.91	1.12	0.83	1.55	1.53	0.61	1.08	0.26	1.88
time (sec)	N/A	0.402	0.425	1.199	0.137	0.102	9.614	0.433	0.234	1.617

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	153	172	133	282	208	82	215	35	341
N.S.	1	0.91	1.02	0.79	1.68	1.24	0.49	1.28	0.21	2.03
time (sec)	N/A	0.429	0.534	1.226	0.114	0.127	20.890	0.440	0.242	2.073

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	141	184	273	327	218	80	0	35	0
N.S.	1	0.97	1.26	1.87	2.24	1.49	0.55	0.00	0.24	0.00
time (sec)	N/A	0.400	0.888	1.257	0.117	0.116	3.326	0.000	0.212	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	111	166	227	242	218	80	0	33	0
N.S.	1	0.97	1.46	1.99	2.12	1.91	0.70	0.00	0.29	0.00
time (sec)	N/A	0.335	0.699	1.236	0.121	0.098	1.942	0.000	0.216	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	95	151	127	121	0	73	0	33	0
N.S.	1	1.01	1.61	1.35	1.29	0.00	0.78	0.00	0.35	0.00
time (sec)	N/A	0.324	0.553	1.206	0.121	0.000	1.385	0.000	0.256	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	40	36	55	34	105	0	35	35
N.S.	1	1.00	0.75	0.68	1.04	0.64	1.98	0.00	0.66	0.66
time (sec)	N/A	0.280	0.302	0.849	0.032	0.122	1.182	0.000	0.217	0.951

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	62	55	90	58	488	0	35	58
N.S.	1	0.99	0.74	0.65	1.07	0.69	5.81	0.00	0.42	0.69
time (sec)	N/A	0.343	0.365	0.860	0.033	0.087	2.030	0.000	0.217	1.054

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	113	86	74	124	82	1120	0	35	105
N.S.	1	0.97	0.74	0.63	1.06	0.70	9.57	0.00	0.30	0.90
time (sec)	N/A	0.404	0.429	0.914	0.031	0.128	2.510	0.000	0.229	1.150

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	0	0	0	80	0	35	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.93	0.00	0.41	0.00
time (sec)	N/A	0.370	10.056	0.000	0.000	0.000	1.849	0.000	0.268	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	0	0	0	80	0	35	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.93	0.00	0.41	0.00
time (sec)	N/A	0.364	10.050	0.000	0.000	0.000	1.678	0.000	0.227	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	78	0	31	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.95	0.00	0.38	0.00
time (sec)	N/A	0.318	10.059	0.000	0.000	0.000	1.151	0.000	0.224	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	78	0	0	0	82	0	31	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.99	0.00	0.37	0.00
time (sec)	N/A	0.325	10.040	0.000	0.000	0.000	1.180	0.000	0.208	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	78	0	0	0	90	0	35	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	1.05	0.00	0.41	0.00
time (sec)	N/A	0.349	10.047	0.000	0.000	0.000	1.493	0.000	0.237	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	105	77	68	117	88	172	113	15	91
N.S.	1	1.04	0.76	0.67	1.16	0.87	1.70	1.12	0.15	0.90
time (sec)	N/A	0.379	0.099	0.786	0.034	0.108	0.554	0.131	0.221	1.047

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	75	56	49	82	64	121	78	15	60
N.S.	1	1.07	0.80	0.70	1.17	0.91	1.73	1.11	0.21	0.86
time (sec)	N/A	0.347	0.089	0.796	0.028	0.098	0.362	0.120	0.220	0.929

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	48	33	30	48	41	70	37	15	32
N.S.	1	1.09	0.75	0.68	1.09	0.93	1.59	0.84	0.34	0.73
time (sec)	N/A	0.306	0.088	0.894	0.030	0.085	0.254	0.115	0.268	0.891

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	118	135	119	119	392	66	115	15	152
N.S.	1	1.05	1.21	1.06	1.06	3.50	0.59	1.03	0.13	1.36
time (sec)	N/A	0.386	0.301	0.883	0.111	0.128	10.499	0.464	0.210	1.179

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	143	168	140	243	594	82	187	15	345
N.S.	1	0.90	1.06	0.88	1.53	3.74	0.52	1.18	0.09	2.17
time (sec)	N/A	0.414	0.758	1.441	0.112	0.150	28.583	0.436	0.202	1.541

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	175	205	170	371	624	80	0	15	0
N.S.	1	1.04	1.22	1.01	2.21	3.71	0.48	0.00	0.09	0.00
time (sec)	N/A	0.466	1.255	1.497	0.118	0.105	15.546	0.000	0.210	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	145	177	148	281	598	80	0	15	0
N.S.	1	1.07	1.30	1.09	2.07	4.40	0.59	0.00	0.11	0.00
time (sec)	N/A	0.419	1.025	1.470	0.112	0.126	4.921	0.000	0.255	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	103	150	136	134	488	71	0	11	0
N.S.	1	1.04	1.52	1.37	1.35	4.93	0.72	0.00	0.11	0.00
time (sec)	N/A	0.330	0.556	0.917	0.108	0.112	3.096	0.000	0.222	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	40	36	52	47	102	0	15	39
N.S.	1	1.00	0.78	0.71	1.02	0.92	2.00	0.00	0.29	0.76
time (sec)	N/A	0.297	0.333	0.847	0.030	0.103	7.610	0.000	0.233	0.825

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	81	62	57	93	69	396	0	15	70
N.S.	1	0.98	0.75	0.69	1.12	0.83	4.77	0.00	0.18	0.84
time (sec)	N/A	0.338	0.440	0.858	0.041	0.091	21.631	0.000	0.218	0.946

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	111	86	74	130	94	920	0	15	103
N.S.	1	0.97	0.75	0.65	1.14	0.82	8.07	0.00	0.13	0.90
time (sec)	N/A	0.402	0.578	0.908	0.030	0.107	41.405	0.000	0.269	1.214

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	96	83	0	0	0	80	0	15	0
N.S.	1	1.08	0.93	0.00	0.00	0.00	0.90	0.00	0.17	0.00
time (sec)	N/A	0.375	10.053	0.000	0.000	0.000	15.169	0.000	0.220	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	96	83	0	0	0	80	0	15	0
N.S.	1	1.08	0.93	0.00	0.00	0.00	0.90	0.00	0.17	0.00
time (sec)	N/A	0.376	10.049	0.000	0.000	0.000	6.332	0.000	0.232	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	95	83	0	0	0	80	0	13	0
N.S.	1	1.12	0.98	0.00	0.00	0.00	0.94	0.00	0.15	0.00
time (sec)	N/A	0.360	10.048	0.000	0.000	0.000	3.431	0.000	0.251	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	0	0	0	82	0	15	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.98	0.00	0.18	0.00
time (sec)	N/A	0.335	10.047	0.000	0.000	0.000	6.313	0.000	0.252	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	72	0	0	0	88	0	15	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	1.02	0.00	0.17	0.00
time (sec)	N/A	0.343	10.048	0.000	0.000	0.000	15.340	0.000	0.214	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	72	0	0	0	94	0	15	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	1.09	0.00	0.17	0.00
time (sec)	N/A	0.345	10.043	0.000	0.000	0.000	39.130	0.000	0.216	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	137	1077	205	851	5418	1331	599	559
N.S.	1	1.00	0.93	7.28	1.39	5.75	36.61	8.99	4.05	3.78
time (sec)	N/A	0.574	0.612	1.493	0.033	0.123	1.729	0.173	0.230	1.473

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	261	91	215	1057	332	176	177
N.S.	1	1.00	0.93	3.68	1.28	3.03	14.89	4.68	2.48	2.49
time (sec)	N/A	0.371	0.154	0.687	0.031	0.134	0.621	0.127	0.234	0.972

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	53	53	92	410	143	91	95
N.S.	1	1.00	0.93	1.18	1.18	2.04	9.11	3.18	2.02	2.11
time (sec)	N/A	0.310	0.080	0.142	0.031	0.084	0.473	0.127	0.248	0.850

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	0	187	0	10	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	2.83	0.00	0.15	0.00
time (sec)	N/A	0.335	0.136	0.000	0.000	0.000	7.763	0.000	0.219	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	1049	0	15	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	11.28	0.00	0.16	0.00
time (sec)	N/A	0.363	0.226	0.000	0.000	0.000	152.790	0.000	0.258	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	0	0	26	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.371	0.398	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	134	113	0	0	0	379	0	804	0
N.S.	1	1.09	0.92	0.00	0.00	0.00	3.08	0.00	6.54	0.00
time (sec)	N/A	0.459	0.932	0.000	0.000	0.000	20.412	0.000	0.266	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	132	111	0	0	0	246	0	491	0
N.S.	1	1.09	0.92	0.00	0.00	0.00	2.03	0.00	4.06	0.00
time (sec)	N/A	0.456	0.391	0.000	0.000	0.000	7.236	0.000	0.294	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	131	110	0	0	0	121	0	258	0
N.S.	1	1.09	0.92	0.00	0.00	0.00	1.01	0.00	2.15	0.00
time (sec)	N/A	0.449	0.411	0.000	0.000	0.000	2.287	0.000	0.282	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	131	110	0	0	0	117	0	107	0
N.S.	1	1.09	0.92	0.00	0.00	0.00	0.98	0.00	0.89	0.00
time (sec)	N/A	0.439	0.290	0.000	0.000	0.000	2.327	0.000	0.228	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	133	113	0	0	0	117	0	27	0
N.S.	1	1.01	0.86	0.00	0.00	0.00	0.89	0.00	0.20	0.00
time (sec)	N/A	0.453	0.514	0.000	0.000	0.000	25.785	0.000	0.212	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	133	113	0	0	0	0	0	38	0
N.S.	1	1.08	0.92	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.454	1.026	0.000	0.000	0.000	0.000	0.000	0.250	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	118	111	0	0	0	246	0	492	0
N.S.	1	1.02	0.96	0.00	0.00	0.00	2.12	0.00	4.24	0.00
time (sec)	N/A	0.419	0.464	0.000	0.000	0.000	12.651	0.000	0.264	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	117	110	0	0	0	121	0	255	0
N.S.	1	1.02	0.96	0.00	0.00	0.00	1.05	0.00	2.22	0.00
time (sec)	N/A	0.426	0.317	0.000	0.000	0.000	5.015	0.000	0.246	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	117	110	0	0	0	121	0	255	0
N.S.	1	1.02	0.96	0.00	0.00	0.00	1.05	0.00	2.22	0.00
time (sec)	N/A	0.423	0.276	0.000	0.000	0.000	3.228	0.000	0.242	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	117	110	0	0	0	117	0	42	0
N.S.	1	1.02	0.96	0.00	0.00	0.00	1.02	0.00	0.37	0.00
time (sec)	N/A	0.425	0.320	0.000	0.000	0.000	1.915	0.000	0.265	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	117	110	0	0	0	117	0	42	0
N.S.	1	1.02	0.96	0.00	0.00	0.00	1.02	0.00	0.37	0.00
time (sec)	N/A	0.422	0.426	0.000	0.000	0.000	2.152	0.000	0.234	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	124	113	0	0	0	117	0	19	0
N.S.	1	1.10	1.00	0.00	0.00	0.00	1.04	0.00	0.17	0.00
time (sec)	N/A	0.415	0.767	0.000	0.000	0.000	25.699	0.000	0.202	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	116	98	234	182	256	0	437	279	232
N.S.	1	0.94	0.80	1.90	1.48	2.08	0.00	3.55	2.27	1.89
time (sec)	N/A	0.457	0.253	1.021	0.036	0.117	0.000	0.144	0.237	1.147

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	84	84	127	123	161	0	204	167	154
N.S.	1	0.95	0.95	1.44	1.40	1.83	0.00	2.32	1.90	1.75
time (sec)	N/A	0.394	0.183	0.892	0.036	0.137	0.000	0.134	0.229	0.878

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	54	48	55	71	88	683	75	84	96
N.S.	1	0.96	0.86	0.98	1.27	1.57	12.20	1.34	1.50	1.71
time (sec)	N/A	0.337	0.147	0.794	0.043	0.103	121.659	0.122	0.215	0.808

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	66	52	0	0	0	76	0	128	0
N.S.	1	0.99	0.78	0.00	0.00	0.00	1.13	0.00	1.91	0.00
time (sec)	N/A	0.310	0.086	0.000	0.000	0.000	13.472	0.000	0.232	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	63	0	0	0	83	0	100	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	1.15	0.00	1.39	0.00
time (sec)	N/A	0.317	0.088	0.000	0.000	0.000	58.294	0.000	0.257	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	78	0	0	0	76	0	894	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.83	0.00	9.72	0.00
time (sec)	N/A	0.403	0.161	0.000	0.000	0.000	96.053	0.000	0.265	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	78	0	0	0	76	0	512	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.83	0.00	5.57	0.00
time (sec)	N/A	0.381	0.155	0.000	0.000	0.000	64.054	0.000	0.228	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	90	0	0	0	75	0	498	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.88	0.00	5.86	0.00
time (sec)	N/A	0.365	0.146	0.000	0.000	0.000	43.093	0.000	0.249	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	79	0	0	0	78	0	501	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.92	0.00	5.89	0.00
time (sec)	N/A	0.373	0.184	0.000	0.000	0.000	42.807	0.000	0.253	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	0	0	0	78	0	512	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.91	0.00	5.95	0.00
time (sec)	N/A	0.368	0.187	0.000	0.000	0.000	46.878	0.000	0.262	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	81	0	0	0	85	0	583	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.96	0.00	6.55	0.00
time (sec)	N/A	0.372	0.187	0.000	0.000	0.000	103.957	0.000	0.224	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	83	0	0	0	0	0	540	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	5.05	0.00
time (sec)	N/A	0.426	0.472	0.000	0.000	0.000	0.000	0.000	0.410	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	100	0	0	0	0	0	525	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	4.82	0.00
time (sec)	N/A	0.429	0.225	0.000	0.000	0.000	0.000	0.000	0.349	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	98	0	0	0	0	0	546	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	5.20	0.00
time (sec)	N/A	0.435	0.424	0.000	0.000	0.000	0.000	0.000	0.376	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	97	0	0	0	0	0	597	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	6.03	0.00
time (sec)	N/A	0.415	0.503	0.000	0.000	0.000	0.000	0.000	0.387	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	96	0	0	0	0	0	446	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	4.42	0.00
time (sec)	N/A	0.420	0.251	0.000	0.000	0.000	0.000	0.000	0.376	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	97	0	0	0	0	0	620	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	6.08	0.00
time (sec)	N/A	0.438	0.577	0.000	0.000	0.000	0.000	0.000	0.478	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	101	0	0	0	0	0	621	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	6.03	0.00
time (sec)	N/A	0.434	0.452	0.000	0.000	0.000	0.000	0.000	0.428	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	121	108	0	0	0	0	0	0	0
N.S.	1	1.02	0.91	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.456	0.133	0.000	0.000	0.000	0.000	0.000	0.262	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	97	81	0	0	0	0	0	119	0
N.S.	1	1.09	0.91	0.00	0.00	0.00	0.00	0.00	1.34	0.00
time (sec)	N/A	0.414	0.165	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	106	19	32	24	0	37	24	18
N.S.	1	1.00	4.82	0.86	1.45	1.09	0.00	1.68	1.09	0.82
time (sec)	N/A	0.255	0.165	3.348	0.073	0.121	0.000	0.134	0.219	1.158

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	87	82	78	84	100	0	88	160	88
N.S.	1	0.97	0.91	0.87	0.93	1.11	0.00	0.98	1.78	0.98
time (sec)	N/A	0.446	0.048	1.188	0.026	0.582	0.000	0.130	0.269	1.272

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	66	66	65	68	72	201	70	130	68
N.S.	1	0.94	0.94	0.93	0.97	1.03	2.87	1.00	1.86	0.97
time (sec)	N/A	0.384	0.035	1.007	0.034	0.232	150.851	0.132	0.215	1.270

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	52	43	43	49	42	144	51	95	51
N.S.	1	0.98	0.81	0.81	0.92	0.79	2.72	0.96	1.79	0.96
time (sec)	N/A	0.356	0.024	0.998	0.026	0.148	2.539	0.139	0.208	1.049

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	44	31	32	41	31	138	51	79	602
N.S.	1	0.98	0.69	0.71	0.91	0.69	3.07	1.13	1.76	13.38
time (sec)	N/A	0.298	0.022	0.986	0.030	0.119	0.769	0.125	0.241	1.030

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	63	54	55	61	54	0	71	107	58
N.S.	1	1.02	0.87	0.89	0.98	0.87	0.00	1.15	1.73	0.94
time (sec)	N/A	0.379	0.031	1.030	0.031	0.446	0.000	0.128	0.250	1.317

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	86	88	82	87	99	0	111	165	87
N.S.	1	0.99	1.01	0.94	1.00	1.14	0.00	1.28	1.90	1.00
time (sec)	N/A	0.444	0.044	1.055	0.026	1.197	0.000	0.125	0.246	1.544

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	304	261	241	366	283	0	351	219	1665
N.S.	1	0.96	0.82	0.76	1.15	0.89	0.00	1.11	0.69	5.25
time (sec)	N/A	1.039	0.209	1.151	0.111	2.608	0.000	0.129	0.241	10.834

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	316	242	228	324	273	0	311	228	1751
N.S.	1	1.05	0.80	0.76	1.08	0.91	0.00	1.03	0.76	5.82
time (sec)	N/A	0.844	0.127	1.141	0.114	0.274	0.000	0.133	0.263	10.700

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	271	238	225	349	228	0	308	190	873
N.S.	1	0.92	0.80	0.76	1.18	0.77	0.00	1.04	0.64	2.95
time (sec)	N/A	0.815	0.125	1.141	0.115	0.116	0.000	0.136	0.224	1.923

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	258	224	207	289	244	573	286	182	1364
N.S.	1	0.90	0.78	0.72	1.00	0.85	1.99	0.99	0.63	4.74
time (sec)	N/A	0.709	0.094	1.126	0.118	0.146	67.206	0.141	0.242	7.885

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	248	224	207	317	199	342	278	170	1265
N.S.	1	0.86	0.78	0.72	1.10	0.69	1.19	0.97	0.59	4.39
time (sec)	N/A	0.735	0.090	1.115	0.117	0.126	9.182	0.141	0.212	6.750

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	258	224	207	265	201	515	290	182	982
N.S.	1	0.90	0.78	0.72	0.92	0.70	1.79	1.01	0.63	3.41
time (sec)	N/A	0.716	0.112	1.122	0.115	0.146	5.562	0.147	0.252	3.218

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	248	224	207	293	254	447	278	188	1364
N.S.	1	0.86	0.78	0.72	1.02	0.88	1.55	0.97	0.65	4.74
time (sec)	N/A	0.673	0.111	0.970	0.118	0.164	57.838	0.137	0.243	7.843

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	314	244	228	300	238	0	305	231	716
N.S.	1	1.05	0.82	0.76	1.00	0.80	0.00	1.02	0.77	2.39
time (sec)	N/A	0.805	0.153	1.253	0.115	0.111	0.000	0.129	0.226	2.306

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	276	259	228	328	301	0	309	257	1829
N.S.	1	0.92	0.86	0.76	1.09	1.00	0.00	1.03	0.85	6.08
time (sec)	N/A	0.827	0.203	1.319	0.113	0.917	0.000	0.137	0.241	10.630

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	355	282	248	341	305	0	328	278	1734
N.S.	1	1.12	0.89	0.78	1.07	0.96	0.00	1.03	0.87	5.45
time (sec)	N/A	0.949	0.265	1.134	0.113	0.720	0.000	0.146	0.239	10.180

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	309	282	248	369	356	0	336	292	1860
N.S.	1	0.96	0.88	0.77	1.15	1.11	0.00	1.05	0.91	5.79
time (sec)	N/A	1.004	0.359	1.139	0.114	0.191	0.000	0.134	0.265	10.421

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	391	304	279	376	332	0	377	312	1814
N.S.	1	1.11	0.86	0.79	1.07	0.94	0.00	1.07	0.89	5.15
time (sec)	N/A	1.124	0.309	1.190	0.120	0.240	0.000	0.150	0.224	10.778

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	99	78	64	69	155	95	82	94	109
N.S.	1	1.02	0.80	0.66	0.71	1.60	0.98	0.85	0.97	1.12
time (sec)	N/A	0.425	0.118	10.101	0.112	0.126	9.463	0.128	0.248	3.339

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	84	66	54	53	130	78	64	73	88
N.S.	1	1.11	0.87	0.71	0.70	1.71	1.03	0.84	0.96	1.16
time (sec)	N/A	0.352	0.094	1.402	0.116	0.118	3.722	0.118	0.210	2.850

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	59	54	44	42	110	61	44	57	71
N.S.	1	1.04	0.95	0.77	0.74	1.93	1.07	0.77	1.00	1.25
time (sec)	N/A	0.307	0.068	1.033	0.106	0.141	1.686	0.125	0.234	2.411

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	69	59	45	0	144	78	50	79	93
N.S.	1	1.06	0.91	0.69	0.00	2.22	1.20	0.77	1.22	1.43
time (sec)	N/A	0.338	0.075	1.826	0.000	0.127	2.777	0.130	0.244	3.252

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	95	88	66	0	189	0	72	165	113
N.S.	1	1.08	1.00	0.75	0.00	2.15	0.00	0.82	1.88	1.28
time (sec)	N/A	0.380	0.182	1.221	0.000	0.105	0.000	0.121	0.232	3.550

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	689	691	133	867	0	2442	0	0	98	0
N.S.	1	1.00	0.19	1.26	0.00	3.54	0.00	0.00	0.14	0.00
time (sec)	N/A	1.228	6.511	1.881	0.000	3.007	0.000	0.000	0.251	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	659	693	63	848	0	2202	0	0	23	0
N.S.	1	1.05	0.10	1.29	0.00	3.34	0.00	0.00	0.03	0.00
time (sec)	N/A	1.148	8.782	1.020	0.000	1.072	0.000	0.000	0.218	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	697	690	136	868	0	2253	0	0	25	0
N.S.	1	0.99	0.20	1.25	0.00	3.23	0.00	0.00	0.04	0.00
time (sec)	N/A	1.214	11.089	1.799	0.000	0.707	0.000	0.000	0.253	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	236	713	0	2387	0	0	95	0
N.S.	1	1.00	3.58	10.80	0.00	36.17	0.00	0.00	1.44	0.00
time (sec)	N/A	0.344	6.368	1.694	0.000	1.774	0.000	0.000	0.226	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	165	696	0	2240	0	0	22	0
N.S.	1	1.00	2.58	10.88	0.00	35.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.314	10.178	1.023	0.000	0.568	0.000	0.000	0.207	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	244	716	0	2361	0	0	25	0
N.S.	1	1.00	3.70	10.85	0.00	35.77	0.00	0.00	0.38	0.00
time (sec)	N/A	0.340	11.159	1.836	0.000	1.151	0.000	0.000	0.205	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	65	54	53	130	78	68	72	88
N.S.	1	1.00	0.83	0.69	0.68	1.67	1.00	0.87	0.92	1.13
time (sec)	N/A	0.398	0.108	1.207	0.127	0.087	9.864	0.128	0.221	3.394

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	56	44	43	108	63	49	36	71
N.S.	1	1.00	0.95	0.75	0.73	1.83	1.07	0.83	0.61	1.20
time (sec)	N/A	0.309	0.074	1.162	0.126	0.083	4.879	0.118	0.265	3.133

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	30	29	86	48	29	36	56
N.S.	1	1.00	1.00	0.75	0.72	2.15	1.20	0.72	0.90	1.40
time (sec)	N/A	0.279	0.054	1.020	0.111	0.150	3.061	0.125	0.211	2.764

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	69	59	45	0	144	80	53	79	94
N.S.	1	1.06	0.91	0.69	0.00	2.22	1.23	0.82	1.22	1.45
time (sec)	N/A	0.339	0.082	1.144	0.000	0.140	3.785	0.124	0.253	3.603

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	95	88	66	0	190	0	72	120	112
N.S.	1	1.08	1.00	0.75	0.00	2.16	0.00	0.82	1.36	1.27
time (sec)	N/A	0.374	0.165	1.260	0.000	0.138	0.000	0.134	0.228	3.875

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	667	701	67	848	0	2268	0	0	36	0
N.S.	1	1.05	0.10	1.27	0.00	3.40	0.00	0.00	0.05	0.00
time (sec)	N/A	1.130	10.043	1.289	0.000	2.064	0.000	0.000	0.208	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	67	416	0	2289	0	0	34	453
N.S.	1	1.00	0.33	2.02	0.00	11.11	0.00	0.00	0.17	2.20
time (sec)	N/A	0.402	10.046	1.112	0.000	0.750	0.000	0.000	0.210	20.074

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	697	697	136	868	0	2293	0	0	36	0
N.S.	1	1.00	0.20	1.25	0.00	3.29	0.00	0.00	0.05	0.00
time (sec)	N/A	1.159	11.098	1.948	0.000	0.864	0.000	0.000	0.243	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	696	0	2300	0	0	36	0
N.S.	1	1.00	1.02	10.55	0.00	34.85	0.00	0.00	0.55	0.00
time (sec)	N/A	0.353	10.039	1.516	0.000	0.592	0.000	0.000	0.239	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	165	416	0	2347	0	0	33	0
N.S.	1	1.00	2.58	6.50	0.00	36.67	0.00	0.00	0.52	0.00
time (sec)	N/A	0.313	10.056	1.046	0.000	0.658	0.000	0.000	0.220	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	243	716	0	2381	0	0	36	0
N.S.	1	1.00	3.68	10.85	0.00	36.08	0.00	0.00	0.55	0.00
time (sec)	N/A	0.342	11.196	1.845	0.000	1.839	0.000	0.000	0.213	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	28	164	0	1019	0	0	24	653
N.S.	1	1.00	0.22	1.29	0.00	8.02	0.00	0.00	0.19	5.14
time (sec)	N/A	0.322	10.030	14.258	0.000	0.153	0.000	0.000	0.236	1.540

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	82	70	96	166	109	100	114	118
N.S.	1	1.00	0.74	0.63	0.86	1.50	0.98	0.90	1.03	1.06
time (sec)	N/A	0.438	0.134	1.451	0.113	0.119	20.710	0.120	0.220	1.602

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	92	71	58	82	144	92	83	95	98
N.S.	1	1.02	0.79	0.64	0.91	1.60	1.02	0.92	1.06	1.09
time (sec)	N/A	0.406	0.109	1.240	0.114	0.129	8.826	0.121	0.235	1.612

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	77	59	47	66	118	75	65	74	78
N.S.	1	1.12	0.86	0.68	0.96	1.71	1.09	0.94	1.07	1.13
time (sec)	N/A	0.337	0.090	1.220	0.108	0.128	4.005	0.123	0.231	1.648

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	52	47	38	56	98	56	43	58	59
N.S.	1	1.04	0.94	0.76	1.12	1.96	1.12	0.86	1.16	1.18
time (sec)	N/A	0.309	0.065	1.122	0.105	0.129	2.092	0.122	0.246	1.595

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	62	53	38	0	133	71	48	80	125
N.S.	1	1.07	0.91	0.66	0.00	2.29	1.22	0.83	1.38	2.16
time (sec)	N/A	0.331	0.066	1.257	0.000	0.110	2.883	0.124	0.227	2.733

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	92	81	65	0	180	0	73	167	69
N.S.	1	1.14	1.00	0.80	0.00	2.22	0.00	0.90	2.06	0.85
time (sec)	N/A	0.385	0.151	1.185	0.000	0.158	0.000	0.128	0.242	1.973

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	122	95	77	0	188	0	100	235	83
N.S.	1	1.14	0.89	0.72	0.00	1.76	0.00	0.93	2.20	0.78
time (sec)	N/A	0.431	0.215	1.197	0.000	0.121	0.000	0.123	0.271	2.134

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	648	656	150	884	0	2442	0	0	117	0
N.S.	1	1.01	0.23	1.36	0.00	3.77	0.00	0.00	0.18	0.00
time (sec)	N/A	1.798	6.378	2.268	0.000	15.991	0.000	0.000	0.241	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	624	626	130	867	0	2428	0	0	100	0
N.S.	1	1.00	0.21	1.39	0.00	3.89	0.00	0.00	0.16	0.00
time (sec)	N/A	1.620	6.474	1.790	0.000	4.316	0.000	0.000	0.230	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	601	629	63	848	0	2194	0	0	24	0
N.S.	1	1.05	0.10	1.41	0.00	3.65	0.00	0.00	0.04	0.00
time (sec)	N/A	2.946	8.903	1.040	0.000	1.168	0.000	0.000	0.239	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	632	625	137	868	0	2219	0	0	26	0
N.S.	1	0.99	0.22	1.37	0.00	3.51	0.00	0.00	0.04	0.00
time (sec)	N/A	1.729	11.100	1.805	0.000	0.460	0.000	0.000	0.261	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	654	657	153	882	0	2401	0	0	26	0
N.S.	1	1.00	0.23	1.35	0.00	3.67	0.00	0.00	0.04	0.00
time (sec)	N/A	1.782	11.116	2.398	0.000	0.747	0.000	0.000	0.212	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	678	687	164	895	0	2436	0	0	26	0
N.S.	1	1.01	0.24	1.32	0.00	3.59	0.00	0.00	0.04	0.00
time (sec)	N/A	1.948	10.127	3.349	0.000	2.447	0.000	0.000	0.247	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	134	93	80	110	188	143	117	133	135
N.S.	1	1.03	0.72	0.62	0.85	1.45	1.10	0.90	1.02	1.04
time (sec)	N/A	0.476	0.149	1.273	0.110	0.119	55.094	0.128	0.302	1.852

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	113	82	69	96	166	122	100	114	115
N.S.	1	1.04	0.75	0.63	0.88	1.52	1.12	0.92	1.05	1.06
time (sec)	N/A	0.436	0.142	1.165	0.109	0.106	30.910	0.126	0.241	1.674

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	99	71	58	82	144	95	83	95	95
N.S.	1	1.12	0.81	0.66	0.93	1.64	1.08	0.94	1.08	1.08
time (sec)	N/A	0.374	0.102	1.158	0.106	0.128	15.181	0.129	0.235	1.731

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	74	59	47	68	118	71	65	74	75
N.S.	1	1.10	0.88	0.70	1.01	1.76	1.06	0.97	1.10	1.12
time (sec)	N/A	0.339	0.097	1.020	0.112	0.117	7.335	0.117	0.268	1.686

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	78	73	52	0	146	92	61	87	89
N.S.	1	1.07	1.00	0.71	0.00	2.00	1.26	0.84	1.19	1.22
time (sec)	N/A	0.382	0.085	1.102	0.000	0.093	4.713	0.124	0.266	3.995

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	86	78	59	0	180	0	64	167	56
N.S.	1	1.10	1.00	0.76	0.00	2.31	0.00	0.82	2.14	0.72
time (sec)	N/A	0.400	0.154	1.167	0.000	0.137	0.000	0.123	0.343	1.944

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	116	95	77	0	212	0	101	235	87
N.S.	1	1.12	0.91	0.74	0.00	2.04	0.00	0.97	2.26	0.84
time (sec)	N/A	0.436	0.203	1.359	0.000	0.117	0.000	0.129	0.320	2.209

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	669	680	163	895	0	2453	0	0	136	0
N.S.	1	1.02	0.24	1.34	0.00	3.67	0.00	0.00	0.20	0.00
time (sec)	N/A	1.846	8.280	2.233	0.000	30.400	0.000	0.000	0.244	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	645	650	150	884	0	2442	0	0	117	0
N.S.	1	1.01	0.23	1.37	0.00	3.79	0.00	0.00	0.18	0.00
time (sec)	N/A	1.741	7.975	1.776	0.000	10.164	0.000	0.000	0.246	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	627	626	127	864	0	2368	0	0	95	0
N.S.	1	1.00	0.20	1.38	0.00	3.78	0.00	0.00	0.15	0.00
time (sec)	N/A	1.608	10.095	1.681	0.000	2.537	0.000	0.000	0.238	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	626	620	137	859	0	0	0	0	102	0
N.S.	1	0.99	0.22	1.37	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	1.617	10.094	1.855	0.000	0.000	0.000	0.000	0.274	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	651	651	154	882	0	2369	0	0	103	0
N.S.	1	1.00	0.24	1.35	0.00	3.64	0.00	0.00	0.16	0.00
time (sec)	N/A	1.750	10.113	2.388	0.000	0.660	0.000	0.000	0.264	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	675	681	167	895	0	2442	0	0	105	0
N.S.	1	1.01	0.25	1.33	0.00	3.62	0.00	0.00	0.16	0.00
time (sec)	N/A	1.864	10.117	3.321	0.000	1.759	0.000	0.000	0.266	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	92	69	58	82	143	92	82	95	98
N.S.	1	1.02	0.77	0.64	0.91	1.59	1.02	0.91	1.06	1.09
time (sec)	N/A	0.406	0.121	1.217	0.108	0.094	20.444	0.127	0.229	1.742

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	73	58	47	66	118	76	69	74	78
N.S.	1	1.03	0.82	0.66	0.93	1.66	1.07	0.97	1.04	1.10
time (sec)	N/A	0.385	0.091	1.120	0.111	0.119	10.445	0.122	0.204	1.638

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	54	49	38	56	100	60	48	37	60
N.S.	1	1.04	0.94	0.73	1.08	1.92	1.15	0.92	0.71	1.15
time (sec)	N/A	0.319	0.062	1.135	0.105	0.105	6.081	0.118	0.221	1.660

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	24	42	75	42	27	37	45
N.S.	1	1.00	1.00	0.73	1.27	2.27	1.27	0.82	1.12	1.36
time (sec)	N/A	0.306	0.046	0.993	0.116	0.093	3.573	0.123	0.259	1.648

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	62	51	38	0	132	75	54	80	47
N.S.	1	1.07	0.88	0.66	0.00	2.28	1.29	0.93	1.38	0.81
time (sec)	N/A	0.348	0.067	1.156	0.000	0.130	3.787	0.129	0.267	1.684

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	92	81	64	0	178	0	73	121	73
N.S.	1	1.14	1.00	0.79	0.00	2.20	0.00	0.90	1.49	0.90
time (sec)	N/A	0.384	0.143	1.211	0.000	0.133	0.000	0.131	0.245	1.841

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	122	95	77	0	210	0	101	220	94
N.S.	1	1.14	0.89	0.72	0.00	1.96	0.00	0.94	2.06	0.88
time (sec)	N/A	0.441	0.224	1.244	0.000	0.144	0.000	0.128	0.251	1.999

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	630	632	130	867	0	2428	0	0	100	0
N.S.	1	1.00	0.21	1.38	0.00	3.85	0.00	0.00	0.16	0.00
time (sec)	N/A	1.604	10.095	2.293	0.000	10.189	0.000	0.000	0.265	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	601	637	67	848	0	2254	0	0	37	0
N.S.	1	1.06	0.11	1.41	0.00	3.75	0.00	0.00	0.06	0.00
time (sec)	N/A	3.021	10.052	1.270	0.000	1.886	0.000	0.000	0.233	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	67	416	0	2285	0	0	35	272
N.S.	1	1.00	0.48	2.95	0.00	16.21	0.00	0.00	0.25	1.93
time (sec)	N/A	1.509	10.042	1.073	0.000	0.486	0.000	0.000	0.230	40.040

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	632	632	137	868	0	2259	0	0	37	0
N.S.	1	1.00	0.22	1.37	0.00	3.57	0.00	0.00	0.06	0.00
time (sec)	N/A	1.662	11.095	1.851	0.000	0.518	0.000	0.000	0.236	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	654	657	152	882	0	2403	0	0	106	0
N.S.	1	1.00	0.23	1.35	0.00	3.67	0.00	0.00	0.16	0.00
time (sec)	N/A	1.785	11.114	2.420	0.000	1.121	0.000	0.000	0.261	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	678	687	167	895	0	2436	0	0	123	0
N.S.	1	1.01	0.25	1.32	0.00	3.59	0.00	0.00	0.18	0.00
time (sec)	N/A	1.854	10.131	3.359	0.000	3.049	0.000	0.000	0.230	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	696	0	2284	0	0	37	0
N.S.	1	1.00	1.02	10.55	0.00	34.61	0.00	0.00	0.56	0.00
time (sec)	N/A	0.361	10.054	1.155	0.000	0.385	0.000	0.000	0.203	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	166	416	0	2319	0	0	34	0
N.S.	1	1.00	2.59	6.50	0.00	36.23	0.00	0.00	0.53	0.00
time (sec)	N/A	0.312	10.167	1.067	0.000	0.578	0.000	0.000	0.249	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	242	716	0	2373	0	0	37	0
N.S.	1	1.00	3.67	10.85	0.00	35.95	0.00	0.00	0.56	0.00
time (sec)	N/A	0.355	11.185	2.392	0.000	0.968	0.000	0.000	0.241	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	94	70	65	82	186	90	82	174	95
N.S.	1	1.04	0.78	0.72	0.91	2.07	1.00	0.91	1.93	1.06
time (sec)	N/A	0.436	0.137	1.418	0.108	0.092	26.754	0.124	0.409	2.186

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	73	59	54	68	158	75	63	153	75
N.S.	1	1.03	0.83	0.76	0.96	2.23	1.06	0.89	2.15	1.06
time (sec)	N/A	0.387	0.105	1.204	0.114	0.094	15.349	0.123	0.446	2.094

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	56	49	36	56	146	56	47	48	60
N.S.	1	1.08	0.94	0.69	1.08	2.81	1.08	0.90	0.92	1.15
time (sec)	N/A	0.318	0.153	1.150	0.114	0.111	11.169	0.126	0.318	2.070

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	59	52	41	58	144	60	48	48	63
N.S.	1	1.07	0.95	0.75	1.05	2.62	1.09	0.87	0.87	1.15
time (sec)	N/A	0.318	0.089	1.029	0.109	0.116	9.147	0.118	0.378	2.040

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	88	69	55	0	206	97	68	46	68
N.S.	1	1.16	0.91	0.72	0.00	2.71	1.28	0.89	0.61	0.89
time (sec)	N/A	0.391	0.121	1.123	0.000	0.114	5.932	0.121	0.241	2.132

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	118	85	76	0	266	0	100	251	88
N.S.	1	1.18	0.85	0.76	0.00	2.66	0.00	1.00	2.51	0.88
time (sec)	N/A	0.435	0.219	1.243	0.000	0.116	0.000	0.121	0.492	2.259

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	148	100	88	0	296	0	118	268	112
N.S.	1	1.16	0.78	0.69	0.00	2.31	0.00	0.92	2.09	0.88
time (sec)	N/A	0.479	0.270	1.256	0.000	0.105	0.000	0.107	0.583	2.625

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	629	629	127	869	0	2384	0	0	48	0
N.S.	1	1.00	0.20	1.38	0.00	3.79	0.00	0.00	0.08	0.00
time (sec)	N/A	1.633	8.855	1.670	0.000	4.521	0.000	0.000	0.360	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	635	635	126	878	0	2525	0	0	48	0
N.S.	1	1.00	0.20	1.38	0.00	3.98	0.00	0.00	0.08	0.00
time (sec)	N/A	1.634	8.645	1.216	0.000	0.798	0.000	0.000	0.370	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	632	631	124	875	0	2525	0	0	46	0
N.S.	1	1.00	0.20	1.38	0.00	4.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.627	10.097	1.046	0.000	0.583	0.000	0.000	0.243	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	653	659	140	890	0	2379	0	0	247	0
N.S.	1	1.01	0.21	1.36	0.00	3.64	0.00	0.00	0.38	0.00
time (sec)	N/A	1.774	11.110	2.447	0.000	0.727	0.000	0.000	0.553	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	675	686	153	911	0	2529	0	0	268	0
N.S.	1	1.02	0.23	1.35	0.00	3.75	0.00	0.00	0.40	0.00
time (sec)	N/A	1.882	11.122	2.956	0.000	2.539	0.000	0.000	0.536	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	699	716	167	930	0	2564	0	0	405	0
N.S.	1	1.02	0.24	1.33	0.00	3.67	0.00	0.00	0.58	0.00
time (sec)	N/A	1.999	10.144	3.872	0.000	4.657	0.000	0.000	0.596	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	233	724	0	2535	0	0	48	0
N.S.	1	1.00	3.53	10.97	0.00	38.41	0.00	0.00	0.73	0.00
time (sec)	N/A	0.361	8.771	1.164	0.000	0.627	0.000	0.000	0.420	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	230	721	0	2483	0	0	45	0
N.S.	1	1.00	3.59	11.27	0.00	38.80	0.00	0.00	0.70	0.00
time (sec)	N/A	0.312	10.164	1.032	0.000	0.525	0.000	0.000	0.236	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	248	736	0	2495	0	0	249	0
N.S.	1	1.00	3.76	11.15	0.00	37.80	0.00	0.00	3.77	0.00
time (sec)	N/A	0.364	11.203	2.388	0.000	1.995	0.000	0.000	0.455	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	737	814	80	977	0	4847	0	0	119	0
N.S.	1	1.10	0.11	1.33	0.00	6.58	0.00	0.00	0.16	0.00
time (sec)	N/A	1.441	10.099	2.467	0.000	4.347	0.000	0.000	0.223	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	757	834	80	924	0	4855	0	0	121	0
N.S.	1	1.10	0.11	1.22	0.00	6.41	0.00	0.00	0.16	0.00
time (sec)	N/A	1.457	10.097	5.192	0.000	4.245	0.000	0.000	0.233	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	774	852	87	926	0	4963	0	0	125	0
N.S.	1	1.10	0.11	1.20	0.00	6.41	0.00	0.00	0.16	0.00
time (sec)	N/A	1.421	10.096	2.032	0.000	4.247	0.000	0.000	0.236	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	768	844	90	983	0	4981	0	0	120	0
N.S.	1	1.10	0.12	1.28	0.00	6.49	0.00	0.00	0.16	0.00
time (sec)	N/A	1.451	10.086	2.004	0.000	4.211	0.000	0.000	0.259	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	738	806	80	977	0	4931	0	0	119	0
N.S.	1	1.09	0.11	1.32	0.00	6.68	0.00	0.00	0.16	0.00
time (sec)	N/A	1.362	10.115	2.615	0.000	4.363	0.000	0.000	0.240	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	758	826	80	924	0	4953	0	0	121	0
N.S.	1	1.09	0.11	1.22	0.00	6.53	0.00	0.00	0.16	0.00
time (sec)	N/A	1.428	10.112	2.704	0.000	4.298	0.000	0.000	0.225	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	774	843	89	926	0	4867	0	0	124	0
N.S.	1	1.09	0.11	1.20	0.00	6.29	0.00	0.00	0.16	0.00
time (sec)	N/A	1.459	10.095	1.970	0.000	4.354	0.000	0.000	0.228	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	768	837	89	983	0	4875	0	0	121	0
N.S.	1	1.09	0.12	1.28	0.00	6.35	0.00	0.00	0.16	0.00
time (sec)	N/A	1.425	10.093	2.016	0.000	4.465	0.000	0.000	0.275	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	83	538	0	5563	0	0	152	0
N.S.	1	1.00	0.26	1.69	0.00	17.49	0.00	0.00	0.48	0.00
time (sec)	N/A	0.520	10.103	1.948	0.000	4.332	0.000	0.000	0.272	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	83	509	0	5587	0	0	151	0
N.S.	1	1.00	0.26	1.57	0.00	17.24	0.00	0.00	0.47	0.00
time (sec)	N/A	0.496	10.194	1.964	0.000	4.524	0.000	0.000	0.238	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	85	510	0	5667	0	0	154	0
N.S.	1	1.00	0.26	1.55	0.00	17.28	0.00	0.00	0.47	0.00
time (sec)	N/A	0.490	10.092	1.925	0.000	4.377	0.000	0.000	0.248	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	87	541	0	5679	0	0	154	0
N.S.	1	1.00	0.26	1.64	0.00	17.21	0.00	0.00	0.47	0.00
time (sec)	N/A	0.525	10.095	1.936	0.000	4.164	0.000	0.000	0.243	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	83	538	0	5631	0	0	152	0
N.S.	1	1.00	0.27	1.74	0.00	18.16	0.00	0.00	0.49	0.00
time (sec)	N/A	0.497	10.127	1.946	0.000	4.967	0.000	0.000	0.237	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	83	509	0	5667	0	0	151	0
N.S.	1	1.00	0.26	1.61	0.00	17.93	0.00	0.00	0.48	0.00
time (sec)	N/A	0.486	10.110	1.941	0.000	4.538	0.000	0.000	0.231	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	84	510	0	5599	0	0	155	0
N.S.	1	1.00	0.26	1.59	0.00	17.50	0.00	0.00	0.48	0.00
time (sec)	N/A	0.494	10.115	1.894	0.000	4.254	0.000	0.000	0.264	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	86	541	0	5599	0	0	153	0
N.S.	1	1.00	0.27	1.68	0.00	17.39	0.00	0.00	0.48	0.00
time (sec)	N/A	0.506	10.128	1.927	0.000	4.380	0.000	0.000	0.238	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	121	124	0	280	173	139	178	176
N.S.	1	1.00	0.97	0.99	0.00	2.24	1.38	1.11	1.42	1.41
time (sec)	N/A	0.500	0.384	4.758	0.000	0.126	11.465	0.122	0.453	4.742

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	98	88	81	0	195	128	96	121	136
N.S.	1	1.05	0.95	0.87	0.00	2.10	1.38	1.03	1.30	1.46
time (sec)	N/A	0.375	0.271	1.243	0.000	0.119	4.916	0.130	0.481	4.676

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	62	0	156	95	66	107	82
N.S.	1	1.00	1.00	0.89	0.00	2.23	1.36	0.94	1.53	1.17
time (sec)	N/A	0.338	0.132	1.080	0.000	0.089	2.800	0.127	0.527	4.775

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	71	0	377	165	79	22	114
N.S.	1	1.00	0.95	0.84	0.00	4.44	1.94	0.93	0.26	1.34
time (sec)	N/A	0.379	0.165	1.242	0.000	0.116	5.383	0.126	0.369	6.705

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	121	107	96	0	507	0	107	24	137
N.S.	1	1.05	0.93	0.83	0.00	4.41	0.00	0.93	0.21	1.19
time (sec)	N/A	0.447	0.466	1.326	0.000	0.131	0.000	0.129	0.725	3.690

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	241	741	0	0	0	0	140	0
N.S.	1	1.00	3.77	11.58	0.00	0.00	0.00	0.00	2.19	0.00
time (sec)	N/A	0.346	7.291	2.260	0.000	0.000	0.000	0.000	0.384	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	857	0	0	0	0	21	0
N.S.	1	1.00	1.02	13.39	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.314	9.538	1.192	0.000	0.000	0.000	0.000	0.291	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	705	0	0	0	0	20	0
N.S.	1	1.00	2.73	11.95	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.298	10.185	1.053	0.000	0.000	0.000	0.000	0.226	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	139	892	0	0	0	0	24	0
N.S.	1	1.00	2.24	14.39	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.338	10.121	2.338	0.000	0.000	0.000	0.000	0.466	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	335	740	0	0	0	0	24	0
N.S.	1	1.00	5.23	11.56	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.331	10.295	2.407	0.000	0.000	0.000	0.000	0.506	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	143	149	0	410	201	193	317	330
N.S.	1	1.00	0.93	0.97	0.00	2.66	1.31	1.25	2.06	2.14
time (sec)	N/A	0.530	0.617	1.766	0.000	0.119	41.591	0.121	0.683	4.606

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	129	111	113	0	297	153	151	225	215
N.S.	1	1.08	0.92	0.94	0.00	2.48	1.28	1.26	1.88	1.79
time (sec)	N/A	0.403	0.331	1.405	0.000	0.126	20.175	0.124	0.693	4.575

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	101	85	94	0	204	124	113	195	143
N.S.	1	1.05	0.89	0.98	0.00	2.12	1.29	1.18	2.03	1.49
time (sec)	N/A	0.371	0.309	1.204	0.000	0.094	9.968	0.121	0.701	4.364

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	108	106	104	0	480	189	112	146	155
N.S.	1	1.04	1.02	1.00	0.00	4.62	1.82	1.08	1.40	1.49
time (sec)	N/A	0.420	0.371	1.227	0.000	0.111	7.821	0.127	0.710	6.394

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	122	108	101	0	532	0	121	886	167
N.S.	1	1.05	0.93	0.87	0.00	4.59	0.00	1.04	7.64	1.44
time (sec)	N/A	0.467	0.442	1.291	0.000	0.107	0.000	0.129	1.157	7.984

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	280	800	0	0	0	0	267	0
N.S.	1	1.00	4.31	12.31	0.00	0.00	0.00	0.00	4.11	0.00
time (sec)	N/A	0.340	8.936	2.375	0.000	0.000	0.000	0.000	0.684	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	149	921	0	0	0	0	190	0
N.S.	1	1.00	2.29	14.17	0.00	0.00	0.00	0.00	2.92	0.00
time (sec)	N/A	0.321	10.169	2.549	0.000	0.000	0.000	0.000	0.514	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	351	767	0	0	0	0	186	0
N.S.	1	1.00	5.85	12.78	0.00	0.00	0.00	0.00	3.10	0.00
time (sec)	N/A	0.319	10.354	2.147	0.000	0.000	0.000	0.000	0.549	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	148	920	0	0	0	0	569	0
N.S.	1	1.00	2.35	14.60	0.00	0.00	0.00	0.00	9.03	0.00
time (sec)	N/A	0.364	10.144	2.905	0.000	0.000	0.000	0.000	0.743	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	343	769	0	0	0	0	581	0
N.S.	1	1.00	5.28	11.83	0.00	0.00	0.00	0.00	8.94	0.00
time (sec)	N/A	0.338	10.388	2.480	0.000	0.000	0.000	0.000	0.750	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	91	79	0	289	0	102	78	121
N.S.	1	1.00	0.88	0.76	0.00	2.78	0.00	0.98	0.75	1.16
time (sec)	N/A	0.430	0.240	1.414	0.000	0.117	0.000	0.122	0.363	4.112

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	75	59	0	205	0	64	38	86
N.S.	1	1.00	1.01	0.80	0.00	2.77	0.00	0.86	0.51	1.16
time (sec)	N/A	0.335	0.113	1.211	0.000	0.119	0.000	0.123	0.321	3.788

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	39	0	130	88	40	38	70
N.S.	1	1.00	1.00	0.76	0.00	2.55	1.73	0.78	0.75	1.37
time (sec)	N/A	0.308	0.048	0.990	0.000	0.100	4.487	0.122	0.323	4.622

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	82	65	0	385	119	71	36	114
N.S.	1	1.00	0.96	0.76	0.00	4.53	1.40	0.84	0.42	1.34
time (sec)	N/A	0.358	0.146	1.219	0.000	0.183	6.121	0.121	0.274	6.163

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	127	109	92	0	518	0	104	153	142
N.S.	1	1.09	0.93	0.79	0.00	4.43	0.00	0.89	1.31	1.21
time (sec)	N/A	0.433	0.292	1.289	0.000	0.317	0.000	0.124	0.583	7.268

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	719	0	0	0	0	38	0
N.S.	1	1.00	1.02	11.23	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.338	10.034	1.245	0.000	0.000	0.000	0.000	0.234	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	429	0	0	0	0	36	0
N.S.	1	1.00	1.02	6.70	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.315	10.030	1.077	0.000	0.000	0.000	0.000	0.238	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	429	0	0	0	0	35	0
N.S.	1	1.00	2.73	7.27	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.298	10.057	1.070	0.000	0.000	0.000	0.000	0.263	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	141	890	0	0	0	0	38	0
N.S.	1	1.00	2.27	14.35	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.344	10.081	2.043	0.000	0.000	0.000	0.000	0.235	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	339	738	0	0	0	0	38	0
N.S.	1	1.00	5.30	11.53	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.338	10.220	2.021	0.000	0.000	0.000	0.000	0.283	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	105	111	100	0	440	0	100	183	115
N.S.	1	0.98	1.04	0.93	0.00	4.11	0.00	0.93	1.71	1.07
time (sec)	N/A	0.487	0.336	1.520	0.000	0.114	0.000	0.127	0.508	5.118

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	80	66	0	326	0	78	62	94
N.S.	1	1.00	0.98	0.80	0.00	3.98	0.00	0.95	0.76	1.15
time (sec)	N/A	0.358	0.197	1.198	0.000	0.120	0.000	0.118	0.425	4.531

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	76	64	0	213	121	73	62	89
N.S.	1	1.00	0.99	0.83	0.00	2.77	1.57	0.95	0.81	1.16
time (sec)	N/A	0.348	0.125	1.027	0.000	0.097	10.782	0.126	0.383	4.395

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	136	110	103	0	741	155	111	60	139
N.S.	1	1.19	0.96	0.90	0.00	6.50	1.36	0.97	0.53	1.22
time (sec)	N/A	0.445	0.404	1.250	0.000	0.205	7.653	0.117	0.282	7.003

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	188	142	141	0	1073	0	173	446	597
N.S.	1	1.19	0.90	0.89	0.00	6.79	0.00	1.09	2.82	3.78
time (sec)	N/A	0.590	0.529	1.464	0.000	0.187	0.000	0.114	0.757	9.432

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	231	749	0	0	0	0	62	0
N.S.	1	1.00	3.45	11.18	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	0.343	8.463	1.533	0.000	0.000	0.000	0.000	0.579	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	142	907	0	0	0	0	60	0
N.S.	1	1.00	2.12	13.54	0.00	0.00	0.00	0.00	0.90	0.00
time (sec)	N/A	0.324	10.093	1.047	0.000	0.000	0.000	0.000	0.251	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	338	753	0	0	0	0	59	0
N.S.	1	1.00	5.45	12.15	0.00	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.299	10.269	1.043	0.000	0.000	0.000	0.000	0.305	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	193	952	0	0	0	0	440	0
N.S.	1	1.00	2.97	14.65	0.00	0.00	0.00	0.00	6.77	0.00
time (sec)	N/A	0.350	10.169	3.100	0.000	0.000	0.000	0.000	1.003	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	408	798	0	0	0	0	442	0
N.S.	1	1.00	6.09	11.91	0.00	0.00	0.00	0.00	6.60	0.00
time (sec)	N/A	0.351	10.417	3.212	0.000	0.000	0.000	0.000	0.980	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	132	91	98	107	216	0	110	194	127
N.S.	1	1.07	0.74	0.80	0.87	1.76	0.00	0.89	1.58	1.03
time (sec)	N/A	0.427	0.159	1.452	0.110	0.115	0.000	0.120	0.440	2.584

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	111	79	79	91	188	0	93	175	107
N.S.	1	1.07	0.76	0.76	0.88	1.81	0.00	0.89	1.68	1.03
time (sec)	N/A	0.372	0.102	1.223	0.104	0.085	0.000	0.127	0.403	2.433

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	90	69	62	79	162	0	69	154	87
N.S.	1	1.08	0.83	0.75	0.95	1.95	0.00	0.83	1.86	1.05
time (sec)	N/A	0.331	0.088	1.218	0.110	0.086	0.000	0.124	0.386	2.427

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	65	61	49	66	146	0	53	136	72
N.S.	1	1.02	0.95	0.77	1.03	2.28	0.00	0.83	2.12	1.12
time (sec)	N/A	0.310	0.068	1.012	0.115	0.102	0.000	0.116	0.397	2.363

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	100	83	78	0	220	0	79	34	76
N.S.	1	1.14	0.94	0.89	0.00	2.50	0.00	0.90	0.39	0.86
time (sec)	N/A	0.375	0.097	1.194	0.000	0.087	0.000	0.119	0.387	2.415

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	142	97	86	0	272	0	113	36	117
N.S.	1	1.15	0.78	0.69	0.00	2.19	0.00	0.91	0.29	0.94
time (sec)	N/A	0.441	0.193	1.278	0.000	0.085	0.000	0.127	0.516	2.724

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	184	112	89	0	304	0	105	36	154
N.S.	1	1.12	0.68	0.54	0.00	1.85	0.00	0.64	0.22	0.94
time (sec)	N/A	0.494	0.239	1.309	0.000	0.121	0.000	0.118	3.233	2.959

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	663	671	176	897	0	2568	0	0	261	0
N.S.	1	1.01	0.27	1.35	0.00	3.87	0.00	0.00	0.39	0.00
time (sec)	N/A	1.841	5.893	3.294	0.000	8.164	0.000	0.000	0.508	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	641	641	167	877	0	2397	0	0	243	0
N.S.	1	1.00	0.26	1.37	0.00	3.74	0.00	0.00	0.38	0.00
time (sec)	N/A	1.657	8.592	1.673	0.000	1.662	0.000	0.000	0.428	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	644	640	164	883	0	2496	0	0	34	0
N.S.	1	0.99	0.25	1.37	0.00	3.88	0.00	0.00	0.05	0.00
time (sec)	N/A	1.615	10.091	1.115	0.000	0.444	0.000	0.000	0.372	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	665	669	179	898	0	2390	0	0	36	0
N.S.	1	1.01	0.27	1.35	0.00	3.59	0.00	0.00	0.05	0.00
time (sec)	N/A	1.743	11.097	3.174	0.000	0.456	0.000	0.000	0.425	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	687	698	199	919	0	2549	0	0	36	0
N.S.	1	1.02	0.29	1.34	0.00	3.71	0.00	0.00	0.05	0.00
time (sec)	N/A	1.878	10.120	3.972	0.000	1.511	0.000	0.000	0.456	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	711	728	209	938	0	2582	0	0	36	0
N.S.	1	1.02	0.29	1.32	0.00	3.63	0.00	0.00	0.05	0.00
time (sec)	N/A	2.029	10.148	5.106	0.000	3.183	0.000	0.000	0.512	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	157	103	109	119	236	0	127	213	147
N.S.	1	1.12	0.74	0.78	0.85	1.69	0.00	0.91	1.52	1.05
time (sec)	N/A	0.460	0.158	1.629	0.105	0.089	0.000	0.128	0.516	2.678

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	135	93	98	107	216	0	111	194	127
N.S.	1	1.12	0.77	0.81	0.88	1.79	0.00	0.92	1.60	1.05
time (sec)	N/A	0.409	0.135	1.248	0.107	0.121	0.000	0.124	0.498	2.615

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	112	81	79	93	189	0	93	175	107
N.S.	1	1.12	0.81	0.79	0.93	1.89	0.00	0.93	1.75	1.07
time (sec)	N/A	0.366	0.115	1.282	0.115	0.078	0.000	0.121	0.555	2.652

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	84	72	63	79	159	0	69	154	87
N.S.	1	1.09	0.94	0.82	1.03	2.06	0.00	0.90	2.00	1.13
time (sec)	N/A	0.343	0.083	1.110	0.107	0.092	0.000	0.131	0.494	2.618

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	94	85	64	0	214	0	70	236	101
N.S.	1	1.11	1.00	0.75	0.00	2.52	0.00	0.82	2.78	1.19
time (sec)	N/A	0.384	0.088	1.185	0.000	0.087	0.000	0.122	0.645	3.444

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	136	97	87	0	274	0	114	381	110
N.S.	1	1.12	0.80	0.72	0.00	2.26	0.00	0.94	3.15	0.91
time (sec)	N/A	0.453	0.201	1.487	0.000	0.093	0.000	0.123	0.905	2.837

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	178	112	97	0	304	0	129	247	151
N.S.	1	1.11	0.70	0.60	0.00	1.89	0.00	0.80	1.53	0.94
time (sec)	N/A	0.515	0.221	1.275	0.000	0.090	0.000	0.123	6.055	3.271

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	681	687	191	921	0	2580	0	0	282	0
N.S.	1	1.01	0.28	1.35	0.00	3.79	0.00	0.00	0.41	0.00
time (sec)	N/A	1.877	6.960	3.230	0.000	16.005	0.000	0.000	0.620	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	657	661	176	897	0	2568	0	0	261	0
N.S.	1	1.01	0.27	1.37	0.00	3.91	0.00	0.00	0.40	0.00
time (sec)	N/A	1.778	9.650	2.971	0.000	3.533	0.000	0.000	0.558	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	638	631	141	874	0	2335	0	0	239	0
N.S.	1	0.99	0.22	1.37	0.00	3.66	0.00	0.00	0.37	0.00
time (sec)	N/A	1.628	10.151	1.126	0.000	1.061	0.000	0.000	0.562	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	547	242	483	0	68	0	0	243	0
N.S.	1	1.05	0.46	0.93	0.00	0.13	0.00	0.00	0.47	0.00
time (sec)	N/A	1.016	11.505	3.180	0.000	0.084	0.000	0.000	0.618	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	684	692	199	919	0	2549	0	0	247	0
N.S.	1	1.01	0.29	1.34	0.00	3.73	0.00	0.00	0.36	0.00
time (sec)	N/A	1.959	10.117	3.899	0.000	0.856	0.000	0.000	0.753	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	708	722	212	938	0	2582	0	0	501	0
N.S.	1	1.02	0.30	1.32	0.00	3.65	0.00	0.00	0.71	0.00
time (sec)	N/A	2.027	10.119	5.082	0.000	1.867	0.000	0.000	0.961	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	105	81	79	93	192	0	93	175	107
N.S.	1	1.03	0.79	0.77	0.91	1.88	0.00	0.91	1.72	1.05
time (sec)	N/A	0.405	0.197	1.351	0.115	0.101	0.000	0.125	0.444	2.733

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	93	70	63	79	164	0	74	154	87
N.S.	1	1.12	0.84	0.76	0.95	1.98	0.00	0.89	1.86	1.05
time (sec)	N/A	0.371	0.130	1.239	0.121	0.103	0.000	0.134	0.389	2.603

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	68	63	50	67	152	0	58	47	72
N.S.	1	1.06	0.98	0.78	1.05	2.38	0.00	0.91	0.73	1.12
time (sec)	N/A	0.328	0.124	1.153	0.117	0.085	0.000	0.119	0.351	2.568

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	71	64	52	72	150	0	59	47	75
N.S.	1	1.06	0.96	0.78	1.07	2.24	0.00	0.88	0.70	1.12
time (sec)	N/A	0.334	0.096	1.004	0.112	0.075	0.000	0.119	0.330	2.606

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	100	83	78	0	220	0	79	45	80
N.S.	1	1.14	0.94	0.89	0.00	2.50	0.00	0.90	0.51	0.91
time (sec)	N/A	0.407	0.148	1.162	0.000	0.086	0.000	0.128	0.273	2.451

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	142	97	87	0	274	0	114	250	117
N.S.	1	1.15	0.78	0.70	0.00	2.21	0.00	0.92	2.02	0.94
time (sec)	N/A	0.474	0.278	1.320	0.000	0.099	0.000	0.131	0.634	2.579

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	184	112	97	0	304	0	128	267	155
N.S.	1	1.12	0.68	0.59	0.00	1.85	0.00	0.78	1.63	0.95
time (sec)	N/A	0.525	0.376	1.260	0.000	0.098	0.000	0.122	3.028	2.769

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	641	641	167	877	0	2397	0	0	47	0
N.S.	1	1.00	0.26	1.37	0.00	3.74	0.00	0.00	0.07	0.00
time (sec)	N/A	1.651	10.169	1.845	0.000	3.396	0.000	0.000	0.418	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	647	646	166	886	0	2538	0	0	47	0
N.S.	1	1.00	0.26	1.37	0.00	3.92	0.00	0.00	0.07	0.00
time (sec)	N/A	1.646	10.128	1.586	0.000	0.392	0.000	0.000	0.384	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	644	641	164	883	0	2540	0	0	45	0
N.S.	1	1.00	0.25	1.37	0.00	3.94	0.00	0.00	0.07	0.00
time (sec)	N/A	1.672	10.120	1.256	0.000	0.480	0.000	0.000	0.264	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	665	669	180	898	0	2391	0	0	246	0
N.S.	1	1.01	0.27	1.35	0.00	3.60	0.00	0.00	0.37	0.00
time (sec)	N/A	1.821	11.168	3.112	0.000	0.600	0.000	0.000	0.498	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	687	698	196	919	0	2549	0	0	267	0
N.S.	1	1.02	0.29	1.34	0.00	3.71	0.00	0.00	0.39	0.00
time (sec)	N/A	1.948	10.180	3.776	0.000	1.945	0.000	0.000	0.529	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	711	728	212	938	0	2582	0	0	402	0
N.S.	1	1.02	0.30	1.32	0.00	3.63	0.00	0.00	0.57	0.00
time (sec)	N/A	2.025	10.219	5.018	0.000	4.438	0.000	0.000	0.685	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	239	723	0	2425	0	0	47	0
N.S.	1	1.00	3.62	10.95	0.00	36.74	0.00	0.00	0.71	0.00
time (sec)	N/A	0.365	10.360	1.739	0.000	0.504	0.000	0.000	0.383	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	237	732	0	2548	0	0	47	0
N.S.	1	1.00	3.59	11.09	0.00	38.61	0.00	0.00	0.71	0.00
time (sec)	N/A	0.353	10.235	1.955	0.000	0.575	0.000	0.000	0.366	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	237	729	0	2498	0	0	44	0
N.S.	1	1.00	3.70	11.39	0.00	39.03	0.00	0.00	0.69	0.00
time (sec)	N/A	0.315	10.199	1.108	0.000	0.639	0.000	0.000	0.254	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	266	744	0	2515	0	0	248	0
N.S.	1	1.00	4.03	11.27	0.00	38.11	0.00	0.00	3.76	0.00
time (sec)	N/A	0.357	10.230	3.256	0.000	1.606	0.000	0.000	0.514	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	279	765	0	2561	0	0	265	0
N.S.	1	1.00	4.23	11.59	0.00	38.80	0.00	0.00	4.02	0.00
time (sec)	N/A	0.357	10.314	4.145	0.000	3.552	0.000	0.000	0.539	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	107	99	75	98	230	0	88	277	111
N.S.	1	1.05	0.97	0.74	0.96	2.25	0.00	0.86	2.72	1.09
time (sec)	N/A	0.391	0.165	1.431	0.130	0.079	0.000	0.120	0.258	2.896

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	95	71	61	81	220	0	72	256	94
N.S.	1	1.16	0.87	0.74	0.99	2.68	0.00	0.88	3.12	1.15
time (sec)	N/A	0.360	0.174	1.176	0.116	0.082	0.000	0.123	0.249	3.155

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	97	73	61	83	220	0	76	58	96
N.S.	1	1.10	0.83	0.69	0.94	2.50	0.00	0.86	0.66	1.09
time (sec)	N/A	0.354	0.152	1.168	0.127	0.088	0.000	0.130	0.237	2.914

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	100	73	62	85	216	0	72	58	97
N.S.	1	1.14	0.83	0.70	0.97	2.45	0.00	0.82	0.66	1.10
time (sec)	N/A	0.356	0.151	1.036	0.112	0.083	0.000	0.122	0.240	2.917

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	126	93	92	0	310	0	93	391	101
N.S.	1	1.19	0.88	0.87	0.00	2.92	0.00	0.88	3.69	0.95
time (sec)	N/A	0.446	0.207	1.218	0.000	0.085	0.000	0.123	0.280	3.012

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	168	109	98	0	362	0	129	428	133
N.S.	1	1.17	0.76	0.69	0.00	2.53	0.00	0.90	2.99	0.93
time (sec)	N/A	0.506	0.342	1.309	0.000	0.091	0.000	0.120	0.297	3.181

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	210	123	108	0	392	0	149	854	171
N.S.	1	1.14	0.66	0.58	0.00	2.12	0.00	0.81	4.62	0.92
time (sec)	N/A	0.561	0.444	1.336	0.000	0.090	0.000	0.123	0.303	3.634

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	668	669	168	910	0	2681	0	0	58	0
N.S.	1	1.00	0.25	1.36	0.00	4.01	0.00	0.00	0.09	0.00
time (sec)	N/A	1.753	10.132	1.989	0.000	0.666	0.000	0.000	0.251	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	671	676	169	910	0	2723	0	0	58	0
N.S.	1	1.01	0.25	1.36	0.00	4.06	0.00	0.00	0.09	0.00
time (sec)	N/A	1.779	10.141	1.675	0.000	0.570	0.000	0.000	0.237	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	665	669	167	904	0	2684	0	0	56	0
N.S.	1	1.01	0.25	1.36	0.00	4.04	0.00	0.00	0.08	0.00
time (sec)	N/A	1.774	10.160	1.113	0.000	0.761	0.000	0.000	0.245	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	686	698	180	920	0	2534	0	0	437	0
N.S.	1	1.02	0.26	1.34	0.00	3.69	0.00	0.00	0.64	0.00
time (sec)	N/A	1.907	11.169	3.933	0.000	0.980	0.000	0.000	0.237	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	708	727	198	943	0	2692	0	0	458	0
N.S.	1	1.03	0.28	1.33	0.00	3.80	0.00	0.00	0.65	0.00
time (sec)	N/A	2.070	10.181	4.797	0.000	3.305	0.000	0.000	0.288	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	732	757	210	962	0	2725	0	0	684	0
N.S.	1	1.03	0.29	1.31	0.00	3.72	0.00	0.00	0.93	0.00
time (sec)	N/A	2.185	10.211	5.796	0.000	5.803	0.000	0.000	0.249	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	66	189	339	0	89	0	0	58	0
N.S.	1	0.26	0.74	1.32	0.00	0.35	0.00	0.00	0.23	0.00
time (sec)	N/A	0.362	10.655	1.949	0.000	0.088	0.000	0.000	0.248	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	242	754	0	2713	0	0	58	0
N.S.	1	1.00	3.67	11.42	0.00	41.11	0.00	0.00	0.88	0.00
time (sec)	N/A	0.358	10.333	1.699	0.000	0.873	0.000	0.000	0.234	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	253	748	0	2640	0	0	55	0
N.S.	1	1.00	3.95	11.69	0.00	41.25	0.00	0.00	0.86	0.00
time (sec)	N/A	0.319	10.284	1.101	0.000	0.777	0.000	0.000	0.235	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	259	764	0	2650	0	0	438	0
N.S.	1	1.00	3.92	11.58	0.00	40.15	0.00	0.00	6.64	0.00
time (sec)	N/A	0.358	10.276	4.058	0.000	1.882	0.000	0.000	0.266	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	283	787	0	2698	0	0	455	0
N.S.	1	1.00	4.29	11.92	0.00	40.88	0.00	0.00	6.89	0.00
time (sec)	N/A	0.353	10.276	4.778	0.000	5.779	0.000	0.000	0.228	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	164	126	129	0	469	0	136	1132	202
N.S.	1	1.24	0.95	0.98	0.00	3.55	0.00	1.03	8.58	1.53
time (sec)	N/A	0.515	0.463	1.908	0.000	0.103	0.000	0.126	1.289	5.745

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	132	98	98	0	334	0	102	1039	152
N.S.	1	1.23	0.92	0.92	0.00	3.12	0.00	0.95	9.71	1.42
time (sec)	N/A	0.425	0.323	1.513	0.000	0.112	0.000	0.122	1.171	5.056

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	65	0	255	0	79	763	125
N.S.	1	1.00	1.00	0.81	0.00	3.19	0.00	0.99	9.54	1.56
time (sec)	N/A	0.344	0.255	1.027	0.000	0.088	0.000	0.126	0.806	4.621

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	126	111	120	0	850	0	114	33	182
N.S.	1	1.04	0.92	0.99	0.00	7.02	0.00	0.94	0.27	1.50
time (sec)	N/A	0.432	0.711	1.300	0.000	0.125	0.000	0.119	0.740	7.504

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	170	132	151	0	825	0	183	35	438
N.S.	1	1.06	0.82	0.94	0.00	5.12	0.00	1.14	0.22	2.72
time (sec)	N/A	0.558	0.978	1.428	0.000	0.145	0.000	0.129	3.506	9.204

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	235	748	0	0	0	0	1371	0
N.S.	1	1.00	3.67	11.69	0.00	0.00	0.00	0.00	21.42	0.00
time (sec)	N/A	0.338	10.252	2.078	0.000	0.000	0.000	0.000	0.999	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	153	908	0	0	0	0	32	0
N.S.	1	1.00	2.39	14.19	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.332	10.126	1.223	0.000	0.000	0.000	0.000	0.688	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	232	753	0	0	0	0	31	0
N.S.	1	1.00	3.93	12.76	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.309	10.320	1.136	0.000	0.000	0.000	0.000	0.683	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	172	920	0	0	0	0	35	0
N.S.	1	1.00	2.77	14.84	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.349	10.177	4.782	0.000	0.000	0.000	0.000	1.122	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	338	766	0	0	0	0	35	0
N.S.	1	1.00	5.28	11.97	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.338	10.247	5.050	0.000	0.000	0.000	0.000	1.105	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	195	162	169	0	443	0	211	1493	331
N.S.	1	1.16	0.96	1.01	0.00	2.64	0.00	1.26	8.89	1.97
time (sec)	N/A	0.543	0.411	2.000	0.000	0.268	0.000	0.129	1.604	6.519

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	163	125	133	0	314	0	173	1367	229
N.S.	1	1.16	0.89	0.94	0.00	2.23	0.00	1.23	9.70	1.62
time (sec)	N/A	0.466	0.247	1.373	0.000	0.209	0.000	0.123	1.552	6.467

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	104	94	108	0	234	0	122	1063	170
N.S.	1	1.11	1.00	1.15	0.00	2.49	0.00	1.30	11.31	1.81
time (sec)	N/A	0.377	0.270	1.091	0.000	0.164	0.000	0.128	1.291	6.272

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	140	122	140	0	680	0	155	1241	214
N.S.	1	1.07	0.93	1.07	0.00	5.19	0.00	1.18	9.47	1.63
time (sec)	N/A	0.495	0.619	1.310	0.000	0.188	0.000	0.129	1.015	8.388

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	197	154	170	0	832	0	216	0	531
N.S.	1	1.16	0.91	1.00	0.00	4.89	0.00	1.27	0.00	3.12
time (sec)	N/A	0.651	0.661	1.421	0.000	0.216	0.000	0.132	4.402	10.134

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	338	808	0	0	0	0	0	0
N.S.	1	1.00	5.20	12.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	10.318	4.666	0.000	0.000	0.000	0.000	1.768	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	177	955	0	0	0	0	1650	0
N.S.	1	1.00	2.72	14.69	0.00	0.00	0.00	0.00	25.38	0.00
time (sec)	N/A	0.333	10.138	1.231	0.000	0.000	0.000	0.000	1.415	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	339	801	0	0	0	0	1642	0
N.S.	1	1.00	5.65	13.35	0.00	0.00	0.00	0.00	27.37	0.00
time (sec)	N/A	0.311	10.261	1.136	0.000	0.000	0.000	0.000	1.460	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	190	970	0	0	0	0	1593	0
N.S.	1	1.00	3.02	15.40	0.00	0.00	0.00	0.00	25.29	0.00
time (sec)	N/A	0.359	10.152	4.806	0.000	0.000	0.000	0.000	2.190	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	370	815	0	0	0	0	1599	0
N.S.	1	1.00	5.69	12.54	0.00	0.00	0.00	0.00	24.60	0.00
time (sec)	N/A	0.347	10.286	5.289	0.000	0.000	0.000	0.000	2.073	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	144	130	132	0	475	0	142	804	160
N.S.	1	1.17	1.06	1.07	0.00	3.86	0.00	1.15	6.54	1.30
time (sec)	N/A	0.493	0.400	1.611	0.000	0.267	0.000	0.119	0.852	6.487

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	98	100	83	0	348	0	116	62	111
N.S.	1	0.99	1.01	0.84	0.00	3.52	0.00	1.17	0.63	1.12
time (sec)	N/A	0.393	0.228	1.171	0.000	0.151	0.000	0.124	0.587	5.734

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	86	86	71	0	302	0	93	62	104
N.S.	1	0.99	0.99	0.82	0.00	3.47	0.00	1.07	0.71	1.20
time (sec)	N/A	0.362	0.318	1.007	0.000	0.129	0.000	0.124	0.489	5.300

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	155	124	140	0	816	0	139	60	162
N.S.	1	1.17	0.94	1.06	0.00	6.18	0.00	1.05	0.45	1.23
time (sec)	N/A	0.491	0.418	1.315	0.000	0.238	0.000	0.126	0.270	8.753

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	213	163	180	0	1189	0	257	446	355
N.S.	1	1.15	0.88	0.97	0.00	6.43	0.00	1.39	2.41	1.92
time (sec)	N/A	0.643	0.699	1.665	0.000	0.353	0.000	0.125	1.560	10.443

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	238	764	0	0	0	0	62	0
N.S.	1	1.00	3.72	11.94	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.354	10.279	2.040	0.000	0.000	0.000	0.000	0.576	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	172	923	0	0	0	0	60	0
N.S.	1	1.00	2.69	14.42	0.00	0.00	0.00	0.00	0.94	0.00
time (sec)	N/A	0.330	10.123	1.195	0.000	0.000	0.000	0.000	0.246	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	392	769	0	0	0	0	59	0
N.S.	1	1.00	6.64	13.03	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.310	10.209	1.138	0.000	0.000	0.000	0.000	0.259	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	226	963	0	0	0	0	438	0
N.S.	1	1.00	3.65	15.53	0.00	0.00	0.00	0.00	7.06	0.00
time (sec)	N/A	0.352	10.203	4.491	0.000	0.000	0.000	0.000	1.036	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	411	809	0	0	0	0	442	0
N.S.	1	1.00	6.42	12.64	0.00	0.00	0.00	0.00	6.91	0.00
time (sec)	N/A	0.345	10.776	4.644	0.000	0.000	0.000	0.000	0.984	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	162	133	146	0	746	0	204	0	367
N.S.	1	1.22	1.00	1.10	0.00	5.61	0.00	1.53	0.00	2.76
time (sec)	N/A	0.533	0.446	1.567	0.000	0.265	0.000	0.124	1.259	6.834

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	138	110	124	0	630	0	181	103	247
N.S.	1	1.14	0.91	1.02	0.00	5.21	0.00	1.50	0.85	2.04
time (sec)	N/A	0.446	0.342	1.272	0.000	0.173	0.000	0.133	0.791	6.599

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	125	101	86	0	428	0	153	103	199
N.S.	1	1.16	0.94	0.80	0.00	3.96	0.00	1.42	0.95	1.84
time (sec)	N/A	0.402	0.288	1.096	0.000	0.173	0.000	0.122	0.653	6.348

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	208	157	199	0	1774	0	226	101	288
N.S.	1	1.21	0.91	1.16	0.00	10.31	0.00	1.31	0.59	1.67
time (sec)	N/A	0.610	0.873	1.354	0.000	0.387	0.000	0.120	0.314	11.210

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	283	223	231	0	2337	0	367	1276	18847
N.S.	1	1.17	0.93	0.96	0.00	9.70	0.00	1.52	5.29	78.20
time (sec)	N/A	0.781	1.376	1.513	0.000	0.669	0.000	0.127	1.621	18.787

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	381	787	0	0	0	0	103	0
N.S.	1	1.00	5.69	11.75	0.00	0.00	0.00	0.00	1.54	0.00
time (sec)	N/A	0.358	10.241	2.133	0.000	0.000	0.000	0.000	1.106	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	216	986	0	0	0	0	101	0
N.S.	1	1.00	3.22	14.72	0.00	0.00	0.00	0.00	1.51	0.00
time (sec)	N/A	0.333	10.202	1.241	0.000	0.000	0.000	0.000	1.896	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	381	830	0	0	0	0	100	0
N.S.	1	1.00	6.15	13.39	0.00	0.00	0.00	0.00	1.61	0.00
time (sec)	N/A	0.314	10.408	1.112	0.000	0.000	0.000	0.000	1.820	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	308	1019	0	0	0	0	1270	0
N.S.	1	1.00	4.74	15.68	0.00	0.00	0.00	0.00	19.54	0.00
time (sec)	N/A	0.362	10.310	6.023	0.000	0.000	0.000	0.000	2.886	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	515	863	0	0	0	0	1269	0
N.S.	1	1.00	7.69	12.88	0.00	0.00	0.00	0.00	18.94	0.00
time (sec)	N/A	0.346	10.716	6.108	0.000	0.000	0.000	0.000	2.788	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	268	308	283	0	325	0	379	278	442
N.S.	1	1.02	1.17	1.07	0.00	1.23	0.00	1.44	1.05	1.67
time (sec)	N/A	0.849	0.598	3.956	0.000	0.200	0.000	0.153	0.355	3.848

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	225	265	235	0	282	0	320	184	336
N.S.	1	1.02	1.20	1.07	0.00	1.28	0.00	1.45	0.84	1.53
time (sec)	N/A	0.681	0.405	1.744	0.000	0.169	0.000	0.144	0.381	3.649

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	207	226	194	0	222	0	276	124	298
N.S.	1	1.11	1.22	1.04	0.00	1.19	0.00	1.48	0.67	1.60
time (sec)	N/A	0.585	0.250	1.675	0.000	0.150	0.000	0.150	0.341	3.328

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	179	205	210	0	206	0	223	107	249
N.S.	1	1.13	1.29	1.32	0.00	1.30	0.00	1.40	0.67	1.57
time (sec)	N/A	0.531	0.193	1.698	0.000	0.125	0.000	0.151	0.322	3.142

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	249	312	244	0	276	0	311	23	1607
N.S.	1	1.01	1.27	0.99	0.00	1.12	0.00	1.26	0.09	6.53
time (sec)	N/A	0.640	0.383	1.879	0.000	0.146	0.000	0.403	0.348	3.654

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	332	351	357	0	426	0	351	25	1917
N.S.	1	1.05	1.11	1.13	0.00	1.34	0.00	1.11	0.08	6.05
time (sec)	N/A	0.757	0.669	1.943	0.000	0.286	0.000	0.408	0.567	9.407

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	345	413	417	0	468	0	455	25	2767
N.S.	1	0.93	1.12	1.13	0.00	1.26	0.00	1.23	0.07	7.48
time (sec)	N/A	0.894	0.852	2.106	0.000	1.468	0.000	0.450	0.623	11.988

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	357	527	426	0	490	0	0	283	0
N.S.	1	1.06	1.57	1.27	0.00	1.46	0.00	0.00	0.84	0.00
time (sec)	N/A	1.002	4.411	2.965	0.000	1.491	0.000	0.000	0.418	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	281	467	379	0	449	0	0	145	0
N.S.	1	1.02	1.69	1.37	0.00	1.63	0.00	0.00	0.53	0.00
time (sec)	N/A	0.719	2.980	2.023	0.000	0.247	0.000	0.000	0.341	0.000

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	240	423	325	0	330	0	0	22	0
N.S.	1	1.03	1.81	1.39	0.00	1.41	0.00	0.00	0.09	0.00
time (sec)	N/A	0.550	2.899	1.369	0.000	0.149	0.000	0.000	0.229	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	181	309	203	0	0	0	0	113	0
N.S.	1	1.08	1.84	1.21	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.467	2.045	1.755	0.000	0.000	0.000	0.000	0.342	0.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	223	333	231	0	0	0	0	134	0
N.S.	1	1.09	1.63	1.13	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.587	1.836	1.828	0.000	0.000	0.000	0.000	0.432	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	278	373	266	0	0	0	0	197	0
N.S.	1	1.08	1.45	1.03	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.763	2.116	1.860	0.000	0.000	0.000	0.000	0.503	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	352	419	306	0	0	0	0	293	0
N.S.	1	1.11	1.32	0.96	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	1.007	2.754	1.985	0.000	0.000	0.000	0.000	0.556	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	281	0	0	0	0	0	277	0
N.S.	1	1.00	4.39	0.00	0.00	0.00	0.00	0.00	4.33	0.00
time (sec)	N/A	0.349	6.843	0.000	0.000	0.000	0.000	0.000	0.390	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	240	0	0	0	0	0	142	0
N.S.	1	1.00	3.75	0.00	0.00	0.00	0.00	0.00	2.22	0.00
time (sec)	N/A	0.354	6.270	0.000	0.000	0.000	0.000	0.000	0.354	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	160	0	0	0	0	0	21	0
N.S.	1	1.00	2.71	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.316	0.029	0.000	0.000	0.000	0.000	0.000	0.253	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	327	0	0	0	0	0	25	0
N.S.	1	1.00	5.11	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.332	10.227	0.000	0.000	0.000	0.000	0.000	0.400	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	289	0	0	0	0	0	25	0
N.S.	1	1.00	4.52	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.351	10.311	0.000	0.000	0.000	0.000	0.000	0.552	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	270	310	272	0	455	0	409	278	490
N.S.	1	1.02	1.17	1.02	0.00	1.71	0.00	1.54	1.05	1.84
time (sec)	N/A	0.760	0.866	1.780	0.000	0.217	0.000	0.155	0.410	3.874

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	227	266	235	0	398	0	350	185	385
N.S.	1	1.02	1.19	1.05	0.00	1.78	0.00	1.57	0.83	1.73
time (sec)	N/A	0.667	0.494	1.741	0.000	0.208	0.000	0.153	0.374	3.615

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	209	228	195	0	353	0	306	125	302
N.S.	1	1.11	1.21	1.04	0.00	1.88	0.00	1.63	0.66	1.61
time (sec)	N/A	0.544	0.359	1.708	0.000	0.195	0.000	0.159	0.360	3.658

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	181	206	211	0	323	0	259	109	238
N.S.	1	1.12	1.27	1.30	0.00	1.99	0.00	1.60	0.67	1.47
time (sec)	N/A	0.493	0.203	1.693	0.000	0.142	0.000	0.145	0.417	3.668

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	248	310	246	0	425	0	341	23	1963
N.S.	1	1.01	1.27	1.00	0.00	1.73	0.00	1.39	0.09	8.01
time (sec)	N/A	0.582	0.483	1.399	0.000	0.176	0.000	0.441	0.309	3.412

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	336	352	293	0	1030	0	395	25	1908
N.S.	1	1.05	1.10	0.92	0.00	3.22	0.00	1.23	0.08	5.96
time (sec)	N/A	0.716	1.024	1.922	0.000	0.257	0.000	0.440	0.555	9.767

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	344	413	314	0	1151	0	488	25	2788
N.S.	1	0.93	1.12	0.85	0.00	3.11	0.00	1.32	0.07	7.54
time (sec)	N/A	0.775	1.003	1.997	0.000	1.213	0.000	0.462	0.627	15.016

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	527	336	0	1164	0	0	278	0
N.S.	1	1.00	1.58	1.01	0.00	3.49	0.00	0.00	0.83	0.00
time (sec)	N/A	0.888	4.819	2.273	0.000	1.208	0.000	0.000	0.421	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	273	466	321	0	1091	0	0	143	0
N.S.	1	1.00	1.71	1.18	0.00	4.01	0.00	0.00	0.53	0.00
time (sec)	N/A	0.624	4.029	2.010	0.000	0.286	0.000	0.000	0.330	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	237	423	326	0	469	0	0	21	0
N.S.	1	1.02	1.82	1.40	0.00	2.01	0.00	0.00	0.09	0.00
time (sec)	N/A	0.499	0.333	1.473	0.000	0.185	0.000	0.000	0.244	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	182	309	207	0	0	0	0	119	0
N.S.	1	1.08	1.83	1.22	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.451	1.851	2.139	0.000	0.000	0.000	0.000	0.375	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	225	334	230	0	0	0	0	134	0
N.S.	1	1.09	1.62	1.12	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.571	2.015	1.848	0.000	0.000	0.000	0.000	0.427	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	279	374	275	0	0	0	0	197	0
N.S.	1	1.09	1.46	1.07	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.757	2.275	1.864	0.000	0.000	0.000	0.000	0.555	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	354	422	307	0	0	0	0	293	0
N.S.	1	1.11	1.32	0.96	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.988	2.803	1.984	0.000	0.000	0.000	0.000	0.496	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	181	0	0	0	0	0	284	0
N.S.	1	1.00	2.83	0.00	0.00	0.00	0.00	0.00	4.44	0.00
time (sec)	N/A	0.362	7.618	0.000	0.000	0.000	0.000	0.000	0.436	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	146	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	2.28	0.00
time (sec)	N/A	0.354	6.987	0.000	0.000	0.000	0.000	0.000	0.339	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	22	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.336	9.632	0.000	0.000	0.000	0.000	0.000	0.255	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	138	0	0	0	0	0	25	0
N.S.	1	1.00	2.23	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.351	10.079	0.000	0.000	0.000	0.000	0.000	0.393	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	181	0	0	0	0	0	25	0
N.S.	1	1.00	2.83	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.350	10.120	0.000	0.000	0.000	0.000	0.000	0.555	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	255	308	260	0	369	0	394	329	477
N.S.	1	1.02	1.23	1.04	0.00	1.47	0.00	1.57	1.31	1.90
time (sec)	N/A	0.780	0.610	1.921	0.000	0.212	0.000	0.153	0.476	4.203

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	239	260	228	0	298	0	348	233	348
N.S.	1	1.13	1.23	1.08	0.00	1.41	0.00	1.65	1.10	1.65
time (sec)	N/A	0.617	0.374	1.720	0.000	0.121	0.000	0.145	0.488	3.805

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	211	221	205	0	246	0	297	200	304
N.S.	1	1.13	1.18	1.10	0.00	1.32	0.00	1.59	1.07	1.63
time (sec)	N/A	0.552	0.263	1.731	0.000	0.124	0.000	0.160	0.476	3.390

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	277	333	293	0	320	0	357	147	796
N.S.	1	1.06	1.28	1.12	0.00	1.23	0.00	1.37	0.56	3.05
time (sec)	N/A	0.676	0.628	1.383	0.000	0.149	0.000	0.459	0.445	5.076

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	383	353	364	0	383	0	394	945	2047
N.S.	1	1.13	1.04	1.07	0.00	1.13	0.00	1.16	2.78	6.02
time (sec)	N/A	0.825	0.816	1.874	0.000	0.305	0.000	0.435	0.820	9.900

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	392	413	448	0	499	0	481	593	2841
N.S.	1	0.97	1.02	1.11	0.00	1.24	0.00	1.19	1.47	7.05
time (sec)	N/A	0.924	1.576	2.075	0.000	1.105	0.000	0.501	1.077	12.690

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	349	526	459	0	547	0	0	281	0
N.S.	1	1.04	1.57	1.37	0.00	1.64	0.00	0.00	0.84	0.00
time (sec)	N/A	1.051	6.806	2.249	0.000	1.250	0.000	0.000	0.440	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	282	469	385	0	396	0	0	194	0
N.S.	1	1.02	1.69	1.39	0.00	1.43	0.00	0.00	0.70	0.00
time (sec)	N/A	0.761	5.200	1.980	0.000	0.270	0.000	0.000	0.436	0.000

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	261	457	336	0	0	0	0	136	0
N.S.	1	1.03	1.80	1.32	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.687	3.670	1.920	0.000	0.000	0.000	0.000	0.331	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	223	328	229	0	0	0	0	216	0
N.S.	1	1.11	1.63	1.14	0.00	0.00	0.00	0.00	1.07	0.00
time (sec)	N/A	0.591	2.035	1.823	0.000	0.000	0.000	0.000	0.588	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	273	369	259	0	0	0	0	248	0
N.S.	1	1.09	1.48	1.04	0.00	0.00	0.00	0.00	0.99	0.00
time (sec)	N/A	0.791	2.728	1.882	0.000	0.000	0.000	0.000	0.739	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	348	419	294	0	0	0	0	347	0
N.S.	1	1.09	1.32	0.92	0.00	0.00	0.00	0.00	1.09	0.00
time (sec)	N/A	1.064	2.542	2.099	0.000	0.000	0.000	0.000	0.784	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	436	478	336	0	0	0	0	415	0
N.S.	1	1.11	1.22	0.86	0.00	0.00	0.00	0.00	1.06	0.00
time (sec)	N/A	1.317	3.232	2.048	0.000	0.000	0.000	0.000	0.882	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	343	0	0	0	0	0	447	0
N.S.	1	1.00	5.28	0.00	0.00	0.00	0.00	0.00	6.88	0.00
time (sec)	N/A	0.351	8.456	0.000	0.000	0.000	0.000	0.000	0.491	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	280	0	0	0	0	0	275	0
N.S.	1	1.00	4.31	0.00	0.00	0.00	0.00	0.00	4.23	0.00
time (sec)	N/A	0.351	8.136	0.000	0.000	0.000	0.000	0.000	0.473	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	346	0	0	0	0	0	190	0
N.S.	1	1.00	5.77	0.00	0.00	0.00	0.00	0.00	3.17	0.00
time (sec)	N/A	0.323	0.211	0.000	0.000	0.000	0.000	0.000	0.373	0.000

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	341	0	0	0	0	0	494	0
N.S.	1	1.00	5.25	0.00	0.00	0.00	0.00	0.00	7.60	0.00
time (sec)	N/A	0.356	10.288	0.000	0.000	0.000	0.000	0.000	0.518	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	286	0	0	0	0	0	587	0
N.S.	1	1.00	4.40	0.00	0.00	0.00	0.00	0.00	9.03	0.00
time (sec)	N/A	0.358	10.338	0.000	0.000	0.000	0.000	0.000	0.881	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	294	307	277	0	1004	0	439	34	438
N.S.	1	1.01	1.06	0.96	0.00	3.46	0.00	1.51	0.12	1.51
time (sec)	N/A	0.821	0.791	1.775	0.000	0.113	0.000	0.150	0.301	3.721

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	248	263	234	0	873	0	371	34	339
N.S.	1	1.02	1.08	0.96	0.00	3.58	0.00	1.52	0.14	1.39
time (sec)	N/A	0.700	0.501	1.684	0.000	0.118	0.000	0.143	0.257	3.543

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	207	231	204	0	768	0	308	34	267
N.S.	1	1.02	1.14	1.00	0.00	3.78	0.00	1.52	0.17	1.32
time (sec)	N/A	0.636	0.385	1.740	0.000	0.113	0.000	0.146	0.256	3.536

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	177	203	186	0	667	0	257	34	219
N.S.	1	1.05	1.21	1.11	0.00	3.97	0.00	1.53	0.20	1.30
time (sec)	N/A	0.488	0.194	1.612	0.000	0.112	0.000	0.147	0.259	3.691

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	149	162	152	0	592	0	226	34	208
N.S.	1	1.03	1.12	1.05	0.00	4.08	0.00	1.56	0.23	1.43
time (sec)	N/A	0.431	0.120	1.103	0.000	0.123	0.000	0.137	0.230	3.593

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	241	309	256	0	628	0	326	31	702
N.S.	1	0.99	1.27	1.05	0.00	2.57	0.00	1.34	0.13	2.88
time (sec)	N/A	0.569	0.484	1.371	0.000	0.109	0.000	0.448	0.210	5.205

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	285	353	283	0	837	0	378	33	1929
N.S.	1	0.96	1.19	0.96	0.00	2.83	0.00	1.28	0.11	6.52
time (sec)	N/A	0.655	0.886	1.954	0.000	0.158	0.000	0.431	0.224	10.660

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	466	313	0	826	0	0	34	0
N.S.	1	1.00	1.71	1.15	0.00	3.03	0.00	0.00	0.12	0.00
time (sec)	N/A	0.636	3.870	2.108	0.000	0.155	0.000	0.000	0.280	0.000

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	229	423	297	0	761	0	0	34	0
N.S.	1	0.98	1.82	1.27	0.00	3.27	0.00	0.00	0.15	0.00
time (sec)	N/A	0.484	2.222	1.392	0.000	0.106	0.000	0.000	0.246	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	255	171	0	0	0	0	30	0
N.S.	1	1.00	1.72	1.16	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.359	0.045	1.205	0.000	0.000	0.000	0.000	0.258	0.000

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	179	314	198	0	0	0	0	33	0
N.S.	1	1.02	1.78	1.12	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.435	1.824	1.912	0.000	0.000	0.000	0.000	0.259	0.000

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	226	340	231	0	0	0	0	33	0
N.S.	1	1.06	1.59	1.08	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.603	1.937	2.004	0.000	0.000	0.000	0.000	0.234	0.000

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	280	374	270	0	0	0	0	33	0
N.S.	1	1.07	1.43	1.03	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.752	2.708	1.880	0.000	0.000	0.000	0.000	0.268	0.000

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	144	0	0	0	0	0	34	0
N.S.	1	1.00	2.25	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.351	7.759	0.000	0.000	0.000	0.000	0.000	0.225	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	34	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.347	7.020	0.000	0.000	0.000	0.000	0.000	0.250	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	32	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.326	9.753	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	141	0	0	0	0	0	33	0
N.S.	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.364	10.089	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	183	0	0	0	0	0	33	0
N.S.	1	1.00	2.86	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.361	10.201	0.000	0.000	0.000	0.000	0.000	0.258	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	246	264	234	0	1322	0	372	34	331
N.S.	1	1.02	1.10	0.97	0.00	5.49	0.00	1.54	0.14	1.37
time (sec)	N/A	0.696	0.532	1.773	0.000	0.112	0.000	0.159	0.252	3.980

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	205	231	204	0	1156	0	308	34	292
N.S.	1	1.02	1.15	1.01	0.00	5.75	0.00	1.53	0.17	1.45
time (sec)	N/A	0.606	0.403	1.762	0.000	0.134	0.000	0.153	0.241	3.683

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	175	202	173	0	1060	0	253	34	232
N.S.	1	1.06	1.22	1.05	0.00	6.42	0.00	1.53	0.21	1.41
time (sec)	N/A	0.516	0.196	1.674	0.000	0.110	0.000	0.145	0.204	3.657

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	149	162	152	0	927	0	221	34	213
N.S.	1	1.03	1.12	1.05	0.00	6.39	0.00	1.52	0.23	1.47
time (sec)	N/A	0.466	0.124	1.112	0.000	0.109	0.000	0.135	0.243	3.868

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	242	308	257	0	468	0	321	31	1413
N.S.	1	0.99	1.26	1.05	0.00	1.91	0.00	1.31	0.13	5.77
time (sec)	N/A	0.609	0.528	1.285	0.000	0.121	0.000	0.427	0.230	3.966

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	287	355	297	0	559	0	377	33	1959
N.S.	1	0.96	1.19	0.99	0.00	1.87	0.00	1.26	0.11	6.55
time (sec)	N/A	0.690	0.763	1.888	0.000	0.538	0.000	0.403	0.210	10.981

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	290	471	329	0	554	0	0	34	0
N.S.	1	1.04	1.69	1.18	0.00	1.99	0.00	0.00	0.12	0.00
time (sec)	N/A	0.692	3.851	2.037	0.000	0.531	0.000	0.000	0.238	0.000

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	232	423	299	0	527	0	0	34	0
N.S.	1	0.99	1.81	1.28	0.00	2.25	0.00	0.00	0.15	0.00
time (sec)	N/A	0.518	2.521	1.341	0.000	0.102	0.000	0.000	0.230	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	255	169	0	0	0	0	32	0
N.S.	1	1.00	1.71	1.13	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.374	1.260	1.145	0.000	0.000	0.000	0.000	0.221	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	178	308	195	0	0	0	0	33	0
N.S.	1	1.03	1.78	1.13	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.446	1.539	1.733	0.000	0.000	0.000	0.000	0.199	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	225	340	232	0	0	0	0	33	0
N.S.	1	1.05	1.58	1.08	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.585	2.313	1.803	0.000	0.000	0.000	0.000	0.253	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	249	0	0	0	0	0	34	0
N.S.	1	1.00	3.89	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.346	8.400	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	34	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.350	7.227	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	30	0
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.315	0.064	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	338	0	0	0	0	0	33	0
N.S.	1	1.00	5.28	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.357	10.215	0.000	0.000	0.000	0.000	0.000	0.267	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	351	349	314	0	1300	0	425	66	564
N.S.	1	1.28	1.27	1.14	0.00	4.73	0.00	1.55	0.24	2.05
time (sec)	N/A	0.939	1.208	1.543	0.000	0.150	0.000	0.146	0.244	4.390

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	258	298	289	0	1141	0	372	66	493
N.S.	1	1.11	1.28	1.25	0.00	4.92	0.00	1.60	0.28	2.12
time (sec)	N/A	0.752	0.915	1.506	0.000	0.158	0.000	0.156	0.213	4.245

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	207	254	222	0	1004	0	332	66	449
N.S.	1	1.02	1.25	1.09	0.00	4.95	0.00	1.64	0.33	2.21
time (sec)	N/A	0.654	0.564	1.481	0.000	0.119	0.000	0.147	0.219	4.166

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	189	227	224	0	872	0	301	66	412
N.S.	1	1.09	1.30	1.29	0.00	5.01	0.00	1.73	0.38	2.37
time (sec)	N/A	0.525	0.377	1.282	0.000	0.131	0.000	0.150	0.238	3.520

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	186	220	197	0	262	0	285	66	389
N.S.	1	1.11	1.32	1.18	0.00	1.57	0.00	1.71	0.40	2.33
time (sec)	N/A	0.519	0.270	1.106	0.000	0.112	0.000	0.137	0.258	3.648

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	293	350	304	0	975	0	389	63	3804
N.S.	1	1.08	1.29	1.12	0.00	3.60	0.00	1.44	0.23	14.04
time (sec)	N/A	0.691	1.080	1.361	0.000	0.134	0.000	0.418	0.238	4.039

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	344	392	350	0	1386	0	481	65	5875
N.S.	1	1.01	1.16	1.03	0.00	4.09	0.00	1.42	0.19	17.33
time (sec)	N/A	0.748	1.293	1.702	0.000	0.777	0.000	0.422	0.261	5.121

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	337	506	488	0	1329	0	0	66	0
N.S.	1	1.05	1.57	1.52	0.00	4.13	0.00	0.00	0.20	0.00
time (sec)	N/A	0.887	8.093	1.819	0.000	0.679	0.000	0.000	0.253	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	283	466	341	0	1127	0	0	66	0
N.S.	1	1.09	1.79	1.31	0.00	4.33	0.00	0.00	0.25	0.00
time (sec)	N/A	0.637	4.560	1.408	0.000	0.117	0.000	0.000	0.225	0.000

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	185	322	217	0	0	0	0	66	0
N.S.	1	1.08	1.87	1.26	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.463	2.260	1.159	0.000	0.000	0.000	0.000	0.234	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	189	328	243	0	0	0	0	62	0
N.S.	1	1.06	1.83	1.36	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.428	0.377	1.136	0.000	0.000	0.000	0.000	0.238	0.000

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	234	358	260	0	0	0	0	65	0
N.S.	1	1.02	1.56	1.14	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.586	2.646	1.580	0.000	0.000	0.000	0.000	0.241	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	289	402	324	0	0	0	0	65	0
N.S.	1	1.01	1.40	1.13	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.762	4.384	1.639	0.000	0.000	0.000	0.000	0.235	0.000

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	361	459	354	0	0	0	0	65	0
N.S.	1	1.03	1.31	1.01	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	1.045	7.335	1.694	0.000	0.000	0.000	0.000	0.281	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	194	0	0	0	0	0	66	0
N.S.	1	1.00	2.90	0.00	0.00	0.00	0.00	0.00	0.99	0.00
time (sec)	N/A	0.367	10.174	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	144	0	0	0	0	0	66	0
N.S.	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	0.99	0.00
time (sec)	N/A	0.370	10.105	0.000	0.000	0.000	0.000	0.000	0.251	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	129	0	0	0	0	0	66	0
N.S.	1	1.00	1.93	0.00	0.00	0.00	0.00	0.00	0.99	0.00
time (sec)	N/A	0.363	9.365	0.000	0.000	0.000	0.000	0.000	0.291	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	141	0	0	0	0	0	64	0
N.S.	1	1.00	2.10	0.00	0.00	0.00	0.00	0.00	0.96	0.00
time (sec)	N/A	0.336	10.100	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	193	0	0	0	0	0	65	0
N.S.	1	1.00	2.97	0.00	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.346	10.161	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	203	202	147	183	186	0	215	113	240
N.S.	1	0.92	0.92	0.67	0.83	0.85	0.00	0.98	0.51	1.09
time (sec)	N/A	0.645	0.267	5.783	0.117	0.096	0.000	0.596	0.288	3.553

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	163	210	136	155	174	0	176	93	219
N.S.	1	0.94	1.21	0.78	0.89	1.00	0.00	1.01	0.53	1.26
time (sec)	N/A	0.553	0.187	2.019	0.114	0.090	0.000	0.583	0.319	3.455

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	171	188	125	153	157	0	174	71	200
N.S.	1	0.99	1.09	0.73	0.89	0.91	0.00	1.01	0.41	1.16
time (sec)	N/A	0.521	0.172	1.700	0.121	0.106	0.000	0.551	0.322	3.495

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	147	169	126	139	144	0	148	51	194
N.S.	1	0.98	1.13	0.84	0.93	0.96	0.00	0.99	0.34	1.29
time (sec)	N/A	0.474	0.160	1.654	0.115	0.092	0.000	0.586	0.273	3.304

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	212	235	182	0	634	0	217	28	345
N.S.	1	0.99	1.10	0.85	0.00	2.96	0.00	1.01	0.13	1.61
time (sec)	N/A	0.547	0.294	1.593	0.000	0.101	0.000	0.935	0.294	3.772

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	279	282	222	0	317	0	243	30	455
N.S.	1	1.04	1.05	0.83	0.00	1.18	0.00	0.91	0.11	1.70
time (sec)	N/A	0.651	0.427	2.033	0.000	0.106	0.000	0.924	0.374	4.401

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	267	316	254	0	341	0	279	30	490
N.S.	1	0.94	1.12	0.90	0.00	1.20	0.00	0.99	0.11	1.73
time (sec)	N/A	0.648	0.461	1.933	0.000	0.112	0.000	0.881	0.347	4.494

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	248	327	242	0	358	0	0	106	0
N.S.	1	0.93	1.22	0.90	0.00	1.34	0.00	0.00	0.40	0.00
time (sec)	N/A	0.746	1.090	7.087	0.000	0.137	0.000	0.000	0.260	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	219	293	243	0	334	0	0	88	0
N.S.	1	0.94	1.26	1.04	0.00	1.43	0.00	0.00	0.38	0.00
time (sec)	N/A	0.585	1.030	2.057	0.000	0.112	0.000	0.000	0.280	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	208	265	219	0	309	0	0	27	0
N.S.	1	1.03	1.32	1.09	0.00	1.54	0.00	0.00	0.13	0.00
time (sec)	N/A	0.493	0.483	1.350	0.000	0.114	0.000	0.000	0.284	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	153	190	150	0	395	0	0	56	0
N.S.	1	0.98	1.22	0.96	0.00	2.53	0.00	0.00	0.36	0.00
time (sec)	N/A	0.420	0.353	1.895	0.000	88.970	0.000	0.000	0.326	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	215	149	0	0	0	0	78	0
N.S.	1	1.00	1.17	0.81	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.553	0.374	1.747	0.000	0.000	0.000	0.000	0.303	0.000

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	208	206	160	0	0	0	0	100	0
N.S.	1	0.99	0.98	0.76	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.654	0.471	1.766	0.000	0.000	0.000	0.000	0.360	0.000

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	243	219	171	0	0	0	0	120	0
N.S.	1	1.03	0.92	0.72	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.793	0.524	1.839	0.000	0.000	0.000	0.000	0.318	0.000

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	521	579	234	0	0	0	0	0	103	0
N.S.	1	1.11	0.45	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	1.432	6.744	0.000	0.000	0.000	0.000	0.000	0.293	0.000

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	494	549	225	0	0	0	0	0	84	0
N.S.	1	1.11	0.46	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	1.249	5.943	0.000	0.000	0.000	0.000	0.000	0.294	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	416	412	431	0	0	0	0	0	26	0
N.S.	1	0.99	1.04	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.807	2.658	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	496	551	231	0	0	0	0	0	30	0
N.S.	1	1.11	0.47	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.225	11.121	0.000	0.000	0.000	0.000	0.000	0.289	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	523	581	243	0	0	0	0	0	30	0
N.S.	1	1.11	0.46	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.385	11.149	0.000	0.000	0.000	0.000	0.000	0.365	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	206	202	150	183	209	0	215	113	261
N.S.	1	0.92	0.91	0.67	0.82	0.94	0.00	0.96	0.51	1.17
time (sec)	N/A	0.615	0.342	1.709	0.116	0.132	0.000	0.613	0.287	4.156

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	166	210	138	155	197	0	176	93	206
N.S.	1	0.94	1.19	0.78	0.88	1.11	0.00	0.99	0.53	1.16
time (sec)	N/A	0.560	0.223	1.659	0.134	0.183	0.000	0.576	0.314	3.982

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	174	188	126	155	181	0	175	71	221
N.S.	1	0.99	1.07	0.72	0.89	1.03	0.00	1.00	0.41	1.26
time (sec)	N/A	0.495	0.242	1.651	0.112	0.096	0.000	0.626	0.282	3.833

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	150	170	126	140	167	0	148	52	186
N.S.	1	0.98	1.11	0.82	0.92	1.09	0.00	0.97	0.34	1.22
time (sec)	N/A	0.448	0.201	1.587	0.110	0.111	0.000	0.583	0.274	4.013

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	212	236	182	0	530	0	217	28	369
N.S.	1	0.99	1.10	0.85	0.00	2.48	0.00	1.01	0.13	1.72
time (sec)	N/A	0.547	0.333	1.207	0.000	0.104	0.000	0.880	0.338	4.290

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	283	282	221	0	612	0	243	30	490
N.S.	1	1.05	1.05	0.82	0.00	2.28	0.00	0.90	0.11	1.82
time (sec)	N/A	0.671	0.481	1.820	0.000	0.111	0.000	0.842	0.310	4.948

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	267	316	254	0	660	0	279	30	513
N.S.	1	0.94	1.11	0.89	0.00	2.32	0.00	0.98	0.11	1.81
time (sec)	N/A	0.668	0.546	1.783	0.000	0.106	0.000	0.842	0.366	4.622

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	263	325	271	0	701	0	0	103	0
N.S.	1	1.00	1.23	1.03	0.00	2.66	0.00	0.00	0.39	0.00
time (sec)	N/A	0.755	1.105	2.187	0.000	0.108	0.000	0.000	0.283	0.000

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	233	291	241	0	653	0	0	84	0
N.S.	1	1.02	1.27	1.05	0.00	2.85	0.00	0.00	0.37	0.00
time (sec)	N/A	0.558	0.755	1.885	0.000	0.098	0.000	0.000	0.292	0.000

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	205	264	219	0	611	0	0	26	0
N.S.	1	1.02	1.32	1.10	0.00	3.06	0.00	0.00	0.13	0.00
time (sec)	N/A	0.467	0.587	1.213	0.000	0.110	0.000	0.000	0.262	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	154	195	156	0	434	0	0	59	0
N.S.	1	0.98	1.24	0.99	0.00	2.76	0.00	0.00	0.38	0.00
time (sec)	N/A	0.420	0.379	1.700	0.000	76.822	0.000	0.000	0.306	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	184	216	150	0	0	0	0	78	0
N.S.	1	1.01	1.19	0.82	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.522	0.413	1.741	0.000	0.000	0.000	0.000	0.288	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	206	159	0	0	0	0	100	0
N.S.	1	1.00	0.99	0.76	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.650	0.746	1.789	0.000	0.000	0.000	0.000	0.316	0.000

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	244	219	172	0	0	0	0	120	0
N.S.	1	1.03	0.93	0.73	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.775	0.722	1.822	0.000	0.000	0.000	0.000	0.367	0.000

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	512	502	147	0	0	0	0	0	106	0
N.S.	1	0.98	0.29	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.278	7.442	0.000	0.000	0.000	0.000	0.000	0.298	0.000

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	485	471	127	0	0	0	0	0	88	0
N.S.	1	0.97	0.26	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	1.102	6.946	0.000	0.000	0.000	0.000	0.000	0.316	0.000

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	457	460	63	0	0	0	0	0	27	0
N.S.	1	1.01	0.14	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.181	9.705	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	483	467	136	0	0	0	0	0	30	0
N.S.	1	0.97	0.28	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.129	11.081	0.000	0.000	0.000	0.000	0.000	0.303	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	512	496	148	0	0	0	0	0	30	0
N.S.	1	0.97	0.29	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.200	11.067	0.000	0.000	0.000	0.000	0.000	0.359	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	131	145	109	119	112	0	134	31	133
N.S.	1	1.03	1.14	0.86	0.94	0.88	0.00	1.06	0.24	1.05
time (sec)	N/A	0.446	0.174	12.563	0.114	0.084	0.000	0.141	0.198	3.271

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	132	139	102	119	108	0	127	31	133
N.S.	1	1.03	1.09	0.80	0.93	0.84	0.00	0.99	0.24	1.04
time (sec)	N/A	0.438	0.147	13.115	0.114	0.086	0.000	0.141	0.210	3.415

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	101	127	109	97	100	0	98	31	111
N.S.	1	1.04	1.31	1.12	1.00	1.03	0.00	1.01	0.32	1.14
time (sec)	N/A	0.403	0.121	8.546	0.113	0.071	0.000	0.134	0.205	3.223

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	102	126	95	97	96	0	98	31	111
N.S.	1	1.04	1.29	0.97	0.99	0.98	0.00	1.00	0.32	1.13
time (sec)	N/A	0.382	0.116	8.716	0.110	0.079	0.000	0.140	0.256	3.408

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	86	104	80	86	84	0	87	31	100
N.S.	1	1.05	1.27	0.98	1.05	1.02	0.00	1.06	0.38	1.22
time (sec)	N/A	0.353	0.073	6.166	0.116	0.081	0.000	0.131	0.212	3.591

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	143	185	145	0	410	0	149	29	256
N.S.	1	1.04	1.35	1.06	0.00	2.99	0.00	1.09	0.21	1.87
time (sec)	N/A	0.473	0.185	3.272	0.000	0.830	0.000	0.126	0.210	3.552

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	169	202	211	0	182	0	163	31	382
N.S.	1	1.08	1.29	1.34	0.00	1.16	0.00	1.04	0.20	2.43
time (sec)	N/A	0.534	0.275	5.134	0.000	0.087	0.000	0.143	0.235	3.634

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	163	220	233	0	198	0	0	31	0
N.S.	1	1.06	1.43	1.51	0.00	1.29	0.00	0.00	0.20	0.00
time (sec)	N/A	0.461	0.512	6.036	0.000	0.086	0.000	0.000	0.225	0.000

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	205	179	0	452	0	0	31	0
N.S.	1	1.00	1.52	1.33	0.00	3.35	0.00	0.00	0.23	0.00
time (sec)	N/A	0.381	0.333	1.671	0.000	0.812	0.000	0.000	0.238	0.000

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	114	95	0	246	0	0	27	0
N.S.	1	1.00	1.30	1.08	0.00	2.80	0.00	0.00	0.31	0.00
time (sec)	N/A	0.293	0.254	4.570	0.000	1.579	0.000	0.000	0.211	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	141	126	0	270	0	0	31	0
N.S.	1	1.00	1.34	1.20	0.00	2.57	0.00	0.00	0.30	0.00
time (sec)	N/A	0.368	0.282	27.207	0.000	1.832	0.000	0.000	0.269	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	129	142	127	0	276	0	0	31	0
N.S.	1	1.04	1.15	1.02	0.00	2.23	0.00	0.00	0.25	0.00
time (sec)	N/A	0.426	0.307	26.221	0.000	1.802	0.000	0.000	0.242	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	155	154	135	0	283	0	0	31	0
N.S.	1	1.10	1.09	0.96	0.00	2.01	0.00	0.00	0.22	0.00
time (sec)	N/A	0.494	0.330	26.476	0.000	1.753	0.000	0.000	0.217	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	271	295	40	0	0	0	0	0	31	0
N.S.	1	1.09	0.15	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.863	10.030	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	254	269	26	0	0	0	0	0	31	0
N.S.	1	1.06	0.10	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.750	10.019	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	233	249	283	0	0	336	0	0	29	0
N.S.	1	1.07	1.21	0.00	0.00	1.44	0.00	0.00	0.12	0.00
time (sec)	N/A	0.707	0.880	0.000	0.000	1.432	0.000	0.000	0.219	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	67	0	0	0	0	0	31	0
N.S.	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.790	11.047	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	289	288	76	0	0	0	0	0	31	0
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.816	11.048	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	129	137	100	119	117	0	127	31	135
N.S.	1	1.03	1.10	0.80	0.95	0.94	0.00	1.02	0.25	1.08
time (sec)	N/A	0.431	0.153	12.053	0.108	0.081	0.000	0.141	0.270	3.668

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	102	127	109	97	110	0	98	31	113
N.S.	1	1.04	1.30	1.11	0.99	1.12	0.00	1.00	0.32	1.15
time (sec)	N/A	0.412	0.130	11.570	0.112	0.076	0.000	0.133	0.202	3.605

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	126	95	97	105	0	98	31	113
N.S.	1	1.04	1.33	1.00	1.02	1.11	0.00	1.03	0.33	1.19
time (sec)	N/A	0.372	0.089	9.305	0.109	0.087	0.000	0.135	0.204	3.692

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	87	102	78	86	94	0	87	31	102
N.S.	1	1.05	1.23	0.94	1.04	1.13	0.00	1.05	0.37	1.23
time (sec)	N/A	0.353	0.081	4.951	0.106	0.097	0.000	0.128	0.209	3.723

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	143	185	145	0	157	0	149	29	344
N.S.	1	1.04	1.35	1.06	0.00	1.15	0.00	1.09	0.21	2.51
time (sec)	N/A	0.461	0.178	2.341	0.000	0.094	0.000	0.138	0.279	3.807

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	167	202	211	0	191	0	163	31	368
N.S.	1	1.06	1.28	1.34	0.00	1.21	0.00	1.03	0.20	2.33
time (sec)	N/A	0.520	0.229	4.926	0.000	0.091	0.000	0.127	0.209	3.681

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	162	222	234	0	208	0	0	31	0
N.S.	1	1.01	1.39	1.46	0.00	1.30	0.00	0.00	0.19	0.00
time (sec)	N/A	0.470	0.509	2.989	0.000	0.098	0.000	0.000	0.211	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	206	179	0	194	0	0	31	0
N.S.	1	1.00	1.48	1.29	0.00	1.40	0.00	0.00	0.22	0.00
time (sec)	N/A	0.397	0.351	1.556	0.000	0.091	0.000	0.000	0.220	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	114	96	0	247	0	0	29	0
N.S.	1	1.00	1.30	1.09	0.00	2.81	0.00	0.00	0.33	0.00
time (sec)	N/A	0.297	0.258	3.840	0.000	1.287	0.000	0.000	0.230	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	143	121	0	266	0	0	31	0
N.S.	1	1.00	1.39	1.17	0.00	2.58	0.00	0.00	0.30	0.00
time (sec)	N/A	0.343	0.360	26.871	0.000	1.133	0.000	0.000	0.219	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	129	141	127	0	277	0	0	31	0
N.S.	1	1.04	1.14	1.02	0.00	2.23	0.00	0.00	0.25	0.00
time (sec)	N/A	0.425	0.563	26.914	0.000	1.116	0.000	0.000	0.252	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	291	314	115	696	0	350	0	0	31	0
N.S.	1	1.08	0.40	2.39	0.00	1.20	0.00	0.00	0.11	0.00
time (sec)	N/A	0.685	10.101	22.779	0.000	1.401	0.000	0.000	0.249	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	294	316	26	0	0	0	0	0	31	0
N.S.	1	1.07	0.09	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.779	10.043	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	293	316	111	0	0	0	0	0	27	0
N.S.	1	1.08	0.38	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.700	10.103	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	294	316	120	695	0	363	0	0	31	0
N.S.	1	1.07	0.41	2.36	0.00	1.23	0.00	0.00	0.11	0.00
time (sec)	N/A	0.730	11.109	73.193	0.000	1.386	0.000	0.000	0.200	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	147	142	138	128	135	0	136	35	148
N.S.	1	1.04	1.01	0.98	0.91	0.96	0.00	0.96	0.25	1.05
time (sec)	N/A	0.457	0.242	8.711	0.106	0.086	0.000	0.143	0.254	3.339

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	134	137	133	119	130	0	120	35	139
N.S.	1	1.03	1.05	1.02	0.92	1.00	0.00	0.92	0.27	1.07
time (sec)	N/A	0.491	0.195	10.867	0.106	0.078	0.000	0.143	0.212	3.347

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	119	132	127	108	125	0	109	35	128
N.S.	1	1.03	1.15	1.10	0.94	1.09	0.00	0.95	0.30	1.11
time (sec)	N/A	0.425	0.166	8.648	0.108	0.082	0.000	0.140	0.208	4.036

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	107	126	122	97	120	0	98	35	117
N.S.	1	1.07	1.26	1.22	0.97	1.20	0.00	0.98	0.35	1.17
time (sec)	N/A	0.381	0.165	8.768	0.105	0.081	0.000	0.131	0.220	4.574

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	106	127	122	97	120	0	98	35	117
N.S.	1	1.06	1.27	1.22	0.97	1.20	0.00	0.98	0.35	1.17
time (sec)	N/A	0.390	0.142	6.993	0.108	0.086	0.000	0.135	0.243	3.323

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	166	198	210	0	198	0	160	32	253
N.S.	1	1.08	1.29	1.36	0.00	1.29	0.00	1.04	0.21	1.64
time (sec)	N/A	0.493	0.282	5.299	0.000	0.089	0.000	0.131	0.239	3.466

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	199	209	267	0	233	0	181	34	399
N.S.	1	1.14	1.19	1.53	0.00	1.33	0.00	1.03	0.19	2.28
time (sec)	N/A	0.540	0.335	4.661	0.000	0.085	0.000	0.144	0.216	3.876

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	186	227	292	0	245	0	0	35	0
N.S.	1	1.07	1.30	1.68	0.00	1.41	0.00	0.00	0.20	0.00
time (sec)	N/A	0.508	0.657	3.919	0.000	0.086	0.000	0.000	0.238	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	160	220	219	0	236	0	0	35	0
N.S.	1	1.05	1.44	1.43	0.00	1.54	0.00	0.00	0.23	0.00
time (sec)	N/A	0.439	0.481	2.661	0.000	0.087	0.000	0.000	0.247	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	108	139	141	0	281	0	0	35	0
N.S.	1	1.02	1.31	1.33	0.00	2.65	0.00	0.00	0.33	0.00
time (sec)	N/A	0.335	0.350	7.830	0.000	1.606	0.000	0.000	0.217	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	109	140	141	0	281	0	0	31	0
N.S.	1	1.03	1.32	1.33	0.00	2.65	0.00	0.00	0.29	0.00
time (sec)	N/A	0.322	0.331	6.177	0.000	1.459	0.000	0.000	0.203	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	126	148	156	0	303	0	0	34	0
N.S.	1	1.02	1.19	1.26	0.00	2.44	0.00	0.00	0.27	0.00
time (sec)	N/A	0.402	0.746	26.736	0.000	1.465	0.000	0.000	0.236	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	153	152	161	0	309	0	0	34	0
N.S.	1	1.06	1.06	1.12	0.00	2.15	0.00	0.00	0.24	0.00
time (sec)	N/A	0.477	0.425	26.461	0.000	1.751	0.000	0.000	0.205	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	178	157	168	0	314	0	0	34	0
N.S.	1	1.10	0.97	1.04	0.00	1.94	0.00	0.00	0.21	0.00
time (sec)	N/A	0.576	0.433	27.636	0.000	1.450	0.000	0.000	0.219	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	292	293	71	0	0	0	0	0	35	0
N.S.	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.799	10.100	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	276	66	0	0	0	0	0	35	0
N.S.	1	1.01	0.24	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.737	10.062	0.000	0.000	0.000	0.000	0.000	0.277	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	277	66	0	0	0	0	0	35	0
N.S.	1	1.01	0.24	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.753	10.056	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	292	45	0	0	0	0	0	33	0
N.S.	1	1.07	0.16	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.840	10.043	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	292	293	76	0	0	0	0	0	34	0
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.801	11.059	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	79	0	0	0	0	0	34	0
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.879	11.097	0.000	0.000	0.000	0.000	0.000	0.285	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	87	88	0	0	256	0	104	46	283
N.S.	1	0.99	1.00	0.00	0.00	2.91	0.00	1.18	0.52	3.22
time (sec)	N/A	0.372	1.029	0.000	0.000	0.094	0.000	0.143	0.439	8.399

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	194	0	54	46	49
N.S.	1	1.00	1.00	0.00	0.00	4.04	0.00	1.12	0.96	1.02
time (sec)	N/A	0.302	0.630	0.000	0.000	0.094	0.000	0.121	0.337	3.757

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	204	0	89	44	136
N.S.	1	1.00	1.00	0.00	0.00	4.25	0.00	1.85	0.92	2.83
time (sec)	N/A	0.309	0.581	0.000	0.000	0.107	0.000	0.131	0.234	6.582

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	90	91	0	0	278	0	413	132	481
N.S.	1	0.99	1.00	0.00	0.00	3.05	0.00	4.54	1.45	5.29
time (sec)	N/A	0.378	1.412	0.000	0.000	0.136	0.000	0.139	0.424	10.004

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	90	0	0	0	0	0	46	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.451	2.183	0.000	0.000	0.000	0.000	0.000	0.263	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	90	0	0	0	0	0	46	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.433	1.818	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	90	0	0	0	0	0	44	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.398	1.738	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	170	0	0	0	0	0	43	0
N.S.	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.369	0.039	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	189	0	0	0	0	0	46	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.457	2.436	0.000	0.000	0.000	0.000	0.000	0.246	0.000

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	365	0	0	0	0	0	46	0
N.S.	1	1.00	4.15	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.449	2.552	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	551	591	69	0	0	34	121	0	37	0
N.S.	1	1.07	0.13	0.00	0.00	0.06	0.22	0.00	0.07	0.00
time (sec)	N/A	1.007	1.306	0.000	0.000	0.075	2.213	0.000	0.225	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	86	0	0	0	0	0	34	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.401	0.264	0.000	0.000	0.000	0.000	0.000	0.243	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	171	154	0	0	0	0	0	454	0
N.S.	1	0.97	0.88	0.00	0.00	0.00	0.00	0.00	2.58	0.00
time (sec)	N/A	0.609	5.132	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	100	104	0	0	0	0	0	202	0
N.S.	1	0.99	1.03	0.00	0.00	0.00	0.00	0.00	2.00	0.00
time (sec)	N/A	0.403	0.100	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	56	61	0	0	0	39	0	67	0
N.S.	1	0.88	0.95	0.00	0.00	0.00	0.61	0.00	1.05	0.00
time (sec)	N/A	0.312	0.031	0.000	0.000	0.000	98.806	0.000	0.220	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	77	75	0	0	0	0	0	35	0
N.S.	1	0.65	0.64	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.374	0.072	0.000	0.000	0.000	0.000	0.000	0.255	0.000

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	102	101	0	0	0	0	0	0	0
N.S.	1	1.48	1.46	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.406	0.077	0.000	0.000	0.000	0.000	0.000	0.621	0.000

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	159	158	0	0	0	0	0	0	0
N.S.	1	1.20	1.19	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.662	0.169	0.000	0.000	0.000	0.000	0.000	11.213	0.000

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	231	486	484	0	0	0	0	0	0	0
N.S.	1	2.10	2.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.257	0.351	0.000	0.000	0.000	0.000	0.000	145.229	0.000

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.448	0.199	0.000	0.000	0.000	0.000	0.000	0.491	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	252	0	0	0	0	0	0	0	28	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.040	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.112	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	108	101	0	0	0	0	0	0	0
N.S.	1	1.04	0.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.476	0.102	0.000	0.000	0.000	0.000	0.000	0.541	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	93	0	0	0	0	0	31	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.442	0.190	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	98	0	0	0	0	0	903	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	8.77	0.00
time (sec)	N/A	0.466	0.115	0.000	0.000	0.000	0.000	0.000	0.384	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [799] had the largest ratio of [.7500000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.03	18	0.167
2	A	4	3	1.10	18	0.167
3	A	4	3	1.03	18	0.167
4	A	4	3	1.14	18	0.167
5	A	2	2	1.00	16	0.125
6	A	2	2	1.00	15	0.133
7	A	2	2	1.00	18	0.111
8	A	2	2	1.00	18	0.111
9	A	2	2	1.00	18	0.111
10	A	2	2	1.00	18	0.111
11	A	4	3	1.10	20	0.150
12	A	5	4	1.09	20	0.200
13	A	4	3	1.02	20	0.150
14	A	4	3	1.02	20	0.150
15	A	2	2	1.00	18	0.111
16	A	2	2	1.00	17	0.118
17	A	2	2	1.00	20	0.100
18	A	2	2	1.00	20	0.100
19	A	2	2	1.00	20	0.100
20	A	2	2	1.00	20	0.100
21	A	2	2	1.00	20	0.100

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	20	0.100
23	A	4	3	1.04	20	0.150
24	A	4	3	1.06	20	0.150
25	A	4	3	1.10	20	0.150
26	A	5	4	0.99	20	0.200
27	A	4	3	1.00	20	0.150
28	A	4	3	1.02	20	0.150
29	A	4	3	1.02	20	0.150
30	A	4	3	1.01	20	0.150
31	A	4	3	0.99	20	0.150
32	A	5	4	0.99	20	0.200
33	A	4	3	1.08	20	0.150
34	A	5	4	1.05	20	0.200
35	A	4	3	1.02	20	0.150
36	A	2	2	1.00	20	0.100
37	A	2	2	1.00	20	0.100
38	A	2	2	1.00	20	0.100
39	A	2	2	1.00	20	0.100
40	A	2	2	1.00	20	0.100
41	A	2	2	1.00	18	0.111
42	A	2	2	1.00	17	0.118
43	A	2	2	1.00	20	0.100
44	A	2	2	1.00	20	0.100
45	A	2	2	1.00	20	0.100
46	A	2	2	1.00	20	0.100
47	A	2	2	1.00	20	0.100
48	A	2	2	1.00	20	0.100
49	A	2	2	1.00	20	0.100
50	A	2	2	1.00	20	0.100
51	A	2	2	1.00	20	0.100
52	A	2	2	1.00	20	0.100
53	A	2	2	1.00	20	0.100

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	20	0.100
55	A	2	2	1.00	20	0.100
56	A	2	2	1.00	20	0.100
57	A	2	2	1.00	20	0.100
58	A	4	3	0.99	20	0.150
59	A	4	3	0.98	20	0.150
60	A	4	3	0.94	20	0.150
61	A	4	3	1.12	20	0.150
62	A	4	3	1.02	20	0.150
63	A	4	3	1.01	20	0.150
64	A	3	3	0.85	20	0.150
65	A	11	10	0.94	20	0.500
66	A	11	10	0.91	20	0.500
67	A	10	9	0.93	18	0.500
68	A	10	9	0.90	17	0.529
69	A	10	9	0.95	20	0.450
70	A	10	9	0.91	20	0.450
71	A	11	10	0.95	20	0.500
72	A	11	10	0.91	20	0.500
73	A	12	11	0.94	20	0.550
74	A	4	3	0.96	20	0.150
75	A	4	3	0.92	20	0.150
76	A	4	3	0.98	20	0.150
77	A	4	3	1.02	20	0.150
78	A	4	3	0.99	20	0.150
79	A	4	3	0.98	20	0.150
80	A	3	3	0.89	20	0.150
81	A	3	3	0.93	20	0.150
82	A	3	3	0.94	20	0.150
83	A	11	10	1.01	20	0.500
84	A	11	10	1.00	20	0.500
85	A	10	9	0.96	18	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	10	9	0.93	17	0.529
87	A	11	10	1.01	20	0.500
88	A	11	10	0.99	20	0.500
89	A	12	11	0.99	20	0.550
90	A	12	11	0.98	20	0.550
91	A	4	3	0.99	20	0.150
92	A	4	3	0.94	20	0.150
93	A	4	3	0.98	20	0.150
94	A	3	2	1.00	20	0.100
95	A	4	3	1.01	20	0.150
96	A	4	3	0.99	20	0.150
97	A	4	3	1.00	20	0.150
98	A	4	4	0.92	20	0.200
99	A	4	4	0.94	20	0.200
100	A	12	11	1.00	20	0.550
101	A	12	11	1.00	20	0.550
102	A	11	10	0.97	20	0.500
103	A	11	10	0.95	20	0.500
104	A	11	10	0.96	18	0.556
105	A	11	10	0.93	17	0.588
106	A	12	11	1.00	20	0.550
107	A	12	11	1.00	20	0.550
108	A	13	12	0.99	20	0.600
109	A	13	12	0.98	20	0.600
110	A	2	2	1.00	20	0.100
111	A	2	2	1.00	20	0.100
112	A	2	2	1.00	20	0.100
113	A	2	2	1.00	20	0.100
114	A	2	2	1.00	20	0.100
115	A	2	2	1.00	20	0.100
116	A	2	2	1.00	20	0.100
117	A	2	2	1.00	22	0.091

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	1.00	22	0.091
119	A	2	2	1.00	22	0.091
120	A	2	2	1.00	22	0.091
121	A	2	2	1.00	22	0.091
122	A	2	2	1.00	22	0.091
123	A	2	2	1.00	22	0.091
124	A	2	2	1.00	22	0.091
125	A	2	2	1.00	22	0.091
126	A	2	2	1.00	22	0.091
127	A	2	2	1.00	22	0.091
128	A	2	2	1.00	22	0.091
129	A	2	2	1.00	22	0.091
130	A	2	2	1.00	22	0.091
131	A	6	5	0.96	22	0.227
132	A	5	4	1.00	22	0.182
133	A	5	4	1.00	22	0.182
134	A	6	5	0.97	22	0.227
135	A	13	12	1.20	22	0.545
136	A	12	11	1.27	22	0.500
137	A	12	11	1.23	22	0.500
138	A	12	11	1.28	22	0.500
139	A	12	11	1.22	22	0.500
140	A	7	6	1.11	22	0.273
141	A	6	5	1.16	22	0.227
142	A	5	4	1.00	22	0.182
143	A	6	5	1.16	22	0.227
144	A	13	12	1.25	22	0.545
145	A	12	11	1.26	22	0.500
146	A	12	11	1.21	22	0.500
147	A	13	12	1.27	22	0.545
148	A	13	12	1.23	22	0.545
149	A	7	6	1.11	22	0.273

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	6	5	1.03	22	0.227
151	A	6	5	1.00	22	0.227
152	A	7	6	1.11	22	0.273
153	A	13	12	1.18	22	0.545
154	A	13	12	1.20	22	0.545
155	A	13	12	1.18	22	0.545
156	A	14	13	1.24	22	0.591
157	A	14	13	1.20	22	0.591
158	A	4	3	1.04	22	0.136
159	A	4	3	1.05	22	0.136
160	A	4	3	1.09	22	0.136
161	A	6	5	1.02	22	0.227
162	A	6	5	1.11	22	0.227
163	A	6	5	1.00	22	0.227
164	A	4	4	0.97	22	0.182
165	A	3	3	0.99	19	0.158
166	A	3	3	0.99	22	0.136
167	A	3	3	0.98	22	0.136
168	A	4	4	0.97	22	0.182
169	A	6	6	0.99	22	0.273
170	A	5	5	1.00	20	0.250
171	A	5	5	1.00	22	0.227
172	A	5	5	1.00	22	0.227
173	A	6	6	0.99	22	0.273
174	A	7	7	0.99	22	0.318
175	A	4	3	1.04	22	0.136
176	A	4	3	1.05	22	0.136
177	A	4	3	1.09	22	0.136
178	A	7	6	1.02	22	0.273
179	A	7	6	1.03	22	0.273
180	A	7	6	1.03	22	0.273
181	A	5	5	0.94	22	0.227

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	4	4	0.96	19	0.211
183	A	4	4	0.98	22	0.182
184	A	4	4	0.98	22	0.182
185	A	4	4	0.97	22	0.182
186	A	7	7	0.98	22	0.318
187	A	6	6	0.98	20	0.300
188	A	6	6	0.99	22	0.273
189	A	6	6	0.99	22	0.273
190	A	6	6	0.99	22	0.273
191	A	7	7	0.99	22	0.318
192	A	4	3	1.02	22	0.136
193	A	4	3	1.03	22	0.136
194	A	4	3	1.04	22	0.136
195	A	5	4	1.00	22	0.182
196	A	5	4	0.98	22	0.182
197	A	6	5	0.98	22	0.227
198	A	3	3	1.00	22	0.136
199	A	2	2	1.00	19	0.105
200	A	2	2	1.00	22	0.091
201	A	3	3	1.00	22	0.136
202	A	5	5	1.01	22	0.227
203	A	4	4	1.01	20	0.200
204	A	4	4	1.02	22	0.182
205	A	5	5	1.00	22	0.227
206	A	6	6	1.00	22	0.273
207	A	4	3	1.00	22	0.136
208	A	4	3	1.00	22	0.136
209	A	4	3	1.00	22	0.136
210	A	5	4	1.00	22	0.182
211	A	6	5	0.98	22	0.227
212	A	7	6	0.97	22	0.273
213	A	4	4	1.00	22	0.182

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	3	3	1.01	22	0.136
215	A	2	2	1.00	19	0.105
216	A	3	3	1.00	22	0.136
217	A	4	4	0.99	22	0.182
218	A	5	5	1.01	22	0.227
219	A	4	4	1.02	20	0.200
220	A	5	5	1.00	22	0.227
221	A	6	6	1.00	22	0.273
222	A	7	7	1.00	22	0.318
223	A	4	3	1.00	22	0.136
224	A	4	3	1.00	22	0.136
225	A	4	3	1.04	22	0.136
226	A	6	5	1.05	22	0.227
227	A	7	6	0.95	22	0.273
228	A	4	4	1.01	22	0.182
229	A	3	3	1.00	22	0.136
230	A	3	3	1.00	19	0.158
231	A	4	4	0.99	22	0.182
232	A	5	5	0.99	22	0.227
233	A	6	6	1.01	22	0.273
234	A	5	5	1.01	22	0.227
235	A	5	5	1.00	20	0.250
236	A	6	6	1.00	22	0.273
237	A	7	7	1.00	22	0.318
238	A	8	7	0.94	26	0.269
239	A	6	5	1.06	26	0.192
240	A	8	7	1.08	26	0.269
241	A	7	6	0.98	26	0.231
242	A	5	4	1.08	26	0.154
243	A	8	7	1.07	26	0.269
244	A	7	6	0.99	26	0.231
245	A	5	4	1.10	26	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	8	7	0.95	24	0.292
247	A	7	6	0.99	24	0.250
248	A	5	4	0.99	24	0.167
249	A	9	8	0.91	26	0.308
250	A	7	6	1.03	26	0.231
251	A	9	8	1.06	26	0.308
252	A	8	7	0.93	26	0.269
253	A	6	5	1.05	26	0.192
254	A	9	8	1.06	26	0.308
255	A	8	7	0.97	26	0.269
256	A	6	5	1.08	26	0.192
257	A	10	9	0.89	26	0.346
258	A	8	7	1.00	26	0.269
259	A	10	9	1.04	26	0.346
260	A	9	8	0.90	26	0.308
261	A	7	6	1.02	26	0.231
262	A	10	9	1.05	26	0.346
263	A	9	8	0.95	26	0.308
264	A	7	6	1.06	26	0.231
265	A	7	6	1.00	26	0.231
266	A	5	4	1.09	26	0.154
267	A	7	6	1.09	26	0.231
268	A	6	5	1.01	26	0.192
269	A	4	3	1.12	26	0.115
270	A	7	6	1.09	26	0.231
271	A	6	5	1.01	26	0.192
272	A	4	3	1.12	26	0.115
273	A	7	6	1.01	26	0.231
274	A	5	4	1.09	26	0.154
275	A	7	6	1.10	26	0.231
276	A	6	5	1.01	26	0.192
277	A	4	3	1.11	26	0.115

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	8	7	1.08	26	0.269
279	A	2	2	1.00	26	0.077
280	A	5	4	1.11	26	0.154
281	A	7	6	0.98	26	0.231
282	A	5	4	1.10	26	0.154
283	A	8	7	1.08	26	0.269
284	A	2	2	1.00	26	0.077
285	A	5	4	1.09	26	0.154
286	A	9	8	1.07	26	0.308
287	A	3	3	1.00	26	0.115
288	A	6	5	1.09	26	0.192
289	A	4	3	1.04	22	0.136
290	A	4	3	1.05	22	0.136
291	A	4	3	1.09	22	0.136
292	A	8	7	1.04	22	0.318
293	A	8	7	0.97	22	0.318
294	A	8	7	0.91	22	0.318
295	A	4	4	0.91	22	0.182
296	A	3	3	0.92	20	0.150
297	A	3	3	0.93	22	0.136
298	A	3	3	1.03	22	0.136
299	A	2	2	1.00	22	0.091
300	A	3	3	0.99	22	0.136
301	A	4	4	0.97	22	0.182
302	A	3	3	1.00	22	0.136
303	A	3	3	1.00	19	0.158
304	A	3	3	1.00	22	0.136
305	A	3	3	1.00	22	0.136
306	A	3	3	1.00	22	0.136
307	A	4	3	1.04	22	0.136
308	A	4	3	1.05	22	0.136
309	A	4	3	1.09	22	0.136

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	8	7	1.03	22	0.318
311	A	8	7	0.95	22	0.318
312	A	8	7	0.90	22	0.318
313	A	4	4	0.90	22	0.182
314	A	3	3	0.91	19	0.158
315	A	3	3	0.93	22	0.136
316	A	3	3	1.03	22	0.136
317	A	2	2	1.00	22	0.091
318	A	3	3	0.99	22	0.136
319	A	4	4	0.97	22	0.182
320	A	3	3	1.00	22	0.136
321	A	3	3	1.00	20	0.150
322	A	3	3	1.00	22	0.136
323	A	3	3	1.00	22	0.136
324	A	3	3	1.00	22	0.136
325	A	3	3	1.00	22	0.136
326	A	4	3	1.04	22	0.136
327	A	4	3	1.05	22	0.136
328	A	4	3	1.09	22	0.136
329	A	7	6	1.02	22	0.273
330	A	7	6	0.92	22	0.273
331	A	8	7	0.90	22	0.318
332	A	3	3	0.96	22	0.136
333	A	2	2	0.96	19	0.105
334	A	2	2	1.01	22	0.091
335	A	2	2	1.00	22	0.091
336	A	3	3	0.99	22	0.136
337	A	4	4	0.97	22	0.182
338	A	3	3	1.00	22	0.136
339	A	3	3	1.00	20	0.150
340	A	3	3	1.00	22	0.136
341	A	3	3	1.00	22	0.136

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	3	3	1.00	22	0.136
343	A	4	3	1.05	22	0.136
344	A	4	3	1.06	22	0.136
345	A	4	3	1.12	22	0.136
346	A	7	6	1.03	22	0.273
347	A	7	6	0.91	22	0.273
348	A	8	7	0.91	22	0.318
349	A	3	3	0.97	22	0.136
350	A	2	2	0.97	20	0.100
351	A	2	2	1.01	22	0.091
352	A	2	2	1.00	22	0.091
353	A	3	3	0.99	22	0.136
354	A	4	4	0.97	22	0.182
355	A	3	3	1.00	22	0.136
356	A	3	3	1.00	22	0.136
357	A	3	3	1.00	19	0.158
358	A	3	3	1.00	22	0.136
359	A	3	3	1.00	22	0.136
360	A	4	3	1.04	22	0.136
361	A	4	3	1.07	22	0.136
362	A	4	3	1.09	22	0.136
363	A	7	6	1.05	22	0.273
364	A	8	7	0.90	22	0.318
365	A	4	4	1.04	22	0.182
366	A	3	3	1.07	22	0.136
367	A	2	2	1.04	19	0.105
368	A	2	2	1.00	22	0.091
369	A	3	3	0.98	22	0.136
370	A	4	4	0.97	22	0.182
371	A	3	3	1.08	22	0.136
372	A	3	3	1.08	22	0.136
373	A	3	3	1.12	20	0.150

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
374	A	3	3	1.00	22	0.136
375	A	3	3	1.00	22	0.136
376	A	3	3	1.00	22	0.136
377	A	2	2	1.00	20	0.100
378	A	2	2	1.00	20	0.100
379	A	2	2	1.00	18	0.111
380	A	2	2	1.00	20	0.100
381	A	2	2	1.00	20	0.100
382	A	2	2	1.00	20	0.100
383	A	3	3	1.09	24	0.125
384	A	3	3	1.09	24	0.125
385	A	3	3	1.09	24	0.125
386	A	3	3	1.09	24	0.125
387	A	3	3	1.01	24	0.125
388	A	3	3	1.08	24	0.125
389	A	3	3	1.02	24	0.125
390	A	3	3	1.02	24	0.125
391	A	3	3	1.02	24	0.125
392	A	3	3	1.02	24	0.125
393	A	3	3	1.02	24	0.125
394	A	3	3	1.10	24	0.125
395	A	4	3	0.94	20	0.150
396	A	4	3	0.95	20	0.150
397	A	4	3	0.96	20	0.150
398	A	4	3	0.99	20	0.150
399	A	4	3	1.00	20	0.150
400	A	3	3	1.00	20	0.150
401	A	3	3	1.00	18	0.167
402	A	3	3	1.00	17	0.176
403	A	3	3	1.00	20	0.150
404	A	3	3	1.00	20	0.150
405	A	3	3	1.00	20	0.150

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	3	3	1.00	24	0.125
407	A	3	3	1.00	24	0.125
408	A	3	3	1.00	24	0.125
409	A	3	3	1.00	24	0.125
410	A	3	3	1.00	24	0.125
411	A	3	3	1.00	24	0.125
412	A	3	3	1.00	24	0.125
413	A	3	3	1.02	22	0.136
414	A	4	3	1.09	24	0.125
415	A	1	1	1.00	34	0.029
416	A	4	3	0.97	22	0.136
417	A	4	3	0.94	22	0.136
418	A	4	3	0.98	22	0.136
419	A	4	3	0.98	22	0.136
420	A	4	3	1.02	22	0.136
421	A	4	3	0.99	22	0.136
422	A	13	12	0.96	22	0.545
423	A	4	4	1.05	22	0.182
424	A	11	10	0.92	22	0.455
425	A	10	9	0.90	22	0.409
426	A	10	9	0.86	22	0.409
427	A	10	9	0.90	20	0.450
428	A	10	9	0.86	19	0.474
429	A	4	4	1.05	22	0.182
430	A	12	11	0.92	22	0.500
431	A	5	5	1.12	22	0.227
432	A	14	13	0.96	22	0.591
433	A	7	7	1.11	22	0.318
434	A	4	3	1.02	26	0.115
435	A	6	5	1.11	26	0.192
436	A	5	4	1.04	26	0.154
437	A	6	5	1.06	26	0.192

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
438	A	8	7	1.08	26	0.269
439	A	4	4	1.00	26	0.154
440	A	5	5	1.05	24	0.208
441	A	4	4	0.99	26	0.154
442	A	2	2	1.00	26	0.077
443	A	2	2	1.00	23	0.087
444	A	2	2	1.00	26	0.077
445	A	4	3	1.00	26	0.115
446	A	5	4	1.00	26	0.154
447	A	4	3	1.00	26	0.115
448	A	6	5	1.06	26	0.192
449	A	8	7	1.08	26	0.269
450	A	5	5	1.05	26	0.192
451	A	1	1	1.00	24	0.042
452	A	4	4	1.00	26	0.154
453	A	2	2	1.00	26	0.077
454	A	2	2	1.00	23	0.087
455	A	2	2	1.00	26	0.077
456	A	1	1	1.00	22	0.045
457	A	4	3	1.00	27	0.111
458	A	4	3	1.02	27	0.111
459	A	6	5	1.12	27	0.185
460	A	5	4	1.04	27	0.148
461	A	6	5	1.07	27	0.185
462	A	8	7	1.14	27	0.259
463	A	10	9	1.14	27	0.333
464	A	6	6	1.01	27	0.222
465	A	4	4	1.00	27	0.148
466	A	13	12	1.05	25	0.480
467	A	4	4	0.99	27	0.148
468	A	6	6	1.00	27	0.222
469	A	8	8	1.01	27	0.296

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
470	A	4	3	1.03	27	0.111
471	A	4	3	1.04	27	0.111
472	A	7	6	1.12	27	0.222
473	A	6	5	1.10	27	0.185
474	A	9	8	1.07	27	0.296
475	A	8	7	1.10	27	0.259
476	A	10	9	1.12	27	0.333
477	A	8	8	1.02	27	0.296
478	A	6	6	1.01	27	0.222
479	A	4	4	1.00	25	0.160
480	A	4	4	0.99	27	0.148
481	A	6	6	1.00	27	0.222
482	A	9	9	1.01	27	0.333
483	A	4	3	1.02	27	0.111
484	A	4	3	1.03	27	0.111
485	A	5	4	1.04	27	0.148
486	A	4	3	1.00	27	0.111
487	A	6	5	1.07	27	0.185
488	A	8	7	1.14	27	0.259
489	A	10	9	1.14	27	0.333
490	A	4	4	1.00	27	0.148
491	A	13	12	1.06	27	0.444
492	A	9	8	1.00	25	0.320
493	A	4	4	1.00	27	0.148
494	A	6	6	1.00	27	0.222
495	A	8	8	1.01	27	0.296
496	A	2	2	1.00	27	0.074
497	A	2	2	1.00	24	0.083
498	A	2	2	1.00	27	0.074
499	A	4	3	1.04	27	0.111
500	A	4	3	1.03	27	0.111
501	A	5	4	1.08	27	0.148

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
502	A	5	4	1.07	27	0.148
503	A	9	8	1.16	27	0.296
504	A	10	9	1.18	27	0.333
505	A	12	11	1.16	27	0.407
506	A	4	4	1.00	27	0.148
507	A	4	4	1.00	27	0.148
508	A	4	4	1.00	25	0.160
509	A	6	6	1.01	27	0.222
510	A	8	8	1.02	27	0.296
511	A	10	10	1.02	27	0.370
512	A	2	2	1.00	27	0.074
513	A	2	2	1.00	24	0.083
514	A	2	2	1.00	27	0.074
515	A	5	5	1.10	33	0.152
516	A	5	5	1.10	35	0.143
517	A	6	6	1.10	35	0.171
518	A	6	6	1.10	37	0.162
519	A	5	5	1.09	33	0.152
520	A	5	5	1.09	35	0.143
521	A	5	5	1.09	36	0.139
522	A	5	5	1.09	36	0.139
523	A	1	1	1.00	33	0.030
524	A	1	1	1.00	35	0.029
525	A	1	1	1.00	35	0.029
526	A	1	1	1.00	37	0.027
527	A	1	1	1.00	33	0.030
528	A	1	1	1.00	35	0.029
529	A	1	1	1.00	36	0.028
530	A	1	1	1.00	36	0.028
531	A	4	3	1.00	24	0.125
532	A	6	5	1.05	24	0.208
533	A	5	4	1.00	24	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
534	A	5	4	1.00	24	0.167
535	A	7	6	1.05	24	0.250
536	A	2	2	1.00	24	0.083
537	A	2	2	1.00	22	0.091
538	A	2	2	1.00	21	0.095
539	A	2	2	1.00	24	0.083
540	A	2	2	1.00	24	0.083
541	A	4	3	1.00	24	0.125
542	A	7	6	1.08	24	0.250
543	A	6	5	1.05	24	0.208
544	A	6	5	1.04	24	0.208
545	A	7	6	1.05	24	0.250
546	A	2	2	1.00	24	0.083
547	A	2	2	1.00	22	0.091
548	A	2	2	1.00	21	0.095
549	A	2	2	1.00	24	0.083
550	A	2	2	1.00	24	0.083
551	A	4	3	1.00	24	0.125
552	A	5	4	1.00	24	0.167
553	A	4	3	1.00	24	0.125
554	A	5	4	1.00	24	0.167
555	A	7	6	1.09	24	0.250
556	A	2	2	1.00	24	0.083
557	A	2	2	1.00	22	0.091
558	A	2	2	1.00	21	0.095
559	A	2	2	1.00	24	0.083
560	A	2	2	1.00	24	0.083
561	A	4	3	0.98	24	0.125
562	A	5	4	1.00	24	0.167
563	A	5	4	1.00	24	0.167
564	A	6	5	1.19	24	0.208
565	A	9	8	1.19	24	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
566	A	2	2	1.00	24	0.083
567	A	2	2	1.00	22	0.091
568	A	2	2	1.00	21	0.095
569	A	2	2	1.00	24	0.083
570	A	2	2	1.00	24	0.083
571	A	10	9	1.07	27	0.333
572	A	8	7	1.07	27	0.259
573	A	6	5	1.08	27	0.185
574	A	5	4	1.02	27	0.148
575	A	8	7	1.14	27	0.259
576	A	10	9	1.15	27	0.333
577	A	12	11	1.12	27	0.407
578	A	6	6	1.01	27	0.222
579	A	4	4	1.00	27	0.148
580	A	4	4	0.99	25	0.160
581	A	6	6	1.01	27	0.222
582	A	9	9	1.02	27	0.333
583	A	11	11	1.02	27	0.407
584	A	11	10	1.12	27	0.370
585	A	9	8	1.12	27	0.296
586	A	7	6	1.12	27	0.222
587	A	6	5	1.09	27	0.185
588	A	8	7	1.11	27	0.259
589	A	10	9	1.12	27	0.333
590	A	12	11	1.11	27	0.407
591	A	8	8	1.01	27	0.296
592	A	6	6	1.01	27	0.222
593	A	4	4	0.99	25	0.160
594	A	6	6	1.05	27	0.222
595	A	9	9	1.01	27	0.333
596	A	11	11	1.02	27	0.407
597	A	7	6	1.03	27	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
598	A	7	6	1.12	27	0.222
599	A	5	4	1.06	27	0.148
600	A	5	4	1.06	27	0.148
601	A	8	7	1.14	27	0.259
602	A	10	9	1.15	27	0.333
603	A	12	11	1.12	27	0.407
604	A	4	4	1.00	27	0.148
605	A	4	4	1.00	27	0.148
606	A	4	4	1.00	25	0.160
607	A	6	6	1.01	27	0.222
608	A	8	8	1.02	27	0.296
609	A	10	10	1.02	27	0.370
610	A	2	2	1.00	27	0.074
611	A	2	2	1.00	27	0.074
612	A	2	2	1.00	24	0.083
613	A	2	2	1.00	27	0.074
614	A	2	2	1.00	27	0.074
615	A	7	6	1.05	27	0.222
616	A	7	6	1.16	27	0.222
617	A	6	5	1.10	27	0.185
618	A	6	5	1.14	27	0.185
619	A	10	9	1.19	27	0.333
620	A	12	11	1.17	27	0.407
621	A	14	13	1.14	27	0.481
622	A	6	6	1.00	27	0.222
623	A	6	6	1.01	27	0.222
624	A	6	6	1.01	25	0.240
625	A	8	8	1.02	27	0.296
626	A	10	10	1.03	27	0.370
627	A	12	12	1.03	27	0.444
628	C	2	2	0.26	27	0.074
629	A	2	2	1.00	27	0.074

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
630	A	2	2	1.00	24	0.083
631	A	2	2	1.00	27	0.074
632	A	2	2	1.00	27	0.074
633	A	8	7	1.24	24	0.292
634	A	6	5	1.23	24	0.208
635	A	5	4	1.00	24	0.167
636	A	7	6	1.04	24	0.250
637	A	9	8	1.06	24	0.333
638	A	2	2	1.00	24	0.083
639	A	2	2	1.00	22	0.091
640	A	2	2	1.00	21	0.095
641	A	2	2	1.00	24	0.083
642	A	2	2	1.00	24	0.083
643	A	9	8	1.16	24	0.333
644	A	7	6	1.16	24	0.250
645	A	6	5	1.11	24	0.208
646	A	7	6	1.07	24	0.250
647	A	8	7	1.16	24	0.292
648	A	2	2	1.00	24	0.083
649	A	2	2	1.00	22	0.091
650	A	2	2	1.00	21	0.095
651	A	2	2	1.00	24	0.083
652	A	2	2	1.00	24	0.083
653	A	7	6	1.17	24	0.250
654	A	5	4	0.99	24	0.167
655	A	5	4	0.99	24	0.167
656	A	7	6	1.17	24	0.250
657	A	8	7	1.15	24	0.292
658	A	2	2	1.00	24	0.083
659	A	2	2	1.00	22	0.091
660	A	2	2	1.00	21	0.095
661	A	2	2	1.00	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
662	A	2	2	1.00	24	0.083
663	A	7	6	1.22	24	0.250
664	A	6	5	1.14	24	0.208
665	A	6	5	1.16	24	0.208
666	A	9	8	1.21	24	0.333
667	A	10	9	1.17	24	0.375
668	A	2	2	1.00	24	0.083
669	A	2	2	1.00	22	0.091
670	A	2	2	1.00	21	0.095
671	A	2	2	1.00	24	0.083
672	A	2	2	1.00	24	0.083
673	A	4	3	1.02	24	0.125
674	A	4	3	1.02	24	0.125
675	A	8	7	1.11	24	0.292
676	A	7	6	1.13	24	0.250
677	A	9	8	1.01	24	0.333
678	A	12	11	1.05	24	0.458
679	A	13	12	0.93	24	0.500
680	A	5	5	1.06	24	0.208
681	A	3	3	1.02	24	0.125
682	A	3	3	1.03	22	0.136
683	A	3	3	1.08	24	0.125
684	A	4	4	1.09	24	0.167
685	A	5	5	1.08	24	0.208
686	A	7	7	1.11	24	0.292
687	A	2	2	1.00	24	0.083
688	A	2	2	1.00	24	0.083
689	A	2	2	1.00	21	0.095
690	A	2	2	1.00	24	0.083
691	A	2	2	1.00	24	0.083
692	A	4	3	1.02	24	0.125
693	A	4	3	1.02	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
694	A	8	7	1.11	24	0.292
695	A	7	6	1.12	24	0.250
696	A	9	8	1.01	24	0.333
697	A	12	11	1.05	24	0.458
698	A	13	12	0.93	24	0.500
699	A	6	6	1.00	24	0.250
700	A	4	4	1.00	24	0.167
701	A	3	3	1.02	21	0.143
702	A	3	3	1.08	24	0.125
703	A	4	4	1.09	24	0.167
704	A	6	6	1.09	24	0.250
705	A	7	7	1.11	24	0.292
706	A	2	2	1.00	24	0.083
707	A	2	2	1.00	24	0.083
708	A	2	2	1.00	22	0.091
709	A	2	2	1.00	24	0.083
710	A	2	2	1.00	24	0.083
711	A	4	3	1.02	24	0.125
712	A	9	8	1.13	24	0.333
713	A	8	7	1.13	24	0.292
714	A	10	9	1.06	24	0.375
715	A	13	12	1.13	24	0.500
716	A	14	13	0.97	24	0.542
717	A	6	6	1.04	24	0.250
718	A	4	4	1.02	22	0.182
719	A	3	3	1.03	24	0.125
720	A	4	4	1.11	24	0.167
721	A	7	7	1.09	24	0.292
722	A	7	7	1.09	24	0.292
723	A	10	10	1.11	24	0.417
724	A	2	2	1.00	24	0.083
725	A	2	2	1.00	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
726	A	2	2	1.00	21	0.095
727	A	2	2	1.00	24	0.083
728	A	2	2	1.00	24	0.083
729	A	4	3	1.01	24	0.125
730	A	4	3	1.02	24	0.125
731	A	4	3	1.02	24	0.125
732	A	7	6	1.05	24	0.250
733	A	6	5	1.03	24	0.208
734	A	9	8	0.99	24	0.333
735	A	11	10	0.96	24	0.417
736	A	4	4	1.00	24	0.167
737	A	3	3	0.98	24	0.125
738	A	1	1	1.00	21	0.048
739	A	3	3	1.02	24	0.125
740	A	5	5	1.06	24	0.208
741	A	6	6	1.07	24	0.250
742	A	2	2	1.00	24	0.083
743	A	2	2	1.00	24	0.083
744	A	2	2	1.00	22	0.091
745	A	2	2	1.00	24	0.083
746	A	2	2	1.00	24	0.083
747	A	4	3	1.02	24	0.125
748	A	4	3	1.02	24	0.125
749	A	7	6	1.06	24	0.250
750	A	6	5	1.03	24	0.208
751	A	9	8	0.99	24	0.333
752	A	11	10	0.96	24	0.417
753	A	3	3	1.04	24	0.125
754	A	3	3	0.99	24	0.125
755	A	1	1	1.00	22	0.045
756	A	4	4	1.03	24	0.167
757	A	5	5	1.05	24	0.208

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
758	A	2	2	1.00	24	0.083
759	A	2	2	1.00	24	0.083
760	A	2	2	1.00	21	0.095
761	A	2	2	1.00	24	0.083
762	A	4	3	1.28	24	0.125
763	A	4	3	1.11	24	0.125
764	A	4	3	1.02	24	0.125
765	A	7	6	1.09	24	0.250
766	A	7	6	1.11	24	0.250
767	A	11	10	1.08	24	0.417
768	A	12	11	1.01	24	0.458
769	A	6	6	1.05	24	0.250
770	A	4	4	1.09	24	0.167
771	A	3	3	1.08	24	0.125
772	A	2	2	1.06	21	0.095
773	A	5	5	1.02	24	0.208
774	A	6	6	1.01	24	0.250
775	A	8	8	1.03	24	0.333
776	A	2	2	1.00	24	0.083
777	A	2	2	1.00	24	0.083
778	A	2	2	1.00	24	0.083
779	A	2	2	1.00	22	0.091
780	A	2	2	1.00	24	0.083
781	A	5	4	0.92	28	0.143
782	A	5	4	0.94	28	0.143
783	A	9	8	0.99	28	0.286
784	A	8	7	0.98	28	0.250
785	A	8	7	0.99	28	0.250
786	A	11	10	1.04	28	0.357
787	A	12	11	0.94	28	0.393
788	A	6	6	0.93	28	0.214
789	A	4	4	0.94	28	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
790	A	4	4	1.03	26	0.154
791	A	3	3	0.98	28	0.107
792	A	5	5	1.00	28	0.179
793	A	7	7	0.99	28	0.250
794	A	10	10	1.03	28	0.357
795	A	21	20	1.11	28	0.714
796	A	19	18	1.11	28	0.643
797	A	11	10	0.99	25	0.400
798	A	19	18	1.11	28	0.643
799	A	22	21	1.11	28	0.750
800	A	5	4	0.92	28	0.143
801	A	5	4	0.94	28	0.143
802	A	9	8	0.99	28	0.286
803	A	8	7	0.98	28	0.250
804	A	8	7	0.99	28	0.250
805	A	11	10	1.05	28	0.357
806	A	12	11	0.94	28	0.393
807	A	7	7	1.00	28	0.250
808	A	5	5	1.02	28	0.179
809	A	4	4	1.02	25	0.160
810	A	3	3	0.98	28	0.107
811	A	5	5	1.01	28	0.179
812	A	7	7	1.00	28	0.250
813	A	10	10	1.03	28	0.357
814	A	6	6	0.98	28	0.214
815	A	4	4	0.97	28	0.143
816	A	17	16	1.01	26	0.615
817	A	4	4	0.97	28	0.143
818	A	7	7	0.97	28	0.250
819	A	4	3	1.03	22	0.136
820	A	4	3	1.03	22	0.136
821	A	4	3	1.04	22	0.136

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
822	A	7	6	1.04	22	0.273
823	A	6	5	1.05	22	0.227
824	A	9	8	1.04	22	0.364
825	A	11	10	1.08	22	0.455
826	A	4	4	1.06	22	0.182
827	A	3	3	1.00	22	0.136
828	A	1	1	1.00	19	0.053
829	A	3	3	1.00	22	0.136
830	A	5	5	1.04	22	0.227
831	A	6	6	1.10	22	0.273
832	A	16	15	1.09	22	0.682
833	A	14	13	1.06	22	0.591
834	A	12	11	1.07	20	0.550
835	A	4	4	1.00	22	0.182
836	A	6	6	1.00	22	0.273
837	A	4	3	1.03	22	0.136
838	A	4	3	1.04	22	0.136
839	A	7	6	1.04	22	0.273
840	A	6	5	1.05	22	0.227
841	A	9	8	1.04	22	0.364
842	A	11	10	1.06	22	0.455
843	A	3	3	1.01	22	0.136
844	A	3	3	1.00	22	0.136
845	A	1	1	1.00	20	0.050
846	A	3	3	1.00	22	0.136
847	A	5	5	1.04	22	0.227
848	A	12	11	1.08	22	0.500
849	A	15	14	1.07	22	0.636
850	A	13	12	1.08	19	0.632
851	A	13	12	1.07	22	0.545
852	A	4	3	1.04	22	0.136
853	A	4	3	1.03	22	0.136

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
854	A	4	3	1.03	22	0.136
855	A	7	6	1.07	22	0.273
856	A	7	6	1.06	22	0.273
857	A	10	9	1.08	22	0.409
858	A	12	11	1.14	22	0.500
859	A	5	5	1.07	22	0.227
860	A	4	4	1.05	22	0.182
861	A	2	2	1.02	22	0.091
862	A	2	2	1.03	19	0.105
863	A	4	4	1.02	22	0.182
864	A	6	6	1.06	22	0.273
865	A	8	8	1.10	22	0.364
866	A	5	5	1.00	22	0.227
867	A	3	3	1.01	22	0.136
868	A	3	3	1.01	22	0.136
869	A	16	15	1.07	20	0.750
870	A	4	4	1.00	22	0.182
871	A	6	6	1.00	22	0.273
872	A	5	4	0.99	26	0.154
873	A	4	3	1.00	26	0.115
874	A	4	3	1.00	26	0.115
875	A	5	4	0.99	26	0.154
876	A	3	3	1.00	26	0.115
877	A	3	3	1.00	26	0.115
878	A	3	3	1.00	24	0.125
879	A	3	3	1.00	23	0.130
880	A	3	3	1.00	26	0.115
881	A	3	3	1.00	26	0.115
882	A	6	5	1.07	27	0.185
883	A	2	2	1.00	24	0.083
884	A	6	5	0.97	26	0.192
885	A	4	3	0.99	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
886	A	3	2	0.88	17	0.118
887	C	2	2	0.65	26	0.077
888	A	2	2	1.48	26	0.077
889	C	2	2	1.20	26	0.077
890	C	2	2	2.10	26	0.077
891	A	3	3	1.00	24	0.125
892	F	0	0	N/A	0.000	N/A
893	F	0	0	N/A	0.000	N/A
894	A	3	3	1.04	30	0.100
895	A	3	3	1.00	26	0.115
896	A	3	3	1.00	30	0.100

CHAPTER 3

Listing of integrals

3.1 $\int x^2(a+bx^3)(A+Bx^3)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int x^2(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{3}aAx^3+\frac{1}{6}(Ab+aB)x^6+\frac{1}{9}bBx^9$$

output
```
1/3*a*A*x^3+1/6*(A*b+B*a)*x^6+1/9*b*B*x^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{3}aAx^3+\frac{1}{6}(Ab+aB)x^6+\frac{1}{9}bBx^9$$

input
```
Integrate[x^2*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(a*A*x^3)/3 + ((A*b + a*B)*x^6)/6 + (b*B*x^9)/9
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {946, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b x^3\right) \left(A+B x^3\right) \, dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{3} \int \left(b x^3+a\right) \left(B x^3+A\right) dx^3$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{3} \int \left(b B x^6+(A b+a B) x^3+a A\right) dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} \left(\frac{1}{2} x^6 (a B+A b)+a A x^3+\frac{1}{3} b B x^9\right)$$

input
```
Int[x^2*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(a*A*x^3 + ((A*b + a*B)*x^6)/2 + (b*B*x^9)/3)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{aA\,x^{3}}{3}+\frac{(Ab+Ba)x^{6}}{6}+\frac{bB\,x^{9}}{9}$	28
norman	$\frac{bB\,x^{9}}{9}+\left(\frac{Ab}{6}+\frac{Ba}{6}\right)x^{6}+\frac{aA\,x^{3}}{3}$	29
gosper	$\frac{1}{9}bB\,x^{9}+\frac{1}{6}x^{6}Ab+\frac{1}{6}x^{6}Ba+\frac{1}{3}aA\,x^{3}$	30
risch	$\frac{1}{9}bB\,x^{9}+\frac{1}{6}x^{6}Ab+\frac{1}{6}x^{6}Ba+\frac{1}{3}aA\,x^{3}$	30
parallelrisch	$\frac{1}{9}bB\,x^{9}+\frac{1}{6}x^{6}Ab+\frac{1}{6}x^{6}Ba+\frac{1}{3}aA\,x^{3}$	30
orering	$\frac{x^{3}\left(2bB\,x^{6}+3Ab\,x^{3}+3Ba\,x^{3}+6Aa\right)}{18}$	32

input
```
int(x^2*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*A*x^3+1/6*(A*b+B*a)*x^6+1/9*b*B*x^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^2\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{1}{9}\,Bbx^9+\frac{1}{6}\,(Ba+Ab)x^6+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/9*B*b*x^9 + 1/6*(B*a + A*b)*x^6 + 1/3*A*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{Aax^3}{3}+\frac{Bbx^9}{9}+x^6\left(\frac{Ab}{6}+\frac{Ba}{6}\right)$$

input
```
integrate(x**2*(b*x**3+a)*(B*x**3+A),x)
```

output
```
A*a*x**3/3 + B*b*x**9/9 + x**6*(A*b/6 + B*a/6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^2(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{9}\,Bbx^9+\frac{1}{6}\,(Ba+Ab)x^6+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/9*B*b*x^9 + 1/6*(B*a + A*b)*x^6 + 1/3*A*a*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{9}\,Bbx^9+\frac{1}{6}\,Bax^6+\frac{1}{6}\,Abx^6+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/9*B*b*x^9 + 1/6*B*a*x^6 + 1/6*A*b*x^6 + 1/3*A*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x^2(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{B\,b\,x^9}{9}+\left(\frac{A\,b}{6}+\frac{B\,a}{6}\right)\,x^6+\frac{A\,a\,x^3}{3}$$

input
```
int(x^2*(A + B*x^3)*(a + b*x^3),x)
```

output
```
x^6*((A*b)/6 + (B*a)/6) + (A*a*x^3)/3 + (B*b*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int x^2(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{x^3(b^2x^6+3ab\,x^3+3a^2)}{9}$$

input
```
int(x^2*(b*x^3+a)*(B*x^3+A),x)
```

output
```
(x**3*(3*a**2 + 3*a*b*x**3 + b**2*x**6))/9
```

3.2 $\int \frac{(a+bx^3)(A+Bx^3)}{x} dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{(a+bx^3)(A+Bx^3)}{x} dx = \frac{1}{3}(Ab+aB)x^3 + \frac{1}{6}bBx^6 + aA\log(x)$$

output
```
1/3*(A*b+B*a)*x^3+1/6*b*B*x^6+a*A*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x} dx = \frac{1}{3}(Ab+aB)x^3 + \frac{1}{6}bBx^6 + aA\log(x)$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/x,x]
```

output
```
((A*b + a*B)*x^3)/3 + (b*B*x^6)/6 + a*A*Log[x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)(Bx^3+A)}{x^3}dx^3$$

$$\Big\downarrow \text{ 85}$$

$$\frac{1}{3}\int \left(bBx^3+Ab+aB+\frac{aA}{x^3}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(x^3(aB+Ab)+aA\log\left(x^3\right)+\frac{1}{2}bBx^6\right)$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/x,x]
```

output
```
((A*b + a*B)*x^3 + (b*B*x^6)/2 + a*A*Log[x^3])/3
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

method	result	size
norman	$\left(\frac{Ab}{3}+\frac{Ba}{3}\right)x^{3}+\frac{bB\,x^{6}}{6}+aA\ln\left(x\right)$	27
default	$\frac{bB\,x^{6}}{6}+\frac{Ab\,x^{3}}{3}+\frac{Ba\,x^{3}}{3}+aA\ln\left(x\right)$	28
parallelrisch	$\frac{bB\,x^{6}}{6}+\frac{Ab\,x^{3}}{3}+\frac{Ba\,x^{3}}{3}+aA\ln\left(x\right)$	28
risch	$\frac{bB\,x^{6}}{6}+\frac{Ab\,x^{3}}{3}+\frac{Ba\,x^{3}}{3}+\frac{b\,A^{2}}{6B}+\frac{Aa}{3}+\frac{B\,a^{2}}{6b}+aA\ln\left(x\right)$	50

input
```
int((b*x^3+a)*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/3*A*b+1/3*B*a)*x^3+1/6*b*B*x^6+a*A*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int\frac{(a+bx^3)\,(A+Bx^3)}{x}\,dx=\frac{1}{6}\,Bbx^{6}+\frac{1}{3}\,(Ba+Ab)x^{3}+Aa\log\left(x\right)$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="fricas")
```

output
```
1/6*B*b*x^6 + 1/3*(B*a + A*b)*x^3 + A*a*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx = Aa\log(x) + \frac{Bbx^6}{6} + x^3\left(\frac{Ab}{3} + \frac{Ba}{3}\right)$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x,x)
```

output
```
A*a*log(x) + B*b*x**6/6 + x**3*(A*b/3 + B*a/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx = \frac{1}{6}Bbx^6 + \frac{1}{3}(Ba+Ab)x^3 + \frac{1}{3}Aa\log(x^3)$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="maxima")
```

output
```
1/6*B*b*x^6 + 1/3*(B*a + A*b)*x^3 + 1/3*A*a*log(x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx = \frac{1}{6}Bbx^6 + \frac{1}{3}Bax^3 + \frac{1}{3}Abx^3 + Aa\log(|x|)$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="giac")
```

output
```
1/6*B*b*x^6 + 1/3*B*a*x^3 + 1/3*A*b*x^3 + A*a*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx = x^3\left(\frac{A\,b}{3}+\frac{B\,a}{3}\right)+\frac{B\,b\,x^6}{6}+A\,a\,\ln(x)$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x,x)
```

output
```
x^3*((A*b)/3 + (B*a)/3) + (B*b*x^6)/6 + A*a*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx = \log(x)\,a^2+\frac{2ab\,x^3}{3}+\frac{b^2x^6}{6}$$

input
```
int((b*x^3+a)*(B*x^3+A)/x,x)
```

output
```
(6*log(x)*a**2 + 4*a*b*x**3 + b**2*x**6)/6
```

3.3 $\int \frac{(a+bx^3)(A+Bx^3)}{x^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4} dx = -\frac{aA}{3x^3} + \frac{1}{3}bBx^3 + (Ab+aB)\log(x)$$

output
```
-1/3*a*A/x^3+1/3*b*B*x^3+(A*b+B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4} dx = -\frac{aA}{3x^3} + \frac{1}{3}bBx^3 + (Ab+aB)\log(x)$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/x^4,x]
```

output
```
-1/3*(a*A)/x^3 + (b*B*x^3)/3 + (A*b + a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)(Bx^3+A)}{x^6}dx^3$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{3}\int \left(\frac{aA}{x^6}+bB+\frac{Ab+aB}{x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\log\left(x^3\right)(aB+Ab)-\frac{aA}{x^3}+bBx^3\right)$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/x^4,x]
```

output
```
(-((a*A)/x^3) + b*B*x^3 + (A*b + a*B)*Log[x^3])/3
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{aA}{3x^{3}}+\frac{bB\,x^{3}}{3}+\left(Ab+Ba\right)\ln\left(x\right)$	26
risch	$-\frac{aA}{3x^{3}}+\frac{bB\,x^{3}}{3}+A\ln\left(x\right)b+B\ln\left(x\right)a$	26
norman	$\frac{-\frac{Aa}{3}+\frac{bB\,x^{6}}{3}}{x^{3}}+\left(Ab+Ba\right)\ln\left(x\right)$	28
parallelrisch	$\frac{bB\,x^{6}+3A\ln\left(x\right)x^{3}b+3B\ln\left(x\right)x^{3}a-Aa}{3x^{3}}$	35

input
```
int((b*x^3+a)*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a*A/x^3+1/3*b*B*x^3+(A*b+B*a)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int\frac{(a+bx^3)\,(A+Bx^3)}{x^4}\,dx=\frac{Bbx^6+3\,(Ba+Ab)x^3\log{(x)}-Aa}{3\,x^3}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="fricas")
```

output
```
1/3*(B*b*x^6 + 3*(B*a + A*b)*x^3*log(x) - A*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx = -\frac{Aa}{3x^3}+\frac{Bbx^3}{3}+(Ab+Ba)\log(x)$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x**4,x)
```

output
```
-A*a/(3*x**3) + B*b*x**3/3 + (A*b + B*a)*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx = \frac{1}{3}\,Bbx^3+\frac{1}{3}\,(Ba+Ab)\log\left(x^3\right)-\frac{Aa}{3\,x^3}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="maxima")
```

output
```
1/3*B*b*x^3 + 1/3*(B*a + A*b)*log(x^3) - 1/3*A*a/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx = \frac{1}{3}\,Bbx^3+(Ba+Ab)\log(|x|)-\frac{Bax^3+Abx^3+Aa}{3\,x^3}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="giac")
```

output
```
1/3*B*b*x^3 + (B*a + A*b)*log(abs(x)) - 1/3*(B*a*x^3 + A*b*x^3 + A*a)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx = \ln(x)\ (A\,b+B\,a) - \frac{A\,a}{3\,x^3} + \frac{B\,b\,x^3}{3}$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x^4,x)
```

output
```
log(x)*(A*b + B*a) - (A*a)/(3*x^3) + (B*b*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx = \frac{6\log(x)\,ab\,x^3 - a^2 + b^2x^6}{3x^3}$$

input
```
int((b*x^3+a)*(B*x^3+A)/x^4,x)
```

output
```
(6*log(x)*a*b*x**3 - a**2 + b**2*x**6)/(3*x**3)
```

3.4 $\int \frac{(a+bx^3)(A+Bx^3)}{x^7} dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7} dx = -\frac{aA}{6x^6} - \frac{Ab+aB}{3x^3} + bB\log(x)$$

output
```
-1/6*a*A/x^6-1/3*(A*b+B*a)/x^3+b*B*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7} dx = -\frac{aA}{6x^6} + \frac{-Ab-aB}{3x^3} + bB\log(x)$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/x^7,x]
```

output
```
-1/6*(a*A)/x^6 + (-(A*b) - a*B)/(3*x^3) + b*B*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)(Bx^3+A)}{x^9}dx^3$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{3}\int \left(\frac{aA}{x^9}+\frac{bB}{x^3}+\frac{Ab+aB}{x^6}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{aB+Ab}{x^3}-\frac{aA}{2x^6}+bB\log\left(x^3\right)\right)$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/x^7,x]
```

output
```
(-1/2*(a*A)/x^6 - (A*b + a*B)/x^3 + b*B*Log[x^3])/3
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{aA}{6x^6}-\frac{Ab+Ba}{3x^3}+bB\ln\left(x\right)$	26
norman	$\frac{\left(-\frac{Ab}{3}-\frac{Ba}{3}\right)x^3-\frac{Aa}{6}}{x^6}+bB\ln\left(x\right)$	29
risch	$\frac{\left(-\frac{Ab}{3}-\frac{Ba}{3}\right)x^3-\frac{Aa}{6}}{x^6}+bB\ln\left(x\right)$	29
parallelrisch	$-\frac{-6Bb\ln(x)x^6+2Ab\,x^3+2Ba\,x^3+Aa}{6x^6}$	33

input
```
int((b*x^3+a)*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a*A/x^6-1/3*(A*b+B*a)/x^3+b*B*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int\frac{(a+bx^3)\left(A+Bx^3\right)}{x^7}\,dx=\frac{6\,Bbx^6\log\left(x\right)-2\,(Ba+Ab)x^3-Aa}{6\,x^6}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="fricas")
```

output
```
1/6*(6*B*b*x^6*log(x) - 2*(B*a + A*b)*x^3 - A*a)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = Bb\log(x) + \frac{-Aa + x^3(-2Ab - 2Ba)}{6x^6}$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x**7,x)
```

output
```
B*b*log(x) + (-A*a + x**3*(-2*A*b - 2*B*a))/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = \frac{1}{3}Bb\log(x^3) - \frac{2(Ba+Ab)x^3 + Aa}{6x^6}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="maxima")
```

output
```
1/3*B*b*log(x^3) - 1/6*(2*(B*a + A*b)*x^3 + A*a)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = Bb\log(|x|) - \frac{3Bbx^6 + 2Bax^3 + 2Abx^3 + Aa}{6x^6}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="giac")
```

output
```
B*b*log(abs(x)) - 1/6*(3*B*b*x^6 + 2*B*a*x^3 + 2*A*b*x^3 + A*a)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = B\,b\,\ln(x) - \frac{\left(\frac{A\,b}{3}+\frac{B\,a}{3}\right)\,x^3+\frac{A\,a}{6}}{x^6}$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x^7,x)
```

output
```
B*b*log(x) - ((A*a)/6 + x^3*((A*b)/3 + (B*a)/3))/x^6
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = \frac{6\log(x)\,b^2x^6-a^2-4ab\,x^3}{6x^6}$$

input
```
int((b*x^3+a)*(B*x^3+A)/x^7,x)
```

output
```
(6*log(x)*b**2*x**6 - a**2 - 4*a*b*x**3)/(6*x**6)
```

3.5 $\int x(a+bx^3)(A+Bx^3)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int x(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{2}aAx^2+\frac{1}{5}(Ab+aB)x^5+\frac{1}{8}bBx^8$$

output
```
1/2*a*A*x^2+1/5*(A*b+B*a)*x^5+1/8*b*B*x^8
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{2}aAx^2+\frac{1}{5}(Ab+aB)x^5+\frac{1}{8}bBx^8$$

input
```
Integrate[x*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(a*A*x^2)/2 + ((A*b + a*B)*x^5)/5 + (b*B*x^8)/8
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \; 950$$

$$\int \left(x^4(aB+Ab)+aAx+bBx^7\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{8}bBx^8$$

input
```
Int[x*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(a*A*x^2)/2 + ((A*b + a*B)*x^5)/5 + (b*B*x^8)/8
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{aA\,x^{2}}{2}+\frac{(Ab+Ba)x^{5}}{5}+\frac{bB\,x^{8}}{8}$	28
norman	$\frac{bB\,x^{8}}{8}+\left(\frac{Ab}{5}+\frac{Ba}{5}\right)x^{5}+\frac{aA\,x^{2}}{2}$	29
gosper	$\frac{1}{8}bB\,x^{8}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}Ba+\frac{1}{2}aA\,x^{2}$	30
risch	$\frac{1}{8}bB\,x^{8}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}Ba+\frac{1}{2}aA\,x^{2}$	30
parallelrisch	$\frac{1}{8}bB\,x^{8}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}Ba+\frac{1}{2}aA\,x^{2}$	30
orering	$\frac{x^{2}\left(5bB\,x^{6}+8Ab\,x^{3}+8Ba\,x^{3}+20Aa\right)}{40}$	32

input
```
int(x*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*A*x^2+1/5*(A*b+B*a)*x^5+1/8*b*B*x^8
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x\left(a+bx^{3}\right)\left(A+Bx^{3}\right)\,dx=\frac{1}{8}\,Bbx^{8}+\frac{1}{5}\,(Ba+Ab)x^{5}+\frac{1}{2}\,Aax^{2}$$

input
```
integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/8*B*b*x^8 + 1/5*(B*a + A*b)*x^5 + 1/2*A*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{Aax^2}{2}+\frac{Bbx^8}{8}+x^5\left(\frac{Ab}{5}+\frac{Ba}{5}\right)$$

input
```
integrate(x*(b*x**3+a)*(B*x**3+A),x)
```

output
```
A*a*x**2/2 + B*b*x**8/8 + x**5*(A*b/5 + B*a/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{8}\,Bbx^8+\frac{1}{5}\,(Ba+Ab)x^5+\frac{1}{2}\,Aax^2$$

input
```
integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/8*B*b*x^8 + 1/5*(B*a + A*b)*x^5 + 1/2*A*a*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{8}\,Bbx^8+\frac{1}{5}\,Bax^5+\frac{1}{5}\,Abx^5+\frac{1}{2}\,Aax^2$$

input
```
integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/8*B*b*x^8 + 1/5*B*a*x^5 + 1/5*A*b*x^5 + 1/2*A*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{B\,b\,x^8}{8}+\left(\frac{A\,b}{5}+\frac{B\,a}{5}\right)\,x^5+\frac{A\,a\,x^2}{2}$$

input
```
int(x*(A + B*x^3)*(a + b*x^3),x)
```

output
```
x^5*((A*b)/5 + (B*a)/5) + (A*a*x^2)/2 + (B*b*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int x(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{x^2(5b^2x^6+16ab\,x^3+20a^2)}{40}$$

input
```
int(x*(b*x^3+a)*(B*x^3+A),x)
```

output
```
(x**2*(20*a**2 + 16*a*b*x**3 + 5*b**2*x**6))/40
```

3.6 $\int (a + bx^3) (A + Bx^3) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int (a + bx^3) (A + Bx^3) \, dx = aAx + \frac{1}{4}(Ab + aB)x^4 + \frac{1}{7}bBx^7$$

output
```
a*A*x+1/4*(A*b+B*a)*x^4+1/7*b*B*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a + bx^3) (A + Bx^3) \, dx = aAx + \frac{1}{4}(Ab + aB)x^4 + \frac{1}{7}bBx^7$$

input
```
Integrate[(a + b*x^3)*(A + B*x^3),x]
```

output
```
a*A*x + ((A*b + a*B)*x^4)/4 + (b*B*x^7)/7
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^3)(A+Bx^3)\,dx$$

$$\Big\downarrow \text{897}$$

$$\int (x^3(aB+Ab)+aA+bBx^6)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(aB+Ab)+aAx+\frac{1}{7}bBx^7$$

input
```
Int[(a + b*x^3)*(A + B*x^3),x]
```

output
```
a*A*x + ((A*b + a*B)*x^4)/4 + (b*B*x^7)/7
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$aAx+\frac{(Ab+Ba)x^4}{4}+\frac{bB\,x^7}{7}$	25
norman	$\frac{bB\,x^7}{7}+\left(\frac{Ab}{4}+\frac{Ba}{4}\right)x^4+aAx$	26
gosper	$\frac{1}{7}bB\,x^7+\frac{1}{4}Ab\,x^4+\frac{1}{4}Ba\,x^4+aAx$	27
risch	$\frac{1}{7}bB\,x^7+\frac{1}{4}Ab\,x^4+\frac{1}{4}Ba\,x^4+aAx$	27
parallelrisch	$\frac{1}{7}bB\,x^7+\frac{1}{4}Ab\,x^4+\frac{1}{4}Ba\,x^4+aAx$	27
orering	$\frac{x\left(4bB\,x^6+7Ab\,x^3+7Ba\,x^3+28Aa\right)}{28}$	30

input
```
int((b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/4*(A*b+B*a)*x^4+1/7*b*B*x^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{1}{7}\,Bbx^7+\frac{1}{4}\,(Ba+Ab)x^4+Aax$$

input
```
integrate((b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/7*B*b*x^7 + 1/4*(B*a + A*b)*x^4 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx^3) (A+Bx^3) \, dx = Aax + \frac{Bbx^7}{7} + x^4\left(\frac{Ab}{4} + \frac{Ba}{4}\right)$$

input
```
integrate((b*x**3+a)*(B*x**3+A),x)
```

output
```
A*a*x + B*b*x**7/7 + x**4*(A*b/4 + B*a/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx^3) (A+Bx^3) \, dx = \frac{1}{7} Bbx^7 + \frac{1}{4} (Ba+Ab)x^4 + Aax$$

input
```
integrate((b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/7*B*b*x^7 + 1/4*(B*a + A*b)*x^4 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx^3) (A+Bx^3) \, dx = \frac{1}{7} Bbx^7 + \frac{1}{4} Bax^4 + \frac{1}{4} Abx^4 + Aax$$

input
```
integrate((b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/7*B*b*x^7 + 1/4*B*a*x^4 + 1/4*A*b*x^4 + A*a*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (a+bx^3) (A+Bx^3) \, dx = \frac{B\,b\,x^7}{7} + \left(\frac{A\,b}{4} + \frac{B\,a}{4}\right) x^4 + A\,a\,x$$

input
```
int((A + B*x^3)*(a + b*x^3),x)
```

output
```
x^4*((A*b)/4 + (B*a)/4) + A*a*x + (B*b*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx^3) (A+Bx^3) \, dx = \frac{x(2b^2x^6 + 7ab\,x^3 + 14a^2)}{14}$$

input
```
int((b*x^3+a)*(B*x^3+A),x)
```

output
```
(x*(14*a**2 + 7*a*b*x**3 + 2*b**2*x**6))/14
```

3.7 $\int \frac{(a+bx^3)(A+Bx^3)}{x^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2} \, dx = -\frac{aA}{x} + \frac{1}{2}(Ab+aB)x^2 + \frac{1}{5}bBx^5$$

output
```
-a*A/x+1/2*(A*b+B*a)*x^2+1/5*b*B*x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2} \, dx = -\frac{aA}{x} + \frac{1}{2}(Ab+aB)x^2 + \frac{1}{5}bBx^5$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/x^2,x]
```

output
```
-((a*A)/x) + ((A*b + a*B)*x^2)/2 + (b*B*x^5)/5
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2}\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x(aB+Ab)+\frac{aA}{x^2}+bBx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2(aB+Ab)-\frac{aA}{x}+\frac{1}{5}bBx^5$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/x^2,x]
```

output
```
-((a*A)/x) + ((A*b + a*B)*x^2)/2 + (b*B*x^5)/5
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{bB\,x^{5}}{5}+\frac{Ab\,x^{2}}{2}+\frac{Ba\,x^{2}}{2}-\frac{aA}{x}$	30
norman	$\frac{\frac{bB\,x^{6}}{5}+\left(\frac{Ab}{2}+\frac{Ba}{2}\right)x^{3}-Aa}{x}$	30
risch	$\frac{bB\,x^{5}}{5}+\frac{Ab\,x^{2}}{2}+\frac{Ba\,x^{2}}{2}-\frac{aA}{x}$	30
gosper	$-\frac{-2bB\,x^{6}-5Ab\,x^{3}-5Ba\,x^{3}+10Aa}{10x}$	32
parallelrisch	$\frac{2bB\,x^{6}+5Ab\,x^{3}+5Ba\,x^{3}-10Aa}{10x}$	32
orering	$-\frac{-2bB\,x^{6}-5Ab\,x^{3}-5Ba\,x^{3}+10Aa}{10x}$	32

input
```
int((b*x^3+a)*(B*x^3+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*b*B*x^5+1/2*A*b*x^2+1/2*B*a*x^2-a*A/x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)\,(A+Bx^3)}{x^2}\,dx=\frac{2\,Bbx^6+5\,(Ba+Ab)x^3-10\,Aa}{10\,x}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="fricas")
```

output
```
1/10*(2*B*b*x^6 + 5*(B*a + A*b)*x^3 - 10*A*a)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2}\,dx = -\frac{Aa}{x} + \frac{Bbx^5}{5} + x^2\left(\frac{Ab}{2} + \frac{Ba}{2}\right)$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x**2,x)
```

output
```
-A*a/x + B*b*x**5/5 + x**2*(A*b/2 + B*a/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2}\,dx = \frac{1}{5}\,Bbx^5 + \frac{1}{2}\,(Ba+Ab)x^2 - \frac{Aa}{x}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="maxima")
```

output
```
1/5*B*b*x^5 + 1/2*(B*a + A*b)*x^2 - A*a/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2}\,dx = \frac{1}{5}\,Bbx^5 + \frac{1}{2}\,Bax^2 + \frac{1}{2}\,Abx^2 - \frac{Aa}{x}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="giac")
```

output
```
1/5*B*b*x^5 + 1/2*B*a*x^2 + 1/2*A*b*x^2 - A*a/x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2}\,dx = x^2\left(\frac{A\,b}{2}+\frac{B\,a}{2}\right)-\frac{A\,a}{x}+\frac{B\,b\,x^5}{5}$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x^2,x)
```

output
```
x^2*((A*b)/2 + (B*a)/2) - (A*a)/x + (B*b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2}\,dx = \frac{b^2x^6+5ab\,x^3-5a^2}{5x}$$

input
```
int((b*x^3+a)*(B*x^3+A)/x^2,x)
```

output
```
( - 5*a**2 + 5*a*b*x**3 + b**2*x**6)/(5*x)
```

3.8 $\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = -\frac{aA}{2x^2}+(Ab+aB)x+\frac{1}{4}bBx^4$$

output
```
-1/2*a*A/x^2+(A*b+B*a)*x+1/4*b*B*x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = -\frac{aA}{2x^2}+(Ab+aB)x+\frac{1}{4}bBx^4$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/x^3,x]
```

output
```
-1/2*(a*A)/x^2 + (A*b + a*B)*x + (b*B*x^4)/4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3} dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(Ab\left(\frac{aB}{Ab}+1\right)+\frac{aA}{x^3}+bBx^3\right) dx$$

$$\Big\downarrow \text{2009}$$

$$x(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{4}bBx^4$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/x^3,x]
```

output
```
-1/2*(a*A)/x^2 + (A*b + a*B)*x + (b*B*x^4)/4
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{bB\,x^{4}}{4}+Abx+Bax-\frac{aA}{2x^{2}}$	24
risch	$\frac{bB\,x^{4}}{4}+Abx+Bax-\frac{aA}{2x^{2}}$	24
norman	$\frac{\frac{bB\,x^{6}}{4}+(Ab+Ba)x^{3}-\frac{Aa}{2}}{x^{2}}$	28
parallelrisch	$\frac{bB\,x^{6}+4Ab\,x^{3}+4Ba\,x^{3}-2Aa}{4x^{2}}$	31
gosper	$-\frac{-bB\,x^{6}-4Ab\,x^{3}-4Ba\,x^{3}+2Aa}{4x^{2}}$	32
orering	$-\frac{-bB\,x^{6}-4Ab\,x^{3}-4Ba\,x^{3}+2Aa}{4x^{2}}$	32

input
```
int((b*x^3+a)*(B*x^3+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*b*B*x^4+A*b*x+B*a*x-1/2*a*A/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)\,(A+Bx^3)}{x^3}\,dx=\frac{Bbx^6+4\,(Ba+Ab)x^3-2\,Aa}{4\,x^2}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="fricas")
```

output
```
1/4*(B*b*x^6 + 4*(B*a + A*b)*x^3 - 2*A*a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = -\frac{Aa}{2x^2} + \frac{Bbx^4}{4} + x(Ab+Ba)$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x**3,x)
```

output
```
-A*a/(2*x**2) + B*b*x**4/4 + x*(A*b + B*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = \frac{1}{4}\,Bbx^4 + (Ba+Ab)x - \frac{Aa}{2\,x^2}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="maxima")
```

output
```
1/4*B*b*x^4 + (B*a + A*b)*x - 1/2*A*a/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = \frac{1}{4}\,Bbx^4 + Bax + Abx - \frac{Aa}{2\,x^2}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="giac")
```

output
```
1/4*B*b*x^4 + B*a*x + A*b*x - 1/2*A*a/x^2
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = x\,(A\,b+B\,a)-\frac{A\,a}{2\,x^2}+\frac{B\,b\,x^4}{4}$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x^3,x)
```

output
```
x*(A*b + B*a) - (A*a)/(2*x^2) + (B*b*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = \frac{b^2x^6+8ab\,x^3-2a^2}{4x^2}$$

input
```
int((b*x^3+a)*(B*x^3+A)/x^3,x)
```

output
```
( - 2*a**2 + 8*a*b*x**3 + b**2*x**6)/(4*x**2)
```

3.9 $\int \frac{(a+bx^3)(A+Bx^3)}{x^5} dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5} dx = -\frac{aA}{4x^4} - \frac{Ab+aB}{x} + \frac{1}{2}bBx^2$$

output
```
-1/4*a*A/x^4-(A*b+B*a)/x+1/2*b*B*x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5} dx = -\frac{aA}{4x^4} + \frac{-Ab-aB}{x} + \frac{1}{2}bBx^2$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/x^5,x]
```

output
```
-1/4*(a*A)/x^4 + (-(A*b) - a*B)/x + (b*B*x^2)/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5}dx$$

$$\Big\downarrow \text{ 950}$$

$$\int \left(\frac{aB+Ab}{x^2}+\frac{aA}{x^5}+bBx\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{aB+Ab}{x}-\frac{aA}{4x^4}+\frac{1}{2}bBx^2$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/x^5,x]
```

output
```
-1/4*(a*A)/x^4 - (A*b + a*B)/x + (b*B*x^2)/2
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{aA}{4x^4}-\frac{Ab+Ba}{x}+\frac{bB\,x^2}{2}$	28
norman	$\frac{\frac{bB\,x^6}{2}+(-Ab-Ba)x^3-\frac{Aa}{4}}{x^4}$	30
gosper	$-\frac{-2bB\,x^6+4Ab\,x^3+4Ba\,x^3+Aa}{4x^4}$	31
risch	$\frac{bB\,x^2}{2}+\frac{(-Ab-Ba)x^3-\frac{Aa}{4}}{x^4}$	31
parallelrisch	$-\frac{-2bB\,x^6+4Ab\,x^3+4Ba\,x^3+Aa}{4x^4}$	31
orering	$-\frac{-2bB\,x^6+4Ab\,x^3+4Ba\,x^3+Aa}{4x^4}$	31

input
```
int((b*x^3+a)*(B*x^3+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a*A/x^4-(A*b+B*a)/x+1/2*b*B*x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)\,(A+Bx^3)}{x^5}\,dx = \frac{2\,Bbx^6-4\,(Ba+Ab)x^3-Aa}{4\,x^4}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="fricas")
```

output
```
1/4*(2*B*b*x^6 - 4*(B*a + A*b)*x^3 - A*a)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx = \frac{Bbx^2}{2} + \frac{-Aa+x^3(-4Ab-4Ba)}{4x^4}$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x**5,x)
```

output
```
B*b*x**2/2 + (-A*a + x**3*(-4*A*b - 4*B*a))/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx = \frac{1}{2}\,Bbx^2 - \frac{4\,(Ba+Ab)x^3+Aa}{4\,x^4}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="maxima")
```

output
```
1/2*B*b*x^2 - 1/4*(4*(B*a + A*b)*x^3 + A*a)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx = \frac{1}{2}\,Bbx^2 - \frac{4\,Bax^3+4\,Abx^3+Aa}{4\,x^4}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="giac")
```

output
```
1/2*B*b*x^2 - 1/4*(4*B*a*x^3 + 4*A*b*x^3 + A*a)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx = \frac{B\,b\,x^2}{2} - \frac{(A\,b+B\,a)\;x^3 + \frac{A\,a}{4}}{x^4}$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x^5,x)
```

output
```
(B*b*x^2)/2 - ((A*a)/4 + x^3*(A*b + B*a))/x^4
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx = \frac{2b^2x^6 - 8ab\,x^3 - a^2}{4x^4}$$

input
```
int((b*x^3+a)*(B*x^3+A)/x^5,x)
```

output
```
( - a**2 - 8*a*b*x**3 + 2*b**2*x**6)/(4*x**4)
```

3.10 $\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = -\frac{aA}{5x^5} - \frac{Ab+aB}{2x^2} + bBx$$

output
```
-1/5*a*A/x^5-1/2*(A*b+B*a)/x^2+b*B*x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = -\frac{aA}{5x^5} + \frac{-Ab-aB}{2x^2} + bBx$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/x^6,x]
```

output
```
-1/5*(a*A)/x^5 + (-(A*b) - a*B)/(2*x^2) + b*B*x
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx$$

$$\Big\downarrow \text{ 950}$$

$$\int \left(\frac{aB+Ab}{x^3}+\frac{aA}{x^6}+bB\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{aB+Ab}{2x^2}-\frac{aA}{5x^5}+bBx$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/x^6,x]
```

output
```
-1/5*(a*A)/x^5 - (A*b + a*B)/(2*x^2) + b*B*x
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{aA}{5x^5}-\frac{Ab+Ba}{2x^2}+bBx$	25
risch	$bBx+\frac{\left(-\frac{Ab}{2}-\frac{Ba}{2}\right)x^3-\frac{Aa}{5}}{x^5}$	28
norman	$\frac{bB\,x^6+\left(-\frac{Ab}{2}-\frac{Ba}{2}\right)x^3-\frac{Aa}{5}}{x^5}$	29
gosper	$-\frac{-10bB\,x^6+5Ab\,x^3+5Ba\,x^3+2Aa}{10x^5}$	32
parallelrisch	$-\frac{-10bB\,x^6+5Ab\,x^3+5Ba\,x^3+2Aa}{10x^5}$	32
orering	$-\frac{-10bB\,x^6+5Ab\,x^3+5Ba\,x^3+2Aa}{10x^5}$	32

input
```
int((b*x^3+a)*(B*x^3+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a*A/x^5-1/2*(A*b+B*a)/x^2+b*B*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = \frac{10\,Bbx^6-5\,(Ba+Ab)x^3-2\,Aa}{10\,x^5}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="fricas")
```

output
```
1/10*(10*B*b*x^6 - 5*(B*a + A*b)*x^3 - 2*A*a)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = Bbx + \frac{-2Aa + x^3(-5Ab - 5Ba)}{10x^5}$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x**6,x)
```

output
```
B*b*x + (-2*A*a + x**3*(-5*A*b - 5*B*a))/(10*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = Bbx - \frac{5\,(Ba+Ab)x^3 + 2\,Aa}{10\,x^5}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="maxima")
```

output
```
B*b*x - 1/10*(5*(B*a + A*b)*x^3 + 2*A*a)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = Bbx - \frac{5\,Bax^3 + 5\,Abx^3 + 2\,Aa}{10\,x^5}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="giac")
```

output
```
B*b*x - 1/10*(5*B*a*x^3 + 5*A*b*x^3 + 2*A*a)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = B\,b\,x - \frac{\left(\frac{A\,b}{2}+\frac{B\,a}{2}\right)\,x^3+\frac{A\,a}{5}}{x^5}$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x^6,x)
```

output
```
B*b*x - ((A*a)/5 + x^3*((A*b)/2 + (B*a)/2))/x^5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = \frac{5b^2x^6-5ab\,x^3-a^2}{5x^5}$$

input
```
int((b*x^3+a)*(B*x^3+A)/x^6,x)
```

output
```
( - a**2 - 5*a*b*x**3 + 5*b**2*x**6)/(5*x**5)
```

3.11 $\int x^2(a+bx^3)^2 (A+Bx^3)\, dx$

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int x^2\left(a+bx^3\right)^2\left(A+Bx^3\right)\, dx=\frac{(Ab-aB)\left(a+bx^3\right)^3}{9b^2}+\frac{B(a+bx^3)^4}{12b^2}$$

output
```
1/9*(A*b-B*a)*(b*x^3+a)^3/b^2+1/12*B*(b*x^3+a)^4/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x^2\left(a+bx^3\right)^2\left(A+Bx^3\right)\, dx=\frac{1}{36}x^3\left(12a^2A+6a(2Ab+aB)x^3+4b(Ab+2aB)x^6+3b^2Bx^9\right)$$

input
```
Integrate[x^2*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(x^3*(12*a^2*A + 6*a*(2*A*b + a*B)*x^3 + 4*b*(A*b + 2*a*B)*x^6 + 3*b^2*B*x
^9))/36
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {946, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \ 946$$

$$\frac{1}{3}\int \left(bx^3+a\right)^2 \left(Bx^3+A\right)dx^3$$

$$\Big\downarrow \ 49$$

$$\frac{1}{3}\int \left(\frac{B\left(bx^3+a\right)^3}{b}+\frac{(Ab-aB)\left(bx^3+a\right)^2}{b}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3\right)^3(Ab-aB)}{3b^2}+\frac{B\left(a+bx^3\right)^4}{4b^2}\right)$$

input
```
Int[x^2*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(((A*b - a*B)*(a + b*x^3)^3)/(3*b^2) + (B*(a + b*x^3)^4)/(4*b^2))/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{b^2 B x^{12}}{12}+\frac{(b^2 A+2abB)x^9}{9}+\frac{(2abA+a^2 B)x^6}{6}+\frac{a^2 A x^3}{3}$	52
norman	$\frac{b^2 B x^{12}}{12}+\left(\frac{1}{9}b^2 A+\frac{2}{9}abB\right)x^9+\left(\frac{1}{3}abA+\frac{1}{6}a^2 B\right)x^6+\frac{a^2 A x^3}{3}$	52
gosper	$\frac{1}{12}b^2 B x^{12}+\frac{1}{9}x^9 b^2 A+\frac{2}{9}x^9 abB+\frac{1}{3}x^6 abA+\frac{1}{6}x^6 a^2 B+\frac{1}{3}a^2 A x^3$	54
risch	$\frac{1}{12}b^2 B x^{12}+\frac{1}{9}x^9 b^2 A+\frac{2}{9}x^9 abB+\frac{1}{3}x^6 abA+\frac{1}{6}x^6 a^2 B+\frac{1}{3}a^2 A x^3$	54
parallelrisch	$\frac{1}{12}b^2 B x^{12}+\frac{1}{9}x^9 b^2 A+\frac{2}{9}x^9 abB+\frac{1}{3}x^6 abA+\frac{1}{6}x^6 a^2 B+\frac{1}{3}a^2 A x^3$	54
orering	$\frac{x^3(3b^2 B x^9+4A b^2 x^6+8Bab x^6+12aAb x^3+6B a^2 x^3+12a^2 A)}{36}$	56

input
```
int(x^2*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/12*b^2*B*x^12+1/9*(A*b^2+2*B*a*b)*x^9+1/6*(2*A*a*b+B*a^2)*x^6+1/3*a^2*A*
x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x^2(a+bx^3)^2(A+Bx^3)\,dx=\frac{1}{12}Bb^2x^{12}+\frac{1}{9}(2Bab+Ab^2)x^9+\frac{1}{6}(Ba^2+2Aab)x^6+\frac{1}{3}Aa^2x^3$$

input
```
integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/12*B*b^2*x^12 + 1/9*(2*B*a*b + A*b^2)*x^9 + 1/6*(B*a^2 + 2*A*a*b)*x^6 +
1/3*A*a^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

$$\int x^2(a+bx^3)^2\,(A+Bx^3)\,\,dx=\frac{Aa^2x^3}{3}+\frac{Bb^2x^{12}}{12}+x^9\left(\frac{Ab^2}{9}+\frac{2Bab}{9}\right)+x^6\left(\frac{Aab}{3}+\frac{Ba^2}{6}\right)$$

input
```
integrate(x**2*(b*x**3+a)**2*(B*x**3+A),x)
```

output
```
A*a**2*x**3/3 + B*b**2*x**12/12 + x**9*(A*b**2/9 + 2*B*a*b/9) + x**6*(A*a*
b/3 + B*a**2/6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x^2(a+bx^3)^2\,(A+Bx^3)\,\,dx=\frac{1}{12}\,Bb^2x^{12}+\frac{1}{9}\,(2\,Bab+Ab^2)x^9+\frac{1}{6}\,(Ba^2+2\,Aab)x^6+\frac{1}{3}\,Aa^2x^3$$

input
```
integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/12*B*b^2*x^12 + 1/9*(2*B*a*b + A*b^2)*x^9 + 1/6*(B*a^2 + 2*A*a*b)*x^6 +
1/3*A*a^2*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int x^2 (a + bx^3)^2 (A + Bx^3) \, dx = \frac{1}{12} Bb^2x^{12} + \frac{2}{9} Babx^9 + \frac{1}{9} Ab^2x^9 + \frac{1}{6} Ba^2x^6 + \frac{1}{3} Aabx^6 + \frac{1}{3} Aa^2x^3$$

input
```
integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

output
```
1/12*B*b^2*x^12 + 2/9*B*a*b*x^9 + 1/9*A*b^2*x^9 + 1/6*B*a^2*x^6 + 1/3*A*a*
b*x^6 + 1/3*A*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x^2 (a + bx^3)^2 (A + Bx^3) \, dx = x^6 \left(\frac{B a^2}{6} + \frac{A b a}{3} \right) + x^9 \left(\frac{A b^2}{9} + \frac{2 B a b}{9} \right) + \frac{A a^2 x^3}{3} + \frac{B b^2 x^{12}}{12}$$

input
```
int(x^2*(A + B*x^3)*(a + b*x^3)^2,x)
```

output
```
x^6*((B*a^2)/6 + (A*a*b)/3) + x^9*((A*b^2)/9 + (2*B*a*b)/9) + (A*a^2*x^3)/
3 + (B*b^2*x^12)/12
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int x^2 \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx = \frac{x^3(b^3x^9+4a\,b^2x^6+6a^2b\,x^3+4a^3)}{12}$$

input
```
int(x^2*(b*x^3+a)^2*(B*x^3+A),x)
```

output
```
(x**3*(4*a**3 + 6*a**2*b*x**3 + 4*a*b**2*x**6 + b**3*x**9))/12
```

3.12 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \frac{(a+bx^3)^2\,(A+Bx^3)}{x}\,dx=\frac{2}{3}aAbx^3+\frac{1}{6}Ab^2x^6+\frac{B(a+bx^3)^3}{9b}+a^2A\log(x)$$

output
```
2/3*a*A*b*x^3+1/6*A*b^2*x^6+1/9*B*(b*x^3+a)^3/b+a^2*A*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^2\,(A+Bx^3)}{x}\,dx=\frac{1}{3}a(2Ab+aB)x^3+\frac{1}{6}b(Ab+2aB)x^6+\frac{1}{9}b^2Bx^9+a^2A\log(x)$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x,x]
```

output
```
(a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^6)/6 + (b^2*B*x^9)/9 + a^2*A*
Log[x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {948, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^2 (Bx^3+A)}{x^3}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(A\int \frac{(bx^3+a)^2}{x^3}dx^3+\frac{B(a+bx^3)^3}{3b}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\left(A\int \left(b^2x^3+2ab+\frac{a^2}{x^3}\right)dx^3+\frac{B(a+bx^3)^3}{3b}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(A\left(a^2\log\left(x^3\right)+2abx^3+\frac{b^2x^6}{2}\right)+\frac{B(a+bx^3)^3}{3b}\right)$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x,x]
```

output
```
((B*(a + b*x^3)^3)/(3*b) + A*(2*a*b*x^3 + (b^2*x^6)/2 + a^2*Log[x^3]))/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

method	result	size
norman	$\left(\frac{1}{6}b^2A+\frac{1}{3}abB\right)x^6+\left(\frac{2}{3}abA+\frac{1}{3}a^2B\right)x^3+\frac{b^2B\,x^9}{9}+a^2A\ln\left(x\right)$	50
default	$\frac{b^2B\,x^9}{9}+\frac{A\,b^2x^6}{6}+\frac{Bab\,x^6}{3}+\frac{2aAb\,x^3}{3}+\frac{B\,a^2x^3}{3}+a^2A\ln\left(x\right)$	52
risch	$\frac{b^2B\,x^9}{9}+\frac{A\,b^2x^6}{6}+\frac{Bab\,x^6}{3}+\frac{2aAb\,x^3}{3}+\frac{B\,a^2x^3}{3}+a^2A\ln\left(x\right)$	52
parallelrisch	$\frac{b^2B\,x^9}{9}+\frac{A\,b^2x^6}{6}+\frac{Bab\,x^6}{3}+\frac{2aAb\,x^3}{3}+\frac{B\,a^2x^3}{3}+a^2A\ln\left(x\right)$	52

input
```
int((b*x^3+a)^2*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/6*b^2*A+1/3*a*b*B)*x^6+(2/3*a*b*A+1/3*a^2*B)*x^3+1/9*b^2*B*x^9+a^2*A*ln
(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x}\,dx = \frac{1}{9}\,Bb^2x^9 + \frac{1}{6}\left(2\,Bab + Ab^2\right)x^6 + \frac{1}{3}\left(Ba^2 + 2\,Aab\right)x^3 + Aa^2\log\left(x\right)$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="fricas")
```

output
```
1/9*B*b^2*x^9 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/3*(B*a^2 + 2*A*a*b)*x^3 + A*
a^2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x}\,dx = Aa^2\log\left(x\right) + \frac{Bb^2x^9}{9} + x^6\left(\frac{Ab^2}{6} + \frac{Bab}{3}\right) + x^3\cdot\left(\frac{2Aab}{3} + \frac{Ba^2}{3}\right)$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x,x)
```

output
```
A*a**2*log(x) + B*b**2*x**9/9 + x**6*(A*b**2/6 + B*a*b/3) + x**3*(2*A*a*b/
3 + B*a**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x}\,dx = \frac{1}{9}Bb^2x^9 + \frac{1}{6}\left(2\,Bab + Ab^2\right)x^6 + \frac{1}{3}\left(Ba^2 + 2\,Aab\right)x^3 + \frac{1}{3}Aa^2\log\left(x^3\right)$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="maxima")
```

output
```
1/9*B*b^2*x^9 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/3*(B*a^2 + 2*A*a*b)*x^3 + 1/
3*A*a^2*log(x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x}\,dx = \frac{1}{9}Bb^2x^9 + \frac{1}{3}Babx^6 + \frac{1}{6}Ab^2x^6 + \frac{1}{3}Ba^2x^3 + \frac{2}{3}Aabx^3 + Aa^2\log\left(|x|\right)$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="giac")
```

output
```
1/9*B*b^2*x^9 + 1/3*B*a*b*x^6 + 1/6*A*b^2*x^6 + 1/3*B*a^2*x^3 + 2/3*A*a*b*
x^3 + A*a^2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x} \, dx = x^3 \left(\frac{B\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+x^6 \left(\frac{A\,b^2}{6}+\frac{B\,a\,b}{3}\right) + \frac{B\,b^2\,x^9}{9}+A\,a^2\,\ln(x)$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x,x)
```

output
```
x^3*((B*a^2)/3 + (2*A*a*b)/3) + x^6*((A*b^2)/6 + (B*a*b)/3) + (B*b^2*x^9)/
9 + A*a^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x} \, dx = \log(x)\,a^3+a^2b\,x^3+\frac{a\,b^2x^6}{2}+\frac{b^3x^9}{9}$$

input
```
int((b*x^3+a)^2*(B*x^3+A)/x,x)
```

output
```
(18*log(x)*a**3 + 18*a**2*b*x**3 + 9*a*b**2*x**6 + 2*b**3*x**9)/18
```

3.13 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^4}\,dx = -\frac{a^2A}{3x^3}+\frac{1}{3}b(Ab+2aB)x^3+\frac{1}{6}b^2Bx^6+a(2Ab+aB)\log(x)$$

output
```
-1/3*a^2*A/x^3+1/3*b*(A*b+2*B*a)*x^3+1/6*b^2*B*x^6+a*(2*A*b+B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^4}\,dx = \frac{1}{6}\left(-\frac{2a^2A}{x^3}+2b(Ab+2aB)x^3+b^2Bx^6 + 6a(2Ab+aB)\log(x)\right)$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^4,x]
```

output
```
((-2*a^2*A)/x^3 + 2*b*(A*b + 2*a*B)*x^3 + b^2*B*x^6 + 6*a*(2*A*b + a*B)*Lo
g[x])/6
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^4}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^2(Bx^3+A)}{x^6}dx^3$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{3}\int \left(b^2Bx^3+b(Ab+2aB)+\frac{a(2Ab+aB)}{x^3}+\frac{a^2A}{x^6}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^2A}{x^3}+bx^3(2aB+Ab)+a\log\left(x^3\right)(aB+2Ab)+\frac{1}{2}b^2Bx^6\right)$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x^4,x]
```

output
```
(-((a^2*A)/x^3) + b*(A*b + 2*a*B)*x^3 + (b^2*B*x^6)/2 + a*(2*A*b + a*B)*Lo
g[x^3])/3
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{b^2 B\,x^6}{6}+\frac{A\,b^2 x^3}{3}+\frac{2 B a b\,x^3}{3}-\frac{a^2 A}{3 x^3}+a(2 A b+B a)\ln(x)$	49
norman	$\frac{\left(\frac{1}{3} b^2 A+\frac{2}{3} a b B\right) x^6-\frac{a^2 A}{3}+\frac{b^2 B\,x^9}{6}}{x^3}+\left(2 a b A+a^2 B\right)\ln(x)$	52
parallelrisch	$\frac{b^2 B\,x^9+2 A\,b^2 x^6+4 B a b\,x^6+12 A\ln(x) x^3 a b+6 B\ln(x) x^3 a^2-2 a^2 A}{6 x^3}$	59
risch	$\frac{b^2 B\,x^6}{6}+\frac{A\,b^2 x^3}{3}+\frac{2 B a b\,x^3}{3}+\frac{A^2 b^2}{6 B}+\frac{2 a b A}{3}+\frac{2 a^2 B}{3}-\frac{a^2 A}{3 x^3}+2 A\ln(x) a b+B\ln(x) a^2$	73

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^2*B*x^6+1/3*A*b^2*x^3+2/3*B*a*b*x^3-1/3*a^2*A/x^3+a*(2*A*b+B*a)*ln(x
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int\frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^4}\,dx$$
$$=\frac{Bb^2x^9+2\,(2\,Bab+Ab^2)x^6+6\,(Ba^2+2\,Aab)x^3\log(x)-2\,Aa^2}{6\,x^3}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="fricas")
```

output
```
1/6*(B*b^2*x^9 + 2*(2*B*a*b + A*b^2)*x^6 + 6*(B*a^2 + 2*A*a*b)*x^3*log(x)
- 2*A*a^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^4} \, dx = -\frac{Aa^2}{3x^3} + \frac{Bb^2x^6}{6} + a(2Ab+Ba)\log(x) + x^3\left(\frac{Ab^2}{3} + \frac{2Bab}{3}\right)$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**4,x)
```

output
```
-A*a**2/(3*x**3) + B*b**2*x**6/6 + a*(2*A*b + B*a)*log(x) + x**3*(A*b**2/3
 + 2*B*a*b/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^4} \, dx = \frac{1}{6}\,Bb^2x^6 + \frac{1}{3}\,(2\,Bab + Ab^2)x^3 + \frac{1}{3}\,(Ba^2 + 2\,Aab)\log\left(x^3\right) - \frac{Aa^2}{3\,x^3}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="maxima")
```

output
```
1/6*B*b^2*x^6 + 1/3*(2*B*a*b + A*b^2)*x^3 + 1/3*(B*a^2 + 2*A*a*b)*log(x^3)
 - 1/3*A*a^2/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^4} \, dx = \frac{1}{6} Bb^2x^6 + \frac{2}{3} Babx^3 + \frac{1}{3} Ab^2x^3 + (Ba^2 + 2 Aab) \log(|x|) - \frac{Ba^2x^3 + 2 Aabx^3 + Aa^2}{3 x^3}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="giac")
```

output
```
1/6*B*b^2*x^6 + 2/3*B*a*b*x^3 + 1/3*A*b^2*x^3 + (B*a^2 + 2*A*a*b)*log(abs(
x)) - 1/3*(B*a^2*x^3 + 2*A*a*b*x^3 + A*a^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^4} \, dx = x^3 \left(\frac{A b^2}{3} + \frac{2 B a b}{3}\right) + \ln(x) \left(B a^2 + 2 A b a\right) - \frac{A a^2}{3 x^3} + \frac{B b^2 x^6}{6}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^4,x)
```

output
```
x^3*((A*b^2)/3 + (2*B*a*b)/3) + log(x)*(B*a^2 + 2*A*a*b) - (A*a^2)/(3*x^3)
 + (B*b^2*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^4} \, dx = \frac{18\log(x)\, a^2 b\, x^3 - 2a^3 + 6a\, b^2 x^6 + b^3 x^9}{6x^3}$$

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^4,x)
```

output
```
(18*log(x)*a**2*b*x**3 - 2*a**3 + 6*a*b**2*x**6 + b**3*x**9)/(6*x**3)
```

3.14 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7}\,dx = -\frac{a^2A}{6x^6} - \frac{a(2Ab+aB)}{3x^3} + \frac{1}{3}b^2Bx^3 + b(Ab+2aB)\log(x)$$

output
```
-1/6*a^2*A/x^6-1/3*a*(2*A*b+B*a)/x^3+1/3*b^2*B*x^3+b*(A*b+2*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7}\,dx = \frac{1}{6}\left(-\frac{4aAb}{x^3} + 2b^2Bx^3 - \frac{a^2(A+2Bx^3)}{x^6} + 6b(Ab+2aB)\log(x)\right)$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^7,x]
```

output
```
((-4*a*A*b)/x^3 + 2*b^2*B*x^3 - (a^2*(A + 2*B*x^3))/x^6 + 6*b*(A*b + 2*a*B
)*Log[x])/6
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^7} \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^2 (Bx^3+A)}{x^9}dx^3$$

$$\Big\downarrow \text{ 85}$$

$$\frac{1}{3}\int \left(\frac{Aa^2}{x^9}+\frac{(2Ab+aB)a}{x^6}+b^2B+\frac{b(Ab+2aB)}{x^3}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{a^2A}{2x^6}-\frac{a(aB+2Ab)}{x^3}+b\log\left(x^3\right)(2aB+Ab)+b^2Bx^3\right)$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x^7,x]
```

output
```
(-1/2*(a^2*A)/x^6 - (a*(2*A*b + a*B))/x^3 + b^2*B*x^3 + b*(A*b + 2*a*B)*Lo
g[x^3])/3
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^{2} A}{6 x^{6}}-\frac{a \left(2 A b +B a \right)}{3 x^{3}}+\frac{B \,b^{2} x^{3}}{3}+b \left(A b +2 B a \right) \ln \left(x \right)$	46
norman	$\frac{\left(-\frac{2}{3} a b A -\frac{1}{3} a^{2} B \right) x^{3}-\frac{a^{2} A}{6}+\frac{b^{2} B \,x^{9}}{3}}{x^{6}}+\left(b^{2} A +2 a b B \right) \ln \left(x \right)$	52
risch	$\frac{B \,b^{2} x^{3}}{3}+\frac{\left(-\frac{2}{3} a b A -\frac{1}{3} a^{2} B \right) x^{3}-\frac{a^{2} A}{6}}{x^{6}}+A \ln \left(x \right) b^{2}+2 B \ln \left(x \right) a b$	52
parallelrisch	$\frac{2 b^{2} B \,x^{9}+6 A \ln \left(x \right) x^{6} b^{2}+12 B \ln \left(x \right) x^{6} a b -4 a A b \,x^{3}-2 B \,a^{2} x^{3}-a^{2} A}{6 x^{6}}$	60

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^2*A/x^6-1/3*a*(2*A*b+B*a)/x^3+1/3*B*b^2*x^3+b*(A*b+2*B*a)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^7}\,dx = \frac{2\,Bb^2x^9 + 6\,(2\,Bab + Ab^2)x^6\log(x) - 2\,(Ba^2 + 2\,Aab)x^3 - Aa^2}{6\,x^6}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="fricas")
```

output
```
1/6*(2*B*b^2*x^9 + 6*(2*B*a*b + A*b^2)*x^6*log(x) - 2*(B*a^2 + 2*A*a*b)*x^
3 - A*a^2)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^7}\,dx = \frac{Bb^2x^3}{3} + b(Ab+2Ba)\log(x) + \frac{-Aa^2 + x^3(-4Aab-2Ba^2)}{6x^6}$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**7,x)
```

output
```
B*b**2*x**3/3 + b*(A*b + 2*B*a)*log(x) + (-A*a**2 + x**3*(-4*A*a*b - 2*B*a
**2))/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^7}\,dx = \frac{1}{3}\,Bb^2x^3 + \frac{1}{3}\left(2\,Bab + Ab^2\right)\log\left(x^3\right) - \frac{2\,(Ba^2 + 2\,Aab)x^3 + Aa^2}{6\,x^6}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="maxima")
```

output
```
1/3*B*b^2*x^3 + 1/3*(2*B*a*b + A*b^2)*log(x^3) - 1/6*(2*(B*a^2 + 2*A*a*b)*
x^3 + A*a^2)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^7} \, dx = \frac{1}{3} Bb^2x^3 + \left(2 Bab + Ab^2\right) \log\left(|x|\right) - \frac{6 Babx^6 + 3 Ab^2x^6 + 2 Ba^2x^3 + 4 Aabx^3 + Aa^2}{6 x^6}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="giac")
```

output
```
1/3*B*b^2*x^3 + (2*B*a*b + A*b^2)*log(abs(x)) - 1/6*(6*B*a*b*x^6 + 3*A*b^2
*x^6 + 2*B*a^2*x^3 + 4*A*a*b*x^3 + A*a^2)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^7} \, dx = \ln(x) \left(A\, b^2 + 2\, B\, a\, b\right) - \frac{x^3 \left(\frac{B\, a^2}{3} + \frac{2\, A\, b\, a}{3}\right) + \frac{A\, a^2}{6}}{x^6} + \frac{B\, b^2\, x^3}{3}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^7,x)
```

output
```
log(x)*(A*b^2 + 2*B*a*b) - (x^3*((B*a^2)/3 + (2*A*a*b)/3) + (A*a^2)/6)/x^6
 + (B*b^2*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^7} \, dx = \frac{18 \log(x)\, a\, b^2x^6 - a^3 - 6a^2b\, x^3 + 2b^3x^9}{6x^6}$$

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^7,x)
```

output
```
(18*log(x)*a*b**2*x**6 - a**3 - 6*a**2*b*x**3 + 2*b**3*x**9)/(6*x**6)
```

3.15 $\int x(a+bx^3)^2\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 55

$$\int x\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{1}{2}a^2Ax^2+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{8}b(Ab+2aB)x^8+\frac{1}{11}b^2Bx^{11}$$

output
```
1/2*a^2*A*x^2+1/5*a*(2*A*b+B*a)*x^5+1/8*b*(A*b+2*B*a)*x^8+1/11*b^2*B*x^11
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{1}{2}a^2Ax^2+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{8}b(Ab+2aB)x^8+\frac{1}{11}b^2Bx^{11}$$

input
```
Integrate[x*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^8)/8 + (b^2*B
*x^11)/11
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^2\left(A+Bx^3\right)dx$$
$$\Big\downarrow \text{950}$$
$$\int \left(a^2Ax+bx^7(2aB+Ab)+ax^4(aB+2Ab)+b^2Bx^{10}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}a^2Ax^2+\frac{1}{8}bx^8(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{11}b^2Bx^{11}$$

input
```
Int[x*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^8)/8 + (b^2*B
*x^11)/11
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{b^2 B\,x^{11}}{11}+\frac{(b^2 A+2abB)x^8}{8}+\frac{(2abA+a^2 B)x^5}{5}+\frac{a^2 A\,x^2}{2}$	52
norman	$\frac{b^2 B\,x^{11}}{11}+\left(\frac{1}{8}b^2 A+\frac{1}{4}abB\right)x^8+\left(\frac{2}{5}abA+\frac{1}{5}a^2 B\right)x^5+\frac{a^2 A\,x^2}{2}$	52
gosper	$\frac{1}{11}b^2 B\,x^{11}+\frac{1}{8}x^8 b^2 A+\frac{1}{4}x^8 abB+\frac{2}{5}x^5 abA+\frac{1}{5}x^5 a^2 B+\frac{1}{2}a^2 A\,x^2$	54
risch	$\frac{1}{11}b^2 B\,x^{11}+\frac{1}{8}x^8 b^2 A+\frac{1}{4}x^8 abB+\frac{2}{5}x^5 abA+\frac{1}{5}x^5 a^2 B+\frac{1}{2}a^2 A\,x^2$	54
parallelrisch	$\frac{1}{11}b^2 B\,x^{11}+\frac{1}{8}x^8 b^2 A+\frac{1}{4}x^8 abB+\frac{2}{5}x^5 abA+\frac{1}{5}x^5 a^2 B+\frac{1}{2}a^2 A\,x^2$	54
orering	$\frac{x^2(40b^2 B\,x^9+55A\,b^2x^6+110Bab\,x^6+176aAb\,x^3+88B\,a^2x^3+220a^2 A)}{440}$	56

input
```
int(x*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*b^2*B*x^11+1/8*(A*b^2+2*B*a*b)*x^8+1/5*(2*A*a*b+B*a^2)*x^5+1/2*a^2*A*
x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x(a+bx^3)^2\left(A+Bx^3\right)\,dx=\frac{1}{11}\,Bb^2x^{11}+\frac{1}{8}\left(2\,Bab+Ab^2\right)x^8+\frac{1}{5}\left(Ba^2+2\,Aab\right)x^5+\frac{1}{2}\,Aa^2x^2$$

input
```
integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/11*B*b^2*x^11 + 1/8*(2*B*a*b + A*b^2)*x^8 + 1/5*(B*a^2 + 2*A*a*b)*x^5 +
1/2*A*a^2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int x(a+bx^3)^2\,(A+Bx^3)\;dx = \frac{Aa^2x^2}{2} + \frac{Bb^2x^{11}}{11} + x^8\left(\frac{Ab^2}{8} + \frac{Bab}{4}\right) + x^5\cdot\left(\frac{2Aab}{5} + \frac{Ba^2}{5}\right)$$

input
```
integrate(x*(b*x**3+a)**2*(B*x**3+A),x)
```

output
```
A*a**2*x**2/2 + B*b**2*x**11/11 + x**8*(A*b**2/8 + B*a*b/4) + x**5*(2*A*a*
b/5 + B*a**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x(a+bx^3)^2\,(A+Bx^3)\;dx = \frac{1}{11}\,Bb^2x^{11} + \frac{1}{8}\,(2\,Bab + Ab^2)x^8 + \frac{1}{5}\,(Ba^2 + 2\,Aab)x^5 + \frac{1}{2}\,Aa^2x^2$$

input
```
integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/11*B*b^2*x^11 + 1/8*(2*B*a*b + A*b^2)*x^8 + 1/5*(B*a^2 + 2*A*a*b)*x^5 +
1/2*A*a^2*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x(a+bx^3)^2\,(A+Bx^3)\;dx = \frac{1}{11}\,Bb^2x^{11} + \frac{1}{4}\,Babx^8 + \frac{1}{8}\,Ab^2x^8 + \frac{1}{5}\,Ba^2x^5 + \frac{2}{5}\,Aabx^5 + \frac{1}{2}\,Aa^2x^2$$

input
```
integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

output
```
1/11*B*b^2*x^11 + 1/4*B*a*b*x^8 + 1/8*A*b^2*x^8 + 1/5*B*a^2*x^5 + 2/5*A*a*
b*x^5 + 1/2*A*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x(a+bx^3)^2(A+Bx^3)\,dx = x^5\left(\frac{B\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right)+x^8\left(\frac{A\,b^2}{8}+\frac{B\,a\,b}{4}\right)+\frac{A\,a^2\,x^2}{2}+\frac{B\,b^2\,x^{11}}{11}$$

input
```
int(x*(A + B*x^3)*(a + b*x^3)^2,x)
```

output
```
x^5*((B*a^2)/5 + (2*A*a*b)/5) + x^8*((A*b^2)/8 + (B*a*b)/4) + (A*a^2*x^2)/
2 + (B*b^2*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

$$\int x(a+bx^3)^2(A+Bx^3)\,dx = \frac{x^2(40b^3x^9+165a\,b^2x^6+264a^2b\,x^3+220a^3)}{440}$$

input
```
int(x*(b*x^3+a)^2*(B*x^3+A),x)
```

output
```
(x**2*(220*a**3 + 264*a**2*b*x**3 + 165*a*b**2*x**6 + 40*b**3*x**9))/440
```

3.16 $\int (a+bx^3)^2 (A+Bx^3)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^3)^2 (A+Bx^3)\, dx = a^2Ax + \frac{1}{4}a(2Ab+aB)x^4 + \frac{1}{7}b(Ab+2aB)x^7 + \frac{1}{10}b^2Bx^{10}$$

output
```
a^2*A*x+1/4*a*(2*A*b+B*a)*x^4+1/7*b*(A*b+2*B*a)*x^7+1/10*b^2*B*x^10
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^2 (A+Bx^3)\, dx = a^2Ax + \frac{1}{4}a(2Ab+aB)x^4 + \frac{1}{7}b(Ab+2aB)x^7 + \frac{1}{10}b^2Bx^{10}$$

input
```
Integrate[(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^4)/4 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*x^10)
/10
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(a^2A+bx^6(2aB+Ab)+ax^3(aB+2Ab)+b^2Bx^9\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2Ax+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{4}ax^4(aB+2Ab)+\frac{1}{10}b^2Bx^{10}$$

input
```
Int[(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^4)/4 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*x^10)
/10
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^2 B\,x^{10}}{10}+\frac{(b^2 A+2abB)x^7}{7}+\frac{(2abA+a^2 B)x^4}{4}+a^2 Ax$	49
norman	$\frac{b^2 B\,x^{10}}{10}+\left(\frac{1}{7}b^2 A+\frac{2}{7}abB\right)x^7+\left(\frac{1}{2}abA+\frac{1}{4}a^2 B\right)x^4+a^2 Ax$	49
gosper	$\frac{1}{10}b^2 B\,x^{10}+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 abB+\frac{1}{2}x^4 abA+\frac{1}{4}x^4 a^2 B+a^2 Ax$	51
risch	$\frac{1}{10}b^2 B\,x^{10}+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 abB+\frac{1}{2}x^4 abA+\frac{1}{4}x^4 a^2 B+a^2 Ax$	51
parallelrisch	$\frac{1}{10}b^2 B\,x^{10}+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 abB+\frac{1}{2}x^4 abA+\frac{1}{4}x^4 a^2 B+a^2 Ax$	51
orering	$\frac{x(14b^2 B\,x^9+20A\,b^2 x^6+40Bab\,x^6+70aAb\,x^3+35B\,a^2 x^3+140a^2 A)}{140}$	54

input
```
int((b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/10*b^2*B*x^10+1/7*(A*b^2+2*B*a*b)*x^7+1/4*(2*A*a*b+B*a^2)*x^4+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^3)^2 (A+Bx^3)\,dx=\frac{1}{10}Bb^2x^{10}+\frac{1}{7}(2Bab+Ab^2)x^7+\frac{1}{4}(Ba^2+2Aab)x^4+Aa^2x$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/10*B*b^2*x^10 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/4*(B*a^2 + 2*A*a*b)*x^4 +
A*a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \left(a+bx^3\right)^2 \left(A+Bx^3\right)\,dx = Aa^2x + \frac{Bb^2x^{10}}{10} + x^7\left(\frac{Ab^2}{7} + \frac{2Bab}{7}\right) + x^4\left(\frac{Aab}{2} + \frac{Ba^2}{4}\right)$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A),x)
```

output
```
A*a**2*x + B*b**2*x**10/10 + x**7*(A*b**2/7 + 2*B*a*b/7) + x**4*(A*a*b/2 +
 B*a**2/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)^2 \left(A+Bx^3\right)\,dx = \frac{1}{10}\,Bb^2x^{10} + \frac{1}{7}\left(2\,Bab + Ab^2\right)x^7 + \frac{1}{4}\left(Ba^2 + 2\,Aab\right)x^4 + Aa^2x$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/10*B*b^2*x^10 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/4*(B*a^2 + 2*A*a*b)*x^4 +
A*a^2*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)^2 \left(A+Bx^3\right)\,dx = \frac{1}{10}\,Bb^2x^{10} + \frac{2}{7}\,Babx^7 + \frac{1}{7}\,Ab^2x^7 + \frac{1}{4}\,Ba^2x^4 + \frac{1}{2}\,Aabx^4 + Aa^2x$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

output
```
1/10*B*b^2*x^10 + 2/7*B*a*b*x^7 + 1/7*A*b^2*x^7 + 1/4*B*a^2*x^4 + 1/2*A*a*
b*x^4 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx = x^4 \left(\frac{B\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^7 \left(\frac{A\,b^2}{7}+\frac{2\,B\,a\,b}{7}\right)+\frac{B\,b^2\,x^{10}}{10}+A\,a^2\,x$$

input
```
int((A + B*x^3)*(a + b*x^3)^2,x)
```

output
```
x^4*((B*a^2)/4 + (A*a*b)/2) + x^7*((A*b^2)/7 + (2*B*a*b)/7) + (B*b^2*x^10)
/10 + A*a^2*x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx = \frac{x(14b^3x^9 + 60a\,b^2x^6 + 105a^2b\,x^3 + 140a^3)}{140}$$

input
```
int((b*x^3+a)^2*(B*x^3+A),x)
```

output
```
(x*(140*a**3 + 105*a**2*b*x**3 + 60*a*b**2*x**6 + 14*b**3*x**9))/140
```

3.17 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^2} dx = -\frac{a^2A}{x} + \frac{1}{2}a(2Ab+aB)x^2 + \frac{1}{5}b(Ab+2aB)x^5 + \frac{1}{8}b^2Bx^8$$

output
```
-a^2*A/x+1/2*a*(2*A*b+B*a)*x^2+1/5*b*(A*b+2*B*a)*x^5+1/8*b^2*B*x^8
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^2} dx = -\frac{a^2A}{x} + \frac{1}{2}a(2Ab+aB)x^2 + \frac{1}{5}b(Ab+2aB)x^5 + \frac{1}{8}b^2Bx^8$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^2,x]
```

output
```
-((a^2*A)/x) + (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*
x^8)/8
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^2} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2 A}{x^2}+bx^4(2aB+Ab)+ax(aB+2Ab)+b^2Bx^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2 A}{x}+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{2}ax^2(aB+2Ab)+\frac{1}{8}b^2Bx^8$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x^2,x]
```

output
```
-((a^2*A)/x) + (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*
x^8)/8
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

method	result	size
norman	$\frac{\frac{b^2 B\,x^9}{8}+\left(\frac{1}{5}b^2 A+\frac{2}{5}abB\right)x^6+\left(abA+\frac{1}{2}a^2 B\right)x^3-a^2 A}{x}$	52
default	$\frac{b^2 B\,x^8}{8}+\frac{A\,b^2 x^5}{5}+\frac{2Bab\,x^5}{5}+aAb\,x^2+\frac{B\,a^2 x^2}{2}-\frac{a^2 A}{x}$	53
risch	$\frac{b^2 B\,x^8}{8}+\frac{A\,b^2 x^5}{5}+\frac{2Bab\,x^5}{5}+aAb\,x^2+\frac{B\,a^2 x^2}{2}-\frac{a^2 A}{x}$	53
gosper	$-\frac{-5b^2 B\,x^9-8A\,b^2 x^6-16Bab\,x^6-40aAb\,x^3-20B\,a^2 x^3+40a^2 A}{40x}$	56
parallelrisch	$\frac{5b^2 B\,x^9+8A\,b^2 x^6+16Bab\,x^6+40aAb\,x^3+20B\,a^2 x^3-40a^2 A}{40x}$	56
orering	$-\frac{-5b^2 B\,x^9-8A\,b^2 x^6-16Bab\,x^6-40aAb\,x^3-20B\,a^2 x^3+40a^2 A}{40x}$	56

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x*(1/8*b^2*B*x^9+(1/5*b^2*A+2/5*a*b*B)*x^6+(a*b*A+1/2*a^2*B)*x^3-a^2*A)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2\,(A+Bx^3)}{x^2}\,dx = \frac{5\,Bb^2x^9+8\,(2\,Bab+Ab^2)x^6+20\,(Ba^2+2\,Aab)x^3-40\,Aa^2}{40\,x}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="fricas")
```

output
```
1/40*(5*B*b^2*x^9 + 8*(2*B*a*b + A*b^2)*x^6 + 20*(B*a^2 + 2*A*a*b)*x^3 - 4
0*A*a^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^2} dx = -\frac{Aa^2}{x} + \frac{Bb^2x^8}{8} + x^5\left(\frac{Ab^2}{5} + \frac{2Bab}{5}\right) + x^2\left(Aab + \frac{Ba^2}{2}\right)$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**2,x)
```

output
```
-A*a**2/x + B*b**2*x**8/8 + x**5*(A*b**2/5 + 2*B*a*b/5) + x**2*(A*a*b + B*
a**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^2} dx = \frac{1}{8} Bb^2x^8 + \frac{1}{5} (2 Bab + Ab^2)x^5 + \frac{1}{2} (Ba^2 + 2 Aab)x^2 - \frac{Aa^2}{x}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="maxima")
```

output
```
1/8*B*b^2*x^8 + 1/5*(2*B*a*b + A*b^2)*x^5 + 1/2*(B*a^2 + 2*A*a*b)*x^2 - A*
a^2/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^2} dx = \frac{1}{8} Bb^2x^8 + \frac{2}{5} Babx^5 + \frac{1}{5} Ab^2x^5 + \frac{1}{2} Ba^2x^2 + Aabx^2 - \frac{Aa^2}{x}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="giac")
```

output
```
1/8*B*b^2*x^8 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 1/2*B*a^2*x^2 + A*a*b*x^2
- A*a^2/x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^2} \, dx = x^2 \left(\frac{B a^2}{2} + A b a\right) + x^5 \left(\frac{A b^2}{5} + \frac{2 B a b}{5}\right) - \frac{A a^2}{x} + \frac{B b^2 x^8}{8}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^2,x)
```

output
```
x^2*((B*a^2)/2 + A*a*b) + x^5*((A*b^2)/5 + (2*B*a*b)/5) - (A*a^2)/x + (B*b
^2*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^2} \, dx = \frac{5b^3x^9 + 24a b^2x^6 + 60a^2b x^3 - 40a^3}{40x}$$

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^2,x)
```

output
```
( - 40*a**3 + 60*a**2*b*x**3 + 24*a*b**2*x**6 + 5*b**3*x**9)/(40*x)
```

3.18 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^3} dx = -\frac{a^2A}{2x^2} + a(2Ab+aB)x + \frac{1}{4}b(Ab+2aB)x^4 + \frac{1}{7}b^2Bx^7$$

output
```
-1/2*a^2*A/x^2+a*(2*A*b+B*a)*x+1/4*b*(A*b+2*B*a)*x^4+1/7*b^2*B*x^7
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^3} dx = -\frac{a^2A}{2x^2} + a(2Ab+aB)x + \frac{1}{4}b(Ab+2aB)x^4 + \frac{1}{7}b^2Bx^7$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^3,x]
```

output
```
-1/2*(a^2*A)/x^2 + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^
7)/7
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^3} \, dx$$

$$\Big\downarrow \ 950$$

$$\int \left(\frac{a^2 A}{x^3}+bx^3(2aB+Ab)+a(aB+2Ab)+b^2Bx^6\right) dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^2 A}{2x^2}+\frac{1}{4}bx^4(2aB+Ab)+ax(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x^3,x]
```

output
```
-1/2*(a^2*A)/x^2 + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^
7)/7
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^2 B x^7}{7}+\frac{A b^2 x^4}{4}+\frac{Bab x^4}{2}+2aAbx+B a^2 x-\frac{a^2 A}{2x^2}$	49
risch	$\frac{b^2 B x^7}{7}+\frac{A b^2 x^4}{4}+\frac{Bab x^4}{2}+2aAbx+B a^2 x-\frac{a^2 A}{2x^2}$	49
norman	$\frac{\frac{b^2 B x^9}{7}+\left(\frac{1}{4}b^2 A+\frac{1}{2}abB\right)x^6+\left(2abA+a^2 B\right)x^3-\frac{a^2 A}{2}}{x^2}$	52
gosper	$-\frac{-4b^2 B x^9-7A b^2 x^6-14Bab x^6-56aAb x^3-28B a^2 x^3+14a^2 A}{28x^2}$	56
parallelrisch	$\frac{4b^2 B x^9+7A b^2 x^6+14Bab x^6+56aAb x^3+28B a^2 x^3-14a^2 A}{28x^2}$	56
orering	$-\frac{-4b^2 B x^9-7A b^2 x^6-14Bab x^6-56aAb x^3-28B a^2 x^3+14a^2 A}{28x^2}$	56

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^2*B*x^7+1/4*A*b^2*x^4+1/2*B*a*b*x^4+2*a*A*b*x+B*a^2*x-1/2*a^2*A/x^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3}\,dx = \frac{4\,Bb^2x^9+7\,(2\,Bab+Ab^2)x^6+28\,(Ba^2+2\,Aab)x^3-14\,Aa^2}{28\,x^2}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="fricas")
```

output
```
1/28*(4*B*b^2*x^9 + 7*(2*B*a*b + A*b^2)*x^6 + 28*(B*a^2 + 2*A*a*b)*x^3 - 1
4*A*a^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3}\,dx = -\frac{Aa^2}{2x^2} + \frac{Bb^2x^7}{7} + x^4\left(\frac{Ab^2}{4} + \frac{Bab}{2}\right) + x\left(2Aab + Ba^2\right)$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**3,x)
```

output
```
-A*a**2/(2*x**2) + B*b**2*x**7/7 + x**4*(A*b**2/4 + B*a*b/2) + x*(2*A*a*b
+ B*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3}\,dx = \frac{1}{7}\,Bb^2x^7 + \frac{1}{4}\,(2\,Bab + Ab^2)x^4 + \left(Ba^2 + 2\,Aab\right)x - \frac{Aa^2}{2\,x^2}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="maxima")
```

output
```
1/7*B*b^2*x^7 + 1/4*(2*B*a*b + A*b^2)*x^4 + (B*a^2 + 2*A*a*b)*x - 1/2*A*a^
2/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3}\,dx = \frac{1}{7}\,Bb^2x^7 + \frac{1}{2}\,Babx^4 + \frac{1}{4}\,Ab^2x^4 + Ba^2x + 2\,Aabx - \frac{Aa^2}{2\,x^2}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="giac")
```

output
```
1/7*B*b^2*x^7 + 1/2*B*a*b*x^4 + 1/4*A*b^2*x^4 + B*a^2*x + 2*A*a*b*x - 1/2*
A*a^2/x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^3} \, dx = x^4 \left(\frac{A\,b^2}{4} + \frac{B\,a\,b}{2}\right) + x\,(B\,a^2 + 2\,A\,b\,a) - \frac{A\,a^2}{2\,x^2} + \frac{B\,b^2\,x^7}{7}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^3,x)
```

output
```
x^4*((A*b^2)/4 + (B*a*b)/2) + x*(B*a^2 + 2*A*a*b) - (A*a^2)/(2*x^2) + (B*b
^2*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^3} \, dx = \frac{4b^3x^9 + 21a\,b^2x^6 + 84a^2b\,x^3 - 14a^3}{28x^2}$$

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^3,x)
```

output
```
( - 14*a**3 + 84*a**2*b*x**3 + 21*a*b**2*x**6 + 4*b**3*x**9)/(28*x**2)
```

3.19 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^5}\,dx = -\frac{a^2A}{4x^4}-\frac{a(2Ab+aB)}{x}+\frac{1}{2}b(Ab+2aB)x^2+\frac{1}{5}b^2Bx^5$$

output
```
-1/4*a^2*A/x^4-a*(2*A*b+B*a)/x+1/2*b*(A*b+2*B*a)*x^2+1/5*b^2*B*x^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^5}\,dx = \frac{-5a^2A-20a(2Ab+aB)x^3+10b(Ab+2aB)x^6+4b^2Bx^9}{20x^4}$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^5,x]
```

output
```
(-5*a^2*A - 20*a*(2*A*b + a*B)*x^3 + 10*b*(A*b + 2*a*B)*x^6 + 4*b^2*B*x^9)
/(20*x^4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^5} dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2A}{x^5}+\frac{a(aB+2Ab)}{x^2}+bx(2aB+Ab)+b^2Bx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{4x^4}+\frac{1}{2}bx^2(2aB+Ab)-\frac{a(aB+2Ab)}{x}+\frac{1}{5}b^2Bx^5$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x^5,x]
```

output
```
-1/4*(a^2*A)/x^4 - (a*(2*A*b + a*B))/x + (b*(A*b + 2*a*B)*x^2)/2 + (b^2*B*
x^5)/5
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{b^{2} B\,x^{5}}{5}+\frac{A\,b^{2} x^{2}}{2}+B a b\,x^{2}-\frac{a^{2} A}{4 x^{4}}-\frac{a \left(2 A b +B a \right)}{x}$	50
norman	$\frac{\frac{b^{2} B\,x^{9}}{5}+\left(\frac{1}{2} b^{2} A +a b B \right) x^{6}+\left(-2 a b A -a^{2} B \right) x^{3}-\frac{a^{2} A}{4}}{x^{4}}$	52
risch	$\frac{b^{2} B\,x^{5}}{5}+\frac{A\,b^{2} x^{2}}{2}+B a b\,x^{2}+\frac{\left(-2 a b A -a^{2} B \right) x^{3}-\frac{a^{2} A}{4}}{x^{4}}$	54
gosper	$-\frac{-4 b^{2} B\,x^{9}-10 A\,b^{2} x^{6}-20 B a b\,x^{6}+40 a A b\,x^{3}+20 B\,a^{2} x^{3}+5 a^{2} A}{20 x^{4}}$	56
parallelrisch	$\frac{4 b^{2} B\,x^{9}+10 A\,b^{2} x^{6}+20 B a b\,x^{6}-40 a A b\,x^{3}-20 B\,a^{2} x^{3}-5 a^{2} A}{20 x^{4}}$	56
orering	$-\frac{-4 b^{2} B\,x^{9}-10 A\,b^{2} x^{6}-20 B a b\,x^{6}+40 a A b\,x^{3}+20 B\,a^{2} x^{3}+5 a^{2} A}{20 x^{4}}$	56

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^2*B*x^5+1/2*A*b^2*x^2+B*a*b*x^2-1/4*a^2*A/x^4-a*(2*A*b+B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^5}\,dx = \frac{4\,Bb^2x^9+10\,(2\,Bab+Ab^2)x^6-20\,(Ba^2+2\,Aab)x^3-5\,Aa^2}{20\,x^4}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="fricas")
```

output
```
1/20*(4*B*b^2*x^9 + 10*(2*B*a*b + A*b^2)*x^6 - 20*(B*a^2 + 2*A*a*b)*x^3 -
5*A*a^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^5}\,dx = \frac{Bb^2x^5}{5}+x^2\left(\frac{Ab^2}{2}+Bab\right)+\frac{-Aa^2+x^3(-8Aab-4Ba^2)}{4x^4}$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**5,x)
```

output
```
B*b**2*x**5/5 + x**2*(A*b**2/2 + B*a*b) + (-A*a**2 + x**3*(-8*A*a*b - 4*B*
a**2))/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^5}\,dx = \frac{1}{5}\,Bb^2x^5+\frac{1}{2}\,(2\,Bab+Ab^2)x^2-\frac{4\,(Ba^2+2\,Aab)x^3+Aa^2}{4\,x^4}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="maxima")
```

output
```
1/5*B*b^2*x^5 + 1/2*(2*B*a*b + A*b^2)*x^2 - 1/4*(4*(B*a^2 + 2*A*a*b)*x^3 +
 A*a^2)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^5}\,dx = \frac{1}{5}\,Bb^2x^5+Babx^2+\frac{1}{2}\,Ab^2x^2-\frac{4\,Ba^2x^3+8\,Aabx^3+Aa^2}{4\,x^4}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="giac")
```

output
```
1/5*B*b^2*x^5 + B*a*b*x^2 + 1/2*A*b^2*x^2 - 1/4*(4*B*a^2*x^3 + 8*A*a*b*x^3
 + A*a^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^5} \, dx = x^2 \left(\frac{A b^2}{2} + B a b\right) - \frac{x^3 (B a^2 + 2 A b a) + \frac{A a^2}{4}}{x^4} + \frac{B b^2 x^5}{5}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^5,x)
```

output
```
x^2*((A*b^2)/2 + B*a*b) - (x^3*(B*a^2 + 2*A*a*b) + (A*a^2)/4)/x^4 + (B*b^2
*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^5} \, dx = \frac{4b^3x^9 + 30a b^2x^6 - 60a^2b x^3 - 5a^3}{20x^4}$$

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^5,x)
```

output
```
( - 5*a**3 - 60*a**2*b*x**3 + 30*a*b**2*x**6 + 4*b**3*x**9)/(20*x**4)
```

3.20 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^6} dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^6} dx = -\frac{a^2A}{5x^5} - \frac{a(2Ab+aB)}{2x^2} + b(Ab+2aB)x + \frac{1}{4}b^2Bx^4$$

output
```
-1/5*a^2*A/x^5-1/2*a*(2*A*b+B*a)/x^2+b*(A*b+2*B*a)*x+1/4*b^2*B*x^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^6} dx = -\frac{a^2A}{5x^5} - \frac{a(2Ab+aB)}{2x^2} + b(Ab+2aB)x + \frac{1}{4}b^2Bx^4$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^6,x]
```

output
```
-1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(2*x^2) + b*(A*b + 2*a*B)*x + (b^2*B*
x^4)/4
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^6}dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2A}{x^6}+\frac{a(aB+2Ab)}{x^3}+b(2aB+Ab)+b^2Bx^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{5x^5}-\frac{a(aB+2Ab)}{2x^2}+bx(2aB+Ab)+\frac{1}{4}b^2Bx^4$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x^6,x]
```

output
```
-1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(2*x^2) + b*(A*b + 2*a*B)*x + (b^2*B*
x^4)/4
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{b^{2} B \,x^{4}}{4}+A \,b^{2} x +2 B a b x -\frac{a^{2} A}{5 x^{5}}-\frac{a \left(2 A b +B a \right)}{2 x^{2}}$	46
risch	$\frac{b^{2} B \,x^{4}}{4}+A \,b^{2} x +2 B a b x +\frac{\left(-a b A -\frac{1}{2} a^{2} B \right) x^{3}-\frac{a^{2} A}{5}}{x^{5}}$	50
norman	$\frac{\frac{b^{2} B \,x^{9}}{4}+\left(b^{2} A +2 a b B \right) x^{6}+\left(-a b A -\frac{1}{2} a^{2} B \right) x^{3}-\frac{a^{2} A}{5}}{x^{5}}$	52
gosper	$-\frac{-5 b^{2} B \,x^{9}-20 A \,b^{2} x^{6}-40 B a b \,x^{6}+20 a A b \,x^{3}+10 B \,a^{2} x^{3}+4 a^{2} A}{20 x^{5}}$	56
parallelrisch	$\frac{5 b^{2} B \,x^{9}+20 A \,b^{2} x^{6}+40 B a b \,x^{6}-20 a A b \,x^{3}-10 B \,a^{2} x^{3}-4 a^{2} A}{20 x^{5}}$	56
orering	$-\frac{-5 b^{2} B \,x^{9}-20 A \,b^{2} x^{6}-40 B a b \,x^{6}+20 a A b \,x^{3}+10 B \,a^{2} x^{3}+4 a^{2} A}{20 x^{5}}$	56

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2*B*x^4+A*b^2*x+2*B*a*b*x-1/5*a^2*A/x^5-1/2*a*(2*A*b+B*a)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^6}\,dx = \frac{5\,Bb^2x^9+20\,(2\,Bab+Ab^2)x^6-10\,(Ba^2+2\,Aab)x^3-4\,Aa^2}{20\,x^5}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="fricas")
```

output
```
1/20*(5*B*b^2*x^9 + 20*(2*B*a*b + A*b^2)*x^6 - 10*(B*a^2 + 2*A*a*b)*x^3 -
4*A*a^2)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^6}\,dx = \frac{Bb^2x^4}{4} + x\left(Ab^2+2Bab\right) + \frac{-2Aa^2+x^3(-10Aab-5Ba^2)}{10x^5}$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**6,x)
```

output
```
B*b**2*x**4/4 + x*(A*b**2 + 2*B*a*b) + (-2*A*a**2 + x**3*(-10*A*a*b - 5*B*
a**2))/(10*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^6}\,dx = \frac{1}{4}\,Bb^2x^4 + \left(2\,Bab+Ab^2\right)x - \frac{5\,(Ba^2+2\,Aab)x^3+2\,Aa^2}{10\,x^5}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="maxima")
```

output
```
1/4*B*b^2*x^4 + (2*B*a*b + A*b^2)*x - 1/10*(5*(B*a^2 + 2*A*a*b)*x^3 + 2*A*
a^2)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^6}\,dx = \frac{1}{4}\,Bb^2x^4 + 2\,Babx + Ab^2x - \frac{5\,Ba^2x^3+10\,Aabx^3+2\,Aa^2}{10\,x^5}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="giac")
```

output
```
1/4*B*b^2*x^4 + 2*B*a*b*x + A*b^2*x - 1/10*(5*B*a^2*x^3 + 10*A*a*b*x^3 + 2
*A*a^2)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^6}\,dx = x\,\left(A\,b^2+2\,B\,a\,b\right)-\frac{x^3\,\left(\frac{B\,a^2}{2}+A\,b\,a\right)+\frac{A\,a^2}{5}}{x^5}+\frac{B\,b^2\,x^4}{4}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^6,x)
```

output
```
x*(A*b^2 + 2*B*a*b) - (x^3*((B*a^2)/2 + A*a*b) + (A*a^2)/5)/x^5 + (B*b^2*x
^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^6}\,dx = \frac{5b^3x^9+60a\,b^2x^6-30a^2b\,x^3-4a^3}{20x^5}$$

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^6,x)
```

output
```
( - 4*a**3 - 30*a**2*b*x**3 + 60*a*b**2*x**6 + 5*b**3*x**9)/(20*x**5)
```

3.21 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8}\,dx = -\frac{a^2A}{7x^7}-\frac{a(2Ab+aB)}{4x^4}-\frac{b(Ab+2aB)}{x}+\frac{1}{2}b^2Bx^2$$

output
```
-1/7*a^2*A/x^7-1/4*a*(2*A*b+B*a)/x^4-b*(A*b+2*B*a)/x+1/2*b^2*B*x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8}\,dx = -\frac{-14b^2x^6(-2A+Bx^3)+14abx^3(A+4Bx^3)+a^2(4A+7Bx^3)}{28x^7}$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^8,x]
```

output
```
-1/28*(-14*b^2*x^6*(-2*A + B*x^3) + 14*a*b*x^3*(A + 4*B*x^3) + a^2*(4*A +
7*B*x^3))/x^7
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^8} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2 A}{x^8}+\frac{a(aB+2Ab)}{x^5}+\frac{b(2aB+Ab)}{x^2}+b^2 Bx\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2 A}{7x^7}-\frac{a(aB+2Ab)}{4x^4}-\frac{b(2aB+Ab)}{x}+\frac{1}{2}b^2 Bx^2$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x^8,x]
```

output
```
-1/7*(a^2*A)/x^7 - (a*(2*A*b + a*B))/(4*x^4) - (b*(A*b + 2*a*B))/x + (b^2*
B*x^2)/2
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{a^2 A}{7x^7}-\frac{a(2Ab+Ba)}{4x^4}-\frac{b(Ab+2Ba)}{x}+\frac{b^2 B\,x^2}{2}$	48
norman	$\frac{\frac{b^2 B\,x^9}{2}+(-b^2 A-2abB)x^6+(-\frac{1}{2}abA-\frac{1}{4}a^2 B)x^3-\frac{a^2 A}{7}}{x^7}$	53
risch	$\frac{b^2 B\,x^2}{2}+\frac{(-b^2 A-2abB)x^6+(-\frac{1}{2}abA-\frac{1}{4}a^2 B)x^3-\frac{a^2 A}{7}}{x^7}$	54
gosper	$-\frac{-14b^2 B\,x^9+28A\,b^2 x^6+56Bab\,x^6+14aAb\,x^3+7B\,a^2 x^3+4a^2 A}{28x^7}$	56
parallelrisch	$-\frac{-14b^2 B\,x^9+28A\,b^2 x^6+56Bab\,x^6+14aAb\,x^3+7B\,a^2 x^3+4a^2 A}{28x^7}$	56
orering	$-\frac{-14b^2 B\,x^9+28A\,b^2 x^6+56Bab\,x^6+14aAb\,x^3+7B\,a^2 x^3+4a^2 A}{28x^7}$	56

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^2*A/x^7-1/4*a*(2*A*b+B*a)/x^4-b*(A*b+2*B*a)/x+1/2*b^2*B*x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8}\,dx = \frac{14\,Bb^2x^9-28\,(2\,Bab+Ab^2)x^6-7\,(Ba^2+2\,Aab)x^3-4\,Aa^2}{28\,x^7}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="fricas")
```

output
```
1/28*(14*B*b^2*x^9 - 28*(2*B*a*b + A*b^2)*x^6 - 7*(B*a^2 + 2*A*a*b)*x^3 -
4*A*a^2)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^8}\,dx = \frac{Bb^2x^2}{2} + \frac{-4Aa^2 + x^6(-28Ab^2 - 56Bab) + x^3(-14Aab - 7Ba^2)}{28x^7}$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**8,x)
```

output
```
B*b**2*x**2/2 + (-4*A*a**2 + x**6*(-28*A*b**2 - 56*B*a*b) + x**3*(-14*A*a*
b - 7*B*a**2))/(28*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^8}\,dx = \frac{1}{2}\,Bb^2x^2 - \frac{28\,(2\,Bab + Ab^2)x^6 + 7\,(Ba^2 + 2\,Aab)x^3 + 4\,Aa^2}{28\,x^7}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="maxima")
```

output
```
1/2*B*b^2*x^2 - 1/28*(28*(2*B*a*b + A*b^2)*x^6 + 7*(B*a^2 + 2*A*a*b)*x^3 +
 4*A*a^2)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^8}\,dx = \frac{1}{2}Bb^2x^2 - \frac{56\,Babx^6 + 28\,Ab^2x^6 + 7\,Ba^2x^3 + 14\,Aabx^3 + 4\,Aa^2}{28\,x^7}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="giac")
```

output
```
1/2*B*b^2*x^2 - 1/28*(56*B*a*b*x^6 + 28*A*b^2*x^6 + 7*B*a^2*x^3 + 14*A*a*b
*x^3 + 4*A*a^2)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^8}\,dx = \frac{B\,b^2\,x^2}{2} - \frac{x^3\left(\frac{B\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^6\,(A\,b^2+2\,B\,a\,b)+\frac{A\,a^2}{7}}{x^7}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^8,x)
```

output
```
(B*b^2*x^2)/2 - (x^3*((B*a^2)/4 + (A*a*b)/2) + x^6*(A*b^2 + 2*B*a*b) + (A*
a^2)/7)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^8}\,dx = \frac{14b^3x^9 - 84a\,b^2x^6 - 21a^2b\,x^3 - 4a^3}{28x^7}$$

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^8,x)
```

output
```
( - 4*a**3 - 21*a**2*b*x**3 - 84*a*b**2*x**6 + 14*b**3*x**9)/(28*x**7)
```

3.22 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx = -\frac{a^2A}{8x^8}-\frac{a(2Ab+aB)}{5x^5}-\frac{b(Ab+2aB)}{2x^2}+b^2Bx$$

output
```
-1/8*a^2*A/x^8-1/5*a*(2*A*b+B*a)/x^5-1/2*b*(A*b+2*B*a)/x^2+b^2*B*x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx = -\frac{a^2A}{8x^8}-\frac{a(2Ab+aB)}{5x^5}-\frac{b(Ab+2aB)}{2x^2}+b^2Bx$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^9,x]
```

output
```
-1/8*(a^2*A)/x^8 - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(2*x^2) +
 b^2*B*x
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^9} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2 A}{x^9}+\frac{a(aB+2Ab)}{x^6}+\frac{b(2aB+Ab)}{x^3}+b^2 B\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2 A}{8x^8}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{2x^2}+b^2 Bx$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x^9,x]
```

output
```
-1/8*(a^2*A)/x^8 - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(2*x^2) +
 b^2*B*x
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^2 A}{8x^8}-\frac{a(2Ab+Ba)}{5x^5}-\frac{b(Ab+2Ba)}{2x^2}+b^2 B x$	45
risch	$b^2 B x+\frac{\left(-\frac{1}{2}b^2 A-abB\right)x^6+\left(-\frac{2}{5}abA-\frac{1}{5}a^2 B\right)x^3-\frac{a^2 A}{8}}{x^8}$	51
norman	$\frac{b^2 B\,x^9+\left(-\frac{1}{2}b^2 A-abB\right)x^6+\left(-\frac{2}{5}abA-\frac{1}{5}a^2 B\right)x^3-\frac{a^2 A}{8}}{x^8}$	52
gosper	$-\frac{-40b^2 B\,x^9+20A\,b^2 x^6+40Bab\,x^6+16aAb\,x^3+8B\,a^2 x^3+5a^2 A}{40x^8}$	56
parallelrisch	$-\frac{-40b^2 B\,x^9+20A\,b^2 x^6+40Bab\,x^6+16aAb\,x^3+8B\,a^2 x^3+5a^2 A}{40x^8}$	56
orering	$-\frac{-40b^2 B\,x^9+20A\,b^2 x^6+40Bab\,x^6+16aAb\,x^3+8B\,a^2 x^3+5a^2 A}{40x^8}$	56

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^2*A/x^8-1/5*a*(2*A*b+B*a)/x^5-1/2*b*(A*b+2*B*a)/x^2+b^2*B*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx = \frac{40\,Bb^2x^9-20\,(2\,Bab+Ab^2)x^6-8\,(Ba^2+2\,Aab)x^3-5\,Aa^2}{40\,x^8}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="fricas")
```

output
```
1/40*(40*B*b^2*x^9 - 20*(2*B*a*b + A*b^2)*x^6 - 8*(B*a^2 + 2*A*a*b)*x^3 -
5*A*a^2)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx = Bb^2x + \frac{-5Aa^2 + x^6(-20Ab^2 - 40Bab) + x^3(-16Aab - 8Ba^2)}{40x^8}$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**9,x)
```

output
```
B*b**2*x + (-5*A*a**2 + x**6*(-20*A*b**2 - 40*B*a*b) + x**3*(-16*A*a*b - 8
*B*a**2))/(40*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx = Bb^2x - \frac{20\,(2\,Bab + Ab^2)x^6 + 8\,(Ba^2 + 2\,Aab)x^3 + 5\,Aa^2}{40\,x^8}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="maxima")
```

output
```
B*b^2*x - 1/40*(20*(2*B*a*b + A*b^2)*x^6 + 8*(B*a^2 + 2*A*a*b)*x^3 + 5*A*a
^2)/x^8
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx = Bb^2x - \frac{40\,Babx^6 + 20\,Ab^2x^6 + 8\,Ba^2x^3 + 16\,Aabx^3 + 5\,Aa^2}{40\,x^8}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="giac")
```

output
```
B*b^2*x - 1/40*(40*B*a*b*x^6 + 20*A*b^2*x^6 + 8*B*a^2*x^3 + 16*A*a*b*x^3 +
 5*A*a^2)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^9} \, dx = B\,b^2\,x - \frac{x^3\left(\frac{B\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right)+x^6\left(\frac{A\,b^2}{2}+B\,a\,b\right)+\frac{A\,a^2}{8}}{x^8}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^9,x)
```

output
```
B*b^2*x - (x^3*((B*a^2)/5 + (2*A*a*b)/5) + x^6*((A*b^2)/2 + B*a*b) + (A*a^
2)/8)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^9} \, dx = \frac{40b^3x^9 - 60a\,b^2x^6 - 24a^2b\,x^3 - 5a^3}{40x^8}$$

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^9,x)
```

output
```
( - 5*a**3 - 24*a**2*b*x**3 - 60*a*b**2*x**6 + 40*b**3*x**9)/(40*x**8)
```

3.23 $\int x^8(a+bx^3)^5 (A+Bx^3) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 95

$$\int x^8 (a+bx^3)^5 (A+Bx^3) \, dx = \frac{a^2(Ab-aB)(a+bx^3)^6}{18b^4} - \frac{a(2Ab-3aB)(a+bx^3)^7}{21b^4} + \frac{(Ab-3aB)(a+bx^3)^8}{24b^4} + \frac{B(a+bx^3)^9}{27b^4}$$

output
```
1/18*a^2*(A*b-B*a)*(b*x^3+a)^6/b^4-1/21*a*(2*A*b-3*B*a)*(b*x^3+a)^7/b^4+1/
24*(A*b-3*B*a)*(b*x^3+a)^8/b^4+1/27*B*(b*x^3+a)^9/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int x^8 (a+bx^3)^5 (A+Bx^3) \, dx = \frac{x^9(168a^5A+126a^4(5Ab+aB)x^3+504a^3b(2Ab+aB)x^6+840a^2b^2(Ab+aB)x^9+360ab^3(Ab+2aB)x}{1512}$$

input
```
Integrate[x^8*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(x^9*(168*a^5*A + 126*a^4*(5*A*b + a*B)*x^3 + 504*a^3*b*(2*A*b + a*B)*x^6
+ 840*a^2*b^2*(A*b + a*B)*x^9 + 360*a*b^3*(A*b + 2*a*B)*x^12 + 63*b^4*(A*b
 + 5*a*B)*x^15 + 56*b^5*B*x^18))/1512
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int x^6 \left(bx^3+a\right)^5 \left(Bx^3+A\right) dx^3$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{3}\int \left(\frac{B\left(bx^3+a\right)^8}{b^3}+\frac{(Ab-3aB)\left(bx^3+a\right)^7}{b^3}+\frac{a(3aB-2Ab)\left(bx^3+a\right)^6}{b^3}-\frac{a^2(aB-Ab)\left(bx^3+a\right)^5}{b^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{a^2\left(a+bx^3\right)^6(Ab-aB)}{6b^4}+\frac{\left(a+bx^3\right)^8(Ab-3aB)}{8b^4}-\frac{a\left(a+bx^3\right)^7(2Ab-3aB)}{7b^4}+\frac{B\left(a+bx^3\right)^9}{9b^4}\right)$$

input
```
Int[x^8*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
((a^2*(A*b - a*B)*(a + b*x^3)^6)/(6*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^3)^
7)/(7*b^4) + ((A*b - 3*a*B)*(a + b*x^3)^8)/(8*b^4) + (B*(a + b*x^3)^9)/(9*
b^4))/3
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.27

method	result
norman	$\frac{b^5 B x^{27}}{27}+\left(\frac{5}{9}a^2b^3A+\frac{5}{9}a^3b^2B\right)x^{18}+\left(\frac{2}{3}a^3b^2A+\frac{1}{3}a^4bB\right)x^{15}+\left(\frac{5}{12}a^4bA+\frac{1}{12}a^5B\right)x^{12}+\left(\frac{5}{21}a\right.$
default	$\frac{b^5 B x^{27}}{27}+\frac{(b^5A+5a b^4B)x^{24}}{24}+\frac{(5a b^4A+10a^2b^3B)x^{21}}{21}+\frac{(10a^2b^3A+10a^3b^2B)x^{18}}{18}+\frac{(10a^3b^2A+5a^4bB)x^{15}}{15}+\frac{(5a}{}$
gosper	$\frac{1}{27}b^5B x^{27}+\frac{5}{9}x^{18}a^2b^3A+\frac{5}{9}x^{18}a^3b^2B+\frac{2}{3}x^{15}a^3b^2A+\frac{1}{3}x^{15}a^4bB+\frac{5}{12}x^{12}a^4bA+\frac{1}{12}x^{12}a^5B+$
risch	$\frac{1}{27}b^5B x^{27}+\frac{5}{9}x^{18}a^2b^3A+\frac{5}{9}x^{18}a^3b^2B+\frac{2}{3}x^{15}a^3b^2A+\frac{1}{3}x^{15}a^4bB+\frac{5}{12}x^{12}a^4bA+\frac{1}{12}x^{12}a^5B+$
parallelrisch	$\frac{1}{27}b^5B x^{27}+\frac{5}{9}x^{18}a^2b^3A+\frac{5}{9}x^{18}a^3b^2B+\frac{2}{3}x^{15}a^3b^2A+\frac{1}{3}x^{15}a^4bB+\frac{5}{12}x^{12}a^4bA+\frac{1}{12}x^{12}a^5B+$
orering	$\frac{x^9(56b^5B x^{18}+63A b^5x^{15}+315Ba b^4x^{15}+360aA b^4x^{12}+720B a^2b^3x^{12}+840a^2A b^3x^9+840B a^3b^2x^9+1008a^3A b^2x^6+504B a}{1512}$

input
```
int(x^8*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/27*b^5*B*x^27+(5/9*a^2*b^3*A+5/9*a^3*b^2*B)*x^18+(2/3*a^3*b^2*A+1/3*a^4*
b*B)*x^15+(5/12*a^4*b*A+1/12*a^5*B)*x^12+(5/21*a*b^4*A+10/21*a^2*b^3*B)*x^
21+(1/24*b^5*A+5/24*a*b^4*B)*x^24+1/9*a^5*A*x^9
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25

$$\int x^8 (a+bx^3)^5 (A+Bx^3)\,dx = \frac{1}{27} Bb^5x^{27} + \frac{1}{24}(5Bab^4 + Ab^5)x^{24} + \frac{5}{21}(2Ba^2b^3 + Aab^4)x^{21} + \frac{5}{9}(Ba^3b^2 + Aa^2b^3)x^{18} + \frac{1}{3}(Ba^4b + 2Aa^3b^2)x^{15} + \frac{1}{9}Aa^5x^9 + \frac{1}{12}(Ba^5 + 5Aa^4b)x^{12}$$

input
```
integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/27*B*b^5*x^27 + 1/24*(5*B*a*b^4 + A*b^5)*x^24 + 5/21*(2*B*a^2*b^3 + A*a*
b^4)*x^21 + 5/9*(B*a^3*b^2 + A*a^2*b^3)*x^18 + 1/3*(B*a^4*b + 2*A*a^3*b^2)
*x^15 + 1/9*A*a^5*x^9 + 1/12*(B*a^5 + 5*A*a^4*b)*x^12
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.43

$$\int x^8 (a+bx^3)^5 (A+Bx^3)\,dx = \frac{Aa^5x^9}{9} + \frac{Bb^5x^{27}}{27} + x^{24}\left(\frac{Ab^5}{24} + \frac{5Bab^4}{24}\right) + x^{21} \cdot \left(\frac{5Aab^4}{21} + \frac{10Ba^2b^3}{21}\right) + x^{18} \cdot \left(\frac{5Aa^2b^3}{9} + \frac{5Ba^3b^2}{9}\right) + x^{15} \cdot \left(\frac{2Aa^3b^2}{3} + \frac{Ba^4b}{3}\right) + x^{12} \cdot \left(\frac{5Aa^4b}{12} + \frac{Ba^5}{12}\right)$$

input
```
integrate(x**8*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x**9/9 + B*b**5*x**27/27 + x**24*(A*b**5/24 + 5*B*a*b**4/24) + x**2
1*(5*A*a*b**4/21 + 10*B*a**2*b**3/21) + x**18*(5*A*a**2*b**3/9 + 5*B*a**3*
b**2/9) + x**15*(2*A*a**3*b**2/3 + B*a**4*b/3) + x**12*(5*A*a**4*b/12 + B*
a**5/12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25

$$\int x^8(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{27}Bb^5x^{27} + \frac{1}{24}(5Bab^4+Ab^5)x^{24} + \frac{5}{21}(2Ba^2b^3+Aab^4)x^{21} + \frac{5}{9}(Ba^3b^2+Aa^2b^3)x^{18} + \frac{1}{3}(Ba^4b+2Aa^3b^2)x^{15} + \frac{1}{9}Aa^5x^9 + \frac{1}{12}(Ba^5+5Aa^4b)x^{12}$$

input
```
integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/27*B*b^5*x^27 + 1/24*(5*B*a*b^4 + A*b^5)*x^24 + 5/21*(2*B*a^2*b^3 + A*a*
b^4)*x^21 + 5/9*(B*a^3*b^2 + A*a^2*b^3)*x^18 + 1/3*(B*a^4*b + 2*A*a^3*b^2)
*x^15 + 1/9*A*a^5*x^9 + 1/12*(B*a^5 + 5*A*a^4*b)*x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.32

$$\int x^8(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{27}Bb^5x^{27} + \frac{5}{24}Bab^4x^{24} + \frac{1}{24}Ab^5x^{24} + \frac{10}{21}Ba^2b^3x^{21} + \frac{5}{21}Aab^4x^{21} + \frac{5}{9}Ba^3b^2x^{18} + \frac{5}{9}Aa^2b^3x^{18} + \frac{1}{3}Ba^4bx^{15} + \frac{2}{3}Aa^3b^2x^{15} + \frac{1}{12}Ba^5x^{12} + \frac{5}{12}Aa^4bx^{12} + \frac{1}{9}Aa^5x^9$$

input
```
integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/27*B*b^5*x^27 + 5/24*B*a*b^4*x^24 + 1/24*A*b^5*x^24 + 10/21*B*a^2*b^3*x^
21 + 5/21*A*a*b^4*x^21 + 5/9*B*a^3*b^2*x^18 + 5/9*A*a^2*b^3*x^18 + 1/3*B*a
^4*b*x^15 + 2/3*A*a^3*b^2*x^15 + 1/12*B*a^5*x^12 + 5/12*A*a^4*b*x^12 + 1/9
*A*a^5*x^9
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int x^8 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx = x^{12} \left(\frac{B\,a^5}{12}+\frac{5\,A\,b\,a^4}{12}\right)+x^{24} \left(\frac{A\,b^5}{24}+\frac{5\,B\,a\,b^4}{24}\right) + \frac{A\,a^5\,x^9}{9}+\frac{B\,b^5\,x^{27}}{27}+\frac{5\,a^2\,b^2\,x^{18}\,(A\,b+B\,a)}{9} + \frac{a^3\,b\,x^{15}\,(2\,A\,b+B\,a)}{3}+\frac{5\,a\,b^3\,x^{21}\,(A\,b+2\,B\,a)}{21}$$

input
```
int(x^8*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^12*((B*a^5)/12 + (5*A*a^4*b)/12) + x^24*((A*b^5)/24 + (5*B*a*b^4)/24) +
(A*a^5*x^9)/9 + (B*b^5*x^27)/27 + (5*a^2*b^2*x^18*(A*b + B*a))/9 + (a^3*b*
x^15*(2*A*b + B*a))/3 + (5*a*b^3*x^21*(A*b + 2*B*a))/21
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int x^8 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx = \frac{x^9(28b^6x^{18}+189a\,b^5x^{15}+540a^2b^4x^{12}+840a^3b^3x^9+756a^4b^2x^6+378a^5b\,x^3+84a^6)}{756}$$

input
```
int(x^8*(b*x^3+a)^5*(B*x^3+A),x)
```

output
```
(x**9*(84*a**6 + 378*a**5*b*x**3 + 756*a**4*b**2*x**6 + 840*a**3*b**3*x**9
 + 540*a**2*b**4*x**12 + 189*a*b**5*x**15 + 28*b**6*x**18))/756
```

3.24 $\int x^5(a+bx^3)^5 (A+Bx^3)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int x^5 (a+bx^3)^5 (A+Bx^3)\,dx = -\frac{a(Ab-aB)(a+bx^3)^6}{18b^3} + \frac{(Ab-2aB)(a+bx^3)^7}{21b^3} + \frac{B(a+bx^3)^8}{24b^3}$$

output
```
-1/18*a*(A*b-B*a)*(b*x^3+a)^6/b^3+1/21*(A*b-2*B*a)*(b*x^3+a)^7/b^3+1/24*B*
(b*x^3+a)^8/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.60

$$\int x^5 (a+bx^3)^5 (A+Bx^3)\,dx = \frac{1}{504}x^6(84a^5A+56a^4(5Ab+aB)x^3+210a^3b(2Ab+aB)x^6 + 336a^2b^2(Ab+aB)x^9+140ab^3(Ab+2aB)x^{12} + 24b^4(Ab+5aB)x^{15}+21b^5Bx^{18})$$

input
```
Integrate[x^5*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(x^6*(84*a^5*A + 56*a^4*(5*A*b + a*B)*x^3 + 210*a^3*b*(2*A*b + a*B)*x^6 +
336*a^2*b^2*(A*b + a*B)*x^9 + 140*a*b^3*(A*b + 2*a*B)*x^12 + 24*b^4*(A*b +
 5*a*B)*x^15 + 21*b^5*B*x^18))/504
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int x^3 \left(bx^3+a\right)^5 \left(Bx^3+A\right) dx^3$$

$$\Big\downarrow \text{ 85}$$

$$\frac{1}{3}\int \left(\frac{B\left(bx^3+a\right)^7}{b^2}+\frac{(Ab-2aB)\left(bx^3+a\right)^6}{b^2}+\frac{a(aB-Ab)\left(bx^3+a\right)^5}{b^2}\right) dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3\right)^7 (Ab-2aB)}{7b^3}-\frac{a\left(a+bx^3\right)^6 (Ab-aB)}{6b^3}+\frac{B\left(a+bx^3\right)^8}{8b^3}\right)$$

input
```
Int[x^5*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(-1/6*(a*(A*b - a*B)*(a + b*x^3)^6)/b^3 + ((A*b - 2*a*B)*(a + b*x^3)^7)/(7
*b^3) + (B*(a + b*x^3)^8)/(8*b^3))/3
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.81

method	result
norman	$\frac{a^5 A x^6}{6}+\left(\frac{5}{9}a^4 bA+\frac{1}{9}a^5 B\right)x^9+\left(\frac{5}{6}a^3 b^2 A+\frac{5}{12}a^4 bB\right)x^{12}+\left(\frac{2}{3}a^2 b^3 A+\frac{2}{3}a^3 b^2 B\right)x^{15}+\left(\frac{5}{18}a\, b^4\right.$
default	$\frac{b^5 B\, x^{24}}{24}+\frac{(b^5 A+5a\, b^4 B)x^{21}}{21}+\frac{(5a\, b^4 A+10a^2 b^3 B)x^{18}}{18}+\frac{(10a^2 b^3 A+10a^3 b^2 B)x^{15}}{15}+\frac{(10a^3 b^2 A+5a^4 bB)x^{12}}{12}+\frac{(5a}{}$
gosper	$\frac{1}{6}a^5 A x^6+\frac{5}{9}x^9 a^4 bA+\frac{1}{9}x^9 a^5 B+\frac{5}{6}x^{12}a^3 b^2 A+\frac{5}{12}x^{12}a^4 bB+\frac{2}{3}x^{15}a^2 b^3 A+\frac{2}{3}x^{15}a^3 b^2 B+\frac{5}{18}x$
risch	$\frac{1}{6}a^5 A x^6+\frac{5}{9}x^9 a^4 bA+\frac{1}{9}x^9 a^5 B+\frac{5}{6}x^{12}a^3 b^2 A+\frac{5}{12}x^{12}a^4 bB+\frac{2}{3}x^{15}a^2 b^3 A+\frac{2}{3}x^{15}a^3 b^2 B+\frac{5}{18}x$
parallelrisch	$\frac{1}{6}a^5 A x^6+\frac{5}{9}x^9 a^4 bA+\frac{1}{9}x^9 a^5 B+\frac{5}{6}x^{12}a^3 b^2 A+\frac{5}{12}x^{12}a^4 bB+\frac{2}{3}x^{15}a^2 b^3 A+\frac{2}{3}x^{15}a^3 b^2 B+\frac{5}{18}x$
orering	$\frac{x^6\left(21b^5 B\, x^{18}+24A\, b^5 x^{15}+120Ba\, b^4 x^{15}+140aA\, b^4 x^{12}+280B\, a^2 b^3 x^{12}+336a^2 A\, b^3 x^9+336B\, a^3 b^2 x^9+420a^3 A\, b^2 x^6+210B\, a\right.}{504}$

input
```
int(x^5*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/6*a^5*A*x^6+(5/9*a^4*b*A+1/9*a^5*B)*x^9+(5/6*a^3*b^2*A+5/12*a^4*b*B)*x^1
2+(2/3*a^2*b^3*A+2/3*a^3*b^2*B)*x^15+(5/18*a*b^4*A+5/9*a^2*b^3*B)*x^18+(1/
21*b^5*A+5/21*a*b^4*B)*x^21+1/24*b^5*B*x^24
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.78

$$\int x^5(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{24}Bb^5x^{24}+\frac{1}{21}(5\,Bab^4+Ab^5)x^{21}+\frac{5}{18}(2\,Ba^2b^3+Aab^4)x^{18}+\frac{2}{3}(Ba^3b^2+Aa^2b^3)x^{15}+\frac{5}{12}(Ba^4b+2\,Aa^3b^2)x^{12}+\frac{1}{6}Aa^5x^6+\frac{1}{9}(Ba^5+5\,Aa^4b)x^9$$

input
```
integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/24*B*b^5*x^24 + 1/21*(5*B*a*b^4 + A*b^5)*x^21 + 5/18*(2*B*a^2*b^3 + A*a*
b^4)*x^18 + 2/3*(B*a^3*b^2 + A*a^2*b^3)*x^15 + 5/12*(B*a^4*b + 2*A*a^3*b^2
)*x^12 + 1/6*A*a^5*x^6 + 1/9*(B*a^5 + 5*A*a^4*b)*x^9
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(60) = 120.

Time = 0.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.06

$$\int x^5(a+bx^3)^5(A+Bx^3)\,dx = \frac{Aa^5x^6}{6}+\frac{Bb^5x^{24}}{24}+x^{21}\left(\frac{Ab^5}{21}+\frac{5Bab^4}{21}\right)+x^{18}\cdot\left(\frac{5Aab^4}{18}+\frac{5Ba^2b^3}{9}\right)+x^{15}\cdot\left(\frac{2Aa^2b^3}{3}+\frac{2Ba^3b^2}{3}\right)+x^{12}\cdot\left(\frac{5Aa^3b^2}{6}+\frac{5Ba^4b}{12}\right)+x^9\cdot\left(\frac{5Aa^4b}{9}+\frac{Ba^5}{9}\right)$$

input
```
integrate(x**5*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x**6/6 + B*b**5*x**24/24 + x**21*(A*b**5/21 + 5*B*a*b**4/21) + x**1
8*(5*A*a*b**4/18 + 5*B*a**2*b**3/9) + x**15*(2*A*a**2*b**3/3 + 2*B*a**3*b*
*2/3) + x**12*(5*A*a**3*b**2/6 + 5*B*a**4*b/12) + x**9*(5*A*a**4*b/9 + B*a
**5/9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.78

$$\int x^5 (a+bx^3)^5 (A+Bx^3)\, dx = \frac{1}{24} Bb^5x^{24} + \frac{1}{21} (5\, Bab^4 + Ab^5)x^{21} + \frac{5}{18} (2\, Ba^2b^3 + Aab^4)x^{18} + \frac{2}{3} (Ba^3b^2 + Aa^2b^3)x^{15} + \frac{5}{12} (Ba^4b + 2\, Aa^3b^2)x^{12} + \frac{1}{6} Aa^5x^6 + \frac{1}{9} (Ba^5 + 5\, Aa^4b)x^9$$

input
```
integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/24*B*b^5*x^24 + 1/21*(5*B*a*b^4 + A*b^5)*x^21 + 5/18*(2*B*a^2*b^3 + A*a*
b^4)*x^18 + 2/3*(B*a^3*b^2 + A*a^2*b^3)*x^15 + 5/12*(B*a^4*b + 2*A*a^3*b^2
)*x^12 + 1/6*A*a^5*x^6 + 1/9*(B*a^5 + 5*A*a^4*b)*x^9
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(62) = 124.

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.87

$$\int x^5 (a+bx^3)^5 (A+Bx^3)\, dx = \frac{1}{24} Bb^5x^{24} + \frac{5}{21} Bab^4x^{21} + \frac{1}{21} Ab^5x^{21} + \frac{5}{9} Ba^2b^3x^{18} + \frac{5}{18} Aab^4x^{18} + \frac{2}{3} Ba^3b^2x^{15} + \frac{2}{3} Aa^2b^3x^{15} + \frac{5}{12} Ba^4bx^{12} + \frac{5}{6} Aa^3b^2x^{12} + \frac{1}{9} Ba^5x^9 + \frac{5}{9} Aa^4bx^9 + \frac{1}{6} Aa^5x^6$$

input
```
integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/24*B*b^5*x^24 + 5/21*B*a*b^4*x^21 + 1/21*A*b^5*x^21 + 5/9*B*a^2*b^3*x^18
 + 5/18*A*a*b^4*x^18 + 2/3*B*a^3*b^2*x^15 + 2/3*A*a^2*b^3*x^15 + 5/12*B*a^
4*b*x^12 + 5/6*A*a^3*b^2*x^12 + 1/9*B*a^5*x^9 + 5/9*A*a^4*b*x^9 + 1/6*A*a^
5*x^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.60

$$\int x^5 (a+bx^3)^5 (A+Bx^3)\,dx = x^9\left(\frac{B\,a^5}{9}+\frac{5\,A\,b\,a^4}{9}\right)+x^{21}\left(\frac{A\,b^5}{21}+\frac{5\,B\,a\,b^4}{21}\right) + \frac{A\,a^5\,x^6}{6}+\frac{B\,b^5\,x^{24}}{24}+\frac{2\,a^2\,b^2\,x^{15}\,(A\,b+B\,a)}{3} + \frac{5\,a^3\,b\,x^{12}\,(2\,A\,b+B\,a)}{12}+\frac{5\,a\,b^3\,x^{18}\,(A\,b+2\,B\,a)}{18}$$

input
```
int(x^5*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^9*((B*a^5)/9 + (5*A*a^4*b)/9) + x^21*((A*b^5)/21 + (5*B*a*b^4)/21) + (A*
a^5*x^6)/6 + (B*b^5*x^24)/24 + (2*a^2*b^2*x^15*(A*b + B*a))/3 + (5*a^3*b*x
^12*(2*A*b + B*a))/12 + (5*a*b^3*x^18*(A*b + 2*B*a))/18
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int x^5 (a+bx^3)^5 (A+Bx^3)\,dx = \frac{x^6(7b^6x^{18}+48a\,b^5x^{15}+140a^2b^4x^{12}+224a^3b^3x^9+210a^4b^2x^6+112a^5b\,x^3+28a^6)}{168}$$

input
```
int(x^5*(b*x^3+a)^5*(B*x^3+A),x)
```

output
```
(x**6*(28*a**6 + 112*a**5*b*x**3 + 210*a**4*b**2*x**6 + 224*a**3*b**3*x**9
 + 140*a**2*b**4*x**12 + 48*a*b**5*x**15 + 7*b**6*x**18))/168
```

3.25 $\int x^2(a+bx^3)^5 (A+Bx^3) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int x^2(a+bx^3)^5 (A+Bx^3) \, dx = \frac{(Ab-aB)(a+bx^3)^6}{18b^2} + \frac{B(a+bx^3)^7}{21b^2}$$

output
```
1/18*(A*b-B*a)*(b*x^3+a)^6/b^2+1/21*B*(b*x^3+a)^7/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 107 vs. 2(42) = 84.

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.55

$$\int x^2(a+bx^3)^5 (A+Bx^3) \, dx = \frac{1}{126}x^3 \left(42a^5A + 21a^4(5Ab+aB)x^3 + 70a^3b(2Ab+aB)x^6 + 105a^2b^2(Ab+aB)x^9 + 42ab^3(Ab+2aB)x^{12} + 7b^4(Ab+5aB)x^{15} + 6b^5Bx^{18}\right)$$

input
```
Integrate[x^2*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(x^3*(42*a^5*A + 21*a^4*(5*A*b + a*B)*x^3 + 70*a^3*b*(2*A*b + a*B)*x^6 + 1
05*a^2*b^2*(A*b + a*B)*x^9 + 42*a*b^3*(A*b + 2*a*B)*x^12 + 7*b^4*(A*b + 5*
a*B)*x^15 + 6*b^5*B*x^18))/126
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {946, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3} \int \left(bx^3+a\right)^5 \left(Bx^3+A\right) dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3} \int \left(\frac{B\left(bx^3+a\right)^6}{b}+\frac{(Ab-aB)\left(bx^3+a\right)^5}{b}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{\left(a+bx^3\right)^6 (Ab-aB)}{6b^2}+\frac{B\left(a+bx^3\right)^7}{7b^2}\right)$$

input
```
Int[x^2*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(((A*b - a*B)*(a + b*x^3)^6)/(6*b^2) + (B*(a + b*x^3)^7)/(7*b^2))/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(38) = 76.

Time = 0.68 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.88

method	result
norman	$\frac{a^5 A x^3}{3}+\left(\frac{5}{6}a^4bA+\frac{1}{6}a^5B\right)x^6+\left(\frac{10}{9}a^3b^2A+\frac{5}{9}a^4bB\right)x^9+\left(\frac{5}{6}a^2b^3A+\frac{5}{6}a^3b^2B\right)x^{12}+\left(\frac{1}{3}a\,b^4A\right.$
default	$\frac{b^5 B x^{21}}{21}+\frac{(b^5A+5a\,b^4B)x^{18}}{18}+\frac{(5a\,b^4A+10a^2b^3B)x^{15}}{15}+\frac{(10a^2b^3A+10a^3b^2B)x^{12}}{12}+\frac{(10a^3b^2A+5a^4bB)x^9}{9}+\frac{(5a^4}{}$
gosper	$\frac{1}{3}a^5A\,x^3+\frac{5}{6}x^6a^4bA+\frac{1}{6}B\,a^5x^6+\frac{10}{9}x^9a^3b^2A+\frac{5}{9}x^9a^4bB+\frac{5}{6}x^{12}a^2b^3A+\frac{5}{6}x^{12}a^3b^2B+\frac{1}{3}x^{15}$
risch	$\frac{1}{3}a^5A\,x^3+\frac{5}{6}x^6a^4bA+\frac{1}{6}B\,a^5x^6+\frac{10}{9}x^9a^3b^2A+\frac{5}{9}x^9a^4bB+\frac{5}{6}x^{12}a^2b^3A+\frac{5}{6}x^{12}a^3b^2B+\frac{1}{3}x^{15}$
parallelrisch	$\frac{1}{3}a^5A\,x^3+\frac{5}{6}x^6a^4bA+\frac{1}{6}B\,a^5x^6+\frac{10}{9}x^9a^3b^2A+\frac{5}{9}x^9a^4bB+\frac{5}{6}x^{12}a^2b^3A+\frac{5}{6}x^{12}a^3b^2B+\frac{1}{3}x^{15}$
orering	$\frac{x^3(6b^5Bx^{18}+7Ab^5x^{15}+35Ba\,b^4x^{15}+42aA\,b^4x^{12}+84B\,a^2b^3x^{12}+105a^2A\,b^3x^9+105B\,a^3b^2x^9+140a^3A\,b^2x^6+70B\,a^4b\,x^6+}{126}$

input
```
int(x^2*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/3*a^5*A*x^3+(5/6*a^4*b*A+1/6*a^5*B)*x^6+(10/9*a^3*b^2*A+5/9*a^4*b*B)*x^9
+(5/6*a^2*b^3*A+5/6*a^3*b^2*B)*x^12+(1/3*a*b^4*A+2/3*a^2*b^3*B)*x^15+(1/18
*b^5*A+5/18*a*b^4*B)*x^18+1/21*b^5*B*x^21
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(38) = 76.

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.83

$$\int x^2(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{21}Bb^5x^{21} + \frac{1}{18}(5Bab^4 + Ab^5)x^{18} + \frac{1}{3}(2Ba^2b^3 + Aab^4)x^{15} + \frac{5}{6}(Ba^3b^2 + Aa^2b^3)x^{12} + \frac{5}{9}(Ba^4b + 2Aa^3b^2)x^9 + \frac{1}{3}Aa^5x^3 + \frac{1}{6}(Ba^5 + 5Aa^4b)x^6$$

input
```
integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/21*B*b^5*x^21 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 1/3*(2*B*a^2*b^3 + A*a*b
^4)*x^15 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 5/9*(B*a^4*b + 2*A*a^3*b^2)*
x^9 + 1/3*A*a^5*x^3 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(36) = 72.

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.24

$$\int x^2(a+bx^3)^5(A+Bx^3)\,dx = \frac{Aa^5x^3}{3} + \frac{Bb^5x^{21}}{21} + x^{18}\left(\frac{Ab^5}{18} + \frac{5Bab^4}{18}\right) + x^{15}\left(\frac{Aab^4}{3} + \frac{2Ba^2b^3}{3}\right) + x^{12}\cdot\left(\frac{5Aa^2b^3}{6} + \frac{5Ba^3b^2}{6}\right) + x^9\cdot\left(\frac{10Aa^3b^2}{9} + \frac{5Ba^4b}{9}\right) + x^6\cdot\left(\frac{5Aa^4b}{6} + \frac{Ba^5}{6}\right)$$

input
```
integrate(x**2*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x**3/3 + B*b**5*x**21/21 + x**18*(A*b**5/18 + 5*B*a*b**4/18) + x**1
5*(A*a*b**4/3 + 2*B*a**2*b**3/3) + x**12*(5*A*a**2*b**3/6 + 5*B*a**3*b**2/
6) + x**9*(10*A*a**3*b**2/9 + 5*B*a**4*b/9) + x**6*(5*A*a**4*b/6 + B*a**5/
6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(38) = 76.

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.83

$$\int x^2 (a + bx^3)^5 (A + Bx^3) \, dx = \frac{1}{21} Bb^5 x^{21} + \frac{1}{18} (5 Bab^4 + Ab^5) x^{18} + \frac{1}{3} (2 Ba^2 b^3 + Aab^4) x^{15} + \frac{5}{6} (Ba^3 b^2 + Aa^2 b^3) x^{12} + \frac{5}{9} (Ba^4 b + 2 Aa^3 b^2) x^9 + \frac{1}{3} Aa^5 x^3 + \frac{1}{6} (Ba^5 + 5 Aa^4 b) x^6$$

input
```
integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/21*B*b^5*x^21 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 1/3*(2*B*a^2*b^3 + A*a*b
^4)*x^15 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 5/9*(B*a^4*b + 2*A*a^3*b^2)*
x^9 + 1/3*A*a^5*x^3 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(38) = 76.

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.98

$$\int x^2 (a + bx^3)^5 (A + Bx^3) \, dx = \frac{1}{21} Bb^5 x^{21} + \frac{5}{18} Bab^4 x^{18} + \frac{1}{18} Ab^5 x^{18} + \frac{2}{3} Ba^2 b^3 x^{15} + \frac{1}{3} Aab^4 x^{15} + \frac{5}{6} Ba^3 b^2 x^{12} + \frac{5}{6} Aa^2 b^3 x^{12} + \frac{5}{9} Ba^4 b x^9 + \frac{10}{9} Aa^3 b^2 x^9 + \frac{1}{6} Ba^5 x^6 + \frac{5}{6} Aa^4 b x^6 + \frac{1}{3} Aa^5 x^3$$

input
```
integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/21*B*b^5*x^21 + 5/18*B*a*b^4*x^18 + 1/18*A*b^5*x^18 + 2/3*B*a^2*b^3*x^15
 + 1/3*A*a*b^4*x^15 + 5/6*B*a^3*b^2*x^12 + 5/6*A*a^2*b^3*x^12 + 5/9*B*a^4*
b*x^9 + 10/9*A*a^3*b^2*x^9 + 1/6*B*a^5*x^6 + 5/6*A*a^4*b*x^6 + 1/3*A*a^5*x
^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.55

$$\int x^2 \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx = x^6\left(\frac{B\,a^5}{6}+\frac{5\,A\,b\,a^4}{6}\right)+x^{18}\left(\frac{A\,b^5}{18}+\frac{5\,B\,a\,b^4}{18}\right) + \frac{A\,a^5\,x^3}{3}+\frac{B\,b^5\,x^{21}}{21}+\frac{5\,a^2\,b^2\,x^{12}\,\left(A\,b+B\,a\right)}{6} + \frac{5\,a^3\,b\,x^9\,\left(2\,A\,b+B\,a\right)}{9}+\frac{a\,b^3\,x^{15}\,\left(A\,b+2\,B\,a\right)}{3}$$

input
```
int(x^2*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^6*((B*a^5)/6 + (5*A*a^4*b)/6) + x^18*((A*b^5)/18 + (5*B*a*b^4)/18) + (A*
a^5*x^3)/3 + (B*b^5*x^21)/21 + (5*a^2*b^2*x^12*(A*b + B*a))/6 + (5*a^3*b*x
^9*(2*A*b + B*a))/9 + (a*b^3*x^15*(A*b + 2*B*a))/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.64

$$\int x^2 \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx = \frac{x^3\left(b^6x^{18}+7a\,b^5x^{15}+21a^2b^4x^{12}+35a^3b^3x^9+35a^4b^2x^6+21a^5b\,x^3+7a^6\right)}{21}$$

input
```
int(x^2*(b*x^3+a)^5*(B*x^3+A),x)
```

output

```
(x**3*(7*a**6 + 21*a**5*b*x**3 + 35*a**4*b**2*x**6 + 35*a**3*b**3*x**9 + 2
1*a**2*b**4*x**12 + 7*a*b**5*x**15 + b**6*x**18))/21
```

3.26 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x} \, dx = \frac{5}{3}a^4Abx^3 + \frac{5}{3}a^3Ab^2x^6 + \frac{10}{9}a^2Ab^3x^9 + \frac{5}{12}aAb^4x^{12} + \frac{1}{15}Ab^5x^{15} + \frac{B(a+bx^3)^6}{18b} + a^5A\log(x)$$

output
```
5/3*a^4*A*b*x^3+5/3*a^3*A*b^2*x^6+10/9*a^2*A*b^3*x^9+5/12*a*A*b^4*x^12+1/1
5*A*b^5*x^15+1/18*B*(b*x^3+a)^6/b+a^5*A*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x} \, dx = \frac{1}{3}a^4(5Ab+aB)x^3 + \frac{5}{6}a^3b(2Ab+aB)x^6 + \frac{10}{9}a^2b^2(Ab+aB)x^9 + \frac{5}{12}ab^3(Ab+2aB)x^{12} + \frac{1}{15}b^4(Ab+5aB)x^{15} + \frac{1}{18}b^5Bx^{18} + a^5A\log(x)$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x,x]
```

output
```
(a^4*(5*A*b + a*B)*x^3)/3 + (5*a^3*b*(2*A*b + a*B)*x^6)/6 + (10*a^2*b^2*(A
*b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B)*x^12)/12 + (b^4*(A*b + 5*a*B)*x^
15)/15 + (b^5*B*x^18)/18 + a^5*A*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {948, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5(Bx^3+A)}{x^3}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(A\int \frac{(bx^3+a)^5}{x^3}dx^3+\frac{B(a+bx^3)^6}{6b}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\left(A\int \left(b^5x^{12}+5ab^4x^9+10a^2b^3x^6+10a^3b^2x^3+5a^4b+\frac{a^5}{x^3}\right)dx^3+\frac{B(a+bx^3)^6}{6b}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(A\left(a^5\log\left(x^3\right)+5a^4bx^3+5a^3b^2x^6+\frac{10}{3}a^2b^3x^9+\frac{5}{4}ab^4x^{12}+\frac{b^5x^{15}}{5}\right)+\frac{B(a+bx^3)^6}{6b}\right)$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x,x]
```

output
```
((B*(a + b*x^3)^6)/(6*b) + A*(5*a^4*b*x^3 + 5*a^3*b^2*x^6 + (10*a^2*b^3*x^
9)/3 + (5*a*b^4*x^12)/4 + (b^5*x^15)/5 + a^5*Log[x^3]))/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.35

method	result
norman	$\left(\frac{1}{15}b^5A+\frac{1}{3}a\,b^4B\right)x^{15}+\left(\frac{5}{12}a\,b^4A+\frac{5}{6}a^2b^3B\right)x^{12}+\left(\frac{10}{9}a^2b^3A+\frac{10}{9}a^3b^2B\right)x^9+\left(\frac{5}{3}a^3b^2A+\right.$
default	$\frac{b^5B\,x^{18}}{18}+\frac{A\,b^5x^{15}}{15}+\frac{Ba\,b^4x^{15}}{3}+\frac{5aA\,b^4x^{12}}{12}+\frac{5B\,a^2b^3x^{12}}{6}+\frac{10a^2A\,b^3x^9}{9}+\frac{10B\,a^3b^2x^9}{9}+\frac{5a^3A\,b^2x^6}{3}+$
risch	$\frac{b^5B\,x^{18}}{18}+\frac{A\,b^5x^{15}}{15}+\frac{Ba\,b^4x^{15}}{3}+\frac{5aA\,b^4x^{12}}{12}+\frac{5B\,a^2b^3x^{12}}{6}+\frac{10a^2A\,b^3x^9}{9}+\frac{10B\,a^3b^2x^9}{9}+\frac{5a^3A\,b^2x^6}{3}+$
parallelrisch	$\frac{b^5B\,x^{18}}{18}+\frac{A\,b^5x^{15}}{15}+\frac{Ba\,b^4x^{15}}{3}+\frac{5aA\,b^4x^{12}}{12}+\frac{5B\,a^2b^3x^{12}}{6}+\frac{10a^2A\,b^3x^9}{9}+\frac{10B\,a^3b^2x^9}{9}+\frac{5a^3A\,b^2x^6}{3}+$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/15*b^5*A+1/3*a*b^4*B)*x^15+(5/12*a*b^4*A+5/6*a^2*b^3*B)*x^12+(10/9*a^2*
b^3*A+10/9*a^3*b^2*B)*x^9+(5/3*a^3*b^2*A+5/6*a^4*b*B)*x^6+(5/3*a^4*b*A+1/3
*a^5*B)*x^3+1/18*b^5*B*x^18+a^5*A*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x} dx = \frac{1}{18} Bb^5x^{18} + \frac{1}{15} (5 Bab^4 + Ab^5)x^{15} + \frac{5}{12} (2 Ba^2b^3 + Aab^4)x^{12} + \frac{10}{9} (Ba^3b^2 + Aa^2b^3)x^9 + \frac{5}{6} (Ba^4b + 2 Aa^3b^2)x^6 + Aa^5 \log(x) + \frac{1}{3} (Ba^5 + 5 Aa^4b)x^3$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="fricas")
```

output
```
1/18*B*b^5*x^18 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/12*(2*B*a^2*b^3 + A*a*
b^4)*x^12 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5/6*(B*a^4*b + 2*A*a^3*b^2)
*x^6 + A*a^5*log(x) + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x} dx = Aa^5 \log(x) + \frac{Bb^5x^{18}}{18} + x^{15} \left(\frac{Ab^5}{15} + \frac{Bab^4}{3}\right) + x^{12} \cdot \left(\frac{5Aab^4}{12} + \frac{5Ba^2b^3}{6}\right) + x^9 \cdot \left(\frac{10Aa^2b^3}{9} + \frac{10Ba^3b^2}{9}\right) + x^6 \cdot \left(\frac{5Aa^3b^2}{3} + \frac{5Ba^4b}{6}\right) + x^3 \cdot \left(\frac{5Aa^4b}{3} + \frac{Ba^5}{3}\right)$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x,x)
```

output
```
A*a**5*log(x) + B*b**5*x**18/18 + x**15*(A*b**5/15 + B*a*b**4/3) + x**12*(
5*A*a*b**4/12 + 5*B*a**2*b**3/6) + x**9*(10*A*a**2*b**3/9 + 10*B*a**3*b**2
/9) + x**6*(5*A*a**3*b**2/3 + 5*B*a**4*b/6) + x**3*(5*A*a**4*b/3 + B*a**5/
3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x}\,dx = \frac{1}{18}\,Bb^5x^{18} + \frac{1}{15}\left(5\,Bab^4 + Ab^5\right)x^{15} + \frac{5}{12}\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + \frac{10}{9}\left(Ba^3b^2 + Aa^2b^3\right)x^9 + \frac{5}{6}\left(Ba^4b + 2\,Aa^3b^2\right)x^6 + \frac{1}{3}\,Aa^5\log\left(x^3\right) + \frac{1}{3}\left(Ba^5 + 5\,Aa^4b\right)x^3$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="maxima")
```

output
```
1/18*B*b^5*x^18 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/12*(2*B*a^2*b^3 + A*a*
b^4)*x^12 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5/6*(B*a^4*b + 2*A*a^3*b^2)
*x^6 + 1/3*A*a^5*log(x^3) + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x}\,dx = \frac{1}{18}\,Bb^5x^{18} + \frac{1}{3}\,Bab^4x^{15} + \frac{1}{15}\,Ab^5x^{15} + \frac{5}{6}\,Ba^2b^3x^{12} + \frac{5}{12}\,Aab^4x^{12} + \frac{10}{9}\,Ba^3b^2x^9 + \frac{10}{9}\,Aa^2b^3x^9 + \frac{5}{6}\,Ba^4bx^6 + \frac{5}{3}\,Aa^3b^2x^6 + \frac{1}{3}\,Ba^5x^3 + \frac{5}{3}\,Aa^4bx^3 + Aa^5\log\left(|x|\right)$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="giac")
```

output
```
1/18*B*b^5*x^18 + 1/3*B*a*b^4*x^15 + 1/15*A*b^5*x^15 + 5/6*B*a^2*b^3*x^12
+ 5/12*A*a*b^4*x^12 + 10/9*B*a^3*b^2*x^9 + 10/9*A*a^2*b^3*x^9 + 5/6*B*a^4*
b*x^6 + 5/3*A*a^3*b^2*x^6 + 1/3*B*a^5*x^3 + 5/3*A*a^4*b*x^3 + A*a^5*log(ab
s(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x} \, dx = x^3 \left(\frac{B\,a^5}{3}+\frac{5\,A\,b\,a^4}{3}\right)+x^{15}\left(\frac{A\,b^5}{15}+\frac{B\,a\,b^4}{3}\right) + \frac{B\,b^5\,x^{18}}{18}+A\,a^5\,\ln(x)+\frac{10\,a^2\,b^2\,x^9\,(A\,b+B\,a)}{9} + \frac{5\,a^3\,b\,x^6\,(2\,A\,b+B\,a)}{6}+\frac{5\,a\,b^3\,x^{12}\,(A\,b+2\,B\,a)}{12}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x,x)
```

output
```
x^3*((B*a^5)/3 + (5*A*a^4*b)/3) + x^15*((A*b^5)/15 + (B*a*b^4)/3) + (B*b^5
*x^18)/18 + A*a^5*log(x) + (10*a^2*b^2*x^9*(A*b + B*a))/9 + (5*a^3*b*x^6*(
2*A*b + B*a))/6 + (5*a*b^3*x^12*(A*b + 2*B*a))/12
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x} \, dx = \log(x)\,a^6+2a^5b\,x^3+\frac{5a^4b^2x^6}{2}+\frac{20a^3b^3x^9}{9} + \frac{5a^2b^4x^{12}}{4}+\frac{2a\,b^5x^{15}}{5}+\frac{b^6x^{18}}{18}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x,x)
```

output
```
(180*log(x)*a**6 + 360*a**5*b*x**3 + 450*a**4*b**2*x**6 + 400*a**3*b**3*x**
*9 + 225*a**2*b**4*x**12 + 72*a*b**5*x**15 + 10*b**6*x**18)/180
```

3.27 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^4}\,dx = -\frac{a^5A}{3x^3}+\frac{5}{3}a^3b(2Ab+aB)x^3+\frac{5}{3}a^2b^2(Ab+aB)x^6 +\frac{5}{9}ab^3(Ab+2aB)x^9+\frac{1}{12}b^4(Ab+5aB)x^{12} +\frac{1}{15}b^5Bx^{15}+a^4(5Ab+aB)\log(x)$$

output
```
-1/3*a^5*A/x^3+5/3*a^3*b*(2*A*b+B*a)*x^3+5/3*a^2*b^2*(A*b+B*a)*x^6+5/9*a*b
^3*(A*b+2*B*a)*x^9+1/12*b^4*(A*b+5*B*a)*x^12+1/15*b^5*B*x^15+a^4*(5*A*b+B*
a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^4}\,dx = -\frac{a^5A}{3x^3}+\frac{5}{3}a^3b(2Ab+aB)x^3+\frac{5}{3}a^2b^2(Ab+aB)x^6 +\frac{5}{9}ab^3(Ab+2aB)x^9+\frac{1}{12}b^4(Ab+5aB)x^{12} +\frac{1}{15}b^5Bx^{15}+\left(5a^4Ab+a^5B\right)\log(x)$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^4,x]
```

output
```
-1/3*(a^5*A)/x^3 + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + (5*a^2*b^2*(A*b + a*B)*
x^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^12)/12 + (b^
5*B*x^15)/15 + (5*a^4*A*b + a^5*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^4} \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{\left(bx^3+a\right)^5 \left(Bx^3+A\right)}{x^6}dx^3$$

$$\Big\downarrow \text{ 85}$$

$$\frac{1}{3}\int \left(b^5Bx^{12}+b^4(Ab+5aB)x^9+5ab^3(Ab+2aB)x^6+10a^2b^2(Ab+aB)x^3+5a^3b(2Ab+aB)+\frac{a^4(5Ab+aB}{x^3}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{a^5A}{x^3}+a^4\log\left(x^3\right)(aB+5Ab)+5a^3bx^3(aB+2Ab)+5a^2b^2x^6(aB+Ab)+\frac{1}{4}b^4x^{12}(5aB+Ab)+\frac{5}{3}ab^3x^9(2\right.$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^4,x]
```

output
```
(-((a^5*A)/x^3) + 5*a^3*b*(2*A*b + a*B)*x^3 + 5*a^2*b^2*(A*b + a*B)*x^6 +
(5*a*b^3*(A*b + 2*a*B)*x^9)/3 + (b^4*(A*b + 5*a*B)*x^12)/4 + (b^5*B*x^15)/
5 + a^4*(5*A*b + a*B)*Log[x^3])/3
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

method	result
default	$\frac{b^5 B x^{15}}{15}+\frac{A b^5 x^{12}}{12}+\frac{5 B a b^4 x^{12}}{12}+\frac{5 a A b^4 x^9}{9}+\frac{10 B a^2 b^3 x^9}{9}+\frac{5 a^2 A b^3 x^6}{3}+\frac{5 B a^3 b^2 x^6}{3}+\frac{10 a^3 A b^2 x^3}{3}+\frac{5 B a^4 b}{3}$
norman	$\frac{\left(\frac{1}{12} b^5 A+\frac{5}{12} a b^4 B\right) x^{15}+\left(\frac{5}{9} a b^4 A+\frac{10}{9} a^2 b^3 B\right) x^{12}+\left(\frac{5}{3} a^2 b^3 A+\frac{5}{3} a^3 b^2 B\right) x^9+\left(\frac{10}{3} a^3 b^2 A+\frac{5}{3} a^4 b B\right) x^6-\frac{a^5 A}{3}+\frac{b^5 B x^{18}}{15}}{x^3}+(5a^4$
risch	$\frac{b^5 B x^{15}}{15}+\frac{A b^5 x^{12}}{12}+\frac{5 B a b^4 x^{12}}{12}+\frac{5 a A b^4 x^9}{9}+\frac{10 B a^2 b^3 x^9}{9}+\frac{5 a^2 A b^3 x^6}{3}+\frac{5 B a^3 b^2 x^6}{3}+\frac{10 a^3 A b^2 x^3}{3}+\frac{5 B a^4 b}{3}$
parallelrisch	$\frac{12 b^5 B x^{18}+15 A b^5 x^{15}+75 B a b^4 x^{15}+100 a A b^4 x^{12}+200 B a^2 b^3 x^{12}+300 a^2 A b^3 x^9+300 B a^3 b^2 x^9+600 a^3 A b^2 x^6+300 B a^4 b x^6}{180 x^3}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/15*b^5*B*x^15+1/12*A*b^5*x^12+5/12*B*a*b^4*x^12+5/9*a*A*b^4*x^9+10/9*B*a
^2*b^3*x^9+5/3*a^2*A*b^3*x^6+5/3*B*a^3*b^2*x^6+10/3*a^3*A*b^2*x^3+5/3*B*a^
4*b*x^3-1/3*a^5*A/x^3+a^4*(5*A*b+B*a)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^4} dx$$
$$= \frac{12 Bb^5x^{18} + 15 (5 Bab^4 + Ab^5)x^{15} + 100 (2 Ba^2b^3 + Aab^4)x^{12} + 300 (Ba^3b^2 + Aa^2b^3)x^9 + 300 (Ba^4b + 2}{180 x^3}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="fricas")
```

output
```
1/180*(12*B*b^5*x^18 + 15*(5*B*a*b^4 + A*b^5)*x^15 + 100*(2*B*a^2*b^3 + A*
a*b^4)*x^12 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 300*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 60*A*a^5 + 180*(B*a^5 + 5*A*a^4*b)*x^3*log(x))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^4} dx = -\frac{Aa^5}{3x^3} + \frac{Bb^5x^{15}}{15} + a^4 \cdot (5Ab + Ba) \log(x) + x^{12} \left(\frac{Ab^5}{12} + \frac{5Bab^4}{12}\right) + x^9 \cdot \left(\frac{5Aab^4}{9} + \frac{10Ba^2b^3}{9}\right) + x^6 \cdot \left(\frac{5Aa^2b^3}{3} + \frac{5Ba^3b^2}{3}\right) + x^3 \cdot \left(\frac{10Aa^3b^2}{3} + \frac{5Ba^4b}{3}\right)$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**4,x)
```

output
```
-A*a**5/(3*x**3) + B*b**5*x**15/15 + a**4*(5*A*b + B*a)*log(x) + x**12*(A*
b**5/12 + 5*B*a*b**4/12) + x**9*(5*A*a*b**4/9 + 10*B*a**2*b**3/9) + x**6*(
5*A*a**2*b**3/3 + 5*B*a**3*b**2/3) + x**3*(10*A*a**3*b**2/3 + 5*B*a**4*b/3
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^4} dx = \frac{1}{15} Bb^5x^{15} + \frac{1}{12} (5 Bab^4 + Ab^5)x^{12} + \frac{5}{9} (2 Ba^2b^3 + Aab^4)x^9 + \frac{5}{3} (Ba^3b^2 + Aa^2b^3)x^6 + \frac{5}{3} (Ba^4b + 2 Aa^3b^2)x^3 - \frac{Aa^5}{3 x^3} + \frac{1}{3} (Ba^5 + 5 Aa^4b) \log (x^3)$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="maxima")
```

output
```
1/15*B*b^5*x^15 + 1/12*(5*B*a*b^4 + A*b^5)*x^12 + 5/9*(2*B*a^2*b^3 + A*a*b
^4)*x^9 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 5/3*(B*a^4*b + 2*A*a^3*b^2)*x^
3 - 1/3*A*a^5/x^3 + 1/3*(B*a^5 + 5*A*a^4*b)*log(x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^4} dx = \frac{1}{15} Bb^5x^{15} + \frac{5}{12} Bab^4x^{12} + \frac{1}{12} Ab^5x^{12} + \frac{10}{9} Ba^2b^3x^9 + \frac{5}{9} Aab^4x^9 + \frac{5}{3} Ba^3b^2x^6 + \frac{5}{3} Aa^2b^3x^6 + \frac{5}{3} Ba^4bx^3 + \frac{10}{3} Aa^3b^2x^3 + (Ba^5 + 5 Aa^4b) \log (|x|) - \frac{Ba^5x^3 + 5 Aa^4bx^3 + Aa^5}{3 x^3}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="giac")
```

output
```
1/15*B*b^5*x^15 + 5/12*B*a*b^4*x^12 + 1/12*A*b^5*x^12 + 10/9*B*a^2*b^3*x^9
 + 5/9*A*a*b^4*x^9 + 5/3*B*a^3*b^2*x^6 + 5/3*A*a^2*b^3*x^6 + 5/3*B*a^4*b*x
^3 + 10/3*A*a^3*b^2*x^3 + (B*a^5 + 5*A*a^4*b)*log(abs(x)) - 1/3*(B*a^5*x^3
 + 5*A*a^4*b*x^3 + A*a^5)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^4}\,dx = x^{12}\left(\frac{A\,b^5}{12}+\frac{5\,B\,a\,b^4}{12}\right)+\ln(x)\,\left(B\,a^5+5\,A\,b\,a^4\right) - \frac{A\,a^5}{3\,x^3}+\frac{B\,b^5\,x^{15}}{15}+\frac{5\,a^2\,b^2\,x^6\,(A\,b+B\,a)}{3} + \frac{5\,a^3\,b\,x^3\,(2\,A\,b+B\,a)}{3}+\frac{5\,a\,b^3\,x^9\,(A\,b+2\,B\,a)}{9}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^4,x)
```

output
```
x^12*((A*b^5)/12 + (5*B*a*b^4)/12) + log(x)*(B*a^5 + 5*A*a^4*b) - (A*a^5)/
(3*x^3) + (B*b^5*x^15)/15 + (5*a^2*b^2*x^6*(A*b + B*a))/3 + (5*a^3*b*x^3*(
2*A*b + B*a))/3 + (5*a*b^3*x^9*(A*b + 2*B*a))/9
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^4}\,dx = \frac{180\log(x)\,a^5b\,x^3-10a^6+150a^4b^2x^6+100a^3b^3x^9+50a^2b^4x^{12}+15a\,b^5x^{15}+2b^6x^{18}}{30x^3}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^4,x)
```

output
```
(180*log(x)*a**5*b*x**3 - 10*a**6 + 150*a**4*b**2*x**6 + 100*a**3*b**3*x**
9 + 50*a**2*b**4*x**12 + 15*a*b**5*x**15 + 2*b**6*x**18)/(30*x**3)
```

3.28 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^7} dx$

Optimal result

Integrand size = 20, antiderivative size = 114

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^7} dx = -\frac{a^5A}{6x^6} - \frac{a^4(5Ab+aB)}{3x^3} + \frac{10}{3}a^2b^2(Ab+aB)x^3 + \frac{5}{6}ab^3(Ab+2aB)x^6 + \frac{1}{9}b^4(Ab+5aB)x^9 + \frac{1}{12}b^5Bx^{12} + 5a^3b(2Ab+aB)\log(x)$$

output
```
-1/6*a^5*A/x^6-1/3*a^4*(5*A*b+B*a)/x^3+10/3*a^2*b^2*(A*b+B*a)*x^3+5/6*a*b^
3*(A*b+2*B*a)*x^6+1/9*b^4*(A*b+5*B*a)*x^9+1/12*b^5*B*x^12+5*a^3*b*(2*A*b+B
*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^7} dx = \frac{1}{36}\left(-\frac{6a^5A}{x^6} - \frac{12a^4(5Ab+aB)}{x^3} + 120a^2b^2(Ab+aB)x^3 + 30ab^3(Ab+2aB)x^6 + 4b^4(Ab+5aB)x^9 + 3b^5Bx^{12} + 180a^3b(2Ab+aB)\log(x)\right)$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^7,x]
```

output
```
((-6*a^5*A)/x^6 - (12*a^4*(5*A*b + a*B))/x^3 + 120*a^2*b^2*(A*b + a*B)*x^3
 + 30*a*b^3*(A*b + 2*a*B)*x^6 + 4*b^4*(A*b + 5*a*B)*x^9 + 3*b^5*B*x^12 + 1
80*a^3*b*(2*A*b + a*B)*Log[x])/36
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^7} dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5 (Bx^3+A)}{x^9}dx^3$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{3}\int \left(b^5Bx^9+b^4(Ab+5aB)x^6+5ab^3(Ab+2aB)x^3+10a^2b^2(Ab+aB)+\frac{5a^3b(2Ab+aB)}{x^3}+\frac{a^4(5Ab+aB)}{x^6}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^5A}{2x^6}-\frac{a^4(aB+5Ab)}{x^3}+5a^3b\log\left(x^3\right)(aB+2Ab)+10a^2b^2x^3(aB+Ab)+\frac{1}{3}b^4x^9(5aB+Ab)+\frac{5}{2}ab^3x^6(2a\right.$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^7,x]
```

output
```
(-1/2*(a^5*A)/x^6 - (a^4*(5*A*b + a*B))/x^3 + 10*a^2*b^2*(A*b + a*B)*x^3 +
 (5*a*b^3*(A*b + 2*a*B)*x^6)/2 + (b^4*(A*b + 5*a*B)*x^9)/3 + (b^5*B*x^12)/
4 + 5*a^3*b*(2*A*b + a*B)*Log[x^3])/3
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03

method	result
default	$\frac{b^5 B x^{12}}{12}+\frac{A b^5 x^9}{9}+\frac{5 B a b^4 x^9}{9}+\frac{5 A a b^4 x^6}{6}+\frac{5 B a^2 b^3 x^6}{3}+\frac{10 a^2 A b^3 x^3}{3}+\frac{10 B a^3 b^2 x^3}{3}-\frac{a^4 (5 A b+B a)}{3 x^3}+5 a^3 b$
norman	$\frac{(\frac{1}{9} b^5 A+\frac{5}{9} a b^4 B) x^{15}+(\frac{5}{6} a b^4 A+\frac{5}{3} a^2 b^3 B) x^{12}+(\frac{10}{3} a^2 b^3 A+\frac{10}{3} a^3 b^2 B) x^9+(-\frac{5}{3} a^4 b A-\frac{1}{3} a^5 B) x^3-\frac{a^5 A}{6}+\frac{b^5 B x^{18}}{12}}{x^6}+(10 a^3 b$
risch	$\frac{b^5 B x^{12}}{12}+\frac{A b^5 x^9}{9}+\frac{5 B a b^4 x^9}{9}+\frac{5 A a b^4 x^6}{6}+\frac{5 B a^2 b^3 x^6}{3}+\frac{10 a^2 A b^3 x^3}{3}+\frac{10 B a^3 b^2 x^3}{3}+\frac{(-\frac{5}{3} a^4 b A-\frac{1}{3} a^5 B) x^3-}{x^6}$
parallelrisch	$\frac{3 b^5 B x^{18}+4 A b^5 x^{15}+20 B a b^4 x^{15}+30 a A b^4 x^{12}+60 B a^2 b^3 x^{12}+120 a^2 A b^3 x^9+120 B a^3 b^2 x^9+360 A \ln(x) x^6 a^3 b^2+180 B \ln(x) x}{36 x^6}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/12*b^5*B*x^12+1/9*A*b^5*x^9+5/9*B*a*b^4*x^9+5/6*A*a*b^4*x^6+5/3*B*a^2*b^
3*x^6+10/3*a^2*A*b^3*x^3+10/3*B*a^3*b^2*x^3-1/3*a^4*(5*A*b+B*a)/x^3+5*a^3*
b*(2*A*b+B*a)*ln(x)-1/6*a^5*A/x^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^7}\,dx$$
$$= \frac{3\,Bb^5x^{18} + 4\,(5\,Bab^4 + Ab^5)x^{15} + 30\,(2\,Ba^2b^3 + Aab^4)x^{12} + 120\,(Ba^3b^2 + Aa^2b^3)x^9 + 180\,(Ba^4b + 2\,Aa}{36\,x^6}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="fricas")
```

output
```
1/36*(3*B*b^5*x^18 + 4*(5*B*a*b^4 + A*b^5)*x^15 + 30*(2*B*a^2*b^3 + A*a*b^
4)*x^12 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 180*(B*a^4*b + 2*A*a^3*b^2)*x^
6*log(x) - 6*A*a^5 - 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^7}\,dx = \frac{Bb^5x^{12}}{12} + 5a^3b(2Ab+Ba)\log(x) + x^9\left(\frac{Ab^5}{9} + \frac{5Bab^4}{9}\right) + x^6\cdot\left(\frac{5Aab^4}{6} + \frac{5Ba^2b^3}{3}\right) + x^3\cdot\left(\frac{10Aa^2b^3}{3} + \frac{10Ba^3b^2}{3}\right) + \frac{-Aa^5 + x^3(-10Aa^4b - 2Ba^5)}{6x^6}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**7,x)
```

output
```
B*b**5*x**12/12 + 5*a**3*b*(2*A*b + B*a)*log(x) + x**9*(A*b**5/9 + 5*B*a*b
**4/9) + x**6*(5*A*a*b**4/6 + 5*B*a**2*b**3/3) + x**3*(10*A*a**2*b**3/3 +
10*B*a**3*b**2/3) + (-A*a**5 + x**3*(-10*A*a**4*b - 2*B*a**5))/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^7}\,dx = \frac{1}{12}\,Bb^5x^{12} + \frac{1}{9}\left(5\,Bab^4 + Ab^5\right)x^9 + \frac{5}{6}\left(2\,Ba^2b^3 + Aab^4\right)x^6 + \frac{10}{3}\left(Ba^3b^2 + Aa^2b^3\right)x^3 + \frac{5}{3}\left(Ba^4b + 2\,Aa^3b^2\right)\log\left(x^3\right) - \frac{Aa^5 + 2\,(Ba^5 + 5\,Aa^4b)x^3}{6\,x^6}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="maxima")
```

output
```
1/12*B*b^5*x^12 + 1/9*(5*B*a*b^4 + A*b^5)*x^9 + 5/6*(2*B*a^2*b^3 + A*a*b^4
)*x^6 + 10/3*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 5/3*(B*a^4*b + 2*A*a^3*b^2)*log
(x^3) - 1/6*(A*a^5 + 2*(B*a^5 + 5*A*a^4*b)*x^3)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^7}\,dx = \frac{1}{12}\,Bb^5x^{12} + \frac{5}{9}\,Bab^4x^9 + \frac{1}{9}\,Ab^5x^9 + \frac{5}{3}\,Ba^2b^3x^6 + \frac{5}{6}\,Aab^4x^6 + \frac{10}{3}\,Ba^3b^2x^3 + \frac{10}{3}\,Aa^2b^3x^3 + 5\left(Ba^4b + 2\,Aa^3b^2\right)\log\left(|x|\right) - \frac{15\,Ba^4bx^6 + 30\,Aa^3b^2x^6 + 2\,Ba^5x^3 + 10\,Aa^4bx^3 + Aa^5}{6\,x^6}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="giac")
```

output
```
1/12*B*b^5*x^12 + 5/9*B*a*b^4*x^9 + 1/9*A*b^5*x^9 + 5/3*B*a^2*b^3*x^6 + 5/
6*A*a*b^4*x^6 + 10/3*B*a^3*b^2*x^3 + 10/3*A*a^2*b^3*x^3 + 5*(B*a^4*b + 2*A
*a^3*b^2)*log(abs(x)) - 1/6*(15*B*a^4*b*x^6 + 30*A*a^3*b^2*x^6 + 2*B*a^5*x
^3 + 10*A*a^4*b*x^3 + A*a^5)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^7} \, dx = \ln(x) \left(5\,B\,a^4\,b+10\,A\,a^3\,b^2\right) - \frac{\frac{A\,a^5}{6}+x^3\left(\frac{B\,a^5}{3}+\frac{5\,A\,b\,a^4}{3}\right)}{x^6} + x^9\left(\frac{A\,b^5}{9}+\frac{5\,B\,a\,b^4}{9}\right) + \frac{B\,b^5\,x^{12}}{12} + \frac{10\,a^2\,b^2\,x^3\,(A\,b+B\,a)}{3} + \frac{5\,a\,b^3\,x^6\,(A\,b+2\,B\,a)}{6}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^7,x)
```

output
```
log(x)*(10*A*a^3*b^2 + 5*B*a^4*b) - ((A*a^5)/6 + x^3*((B*a^5)/3 + (5*A*a^4
*b)/3))/x^6 + x^9*((A*b^5)/9 + (5*B*a*b^4)/9) + (B*b^5*x^12)/12 + (10*a^2*
b^2*x^3*(A*b + B*a))/3 + (5*a*b^3*x^6*(A*b + 2*B*a))/6
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^7} \, dx = \frac{180\log(x)\,a^4b^2x^6 - 2a^6 - 24a^5b\,x^3 + 80a^3b^3x^9 + 30a^2b^4x^{12} + 8a\,b^5x^{15} + b^6x^{18}}{12x^6}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^7,x)
```

output
```
(180*log(x)*a**4*b**2*x**6 - 2*a**6 - 24*a**5*b*x**3 + 80*a**3*b**3*x**9 +
 30*a**2*b**4*x**12 + 8*a*b**5*x**15 + b**6*x**18)/(12*x**6)
```

3.29 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{10}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 114

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{10}}\,dx = -\frac{a^5A}{9x^9}-\frac{a^4(5Ab+aB)}{6x^6}-\frac{5a^3b(2Ab+aB)}{3x^3}+\frac{5}{3}ab^3(Ab+2aB)x^3+\frac{1}{6}b^4(Ab+5aB)x^6+\frac{1}{9}b^5Bx^9+10a^2b^2(Ab+aB)\log(x)$$

output
```
-1/9*a^5*A/x^9-1/6*a^4*(5*A*b+B*a)/x^6-5/3*a^3*b*(2*A*b+B*a)/x^3+5/3*a*b^3
*(A*b+2*B*a)*x^3+1/6*b^4*(A*b+5*B*a)*x^6+1/9*b^5*B*x^9+10*a^2*b^2*(A*b+B*a
)*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{10}}\,dx = \frac{1}{18}\left(-\frac{2a^5A}{x^9}-\frac{3a^4(5Ab+aB)}{x^6}-\frac{30a^3b(2Ab+aB)}{x^3}+30ab^3(Ab+2aB)x^3+3b^4(Ab+5aB)x^6+2b^5Bx^9+180a^2b^2(Ab+aB)\log(x)\right)$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^10,x]
```

output
```
((-2*a^5*A)/x^9 - (3*a^4*(5*A*b + a*B))/x^6 - (30*a^3*b*(2*A*b + a*B))/x^3
 + 30*a*b^3*(A*b + 2*a*B)*x^3 + 3*b^4*(A*b + 5*a*B)*x^6 + 2*b^5*B*x^9 + 18
0*a^2*b^2*(A*b + a*B)*Log[x])/18
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5 (Bx^3+A)}{x^{12}}dx^3$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{3}\int \left(b^5Bx^6+b^4(Ab+5aB)x^3+5ab^3(Ab+2aB)+\frac{10a^2b^2(Ab+aB)}{x^3}+\frac{5a^3b(2Ab+aB)}{x^6}+\frac{a^4(5Ab+aB)}{x^9}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^5A}{3x^9}-\frac{a^4(aB+5Ab)}{2x^6}-\frac{5a^3b(aB+2Ab)}{x^3}+10a^2b^2\log\left(x^3\right)(aB+Ab)+\frac{1}{2}b^4x^6(5aB+Ab)+5ab^3x^3(2aB\right.$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^10,x]
```

output
```
(-1/3*(a^5*A)/x^9 - (a^4*(5*A*b + a*B))/(2*x^6) - (5*a^3*b*(2*A*b + a*B))/
x^3 + 5*a*b^3*(A*b + 2*a*B)*x^3 + (b^4*(A*b + 5*a*B)*x^6)/2 + (b^5*B*x^9)/
3 + 10*a^2*b^2*(A*b + a*B)*Log[x^3])/3
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^5 B x^9}{9}+\frac{b^5 A x^6}{6}+\frac{5 B a b^4 x^6}{6}+\frac{5 A a b^4 x^3}{3}+\frac{10 B a^2 b^3 x^3}{3}-\frac{5 a^3 b (2 A b+B a)}{3 x^3}+10 a^2 b^2 (A b+B a) \ln(x)-$
norman	$\frac{(\frac{1}{6} b^5 A+\frac{5}{6} a b^4 B) x^{15}+(\frac{5}{3} a b^4 A+\frac{10}{3} a^2 b^3 B) x^{12}+(-\frac{10}{3} a^3 b^2 A-\frac{5}{3} a^4 b B) x^6+(-\frac{5}{6} a^4 b A-\frac{1}{6} a^5 B) x^3-\frac{a^5 A}{9}+\frac{b^5 B x^{18}}{9}}{x^9}+(10 a^2$
risch	$\frac{b^5 B x^9}{9}+\frac{b^5 A x^6}{6}+\frac{5 B a b^4 x^6}{6}+\frac{5 A a b^4 x^3}{3}+\frac{10 B a^2 b^3 x^3}{3}+\frac{(-\frac{10}{3} a^3 b^2 A-\frac{5}{3} a^4 b B) x^6+(-\frac{5}{6} a^4 b A-\frac{1}{6} a^5 B) x^3-\frac{a^5 A}{9}}{x^9}+$
parallelrisch	$\frac{2 b^5 B x^{18}+3 A b^5 x^{15}+15 B a b^4 x^{15}+30 a A b^4 x^{12}+60 B a^2 b^3 x^{12}+180 A \ln(x) x^9 a^2 b^3+180 B \ln(x) x^9 a^3 b^2-60 a^3 A b^2 x^6-30 B a^4 b}{18 x^9}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/9*b^5*B*x^9+1/6*b^5*A*x^6+5/6*B*a*b^4*x^6+5/3*A*a*b^4*x^3+10/3*B*a^2*b^3
*x^3-5/3*a^3*b*(2*A*b+B*a)/x^3+10*a^2*b^2*(A*b+B*a)*ln(x)-1/6*a^4*(5*A*b+B
*a)/x^6-1/9*a^5*A/x^9
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}} dx = \frac{2 Bb^5x^{18} + 3 (5 Bab^4 + Ab^5)x^{15} + 30 (2 Ba^2b^3 + Aab^4)x^{12} + 180 (Ba^3b^2 + Aa^2b^3)x^9 \log(x) - 30 (Ba^4b -}{18 x^9}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="fricas")
```

output
```
1/18*(2*B*b^5*x^18 + 3*(5*B*a*b^4 + A*b^5)*x^15 + 30*(2*B*a^2*b^3 + A*a*b^
4)*x^12 + 180*(B*a^3*b^2 + A*a^2*b^3)*x^9*log(x) - 30*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 2*A*a^5 - 3*(B*a^5 + 5*A*a^4*b)*x^3)/x^9
```

Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}} dx = \frac{Bb^5x^9}{9} + 10a^2b^2(Ab+Ba)\log(x) + x^6\left(\frac{Ab^5}{6} + \frac{5Bab^4}{6}\right) + x^3\cdot\left(\frac{5Aab^4}{3} + \frac{10Ba^2b^3}{3}\right) + \frac{-2Aa^5 + x^6(-60Aa^3b^2 - 30Ba^4b) + x^3(-15Aa^4b - 3Ba^5)}{18x^9}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**10,x)
```

output
```
B*b**5*x**9/9 + 10*a**2*b**2*(A*b + B*a)*log(x) + x**6*(A*b**5/6 + 5*B*a*b
**4/6) + x**3*(5*A*a*b**4/3 + 10*B*a**2*b**3/3) + (-2*A*a**5 + x**6*(-60*A
*a**3*b**2 - 30*B*a**4*b) + x**3*(-15*A*a**4*b - 3*B*a**5))/(18*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}}\,dx = \frac{1}{9}Bb^5x^9 + \frac{1}{6}\left(5\,Bab^4 + Ab^5\right)x^6 + \frac{5}{3}\left(2\,Ba^2b^3 + Aab^4\right)x^3 + \frac{10}{3}\left(Ba^3b^2 + Aa^2b^3\right)\log\left(x^3\right) - \frac{30\,(Ba^4b + 2\,Aa^3b^2)x^6 + 2\,Aa^5 + 3\,(Ba^5 + 5\,Aa^4b)x^3}{18\,x^9}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="maxima")
```

output
```
1/9*B*b^5*x^9 + 1/6*(5*B*a*b^4 + A*b^5)*x^6 + 5/3*(2*B*a^2*b^3 + A*a*b^4)*
x^3 + 10/3*(B*a^3*b^2 + A*a^2*b^3)*log(x^3) - 1/18*(30*(B*a^4*b + 2*A*a^3*
b^2)*x^6 + 2*A*a^5 + 3*(B*a^5 + 5*A*a^4*b)*x^3)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}}\,dx = \frac{1}{9}Bb^5x^9 + \frac{5}{6}Bab^4x^6 + \frac{1}{6}Ab^5x^6 + \frac{10}{3}Ba^2b^3x^3 + \frac{5}{3}Aab^4x^3 + 10\left(Ba^3b^2 + Aa^2b^3\right)\log\left(|x|\right) - \frac{110\,Ba^3b^2x^9 + 110\,Aa^2b^3x^9 + 30\,Ba^4bx^6 + 60\,Aa^3b^2x^6 + 3\,Ba^5x^3 + 15\,Aa^4bx^3 + 2\,Aa^5}{18\,x^9}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="giac")
```

output
```
1/9*B*b^5*x^9 + 5/6*B*a*b^4*x^6 + 1/6*A*b^5*x^6 + 10/3*B*a^2*b^3*x^3 + 5/3
*A*a*b^4*x^3 + 10*(B*a^3*b^2 + A*a^2*b^3)*log(abs(x)) - 1/18*(110*B*a^3*b^
2*x^9 + 110*A*a^2*b^3*x^9 + 30*B*a^4*b*x^6 + 60*A*a^3*b^2*x^6 + 3*B*a^5*x^
3 + 15*A*a^4*b*x^3 + 2*A*a^5)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}} dx = x^6 \left(\frac{A b^5}{6} + \frac{5 B a b^4}{6}\right) - \frac{\frac{A a^5}{9} + x^6 \left(\frac{5 B a^4 b}{3} + \frac{10 A a^3 b^2}{3}\right) + x^3 \left(\frac{B a^5}{6} + \frac{5 A b a^4}{6}\right)}{x^9} + \ln(x) \left(10 B a^3 b^2 + 10 A a^2 b^3\right) + \frac{B b^5 x^9}{9} + \frac{5 a b^3 x^3 (A b + 2 B a)}{3}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^10,x)
```

output
```
x^6*((A*b^5)/6 + (5*B*a*b^4)/6) - ((A*a^5)/9 + x^6*((10*A*a^3*b^2)/3 + (5*
B*a^4*b)/3) + x^3*((B*a^5)/6 + (5*A*a^4*b)/6))/x^9 + log(x)*(10*A*a^2*b^3
+ 10*B*a^3*b^2) + (B*b^5*x^9)/9 + (5*a*b^3*x^3*(A*b + 2*B*a))/3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}} dx = \frac{180 \log(x) a^3 b^3 x^9 - a^6 - 9a^5 b x^3 - 45a^4 b^2 x^6 + 45a^2 b^4 x^{12} + 9a b^5 x^{15} + b^6 x^{18}}{9x^9}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^10,x)
```

output
```
(180*log(x)*a**3*b**3*x**9 - a**6 - 9*a**5*b*x**3 - 45*a**4*b**2*x**6 + 45
*a**2*b**4*x**12 + 9*a*b**5*x**15 + b**6*x**18)/(9*x**9)
```

3.30 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{13}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 114

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{13}}\,dx = -\frac{a^5A}{12x^{12}}-\frac{a^4(5Ab+aB)}{9x^9}-\frac{5a^3b(2Ab+aB)}{6x^6} -\frac{10a^2b^2(Ab+aB)}{3x^3}+\frac{1}{3}b^4(Ab+5aB)x^3 +\frac{1}{6}b^5Bx^6+5ab^3(Ab+2aB)\log(x)$$

output
```
-1/12*a^5*A/x^12-1/9*a^4*(5*A*b+B*a)/x^9-5/6*a^3*b*(2*A*b+B*a)/x^6-10/3*a^
2*b^2*(A*b+B*a)/x^3+1/3*b^4*(A*b+5*B*a)*x^3+1/6*b^5*B*x^6+5*a*b^3*(A*b+2*B
*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{13}}\,dx = -\frac{120a^2Ab^3x^9-60ab^4Bx^{15}-6b^5x^{15}(2A+Bx^3)+60a^3b^2x^6(A+2Bx^3)+10a^4bx^3(2A+3Bx^3)+a^5(3}{36x^{12}}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^13,x]
```

output
```
-1/36*(120*a^2*A*b^3*x^9 - 60*a*b^4*B*x^15 - 6*b^5*x^15*(2*A + B*x^3) + 60
*a^3*b^2*x^6*(A + 2*B*x^3) + 10*a^4*b*x^3*(2*A + 3*B*x^3) + a^5*(3*A + 4*B
*x^3) - 180*a*b^3*(A*b + 2*a*B)*x^12*Log[x])/x^12
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}} \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5 (Bx^3+A)}{x^{15}}dx^3$$

$$\Big\downarrow \text{ 85}$$

$$\frac{1}{3}\int \left(\frac{Aa^5}{x^{15}}+\frac{(5Ab+aB)a^4}{x^{12}}+\frac{5b(2Ab+aB)a^3}{x^9}+\frac{10b^2(Ab+aB)a^2}{x^6}+\frac{5b^3(Ab+2aB)a}{x^3}+b^5Bx^3+b^4(Ab+5a$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{a^5A}{4x^{12}}-\frac{a^4(aB+5Ab)}{3x^9}-\frac{5a^3b(aB+2Ab)}{2x^6}-\frac{10a^2b^2(aB+Ab)}{x^3}+b^4x^3(5aB+Ab)+5ab^3\log\left(x^3\right)(2aB+$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^13,x]
```

output
```
(-1/4*(a^5*A)/x^12 - (a^4*(5*A*b + a*B))/(3*x^9) - (5*a^3*b*(2*A*b + a*B))
/(2*x^6) - (10*a^2*b^2*(A*b + a*B))/x^3 + b^4*(A*b + 5*a*B)*x^3 + (b^5*B*x
^6)/2 + 5*a*b^3*(A*b + 2*a*B)*Log[x^3])/3
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93

method	result
default	$\frac{B\,b^5x^6}{6}+\frac{A\,b^5x^3}{3}+\frac{5Ba\,b^4x^3}{3}-\frac{10a^2b^2(Ab+Ba)}{3x^3}+5a\,b^3(Ab+2Ba)\ln(x)-\frac{5a^3b(2Ab+Ba)}{6x^6}-\frac{a^4(5Ab+B}{9x^9}$
norman	$\frac{(\frac{1}{3}b^5A+\frac{5}{3}a\,b^4B)x^{15}+(-\frac{10}{3}a^2b^3A-\frac{10}{3}a^3b^2B)x^9+(-\frac{5}{3}a^3b^2A-\frac{5}{6}a^4bB)x^6+(-\frac{5}{9}a^4bA-\frac{1}{9}a^5B)x^3-\frac{a^5A}{12}+\frac{b^5Bx^{18}}{6}}{x^{12}}+(5a\,b$
parallelrisch	$\frac{6b^5Bx^{18}+12A\,b^5x^{15}+60Ba\,b^4x^{15}+180A\ln(x)x^{12}a\,b^4+360B\ln(x)x^{12}a^2b^3-120a^2A\,b^3x^9-120B\,a^3b^2x^9-60a^3A\,b^2x^6-30B}{36x^{12}}$
risch	$\frac{B\,b^5x^6}{6}+\frac{A\,b^5x^3}{3}+\frac{5Ba\,b^4x^3}{3}+\frac{b^5A^2}{6B}+\frac{5a\,b^4A}{3}+\frac{25a^2b^3B}{6}+\frac{(-\frac{10}{3}a^2b^3A-\frac{10}{3}a^3b^2B)x^9+(-\frac{5}{3}a^3b^2A-\frac{5}{6}a^4bB)x^6}{x^{12}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^13,x,method=_RETURNVERBOSE)
```

output
```
1/6*B*b^5*x^6+1/3*A*b^5*x^3+5/3*B*a*b^4*x^3-10/3*a^2*b^2*(A*b+B*a)/x^3+5*a
*b^3*(A*b+2*B*a)*ln(x)-5/6*a^3*b*(2*A*b+B*a)/x^6-1/9*a^4*(5*A*b+B*a)/x^9-1
/12*a^5*A/x^12
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}}\,dx = \frac{6\,Bb^5x^{18} + 12\,(5\,Bab^4 + Ab^5)x^{15} + 180\,(2\,Ba^2b^3 + Aab^4)x^{12}\log(x) - 120\,(Ba^3b^2 + Aa^2b^3)x^9 - 30\,(Ba^4}{36\,x^{12}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="fricas")
```

output
```
1/36*(6*B*b^5*x^18 + 12*(5*B*a*b^4 + A*b^5)*x^15 + 180*(2*B*a^2*b^3 + A*a*
b^4)*x^12*log(x) - 120*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 30*(B*a^4*b + 2*A*a^3
*b^2)*x^6 - 3*A*a^5 - 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^12
```

Sympy [A] (verification not implemented)

Time = 3.67 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}}\,dx = \frac{Bb^5x^6}{6} + 5ab^3(Ab+2Ba)\log(x) + x^3\left(\frac{Ab^5}{3} + \frac{5Bab^4}{3}\right) + \frac{-3Aa^5 + x^9(-120Aa^2b^3 - 120Ba^3b^2) + x^6(-60Aa^3b^2 - 30Ba^4b) + x^3(-20Aa^4b - 4Ba^5)}{36x^{12}}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**13,x)
```

output
```
B*b**5*x**6/6 + 5*a*b**3*(A*b + 2*B*a)*log(x) + x**3*(A*b**5/3 + 5*B*a*b**
4/3) + (-3*A*a**5 + x**9*(-120*A*a**2*b**3 - 120*B*a**3*b**2) + x**6*(-60*
A*a**3*b**2 - 30*B*a**4*b) + x**3*(-20*A*a**4*b - 4*B*a**5))/(36*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}}\,dx$$
$$= \frac{1}{6}\,Bb^5x^6 + \frac{1}{3}\left(5\,Bab^4 + Ab^5\right)x^3 + \frac{5}{3}\left(2\,Ba^2b^3 + Aab^4\right)\log\left(x^3\right)$$
$$- \frac{120\,(Ba^3b^2 + Aa^2b^3)x^9 + 30\,(Ba^4b + 2\,Aa^3b^2)x^6 + 3\,Aa^5 + 4\,(Ba^5 + 5\,Aa^4b)x^3}{36\,x^{12}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="maxima")
```

output
```
1/6*B*b^5*x^6 + 1/3*(5*B*a*b^4 + A*b^5)*x^3 + 5/3*(2*B*a^2*b^3 + A*a*b^4)*
log(x^3) - 1/36*(120*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 30*(B*a^4*b + 2*A*a^3*b
^2)*x^6 + 3*A*a^5 + 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}}\,dx$$
$$= \frac{1}{6}\,Bb^5x^6 + \frac{5}{3}\,Bab^4x^3 + \frac{1}{3}\,Ab^5x^3 + 5\left(2\,Ba^2b^3 + Aab^4\right)\log\left(|x|\right)$$
$$- \frac{250\,Ba^2b^3x^{12} + 125\,Aab^4x^{12} + 120\,Ba^3b^2x^9 + 120\,Aa^2b^3x^9 + 30\,Ba^4bx^6 + 60\,Aa^3b^2x^6 + 4\,Ba^5x^3 + 20}{36\,x^{12}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="giac")
```

output
```
1/6*B*b^5*x^6 + 5/3*B*a*b^4*x^3 + 1/3*A*b^5*x^3 + 5*(2*B*a^2*b^3 + A*a*b^4
)*log(abs(x)) - 1/36*(250*B*a^2*b^3*x^12 + 125*A*a*b^4*x^12 + 120*B*a^3*b^
2*x^9 + 120*A*a^2*b^3*x^9 + 30*B*a^4*b*x^6 + 60*A*a^3*b^2*x^6 + 4*B*a^5*x^
3 + 20*A*a^4*b*x^3 + 3*A*a^5)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}} dx$$
$$= \ln(x) \left(10 B a^2 b^3 + 5 A a b^4\right) - \frac{\frac{A a^5}{12} + x^6 \left(\frac{5 B a^4 b}{6} + \frac{5 A a^3 b^2}{3}\right) + x^3 \left(\frac{B a^5}{9} + \frac{5 A b a^4}{9}\right) + x^9 \left(\frac{10 B a^3 b^2}{3} + \frac{10 A a^2 b^3}{3}\right)}{x^{12}} + x^3 \left(\frac{A b^5}{3} + \frac{5 B a b^4}{3}\right) + \frac{B b^5 x^6}{6}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^13,x)
```

output
```
log(x)*(10*B*a^2*b^3 + 5*A*a*b^4) - ((A*a^5)/12 + x^6*((5*A*a^3*b^2)/3 + (
5*B*a^4*b)/6) + x^3*((B*a^5)/9 + (5*A*a^4*b)/9) + x^9*((10*A*a^2*b^3)/3 +
(10*B*a^3*b^2)/3))/x^12 + x^3*((A*b^5)/3 + (5*B*a*b^4)/3) + (B*b^5*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}} dx$$
$$= \frac{180 \log(x) a^2 b^4 x^{12} - a^6 - 8a^5 b x^3 - 30a^4 b^2 x^6 - 80a^3 b^3 x^9 + 24a b^5 x^{15} + 2b^6 x^{18}}{12x^{12}}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^13,x)
```

output
```
(180*log(x)*a**2*b**4*x**12 - a**6 - 8*a**5*b*x**3 - 30*a**4*b**2*x**6 - 8
0*a**3*b**3*x**9 + 24*a*b**5*x**15 + 2*b**6*x**18)/(12*x**12)
```

3.31 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}}\,dx = -\frac{a^5A}{15x^{15}} - \frac{a^4(5Ab+aB)}{12x^{12}} - \frac{5a^3b(2Ab+aB)}{9x^9} - \frac{5a^2b^2(Ab+aB)}{3x^6} - \frac{5ab^3(Ab+2aB)}{3x^3} + \frac{1}{3}b^5Bx^3 + b^4(Ab+5aB)\log(x)$$

output
```
-1/15*a^5*A/x^15-1/12*a^4*(5*A*b+B*a)/x^12-5/9*a^3*b*(2*A*b+B*a)/x^9-5/3*a
^2*b^2*(A*b+B*a)/x^6-5/3*a*b^3*(A*b+2*B*a)/x^3+1/3*b^5*B*x^3+b^4*(A*b+5*B*
a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}}\,dx = -\frac{300aAb^4x^{12} - 60b^5Bx^{18} + 300a^2b^3x^9(A+2Bx^3) + 100a^3b^2x^6(2A+3Bx^3) + 25a^4bx^3(3A+4Bx^3) +}{180x^{15}} + b^4(Ab+5aB)\log(x)$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^16,x]
```

output
```
-1/180*(300*a*A*b^4*x^12 - 60*b^5*B*x^18 + 300*a^2*b^3*x^9*(A + 2*B*x^3) +
 100*a^3*b^2*x^6*(2*A + 3*B*x^3) + 25*a^4*b*x^3*(3*A + 4*B*x^3) + 3*a^5*(4
*A + 5*B*x^3))/x^15 + b^4*(A*b + 5*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5(Bx^3+A)}{x^{18}}dx^3$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{3}\int \left(\frac{Aa^5}{x^{18}}+\frac{(5Ab+aB)a^4}{x^{15}}+\frac{5b(2Ab+aB)a^3}{x^{12}}+\frac{10b^2(Ab+aB)a^2}{x^9}+\frac{5b^3(Ab+2aB)a}{x^6}+b^5B+\frac{b^4(Ab+5aB)}{x^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^5A}{5x^{15}}-\frac{a^4(aB+5Ab)}{4x^{12}}-\frac{5a^3b(aB+2Ab)}{3x^9}-\frac{5a^2b^2(aB+Ab)}{x^6}+b^4\log(x^3)(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x^3}\right.$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^16,x]
```

output
```
(-1/5*(a^5*A)/x^15 - (a^4*(5*A*b + a*B))/(4*x^12) - (5*a^3*b*(2*A*b + a*B)
)/(3*x^9) - (5*a^2*b^2*(A*b + a*B))/x^6 - (5*a*b^3*(A*b + 2*a*B))/x^3 + b^
5*B*x^3 + b^4*(A*b + 5*a*B)*Log[x^3])/3
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a^{5} A}{15 x^{15}}-\frac{a^{4} (5 A b+B a)}{12 x^{12}}-\frac{5 a^{3} b (2 A b+B a)}{9 x^{9}}-\frac{5 a^{2} b^{2} (A b+B a)}{3 x^{6}}-\frac{5 a \,b^{3} (A b+2 B a)}{3 x^{3}}+\frac{b^{5} B \,x^{3}}{3}+b^{4} (A b+5 B a) \mathrm{l}$
norman	$\frac{\left(-\frac{5}{3} a \,b^{4} A-\frac{10}{3} a^{2} b^{3} B\right) x^{12}+\left(-\frac{5}{3} a^{2} b^{3} A-\frac{5}{3} a^{3} b^{2} B\right) x^{9}+\left(-\frac{10}{9} a^{3} b^{2} A-\frac{5}{9} a^{4} b B\right) x^{6}+\left(-\frac{5}{12} a^{4} b A-\frac{1}{12} a^{5} B\right) x^{3}-\frac{a^{5} A}{15}+\frac{b^{5} B \,x^{18}}{3}}{x^{15}}+$
risch	$\frac{b^{5} B \,x^{3}}{3}+\frac{-\frac{a^{5} A}{15}+\left(-\frac{5}{12} a^{4} b A-\frac{1}{12} a^{5} B\right) x^{3}+\left(-\frac{10}{9} a^{3} b^{2} A-\frac{5}{9} a^{4} b B\right) x^{6}+\left(-\frac{5}{3} a^{2} b^{3} A-\frac{5}{3} a^{3} b^{2} B\right) x^{9}+\left(-\frac{5}{3} a \,b^{4} A-\frac{10}{3} a^{2} b^{3} B\right) x^{12}}{x^{15}}$
parallelrisch	$\frac{60 b^{5} B \,x^{18}+180 A \ln (x) x^{15} b^{5}+900 B \ln (x) x^{15} a \,b^{4}-300 a A \,b^{4} x^{12}-600 B \,a^{2} b^{3} x^{12}-300 a^{2} A \,b^{3} x^{9}-300 B \,a^{3} b^{2} x^{9}-200 a^{3} A \,b^{2} x^{6}-}{180 x^{15}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/15*a^5*A/x^15-1/12*a^4*(5*A*b+B*a)/x^12-5/9*a^3*b*(2*A*b+B*a)/x^9-5/3*a
^2*b^2*(A*b+B*a)/x^6-5/3*a*b^3*(A*b+2*B*a)/x^3+1/3*b^5*B*x^3+b^4*(A*b+5*B*
a)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{16}}\,dx$$
$$= \frac{60\,Bb^5x^{18} + 180\,(5\,Bab^4 + Ab^5)x^{15}\log(x) - 300\,(2\,Ba^2b^3 + Aab^4)x^{12} - 300\,(Ba^3b^2 + Aa^2b^3)x^9 - 100\,(\ldots}{180\,x^{15}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="fricas")
```

output
```
1/180*(60*B*b^5*x^18 + 180*(5*B*a*b^4 + A*b^5)*x^15*log(x) - 300*(2*B*a^2*
b^3 + A*a*b^4)*x^12 - 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 100*(B*a^4*b + 2*A
*a^3*b^2)*x^6 - 12*A*a^5 - 15*(B*a^5 + 5*A*a^4*b)*x^3)/x^15
```

Sympy [A] (verification not implemented)

Time = 19.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{16}}\,dx = \frac{Bb^5x^3}{3} + b^4(Ab+5Ba)\log(x)$$
$$+ \frac{-12Aa^5 + x^{12}(-300Aab^4 - 600Ba^2b^3) + x^9(-300Aa^2b^3 - 300Ba^3b^2) + x^6(-200Aa^3b^2 - 100Ba^4b) \ldots}{180x^{15}}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**16,x)
```

output
```
B*b**5*x**3/3 + b**4*(A*b + 5*B*a)*log(x) + (-12*A*a**5 + x**12*(-300*A*a*
b**4 - 600*B*a**2*b**3) + x**9*(-300*A*a**2*b**3 - 300*B*a**3*b**2) + x**6
*(-200*A*a**3*b**2 - 100*B*a**4*b) + x**3*(-75*A*a**4*b - 15*B*a**5))/(180
*x**15)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{16}} dx = \frac{1}{3} Bb^5x^3 + \frac{1}{3} (5 Bab^4 + Ab^5) \log(x^3) - \frac{300 (2 Ba^2b^3 + Aab^4)x^{12} + 300 (Ba^3b^2 + Aa^2b^3)x^9 + 100 (Ba^4b + 2 Aa^3b^2)x^6 + 12 Aa^5 + 15 (Ba^5 + 5}{180 x^{15}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="maxima")
```

output
```
1/3*B*b^5*x^3 + 1/3*(5*B*a*b^4 + A*b^5)*log(x^3) - 1/180*(300*(2*B*a^2*b^3
 + A*a*b^4)*x^12 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 100*(B*a^4*b + 2*A*a^
3*b^2)*x^6 + 12*A*a^5 + 15*(B*a^5 + 5*A*a^4*b)*x^3)/x^15
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{16}} dx = \frac{1}{3} Bb^5x^3 + (5 Bab^4 + Ab^5) \log(|x|) - \frac{685 Bab^4x^{15} + 137 Ab^5x^{15} + 600 Ba^2b^3x^{12} + 300 Aab^4x^{12} + 300 Ba^3b^2x^9 + 300 Aa^2b^3x^9 + 100 Ba^4bx^6}{180 x^{15}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="giac")
```

output
```
1/3*B*b^5*x^3 + (5*B*a*b^4 + A*b^5)*log(abs(x)) - 1/180*(685*B*a*b^4*x^15
+ 137*A*b^5*x^15 + 600*B*a^2*b^3*x^12 + 300*A*a*b^4*x^12 + 300*B*a^3*b^2*x
^9 + 300*A*a^2*b^3*x^9 + 100*B*a^4*b*x^6 + 200*A*a^3*b^2*x^6 + 15*B*a^5*x^
3 + 75*A*a^4*b*x^3 + 12*A*a^5)/x^15
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{16}} dx = \ln(x) \left(A b^5 + 5 B a b^4\right) - \frac{\frac{A a^5}{15} + x^{12} \left(\frac{10 B a^2 b^3}{3} + \frac{5 A a b^4}{3}\right) + x^6 \left(\frac{5 B a^4 b}{9} + \frac{10 A a^3 b^2}{9}\right) + x^3 \left(\frac{B a^5}{12} + \frac{5 A b a^4}{12}\right) + x^9 \left(\frac{5 B a^3 b^2}{3} + \frac{5 A a^2 b^3}{3}\right)}{x^{15}} + \frac{B b^5 x^3}{3}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^16,x)
```

output
```
log(x)*(A*b^5 + 5*B*a*b^4) - ((A*a^5)/15 + x^12*((10*B*a^2*b^3)/3 + (5*A*a
*b^4)/3) + x^6*((10*A*a^3*b^2)/9 + (5*B*a^4*b)/9) + x^3*((B*a^5)/12 + (5*A
*a^4*b)/12) + x^9*((5*A*a^2*b^3)/3 + (5*B*a^3*b^2)/3))/x^15 + (B*b^5*x^3)/
3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{16}} dx = \frac{180 \log(x) a b^5 x^{15} - 2a^6 - 15a^5 b x^3 - 50a^4 b^2 x^6 - 100a^3 b^3 x^9 - 150a^2 b^4 x^{12} + 10b^6 x^{18}}{30x^{15}}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^16,x)
```

output
```
(180*log(x)*a*b**5*x**15 - 2*a**6 - 15*a**5*b*x**3 - 50*a**4*b**2*x**6 - 1
00*a**3*b**3*x**9 - 150*a**2*b**4*x**12 + 10*b**6*x**18)/(30*x**15)
```

3.32 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{19}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 91

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{19}}\,dx = -\frac{a^5B}{15x^{15}}-\frac{5a^4bB}{12x^{12}}-\frac{10a^3b^2B}{9x^9}-\frac{5a^2b^3B}{3x^6}-\frac{5ab^4B}{3x^3}-\frac{A(a+bx^3)^6}{18ax^{18}}+b^5B\log(x)$$

output
```
-1/15*a^5*B/x^15-5/12*a^4*b*B/x^12-10/9*a^3*b^2*B/x^9-5/3*a^2*b^3*B/x^6-5/
3*a*b^4*B/x^3-1/18*A*(b*x^3+a)^6/a/x^18+b^5*B*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{19}}\,dx = -\frac{60Ab^5x^{15}+150ab^4x^{12}(A+2Bx^3)+100a^2b^3x^9(2A+3Bx^3)+50a^3b^2x^6(3A+4Bx^3)+15a^4bx^3(4A+}{180x^{18}}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^19,x]
```

output
```
-1/180*(60*A*b^5*x^15 + 150*a*b^4*x^12*(A + 2*B*x^3) + 100*a^2*b^3*x^9*(2*
A + 3*B*x^3) + 50*a^3*b^2*x^6*(3*A + 4*B*x^3) + 15*a^4*b*x^3*(4*A + 5*B*x^
3) + 2*a^5*(5*A + 6*B*x^3) - 180*b^5*B*x^18*Log[x])/x^18
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {948, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5 (Bx^3+A)}{x^{21}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(B\int \frac{(bx^3+a)^5}{x^{18}}dx^3-\frac{A(a+bx^3)^6}{6ax^{18}}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\left(B\int \left(\frac{a^5}{x^{18}}+\frac{5ba^4}{x^{15}}+\frac{10b^2a^3}{x^{12}}+\frac{10b^3a^2}{x^9}+\frac{5b^4a}{x^6}+\frac{b^5}{x^3}\right)dx^3-\frac{A(a+bx^3)^6}{6ax^{18}}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(B\left(-\frac{a^5}{5x^{15}}-\frac{5a^4b}{4x^{12}}-\frac{10a^3b^2}{3x^9}-\frac{5a^2b^3}{x^6}-\frac{5ab^4}{x^3}+b^5\log\left(x^3\right)\right)-\frac{A(a+bx^3)^6}{6ax^{18}}\right)$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^19,x]
```

output
```
(-1/6*(A*(a + b*x^3)^6)/(a*x^18) + B*(-1/5*a^5/x^15 - (5*a^4*b)/(4*x^12) -
 (10*a^3*b^2)/(3*x^9) - (5*a^2*b^3)/x^6 - (5*a*b^4)/x^3 + b^5*Log[x^3]))/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

method	result
default	$-\frac{b^4(Ab+5Ba)}{3x^3}-\frac{a^4(5Ab+Ba)}{15x^{15}}+b^5B\ln(x)-\frac{5a\,b^3(Ab+2Ba)}{6x^6}-\frac{a^5A}{18x^{18}}-\frac{10a^2b^2(Ab+Ba)}{9x^9}-\frac{5a^3b(2Ab+Ba)}{12x^{12}}$
norman	$\frac{(-\frac{1}{3}b^5A-\frac{5}{3}a\,b^4B)x^{15}+(-\frac{5}{6}a\,b^4A-\frac{5}{3}a^2b^3B)x^{12}+(-\frac{10}{9}a^2b^3A-\frac{10}{9}a^3b^2B)x^9+(-\frac{5}{6}a^3b^2A-\frac{5}{12}a^4bB)x^6+(-\frac{1}{3}a^4bA-\frac{1}{15}a^5B}{x^{18}}$
risch	$\frac{(-\frac{1}{3}b^5A-\frac{5}{3}a\,b^4B)x^{15}+(-\frac{5}{6}a\,b^4A-\frac{5}{3}a^2b^3B)x^{12}+(-\frac{10}{9}a^2b^3A-\frac{10}{9}a^3b^2B)x^9+(-\frac{5}{6}a^3b^2A-\frac{5}{12}a^4bB)x^6+(-\frac{1}{3}a^4bA-\frac{1}{15}a^5B}{x^{18}}$
parallelrisch	$-\frac{-180b^5B\ln(x)x^{18}+60A\,b^5x^{15}+300Ba\,b^4x^{15}+150aA\,b^4x^{12}+300B\,a^2b^3x^{12}+200a^2A\,b^3x^9+200B\,a^3b^2x^9+150a^3A\,b^2x^6+}{180x^{18}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^19,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^4*(A*b+5*B*a)/x^3-1/15*a^4*(5*A*b+B*a)/x^15+b^5*B*ln(x)-5/6*a*b^3*(
A*b+2*B*a)/x^6-1/18*a^5*A/x^18-10/9*a^2*b^2*(A*b+B*a)/x^9-5/12*a^3*b*(2*A*
b+B*a)/x^12
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}}\,dx$$
$$= \frac{180\,Bb^5x^{18}\log(x) - 60\,(5\,Bab^4 + Ab^5)x^{15} - 150\,(2\,Ba^2b^3 + Aab^4)x^{12} - 200\,(Ba^3b^2 + Aa^2b^3)x^9 - 75\,(B}{180\,x^{18}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="fricas")
```

output
```
1/180*(180*B*b^5*x^18*log(x) - 60*(5*B*a*b^4 + A*b^5)*x^15 - 150*(2*B*a^2*
b^3 + A*a*b^4)*x^12 - 200*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 75*(B*a^4*b + 2*A*
a^3*b^2)*x^6 - 10*A*a^5 - 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^18
```

Sympy [A] (verification not implemented)

Time = 84.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}}\,dx = Bb^5\log(x)$$
$$+ \frac{-10Aa^5 + x^{15}(-60Ab^5 - 300Bab^4) + x^{12}(-150Aab^4 - 300Ba^2b^3) + x^9(-200Aa^2b^3 - 200Ba^3b^2) + x}{180x^{18}}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**19,x)
```

output
```
B*b**5*log(x) + (-10*A*a**5 + x**15*(-60*A*b**5 - 300*B*a*b**4) + x**12*(-
150*A*a*b**4 - 300*B*a**2*b**3) + x**9*(-200*A*a**2*b**3 - 200*B*a**3*b**2
) + x**6*(-150*A*a**3*b**2 - 75*B*a**4*b) + x**3*(-60*A*a**4*b - 12*B*a**5
))/(180*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}}\,dx = \frac{1}{3}\,Bb^5\log\left(x^3\right) - \frac{60\,(5\,Bab^4+Ab^5)x^{15}+150\,(2\,Ba^2b^3+Aab^4)x^{12}+200\,(Ba^3b^2+Aa^2b^3)x^9+75\,(Ba^4b+2\,Aa^3b^2)x^6+}{180\,x^{18}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="maxima")
```

output
```
1/3*B*b^5*log(x^3) - 1/180*(60*(5*B*a*b^4 + A*b^5)*x^15 + 150*(2*B*a^2*b^3
 + A*a*b^4)*x^12 + 200*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 75*(B*a^4*b + 2*A*a^3
*b^2)*x^6 + 10*A*a^5 + 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^18
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}}\,dx = Bb^5\log\left(|x|\right) - \frac{147\,Bb^5x^{18}+300\,Bab^4x^{15}+60\,Ab^5x^{15}+300\,Ba^2b^3x^{12}+150\,Aab^4x^{12}+200\,Ba^3b^2x^9+200\,Aa^2b^3x^9}{180\,x^{18}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="giac")
```

output
```
B*b^5*log(abs(x)) - 1/180*(147*B*b^5*x^18 + 300*B*a*b^4*x^15 + 60*A*b^5*x^
15 + 300*B*a^2*b^3*x^12 + 150*A*a*b^4*x^12 + 200*B*a^3*b^2*x^9 + 200*A*a^2
*b^3*x^9 + 75*B*a^4*b*x^6 + 150*A*a^3*b^2*x^6 + 12*B*a^5*x^3 + 60*A*a^4*b*
x^3 + 10*A*a^5)/x^18
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}}\,dx = B\,b^5\,\ln(x) - \frac{\frac{A\,a^5}{18} + x^{12}\left(\frac{5\,B\,a^2\,b^3}{3} + \frac{5\,A\,a\,b^4}{6}\right) + x^6\left(\frac{5\,B\,a^4\,b}{12} + \frac{5\,A\,a^3\,b^2}{6}\right) + x^3\left(\frac{B\,a^5}{15} + \frac{A\,b\,a^4}{3}\right) + x^{15}\left(\frac{A\,b^5}{3} + \frac{5\,B\,a\,b^4}{3}\right) + x^9\,(}{x^{18}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^19,x)
```

output
```
B*b^5*log(x) - ((A*a^5)/18 + x^12*((5*B*a^2*b^3)/3 + (5*A*a*b^4)/6) + x^6*
((5*A*a^3*b^2)/6 + (5*B*a^4*b)/12) + x^3*((B*a^5)/15 + (A*a^4*b)/3) + x^15
*((A*b^5)/3 + (5*B*a*b^4)/3) + x^9*((10*A*a^2*b^3)/9 + (10*B*a^3*b^2)/9))/
x^18
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}}\,dx = \frac{180\log(x)\,b^6x^{18} - 10a^6 - 72a^5b\,x^3 - 225a^4b^2x^6 - 400a^3b^3x^9 - 450a^2b^4x^{12} - 360a\,b^5x^{15}}{180x^{18}}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^19,x)
```

output
```
(180*log(x)*b**6*x**18 - 10*a**6 - 72*a**5*b*x**3 - 225*a**4*b**2*x**6 - 4
00*a**3*b**3*x**9 - 450*a**2*b**4*x**12 - 360*a*b**5*x**15)/(180*x**18)
```

3.33 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{22}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{22}}\,dx = -\frac{A(a+bx^3)^6}{21ax^{21}} + \frac{(Ab-7aB)(a+bx^3)^6}{126a^2x^{18}}$$

output
```
-1/21*A*(b*x^3+a)^6/a/x^21+1/126*(A*b-7*B*a)*(b*x^3+a)^6/a^2/x^18
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 118 vs. 2(48) = 96.

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.46

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{22}}\,dx = -\frac{21b^5x^{15}(A+2Bx^3)+35ab^4x^{12}(2A+3Bx^3)+35a^2b^3x^9(3A+4Bx^3)+21a^3b^2x^6(4A+5Bx^3)+7a^4b\cdots}{126x^{21}}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^22,x]
```

output
```
-1/126*(21*b^5*x^15*(A + 2*B*x^3) + 35*a*b^4*x^12*(2*A + 3*B*x^3) + 35*a^2
*b^3*x^9*(3*A + 4*B*x^3) + 21*a^3*b^2*x^6*(4*A + 5*B*x^3) + 7*a^4*b*x^3*(5
*A + 6*B*x^3) + a^5*(6*A + 7*B*x^3))/x^21
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{22}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5 (Bx^3+A)}{x^{24}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(-\frac{(Ab-7aB)\int \frac{(bx^3+a)^5}{x^{21}}dx^3}{7a}-\frac{A(a+bx^3)^6}{7ax^{21}}\right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{3}\left(\frac{(a+bx^3)^6 (Ab-7aB)}{42a^2x^{18}}-\frac{A(a+bx^3)^6}{7ax^{21}}\right)$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^22,x]
```

output
```
(-1/7*(A*(a + b*x^3)^6)/(a*x^21) + ((A*b - 7*a*B)*(a + b*x^3)^6)/(42*a^2*x
^18))/3
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(44) = 88$.

Time = 0.55 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.17

method	result
default	$-\frac{b^5 B}{3x^3}-\frac{a^5 A}{21x^{21}}-\frac{a^3 b(2Ab+Ba)}{3x^{15}}-\frac{b^4(Ab+5Ba)}{6x^6}-\frac{a^4(5Ab+Ba)}{18x^{18}}-\frac{5a\,b^3(Ab+2Ba)}{9x^9}-\frac{5a^2b^2(Ab+Ba)}{6x^{12}}$
norman	$\frac{-\frac{a^5A}{21}+(-\frac{5}{18}a^4bA-\frac{1}{18}a^5B)x^3+(-\frac{2}{3}a^3b^2A-\frac{1}{3}a^4bB)x^6+(-\frac{5}{6}a^2b^3A-\frac{5}{6}a^3b^2B)x^9+(-\frac{5}{9}a\,b^4A-\frac{10}{9}a^2b^3B)x^{12}+(-\frac{1}{6}b^5A-\frac{5}{6}}{x^{21}}$
risch	$\frac{-\frac{a^5A}{21}+(-\frac{5}{18}a^4bA-\frac{1}{18}a^5B)x^3+(-\frac{2}{3}a^3b^2A-\frac{1}{3}a^4bB)x^6+(-\frac{5}{6}a^2b^3A-\frac{5}{6}a^3b^2B)x^9+(-\frac{5}{9}a\,b^4A-\frac{10}{9}a^2b^3B)x^{12}+(-\frac{1}{6}b^5A-\frac{5}{6}}{x^{21}}$
gosper	$-\frac{42b^5B\,x^{18}+21A\,b^5x^{15}+105Ba\,b^4x^{15}+70aA\,b^4x^{12}+140B\,a^2b^3x^{12}+105a^2A\,b^3x^9+105B\,a^3b^2x^9+84a^3A\,b^2x^6+42B\,a^4b\,x^6}{126x^{21}}$
parallelrisch	$-\frac{42b^5B\,x^{18}+21A\,b^5x^{15}+105Ba\,b^4x^{15}+70aA\,b^4x^{12}+140B\,a^2b^3x^{12}+105a^2A\,b^3x^9+105B\,a^3b^2x^9+84a^3A\,b^2x^6+42B\,a^4b\,x^6}{126x^{21}}$
orering	$-\frac{42b^5B\,x^{18}+21A\,b^5x^{15}+105Ba\,b^4x^{15}+70aA\,b^4x^{12}+140B\,a^2b^3x^{12}+105a^2A\,b^3x^9+105B\,a^3b^2x^9+84a^3A\,b^2x^6+42B\,a^4b\,x^6}{126x^{21}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^22,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^5*B/x^3-1/21*a^5*A/x^21-1/3*a^3*b*(2*A*b+B*a)/x^15-1/6*b^4*(A*b+5*B
*a)/x^6-1/18*a^4*(5*A*b+B*a)/x^18-5/9*a*b^3*(A*b+2*B*a)/x^9-5/6*a^2*b^2*(A
*b+B*a)/x^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(45) = 90$.

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.52

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{22}}\,dx = -\frac{42\,Bb^5x^{18} + 21\,(5\,Bab^4+Ab^5)x^{15} + 70\,(2\,Ba^2b^3+Aab^4)x^{12} + 105\,(Ba^3b^2+Aa^2b^3)x^9 + 42\,(Ba^4b + 2}{126\,x^{21}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="fricas")
```

output
```
-1/126*(42*B*b^5*x^18 + 21*(5*B*a*b^4 + A*b^5)*x^15 + 70*(2*B*a^2*b^3 + A*
a*b^4)*x^12 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 42*(B*a^4*b + 2*A*a^3*b^2)
*x^6 + 6*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^21
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{22}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**22,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(45) = 90$.

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.52

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{22}}\,dx =$$
$$-\frac{42\,Bb^5x^{18} + 21\,(5\,Bab^4 + Ab^5)x^{15} + 70\,(2\,Ba^2b^3 + Aab^4)x^{12} + 105\,(Ba^3b^2 + Aa^2b^3)x^9 + 42\,(Ba^4b + 2}{126\,x^{21}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="maxima")
```

output
```
-1/126*(42*B*b^5*x^18 + 21*(5*B*a*b^4 + A*b^5)*x^15 + 70*(2*B*a^2*b^3 + A*
a*b^4)*x^12 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 42*(B*a^4*b + 2*A*a^3*b^2)
*x^6 + 6*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^21
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(45) = 90$.

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.65

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{22}}\,dx =$$
$$-\frac{42\,Bb^5x^{18} + 105\,Bab^4x^{15} + 21\,Ab^5x^{15} + 140\,Ba^2b^3x^{12} + 70\,Aab^4x^{12} + 105\,Ba^3b^2x^9 + 105\,Aa^2b^3x^9 + 4}{126\,x^{21}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="giac")
```

output
```
-1/126*(42*B*b^5*x^18 + 105*B*a*b^4*x^15 + 21*A*b^5*x^15 + 140*B*a^2*b^3*x
^12 + 70*A*a*b^4*x^12 + 105*B*a^3*b^2*x^9 + 105*A*a^2*b^3*x^9 + 42*B*a^4*b
*x^6 + 84*A*a^3*b^2*x^6 + 7*B*a^5*x^3 + 35*A*a^4*b*x^3 + 6*A*a^5)/x^21
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.54

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{22}} \, dx = -\frac{\frac{A\,a^5}{21} + x^6 \left(\frac{B\,a^4\,b}{3} + \frac{2\,A\,a^3\,b^2}{3}\right) + x^{12} \left(\frac{10\,B\,a^2\,b^3}{9} + \frac{5\,A\,a\,b^4}{9}\right) + x^3 \left(\frac{B\,a^5}{18} + \frac{5\,A\,b\,a^4}{18}\right) + x^{15} \left(\frac{A\,b^5}{6} + \frac{5\,B\,a\,b^4}{6}\right) + x^9}{x^{21}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^22,x)
```

output
```
-((A*a^5)/21 + x^6*((2*A*a^3*b^2)/3 + (B*a^4*b)/3) + x^12*((10*B*a^2*b^3)/
9 + (5*A*a*b^4)/9) + x^3*((B*a^5)/18 + (5*A*a^4*b)/18) + x^15*((A*b^5)/6 +
 (5*B*a*b^4)/6) + x^9*((5*A*a^2*b^3)/6 + (5*B*a^3*b^2)/6) + (B*b^5*x^18)/3
)/x^21
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{22}} \, dx = \frac{-7b^6x^{18} - 21a\,b^5x^{15} - 35a^2b^4x^{12} - 35a^3b^3x^9 - 21a^4b^2x^6 - 7a^5b\,x^3 - a^6}{21x^{21}}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^22,x)
```

output
```
( - a**6 - 7*a**5*b*x**3 - 21*a**4*b**2*x**6 - 35*a**3*b**3*x**9 - 35*a**2
*b**4*x**12 - 21*a*b**5*x**15 - 7*b**6*x**18)/(21*x**21)
```

3.34 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{25}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{25}}\,dx = -\frac{A(a+bx^3)^6}{24ax^{24}} + \frac{(Ab-4aB)(a+bx^3)^6}{84a^2x^{21}} - \frac{b(Ab-4aB)(a+bx^3)^6}{504a^3x^{18}}$$

output
```
-1/24*A*(b*x^3+a)^6/a/x^24+1/84*(A*b-4*B*a)*(b*x^3+a)^6/a^2/x^21-1/504*b*(
A*b-4*B*a)*(b*x^3+a)^6/a^3/x^18
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{25}}\,dx = -\frac{28b^5x^{15}(2A+3Bx^3) + 70ab^4x^{12}(3A+4Bx^3) + 84a^2b^3x^9(4A+5Bx^3) + 56a^3b^2x^6(5A+6Bx^3) + 20a}{504x^{24}}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^25,x]
```

output
```
-1/504*(28*b^5*x^15*(2*A + 3*B*x^3) + 70*a*b^4*x^12*(3*A + 4*B*x^3) + 84*a
^2*b^3*x^9*(4*A + 5*B*x^3) + 56*a^3*b^2*x^6*(5*A + 6*B*x^3) + 20*a^4*b*x^3
*(6*A + 7*B*x^3) + 3*a^5*(7*A + 8*B*x^3))/x^24
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {948, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{25}} \, dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3} \int \frac{(bx^3+a)^5 (Bx^3+A)}{x^{27}} dx^3$$

$$\Big\downarrow \; 87$$

$$\frac{1}{3} \left(-\frac{(Ab-4aB) \int \frac{(bx^3+a)^5}{x^{24}} dx^3}{4a} - \frac{A(a+bx^3)^6}{8ax^{24}} \right)$$

$$\Big\downarrow \; 55$$

$$\frac{1}{3} \left(-\frac{(Ab-4aB) \left(-\frac{b \int \frac{(bx^3+a)^5}{x^{21}} dx^3}{7a} - \frac{(a+bx^3)^6}{7ax^{21}} \right)}{4a} - \frac{A(a+bx^3)^6}{8ax^{24}} \right)$$

$$\Big\downarrow \; 48$$

$$\frac{1}{3} \left(-\frac{\left(\frac{b(a+bx^3)^6}{42a^2x^{18}} - \frac{(a+bx^3)^6}{7ax^{21}} \right) (Ab-4aB)}{4a} - \frac{A(a+bx^3)^6}{8ax^{24}} \right)$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^25,x]
```

output
```
(-1/8*(A*(a + b*x^3)^6)/(a*x^24) - ((A*b - 4*a*B)*(-1/7*(a + b*x^3)^6/(a*x
^21) + (b*(a + b*x^3)^6)/(42*a^2*x^18)))/(4*a))/3
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.37

method	result
default	$-\frac{a^4(5Ab+Ba)}{21x^{21}}-\frac{2a^2b^2(Ab+Ba)}{3x^{15}}-\frac{a^5A}{24x^{24}}-\frac{b^5B}{6x^6}-\frac{5a^3b(2Ab+Ba)}{18x^{18}}-\frac{b^4(Ab+5Ba)}{9x^9}-\frac{5ab^3(Ab+2Ba)}{12x^{12}}$
norman	$\frac{-\frac{a^5A}{24}+(-\frac{5}{21}a^4bA-\frac{1}{21}a^5B)x^3+(-\frac{5}{9}a^3b^2A-\frac{5}{18}a^4bB)x^6+(-\frac{2}{3}a^2b^3A-\frac{2}{3}a^3b^2B)x^9+(-\frac{5}{12}ab^4A-\frac{5}{6}a^2b^3B)x^{12}+(-\frac{1}{9}b^5A-}{x^{24}}$
risch	$\frac{-\frac{a^5A}{24}+(-\frac{5}{21}a^4bA-\frac{1}{21}a^5B)x^3+(-\frac{5}{9}a^3b^2A-\frac{5}{18}a^4bB)x^6+(-\frac{2}{3}a^2b^3A-\frac{2}{3}a^3b^2B)x^9+(-\frac{5}{12}ab^4A-\frac{5}{6}a^2b^3B)x^{12}+(-\frac{1}{9}b^5A-}{x^{24}}$
gosper	$-\frac{84b^5Bx^{18}+56Ab^5x^{15}+280Bab^4x^{15}+210aAb^4x^{12}+420Ba^2b^3x^{12}+336a^2Ab^3x^9+336Ba^3b^2x^9+280a^3Ab^2x^6+140Ba^4}{504x^{24}}$
parallelrisch	$-\frac{84b^5Bx^{18}+56Ab^5x^{15}+280Bab^4x^{15}+210aAb^4x^{12}+420Ba^2b^3x^{12}+336a^2Ab^3x^9+336Ba^3b^2x^9+280a^3Ab^2x^6+140Ba^4}{504x^{24}}$
orering	$-\frac{84b^5Bx^{18}+56Ab^5x^{15}+280Bab^4x^{15}+210aAb^4x^{12}+420Ba^2b^3x^{12}+336a^2Ab^3x^9+336Ba^3b^2x^9+280a^3Ab^2x^6+140Ba^4}{504x^{24}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^25,x,method=_RETURNVERBOSE)
```

output
```
-1/21*a^4*(5*A*b+B*a)/x^21-2/3*a^2*b^2*(A*b+B*a)/x^15-1/24*a^5*A/x^24-1/6*
b^5*B/x^6-5/18*a^3*b*(2*A*b+B*a)/x^18-1/9*b^4*(A*b+5*B*a)/x^9-5/12*a*b^3*(
A*b+2*B*a)/x^12
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{25}}\,dx = -\frac{84\,Bb^5x^{18}+56\,(5\,Bab^4+Ab^5)x^{15}+210\,(2\,Ba^2b^3+Aab^4)x^{12}+336\,(Ba^3b^2+Aa^2b^3)x^9+140\,(Ba^4b-}{504\,x^{24}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^25,x, algorithm="fricas")
```

output
```
-1/504*(84*B*b^5*x^18 + 56*(5*B*a*b^4 + A*b^5)*x^15 + 210*(2*B*a^2*b^3 + A
*a*b^4)*x^12 + 336*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 140*(B*a^4*b + 2*A*a^3*b^
2)*x^6 + 21*A*a^5 + 24*(B*a^5 + 5*A*a^4*b)*x^3)/x^24
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{25}} dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**25,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{25}} dx = -\frac{84 Bb^5x^{18} + 56 (5 Bab^4 + Ab^5)x^{15} + 210 (2 Ba^2b^3 + Aab^4)x^{12} + 336 (Ba^3b^2 + Aa^2b^3)x^9 + 140 (Ba^4b}{504 x^{24}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^25,x, algorithm="maxima")
```

output
```
-1/504*(84*B*b^5*x^18 + 56*(5*B*a*b^4 + A*b^5)*x^15 + 210*(2*B*a^2*b^3 + A
*a*b^4)*x^12 + 336*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 140*(B*a^4*b + 2*A*a^3*b^
2)*x^6 + 21*A*a^5 + 24*(B*a^5 + 5*A*a^4*b)*x^3)/x^24
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{25}} dx = -\frac{84 Bb^5x^{18} + 280 Bab^4x^{15} + 56 Ab^5x^{15} + 420 Ba^2b^3x^{12} + 210 Aab^4x^{12} + 336 Ba^3b^2x^9 + 336 Aa^2b^3x^9 +}{504 x^{24}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^25,x, algorithm="giac")
```

output
```
-1/504*(84*B*b^5*x^18 + 280*B*a*b^4*x^15 + 56*A*b^5*x^15 + 420*B*a^2*b^3*x
^12 + 210*A*a*b^4*x^12 + 336*B*a^3*b^2*x^9 + 336*A*a^2*b^3*x^9 + 140*B*a^4
*b*x^6 + 280*A*a^3*b^2*x^6 + 24*B*a^5*x^3 + 120*A*a^4*b*x^3 + 21*A*a^5)/x^
24
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{25}}\,dx = -\frac{\frac{A\,a^5}{24}+x^{12}\left(\frac{5\,B\,a^2\,b^3}{6}+\frac{5\,A\,a\,b^4}{12}\right)+x^6\left(\frac{5\,B\,a^4\,b}{18}+\frac{5\,A\,a^3\,b^2}{9}\right)+x^3\left(\frac{B\,a^5}{21}+\frac{5\,A\,b\,a^4}{21}\right)+x^{15}\left(\frac{A\,b^5}{9}+\frac{5\,B\,a\,b^4}{9}\right)+x^9}{x^{24}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^25,x)
```

output
```
-((A*a^5)/24 + x^12*((5*B*a^2*b^3)/6 + (5*A*a*b^4)/12) + x^6*((5*A*a^3*b^2
)/9 + (5*B*a^4*b)/18) + x^3*((B*a^5)/21 + (5*A*a^4*b)/21) + x^15*((A*b^5)/
9 + (5*B*a*b^4)/9) + x^9*((2*A*a^2*b^3)/3 + (2*B*a^3*b^2)/3) + (B*b^5*x^18
)/6)/x^24
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{25}}\,dx = \frac{-28b^6x^{18}-112a\,b^5x^{15}-210a^2b^4x^{12}-224a^3b^3x^9-140a^4b^2x^6-48a^5b\,x^3-7a^6}{168x^{24}}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^25,x)
```

output
```
( - 7*a**6 - 48*a**5*b*x**3 - 140*a**4*b**2*x**6 - 224*a**3*b**3*x**9 - 21
0*a**2*b**4*x**12 - 112*a*b**5*x**15 - 28*b**6*x**18)/(168*x**24)
```

3.35 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{28}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{28}}\,dx = -\frac{a^5A}{27x^{27}} - \frac{a^4(5Ab+aB)}{24x^{24}} - \frac{5a^3b(2Ab+aB)}{21x^{21}} - \frac{5a^2b^2(Ab+aB)}{9x^{18}} - \frac{ab^3(Ab+2aB)}{3x^{15}} - \frac{b^4(Ab+5aB)}{12x^{12}} - \frac{b^5B}{9x^9}$$

output
```
-1/27*a^5*A/x^27-1/24*a^4*(5*A*b+B*a)/x^24-5/21*a^3*b*(2*A*b+B*a)/x^21-5/9
*a^2*b^2*(A*b+B*a)/x^18-1/3*a*b^3*(A*b+2*B*a)/x^15-1/12*b^4*(A*b+5*B*a)/x^
12-1/9*b^5*B/x^9
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{28}}\,dx = -\frac{42b^5x^{15}(3A+4Bx^3)+126ab^4x^{12}(4A+5Bx^3)+168a^2b^3x^9(5A+6Bx^3)+120a^3b^2x^6(6A+7Bx^3)+}{1512x^{27}}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^28,x]
```

output
```
-1/1512*(42*b^5*x^15*(3*A + 4*B*x^3) + 126*a*b^4*x^12*(4*A + 5*B*x^3) + 16
8*a^2*b^3*x^9*(5*A + 6*B*x^3) + 120*a^3*b^2*x^6*(6*A + 7*B*x^3) + 45*a^4*b
*x^3*(7*A + 8*B*x^3) + 7*a^5*(8*A + 9*B*x^3))/x^27
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{28}} dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5 (Bx^3+A)}{x^{30}}dx^3$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{3}\int \left(\frac{Aa^5}{x^{30}}+\frac{(5Ab+aB)a^4}{x^{27}}+\frac{5b(2Ab+aB)a^3}{x^{24}}+\frac{10b^2(Ab+aB)a^2}{x^{21}}+\frac{5b^3(Ab+2aB)a}{x^{18}}+\frac{b^5B}{x^{12}}+\frac{b^4(Ab+5aB}{x^{15}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^5A}{9x^{27}}-\frac{a^4(aB+5Ab)}{8x^{24}}-\frac{5a^3b(aB+2Ab)}{7x^{21}}-\frac{5a^2b^2(aB+Ab)}{3x^{18}}-\frac{b^4(5aB+Ab)}{4x^{12}}-\frac{ab^3(2aB+Ab)}{x^{15}}-\frac{b^5B}{3x^9}\right)$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^28,x]
```

output
```
(-1/9*(a^5*A)/x^27 - (a^4*(5*A*b + a*B))/(8*x^24) - (5*a^3*b*(2*A*b + a*B)
)/(7*x^21) - (5*a^2*b^2*(A*b + a*B))/(3*x^18) - (a*b^3*(A*b + 2*a*B))/x^15
 - (b^4*(A*b + 5*a*B))/(4*x^12) - (b^5*B)/(3*x^9))/3
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^5 A}{27x^{27}}-\frac{a^4(5Ab+Ba)}{24x^{24}}-\frac{5a^3b(2Ab+Ba)}{21x^{21}}-\frac{5a^2b^2(Ab+Ba)}{9x^{18}}-\frac{a\,b^3(Ab+2Ba)}{3x^{15}}-\frac{b^4(Ab+5Ba)}{12x^{12}}-\frac{b^5 B}{9x^9}$
norman	$\frac{-\frac{a^5A}{27}+\left(-\frac{5}{24}a^4bA-\frac{1}{24}a^5B\right)x^3+\left(-\frac{10}{21}a^3b^2A-\frac{5}{21}a^4bB\right)x^6+\left(-\frac{5}{9}a^2b^3A-\frac{5}{9}a^3b^2B\right)x^9+\left(-\frac{1}{3}a\,b^4A-\frac{2}{3}a^2b^3B\right)x^{12}+\left(-\frac{1}{12}b^5A-\dots\right.}{x^{27}}$
risch	$\frac{-\frac{a^5A}{27}+\left(-\frac{5}{24}a^4bA-\frac{1}{24}a^5B\right)x^3+\left(-\frac{10}{21}a^3b^2A-\frac{5}{21}a^4bB\right)x^6+\left(-\frac{5}{9}a^2b^3A-\frac{5}{9}a^3b^2B\right)x^9+\left(-\frac{1}{3}a\,b^4A-\frac{2}{3}a^2b^3B\right)x^{12}+\left(-\frac{1}{12}b^5A-\dots\right.}{x^{27}}$
gosper	$-\frac{168b^5Bx^{18}+126Ab^5x^{15}+630Bab^4x^{15}+504aAb^4x^{12}+1008Ba^2b^3x^{12}+840a^2Ab^3x^9+840Ba^3b^2x^9+720a^3Ab^2x^6+360B\dots}{1512x^{27}}$
parallelrisch	$-\frac{168b^5Bx^{18}+126Ab^5x^{15}+630Bab^4x^{15}+504aAb^4x^{12}+1008Ba^2b^3x^{12}+840a^2Ab^3x^9+840Ba^3b^2x^9+720a^3Ab^2x^6+360B\dots}{1512x^{27}}$
orering	$-\frac{168b^5Bx^{18}+126Ab^5x^{15}+630Bab^4x^{15}+504aAb^4x^{12}+1008Ba^2b^3x^{12}+840a^2Ab^3x^9+840Ba^3b^2x^9+720a^3Ab^2x^6+360B\dots}{1512x^{27}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^28,x,method=_RETURNVERBOSE)
```

output
```
-1/27*a^5*A/x^27-1/24*a^4*(5*A*b+B*a)/x^24-5/21*a^3*b*(2*A*b+B*a)/x^21-5/9
*a^2*b^2*(A*b+B*a)/x^18-1/3*a*b^3*(A*b+2*B*a)/x^15-1/12*b^4*(A*b+5*B*a)/x^
12-1/9*b^5*B/x^9
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{28}}\,dx = -\frac{168\,Bb^5x^{18}+126\,(5\,Bab^4+Ab^5)x^{15}+504\,(2\,Ba^2b^3+Aab^4)x^{12}+840\,(Ba^3b^2+Aa^2b^3)x^9+360\,(Ba^4}{1512\,x^{27}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^28,x, algorithm="fricas")
```

output
```
-1/1512*(168*B*b^5*x^18 + 126*(5*B*a*b^4 + A*b^5)*x^15 + 504*(2*B*a^2*b^3
+ A*a*b^4)*x^12 + 840*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 360*(B*a^4*b + 2*A*a^3
*b^2)*x^6 + 56*A*a^5 + 63*(B*a^5 + 5*A*a^4*b)*x^3)/x^27
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{28}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**28,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{28}} \, dx = -\frac{168 \, Bb^5x^{18} + 126 \, (5 \, Bab^4 + Ab^5)x^{15} + 504 \, (2 \, Ba^2b^3 + Aab^4)x^{12} + 840 \, (Ba^3b^2 + Aa^2b^3)x^9 + 360 \, (Ba^4}{1512 \, x^{27}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^28,x, algorithm="maxima")
```

output
```
-1/1512*(168*B*b^5*x^18 + 126*(5*B*a*b^4 + A*b^5)*x^15 + 504*(2*B*a^2*b^3
+ A*a*b^4)*x^12 + 840*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 360*(B*a^4*b + 2*A*a^3
*b^2)*x^6 + 56*A*a^5 + 63*(B*a^5 + 5*A*a^4*b)*x^3)/x^27
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{28}} \, dx = -\frac{168 \, Bb^5x^{18} + 630 \, Bab^4x^{15} + 126 \, Ab^5x^{15} + 1008 \, Ba^2b^3x^{12} + 504 \, Aab^4x^{12} + 840 \, Ba^3b^2x^9 + 840 \, Aa^2b^3x}{1512 \, x^{27}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^28,x, algorithm="giac")
```

output
```
-1/1512*(168*B*b^5*x^18 + 630*B*a*b^4*x^15 + 126*A*b^5*x^15 + 1008*B*a^2*b
^3*x^12 + 504*A*a*b^4*x^12 + 840*B*a^3*b^2*x^9 + 840*A*a^2*b^3*x^9 + 360*B
*a^4*b*x^6 + 720*A*a^3*b^2*x^6 + 63*B*a^5*x^3 + 315*A*a^4*b*x^3 + 56*A*a^5
)/x^27
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{28}} \, dx =$$
$$-\frac{\frac{A\,a^5}{27} + x^{12}\left(\frac{2\,B\,a^2\,b^3}{3} + \frac{A\,a\,b^4}{3}\right) + x^6\left(\frac{5\,B\,a^4\,b}{21} + \frac{10\,A\,a^3\,b^2}{21}\right) + x^3\left(\frac{B\,a^5}{24} + \frac{5\,A\,b\,a^4}{24}\right) + x^{15}\left(\frac{A\,b^5}{12} + \frac{5\,B\,a\,b^4}{12}\right) + x^9}{x^{27}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^28,x)
```

output
```
-((A*a^5)/27 + x^12*((2*B*a^2*b^3)/3 + (A*a*b^4)/3) + x^6*((10*A*a^3*b^2)/
21 + (5*B*a^4*b)/21) + x^3*((B*a^5)/24 + (5*A*a^4*b)/24) + x^15*((A*b^5)/1
2 + (5*B*a*b^4)/12) + x^9*((5*A*a^2*b^3)/9 + (5*B*a^3*b^2)/9) + (B*b^5*x^1
8)/9)/x^27
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{28}} \, dx$$
$$= \frac{-84b^6x^{18} - 378a\,b^5x^{15} - 756a^2b^4x^{12} - 840a^3b^3x^9 - 540a^4b^2x^6 - 189a^5b\,x^3 - 28a^6}{756x^{27}}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^28,x)
```

output
```
( - 28*a**6 - 189*a**5*b*x**3 - 540*a**4*b**2*x**6 - 840*a**3*b**3*x**9 -
756*a**2*b**4*x**12 - 378*a*b**5*x**15 - 84*b**6*x**18)/(756*x**27)
```

3.36 $\int x^9(a+bx^3)^5(A+Bx^3)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int x^9\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{1}{10}a^5Ax^{10}+\frac{1}{13}a^4(5Ab+aB)x^{13}+\frac{5}{16}a^3b(2Ab+aB)x^{16}+\frac{10}{19}a^2b^2(Ab+aB)x^{19}+\frac{5}{22}ab^3(Ab+2aB)x^{22}+\frac{1}{25}b^4(Ab+5aB)x^{25}+\frac{1}{28}b^5Bx^{28}$$

output

```
1/10*a^5*A*x^10+1/13*a^4*(5*A*b+B*a)*x^13+5/16*a^3*b*(2*A*b+B*a)*x^16+10/1
9*a^2*b^2*(A*b+B*a)*x^19+5/22*a*b^3*(A*b+2*B*a)*x^22+1/25*b^4*(A*b+5*B*a)*
x^25+1/28*b^5*B*x^28
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int x^9\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{1}{10}a^5Ax^{10}+\frac{1}{13}a^4(5Ab+aB)x^{13}+\frac{5}{16}a^3b(2Ab+aB)x^{16}+\frac{10}{19}a^2b^2(Ab+aB)x^{19}+\frac{5}{22}ab^3(Ab+2aB)x^{22}+\frac{1}{25}b^4(Ab+5aB)x^{25}+\frac{1}{28}b^5Bx^{28}$$

input
```
Integrate[x^9*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^13)/13 + (5*a^3*b*(2*A*b + a*B)*x^1
6)/16 + (10*a^2*b^2*(A*b + a*B)*x^19)/19 + (5*a*b^3*(A*b + 2*a*B)*x^22)/22
 + (b^4*(A*b + 5*a*B)*x^25)/25 + (b^5*B*x^28)/28
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \; 950$$

$$\int \left(a^5 A x^9 + a^4 x^{12}(aB+5Ab) + 5a^3 b x^{15}(aB+2Ab) + 10a^2 b^2 x^{18}(aB+Ab) + b^4 x^{24}(5aB+Ab) + 5ab^3 x^{21}(2aB$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{10}a^5 A x^{10} + \frac{1}{13}a^4 x^{13}(aB+5Ab) + \frac{5}{16}a^3 b x^{16}(aB+2Ab) + \frac{10}{19}a^2 b^2 x^{19}(aB+Ab) + \frac{1}{25}b^4 x^{25}(5aB+Ab) + \frac{5}{22}ab^3 x^{22}(2aB+Ab) + \frac{1}{28}b^5 B x^{28}$$

input
```
Int[x^9*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^13)/13 + (5*a^3*b*(2*A*b + a*B)*x^1
6)/16 + (10*a^2*b^2*(A*b + a*B)*x^19)/19 + (5*a*b^3*(A*b + 2*a*B)*x^22)/22
 + (b^4*(A*b + 5*a*B)*x^25)/25 + (b^5*B*x^28)/28
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
norman	$\frac{b^5 B\,x^{28}}{28}+\left(\frac{1}{25}b^5A+\frac{1}{5}a\,b^4B\right)x^{25}+\left(\frac{5}{22}a\,b^4A+\frac{5}{11}a^2b^3B\right)x^{22}+\frac{a^5A\,x^{10}}{10}+\left(\frac{5}{13}a^4bA+\frac{1}{13}a^5B\right)x$
default	$\frac{b^5B\,x^{28}}{28}+\frac{(b^5A+5a\,b^4B)x^{25}}{25}+\frac{(5a\,b^4A+10a^2b^3B)x^{22}}{22}+\frac{(10a^2b^3A+10a^3b^2B)x^{19}}{19}+\frac{(10a^3b^2A+5a^4bB)x^{16}}{16}+(5a$
gosper	$\frac{1}{28}b^5B\,x^{28}+\frac{1}{25}x^{25}b^5A+\frac{1}{5}x^{25}a\,b^4B+\frac{5}{22}x^{22}a\,b^4A+\frac{5}{11}x^{22}a^2b^3B+\frac{1}{10}a^5A\,x^{10}+\frac{5}{13}x^{13}a^4bA+$
risch	$\frac{1}{28}b^5B\,x^{28}+\frac{1}{25}x^{25}b^5A+\frac{1}{5}x^{25}a\,b^4B+\frac{5}{22}x^{22}a\,b^4A+\frac{5}{11}x^{22}a^2b^3B+\frac{1}{10}a^5A\,x^{10}+\frac{5}{13}x^{13}a^4bA+$
parallelrisch	$\frac{1}{28}b^5B\,x^{28}+\frac{1}{25}x^{25}b^5A+\frac{1}{5}x^{25}a\,b^4B+\frac{5}{22}x^{22}a\,b^4A+\frac{5}{11}x^{22}a^2b^3B+\frac{1}{10}a^5A\,x^{10}+\frac{5}{13}x^{13}a^4bA+$
orering	$\frac{x^{10}(271700b^5B\,x^{18}+304304A\,b^5x^{15}+1521520Ba\,b^4x^{15}+1729000aA\,b^4x^{12}+3458000B\,a^2b^3x^{12}+4004000a^2A\,b^3x^9+400400}{7607600}$

input
```
int(x^9*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/28*b^5*B*x^28+(1/25*b^5*A+1/5*a*b^4*B)*x^25+(5/22*a*b^4*A+5/11*a^2*b^3*B
)*x^22+1/10*a^5*A*x^10+(5/13*a^4*b*A+1/13*a^5*B)*x^13+(5/8*a^3*b^2*A+5/16*
a^4*b*B)*x^16+(10/19*a^2*b^3*A+10/19*a^3*b^2*B)*x^19
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^9\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{1}{28}\,Bb^5x^{28}+\frac{1}{25}\left(5\,Bab^4+Ab^5\right)x^{25}+\frac{5}{22}\left(2\,Ba^2b^3+Aab^4\right)x^{22}+\frac{10}{19}\left(Ba^3b^2+Aa^2b^3\right)x^{19}+\frac{5}{16}\left(Ba^4b+2\,Aa^3b^2\right)x^{16}+\frac{1}{10}\,Aa^5x^{10}+\frac{1}{13}\left(Ba^5+5\,Aa^4b\right)x^{13}$$

input
```
integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/28*B*b^5*x^28 + 1/25*(5*B*a*b^4 + A*b^5)*x^25 + 5/22*(2*B*a^2*b^3 + A*a*
b^4)*x^22 + 10/19*(B*a^3*b^2 + A*a^2*b^3)*x^19 + 5/16*(B*a^4*b + 2*A*a^3*b
^2)*x^16 + 1/10*A*a^5*x^10 + 1/13*(B*a^5 + 5*A*a^4*b)*x^13
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16

$$\int x^9\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{Aa^5x^{10}}{10}+\frac{Bb^5x^{28}}{28}+x^{25}\left(\frac{Ab^5}{25}+\frac{Bab^4}{5}\right)+x^{22}\cdot\left(\frac{5Aab^4}{22}+\frac{5Ba^2b^3}{11}\right)+x^{19}\cdot\left(\frac{10Aa^2b^3}{19}+\frac{10Ba^3b^2}{19}\right)+x^{16}\cdot\left(\frac{5Aa^3b^2}{8}+\frac{5Ba^4b}{16}\right)+x^{13}\cdot\left(\frac{5Aa^4b}{13}+\frac{Ba^5}{13}\right)$$

input
```
integrate(x**9*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x**10/10 + B*b**5*x**28/28 + x**25*(A*b**5/25 + B*a*b**4/5) + x**22
*(5*A*a*b**4/22 + 5*B*a**2*b**3/11) + x**19*(10*A*a**2*b**3/19 + 10*B*a**3
*b**2/19) + x**16*(5*A*a**3*b**2/8 + 5*B*a**4*b/16) + x**13*(5*A*a**4*b/13
 + B*a**5/13)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^9 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx = \frac{1}{28} Bb^5x^{28} + \frac{1}{25} \left(5 Bab^4 + Ab^5\right)x^{25} + \frac{5}{22} \left(2 Ba^2b^3 + Aab^4\right)x^{22} + \frac{10}{19} \left(Ba^3b^2 + Aa^2b^3\right)x^{19} + \frac{5}{16} \left(Ba^4b + 2 Aa^3b^2\right)x^{16} + \frac{1}{10} Aa^5x^{10} + \frac{1}{13} \left(Ba^5 + 5 Aa^4b\right)x^{13}$$

input
```
integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/28*B*b^5*x^28 + 1/25*(5*B*a*b^4 + A*b^5)*x^25 + 5/22*(2*B*a^2*b^3 + A*a*
b^4)*x^22 + 10/19*(B*a^3*b^2 + A*a^2*b^3)*x^19 + 5/16*(B*a^4*b + 2*A*a^3*b
^2)*x^16 + 1/10*A*a^5*x^10 + 1/13*(B*a^5 + 5*A*a^4*b)*x^13
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int x^9 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx = \frac{1}{28} Bb^5x^{28} + \frac{1}{5} Bab^4x^{25} + \frac{1}{25} Ab^5x^{25} + \frac{5}{11} Ba^2b^3x^{22} + \frac{5}{22} Aab^4x^{22} + \frac{10}{19} Ba^3b^2x^{19} + \frac{10}{19} Aa^2b^3x^{19} + \frac{5}{16} Ba^4bx^{16} + \frac{5}{8} Aa^3b^2x^{16} + \frac{1}{13} Ba^5x^{13} + \frac{5}{13} Aa^4bx^{13} + \frac{1}{10} Aa^5x^{10}$$

input
```
integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/28*B*b^5*x^28 + 1/5*B*a*b^4*x^25 + 1/25*A*b^5*x^25 + 5/11*B*a^2*b^3*x^22
 + 5/22*A*a*b^4*x^22 + 10/19*B*a^3*b^2*x^19 + 10/19*A*a^2*b^3*x^19 + 5/16*
B*a^4*b*x^16 + 5/8*A*a^3*b^2*x^16 + 1/13*B*a^5*x^13 + 5/13*A*a^4*b*x^13 +
1/10*A*a^5*x^10
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^9(a+bx^3)^5\,(A+Bx^3)\;dx = x^{13}\left(\frac{B\,a^5}{13}+\frac{5\,A\,b\,a^4}{13}\right)+x^{25}\left(\frac{A\,b^5}{25}+\frac{B\,a\,b^4}{5}\right) + \frac{A\,a^5\,x^{10}}{10}+\frac{B\,b^5\,x^{28}}{28}+\frac{10\,a^2\,b^2\,x^{19}\,(A\,b+B\,a)}{19} + \frac{5\,a^3\,b\,x^{16}\,(2\,A\,b+B\,a)}{16}+\frac{5\,a\,b^3\,x^{22}\,(A\,b+2\,B\,a)}{22}$$

input
```
int(x^9*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^13*((B*a^5)/13 + (5*A*a^4*b)/13) + x^25*((A*b^5)/25 + (B*a*b^4)/5) + (A*
a^5*x^10)/10 + (B*b^5*x^28)/28 + (10*a^2*b^2*x^19*(A*b + B*a))/19 + (5*a^3
*b*x^16*(2*A*b + B*a))/16 + (5*a*b^3*x^22*(A*b + 2*B*a))/22
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int x^9(a+bx^3)^5\,(A+Bx^3)\;dx = \frac{x^{10}(271700b^6x^{18}+1825824a\,b^5x^{15}+5187000a^2b^4x^{12}+8008000a^3b^3x^9+7132125a^4b^2x^6+3511200a^5b\,x}{7607600}$$

input
```
int(x^9*(b*x^3+a)^5*(B*x^3+A),x)
```

output
```
(x**10*(760760*a**6 + 3511200*a**5*b*x**3 + 7132125*a**4*b**2*x**6 + 80080
00*a**3*b**3*x**9 + 5187000*a**2*b**4*x**12 + 1825824*a*b**5*x**15 + 27170
0*b**6*x**18))/7607600
```

3.37 $\int x^7(a+bx^3)^5(A+Bx^3)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int x^7(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{8}a^5Ax^8+\frac{1}{11}a^4(5Ab+aB)x^{11}+\frac{5}{14}a^3b(2Ab+aB)x^{14} +\frac{10}{17}a^2b^2(Ab+aB)x^{17}+\frac{1}{4}ab^3(Ab+2aB)x^{20} +\frac{1}{23}b^4(Ab+5aB)x^{23}+\frac{1}{26}b^5Bx^{26}$$

output
```
1/8*a^5*A*x^8+1/11*a^4*(5*A*b+B*a)*x^11+5/14*a^3*b*(2*A*b+B*a)*x^14+10/17*
a^2*b^2*(A*b+B*a)*x^17+1/4*a*b^3*(A*b+2*B*a)*x^20+1/23*b^4*(A*b+5*B*a)*x^2
3+1/26*b^5*B*x^26
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int x^7(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{8}a^5Ax^8+\frac{1}{11}a^4(5Ab+aB)x^{11}+\frac{5}{14}a^3b(2Ab+aB)x^{14} +\frac{10}{17}a^2b^2(Ab+aB)x^{17}+\frac{1}{4}ab^3(Ab+2aB)x^{20} +\frac{1}{23}b^4(Ab+5aB)x^{23}+\frac{1}{26}b^5Bx^{26}$$

input
```
Integrate[x^7*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^14)
/14 + (10*a^2*b^2*(A*b + a*B)*x^17)/17 + (a*b^3*(A*b + 2*a*B)*x^20)/4 + (b
^4*(A*b + 5*a*B)*x^23)/23 + (b^5*B*x^26)/26
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 (a + bx^3)^5 (A + Bx^3) \, dx$$

$$\Big\downarrow \text{950}$$

$$\int (a^5 A x^7 + a^4 x^{10}(aB + 5Ab) + 5a^3 b x^{13}(aB + 2Ab) + 10a^2 b^2 x^{16}(aB + Ab) + b^4 x^{22}(5aB + Ab) + 5ab^3 x^{19}(2aB$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}a^5 A x^8 + \frac{1}{11}a^4 x^{11}(aB + 5Ab) + \frac{5}{14}a^3 b x^{14}(aB + 2Ab) + \frac{10}{17}a^2 b^2 x^{17}(aB + Ab) + \frac{1}{23}b^4 x^{23}(5aB + Ab) + \frac{1}{4}ab^3 x^{20}(2aB + Ab) + \frac{1}{26}b^5 B x^{26}$$

input
```
Int[x^7*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^14)
/14 + (10*a^2*b^2*(A*b + a*B)*x^17)/17 + (a*b^3*(A*b + 2*a*B)*x^20)/4 + (b
^4*(A*b + 5*a*B)*x^23)/23 + (b^5*B*x^26)/26
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
norman	$\left(\frac{5}{11}a^4bA+\frac{1}{11}a^5B\right)x^{11}+\left(\frac{5}{7}a^3b^2A+\frac{5}{14}a^4bB\right)x^{14}+\frac{a^5Ax^8}{8}+\left(\frac{10}{17}a^2b^3A+\frac{10}{17}a^3b^2B\right)x^{17}+$
default	$\frac{b^5Bx^{26}}{26}+\frac{(b^5A+5ab^4B)x^{23}}{23}+\frac{(5ab^4A+10a^2b^3B)x^{20}}{20}+\frac{(10a^2b^3A+10a^3b^2B)x^{17}}{17}+\frac{(10a^3b^2A+5a^4bB)x^{14}}{14}+$
gosper	$\frac{5}{11}x^{11}a^4bA+\frac{1}{11}x^{11}a^5B+\frac{5}{7}x^{14}a^3b^2A+\frac{5}{14}x^{14}a^4bB+\frac{1}{8}a^5Ax^8+\frac{10}{17}x^{17}a^2b^3A+\frac{10}{17}x^{17}a^3b^2B$
risch	$\frac{5}{11}x^{11}a^4bA+\frac{1}{11}x^{11}a^5B+\frac{5}{7}x^{14}a^3b^2A+\frac{5}{14}x^{14}a^4bB+\frac{1}{8}a^5Ax^8+\frac{10}{17}x^{17}a^2b^3A+\frac{10}{17}x^{17}a^3b^2B$
parallelrisch	$\frac{5}{11}x^{11}a^4bA+\frac{1}{11}x^{11}a^5B+\frac{5}{7}x^{14}a^3b^2A+\frac{5}{14}x^{14}a^4bB+\frac{1}{8}a^5Ax^8+\frac{10}{17}x^{17}a^2b^3A+\frac{10}{17}x^{17}a^3b^2B$
orering	$\frac{x^8(120428b^5Bx^{18}+136136Ab^5x^{15}+680680Bab^4x^{15}+782782aAb^4x^{12}+1565564Ba^2b^3x^{12}+1841840a^2Ab^3x^9+1841840B}{3131128}$

input
```
int(x^7*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
(5/11*a^4*b*A+1/11*a^5*B)*x^11+(5/7*a^3*b^2*A+5/14*a^4*b*B)*x^14+1/8*a^5*A
*x^8+(10/17*a^2*b^3*A+10/17*a^3*b^2*B)*x^17+1/26*b^5*B*x^26+(1/4*a*b^4*A+1
/2*a^2*b^3*B)*x^20+(1/23*b^5*A+5/23*a*b^4*B)*x^23
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^7 (a+bx^3)^5 (A+Bx^3)\, dx = \frac{1}{26} Bb^5x^{26} + \frac{1}{23} (5\,Bab^4 + Ab^5)x^{23} + \frac{1}{4} (2\,Ba^2b^3 + Aab^4)x^{20} + \frac{10}{17} (Ba^3b^2 + Aa^2b^3)x^{17} + \frac{5}{14} (Ba^4b + 2\,Aa^3b^2)x^{14} + \frac{1}{8} Aa^5x^8 + \frac{1}{11} (Ba^5 + 5\,Aa^4b)x^{11}$$

input
```
integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/26*B*b^5*x^26 + 1/23*(5*B*a*b^4 + A*b^5)*x^23 + 1/4*(2*B*a^2*b^3 + A*a*b
^4)*x^20 + 10/17*(B*a^3*b^2 + A*a^2*b^3)*x^17 + 5/14*(B*a^4*b + 2*A*a^3*b^
2)*x^14 + 1/8*A*a^5*x^8 + 1/11*(B*a^5 + 5*A*a^4*b)*x^11
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.15

$$\int x^7 (a+bx^3)^5 (A+Bx^3)\, dx = \frac{Aa^5x^8}{8} + \frac{Bb^5x^{26}}{26} + x^{23}\left(\frac{Ab^5}{23} + \frac{5Bab^4}{23}\right) + x^{20}\left(\frac{Aab^4}{4} + \frac{Ba^2b^3}{2}\right) + x^{17} \cdot \left(\frac{10Aa^2b^3}{17} + \frac{10Ba^3b^2}{17}\right) + x^{14} \cdot \left(\frac{5Aa^3b^2}{7} + \frac{5Ba^4b}{14}\right) + x^{11} \cdot \left(\frac{5Aa^4b}{11} + \frac{Ba^5}{11}\right)$$

input
```
integrate(x**7*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x**8/8 + B*b**5*x**26/26 + x**23*(A*b**5/23 + 5*B*a*b**4/23) + x**2
0*(A*a*b**4/4 + B*a**2*b**3/2) + x**17*(10*A*a**2*b**3/17 + 10*B*a**3*b**2
/17) + x**14*(5*A*a**3*b**2/7 + 5*B*a**4*b/14) + x**11*(5*A*a**4*b/11 + B*
a**5/11)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^7 (a + bx^3)^5 (A + Bx^3) \, dx = \frac{1}{26} Bb^5x^{26} + \frac{1}{23} (5 Bab^4 + Ab^5)x^{23} + \frac{1}{4} (2 Ba^2b^3 + Aab^4)x^{20} + \frac{10}{17} (Ba^3b^2 + Aa^2b^3)x^{17} + \frac{5}{14} (Ba^4b + 2 Aa^3b^2)x^{14} + \frac{1}{8} Aa^5x^8 + \frac{1}{11} (Ba^5 + 5 Aa^4b)x^{11}$$

input
```
integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/26*B*b^5*x^26 + 1/23*(5*B*a*b^4 + A*b^5)*x^23 + 1/4*(2*B*a^2*b^3 + A*a*b
^4)*x^20 + 10/17*(B*a^3*b^2 + A*a^2*b^3)*x^17 + 5/14*(B*a^4*b + 2*A*a^3*b^
2)*x^14 + 1/8*A*a^5*x^8 + 1/11*(B*a^5 + 5*A*a^4*b)*x^11
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int x^7 (a + bx^3)^5 (A + Bx^3) \, dx = \frac{1}{26} Bb^5x^{26} + \frac{5}{23} Bab^4x^{23} + \frac{1}{23} Ab^5x^{23} + \frac{1}{2} Ba^2b^3x^{20} + \frac{1}{4} Aab^4x^{20} + \frac{10}{17} Ba^3b^2x^{17} + \frac{10}{17} Aa^2b^3x^{17} + \frac{5}{14} Ba^4bx^{14} + \frac{5}{7} Aa^3b^2x^{14} + \frac{1}{11} Ba^5x^{11} + \frac{5}{11} Aa^4bx^{11} + \frac{1}{8} Aa^5x^8$$

input
```
integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/26*B*b^5*x^26 + 5/23*B*a*b^4*x^23 + 1/23*A*b^5*x^23 + 1/2*B*a^2*b^3*x^20
 + 1/4*A*a*b^4*x^20 + 10/17*B*a^3*b^2*x^17 + 10/17*A*a^2*b^3*x^17 + 5/14*B
*a^4*b*x^14 + 5/7*A*a^3*b^2*x^14 + 1/11*B*a^5*x^11 + 5/11*A*a^4*b*x^11 + 1
/8*A*a^5*x^8
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^7 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx = x^{11} \left(\frac{B\,a^5}{11}+\frac{5\,A\,b\,a^4}{11}\right)+x^{23} \left(\frac{A\,b^5}{23}+\frac{5\,B\,a\,b^4}{23}\right) + \frac{A\,a^5\,x^8}{8}+\frac{B\,b^5\,x^{26}}{26}+\frac{10\,a^2\,b^2\,x^{17}\,(A\,b+B\,a)}{17} + \frac{5\,a^3\,b\,x^{14}\,(2\,A\,b+B\,a)}{14}+\frac{a\,b^3\,x^{20}\,(A\,b+2\,B\,a)}{4}$$

input
```
int(x^7*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^11*((B*a^5)/11 + (5*A*a^4*b)/11) + x^23*((A*b^5)/23 + (5*B*a*b^4)/23) +
(A*a^5*x^8)/8 + (B*b^5*x^26)/26 + (10*a^2*b^2*x^17*(A*b + B*a))/17 + (5*a^
3*b*x^14*(2*A*b + B*a))/14 + (a*b^3*x^20*(A*b + 2*B*a))/4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int x^7 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx = \frac{x^8(120428b^6x^{18}+816816a\,b^5x^{15}+2348346a^2b^4x^{12}+3683680a^3b^3x^9+3354780a^4b^2x^6+1707888a^5b\,x^3}{3131128}$$

input
```
int(x^7*(b*x^3+a)^5*(B*x^3+A),x)
```

output
```
(x**8*(391391*a**6 + 1707888*a**5*b*x**3 + 3354780*a**4*b**2*x**6 + 368368
0*a**3*b**3*x**9 + 2348346*a**2*b**4*x**12 + 816816*a*b**5*x**15 + 120428*
b**6*x**18))/3131128
```

3.38 $\int x^6(a+bx^3)^5 (A+Bx^3) dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int x^6(a+bx^3)^5 (A+Bx^3) dx = \frac{1}{7}a^5Ax^7 + \frac{1}{10}a^4(5Ab+aB)x^{10} + \frac{5}{13}a^3b(2Ab+aB)x^{13} + \frac{5}{8}a^2b^2(Ab+aB)x^{16} + \frac{5}{19}ab^3(Ab+2aB)x^{19} + \frac{1}{22}b^4(Ab+5aB)x^{22} + \frac{1}{25}b^5Bx^{25}$$

output
```
1/7*a^5*A*x^7+1/10*a^4*(5*A*b+B*a)*x^10+5/13*a^3*b*(2*A*b+B*a)*x^13+5/8*a^
2*b^2*(A*b+B*a)*x^16+5/19*a*b^3*(A*b+2*B*a)*x^19+1/22*b^4*(A*b+5*B*a)*x^22
+1/25*b^5*B*x^25
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int x^6(a+bx^3)^5 (A+Bx^3) dx = \frac{1}{7}a^5Ax^7 + \frac{1}{10}a^4(5Ab+aB)x^{10} + \frac{5}{13}a^3b(2Ab+aB)x^{13} + \frac{5}{8}a^2b^2(Ab+aB)x^{16} + \frac{5}{19}ab^3(Ab+2aB)x^{19} + \frac{1}{22}b^4(Ab+5aB)x^{22} + \frac{1}{25}b^5Bx^{25}$$

input
```
Integrate[x^6*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^13)
/13 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^19)/19 + (
b^4*(A*b + 5*a*B)*x^22)/22 + (b^5*B*x^25)/25
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \; 950$$

$$\int \left(a^5Ax^6+a^4x^9(aB+5Ab)+5a^3bx^{12}(aB+2Ab)+10a^2b^2x^{15}(aB+Ab)+b^4x^{21}(5aB+Ab)+5ab^3x^{18}(2aB\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{7}a^5Ax^7+\frac{1}{10}a^4x^{10}(aB+5Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{1}{22}b^4x^{22}(5aB+Ab)+\frac{5}{19}ab^3x^{19}(2aB+Ab)+\frac{1}{25}b^5Bx^{25}$$

input
```
Int[x^6*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^13)
/13 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^19)/19 + (
b^4*(A*b + 5*a*B)*x^22)/22 + (b^5*B*x^25)/25
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A x^7}{7}+\left(\frac{1}{2}a^4bA+\frac{1}{10}a^5B\right)x^{10}+\left(\frac{10}{13}a^3b^2A+\frac{5}{13}a^4bB\right)x^{13}+\left(\frac{5}{8}a^2b^3A+\frac{5}{8}a^3b^2B\right)x^{16}+\left(\frac{5}{19}a\right.$
default	$\frac{b^5 B x^{25}}{25}+\frac{(b^5A+5a b^4B)x^{22}}{22}+\frac{(5a b^4A+10a^2b^3B)x^{19}}{19}+\frac{(10a^2b^3A+10a^3b^2B)x^{16}}{16}+\frac{(10a^3b^2A+5a^4bB)x^{13}}{13}+\frac{(5a}{}$
gosper	$\frac{1}{7}a^5Ax^7+\frac{1}{2}x^{10}a^4bA+\frac{1}{10}x^{10}a^5B+\frac{10}{13}x^{13}a^3b^2A+\frac{5}{13}x^{13}a^4bB+\frac{5}{8}x^{16}a^2b^3A+\frac{5}{8}x^{16}a^3b^2B+$
risch	$\frac{1}{7}a^5Ax^7+\frac{1}{2}x^{10}a^4bA+\frac{1}{10}x^{10}a^5B+\frac{10}{13}x^{13}a^3b^2A+\frac{5}{13}x^{13}a^4bB+\frac{5}{8}x^{16}a^2b^3A+\frac{5}{8}x^{16}a^3b^2B+$
parallelrisch	$\frac{1}{7}a^5Ax^7+\frac{1}{2}x^{10}a^4bA+\frac{1}{10}x^{10}a^5B+\frac{10}{13}x^{13}a^3b^2A+\frac{5}{13}x^{13}a^4bB+\frac{5}{8}x^{16}a^2b^3A+\frac{5}{8}x^{16}a^3b^2B+$
orering	$\frac{x^7\left(152152b^5Bx^{18}+172900Ab^5x^{15}+864500Bab^4x^{15}+1001000aAb^4x^{12}+2002000Ba^2b^3x^{12}+2377375a^2Ab^3x^9+2377375B\right.}{3803800}$

input
```
int(x^6*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*a^5*A*x^7+(1/2*a^4*b*A+1/10*a^5*B)*x^10+(10/13*a^3*b^2*A+5/13*a^4*b*B)
*x^13+(5/8*a^2*b^3*A+5/8*a^3*b^2*B)*x^16+(5/19*a*b^4*A+10/19*a^2*b^3*B)*x^
19+(1/22*b^5*A+5/22*a*b^4*B)*x^22+1/25*b^5*B*x^25
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^6 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx = \frac{1}{25} \, Bb^5x^{25} + \frac{1}{22} \, \left(5 \, Bab^4 + Ab^5\right)x^{22} + \frac{5}{19} \, \left(2 \, Ba^2b^3 + Aab^4\right)x^{19} + \frac{5}{8} \, \left(Ba^3b^2 + Aa^2b^3\right)x^{16} + \frac{5}{13} \, \left(Ba^4b + 2 \, Aa^3b^2\right)x^{13} + \frac{1}{7} \, Aa^5x^7 + \frac{1}{10} \, \left(Ba^5 + 5 \, Aa^4b\right)x^{10}$$

input
```
integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/25*B*b^5*x^25 + 1/22*(5*B*a*b^4 + A*b^5)*x^22 + 5/19*(2*B*a^2*b^3 + A*a*
b^4)*x^19 + 5/8*(B*a^3*b^2 + A*a^2*b^3)*x^16 + 5/13*(B*a^4*b + 2*A*a^3*b^2
)*x^13 + 1/7*A*a^5*x^7 + 1/10*(B*a^5 + 5*A*a^4*b)*x^10
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16

$$\int x^6 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx = \frac{Aa^5x^7}{7} + \frac{Bb^5x^{25}}{25} + x^{22}\left(\frac{Ab^5}{22} + \frac{5Bab^4}{22}\right) + x^{19} \cdot \left(\frac{5Aab^4}{19} + \frac{10Ba^2b^3}{19}\right) + x^{16} \cdot \left(\frac{5Aa^2b^3}{8} + \frac{5Ba^3b^2}{8}\right) + x^{13} \cdot \left(\frac{10Aa^3b^2}{13} + \frac{5Ba^4b}{13}\right) + x^{10}\left(\frac{Aa^4b}{2} + \frac{Ba^5}{10}\right)$$

input
```
integrate(x**6*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x**7/7 + B*b**5*x**25/25 + x**22*(A*b**5/22 + 5*B*a*b**4/22) + x**1
9*(5*A*a*b**4/19 + 10*B*a**2*b**3/19) + x**16*(5*A*a**2*b**3/8 + 5*B*a**3*
b**2/8) + x**13*(10*A*a**3*b**2/13 + 5*B*a**4*b/13) + x**10*(A*a**4*b/2 +
B*a**5/10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^6 (a+bx^3)^5 (A+Bx^3)\,dx = \frac{1}{25}Bb^5x^{25} + \frac{1}{22}(5Bab^4+Ab^5)x^{22} + \frac{5}{19}(2Ba^2b^3+Aab^4)x^{19} + \frac{5}{8}(Ba^3b^2+Aa^2b^3)x^{16} + \frac{5}{13}(Ba^4b+2Aa^3b^2)x^{13} + \frac{1}{7}Aa^5x^7 + \frac{1}{10}(Ba^5+5Aa^4b)x^{10}$$

input
```
integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/25*B*b^5*x^25 + 1/22*(5*B*a*b^4 + A*b^5)*x^22 + 5/19*(2*B*a^2*b^3 + A*a*
b^4)*x^19 + 5/8*(B*a^3*b^2 + A*a^2*b^3)*x^16 + 5/13*(B*a^4*b + 2*A*a^3*b^2
)*x^13 + 1/7*A*a^5*x^7 + 1/10*(B*a^5 + 5*A*a^4*b)*x^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int x^6 (a+bx^3)^5 (A+Bx^3)\,dx = \frac{1}{25}Bb^5x^{25} + \frac{5}{22}Bab^4x^{22} + \frac{1}{22}Ab^5x^{22} + \frac{10}{19}Ba^2b^3x^{19} + \frac{5}{19}Aab^4x^{19} + \frac{5}{8}Ba^3b^2x^{16} + \frac{5}{8}Aa^2b^3x^{16} + \frac{5}{13}Ba^4bx^{13} + \frac{10}{13}Aa^3b^2x^{13} + \frac{1}{10}Ba^5x^{10} + \frac{1}{2}Aa^4bx^{10} + \frac{1}{7}Aa^5x^7$$

input
```
integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/25*B*b^5*x^25 + 5/22*B*a*b^4*x^22 + 1/22*A*b^5*x^22 + 10/19*B*a^2*b^3*x^
19 + 5/19*A*a*b^4*x^19 + 5/8*B*a^3*b^2*x^16 + 5/8*A*a^2*b^3*x^16 + 5/13*B*
a^4*b*x^13 + 10/13*A*a^3*b^2*x^13 + 1/10*B*a^5*x^10 + 1/2*A*a^4*b*x^10 + 1
/7*A*a^5*x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^6 (a+bx^3)^5 (A+Bx^3)\, dx = x^{10} \left(\frac{B\,a^5}{10}+\frac{A\,b\,a^4}{2}\right)+x^{22} \left(\frac{A\,b^5}{22}+\frac{5\,B\,a\,b^4}{22}\right) +\frac{A\,a^5\,x^7}{7}+\frac{B\,b^5\,x^{25}}{25}+\frac{5\,a^2\,b^2\,x^{16}\,(A\,b+B\,a)}{8} +\frac{5\,a^3\,b\,x^{13}\,(2\,A\,b+B\,a)}{13}+\frac{5\,a\,b^3\,x^{19}\,(A\,b+2\,B\,a)}{19}$$

input
```
int(x^6*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^10*((B*a^5)/10 + (A*a^4*b)/2) + x^22*((A*b^5)/22 + (5*B*a*b^4)/22) + (A*
a^5*x^7)/7 + (B*b^5*x^25)/25 + (5*a^2*b^2*x^16*(A*b + B*a))/8 + (5*a^3*b*x
^13*(2*A*b + B*a))/13 + (5*a*b^3*x^19*(A*b + 2*B*a))/19
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int x^6 (a+bx^3)^5 (A+Bx^3)\, dx = \frac{x^7(76076b^6x^{18}+518700a\,b^5x^{15}+1501500a^2b^4x^{12}+2377375a^3b^3x^9+2194500a^4b^2x^6+1141140a^5b\,x^3+}{1901900}$$

input
```
int(x^6*(b*x^3+a)^5*(B*x^3+A),x)
```

output
```
(x**7*(271700*a**6 + 1141140*a**5*b*x**3 + 2194500*a**4*b**2*x**6 + 237737
5*a**3*b**3*x**9 + 1501500*a**2*b**4*x**12 + 518700*a*b**5*x**15 + 76076*b
**6*x**18))/1901900
```

3.39 $\int x^4(a+bx^3)^5 (A+Bx^3)\, dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int x^4\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{1}{5}a^5Ax^5+\frac{1}{8}a^4(5Ab+aB)x^8+\frac{5}{11}a^3b(2Ab+aB)x^{11}+\frac{5}{7}a^2b^2(Ab+aB)x^{14}+\frac{5}{17}ab^3(Ab+2aB)x^{17}+\frac{1}{20}b^4(Ab+5aB)x^{20}+\frac{1}{23}b^5Bx^{23}$$

output
```
1/5*a^5*A*x^5+1/8*a^4*(5*A*b+B*a)*x^8+5/11*a^3*b*(2*A*b+B*a)*x^11+5/7*a^2*
b^2*(A*b+B*a)*x^14+5/17*a*b^3*(A*b+2*B*a)*x^17+1/20*b^4*(A*b+5*B*a)*x^20+1
/23*b^5*B*x^23
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int x^4\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{1}{5}a^5Ax^5+\frac{1}{8}a^4(5Ab+aB)x^8+\frac{5}{11}a^3b(2Ab+aB)x^{11}+\frac{5}{7}a^2b^2(Ab+aB)x^{14}+\frac{5}{17}ab^3(Ab+2aB)x^{17}+\frac{1}{20}b^4(Ab+5aB)x^{20}+\frac{1}{23}b^5Bx^{23}$$

input
```
Integrate[x^4*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^8)/8 + (5*a^3*b*(2*A*b + a*B)*x^11)/1
1 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^
4*(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^23)/23
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \; 950$$

$$\int \left(a^5 A x^4 + a^4 x^7 (aB+5Ab) + 5a^3 b x^{10} (aB+2Ab) + 10a^2 b^2 x^{13} (aB+Ab) + b^4 x^{19} (5aB+Ab) + 5ab^3 x^{16} (2aB$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{5}a^5 A x^5 + \frac{1}{8}a^4 x^8 (aB+5Ab) + \frac{5}{11}a^3 b x^{11} (aB+2Ab) + \frac{5}{7}a^2 b^2 x^{14} (aB+Ab) + \frac{1}{20}b^4 x^{20} (5aB+Ab) + \frac{5}{17}ab^3 x^{17} (2aB+Ab) + \frac{1}{23}b^5 B x^{23}$$

input
```
Int[x^4*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^8)/8 + (5*a^3*b*(2*A*b + a*B)*x^11)/1
1 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^
4*(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^23)/23
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A x^5}{5}+\left(\frac{5}{8}a^4bA+\frac{1}{8}a^5B\right)x^8+\left(\frac{10}{11}a^3b^2A+\frac{5}{11}a^4bB\right)x^{11}+\left(\frac{5}{7}a^2b^3A+\frac{5}{7}a^3b^2B\right)x^{14}+\left(\frac{5}{17}a\,b\right.$
default	$\frac{b^5 B\,x^{23}}{23}+\frac{\left(b^5A+5a\,b^4B\right)x^{20}}{20}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{17}}{17}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{14}}{14}+\frac{\left(10a^3b^2A+5a^4bB\right)x^{11}}{11}+\frac{(5a}{}$
gosper	$\frac{1}{5}a^5A\,x^5+\frac{5}{8}x^8a^4bA+\frac{1}{8}x^8a^5B+\frac{10}{11}x^{11}a^3b^2A+\frac{5}{11}x^{11}a^4bB+\frac{5}{7}x^{14}a^2b^3A+\frac{5}{7}x^{14}a^3b^2B+\frac{5}{17}$
risch	$\frac{1}{5}a^5A\,x^5+\frac{5}{8}x^8a^4bA+\frac{1}{8}x^8a^5B+\frac{10}{11}x^{11}a^3b^2A+\frac{5}{11}x^{11}a^4bB+\frac{5}{7}x^{14}a^2b^3A+\frac{5}{7}x^{14}a^3b^2B+\frac{5}{17}$
parallelrisch	$\frac{1}{5}a^5A\,x^5+\frac{5}{8}x^8a^4bA+\frac{1}{8}x^8a^5B+\frac{10}{11}x^{11}a^3b^2A+\frac{5}{11}x^{11}a^4bB+\frac{5}{7}x^{14}a^2b^3A+\frac{5}{7}x^{14}a^3b^2B+\frac{5}{17}$
orering	$\frac{x^5\left(52360b^5B\,x^{18}+60214A\,b^5x^{15}+301070Ba\,b^4x^{15}+354200aA\,b^4x^{12}+708400B\,a^2b^3x^{12}+860200a^2A\,b^3x^9+860200B\,a^3b^2x\right.}{1204280}$

input
```
int(x^4*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/5*a^5*A*x^5+(5/8*a^4*b*A+1/8*a^5*B)*x^8+(10/11*a^3*b^2*A+5/11*a^4*b*B)*x
^11+(5/7*a^2*b^3*A+5/7*a^3*b^2*B)*x^14+(5/17*a*b^4*A+10/17*a^2*b^3*B)*x^17
+(1/20*b^5*A+1/4*a*b^4*B)*x^20+1/23*b^5*B*x^23
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^4 \left(a+b x^3\right)^5 \left(A+B x^3\right) dx = \frac{1}{23} B b^5 x^{23} + \frac{1}{20} \left(5 B a b^4 + A b^5\right) x^{20} + \frac{5}{17} \left(2 B a^2 b^3 + A a b^4\right) x^{17} + \frac{5}{7} \left(B a^3 b^2 + A a^2 b^3\right) x^{14} + \frac{5}{11} \left(B a^4 b + 2 A a^3 b^2\right) x^{11} + \frac{1}{5} A a^5 x^5 + \frac{1}{8} \left(B a^5 + 5 A a^4 b\right) x^8$$

input
```
integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/23*B*b^5*x^23 + 1/20*(5*B*a*b^4 + A*b^5)*x^20 + 5/17*(2*B*a^2*b^3 + A*a*
b^4)*x^17 + 5/7*(B*a^3*b^2 + A*a^2*b^3)*x^14 + 5/11*(B*a^4*b + 2*A*a^3*b^2
)*x^11 + 1/5*A*a^5*x^5 + 1/8*(B*a^5 + 5*A*a^4*b)*x^8
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16

$$\int x^4 \left(a+b x^3\right)^5 \left(A+B x^3\right) dx = \frac{A a^5 x^5}{5} + \frac{B b^5 x^{23}}{23} + x^{20} \left(\frac{A b^5}{20} + \frac{B a b^4}{4}\right) + x^{17} \cdot \left(\frac{5 A a b^4}{17} + \frac{10 B a^2 b^3}{17}\right) + x^{14} \cdot \left(\frac{5 A a^2 b^3}{7} + \frac{5 B a^3 b^2}{7}\right) + x^{11} \cdot \left(\frac{10 A a^3 b^2}{11} + \frac{5 B a^4 b}{11}\right) + x^8 \cdot \left(\frac{5 A a^4 b}{8} + \frac{B a^5}{8}\right)$$

input
```
integrate(x**4*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x**5/5 + B*b**5*x**23/23 + x**20*(A*b**5/20 + B*a*b**4/4) + x**17*(
5*A*a*b**4/17 + 10*B*a**2*b**3/17) + x**14*(5*A*a**2*b**3/7 + 5*B*a**3*b**
2/7) + x**11*(10*A*a**3*b**2/11 + 5*B*a**4*b/11) + x**8*(5*A*a**4*b/8 + B*
a**5/8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^4(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{23}Bb^5x^{23} + \frac{1}{20}(5Bab^4 + Ab^5)x^{20} + \frac{5}{17}(2Ba^2b^3 + Aab^4)x^{17} + \frac{5}{7}(Ba^3b^2 + Aa^2b^3)x^{14} + \frac{5}{11}(Ba^4b + 2Aa^3b^2)x^{11} + \frac{1}{5}Aa^5x^5 + \frac{1}{8}(Ba^5 + 5Aa^4b)x^8$$

input
```
integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/23*B*b^5*x^23 + 1/20*(5*B*a*b^4 + A*b^5)*x^20 + 5/17*(2*B*a^2*b^3 + A*a*
b^4)*x^17 + 5/7*(B*a^3*b^2 + A*a^2*b^3)*x^14 + 5/11*(B*a^4*b + 2*A*a^3*b^2
)*x^11 + 1/5*A*a^5*x^5 + 1/8*(B*a^5 + 5*A*a^4*b)*x^8
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int x^4(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{23}Bb^5x^{23} + \frac{1}{4}Bab^4x^{20} + \frac{1}{20}Ab^5x^{20} + \frac{10}{17}Ba^2b^3x^{17} + \frac{5}{17}Aab^4x^{17} + \frac{5}{7}Ba^3b^2x^{14} + \frac{5}{7}Aa^2b^3x^{14} + \frac{5}{11}Ba^4bx^{11} + \frac{10}{11}Aa^3b^2x^{11} + \frac{1}{8}Ba^5x^8 + \frac{5}{8}Aa^4bx^8 + \frac{1}{5}Aa^5x^5$$

input
```
integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/23*B*b^5*x^23 + 1/4*B*a*b^4*x^20 + 1/20*A*b^5*x^20 + 10/17*B*a^2*b^3*x^1
7 + 5/17*A*a*b^4*x^17 + 5/7*B*a^3*b^2*x^14 + 5/7*A*a^2*b^3*x^14 + 5/11*B*a
^4*b*x^11 + 10/11*A*a^3*b^2*x^11 + 1/8*B*a^5*x^8 + 5/8*A*a^4*b*x^8 + 1/5*A
*a^5*x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^4 (a+bx^3)^5 (A+Bx^3)\,dx = x^8\left(\frac{B\,a^5}{8}+\frac{5\,A\,b\,a^4}{8}\right)+x^{20}\left(\frac{A\,b^5}{20}+\frac{B\,a\,b^4}{4}\right) + \frac{A\,a^5\,x^5}{5}+\frac{B\,b^5\,x^{23}}{23}+\frac{5\,a^2\,b^2\,x^{14}\,(A\,b+B\,a)}{7} + \frac{5\,a^3\,b\,x^{11}\,(2\,A\,b+B\,a)}{11}+\frac{5\,a\,b^3\,x^{17}\,(A\,b+2\,B\,a)}{17}$$

input
```
int(x^4*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^8*((B*a^5)/8 + (5*A*a^4*b)/8) + x^20*((A*b^5)/20 + (B*a*b^4)/4) + (A*a^5
*x^5)/5 + (B*b^5*x^23)/23 + (5*a^2*b^2*x^14*(A*b + B*a))/7 + (5*a^3*b*x^11
*(2*A*b + B*a))/11 + (5*a*b^3*x^17*(A*b + 2*B*a))/17
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int x^4 (a+bx^3)^5 (A+Bx^3)\,dx = \frac{x^5(26180b^6x^{18}+180642a\,b^5x^{15}+531300a^2b^4x^{12}+860200a^3b^3x^9+821100a^4b^2x^6+451605a^5b\,x^3+120}{602140}$$

input
```
int(x^4*(b*x^3+a)^5*(B*x^3+A),x)
```

output
```
(x**5*(120428*a**6 + 451605*a**5*b*x**3 + 821100*a**4*b**2*x**6 + 860200*a
**3*b**3*x**9 + 531300*a**2*b**4*x**12 + 180642*a*b**5*x**15 + 26180*b**6*
x**18))/602140
```

3.40 $\int x^3(a+bx^3)^5(A+Bx^3)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int x^3(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{4}a^5Ax^4+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{1}{2}a^3b(2Ab+aB)x^{10} + \frac{10}{13}a^2b^2(Ab+aB)x^{13}+\frac{5}{16}ab^3(Ab+2aB)x^{16} + \frac{1}{19}b^4(Ab+5aB)x^{19}+\frac{1}{22}b^5Bx^{22}$$

output
```
1/4*a^5*A*x^4+1/7*a^4*(5*A*b+B*a)*x^7+1/2*a^3*b*(2*A*b+B*a)*x^10+10/13*a^2
*b^2*(A*b+B*a)*x^13+5/16*a*b^3*(A*b+2*B*a)*x^16+1/19*b^4*(A*b+5*B*a)*x^19+
1/22*b^5*B*x^22
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int x^3(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{4}a^5Ax^4+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{1}{2}a^3b(2Ab+aB)x^{10} + \frac{10}{13}a^2b^2(Ab+aB)x^{13}+\frac{5}{16}ab^3(Ab+2aB)x^{16} + \frac{1}{19}b^4(Ab+5aB)x^{19}+\frac{1}{22}b^5Bx^{22}$$

input
```
Integrate[x^3*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^7)/7 + (a^3*b*(2*A*b + a*B)*x^10)/2 +
 (10*a^2*b^2*(A*b + a*B)*x^13)/13 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^4
*(A*b + 5*a*B)*x^19)/19 + (b^5*B*x^22)/22
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \; 950$$

$$\int \left(a^5Ax^3+a^4x^6(aB+5Ab)+5a^3bx^9(aB+2Ab)+10a^2b^2x^{12}(aB+Ab)+b^4x^{18}(5aB+Ab)+5ab^3x^{15}(2aB+\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}a^5Ax^4+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{1}{2}a^3bx^{10}(aB+2Ab)+\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{16}ab^3x^{16}(2aB+Ab)+\frac{1}{22}b^5Bx^{22}$$

input
```
Int[x^3*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^7)/7 + (a^3*b*(2*A*b + a*B)*x^10)/2 +
 (10*a^2*b^2*(A*b + a*B)*x^13)/13 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^4
*(A*b + 5*a*B)*x^19)/19 + (b^5*B*x^22)/22
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A x^4}{4}+\left(\frac{5}{7}a^4 bA+\frac{1}{7}a^5 B\right)x^7+\left(a^3 b^2 A+\frac{1}{2}a^4 bB\right)x^{10}+\left(\frac{10}{13}a^2 b^3 A+\frac{10}{13}a^3 b^2 B\right)x^{13}+\left(\frac{5}{16}a\, b^4\right.$
default	$\frac{b^5 B\, x^{22}}{22}+\frac{(b^5 A+5a\, b^4 B)x^{19}}{19}+\frac{(5a\, b^4 A+10a^2 b^3 B)x^{16}}{16}+\frac{(10a^2 b^3 A+10a^3 b^2 B)x^{13}}{13}+\frac{(10a^3 b^2 A+5a^4 bB)x^{10}}{10}+\frac{(5a}{}$
gosper	$\frac{1}{4}a^5 A\, x^4+\frac{5}{7}x^7 a^4 bA+\frac{1}{7}x^7 a^5 B+x^{10}a^3 b^2 A+\frac{1}{2}x^{10}a^4 bB+\frac{10}{13}x^{13}a^2 b^3 A+\frac{10}{13}x^{13}a^3 b^2 B+\frac{5}{16}x$
risch	$\frac{1}{4}a^5 A\, x^4+\frac{5}{7}x^7 a^4 bA+\frac{1}{7}x^7 a^5 B+x^{10}a^3 b^2 A+\frac{1}{2}x^{10}a^4 bB+\frac{10}{13}x^{13}a^2 b^3 A+\frac{10}{13}x^{13}a^3 b^2 B+\frac{5}{16}x$
parallelrisch	$\frac{1}{4}a^5 A\, x^4+\frac{5}{7}x^7 a^4 bA+\frac{1}{7}x^7 a^5 B+x^{10}a^3 b^2 A+\frac{1}{2}x^{10}a^4 bB+\frac{10}{13}x^{13}a^2 b^3 A+\frac{10}{13}x^{13}a^3 b^2 B+\frac{5}{16}x$
orering	$\frac{x^4(13832b^5 B\, x^{18}+16016A\, b^5 x^{15}+80080Ba\, b^4 x^{15}+95095aA\, b^4 x^{12}+190190B\, a^2 b^3 x^{12}+234080a^2 A\, b^3 x^9+234080B\, a^3 b^2 x^9+}{304304}$

input
```
int(x^3*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/4*a^5*A*x^4+(5/7*a^4*b*A+1/7*a^5*B)*x^7+(a^3*b^2*A+1/2*a^4*b*B)*x^10+(10
/13*a^2*b^3*A+10/13*a^3*b^2*B)*x^13+(5/16*a*b^4*A+5/8*a^2*b^3*B)*x^16+(1/1
9*b^5*A+5/19*a*b^4*B)*x^19+1/22*b^5*B*x^22
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^3 (a+bx^3)^5 (A+Bx^3) \, dx = \frac{1}{22} Bb^5x^{22} + \frac{1}{19} (5 Bab^4 + Ab^5)x^{19} + \frac{5}{16} (2 Ba^2b^3 + Aab^4)x^{16} + \frac{10}{13} (Ba^3b^2 + Aa^2b^3)x^{13} + \frac{1}{2} (Ba^4b + 2 Aa^3b^2)x^{10} + \frac{1}{4} Aa^5x^4 + \frac{1}{7} (Ba^5 + 5 Aa^4b)x^7$$

input
```
integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/22*B*b^5*x^22 + 1/19*(5*B*a*b^4 + A*b^5)*x^19 + 5/16*(2*B*a^2*b^3 + A*a*
b^4)*x^16 + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^13 + 1/2*(B*a^4*b + 2*A*a^3*b^
2)*x^10 + 1/4*A*a^5*x^4 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14

$$\int x^3 (a+bx^3)^5 (A+Bx^3) \, dx = \frac{Aa^5x^4}{4} + \frac{Bb^5x^{22}}{22} + x^{19} \left(\frac{Ab^5}{19} + \frac{5Bab^4}{19}\right) + x^{16} \cdot \left(\frac{5Aab^4}{16} + \frac{5Ba^2b^3}{8}\right) + x^{13} \cdot \left(\frac{10Aa^2b^3}{13} + \frac{10Ba^3b^2}{13}\right) + x^{10} \left(Aa^3b^2 + \frac{Ba^4b}{2}\right) + x^7 \cdot \left(\frac{5Aa^4b}{7} + \frac{Ba^5}{7}\right)$$

input
```
integrate(x**3*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x**4/4 + B*b**5*x**22/22 + x**19*(A*b**5/19 + 5*B*a*b**4/19) + x**1
6*(5*A*a*b**4/16 + 5*B*a**2*b**3/8) + x**13*(10*A*a**2*b**3/13 + 10*B*a**3
*b**2/13) + x**10*(A*a**3*b**2 + B*a**4*b/2) + x**7*(5*A*a**4*b/7 + B*a**5
/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^3(a+bx^3)^5(A+Bx^3)\,dx=\frac{1}{22}Bb^5x^{22}+\frac{1}{19}(5Bab^4+Ab^5)x^{19}+\frac{5}{16}(2Ba^2b^3+Aab^4)x^{16}+\frac{10}{13}(Ba^3b^2+Aa^2b^3)x^{13}+\frac{1}{2}(Ba^4b+2Aa^3b^2)x^{10}+\frac{1}{4}Aa^5x^4+\frac{1}{7}(Ba^5+5Aa^4b)x^7$$

input
```
integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/22*B*b^5*x^22 + 1/19*(5*B*a*b^4 + A*b^5)*x^19 + 5/16*(2*B*a^2*b^3 + A*a*
b^4)*x^16 + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^13 + 1/2*(B*a^4*b + 2*A*a^3*b^
2)*x^10 + 1/4*A*a^5*x^4 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int x^3(a+bx^3)^5(A+Bx^3)\,dx=\frac{1}{22}Bb^5x^{22}+\frac{5}{19}Bab^4x^{19}+\frac{1}{19}Ab^5x^{19}+\frac{5}{8}Ba^2b^3x^{16}+\frac{5}{16}Aab^4x^{16}+\frac{10}{13}Ba^3b^2x^{13}+\frac{10}{13}Aa^2b^3x^{13}+\frac{1}{2}Ba^4bx^{10}+Aa^3b^2x^{10}+\frac{1}{7}Ba^5x^7+\frac{5}{7}Aa^4bx^7+\frac{1}{4}Aa^5x^4$$

input
```
integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/22*B*b^5*x^22 + 5/19*B*a*b^4*x^19 + 1/19*A*b^5*x^19 + 5/8*B*a^2*b^3*x^16
 + 5/16*A*a*b^4*x^16 + 10/13*B*a^3*b^2*x^13 + 10/13*A*a^2*b^3*x^13 + 1/2*B
*a^4*b*x^10 + A*a^3*b^2*x^10 + 1/7*B*a^5*x^7 + 5/7*A*a^4*b*x^7 + 1/4*A*a^5
*x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^3(a+bx^3)^5\left(A+Bx^3\right)\,dx = x^7\left(\frac{B\,a^5}{7}+\frac{5\,A\,b\,a^4}{7}\right)+x^{19}\left(\frac{A\,b^5}{19}+\frac{5\,B\,a\,b^4}{19}\right)+\frac{A\,a^5\,x^4}{4}+\frac{B\,b^5\,x^{22}}{22}+\frac{10\,a^2\,b^2\,x^{13}\,(A\,b+B\,a)}{13}+\frac{a^3\,b\,x^{10}\,(2\,A\,b+B\,a)}{2}+\frac{5\,a\,b^3\,x^{16}\,(A\,b+2\,B\,a)}{16}$$

input
```
int(x^3*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^7*((B*a^5)/7 + (5*A*a^4*b)/7) + x^19*((A*b^5)/19 + (5*B*a*b^4)/19) + (A*
a^5*x^4)/4 + (B*b^5*x^22)/22 + (10*a^2*b^2*x^13*(A*b + B*a))/13 + (a^3*b*x
^10*(2*A*b + B*a))/2 + (5*a*b^3*x^16*(A*b + 2*B*a))/16
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int x^3(a+bx^3)^5\left(A+Bx^3\right)\,dx = \frac{x^4(13832b^6x^{18}+96096a\,b^5x^{15}+285285a^2b^4x^{12}+468160a^3b^3x^9+456456a^4b^2x^6+260832a^5b\,x^3+7607}{304304}$$

input
```
int(x^3*(b*x^3+a)^5*(B*x^3+A),x)
```

output
```
(x**4*(76076*a**6 + 260832*a**5*b*x**3 + 456456*a**4*b**2*x**6 + 468160*a*
*3*b**3*x**9 + 285285*a**2*b**4*x**12 + 96096*a*b**5*x**15 + 13832*b**6*x*
*18))/304304
```

3.41 $\int x(a+bx^3)^5 (A+Bx^3)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 117

$$\int x\left(a+bx^3\right)^5 \left(A+Bx^3\right)\, dx = \frac{1}{2}a^5Ax^2+\frac{1}{5}a^4(5Ab+aB)x^5+\frac{5}{8}a^3b(2Ab+aB)x^8 + \frac{10}{11}a^2b^2(Ab+aB)x^{11}+\frac{5}{14}ab^3(Ab+2aB)x^{14} + \frac{1}{17}b^4(Ab+5aB)x^{17}+\frac{1}{20}b^5Bx^{20}$$

output
```
1/2*a^5*A*x^2+1/5*a^4*(5*A*b+B*a)*x^5+5/8*a^3*b*(2*A*b+B*a)*x^8+10/11*a^2*
b^2*(A*b+B*a)*x^11+5/14*a*b^3*(A*b+2*B*a)*x^14+1/17*b^4*(A*b+5*B*a)*x^17+1
/20*b^5*B*x^20
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int x\left(a+bx^3\right)^5 \left(A+Bx^3\right)\, dx = \frac{1}{2}a^5Ax^2+\frac{1}{5}a^4(5Ab+aB)x^5+\frac{5}{8}a^3b(2Ab+aB)x^8 + \frac{10}{11}a^2b^2(Ab+aB)x^{11}+\frac{5}{14}ab^3(Ab+2aB)x^{14} + \frac{1}{17}b^4(Ab+5aB)x^{17}+\frac{1}{20}b^5Bx^{20}$$

input
```
Integrate[x*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^2)/2 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^8)/8
+ (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^14)/14 + (b^
4*(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^20)/20
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^5Ax+a^4x^4(aB+5Ab)+5a^3bx^7(aB+2Ab)+10a^2b^2x^{10}(aB+Ab)+b^4x^{16}(5aB+Ab)+5ab^3x^{13}(2aB+$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a^5Ax^2+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{5}{8}a^3bx^8(aB+2Ab)+\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{5}{14}ab^3x^{14}(2aB+Ab)+\frac{1}{20}b^5Bx^{20}$$

input
```
Int[x*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^2)/2 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^8)/8
+ (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^14)/14 + (b^
4*(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^20)/20
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A x^2}{2}+\left(a^4 b A+\frac{1}{5} a^5 B\right) x^5+\left(\frac{5}{4} a^3 b^2 A+\frac{5}{8} a^4 b B\right) x^8+\left(\frac{10}{11} a^2 b^3 A+\frac{10}{11} a^3 b^2 B\right) x^{11}+\left(\frac{5}{14} a\, b^4 A\right.$
default	$\frac{b^5 B x^{20}}{20}+\frac{(b^5 A+5 a\, b^4 B) x^{17}}{17}+\frac{(5 a\, b^4 A+10 a^2 b^3 B) x^{14}}{14}+\frac{(10 a^2 b^3 A+10 a^3 b^2 B) x^{11}}{11}+\frac{(10 a^3 b^2 A+5 a^4 b B) x^8}{8}+\frac{(5a^4}{}$
gosper	$\frac{1}{2} a^5 A x^2+x^5 a^4 b A+\frac{1}{5} x^5 a^5 B+\frac{5}{4} x^8 a^3 b^2 A+\frac{5}{8} B a^4 b x^8+\frac{10}{11} x^{11} a^2 b^3 A+\frac{10}{11} x^{11} a^3 b^2 B+\frac{5}{14} x^{14}$
risch	$\frac{1}{2} a^5 A x^2+x^5 a^4 b A+\frac{1}{5} x^5 a^5 B+\frac{5}{4} x^8 a^3 b^2 A+\frac{5}{8} B a^4 b x^8+\frac{10}{11} x^{11} a^2 b^3 A+\frac{10}{11} x^{11} a^3 b^2 B+\frac{5}{14} x^{14}$
parallelrisch	$\frac{1}{2} a^5 A x^2+x^5 a^4 b A+\frac{1}{5} x^5 a^5 B+\frac{5}{4} x^8 a^3 b^2 A+\frac{5}{8} B a^4 b x^8+\frac{10}{11} x^{11} a^2 b^3 A+\frac{10}{11} x^{11} a^3 b^2 B+\frac{5}{14} x^{14}$
orering	$\frac{x^2 (2618 b^5 B x^{18}+3080 A b^5 x^{15}+15400 B a b^4 x^{15}+18700 a A b^4 x^{12}+37400 B a^2 b^3 x^{12}+47600 a^2 A b^3 x^9+47600 B a^3 b^2 x^9+65450}{52360}$

input
```
int(x*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/2*a^5*A*x^2+(a^4*b*A+1/5*a^5*B)*x^5+(5/4*a^3*b^2*A+5/8*a^4*b*B)*x^8+(10/
11*a^2*b^3*A+10/11*a^3*b^2*B)*x^11+(5/14*a*b^4*A+5/7*a^2*b^3*B)*x^14+(1/17
*b^5*A+5/17*a*b^4*B)*x^17+1/20*b^5*B*x^20
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx = \frac{1}{20}\,Bb^5x^{20}+\frac{1}{17}\left(5\,Bab^4+Ab^5\right)x^{17}+\frac{5}{14}\left(2\,Ba^2b^3+Aab^4\right)x^{14}+\frac{10}{11}\left(Ba^3b^2+Aa^2b^3\right)x^{11}+\frac{5}{8}\left(Ba^4b+2\,Aa^3b^2\right)x^8+\frac{1}{2}\,Aa^5x^2+\frac{1}{5}\left(Ba^5+5\,Aa^4b\right)x^5$$

input
```
integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/20*B*b^5*x^20 + 1/17*(5*B*a*b^4 + A*b^5)*x^17 + 5/14*(2*B*a^2*b^3 + A*a*
b^4)*x^14 + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^11 + 5/8*(B*a^4*b + 2*A*a^3*b^
2)*x^8 + 1/2*A*a^5*x^2 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.15

$$\int x\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx = \frac{Aa^5x^2}{2}+\frac{Bb^5x^{20}}{20}+x^{17}\left(\frac{Ab^5}{17}+\frac{5Bab^4}{17}\right)+x^{14}\cdot\left(\frac{5Aab^4}{14}+\frac{5Ba^2b^3}{7}\right)+x^{11}\cdot\left(\frac{10Aa^2b^3}{11}+\frac{10Ba^3b^2}{11}\right)+x^8\cdot\left(\frac{5Aa^3b^2}{4}+\frac{5Ba^4b}{8}\right)+x^5\left(Aa^4b+\frac{Ba^5}{5}\right)$$

input
```
integrate(x*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x**2/2 + B*b**5*x**20/20 + x**17*(A*b**5/17 + 5*B*a*b**4/17) + x**1
4*(5*A*a*b**4/14 + 5*B*a**2*b**3/7) + x**11*(10*A*a**2*b**3/11 + 10*B*a**3
*b**2/11) + x**8*(5*A*a**3*b**2/4 + 5*B*a**4*b/8) + x**5*(A*a**4*b + B*a**
5/5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{20}Bb^5x^{20} + \frac{1}{17}(5Bab^4 + Ab^5)x^{17} + \frac{5}{14}(2Ba^2b^3 + Aab^4)x^{14} + \frac{10}{11}(Ba^3b^2 + Aa^2b^3)x^{11} + \frac{5}{8}(Ba^4b + 2Aa^3b^2)x^8 + \frac{1}{2}Aa^5x^2 + \frac{1}{5}(Ba^5 + 5Aa^4b)x^5$$

input
```
integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/20*B*b^5*x^20 + 1/17*(5*B*a*b^4 + A*b^5)*x^17 + 5/14*(2*B*a^2*b^3 + A*a*
b^4)*x^14 + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^11 + 5/8*(B*a^4*b + 2*A*a^3*b^
2)*x^8 + 1/2*A*a^5*x^2 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int x(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{20}Bb^5x^{20} + \frac{5}{17}Bab^4x^{17} + \frac{1}{17}Ab^5x^{17} + \frac{5}{7}Ba^2b^3x^{14} + \frac{5}{14}Aab^4x^{14} + \frac{10}{11}Ba^3b^2x^{11} + \frac{10}{11}Aa^2b^3x^{11} + \frac{5}{8}Ba^4bx^8 + \frac{5}{4}Aa^3b^2x^8 + \frac{1}{5}Ba^5x^5 + Aa^4bx^5 + \frac{1}{2}Aa^5x^2$$

input
```
integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/20*B*b^5*x^20 + 5/17*B*a*b^4*x^17 + 1/17*A*b^5*x^17 + 5/7*B*a^2*b^3*x^14
 + 5/14*A*a*b^4*x^14 + 10/11*B*a^3*b^2*x^11 + 10/11*A*a^2*b^3*x^11 + 5/8*B
*a^4*b*x^8 + 5/4*A*a^3*b^2*x^8 + 1/5*B*a^5*x^5 + A*a^4*b*x^5 + 1/2*A*a^5*x
^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int x(a+bx^3)^5 (A+Bx^3)\,dx = x^5\left(\frac{B\,a^5}{5}+A\,b\,a^4\right)+x^{17}\left(\frac{A\,b^5}{17}+\frac{5\,B\,a\,b^4}{17}\right)+\frac{A\,a^5\,x^2}{2}+\frac{B\,b^5\,x^{20}}{20}+\frac{10\,a^2\,b^2\,x^{11}\,(A\,b+B\,a)}{11}+\frac{5\,a^3\,b\,x^8\,(2\,A\,b+B\,a)}{8}+\frac{5\,a\,b^3\,x^{14}\,(A\,b+2\,B\,a)}{14}$$

input
```
int(x*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^5*((B*a^5)/5 + A*a^4*b) + x^17*((A*b^5)/17 + (5*B*a*b^4)/17) + (A*a^5*x^
2)/2 + (B*b^5*x^20)/20 + (10*a^2*b^2*x^11*(A*b + B*a))/11 + (5*a^3*b*x^8*(
2*A*b + B*a))/8 + (5*a*b^3*x^14*(A*b + 2*B*a))/14
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int x(a+bx^3)^5 (A+Bx^3)\,dx = \frac{x^2(2618b^6x^{18}+18480a\,b^5x^{15}+56100a^2b^4x^{12}+95200a^3b^3x^9+98175a^4b^2x^6+62832a^5b\,x^3+26180a^6)}{52360}$$

input
```
int(x*(b*x^3+a)^5*(B*x^3+A),x)
```

output
```
(x**2*(26180*a**6 + 62832*a**5*b*x**3 + 98175*a**4*b**2*x**6 + 95200*a**3*
b**3*x**9 + 56100*a**2*b**4*x**12 + 18480*a*b**5*x**15 + 2618*b**6*x**18))
/52360
```

3.42 $\int (a+bx^3)^5 (A+Bx^3) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 109

$$\int (a+bx^3)^5 (A+Bx^3) \, dx = a^5Ax + \frac{1}{4}a^4(5Ab+aB)x^4 + \frac{5}{7}a^3b(2Ab+aB)x^7 + a^2b^2(Ab+aB)x^{10} + \frac{5}{13}ab^3(Ab+2aB)x^{13} + \frac{1}{16}b^4(Ab+5aB)x^{16} + \frac{1}{19}b^5Bx^{19}$$

output
```
a^5*A*x+1/4*a^4*(5*A*b+B*a)*x^4+5/7*a^3*b*(2*A*b+B*a)*x^7+a^2*b^2*(A*b+B*a
)*x^10+5/13*a*b^3*(A*b+2*B*a)*x^13+1/16*b^4*(A*b+5*B*a)*x^16+1/19*b^5*B*x^
19
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^5 (A+Bx^3) \, dx = a^5Ax + \frac{1}{4}a^4(5Ab+aB)x^4 + \frac{5}{7}a^3b(2Ab+aB)x^7 + a^2b^2(Ab+aB)x^{10} + \frac{5}{13}ab^3(Ab+2aB)x^{13} + \frac{1}{16}b^4(Ab+5aB)x^{16} + \frac{1}{19}b^5Bx^{19}$$

input
```
Integrate[(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
a^5*A*x + (a^4*(5*A*b + a*B)*x^4)/4 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 + a^2*
b^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*(A*b + 5*a*B
)*x^16)/16 + (b^5*B*x^19)/19
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$$

$$\Big\downarrow \text{ 897}$$

$$\int \left(a^5A+a^4x^3(aB+5Ab)+5a^3bx^6(aB+2Ab)+10a^2b^2x^9(aB+Ab)+b^4x^{15}(5aB+Ab)+5ab^3x^{12}(2aB+Ab$$

$$\Big\downarrow \text{ 2009}$$

$$a^5Ax+\frac{1}{4}a^4x^4(aB+5Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+a^2b^2x^{10}(aB+Ab)+\frac{1}{16}b^4x^{16}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+\frac{1}{19}b^5Bx^{19}$$

input
```
Int[(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
a^5*A*x + (a^4*(5*A*b + a*B)*x^4)/4 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 + a^2*
b^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*(A*b + 5*a*B
)*x^16)/16 + (b^5*B*x^19)/19
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06

method	result
norman	$a^5Ax+\left(\frac{5}{4}a^4bA+\frac{1}{4}a^5B\right)x^4+\left(\frac{10}{7}a^3b^2A+\frac{5}{7}a^4bB\right)x^7+\left(a^2b^3A+a^3b^2B\right)x^{10}+\left(\frac{5}{13}a\,b^4A+\right.$
gosper	$a^5Ax+\frac{5}{4}x^4a^4bA+\frac{1}{4}x^4a^5B+\frac{10}{7}x^7a^3b^2A+\frac{5}{7}x^7a^4bB+A\,a^2b^3x^{10}+B\,a^3b^2x^{10}+\frac{5}{13}x^{13}a\,b^4A$
default	$\frac{b^5B\,x^{19}}{19}+\frac{(b^5A+5a\,b^4B)x^{16}}{16}+\frac{(5a\,b^4A+10a^2b^3B)x^{13}}{13}+\frac{(10a^2b^3A+10a^3b^2B)x^{10}}{10}+\frac{(10a^3b^2A+5a^4bB)x^7}{7}+\frac{(5a^4}{}$
risch	$a^5Ax+\frac{5}{4}x^4a^4bA+\frac{1}{4}x^4a^5B+\frac{10}{7}x^7a^3b^2A+\frac{5}{7}x^7a^4bB+A\,a^2b^3x^{10}+B\,a^3b^2x^{10}+\frac{5}{13}x^{13}a\,b^4A$
parallelrisch	$a^5Ax+\frac{5}{4}x^4a^4bA+\frac{1}{4}x^4a^5B+\frac{10}{7}x^7a^3b^2A+\frac{5}{7}x^7a^4bB+A\,a^2b^3x^{10}+B\,a^3b^2x^{10}+\frac{5}{13}x^{13}a\,b^4A$
orering	$\frac{x(1456b^5B\,x^{18}+1729A\,b^5x^{15}+8645Ba\,b^4x^{15}+10640aA\,b^4x^{12}+21280B\,a^2b^3x^{12}+27664a^2A\,b^3x^9+27664B\,a^3b^2x^9+39520a^3}{27664}$

input
```
int((b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
a^5*A*x+(5/4*a^4*b*A+1/4*a^5*B)*x^4+(10/7*a^3*b^2*A+5/7*a^4*b*B)*x^7+(A*a^
2*b^3+B*a^3*b^2)*x^10+(5/13*a*b^4*A+10/13*a^2*b^3*B)*x^13+(1/16*b^5*A+5/16
*a*b^4*B)*x^16+1/19*b^5*B*x^19
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx = \frac{1}{19} Bb^5x^{19} + \frac{1}{16} \left(5 \, Bab^4 + Ab^5\right)x^{16} + \frac{5}{13} \left(2 \, Ba^2b^3 + Aab^4\right)x^{13} + \left(Ba^3b^2 + Aa^2b^3\right)x^{10} + \frac{5}{7} \left(Ba^4b + 2 \, Aa^3b^2\right)x^7 + Aa^5x + \frac{1}{4} \left(Ba^5 + 5 \, Aa^4b\right)x^4$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/19*B*b^5*x^19 + 1/16*(5*B*a*b^4 + A*b^5)*x^16 + 5/13*(2*B*a^2*b^3 + A*a*
b^4)*x^13 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 5/7*(B*a^4*b + 2*A*a^3*b^2)*x^7
 + A*a^5*x + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.17

$$\int \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx = Aa^5x + \frac{Bb^5x^{19}}{19} + x^{16}\left(\frac{Ab^5}{16} + \frac{5Bab^4}{16}\right) + x^{13} \cdot \left(\frac{5Aab^4}{13} + \frac{10Ba^2b^3}{13}\right) + x^{10}\left(Aa^2b^3 + Ba^3b^2\right) + x^7 \cdot \left(\frac{10Aa^3b^2}{7} + \frac{5Ba^4b}{7}\right) + x^4 \cdot \left(\frac{5Aa^4b}{4} + \frac{Ba^5}{4}\right)$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x + B*b**5*x**19/19 + x**16*(A*b**5/16 + 5*B*a*b**4/16) + x**13*(5*
A*a*b**4/13 + 10*B*a**2*b**3/13) + x**10*(A*a**2*b**3 + B*a**3*b**2) + x**
7*(10*A*a**3*b**2/7 + 5*B*a**4*b/7) + x**4*(5*A*a**4*b/4 + B*a**5/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx = \frac{1}{19}Bb^5x^{19} + \frac{1}{16}\left(5\,Bab^4 + Ab^5\right)x^{16} + \frac{5}{13}\left(2\,Ba^2b^3 + Aab^4\right)x^{13} + \left(Ba^3b^2 + Aa^2b^3\right)x^{10} + \frac{5}{7}\left(Ba^4b + 2\,Aa^3b^2\right)x^7 + Aa^5x + \frac{1}{4}\left(Ba^5 + 5\,Aa^4b\right)x^4$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/19*B*b^5*x^19 + 1/16*(5*B*a*b^4 + A*b^5)*x^16 + 5/13*(2*B*a^2*b^3 + A*a*
b^4)*x^13 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 5/7*(B*a^4*b + 2*A*a^3*b^2)*x^7
 + A*a^5*x + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.10

$$\int \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx = \frac{1}{19}Bb^5x^{19} + \frac{5}{16}Bab^4x^{16} + \frac{1}{16}Ab^5x^{16} + \frac{10}{13}Ba^2b^3x^{13} + \frac{5}{13}Aab^4x^{13} + Ba^3b^2x^{10} + Aa^2b^3x^{10} + \frac{5}{7}Ba^4bx^7 + \frac{10}{7}Aa^3b^2x^7 + \frac{1}{4}Ba^5x^4 + \frac{5}{4}Aa^4bx^4 + Aa^5x$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/19*B*b^5*x^19 + 5/16*B*a*b^4*x^16 + 1/16*A*b^5*x^16 + 10/13*B*a^2*b^3*x^
13 + 5/13*A*a*b^4*x^13 + B*a^3*b^2*x^10 + A*a^2*b^3*x^10 + 5/7*B*a^4*b*x^7
 + 10/7*A*a^3*b^2*x^7 + 1/4*B*a^5*x^4 + 5/4*A*a^4*b*x^4 + A*a^5*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int (a+bx^3)^5 (A+Bx^3)\,dx = x^4\left(\frac{B\,a^5}{4}+\frac{5\,A\,b\,a^4}{4}\right)+x^{16}\left(\frac{A\,b^5}{16}+\frac{5\,B\,a\,b^4}{16}\right)+\frac{B\,b^5\,x^{19}}{19}+A\,a^5\,x+a^2\,b^2\,x^{10}\,(A\,b+B\,a)+\frac{5\,a^3\,b\,x^7\,(2\,A\,b+B\,a)}{7}+\frac{5\,a\,b^3\,x^{13}\,(A\,b+2\,B\,a)}{13}$$

input
```
int((A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^4*((B*a^5)/4 + (5*A*a^4*b)/4) + x^16*((A*b^5)/16 + (5*B*a*b^4)/16) + (B*
b^5*x^19)/19 + A*a^5*x + a^2*b^2*x^10*(A*b + B*a) + (5*a^3*b*x^7*(2*A*b +
B*a))/7 + (5*a*b^3*x^13*(A*b + 2*B*a))/13
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.62

$$\int (a+bx^3)^5 (A+Bx^3)\,dx = \frac{x(728b^6x^{18}+5187a\,b^5x^{15}+15960a^2b^4x^{12}+27664a^3b^3x^9+29640a^4b^2x^6+20748a^5b\,x^3+13832a^6)}{13832}$$

input
```
int((b*x^3+a)^5*(B*x^3+A),x)
```

output
```
(x*(13832*a**6 + 20748*a**5*b*x**3 + 29640*a**4*b**2*x**6 + 27664*a**3*b**
3*x**9 + 15960*a**2*b**4*x**12 + 5187*a*b**5*x**15 + 728*b**6*x**18))/1383
2
```

3.43 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^2}\,dx = -\frac{a^5A}{x}+\frac{1}{2}a^4(5Ab+aB)x^2+a^3b(2Ab+aB)x^5 + \frac{5}{4}a^2b^2(Ab+aB)x^8+\frac{5}{11}ab^3(Ab+2aB)x^{11} + \frac{1}{14}b^4(Ab+5aB)x^{14}+\frac{1}{17}b^5Bx^{17}$$

output
```
-a^5*A/x+1/2*a^4*(5*A*b+B*a)*x^2+a^3*b*(2*A*b+B*a)*x^5+5/4*a^2*b^2*(A*b+B*
a)*x^8+5/11*a*b^3*(A*b+2*B*a)*x^11+1/14*b^4*(A*b+5*B*a)*x^14+1/17*b^5*B*x^
17
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^2}\,dx = -\frac{a^5A}{x}+\frac{1}{2}a^4(5Ab+aB)x^2+a^3b(2Ab+aB)x^5 + \frac{5}{4}a^2b^2(Ab+aB)x^8+\frac{5}{11}ab^3(Ab+2aB)x^{11} + \frac{1}{14}b^4(Ab+5aB)x^{14}+\frac{1}{17}b^5Bx^{17}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^2,x]
```

output
```
-((a^5*A)/x) + (a^4*(5*A*b + a*B)*x^2)/2 + a^3*b*(2*A*b + a*B)*x^5 + (5*a^
2*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A*b + 5
*a*B)*x^14)/14 + (b^5*B*x^17)/17
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^2} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5 A}{x^2} + a^4 x (aB+5Ab) + 5a^3 b x^4 (aB+2Ab) + 10a^2 b^2 x^7 (aB+Ab) + b^4 x^{13} (5aB+Ab) + 5ab^3 x^{10} (2aB+A$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5 A}{x} + \frac{1}{2} a^4 x^2 (aB+5Ab) + a^3 b x^5 (aB+2Ab) + \frac{5}{4} a^2 b^2 x^8 (aB+Ab) + \frac{1}{14} b^4 x^{14} (5aB+Ab) + \frac{5}{11} ab^3 x^{11} (2aB+Ab) + \frac{1}{17} b^5 B x^{17}$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^2,x]
```

output
```
-((a^5*A)/x) + (a^4*(5*A*b + a*B)*x^2)/2 + a^3*b*(2*A*b + a*B)*x^5 + (5*a^
2*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A*b + 5
*a*B)*x^14)/14 + (b^5*B*x^17)/17
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

method	result
norman	$\frac{-a^5A+(\frac{5}{2}a^4bA+\frac{1}{2}a^5B)x^3+(2a^3b^2A+a^4bB)x^6+(\frac{5}{4}a^2b^3A+\frac{5}{4}a^3b^2B)x^9+(\frac{5}{11}ab^4A+\frac{10}{11}a^2b^3B)x^{12}+(\frac{1}{14}b^5A+\frac{5}{14}ab^4B)x^{15}}{x}$
default	$\frac{b^5Bx^{17}}{17}+\frac{Ab^5x^{14}}{14}+\frac{5Bab^4x^{14}}{14}+\frac{5Aab^4x^{11}}{11}+\frac{10Ba^2b^3x^{11}}{11}+\frac{5Aa^2b^3x^8}{4}+\frac{5Ba^3b^2x^8}{4}+2a^3Ab^2x^5+B$
risch	$\frac{b^5Bx^{17}}{17}+\frac{Ab^5x^{14}}{14}+\frac{5Bab^4x^{14}}{14}+\frac{5Aab^4x^{11}}{11}+\frac{10Ba^2b^3x^{11}}{11}+\frac{5Aa^2b^3x^8}{4}+\frac{5Ba^3b^2x^8}{4}+2a^3Ab^2x^5+B$
gosper	$-\frac{-308b^5Bx^{18}-374Ab^5x^{15}-1870Bab^4x^{15}-2380aAb^4x^{12}-4760Ba^2b^3x^{12}-6545a^2Ab^3x^9-6545Ba^3b^2x^9-10472a^3Ab^2}{5236x}$
parallelrisch	$\frac{308b^5Bx^{18}+374Ab^5x^{15}+1870Bab^4x^{15}+2380aAb^4x^{12}+4760Ba^2b^3x^{12}+6545a^2Ab^3x^9+6545Ba^3b^2x^9+10472a^3Ab^2x^6+}{5236x}$
orering	$-\frac{-308b^5Bx^{18}-374Ab^5x^{15}-1870Bab^4x^{15}-2380aAb^4x^{12}-4760Ba^2b^3x^{12}-6545a^2Ab^3x^9-6545Ba^3b^2x^9-10472a^3Ab^2}{5236x}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x*(-a^5*A+(5/2*a^4*b*A+1/2*a^5*B)*x^3+(2*A*a^3*b^2+B*a^4*b)*x^6+(5/4*a^2
*b^3*A+5/4*a^3*b^2*B)*x^9+(5/11*a*b^4*A+10/11*a^2*b^3*B)*x^12+(1/14*b^5*A+
5/14*a*b^4*B)*x^15+1/17*b^5*B*x^18)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^2}\,dx$$
$$= \frac{308\,Bb^5x^{18} + 374\,(5\,Bab^4 + Ab^5)x^{15} + 2380\,(2\,Ba^2b^3 + Aab^4)x^{12} + 6545\,(Ba^3b^2 + Aa^2b^3)x^9 + 5236\,(Ba}{5236\,x}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="fricas")
```

output
```
1/5236*(308*B*b^5*x^18 + 374*(5*B*a*b^4 + A*b^5)*x^15 + 2380*(2*B*a^2*b^3
+ A*a*b^4)*x^12 + 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5236*(B*a^4*b + 2*A*a
^3*b^2)*x^6 - 5236*A*a^5 + 2618*(B*a^5 + 5*A*a^4*b)*x^3)/x
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^2}\,dx = -\frac{Aa^5}{x} + \frac{Bb^5x^{17}}{17} + x^{14}\left(\frac{Ab^5}{14} + \frac{5Bab^4}{14}\right) + x^{11} \cdot \left(\frac{5Aab^4}{11} + \frac{10Ba^2b^3}{11}\right) + x^8 \cdot \left(\frac{5Aa^2b^3}{4} + \frac{5Ba^3b^2}{4}\right) + x^5 \cdot \left(2Aa^3b^2 + Ba^4b\right) + x^2 \cdot \left(\frac{5Aa^4b}{2} + \frac{Ba^5}{2}\right)$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**2,x)
```

output
```
-A*a**5/x + B*b**5*x**17/17 + x**14*(A*b**5/14 + 5*B*a*b**4/14) + x**11*(5
*A*a*b**4/11 + 10*B*a**2*b**3/11) + x**8*(5*A*a**2*b**3/4 + 5*B*a**3*b**2/
4) + x**5*(2*A*a**3*b**2 + B*a**4*b) + x**2*(5*A*a**4*b/2 + B*a**5/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^2}\,dx = \frac{1}{17}Bb^5x^{17} + \frac{1}{14}\left(5\,Bab^4 + Ab^5\right)x^{14} + \frac{5}{11}\left(2\,Ba^2b^3 + Aab^4\right)x^{11} + \frac{5}{4}\left(Ba^3b^2 + Aa^2b^3\right)x^8 + \left(Ba^4b + 2\,Aa^3b^2\right)x^5 - \frac{Aa^5}{x} + \frac{1}{2}\left(Ba^5 + 5\,Aa^4b\right)x^2$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="maxima")
```

output
```
1/17*B*b^5*x^17 + 1/14*(5*B*a*b^4 + A*b^5)*x^14 + 5/11*(2*B*a^2*b^3 + A*a*
b^4)*x^11 + 5/4*(B*a^3*b^2 + A*a^2*b^3)*x^8 + (B*a^4*b + 2*A*a^3*b^2)*x^5
- A*a^5/x + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^2}\,dx = \frac{1}{17}Bb^5x^{17} + \frac{5}{14}Bab^4x^{14} + \frac{1}{14}Ab^5x^{14} + \frac{10}{11}Ba^2b^3x^{11} + \frac{5}{11}Aab^4x^{11} + \frac{5}{4}Ba^3b^2x^8 + \frac{5}{4}Aa^2b^3x^8 + Ba^4bx^5 + 2\,Aa^3b^2x^5 + \frac{1}{2}Ba^5x^2 + \frac{5}{2}Aa^4bx^2 - \frac{Aa^5}{x}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="giac")
```

output
```
1/17*B*b^5*x^17 + 5/14*B*a*b^4*x^14 + 1/14*A*b^5*x^14 + 10/11*B*a^2*b^3*x^
11 + 5/11*A*a*b^4*x^11 + 5/4*B*a^3*b^2*x^8 + 5/4*A*a^2*b^3*x^8 + B*a^4*b*x
^5 + 2*A*a^3*b^2*x^5 + 1/2*B*a^5*x^2 + 5/2*A*a^4*b*x^2 - A*a^5/x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^2}\,dx = x^2\left(\frac{B\,a^5}{2}+\frac{5\,A\,b\,a^4}{2}\right)+x^{14}\left(\frac{A\,b^5}{14}+\frac{5\,B\,a\,b^4}{14}\right) - \frac{A\,a^5}{x}+\frac{B\,b^5\,x^{17}}{17}+\frac{5\,a^2\,b^2\,x^8\,(A\,b+B\,a)}{4} + a^3\,b\,x^5\,(2\,A\,b+B\,a)+\frac{5\,a\,b^3\,x^{11}\,(A\,b+2\,B\,a)}{11}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^2,x)
```

output
```
x^2*((B*a^5)/2 + (5*A*a^4*b)/2) + x^14*((A*b^5)/14 + (5*B*a*b^4)/14) - (A*
a^5)/x + (B*b^5*x^17)/17 + (5*a^2*b^2*x^8*(A*b + B*a))/4 + a^3*b*x^5*(2*A*
b + B*a) + (5*a*b^3*x^11*(A*b + 2*B*a))/11
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^2}\,dx = \frac{154b^6x^{18}+1122a\,b^5x^{15}+3570a^2b^4x^{12}+6545a^3b^3x^9+7854a^4b^2x^6+7854a^5b\,x^3-2618a^6}{2618x}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^2,x)
```

output
```
( - 2618*a**6 + 7854*a**5*b*x**3 + 7854*a**4*b**2*x**6 + 6545*a**3*b**3*x*
*9 + 3570*a**2*b**4*x**12 + 1122*a*b**5*x**15 + 154*b**6*x**18)/(2618*x)
```

3.44 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^3} dx = -\frac{a^5A}{2x^2} + a^4(5Ab+aB)x + \frac{5}{4}a^3b(2Ab+aB)x^4 + \frac{10}{7}a^2b^2(Ab+aB)x^7 + \frac{1}{2}ab^3(Ab+2aB)x^{10} + \frac{1}{13}b^4(Ab+5aB)x^{13} + \frac{1}{16}b^5Bx^{16}$$

output
```
-1/2*a^5*A/x^2+a^4*(5*A*b+B*a)*x+5/4*a^3*b*(2*A*b+B*a)*x^4+10/7*a^2*b^2*(A
*b+B*a)*x^7+1/2*a*b^3*(A*b+2*B*a)*x^10+1/13*b^4*(A*b+5*B*a)*x^13+1/16*b^5*
B*x^16
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^3} dx = -\frac{a^5A}{2x^2} + a^4(5Ab+aB)x + \frac{5}{4}a^3b(2Ab+aB)x^4 + \frac{10}{7}a^2b^2(Ab+aB)x^7 + \frac{1}{2}ab^3(Ab+2aB)x^{10} + \frac{1}{13}b^4(Ab+5aB)x^{13} + \frac{1}{16}b^5Bx^{16}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^3,x]
```

output
```
-1/2*(a^5*A)/x^2 + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (
10*a^2*b^2*(A*b + a*B)*x^7)/7 + (a*b^3*(A*b + 2*a*B)*x^10)/2 + (b^4*(A*b +
 5*a*B)*x^13)/13 + (b^5*B*x^16)/16
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5A}{x^3}+a^4(aB+5Ab)+5a^3bx^3(aB+2Ab)+10a^2b^2x^6(aB+Ab)+b^4x^{12}(5aB+Ab)+5ab^3x^9(2aB+Ab)\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{2x^2}+a^4x(aB+5Ab)+\frac{5}{4}a^3bx^4(aB+2Ab)+\frac{10}{7}a^2b^2x^7(aB+Ab)+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{1}{2}ab^3x^{10}(2aB+Ab)+\frac{1}{16}b^5Bx^{16}$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^3,x]
```

output
```
-1/2*(a^5*A)/x^2 + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (
10*a^2*b^2*(A*b + a*B)*x^7)/7 + (a*b^3*(A*b + 2*a*B)*x^10)/2 + (b^4*(A*b +
 5*a*B)*x^13)/13 + (b^5*B*x^16)/16
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07

method	result
default	$\frac{b^5 B x^{16}}{16}+\frac{A b^5 x^{13}}{13}+\frac{5 B a b^4 x^{13}}{13}+\frac{A a b^4 x^{10}}{2}+B a^2 b^3 x^{10}+\frac{10 a^2 A b^3 x^7}{7}+\frac{10 B a^3 b^2 x^7}{7}+\frac{5 a^3 A b^2 x^4}{2}+$
norman	$\frac{-\frac{a^5 A}{2}+(5 a^4 b A+a^5 B) x^3+(\frac{5}{2} a^3 b^2 A+\frac{5}{4} a^4 b B) x^6+(\frac{10}{7} a^2 b^3 A+\frac{10}{7} a^3 b^2 B) x^9+(\frac{1}{2} a b^4 A+a^2 b^3 B) x^{12}+(\frac{1}{13} b^5 A+\frac{5}{13} a b^4 B) x^{15}+}{x^2}$
risch	$\frac{b^5 B x^{16}}{16}+\frac{A b^5 x^{13}}{13}+\frac{5 B a b^4 x^{13}}{13}+\frac{A a b^4 x^{10}}{2}+B a^2 b^3 x^{10}+\frac{10 a^2 A b^3 x^7}{7}+\frac{10 B a^3 b^2 x^7}{7}+\frac{5 a^3 A b^2 x^4}{2}+$
gosper	$-\frac{-91 b^5 B x^{18}-112 A b^5 x^{15}-560 B a b^4 x^{15}-728 a A b^4 x^{12}-1456 B a^2 b^3 x^{12}-2080 a^2 A b^3 x^9-2080 B a^3 b^2 x^9-3640 a^3 A b^2 x^6-}{1456 x^2}$
parallelrisch	$\frac{91 b^5 B x^{18}+112 A b^5 x^{15}+560 B a b^4 x^{15}+728 a A b^4 x^{12}+1456 B a^2 b^3 x^{12}+2080 a^2 A b^3 x^9+2080 B a^3 b^2 x^9+3640 a^3 A b^2 x^6+1820}{1456 x^2}$
orering	$-\frac{-91 b^5 B x^{18}-112 A b^5 x^{15}-560 B a b^4 x^{15}-728 a A b^4 x^{12}-1456 B a^2 b^3 x^{12}-2080 a^2 A b^3 x^9-2080 B a^3 b^2 x^9-3640 a^3 A b^2 x^6-}{1456 x^2}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*b^5*B*x^16+1/13*A*b^5*x^13+5/13*B*a*b^4*x^13+1/2*A*a*b^4*x^10+B*a^2*b
^3*x^10+10/7*a^2*A*b^3*x^7+10/7*B*a^3*b^2*x^7+5/2*a^3*A*b^2*x^4+5/4*B*a^4*
b*x^4+5*a^4*A*b*x+B*a^5*x-1/2*a^5*A/x^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3}\,dx = \frac{91\,Bb^5x^{18}+112\,(5\,Bab^4+Ab^5)x^{15}+728\,(2\,Ba^2b^3+Aab^4)x^{12}+2080\,(Ba^3b^2+Aa^2b^3)x^9+1820\,(Ba^4b}{1456\,x^2}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="fricas")
```

output
```
1/1456*(91*B*b^5*x^18 + 112*(5*B*a*b^4 + A*b^5)*x^15 + 728*(2*B*a^2*b^3 +
A*a*b^4)*x^12 + 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1820*(B*a^4*b + 2*A*a^3
*b^2)*x^6 - 728*A*a^5 + 1456*(B*a^5 + 5*A*a^4*b)*x^3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3}\,dx = -\frac{Aa^5}{2x^2}+\frac{Bb^5x^{16}}{16}+x^{13}\left(\frac{Ab^5}{13}+\frac{5Bab^4}{13}\right) + x^{10}\left(\frac{Aab^4}{2}+Ba^2b^3\right)+x^7\cdot\left(\frac{10Aa^2b^3}{7}+\frac{10Ba^3b^2}{7}\right) + x^4\cdot\left(\frac{5Aa^3b^2}{2}+\frac{5Ba^4b}{4}\right)+x\left(5Aa^4b+Ba^5\right)$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**3,x)
```

output
```
-A*a**5/(2*x**2) + B*b**5*x**16/16 + x**13*(A*b**5/13 + 5*B*a*b**4/13) + x
**10*(A*a*b**4/2 + B*a**2*b**3) + x**7*(10*A*a**2*b**3/7 + 10*B*a**3*b**2/
7) + x**4*(5*A*a**3*b**2/2 + 5*B*a**4*b/4) + x*(5*A*a**4*b + B*a**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^3}\,dx = \frac{1}{16}\,Bb^5x^{16} + \frac{1}{13}\,\left(5\,Bab^4 + Ab^5\right)x^{13} + \frac{1}{2}\,\left(2\,Ba^2b^3 + Aab^4\right)x^{10} + \frac{10}{7}\,\left(Ba^3b^2 + Aa^2b^3\right)x^7 + \frac{5}{4}\,\left(Ba^4b + 2\,Aa^3b^2\right)x^4 - \frac{Aa^5}{2\,x^2} + \left(Ba^5 + 5\,Aa^4b\right)x$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="maxima")
```

output
```
1/16*B*b^5*x^16 + 1/13*(5*B*a*b^4 + A*b^5)*x^13 + 1/2*(2*B*a^2*b^3 + A*a*b
^4)*x^10 + 10/7*(B*a^3*b^2 + A*a^2*b^3)*x^7 + 5/4*(B*a^4*b + 2*A*a^3*b^2)*
x^4 - 1/2*A*a^5/x^2 + (B*a^5 + 5*A*a^4*b)*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^3}\,dx = \frac{1}{16}\,Bb^5x^{16} + \frac{5}{13}\,Bab^4x^{13} + \frac{1}{13}\,Ab^5x^{13} + Ba^2b^3x^{10} + \frac{1}{2}\,Aab^4x^{10} + \frac{10}{7}\,Ba^3b^2x^7 + \frac{10}{7}\,Aa^2b^3x^7 + \frac{5}{4}\,Ba^4bx^4 + \frac{5}{2}\,Aa^3b^2x^4 + Ba^5x + 5\,Aa^4bx - \frac{Aa^5}{2\,x^2}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="giac")
```

output
```
1/16*B*b^5*x^16 + 5/13*B*a*b^4*x^13 + 1/13*A*b^5*x^13 + B*a^2*b^3*x^10 + 1
/2*A*a*b^4*x^10 + 10/7*B*a^3*b^2*x^7 + 10/7*A*a^2*b^3*x^7 + 5/4*B*a^4*b*x^
4 + 5/2*A*a^3*b^2*x^4 + B*a^5*x + 5*A*a^4*b*x - 1/2*A*a^5/x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3} dx = x (B a^5 + 5 A b a^4) + x^{13} \left(\frac{A b^5}{13} + \frac{5 B a b^4}{13}\right) - \frac{A a^5}{2 x^2} + \frac{B b^5 x^{16}}{16} + \frac{10 a^2 b^2 x^7 (A b + B a)}{7} + \frac{5 a^3 b x^4 (2 A b + B a)}{4} + \frac{a b^3 x^{10} (A b + 2 B a)}{2}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^3,x)
```

output
```
x*(B*a^5 + 5*A*a^4*b) + x^13*((A*b^5)/13 + (5*B*a*b^4)/13) - (A*a^5)/(2*x^
2) + (B*b^5*x^16)/16 + (10*a^2*b^2*x^7*(A*b + B*a))/7 + (5*a^3*b*x^4*(2*A*
b + B*a))/4 + (a*b^3*x^10*(A*b + 2*B*a))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3} dx = \frac{91b^6x^{18} + 672a b^5x^{15} + 2184a^2b^4x^{12} + 4160a^3b^3x^9 + 5460a^4b^2x^6 + 8736a^5b x^3 - 728a^6}{1456x^2}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^3,x)
```

output
```
( - 728*a**6 + 8736*a**5*b*x**3 + 5460*a**4*b**2*x**6 + 4160*a**3*b**3*x**
9 + 2184*a**2*b**4*x**12 + 672*a*b**5*x**15 + 91*b**6*x**18)/(1456*x**2)
```

3.45 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5}\,dx = -\frac{a^5A}{4x^4}-\frac{a^4(5Ab+aB)}{x}+\frac{5}{2}a^3b(2Ab+aB)x^2 + 2a^2b^2(Ab+aB)x^5+\frac{5}{8}ab^3(Ab+2aB)x^8 + \frac{1}{11}b^4(Ab+5aB)x^{11}+\frac{1}{14}b^5Bx^{14}$$

output
```
-1/4*a^5*A/x^4-a^4*(5*A*b+B*a)/x+5/2*a^3*b*(2*A*b+B*a)*x^2+2*a^2*b^2*(A*b+
B*a)*x^5+5/8*a*b^3*(A*b+2*B*a)*x^8+1/11*b^4*(A*b+5*B*a)*x^11+1/14*b^5*B*x^
14
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5}\,dx = -\frac{a^5A}{4x^4}+\frac{-5a^4Ab-a^5B}{x}+\frac{5}{2}a^3b(2Ab+aB)x^2 + 2a^2b^2(Ab+aB)x^5+\frac{5}{8}ab^3(Ab+2aB)x^8 + \frac{1}{11}b^4(Ab+5aB)x^{11}+\frac{1}{14}b^5Bx^{14}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^5,x]
```

output
```
-1/4*(a^5*A)/x^4 + (-5*a^4*A*b - a^5*B)/x + (5*a^3*b*(2*A*b + a*B)*x^2)/2
+ 2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b +
5*a*B)*x^11)/11 + (b^5*B*x^14)/14
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^5} \, dx$$

$$\Big\downarrow \; 950$$

$$\int \left(\frac{a^5A}{x^5}+\frac{a^4(aB+5Ab)}{x^2}+5a^3bx(aB+2Ab)+10a^2b^2x^4(aB+Ab)+b^4x^{10}(5aB+Ab)+5ab^3x^7(2aB+Ab)\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^5A}{4x^4}-\frac{a^4(aB+5Ab)}{x}+\frac{5}{2}a^3bx^2(aB+2Ab)+2a^2b^2x^5(aB+Ab)+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{8}ab^3x^8(2aB+Ab)+\frac{1}{14}b^5Bx^{14}$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^5,x]
```

output
```
-1/4*(a^5*A)/x^4 - (a^4*(5*A*b + a*B))/x + (5*a^3*b*(2*A*b + a*B)*x^2)/2 +
 2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5
*a*B)*x^11)/11 + (b^5*B*x^14)/14
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.08

method	result
norman	$\frac{-\frac{a^5 A}{4}+(-5a^4bA-a^5B)x^3+(5a^3b^2A+\frac{5}{2}a^4bB)x^6+(2a^2b^3A+2a^3b^2B)x^9+(\frac{5}{8}ab^4A+\frac{5}{4}a^2b^3B)x^{12}+(\frac{1}{11}b^5A+\frac{5}{11}ab^4B)x^{15}+}{x^4}$
default	$\frac{b^5Bx^{14}}{14}+\frac{Ab^5x^{11}}{11}+\frac{5Bab^4x^{11}}{11}+\frac{5aAb^4x^8}{8}+\frac{5Ba^2b^3x^8}{4}+2a^2Ab^3x^5+2Ba^3b^2x^5+5a^3Ab^2x^2+$
risch	$\frac{b^5Bx^{14}}{14}+\frac{Ab^5x^{11}}{11}+\frac{5Bab^4x^{11}}{11}+\frac{5aAb^4x^8}{8}+\frac{5Ba^2b^3x^8}{4}+2a^2Ab^3x^5+2Ba^3b^2x^5+5a^3Ab^2x^2+$
gosper	$-\frac{-44b^5Bx^{18}-56Ab^5x^{15}-280Bab^4x^{15}-385aAb^4x^{12}-770Ba^2b^3x^{12}-1232a^2Ab^3x^9-1232Ba^3b^2x^9-3080a^3Ab^2x^6-154}{616x^4}$
parallelrisch	$\frac{44b^5Bx^{18}+56Ab^5x^{15}+280Bab^4x^{15}+385aAb^4x^{12}+770Ba^2b^3x^{12}+1232a^2Ab^3x^9+1232Ba^3b^2x^9+3080a^3Ab^2x^6+1540B}{616x^4}$
orering	$-\frac{-44b^5Bx^{18}-56Ab^5x^{15}-280Bab^4x^{15}-385aAb^4x^{12}-770Ba^2b^3x^{12}-1232a^2Ab^3x^9-1232Ba^3b^2x^9-3080a^3Ab^2x^6-154}{616x^4}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/x^4*(-1/4*a^5*A+(-5*A*a^4*b-B*a^5)*x^3+(5*a^3*b^2*A+5/2*a^4*b*B)*x^6+(2*
A*a^2*b^3+2*B*a^3*b^2)*x^9+(5/8*a*b^4*A+5/4*a^2*b^3*B)*x^12+(1/11*b^5*A+5/
11*a*b^4*B)*x^15+1/14*b^5*B*x^18)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^5} dx = \frac{44 Bb^5x^{18} + 56 (5 Bab^4 + Ab^5)x^{15} + 385 (2 Ba^2b^3 + Aab^4)x^{12} + 1232 (Ba^3b^2 + Aa^2b^3)x^9 + 1540 (Ba^4b -}{616 x^4}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="fricas")
```

output
```
1/616*(44*B*b^5*x^18 + 56*(5*B*a*b^4 + A*b^5)*x^15 + 385*(2*B*a^2*b^3 + A*
a*b^4)*x^12 + 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1540*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 154*A*a^5 - 616*(B*a^5 + 5*A*a^4*b)*x^3)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^5} dx = \frac{Bb^5x^{14}}{14} + x^{11}\left(\frac{Ab^5}{11} + \frac{5Bab^4}{11}\right) + x^8 \cdot \left(\frac{5Aab^4}{8} + \frac{5Ba^2b^3}{4}\right) + x^5 \cdot \left(2Aa^2b^3 + 2Ba^3b^2\right) + x^2 \cdot \left(5Aa^3b^2 + \frac{5Ba^4b}{2}\right) + \frac{-Aa^5 + x^3(-20Aa^4b - 4Ba^5)}{4x^4}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**5,x)
```

output
```
B*b**5*x**14/14 + x**11*(A*b**5/11 + 5*B*a*b**4/11) + x**8*(5*A*a*b**4/8 +
 5*B*a**2*b**3/4) + x**5*(2*A*a**2*b**3 + 2*B*a**3*b**2) + x**2*(5*A*a**3*
b**2 + 5*B*a**4*b/2) + (-A*a**5 + x**3*(-20*A*a**4*b - 4*B*a**5))/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5}\,dx = \frac{1}{14}Bb^5x^{14} + \frac{1}{11}\left(5\,Bab^4 + Ab^5\right)x^{11} + \frac{5}{8}\left(2\,Ba^2b^3 + Aab^4\right)x^8 + 2\left(Ba^3b^2 + Aa^2b^3\right)x^5 + \frac{5}{2}\left(Ba^4b + 2\,Aa^3b^2\right)x^2 - \frac{Aa^5 + 4\,(Ba^5 + 5\,Aa^4b)x^3}{4\,x^4}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="maxima")
```

output
```
1/14*B*b^5*x^14 + 1/11*(5*B*a*b^4 + A*b^5)*x^11 + 5/8*(2*B*a^2*b^3 + A*a*b
^4)*x^8 + 2*(B*a^3*b^2 + A*a^2*b^3)*x^5 + 5/2*(B*a^4*b + 2*A*a^3*b^2)*x^2
- 1/4*(A*a^5 + 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5}\,dx = \frac{1}{14}Bb^5x^{14} + \frac{5}{11}Bab^4x^{11} + \frac{1}{11}Ab^5x^{11} + \frac{5}{4}Ba^2b^3x^8 + \frac{5}{8}Aab^4x^8 + 2\,Ba^3b^2x^5 + 2\,Aa^2b^3x^5 + \frac{5}{2}Ba^4bx^2 + 5\,Aa^3b^2x^2 - \frac{4\,Ba^5x^3 + 20\,Aa^4bx^3 + Aa^5}{4\,x^4}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="giac")
```

output
```
1/14*B*b^5*x^14 + 5/11*B*a*b^4*x^11 + 1/11*A*b^5*x^11 + 5/4*B*a^2*b^3*x^8
+ 5/8*A*a*b^4*x^8 + 2*B*a^3*b^2*x^5 + 2*A*a^2*b^3*x^5 + 5/2*B*a^4*b*x^2 +
5*A*a^3*b^2*x^2 - 1/4*(4*B*a^5*x^3 + 20*A*a^4*b*x^3 + A*a^5)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5}\,dx = x^{11}\left(\frac{A\,b^5}{11}+\frac{5\,B\,a\,b^4}{11}\right)-\frac{\frac{A\,a^5}{4}+x^3\,(B\,a^5+5\,A\,b\,a^4)}{x^4} + \frac{B\,b^5\,x^{14}}{14}+2\,a^2\,b^2\,x^5\,(A\,b+B\,a) + \frac{5\,a^3\,b\,x^2\,(2\,A\,b+B\,a)}{2}+\frac{5\,a\,b^3\,x^8\,(A\,b+2\,B\,a)}{8}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^5,x)
```

output
```
x^11*((A*b^5)/11 + (5*B*a*b^4)/11) - ((A*a^5)/4 + x^3*(B*a^5 + 5*A*a^4*b))
/x^4 + (B*b^5*x^14)/14 + 2*a^2*b^2*x^5*(A*b + B*a) + (5*a^3*b*x^2*(2*A*b +
 B*a))/2 + (5*a*b^3*x^8*(A*b + 2*B*a))/8
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5}\,dx = \frac{44b^6x^{18}+336a\,b^5x^{15}+1155a^2b^4x^{12}+2464a^3b^3x^9+4620a^4b^2x^6-3696a^5b\,x^3-154a^6}{616x^4}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^5,x)
```

output
```
( - 154*a**6 - 3696*a**5*b*x**3 + 4620*a**4*b**2*x**6 + 2464*a**3*b**3*x**
9 + 1155*a**2*b**4*x**12 + 336*a*b**5*x**15 + 44*b**6*x**18)/(616*x**4)
```

3.46 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^6}\,dx = -\frac{a^5A}{5x^5}-\frac{a^4(5Ab+aB)}{2x^2}+5a^3b(2Ab+aB)x+\frac{5}{2}a^2b^2(Ab+aB)x^4+\frac{5}{7}ab^3(Ab+2aB)x^7+\frac{1}{10}b^4(Ab+5aB)x^{10}+\frac{1}{13}b^5Bx^{13}$$

output
```
-1/5*a^5*A/x^5-1/2*a^4*(5*A*b+B*a)/x^2+5*a^3*b*(2*A*b+B*a)*x+5/2*a^2*b^2*(
A*b+B*a)*x^4+5/7*a*b^3*(A*b+2*B*a)*x^7+1/10*b^4*(A*b+5*B*a)*x^10+1/13*b^5*
B*x^13
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^6}\,dx = -\frac{a^5A}{5x^5}-\frac{a^4(5Ab+aB)}{2x^2}+5a^3b(2Ab+aB)x+\frac{5}{2}a^2b^2(Ab+aB)x^4+\frac{5}{7}ab^3(Ab+2aB)x^7+\frac{1}{10}b^4(Ab+5aB)x^{10}+\frac{1}{13}b^5Bx^{13}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^6,x]
```

output
```
-1/5*(a^5*A)/x^5 - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^3*b*(2*A*b + a*B)*x +
 (5*a^2*b^2*(A*b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b
 + 5*a*B)*x^10)/10 + (b^5*B*x^13)/13
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5 A}{x^6}+\frac{a^4(aB+5Ab)}{x^3}+5a^3b(aB+2Ab)+10a^2b^2x^3(aB+Ab)+b^4x^9(5aB+Ab)+5ab^3x^6(2aB+Ab)+b$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5 A}{5x^5}-\frac{a^4(aB+5Ab)}{2x^2}+5a^3bx(aB+2Ab)+\frac{5}{2}a^2b^2x^4(aB+Ab)+\frac{1}{10}b^4x^{10}(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^6,x]
```

output
```
-1/5*(a^5*A)/x^5 - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^3*b*(2*A*b + a*B)*x +
 (5*a^2*b^2*(A*b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b
 + 5*a*B)*x^10)/10 + (b^5*B*x^13)/13
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

method	result
default	$\frac{b^5Bx^{13}}{13}+\frac{Ab^5x^{10}}{10}+\frac{Bab^4x^{10}}{2}+\frac{5Aab^4x^7}{7}+\frac{10Ba^2b^3x^7}{7}+\frac{5a^2Ab^3x^4}{2}+\frac{5Ba^3b^2x^4}{2}+10a^3b^2Ax+5a^4bI$
norman	$\frac{-\frac{a^5A}{5}+(-\frac{5}{2}a^4bA-\frac{1}{2}a^5B)x^3+(10a^3b^2A+5a^4bB)x^6+(\frac{5}{2}a^2b^3A+\frac{5}{2}a^3b^2B)x^9+(\frac{5}{7}ab^4A+\frac{10}{7}a^2b^3B)x^{12}+(\frac{1}{10}b^5A+\frac{1}{2}ab^4B)x}{x^5}$
risch	$\frac{b^5Bx^{13}}{13}+\frac{Ab^5x^{10}}{10}+\frac{Bab^4x^{10}}{2}+\frac{5Aab^4x^7}{7}+\frac{10Ba^2b^3x^7}{7}+\frac{5a^2Ab^3x^4}{2}+\frac{5Ba^3b^2x^4}{2}+10a^3b^2Ax+5a^4bI$
gosper	$-\frac{-70b^5Bx^{18}-91Ab^5x^{15}-455Bab^4x^{15}-650aAb^4x^{12}-1300Ba^2b^3x^{12}-2275a^2Ab^3x^9-2275Ba^3b^2x^9-9100a^3Ab^2x^6-45}{910x^5}$
parallelrisch	$\frac{70b^5Bx^{18}+91Ab^5x^{15}+455Bab^4x^{15}+650aAb^4x^{12}+1300Ba^2b^3x^{12}+2275a^2Ab^3x^9+2275Ba^3b^2x^9+9100a^3Ab^2x^6+4550B}{910x^5}$
orering	$-\frac{-70b^5Bx^{18}-91Ab^5x^{15}-455Bab^4x^{15}-650aAb^4x^{12}-1300Ba^2b^3x^{12}-2275a^2Ab^3x^9-2275Ba^3b^2x^9-9100a^3Ab^2x^6-45}{910x^5}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/13*b^5*B*x^13+1/10*A*b^5*x^10+1/2*B*a*b^4*x^10+5/7*A*a*b^4*x^7+10/7*B*a^
2*b^3*x^7+5/2*a^2*A*b^3*x^4+5/2*B*a^3*b^2*x^4+10*a^3*b^2*A*x+5*a^4*b*B*x-1
/5*a^5*A/x^5-1/2*a^4*(5*A*b+B*a)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} \, dx$$
$$= \frac{70\, Bb^5x^{18} + 91\,(5\, Bab^4 + Ab^5)x^{15} + 650\,(2\, Ba^2b^3 + Aab^4)x^{12} + 2275\,(Ba^3b^2 + Aa^2b^3)x^9 + 4550\,(Ba^4b -}{910\, x^5}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="fricas")
```

output
```
1/910*(70*B*b^5*x^18 + 91*(5*B*a*b^4 + A*b^5)*x^15 + 650*(2*B*a^2*b^3 + A*
a*b^4)*x^12 + 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 4550*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 182*A*a^5 - 455*(B*a^5 + 5*A*a^4*b)*x^3)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} \, dx = \frac{Bb^5x^{13}}{13} + x^{10}\left(\frac{Ab^5}{10} + \frac{Bab^4}{2}\right) + x^7 \cdot \left(\frac{5Aab^4}{7} + \frac{10Ba^2b^3}{7}\right) + x^4 \cdot \left(\frac{5Aa^2b^3}{2} + \frac{5Ba^3b^2}{2}\right) + x\left(10Aa^3b^2 + 5Ba^4b\right) + \frac{-2Aa^5 + x^3(-25Aa^4b - 5Ba^5)}{10x^5}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**6,x)
```

output
```
B*b**5*x**13/13 + x**10*(A*b**5/10 + B*a*b**4/2) + x**7*(5*A*a*b**4/7 + 10
*B*a**2*b**3/7) + x**4*(5*A*a**2*b**3/2 + 5*B*a**3*b**2/2) + x*(10*A*a**3*
b**2 + 5*B*a**4*b) + (-2*A*a**5 + x**3*(-25*A*a**4*b - 5*B*a**5))/(10*x**5
)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} \, dx = \frac{1}{13} Bb^5x^{13} + \frac{1}{10} \left(5 Bab^4 + Ab^5\right)x^{10} + \frac{5}{7} \left(2 Ba^2b^3 + Aab^4\right)x^7 + \frac{5}{2} \left(Ba^3b^2 + Aa^2b^3\right)x^4 + 5 \left(Ba^4b + 2 Aa^3b^2\right)x - \frac{2 Aa^5 + 5 (Ba^5 + 5 Aa^4b)x^3}{10 x^5}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="maxima")
```

output
```
1/13*B*b^5*x^13 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/7*(2*B*a^2*b^3 + A*a*b
^4)*x^7 + 5/2*(B*a^3*b^2 + A*a^2*b^3)*x^4 + 5*(B*a^4*b + 2*A*a^3*b^2)*x -
1/10*(2*A*a^5 + 5*(B*a^5 + 5*A*a^4*b)*x^3)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} \, dx = \frac{1}{13} Bb^5x^{13} + \frac{1}{2} Bab^4x^{10} + \frac{1}{10} Ab^5x^{10} + \frac{10}{7} Ba^2b^3x^7 + \frac{5}{7} Aab^4x^7 + \frac{5}{2} Ba^3b^2x^4 + \frac{5}{2} Aa^2b^3x^4 + 5 Ba^4bx + 10 Aa^3b^2x - \frac{5 Ba^5x^3 + 25 Aa^4bx^3 + 2 Aa^5}{10 x^5}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="giac")
```

output
```
1/13*B*b^5*x^13 + 1/2*B*a*b^4*x^10 + 1/10*A*b^5*x^10 + 10/7*B*a^2*b^3*x^7
+ 5/7*A*a*b^4*x^7 + 5/2*B*a^3*b^2*x^4 + 5/2*A*a^2*b^3*x^4 + 5*B*a^4*b*x +
10*A*a^3*b^2*x - 1/10*(5*B*a^5*x^3 + 25*A*a^4*b*x^3 + 2*A*a^5)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} dx = x^{10} \left(\frac{A b^5}{10} + \frac{B a b^4}{2}\right) - \frac{\frac{A a^5}{5} + x^3 \left(\frac{B a^5}{2} + \frac{5 A b a^4}{2}\right)}{x^5} + \frac{B b^5 x^{13}}{13} + \frac{5 a^2 b^2 x^4 (A b + B a)}{2} + 5 a^3 b x (2 A b + B a) + \frac{5 a b^3 x^7 (A b + 2 B a)}{7}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^6,x)
```

output
```
x^10*((A*b^5)/10 + (B*a*b^4)/2) - ((A*a^5)/5 + x^3*((B*a^5)/2 + (5*A*a^4*b
)/2))/x^5 + (B*b^5*x^13)/13 + (5*a^2*b^2*x^4*(A*b + B*a))/2 + 5*a^3*b*x*(2
*A*b + B*a) + (5*a*b^3*x^7*(A*b + 2*B*a))/7
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} dx = \frac{35b^6x^{18} + 273a b^5x^{15} + 975a^2b^4x^{12} + 2275a^3b^3x^9 + 6825a^4b^2x^6 - 1365a^5b x^3 - 91a^6}{455x^5}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^6,x)
```

output
```
( - 91*a**6 - 1365*a**5*b*x**3 + 6825*a**4*b**2*x**6 + 2275*a**3*b**3*x**9
 + 975*a**2*b**4*x**12 + 273*a*b**5*x**15 + 35*b**6*x**18)/(455*x**5)
```

3.47 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^8}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^8}\,dx = -\frac{a^5A}{7x^7}-\frac{a^4(5Ab+aB)}{4x^4}-\frac{5a^3b(2Ab+aB)}{x} + 5a^2b^2(Ab+aB)x^2+ab^3(Ab+2aB)x^5 + \frac{1}{8}b^4(Ab+5aB)x^8+\frac{1}{11}b^5Bx^{11}$$

output
```
-1/7*a^5*A/x^7-1/4*a^4*(5*A*b+B*a)/x^4-5*a^3*b*(2*A*b+B*a)/x+5*a^2*b^2*(A*
b+B*a)*x^2+a*b^3*(A*b+2*B*a)*x^5+1/8*b^4*(A*b+5*B*a)*x^8+1/11*b^5*B*x^11
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^8}\,dx = -\frac{a^5A}{7x^7}-\frac{a^4(5Ab+aB)}{4x^4}-\frac{5a^3b(2Ab+aB)}{x} + 5a^2b^2(Ab+aB)x^2+ab^3(Ab+2aB)x^5 + \frac{1}{8}b^4(Ab+5aB)x^8+\frac{1}{11}b^5Bx^{11}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^8,x]
```

output
```
-1/7*(a^5*A)/x^7 - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/x
 + 5*a^2*b^2*(A*b + a*B)*x^2 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B
)*x^8)/8 + (b^5*B*x^11)/11
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5 A}{x^8}+\frac{a^4(aB+5Ab)}{x^5}+\frac{5a^3b(aB+2Ab)}{x^2}+10a^2b^2x(aB+Ab)+b^4x^7(5aB+Ab)+5ab^3x^4(2aB+Ab)+b$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{7x^7}-\frac{a^4(aB+5Ab)}{4x^4}-\frac{5a^3b(aB+2Ab)}{x}+5a^2b^2x^2(aB+Ab)+\frac{1}{8}b^4x^8(5aB+Ab)+ ab^3x^5(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^8,x]
```

output

```
-1/7*(a^5*A)/x^7 - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/x
 + 5*a^2*b^2*(A*b + a*B)*x^2 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B
)*x^8)/8 + (b^5*B*x^11)/11
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.06

method	result
default	$\frac{b^5 B x^{11}}{11}+\frac{A b^5 x^8}{8}+\frac{5 B a b^4 x^8}{8}+a A b^4 x^5+2 B a^2 b^3 x^5+5 A a^2 b^3 x^2+5 B a^3 b^2 x^2-\frac{a^5 A}{7 x^7}-\frac{a^4(5Ab+\cdots}{4x^4}$
norman	$\frac{-\frac{a^5 A}{7}+\left(-\frac{5}{4} a^4 b A-\frac{1}{4} a^5 B\right) x^3+\left(-10 a^3 b^2 A-5 a^4 b B\right) x^6+\left(5 a^2 b^3 A+5 a^3 b^2 B\right) x^9+\left(a b^4 A+2 a^2 b^3 B\right) x^{12}+\left(\frac{1}{8} b^5 A+\frac{5}{8} a b^4 B\right) x^{15}\cdots}{x^7}$
risch	$\frac{b^5 B x^{11}}{11}+\frac{A b^5 x^8}{8}+\frac{5 B a b^4 x^8}{8}+a A b^4 x^5+2 B a^2 b^3 x^5+5 A a^2 b^3 x^2+5 B a^3 b^2 x^2+\frac{(-10 a^3 b^2 A-5 a^4 b\cdots}{\cdots}$
gosper	$-\frac{-56 b^5 B x^{18}-77 A b^5 x^{15}-385 B a b^4 x^{15}-616 a A b^4 x^{12}-1232 B a^2 b^3 x^{12}-3080 a^2 A b^3 x^9-3080 B a^3 b^2 x^9+6160 a^3 A b^2 x^6+30\cdots}{616 x^7}$
parallelrisch	$\frac{56 b^5 B x^{18}+77 A b^5 x^{15}+385 B a b^4 x^{15}+616 a A b^4 x^{12}+1232 B a^2 b^3 x^{12}+3080 a^2 A b^3 x^9+3080 B a^3 b^2 x^9-6160 a^3 A b^2 x^6-3080 B\cdots}{616 x^7}$
orering	$-\frac{-56 b^5 B x^{18}-77 A b^5 x^{15}-385 B a b^4 x^{15}-616 a A b^4 x^{12}-1232 B a^2 b^3 x^{12}-3080 a^2 A b^3 x^9-3080 B a^3 b^2 x^9+6160 a^3 A b^2 x^6+30\cdots}{616 x^7}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/11*b^5*B*x^11+1/8*A*b^5*x^8+5/8*B*a*b^4*x^8+a*A*b^4*x^5+2*B*a^2*b^3*x^5+
5*A*a^2*b^3*x^2+5*B*a^3*b^2*x^2-1/7*a^5*A/x^7-1/4*a^4*(5*A*b+B*a)/x^4-5*a^
3*b*(2*A*b+B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8} dx = \frac{56 Bb^5x^{18} + 77 (5 Bab^4 + Ab^5)x^{15} + 616 (2 Ba^2b^3 + Aab^4)x^{12} + 3080 (Ba^3b^2 + Aa^2b^3)x^9 - 3080 (Ba^4b -}{616 x^7}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="fricas")
```

output
```
1/616*(56*B*b^5*x^18 + 77*(5*B*a*b^4 + A*b^5)*x^15 + 616*(2*B*a^2*b^3 + A*
a*b^4)*x^12 + 3080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 3080*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 88*A*a^5 - 154*(B*a^5 + 5*A*a^4*b)*x^3)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8} dx = \frac{Bb^5x^{11}}{11} + x^8\left(\frac{Ab^5}{8} + \frac{5Bab^4}{8}\right) + x^5\left(Aab^4 + 2Ba^2b^3\right) + x^2 \cdot \left(5Aa^2b^3 + 5Ba^3b^2\right) + \frac{-4Aa^5 + x^6(-280Aa^3b^2 - 140Ba^4b) + x^3(-35Aa^4b - 7Ba^5)}{28x^7}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**8,x)
```

output
```
B*b**5*x**11/11 + x**8*(A*b**5/8 + 5*B*a*b**4/8) + x**5*(A*a*b**4 + 2*B*a*
*2*b**3) + x**2*(5*A*a**2*b**3 + 5*B*a**3*b**2) + (-4*A*a**5 + x**6*(-280*
A*a**3*b**2 - 140*B*a**4*b) + x**3*(-35*A*a**4*b - 7*B*a**5))/(28*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8}\,dx = \frac{1}{11}Bb^5x^{11} + \frac{1}{8}\left(5\,Bab^4 + Ab^5\right)x^8 + \left(2\,Ba^2b^3 + Aab^4\right)x^5 + 5\left(Ba^3b^2 + Aa^2b^3\right)x^2 - \frac{140\,(Ba^4b + 2\,Aa^3b^2)x^6 + 4\,Aa^5 + 7\,(Ba^5 + 5\,Aa^4b)x^3}{28\,x^7}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="maxima")
```

output
```
1/11*B*b^5*x^11 + 1/8*(5*B*a*b^4 + A*b^5)*x^8 + (2*B*a^2*b^3 + A*a*b^4)*x^
5 + 5*(B*a^3*b^2 + A*a^2*b^3)*x^2 - 1/28*(140*(B*a^4*b + 2*A*a^3*b^2)*x^6
+ 4*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^7
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8}\,dx = \frac{1}{11}Bb^5x^{11} + \frac{5}{8}Bab^4x^8 + \frac{1}{8}Ab^5x^8 + 2\,Ba^2b^3x^5 + Aab^4x^5 + 5\,Ba^3b^2x^2 + 5\,Aa^2b^3x^2 - \frac{140\,Ba^4bx^6 + 280\,Aa^3b^2x^6 + 7\,Ba^5x^3 + 35\,Aa^4bx^3 + 4\,Aa^5}{28\,x^7}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="giac")
```

output
```
1/11*B*b^5*x^11 + 5/8*B*a*b^4*x^8 + 1/8*A*b^5*x^8 + 2*B*a^2*b^3*x^5 + A*a*
b^4*x^5 + 5*B*a^3*b^2*x^2 + 5*A*a^2*b^3*x^2 - 1/28*(140*B*a^4*b*x^6 + 280*
A*a^3*b^2*x^6 + 7*B*a^5*x^3 + 35*A*a^4*b*x^3 + 4*A*a^5)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8} \, dx = x^8 \left(\frac{A\,b^5}{8}+\frac{5\,B\,a\,b^4}{8}\right) - \frac{\frac{A\,a^5}{7}+x^6\,(5\,B\,a^4\,b+10\,A\,a^3\,b^2)+x^3\left(\frac{B\,a^5}{4}+\frac{5\,A\,b\,a^4}{4}\right)}{x^7} + \frac{B\,b^5\,x^{11}}{11}+5\,a^2\,b^2\,x^2\,(A\,b+B\,a)+a\,b^3\,x^5\,(A\,b+2\,B\,a)$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^8,x)
```

output
```
x^8*((A*b^5)/8 + (5*B*a*b^4)/8) - ((A*a^5)/7 + x^6*(10*A*a^3*b^2 + 5*B*a^4
*b) + x^3*((B*a^5)/4 + (5*A*a^4*b)/4))/x^7 + (B*b^5*x^11)/11 + 5*a^2*b^2*x
^2*(A*b + B*a) + a*b^3*x^5*(A*b + 2*B*a)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8} \, dx = \frac{28b^6x^{18}+231a\,b^5x^{15}+924a^2b^4x^{12}+3080a^3b^3x^9-4620a^4b^2x^6-462a^5b\,x^3-44a^6}{308x^7}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^8,x)
```

output
```
( - 44*a**6 - 462*a**5*b*x**3 - 4620*a**4*b**2*x**6 + 3080*a**3*b**3*x**9
+ 924*a**2*b**4*x**12 + 231*a*b**5*x**15 + 28*b**6*x**18)/(308*x**7)
```

3.48 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^9}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^9}\,dx=-\frac{a^5A}{8x^8}-\frac{a^4(5Ab+aB)}{5x^5}-\frac{5a^3b(2Ab+aB)}{2x^2}+10a^2b^2(Ab+aB)x+\frac{5}{4}ab^3(Ab+2aB)x^4+\frac{1}{7}b^4(Ab+5aB)x^7+\frac{1}{10}b^5Bx^{10}$$

output
```
-1/8*a^5*A/x^8-1/5*a^4*(5*A*b+B*a)/x^5-5/2*a^3*b*(2*A*b+B*a)/x^2+10*a^2*b^
2*(A*b+B*a)*x+5/4*a*b^3*(A*b+2*B*a)*x^4+1/7*b^4*(A*b+5*B*a)*x^7+1/10*b^5*B
*x^10
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^9}\,dx=-\frac{a^5A}{8x^8}-\frac{a^4(5Ab+aB)}{5x^5}-\frac{5a^3b(2Ab+aB)}{2x^2}+10a^2b^2(Ab+aB)x+\frac{5}{4}ab^3(Ab+2aB)x^4+\frac{1}{7}b^4(Ab+5aB)x^7+\frac{1}{10}b^5Bx^{10}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^9,x]
```

output
```
-1/8*(a^5*A)/x^8 - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(
2*x^2) + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(
A*b + 5*a*B)*x^7)/7 + (b^5*B*x^10)/10
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^9}dx$$

$$\Big\downarrow \text{ 950}$$

$$\int \left(\frac{a^5A}{x^9}+\frac{a^4(aB+5Ab)}{x^6}+\frac{5a^3b(aB+2Ab)}{x^3}+10a^2b^2(aB+Ab)+b^4x^6(5aB+Ab)+5ab^3x^3(2aB+Ab)+b^5\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^5A}{8x^8}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{5a^3b(aB+2Ab)}{2x^2}+10a^2b^2x(aB+Ab)+\frac{1}{7}b^4x^7(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{10}b^5Bx^{10}$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^9,x]
```

output
```
-1/8*(a^5*A)/x^8 - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(
2*x^2) + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(
A*b + 5*a*B)*x^7)/7 + (b^5*B*x^10)/10
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01

method	result
default	$\frac{b^5 B x^{10}}{10}+\frac{A b^5 x^7}{7}+\frac{5 B a b^4 x^7}{7}+\frac{5 a A b^4 x^4}{4}+\frac{5 B a^2 b^3 x^4}{2}+10 A a^2 b^3 x+10 B a^3 b^2 x-\frac{a^4 (5 A b+B a)}{5 x^5}-\frac{5 a$
norman	$\frac{-\frac{a^5 A}{8}+\left(-a^4 b A-\frac{1}{5} a^5 B\right) x^3+\left(-5 a^3 b^2 A-\frac{5}{2} a^4 b B\right) x^6+\left(10 a^2 b^3 A+10 a^3 b^2 B\right) x^9+\left(\frac{5}{4} a b^4 A+\frac{5}{2} a^2 b^3 B\right) x^{12}+\left(\frac{1}{7} b^5 A+\frac{5}{7} a b^4 B\right) x^1}{x^8}$
risch	$\frac{b^5 B x^{10}}{10}+\frac{A b^5 x^7}{7}+\frac{5 B a b^4 x^7}{7}+\frac{5 a A b^4 x^4}{4}+\frac{5 B a^2 b^3 x^4}{2}+10 A a^2 b^3 x+10 B a^3 b^2 x+\frac{\left(-5 a^3 b^2 A-\frac{5}{2} a^4 b B\right)}{}$
gosper	$-\frac{-28 b^5 B x^{18}-40 A b^5 x^{15}-200 B a b^4 x^{15}-350 a A b^4 x^{12}-700 B a^2 b^3 x^{12}-2800 a^2 A b^3 x^9-2800 B a^3 b^2 x^9+1400 a^3 A b^2 x^6+700}{280 x^8}$
parallelrisch	$\frac{28 b^5 B x^{18}+40 A b^5 x^{15}+200 B a b^4 x^{15}+350 a A b^4 x^{12}+700 B a^2 b^3 x^{12}+2800 a^2 A b^3 x^9+2800 B a^3 b^2 x^9-1400 a^3 A b^2 x^6-700 B a}{280 x^8}$
orering	$-\frac{-28 b^5 B x^{18}-40 A b^5 x^{15}-200 B a b^4 x^{15}-350 a A b^4 x^{12}-700 B a^2 b^3 x^{12}-2800 a^2 A b^3 x^9-2800 B a^3 b^2 x^9+1400 a^3 A b^2 x^6+700}{280 x^8}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/10*b^5*B*x^10+1/7*A*b^5*x^7+5/7*B*a*b^4*x^7+5/4*a*A*b^4*x^4+5/2*B*a^2*b^
3*x^4+10*A*a^2*b^3*x+10*B*a^3*b^2*x-1/5*a^4*(5*A*b+B*a)/x^5-5/2*a^3*b*(2*A
*b+B*a)/x^2-1/8*a^5*A/x^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^9}\,dx$$
$$= \frac{28\,Bb^5x^{18} + 40\,(5\,Bab^4 + Ab^5)x^{15} + 350\,(2\,Ba^2b^3 + Aab^4)x^{12} + 2800\,(Ba^3b^2 + Aa^2b^3)x^9 - 700\,(Ba^4b +}{280\,x^8}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="fricas")
```

output
```
1/280*(28*B*b^5*x^18 + 40*(5*B*a*b^4 + A*b^5)*x^15 + 350*(2*B*a^2*b^3 + A*
a*b^4)*x^12 + 2800*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 700*(B*a^4*b + 2*A*a^3*b^
2)*x^6 - 35*A*a^5 - 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^9}\,dx$$
$$= \frac{Bb^5x^{10}}{10} + x^7\left(\frac{Ab^5}{7} + \frac{5Bab^4}{7}\right) + x^4\cdot\left(\frac{5Aab^4}{4} + \frac{5Ba^2b^3}{2}\right) + x\left(10Aa^2b^3 + 10Ba^3b^2\right)$$
$$+ \frac{-5Aa^5 + x^6(-200Aa^3b^2 - 100Ba^4b) + x^3(-40Aa^4b - 8Ba^5)}{40x^8}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**9,x)
```

output
```
B*b**5*x**10/10 + x**7*(A*b**5/7 + 5*B*a*b**4/7) + x**4*(5*A*a*b**4/4 + 5*
B*a**2*b**3/2) + x*(10*A*a**2*b**3 + 10*B*a**3*b**2) + (-5*A*a**5 + x**6*(
-200*A*a**3*b**2 - 100*B*a**4*b) + x**3*(-40*A*a**4*b - 8*B*a**5))/(40*x**
8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^9}\,dx = \frac{1}{10}Bb^5x^{10} + \frac{1}{7}(5Bab^4 + Ab^5)x^7 + \frac{5}{4}(2Ba^2b^3 + Aab^4)x^4 + 10(Ba^3b^2 + Aa^2b^3)x - \frac{100(Ba^4b + 2Aa^3b^2)x^6 + 5Aa^5 + 8(Ba^5 + 5Aa^4b)x^3}{40x^8}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="maxima")
```

output
```
1/10*B*b^5*x^10 + 1/7*(5*B*a*b^4 + A*b^5)*x^7 + 5/4*(2*B*a^2*b^3 + A*a*b^4
)*x^4 + 10*(B*a^3*b^2 + A*a^2*b^3)*x - 1/40*(100*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + 5*A*a^5 + 8*(B*a^5 + 5*A*a^4*b)*x^3)/x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^9}\,dx = \frac{1}{10}Bb^5x^{10} + \frac{5}{7}Bab^4x^7 + \frac{1}{7}Ab^5x^7 + \frac{5}{2}Ba^2b^3x^4 + \frac{5}{4}Aab^4x^4 + 10Ba^3b^2x + 10Aa^2b^3x - \frac{100Ba^4bx^6 + 200Aa^3b^2x^6 + 8Ba^5x^3 + 40Aa^4bx^3 + 5Aa^5}{40x^8}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="giac")
```

output
```
1/10*B*b^5*x^10 + 5/7*B*a*b^4*x^7 + 1/7*A*b^5*x^7 + 5/2*B*a^2*b^3*x^4 + 5/
4*A*a*b^4*x^4 + 10*B*a^3*b^2*x + 10*A*a^2*b^3*x - 1/40*(100*B*a^4*b*x^6 +
200*A*a^3*b^2*x^6 + 8*B*a^5*x^3 + 40*A*a^4*b*x^3 + 5*A*a^5)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^9} dx = x^7 \left(\frac{A b^5}{7} + \frac{5 B a b^4}{7}\right) - \frac{\frac{A a^5}{8} + x^6 \left(\frac{5 B a^4 b}{2} + 5 A a^3 b^2\right) + x^3 \left(\frac{B a^5}{5} + A b a^4\right)}{x^8} + \frac{B b^5 x^{10}}{10} + 10 a^2 b^2 x (A b + B a) + \frac{5 a b^3 x^4 (A b + 2 B a)}{4}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^9,x)
```

output
```
x^7*((A*b^5)/7 + (5*B*a*b^4)/7) - ((A*a^5)/8 + x^6*(5*A*a^3*b^2 + (5*B*a^4
*b)/2) + x^3*((B*a^5)/5 + A*a^4*b))/x^8 + (B*b^5*x^10)/10 + 10*a^2*b^2*x*(
A*b + B*a) + (5*a*b^3*x^4*(A*b + 2*B*a))/4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^9} dx = \frac{28b^6x^{18} + 240a b^5 x^{15} + 1050a^2b^4x^{12} + 5600a^3b^3x^9 - 2100a^4b^2x^6 - 336a^5 b x^3 - 35a^6}{280x^8}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^9,x)
```

output
```
( - 35*a**6 - 336*a**5*b*x**3 - 2100*a**4*b**2*x**6 + 5600*a**3*b**3*x**9
+ 1050*a**2*b**4*x**12 + 240*a*b**5*x**15 + 28*b**6*x**18)/(280*x**8)
```

3.49 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 115

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}}\,dx = -\frac{a^5A}{10x^{10}} - \frac{a^4(5Ab+aB)}{7x^7} - \frac{5a^3b(2Ab+aB)}{4x^4} - \frac{10a^2b^2(Ab+aB)}{x} + \frac{5}{2}ab^3(Ab+2aB)x^2 + \frac{1}{5}b^4(Ab+5aB)x^5 + \frac{1}{8}b^5Bx^8$$

output
```
-1/10*a^5*A/x^10-1/7*a^4*(5*A*b+B*a)/x^7-5/4*a^3*b*(2*A*b+B*a)/x^4-10*a^2*
b^2*(A*b+B*a)/x+5/2*a*b^3*(A*b+2*B*a)*x^2+1/5*b^4*(A*b+5*B*a)*x^5+1/8*b^5*
B*x^8
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}}\,dx = \frac{1400a^2b^3x^9(-2A+Bx^3) + 140ab^4x^{12}(5A+2Bx^3) - 700a^3b^2x^6(A+4Bx^3) + 7b^5x^{15}(8A+5Bx^3) - 50}{280x^{10}}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^11,x]
```

output
```
(1400*a^2*b^3*x^9*(-2*A + B*x^3) + 140*a*b^4*x^12*(5*A + 2*B*x^3) - 700*a^
3*b^2*x^6*(A + 4*B*x^3) + 7*b^5*x^15*(8*A + 5*B*x^3) - 50*a^4*b*x^3*(4*A +
 7*B*x^3) - 4*a^5*(7*A + 10*B*x^3))/(280*x^10)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{11}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5 A}{x^{11}}+\frac{a^4(aB+5Ab)}{x^8}+\frac{5a^3b(aB+2Ab)}{x^5}+\frac{10a^2b^2(aB+Ab)}{x^2}+b^4x^4(5aB+Ab)+5ab^3x(2aB+Ab)+b^5 B$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5 A}{10x^{10}}-\frac{a^4(aB+5Ab)}{7x^7}-\frac{5a^3b(aB+2Ab)}{4x^4}-\frac{10a^2b^2(aB+Ab)}{x}+\frac{1}{5}b^4x^5(5aB+Ab)+\frac{5}{2}ab^3x^2(2aB+Ab)+\frac{1}{8}b^5Bx^8$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^11,x]
```

output
```
-1/10*(a^5*A)/x^10 - (a^4*(5*A*b + a*B))/(7*x^7) - (5*a^3*b*(2*A*b + a*B))
/(4*x^4) - (10*a^2*b^2*(A*b + a*B))/x + (5*a*b^3*(A*b + 2*a*B)*x^2)/2 + (b
^4*(A*b + 5*a*B)*x^5)/5 + (b^5*B*x^8)/8
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^5 B x^8}{8}+\frac{A b^5 x^5}{5}+B a b^4 x^5+\frac{5 A a b^4 x^2}{2}+5 B a^2 b^3 x^2-\frac{a^4 (5 A b+B a)}{7 x^7}-\frac{5 a^3 b (2 A b+B a)}{4 x^4}-\frac{a^5 A}{10 x^{10}}-\frac{10 a^2 b$
norman	$\frac{-\frac{a^5 A}{10}+\left(-\frac{5}{7} a^4 b A-\frac{1}{7} a^5 B\right) x^3+\left(-\frac{5}{2} a^3 b^2 A-\frac{5}{4} a^4 b B\right) x^6+\left(-10 a^2 b^3 A-10 a^3 b^2 B\right) x^9+\left(\frac{5}{2} a b^4 A+5 a^2 b^3 B\right) x^{12}+\left(\frac{1}{5} b^5 A+a b^4 B\right)}{x^{10}}$
risch	$\frac{b^5 B x^8}{8}+\frac{A b^5 x^5}{5}+B a b^4 x^5+\frac{5 A a b^4 x^2}{2}+5 B a^2 b^3 x^2+\frac{\left(-10 a^2 b^3 A-10 a^3 b^2 B\right) x^9+\left(-\frac{5}{2} a^3 b^2 A-\frac{5}{4} a^4 b B\right) x^6+(}{x^{10}}$
gosper	$-\frac{-35 b^5 B x^{18}-56 A b^5 x^{15}-280 B a b^4 x^{15}-700 a A b^4 x^{12}-1400 B a^2 b^3 x^{12}+2800 a^2 A b^3 x^9+2800 B a^3 b^2 x^9+700 a^3 A b^2 x^6+350}{280 x^{10}}$
parallelrisch	$\frac{35 b^5 B x^{18}+56 A b^5 x^{15}+280 B a b^4 x^{15}+700 a A b^4 x^{12}+1400 B a^2 b^3 x^{12}-2800 a^2 A b^3 x^9-2800 B a^3 b^2 x^9-700 a^3 A b^2 x^6-350 B a}{280 x^{10}}$
orering	$-\frac{-35 b^5 B x^{18}-56 A b^5 x^{15}-280 B a b^4 x^{15}-700 a A b^4 x^{12}-1400 B a^2 b^3 x^{12}+2800 a^2 A b^3 x^9+2800 B a^3 b^2 x^9+700 a^3 A b^2 x^6+350}{280 x^{10}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^11,x,method=_RETURNVERBOSE)
```

output
```
1/8*b^5*B*x^8+1/5*A*b^5*x^5+B*a*b^4*x^5+5/2*A*a*b^4*x^2+5*B*a^2*b^3*x^2-1/
7*a^4*(5*A*b+B*a)/x^7-5/4*a^3*b*(2*A*b+B*a)/x^4-1/10*a^5*A/x^10-10*a^2*b^2
*(A*b+B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}}\,dx$$
$$= \frac{35\,Bb^5x^{18} + 56\,(5\,Bab^4 + Ab^5)x^{15} + 700\,(2\,Ba^2b^3 + Aab^4)x^{12} - 2800\,(Ba^3b^2 + Aa^2b^3)x^9 - 350\,(Ba^4b +}{280\,x^{10}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="fricas")
```

output
```
1/280*(35*B*b^5*x^18 + 56*(5*B*a*b^4 + A*b^5)*x^15 + 700*(2*B*a^2*b^3 + A*
a*b^4)*x^12 - 2800*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 350*(B*a^4*b + 2*A*a^3*b^
2)*x^6 - 28*A*a^5 - 40*(B*a^5 + 5*A*a^4*b)*x^3)/x^10
```

Sympy [A] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}}\,dx = \frac{Bb^5x^8}{8} + x^5\left(\frac{Ab^5}{5} + Bab^4\right) + x^2\cdot\left(\frac{5Aab^4}{2} + 5Ba^2b^3\right)$$
$$+ \frac{-14Aa^5 + x^9(-1400Aa^2b^3 - 1400Ba^3b^2) + x^6(-350Aa^3b^2 - 175Ba^4b) + x^3(-100Aa^4b - 20Ba^5)}{140x^{10}}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**11,x)
```

output
```
B*b**5*x**8/8 + x**5*(A*b**5/5 + B*a*b**4) + x**2*(5*A*a*b**4/2 + 5*B*a**2
*b**3) + (-14*A*a**5 + x**9*(-1400*A*a**2*b**3 - 1400*B*a**3*b**2) + x**6*
(-350*A*a**3*b**2 - 175*B*a**4*b) + x**3*(-100*A*a**4*b - 20*B*a**5))/(140
*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{11}}\,dx = \frac{1}{8}\,Bb^5x^8 + \frac{1}{5}\,\left(5\,Bab^4 + Ab^5\right)x^5 + \frac{5}{2}\,\left(2\,Ba^2b^3 + Aab^4\right)x^2 - \frac{1400\,(Ba^3b^2 + Aa^2b^3)x^9 + 175\,(Ba^4b + 2\,Aa^3b^2)x^6 + 14\,Aa^5 + 20\,(Ba^5 + 5\,Aa^4b)x^3}{140\,x^{10}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="maxima")
```

output
```
1/8*B*b^5*x^8 + 1/5*(5*B*a*b^4 + A*b^5)*x^5 + 5/2*(2*B*a^2*b^3 + A*a*b^4)*
x^2 - 1/140*(1400*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 175*(B*a^4*b + 2*A*a^3*b^2
)*x^6 + 14*A*a^5 + 20*(B*a^5 + 5*A*a^4*b)*x^3)/x^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{11}}\,dx = \frac{1}{8}\,Bb^5x^8 + Bab^4x^5 + \frac{1}{5}\,Ab^5x^5 + 5\,Ba^2b^3x^2 + \frac{5}{2}\,Aab^4x^2 - \frac{1400\,Ba^3b^2x^9 + 1400\,Aa^2b^3x^9 + 175\,Ba^4bx^6 + 350\,Aa^3b^2x^6 + 20\,Ba^5x^3 + 100\,Aa^4bx^3 + 14\,Aa^5}{140\,x^{10}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="giac")
```

output
```
1/8*B*b^5*x^8 + B*a*b^4*x^5 + 1/5*A*b^5*x^5 + 5*B*a^2*b^3*x^2 + 5/2*A*a*b^
4*x^2 - 1/140*(1400*B*a^3*b^2*x^9 + 1400*A*a^2*b^3*x^9 + 175*B*a^4*b*x^6 +
 350*A*a^3*b^2*x^6 + 20*B*a^5*x^3 + 100*A*a^4*b*x^3 + 14*A*a^5)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{11}} \, dx$$
$$= x^5 \left(\frac{A\,b^5}{5}+B\,a\,b^4\right) - \frac{\frac{A\,a^5}{10}+x^6\left(\frac{5\,B\,a^4\,b}{4}+\frac{5\,A\,a^3\,b^2}{2}\right)+x^3\left(\frac{B\,a^5}{7}+\frac{5\,A\,b\,a^4}{7}\right)+x^9\left(10\,B\,a^3\,b^2+10\,A\,a^2\,b^3\right)}{x^{10}} + \frac{B\,b^5\,x^8}{8}+\frac{5\,a\,b^3\,x^2\,(A\,b+2\,B\,a)}{2}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^11,x)
```

output
```
x^5*((A*b^5)/5 + B*a*b^4) - ((A*a^5)/10 + x^6*((5*A*a^3*b^2)/2 + (5*B*a^4*
b)/4) + x^3*((B*a^5)/7 + (5*A*a^4*b)/7) + x^9*(10*A*a^2*b^3 + 10*B*a^3*b^2
))/x^10 + (B*b^5*x^8)/8 + (5*a*b^3*x^2*(A*b + 2*B*a))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{11}} \, dx$$
$$= \frac{35b^6x^{18}+336a\,b^5x^{15}+2100a^2b^4x^{12}-5600a^3b^3x^9-1050a^4b^2x^6-240a^5b\,x^3-28a^6}{280x^{10}}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^11,x)
```

output
```
( - 28*a**6 - 240*a**5*b*x**3 - 1050*a**4*b**2*x**6 - 5600*a**3*b**3*x**9
+ 2100*a**2*b**4*x**12 + 336*a*b**5*x**15 + 35*b**6*x**18)/(280*x**10)
```

3.50 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 109

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}}\,dx = -\frac{a^5A}{11x^{11}} - \frac{a^4(5Ab+aB)}{8x^8} - \frac{a^3b(2Ab+aB)}{x^5} - \frac{5a^2b^2(Ab+aB)}{x^2} + 5ab^3(Ab+2aB)x + \frac{1}{4}b^4(Ab+5aB)x^4 + \frac{1}{7}b^5Bx^7$$

output
```
-1/11*a^5*A/x^11-1/8*a^4*(5*A*b+B*a)/x^8-a^3*b*(2*A*b+B*a)/x^5-5*a^2*b^2*(
A*b+B*a)/x^2+5*a*b^3*(A*b+2*B*a)*x+1/4*b^4*(A*b+5*B*a)*x^4+1/7*b^5*B*x^7
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}}\,dx = -\frac{a^5A}{11x^{11}} - \frac{a^4(5Ab+aB)}{8x^8} - \frac{a^3b(2Ab+aB)}{x^5} - \frac{5a^2b^2(Ab+aB)}{x^2} + 5ab^3(Ab+2aB)x + \frac{1}{4}b^4(Ab+5aB)x^4 + \frac{1}{7}b^5Bx^7$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^12,x]
```

output
```
-1/11*(a^5*A)/x^11 - (a^4*(5*A*b + a*B))/(8*x^8) - (a^3*b*(2*A*b + a*B))/x
^5 - (5*a^2*b^2*(A*b + a*B))/x^2 + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5
*a*B)*x^4)/4 + (b^5*B*x^7)/7
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{12}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5A}{x^{12}}+\frac{a^4(aB+5Ab)}{x^9}+\frac{5a^3b(aB+2Ab)}{x^6}+\frac{10a^2b^2(aB+Ab)}{x^3}+b^4x^3(5aB+Ab)+5ab^3(2aB+Ab)+b^5B$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{11x^{11}}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{5a^2b^2(aB+Ab)}{x^2}+\frac{1}{4}b^4x^4(5aB+Ab)+ 5ab^3x(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^12,x]
```

output
```
-1/11*(a^5*A)/x^11 - (a^4*(5*A*b + a*B))/(8*x^8) - (a^3*b*(2*A*b + a*B))/x
^5 - (5*a^2*b^2*(A*b + a*B))/x^2 + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5
*a*B)*x^4)/4 + (b^5*B*x^7)/7
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

method	result
default	$\frac{b^5 B x^7}{7}+\frac{A b^5 x^4}{4}+\frac{5 B a b^4 x^4}{4}+5 a b^4 A x+10 a^2 b^3 B x-\frac{a^3 b (2 A b+B a)}{x^5}-\frac{a^5 A}{11 x^{11}}-\frac{5 a^2 b^2 (A b+B a)}{x^2}-\frac{a^4(5$
risch	$\frac{b^5 B x^7}{7}+\frac{A b^5 x^4}{4}+\frac{5 B a b^4 x^4}{4}+5 a b^4 A x+10 a^2 b^3 B x+\frac{(-5 a^2 b^3 A-5 a^3 b^2 B) x^9+(-2 a^3 b^2 A-a^4 b B) x^6+(-\frac{5}{8} a}{x^{11}}$
norman	$\frac{-\frac{a^5 A}{11}+(-\frac{5}{8} a^4 b A-\frac{1}{8} a^5 B) x^3+(-2 a^3 b^2 A-a^4 b B) x^6+(-5 a^2 b^3 A-5 a^3 b^2 B) x^9+(5 a b^4 A+10 a^2 b^3 B) x^{12}+(\frac{1}{4} b^5 A+\frac{5}{4} a b^4 B) x^1}{x^{11}}$
gosper	$-\frac{-88 b^5 B x^{18}-154 A b^5 x^{15}-770 B a b^4 x^{15}-3080 a A b^4 x^{12}-6160 B a^2 b^3 x^{12}+3080 a^2 A b^3 x^9+3080 B a^3 b^2 x^9+1232 a^3 A b^2 x^6+}{616 x^{11}}$
parallelrisch	$\frac{88 b^5 B x^{18}+154 A b^5 x^{15}+770 B a b^4 x^{15}+3080 a A b^4 x^{12}+6160 B a^2 b^3 x^{12}-3080 a^2 A b^3 x^9-3080 B a^3 b^2 x^9-1232 a^3 A b^2 x^6-616}{616 x^{11}}$
orering	$-\frac{-88 b^5 B x^{18}-154 A b^5 x^{15}-770 B a b^4 x^{15}-3080 a A b^4 x^{12}-6160 B a^2 b^3 x^{12}+3080 a^2 A b^3 x^9+3080 B a^3 b^2 x^9+1232 a^3 A b^2 x^6+}{616 x^{11}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^12,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^5*B*x^7+1/4*A*b^5*x^4+5/4*B*a*b^4*x^4+5*a*b^4*A*x+10*a^2*b^3*B*x-a^3
*b*(2*A*b+B*a)/x^5-1/11*a^5*A/x^11-5*a^2*b^2*(A*b+B*a)/x^2-1/8*a^4*(5*A*b+
B*a)/x^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{12}}\,dx = \frac{88\,Bb^5x^{18} + 154\,(5\,Bab^4 + Ab^5)x^{15} + 3080\,(2\,Ba^2b^3 + Aab^4)x^{12} - 3080\,(Ba^3b^2 + Aa^2b^3)x^9 - 616\,(Ba^4b}{616\,x^{11}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="fricas")
```

output
```
1/616*(88*B*b^5*x^18 + 154*(5*B*a*b^4 + A*b^5)*x^15 + 3080*(2*B*a^2*b^3 +
A*a*b^4)*x^12 - 3080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 616*(B*a^4*b + 2*A*a^3*
b^2)*x^6 - 56*A*a^5 - 77*(B*a^5 + 5*A*a^4*b)*x^3)/x^11
```

Sympy [A] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{12}}\,dx = \frac{Bb^5x^7}{7} + x^4\left(\frac{Ab^5}{4} + \frac{5Bab^4}{4}\right) + x\left(5Aab^4 + 10Ba^2b^3\right) + \frac{-8Aa^5 + x^9(-440Aa^2b^3 - 440Ba^3b^2) + x^6(-176Aa^3b^2 - 88Ba^4b) + x^3(-55Aa^4b - 11Ba^5)}{88x^{11}}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**12,x)
```

output
```
B*b**5*x**7/7 + x**4*(A*b**5/4 + 5*B*a*b**4/4) + x*(5*A*a*b**4 + 10*B*a**2
*b**3) + (-8*A*a**5 + x**9*(-440*A*a**2*b**3 - 440*B*a**3*b**2) + x**6*(-1
76*A*a**3*b**2 - 88*B*a**4*b) + x**3*(-55*A*a**4*b - 11*B*a**5))/(88*x**11
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{12}}\,dx$$
$$=\frac{1}{7}Bb^5x^7+\frac{1}{4}\left(5\,Bab^4+Ab^5\right)x^4+5\left(2\,Ba^2b^3+Aab^4\right)x$$
$$-\frac{440\,(Ba^3b^2+Aa^2b^3)x^9+88\,(Ba^4b+2\,Aa^3b^2)x^6+8\,Aa^5+11\,(Ba^5+5\,Aa^4b)x^3}{88\,x^{11}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="maxima")
```

output
```
1/7*B*b^5*x^7 + 1/4*(5*B*a*b^4 + A*b^5)*x^4 + 5*(2*B*a^2*b^3 + A*a*b^4)*x
- 1/88*(440*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 88*(B*a^4*b + 2*A*a^3*b^2)*x^6 +
 8*A*a^5 + 11*(B*a^5 + 5*A*a^4*b)*x^3)/x^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{12}}\,dx=\frac{1}{7}Bb^5x^7+\frac{5}{4}Bab^4x^4+\frac{1}{4}Ab^5x^4+10\,Ba^2b^3x+5\,Aab^4x$$
$$-\frac{440\,Ba^3b^2x^9+440\,Aa^2b^3x^9+88\,Ba^4bx^6+176\,Aa^3b^2x^6+11\,Ba^5x^3+55\,Aa^4bx^3+8\,Aa^5}{88\,x^{11}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="giac")
```

output
```
1/7*B*b^5*x^7 + 5/4*B*a*b^4*x^4 + 1/4*A*b^5*x^4 + 10*B*a^2*b^3*x + 5*A*a*b
^4*x - 1/88*(440*B*a^3*b^2*x^9 + 440*A*a^2*b^3*x^9 + 88*B*a^4*b*x^6 + 176*
A*a^3*b^2*x^6 + 11*B*a^5*x^3 + 55*A*a^4*b*x^3 + 8*A*a^5)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{12}} \, dx$$
$$= x^4 \left(\frac{A\,b^5}{4}+\frac{5\,B\,a\,b^4}{4}\right)$$
$$-\frac{\frac{A\,a^5}{11}+x^6\,\left(B\,a^4\,b+2\,A\,a^3\,b^2\right)+x^3\,\left(\frac{B\,a^5}{8}+\frac{5\,A\,b\,a^4}{8}\right)+x^9\,\left(5\,B\,a^3\,b^2+5\,A\,a^2\,b^3\right)}{x^{11}}$$
$$+\frac{B\,b^5\,x^7}{7}+5\,a\,b^3\,x\,\left(A\,b+2\,B\,a\right)$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^12,x)
```

output
```
x^4*((A*b^5)/4 + (5*B*a*b^4)/4) - ((A*a^5)/11 + x^6*(2*A*a^3*b^2 + B*a^4*b
) + x^3*((B*a^5)/8 + (5*A*a^4*b)/8) + x^9*(5*A*a^2*b^3 + 5*B*a^3*b^2))/x^1
1 + (B*b^5*x^7)/7 + 5*a*b^3*x*(A*b + 2*B*a)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{12}} \, dx$$
$$= \frac{44b^6x^{18}+462a\,b^5x^{15}+4620a^2b^4x^{12}-3080a^3b^3x^9-924a^4b^2x^6-231a^5b\,x^3-28a^6}{308x^{11}}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^12,x)
```

output
```
( - 28*a**6 - 231*a**5*b*x**3 - 924*a**4*b**2*x**6 - 3080*a**3*b**3*x**9 +
 4620*a**2*b**4*x**12 + 462*a*b**5*x**15 + 44*b**6*x**18)/(308*x**11)
```

3.51 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{14}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 115

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} \, dx = -\frac{a^5 A}{13x^{13}} - \frac{a^4(5Ab+aB)}{10x^{10}} - \frac{5a^3b(2Ab+aB)}{7x^7} - \frac{5a^2b^2(Ab+aB)}{2x^4} - \frac{5ab^3(Ab+2aB)}{x} + \frac{1}{2}b^4(Ab+5aB)x^2 + \frac{1}{5}b^5Bx^5$$

output
```
-1/13*a^5*A/x^13-1/10*a^4*(5*A*b+B*a)/x^10-5/7*a^3*b*(2*A*b+B*a)/x^7-5/2*a
^2*b^2*(A*b+B*a)/x^4-5*a*b^3*(A*b+2*B*a)/x+1/2*b^4*(A*b+5*B*a)*x^2+1/5*b^5
*B*x^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} \, dx = -\frac{-2275ab^4x^{12}(-2A+Bx^3) - 91b^5x^{15}(5A+2Bx^3) + 2275a^2b^3x^9(A+4Bx^3) + 325a^3b^2x^6(4A+7Bx^3}{910x^{13}}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^14,x]
```

output
```
-1/910*(-2275*a*b^4*x^12*(-2*A + B*x^3) - 91*b^5*x^15*(5*A + 2*B*x^3) + 22
75*a^2*b^3*x^9*(A + 4*B*x^3) + 325*a^3*b^2*x^6*(4*A + 7*B*x^3) + 65*a^4*b*
x^3*(7*A + 10*B*x^3) + a^5*(70*A + 91*B*x^3))/x^13
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5A}{x^{14}} + \frac{a^4(aB+5Ab)}{x^{11}} + \frac{5a^3b(aB+2Ab)}{x^8} + \frac{10a^2b^2(aB+Ab)}{x^5} + b^4x(5aB+Ab) + \frac{5ab^3(2aB+Ab)}{x^2} + b^5Bx\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{13x^{13}} - \frac{a^4(aB+5Ab)}{10x^{10}} - \frac{5a^3b(aB+2Ab)}{7x^7} - \frac{5a^2b^2(aB+Ab)}{2x^4} + \frac{1}{2}b^4x^2(5aB+Ab) - \frac{5ab^3(2aB+Ab)}{x} + \frac{1}{5}b^5Bx^5$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^14,x]
```

output
```
-1/13*(a^5*A)/x^13 - (a^4*(5*A*b + a*B))/(10*x^10) - (5*a^3*b*(2*A*b + a*B
))/(7*x^7) - (5*a^2*b^2*(A*b + a*B))/(2*x^4) - (5*a*b^3*(A*b + 2*a*B))/x +
 (b^4*(A*b + 5*a*B)*x^2)/2 + (b^5*B*x^5)/5
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

method	result
default	$\frac{b^5 B x^5}{5}+\frac{A b^5 x^2}{2}+\frac{5 B a b^4 x^2}{2}-\frac{5 a^3 b (2 A b+B a)}{7 x^7}-\frac{5 a^2 b^2 (A b+B a)}{2 x^4}-\frac{a^5 A}{13 x^{13}}-\frac{a^4 (5 A b+B a)}{10 x^{10}}-\frac{5 a b^3 (A b+2 B a)}{x}$
norman	$\frac{-\frac{a^5 A}{13}+\left(-\frac{1}{2} a^4 b A-\frac{1}{10} a^5 B\right) x^3+\left(-\frac{10}{7} a^3 b^2 A-\frac{5}{7} a^4 b B\right) x^6+\left(-\frac{5}{2} a^2 b^3 A-\frac{5}{2} a^3 b^2 B\right) x^9+\left(-5 a b^4 A-10 a^2 b^3 B\right) x^{12}+\left(\frac{1}{2} b^5 A+\frac{5}{2} a b \right.}{x^{13}}$
risch	$\frac{b^5 B x^5}{5}+\frac{A b^5 x^2}{2}+\frac{5 B a b^4 x^2}{2}+\frac{\left(-5 a b^4 A-10 a^2 b^3 B\right) x^{12}+\left(-\frac{5}{2} a^2 b^3 A-\frac{5}{2} a^3 b^2 B\right) x^9+\left(-\frac{10}{7} a^3 b^2 A-\frac{5}{7} a^4 b B\right) x^6+\left(-\frac{1}{2} a^4 b\right.}{x^{13}}$
gosper	$-\frac{-182 b^5 B x^{18}-455 A b^5 x^{15}-2275 B a b^4 x^{15}+4550 a A b^4 x^{12}+9100 B a^2 b^3 x^{12}+2275 a^2 A b^3 x^9+2275 B a^3 b^2 x^9+1300 a^3 A b^2 x}{910 x^{13}}$
parallelrisch	$\frac{182 b^5 B x^{18}+455 A b^5 x^{15}+2275 B a b^4 x^{15}-4550 a A b^4 x^{12}-9100 B a^2 b^3 x^{12}-2275 a^2 A b^3 x^9-2275 B a^3 b^2 x^9-1300 a^3 A b^2 x^6-65}{910 x^{13}}$
orering	$-\frac{-182 b^5 B x^{18}-455 A b^5 x^{15}-2275 B a b^4 x^{15}+4550 a A b^4 x^{12}+9100 B a^2 b^3 x^{12}+2275 a^2 A b^3 x^9+2275 B a^3 b^2 x^9+1300 a^3 A b^2 x}{910 x^{13}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^14,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^5*B*x^5+1/2*A*b^5*x^2+5/2*B*a*b^4*x^2-5/7*a^3*b*(2*A*b+B*a)/x^7-5/2*
a^2*b^2*(A*b+B*a)/x^4-1/13*a^5*A/x^13-1/10*a^4*(5*A*b+B*a)/x^10-5*a*b^3*(A
*b+2*B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} \, dx$$
$$= \frac{182 \, Bb^5x^{18} + 455 \, (5 \, Bab^4 + Ab^5)x^{15} - 4550 \, (2 \, Ba^2b^3 + Aab^4)x^{12} - 2275 \, (Ba^3b^2 + Aa^2b^3)x^9 - 650 \, (Ba^4}{910 \, x^{13}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="fricas")
```

output
```
1/910*(182*B*b^5*x^18 + 455*(5*B*a*b^4 + A*b^5)*x^15 - 4550*(2*B*a^2*b^3 +
 A*a*b^4)*x^12 - 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 650*(B*a^4*b + 2*A*a^3
*b^2)*x^6 - 70*A*a^5 - 91*(B*a^5 + 5*A*a^4*b)*x^3)/x^13
```

Sympy [A] (verification not implemented)

Time = 7.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} \, dx = \frac{Bb^5x^5}{5} + x^2\left(\frac{Ab^5}{2} + \frac{5Bab^4}{2}\right)$$
$$+ \frac{-70Aa^5 + x^{12}(-4550Aab^4 - 9100Ba^2b^3) + x^9(-2275Aa^2b^3 - 2275Ba^3b^2) + x^6(-1300Aa^3b^2 - 650B}{910x^{13}}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**14,x)
```

output
```
B*b**5*x**5/5 + x**2*(A*b**5/2 + 5*B*a*b**4/2) + (-70*A*a**5 + x**12*(-455
0*A*a*b**4 - 9100*B*a**2*b**3) + x**9*(-2275*A*a**2*b**3 - 2275*B*a**3*b**
2) + x**6*(-1300*A*a**3*b**2 - 650*B*a**4*b) + x**3*(-455*A*a**4*b - 91*B*
a**5))/(910*x**13)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} \, dx = \frac{1}{5} Bb^5x^5 + \frac{1}{2} (5 Bab^4 + Ab^5)x^2 - \frac{4550 (2 Ba^2b^3 + Aab^4)x^{12} + 2275 (Ba^3b^2 + Aa^2b^3)x^9 + 650 (Ba^4b + 2 Aa^3b^2)x^6 + 70 Aa^5 + 91 (Ba^5 +}{910 x^{13}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="maxima")
```

output
```
1/5*B*b^5*x^5 + 1/2*(5*B*a*b^4 + A*b^5)*x^2 - 1/910*(4550*(2*B*a^2*b^3 + A
*a*b^4)*x^12 + 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 650*(B*a^4*b + 2*A*a^3*b
^2)*x^6 + 70*A*a^5 + 91*(B*a^5 + 5*A*a^4*b)*x^3)/x^13
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} \, dx = \frac{1}{5} Bb^5x^5 + \frac{5}{2} Bab^4x^2 + \frac{1}{2} Ab^5x^2 - \frac{9100 Ba^2b^3x^{12} + 4550 Aab^4x^{12} + 2275 Ba^3b^2x^9 + 2275 Aa^2b^3x^9 + 650 Ba^4bx^6 + 1300 Aa^3b^2x^6 + 91 B}{910 x^{13}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="giac")
```

output
```
1/5*B*b^5*x^5 + 5/2*B*a*b^4*x^2 + 1/2*A*b^5*x^2 - 1/910*(9100*B*a^2*b^3*x^
12 + 4550*A*a*b^4*x^12 + 2275*B*a^3*b^2*x^9 + 2275*A*a^2*b^3*x^9 + 650*B*a
^4*b*x^6 + 1300*A*a^3*b^2*x^6 + 91*B*a^5*x^3 + 455*A*a^4*b*x^3 + 70*A*a^5)
/x^13
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} \, dx = x^2 \left(\frac{A\,b^5}{2} + \frac{5\,B\,a\,b^4}{2}\right) - \frac{\frac{A\,a^5}{13} + x^{12}\,(10\,B\,a^2\,b^3 + 5\,A\,a\,b^4) + x^6\left(\frac{5\,B\,a^4\,b}{7} + \frac{10\,A\,a^3\,b^2}{7}\right) + x^3\left(\frac{B\,a^5}{10} + \frac{A\,b\,a^4}{2}\right) + x^9\left(\frac{5\,B\,a^3\,b^2}{2} + \frac{5\,A\,a^2\,b^3}{2}\right)}{x^{13}} + \frac{B\,b^5\,x^5}{5}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^14,x)
```

output
```
x^2*((A*b^5)/2 + (5*B*a*b^4)/2) - ((A*a^5)/13 + x^12*(10*B*a^2*b^3 + 5*A*a
*b^4) + x^6*((10*A*a^3*b^2)/7 + (5*B*a^4*b)/7) + x^3*((B*a^5)/10 + (A*a^4*
b)/2) + x^9*((5*A*a^2*b^3)/2 + (5*B*a^3*b^2)/2))/x^13 + (B*b^5*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} \, dx = \frac{91b^6x^{18} + 1365a\,b^5x^{15} - 6825a^2b^4x^{12} - 2275a^3b^3x^9 - 975a^4b^2x^6 - 273a^5b\,x^3 - 35a^6}{455x^{13}}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^14,x)
```

output
```
( - 35*a**6 - 273*a**5*b*x**3 - 975*a**4*b**2*x**6 - 2275*a**3*b**3*x**9 -
 6825*a**2*b**4*x**12 + 1365*a*b**5*x**15 + 91*b**6*x**18)/(455*x**13)
```

3.52 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{15}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{15}}\,dx = -\frac{a^5A}{14x^{14}}-\frac{a^4(5Ab+aB)}{11x^{11}}-\frac{5a^3b(2Ab+aB)}{8x^8}-\frac{2a^2b^2(Ab+aB)}{x^5}-\frac{5ab^3(Ab+2aB)}{2x^2}+b^4(Ab+5aB)x+\frac{1}{4}b^5Bx^4$$

output
```
-1/14*a^5*A/x^14-1/11*a^4*(5*A*b+B*a)/x^11-5/8*a^3*b*(2*A*b+B*a)/x^8-2*a^2
*b^2*(A*b+B*a)/x^5-5/2*a*b^3*(A*b+2*B*a)/x^2+b^4*(A*b+5*B*a)*x+1/4*b^5*B*x
^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{15}}\,dx = -\frac{a^5A}{14x^{14}}-\frac{a^4(5Ab+aB)}{11x^{11}}-\frac{5a^3b(2Ab+aB)}{8x^8}-\frac{2a^2b^2(Ab+aB)}{x^5}-\frac{5ab^3(Ab+2aB)}{2x^2}+b^4(Ab+5aB)x+\frac{1}{4}b^5Bx^4$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^15,x]
```

output
```
-1/14*(a^5*A)/x^14 - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B
))/(8*x^8) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(2*x^2)
 + b^4*(A*b + 5*a*B)*x + (b^5*B*x^4)/4
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5A}{x^{15}}+\frac{a^4(aB+5Ab)}{x^{12}}+\frac{5a^3b(aB+2Ab)}{x^9}+\frac{10a^2b^2(aB+Ab)}{x^6}+b^4(5aB+Ab)+\frac{5ab^3(2aB+Ab)}{x^3}+b^5Bx^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{14x^{14}}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{8x^8}-\frac{2a^2b^2(aB+Ab)}{x^5}+b^4x(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{2x^2}+\frac{1}{4}b^5Bx^4$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^15,x]
```

output
```
-1/14*(a^5*A)/x^14 - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B
))/(8*x^8) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(2*x^2)
 + b^4*(A*b + 5*a*B)*x + (b^5*B*x^4)/4
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

method	result
default	$\frac{b^5 B x^4}{4}+b^5 A x+5 a b^4 B x-\frac{2 a^2 b^2 (A b+B a)}{x^5}-\frac{a^4 (5 A b+B a)}{11 x^{11}}-\frac{5 a b^3 (A b+2 B a)}{2 x^2}-\frac{5 a^3 b (2 A b+B a)}{8 x^8}-\frac{a^5 A}{14 x^{14}}$
risch	$\frac{b^5 B x^4}{4}+b^5 A x+5 a b^4 B x+\frac{\left(-\frac{5}{2} a b^4 A-5 a^2 b^3 B\right) x^{12}+\left(-2 a^2 b^3 A-2 a^3 b^2 B\right) x^9+\left(-\frac{5}{4} a^3 b^2 A-\frac{5}{8} a^4 b B\right) x^6+\left(-\frac{5}{11} a^4 b\right.}{x^{14}}$
norman	$\frac{-\frac{a^5 A}{14}+\left(-\frac{5}{11} a^4 b A-\frac{1}{11} a^5 B\right) x^3+\left(-\frac{5}{4} a^3 b^2 A-\frac{5}{8} a^4 b B\right) x^6+\left(-2 a^2 b^3 A-2 a^3 b^2 B\right) x^9+\left(-\frac{5}{2} a b^4 A-5 a^2 b^3 B\right) x^{12}+\left(b^5 A+5 a b^4 B\right.}{x^{14}}$
gosper	$-\frac{-154 b^5 B x^{18}-616 A b^5 x^{15}-3080 B a b^4 x^{15}+1540 a A b^4 x^{12}+3080 B a^2 b^3 x^{12}+1232 a^2 A b^3 x^9+1232 B a^3 b^2 x^9+770 a^3 A b^2 x^6}{616 x^{14}}$
parallelrisch	$\frac{154 b^5 B x^{18}+616 A b^5 x^{15}+3080 B a b^4 x^{15}-1540 a A b^4 x^{12}-3080 B a^2 b^3 x^{12}-1232 a^2 A b^3 x^9-1232 B a^3 b^2 x^9-770 a^3 A b^2 x^6-385}{616 x^{14}}$
orering	$-\frac{-154 b^5 B x^{18}-616 A b^5 x^{15}-3080 B a b^4 x^{15}+1540 a A b^4 x^{12}+3080 B a^2 b^3 x^{12}+1232 a^2 A b^3 x^9+1232 B a^3 b^2 x^9+770 a^3 A b^2 x^6}{616 x^{14}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^15,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^5*B*x^4+b^5*A*x+5*a*b^4*B*x-2*a^2*b^2*(A*b+B*a)/x^5-1/11*a^4*(5*A*b+
B*a)/x^11-5/2*a*b^3*(A*b+2*B*a)/x^2-5/8*a^3*b*(2*A*b+B*a)/x^8-1/14*a^5*A/x
^14
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}}\,dx$$
$$= \frac{154\,Bb^5x^{18}+616\,(5\,Bab^4+Ab^5)x^{15}-1540\,(2\,Ba^2b^3+Aab^4)x^{12}-1232\,(Ba^3b^2+Aa^2b^3)x^9-385\,(Ba^4}{616\,x^{14}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="fricas")
```

output
```
1/616*(154*B*b^5*x^18 + 616*(5*B*a*b^4 + A*b^5)*x^15 - 1540*(2*B*a^2*b^3 +
 A*a*b^4)*x^12 - 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 385*(B*a^4*b + 2*A*a^3
*b^2)*x^6 - 44*A*a^5 - 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^14
```

Sympy [A] (verification not implemented)

Time = 15.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}}\,dx = \frac{Bb^5x^4}{4}+x\left(Ab^5+5Bab^4\right)$$
$$+\frac{-44Aa^5+x^{12}(-1540Aab^4-3080Ba^2b^3)+x^9(-1232Aa^2b^3-1232Ba^3b^2)+x^6(-770Aa^3b^2-385Ba}{616x^{14}}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**15,x)
```

output
```
B*b**5*x**4/4 + x*(A*b**5 + 5*B*a*b**4) + (-44*A*a**5 + x**12*(-1540*A*a*b
**4 - 3080*B*a**2*b**3) + x**9*(-1232*A*a**2*b**3 - 1232*B*a**3*b**2) + x*
*6*(-770*A*a**3*b**2 - 385*B*a**4*b) + x**3*(-280*A*a**4*b - 56*B*a**5))/(
616*x**14)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}}\,dx = \frac{1}{4}\,Bb^5x^4 + \left(5\,Bab^4 + Ab^5\right)x$$
$$-\frac{1540\,(2\,Ba^2b^3 + Aab^4)x^{12} + 1232\,(Ba^3b^2 + Aa^2b^3)x^9 + 385\,(Ba^4b + 2\,Aa^3b^2)x^6 + 44\,Aa^5 + 56\,(Ba^5 +}{616\,x^{14}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="maxima")
```

output
```
1/4*B*b^5*x^4 + (5*B*a*b^4 + A*b^5)*x - 1/616*(1540*(2*B*a^2*b^3 + A*a*b^4
)*x^12 + 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 385*(B*a^4*b + 2*A*a^3*b^2)*x^
6 + 44*A*a^5 + 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^14
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}}\,dx = \frac{1}{4}\,Bb^5x^4 + 5\,Bab^4x + Ab^5x$$
$$-\frac{3080\,Ba^2b^3x^{12} + 1540\,Aab^4x^{12} + 1232\,Ba^3b^2x^9 + 1232\,Aa^2b^3x^9 + 385\,Ba^4bx^6 + 770\,Aa^3b^2x^6 + 56\,Ba}{616\,x^{14}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="giac")
```

output
```
1/4*B*b^5*x^4 + 5*B*a*b^4*x + A*b^5*x - 1/616*(3080*B*a^2*b^3*x^12 + 1540*
A*a*b^4*x^12 + 1232*B*a^3*b^2*x^9 + 1232*A*a^2*b^3*x^9 + 385*B*a^4*b*x^6 +
 770*A*a^3*b^2*x^6 + 56*B*a^5*x^3 + 280*A*a^4*b*x^3 + 44*A*a^5)/x^14
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}} \, dx = x \left(A b^5 + 5 B a b^4\right) - \frac{\frac{A a^5}{14} + x^{12} \left(5 B a^2 b^3 + \frac{5 A a b^4}{2}\right) + x^6 \left(\frac{5 B a^4 b}{8} + \frac{5 A a^3 b^2}{4}\right) + x^3 \left(\frac{B a^5}{11} + \frac{5 A b a^4}{11}\right) + x^9 \left(2 B a^3 b^2 + 2 A a^2 b\right.}{x^{14}} + \frac{B b^5 x^4}{4}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^15,x)
```

output
```
x*(A*b^5 + 5*B*a*b^4) - ((A*a^5)/14 + x^12*(5*B*a^2*b^3 + (5*A*a*b^4)/2) +
 x^6*((5*A*a^3*b^2)/4 + (5*B*a^4*b)/8) + x^3*((B*a^5)/11 + (5*A*a^4*b)/11)
 + x^9*(2*A*a^2*b^3 + 2*B*a^3*b^2))/x^14 + (B*b^5*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}} \, dx = \frac{154b^6x^{18} + 3696a b^5x^{15} - 4620a^2b^4x^{12} - 2464a^3b^3x^9 - 1155a^4b^2x^6 - 336a^5b x^3 - 44a^6}{616x^{14}}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^15,x)
```

output
```
( - 44*a**6 - 336*a**5*b*x**3 - 1155*a**4*b**2*x**6 - 2464*a**3*b**3*x**9
- 4620*a**2*b**4*x**12 + 3696*a*b**5*x**15 + 154*b**6*x**18)/(616*x**14)
```

3.53 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 115

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx = -\frac{a^5A}{16x^{16}} - \frac{a^4(5Ab+aB)}{13x^{13}} - \frac{a^3b(2Ab+aB)}{2x^{10}} - \frac{10a^2b^2(Ab+aB)}{7x^7} - \frac{5ab^3(Ab+2aB)}{4x^4} - \frac{b^4(Ab+5aB)}{x} + \frac{1}{2}b^5Bx^2$$

output
```
-1/16*a^5*A/x^16-1/13*a^4*(5*A*b+B*a)/x^13-1/2*a^3*b*(2*A*b+B*a)/x^10-10/7
*a^2*b^2*(A*b+B*a)/x^7-5/4*a*b^3*(A*b+2*B*a)/x^4-b^4*(A*b+5*B*a)/x+1/2*b^5
*B*x^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx = -\frac{-728b^5x^{15}(-2A+Bx^3)+1820ab^4x^{12}(A+4Bx^3)+520a^2b^3x^9(4A+7Bx^3)+208a^3b^2x^6(7A+10Bx}{1456x^{16}}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^17,x]
```

output
```
-1/1456*(-728*b^5*x^15*(-2*A + B*x^3) + 1820*a*b^4*x^12*(A + 4*B*x^3) + 52
0*a^2*b^3*x^9*(4*A + 7*B*x^3) + 208*a^3*b^2*x^6*(7*A + 10*B*x^3) + 56*a^4*
b*x^3*(10*A + 13*B*x^3) + 7*a^5*(13*A + 16*B*x^3))/x^16
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{17}} dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5A}{x^{17}}+\frac{a^4(aB+5Ab)}{x^{14}}+\frac{5a^3b(aB+2Ab)}{x^{11}}+\frac{10a^2b^2(aB+Ab)}{x^8}+\frac{b^4(5aB+Ab)}{x^2}+\frac{5ab^3(2aB+Ab)}{x^5}+b^5Bx\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{16x^{16}}-\frac{a^4(aB+5Ab)}{13x^{13}}-\frac{a^3b(aB+2Ab)}{2x^{10}}-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{x}-\frac{5ab^3(2aB+Ab)}{4x^4}+\frac{1}{2}b^5Bx^2$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^17,x]
```

output
```
-1/16*(a^5*A)/x^16 - (a^4*(5*A*b + a*B))/(13*x^13) - (a^3*b*(2*A*b + a*B))
/(2*x^10) - (10*a^2*b^2*(A*b + a*B))/(7*x^7) - (5*a*b^3*(A*b + 2*a*B))/(4*
x^4) - (b^4*(A*b + 5*a*B))/x + (b^5*B*x^2)/2
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a^5 A}{16x^{16}}-\frac{a^4(5Ab+Ba)}{13x^{13}}-\frac{a^3b(2Ab+Ba)}{2x^{10}}-\frac{10a^2b^2(Ab+Ba)}{7x^7}-\frac{5a\,b^3(Ab+2Ba)}{4x^4}-\frac{b^4(Ab+5Ba)}{x}+\frac{b^5B\,x^2}{2}$
norman	$\frac{-\frac{a^5A}{16}+\left(-\frac{5}{13}a^4bA-\frac{1}{13}a^5B\right)x^3+\left(-a^3b^2A-\frac{1}{2}a^4bB\right)x^6+\left(-\frac{10}{7}a^2b^3A-\frac{10}{7}a^3b^2B\right)x^9+\left(-\frac{5}{4}a\,b^4A-\frac{5}{2}a^2b^3B\right)x^{12}+(-b^5A-5a\,b}{x^{16}}$
risch	$\frac{b^5B\,x^2}{2}+\frac{-\frac{a^5A}{16}+\left(-\frac{5}{13}a^4bA-\frac{1}{13}a^5B\right)x^3+\left(-a^3b^2A-\frac{1}{2}a^4bB\right)x^6+\left(-\frac{10}{7}a^2b^3A-\frac{10}{7}a^3b^2B\right)x^9+\left(-\frac{5}{4}a\,b^4A-\frac{5}{2}a^2b^3B\right)x^{12}+(}{x^{16}}$
gosper	$-\frac{-728b^5B\,x^{18}+1456A\,b^5x^{15}+7280Ba\,b^4x^{15}+1820aA\,b^4x^{12}+3640B\,a^2b^3x^{12}+2080a^2A\,b^3x^9+2080B\,a^3b^2x^9+1456a^3A\,b^2}{1456x^{16}}$
parallelrisch	$-\frac{-728b^5B\,x^{18}+1456A\,b^5x^{15}+7280Ba\,b^4x^{15}+1820aA\,b^4x^{12}+3640B\,a^2b^3x^{12}+2080a^2A\,b^3x^9+2080B\,a^3b^2x^9+1456a^3A\,b^2}{1456x^{16}}$
orering	$-\frac{-728b^5B\,x^{18}+1456A\,b^5x^{15}+7280Ba\,b^4x^{15}+1820aA\,b^4x^{12}+3640B\,a^2b^3x^{12}+2080a^2A\,b^3x^9+2080B\,a^3b^2x^9+1456a^3A\,b^2}{1456x^{16}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^17,x,method=_RETURNVERBOSE)
```

output
```
-1/16*a^5*A/x^16-1/13*a^4*(5*A*b+B*a)/x^13-1/2*a^3*b*(2*A*b+B*a)/x^10-10/7
*a^2*b^2*(A*b+B*a)/x^7-5/4*a*b^3*(A*b+2*B*a)/x^4-b^4*(A*b+5*B*a)/x+1/2*b^5
*B*x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{17}}\,dx = \frac{728\,Bb^5x^{18} - 1456\,(5\,Bab^4 + Ab^5)x^{15} - 1820\,(2\,Ba^2b^3 + Aab^4)x^{12} - 2080\,(Ba^3b^2 + Aa^2b^3)x^9 - 728\,(Ba}{1456\,x^{16}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="fricas")
```

output
```
1/1456*(728*B*b^5*x^18 - 1456*(5*B*a*b^4 + A*b^5)*x^15 - 1820*(2*B*a^2*b^3
 + A*a*b^4)*x^12 - 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 728*(B*a^4*b + 2*A*a
^3*b^2)*x^6 - 91*A*a^5 - 112*(B*a^5 + 5*A*a^4*b)*x^3)/x^16
```

Sympy [A] (verification not implemented)

Time = 32.74 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{17}}\,dx = \frac{Bb^5x^2}{2} + \frac{-91Aa^5 + x^{15}(-1456Ab^5 - 7280Bab^4) + x^{12}(-1820Aab^4 - 3640Ba^2b^3) + x^9(-2080Aa^2b^3 - 2080Ba}{1456x^{16}}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**17,x)
```

output
```
B*b**5*x**2/2 + (-91*A*a**5 + x**15*(-1456*A*b**5 - 7280*B*a*b**4) + x**12
*(-1820*A*a*b**4 - 3640*B*a**2*b**3) + x**9*(-2080*A*a**2*b**3 - 2080*B*a*
*3*b**2) + x**6*(-1456*A*a**3*b**2 - 728*B*a**4*b) + x**3*(-560*A*a**4*b -
 112*B*a**5))/(1456*x**16)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx = \frac{1}{2}Bb^5x^2 - \frac{1456\,(5\,Bab^4+Ab^5)x^{15}+1820\,(2\,Ba^2b^3+Aab^4)x^{12}+2080\,(Ba^3b^2+Aa^2b^3)x^9+728\,(Ba^4b+2\,Aa^3b}{1456\,x^{16}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="maxima")
```

output
```
1/2*B*b^5*x^2 - 1/1456*(1456*(5*B*a*b^4 + A*b^5)*x^15 + 1820*(2*B*a^2*b^3
+ A*a*b^4)*x^12 + 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 728*(B*a^4*b + 2*A*a^
3*b^2)*x^6 + 91*A*a^5 + 112*(B*a^5 + 5*A*a^4*b)*x^3)/x^16
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx = \frac{1}{2}Bb^5x^2 - \frac{7280\,Bab^4x^{15}+1456\,Ab^5x^{15}+3640\,Ba^2b^3x^{12}+1820\,Aab^4x^{12}+2080\,Ba^3b^2x^9+2080\,Aa^2b^3x^9+728}{1456\,x^{16}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="giac")
```

output
```
1/2*B*b^5*x^2 - 1/1456*(7280*B*a*b^4*x^15 + 1456*A*b^5*x^15 + 3640*B*a^2*b
^3*x^12 + 1820*A*a*b^4*x^12 + 2080*B*a^3*b^2*x^9 + 2080*A*a^2*b^3*x^9 + 72
8*B*a^4*b*x^6 + 1456*A*a^3*b^2*x^6 + 112*B*a^5*x^3 + 560*A*a^4*b*x^3 + 91*
A*a^5)/x^16
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{17}}\,dx = \frac{B\,b^5\,x^2}{2} - \frac{\frac{A\,a^5}{16} + x^6\left(\frac{B\,a^4\,b}{2} + A\,a^3\,b^2\right) + x^{12}\left(\frac{5\,B\,a^2\,b^3}{2} + \frac{5\,A\,a\,b^4}{4}\right) + x^3\left(\frac{B\,a^5}{13} + \frac{5\,A\,b\,a^4}{13}\right) + x^{15}\left(A\,b^5 + 5\,B\,a\,b^4\right) + x^9}{x^{16}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^17,x)
```

output
```
(B*b^5*x^2)/2 - ((A*a^5)/16 + x^6*(A*a^3*b^2 + (B*a^4*b)/2) + x^12*((5*B*a
^2*b^3)/2 + (5*A*a*b^4)/4) + x^3*((B*a^5)/13 + (5*A*a^4*b)/13) + x^15*(A*b
^5 + 5*B*a*b^4) + x^9*((10*A*a^2*b^3)/7 + (10*B*a^3*b^2)/7))/x^16
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{17}}\,dx = \frac{728b^6x^{18} - 8736a\,b^5x^{15} - 5460a^2b^4x^{12} - 4160a^3b^3x^9 - 2184a^4b^2x^6 - 672a^5b\,x^3 - 91a^6}{1456x^{16}}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^17,x)
```

output
```
( - 91*a**6 - 672*a**5*b*x**3 - 2184*a**4*b**2*x**6 - 4160*a**3*b**3*x**9
- 5460*a**2*b**4*x**12 - 8736*a*b**5*x**15 + 728*b**6*x**18)/(1456*x**16)
```

3.54 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{18}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{18}}\,dx = -\frac{a^5A}{17x^{17}}-\frac{a^4(5Ab+aB)}{14x^{14}}-\frac{5a^3b(2Ab+aB)}{11x^{11}}-\frac{5a^2b^2(Ab+aB)}{4x^8}-\frac{ab^3(Ab+2aB)}{x^5}-\frac{b^4(Ab+5aB)}{2x^2}+b^5Bx$$

output
```
-1/17*a^5*A/x^17-1/14*a^4*(5*A*b+B*a)/x^14-5/11*a^3*b*(2*A*b+B*a)/x^11-5/4
*a^2*b^2*(A*b+B*a)/x^8-a*b^3*(A*b+2*B*a)/x^5-1/2*b^4*(A*b+5*B*a)/x^2+b^5*B
*x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{18}}\,dx = -\frac{a^5A}{17x^{17}}-\frac{a^4(5Ab+aB)}{14x^{14}}-\frac{5a^3b(2Ab+aB)}{11x^{11}}-\frac{5a^2b^2(Ab+aB)}{4x^8}-\frac{ab^3(Ab+2aB)}{x^5}-\frac{b^4(Ab+5aB)}{2x^2}+b^5Bx$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^18,x]
```

output
```
-1/17*(a^5*A)/x^17 - (a^4*(5*A*b + a*B))/(14*x^14) - (5*a^3*b*(2*A*b + a*B
))/(11*x^11) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (a*b^3*(A*b + 2*a*B))/x^5
 - (b^4*(A*b + 5*a*B))/(2*x^2) + b^5*B*x
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5A}{x^{18}}+\frac{a^4(aB+5Ab)}{x^{15}}+\frac{5a^3b(aB+2Ab)}{x^{12}}+\frac{10a^2b^2(aB+Ab)}{x^9}+\frac{b^4(5aB+Ab)}{x^3}+\frac{5ab^3(2aB+Ab)}{x^6}+b^5B\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{17x^{17}}-\frac{a^4(aB+5Ab)}{14x^{14}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{5a^2b^2(aB+Ab)}{4x^8}-\frac{b^4(5aB+Ab)}{2x^2}-\frac{ab^3(2aB+Ab)}{x^5}+b^5Bx$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^18,x]
```

output
```
-1/17*(a^5*A)/x^17 - (a^4*(5*A*b + a*B))/(14*x^14) - (5*a^3*b*(2*A*b + a*B
))/(11*x^11) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (a*b^3*(A*b + 2*a*B))/x^5
 - (b^4*(A*b + 5*a*B))/(2*x^2) + b^5*B*x
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a^{5} A}{17 x^{17}}-\frac{a^{4} \left(5 A b +B a \right)}{14 x^{14}}-\frac{5 a^{3} b \left(2 A b +B a \right)}{11 x^{11}}-\frac{5 a^{2} b^{2} \left(A b +B a \right)}{4 x^{8}}-\frac{a \,b^{3} \left(A b +2 B a \right)}{x^{5}}-\frac{b^{4} \left(A b +5 B a \right)}{2 x^{2}}+b^{5} B x$
risch	$b^{5} B x +\frac{-\frac{a^{5} A}{17}+\left(-\frac{5}{14} a^{4} b A -\frac{1}{14} a^{5} B \right) x^{3}+\left(-\frac{10}{11} a^{3} b^{2} A -\frac{5}{11} a^{4} b B \right) x^{6}+\left(-\frac{5}{4} a^{2} b^{3} A -\frac{5}{4} a^{3} b^{2} B \right) x^{9}+\left(-a \,b^{4} A -2 a^{2} b^{3} B \right) x^{12}+(\cdots}{x^{17}}$
norman	$\frac{-\frac{a^{5} A}{17}+\left(-\frac{5}{14} a^{4} b A -\frac{1}{14} a^{5} B \right) x^{3}+\left(-\frac{10}{11} a^{3} b^{2} A -\frac{5}{11} a^{4} b B \right) x^{6}+\left(-\frac{5}{4} a^{2} b^{3} A -\frac{5}{4} a^{3} b^{2} B \right) x^{9}+\left(-a \,b^{4} A -2 a^{2} b^{3} B \right) x^{12}+\left(-\frac{1}{2} b^{5} A -\frac{5}{2} \cdots\right.}{x^{17}}$
gosper	$-\frac{-5236 b^{5} B \,x^{18}+2618 A \,b^{5} x^{15}+13090 B a \,b^{4} x^{15}+5236 a A \,b^{4} x^{12}+10472 B \,a^{2} b^{3} x^{12}+6545 a^{2} A \,b^{3} x^{9}+6545 B \,a^{3} b^{2} x^{9}+4760 a^{3} \cdots}{5236 x^{17}}$
parallelrisch	$-\frac{-5236 b^{5} B \,x^{18}+2618 A \,b^{5} x^{15}+13090 B a \,b^{4} x^{15}+5236 a A \,b^{4} x^{12}+10472 B \,a^{2} b^{3} x^{12}+6545 a^{2} A \,b^{3} x^{9}+6545 B \,a^{3} b^{2} x^{9}+4760 a^{3} \cdots}{5236 x^{17}}$
orering	$-\frac{-5236 b^{5} B \,x^{18}+2618 A \,b^{5} x^{15}+13090 B a \,b^{4} x^{15}+5236 a A \,b^{4} x^{12}+10472 B \,a^{2} b^{3} x^{12}+6545 a^{2} A \,b^{3} x^{9}+6545 B \,a^{3} b^{2} x^{9}+4760 a^{3} \cdots}{5236 x^{17}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^18,x,method=_RETURNVERBOSE)
```

output
```
-1/17*a^5*A/x^17-1/14*a^4*(5*A*b+B*a)/x^14-5/11*a^3*b*(2*A*b+B*a)/x^11-5/4
*a^2*b^2*(A*b+B*a)/x^8-a*b^3*(A*b+2*B*a)/x^5-1/2*b^4*(A*b+5*B*a)/x^2+b^5*B
*x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}}\,dx = \frac{5236\,Bb^5x^{18} - 2618\,(5\,Bab^4 + Ab^5)x^{15} - 5236\,(2\,Ba^2b^3 + Aab^4)x^{12} - 6545\,(Ba^3b^2 + Aa^2b^3)x^9 - 2380\,(\ldots}{5236\,x^{17}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="fricas")
```

output
```
1/5236*(5236*B*b^5*x^18 - 2618*(5*B*a*b^4 + A*b^5)*x^15 - 5236*(2*B*a^2*b^
3 + A*a*b^4)*x^12 - 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 2380*(B*a^4*b + 2*A
*a^3*b^2)*x^6 - 308*A*a^5 - 374*(B*a^5 + 5*A*a^4*b)*x^3)/x^17
```

Sympy [A] (verification not implemented)

Time = 102.68 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}}\,dx = Bb^5x + \frac{-308Aa^5 + x^{15}(-2618Ab^5 - 13090Bab^4) + x^{12}(-5236Aab^4 - 10472Ba^2b^3) + x^9(-6545Aa^2b^3 - 654\ldots}{5236x^{17}}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**18,x)
```

output
```
B*b**5*x + (-308*A*a**5 + x**15*(-2618*A*b**5 - 13090*B*a*b**4) + x**12*(-
5236*A*a*b**4 - 10472*B*a**2*b**3) + x**9*(-6545*A*a**2*b**3 - 6545*B*a**3
*b**2) + x**6*(-4760*A*a**3*b**2 - 2380*B*a**4*b) + x**3*(-1870*A*a**4*b -
 374*B*a**5))/(5236*x**17)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}}\,dx = Bb^5x - \frac{2618\,(5\,Bab^4+Ab^5)x^{15}+5236\,(2\,Ba^2b^3+Aab^4)x^{12}+6545\,(Ba^3b^2+Aa^2b^3)x^9+2380\,(Ba^4b+2\,Aa^3}{5236\,x^{17}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="maxima")
```

output
```
B*b^5*x - 1/5236*(2618*(5*B*a*b^4 + A*b^5)*x^15 + 5236*(2*B*a^2*b^3 + A*a*
b^4)*x^12 + 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 2380*(B*a^4*b + 2*A*a^3*b^2
)*x^6 + 308*A*a^5 + 374*(B*a^5 + 5*A*a^4*b)*x^3)/x^17
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}}\,dx = Bb^5x - \frac{13090\,Bab^4x^{15}+2618\,Ab^5x^{15}+10472\,Ba^2b^3x^{12}+5236\,Aab^4x^{12}+6545\,Ba^3b^2x^9+6545\,Aa^2b^3x^9+2}{5236\,x^{17}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="giac")
```

output
```
B*b^5*x - 1/5236*(13090*B*a*b^4*x^15 + 2618*A*b^5*x^15 + 10472*B*a^2*b^3*x
^12 + 5236*A*a*b^4*x^12 + 6545*B*a^3*b^2*x^9 + 6545*A*a^2*b^3*x^9 + 2380*B
*a^4*b*x^6 + 4760*A*a^3*b^2*x^6 + 374*B*a^5*x^3 + 1870*A*a^4*b*x^3 + 308*A
*a^5)/x^17
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}}\,dx = B\,b^5\,x$$
$$-\frac{\frac{A\,a^5}{17}+x^{12}\left(2\,B\,a^2\,b^3+A\,a\,b^4\right)+x^6\left(\frac{5\,B\,a^4\,b}{11}+\frac{10\,A\,a^3\,b^2}{11}\right)+x^3\left(\frac{B\,a^5}{14}+\frac{5\,A\,b\,a^4}{14}\right)+x^{15}\left(\frac{A\,b^5}{2}+\frac{5\,B\,a\,b^4}{2}\right)+x}{x^{17}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^18,x)
```

output
```
B*b^5*x - ((A*a^5)/17 + x^12*(2*B*a^2*b^3 + A*a*b^4) + x^6*((10*A*a^3*b^2)
/11 + (5*B*a^4*b)/11) + x^3*((B*a^5)/14 + (5*A*a^4*b)/14) + x^15*((A*b^5)/
2 + (5*B*a*b^4)/2) + x^9*((5*A*a^2*b^3)/4 + (5*B*a^3*b^2)/4))/x^17
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}}\,dx$$
$$=\frac{2618b^6x^{18}-7854a\,b^5x^{15}-7854a^2b^4x^{12}-6545a^3b^3x^9-3570a^4b^2x^6-1122a^5b\,x^3-154a^6}{2618x^{17}}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^18,x)
```

output
```
( - 154*a**6 - 1122*a**5*b*x**3 - 3570*a**4*b**2*x**6 - 6545*a**3*b**3*x**
9 - 7854*a**2*b**4*x**12 - 7854*a*b**5*x**15 + 2618*b**6*x**18)/(2618*x**1
7)
```

3.55 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{20}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{20}}\,dx = -\frac{a^5A}{19x^{19}} - \frac{a^4(5Ab+aB)}{16x^{16}} - \frac{5a^3b(2Ab+aB)}{13x^{13}} - \frac{a^2b^2(Ab+aB)}{x^{10}} - \frac{5ab^3(Ab+2aB)}{7x^7} - \frac{b^4(Ab+5aB)}{4x^4} - \frac{b^5B}{x}$$

output
```
-1/19*a^5*A/x^19-1/16*a^4*(5*A*b+B*a)/x^16-5/13*a^3*b*(2*A*b+B*a)/x^13-a^2
*b^2*(A*b+B*a)/x^10-5/7*a*b^3*(A*b+2*B*a)/x^7-1/4*b^4*(A*b+5*B*a)/x^4-b^5*
B/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{20}}\,dx = -\frac{6916b^5x^{15}(A+4Bx^3)+4940ab^4x^{12}(4A+7Bx^3)+3952a^2b^3x^9(7A+10Bx^3)+2128a^3b^2x^6(10A+13}{27664x^{19}}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^20,x]
```

output
```
-1/27664*(6916*b^5*x^15*(A + 4*B*x^3) + 4940*a*b^4*x^12*(4*A + 7*B*x^3) +
3952*a^2*b^3*x^9*(7*A + 10*B*x^3) + 2128*a^3*b^2*x^6*(10*A + 13*B*x^3) + 6
65*a^4*b*x^3*(13*A + 16*B*x^3) + 91*a^5*(16*A + 19*B*x^3))/x^19
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5A}{x^{20}}+\frac{a^4(aB+5Ab)}{x^{17}}+\frac{5a^3b(aB+2Ab)}{x^{14}}+\frac{10a^2b^2(aB+Ab)}{x^{11}}+\frac{b^4(5aB+Ab)}{x^5}+\frac{5ab^3(2aB+Ab)}{x^8}+\frac{b^5B}{x^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{19x^{19}}-\frac{a^4(aB+5Ab)}{16x^{16}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{a^2b^2(aB+Ab)}{x^{10}}-\frac{b^4(5aB+Ab)}{4x^4}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^5B}{x}$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^20,x]
```

output
```
-1/19*(a^5*A)/x^19 - (a^4*(5*A*b + a*B))/(16*x^16) - (5*a^3*b*(2*A*b + a*B
))/(13*x^13) - (a^2*b^2*(A*b + a*B))/x^10 - (5*a*b^3*(A*b + 2*a*B))/(7*x^7
) - (b^4*(A*b + 5*a*B))/(4*x^4) - (b^5*B)/x
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a^{5}A}{19x^{19}}-\frac{a^{4}(5Ab+Ba)}{16x^{16}}-\frac{5a^{3}b(2Ab+Ba)}{13x^{13}}-\frac{a^{2}b^{2}(Ab+Ba)}{x^{10}}-\frac{5a\,b^{3}(Ab+2Ba)}{7x^{7}}-\frac{b^{4}(Ab+5Ba)}{4x^{4}}-\frac{b^{5}B}{x}$
norman	$\frac{-\frac{a^{5}A}{19}+\left(-\frac{5}{16}a^{4}bA-\frac{1}{16}a^{5}B\right)x^{3}+\left(-\frac{10}{13}a^{3}b^{2}A-\frac{5}{13}a^{4}bB\right)x^{6}+\left(-a^{2}b^{3}A-a^{3}b^{2}B\right)x^{9}+\left(-\frac{5}{7}a\,b^{4}A-\frac{10}{7}a^{2}b^{3}B\right)x^{12}+\left(-\frac{1}{4}b^{5}A-\frac{5}{4}\right.}{x^{19}}$
risch	$\frac{-\frac{a^{5}A}{19}+\left(-\frac{5}{16}a^{4}bA-\frac{1}{16}a^{5}B\right)x^{3}+\left(-\frac{10}{13}a^{3}b^{2}A-\frac{5}{13}a^{4}bB\right)x^{6}+\left(-a^{2}b^{3}A-a^{3}b^{2}B\right)x^{9}+\left(-\frac{5}{7}a\,b^{4}A-\frac{10}{7}a^{2}b^{3}B\right)x^{12}+\left(-\frac{1}{4}b^{5}A-\frac{5}{4}\right.}{x^{19}}$
gosper	$-\frac{27664b^{5}B\,x^{18}+6916A\,b^{5}x^{15}+34580Ba\,b^{4}x^{15}+19760aA\,b^{4}x^{12}+39520B\,a^{2}b^{3}x^{12}+27664a^{2}A\,b^{3}x^{9}+27664B\,a^{3}b^{2}x^{9}+21280}{27664x^{19}}$
parallelrisch	$-\frac{27664b^{5}B\,x^{18}+6916A\,b^{5}x^{15}+34580Ba\,b^{4}x^{15}+19760aA\,b^{4}x^{12}+39520B\,a^{2}b^{3}x^{12}+27664a^{2}A\,b^{3}x^{9}+27664B\,a^{3}b^{2}x^{9}+21280}{27664x^{19}}$
orering	$-\frac{27664b^{5}B\,x^{18}+6916A\,b^{5}x^{15}+34580Ba\,b^{4}x^{15}+19760aA\,b^{4}x^{12}+39520B\,a^{2}b^{3}x^{12}+27664a^{2}A\,b^{3}x^{9}+27664B\,a^{3}b^{2}x^{9}+21280}{27664x^{19}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^20,x,method=_RETURNVERBOSE)
```

output
```
-1/19*a^5*A/x^19-1/16*a^4*(5*A*b+B*a)/x^16-5/13*a^3*b*(2*A*b+B*a)/x^13-a^2
*b^2*(A*b+B*a)/x^10-5/7*a*b^3*(A*b+2*B*a)/x^7-1/4*b^4*(A*b+5*B*a)/x^4-b^5*
B/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}}\,dx = -\frac{27664\,Bb^5x^{18} + 6916\,(5\,Bab^4 + Ab^5)x^{15} + 19760\,(2\,Ba^2b^3 + Aab^4)x^{12} + 27664\,(Ba^3b^2 + Aa^2b^3)x^9 + 1}{27664\,x^{19}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="fricas")
```

output
```
-1/27664*(27664*B*b^5*x^18 + 6916*(5*B*a*b^4 + A*b^5)*x^15 + 19760*(2*B*a^
2*b^3 + A*a*b^4)*x^12 + 27664*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 10640*(B*a^4*b
 + 2*A*a^3*b^2)*x^6 + 1456*A*a^5 + 1729*(B*a^5 + 5*A*a^4*b)*x^3)/x^19
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**20,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}}\,dx = -\frac{27664\,Bb^5x^{18} + 6916\,(5\,Bab^4 + Ab^5)x^{15} + 19760\,(2\,Ba^2b^3 + Aab^4)x^{12} + 27664\,(Ba^3b^2 + Aa^2b^3)x^9 + 1}{27664\,x^{19}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="maxima")
```

output
```
-1/27664*(27664*B*b^5*x^18 + 6916*(5*B*a*b^4 + A*b^5)*x^15 + 19760*(2*B*a^
2*b^3 + A*a*b^4)*x^12 + 27664*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 10640*(B*a^4*b
 + 2*A*a^3*b^2)*x^6 + 1456*A*a^5 + 1729*(B*a^5 + 5*A*a^4*b)*x^3)/x^19
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}} \, dx = -\frac{27664 \, Bb^5x^{18} + 34580 \, Bab^4x^{15} + 6916 \, Ab^5x^{15} + 39520 \, Ba^2b^3x^{12} + 19760 \, Aab^4x^{12} + 27664 \, Ba^3b^2x^9 + }{27664 \, x^{19}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="giac")
```

output
```
-1/27664*(27664*B*b^5*x^18 + 34580*B*a*b^4*x^15 + 6916*A*b^5*x^15 + 39520*
B*a^2*b^3*x^12 + 19760*A*a*b^4*x^12 + 27664*B*a^3*b^2*x^9 + 27664*A*a^2*b^
3*x^9 + 10640*B*a^4*b*x^6 + 21280*A*a^3*b^2*x^6 + 1729*B*a^5*x^3 + 8645*A*
a^4*b*x^3 + 1456*A*a^5)/x^19
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}} \, dx = -\frac{\frac{A\,a^5}{19} + x^{12}\left(\frac{10\,B\,a^2\,b^3}{7} + \frac{5\,A\,a\,b^4}{7}\right) + x^6\left(\frac{5\,B\,a^4\,b}{13} + \frac{10\,A\,a^3\,b^2}{13}\right) + x^3\left(\frac{B\,a^5}{16} + \frac{5\,A\,b\,a^4}{16}\right) + x^{15}\left(\frac{A\,b^5}{4} + \frac{5\,B\,a\,b^4}{4}\right) + }{x^{19}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^20,x)
```

output
```
-((A*a^5)/19 + x^12*((10*B*a^2*b^3)/7 + (5*A*a*b^4)/7) + x^6*((10*A*a^3*b^
2)/13 + (5*B*a^4*b)/13) + x^3*((B*a^5)/16 + (5*A*a^4*b)/16) + x^15*((A*b^5
)/4 + (5*B*a*b^4)/4) + x^9*(A*a^2*b^3 + B*a^3*b^2) + B*b^5*x^18)/x^19
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}} \, dx = \frac{-13832b^6x^{18} - 20748a\,b^5x^{15} - 29640a^2b^4x^{12} - 27664a^3b^3x^9 - 15960a^4b^2x^6 - 5187a^5b\,x^3 - 728a^6}{13832x^{19}}$$

input

```
int((b*x^3+a)^5*(B*x^3+A)/x^20,x)
```

output

```
( - 728*a**6 - 5187*a**5*b*x**3 - 15960*a**4*b**2*x**6 - 27664*a**3*b**3*x
**9 - 29640*a**2*b**4*x**12 - 20748*a*b**5*x**15 - 13832*b**6*x**18)/(1383
2*x**19)
```

3.56 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{21}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{21}}\,dx = -\frac{a^5A}{20x^{20}} - \frac{a^4(5Ab+aB)}{17x^{17}} - \frac{5a^3b(2Ab+aB)}{14x^{14}} - \frac{10a^2b^2(Ab+aB)}{11x^{11}} - \frac{5ab^3(Ab+2aB)}{8x^8} - \frac{b^4(Ab+5aB)}{5x^5} - \frac{b^5B}{2x^2}$$

output
```
-1/20*a^5*A/x^20-1/17*a^4*(5*A*b+B*a)/x^17-5/14*a^3*b*(2*A*b+B*a)/x^14-10/
11*a^2*b^2*(A*b+B*a)/x^11-5/8*a*b^3*(A*b+2*B*a)/x^8-1/5*b^4*(A*b+5*B*a)/x^
5-1/2*b^5*B/x^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{21}}\,dx = -\frac{5236b^5x^{15}(2A+5Bx^3)+6545ab^4x^{12}(5A+8Bx^3)+5950a^2b^3x^9(8A+11Bx^3)+3400a^3b^2x^6(11A+1}{52360x^{20}}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^21,x]
```

output
```
-1/52360*(5236*b^5*x^15*(2*A + 5*B*x^3) + 6545*a*b^4*x^12*(5*A + 8*B*x^3)
+ 5950*a^2*b^3*x^9*(8*A + 11*B*x^3) + 3400*a^3*b^2*x^6*(11*A + 14*B*x^3) +
 1100*a^4*b*x^3*(14*A + 17*B*x^3) + 154*a^5*(17*A + 20*B*x^3))/x^20
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{21}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5A}{x^{21}}+\frac{a^4(aB+5Ab)}{x^{18}}+\frac{5a^3b(aB+2Ab)}{x^{15}}+\frac{10a^2b^2(aB+Ab)}{x^{12}}+\frac{b^4(5aB+Ab)}{x^6}+\frac{5ab^3(2aB+Ab)}{x^9}+\frac{b^5B}{x^3}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{20x^{20}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{5a^3b(aB+2Ab)}{14x^{14}}-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{5ab^3(2aB+Ab)}{8x^8}-\frac{b^5B}{2x^2}$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^21,x]
```

output
```
-1/20*(a^5*A)/x^20 - (a^4*(5*A*b + a*B))/(17*x^17) - (5*a^3*b*(2*A*b + a*B
))/(14*x^14) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a*b^3*(A*b + 2*a*B)
)/(8*x^8) - (b^4*(A*b + 5*a*B))/(5*x^5) - (b^5*B)/(2*x^2)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^5A}{20x^{20}}-\frac{a^4(5Ab+Ba)}{17x^{17}}-\frac{5a^3b(2Ab+Ba)}{14x^{14}}-\frac{10a^2b^2(Ab+Ba)}{11x^{11}}-\frac{5a\,b^3(Ab+2Ba)}{8x^8}-\frac{b^4(Ab+5Ba)}{5x^5}-\frac{b^5B}{2x^2}$
norman	$\frac{-\frac{a^5A}{20}+(-\frac{5}{17}a^4bA-\frac{1}{17}a^5B)x^3+(-\frac{5}{7}a^3b^2A-\frac{5}{14}a^4bB)x^6+(-\frac{10}{11}a^2b^3A-\frac{10}{11}a^3b^2B)x^9+(-\frac{5}{8}a\,b^4A-\frac{5}{4}a^2b^3B)x^{12}+(-\frac{1}{5}b^5A-}{x^{20}}$
risch	$\frac{-\frac{a^5A}{20}+(-\frac{5}{17}a^4bA-\frac{1}{17}a^5B)x^3+(-\frac{5}{7}a^3b^2A-\frac{5}{14}a^4bB)x^6+(-\frac{10}{11}a^2b^3A-\frac{10}{11}a^3b^2B)x^9+(-\frac{5}{8}a\,b^4A-\frac{5}{4}a^2b^3B)x^{12}+(-\frac{1}{5}b^5A-}{x^{20}}$
gosper	$-\frac{26180b^5B\,x^{18}+10472A\,b^5x^{15}+52360Ba\,b^4x^{15}+32725aA\,b^4x^{12}+65450B\,a^2b^3x^{12}+47600a^2A\,b^3x^9+47600B\,a^3b^2x^9+374}{52360x^{20}}$
parallelrisch	$-\frac{26180b^5B\,x^{18}+10472A\,b^5x^{15}+52360Ba\,b^4x^{15}+32725aA\,b^4x^{12}+65450B\,a^2b^3x^{12}+47600a^2A\,b^3x^9+47600B\,a^3b^2x^9+374}{52360x^{20}}$
orering	$-\frac{26180b^5B\,x^{18}+10472A\,b^5x^{15}+52360Ba\,b^4x^{15}+32725aA\,b^4x^{12}+65450B\,a^2b^3x^{12}+47600a^2A\,b^3x^9+47600B\,a^3b^2x^9+374}{52360x^{20}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^21,x,method=_RETURNVERBOSE)
```

output
```
-1/20*a^5*A/x^20-1/17*a^4*(5*A*b+B*a)/x^17-5/14*a^3*b*(2*A*b+B*a)/x^14-10/
11*a^2*b^2*(A*b+B*a)/x^11-5/8*a*b^3*(A*b+2*B*a)/x^8-1/5*b^4*(A*b+5*B*a)/x^
5-1/2*b^5*B/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{21}} dx = -\frac{26180 Bb^5x^{18} + 10472 (5 Bab^4 + Ab^5)x^{15} + 32725 (2 Ba^2b^3 + Aab^4)x^{12} + 47600 (Ba^3b^2 + Aa^2b^3)x^9 +}{52360 x^{20}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="fricas")
```

output
```
-1/52360*(26180*B*b^5*x^18 + 10472*(5*B*a*b^4 + A*b^5)*x^15 + 32725*(2*B*a
^2*b^3 + A*a*b^4)*x^12 + 47600*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 18700*(B*a^4*
b + 2*A*a^3*b^2)*x^6 + 2618*A*a^5 + 3080*(B*a^5 + 5*A*a^4*b)*x^3)/x^20
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{21}} dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**21,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{21}} dx = -\frac{26180 Bb^5x^{18} + 10472 (5 Bab^4 + Ab^5)x^{15} + 32725 (2 Ba^2b^3 + Aab^4)x^{12} + 47600 (Ba^3b^2 + Aa^2b^3)x^9 +}{52360 x^{20}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="maxima")
```

output
```
-1/52360*(26180*B*b^5*x^18 + 10472*(5*B*a*b^4 + A*b^5)*x^15 + 32725*(2*B*a
^2*b^3 + A*a*b^4)*x^12 + 47600*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 18700*(B*a^4*
b + 2*A*a^3*b^2)*x^6 + 2618*A*a^5 + 3080*(B*a^5 + 5*A*a^4*b)*x^3)/x^20
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{21}}\,dx = -\frac{26180\,Bb^5x^{18}+52360\,Bab^4x^{15}+10472\,Ab^5x^{15}+65450\,Ba^2b^3x^{12}+32725\,Aab^4x^{12}+47600\,Ba^3b^2x^9}{52360\,x^2}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="giac")
```

output
```
-1/52360*(26180*B*b^5*x^18 + 52360*B*a*b^4*x^15 + 10472*A*b^5*x^15 + 65450
*B*a^2*b^3*x^12 + 32725*A*a*b^4*x^12 + 47600*B*a^3*b^2*x^9 + 47600*A*a^2*b
^3*x^9 + 18700*B*a^4*b*x^6 + 37400*A*a^3*b^2*x^6 + 3080*B*a^5*x^3 + 15400*
A*a^4*b*x^3 + 2618*A*a^5)/x^20
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{21}}\,dx = -\frac{\frac{A\,a^5}{20}+x^{12}\left(\frac{5\,B\,a^2\,b^3}{4}+\frac{5\,A\,a\,b^4}{8}\right)+x^6\left(\frac{5\,B\,a^4\,b}{14}+\frac{5\,A\,a^3\,b^2}{7}\right)+x^3\left(\frac{B\,a^5}{17}+\frac{5\,A\,b\,a^4}{17}\right)+x^{15}\left(\frac{A\,b^5}{5}+B\,a\,b^4\right)+x^9}{x^{20}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^21,x)
```

output
```
-((A*a^5)/20 + x^12*((5*B*a^2*b^3)/4 + (5*A*a*b^4)/8) + x^6*((5*A*a^3*b^2)
/7 + (5*B*a^4*b)/14) + x^3*((B*a^5)/17 + (5*A*a^4*b)/17) + x^15*((A*b^5)/5
 + B*a*b^4) + x^9*((10*A*a^2*b^3)/11 + (10*B*a^3*b^2)/11) + (B*b^5*x^18)/2
)/x^20
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{21}}\,dx = \frac{-26180b^6x^{18}-62832a\,b^5x^{15}-98175a^2b^4x^{12}-95200a^3b^3x^9-56100a^4b^2x^6-18480a^5b\,x^3-2618a^6}{52360x^{20}}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^21,x)
```

output
```
( - 2618*a**6 - 18480*a**5*b*x**3 - 56100*a**4*b**2*x**6 - 95200*a**3*b**3
*x**9 - 98175*a**2*b**4*x**12 - 62832*a*b**5*x**15 - 26180*b**6*x**18)/(52
360*x**20)
```

3.57 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{23}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{23}}\,dx = -\frac{a^5A}{22x^{22}} - \frac{a^4(5Ab+aB)}{19x^{19}} - \frac{5a^3b(2Ab+aB)}{16x^{16}} - \frac{10a^2b^2(Ab+aB)}{13x^{13}} - \frac{ab^3(Ab+2aB)}{2x^{10}} - \frac{b^4(Ab+5aB)}{7x^7} - \frac{b^5B}{4x^4}$$

output

```
-1/22*a^5*A/x^22-1/19*a^4*(5*A*b+B*a)/x^19-5/16*a^3*b*(2*A*b+B*a)/x^16-10/
13*a^2*b^2*(A*b+B*a)/x^13-1/2*a*b^3*(A*b+2*B*a)/x^10-1/7*b^4*(A*b+5*B*a)/x
^7-1/4*b^5*B/x^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{23}}\,dx = -\frac{a^5A}{22x^{22}} - \frac{a^4(5Ab+aB)}{19x^{19}} - \frac{5a^3b(2Ab+aB)}{16x^{16}} - \frac{10a^2b^2(Ab+aB)}{13x^{13}} - \frac{ab^3(Ab+2aB)}{2x^{10}} - \frac{b^4(Ab+5aB)}{7x^7} - \frac{b^5B}{4x^4}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^23,x]
```

output
```
-1/22*(a^5*A)/x^22 - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B
))/(16*x^16) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (a*b^3*(A*b + 2*a*B))/
(2*x^10) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(4*x^4)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}} \, dx$$

$$\Big\downarrow \text{ 950}$$

$$\int \left(\frac{a^5 A}{x^{23}}+\frac{a^4(aB+5Ab)}{x^{20}}+\frac{5a^3b(aB+2Ab)}{x^{17}}+\frac{10a^2b^2(aB+Ab)}{x^{14}}+\frac{b^4(5aB+Ab)}{x^8}+\frac{5ab^3(2aB+Ab)}{x^{11}}+\frac{b^5B}{x^5}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^5 A}{22x^{22}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{16x^{16}}-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{ab^3(2aB+Ab)}{2x^{10}}-\frac{b^5B}{4x^4}$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^23,x]
```

output
```
-1/22*(a^5*A)/x^22 - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B
))/(16*x^16) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (a*b^3*(A*b + 2*a*B))/
(2*x^10) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(4*x^4)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^5 A}{22x^{22}}-\frac{a^4(5Ab+Ba)}{19x^{19}}-\frac{5a^3b(2Ab+Ba)}{16x^{16}}-\frac{10a^2b^2(Ab+Ba)}{13x^{13}}-\frac{a\,b^3(Ab+2Ba)}{2x^{10}}-\frac{b^4(Ab+5Ba)}{7x^7}-\frac{b^5 B}{4x^4}$
norman	$\frac{-\frac{a^5 A}{22}+\left(-\frac{5}{19}a^4bA-\frac{1}{19}a^5B\right)x^3+\left(-\frac{5}{8}a^3b^2A-\frac{5}{16}a^4bB\right)x^6+\left(-\frac{10}{13}a^2b^3A-\frac{10}{13}a^3b^2B\right)x^9+\left(-\frac{1}{2}a\,b^4A-a^2b^3B\right)x^{12}+\left(-\frac{1}{7}b^5A-\right.}{x^{22}}$
risch	$\frac{-\frac{a^5 A}{22}+\left(-\frac{5}{19}a^4bA-\frac{1}{19}a^5B\right)x^3+\left(-\frac{5}{8}a^3b^2A-\frac{5}{16}a^4bB\right)x^6+\left(-\frac{10}{13}a^2b^3A-\frac{10}{13}a^3b^2B\right)x^9+\left(-\frac{1}{2}a\,b^4A-a^2b^3B\right)x^{12}+\left(-\frac{1}{7}b^5A-\right.}{x^{22}}$
gosper	$-\frac{76076b^5B\,x^{18}+43472A\,b^5x^{15}+217360Ba\,b^4x^{15}+152152aA\,b^4x^{12}+304304B\,a^2b^3x^{12}+234080a^2A\,b^3x^9+234080B\,a^3b^2x^9}{304304x^{22}}$
parallelrisch	$-\frac{76076b^5B\,x^{18}+43472A\,b^5x^{15}+217360Ba\,b^4x^{15}+152152aA\,b^4x^{12}+304304B\,a^2b^3x^{12}+234080a^2A\,b^3x^9+234080B\,a^3b^2x^9}{304304x^{22}}$
orering	$-\frac{76076b^5B\,x^{18}+43472A\,b^5x^{15}+217360Ba\,b^4x^{15}+152152aA\,b^4x^{12}+304304B\,a^2b^3x^{12}+234080a^2A\,b^3x^9+234080B\,a^3b^2x^9}{304304x^{22}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^23,x,method=_RETURNVERBOSE)
```

output
```
-1/22*a^5*A/x^22-1/19*a^4*(5*A*b+B*a)/x^19-5/16*a^3*b*(2*A*b+B*a)/x^16-10/
13*a^2*b^2*(A*b+B*a)/x^13-1/2*a*b^3*(A*b+2*B*a)/x^10-1/7*b^4*(A*b+5*B*a)/x
^7-1/4*b^5*B/x^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}} dx = -\frac{76076 Bb^5x^{18} + 43472 (5 Bab^4 + Ab^5)x^{15} + 152152 (2 Ba^2b^3 + Aab^4)x^{12} + 234080 (Ba^3b^2 + Aa^2b^3)x^9}{304304 x^{22}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="fricas")
```

output
```
-1/304304*(76076*B*b^5*x^18 + 43472*(5*B*a*b^4 + A*b^5)*x^15 + 152152*(2*B
*a^2*b^3 + A*a*b^4)*x^12 + 234080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 95095*(B*a
^4*b + 2*A*a^3*b^2)*x^6 + 13832*A*a^5 + 16016*(B*a^5 + 5*A*a^4*b)*x^3)/x^2
2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}} dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**23,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}} dx = -\frac{76076 Bb^5x^{18} + 43472 (5 Bab^4 + Ab^5)x^{15} + 152152 (2 Ba^2b^3 + Aab^4)x^{12} + 234080 (Ba^3b^2 + Aa^2b^3)x^9}{304304 x^{22}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="maxima")
```

output
```
-1/304304*(76076*B*b^5*x^18 + 43472*(5*B*a*b^4 + A*b^5)*x^15 + 152152*(2*B
*a^2*b^3 + A*a*b^4)*x^12 + 234080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 95095*(B*a
^4*b + 2*A*a^3*b^2)*x^6 + 13832*A*a^5 + 16016*(B*a^5 + 5*A*a^4*b)*x^3)/x^2
2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}} dx = -\frac{76076 Bb^5x^{18} + 217360 Bab^4x^{15} + 43472 Ab^5x^{15} + 304304 Ba^2b^3x^{12} + 152152 Aab^4x^{12} + 234080 Ba^3b}{3043}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="giac")
```

output
```
-1/304304*(76076*B*b^5*x^18 + 217360*B*a*b^4*x^15 + 43472*A*b^5*x^15 + 304
304*B*a^2*b^3*x^12 + 152152*A*a*b^4*x^12 + 234080*B*a^3*b^2*x^9 + 234080*A
*a^2*b^3*x^9 + 95095*B*a^4*b*x^6 + 190190*A*a^3*b^2*x^6 + 16016*B*a^5*x^3
+ 80080*A*a^4*b*x^3 + 13832*A*a^5)/x^22
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}} dx = -\frac{\frac{A a^5}{22} + x^{12} \left(B a^2 b^3 + \frac{A a b^4}{2}\right) + x^6 \left(\frac{5 B a^4 b}{16} + \frac{5 A a^3 b^2}{8}\right) + x^3 \left(\frac{B a^5}{19} + \frac{5 A b a^4}{19}\right) + x^{15} \left(\frac{A b^5}{7} + \frac{5 B a b^4}{7}\right) + x^9}{x^{22}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^23,x)
```

output
```
-((A*a^5)/22 + x^12*(B*a^2*b^3 + (A*a*b^4)/2) + x^6*((5*A*a^3*b^2)/8 + (5*
B*a^4*b)/16) + x^3*((B*a^5)/19 + (5*A*a^4*b)/19) + x^15*((A*b^5)/7 + (5*B*
a*b^4)/7) + x^9*((10*A*a^2*b^3)/13 + (10*B*a^3*b^2)/13) + (B*b^5*x^18)/4)/
x^22
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}}\,dx$$
$$= \frac{-76076b^6x^{18} - 260832a\,b^5x^{15} - 456456a^2b^4x^{12} - 468160a^3b^3x^9 - 285285a^4b^2x^6 - 96096a^5b\,x^3 - 13832}{304304x^{22}}$$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^23,x)
```

output
```
( - 13832*a**6 - 96096*a**5*b*x**3 - 285285*a**4*b**2*x**6 - 468160*a**3*b
**3*x**9 - 456456*a**2*b**4*x**12 - 260832*a*b**5*x**15 - 76076*b**6*x**18
)/(304304*x**22)
```

3.58 $\int \frac{x^8(A+Bx^3)}{a+bx^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 75

$$\int \frac{x^8(A+Bx^3)}{a+bx^3} dx = -\frac{a(Ab-aB)x^3}{3b^3} + \frac{(Ab-aB)x^6}{6b^2} + \frac{Bx^9}{9b} + \frac{a^2(Ab-aB)\log(a+bx^3)}{3b^4}$$

output
```
-1/3*a*(A*b-B*a)*x^3/b^3+1/6*(A*b-B*a)*x^6/b^2+1/9*B*x^9/b+1/3*a^2*(A*b-B*
a)*ln(b*x^3+a)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{x^8(A+Bx^3)}{a+bx^3} dx = \frac{bx^3(6a^2B-3ab(2A+Bx^3)+b^2x^3(3A+2Bx^3))+6a^2(Ab-aB)\log(a+bx^3)}{18b^4}$$

input
```
Integrate[(x^8*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(b*x^3*(6*a^2*B - 3*a*b*(2*A + B*x^3) + b^2*x^3*(3*A + 2*B*x^3)) + 6*a^2*(
A*b - a*B)*Log[a + b*x^3])/(18*b^4)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (A+Bx^3)}{a+bx^3} \, dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3} \int \frac{x^6 (Bx^3+A)}{bx^3+a} dx^3$$

$$\Big\downarrow \; 86$$

$$\frac{1}{3} \int \left(\frac{Bx^6}{b} + \frac{(Ab-aB)x^3}{b^2} + \frac{a(aB-Ab)}{b^3} - \frac{a^2(aB-Ab)}{b^3 (bx^3+a)}\right) dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(\frac{a^2(Ab-aB)\log\left(a+bx^3\right)}{b^4} - \frac{ax^3(Ab-aB)}{b^3} + \frac{x^6(Ab-aB)}{2b^2} + \frac{Bx^9}{3b}\right)$$

input
```
Int[(x^8*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(-((a*(A*b - a*B)*x^3)/b^3) + ((A*b - a*B)*x^6)/(2*b^2) + (B*x^9)/(3*b) +
(a^2*(A*b - a*B)*Log[a + b*x^3])/b^4)/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

method	result	size
norman	$-\frac{a(Ab-Ba)x^3}{3b^3}+\frac{(Ab-Ba)x^6}{6b^2}+\frac{B\,x^9}{9b}+\frac{a^2(Ab-Ba)\ln(b\,x^3+a)}{3b^4}$	68
default	$-\frac{-\frac{1}{3}b^2B\,x^9-\frac{1}{2}A\,b^2x^6+\frac{1}{2}Bab\,x^6+aAb\,x^3-B\,a^2x^3}{3b^3}+\frac{a^2(Ab-Ba)\ln(b\,x^3+a)}{3b^4}$	74
parallelrisch	$\frac{2b^3B\,x^9+3A\,b^3x^6-3Ba\,b^2x^6-6aA\,b^2x^3+6B\,a^2b\,x^3+6A\ln(b\,x^3+a)a^2b-6B\ln(b\,x^3+a)a^3}{18b^4}$	84
risch	$\frac{B\,x^9}{9b}+\frac{A\,x^6}{6b}-\frac{Ba\,x^6}{6b^2}-\frac{aA\,x^3}{3b^2}+\frac{B\,a^2x^3}{3b^3}+\frac{a^2\ln(b\,x^3+a)A}{3b^3}-\frac{a^3\ln(b\,x^3+a)B}{3b^4}$	86

input
```
int(x^8*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*a*(A*b-B*a)*x^3/b^3+1/6*(A*b-B*a)*x^6/b^2+1/9*B*x^9/b+1/3*a^2*(A*b-B*
a)*ln(b*x^3+a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{x^8(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,Bb^3x^9-3\,(Bab^2-Ab^3)x^6+6\,(Ba^2b-Aab^2)x^3-6\,(Ba^3-Aa^2b)\log\left(bx^3+a\right)}{18\,b^4}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/18*(2*B*b^3*x^9 - 3*(B*a*b^2 - A*b^3)*x^6 + 6*(B*a^2*b - A*a*b^2)*x^3 -
6*(B*a^3 - A*a^2*b)*log(b*x^3 + a))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int \frac{x^8(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^9}{9b} - \frac{a^2(-Ab+Ba)\log(a+bx^3)}{3b^4} + x^6\left(\frac{A}{6b} - \frac{Ba}{6b^2}\right) + x^3\left(-\frac{Aa}{3b^2} + \frac{Ba^2}{3b^3}\right)$$

input
```
integrate(x**8*(B*x**3+A)/(b*x**3+a),x)
```

output
```
B*x**9/(9*b) - a**2*(-A*b + B*a)*log(a + b*x**3)/(3*b**4) + x**6*(A/(6*b)
- B*a/(6*b**2)) + x**3*(-A*a/(3*b**2) + B*a**2/(3*b**3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{x^8(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,Bb^2x^9 - 3\,(Bab-Ab^2)x^6 + 6\,(Ba^2-Aab)x^3}{18\,b^3} - \frac{(Ba^3-Aa^2b)\log(bx^3+a)}{3\,b^4}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/18*(2*B*b^2*x^9 - 3*(B*a*b - A*b^2)*x^6 + 6*(B*a^2 - A*a*b)*x^3)/b^3 - 1
/3*(B*a^3 - A*a^2*b)*log(b*x^3 + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int \frac{x^8(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,Bb^2x^9-3\,Babx^6+3\,Ab^2x^6+6\,Ba^2x^3-6\,Aabx^3}{18\,b^3} - \frac{(Ba^3-Aa^2b)\log\left(|bx^3+a|\right)}{3\,b^4}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/18*(2*B*b^2*x^9 - 3*B*a*b*x^6 + 3*A*b^2*x^6 + 6*B*a^2*x^3 - 6*A*a*b*x^3)
/b^3 - 1/3*(B*a^3 - A*a^2*b)*log(abs(b*x^3 + a))/b^4
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int \frac{x^8(A+Bx^3)}{a+bx^3}\,dx = x^6\left(\frac{A}{6\,b}-\frac{B\,a}{6\,b^2}\right)+\frac{B\,x^9}{9\,b} - \frac{\ln\left(b\,x^3+a\right)\,\left(B\,a^3-A\,a^2\,b\right)}{3\,b^4} - \frac{a\,x^3\,\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{3\,b}$$

input
```
int((x^8*(A + B*x^3))/(a + b*x^3),x)
```

output
```
x^6*(A/(6*b) - (B*a)/(6*b^2)) + (B*x^9)/(9*b) - (log(a + b*x^3)*(B*a^3 - A
*a^2*b))/(3*b^4) - (a*x^3*(A/b - (B*a)/b^2))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.07

$$\int \frac{x^8(A+Bx^3)}{a+bx^3}\,dx=\frac{x^9}{9}$$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a),x)
```

output
```
x**9/9
```

3.59 $\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx = \frac{(Ab-aB)x^3}{3b^2}+\frac{Bx^6}{6b}-\frac{a(Ab-aB)\log(a+bx^3)}{3b^3}$$

output
```
1/3*(A*b-B*a)*x^3/b^2+1/6*B*x^6/b-1/3*a*(A*b-B*a)*ln(b*x^3+a)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx = \frac{bx^3(2Ab-2aB+bBx^3)+2a(-Ab+aB)\log(a+bx^3)}{6b^3}$$

input
```
Integrate[(x^5*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(b*x^3*(2*A*b - 2*a*B + b*B*x^3) + 2*a*(-(A*b) + a*B)*Log[a + b*x^3])/(6*b
^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^3)}{a+bx^3} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^3 (Bx^3+A)}{bx^3+a} dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3} \int \left(\frac{Bx^3}{b} + \frac{Ab-aB}{b^2} + \frac{a(aB-Ab)}{b^2 (bx^3+a)}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{a(Ab-aB)\log(a+bx^3)}{b^3} + \frac{x^3(Ab-aB)}{b^2} + \frac{Bx^6}{2b}\right)$$

input
```
Int[(x^5*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(((A*b - a*B)*x^3)/b^2 + (B*x^6)/(2*b) - (a*(A*b - a*B)*Log[a + b*x^3])/b^
3)/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

method	result	size
norman	$\frac{(Ab-Ba)x^3}{3b^2}+\frac{B\,x^6}{6b}-\frac{a(Ab-Ba)\ln\left(b\,x^3+a\right)}{3b^3}$	49
default	$\frac{\frac{1}{2}bB\,x^6+Ab\,x^3-Ba\,x^3}{3b^2}-\frac{a(Ab-Ba)\ln\left(b\,x^3+a\right)}{3b^3}$	50
parallelrisch	$-\frac{-b^2B\,x^6-2A\,b^2x^3+2Bab\,x^3+2A\ln\left(b\,x^3+a\right)ab-2B\ln\left(b\,x^3+a\right)a^2}{6b^3}$	60
risch	$\frac{B\,x^6}{6b}+\frac{A\,x^3}{3b}-\frac{Ba\,x^3}{3b^2}+\frac{A^2}{6Bb}-\frac{Aa}{3b^2}+\frac{B\,a^2}{6b^3}-\frac{a\ln\left(b\,x^3+a\right)A}{3b^2}+\frac{a^2\ln\left(b\,x^3+a\right)B}{3b^3}$	89

input
```
int(x^5*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*(A*b-B*a)*x^3/b^2+1/6*B*x^6/b-1/3*a*(A*b-B*a)*ln(b*x^3+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx=\frac{Bb^2x^6-2\,(Bab-Ab^2)x^3+2\,(Ba^2-Aab)\log\left(bx^3+a\right)}{6\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(B*b^2*x^6 - 2*(B*a*b - A*b^2)*x^3 + 2*(B*a^2 - A*a*b)*log(b*x^3 + a))
/b^3
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^6}{6b} + \frac{a(-Ab+Ba)\log\left(a+bx^3\right)}{3b^3} + x^3\left(\frac{A}{3b} - \frac{Ba}{3b^2}\right)$$

input
```
integrate(x**5*(B*x**3+A)/(b*x**3+a),x)
```

output
```
B*x**6/(6*b) + a*(-A*b + B*a)*log(a + b*x**3)/(3*b**3) + x**3*(A/(3*b) - B
*a/(3*b**2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx = \frac{Bbx^6 - 2\,(Ba-Ab)x^3}{6\,b^2} + \frac{(Ba^2-Aab)\log\left(bx^3+a\right)}{3\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/6*(B*b*x^6 - 2*(B*a - A*b)*x^3)/b^2 + 1/3*(B*a^2 - A*a*b)*log(b*x^3 + a)
/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx = \frac{Bbx^6 - 2\,Bax^3 + 2\,Abx^3}{6\,b^2} + \frac{(Ba^2-Aab)\log\left(|bx^3+a|\right)}{3\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/6*(B*b*x^6 - 2*B*a*x^3 + 2*A*b*x^3)/b^2 + 1/3*(B*a^2 - A*a*b)*log(abs(b*
x^3 + a))/b^3
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx = x^3\left(\frac{A}{3\,b}-\frac{B\,a}{3\,b^2}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(B\,a^2-A\,a\,b\right)}{3\,b^3}+\frac{B\,x^6}{6\,b}$$

input
```
int((x^5*(A + B*x^3))/(a + b*x^3),x)
```

output
```
x^3*(A/(3*b) - (B*a)/(3*b^2)) + (log(a + b*x^3)*(B*a^2 - A*a*b))/(3*b^3) +
 (B*x^6)/(6*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.09

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx = \frac{x^6}{6}$$

input
```
int(x^5*(B*x^3+A)/(b*x^3+a),x)
```

output
```
x**6/6
```

3.60 $\int \frac{x^2(A+Bx^3)}{a+bx^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 35

$$\int \frac{x^2(A+Bx^3)}{a+bx^3} \, dx = \frac{Bx^3}{3b} + \frac{(Ab-aB)\log(a+bx^3)}{3b^2}$$

output
```
1/3*B*x^3/b+1/3*(A*b-B*a)*ln(b*x^3+a)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx^3)}{a+bx^3} \, dx = \frac{bBx^3+(Ab-aB)\log(a+bx^3)}{3b^2}$$

input
```
Integrate[(x^2*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(b*B*x^3 + (A*b - a*B)*Log[a + b*x^3])/(3*b^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {946, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^3)}{a+bx^3} \, dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{3} \int \frac{Bx^3+A}{bx^3+a} dx^3$$

$$\Big\downarrow \; 49$$

$$\frac{1}{3} \int \left(\frac{B}{b}+\frac{Ab-aB}{b\left(bx^3+a\right)}\right) dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3} \left(\frac{(Ab-aB) \log \left(a+bx^3\right)}{b^2}+\frac{Bx^3}{b}\right)$$

input
```
Int[(x^2*(A + B*x^3))/(a + b*x^3),x]
```

output
```
((B*x^3)/b + ((A*b - a*B)*Log[a + b*x^3])/b^2)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{B\,x^{3}}{3 b}+\frac{\left(A b -B a \right) \ln \left(b \,x^{3}+a \right)}{3 b^{2}}$	32
norman	$\frac{B\,x^{3}}{3 b}+\frac{\left(A b -B a \right) \ln \left(b \,x^{3}+a \right)}{3 b^{2}}$	32
parallelrisch	$\frac{b B \,x^{3}+A \ln \left(b \,x^{3}+a \right) b -B \ln \left(b \,x^{3}+a \right) a}{3 b^{2}}$	36
risch	$\frac{B\,x^{3}}{3 b}+\frac{\ln \left(b \,x^{3}+a \right) A}{3 b}-\frac{\ln \left(b \,x^{3}+a \right) B a}{3 b^{2}}$	40

input
```
int(x^2*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*B*x^3/b+1/3*(A*b-B*a)*ln(b*x^3+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx=\frac{Bbx^3-(Ba-Ab)\log\left(bx^3+a\right)}{3\,b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/3*(B*b*x^3 - (B*a - A*b)*log(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^3}{3b} - \frac{(-Ab+Ba)\log\left(a+bx^3\right)}{3b^2}$$

input
```
integrate(x**2*(B*x**3+A)/(b*x**3+a),x)
```

output
```
B*x**3/(3*b) - (-A*b + B*a)*log(a + b*x**3)/(3*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^3}{3\,b} - \frac{(Ba-Ab)\log\left(bx^3+a\right)}{3\,b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*B*x^3/b - 1/3*(B*a - A*b)*log(b*x^3 + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^3}{3\,b} - \frac{(Ba-Ab)\log\left(|bx^3+a|\right)}{3\,b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*B*x^3/b - 1/3*(B*a - A*b)*log(abs(b*x^3 + a))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx = \frac{B\,x^3}{3\,b} + \frac{\ln\left(b\,x^3+a\right)\,\left(A\,b-B\,a\right)}{3\,b^2}$$

input
```
int((x^2*(A + B*x^3))/(a + b*x^3),x)
```

output
```
(B*x^3)/(3*b) + (log(a + b*x^3)*(A*b - B*a))/(3*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.14

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx = \frac{x^3}{3}$$

input
```
int(x^2*(B*x^3+A)/(b*x^3+a),x)
```

output
```
x**3/3
```

3.61 $\int \frac{A+Bx^3}{x(a+bx^3)} dx$

Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \frac{A+Bx^3}{x(a+bx^3)} dx = \frac{A\log(x)}{a} - \frac{(Ab-aB)\log(a+bx^3)}{3ab}$$

output
```
A*ln(x)/a-1/3*(A*b-B*a)*ln(b*x^3+a)/a/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x(a+bx^3)} dx = \frac{A\log(x)}{a} + \frac{(-Ab+aB)\log(a+bx^3)}{3ab}$$

input
```
Integrate[(A + B*x^3)/(x*(a + b*x^3)),x]
```

output
```
(A*Log[x])/a + ((-(A*b) + a*B)*Log[a + b*x^3])/(3*a*b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^3\left(bx^3+a\right)}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(\frac{A}{ax^3}+\frac{aB-Ab}{a\left(bx^3+a\right)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{A\log\left(x^3\right)}{a}-\frac{(Ab-aB)\log\left(a+bx^3\right)}{ab}\right)$$

input
```
Int[(A + B*x^3)/(x*(a + b*x^3)),x]
```

output
```
((A*Log[x^3])/a - ((A*b - a*B)*Log[a + b*x^3])/(a*b))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{A\ln(x)}{a}-\frac{(Ab-Ba)\ln(b\,x^3+a)}{3ab}$	33
norman	$\frac{A\ln(x)}{a}-\frac{(Ab-Ba)\ln(b\,x^3+a)}{3ab}$	33
risch	$\frac{A\ln(x)}{a}-\frac{\ln(b\,x^3+a)A}{3a}+\frac{\ln(b\,x^3+a)B}{3b}$	37
parallelrisch	$\frac{3A\ln(x)b-A\ln(b\,x^3+a)b+B\ln(b\,x^3+a)a}{3ab}$	39

input
```
int((B*x^3+A)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
A*ln(x)/a-1/3*(A*b-B*a)*ln(b*x^3+a)/a/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^3}{x\,(a+bx^3)}\,dx=\frac{3\,Ab\log{(x)}+(Ba-Ab)\log{(bx^3+a)}}{3\,ab}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/3*(3*A*b*log(x) + (B*a - A*b)*log(b*x^3 + a))/(a*b)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)}\,dx=\frac{A\log\left(x\right)}{a}+\frac{\left(-Ab+Ba\right)\log\left(\frac{a}{b}+x^3\right)}{3ab}$$

input
```
integrate((B*x**3+A)/x/(b*x**3+a),x)
```

output
```
A*log(x)/a + (-A*b + B*a)*log(a/b + x**3)/(3*a*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)}\,dx=\frac{A\log\left(x^3\right)}{3\,a}+\frac{\left(Ba-Ab\right)\log\left(bx^3+a\right)}{3\,ab}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*A*log(x^3)/a + 1/3*(B*a - A*b)*log(b*x^3 + a)/(a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)}\,dx=\frac{A\log\left(|x|\right)}{a}+\frac{\left(Ba-Ab\right)\log\left(|bx^3+a|\right)}{3\,ab}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="giac")
```

output
```
A*log(abs(x))/a + 1/3*(B*a - A*b)*log(abs(b*x^3 + a))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x\,(a+bx^3)}\,dx=\frac{B\,\ln\left(b\,x^3+a\right)}{3\,b}-\frac{A\,\ln\left(b\,x^3+a\right)}{3\,a}+\frac{A\,\ln\left(x\right)}{a}$$

input
```
int((A + B*x^3)/(x*(a + b*x^3)),x)
```

output
```
(B*log(a + b*x^3))/(3*b) - (A*log(a + b*x^3))/(3*a) + (A*log(x))/a
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.06

$$\int \frac{A+Bx^3}{x\,(a+bx^3)}\,dx=\log(x)$$

input
```
int((B*x^3+A)/x/(b*x^3+a),x)
```

output
```
log(x)
```

3.62 $\int \frac{A+Bx^3}{x^4(a+bx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx=-\frac{A}{3ax^3}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}$$

output
```
-1/3*A/a/x^3-(A*b-B*a)*ln(x)/a^2+1/3*(A*b-B*a)*ln(b*x^3+a)/a^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx=-\frac{A}{3ax^3}+\frac{(-Ab+aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}$$

input
```
Integrate[(A + B*x^3)/(x^4*(a + b*x^3)),x]
```

output
```
-1/3*A/(a*x^3) + ((-(A*b) + a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^3]
)/(3*a^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^4 (a+bx^3)} dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3} \int \frac{Bx^3+A}{x^6 (bx^3+a)} dx^3$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{3} \int \left(\frac{A}{ax^6} - \frac{b(aB-Ab)}{a^2 (bx^3+a)} + \frac{aB-Ab}{a^2x^3}\right) dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{\log (x^3) (Ab-aB)}{a^2} + \frac{(Ab-aB)\log (a+bx^3)}{a^2} - \frac{A}{ax^3}\right)$$

input
```
Int[(A + B*x^3)/(x^4*(a + b*x^3)),x]
```

output
```
(-(A/(a*x^3)) - ((A*b - a*B)*Log[x^3])/a^2 + ((A*b - a*B)*Log[a + b*x^3])/
a^2)/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{A}{3a\,x^{3}}+\frac{(-Ab+Ba)\ln(x)}{a^{2}}+\frac{(Ab-Ba)\ln(b\,x^{3}+a)}{3a^{2}}$	46
norman	$-\frac{A}{3a\,x^{3}}-\frac{(Ab-Ba)\ln(x)}{a^{2}}+\frac{(Ab-Ba)\ln(b\,x^{3}+a)}{3a^{2}}$	47
parallelrisch	$-\frac{3A\ln(x)x^{3}b-A\ln(b\,x^{3}+a)x^{3}b-3B\ln(x)x^{3}a+B\ln(b\,x^{3}+a)x^{3}a+Aa}{3x^{3}a^{2}}$	60
risch	$-\frac{A}{3a\,x^{3}}-\frac{\ln(x)Ab}{a^{2}}+\frac{\ln(x)B}{a}+\frac{\ln(-b\,x^{3}-a)Ab}{3a^{2}}-\frac{\ln(-b\,x^{3}-a)B}{3a}$	62

input
```
int((B*x^3+A)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/a/x^3+1/a^2*(-A*b+B*a)*ln(x)+1/3*(A*b-B*a)*ln(b*x^3+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx=-\frac{(Ba-Ab)x^3\log\left(bx^3+a\right)-3\,(Ba-Ab)x^3\log\left(x\right)+Aa}{3\,a^2x^3}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/3*((B*a - A*b)*x^3*log(b*x^3 + a) - 3*(B*a - A*b)*x^3*log(x) + A*a)/(a^
2*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx^3}{x^4\,(a+bx^3)}\,dx = -\frac{A}{3ax^3}+\frac{(-Ab+Ba)\log{(x)}}{a^2}-\frac{(-Ab+Ba)\log{\left(\frac{a}{b}+x^3\right)}}{3a^2}$$

input
```
integrate((B*x**3+A)/x**4/(b*x**3+a),x)
```

output
```
-A/(3*a*x**3) + (-A*b + B*a)*log(x)/a**2 - (-A*b + B*a)*log(a/b + x**3)/(3
*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^3}{x^4\,(a+bx^3)}\,dx = -\frac{(Ba-Ab)\log{(bx^3+a)}}{3\,a^2}+\frac{(Ba-Ab)\log{(x^3)}}{3\,a^2}-\frac{A}{3\,ax^3}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*(B*a - A*b)*log(b*x^3 + a)/a^2 + 1/3*(B*a - A*b)*log(x^3)/a^2 - 1/3*A
/(a*x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx^3}{x^4\,(a+bx^3)}\,dx = \frac{(Ba-Ab)\log{(|x|)}}{a^2}-\frac{(Bab-Ab^2)\log{(|bx^3+a|)}}{3\,a^2b} - \frac{Bax^3-Abx^3+Aa}{3\,a^2x^3}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="giac")
```

output
```
(B*a - A*b)*log(abs(x))/a^2 - 1/3*(B*a*b - A*b^2)*log(abs(b*x^3 + a))/(a^2
*b) - 1/3*(B*a*x^3 - A*b*x^3 + A*a)/(a^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x^4(a+bx^3)}\,dx = \frac{\ln (b\,x^3+a)\,\left(A\,b-B\,a\right)}{3\,a^2} - \frac{A}{3\,a\,x^3} - \frac{\ln (x)\,\left(A\,b-B\,a\right)}{a^2}$$

input
```
int((A + B*x^3)/(x^4*(a + b*x^3)),x)
```

output
```
(log(a + b*x^3)*(A*b - B*a))/(3*a^2) - A/(3*a*x^3) - (log(x)*(A*b - B*a))/
a^2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.10

$$\int \frac{A+Bx^3}{x^4(a+bx^3)}\,dx = -\frac{1}{3x^3}$$

input
```
int((B*x^3+A)/x^4/(b*x^3+a),x)
```

output
```
( - 1)/(3*x**3)
```

3.63 $\int \frac{A+Bx^3}{x^7(a+bx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx=-\frac{A}{6ax^6}+\frac{Ab-aB}{3a^2x^3}+\frac{b(Ab-aB)\log(x)}{a^3}-\frac{b(Ab-aB)\log\left(a+bx^3\right)}{3a^3}$$

output
```
-1/6*A/a/x^6+1/3*(A*b-B*a)/a^2/x^3+b*(A*b-B*a)*ln(x)/a^3-1/3*b*(A*b-B*a)*l
n(b*x^3+a)/a^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx$$
$$=\frac{-a(aA-2Abx^3+2aBx^3)+6b(Ab-aB)x^6\log(x)+2b(-Ab+aB)x^6\log\left(a+bx^3\right)}{6a^3x^6}$$

input
```
Integrate[(A + B*x^3)/(x^7*(a + b*x^3)),x]
```

output
```
(-(a*(a*A - 2*A*b*x^3 + 2*a*B*x^3)) + 6*b*(A*b - a*B)*x^6*Log[x] + 2*b*(-(
A*b) + a*B)*x^6*Log[a + b*x^3])/(6*a^3*x^6)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^7 (a+bx^3)} dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^9 (bx^3+a)}dx^3$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{3}\int \left(\frac{(aB-Ab)b^2}{a^3 (bx^3+a)}-\frac{(aB-Ab)b}{a^3x^3}+\frac{aB-Ab}{a^2x^6}+\frac{A}{ax^9}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{b\log (x^3) (Ab-aB)}{a^3}-\frac{b(Ab-aB)\log (a+bx^3)}{a^3}+\frac{Ab-aB}{a^2x^3}-\frac{A}{2ax^6}\right)$$

input
```
Int[(A + B*x^3)/(x^7*(a + b*x^3)),x]
```

output
```
(-1/2*A/(a*x^6) + (A*b - a*B)/(a^2*x^3) + (b*(A*b - a*B)*Log[x^3])/a^3 - (
b*(A*b - a*B)*Log[a + b*x^3])/a^3)/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{A}{6a\,x^{6}}-\frac{-Ab+Ba}{3a^{2}x^{3}}+\frac{b(Ab-Ba)\ln(x)}{a^{3}}-\frac{b(Ab-Ba)\ln\left(b\,x^{3}+a\right)}{3a^{3}}$	64
norman	$\frac{-\frac{A}{6a}+\frac{(Ab-Ba)x^{3}}{3a^{2}}}{x^{6}}+\frac{b(Ab-Ba)\ln(x)}{a^{3}}-\frac{b(Ab-Ba)\ln\left(b\,x^{3}+a\right)}{3a^{3}}$	66
risch	$\frac{-\frac{A}{6a}+\frac{(Ab-Ba)x^{3}}{3a^{2}}}{x^{6}}+\frac{b^{2}\ln(x)A}{a^{3}}-\frac{b\ln(x)B}{a^{2}}-\frac{b^{2}\ln\left(b\,x^{3}+a\right)A}{3a^{3}}+\frac{b\ln\left(b\,x^{3}+a\right)B}{3a^{2}}$	80
parallelrisch	$\frac{6A\ln(x)x^{6}b^{2}-2A\ln\left(b\,x^{3}+a\right)x^{6}b^{2}-6B\ln(x)x^{6}ab+2B\ln\left(b\,x^{3}+a\right)x^{6}ab+2aAb\,x^{3}-2B\,a^{2}x^{3}-a^{2}A}{6a^{3}x^{6}}$	87

input
```
int((B*x^3+A)/x^7/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6*A/a/x^6-1/3*(-A*b+B*a)/a^2/x^3+b*(A*b-B*a)*ln(x)/a^3-1/3*b*(A*b-B*a)*
ln(b*x^3+a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x^7\,(a+bx^3)}\,dx = \frac{2\,(Bab-Ab^2)x^6\log\left(bx^3+a\right)-6\,(Bab-Ab^2)x^6\log\left(x\right)-2\,(Ba^2-Aab)x^3-Aa^2}{6\,a^3x^6}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(2*(B*a*b - A*b^2)*x^6*log(b*x^3 + a) - 6*(B*a*b - A*b^2)*x^6*log(x) -
 2*(B*a^2 - A*a*b)*x^3 - A*a^2)/(a^3*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx = \frac{-Aa+x^3\cdot(2Ab-2Ba)}{6a^2x^6} - \frac{b(-Ab+Ba)\log{(x)}}{a^3} + \frac{b(-Ab+Ba)\log{\left(\frac{a}{b}+x^3\right)}}{3a^3}$$

input
```
integrate((B*x**3+A)/x**7/(b*x**3+a),x)
```

output
```
(-A*a + x**3*(2*A*b - 2*B*a))/(6*a**2*x**6) - b*(-A*b + B*a)*log(x)/a**3 +
 b*(-A*b + B*a)*log(a/b + x**3)/(3*a**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx = \frac{(Bab-Ab^2)\log{(bx^3+a)}}{3\,a^3} - \frac{(Bab-Ab^2)\log{(x^3)}}{3\,a^3} - \frac{2\,(Ba-Ab)x^3+Aa}{6\,a^2x^6}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*(B*a*b - A*b^2)*log(b*x^3 + a)/a^3 - 1/3*(B*a*b - A*b^2)*log(x^3)/a^3
- 1/6*(2*(B*a - A*b)*x^3 + A*a)/(a^2*x^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.43

$$\int \frac{A+Bx^3}{x^7(a+bx^3)}\,dx = -\frac{(Bab-Ab^2)\log(|x|)}{a^3} + \frac{(Bab^2-Ab^3)\log(|bx^3+a|)}{3\,a^3b} + \frac{3\,Babx^6-3\,Ab^2x^6-2\,Ba^2x^3+2\,Aabx^3-Aa^2}{6\,a^3x^6}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="giac")
```

output
```
-(B*a*b - A*b^2)*log(abs(x))/a^3 + 1/3*(B*a*b^2 - A*b^3)*log(abs(b*x^3 + a
))/(a^3*b) + 1/6*(3*B*a*b*x^6 - 3*A*b^2*x^6 - 2*B*a^2*x^3 + 2*A*a*b*x^3 -
A*a^2)/(a^3*x^6)
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^3}{x^7(a+bx^3)}\,dx = \frac{\ln(x)\,(A\,b^2-B\,a\,b)}{a^3} - \frac{\ln(b\,x^3+a)\,(A\,b^2-B\,a\,b)}{3\,a^3} - \frac{\frac{A}{6\,a}-\frac{x^3\,(A\,b-B\,a)}{3\,a^2}}{x^6}$$

input
```
int((A + B*x^3)/(x^7*(a + b*x^3)),x)
```

output
```
(log(x)*(A*b^2 - B*a*b))/a^3 - (log(a + b*x^3)*(A*b^2 - B*a*b))/(3*a^3) -
(A/(6*a) - (x^3*(A*b - B*a))/(3*a^2))/x^6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.07

$$\int \frac{A+Bx^3}{x^7(a+bx^3)}\,dx = -\frac{1}{6x^6}$$

input
```
int((B*x^3+A)/x^7/(b*x^3+a),x)
```

output
```
( - 1)/(6*x**6)
```

3.64 $\int \frac{x^6(A+Bx^3)}{a+bx^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 183

$$\begin{aligned}\int \frac{x^6(A+Bx^3)}{a+bx^3} \, dx = & -\frac{a(Ab-aB)x}{b^3}+\frac{(Ab-aB)x^4}{4b^2}+\frac{Bx^7}{7b} \\ & -\frac{a^{4/3}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{10/3}} \\ & +\frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}} \\ & -\frac{a^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}}\end{aligned}$$

output
```
-a*(A*b-B*a)*x/b^3+1/4*(A*b-B*a)*x^4/b^2+1/7*B*x^7/b-1/3*a^(4/3)*(A*b-B*a)
*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/b^(10/3)+1/3*a^
(4/3)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(10/3)-1/6*a^(4/3)*(A*b-B*a)*ln(a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(10/3)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.93

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx$$

$$= \frac{84a\sqrt[3]{b}(-Ab+aB)x+21b^{4/3}(Ab-aB)x^4+12b^{7/3}Bx^7+28\sqrt{3}a^{4/3}(-Ab+aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2}{84b^{10/3}}$$

input
```
Integrate[(x^6*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(84*a*b^(1/3)*(-(A*b) + a*B)*x + 21*b^(4/3)*(A*b - a*B)*x^4 + 12*b^(7/3)*B
*x^7 + 28*Sqrt[3]*a^(4/3)*(-(A*b) + a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]] - 28*a^(4/3)*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*a^(4/
3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(84*b^(1
0/3))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {959, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(Ab-aB)\int \frac{x^6}{bx^3+a}dx}{b}+\frac{Bx^7}{7b}$$

$$\Big\downarrow \text{831}$$

$$\frac{(Ab-aB)\int\left(\frac{x^3}{b}+\frac{a^2}{b^2(bx^3+a)}-\frac{a}{b^2}\right)dx}{b}+\frac{Bx^7}{7b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(Ab-aB)\left(-\frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{7/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}}-\frac{ax}{b^2}+\frac{x^4}{4b}\right)}{b}+\frac{Bx^7}{7b}$$

input
```
Int[(x^6*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(B*x^7)/(7*b) + ((A*b - a*B)*(-((a*x)/b^2) + x^4/(4*b) - (a^(4/3)*ArcTan[(
a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) + (a^(4/3)*Lo
g[a^(1/3) + b^(1/3)*x])/(3*b^(7/3)) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(6*b^(7/3))))/b
```

Defintions of rubi rules used

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.82 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.46

method	result
risch	$\frac{B\,x^7}{7b}+\frac{A\,x^4}{4b}-\frac{Ba\,x^4}{4b^2}-\frac{aAx}{b^2}+\frac{B\,a^2x}{b^3}+\frac{a^2\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab-Ba)\ln\left(x-\_R\right)}{\_R^2}\right)}{3b^4}$
default	$-\frac{-\frac{1}{7}b^2B\,x^7-\frac{1}{4}A\,b^2x^4+\frac{1}{4}Bab\,x^4+aAbx-B\,a^2x}{b^3}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b^3}$

input
```
int(x^6*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/7*B*x^7/b+1/4/b*A*x^4-1/4/b^2*B*a*x^4-1/b^2*a*A*x+1/b^3*B*a^2*x+1/3/b^4*
a^2*sum((A*b-B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91

$$\int\frac{x^6(A+Bx^3)}{a+bx^3}\,dx$$
$$=\frac{12\,Bb^2x^7-21\,(Bab-Ab^2)x^4-28\,\sqrt{3}(Ba^2-Aab)\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right)+14\,(Ba^2-Aab)\left(\frac{a}{b}\right)}{84\,b^3}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/84*(12*B*b^2*x^7 - 21*(B*a*b - A*b^2)*x^4 - 28*sqrt(3)*(B*a^2 - A*a*b)*(
a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 14*(B*a
^2 - A*a*b)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 28*(B*a^2
 - A*a*b)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 84*(B*a^2 - A*a*b)*x)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.62

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^7}{7b} + x^4\left(\frac{A}{4b} - \frac{Ba}{4b^2}\right) + x\left(-\frac{Aa}{b^2} + \frac{Ba^2}{b^3}\right) + \operatorname{RootSum}\left(27t^3b^{10} - A^3a^4b^3 + 3A^2Ba^5b^2 - 3AB^2a^6b + B^3a^7, \left(t \mapsto t\log\left(-\frac{3tb^3}{-Aab+Ba^2} + x\right)\right)\right)$$

input
```
integrate(x**6*(B*x**3+A)/(b*x**3+a),x)
```

output
```
B*x**7/(7*b) + x**4*(A/(4*b) - B*a/(4*b**2)) + x*(-A*a/b**2 + B*a**2/b**3)
 + RootSum(27*_t**3*b**10 - A**3*a**4*b**3 + 3*A**2*B*a**5*b**2 - 3*A*B**2
*a**6*b + B**3*a**7, Lambda(_t, _t*log(-3*_t*b**3/(-A*a*b + B*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.99

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx = \frac{4\,Bb^2x^7 - 7\,(Bab - Ab^2)x^4 + 28\,(Ba^2 - Aab)x}{28\,b^3} - \frac{\sqrt{3}(Ba^3 - Aa^2b)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba^3 - Aa^2b)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba^3 - Aa^2b)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/28*(4*B*b^2*x^7 - 7*(B*a*b - A*b^2)*x^4 + 28*(B*a^2 - A*a*b)*x)/b^3 - 1/
3*sqrt(3)*(B*a^3 - A*a^2*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(
1/3))/(b^4*(a/b)^(2/3)) + 1/6*(B*a^3 - A*a^2*b)*log(x^2 - x*(a/b)^(1/3) +
(a/b)^(2/3))/(b^4*(a/b)^(2/3)) - 1/3*(B*a^3 - A*a^2*b)*log(x + (a/b)^(1/3)
)/(b^4*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.19

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx = -\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba^2-\left(-ab^2\right)^{\frac{1}{3}}Aab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba^2-\left(-ab^2\right)^{\frac{1}{3}}Aab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4} + \frac{\left(Ba^3b^4-Aa^2b^5\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^7} + \frac{4\,Bb^6x^7-7\,Bab^5x^4+7\,Ab^6x^4+28\,Ba^2b^4x-28\,Aab^5x}{28\,b^7}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a^2 - (-a*b^2)^(1/3)*A*a*b)*arctan(1/3*sqrt
(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)*B*a^2 - (
-a*b^2)^(1/3)*A*a*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/3*(B
*a^3*b^4 - A*a^2*b^5)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1/
28*(4*B*b^6*x^7 - 7*B*a*b^5*x^4 + 7*A*b^6*x^4 + 28*B*a^2*b^4*x - 28*A*a*b^
5*x)/b^7
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.90

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx = x^4\left(\frac{A}{4\,b}-\frac{B\,a}{4\,b^2}\right)+\frac{B\,x^7}{7\,b} + \frac{a^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,b^{10/3}} - \frac{a\,x\,\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{b} - \frac{a^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,b^{10/3}} + \frac{a^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,b^{10/3}}$$

input
```
int((x^6*(A + B*x^3))/(a + b*x^3),x)
```

output
```
x^4*(A/(4*b) - (B*a)/(4*b^2)) + (B*x^7)/(7*b) + (a^(4/3)*log(b^(1/3)*x + a
^(1/3))*(A*b - B*a))/(3*b^(10/3)) - (a*x*(A/b - (B*a)/b^2))/b - (a^(4/3)*l
og(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b
 - B*a))/(3*b^(10/3)) + (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^
(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*b^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.03

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx = \frac{x^7}{7}$$

input
```
int(x^6*(B*x^3+A)/(b*x^3+a),x)
```

output
```
x**7/7
```

3.65 $\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 167

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx = \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}} + \frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} - \frac{a^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}}$$

output

```
1/2*(A*b-B*a)*x^2/b^2+1/5*B*x^5/b+1/3*a^(2/3)*(A*b-B*a)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/b^(8/3)+1/3*a^(2/3)*(A*b-B*a)*ln(a
^(1/3)+b^(1/3)*x)/b^(8/3)-1/6*a^(2/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/b^(8/3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.92

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx$$

$$= \frac{15b^{2/3}(Ab-aB)x^2+6b^{5/3}Bx^5-10\sqrt{3}a^{2/3}(-Ab+aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-10a^{2/3}(-Ab+aB)\log\left(\sqrt[\ldots]{\ldots}\right.}{30b^{8/3}}$$

input
```
Integrate[(x^4*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(15*b^(2/3)*(A*b - a*B)*x^2 + 6*b^(5/3)*B*x^5 - 10*Sqrt[3]*a^(2/3)*(-(A*b)
 + a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 10*a^(2/3)*(-(A*b) +
 a*B)*Log[a^(1/3) + b^(1/3)*x] + 5*a^(2/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(30*b^(8/3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {959, 843, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(Ab-aB)\int \frac{x^4}{bx^3+a}dx}{b}+\frac{Bx^5}{5b}$$

$$\Big\downarrow \; 843$$

$$\frac{(Ab-aB)\left(\frac{x^2}{2b}-\frac{a\int \frac{x}{bx^3+a}dx}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$$\Big\downarrow \text{821}$$

$$\frac{(Ab-aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$$\Big\downarrow \text{16}$$

$$\frac{(Ab-aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$$\Big\downarrow \text{1142}$$

$$\frac{(Ab-aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$$\Big\downarrow \text{25}$$

$$\frac{(Ab-aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$$\Big\downarrow \text{27}$$

$$\frac{(Ab-aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(Ab-aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$\Big\downarrow$ 217

$$\frac{(Ab-aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$\Big\downarrow$ 1103

$$\frac{(Ab-aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

input
```
Int[(x^4*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(B*x^5)/(5*b) + ((A*b - a*B)*(x^2/(2*b) - (a*(-1/3*Log[a^(1/3) + b^(1/3)*x
]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3))
)/(3*a^(1/3)*b^(1/3))))/b))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.81 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{B\,x^{5}}{5b}+\frac{A\,x^{2}}{2b}-\frac{B a\,x^{2}}{2b^{2}}+\frac{a\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-A b+B a\right)\ln\left(x-\_R\right)}{\_R}\right)}{3b^{3}}$	65
default	$\frac{\frac{bB\,x^{5}}{5}+\frac{(Ab-Ba)x^{2}}{2}}{b^{2}}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a(Ab-Ba)}{b^{2}}$	131

input
```
int(x^4*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/5*B*x^5/b+1/2/b*A*x^2-1/2/b^2*B*a*x^2+1/3/b^3*a*sum((-A*b+B*a)/_R*ln(x-_
R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.97

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx$$
$$=\frac{6\,Bbx^{5}-15\,(Ba-Ab)x^{2}+10\,\sqrt{3}(Ba-Ab)\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)+5\,(Ba-Ab)\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log}{30\,b^{2}}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/30*(6*B*b*x^5 - 15*(B*a - A*b)*x^2 + 10*sqrt(3)*(B*a - A*b)*(a^2/b^2)^(1
/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*(B*a - A
*b)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) -
 10*(B*a - A*b)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.68

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^5}{5b} + x^2\left(\frac{A}{2b} - \frac{Ba}{2b^2}\right)$$
$$+\operatorname{RootSum}\left(27t^3b^8 - A^3a^2b^3 + 3A^2Ba^3b^2 - 3AB^2a^4b + B^3a^5, \left(t \mapsto t\log\left(\frac{9t^2b^5}{A^2ab^2 - 2ABa^2b + B^2a^3} +\right.\right.\right.$$

input
```
integrate(x**4*(B*x**3+A)/(b*x**3+a),x)
```

output
```
B*x**5/(5*b) + x**2*(A/(2*b) - B*a/(2*b**2)) + RootSum(27*_t**3*b**8 - A**
3*a**2*b**3 + 3*A**2*B*a**3*b**2 - 3*A*B**2*a**4*b + B**3*a**5, Lambda(_t,
 _t*log(9*_t**2*b**5/(A**2*a*b**2 - 2*A*B*a**2*b + B**2*a**3) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.94

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx = \frac{\sqrt{3}(Ba^2 - Aab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{2\,Bbx^5 - 5\,(Ba - Ab)x^2}{10\,b^2} + \frac{(Ba^2 - Aab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba^2 - Aab)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(B*a^2 - A*a*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(
1/3))/(b^3*(a/b)^(1/3)) + 1/10*(2*B*b*x^5 - 5*(B*a - A*b)*x^2)/b^2 + 1/6*(
B*a^2 - A*a*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/3)) -
1/3*(B*a^2 - A*a*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.24

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx = -\frac{\sqrt{3}\left((-ab^2)^{\frac{2}{3}}\,Ba-(-ab^2)^{\frac{2}{3}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4} + \frac{\left((-ab^2)^{\frac{2}{3}}\,Ba-(-ab^2)^{\frac{2}{3}}\,Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4} - \frac{\left(Ba^2b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Aab^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^5} + \frac{2\,Bb^4x^5-5\,Bab^3x^2+5\,Ab^4x^2}{10\,b^5}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 + 1/6*((-a*b^2)^(2/3)*B*a - (-a*b^2
)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 - 1/3*(B*a^2*b^3
*(-a/b)^(1/3) - A*a*b^4*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3
)))/(a*b^5) + 1/10*(2*B*b^4*x^5 - 5*B*a*b^3*x^2 + 5*A*b^4*x^2)/b^5
```

Mupad [B] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.86

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx = x^2\left(\frac{A}{2\,b}-\frac{B\,a}{2\,b^2}\right)+\frac{B\,x^5}{5\,b}+\frac{a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,b^{8/3}} + \frac{a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,b^{8/3}} - \frac{a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,b^{8/3}}$$

input
```
int((x^4*(A + B*x^3))/(a + b*x^3),x)
```

output
```
x^2*(A/(2*b) - (B*a)/(2*b^2)) + (B*x^5)/(5*b) + (a^(2/3)*log(b^(1/3)*x + a
^(1/3))*(A*b - B*a))/(3*b^(8/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(
1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*b^(8/3)) - (a^(2/
3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(A*b - B*a))/(3*b^(8/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.03

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx = \frac{x^5}{5}$$

input
```
int(x^4*(B*x^3+A)/(b*x^3+a),x)
```

output
```
x**5/5
```

3.66 $\int \frac{x^3(A+Bx^3)}{a+bx^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 162

$$\int \frac{x^3(A+Bx^3)}{a+bx^3} dx = \frac{(Ab-aB)x}{b^2} + \frac{Bx^4}{4b} + \frac{\sqrt[3]{a}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}} - \frac{\sqrt[3]{a}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}} + \frac{\sqrt[3]{a}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{7/3}}$$

output
```
(A*b-B*a)*x/b^2+1/4*B*x^4/b+1/3*a^(1/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/b^(7/3)-1/3*a^(1/3)*(A*b-B*a)*ln(a^(1/3)
+b^(1/3)*x)/b^(7/3)+1/6*a^(1/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/b^(7/3)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.94

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx$$

$$= \frac{12\sqrt[3]{b}(Ab-aB)x+3b^{4/3}Bx^4-4\sqrt{3}\sqrt[3]{a}(-Ab+aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{a}(-Ab+aB)\log\left(\sqrt[3]{a}+\right.}{12b^{7/3}}$$

input
```
Integrate[(x^3*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(12*b^(1/3)*(A*b - a*B)*x + 3*b^(4/3)*B*x^4 - 4*Sqrt[3]*a^(1/3)*(-(A*b) +
a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*(-(A*b) + a*B
)*Log[a^(1/3) + b^(1/3)*x] - 2*a^(1/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(12*b^(7/3))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {959, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(Ab-aB)\int \frac{x^3}{bx^3+a}dx}{b}+\frac{Bx^4}{4b}$$

$$\Big\downarrow \; 843$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^3+a}dx}{b}\right)}{b}+\frac{Bx^4}{4b}$$

$$\Big\downarrow \; 750$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{b}\right)}{b}+\frac{Bx^4}{4b}$$

$$\Big\downarrow \; 16$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{b}+\frac{Bx^4}{4b}$$

$$\Big\downarrow \; 1142$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{b}+\frac{Bx^4}{4b}$$

$$\Big\downarrow \; 25$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{b}+\frac{Bx^4}{4b}$$

$$\Big\downarrow \; 27$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{b}+\frac{Bx^4}{4b}$$

$$\Big\downarrow \; 1082$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{b}+\frac{Bx^4}{4b}$$

$\Big\downarrow$ 217

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{b}+\frac{Bx^4}{4b}$$

$\Big\downarrow$ 1103

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{b}+\frac{Bx^4}{4b}$$

input
```
Int[(x^3*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(B*x^4)/(4*b) + ((A*b - a*B)*(x/b - (a*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3
)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1
/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2
/3))))/b))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.82 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.37

method	result	size
risch	$\frac{B\,x^{4}}{4b}+\frac{Ax}{b}-\frac{Bax}{b^{2}}+\frac{a\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-Ab+Ba\right)\ln\left(x-\_R\right)}{\_R^{2}}\right)}{3b^{3}}$	60
default	$\frac{\frac{1}{4}bB\,x^{4}+Abx-Bax}{b^{2}}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a\left(Ab-Ba\right)}{b^{2}}$	127

input
```
int(x^3*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*B*x^4/b+1/b*A*x-1/b^2*B*a*x+1/3/b^3*a*sum((-A*b+B*a)/_R^2*ln(x-_R),_R=
RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx$$
$$=\frac{3\,Bbx^4-4\,\sqrt{3}(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right)+2\,(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(\cdots\right.\right.}{12\,b^2}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/12*(3*B*b*x^4 - 4*sqrt(3)*(B*a - A*b)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)
*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 2*(B*a - A*b)*(-a/b)^(1/3)*log(x^2 + x
*(-a/b)^(1/3) + (-a/b)^(2/3)) - 4*(B*a - A*b)*(-a/b)^(1/3)*log(x - (-a/b)^
(1/3)) - 12*(B*a - A*b)*x)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.54

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^4}{4b} + x\left(\frac{A}{b} - \frac{Ba}{b^2}\right) + \operatorname{RootSum}\left(27t^3b^7 + A^3ab^3 - 3A^2Ba^2b^2 + 3AB^2a^3b - B^3a^4, \left(t \mapsto t\log\left(\frac{3tb^2}{-Ab+Ba} + x\right)\right)\right)$$

input
```
integrate(x**3*(B*x**3+A)/(b*x**3+a),x)
```

output
```
B*x**4/(4*b) + x*(A/b - B*a/b**2) + RootSum(27*_t**3*b**7 + A**3*a*b**3 -
3*A**2*B*a**2*b**2 + 3*A*B**2*a**3*b - B**3*a**4, Lambda(_t, _t*log(3*_t*b
**2/(-A*b + B*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.95

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx = \frac{Bbx^4 - 4\,(Ba-Ab)x}{4\,b^2} + \frac{\sqrt{3}(Ba^2-Aab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba^2-Aab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba^2-Aab)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/4*(B*b*x^4 - 4*(B*a - A*b)*x)/b^2 + 1/3*sqrt(3)*(B*a^2 - A*a*b)*arctan(1
/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) - 1/6*(B*a^2
 - A*a*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/3*(
B*a^2 - A*a*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.15

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx = \frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}\,Ba-(-ab^2)^{\frac{1}{3}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3} + \frac{\left((-ab^2)^{\frac{1}{3}}\,Ba-(-ab^2)^{\frac{1}{3}}\,Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3} - \frac{(Ba^2b^2-Aab^3)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^4} + \frac{Bb^3x^4-4\,Bab^2x+4\,Ab^3x}{4\,b^4}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)
^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 - 1/3*(B*a^2*b^2
- A*a*b^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) + 1/4*(B*b^3*x^
4 - 4*B*a*b^2*x + 4*A*b^3*x)/b^4
```

Mupad [B] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx$$
$$= x\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)+\frac{B\,x^4}{4\,b}+\frac{(-a)^{1/3}\ln\left((-a)^{4/3}+a\,b^{1/3}\,x\right)(A\,b-B\,a)}{3\,b^{7/3}}$$
$$-\frac{(-a)^{1/3}\ln\left(2\,a\,b^{1/3}\,x-(-a)^{4/3}-\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(A\,b-B\,a)}{3\,b^{7/3}}$$
$$+\frac{(-a)^{1/3}\ln\left(2\,a\,b^{1/3}\,x-(-a)^{4/3}+\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(A\,b-B\,a)}{3\,b^{7/3}}$$

input
```
int((x^3*(A + B*x^3))/(a + b*x^3),x)
```

output
```
x*(A/b - (B*a)/b^2) + (B*x^4)/(4*b) + ((-a)^(1/3)*log((-a)^(4/3) + a*b^(1/
3)*x)*(A*b - B*a))/(3*b^(7/3)) - ((-a)^(1/3)*log(2*a*b^(1/3)*x - 3^(1/2)*(
-a)^(4/3)*1i - (-a)^(4/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*b^(7/3))
 + ((-a)^(1/3)*log(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3
^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*b^(7/3))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.03

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx = \frac{x^4}{4}$$

input
```
int(x^3*(B*x^3+A)/(b*x^3+a),x)
```

output
```
x**4/4
```

3.67 $\int \frac{x(A+Bx^3)}{a+bx^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 150

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^2}{2b} - \frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{5/3}}$$

output
```
1/2*B*x^2/b-1/3*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3)
)*3^(1/2)/a^(1/3)/b^(5/3)-1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(5
/3)+1/6*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(5/3
)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.01

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^2}{2b} - \frac{(-Ab+aB)\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}} + \frac{(-Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}} - \frac{(-Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{5/3}}$$

input
```
Integrate[(x*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(B*x^2)/(2*b) - ((-(A*b) + a*B)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a
^(1/3))])/(Sqrt[3]*a^(1/3)*b^(5/3)) + ((-(A*b) + a*B)*Log[a^(1/3) + b^(1/3
)*x]])/(3*a^(1/3)*b^(5/3)) - ((-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(6*a^(1/3)*b^(5/3))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {959, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^3)}{a+bx^3}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(Ab-aB)\int \frac{x}{bx^3+a}dx}{b} + \frac{Bx^2}{2b}$$

$$\Big\downarrow \text{821}$$

$$\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{b}+\frac{Bx^2}{2b}$$

$$\Big\downarrow \; 16$$

$$\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}+\frac{Bx^2}{2b}$$

$$\Big\downarrow \; 1142$$

$$\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}+\frac{Bx^2}{2b}$$

$$\Big\downarrow \; 25$$

$$\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}+\frac{Bx^2}{2b}$$

$$\Big\downarrow \; 27$$

$$\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}+\frac{Bx^2}{2b}$$

$$\Big\downarrow \; 1082$$

$$\frac{(Ab-aB)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}+\frac{Bx^2}{2b}$$

$\downarrow$ 217

$$\frac{(Ab-aB)\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}+\frac{Bx^2}{2b}$$

$\downarrow$ 1103

$$\frac{(Ab-aB)\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}+\frac{Bx^2}{2b}$$

input
```
Int[(x*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(B*x^2)/(2*b) + ((A*b - a*B)*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/
3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) +
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(
1/3))))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.30

method	result	size
risch	$\frac{B x^2}{2b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab-Ba)\ln\left(x-\_R\right)}{\_R}}{3b^2}$	45
default	$\frac{B x^2}{2b}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)(Ab-Ba)}{b}$	113

input
```
int(x*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x^2/b+1/3/b^2*sum((A*b-B*a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.55

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \left[\frac{3\,Bab^2x^2 - 3\,\sqrt{\frac{1}{3}}(Ba^2b - Aab^2)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3 - ab + 3\,\sqrt{\frac{1}{3}}\left(abx + 2\,(-ab^2)^{\frac{2}{3}}x^2 + (-ab^2)^{\frac{1}{3}}a\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}} - 3\,(-ab^2)}{bx^3+a}\right.}{} \right.$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(3*B*a*b^2*x^2 - 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt((-a*b^2)^(1/3)/
a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*
b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) -
(-a*b^2)^(2/3)*(B*a - A*b)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/
3)) + 2*(-a*b^2)^(2/3)*(B*a - A*b)*log(b*x - (-a*b^2)^(1/3)))/(a*b^3), 1/6
*(3*B*a*b^2*x^2 - 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt(-(-a*b^2)^(1/3)/a)*
arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - (-a
*b^2)^(2/3)*(B*a - A*b)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3))
 + 2*(-a*b^2)^(2/3)*(B*a - A*b)*log(b*x - (-a*b^2)^(1/3)))/(a*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.61

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^2}{2b} + \operatorname{RootSum}\left(27t^3ab^5 + A^3b^3 - 3A^2Bab^2 + 3AB^2a^2b - B^3a^3, \left(t \mapsto t\log\left(\frac{9t^2ab^3}{A^2b^2 - 2ABab + B^2a^2} + x\right)\right.\right.$$

input
```
integrate(x*(B*x**3+A)/(b*x**3+a),x)
```

output
```
B*x**2/(2*b) + RootSum(27*_t**3*a*b**5 + A**3*b**3 - 3*A**2*B*a*b**2 + 3*A
*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(9*_t**2*a*b**3/(A**2*b**2 - 2*
A*B*a*b + B**2*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^2}{2\,b} - \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/2*B*x^2/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3
))/(a/b)^(1/3))/(b^2*(a/b)^(1/3)) - 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(b^2*(a/b)^(1/3)) + 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/
(b^2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^2}{2\,b} - \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{1}{3}}\,b} + \frac{(Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{1}{3}}\,b} + \frac{\left(Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/2*B*x^2/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b) + 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^
(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b) + 1/3*(B*a*b*(-a/b)^(1/3) - A*b^2
*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{B\,x^2}{2\,b} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(A\,b-B\,a)}{3\,a^{1/3}\,b^{5/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-B\,a)}{3\,a^{1/3}\,b^{5/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-B\,a)}{3\,a^{1/3}\,b^{5/3}}$$

input
```
int((x*(A + B*x^3))/(a + b*x^3),x)
```

output
```
(B*x^2)/(2*b) - (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(1/3)*b^(5/3))
 - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)
*(A*b - B*a))/(3*a^(1/3)*b^(5/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x
- a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(1/3)*b^(5/3))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.03

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{x^2}{2}$$

input
```
int(x*(B*x^3+A)/(b*x^3+a),x)
```

output
```
x**2/2
```

3.68 $\int \frac{A+Bx^3}{a+bx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 145

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = \frac{Bx}{b} - \frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}$$

output
```
B*x/b-1/3*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1
/2)/a^(2/3)/b^(4/3)+1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(4/3)-1/
6*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = \frac{6a^{2/3}\sqrt[3]{b}Bx - 2\sqrt{3}(Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) - (Ab-aB)\log\left(a^{2/3}\right.}{6a^{2/3}b^{4/3}}$$

input
```
Integrate[(A + B*x^3)/(a + b*x^3),x]
```

output
```
(6*a^(2/3)*b^(1/3)*B*x - 2*Sqrt[3]*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x] - (A*b - a*B)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {913, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{a+bx^3}dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(Ab-aB)\int \frac{1}{bx^3+a}dx}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{750}$$

$$\frac{(Ab-aB)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{(Ab-aB)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{1142}$$

$$\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \; 25$$

$$\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \; 1082$$

$$\frac{(Ab-aB)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \; 217$$

$$\frac{(Ab-aB)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \; 1103$$

$$\frac{(Ab-aB)\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{Bx}{b}$$

input
```
Int[(A + B*x^3)/(a + b*x^3),x]
```

output
```
(B*x)/b + ((A*b - a*B)*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-(
(Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.93 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{Bx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(Ab-Ba)\ln\left(x-\_R\right)}{\_R^{2}}}{3b^{2}}$	42
default	$\frac{Bx}{b}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)(Ab-Ba)}{b}$	110

input

```
int((B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output

```
B*x/b+1/3/b^2*sum((A*b-B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.54

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = \left[\frac{6\,Ba^2bx-3\sqrt{\tfrac{1}{3}}(Ba^2b-Aab^2)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\sqrt{\tfrac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right.}{6\,a^2b^2}\right.$$

input
```
integrate((B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(6*B*a^2*b*x - 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt(-(a^2*b)^(1/3)/b)
*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^
2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + (a^
2*b)^(2/3)*(B*a - A*b)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) -
2*(a^2*b)^(2/3)*(B*a - A*b)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^2), 1/6*(6*
B*a^2*b*x - 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt((a^2*b)^(1/3)/b)*arctan(s
qrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2)
+ (a^2*b)^(2/3)*(B*a - A*b)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*
a) - 2*(a^2*b)^(2/3)*(B*a - A*b)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.49

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = \frac{Bx}{b} + \operatorname{RootSum}\left(27t^3a^2b^4 - A^3b^3 + 3A^2Bab^2 - 3AB^2a^2b + B^3a^3, \left(t \mapsto t\log\left(-\frac{3tab}{-Ab+Ba} + x\right)\right)\right)$$

input
```
integrate((B*x**3+A)/(b*x**3+a),x)
```

output
```
B*x/b + RootSum(27*_t**3*a**2*b**4 - A**3*b**3 + 3*A**2*B*a*b**2 - 3*A*B**
2*a**2*b + B**3*a**3, Lambda(_t, _t*log(-3*_t*a*b/(-A*b + B*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = \frac{Bx}{b} - \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba-Ab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
B*x/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/
b)^(1/3))/(b^2*(a/b)^(2/3)) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a
/b)^(2/3))/(b^2*(a/b)^(2/3)) - 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(b^2*(
a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^2\right)^{\frac{2}{3}}} + \frac{(Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^2\right)^{\frac{2}{3}}} + \frac{Bx}{b} + \frac{(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

input
```
integrate((B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/
3))/(-a*b^2)^(2/3) + 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/
3))/(-a*b^2)^(2/3) + B*x/b + 1/3*(B*a - A*b)*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = \frac{B\,x}{b} + \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{2/3}\,b^{4/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{2/3}\,b^{4/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{2/3}\,b^{4/3}}$$

input
```
int((A + B*x^3)/(a + b*x^3),x)
```

output
```
(B*x)/b + (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(2/3)*b^(4/3)) - (lo
g(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b
- B*a))/(3*a^(2/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1
/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*a^(2/3)*b^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = x$$

input
```
int((B*x^3+A)/(b*x^3+a),x)
```

output
```
x
```

3.69 $\int \frac{A+Bx^3}{x^2(a+bx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 147

$$\int \frac{A+Bx^3}{x^2(a+bx^3)}\,dx = -\frac{A}{ax}+\frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{2/3}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}} - \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{2/3}}$$

output
```
-A/a/x+1/3*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(
1/2)/a^(4/3)/b^(2/3)+1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(2/3)-1
/6*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx$$
$$=\frac{-6\sqrt[3]{a}Ab^{2/3}+2\sqrt{3}(Ab-aB)x\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2(Ab-aB)x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-(Ab-aB)x\log}{6a^{4/3}b^{2/3}x}$$

input
```
Integrate[(A + B*x^3)/(x^2*(a + b*x^3)),x]
```

output
```
(-6*a^(1/3)*A*b^(2/3) + 2*Sqrt[3]*(A*b - a*B)*x*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]] + 2*(A*b - a*B)*x*Log[a^(1/3) + b^(1/3)*x] - (A*b - a*B)
*x*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(2/3)*x)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {955, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx$$
$$\Big\downarrow \text{ 955}$$
$$-\frac{(Ab-aB)\int \frac{x}{bx^3+a}dx}{a}-\frac{A}{ax}$$
$$\Big\downarrow \text{ 821}$$

$$-\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{A}{ax}$$

$\downarrow$ 16

$$-\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{A}{ax}$$

$\downarrow$ 1142

$$-\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{A}{ax}$$

$\downarrow$ 25

$$-\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{A}{ax}$$

$\downarrow$ 27

$$-\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{A}{ax}$$

$\downarrow$ 1082

$$-\frac{(Ab-aB)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{A}{ax}$$

$$\Big\downarrow \textbf{217}$$

$$-\frac{(Ab-aB)\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{A}{ax}$$

$$\Big\downarrow \textbf{1103}$$

$$-\frac{(Ab-aB)\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{A}{ax}$$

input
```
Int[(A + B*x^3)/(x^2*(a + b*x^3)),x]
```

output
```
-(A/(a*x)) - ((A*b - a*B)*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3))
 + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3
))))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.78

method	result
default	$-\frac{A}{ax}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)(Ab-Ba)}{a}$
risch	$-\frac{A}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}b^{2}a^{4}-A^{3}b^{3}+3A^{2}Ba\,b^{2}-3A\,B^{2}a^{2}b+B^{3}a^{3}\right)}\_R\ln\left(\left(-4\_R^{3}a^{4}b^{2}+3A^{3}b^{3}-9A^{2}Ba\,b^{2}+9A\,B^{2}a^{2}b-3B^{3}\right.\right.\right.}{3}$

input
```
int((B*x^3+A)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-A/a/x-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b
)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x-1)))/a*(A*b-B*a)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.53

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx$$

$$= \left[-\frac{6\,Aab^2+3\,\sqrt{\frac{1}{3}}(Ba^2b-Aab^2)x\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab-3\,\sqrt{\frac{1}{3}}\left(abx+2\,(ab^2)^{\frac{2}{3}}x^2-(ab^2)^{\frac{1}{3}}a\right)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}-3\,(ab^2)^{\frac{2}{3}}x}{bx^3+a}\right.}{6\,a^2b}\right.$$

$$-\frac{6\,Aab^2+6\,\sqrt{\frac{1}{3}}(Ba^2b-Aab^2)x\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2\,bx-(ab^2)^{\frac{1}{3}}\right)\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}}{b}\right)-(ab^2)^{\frac{2}{3}}\,(Ba-Ab)x\log}{6\,a^2b^2x}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/6*(6*A*a*b^2 + 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x*sqrt(-(a*b^2)^(1/3)/a
)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2
)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) - (a*b
^2)^(2/3)*(B*a - A*b)*x*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) +
 2*(a*b^2)^(2/3)*(B*a - A*b)*x*log(b*x + (a*b^2)^(1/3)))/(a^2*b^2*x), -1/6
*(6*A*a*b^2 + 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x*sqrt((a*b^2)^(1/3)/a)*arct
an(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - (a*b^2)^(
2/3)*(B*a - A*b)*x*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*(a
*b^2)^(2/3)*(B*a - A*b)*x*log(b*x + (a*b^2)^(1/3)))/(a^2*b^2*x)]
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx^3}{x^2(a+bx^3)}\,dx = -\frac{A}{ax} + \operatorname{RootSum}\left(27t^3a^4b^2 - A^3b^3 + 3A^2Bab^2 - 3AB^2a^2b + B^3a^3, \left(t \mapsto t\log\left(\frac{9t^2a^3b}{A^2b^2 - 2ABab + B^2a^2} + x\right.\right.\right.$$

input
```
integrate((B*x**3+A)/x**2/(b*x**3+a),x)
```

output
```
-A/(a*x) + RootSum(27*_t**3*a**4*b**2 - A**3*b**3 + 3*A**2*B*a*b**2 - 3*A*
B**2*a**2*b + B**3*a**3, Lambda(_t, _t*log(9*_t**2*a**3*b/(A**2*b**2 - 2*A
*B*a*b + B**2*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^3}{x^2(a+bx^3)}\,dx = \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{A}{ax}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/(a*b*(a/b)^(1/3)) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3
))/(a*b*(a/b)^(1/3)) - 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(1/
3)) - A/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3}{x^2\,(a+bx^3)}\,dx = \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^2\right)^{\frac{1}{3}}\,a} - \frac{(Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^2\right)^{\frac{1}{3}}\,a} - \frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2} - \frac{A}{ax}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/
3))/((-a*b^2)^(1/3)*a) - 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)
^(2/3))/((-a*b^2)^(1/3)*a) - 1/3*(B*a*(-a/b)^(1/3) - A*b*(-a/b)^(1/3))*(-a
/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - A/(a*x)
```

Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^3}{x^2\,(a+bx^3)}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{4/3}\,b^{2/3}} - \frac{A}{a\,x} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{4/3}\,b^{2/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{4/3}\,b^{2/3}}$$

input
```
int((A + B*x^3)/(x^2*(a + b*x^3)),x)
```

output
```
(log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(4/3)*b^(2/3)) - A/(a*x) + (lo
g(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b
- B*a))/(3*a^(4/3)*b^(2/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1
/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(4/3)*b^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.03

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx=-\frac{1}{x}$$

input
```
int((B*x^3+A)/x^2/(b*x^3+a),x)
```

output
```
( - 1)/x
```

3.70 $\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 149

$$\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx = -\frac{A}{2ax^2}+\frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}}$$

output
```
-1/2*A/a/x^2+1/3*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3
))*3^(1/2)/a^(5/3)/b^(1/3)-1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(
1/3)+1/6*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(1/
3)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx$$
$$= \frac{-\frac{3a^{2/3}A}{x^2} + \frac{2\sqrt{3}(Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{2(-Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}}{6a^{5/3}}$$

input
```
Integrate[(A + B*x^3)/(x^3*(a + b*x^3)),x]
```

output
```
((-3*a^(2/3)*A)/x^2 + (2*Sqrt[3]*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]])/b^(1/3) + (2*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(1
/3) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3)
)/(6*a^(5/3))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {955, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx$$
$$\Big\downarrow \text{ 955}$$
$$-\frac{(Ab-aB)\int \frac{1}{bx^3+a}dx}{a} - \frac{A}{2ax^2}$$
$$\Big\downarrow \text{ 750}$$

$$-\frac{(Ab-aB)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{A}{2ax^2}$$

$$\Big\downarrow \ 16$$

$$-\frac{(Ab-aB)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{A}{2ax^2}$$

$$\Big\downarrow \ 1142$$

$$-\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{A}{2ax^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{A}{2ax^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{A}{2ax^2}$$

$$\Big\downarrow \ 1082$$

$$-\frac{(Ab-aB)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{A}{2ax^2}$$

$$\Big\downarrow \; 217$$

$$-\frac{(Ab-aB)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{A}{2ax^2}$$

$$\Big\downarrow \; 1103$$

$$-\frac{(Ab-aB)\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{A}{2ax^2}$$

input
```
Int[(A + B*x^3)/(x^3*(a + b*x^3)),x]
```

output
```
-1/2*A/(a*x^2) - ((A*b - a*B)*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)
) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.76

method	result
default	$-\frac{A}{2 a x^{2}}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}\right) (-A b+B a)}{a}$
risch	$-\frac{A}{2 a x^{2}}+\frac{\Big(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{3} b a^{5}+A^{3} b^{3}-3 A^{2} B a b^{2}+3 A B^{2} a^{2} b-B^{3} a^{3}\right)} \_R \ln\big(\big(-4 \_R^{3} a^{5} b-3 A^{3} b^{3}+9 A^{2} B a b^{2}-9 A B^{2} a^{2} b+3 B^{3} \cdots}{3}$

input
```
int((B*x^3+A)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/a/x^2+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2
-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1)))*(-A*b+B*a)/a
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.76

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx$$
$$= \left[-\frac{3\sqrt{\frac{1}{3}}(Ba^2b-Aab^2)x^2\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3+3\,(-a^2b)^{\frac{1}{3}}\,ax-a^2-3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+(-a^2b)^{\frac{2}{3}}\,x+(-a^2b)^{\frac{1}{3}}\,a\right)\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)+}{}\right.$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/6*(3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x^2*sqrt((-a^2*b)^(1/3)/b)*log((2*a
*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2
/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) + (-a^2*b)^
(2/3)*(B*a - A*b)*x^2*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) -
 2*(-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x + (-a^2*b)^(2/3)) + 3*A*a^2*b)
/(a^3*b*x^2), 1/6*(6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x^2*sqrt(-(-a^2*b)^(1/3
)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*
b)^(1/3)/b)/a^2) - (-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x^2 - (-a^2*b)^(
2/3)*x - (-a^2*b)^(1/3)*a) + 2*(-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x +
(-a^2*b)^(2/3)) - 3*A*a^2*b)/(a^3*b*x^2)]
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.49

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx = -\frac{A}{2ax^2}$$
$$+\operatorname{RootSum}\left(27t^3a^5b+A^3b^3-3A^2Bab^2+3AB^2a^2b-B^3a^3,\left(t\mapsto t\log\left(\frac{3ta^2}{-Ab+Ba}+x\right)\right)\right)$$

input
```
integrate((B*x**3+A)/x**3/(b*x**3+a),x)
```

output
```
-A/(2*a*x**2) + RootSum(27*_t**3*a**5*b + A**3*b**3 - 3*A**2*B*a*b**2 + 3*
A*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(3*_t*a**2/(-A*b + B*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx = \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{A}{2\,ax^2}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/(a*b*(a/b)^(2/3)) - 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3
))/(a*b*(a/b)^(2/3)) + 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/
3)) - 1/2*A/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx = -\frac{(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b} - \frac{A}{2\,ax^2}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*(B*a - A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*sqrt(3)
*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/(a^2*b) + 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*
A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) - 1/2*A/(a*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx = -\frac{A}{2\,a\,x^2} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{5/3}\,b^{1/3}} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{5/3}\,b^{1/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{5/3}\,b^{1/3}}$$

input
```
int((A + B*x^3)/(x^3*(a + b*x^3)),x)
```

output
```
(log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A
*b - B*a))/(3*a^(5/3)*b^(1/3)) - (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3
*a^(5/3)*b^(1/3)) - A/(2*a*x^2) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x -
a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*a^(5/3)*b^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.03

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx=-\frac{1}{2x^2}$$

input
```
int((B*x^3+A)/x^3/(b*x^3+a),x)
```

output
```
( - 1)/(2*x**2)
```

3.71 $\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 165

$$\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx = -\frac{A}{4ax^4}+\frac{Ab-aB}{a^2x}-\frac{\sqrt[3]{b}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}} - \frac{\sqrt[3]{b}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}} + \frac{\sqrt[3]{b}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}}$$

output
```
-1/4*A/a/x^4+(A*b-B*a)/a^2/x-1/3*b^(1/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b
^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(7/3)-1/3*b^(1/3)*(A*b-B*a)*ln(a^(1/3
)+b^(1/3)*x)/a^(7/3)+1/6*b^(1/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/a^(7/3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx$$

$$= \frac{-\frac{3a^{4/3}A}{x^4}+\frac{12\sqrt[3]{a}(Ab-aB)}{x}-4\sqrt{3}\sqrt[3]{b}(Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{b}(-Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+}{12a^{7/3}}$$

input
```
Integrate[(A + B*x^3)/(x^5*(a + b*x^3)),x]
```

output
```
((-3*a^(4/3)*A)/x^4 + (12*a^(1/3)*(A*b - a*B))/x - 4*Sqrt[3]*b^(1/3)*(A*b
- a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*(-(A*b) + a
*B)*Log[a^(1/3) + b^(1/3)*x] + 2*b^(1/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/(12*a^(7/3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {955, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(Ab-aB)\int \frac{1}{x^2(bx^3+a)}dx}{a}-\frac{A}{4ax^4}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{(Ab-aB)\left(-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{4ax^4}$$

$$\Big\downarrow \text{821}$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{4ax^4}$$

$$\Big\downarrow \text{16}$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{4ax^4}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{4ax^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{4ax^4}$$

$\Big\downarrow$ 27

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{4ax^4}$$

$\Big\downarrow$ 1082

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{4ax^4}$$

$\downarrow$ 217

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{4ax^4}$$

$\downarrow$ 1103

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{4ax^4}$$

input
```
Int[(A + B*x^3)/(x^5*(a + b*x^3)),x]
```

output
```
-1/4*A/(a*x^4) - ((A*b - a*B)*(-(1/(a*x)) - (b*(-1/3*Log[a^(1/3) + b^(1/3)
*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3
)))/(3*a^(1/3)*b^(1/3))))/a))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.79

method	result
default	$-\frac{A}{4a\,x^{4}}-\frac{-Ab+Ba}{a^{2}x}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)(Ab-Ba)b}{a^{2}}$
risch	$\frac{\frac{(Ab-Ba)x^{3}}{a^{2}}-\frac{A}{4a}}{x^{4}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{7}\_Z^{3}+A^{3}b^{4}-3A^{2}Ba\,b^{3}+3A\,B^{2}a^{2}b^{2}-B^{3}a^{3}b\right)}\_R\ln\left(\left(-4a^{7}\_R^{3}-3A^{3}b^{4}+9A^{2}Ba\,b^{3}-9A\,B^{2}a\right.\right.\right.}{3}$

input
```
int((B*x^3+A)/x^5/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/a/x^4-1/a^2*(-A*b+B*a)/x+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/
b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*(A*b-B*a)/a^2*b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^3}{x^5\,(a+bx^3)}\,dx =$$
$$-\frac{4\sqrt{3}(Ba-Ab)x^4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-2\,(Ba-Ab)x^4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right.}{12\,a^2x^4}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/12*(4*sqrt(3)*(B*a - A*b)*x^4*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^
(1/3) + 1/3*sqrt(3)) - 2*(B*a - A*b)*x^4*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/
a)^(2/3) - a*(-b/a)^(1/3)) + 4*(B*a - A*b)*x^4*(-b/a)^(1/3)*log(b*x + a*(-
b/a)^(2/3)) + 12*(B*a - A*b)*x^3 + 3*A*a)/(a^2*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^3}{x^5\,(a+bx^3)}\,dx = \operatorname{RootSum}\left(27t^3a^7+A^3b^4-3A^2Bab^3+3AB^2a^2b^2-B^3a^3b,\left(t\mapsto t\log\left(\frac{9t^2a^5}{A^2b^3-2ABab^2+B^2a^2b}+x\right)\right)\right) + \frac{-Aa+x^3\cdot(4Ab-4Ba)}{4a^2x^4}$$

input
```
integrate((B*x**3+A)/x**5/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**7 + A**3*b**4 - 3*A**2*B*a*b**3 + 3*A*B**2*a**2*b**2 -
 B**3*a**3*b, Lambda(_t, _t*log(9*_t**2*a**5/(A**2*b**3 - 2*A*B*a*b**2 + B
**2*a**2*b) + x))) + (-A*a + x**3*(4*A*b - 4*B*a))/(4*a**2*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{x^5\,(a+bx^3)}\,dx = -\frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{4\,(Ba-Ab)x^3+Aa}{4\,a^2x^4}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(a^2*(a/b)^(1/3)) - 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/
3))/(a^2*(a/b)^(1/3)) + 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a^2*(a/b)^(1
/3)) - 1/4*(4*(B*a - A*b)*x^3 + A*a)/(a^2*x^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)}\,dx = \frac{\left(Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}} - Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}Ba - \left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3b} - \frac{\left(\left(-ab^2\right)^{\frac{2}{3}}Ba - \left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^3b} - \frac{4\,Bax^3-4\,Abx^3+Aa}{4\,a^2x^4}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*(B*a*b*(-a/b)^(1/3) - A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a
/b)^(1/3)))/a^3 + 1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*ar
ctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) - 1/6*((-a*b^2
)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/(a^3*b) - 1/4*(4*B*a*x^3 - 4*A*b*x^3 + A*a)/(a^2*x^4)
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx$$
$$= \frac{(-b)^{1/3}\ln\left(a^{1/3}(-b)^{8/3}+b^3x\right)(A\,b-B\,a)}{3\,a^{7/3}} - \frac{\frac{A}{4\,a}-\frac{x^3(A\,b-B\,a)}{a^2}}{x^4}$$
$$+ \frac{(-b)^{1/3}\ln\left(a^{1/3}(-b)^{8/3}-2\,b^3\,x+\sqrt{3}\,a^{1/3}(-b)^{8/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(A\,b-B\,a)}{3\,a^{7/3}}$$
$$- \frac{(-b)^{1/3}\ln\left(2\,b^3\,x-a^{1/3}(-b)^{8/3}+\sqrt{3}\,a^{1/3}(-b)^{8/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(A\,b-B\,a)}{3\,a^{7/3}}$$

input
```
int((A + B*x^3)/(x^5*(a + b*x^3)),x)
```

output
```
((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) + b^3*x)*(A*b - B*a))/(3*a^(7/3)) - (A/
(4*a) - (x^3*(A*b - B*a))/a^2)/x^4 + ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) -
2*b^3*x + 3^(1/2)*a^(1/3)*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a
))/(3*a^(7/3)) - ((-b)^(1/3)*log(2*b^3*x - a^(1/3)*(-b)^(8/3) + 3^(1/2)*a^
(1/3)*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(7/3))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.03

$$\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx = -\frac{1}{4x^4}$$

input
```
int((B*x^3+A)/x^5/(b*x^3+a),x)
```

output
```
( - 1)/(4*x**4)
```

3.72 $\int \frac{A+Bx^3}{x^6(a+bx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 168

$$\int \frac{A+Bx^3}{x^6(a+bx^3)}\,dx = -\frac{A}{5ax^5}+\frac{Ab-aB}{2a^2x^2}-\frac{b^{2/3}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}} + \frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}} - \frac{b^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}}$$

output
```
-1/5*A/a/x^5+1/2*(A*b-B*a)/a^2/x^2-1/3*b^(2/3)*(A*b-B*a)*arctan(1/3*(a^(1/
3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3)+1/3*b^(2/3)*(A*b-B*a)*ln(
a^(1/3)+b^(1/3)*x)/a^(8/3)-1/6*b^(2/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/a^(8/3)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x^6(a+bx^3)}\,dx$$
$$= \frac{-\frac{6a^{5/3}A}{x^5}+\frac{15a^{2/3}(Ab-aB)}{x^2}-10\sqrt{3}b^{2/3}(Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+10b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{30a^{8/3}}$$

input
```
Integrate[(A + B*x^3)/(x^6*(a + b*x^3)),x]
```

output
```
((-6*a^(5/3)*A)/x^5 + (15*a^(2/3)*(A*b - a*B))/x^2 - 10*Sqrt[3]*b^(2/3)*(A
*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*b^(2/3)*(A*b -
a*B)*Log[a^(1/3) + b^(1/3)*x] + 5*b^(2/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(30*a^(8/3))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {955, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^6(a+bx^3)}\,dx$$
$$\Big\downarrow \text{955}$$
$$-\frac{(Ab-aB)\int \frac{1}{x^3(bx^3+a)}dx}{a}-\frac{A}{5ax^5}$$
$$\Big\downarrow \text{847}$$
$$-\frac{(Ab-aB)\left(-\frac{b\int \frac{1}{bx^3+a}dx}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \ 750$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \ 16$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \ 1142$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \ 25$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \; 27$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \; 1082$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{A}{5ax^5}$$

$\downarrow$ 217

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{A}{5ax^5}$$

$\downarrow$ 1103

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{A}{5ax^5}$$

input
```
Int[(A + B*x^3)/(x^6*(a + b*x^3)),x]
```

output
```
-1/5*A/(a*x^5) - ((A*b - a*B)*(-1/2*1/(a*x^2) - (b*(Log[a^(1/3) + b^(1/3)*
x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/
3)))/(3*a^(2/3))))/a))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.77

method	result
default	$-\frac{A}{5a\,x^5}-\frac{-Ab+Ba}{2x^2a^2}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)(Ab-Ba)b}{a^2}$
risch	$\frac{\frac{(Ab-Ba)x^3}{2a^2}-\frac{A}{5a}}{x^5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^8\_Z^3-A^3b^5+3A^2Ba\,b^4-3A\,B^2a^2b^3+B^3a^3b^2\right)}\_R\ln\left(\left(-4\_R^3a^8+3A^3b^5-9A^2Ba\,b^4+9A\,B^2\right.\right.\right.}{3}$

input
```
int((B*x^3+A)/x^6/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/5*A/a/x^5-1/2*(-A*b+B*a)/x^2/a^2+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1
/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*(A*b-B*a)/a^2*b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx =$$

$$-\frac{10\sqrt{3}(Ba-Ab)x^5\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)-5\,(Ba-Ab)x^5\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+\right.}{30\,a^2x^5}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/30*(10*sqrt(3)*(B*a - A*b)*x^5*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*
x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*(B*a - A*b)*x^5*(b^2/a^2)^(1/3)*log(
b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 10*(B*a - A*b)*x^
5*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 15*(B*a - A*b)*x^3 + 6*A*
a)/(a^2*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx^3}{x^6\,(a+bx^3)}\,dx$$
$$=\operatorname{RootSum}\left(27t^3a^8-A^3b^5+3A^2Bab^4-3AB^2a^2b^3+B^3a^3b^2,\left(t\mapsto t\log\left(-\frac{3ta^3}{-Ab^2+Bab}+x\right)\right)\right)$$
$$+\frac{-2Aa+x^3\cdot(5Ab-5Ba)}{10a^2x^5}$$

input
```
integrate((B*x**3+A)/x**6/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**8 - A**3*b**5 + 3*A**2*B*a*b**4 - 3*A*B**2*a**2*b**3 +
 B**3*a**3*b**2, Lambda(_t, _t*log(-3*_t*a**3/(-A*b**2 + B*a*b) + x))) + (
-2*A*a + x**3*(5*A*b - 5*B*a))/(10*a**2*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx = -\frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5(Ba-Ab)x^3+2Aa}{10a^2x^5}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(a^2*(a/b)^(2/3)) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/
3))/(a^2*(a/b)^(2/3)) - 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a^2*(a/b)^(2
/3)) - 1/10*(5*(B*a - A*b)*x^3 + 2*A*a)/(a^2*x^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx = -\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3} + \frac{\left(Bab-Ab^2\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^3} - \frac{5Bax^3-5Abx^3+2Aa}{10a^2x^5}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^3 + 1/3*(B*a*b - A*b^2)*(-a/b)^(1/3)*
log(abs(x - (-a/b)^(1/3)))/a^3 - 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*
A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^3 - 1/10*(5*B*a*x^3 - 5*A*
b*x^3 + 2*A*a)/(a^2*x^5)
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^3}{x^6\,(a+bx^3)}\,dx = \frac{b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{8/3}}-\frac{\frac{A}{5\,a}-\frac{x^3\,\left(A\,b-B\,a\right)}{2\,a^2}}{x^5} - \frac{b^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{8/3}} + \frac{b^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{8/3}}$$

input
```
int((A + B*x^3)/(x^6*(a + b*x^3)),x)
```

output
```
(b^(2/3)*log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(8/3)) - (A/(5*a) - (x
^3*(A*b - B*a))/(2*a^2))/x^5 - (b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)
*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(8/3)) + (b^(2/3)*l
og(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b
 - B*a))/(3*a^(8/3))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.03

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx=-\frac{1}{5x^5}$$

input
```
int((B*x^3+A)/x^6/(b*x^3+a),x)
```

output
```
( - 1)/(5*x**5)
```

3.73 $\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 184

$$\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx = -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x} + \frac{b^{4/3}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}} + \frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}} - \frac{b^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}}$$

output
```
-1/7*A/a/x^7+1/4*(A*b-B*a)/a^2/x^4-b*(A*b-B*a)/a^3/x+1/3*b^(4/3)*(A*b-B*a)
*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(10/3)+1/3*b^
(4/3)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)-1/6*b^(4/3)*(A*b-B*a)*ln(a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)}\,dx = \frac{-\frac{12a^{7/3}A}{x^7}+\frac{21a^{4/3}(Ab-aB)}{x^4}+\frac{84\sqrt[3]{a}b(-Ab+aB)}{x}+28\sqrt{3}b^{4/3}(Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+28b^{4/3}(Ab-aB)}{84a^{10/3}}$$

input
```
Integrate[(A + B*x^3)/(x^8*(a + b*x^3)),x]
```

output
```
((-12*a^(7/3)*A)/x^7 + (21*a^(4/3)*(A*b - a*B))/x^4 + (84*a^(1/3)*b*(-(A*b
) + a*B))/x + 28*Sqrt[3]*b^(4/3)*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]] + 28*b^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*b^(4
/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(84*a^(
10/3))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {955, 847, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)}\,dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(Ab-aB)\int \frac{1}{x^5\left(bx^3+a\right)}dx}{a}-\frac{A}{7ax^7}$$

$$\Big\downarrow \; 847$$

$$-\frac{(Ab-aB)\left(-\frac{b\int \frac{1}{x^2(bx^3+a)}dx}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

$\big\downarrow$ 847

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

$\big\downarrow$ 821

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

$\big\downarrow$ 16

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

$\Big\downarrow$ 1142

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

$$\Big\downarrow \ 25$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

$\downarrow$ 27

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

$\Big\downarrow$ 1082

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

$\downarrow$ 217

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

$\downarrow$ 1103

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

input
```
Int[(A + B*x^3)/(x^8*(a + b*x^3)),x]
```

output
```
-1/7*A/(a*x^7) - ((A*b - a*B)*(-1/4*1/(a*x^4) - (b*(-(1/(a*x)) - (b*(-1/3*
Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a))/a))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.82

method	result
default	$-\frac{A}{7a\,x^{7}}-\frac{-Ab+Ba}{4a^{2}x^{4}}-\frac{b(Ab-Ba)}{a^{3}x}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^{2}(Ab-Ba}{a^{3}}$
risch	$\frac{-\frac{(Ab-Ba)b\,x^{6}}{a^{3}}+\frac{(Ab-Ba)x^{3}}{4a^{2}}-\frac{A}{7a}}{x^{7}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}\_Z^{3}-A^{3}b^{7}+3A^{2}Ba\,b^{6}-3A\,B^{2}a^{2}b^{5}+B^{3}a^{3}b^{4}\right)}\_R\ln\left(\left(-4a^{10}\_R^{3}+3A^{3}b^{7}-\right.\right.\right.}{3}$

input
```
int((B*x^3+A)/x^8/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/7*A/a/x^7-1/4*(-A*b+B*a)/a^2/x^4-b*(A*b-B*a)/a^3/x-(-1/3/b/(a/b)^(1/3)*
ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*
3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b^2*(A*b-B*
a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.98

$$\int\frac{A+Bx^{3}}{x^{8}\left(a+bx^{3}\right)}\,dx$$
$$=\frac{28\sqrt{3}(Bab-Ab^{2})x^{7}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+14\,(Bab-Ab^{2})x^{7}\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\right.}{84\,a^{3}}$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/84*(28*sqrt(3)*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a
)^(1/3) - 1/3*sqrt(3)) + 14*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*log(b*x^2 - a*
x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 28*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*log(b*
x + a*(b/a)^(2/3)) + 84*(B*a*b - A*b^2)*x^6 - 21*(B*a^2 - A*a*b)*x^3 - 12*
A*a^2)/(a^3*x^7)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.76

$$\int \frac{A + Bx^3}{x^8 (a + bx^3)} \, dx$$
$$= \text{RootSum}\left(27t^3a^{10} - A^3b^7 + 3A^2Bab^6 - 3AB^2a^2b^5 + B^3a^3b^4, \left(t \mapsto t\log\left(\frac{9t^2a^7}{A^2b^5 - 2ABab^4 + B^2a^2b^3} + \right.\right.\right.$$
$$+ \frac{-4Aa^2 + x^6(-28Ab^2 + 28Bab) + x^3 \cdot (7Aab - 7Ba^2)}{28a^3x^7}$$

input
```
integrate((B*x**3+A)/x**8/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**10 - A**3*b**7 + 3*A**2*B*a*b**6 - 3*A*B**2*a**2*b**5
+ B**3*a**3*b**4, Lambda(_t, _t*log(9*_t**2*a**7/(A**2*b**5 - 2*A*B*a*b**4
 + B**2*a**2*b**3) + x))) + (-4*A*a**2 + x**6*(-28*A*b**2 + 28*B*a*b) + x*
*3*(7*A*a*b - 7*B*a**2))/(28*a**3*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx = \frac{\sqrt{3}(Bab-Ab^2)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Bab-Ab^2)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Bab-Ab^2)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{28(Bab-Ab^2)x^6-7(Ba^2-Aab)x^3-4Aa^2}{28a^3x^7}$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(B*a*b - A*b^2)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(
1/3))/(a^3*(a/b)^(1/3)) + 1/6*(B*a*b - A*b^2)*log(x^2 - x*(a/b)^(1/3) + (a
/b)^(2/3))/(a^3*(a/b)^(1/3)) - 1/3*(B*a*b - A*b^2)*log(x + (a/b)^(1/3))/(a
^3*(a/b)^(1/3)) + 1/28*(28*(B*a*b - A*b^2)*x^6 - 7*(B*a^2 - A*a*b)*x^3 - 4
*A*a^2)/(a^3*x^7)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx = -\frac{\sqrt{3}\left((-ab^2)^{\frac{2}{3}}Ba-(-ab^2)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^4} - \frac{\left(Bab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^4} + \frac{\left((-ab^2)^{\frac{2}{3}}Ba-(-ab^2)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^4} + \frac{28Babx^6-28Ab^2x^6-7Ba^2x^3+7Aabx^3-4Aa^2}{28a^3x^7}$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4 - 1/3*(B*a*b^2*(-a/b)^(1/3) - A*b^3
*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 + 1/6*((-a*b^2)
^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/
a^4 + 1/28*(28*B*a*b*x^6 - 28*A*b^2*x^6 - 7*B*a^2*x^3 + 7*A*a*b*x^3 - 4*A*
a^2)/(a^3*x^7)
```

Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx = \frac{b^{4/3}\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(A\,b-B\,a)}{3\,a^{10/3}} - \frac{\frac{A}{7\,a}-\frac{x^3\,(A\,b-B\,a)}{4\,a^2}+\frac{b\,x^6\,(A\,b-B\,a)}{a^3}}{x^7} + \frac{b^{4/3}\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-B\,a)}{3\,a^{10/3}} - \frac{b^{4/3}\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-B\,a)}{3\,a^{10/3}}$$

input
```
int((A + B*x^3)/(x^8*(a + b*x^3)),x)
```

output
```
(b^(4/3)*log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(10/3)) - (A/(7*a) - (
x^3*(A*b - B*a))/(4*a^2) + (b*x^6*(A*b - B*a))/a^3)/x^7 + (b^(4/3)*log(3^(
1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a
))/(3*a^(10/3)) - (b^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))
*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.03

$$\int \frac{A + Bx^3}{x^8 (a + bx^3)} \, dx = -\frac{1}{7x^7}$$

input
```
int((B*x^3+A)/x^8/(b*x^3+a),x)
```

output
```
( - 1)/(7*x**7)
```

3.74 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 82

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2} dx = \frac{(Ab-2aB)x^3}{3b^3} + \frac{Bx^6}{6b^2} - \frac{a^2(Ab-aB)}{3b^4(a+bx^3)} - \frac{a(2Ab-3aB)\log(a+bx^3)}{3b^4}$$

output
```
1/3*(A*b-2*B*a)*x^3/b^3+1/6*B*x^6/b^2-1/3*a^2*(A*b-B*a)/b^4/(b*x^3+a)-1/3*
a*(2*A*b-3*B*a)*ln(b*x^3+a)/b^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2} dx = \frac{2b(Ab-2aB)x^3 + b^2Bx^6 + \frac{2a^2(-Ab+aB)}{a+bx^3} + 2a(-2Ab+3aB)\log(a+bx^3)}{6b^4}$$

input
```
Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(2*b*(A*b - 2*a*B)*x^3 + b^2*B*x^6 + (2*a^2*(-(A*b) + a*B))/(a + b*x^3) +
2*a*(-2*A*b + 3*a*B)*Log[a + b*x^3])/(6*b^4)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6(Bx^3+A)}{(bx^3+a)^2}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(\frac{Bx^3}{b^2}+\frac{Ab-2aB}{b^3}+\frac{a(3aB-2Ab)}{b^3(bx^3+a)}-\frac{a^2(aB-Ab)}{b^3(bx^3+a)^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^2(Ab-aB)}{b^4(a+bx^3)}-\frac{a(2Ab-3aB)\log(a+bx^3)}{b^4}+\frac{x^3(Ab-2aB)}{b^3}+\frac{Bx^6}{2b^2}\right)$$

input
```
Int[(x^8*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(((A*b - 2*a*B)*x^3)/b^3 + (B*x^6)/(2*b^2) - (a^2*(A*b - a*B))/(b^4*(a + b
*x^3)) - (a*(2*A*b - 3*a*B)*Log[a + b*x^3])/b^4)/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

method	result
default	$\frac{\left(bB\,x^{3}+Ab-2Ba\right)^{2}}{6b^{4}B}-\frac{a\left(\frac{a(Ab-Ba)}{b\left(b\,x^{3}+a\right)}+\frac{(2Ab-3Ba)\ln\left(b\,x^{3}+a\right)}{b}\right)}{3b^{3}}$
norman	$\frac{\frac{B\,x^{9}}{6b}-\frac{a\left(2abA-3a^{2}B\right)}{3b^{4}}+\frac{(2Ab-3Ba)x^{6}}{6b^{2}}}{b\,x^{3}+a}-\frac{a(2Ab-3Ba)\ln\left(b\,x^{3}+a\right)}{3b^{4}}$
parallelrisch	$-\frac{-b^{3}B\,x^{9}-2A\,b^{3}x^{6}+3Ba\,b^{2}x^{6}+4A\ln\left(b\,x^{3}+a\right)x^{3}a\,b^{2}-6B\ln\left(b\,x^{3}+a\right)x^{3}a^{2}b+4A\ln\left(b\,x^{3}+a\right)a^{2}b-6B\ln\left(b\,x^{3}+a\right)a^{3}+4a^{2}bA}{6b^{4}\left(b\,x^{3}+a\right)}$
risch	$\frac{B\,x^{6}}{6b^{2}}+\frac{A\,x^{3}}{3b^{2}}-\frac{2Ba\,x^{3}}{3b^{3}}+\frac{A^{2}}{6b^{2}B}-\frac{2Aa}{3b^{3}}+\frac{2B\,a^{2}}{3b^{4}}-\frac{a^{2}A}{3b^{3}\left(b\,x^{3}+a\right)}+\frac{a^{3}B}{3b^{4}\left(b\,x^{3}+a\right)}-\frac{2a\ln\left(b\,x^{3}+a\right)A}{3b^{3}}+\frac{a^{2}\ln\left(b\,x^{3}\right.}{b^{4}}$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(B*b*x^3+A*b-2*B*a)^2/b^4/B-1/3/b^3*a*(a*(A*b-B*a)/b/(b*x^3+a)+(2*A*b-
3*B*a)/b*ln(b*x^3+a))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.48

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx$$
$$= \frac{Bb^3x^9-(3\,Bab^2-2\,Ab^3)x^6+2\,Ba^3-2\,Aa^2b-2\,(2\,Ba^2b-Aab^2)x^3+2\,(3\,Ba^3-2\,Aa^2b+(3\,Ba^2b-}{6\,(b^5x^3+ab^4)}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/6*(B*b^3*x^9 - (3*B*a*b^2 - 2*A*b^3)*x^6 + 2*B*a^3 - 2*A*a^2*b - 2*(2*B*
a^2*b - A*a*b^2)*x^3 + 2*(3*B*a^3 - 2*A*a^2*b + (3*B*a^2*b - 2*A*a*b^2)*x^
3)*log(b*x^3 + a))/(b^5*x^3 + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^6}{6b^2}+\frac{a(-2Ab+3Ba)\log(a+bx^3)}{3b^4} + x^3\left(\frac{A}{3b^2}-\frac{2Ba}{3b^3}\right)+\frac{-Aa^2b+Ba^3}{3ab^4+3b^5x^3}$$

input
```
integrate(x**8*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
B*x**6/(6*b**2) + a*(-2*A*b + 3*B*a)*log(a + b*x**3)/(3*b**4) + x**3*(A/(3
*b**2) - 2*B*a/(3*b**3)) + (-A*a**2*b + B*a**3)/(3*a*b**4 + 3*b**5*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Ba^3-Aa^2b}{3\,(b^5x^3+ab^4)} + \frac{Bbx^6-2\,(2\,Ba-Ab)x^3}{6\,b^3} + \frac{(3\,Ba^2-2\,Aab)\log(bx^3+a)}{3\,b^4}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(B*a^3 - A*a^2*b)/(b^5*x^3 + a*b^4) + 1/6*(B*b*x^6 - 2*(2*B*a - A*b)*x
^3)/b^3 + 1/3*(3*B*a^2 - 2*A*a*b)*log(b*x^3 + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.29

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(3\,Ba^2-2\,Aab)\log(|bx^3+a|)}{3\,b^4} + \frac{Bb^2x^6-4\,Babx^3+2\,Ab^2x^3}{6\,b^4} - \frac{3\,Ba^2bx^3-2\,Aab^2x^3+2\,Ba^3-Aa^2b}{3\,(bx^3+a)b^4}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*(3*B*a^2 - 2*A*a*b)*log(abs(b*x^3 + a))/b^4 + 1/6*(B*b^2*x^6 - 4*B*a*b
*x^3 + 2*A*b^2*x^3)/b^4 - 1/3*(3*B*a^2*b*x^3 - 2*A*a*b^2*x^3 + 2*B*a^3 - A
*a^2*b)/((b*x^3 + a)*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx = x^3\left(\frac{A}{3\,b^2}-\frac{2\,B\,a}{3\,b^3}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(3\,B\,a^2-2\,A\,a\,b\right)}{3\,b^4} + \frac{B\,x^6}{6\,b^2}+\frac{B\,a^3-A\,a^2\,b}{3\,b\,\left(b^4\,x^3+a\,b^3\right)}$$

input
```
int((x^8*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
x^3*(A/(3*b^2) - (2*B*a)/(3*b^3)) + (log(a + b*x^3)*(3*B*a^2 - 2*A*a*b))/(
3*b^4) + (B*x^6)/(6*b^2) + (B*a^3 - A*a^2*b)/(3*b*(a*b^3 + b^4*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^2+2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a^2-2ab\,x^3+b^2x^6}{6b^3}$$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^2,x)
```

output
```
(2*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2 + 2*log(a**(1/
3) + b**(1/3)*x)*a**2 - 2*a*b*x**3 + b**2*x**6)/(6*b**3)
```

3.75 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2} dx = \frac{Bx^3}{3b^2} + \frac{a(Ab-aB)}{3b^3(a+bx^3)} + \frac{(Ab-2aB)\log(a+bx^3)}{3b^3}$$

output
```
1/3*B*x^3/b^2+1/3*a*(A*b-B*a)/b^3/(b*x^3+a)+1/3*(A*b-2*B*a)*ln(b*x^3+a)/b^
3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2} dx = \frac{bBx^3 + \frac{a(Ab-aB)}{a+bx^3} + (Ab-2aB)\log(a+bx^3)}{3b^3}$$

input
```
Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(b*B*x^3 + (a*(A*b - a*B))/(a + b*x^3) + (A*b - 2*a*B)*Log[a + b*x^3])/(3*
b^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^3)}{(a+bx^3)^2} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^3 (Bx^3+A)}{(bx^3+a)^2} dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3} \int \left(\frac{B}{b^2}+\frac{Ab-2aB}{b^2 (bx^3+a)}+\frac{a(aB-Ab)}{b^2 (bx^3+a)^2}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{a(Ab-aB)}{b^3 (a+bx^3)}+\frac{(Ab-2aB) \log (a+bx^3)}{b^3}+\frac{Bx^3}{b^2}\right)$$

input
```
Int[(x^5*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((B*x^3)/b^2 + (a*(A*b - a*B))/(b^3*(a + b*x^3)) + ((A*b - 2*a*B)*Log[a +
b*x^3])/b^3)/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

method	result	size
norman	$\frac{\frac{B\,x^{6}}{3b}+\frac{a(Ab-2Ba)}{3b^{3}}}{b\,x^{3}+a}+\frac{(Ab-2Ba)\ln\left(b\,x^{3}+a\right)}{3b^{3}}$	57
default	$\frac{B\,x^{3}}{3b^{2}}+\frac{\frac{a(Ab-Ba)}{b\left(b\,x^{3}+a\right)}+\frac{(Ab-2Ba)\ln\left(b\,x^{3}+a\right)}{b}}{3b^{2}}$	59
risch	$\frac{B\,x^{3}}{3b^{2}}+\frac{aA}{3b^{2}(b\,x^{3}+a)}-\frac{a^{2}B}{3b^{3}(b\,x^{3}+a)}+\frac{\ln(b\,x^{3}+a)A}{3b^{2}}-\frac{2\ln(b\,x^{3}+a)Ba}{3b^{3}}$	74
parallelrisch	$\frac{b^{2}B\,x^{6}+A\ln(b\,x^{3}+a)x^{3}b^{2}-2B\ln(b\,x^{3}+a)x^{3}ab+A\ln(b\,x^{3}+a)ab-2B\ln(b\,x^{3}+a)a^{2}+abA-2a^{2}B}{3b^{3}(b\,x^{3}+a)}$	92

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/3*B*x^6/b+1/3*a*(A*b-2*B*a)/b^3)/(b*x^3+a)+1/3*(A*b-2*B*a)*ln(b*x^3+a)/
b^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.35

$$\int\frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx$$
$$=\frac{Bb^2x^6+Babx^3-Ba^2+Aab-((2\,Bab-Ab^2)x^3+2\,Ba^2-Aab)\log\left(bx^3+a\right)}{3\left(b^4x^3+ab^3\right)}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/3*(B*b^2*x^6 + B*a*b*x^3 - B*a^2 + A*a*b - ((2*B*a*b - A*b^2)*x^3 + 2*B*
a^2 - A*a*b)*log(b*x^3 + a))/(b^4*x^3 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^3}{3b^2} + \frac{Aab-Ba^2}{3ab^3+3b^4x^3} - \frac{(-Ab+2Ba)\log(a+bx^3)}{3b^3}$$

input
```
integrate(x**5*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
B*x**3/(3*b**2) + (A*a*b - B*a**2)/(3*a*b**3 + 3*b**4*x**3) - (-A*b + 2*B*
a)*log(a + b*x**3)/(3*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^3}{3\,b^2} - \frac{Ba^2-Aab}{3\,(b^4x^3+ab^3)} - \frac{(2\,Ba-Ab)\log(bx^3+a)}{3\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*B*x^3/b^2 - 1/3*(B*a^2 - A*a*b)/(b^4*x^3 + a*b^3) - 1/3*(2*B*a - A*b)*
log(b*x^3 + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.52

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{\frac{(bx^3+a)B}{b^2} + \frac{(2\,Ba-Ab)\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b^2} - \frac{\frac{Ba^2b}{bx^3+a} - \frac{Aab^2}{bx^3+a}}{b^3}}{3\,b}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*((b*x^3 + a)*B/b^2 + (2*B*a - A*b)*log(abs(b*x^3 + a)/((b*x^3 + a)^2*a
bs(b)))/b^2 - (B*a^2*b/(b*x^3 + a) - A*a*b^2/(b*x^3 + a))/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{B\,x^3}{3\,b^2} + \frac{\ln\left(b\,x^3+a\right)\,\left(A\,b-2\,B\,a\right)}{3\,b^3} - \frac{B\,a^2-A\,a\,b}{3\,b\,\left(b^3\,x^3+a\,b^2\right)}$$

input
```
int((x^5*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
(B*x^3)/(3*b^2) + (log(a + b*x^3)*(A*b - 2*B*a))/(3*b^3) - (B*a^2 - A*a*b)
/(3*b*(a*b^2 + b^3*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a-\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a+b\,x^3}{3b^2}$$

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^2,x)
```

output
```
( - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a - log(a**(1/3) +
 b**(1/3)*x)*a + b*x**3)/(3*b**2)
```

3.76 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 41

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx=\frac{-Ab+aB}{3b^2(a+bx^3)}+\frac{B\log(a+bx^3)}{3b^2}$$

output
```
1/3*(-A*b+B*a)/b^2/(b*x^3+a)+1/3*B*ln(b*x^3+a)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx=\frac{-Ab+aB}{3b^2(a+bx^3)}+\frac{B\log(a+bx^3)}{3b^2}$$

input
```
Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(-(A*b) + a*B)/(3*b^2*(a + b*x^3)) + (B*Log[a + b*x^3])/(3*b^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {946, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^3)}{(a+bx^3)^2} \, dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{3} \int \frac{Bx^3+A}{(bx^3+a)^2}dx^3$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{3} \int \left(\frac{B}{b (bx^3+a)}+\frac{Ab-aB}{b (bx^3+a)^2}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} \left(\frac{B \log (a+bx^3)}{b^2}-\frac{Ab-aB}{b^2 (a+bx^3)}\right)$$

input
```
Int[(x^2*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(-((A*b - a*B)/(b^2*(a + b*x^3))) + (B*Log[a + b*x^3])/b^2)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{Ab-Ba}{3b^2(b\,x^3+a)}+\frac{B\ln(b\,x^3+a)}{3b^2}$	38
norman	$-\frac{Ab-Ba}{3b^2(b\,x^3+a)}+\frac{B\ln(b\,x^3+a)}{3b^2}$	38
risch	$-\frac{A}{3b(b\,x^3+a)}+\frac{Ba}{3b^2(b\,x^3+a)}+\frac{B\ln(b\,x^3+a)}{3b^2}$	47
parallelrisch	$-\frac{-B\ln(b\,x^3+a)x^3b-B\ln(b\,x^3+a)a+Ab-Ba}{3b^2(b\,x^3+a)}$	50

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(A*b-B*a)/b^2/(b*x^3+a)+1/3*B*ln(b*x^3+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx=\frac{Ba-Ab+(Bbx^3+Ba)\log(bx^3+a)}{3(b^3x^3+ab^2)}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/3*(B*a - A*b + (B*b*x^3 + B*a)*log(b*x^3 + a))/(b^3*x^3 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{B\log(a+bx^3)}{3b^2} + \frac{-Ab+Ba}{3ab^2+3b^3x^3}$$

input
```
integrate(x**2*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
B*log(a + b*x**3)/(3*b**2) + (-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Ba-Ab}{3\,(b^3x^3+ab^2)} + \frac{B\log(bx^3+a)}{3\,b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(B*a - A*b)/(b^3*x^3 + a*b^2) + 1/3*B*log(b*x^3 + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{B\left(\frac{\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b} - \frac{a}{(bx^3+a)b}\right)}{3\,b} - \frac{A}{3\,(bx^3+a)b}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/3*B*(log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b - a/((b*x^3 + a)*b))/
b - 1/3*A/((b*x^3 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{B\,\ln\left(b\,x^3+a\right)}{3\,b^2} - \frac{A\,b-B\,a}{3\,b^2\,\left(b\,x^3+a\right)}$$

input
```
int((x^2*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
(B*log(a + b*x^3))/(3*b^2) - (A*b - B*a)/(3*b^2*(a + b*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)+\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b}$$

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^2,x)
```

output
```
(log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) + log(a**(1/3) + b**(
1/3)*x))/(3*b)
```

3.77 $\int \frac{A+Bx^3}{x(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{A+Bx^3}{x(a+bx^3)^2}\,dx = \frac{Ab-aB}{3ab(a+bx^3)} + \frac{A\log(x)}{a^2} - \frac{A\log(a+bx^3)}{3a^2}$$

output
```
1/3*(A*b-B*a)/a/b/(b*x^3+a)+A*ln(x)/a^2-1/3*A*ln(b*x^3+a)/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^3}{x(a+bx^3)^2}\,dx = \frac{\frac{a(Ab-aB)}{b(a+bx^3)} + 3A\log(x) - A\log(a+bx^3)}{3a^2}$$

input
```
Integrate[(A + B*x^3)/(x*(a + b*x^3)^2),x]
```

output
```
((a*(A*b - a*B))/(b*(a + b*x^3)) + 3*A*Log[x] - A*Log[a + b*x^3])/(3*a^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^3\left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(-\frac{bA}{a^2\left(bx^3+a\right)}+\frac{A}{a^2x^3}+\frac{aB-Ab}{a\left(bx^3+a\right)^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{A\log\left(a+bx^3\right)}{a^2}+\frac{A\log\left(x^3\right)}{a^2}+\frac{Ab-aB}{ab\left(a+bx^3\right)}\right)$$

input
```
Int[(A + B*x^3)/(x*(a + b*x^3)^2),x]
```

output
```
((A*b - a*B)/(a*b*(a + b*x^3)) + (A*Log[x^3])/a^2 - (A*Log[a + b*x^3])/a^2
)/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{A\ln(x)}{a^2}-\frac{-\frac{a(Ab-Ba)}{b\left(b\,x^3+a\right)}+A\ln\left(b\,x^3+a\right)}{3a^2}$	48
norman	$-\frac{(Ab-Ba)x^3}{3a^2(b\,x^3+a)}+\frac{A\ln(x)}{a^2}-\frac{A\ln\left(b\,x^3+a\right)}{3a^2}$	48
risch	$\frac{A}{3a(b\,x^3+a)}-\frac{B}{3b(b\,x^3+a)}+\frac{A\ln(x)}{a^2}-\frac{A\ln\left(b\,x^3+a\right)}{3a^2}$	53
parallelrisch	$\frac{3A\ln(x)x^3b-A\ln\left(b\,x^3+a\right)x^3b-Ab\,x^3+Ba\,x^3+3aA\ln(x)-A\ln\left(b\,x^3+a\right)a}{3a^2(b\,x^3+a)}$	71

input
```
int((B*x^3+A)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
A*ln(x)/a^2-1/3/a^2*(-a*(A*b-B*a)/b/(b*x^3+a)+A*ln(b*x^3+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx = -\frac{Ba^2-Aab+\left(Ab^2x^3+Aab\right)\log\left(bx^3+a\right)-3\left(Ab^2x^3+Aab\right)\log\left(x\right)}{3\left(a^2b^2x^3+a^3b\right)}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/3*(B*a^2 - A*a*b + (A*b^2*x^3 + A*a*b)*log(b*x^3 + a) - 3*(A*b^2*x^3 +
A*a*b)*log(x))/(a^2*b^2*x^3 + a^3*b)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{A + Bx^3}{x\left(a + bx^3\right)^2} \, dx = \frac{A\log\left(x\right)}{a^2} - \frac{A\log\left(\frac{a}{b} + x^3\right)}{3a^2} + \frac{Ab - Ba}{3a^2b + 3ab^2x^3}$$

input
```
integrate((B*x**3+A)/x/(b*x**3+a)**2,x)
```

output
```
A*log(x)/a**2 - A*log(a/b + x**3)/(3*a**2) + (A*b - B*a)/(3*a**2*b + 3*a*b
**2*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{A + Bx^3}{x\left(a + bx^3\right)^2} \, dx = -\frac{Ba - Ab}{3\left(ab^2x^3 + a^2b\right)} - \frac{A\log\left(bx^3 + a\right)}{3\,a^2} + \frac{A\log\left(x^3\right)}{3\,a^2}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(B*a - A*b)/(a*b^2*x^3 + a^2*b) - 1/3*A*log(b*x^3 + a)/a^2 + 1/3*A*lo
g(x^3)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx^3}{x(a+bx^3)^2}\,dx = -\frac{A\log(|bx^3+a|)}{3a^2} + \frac{A\log(|x|)}{a^2} + \frac{Ab^2x^3 - Ba^2 + 2Aab}{3(bx^3+a)a^2b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/3*A*log(abs(b*x^3 + a))/a^2 + A*log(abs(x))/a^2 + 1/3*(A*b^2*x^3 - B*a^
2 + 2*A*a*b)/((b*x^3 + a)*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x(a+bx^3)^2}\,dx = \frac{A\ln(x)}{a^2} - \frac{A\ln(b\,x^3+a)}{3a^2} + \frac{A\,b - B\,a}{3\,a\,b\,(b\,x^3+a)}$$

input
```
int((A + B*x^3)/(x*(a + b*x^3)^2),x)
```

output
```
(A*log(x))/a^2 - (A*log(a + b*x^3))/(3*a^2) + (A*b - B*a)/(3*a*b*(a + b*x^
3))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x(a+bx^3)^2}\,dx = \frac{-\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right) - \log\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) + 3\log(x)}{3a}$$

input
```
int((B*x^3+A)/x/(b*x^3+a)^2,x)
```

output
```
( - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) - log(a**(1/3) + b
**(1/3)*x) + 3*log(x))/(3*a)
```

3.78 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx=-\frac{A}{3a^2x^3}-\frac{Ab-aB}{3a^2\left(a+bx^3\right)}-\frac{(2Ab-aB)\log(x)}{a^3}+\frac{(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}$$

output
```
-1/3*A/a^2/x^3-1/3*(A*b-B*a)/a^2/(b*x^3+a)-(2*A*b-B*a)*ln(x)/a^3+1/3*(2*A*
b-B*a)*ln(b*x^3+a)/a^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx=\frac{-\frac{aA}{x^3}+\frac{a(-Ab+aB)}{a+bx^3}+3(-2Ab+aB)\log(x)+(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}$$

input
```
Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^2),x]
```

output
```
(-((a*A)/x^3) + (a*(-(A*b) + a*B))/(a + b*x^3) + 3*(-2*A*b + a*B)*Log[x] +
 (2*A*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^6\left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \ 86$$

$$\frac{1}{3}\int \left(\frac{A}{a^2x^6}-\frac{b(aB-2Ab)}{a^3\left(bx^3+a\right)}-\frac{b(aB-Ab)}{a^2\left(bx^3+a\right)^2}+\frac{aB-2Ab}{a^3x^3}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(-\frac{\log\left(x^3\right)(2Ab-aB)}{a^3}+\frac{(2Ab-aB)\log\left(a+bx^3\right)}{a^3}-\frac{Ab-aB}{a^2\left(a+bx^3\right)}-\frac{A}{a^2x^3}\right)$$

input
```
Int[(A + B*x^3)/(x^4*(a + b*x^3)^2),x]
```

output
```
(-(A/(a^2*x^3)) - (A*b - a*B)/(a^2*(a + b*x^3)) - ((2*A*b - a*B)*Log[x^3])
/a^3 + ((2*A*b - a*B)*Log[a + b*x^3])/a^3)/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

method	result
default	$-\frac{A}{3a^2x^3}+\frac{(-2Ab+Ba)\ln(x)}{a^3}+\frac{b\left(-\frac{a(Ab-Ba)}{b\left(b\,x^3+a\right)}+\frac{(2Ab-Ba)\ln\left(b\,x^3+a\right)}{b}\right)}{3a^3}$
norman	$\frac{-\frac{A}{3a}+\frac{b(2Ab-Ba)x^6}{3a^3}}{x^3(b\,x^3+a)}-\frac{(2Ab-Ba)\ln(x)}{a^3}+\frac{(2Ab-Ba)\ln(b\,x^3+a)}{3a^3}$
risch	$\frac{-\frac{(2Ab-Ba)x^3}{3a^2}-\frac{A}{3a}}{x^3(b\,x^3+a)}-\frac{2\ln(x)Ab}{a^3}+\frac{\ln(x)B}{a^2}+\frac{2\ln\left(-b\,x^3-a\right)Ab}{3a^3}-\frac{\ln\left(-b\,x^3-a\right)B}{3a^2}$
parallelrisch	$-\frac{6A\ln(x)x^6b^2-2A\ln\left(b\,x^3+a\right)x^6b^2-3B\ln(x)x^6ab+B\ln\left(b\,x^3+a\right)x^6ab-2A\,b^2x^6+Bab\,x^6+6A\ln(x)x^3ab-2A\ln\left(b\,x^3+a\right)x^3}{3a^3x^3(b\,x^3+a)}$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/a^2/x^3+(-2*A*b+B*a)/a^3*ln(x)+1/3/a^3*b*(-a*(A*b-B*a)/b/(b*x^3+a)+
(2*A*b-B*a)/b*ln(b*x^3+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.55

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx$$
$$= \frac{(Ba^2 - 2\,Aab)x^3 - Aa^2 - ((Bab - 2\,Ab^2)x^6 + (Ba^2 - 2\,Aab)x^3)\log\left(bx^3+a\right) + 3\,((Bab - 2\,Ab^2)x^6 + (}{3\left(a^3bx^6 + a^4x^3\right)}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/3*((B*a^2 - 2*A*a*b)*x^3 - A*a^2 - ((B*a*b - 2*A*b^2)*x^6 + (B*a^2 - 2*A
*a*b)*x^3)*log(b*x^3 + a) + 3*((B*a*b - 2*A*b^2)*x^6 + (B*a^2 - 2*A*a*b)*x
^3)*log(x))/(a^3*b*x^6 + a^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx = \frac{-Aa + x^3(-2Ab+Ba)}{3a^3x^3+3a^2bx^6} + \frac{(-2Ab+Ba)\log(x)}{a^3} - \frac{(-2Ab+Ba)\log\left(\frac{a}{b}+x^3\right)}{3a^3}$$

input
```
integrate((B*x**3+A)/x**4/(b*x**3+a)**2,x)
```

output
```
(-A*a + x**3*(-2*A*b + B*a))/(3*a**3*x**3 + 3*a**2*b*x**6) + (-2*A*b + B*a
)*log(x)/a**3 - (-2*A*b + B*a)*log(a/b + x**3)/(3*a**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{A + Bx^3}{x^4 (a + bx^3)^2} \, dx = \frac{(Ba - 2\,Ab)x^3 - Aa}{3\,(a^2bx^6 + a^3x^3)} - \frac{(Ba - 2\,Ab)\log(bx^3 + a)}{3\,a^3} + \frac{(Ba - 2\,Ab)\log(x^3)}{3\,a^3}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*((B*a - 2*A*b)*x^3 - A*a)/(a^2*b*x^6 + a^3*x^3) - 1/3*(B*a - 2*A*b)*lo
g(b*x^3 + a)/a^3 + 1/3*(B*a - 2*A*b)*log(x^3)/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{A + Bx^3}{x^4 (a + bx^3)^2} \, dx = \frac{(Ba - 2\,Ab)\log(|x|)}{a^3} + \frac{Bax^3 - 2\,Abx^3 - Aa}{3\,(bx^6 + ax^3)a^2} - \frac{(Bab - 2\,Ab^2)\log(|bx^3 + a|)}{3\,a^3b}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
(B*a - 2*A*b)*log(abs(x))/a^3 + 1/3*(B*a*x^3 - 2*A*b*x^3 - A*a)/((b*x^6 +
a*x^3)*a^2) - 1/3*(B*a*b - 2*A*b^2)*log(abs(b*x^3 + a))/(a^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx$$
$$=\frac{\ln\left(b\,x^3+a\right)\,\left(2\,A\,b-B\,a\right)}{3\,a^3}-\frac{\frac{A}{3\,a}+\frac{x^3\,\left(2\,A\,b-B\,a\right)}{3\,a^2}}{b\,x^6+a\,x^3}-\frac{\ln\left(x\right)\,\left(2\,A\,b-B\,a\right)}{a^3}$$

input
```
int((A + B*x^3)/(x^4*(a + b*x^3)^2),x)
```

output
```
(log(a + b*x^3)*(2*A*b - B*a))/(3*a^3) - (A/(3*a) + (x^3*(2*A*b - B*a))/(3
*a^2))/(a*x^3 + b*x^6) - (log(x)*(2*A*b - B*a))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx=\frac{\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)b\,x^3+\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)b\,x^3-3\log(x)\,b\,x^3-a}{3a^2x^3}$$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^2,x)
```

output
```
(log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**3 + log(a**(1/3)
 + b**(1/3)*x)*b*x**3 - 3*log(x)*b*x**3 - a)/(3*a**2*x**3)
```

3.79 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 97

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx = -\frac{A}{6a^2x^6}+\frac{2Ab-aB}{3a^3x^3}+\frac{b(Ab-aB)}{3a^3\left(a+bx^3\right)} + \frac{b(3Ab-2aB)\log(x)}{a^4}-\frac{b(3Ab-2aB)\log\left(a+bx^3\right)}{3a^4}$$

output
```
-1/6*A/a^2/x^6+1/3*(2*A*b-B*a)/a^3/x^3+1/3*b*(A*b-B*a)/a^3/(b*x^3+a)+b*(3*
A*b-2*B*a)*ln(x)/a^4-1/3*b*(3*A*b-2*B*a)*ln(b*x^3+a)/a^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx = -\frac{\frac{a^2A}{x^6}+\frac{2a(-2Ab+aB)}{x^3}+\frac{2ab(-Ab+aB)}{a+bx^3}-6b(3Ab-2aB)\log(x)+2b(3Ab-2aB)\log\left(a+bx^3\right)}{6a^4}$$

input
```
Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^2),x]
```

output
```
-1/6*((a^2*A)/x^6 + (2*a*(-2*A*b + a*B))/x^3 + (2*a*b*(-(A*b) + a*B))/(a +
 b*x^3) - 6*b*(3*A*b - 2*a*B)*Log[x] + 2*b*(3*A*b - 2*a*B)*Log[a + b*x^3])
/a^4
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^7 (a+bx^3)^2} \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3} \int \frac{Bx^3+A}{x^9 (bx^3+a)^2} dx^3$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{3} \int \left(\frac{(2aB-3Ab)b^2}{a^4 (bx^3+a)} + \frac{(aB-Ab)b^2}{a^3 (bx^3+a)^2} - \frac{(2aB-3Ab)b}{a^4x^3} + \frac{aB-2Ab}{a^3x^6} + \frac{A}{a^2x^9}\right) dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{b\log(x^3)(3Ab-2aB)}{a^4} - \frac{b(3Ab-2aB)\log(a+bx^3)}{a^4} + \frac{b(Ab-aB)}{a^3(a+bx^3)} + \frac{2Ab-aB}{a^3x^3} - \frac{A}{2a^2x^6}\right)$$

input
```
Int[(A + B*x^3)/(x^7*(a + b*x^3)^2),x]
```

output
```
(-1/2*A/(a^2*x^6) + (2*A*b - a*B)/(a^3*x^3) + (b*(A*b - a*B))/(a^3*(a + b*
x^3)) + (b*(3*A*b - 2*a*B)*Log[x^3])/a^4 - (b*(3*A*b - 2*a*B)*Log[a + b*x^
3])/a^4)/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

method	result
default	$-\frac{A}{6a^2x^6}-\frac{-2Ab+Ba}{3a^3x^3}+\frac{b(3Ab-2Ba)\ln(x)}{a^4}-\frac{b^2\left(-\frac{a(Ab-Ba)}{b\left(b\,x^3+a\right)}+\frac{(3Ab-2Ba)\ln\left(b\,x^3+a\right)}{b}\right)}{3a^4}$
norman	$\frac{-\frac{A}{6a}+\frac{(3Ab-2Ba)x^3}{6a^2}-\frac{b\left(3b^2A-2abB\right)x^9}{3a^4}}{x^6(b\,x^3+a)}+\frac{b(3Ab-2Ba)\ln(x)}{a^4}-\frac{b(3Ab-2Ba)\ln(b\,x^3+a)}{3a^4}$
risch	$\frac{\frac{b(3Ab-2Ba)x^6}{3a^3}+\frac{(3Ab-2Ba)x^3}{6a^2}-\frac{A}{6a}}{x^6(b\,x^3+a)}+\frac{3b^2\ln(x)A}{a^4}-\frac{2b\ln(x)B}{a^3}-\frac{b^2\ln\left(b\,x^3+a\right)A}{a^4}+\frac{2b\ln\left(b\,x^3+a\right)B}{3a^3}$
parallelrisch	$\frac{18A\ln(x)x^9b^3-6A\ln\left(b\,x^3+a\right)x^9b^3-12B\ln(x)x^9a\,b^2+4B\ln\left(b\,x^3+a\right)x^9a\,b^2-6A\,x^9b^3+4B\,x^9a\,b^2+18A\ln(x)x^6a\,b^2-6A\ln(b \cdots}{6a^4x^6(b\,x^3+a)}$

input
```
int((B*x^3+A)/x^7/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*A/a^2/x^6-1/3*(-2*A*b+B*a)/a^3/x^3+b*(3*A*b-2*B*a)*ln(x)/a^4-1/3/a^4*
b^2*(-a*(A*b-B*a)/b/(b*x^3+a)+(3*A*b-2*B*a)/b*ln(b*x^3+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx = -\frac{2\,(2\,Ba^2b-3\,Aab^2)x^6+Aa^3+(2\,Ba^3-3\,Aa^2b)x^3-2\,((2\,Bab^2-3\,Ab^3)x^9+(2\,Ba^2b-3\,Aab^2)x^6)\,\mathrm{lo}}{6\,(a^4bx^9+a^5x^6)}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/6*(2*(2*B*a^2*b - 3*A*a*b^2)*x^6 + A*a^3 + (2*B*a^3 - 3*A*a^2*b)*x^3 -
2*((2*B*a*b^2 - 3*A*b^3)*x^9 + (2*B*a^2*b - 3*A*a*b^2)*x^6)*log(b*x^3 + a)
 + 6*((2*B*a*b^2 - 3*A*b^3)*x^9 + (2*B*a^2*b - 3*A*a*b^2)*x^6)*log(x))/(a^
4*b*x^9 + a^5*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx = \frac{-Aa^2+x^6\cdot(6Ab^2-4Bab)+x^3\cdot(3Aab-2Ba^2)}{6a^4x^6+6a^3bx^9} - \frac{b(-3Ab+2Ba)\log{(x)}}{a^4} + \frac{b(-3Ab+2Ba)\log{\left(\frac{a}{b}+x^3\right)}}{3a^4}$$

input
```
integrate((B*x**3+A)/x**7/(b*x**3+a)**2,x)
```

output
```
(-A*a**2 + x**6*(6*A*b**2 - 4*B*a*b) + x**3*(3*A*a*b - 2*B*a**2))/(6*a**4*
x**6 + 6*a**3*b*x**9) - b*(-3*A*b + 2*B*a)*log(x)/a**4 + b*(-3*A*b + 2*B*a
)*log(a/b + x**3)/(3*a**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx = -\frac{2\,(2\,Bab-3\,Ab^2)x^6+(2\,Ba^2-3\,Aab)x^3+Aa^2}{6\,(a^3bx^9+a^4x^6)} + \frac{(2\,Bab-3\,Ab^2)\log\left(bx^3+a\right)}{3\,a^4} - \frac{(2\,Bab-3\,Ab^2)\log\left(x^3\right)}{3\,a^4}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/6*(2*(2*B*a*b - 3*A*b^2)*x^6 + (2*B*a^2 - 3*A*a*b)*x^3 + A*a^2)/(a^3*b*
x^9 + a^4*x^6) + 1/3*(2*B*a*b - 3*A*b^2)*log(b*x^3 + a)/a^4 - 1/3*(2*B*a*b
 - 3*A*b^2)*log(x^3)/a^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.54

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx = -\frac{(2\,Bab-3\,Ab^2)\log\left(|x|\right)}{a^4} + \frac{(2\,Bab^2-3\,Ab^3)\log\left(|bx^3+a|\right)}{3\,a^4b} - \frac{2\,Bab^2x^3-3\,Ab^3x^3+3\,Ba^2b-4\,Aab^2}{3\,(bx^3+a)a^4} + \frac{6\,Babx^6-9\,Ab^2x^6-2\,Ba^2x^3+4\,Aabx^3-Aa^2}{6\,a^4x^6}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-(2*B*a*b - 3*A*b^2)*log(abs(x))/a^4 + 1/3*(2*B*a*b^2 - 3*A*b^3)*log(abs(b
*x^3 + a))/(a^4*b) - 1/3*(2*B*a*b^2*x^3 - 3*A*b^3*x^3 + 3*B*a^2*b - 4*A*a*
b^2)/((b*x^3 + a)*a^4) + 1/6*(6*B*a*b*x^6 - 9*A*b^2*x^6 - 2*B*a^2*x^3 + 4*
A*a*b*x^3 - A*a^2)/(a^4*x^6)
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^3}{x^7 (a+bx^3)^2}\,dx = \frac{\frac{x^3\,(3\,A\,b-2\,B\,a)}{6\,a^2}-\frac{A}{6\,a}+\frac{b\,x^6\,(3\,A\,b-2\,B\,a)}{3\,a^3}}{b\,x^9+a\,x^6} - \frac{\ln\left(b\,x^3+a\right)\,\left(3\,A\,b^2-2\,B\,a\,b\right)}{3\,a^4} + \frac{\ln\left(x\right)\,\left(3\,A\,b^2-2\,B\,a\,b\right)}{a^4}$$

input
```
int((A + B*x^3)/(x^7*(a + b*x^3)^2),x)
```

output
```
((x^3*(3*A*b - 2*B*a))/(6*a^2) - A/(6*a) + (b*x^6*(3*A*b - 2*B*a))/(3*a^3)
)/(a*x^6 + b*x^9) - (log(a + b*x^3)*(3*A*b^2 - 2*B*a*b))/(3*a^4) + (log(x)
*(3*A*b^2 - 2*B*a*b))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx^3}{x^7 (a+bx^3)^2}\,dx = \frac{-2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)b^2x^6-2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)b^2x^6+6\log(x)\,b^2x^6-a^2+2ab\,x^3}{6a^3x^6}$$

input
```
int((B*x^3+A)/x^7/(b*x^3+a)^2,x)
```

output
```
( - 2*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**2*x**6 - 2*lo
g(a**(1/3) + b**(1/3)*x)*b**2*x**6 + 6*log(x)*b**2*x**6 - a**2 + 2*a*b*x**
3)/(6*a**3*x**6)
```

3.80 $\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 217

$$\begin{aligned}\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx = & -\frac{a(2Ab-3aB)x}{b^4}+\frac{(Ab-2aB)x^4}{4b^3}+\frac{Bx^7}{7b^2} \\ & -\frac{a^2(Ab-aB)x}{3b^4(a+bx^3)}-\frac{a^{4/3}(7Ab-10aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{13/3}} \\ & +\frac{a^{4/3}(7Ab-10aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{13/3}} \\ & -\frac{a^{4/3}(7Ab-10aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{13/3}}\end{aligned}$$

output
```
-a*(2*A*b-3*B*a)*x/b^4+1/4*(A*b-2*B*a)*x^4/b^3+1/7*B*x^7/b^2-1/3*a^2*(A*b-
B*a)*x/b^4/(b*x^3+a)-1/9*a^(4/3)*(7*A*b-10*B*a)*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/b^(13/3)+1/9*a^(4/3)*(7*A*b-10*B*a)*ln(a^(
1/3)+b^(1/3)*x)/b^(13/3)-1/18*a^(4/3)*(7*A*b-10*B*a)*ln(a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/b^(13/3)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{252a\sqrt[3]{b}(-2Ab+3aB)x+63b^{4/3}(Ab-2aB)x^4+36b^{7/3}Bx^7+\frac{84a^2\sqrt[3]{b}(-Ab+aB)x}{a+bx^3}+28\sqrt{3}a^{4/3}(-7Ab+10\cdots}{\cdots}$$

input
```
Integrate[(x^9*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(252*a*b^(1/3)*(-2*A*b + 3*a*B)*x + 63*b^(4/3)*(A*b - 2*a*B)*x^4 + 36*b^(7
/3)*B*x^7 + (84*a^2*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3) + 28*Sqrt[3]*a^(
4/3)*(-7*A*b + 10*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 28*a^
(4/3)*(-7*A*b + 10*a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*a^(4/3)*(-7*A*b + 10
*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(252*b^(13/3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {957, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^{10}(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(7Ab-10aB)\int \frac{x^9}{bx^3+a}dx}{3ab}$$

$$\Big\downarrow \text{831}$$

$$\frac{x^{10}(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(7Ab-10aB)\int\left(\frac{x^6}{b}-\frac{ax^3}{b^2}-\frac{a^3}{b^3(bx^3+a)}+\frac{a^2}{b^3}\right)dx}{3ab}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^{10}(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(7Ab-10aB)\left(\frac{a^{7/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{10/3}}+\frac{a^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}}-\frac{a^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}}+\frac{a^2x}{b^3}-\frac{ax^4}{4b^2}+\frac{x^7}{7b}\right)}{3ab}$$

input
```
Int[(x^9*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^10)/(3*a*b*(a + b*x^3)) - ((7*A*b - 10*a*B)*((a^2*x)/b^3 -
(a*x^4)/(4*b^2) + x^7/(7*b) + (a^(7/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*b^(10/3)) - (a^(7/3)*Log[a^(1/3) + b^(1/3)*x])/(3
*b^(10/3)) + (a^(7/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b
^(10/3))))/(3*a*b)
```

Defintions of rubi rules used

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.53

method	result
risch	$\frac{B x^7}{7b^2}+\frac{A x^4}{4b^2}-\frac{B a x^4}{2b^3}-\frac{2aAx}{b^3}+\frac{3B a^2 x}{b^4}+\frac{\left(-\frac{1}{3}a^2 bA+\frac{1}{3}a^3 B\right)x}{b^4\left(b x^3+a\right)}+\frac{a^2\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(7Ab-10Ba)\ln\left(x-\_R\right)}{\_R^2}\right)}{9b^5}$
default	$-\frac{-\frac{1}{7}b^2 B x^7-\frac{1}{4}A b^2 x^4+\frac{1}{2}Bab x^4+2aAbx-3B a^2 x}{b^4}+\frac{a^2\left(\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x}{b x^3+a}+\frac{(7Ab-10Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\right.}{3}\right.}{b^4}$

input
```
int(x^9*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*B*x^7/b^2+1/4/b^2*A*x^4-1/2/b^3*B*a*x^4-2/b^3*a*A*x+3/b^4*B*a^2*x+(-1/
3*a^2*b*A+1/3*a^3*B)*x/b^4/(b*x^3+a)+1/9/b^5*a^2*sum((7*A*b-10*B*a)/_R^2*l
n(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.25

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx$$
$$= \frac{36\,Bb^3x^{10} - 9\,(10\,Bab^2 - 7\,Ab^3)x^7 + 63\,(10\,Ba^2b - 7\,Aab^2)x^4 - 28\,\sqrt{3}(10\,Ba^3 - 7\,Aa^2b + (10\,Ba^2b - 7}{}$$

input
```
integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/252*(36*B*b^3*x^10 - 9*(10*B*a*b^2 - 7*A*b^3)*x^7 + 63*(10*B*a^2*b - 7*A
*a*b^2)*x^4 - 28*sqrt(3)*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A*a*b^2)*
x^3)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 1
4*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A*a*b^2)*x^3)*(a/b)^(1/3)*log(x^
2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 28*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b
- 7*A*a*b^2)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 84*(10*B*a^3 - 7*A*a^
2*b)*x)/(b^5*x^3 + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.72

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^7}{7b^2} + x^4\left(\frac{A}{4b^2} - \frac{Ba}{2b^3}\right) + x\left(-\frac{2Aa}{b^3} + \frac{3Ba^2}{b^4}\right) + \frac{x(-Aa^2b + Ba^3)}{3ab^4 + 3b^5x^3}$$
$$+\mathrm{RootSum}\left(729t^3b^{13} - 343A^3a^4b^3 + 1470A^2Ba^5b^2 - 2100AB^2a^6b + 1000B^3a^7, \left(t \mapsto t\log\left(-\frac{9}{-7Aab}\right.\right.\right.$$

input
```
integrate(x**9*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
B*x**7/(7*b**2) + x**4*(A/(4*b**2) - B*a/(2*b**3)) + x*(-2*A*a/b**3 + 3*B*
a**2/b**4) + x*(-A*a**2*b + B*a**3)/(3*a*b**4 + 3*b**5*x**3) + RootSum(729
*_t**3*b**13 - 343*A**3*a**4*b**3 + 1470*A**2*B*a**5*b**2 - 2100*A*B**2*a*
*6*b + 1000*B**3*a**7, Lambda(_t, _t*log(-9*_t*b**4/(-7*A*a*b + 10*B*a**2)
 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ba^3-Aa^2b)x}{3\,(b^5x^3+ab^4)} + \frac{4\,Bb^2x^7-7\,(2\,Bab-Ab^2)x^4+28\,(3\,Ba^2-2\,Aab)x}{28\,b^4} - \frac{\sqrt{3}(10\,Ba^3-7\,Aa^2b)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(10\,Ba^3-7\,Aa^2b)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(10\,Ba^3-7\,Aa^2b)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input

```
integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output

```
1/3*(B*a^3 - A*a^2*b)*x/(b^5*x^3 + a*b^4) + 1/28*(4*B*b^2*x^7 - 7*(2*B*a*b
 - A*b^2)*x^4 + 28*(3*B*a^2 - 2*A*a*b)*x)/b^4 - 1/9*sqrt(3)*(10*B*a^3 - 7*
A*a^2*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(2
/3)) + 1/18*(10*B*a^3 - 7*A*a^2*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/
(b^5*(a/b)^(2/3)) - 1/9*(10*B*a^3 - 7*A*a^2*b)*log(x + (a/b)^(1/3))/(b^5*(
a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.12

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx$$
$$= -\frac{\sqrt{3}\left(10\left(-ab^2\right)^{\frac{1}{3}}Ba^2-7\left(-ab^2\right)^{\frac{1}{3}}Aab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5}$$
$$+\frac{(10\,Ba^3-7\,Aa^2b)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^4}$$
$$-\frac{\left(10\left(-ab^2\right)^{\frac{1}{3}}Ba^2-7\left(-ab^2\right)^{\frac{1}{3}}Aab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5}$$
$$+\frac{Ba^3x-Aa^2bx}{3\,(bx^3+a)b^4}+\frac{4\,Bb^{12}x^7-14\,Bab^{11}x^4+7\,Ab^{12}x^4+84\,Ba^2b^{10}x-56\,Aab^{11}x}{28\,b^{14}}$$

input
```
integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(10*(-a*b^2)^(1/3)*B*a^2 - 7*(-a*b^2)^(1/3)*A*a*b)*arctan(1/3
*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 + 1/9*(10*B*a^3 - 7*A*a^2*
b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) - 1/18*(10*(-a*b^2)^(1/
3)*B*a^2 - 7*(-a*b^2)^(1/3)*A*a*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)
)/b^5 + 1/3*(B*a^3*x - A*a^2*b*x)/((b*x^3 + a)*b^4) + 1/28*(4*B*b^12*x^7 -
 14*B*a*b^11*x^4 + 7*A*b^12*x^4 + 84*B*a^2*b^10*x - 56*A*a*b^11*x)/b^14
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.96

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx$$
$$= x^4\left(\frac{A}{4\,b^2}-\frac{B\,a}{2\,b^3}\right)-x\left(\frac{2\,a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)+\frac{B\,x^7}{7\,b^2}$$
$$+\frac{x\,\left(\frac{B\,a^3}{3}-\frac{A\,a^2\,b}{3}\right)}{b^5\,x^3+a\,b^4}+\frac{a^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(7\,A\,b-10\,B\,a)}{9\,b^{13/3}}$$
$$-\frac{a^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(7\,A\,b-10\,B\,a)}{9\,b^{13/3}}$$
$$+\frac{a^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(7\,A\,b-10\,B\,a)}{9\,b^{13/3}}$$

input
```
int((x^9*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
x^4*(A/(4*b^2) - (B*a)/(2*b^3)) - x*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^
2)/b^4) + (B*x^7)/(7*b^2) + (x*((B*a^3)/3 - (A*a^2*b)/3))/(a*b^4 + b^5*x^3
) + (a^(4/3)*log(b^(1/3)*x + a^(1/3))*(7*A*b - 10*B*a))/(9*b^(13/3)) - (a^
(4/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/
2)*(7*A*b - 10*B*a))/(9*b^(13/3)) + (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^
(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - 10*B*a))/(9*b^(13/3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.46

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx$$
$$=\frac{28a^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)+14a^{\frac{7}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-28a^{\frac{7}{3}}\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)+84b^{\frac{1}{3}}a^2x-21b^{\frac{4}{3}}a\,x^4}{84b^{\frac{10}{3}}}$$

input
```
int(x^9*(B*x^3+A)/(b*x^3+a)^2,x)
```

output
```
(28*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*
*2 + 14*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2
- 28*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**2 + 84*b**(1/3)*a**2*x - 21*b*
*(1/3)*a*b*x**4 + 12*b**(1/3)*b**2*x**7)/(84*b**(1/3)*b**3)
```

3.81 $\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 201

$$\begin{aligned}\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx &= \frac{(Ab-2aB)x^2}{2b^3}+\frac{Bx^5}{5b^2}+\frac{a(Ab-aB)x^2}{3b^3(a+bx^3)}\\ &\quad+\frac{a^{2/3}(5Ab-8aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{11/3}}\\ &\quad+\frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{11/3}}\\ &\quad-\frac{a^{2/3}(5Ab-8aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{11/3}}\end{aligned}$$

output
```
1/2*(A*b-2*B*a)*x^2/b^3+1/5*B*x^5/b^2+1/3*a*(A*b-B*a)*x^2/b^3/(b*x^3+a)+1/
9*a^(2/3)*(5*A*b-8*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*
3^(1/2)/b^(11/3)+1/9*a^(2/3)*(5*A*b-8*B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(11/3)-
1/18*a^(2/3)*(5*A*b-8*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(11
/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.92

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{45b^{2/3}(Ab-2aB)x^2+18b^{5/3}Bx^5+\frac{30ab^{2/3}(Ab-aB)x^2}{a+bx^3}-10\sqrt{3}a^{2/3}(-5Ab+8aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-10}{90b^{11/3}}$$

input
```
Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(45*b^(2/3)*(A*b - 2*a*B)*x^2 + 18*b^(5/3)*B*x^5 + (30*a*b^(2/3)*(A*b - a*
B)*x^2)/(a + b*x^3) - 10*Sqrt[3]*a^(2/3)*(-5*A*b + 8*a*B)*ArcTan[(1 - (2*b
^(1/3)*x)/a^(1/3))/Sqrt[3]] - 10*a^(2/3)*(-5*A*b + 8*a*B)*Log[a^(1/3) + b^
(1/3)*x] + 5*a^(2/3)*(-5*A*b + 8*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(90*b^(11/3))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {957, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \; 957$$

$$\frac{x^8(Ab-aB)}{3ab(a+bx^3)}-\frac{(5Ab-8aB)\int \frac{x^7}{bx^3+a}dx}{3ab}$$

$$\Big\downarrow \; 831$$

$$\frac{x^8(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(5Ab-8aB)\int\left(\frac{x^4}{b}+\frac{a^2x}{b^2(bx^3+a)}-\frac{ax}{b^2}\right)dx}{3ab}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^8(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(5Ab-8aB)\left(-\frac{a^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}}+\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}}-\frac{ax^2}{2b^2}+\frac{x^5}{5b}\right)}{3ab}$$

input
```
Int[(x^7*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^8)/(3*a*b*(a + b*x^3)) - ((5*A*b - 8*a*B)*(-1/2*(a*x^2)/b^2
 + x^5/(5*b) - (a^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])
/(Sqrt[3]*b^(8/3)) - (a^(5/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(8/3)) + (a^(
5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(8/3))))/(3*a*b)
```

Defintions of rubi rules used

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.81 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.47

method	result
risch	$\frac{B x^5}{5b^2}+\frac{A x^2}{2b^2}-\frac{B a x^2}{b^3}+\frac{\left(\frac{1}{3}abA-\frac{1}{3}a^2B\right)x^2}{b^3(b x^3+a)}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(-5Ab+8Ba)\ln\left(x-\_R\right)}{\_R}\right)}{9b^4}$
default	$\frac{\frac{bBx^5}{5}+\frac{(Ab-2Ba)x^2}{2}}{b^3}-\frac{a\left(\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x^2}{bx^3+a}+\left(\frac{5Ab}{3}-\frac{8Ba}{3}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\right)}{b^3}$

input
```
int(x^7*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*B*x^5/b^2+1/2/b^2*A*x^2-1/b^3*B*a*x^2+(1/3*a*b*A-1/3*a^2*B)*x^2/b^3/(b
*x^3+a)+1/9/b^4*a*sum((-5*A*b+8*B*a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.28

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{18\,Bb^2x^8-9\,(8\,Bab-5\,Ab^2)x^5-15\,(8\,Ba^2-5\,Aab)x^2+10\,\sqrt{3}((8\,Bab-5\,Ab^2)x^3+8\,Ba^2-5\,Aab)\left(\ldots\right.}{\ldots}$$

input
```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/90*(18*B*b^2*x^8 - 9*(8*B*a*b - 5*A*b^2)*x^5 - 15*(8*B*a^2 - 5*A*a*b)*x^
2 + 10*sqrt(3)*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*(a^2/b^2)^(1/
3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*((8*B*a*b
 - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/
b^2)^(2/3) + a*(a^2/b^2)^(1/3)) - 10*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 -
5*A*a*b)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/(b^4*x^3 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.75

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^5}{5b^2} + x^2\left(\frac{A}{2b^2} - \frac{Ba}{b^3}\right) + \frac{x^2(Aab-Ba^2)}{3ab^3+3b^4x^3}$$
$$+\text{RootSum}\left(729t^3b^{11} - 125A^3a^2b^3 + 600A^2Ba^3b^2 - 960AB^2a^4b + 512B^3a^5, \left(t \mapsto t\log\left(\frac{8}{25A^2ab^2 - 80}\right.\right.\right.$$

input
```
integrate(x**7*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
B*x**5/(5*b**2) + x**2*(A/(2*b**2) - B*a/b**3) + x**2*(A*a*b - B*a**2)/(3*
a*b**3 + 3*b**4*x**3) + RootSum(729*_t**3*b**11 - 125*A**3*a**2*b**3 + 600
*A**2*B*a**3*b**2 - 960*A*B**2*a**4*b + 512*B**3*a**5, Lambda(_t, _t*log(8
1*_t**2*b**7/(25*A**2*a*b**2 - 80*A*B*a**2*b + 64*B**2*a**3) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.96

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(Ba^2-Aab)x^2}{3\,(b^4x^3+ab^3)} + \frac{\sqrt{3}(8\,Ba^2-5\,Aab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{2\,Bbx^5-5\,(2\,Ba-Ab)x^2}{10\,b^3} + \frac{(8\,Ba^2-5\,Aab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(8\,Ba^2-5\,Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(B*a^2 - A*a*b)*x^2/(b^4*x^3 + a*b^3) + 1/9*sqrt(3)*(8*B*a^2 - 5*A*a*
b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(1/3)) +
 1/10*(2*B*b*x^5 - 5*(2*B*a - A*b)*x^2)/b^3 + 1/18*(8*B*a^2 - 5*A*a*b)*log
(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(1/3)) - 1/9*(8*B*a^2 - 5*A
*a*b)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.17

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{\left(8\,Ba^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 5\,Aab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^3} - \frac{\sqrt{3}\left(8\left(-ab^2\right)^{\frac{2}{3}}Ba - 5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5} - \frac{Ba^2x^2 - Aabx^2}{3\,(bx^3+a)b^3} + \frac{\left(8\left(-ab^2\right)^{\frac{2}{3}}Ba - 5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5} + \frac{2\,Bb^8x^5 - 10\,Bab^7x^2 + 5\,Ab^8x^2}{10\,b^{10}}$$

input
```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*(8*B*a^2*(-a/b)^(1/3) - 5*A*a*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/(a*b^3) - 1/9*sqrt(3)*(8*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^
(2/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 - 1/3
*(B*a^2*x^2 - A*a*b*x^2)/((b*x^3 + a)*b^3) + 1/18*(8*(-a*b^2)^(2/3)*B*a -
5*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^5 + 1/10*
(2*B*b^8*x^5 - 10*B*a*b^7*x^2 + 5*A*b^8*x^2)/b^10
```

Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.89

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx$$
$$= x^2\left(\frac{A}{2\,b^2}-\frac{B\,a}{b^3}\right)+\frac{B\,x^5}{5\,b^2}-\frac{x^2\left(\frac{B\,a^2}{3}-\frac{A\,a\,b}{3}\right)}{b^4\,x^3+a\,b^3}+\frac{a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(5\,A\,b-8\,B\,a)}{9\,b^{11/3}}$$
$$+\frac{a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-8\,B\,a)}{9\,b^{11/3}}$$
$$-\frac{a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-8\,B\,a)}{9\,b^{11/3}}$$

input
```
int((x^7*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
x^2*(A/(2*b^2) - (B*a)/b^3) + (B*x^5)/(5*b^2) - (x^2*((B*a^2)/3 - (A*a*b)/
3))/(a*b^3 + b^4*x^3) + (a^(2/3)*log(b^(1/3)*x + a^(1/3))*(5*A*b - 8*B*a))
/(9*b^(11/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*(
(3^(1/2)*1i)/2 - 1/2)*(5*A*b - 8*B*a))/(9*b^(11/3)) - (a^(2/3)*log(3^(1/2)
*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b - 8*B*a
))/(9*b^(11/3))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.49

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx$$
$$= \frac{-10\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a^2-15b^{\frac{2}{3}}a^{\frac{4}{3}}x^2+6b^{\frac{5}{3}}a^{\frac{1}{3}}x^5+5\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^2-10\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a}{30b^{\frac{8}{3}}a^{\frac{1}{3}}}$$

input
```
int(x^7*(B*x^3+A)/(b*x^3+a)^2,x)
```

output
```
( - 10*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2 - 1
5*b**(2/3)*a**(1/3)*a*x**2 + 6*b**(2/3)*a**(1/3)*b*x**5 + 5*log(a**(2/3) -
 b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2 - 10*log(a**(1/3) + b**(1/3)*x)
*a**2)/(30*b**(2/3)*a**(1/3)*b**2)
```

3.82 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 194

$$\begin{aligned}\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx &= \frac{(Ab-2aB)x}{b^3}+\frac{Bx^4}{4b^2}+\frac{a(Ab-aB)x}{3b^3(a+bx^3)} \\ &+ \frac{\sqrt[3]{a}(4Ab-7aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{10/3}} \\ &- \frac{\sqrt[3]{a}(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{10/3}} \\ &+ \frac{\sqrt[3]{a}(4Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{10/3}}\end{aligned}$$

output
```
(A*b-2*B*a)*x/b^3+1/4*B*x^4/b^2+1/3*a*(A*b-B*a)*x/b^3/(b*x^3+a)+1/9*a^(1/3
)*(4*A*b-7*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/
b^(10/3)-1/9*a^(1/3)*(4*A*b-7*B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(10/3)+1/18*a^(
1/3)*(4*A*b-7*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(10/3)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.93

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{36\sqrt[3]{b}(Ab-2aB)x+9b^{4/3}Bx^4+\frac{12a\sqrt[3]{b}(Ab-aB)x}{a+bx^3}-4\sqrt{3}\sqrt[3]{a}(-4Ab+7aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{a}(-\cdots}{36b^{10/3}}$$

input
```
Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(36*b^(1/3)*(A*b - 2*a*B)*x + 9*b^(4/3)*B*x^4 + (12*a*b^(1/3)*(A*b - a*B)*
x)/(a + b*x^3) - 4*Sqrt[3]*a^(1/3)*(-4*A*b + 7*a*B)*ArcTan[(1 - (2*b^(1/3)
*x)/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*(-4*A*b + 7*a*B)*Log[a^(1/3) + b^(1/3)*x
] - 2*a^(1/3)*(-4*A*b + 7*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(36*b^(10/3))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {957, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^7(Ab-aB)}{3ab(a+bx^3)}-\frac{(4Ab-7aB)\int \frac{x^6}{bx^3+a}dx}{3ab}$$

$$\Big\downarrow \text{831}$$

$$\frac{x^7(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(4Ab-7aB)\int\left(\frac{x^3}{b}+\frac{a^2}{b^2(bx^3+a)}-\frac{a}{b^2}\right)dx}{3ab}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^7(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(4Ab-7aB)\left(-\frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{7/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}}-\frac{ax}{b^2}+\frac{x^4}{4b}\right)}{3ab}$$

input
```
Int[(x^6*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^7)/(3*a*b*(a + b*x^3)) - ((4*A*b - 7*a*B)*(-((a*x)/b^2) + x
^4/(4*b) - (a^(4/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^(7/3)) + (a^(4/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(7/3)) - (a^(4/3)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(7/3))))/(3*a*b)
```

Defintions of rubi rules used

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.82 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.45

method	result
risch	$\frac{B\,x^{4}}{4 b^{2}}+\frac{A x}{b^{2}}-\frac{2 B a x}{b^{3}}+\frac{\left(\frac{1}{3} a b A-\frac{1}{3} a^{2} B\right) x}{b^{3} \left(b\,x^{3}+a\right)}+\frac{a \left(\sum_{\_R =\operatorname{RootOf}\left(b\,\_Z^{3}+a\right)}\frac{\left(-4 A b+7 B a\right) \ln\left(x-\_R\right)}{\_R^{2}}\right)}{9 b^{4}}$
default	$\frac{\frac{1}{4} b B\,x^{4}+A b x-2 B a x}{b^{3}}-\frac{a \left(\frac{\left(-\frac{A b}{3}+\frac{B a}{3}\right) x}{b\,x^{3}+a}+\frac{\left(4 A b-7 B a\right) \left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}\right)}{b^{3}}$

input
```
int(x^6*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*B*x^4/b^2+1/b^2*A*x-2/b^3*B*a*x+(1/3*a*b*A-1/3*a^2*B)*x/b^3/(b*x^3+a)+
1/9/b^4*a*sum((-4*A*b+7*B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.24

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{9\,Bb^2x^7 - 9\,(7\,Bab - 4\,Ab^2)x^4 - 4\,\sqrt{3}((7\,Bab - 4\,Ab^2)x^3 + 7\,Ba^2 - 4\,Aab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}} - \cdots}{3\,a}\right.}{}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/36*(9*B*b^2*x^7 - 9*(7*B*a*b - 4*A*b^2)*x^4 - 4*sqrt(3)*((7*B*a*b - 4*A*
b^2)*x^3 + 7*B*a^2 - 4*A*a*b)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b
)^(2/3) - sqrt(3)*a)/a) + 2*((7*B*a*b - 4*A*b^2)*x^3 + 7*B*a^2 - 4*A*a*b)*
(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) - 4*((7*B*a*b - 4*A*
b^2)*x^3 + 7*B*a^2 - 4*A*a*b)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) - 12*(7*B
*a^2 - 4*A*a*b)*x)/(b^4*x^3 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.65

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^4}{4b^2} + x\left(\frac{A}{b^2} - \frac{2Ba}{b^3}\right) + \frac{x(Aab - Ba^2)}{3ab^3 + 3b^4x^3}$$

$$+\text{RootSum}\left(729t^3b^{10} + 64A^3ab^3 - 336A^2Ba^2b^2 + 588AB^2a^3b - 343B^3a^4, \left(t \mapsto t\log\left(\frac{9tb^3}{-4Ab+7Ba} + \cdots\right.\right.\right.$$

input
```
integrate(x**6*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
B*x**4/(4*b**2) + x*(A/b**2 - 2*B*a/b**3) + x*(A*a*b - B*a**2)/(3*a*b**3 +
 3*b**4*x**3) + RootSum(729*_t**3*b**10 + 64*A**3*a*b**3 - 336*A**2*B*a**2
*b**2 + 588*A*B**2*a**3*b - 343*B**3*a**4, Lambda(_t, _t*log(9*_t*b**3/(-4
*A*b + 7*B*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.96

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(Ba^2-Aab)x}{3\,(b^4x^3+ab^3)} + \frac{Bbx^4-4\,(2\,Ba-Ab)x}{4\,b^3} + \frac{\sqrt{3}(7\,Ba^2-4\,Aab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(7\,Ba^2-4\,Aab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(7\,Ba^2-4\,Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(B*a^2 - A*a*b)*x/(b^4*x^3 + a*b^3) + 1/4*(B*b*x^4 - 4*(2*B*a - A*b)*
x)/b^3 + 1/9*sqrt(3)*(7*B*a^2 - 4*A*a*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(
1/3))/(a/b)^(1/3))/(b^4*(a/b)^(2/3)) - 1/18*(7*B*a^2 - 4*A*a*b)*log(x^2 -
x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1/9*(7*B*a^2 - 4*A*a*b)*l
og(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.09

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{\sqrt{3}\left(7\left(-ab^2\right)^{\frac{1}{3}}Ba-4\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^4} - \frac{(7\,Ba^2-4\,Aab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^3} + \frac{\left(7\left(-ab^2\right)^{\frac{1}{3}}Ba-4\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^4} - \frac{Ba^2x-Aabx}{3\,(bx^3+a)b^3} + \frac{Bb^6x^4-8\,Bab^5x+4\,Ab^6x}{4\,b^8}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*sqrt(3)*(7*(-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/9*(7*B*a^2 - 4*A*a*b)*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3) + 1/18*(7*(-a*b^2)^(1/3)*B*a - 4
*(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 - 1/3*(B
*a^2*x - A*a*b*x)/((b*x^3 + a)*b^3) + 1/4*(B*b^6*x^4 - 8*B*a*b^5*x + 4*A*b
^6*x)/b^8
```

Mupad [B] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.99

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx$$
$$= x\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)-\frac{x\left(\frac{B\,a^2}{3}-\frac{A\,a\,b}{3}\right)}{b^4\,x^3+a\,b^3}+\frac{B\,x^4}{4\,b^2}$$
$$+\frac{(-a)^{1/3}\,\ln\left((-a)^{4/3}+a\,b^{1/3}\,x\right)\,(4\,A\,b-7\,B\,a)}{9\,b^{10/3}}$$
$$-\frac{(-a)^{1/3}\,\ln\left((-a)^{4/3}-2\,a\,b^{1/3}\,x+\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(4\,A\,b-7\,B\,a)}{9\,b^{10/3}}$$
$$+\frac{(-a)^{1/3}\,\ln\left(2\,a\,b^{1/3}\,x-(-a)^{4/3}+\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(4\,A\,b-7\,B\,a)}{9\,b^{10/3}}$$

input
```
int((x^6*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
x*(A/b^2 - (2*B*a)/b^3) - (x*((B*a^2)/3 - (A*a*b)/3))/(a*b^3 + b^4*x^3) +
(B*x^4)/(4*b^2) + ((-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*x)*(4*A*b - 7*B*a
))/(9*b^(10/3)) - ((-a)^(1/3)*log((-a)^(4/3) + 3^(1/2)*(-a)^(4/3)*1i - 2*a
*b^(1/3)*x)*((3^(1/2)*1i)/2 + 1/2)*(4*A*b - 7*B*a))/(9*b^(10/3)) + ((-a)^(
1/3)*log(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)
/2 - 1/2)*(4*A*b - 7*B*a))/(9*b^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.45

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx$$
$$= \frac{-4a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)-2a^{\frac{4}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)+4a^{\frac{4}{3}}\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)-12b^{\frac{1}{3}}ax+3b^{\frac{4}{3}}x^4}{12b^{\frac{7}{3}}}$$

input
```
int(x^6*(B*x^3+A)/(b*x^3+a)^2,x)
```

output
```
( - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
a - 2*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a + 4*a
**(1/3)*log(a**(1/3) + b**(1/3)*x)*a - 12*b**(1/3)*a*x + 3*b**(1/3)*b*x**4
)/(12*b**(1/3)*b**2)
```

3.83 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 182

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2} dx = \frac{Bx^2}{2b^2} - \frac{(Ab-aB)x^2}{3b^2(a+bx^3)} - \frac{(2Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{8/3}} - \frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{8/3}} + \frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}b^{8/3}}$$

output
```
1/2*B*x^2/b^2-1/3*(A*b-B*a)*x^2/b^2/(b*x^3+a)-1/9*(2*A*b-5*B*a)*arctan(1/3
*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(1/3)/b^(8/3)-1/9*(2*A*b
-5*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(8/3)+1/18*(2*A*b-5*B*a)*ln(a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(8/3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.91

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{9b^{2/3}Bx^2 - \frac{6b^{2/3}(Ab-aB)x^2}{a+bx^3} + \frac{2\sqrt{3}(-2Ab+5aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{2(-2Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} + \frac{(2Ab-5aB)\log\left(a^{2/}\right.\cdots}{\cdots}}{18b^{8/3}}$$

input
```
Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(9*b^(2/3)*B*x^2 - (6*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3) + (2*Sqrt[3]*(-
2*A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (2*(
-2*A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + ((2*A*b - 5*a*B)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(18*b^(8/3))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {957, 843, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{x^5(Ab-aB)}{3ab\,(a+bx^3)} - \frac{(2Ab-5aB)\int \frac{x^4}{bx^3+a}dx}{3ab}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x^5(Ab-aB)}{3ab(a+bx^3)}-\frac{(2Ab-5aB)\left(\frac{x^2}{2b}-\frac{a\int \frac{x}{bx^3+a}dx}{b}\right)}{3ab}$$

$\Big\downarrow$ 821

$$\frac{x^5(Ab-aB)}{3ab(a+bx^3)}-\frac{(2Ab-5aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{b}\right)}{3ab}$$

$\Big\downarrow$ 16

$$\frac{x^5(Ab-aB)}{3ab(a+bx^3)}-\frac{(2Ab-5aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3ab}$$

$\Big\downarrow$ 1142

$$\frac{x^5(Ab-aB)}{3ab(a+bx^3)}-\frac{(2Ab-5aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \; 25$$

$$\frac{x^5(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(2Ab-5aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \; 27$$

$$\frac{x^5(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(2Ab-5aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \; 1082$$

$$\frac{x^5(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(2Ab-5aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3ab}$$

$\downarrow$ 217

$$\frac{x^5(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(2Ab-5aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3ab}$$

$\downarrow$ 1103

$$\frac{x^5(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(2Ab-5aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3ab}$$

input
```
Int[(x^4*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^5)/(3*a*b*(a + b*x^3)) - ((2*A*b - 5*a*B)*(x^2/(2*b) - (a*(
-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 -
(2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/b))/(3*a*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.84 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{B\,x^{2}}{2 b^{2}}+\frac{\left(-\frac{A b}{3}+\frac{B a}{3}\right) x^{2}}{b^{2} \left(b\,x^{3}+a \right)}+\frac{\sum_{\_R =\operatorname{RootOf}\left(b\,\_Z^{3}+a \right)}\frac{\left(2 A b -5 B a \right) \ln \left(x -\_R \right)}{\_R}}{9 b^{3}}$	71
default	$\frac{B\,x^{2}}{2 b^{2}}+\frac{\frac{\left(-\frac{A b}{3}+\frac{B a}{3}\right) x^{2}}{b\,x^{3}+a}+\left(-\frac{5 B a}{3}+\frac{2 A b}{3}\right) \left(-\frac{\ln \left(x +\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln \left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x +\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}}$	138

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x^2/b^2+(-1/3*A*b+1/3*B*a)*x^2/b^2/(b*x^3+a)+1/9/b^3*sum((2*A*b-5*B*
a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 578, normalized size of antiderivative = 3.18

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \left[\frac{9\,Bab^3x^5 + 3\,(5\,Ba^2b^2 - 2\,Aab^3)x^2 - 3\,\sqrt{\frac{1}{3}}(5\,Ba^3b - 2\,Aa^2b^2 + (5\,Ba^2b^2 - 2\,Aab^3)x^3)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\right.}{} \right.$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/18*(9*B*a*b^3*x^5 + 3*(5*B*a^2*b^2 - 2*A*a*b^3)*x^2 - 3*sqrt(1/3)*(5*B*
a^3*b - 2*A*a^2*b^2 + (5*B*a^2*b^2 - 2*A*a*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b
^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - (
(5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 +
(-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^
2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^5*x^3 + a^2*b^
4), 1/18*(9*B*a*b^3*x^5 + 3*(5*B*a^2*b^2 - 2*A*a*b^3)*x^2 - 6*sqrt(1/3)*(5
*B*a^3*b - 2*A*a^2*b^2 + (5*B*a^2*b^2 - 2*A*a*b^3)*x^3)*sqrt(-(-a*b^2)^(1/
3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b)
 - ((5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^
2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b - 2*A*b^2)*x^3 + 5*
B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^5*x^3 + a^
2*b^4)]
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.69

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^2}{2b^2} + \frac{x^2(-Ab+Ba)}{3ab^2+3b^3x^3} + \operatorname{RootSum}\left(729t^3ab^8+8A^3b^3-60A^2Bab^2+150AB^2a^2b-125B^3a^3, \left(t\mapsto t\log\left(\frac{81t^2ab^5}{4A^2b^2-20ABab+\cdots}\right.\right.\right.$$

input
```
integrate(x**4*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
B*x**2/(2*b**2) + x**2*(-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3) + RootSum(729
*_t**3*a*b**8 + 8*A**3*b**3 - 60*A**2*B*a*b**2 + 150*A*B**2*a**2*b - 125*B
**3*a**3, Lambda(_t, _t*log(81*_t**2*a*b**5/(4*A**2*b**2 - 20*A*B*a*b + 25
*B**2*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.89

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ba-Ab)x^2}{3\,(b^3x^3+ab^2)} + \frac{Bx^2}{2\,b^2} - \frac{\sqrt{3}(5\,Ba-2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(5\,Ba-2\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(5\,Ba-2\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(B*a - A*b)*x^2/(b^3*x^3 + a*b^2) + 1/2*B*x^2/b^2 - 1/9*sqrt(3)*(5*B*a
 - 2*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(
1/3)) - 1/18*(5*B*a - 2*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(
a/b)^(1/3)) + 1/9*(5*B*a - 2*A*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.04

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^2}{2\,b^2} - \frac{\sqrt{3}(5\,Ba-2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{1}{3}}\,b^2} + \frac{(5\,Ba-2\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{1}{3}}\,b^2} + \frac{\left(5\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^2} + \frac{Bax^2-Abx^2}{3\,(bx^3+a)b^2}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/2*B*x^2/b^2 - 1/9*sqrt(3)*(5*B*a - 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^2) + 1/18*(5*B*a - 2*A*b)*log(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b^2) + 1/9*(5*B*a*(-a/b
)^(1/3) - 2*A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b
^2) + 1/3*(B*a*x^2 - A*b*x^2)/((b*x^3 + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.87

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{B\,x^2}{2\,b^2} - \frac{x^2\left(\frac{A\,b}{3}-\frac{B\,a}{3}\right)}{b^3\,x^3+a\,b^2} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(2\,A\,b-5\,B\,a)}{9\,a^{1/3}\,b^{8/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b-5\,B\,a)}{9\,a^{1/3}\,b^{8/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b-5\,B\,a)}{9\,a^{1/3}\,b^{8/3}}$$

input
```
int((x^4*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
(B*x^2)/(2*b^2) - (x^2*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) - (log(b^(1/
3)*x + a^(1/3))*(2*A*b - 5*B*a))/(9*a^(1/3)*b^(8/3)) - (log(3^(1/2)*a^(1/3
)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*A*b - 5*B*a))/(9*a
^(1/3)*b^(8/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/
2)*1i)/2 + 1/2)*(2*A*b - 5*B*a))/(9*a^(1/3)*b^(8/3))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.45

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a+3b^{\frac{2}{3}}a^{\frac{1}{3}}x^2-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a+2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a}{6b^{\frac{5}{3}}a^{\frac{1}{3}}}$$

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^2,x)
```

output
```
(2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a + 3*b**(2/
3)*a**(1/3)*x**2 - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a +
 2*log(a**(1/3) + b**(1/3)*x)*a)/(6*b**(2/3)*a**(1/3)*b)
```

3.84 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 172

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2} dx = \frac{Bx}{b^2} - \frac{(Ab-aB)x}{3b^2(a+bx^3)} - \frac{(Ab-4aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{7/3}} + \frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}} - \frac{(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{7/3}}$$

output
```
B*x/b^2-1/3*(A*b-B*a)*x/b^2/(b*x^3+a)-1/9*(A*b-4*B*a)*arctan(1/3*(a^(1/3)-
2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)/b^(7/3)+1/9*(A*b-4*B*a)*ln(a
^(1/3)+b^(1/3)*x)/a^(2/3)/b^(7/3)-1/18*(A*b-4*B*a)*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(7/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.93

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{18\sqrt[3]{b}Bx - \frac{6\sqrt[3]{b}(Ab-aB)x}{a+bx^3} + \frac{2\sqrt{3}(-Ab+4aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{2(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{(-Ab+4aB)\log\left(a^{2/3}-\sqrt[3]{}\right.}{a^{2/3}}}{18b^{7/3}}$$

input
```
Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(18*b^(1/3)*B*x - (6*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3) + (2*Sqrt[3]*(-(A*
b) + 4*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*(A*b
 - 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + ((-(A*b) + 4*a*B)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(18*b^(7/3))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {957, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^4(Ab-aB)}{3ab(a+bx^3)} - \frac{(Ab-4aB)\int \frac{x^3}{bx^3+a}dx}{3ab}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\left(\frac{x}{b}-\frac{a\int\frac{1}{bx^3+a}dx}{b}\right)}{3ab}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\left(\frac{x}{b}-\frac{a\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\left(\frac{x}{b}-\frac{a\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \text{1082}$$

$$\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3ab}$$

$\downarrow$ 217

$$\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3ab}$$

$\downarrow$ 1103

$$\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\left(\frac{x}{b}-\frac{a\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3ab}$$

input
```
Int[(x^3*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^4)/(3*a*b*(a + b*x^3)) - ((A*b - 4*a*B)*(x/b - (a*(Log[a^(1
/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*
x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/b))/(3*a*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.82 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.38

method	result	size
risch	$\frac{Bx}{b^2}+\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x}{b^2(b\,x^3+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab-4Ba)\ln\left(x-\_R\right)}{\_R^2}}{9b^3}$	65
default	$\frac{Bx}{b^2}+\frac{\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x}{b\,x^3+a}+\frac{(Ab-4Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}}{b^2}$	133

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
B*x/b^2+(-1/3*A*b+1/3*B*a)*x/b^2/(b*x^3+a)+1/9/b^3*sum((A*b-4*B*a)/_R^2*ln
(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.33

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \left[\frac{18\,Ba^2b^2x^4 - 3\,\sqrt{\tfrac{1}{3}}(4\,Ba^3b - Aa^2b^2 + (4\,Ba^2b^2 - Aab^3)x^3)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3 - 3\,(a^2b)^{\frac{1}{3}}ax - a^2 + 3\,\sqrt{\tfrac{1}{3}}\left(2\,ab\right.}{bx}\right.}{} \right.$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/18*(18*B*a^2*b^2*x^4 - 3*sqrt(1/3)*(4*B*a^3*b - A*a^2*b^2 + (4*B*a^2*b^
2 - A*a*b^3)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*
a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sq
rt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + ((4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*
a*b)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((
4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^
(2/3)) + 6*(4*B*a^3*b - A*a^2*b^2)*x)/(a^2*b^4*x^3 + a^3*b^3), 1/18*(18*B*
a^2*b^2*x^4 - 6*sqrt(1/3)*(4*B*a^3*b - A*a^2*b^2 + (4*B*a^2*b^2 - A*a*b^3)
*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^
(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + ((4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A
*a*b)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*(
(4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)
^(2/3)) + 6*(4*B*a^3*b - A*a^2*b^2)*x)/(a^2*b^4*x^3 + a^3*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.59

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx}{b^2} + \frac{x(-Ab+Ba)}{3ab^2+3b^3x^3} + \operatorname{RootSum}\left(729t^3a^2b^7 - A^3b^3 + 12A^2Bab^2 - 48AB^2a^2b + 64B^3a^3, \left(t \mapsto t\log\left(-\frac{9tab^2}{-Ab+4Ba}+x\right)\right)\right)$$

input
```
integrate(x**3*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
B*x/b**2 + x*(-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3) + RootSum(729*_t**3*a**
2*b**7 - A**3*b**3 + 12*A**2*B*a*b**2 - 48*A*B**2*a**2*b + 64*B**3*a**3, L
ambda(_t, _t*log(-9*_t*a*b**2/(-A*b + 4*B*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.91

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ba-Ab)x}{3\,(b^3x^3+ab^2)} + \frac{Bx}{b^2} - \frac{\sqrt{3}(4\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(4\,Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(4\,Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(B*a - A*b)*x/(b^3*x^3 + a*b^2) + B*x/b^2 - 1/9*sqrt(3)*(4*B*a - A*b)*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 1/
18*(4*B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3))
- 1/9*(4*B*a - A*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.97

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{\sqrt{3}(4\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}b} + \frac{(4\,Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}b} + \frac{Bx}{b^2} + \frac{(4\,Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^2} + \frac{Bax-Abx}{3\,(bx^3+a)b^2}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*sqrt(3)*(4*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(2/3)*b) + 1/18*(4*B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (
-a/b)^(2/3))/((-a*b^2)^(2/3)*b) + B*x/b^2 + 1/9*(4*B*a - A*b)*(-a/b)^(1/3)
*log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*b^2
)
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.87

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{B\,x}{b^2} - \frac{x\left(\frac{A\,b}{3}-\frac{B\,a}{3}\right)}{b^3\,x^3+a\,b^2} + \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-4\,B\,a\right)}{9\,a^{2/3}\,b^{7/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-4\,B\,a\right)}{9\,a^{2/3}\,b^{7/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-4\,B\,a\right)}{9\,a^{2/3}\,b^{7/3}}$$

input
```
int((x^3*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
(B*x)/b^2 - (x*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) + (log(b^(1/3)*x + a
^(1/3))*(A*b - 4*B*a))/(9*a^(2/3)*b^(7/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b
^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - 4*B*a))/(9*a^(2/3)*b^(7/
3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1
/2)*(A*b - 4*B*a))/(9*a^(2/3)*b^(7/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.45

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)+a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-2a^{\frac{1}{3}}\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)+6b^{\frac{1}{3}}x}{6b^{\frac{4}{3}}}$$

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^2,x)
```

output
```
(2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3))) + a
**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) - 2*a**(1/3)*l
og(a**(1/3) + b**(1/3)*x) + 6*b**(1/3)*x)/(6*b**(1/3)*b)
```

3.85 $\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 171

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ab-aB)x^2}{3ab\,(a+bx^3)} - \frac{(Ab+2aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{5/3}} - \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{5/3}} + \frac{(Ab+2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{5/3}}$$

output
```
1/3*(A*b-B*a)*x^2/a/b/(b*x^3+a)-1/9*(A*b+2*B*a)*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(4/3)/b^(5/3)-1/9*(A*b+2*B*a)*ln(a^(1/3)
+b^(1/3)*x)/a^(4/3)/b^(5/3)+1/18*(A*b+2*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/a^(4/3)/b^(5/3)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.85

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx$$
$$= \frac{-\frac{6\sqrt[3]{a}b^{2/3}(-Ab+aB)x^2}{a+bx^3} - 2\sqrt{3}(Ab+2aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) + (Ab+2a}{18a^{4/3}b^{5/3}}$$

input
```
Integrate[(x*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((-6*a^(1/3)*b^(2/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3) - 2*Sqrt[3]*(A*b + 2*
a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(A*b + 2*a*B)*Log[a^(
1/3) + b^(1/3)*x] + (A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(18*a^(4/3)*b^(5/3))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {957, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}dx$$
$$\Big\downarrow \ 957$$
$$\frac{(2aB+Ab)\int \frac{x}{bx^3+a}dx}{3ab} + \frac{x^2(Ab-aB)}{3ab(a+bx^3)}$$
$$\Big\downarrow \ 821$$

$$\frac{(2aB+Ab)\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3ab}+\frac{x^2(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$\Big\downarrow$ 16

$$\frac{(2aB+Ab)\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3ab}+\frac{x^2(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$\Big\downarrow$ 1142

$$\frac{(2aB+Ab)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3ab}+\frac{x^2(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$\Big\downarrow$ 25

$$\frac{(2aB+Ab)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3ab}+\frac{x^2(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$\Big\downarrow$ 27

$$\frac{(2aB+Ab)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3ab}+\frac{x^2(Ab-aB)}{3ab\,(a+bx^3)}$$

$$\Big\downarrow \; 1082$$

$$\frac{(2aB+Ab)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3ab}+\frac{x^2(Ab-aB)}{3ab\,(a+bx^3)}$$

$$\Big\downarrow \; 217$$

$$\frac{(2aB+Ab)\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3ab}+\frac{x^2(Ab-aB)}{3ab\,(a+bx^3)}$$

$$\Big\downarrow \; 1103$$

$$\frac{(2aB+Ab)\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3ab}+\frac{x^2(Ab-aB)}{3ab\left(a+bx^3\right)}$$

input
```
Int[(x*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^2)/(3*a*b*(a + b*x^3)) + ((A*b + 2*a*B)*(-1/3*Log[a^(1/3) +
 b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(
2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/(3*a*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 957

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{(Ab-Ba)x^2}{3ab(b\,x^3+a)}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab+2Ba)\ln\left(x-\_R\right)}{\_R}}{9a\,b^2}$	67
default	$\frac{(Ab-Ba)x^2}{3ab(b\,x^3+a)}+\frac{(Ab+2Ba)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab}$	136

input
```
int(x*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(A*b-B*a)*x^2/a/b/(b*x^3+a)+1/9/a/b^2*sum((A*b+2*B*a)/_R*ln(x-_R),_R=R
ootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.20

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \left[-\frac{6\,(Ba^2b^2 - Aab^3)x^2 - 3\,\sqrt{\frac{1}{3}}(2\,Ba^3b + Aa^2b^2 + (2\,Ba^2b^2 + Aab^3)x^3)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3 - ab + 3\,\sqrt{\frac{1}{3}}\left(abx\right.}{}\right.}{} \right.$$

$$-\frac{6\,(Ba^2b^2 - Aab^3)x^2 - 6\,\sqrt{\frac{1}{3}}(2\,Ba^3b + Aa^2b^2 + (2\,Ba^2b^2 + Aab^3)x^3)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx + (-ab\right.}{}\right.}{}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/18*(6*(B*a^2*b^2 - A*a*b^3)*x^2 - 3*sqrt(1/3)*(2*B*a^3*b + A*a^2*b^2 +
 (2*B*a^2*b^2 + A*a*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b
+ 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b
^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((2*B*a*b + A*b^2)*x^3 +
 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^
2)^(2/3)) + 2*((2*B*a*b + A*b^2)*x^3 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*log
(b*x - (-a*b^2)^(1/3)))/(a^2*b^4*x^3 + a^3*b^3), -1/18*(6*(B*a^2*b^2 - A*a
*b^3)*x^2 - 6*sqrt(1/3)*(2*B*a^3*b + A*a^2*b^2 + (2*B*a^2*b^2 + A*a*b^3)*x
^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt
(-(-a*b^2)^(1/3)/a)/b) - ((2*B*a*b + A*b^2)*x^3 + 2*B*a^2 + A*a*b)*(-a*b^2
)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((2*B*a*b +
 A*b^2)*x^3 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(
a^2*b^4*x^3 + a^3*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.68

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{x^2(Ab-Ba)}{3a^2b+3ab^2x^3} + \text{RootSum}\left(729t^3a^4b^5+A^3b^3+6A^2Bab^2+12AB^2a^2b+8B^3a^3, \left(t\mapsto t\log\left(\frac{81t^2a^3b^3}{A^2b^2+4ABab+4B^2a^2}\right.\right.\right.$$

input
```
integrate(x*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
x**2*(A*b - B*a)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**4*b**5
+ A**3*b**3 + 6*A**2*B*a*b**2 + 12*A*B**2*a**2*b + 8*B**3*a**3, Lambda(_t,
 _t*log(81*_t**2*a**3*b**3/(A**2*b**2 + 4*A*B*a*b + 4*B**2*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.94

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(Ba-Ab)x^2}{3\,(ab^2x^3+a^2b)} + \frac{\sqrt{3}(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(2\,Ba+Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(2\,Ba+Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(B*a - A*b)*x^2/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(2*B*a + A*b)*arcta
n(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(1/3)) + 1/18*
(2*B*a + A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(1/3)) -
 1/9*(2*B*a + A*b)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.09

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{\sqrt{3}(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}ab} - \frac{(2\,Ba+Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}ab} - \frac{\left(2\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}+Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b} - \frac{Bax^2-Abx^2}{3\,(bx^3+a)ab}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*sqrt(3)*(2*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(1/3)*a*b) - 1/18*(2*B*a + A*b)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/((-a*b^2)^(1/3)*a*b) - 1/9*(2*B*a*(-a/b)^(1/3) + A*b*(-a/b)
^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) - 1/3*(B*a*x^2 - A
*b*x^2)/((b*x^3 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{x^2\,(A\,b-B\,a)}{3\,a\,b\,(b\,x^3+a)} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b+2\,B\,a)}{9\,a^{4/3}\,b^{5/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b+2\,B\,a)}{9\,a^{4/3}\,b^{5/3}} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(A\,b+2\,B\,a)}{9\,a^{4/3}\,b^{5/3}}$$

input
```
int((x*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
(log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A
*b + 2*B*a))/(9*a^(4/3)*b^(5/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x +
 a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b + 2*B*a))/(9*a^(4/3)*b^(5/3)) - (log
(b^(1/3)*x + a^(1/3))*(A*b + 2*B*a))/(9*a^(4/3)*b^(5/3)) + (x^2*(A*b - B*a
))/(3*a*b*(a + b*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.38

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)+\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{6b^{\frac{2}{3}}a^{\frac{1}{3}}}$$

input
```
int(x*(B*x^3+A)/(b*x^3+a)^2,x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3))) + log(a**
(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) - 2*log(a**(1/3) + b**(1/3)*x
))/(6*b**(2/3)*a**(1/3))
```

3.86 $\int \frac{A+Bx^3}{\left(a+bx^3\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 169

$$\begin{aligned}\int \frac{A+Bx^3}{\left(a+bx^3\right)^2}\,dx &= \frac{(Ab-aB)x}{3ab\left(a+bx^3\right)}-\frac{(2Ab+aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}}\\ &\quad+\frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}\\ &\quad-\frac{(2Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}\end{aligned}$$

output
```
1/3*(A*b-B*a)*x/a/b/(b*x^3+a)-1/9*(2*A*b+B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3
)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/b^(4/3)+1/9*(2*A*b+B*a)*ln(a^(1/3)+b
^(1/3)*x)/a^(5/3)/b^(4/3)-1/18*(2*A*b+B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/a^(5/3)/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx = \frac{-\frac{6a^{2/3}\sqrt[3]{b}(-Ab+aB)x}{a+bx^3}-2\sqrt{3}(2Ab+aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-(2Ab+a}{18a^{5/3}b^{4/3}}$$

input
```
Integrate[(A + B*x^3)/(a + b*x^3)^2,x]
```

output
```
((-6*a^(2/3)*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3) - 2*Sqrt[3]*(2*A*b + a*
B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(2*A*b + a*B)*Log[a^(1/
3) + b^(1/3)*x] - (2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(18*a^(5/3)*b^(4/3))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {910, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(aB+2Ab)\int \frac{1}{bx^3+a}dx}{3ab}+\frac{x(Ab-aB)}{3ab\,(a+bx^3)}$$

$$\Big\downarrow \text{750}$$

$$\frac{(aB+2Ab)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3ab}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{16}$$

$$\frac{(aB+2Ab)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{(aB+2Ab)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{(aB+2Ab)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(aB+2Ab)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(aB+2Ab)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{(aB+2Ab)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{(aB+2Ab)\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

input
```
Int[(A + B*x^3)/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x)/(3*a*b*(a + b*x^3)) + ((2*A*b + a*B)*(Log[a^(1/3) + b^(1/3
)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(
1/3)))/(3*a^(2/3))))/(3*a*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.38

method	result	size
risch	$\frac{(Ab-Ba)x}{3ab(b\,x^3+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^3+a\right)}\frac{(2Ab+Ba)\ln\left(x-\_R\right)}{\_R^2}}{9a\,b^2}$	65
default	$\frac{(Ab-Ba)x}{3ab(b\,x^3+a)}+\frac{(2Ab+Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3ab}$	134

input
```
int((B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(A*b-B*a)*x/a/b/(b*x^3+a)+1/9/a/b^2*sum((2*A*b+B*a)/_R^2*ln(x-_R),_R=R
ootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.18

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx$$
$$= \left[\frac{3\sqrt{\frac{1}{3}}(Ba^3b+2\,Aa^2b^2+(Ba^2b^2+2\,Aab^3)x^3)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b\right.}{bx^3+a}\right.}{} \right.$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(1/3)*(B*a^3*b + 2*A*a^2*b^2 + (B*a^2*b^2 + 2*A*a*b^3)*x^3)*s
qrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(
1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b
))/(b*x^3 + a)) - ((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*
log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b + 2*A*b^2)*x^
3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^3*b
 - A*a^2*b^2)*x)/(a^3*b^3*x^3 + a^4*b^2), 1/18*(6*sqrt(1/3)*(B*a^3*b + 2*A
*a^2*b^2 + (B*a^2*b^2 + 2*A*a*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(
1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((
B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b
)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)
*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^3*b - A*a^2*b^2)*x)/(a^
3*b^3*x^3 + a^4*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.57

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx = \frac{x(Ab-Ba)}{3a^2b+3ab^2x^3} + \operatorname{RootSum}\left(729t^3a^5b^4-8A^3b^3-12A^2Bab^2-6AB^2a^2b-B^3a^3, \left(t\mapsto t\log\left(\frac{9ta^2b}{2Ab+Ba}+x\right)\right)\right)$$

input
```
integrate((B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
x*(A*b - B*a)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**5*b**4 - 8
*A**3*b**3 - 12*A**2*B*a*b**2 - 6*A*B**2*a**2*b - B**3*a**3, Lambda(_t, _t
*log(9*_t*a**2*b/(2*A*b + B*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx = -\frac{(Ba-Ab)x}{3\,(ab^2x^3+a^2b)} + \frac{\sqrt{3}(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba+2\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba+2\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(B*a - A*b)*x/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(B*a + 2*A*b)*arctan(
1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/18*(B
*a + 2*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1
/9*(B*a + 2*A*b)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}a} - \frac{(Ba+2\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}a} - \frac{(Ba+2\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b} - \frac{Bax-Abx}{3\,(bx^3+a)ab}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(B*a + 2*A*b)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(B*a + 2*A*b)*(-a/b)^(1/3)*log(abs(
x - (-a/b)^(1/3)))/(a^2*b) - 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(2\,A\,b+B\,a)}{9\,a^{5/3}\,b^{4/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b+B\,a)}{9\,a^{5/3}\,b^{4/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b+B\,a)}{9\,a^{5/3}\,b^{4/3}} + \frac{x\,(A\,b-B\,a)}{3\,a\,b\,\left(b\,x^3+a\right)}$$

input
```
int((A + B*x^3)/(a + b*x^3)^2,x)
```

output
```
(log(b^(1/3)*x + a^(1/3))*(2*A*b + B*a))/(9*a^(5/3)*b^(4/3)) - (log(3^(1/2
)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*A*b + B*a)
)/(9*a^(5/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*(
(3^(1/2)*1i)/2 - 1/2)*(2*A*b + B*a))/(9*a^(5/3)*b^(4/3)) + (x*(A*b - B*a))
/(3*a*b*(a + b*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)+2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{6a^{\frac{2}{3}}b^{\frac{1}{3}}}$$

input
```
int((B*x^3+A)/(b*x^3+a)^2,x)
```

output
```
(a**(1/3)*( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))
 - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) + 2*log(a**(1/3) +
b**(1/3)*x)))/(6*b**(1/3)*a)
```

3.87 $\int \frac{A+Bx^3}{x^2(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 180

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^2}\,dx = -\frac{A}{a^2x} - \frac{(Ab-aB)x^2}{3a^2(a+bx^3)} + \frac{(4Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{2/3}} + \frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{2/3}} - \frac{(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{2/3}}$$

output

```
-A/a^2/x-1/3*(A*b-B*a)*x^2/a^2/(b*x^3+a)+1/9*(4*A*b-B*a)*arctan(1/3*(a^(1/
3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(7/3)/b^(2/3)+1/9*(4*A*b-B*a)*l
n(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(2/3)-1/18*(4*A*b-B*a)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^2}\,dx$$

$$=\frac{-\frac{18\sqrt[3]{a}A}{x}+\frac{6\sqrt[3]{a}(-Ab+aB)x^2}{a+bx^3}+\frac{2\sqrt{3}(4Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{2(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{(-4Ab+aB)\log\left(a^{2/3}-\right.}{b^{2/3}}}{18a^{7/3}}$$

input
```
Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^2),x]
```

output
```
((-18*a^(1/3)*A)/x + (6*a^(1/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3) + (2*Sqrt[
3]*(4*A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2
*(4*A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-4*A*b + a*B)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(18*a^(7/3))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {957, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(4Ab-aB)\int \frac{1}{x^2(bx^3+a)}dx}{3ab}+\frac{Ab-aB}{3abx(a+bx^3)}$$

$$\Big\downarrow \text{847}$$

$$\frac{(4Ab-aB)\left(-\frac{b\int\frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{3ab}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$

$\downarrow$ 821

$$\frac{(4Ab-aB)\left(-\frac{b\left(\frac{\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{3ab}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$

$\downarrow$ 16

$$\frac{(4Ab-aB)\left(-\frac{b\left(\frac{\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3ab}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$

$\downarrow$ 1142

$$\frac{(4Ab-aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3ab}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$

$\Big\downarrow$ 25

$$\frac{(4Ab-aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3ab}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$

$\Big\downarrow$ 27

$$\frac{(4Ab-aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3ab}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$

$\Big\downarrow$ 1082

$$\frac{(4Ab-aB)\left(-\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3ab}+$$

$$\frac{Ab-aB}{3abx\,(a+bx^3)}$$

$\big\downarrow$ 217

$$\frac{(4Ab-aB)\left(-\frac{b\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3ab}+$$

$$\frac{Ab-aB}{3abx\,(a+bx^3)}$$

$\big\downarrow$ 1103

$$\frac{(4Ab-aB)\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3ab}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$

input
```
Int[(A + B*x^3)/(x^2*(a + b*x^3)^2),x]
```

output
```
(A*b - a*B)/(3*a*b*x*(a + b*x^3)) + ((4*A*b - a*B)*(-(1/(a*x)) - (b*(-1/3*
Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a))/(3*a*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.77

method	result
default	$-\frac{A}{a^{2} x}-\frac{\frac{\left(\frac{A b}{3}-\frac{B a}{3}\right) x^{2}}{b\,x^{3}+a}+\left(\frac{4 A b}{3}-\frac{B a}{3}\right) \left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\, \arctan\left(\frac{\sqrt{3}\, \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{2}}$
risch	$\frac{-\frac{(4 A b-B a) x^{3}}{3 a^{2}}-\frac{A}{a}}{x\left(b\,x^{3}+a\right)}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{7} b^{2} \_Z^{3}-64 A^{3} b^{3}+48 A^{2} B a\, b^{2}-12 A\, B^{2} a^{2} b+B^{3} a^{3}\right)}\_R \ln\left(\left(-4 \_R^{3} a^{7} b^{2}+192 A^{3} b^{3}-144 A^{2} B\right.\right.\right.}{9}$

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-A/a^2/x-1/a^2*((1/3*A*b-1/3*B*a)*x^2/(b*x^3+a)+(4/3*A*b-1/3*B*a)*(-1/3/b/
(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))
)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 570, normalized size of antiderivative = 3.17

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/18*(18*A*a^2*b^2 - 6*(B*a^2*b^2 - 4*A*a*b^3)*x^3 + 3*sqrt(1/3)*((B*a^2
*b^2 - 4*A*a*b^3)*x^4 + (B*a^3*b - 4*A*a^2*b^2)*x)*sqrt(-(a*b^2)^(1/3)/a)*
log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^
(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((B*a*
b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2
)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b
)*x)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^3*x^4 + a^4*b^2*x), -1
/18*(18*A*a^2*b^2 - 6*(B*a^2*b^2 - 4*A*a*b^3)*x^3 + 6*sqrt(1/3)*((B*a^2*b^
2 - 4*A*a*b^3)*x^4 + (B*a^3*b - 4*A*a^2*b^2)*x)*sqrt((a*b^2)^(1/3)/a)*arct
an(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - ((B*a*b -
 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(
1/3)*b*x + (a*b^2)^(2/3)) + 2*((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x
)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^3*x^4 + a^4*b^2*x)]
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx = \frac{-3Aa+x^3(-4Ab+Ba)}{3a^3x+3a^2bx^4}$$
$$+\text{RootSum}\left(729t^3a^7b^2-64A^3b^3+48A^2Bab^2-12AB^2a^2b+B^3a^3,\left(t\mapsto t\log\left(\frac{81t^2a^5b}{16A^2b^2-8ABab+B\right.\right.\right.$$

input
```
integrate((B*x**3+A)/x**2/(b*x**3+a)**2,x)
```

output
```
(-3*A*a + x**3*(-4*A*b + B*a))/(3*a**3*x + 3*a**2*b*x**4) + RootSum(729*_t
**3*a**7*b**2 - 64*A**3*b**3 + 48*A**2*B*a*b**2 - 12*A*B**2*a**2*b + B**3*
a**3, Lambda(_t, _t*log(81*_t**2*a**5*b/(16*A**2*b**2 - 8*A*B*a*b + B**2*a
**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^2}\,dx = \frac{(Ba-4\,Ab)x^3-3\,Aa}{3\,(a^2bx^4+a^3x)} + \frac{\sqrt{3}(Ba-4\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba-4\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba-4\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*((B*a - 4*A*b)*x^3 - 3*A*a)/(a^2*b*x^4 + a^3*x) + 1/9*sqrt(3)*(B*a - 4
*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(1/
3)) + 1/18*(B*a - 4*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/
b)^(1/3)) - 1/9*(B*a - 4*A*b)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx = \frac{\sqrt{3}(Ba-4\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}a^2} - \frac{(Ba-4\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}a^2} - \frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3} + \frac{Bax^3-4\,Abx^3-3\,Aa}{3\,(bx^4+ax)a^2}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*sqrt(3)*(B*a - 4*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(1/3)*a^2) - 1/18*(B*a - 4*A*b)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2) - 1/9*(B*a*(-a/b)^(1/3) - 4*A*b*(-a/b)
^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/3*(B*a*x^3 - 4*A*b
*x^3 - 3*A*a)/((b*x^4 + a*x)*a^2)
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(4\,A\,b-B\,a)}{9\,a^{7/3}\,b^{2/3}} - \frac{\frac{A}{a}+\frac{x^3\,(4\,A\,b-B\,a)}{3\,a^2}}{b\,x^4+a\,x} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(4\,A\,b-B\,a)}{9\,a^{7/3}\,b^{2/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(4\,A\,b-B\,a)}{9\,a^{7/3}\,b^{2/3}}$$

input
```
int((A + B*x^3)/(x^2*(a + b*x^3)^2),x)
```

output
```
(log(b^(1/3)*x + a^(1/3))*(4*A*b - B*a))/(9*a^(7/3)*b^(2/3)) - (A/a + (x^3
*(4*A*b - B*a))/(3*a^2))/(a*x + b*x^4) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/
3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*A*b - B*a))/(9*a^(7/3)*b^(2/3))
- (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(4*A*b - B*a))/(9*a^(7/3)*b^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.47

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx$$
$$=\frac{2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)bx-6b^{\frac{2}{3}}a^{\frac{1}{3}}-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)bx+2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)bx}{6b^{\frac{2}{3}}a^{\frac{4}{3}}x}$$

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^2,x)
```

output
```
(2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b*x - 6*b**(
2/3)*a**(1/3) - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x +
2*log(a**(1/3) + b**(1/3)*x)*b*x)/(6*b**(2/3)*a**(1/3)*a*x)
```

3.88 $\int \frac{A+Bx^3}{x^3(a+bx^3)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 180

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^2} dx = -\frac{A}{2a^2x^2} - \frac{(Ab-aB)x}{3a^2(a+bx^3)} + \frac{(5Ab-2aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}} - \frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}} + \frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}\sqrt[3]{b}}$$

output
```
-1/2*A/a^2/x^2-1/3*(A*b-B*a)*x/a^2/(b*x^3+a)+1/9*(5*A*b-2*B*a)*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3)/b^(1/3)-1/9*(5*A*b-
2*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(1/3)+1/18*(5*A*b-2*B*a)*ln(a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx$$
$$=\frac{-\frac{9a^{2/3}A}{x^2}+\frac{6a^{2/3}(-Ab+aB)x}{a+bx^3}+\frac{2\sqrt{3}(5Ab-2aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{2(-5Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{(5Ab-2aB)\log\left(a^{2/3}-\right.}{\sqrt[3]{\ }}}{18a^{8/3}}$$

input
```
Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^2),x]
```

output
```
((-9*a^(2/3)*A)/x^2 + (6*a^(2/3)*(-(A*b) + a*B)*x)/(a + b*x^3) + (2*Sqrt[3
]*(5*A*b - 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (
2*(-5*A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + ((5*A*b - 2*a*B)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(18*a^(8/3))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {957, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{(5Ab-2aB)\int \frac{1}{x^3\left(bx^3+a\right)}dx}{3ab}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{(5Ab-2aB)\left(-\frac{b\int \frac{1}{bx^3+a}dx}{a}-\frac{1}{2ax^2}\right)}{3ab}+\frac{Ab-aB}{3abx^2(a+bx^3)}$$

$$\Big\downarrow \text{750}$$

$$\frac{(5Ab-2aB)\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}\right)}{3ab}+\frac{Ab-aB}{3abx^2(a+bx^3)}$$

$$\Big\downarrow \text{16}$$

$$\frac{(5Ab-2aB)\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3ab}+\frac{Ab-aB}{3abx^2(a+bx^3)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{(5Ab-2aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3ab}+\frac{Ab-aB}{3abx^2(a+bx^3)}$$

$$\Big\downarrow \; 25$$

$$\frac{(5Ab-2aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3ab}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{(5Ab-2aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3ab}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{(5Ab-2aB)\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3ab}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{(5Ab-2aB)\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3ab}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{(5Ab-2aB)\left(-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3ab}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

input
```
Int[(A + B*x^3)/(x^3*(a + b*x^3)^2),x]
```

output
```
(A*b - a*B)/(3*a*b*x^2*(a + b*x^3)) + ((5*A*b - 2*a*B)*(-1/2*1/(a*x^2) - (
b*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 -
(2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/a))/(3*a*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.77

method	result
default	$-\frac{A}{2a^2x^2}-\frac{\frac{\left(\frac{Ab}{3}-\frac{Ba}{3}\right)x}{b\,x^3+a}+\frac{(5Ab-2Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}}{a^2}$
risch	$\frac{-\frac{(5Ab-2Ba)x^3}{6a^2}-\frac{A}{2a}}{x^2(b\,x^3+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^8b\_Z^3+125A^3b^3-150A^2Ba\,b^2+60A\,B^2a^2b-8B^3a^3\right)}\_R\ln\left(\left(-4\_R^3a^8b-375A^3b^3+450A\right.\right.\right.}{9}$

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/a^2/x^2-1/a^2*((1/3*A*b-1/3*B*a)*x/(b*x^3+a)+1/3*(5*A*b-2*B*a)*(1/3
/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(139) = 278$.

Time = 0.11 (sec) , antiderivative size = 618, normalized size of antiderivative = 3.43

$$\int \frac{A + Bx^3}{x^3 (a + bx^3)^2} \, dx = \text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/18*(9*A*a^3*b - 3*(2*B*a^3*b - 5*A*a^2*b^2)*x^3 + 3*sqrt(1/3)*((2*B*a^
2*b^2 - 5*A*a*b^3)*x^5 + (2*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt((-a^2*b)^(1/3
)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2
+ (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)
) + ((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*log
(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*((2*B*a*b - 5*A*b^2)*x
^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/
(a^4*b^2*x^5 + a^5*b*x^2), -1/18*(9*A*a^3*b - 3*(2*B*a^3*b - 5*A*a^2*b^2)*
x^3 - 6*sqrt(1/3)*((2*B*a^2*b^2 - 5*A*a*b^3)*x^5 + (2*B*a^3*b - 5*A*a^2*b^
2)*x^2)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a
^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) + ((2*B*a*b - 5*A*b^2)*x^5 + (
2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-
a^2*b)^(1/3)*a) - 2*((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-
a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^2*x^5 + a^5*b*x^2)]
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.61

$$\int \frac{A + Bx^3}{x^3 (a + bx^3)^2} \, dx = \frac{-3Aa + x^3(-5Ab + 2Ba)}{6a^3x^2 + 6a^2bx^5} + \operatorname{RootSum}\left(729t^3a^8b + 125A^3b^3 - 150A^2Bab^2 + 60AB^2a^2b - 8B^3a^3, \left(t \mapsto t\log\left(\frac{9ta^3}{-5Ab + 2Ba} + x\right)\right)\right)$$

input
```
integrate((B*x**3+A)/x**3/(b*x**3+a)**2,x)
```

output
```
(-3*A*a + x**3*(-5*A*b + 2*B*a))/(6*a**3*x**2 + 6*a**2*b*x**5) + RootSum(7
29*_t**3*a**8*b + 125*A**3*b**3 - 150*A**2*B*a*b**2 + 60*A*B**2*a**2*b - 8
*B**3*a**3, Lambda(_t, _t*log(9*_t*a**3/(-5*A*b + 2*B*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx = \frac{(2\,Ba-5\,Ab)x^3-3\,Aa}{6\,(a^2bx^5+a^3x^2)} + \frac{\sqrt{3}(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(2\,Ba-5\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(2\,Ba-5\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/6*((2*B*a - 5*A*b)*x^3 - 3*A*a)/(a^2*b*x^5 + a^3*x^2) + 1/9*sqrt(3)*(2*B
*a - 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/
b)^(2/3)) - 1/18*(2*B*a - 5*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a
^2*b*(a/b)^(2/3)) + 1/9*(2*B*a - 5*A*b)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^
(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{x^3 (a+bx^3)^2}\,dx = -\frac{(2\,Ba-5\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3} + \frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}Ba-5\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b} + \frac{Bax-Abx}{3\,(bx^3+a)a^2} + \frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}Ba-5\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b} - \frac{A}{2\,a^2x^2}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*(2*B*a - 5*A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/9*sqr
t(3)*(2*(-a*b^2)^(1/3)*B*a - 5*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x
 + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) + 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*
a^2) + 1/18*(2*(-a*b^2)^(1/3)*B*a - 5*(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a^3*b) - 1/2*A/(a^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^3 (a+bx^3)^2}\,dx = -\frac{\frac{A}{2\,a}+\frac{x^3\,(5\,A\,b-2\,B\,a)}{6\,a^2}}{b\,x^5+a\,x^2} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(5\,A\,b-2\,B\,a)}{9\,a^{8/3}\,b^{1/3}} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-2\,B\,a)}{9\,a^{8/3}\,b^{1/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-2\,B\,a)}{9\,a^{8/3}\,b^{1/3}}$$

input
```
int((A + B*x^3)/(x^3*(a + b*x^3)^2),x)
```

output
```
(log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5
*A*b - 2*B*a))/(9*a^(8/3)*b^(1/3)) - (log(b^(1/3)*x + a^(1/3))*(5*A*b - 2*
B*a))/(9*a^(8/3)*b^(1/3)) - (A/(2*a) + (x^3*(5*A*b - 2*B*a))/(6*a^2))/(a*x
^2 + b*x^5) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1
i)/2 - 1/2)*(5*A*b - 2*B*a))/(9*a^(8/3)*b^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.53

$$\int \frac{A+Bx^3}{x^3 (a+bx^3)^2} \, dx = \frac{2a^{\frac{1}{3}}\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) b\, x^2 + a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right) b\, x^2 - 2a^{\frac{1}{3}}\log\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) b\, x^2 - 3b^{\frac{1}{3}}a}{6b^{\frac{1}{3}}a^2x^2}$$

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^2,x)
```

output
```
(2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b*x
**2 + a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**2
- 2*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b*x**2 - 3*b**(1/3)*a)/(6*b**(1/3)
*a**2*x**2)
```

3.89 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 199

$$\begin{aligned}\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx =& -\frac{A}{4a^2x^4}+\frac{2Ab-aB}{a^3x}+\frac{b(Ab-aB)x^2}{3a^3\left(a+bx^3\right)} \\ &-\frac{\sqrt[3]{b}(7Ab-4aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}} \\ &-\frac{\sqrt[3]{b}(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}} \\ &+\frac{\sqrt[3]{b}(7Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{10/3}}\end{aligned}$$

output
```
-1/4*A/a^2/x^4+(2*A*b-B*a)/a^3/x+1/3*b*(A*b-B*a)*x^2/a^3/(b*x^3+a)-1/9*b^(
1/3)*(7*A*b-4*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/
2)/a^(10/3)-1/9*b^(1/3)*(7*A*b-4*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)+1/18*
b^(1/3)*(7*A*b-4*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^3}{x^5 (a+bx^3)^2}\,dx$$
$$= \frac{-\frac{9a^{4/3}A}{x^4} - \frac{36\sqrt[3]{a}(-2Ab+aB)}{x} - \frac{12\sqrt[3]{a}b(-Ab+aB)x^2}{a+bx^3} - 4\sqrt{3}\sqrt[3]{b}(7Ab-4aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 4\sqrt[3]{b}(-7Ab+}{36a^{10/3}}$$

input
```
Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^2),x]
```

output
```
((-9*a^(4/3)*A)/x^4 - (36*a^(1/3)*(-2*A*b + a*B))/x - (12*a^(1/3)*b*(-(A*b
) + a*B)*x^2)/(a + b*x^3) - 4*Sqrt[3]*b^(1/3)*(7*A*b - 4*a*B)*ArcTan[(1 -
(2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*(-7*A*b + 4*a*B)*Log[a^(1/3) +
 b^(1/3)*x] + 2*b^(1/3)*(7*A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(36*a^(10/3))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {957, 847, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^5 (a+bx^3)^2}\,dx$$

$$\Big\downarrow \; 957$$

$$\frac{(7Ab-4aB)\int \frac{1}{x^5(bx^3+a)}dx}{3ab} + \frac{Ab-aB}{3abx^4(a+bx^3)}$$

$$\Big\downarrow \; 847$$

$$\frac{(7Ab-4aB)\left(-\frac{b\int \frac{1}{x^2(bx^3+a)}dx}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4(a+bx^3)}$$

$\Big\downarrow$ 847

$$\frac{(7Ab-4aB)\left(-\frac{b\left(-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4(a+bx^3)}$$

$\Big\downarrow$ 821

$$\frac{(7Ab-4aB)\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4(a+bx^3)}$$

$\Big\downarrow$ 16

$$\frac{(7Ab-4aB)\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{(7Ab-4aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{(7Ab-4aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

$\Big\downarrow$ 27

$$\frac{(7Ab-4aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{(7Ab-4aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{(7Ab-4aB)\left(-\frac{b\left(-\frac{b\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{(7Ab-4aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

input
```
Int[(A + B*x^3)/(x^5*(a + b*x^3)^2),x]
```

output
```
(A*b - a*B)/(3*a*b*x^4*(a + b*x^3)) + ((7*A*b - 4*a*B)*(-1/4*1/(a*x^4) - (
b*(-(1/(a*x)) - (b*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((
Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a)
)/a))/(3*a*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.78

method	result
default	$-\frac{A}{4a^2x^4}-\frac{-2Ab+Ba}{a^3x}+\frac{b\left(\frac{\left(\frac{Ab}{3}-\frac{Ba}{3}\right)x^2}{b\,x^3+a}+\left(\frac{7Ab}{3}-\frac{4Ba}{3}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\right.}{3}\right.}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.\right.}{a^3}$
risch	$\frac{\frac{b(7Ab-4Ba)x^6}{3a^3}+\frac{(7Ab-4Ba)x^3}{4a^2}-\frac{A}{4a}}{x^4(b\,x^3+a)}+\frac{\left(\sum_{\_R=\text{RootOf}\left(a^{10}\_Z^3+343A^3b^4-588A^2Ba\,b^3+336A\,B^2a^2b^2-64B^3a^3b\right)}\_R\ln\left(\left(-4a^{10}\_R^3\right.\right.\right.}{9}$

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/a^2/x^4-(-2*A*b+B*a)/a^3/x+1/a^3*b*((1/3*A*b-1/3*B*a)*x^2/(b*x^3+a)
+(7/3*A*b-4/3*B*a)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)
*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.30

$$\int\frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx=$$

$$-\frac{12\,(4\,Bab-7\,Ab^2)x^6+9\,(4\,Ba^2-7\,Aab)x^3+9\,Aa^2+4\,\sqrt{3}((4\,Bab-7\,Ab^2)x^7+(4\,Ba^2-7\,Aab)x^4)}{}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/36*(12*(4*B*a*b - 7*A*b^2)*x^6 + 9*(4*B*a^2 - 7*A*a*b)*x^3 + 9*A*a^2 +
4*sqrt(3)*((4*B*a*b - 7*A*b^2)*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a)^(1/3)
*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 2*((4*B*a*b - 7*A*b^2)
*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3)
- a*(-b/a)^(1/3)) + 4*((4*B*a*b - 7*A*b^2)*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*
(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^3*b*x^7 + a^4*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$$
$$= \text{RootSum}\left(729t^3a^{10}+343A^3b^4-588A^2Bab^3+336AB^2a^2b^2-64B^3a^3b,\left(t\mapsto t\log\left(\frac{81t^2a}{49A^2b^3-56ABa}\right.\right.\right.$$
$$+\frac{-3Aa^2+x^6\cdot(28Ab^2-16Bab)+x^3\cdot(21Aab-12Ba^2)}{12a^4x^4+12a^3bx^7}$$

input
```
integrate((B*x**3+A)/x**5/(b*x**3+a)**2,x)
```

output
```
RootSum(729*_t**3*a**10 + 343*A**3*b**4 - 588*A**2*B*a*b**3 + 336*A*B**2*a
**2*b**2 - 64*B**3*a**3*b, Lambda(_t, _t*log(81*_t**2*a**7/(49*A**2*b**3 -
 56*A*B*a*b**2 + 16*B**2*a**2*b) + x))) + (-3*A*a**2 + x**6*(28*A*b**2 - 1
6*B*a*b) + x**3*(21*A*a*b - 12*B*a**2))/(12*a**4*x**4 + 12*a**3*b*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^3}{x^5(a+bx^3)^2}\,dx = -\frac{4\,(4\,Bab-7\,Ab^2)x^6+3\,(4\,Ba^2-7\,Aab)x^3+3\,Aa^2}{12\,(a^3bx^7+a^4x^4)} - \frac{\sqrt{3}(4\,Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(4\,Ba-7\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(4\,Ba-7\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/12*(4*(4*B*a*b - 7*A*b^2)*x^6 + 3*(4*B*a^2 - 7*A*a*b)*x^3 + 3*A*a^2)/(a
^3*b*x^7 + a^4*x^4) - 1/9*sqrt(3)*(4*B*a - 7*A*b)*arctan(1/3*sqrt(3)*(2*x
 - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*(a/b)^(1/3)) - 1/18*(4*B*a - 7*A*b)*log(x
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(1/3)) + 1/9*(4*B*a - 7*A*b)*
log(x + (a/b)^(1/3))/(a^3*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx = \frac{\left(4\,Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7\,Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4} + \frac{\sqrt{3}\left(4\left(-ab^2\right)^{\frac{2}{3}}Ba-7\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4b} - \frac{Babx^2-Ab^2x^2}{3\left(bx^3+a\right)a^3} - \frac{\left(4\left(-ab^2\right)^{\frac{2}{3}}Ba-7\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^4b} - \frac{4\,Bax^3-8\,Abx^3+Aa}{4\,a^3x^4}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*(4*B*a*b*(-a/b)^(1/3) - 7*A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/a^4 + 1/9*sqrt(3)*(4*(-a*b^2)^(2/3)*B*a - 7*(-a*b^2)^(2/3)
*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b) - 1/3*
(B*a*b*x^2 - A*b^2*x^2)/((b*x^3 + a)*a^3) - 1/18*(4*(-a*b^2)^(2/3)*B*a - 7
*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/
4*(4*B*a*x^3 - 8*A*b*x^3 + A*a)/(a^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3}{x^5(a+bx^3)^2}\,dx$$
$$= \frac{\frac{x^3(7Ab-4Ba)}{4a^2}-\frac{A}{4a}+\frac{bx^6(7Ab-4Ba)}{3a^3}}{bx^7+ax^4}+\frac{(-b)^{1/3}\ln\left(a^{1/3}(-b)^{8/3}+b^3x\right)(7Ab-4Ba)}{9a^{10/3}}$$
$$+\frac{(-b)^{1/3}\ln\left(a^{1/3}(-b)^{8/3}-2b^3x+\sqrt{3}\,a^{1/3}(-b)^{8/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(7Ab-4Ba)}{9a^{10/3}}$$
$$-\frac{(-b)^{1/3}\ln\left(2b^3x-a^{1/3}(-b)^{8/3}+\sqrt{3}\,a^{1/3}(-b)^{8/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(7Ab-4Ba)}{9a^{10/3}}$$

input
```
int((A + B*x^3)/(x^5*(a + b*x^3)^2),x)
```

output
```
((x^3*(7*A*b - 4*B*a))/(4*a^2) - A/(4*a) + (b*x^6*(7*A*b - 4*B*a))/(3*a^3)
)/(a*x^4 + b*x^7) + ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) + b^3*x)*(7*A*b - 4
*B*a))/(9*a^(10/3)) + ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) - 2*b^3*x + 3^(1/
2)*a^(1/3)*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - 4*B*a))/(9*a^(10
/3)) - ((-b)^(1/3)*log(2*b^3*x - a^(1/3)*(-b)^(8/3) + 3^(1/2)*a^(1/3)*(-b)
^(8/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - 4*B*a))/(9*a^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.54

$$\int \frac{A+Bx^3}{x^5(a+bx^3)^2}\,dx$$
$$= \frac{-4\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2x^4-3b^{\frac{2}{3}}a^{\frac{4}{3}}+12b^{\frac{5}{3}}a^{\frac{1}{3}}x^3+2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)b^2x^4-4\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)b}{12b^{\frac{2}{3}}a^{\frac{7}{3}}x^4}$$

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^2,x)
```

output
```
( - 4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**2*x**4
 - 3*b**(2/3)*a**(1/3)*a + 12*b**(2/3)*a**(1/3)*b*x**3 + 2*log(a**(2/3) -
b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**2*x**4 - 4*log(a**(1/3) + b**(1/3)
*x)*b**2*x**4)/(12*b**(2/3)*a**(1/3)*a**2*x**4)
```

3.90 $\int \frac{A+Bx^3}{x^6(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 200

$$\int \frac{A+Bx^3}{x^6(a+bx^3)^2}\,dx = -\frac{A}{5a^2x^5}+\frac{2Ab-aB}{2a^3x^2}+\frac{b(Ab-aB)x}{3a^3(a+bx^3)} - \frac{b^{2/3}(8Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}} + \frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{11/3}} - \frac{b^{2/3}(8Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{11/3}}$$

output
```
-1/5*A/a^2/x^5+1/2*(2*A*b-B*a)/a^3/x^2+1/3*b*(A*b-B*a)*x/a^3/(b*x^3+a)-1/9
*b^(2/3)*(8*A*b-5*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3
^(1/2)/a^(11/3)+1/9*b^(2/3)*(8*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)-1
/18*b^(2/3)*(8*A*b-5*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/
3)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^2} dx$$

$$= \frac{-\frac{18a^{5/3}A}{x^5} - \frac{45a^{2/3}(-2Ab+aB)}{x^2} - \frac{30a^{2/3}b(-Ab+aB)x}{a+bx^3} - 10\sqrt{3}b^{2/3}(8Ab-5aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 10b^{2/3}(8Ab}{90a^{11/3}}$$

input
```
Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^2),x]
```

output
```
((-18*a^(5/3)*A)/x^5 - (45*a^(2/3)*(-2*A*b + a*B))/x^2 - (30*a^(2/3)*b*(-(
A*b) + a*B)*x)/(a + b*x^3) - 10*Sqrt[3]*b^(2/3)*(8*A*b - 5*a*B)*ArcTan[(1
- (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*b^(2/3)*(8*A*b - 5*a*B)*Log[a^(1/3)
 + b^(1/3)*x] + 5*b^(2/3)*(-8*A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2]])/(90*a^(11/3))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {957, 847, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^2} dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{(8Ab-5aB)\int \frac{1}{x^6(bx^3+a)}dx}{3ab} + \frac{Ab-aB}{3abx^5(a+bx^3)}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{(8Ab-5aB)\left(-\frac{b\int \frac{1}{x^3(bx^3+a)}dx}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5(a+bx^3)}$$

$\big\downarrow$ 847

$$\frac{(8Ab-5aB)\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^3+a}dx}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5(a+bx^3)}$$

$\big\downarrow$ 750

$$\frac{(8Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5(a+bx^3)}$$

$\big\downarrow$ 16

$$\frac{(8Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

$\big\downarrow$ 1142

$$\frac{(8Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

$\downarrow$ 25

$$\frac{(8Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

$\Big\downarrow$ 27

$$\frac{(8Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{(8Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{(8Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3ab}+$$

$$\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

$\downarrow$ 1103

$$\frac{(8Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

input
```
Int[(A + B*x^3)/(x^6*(a + b*x^3)^2),x]
```

output
```
(A*b - a*B)/(3*a*b*x^5*(a + b*x^3)) + ((8*A*b - 5*a*B)*(-1/5*1/(a*x^5) - (
b*(-1/2*1/(a*x^2) - (b*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-(
(Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/a))/a))/(
3*a*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.77

method	result
default	$-\frac{A}{5a^2x^5}-\frac{-2Ab+Ba}{2x^2a^3}+\frac{b\left(\frac{\left(\frac{Ab}{3}-\frac{Ba}{3}\right)x}{b\,x^3+a}+\frac{(8Ab-5Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}\right)}{a^3}$
risch	$\frac{\frac{b(8Ab-5Ba)x^6}{6a^3}+\frac{(8Ab-5Ba)x^3}{10a^2}-\frac{A}{5a}}{x^5(b\,x^3+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}\_Z^3-512A^3b^5+960A^2Ba\,b^4-600A\,B^2a^2b^3+125B^3a^3b^2\right)}\_R\ln\left(\left(-4\_R^3a\right.\right.\right.}{}$

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*A/a^2/x^5-1/2*(-2*A*b+B*a)/x^2/a^3+1/a^3*b*((1/3*A*b-1/3*B*a)*x/(b*x^
3+a)+1/3*(8*A*b-5*B*a)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx =$$

$$-\frac{15\,(5\,Bab-8\,Ab^2)x^6+9\,(5\,Ba^2-8\,Aab)x^3+18\,Aa^2+10\,\sqrt{3}((5\,Bab-8\,Ab^2)x^8+(5\,Ba^2-8\,Aab)x}{}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/90*(15*(5*B*a*b - 8*A*b^2)*x^6 + 9*(5*B*a^2 - 8*A*a*b)*x^3 + 18*A*a^2 +
 10*sqrt(3)*((5*B*a*b - 8*A*b^2)*x^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^
(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*((5*B*
a*b - 8*A*b^2)*x^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1/3)*log(b^2*x^2
- a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 10*((5*B*a*b - 8*A*b^2)*x
^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3))
)/(a^3*b*x^8 + a^4*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx$$
$$= \text{RootSum}\left(729t^3a^{11} - 512A^3b^5 + 960A^2Bab^4 - 600AB^2a^2b^3 + 125B^3a^3b^2, \left(t \mapsto t\log\left(-\frac{9ta^4}{-8Ab^2 + 5Ba\right.\right.\right.$$
$$+ \frac{-6Aa^2 + x^6\cdot(40Ab^2 - 25Bab) + x^3\cdot(24Aab - 15Ba^2)}{30a^4x^5 + 30a^3bx^8}$$

input
```
integrate((B*x**3+A)/x**6/(b*x**3+a)**2,x)
```

output
```
RootSum(729*_t**3*a**11 - 512*A**3*b**5 + 960*A**2*B*a*b**4 - 600*A*B**2*a
**2*b**3 + 125*B**3*a**3*b**2, Lambda(_t, _t*log(-9*_t*a**4/(-8*A*b**2 + 5
*B*a*b) + x))) + (-6*A*a**2 + x**6*(40*A*b**2 - 25*B*a*b) + x**3*(24*A*a*b
 - 15*B*a**2))/(30*a**4*x**5 + 30*a**3*b*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^3}{x^6\,(a+bx^3)^2}\,dx = -\frac{5\,(5\,Bab-8\,Ab^2)x^6+3\,(5\,Ba^2-8\,Aab)x^3+6\,Aa^2}{30\,(a^3bx^8+a^4x^5)} - \frac{\sqrt{3}(5\,Ba-8\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(5\,Ba-8\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(5\,Ba-8\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/30*(5*(5*B*a*b - 8*A*b^2)*x^6 + 3*(5*B*a^2 - 8*A*a*b)*x^3 + 6*A*a^2)/(a
^3*b*x^8 + a^4*x^5) - 1/9*sqrt(3)*(5*B*a - 8*A*b)*arctan(1/3*sqrt(3)*(2*x
- (a/b)^(1/3))/(a/b)^(1/3))/(a^3*(a/b)^(2/3)) + 1/18*(5*B*a - 8*A*b)*log(x
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(2/3)) - 1/9*(5*B*a - 8*A*b)*
log(x + (a/b)^(1/3))/(a^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^3}{x^6(a+bx^3)^2}\,dx = -\frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba-8\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^4} + \frac{\left(5Bab-8Ab^2\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^4} - \frac{\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba-8\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^4} - \frac{Babx-Ab^2x}{3\left(bx^3+a\right)a^3} - \frac{5Bax^3-10Abx^3+2Aa}{10a^3x^5}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 8*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt
(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4 + 1/9*(5*B*a*b - 8*A*b^2)*(-a/b
)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 - 1/18*(5*(-a*b^2)^(1/3)*B*a - 8*(-
a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^4 - 1/3*(B*a*
b*x - A*b^2*x)/((b*x^3 + a)*a^3) - 1/10*(5*B*a*x^3 - 10*A*b*x^3 + 2*A*a)/(
a^3*x^5)
```

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^6(a+bx^3)^2}\,dx = \frac{\frac{x^3\,(8\,A\,b-5\,B\,a)}{10\,a^2}-\frac{A}{5\,a}+\frac{b\,x^6\,(8\,A\,b-5\,B\,a)}{6\,a^3}}{b\,x^8+a\,x^5} + \frac{b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(8\,A\,b-5\,B\,a)}{9\,a^{11/3}} - \frac{b^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(8\,A\,b-5\,B\,a)}{9\,a^{11/3}} + \frac{b^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(8\,A\,b-5\,B\,a)}{9\,a^{11/3}}$$

input
```
int((A + B*x^3)/(x^6*(a + b*x^3)^2),x)
```

output
```
((x^3*(8*A*b - 5*B*a))/(10*a^2) - A/(5*a) + (b*x^6*(8*A*b - 5*B*a))/(6*a^3
))/(a*x^5 + b*x^8) + (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(8*A*b - 5*B*a))/(9
*a^(11/3)) - (b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^
(1/2)*1i)/2 + 1/2)*(8*A*b - 5*B*a))/(9*a^(11/3)) + (b^(2/3)*log(3^(1/2)*a^
(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(8*A*b - 5*B*a))/
(9*a^(11/3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.57

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^2}\,dx = \frac{-10a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2x^5 - 5a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)b^2x^5 + 10a^{\frac{1}{3}}\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)b^2x^5 - 6b^{\frac{1}{3}}a^2}{30b^{\frac{1}{3}}a^3x^5}$$

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^2,x)
```

output
```
( - 10*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))
*b**2*x**5 - 5*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2
)*b**2*x**5 + 10*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b**2*x**5 - 6*b**(1/3
)*a**2 + 15*b**(1/3)*a*b*x**3)/(30*b**(1/3)*a**3*x**5)
```

3.91 $\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 107

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ab-3aB)x^3}{3b^4}+\frac{Bx^6}{6b^3}+\frac{a^3(Ab-aB)}{6b^5(a+bx^3)^2} - \frac{a^2(3Ab-4aB)}{3b^5(a+bx^3)}-\frac{a(Ab-2aB)\log(a+bx^3)}{b^5}$$

output
```
1/3*(A*b-3*B*a)*x^3/b^4+1/6*B*x^6/b^3+1/6*a^3*(A*b-B*a)/b^5/(b*x^3+a)^2-1/
3*a^2*(3*A*b-4*B*a)/b^5/(b*x^3+a)-a*(A*b-2*B*a)*ln(b*x^3+a)/b^5
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.88

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2b(Ab-3aB)x^3+b^2Bx^6+\frac{a^3(Ab-aB)}{(a+bx^3)^2}+\frac{2a^2(-3Ab+4aB)}{a+bx^3}+6a(-Ab+2aB)\log(a+bx^3)}{6b^5}$$

input
```
Integrate[(x^11*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
(2*b*(A*b - 3*a*B)*x^3 + b^2*B*x^6 + (a^3*(A*b - a*B))/(a + b*x^3)^2 + (2*
a^2*(-3*A*b + 4*a*B))/(a + b*x^3) + 6*a*(-(A*b) + 2*a*B)*Log[a + b*x^3])/(
6*b^5)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{x^9(Bx^3+A)}{(bx^3+a)^3}dx^3$$

$$\Big\downarrow \ 86$$

$$\frac{1}{3}\int \left(\frac{(aB-Ab)a^3}{b^4(bx^3+a)^3}-\frac{(4aB-3Ab)a^2}{b^4(bx^3+a)^2}+\frac{3(2aB-Ab)a}{b^4(bx^3+a)}+\frac{Bx^3}{b^3}+\frac{Ab-3aB}{b^4}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(\frac{a^3(Ab-aB)}{2b^5(a+bx^3)^2}-\frac{a^2(3Ab-4aB)}{b^5(a+bx^3)}-\frac{3a(Ab-2aB)\log(a+bx^3)}{b^5}+\frac{x^3(Ab-3aB)}{b^4}+\frac{Bx^6}{2b^3}\right)$$

input
```
Int[(x^11*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
(((A*b - 3*a*B)*x^3)/b^4 + (B*x^6)/(2*b^3) + (a^3*(A*b - a*B))/(2*b^5*(a +
 b*x^3)^2) - (a^2*(3*A*b - 4*a*B))/(b^5*(a + b*x^3)) - (3*a*(A*b - 2*a*B)*
Log[a + b*x^3])/b^5)/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

method	result
norman	$\frac{\frac{B\,x^{12}}{6b}-\frac{a^{2}\left(3abA-6a^{2}B\right)}{2b^{5}}+\frac{(Ab-2Ba)x^{9}}{3b^{2}}-\frac{2a\left(abA-2a^{2}B\right)x^{3}}{b^{4}}}{\left(b\,x^{3}+a\right)^{2}}-\frac{a(Ab-2Ba)\ln\left(b\,x^{3}+a\right)}{b^{5}}$
default	$\frac{\left(bB\,x^{3}+Ab-3Ba\right)^{2}}{6b^{5}B}-\frac{a\left(-\frac{a^{2}(Ab-Ba)}{2b\left(b\,x^{3}+a\right)^{2}}+\frac{a(3Ab-4Ba)}{b\left(b\,x^{3}+a\right)}+\frac{(3Ab-6Ba)\ln\left(b\,x^{3}+a\right)}{b}\right)}{3b^{4}}$
risch	$\frac{B\,x^{6}}{6b^{3}}+\frac{A\,x^{3}}{3b^{3}}-\frac{Ba\,x^{3}}{b^{4}}+\frac{A^{2}}{6b^{3}B}-\frac{Aa}{b^{4}}+\frac{3B\,a^{2}}{2b^{5}}+\frac{\left(-a^{2}bA+\frac{4}{3}a^{3}B\right)x^{3}-\frac{a^{3}(5Ab-7Ba)}{6b}}{b^{4}\left(b\,x^{3}+a\right)^{2}}-\frac{a\ln\left(b\,x^{3}+a\right)A}{b^{4}}+\frac{2a^{2}\ln(\ldots$
parallelrisch	$-\frac{-B\,x^{12}b^{4}-2A\,x^{9}b^{4}+4B\,x^{9}a\,b^{3}+6A\ln\left(b\,x^{3}+a\right)x^{6}a\,b^{3}-12B\ln\left(b\,x^{3}+a\right)x^{6}a^{2}b^{2}+12A\ln\left(b\,x^{3}+a\right)x^{3}a^{2}b^{2}-24B\ln\left(b\,x^{3}+a\right)x\ldots}{6b^{5}\left(b\,x^{3}+a\right)^{2}}$

input
```
int(x^11*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/6*B/b*x^12-1/2*a^2*(3*A*a*b-6*B*a^2)/b^5+1/3*(A*b-2*B*a)/b^2*x^9-2*a*(A
*a*b-2*B*a^2)/b^4*x^3)/(b*x^3+a)^2-a*(A*b-2*B*a)*ln(b*x^3+a)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.67

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx$$
$$= \frac{Bb^4x^{12}-2\,(2\,Bab^3-Ab^4)x^9-(11\,Ba^2b^2-4\,Aab^3)x^6+7\,Ba^4-5\,Aa^3b+2\,(Ba^3b-2\,Aa^2b^2)x^3+6\,((2}{6\,(b^7x^6+2\,ab^6x^3+a^2b^5)}$$

input
```
integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/6*(B*b^4*x^12 - 2*(2*B*a*b^3 - A*b^4)*x^9 - (11*B*a^2*b^2 - 4*A*a*b^3)*x
^6 + 7*B*a^4 - 5*A*a^3*b + 2*(B*a^3*b - 2*A*a^2*b^2)*x^3 + 6*((2*B*a^2*b^2
 - A*a*b^3)*x^6 + 2*B*a^4 - A*a^3*b + 2*(2*B*a^3*b - A*a^2*b^2)*x^3)*log(b
*x^3 + a))/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5)
```

Sympy [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^6}{6b^3}+\frac{a(-Ab+2Ba)\log{(a+bx^3)}}{b^5}+x^3\left(\frac{A}{3b^3}-\frac{Ba}{b^4}\right) + \frac{-5Aa^3b+7Ba^4+x^3(-6Aa^2b^2+8Ba^3b)}{6a^2b^5+12ab^6x^3+6b^7x^6}$$

input
```
integrate(x**11*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
B*x**6/(6*b**3) + a*(-A*b + 2*B*a)*log(a + b*x**3)/b**5 + x**3*(A/(3*b**3)
 - B*a/b**4) + (-5*A*a**3*b + 7*B*a**4 + x**3*(-6*A*a**2*b**2 + 8*B*a**3*b
))/(6*a**2*b**5 + 12*a*b**6*x**3 + 6*b**7*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{7\,Ba^4-5\,Aa^3b+2\,(4\,Ba^3b-3\,Aa^2b^2)x^3}{6\,(b^7x^6+2\,ab^6x^3+a^2b^5)} + \frac{Bbx^6-2\,(3\,Ba-Ab)x^3}{6\,b^4} + \frac{(2\,Ba^2-Aab)\log{(bx^3+a)}}{b^5}$$

input
```
integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/6*(7*B*a^4 - 5*A*a^3*b + 2*(4*B*a^3*b - 3*A*a^2*b^2)*x^3)/(b^7*x^6 + 2*a
*b^6*x^3 + a^2*b^5) + 1/6*(B*b*x^6 - 2*(3*B*a - A*b)*x^3)/b^4 + (2*B*a^2 -
 A*a*b)*log(b*x^3 + a)/b^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.22

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(2\,Ba^2-Aab)\log{(|bx^3+a|)}}{b^5} + \frac{Bb^3x^6-6\,Bab^2x^3+2\,Ab^3x^3}{6\,b^6} - \frac{18\,Ba^2b^2x^6-9\,Aab^3x^6+28\,Ba^3bx^3-12\,Aa^2b^2x^3+11\,Ba^4-4\,Aa^3b}{6\,(bx^3+a)^2b^5}$$

input
```
integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
(2*B*a^2 - A*a*b)*log(abs(b*x^3 + a))/b^5 + 1/6*(B*b^3*x^6 - 6*B*a*b^2*x^3
 + 2*A*b^3*x^3)/b^6 - 1/6*(18*B*a^2*b^2*x^6 - 9*A*a*b^3*x^6 + 28*B*a^3*b*x
^3 - 12*A*a^2*b^2*x^3 + 11*B*a^4 - 4*A*a^3*b)/((b*x^3 + a)^2*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\frac{7\,B\,a^4-5\,A\,a^3\,b}{6\,b}+x^3\,\left(\frac{4\,B\,a^3}{3}-A\,a^2\,b\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+x^3\,\left(\frac{A}{3\,b^3}-\frac{B\,a}{b^4}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(2\,B\,a^2-A\,a\,b\right)}{b^5}+\frac{B\,x^6}{6\,b^3}$$

input
```
int((x^11*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
((7*B*a^4 - 5*A*a^3*b)/(6*b) + x^3*((4*B*a^3)/3 - A*a^2*b))/(a^2*b^4 + b^6
*x^6 + 2*a*b^5*x^3) + x^3*(A/(3*b^3) - (B*a)/b^4) + (log(a + b*x^3)*(2*B*a
^2 - A*a*b))/b^5 + (B*x^6)/(6*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.21

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{6\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^3+6\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^2b\,x^3+6\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a^3+6\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\right.}{6b^4\left(b\,x^3+a\right)}$$

input
```
int(x^11*(B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
(6*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3 + 6*log(a**(2/
3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b*x**3 + 6*log(a**(1/3) + b
**(1/3)*x)*a**3 + 6*log(a**(1/3) + b**(1/3)*x)*a**2*b*x**3 - 6*a**2*b*x**3
 - 3*a*b**2*x**6 + b**3*x**9)/(6*b**4*(a + b*x**3))
```

3.92 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^3}{3b^3} - \frac{a^2(Ab-aB)}{6b^4(a+bx^3)^2} + \frac{a(2Ab-3aB)}{3b^4(a+bx^3)} + \frac{(Ab-3aB)\log(a+bx^3)}{3b^4}$$

output
```
1/3*B*x^3/b^3-1/6*a^2*(A*b-B*a)/b^4/(b*x^3+a)^2+1/3*a*(2*A*b-3*B*a)/b^4/(b
*x^3+a)+1/3*(A*b-3*B*a)*ln(b*x^3+a)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^3}{3b^3} + \frac{-a^2Ab+a^3B}{6b^4(a+bx^3)^2} + \frac{2aAb-3a^2B}{3b^4(a+bx^3)} + \frac{(Ab-3aB)\log(a+bx^3)}{3b^4}$$

input
```
Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
(B*x^3)/(3*b^3) + (-(a^2*A*b) + a^3*B)/(6*b^4*(a + b*x^3)^2) + (2*a*A*b -
3*a^2*B)/(3*b^4*(a + b*x^3)) + ((A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (A + Bx^3)}{(a + bx^3)^3} \, dx$$
$$\Big\downarrow \; 948$$
$$\frac{1}{3} \int \frac{x^6 (Bx^3 + A)}{(bx^3 + a)^3} dx^3$$
$$\Big\downarrow \; 86$$
$$\frac{1}{3} \int \left(-\frac{(aB - Ab)a^2}{b^3 (bx^3 + a)^3} + \frac{(3aB - 2Ab)a}{b^3 (bx^3 + a)^2} + \frac{B}{b^3} + \frac{Ab - 3aB}{b^3 (bx^3 + a)} \right) dx^3$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{3} \left(-\frac{a^2 (Ab - aB)}{2b^4 (a + bx^3)^2} + \frac{a(2Ab - 3aB)}{b^4 (a + bx^3)} + \frac{(Ab - 3aB) \log (a + bx^3)}{b^4} + \frac{Bx^3}{b^3} \right)$$

input
```
Int[(x^8*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((B*x^3)/b^3 - (a^2*(A*b - a*B))/(2*b^4*(a + b*x^3)^2) + (a*(2*A*b - 3*a*B
))/(b^4*(a + b*x^3)) + ((A*b - 3*a*B)*Log[a + b*x^3])/b^4)/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

method	result
norman	$\frac{\frac{B\,x^{9}}{3b}+\frac{a^{2}(Ab-3Ba)}{2b^{4}}+\frac{2a(Ab-3Ba)x^{3}}{3b^{3}}}{(b\,x^{3}+a)^{2}}+\frac{(Ab-3Ba)\ln(b\,x^{3}+a)}{3b^{4}}$
default	$\frac{B\,x^{3}}{3b^{3}}+\frac{-\frac{a^{2}(Ab-Ba)}{2b\left(b\,x^{3}+a\right)^{2}}+\frac{a(2Ab-3Ba)}{b\left(b\,x^{3}+a\right)}+\frac{(Ab-3Ba)\ln\left(b\,x^{3}+a\right)}{b}}{3b^{3}}$
risch	$\frac{B\,x^{3}}{3b^{3}}+\frac{\left(\frac{2}{3}abA-a^{2}B\right)x^{3}+\frac{a^{2}(3Ab-5Ba)}{6b}}{b^{3}(b\,x^{3}+a)^{2}}+\frac{\ln(b\,x^{3}+a)A}{3b^{3}}-\frac{\ln(b\,x^{3}+a)Ba}{b^{4}}$
parallelrisch	$\frac{2b^{3}B\,x^{9}+2A\ln(b\,x^{3}+a)x^{6}b^{3}-6B\ln(b\,x^{3}+a)x^{6}a\,b^{2}+4A\ln(b\,x^{3}+a)x^{3}a\,b^{2}-12B\ln(b\,x^{3}+a)x^{3}a^{2}b+4aA\,b^{2}x^{3}-12B\,a^{2}b\,x^{3}+}{6b^{4}(b\,x^{3}+a)^{2}}$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/3*B*x^9/b+1/2*a^2*(A*b-3*B*a)/b^4+2/3*a*(A*b-3*B*a)/b^3*x^3)/(b*x^3+a)^
2+1/3*(A*b-3*B*a)*ln(b*x^3+a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.61

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2\,Bb^3x^9+4\,Bab^2x^6-5\,Ba^3+3\,Aa^2b-4\,(Ba^2b-Aab^2)x^3-2\,((3\,Bab^2-Ab^3)x^6+3\,Ba^3-Aa^2b+2}{6\,(b^6x^6+2\,ab^5x^3+a^2b^4)}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/6*(2*B*b^3*x^9 + 4*B*a*b^2*x^6 - 5*B*a^3 + 3*A*a^2*b - 4*(B*a^2*b - A*a*
b^2)*x^3 - 2*((3*B*a*b^2 - A*b^3)*x^6 + 3*B*a^3 - A*a^2*b + 2*(3*B*a^2*b -
 A*a*b^2)*x^3)*log(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^3}{3b^3} + \frac{3Aa^2b - 5Ba^3 + x^3\cdot(4Aab^2 - 6Ba^2b)}{6a^2b^4 + 12ab^5x^3 + 6b^6x^6} - \frac{(-Ab+3Ba)\log(a+bx^3)}{3b^4}$$

input
```
integrate(x**8*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
B*x**3/(3*b**3) + (3*A*a**2*b - 5*B*a**3 + x**3*(4*A*a*b**2 - 6*B*a**2*b))
/(6*a**2*b**4 + 12*a*b**5*x**3 + 6*b**6*x**6) - (-A*b + 3*B*a)*log(a + b*x
**3)/(3*b**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^3}{3\,b^3} - \frac{5\,Ba^3 - 3\,Aa^2b + 2\,(3\,Ba^2b - 2\,Aab^2)x^3}{6\,(b^6x^6 + 2\,ab^5x^3 + a^2b^4)} - \frac{(3\,Ba - Ab)\log(bx^3+a)}{3\,b^4}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/3*B*x^3/b^3 - 1/6*(5*B*a^3 - 3*A*a^2*b + 2*(3*B*a^2*b - 2*A*a*b^2)*x^3)/
(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4) - 1/3*(3*B*a - A*b)*log(b*x^3 + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.06

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^3}{3\,b^3} - \frac{(3\,Ba-Ab)\log\left(|bx^3+a|\right)}{3\,b^4} + \frac{9\,Bab^2x^6-3\,Ab^3x^6+12\,Ba^2bx^3-2\,Aab^2x^3+4\,Ba^3}{6\,(bx^3+a)^2b^4}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/3*B*x^3/b^3 - 1/3*(3*B*a - A*b)*log(abs(b*x^3 + a))/b^4 + 1/6*(9*B*a*b^2
*x^6 - 3*A*b^3*x^6 + 12*B*a^2*b*x^3 - 2*A*a*b^2*x^3 + 4*B*a^3)/((b*x^3 + a
)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{B\,x^3}{3\,b^3} - \frac{x^3\left(B\,a^2-\frac{2\,A\,a\,b}{3}\right)+\frac{5\,B\,a^3-3\,A\,a^2\,b}{6\,b}}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6} + \frac{\ln\left(b\,x^3+a\right)\,\left(A\,b-3\,B\,a\right)}{3\,b^4}$$

input
```
int((x^8*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
(B*x^3)/(3*b^3) - (x^3*(B*a^2 - (2*A*a*b)/3) + (5*B*a^3 - 3*A*a^2*b)/(6*b)
)/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (log(a + b*x^3)*(A*b - 3*B*a))/(3*b^
4)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx$$
$$= \frac{-2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^2-2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)ab\,x^3-2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a^2-2\log\left(a^{\frac{1}{3}}+b\right.}{3b^3\,(b\,x^3+a)}$$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
( - 2*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2 - 2*log(a**
(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b*x**3 - 2*log(a**(1/3) + b
**(1/3)*x)*a**2 - 2*log(a**(1/3) + b**(1/3)*x)*a*b*x**3 + 2*a*b*x**3 + b**
2*x**6)/(3*b**3*(a + b*x**3))
```

3.93 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{a(Ab-aB)}{6b^3(a+bx^3)^2} - \frac{Ab-2aB}{3b^3(a+bx^3)} + \frac{B\log(a+bx^3)}{3b^3}$$

output
```
1/6*a*(A*b-B*a)/b^3/(b*x^3+a)^2-1/3*(A*b-2*B*a)/b^3/(b*x^3+a)+1/3*B*ln(b*x
^3+a)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{3a^2B-2Ab^2x^3-ab(A-4Bx^3)+2B(a+bx^3)^2\log(a+bx^3)}{6b^3(a+bx^3)^2}$$

input
```
Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
(3*a^2*B - 2*A*b^2*x^3 - a*b*(A - 4*B*x^3) + 2*B*(a + b*x^3)^2*Log[a + b*x
^3])/(6*b^3*(a + b*x^3)^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^3)}{(a+bx^3)^3} \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3} \int \frac{x^3 (Bx^3+A)}{(bx^3+a)^3} dx^3$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{3} \int \left(\frac{B}{b^2 (bx^3+a)}+\frac{Ab-2aB}{b^2 (bx^3+a)^2}+\frac{a(aB-Ab)}{b^2 (bx^3+a)^3}\right) dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} \left(-\frac{Ab-2aB}{b^3 (a+bx^3)}+\frac{a(Ab-aB)}{2b^3 (a+bx^3)^2}+\frac{B \log (a+bx^3)}{b^3}\right)$$

input
```
Int[(x^5*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((a*(A*b - a*B))/(2*b^3*(a + b*x^3)^2) - (A*b - 2*a*B)/(b^3*(a + b*x^3)) +
 (B*Log[a + b*x^3])/b^3)/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result	size
norman	$\frac{-\frac{a(Ab-3Ba)}{6b^3}-\frac{(Ab-2Ba)x^3}{3b^2}}{(b\,x^3+a)^2}+\frac{B\ln(b\,x^3+a)}{3b^3}$	57
risch	$\frac{-\frac{a(Ab-3Ba)}{6b^3}-\frac{(Ab-2Ba)x^3}{3b^2}}{(b\,x^3+a)^2}+\frac{B\ln(b\,x^3+a)}{3b^3}$	57
default	$\frac{a(Ab-Ba)}{6b^3(b\,x^3+a)^2}-\frac{Ab-2Ba}{3b^3(b\,x^3+a)}+\frac{B\ln(b\,x^3+a)}{3b^3}$	61
parallelrisch	$-\frac{-2B\ln(b\,x^3+a)x^6b^2-4B\ln(b\,x^3+a)x^3ab+2A\,b^2x^3-4Bab\,x^3-2B\ln(b\,x^3+a)a^2+abA-3a^2B}{6b^3(b\,x^3+a)^2}$	90

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/6*a*(A*b-3*B*a)/b^3-1/3*(A*b-2*B*a)/b^2*x^3)/(b*x^3+a)^2+1/3*B*ln(b*x^
3+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.35

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2\,(2\,Bab-Ab^2)x^3+3\,Ba^2-Aab+2\,(Bb^2x^6+2\,Babx^3+Ba^2)\log(bx^3+a)}{6\,(b^5x^6+2\,ab^4x^3+a^2b^3)}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/6*(2*(2*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b + 2*(B*b^2*x^6 + 2*B*a*b*x^
3 + B*a^2)*log(b*x^3 + a))/(b^5*x^6 + 2*a*b^4*x^3 + a^2*b^3)
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{B\log\left(a+bx^3\right)}{3b^3} + \frac{-Aab+3Ba^2+x^3(-2Ab^2+4Bab)}{6a^2b^3+12ab^4x^3+6b^5x^6}$$

input
```
integrate(x**5*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
B*log(a + b*x**3)/(3*b**3) + (-A*a*b + 3*B*a**2 + x**3*(-2*A*b**2 + 4*B*a*
b))/(6*a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2\,(2\,Bab-Ab^2)x^3+3\,Ba^2-Aab}{6\,(b^5x^6+2\,ab^4x^3+a^2b^3)} + \frac{B\log\left(bx^3+a\right)}{3\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/6*(2*(2*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b)/(b^5*x^6 + 2*a*b^4*x^3 + a
^2*b^3) + 1/3*B*log(b*x^3 + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{B\log(|bx^3+a|)}{3\,b^3} + \frac{2\,(2\,Ba-Ab)x^3+\frac{3\,Ba^2-Aab}{b}}{6\,(bx^3+a)^2b^2}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/3*B*log(abs(b*x^3 + a))/b^3 + 1/6*(2*(2*B*a - A*b)*x^3 + (3*B*a^2 - A*a*
b)/b)/((b*x^3 + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\frac{3\,B\,a^2-A\,a\,b}{6\,b^3}-\frac{x^3\,(A\,b-2\,B\,a)}{3\,b^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6} + \frac{B\,\ln(b\,x^3+a)}{3\,b^3}$$

input
```
int((x^5*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
((3*B*a^2 - A*a*b)/(6*b^3) - (x^3*(A*b - 2*B*a))/(3*b^2))/(a^2 + b^2*x^6 +
 2*a*b*x^3) + (B*log(a + b*x^3))/(3*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.47

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx$$
$$= \frac{\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a+\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)b\,x^3+\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a+\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)b\,x^3-}{3b^2\,(b\,x^3+a)}$$

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
(log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a + log(a**(2/3) - b*
*(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**3 + log(a**(1/3) + b**(1/3)*x)*a +
 log(a**(1/3) + b**(1/3)*x)*b*x**3 - b*x**3)/(3*b**2*(a + b*x**3))
```

3.94 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 32

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3} dx = -\frac{(A+Bx^3)^2}{6(Ab-aB)(a+bx^3)^2}$$

output
```
-1/6*(B*x^3+A)^2/(A*b-B*a)/(b*x^3+a)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3} dx = -\frac{Ab+B(a+2bx^3)}{6b^2(a+bx^3)^2}$$

input
```
Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
-1/6*(A*b + B*(a + 2*b*x^3))/(b^2*(a + b*x^3)^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {946, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^3)}{(a+bx^3)^3} \, dx$$

$$\Big\downarrow \ 946$$

$$\frac{1}{3}\int \frac{Bx^3+A}{(bx^3+a)^3}dx^3$$

$$\Big\downarrow \ 48$$

$$-\frac{(A+Bx^3)^2}{6\,(a+bx^3)^2\,(Ab-aB)}$$

input
```
Int[(x^2*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
-1/6*(A + B*x^3)^2/((A*b - a*B)*(a + b*x^3)^2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
gosper	$-\frac{2bB\,x^3+Ab+Ba}{6\left(b\,x^3+a\right)^2 b^2}$	29
parallelrisch	$-\frac{2bB\,x^3+Ab+Ba}{6\left(b\,x^3+a\right)^2 b^2}$	29
orering	$-\frac{2bB\,x^3+Ab+Ba}{6\left(b\,x^3+a\right)^2 b^2}$	29
norman	$\frac{-\frac{B\,x^3}{3b}-\frac{Ab+Ba}{6b^2}}{\left(b\,x^3+a\right)^2}$	33
risch	$\frac{-\frac{B\,x^3}{3b}-\frac{Ab+Ba}{6b^2}}{\left(b\,x^3+a\right)^2}$	33
default	$-\frac{Ab-Ba}{6b^2\left(b\,x^3+a\right)^2}-\frac{B}{3b^2\left(b\,x^3+a\right)}$	39

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*B*b*x^3+A*b+B*a)/(b*x^3+a)^2/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx=-\frac{2\,Bbx^3+Ba+Ab}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/6*(2*B*b*x^3 + B*a + A*b)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{-Ab-Ba-2Bbx^3}{6a^2b^2+12ab^3x^3+6b^4x^6}$$

input
```
integrate(x**2*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
(-A*b - B*a - 2*B*b*x**3)/(6*a**2*b**2 + 12*a*b**3*x**3 + 6*b**4*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{2\,Bbx^3+Ba+Ab}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/6*(2*B*b*x^3 + B*a + A*b)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{2\,Bbx^3+Ba+Ab}{6\,(bx^3+a)^2b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/6*(2*B*b*x^3 + B*a + A*b)/((b*x^3 + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{\frac{A\,b+B\,a}{6\,b^2}+\frac{B\,x^3}{3\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

input
```
int((x^2*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
-((A*b + B*a)/(6*b^2) + (B*x^3)/(3*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^3}{3a\,(b\,x^3+a)}$$

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
x**3/(3*a*(a + b*x**3))
```

3.95 $\int \frac{A+Bx^3}{x(a+bx^3)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \frac{A+Bx^3}{x(a+bx^3)^3} dx = \frac{Ab-aB}{6ab(a+bx^3)^2} + \frac{A}{3a^2(a+bx^3)} + \frac{A\log(x)}{a^3} - \frac{A\log(a+bx^3)}{3a^3}$$

output
```
1/6*(A*b-B*a)/a/b/(b*x^3+a)^2+1/3*A/a^2/(b*x^3+a)+A*ln(x)/a^3-1/3*A*ln(b*x
^3+a)/a^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{x(a+bx^3)^3} dx = \frac{\frac{a(3aAb-a^2B+2Ab^2x^3)}{b(a+bx^3)^2} + 6A\log(x) - 2A\log(a+bx^3)}{6a^3}$$

input
```
Integrate[(A + B*x^3)/(x*(a + b*x^3)^3),x]
```

output
```
((a*(3*a*A*b - a^2*B + 2*A*b^2*x^3))/(b*(a + b*x^3)^2) + 6*A*Log[x] - 2*A*
Log[a + b*x^3])/(6*a^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^3\left(bx^3+a\right)^3}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(-\frac{bA}{a^3\left(bx^3+a\right)}-\frac{bA}{a^2\left(bx^3+a\right)^2}+\frac{A}{a^3x^3}+\frac{aB-Ab}{a\left(bx^3+a\right)^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{A\log\left(a+bx^3\right)}{a^3}+\frac{A\log\left(x^3\right)}{a^3}+\frac{A}{a^2\left(a+bx^3\right)}+\frac{Ab-aB}{2ab\left(a+bx^3\right)^2}\right)$$

input
```
Int[(A + B*x^3)/(x*(a + b*x^3)^3),x]
```

output
```
((A*b - a*B)/(2*a*b*(a + b*x^3)^2) + A/(a^2*(a + b*x^3)) + (A*Log[x^3])/a^
3 - (A*Log[a + b*x^3])/a^3)/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

method	result
risch	$\frac{\frac{A b\,x^{3}}{3 a^{2}}+\frac{3 A b -B a}{6 a b}}{\left(b\,x^{3}+a \right)^{2}}+\frac{A \ln \left(x \right)}{a^{3}}-\frac{A \ln \left(b\,x^{3}+a \right)}{3 a^{3}}$
default	$\frac{A \ln \left(x \right)}{a^{3}}-\frac{-\frac{a^{2} \left(A b -B a \right)}{2 b \left(b\,x^{3}+a \right)^{2}}-\frac{A a}{b\,x^{3}+a}+A \ln \left(b\,x^{3}+a \right)}{3 a^{3}}$
norman	$\frac{-\frac{\left(2 A b -B a \right) x^{3}}{3 a^{2}}-\frac{b \left(3 A b -B a \right) x^{6}}{6 a^{3}}}{\left(b\,x^{3}+a \right)^{2}}+\frac{A \ln \left(x \right)}{a^{3}}-\frac{A \ln \left(b\,x^{3}+a \right)}{3 a^{3}}$
parallelrisch	$\frac{6 A \ln \left(x \right) x^{6} b^{2}-2 A \ln \left(b\,x^{3}+a \right) x^{6} b^{2}-3 A \,b^{2} x^{6}+B a b\,x^{6}+12 A \ln \left(x \right) x^{3} a b -4 A \ln \left(b\,x^{3}+a \right) x^{3} a b -4 a A b\,x^{3}+2 B \,a^{2} x^{3}+6 a^{2} A \ln \left(x \right)-}{6 a^{3} \left(b\,x^{3}+a \right)^{2}}$

input
```
int((B*x^3+A)/x/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/3/a^2*A*b*x^3+1/6*(3*A*b-B*a)/a/b)/(b*x^3+a)^2+A*ln(x)/a^3-1/3*A*ln(b*x
^3+a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx^3}{x\,(a+bx^3)^3}\,dx$$
$$=\frac{2\,Aab^2x^3-Ba^3+3\,Aa^2b-2\,(Ab^3x^6+2\,Aab^2x^3+Aa^2b)\log\left(bx^3+a\right)+6\,(Ab^3x^6+2\,Aab^2x^3+Aa^2b)\,\mathrm{l}}{6\,(a^3b^3x^6+2\,a^4b^2x^3+a^5b)}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/6*(2*A*a*b^2*x^3 - B*a^3 + 3*A*a^2*b - 2*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*
a^2*b)*log(b*x^3 + a) + 6*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a^2*b)*log(x))/(a
^3*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^3}{x(a+bx^3)^3}\,dx = \frac{A\log(x)}{a^3} - \frac{A\log\left(\frac{a}{b}+x^3\right)}{3a^3} + \frac{3Aab+2Ab^2x^3-Ba^2}{6a^4b+12a^3b^2x^3+6a^2b^3x^6}$$

input
```
integrate((B*x**3+A)/x/(b*x**3+a)**3,x)
```

output
```
A*log(x)/a**3 - A*log(a/b + x**3)/(3*a**3) + (3*A*a*b + 2*A*b**2*x**3 - B*
a**2)/(6*a**4*b + 12*a**3*b**2*x**3 + 6*a**2*b**3*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx^3}{x(a+bx^3)^3}\,dx = \frac{2\,Ab^2x^3-Ba^2+3\,Aab}{6\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)} - \frac{A\log(bx^3+a)}{3\,a^3} + \frac{A\log(x^3)}{3\,a^3}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/6*(2*A*b^2*x^3 - B*a^2 + 3*A*a*b)/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)
- 1/3*A*log(b*x^3 + a)/a^3 + 1/3*A*log(x^3)/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^3}{x\,(a+bx^3)^3}\,dx = -\frac{A\log\left(|bx^3+a|\right)}{3\,a^3} + \frac{A\log\left(|x|\right)}{a^3} + \frac{3\,Ab^3x^6+8\,Aab^2x^3-Ba^3+6\,Aa^2b}{6\,(bx^3+a)^2a^3b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/3*A*log(abs(b*x^3 + a))/a^3 + A*log(abs(x))/a^3 + 1/6*(3*A*b^3*x^6 + 8*
A*a*b^2*x^3 - B*a^3 + 6*A*a^2*b)/((b*x^3 + a)^2*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{x\,(a+bx^3)^3}\,dx = \frac{\frac{3\,A\,b-B\,a}{6\,a\,b}+\frac{A\,b\,x^3}{3\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6} - \frac{A\,\ln\left(b\,x^3+a\right)}{3\,a^3} + \frac{A\,\ln\left(x\right)}{a^3}$$

input
```
int((A + B*x^3)/(x*(a + b*x^3)^3),x)
```

output
```
((3*A*b - B*a)/(6*a*b) + (A*b*x^3)/(3*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) -
(A*log(a + b*x^3))/(3*a^3) + (A*log(x))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.68

$$\int \frac{A+Bx^3}{x\,(a+bx^3)^3}\,dx = \frac{-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)b\,x^3-\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a-\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)b\,x^3}{3a^2\left(b\,x^3+a\right)}$$

input
```
int((B*x^3+A)/x/(b*x^3+a)^3,x)
```

output
```
( - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a - log(a**(2/3) -
 b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**3 - log(a**(1/3) + b**(1/3)*x)*
a - log(a**(1/3) + b**(1/3)*x)*b*x**3 + 3*log(x)*a + 3*log(x)*b*x**3 - b*x
**3)/(3*a**2*(a + b*x**3))
```

3.96 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx = -\frac{A}{3a^3x^3}-\frac{Ab-aB}{6a^2\left(a+bx^3\right)^2}-\frac{2Ab-aB}{3a^3\left(a+bx^3\right)} -\frac{(3Ab-aB)\log(x)}{a^4}+\frac{(3Ab-aB)\log\left(a+bx^3\right)}{3a^4}$$

output
```
-1/3*A/a^3/x^3-1/6*(A*b-B*a)/a^2/(b*x^3+a)^2-1/3*(2*A*b-B*a)/a^3/(b*x^3+a)
-(3*A*b-B*a)*ln(x)/a^4+1/3*(3*A*b-B*a)*ln(b*x^3+a)/a^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx = \frac{-\frac{2aA}{x^3}+\frac{a^2(-Ab+aB)}{(a+bx^3)^2}+\frac{2a(-2Ab+aB)}{a+bx^3}+6(-3Ab+aB)\log(x)+2(3Ab-aB)\log\left(a+bx^3\right)}{6a^4}$$

input
```
Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^3),x]
```

output
```
((-2*a*A)/x^3 + (a^2*(-(A*b) + a*B))/(a + b*x^3)^2 + (2*a*(-2*A*b + a*B))/
(a + b*x^3) + 6*(-3*A*b + a*B)*Log[x] + 2*(3*A*b - a*B)*Log[a + b*x^3])/(6
*a^4)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^6\left(bx^3+a\right)^3}dx^3$$

$$\Big\downarrow \; 86$$

$$\frac{1}{3}\int \left(\frac{A}{a^3x^6}-\frac{b(aB-3Ab)}{a^4\left(bx^3+a\right)}-\frac{b(aB-2Ab)}{a^3\left(bx^3+a\right)^2}+\frac{aB-3Ab}{a^4x^3}-\frac{b(aB-Ab)}{a^2\left(bx^3+a\right)^3}\right)dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(-\frac{\log\left(x^3\right)(3Ab-aB)}{a^4}+\frac{(3Ab-aB)\log\left(a+bx^3\right)}{a^4}-\frac{2Ab-aB}{a^3\left(a+bx^3\right)}-\frac{A}{a^3x^3}-\frac{Ab-aB}{2a^2\left(a+bx^3\right)^2}\right)$$

input
```
Int[(A + B*x^3)/(x^4*(a + b*x^3)^3),x]
```

output
```
(-(A/(a^3*x^3)) - (A*b - a*B)/(2*a^2*(a + b*x^3)^2) - (2*A*b - a*B)/(a^3*(
a + b*x^3)) - ((3*A*b - a*B)*Log[x^3])/a^4 + ((3*A*b - a*B)*Log[a + b*x^3]
)/a^4)/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

method	result
norman	$\frac{-\frac{A}{3a}+\frac{2b(3Ab-Ba)x^6}{3a^3}+\frac{b^2(3Ab-Ba)x^9}{2a^4}}{x^3(b\,x^3+a)^2}-\frac{(3Ab-Ba)\ln(x)}{a^4}+\frac{(3Ab-Ba)\ln\left(b\,x^3+a\right)}{3a^4}$
default	$-\frac{A}{3a^3x^3}+\frac{(-3Ab+Ba)\ln(x)}{a^4}+\frac{b\left(-\frac{a^2(Ab-Ba)}{2b\left(b\,x^3+a\right)^2}-\frac{a(2Ab-Ba)}{b\left(b\,x^3+a\right)}+\frac{(3Ab-Ba)\ln\left(b\,x^3+a\right)}{b}\right)}{3a^4}$
risch	$\frac{-\frac{b(3Ab-Ba)x^6}{3a^3}-\frac{(3Ab-Ba)x^3}{2a^2}-\frac{A}{3a}}{x^3(b\,x^3+a)^2}-\frac{3\ln(x)Ab}{a^4}+\frac{\ln(x)B}{a^3}+\frac{\ln(-b\,x^3-a)Ab}{a^4}-\frac{\ln(-b\,x^3-a)B}{3a^3}$
parallelrisch	$-\frac{18A\ln(x)x^9b^3-6A\ln\left(b\,x^3+a\right)x^9b^3-6B\ln(x)x^9a\,b^2+2B\ln\left(b\,x^3+a\right)x^9a\,b^2-9A\,x^9b^3+3B\,x^9a\,b^2+36A\ln(x)x^6a\,b^2-12A\ln}{}$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*A/a+2/3*b*(3*A*b-B*a)/a^3*x^6+1/2*b^2*(3*A*b-B*a)/a^4*x^9)/x^3/(b*x^
3+a)^2-(3*A*b-B*a)*ln(x)/a^4+1/3*(3*A*b-B*a)*ln(b*x^3+a)/a^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(89) = 178.

Time = 0.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.95

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx$$
$$= \frac{2\left(Ba^2b-3\,Aab^2\right)x^6-2\,Aa^3+3\left(Ba^3-3\,Aa^2b\right)x^3-2\left(\left(Bab^2-3\,Ab^3\right)x^9+2\left(Ba^2b-3\,Aab^2\right)x^6+(Ba\right.}{6\left(a^4b^2x^9+2\,a\right.}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/6*(2*(B*a^2*b - 3*A*a*b^2)*x^6 - 2*A*a^3 + 3*(B*a^3 - 3*A*a^2*b)*x^3 - 2
*((B*a*b^2 - 3*A*b^3)*x^9 + 2*(B*a^2*b - 3*A*a*b^2)*x^6 + (B*a^3 - 3*A*a^2
*b)*x^3)*log(b*x^3 + a) + 6*((B*a*b^2 - 3*A*b^3)*x^9 + 2*(B*a^2*b - 3*A*a*
b^2)*x^6 + (B*a^3 - 3*A*a^2*b)*x^3)*log(x))/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a
^6*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx = \frac{-2Aa^2+x^6(-6Ab^2+2Bab)+x^3(-9Aab+3Ba^2)}{6a^5x^3+12a^4bx^6+6a^3b^2x^9} + \frac{(-3Ab+Ba)\log{(x)}}{a^4} - \frac{(-3Ab+Ba)\log{\left(\frac{a}{b}+x^3\right)}}{3a^4}$$

input
```
integrate((B*x**3+A)/x**4/(b*x**3+a)**3,x)
```

output
```
(-2*A*a**2 + x**6*(-6*A*b**2 + 2*B*a*b) + x**3*(-9*A*a*b + 3*B*a**2))/(6*a
**5*x**3 + 12*a**4*b*x**6 + 6*a**3*b**2*x**9) + (-3*A*b + B*a)*log(x)/a**4
 - (-3*A*b + B*a)*log(a/b + x**3)/(3*a**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^3}{x^4(a+bx^3)^3}\,dx = \frac{2\,(Bab-3\,Ab^2)x^6+3\,(Ba^2-3\,Aab)x^3-2\,Aa^2}{6\,(a^3b^2x^9+2\,a^4bx^6+a^5x^3)} - \frac{(Ba-3\,Ab)\log(bx^3+a)}{3\,a^4} + \frac{(Ba-3\,Ab)\log(x^3)}{3\,a^4}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/6*(2*(B*a*b - 3*A*b^2)*x^6 + 3*(B*a^2 - 3*A*a*b)*x^3 - 2*A*a^2)/(a^3*b^2
*x^9 + 2*a^4*b*x^6 + a^5*x^3) - 1/3*(B*a - 3*A*b)*log(b*x^3 + a)/a^4 + 1/3
*(B*a - 3*A*b)*log(x^3)/a^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx^3}{x^4(a+bx^3)^3}\,dx = \frac{(Ba-3\,Ab)\log(|x|)}{a^4} - \frac{(Bab-3\,Ab^2)\log(|bx^3+a|)}{3\,a^4b} + \frac{3\,Bab^2x^6-9\,Ab^3x^6+8\,Ba^2bx^3-22\,Aab^2x^3+6\,Ba^3-14\,Aa^2b}{6\,(bx^3+a)^2a^4} - \frac{Bax^3-3\,Abx^3+Aa}{3\,a^4x^3}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
(B*a - 3*A*b)*log(abs(x))/a^4 - 1/3*(B*a*b - 3*A*b^2)*log(abs(b*x^3 + a))/
(a^4*b) + 1/6*(3*B*a*b^2*x^6 - 9*A*b^3*x^6 + 8*B*a^2*b*x^3 - 22*A*a*b^2*x^
3 + 6*B*a^3 - 14*A*a^2*b)/((b*x^3 + a)^2*a^4) - 1/3*(B*a*x^3 - 3*A*b*x^3 +
 A*a)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx = \frac{\ln\left(b\,x^3+a\right)\,\left(3\,A\,b-B\,a\right)}{3\,a^4} - \frac{\frac{A}{3\,a}+\frac{x^3\,\left(3\,A\,b-B\,a\right)}{2\,a^2}+\frac{b\,x^6\,\left(3\,A\,b-B\,a\right)}{3\,a^3}}{a^2\,x^3+2\,a\,b\,x^6+b^2\,x^9} - \frac{\ln\left(x\right)\,\left(3\,A\,b-B\,a\right)}{a^4}$$

input
```
int((A + B*x^3)/(x^4*(a + b*x^3)^3),x)
```

output
```
(log(a + b*x^3)*(3*A*b - B*a))/(3*a^4) - (A/(3*a) + (x^3*(3*A*b - B*a))/(2
*a^2) + (b*x^6*(3*A*b - B*a))/(3*a^3))/(a^2*x^3 + b^2*x^9 + 2*a*b*x^6) - (
log(x)*(3*A*b - B*a))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.41

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx = \frac{2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)ab\,x^3+2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)b^2x^6+2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)ab\,x^3+2\log\left(a^{\frac{1}{3}}\right.}{3a^3x^3\left(b\,x^3+a\right)}$$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^3,x)
```

output
```
(2*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b*x**3 + 2*log(a*
*(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**2*x**6 + 2*log(a**(1/3) +
 b**(1/3)*x)*a*b*x**3 + 2*log(a**(1/3) + b**(1/3)*x)*b**2*x**6 - 6*log(x)*
a*b*x**3 - 6*log(x)*b**2*x**6 - a**2 + 2*b**2*x**6)/(3*a**3*x**3*(a + b*x*
*3))
```

3.97 $\int \frac{A+Bx^3}{x^7 \left(a+bx^3\right)^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 122

$$\int \frac{A+Bx^3}{x^7 \left(a+bx^3\right)^3} \, dx = -\frac{A}{6a^3x^6}+\frac{3Ab-aB}{3a^4x^3}+\frac{b(Ab-aB)}{6a^3\left(a+bx^3\right)^2}+\frac{b(3Ab-2aB)}{3a^4\left(a+bx^3\right)} +\frac{3b(2Ab-aB)\log(x)}{a^5}-\frac{b(2Ab-aB)\log\left(a+bx^3\right)}{a^5}$$

output
```
-1/6*A/a^3/x^6+1/3*(3*A*b-B*a)/a^4/x^3+1/6*b*(A*b-B*a)/a^3/(b*x^3+a)^2+1/3
*b*(3*A*b-2*B*a)/a^4/(b*x^3+a)+3*b*(2*A*b-B*a)*ln(x)/a^5-b*(2*A*b-B*a)*ln(
b*x^3+a)/a^5
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{x^7 \left(a+bx^3\right)^3} \, dx = \frac{-\frac{a^2A}{x^6}-\frac{2a(-3Ab+aB)}{x^3}+\frac{a^2b(Ab-aB)}{\left(a+bx^3\right)^2}+\frac{2ab(3Ab-2aB)}{a+bx^3}+18b(2Ab-aB)\log(x)+6b(-2Ab+aB)\log\left(a+bx^3\right)}{6a^5}$$

input
```
Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^3),x]
```

output
```
(-((a^2*A)/x^6) - (2*a*(-3*A*b + a*B))/x^3 + (a^2*b*(A*b - a*B))/(a + b*x^
3)^2 + (2*a*b*(3*A*b - 2*a*B))/(a + b*x^3) + 18*b*(2*A*b - a*B)*Log[x] + 6
*b*(-2*A*b + a*B)*Log[a + b*x^3])/(6*a^5)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^7 (a+bx^3)^3} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^9 (bx^3+a)^3}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(\frac{3(aB-2Ab)b^2}{a^5 (bx^3+a)}+\frac{(2aB-3Ab)b^2}{a^4 (bx^3+a)^2}+\frac{(aB-Ab)b^2}{a^3 (bx^3+a)^3}-\frac{3(aB-2Ab)b}{a^5x^3}+\frac{aB-3Ab}{a^4x^6}+\frac{A}{a^3x^9}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3b\log (x^3) (2Ab-aB)}{a^5}-\frac{3b(2Ab-aB)\log (a+bx^3)}{a^5}+\frac{b(3Ab-2aB)}{a^4 (a+bx^3)}+\frac{3Ab-aB}{a^4x^3}+\frac{b(Ab-aB)}{2a^3 (a+bx^3)^2}-\frac{A}{2a^3 \cdots}\right.$$

input
```
Int[(A + B*x^3)/(x^7*(a + b*x^3)^3),x]
```

output
```
(-1/2*A/(a^3*x^6) + (3*A*b - a*B)/(a^4*x^3) + (b*(A*b - a*B))/(2*a^3*(a +
b*x^3)^2) + (b*(3*A*b - 2*a*B))/(a^4*(a + b*x^3)) + (3*b*(2*A*b - a*B)*Log
[x^3])/a^5 - (3*b*(2*A*b - a*B)*Log[a + b*x^3])/a^5)/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01

method	result
default	$-\frac{A}{6a^3x^6}-\frac{-3Ab+Ba}{3a^4x^3}+\frac{3b(2Ab-Ba)\ln(x)}{a^5}-\frac{b^2\left(-\frac{a^2(Ab-Ba)}{2b\left(bx^3+a\right)^2}-\frac{a(3Ab-2Ba)}{b\left(bx^3+a\right)}+\frac{(6Ab-3Ba)\ln\left(bx^3+a\right)}{b}\right)}{3a^5}$
norman	$\frac{-\frac{A}{6a}+\frac{(2Ab-Ba)x^3}{3a^2}-\frac{2b\left(2b^2A-abB\right)x^9}{a^4}-\frac{b^2\left(6b^2A-3abB\right)x^{12}}{2a^5}}{x^6\left(bx^3+a\right)^2}+\frac{3b(2Ab-Ba)\ln(x)}{a^5}-\frac{b(2Ab-Ba)\ln\left(bx^3+a\right)}{a^5}$
risch	$\frac{\frac{b^2(2Ab-Ba)x^9}{a^4}+\frac{3b(2Ab-Ba)x^6}{2a^3}+\frac{(2Ab-Ba)x^3}{3a^2}-\frac{A}{6a}}{x^6\left(bx^3+a\right)^2}+\frac{6b^2\ln(x)A}{a^5}-\frac{3b\ln(x)B}{a^4}-\frac{2b^2\ln\left(bx^3+a\right)A}{a^5}+\frac{b\ln\left(bx^3+a\right)B}{a^4}$
parallelrisch	$36A\ln(x)x^{12}b^4-12A\ln\left(bx^3+a\right)x^{12}b^4-18B\ln(x)x^{12}ab^3+6B\ln\left(bx^3+a\right)x^{12}ab^3-18Ax^{12}b^4+9Bx^{12}ab^3+72A\ln(x)x^9ab^3$

input
```
int((B*x^3+A)/x^7/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6*A/a^3/x^6-1/3*(-3*A*b+B*a)/a^4/x^3+3*b*(2*A*b-B*a)*ln(x)/a^5-1/3/a^5*
b^2*(-1/2*a^2*(A*b-B*a)/b/(b*x^3+a)^2-a*(3*A*b-2*B*a)/b/(b*x^3+a)+(6*A*b-3
*B*a)/b*ln(b*x^3+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(110) = 220$.

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.88

$$\int \frac{A+Bx^3}{x^7 (a+bx^3)^3}\,dx = -\frac{6\,(Ba^2b^2-2\,Aab^3)x^9+9\,(Ba^3b-2\,Aa^2b^2)x^6+Aa^4+2\,(Ba^4-2\,Aa^3b)x^3-6\,((Bab^3-2\,Ab^4)x^{12}+$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/6*(6*(B*a^2*b^2 - 2*A*a*b^3)*x^9 + 9*(B*a^3*b - 2*A*a^2*b^2)*x^6 + A*a^
4 + 2*(B*a^4 - 2*A*a^3*b)*x^3 - 6*((B*a*b^3 - 2*A*b^4)*x^12 + 2*(B*a^2*b^2
 - 2*A*a*b^3)*x^9 + (B*a^3*b - 2*A*a^2*b^2)*x^6)*log(b*x^3 + a) + 18*((B*a
*b^3 - 2*A*b^4)*x^12 + 2*(B*a^2*b^2 - 2*A*a*b^3)*x^9 + (B*a^3*b - 2*A*a^2*
b^2)*x^6)*log(x))/(a^5*b^2*x^12 + 2*a^6*b*x^9 + a^7*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^3}{x^7 (a+bx^3)^3}\,dx = \frac{-Aa^3+x^9\cdot(12Ab^3-6Bab^2)+x^6\cdot(18Aab^2-9Ba^2b)+x^3\cdot(4Aa^2b-2Ba^3)}{6a^6x^6+12a^5bx^9+6a^4b^2x^{12}} - \frac{3b(-2Ab+Ba)\log(x)}{a^5} + \frac{b(-2Ab+Ba)\log\left(\frac{a}{b}+x^3\right)}{a^5}$$

input
```
integrate((B*x**3+A)/x**7/(b*x**3+a)**3,x)
```

output
```
(-A*a**3 + x**9*(12*A*b**3 - 6*B*a*b**2) + x**6*(18*A*a*b**2 - 9*B*a**2*b)
 + x**3*(4*A*a**2*b - 2*B*a**3))/(6*a**6*x**6 + 12*a**5*b*x**9 + 6*a**4*b*
*2*x**12) - 3*b*(-2*A*b + B*a)*log(x)/a**5 + b*(-2*A*b + B*a)*log(a/b + x*
*3)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx = -\frac{6\,(Bab^2-2\,Ab^3)x^9+9\,(Ba^2b-2\,Aab^2)x^6+Aa^3+2\,(Ba^3-2\,Aa^2b)x^3}{6\,(a^4b^2x^{12}+2\,a^5bx^9+a^6x^6)} + \frac{(Bab-2\,Ab^2)\log\left(bx^3+a\right)}{a^5} - \frac{(Bab-2\,Ab^2)\log\left(x^3\right)}{a^5}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/6*(6*(B*a*b^2 - 2*A*b^3)*x^9 + 9*(B*a^2*b - 2*A*a*b^2)*x^6 + A*a^3 + 2*
(B*a^3 - 2*A*a^2*b)*x^3)/(a^4*b^2*x^12 + 2*a^5*b*x^9 + a^6*x^6) + (B*a*b -
 2*A*b^2)*log(b*x^3 + a)/a^5 - (B*a*b - 2*A*b^2)*log(x^3)/a^5
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx = -\frac{3\,(Bab-2\,Ab^2)\log\left(|x|\right)}{a^5} + \frac{(Bab^2-2\,Ab^3)\log\left(|bx^3+a|\right)}{a^5b} - \frac{6\,Bab^2x^9-12\,Ab^3x^9+9\,Ba^2bx^6-18\,Aab^2x^6+2\,Ba^3x^3-4\,Aa^2bx^3+Aa^3}{6\,\left(bx^6+ax^3\right)^2a^4}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-3*(B*a*b - 2*A*b^2)*log(abs(x))/a^5 + (B*a*b^2 - 2*A*b^3)*log(abs(b*x^3 +
 a))/(a^5*b) - 1/6*(6*B*a*b^2*x^9 - 12*A*b^3*x^9 + 9*B*a^2*b*x^6 - 18*A*a*
b^2*x^6 + 2*B*a^3*x^3 - 4*A*a^2*b*x^3 + A*a^3)/((b*x^6 + a*x^3)^2*a^4)
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^3}{x^7 (a+bx^3)^3}\,dx = \frac{\frac{x^3\,(2\,A\,b-B\,a)}{3\,a^2}-\frac{A}{6\,a}+\frac{b^2\,x^9\,(2\,A\,b-B\,a)}{a^4}+\frac{3\,b\,x^6\,(2\,A\,b-B\,a)}{2\,a^3}}{a^2\,x^6+2\,a\,b\,x^9+b^2\,x^{12}} - \frac{\ln\left(b\,x^3+a\right)\,\left(2\,A\,b^2-B\,a\,b\right)}{a^5}+\frac{\ln\left(x\right)\,\left(6\,A\,b^2-3\,B\,a\,b\right)}{a^5}$$

input
```
int((A + B*x^3)/(x^7*(a + b*x^3)^3),x)
```

output
```
((x^3*(2*A*b - B*a))/(3*a^2) - A/(6*a) + (b^2*x^9*(2*A*b - B*a))/a^4 + (3*
b*x^6*(2*A*b - B*a))/(2*a^3))/(a^2*x^6 + b^2*x^12 + 2*a*b*x^9) - (log(a +
b*x^3)*(2*A*b^2 - B*a*b))/a^5 + (log(x)*(6*A*b^2 - 3*B*a*b))/a^5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx^3}{x^7 (a+bx^3)^3}\,dx = \frac{-6\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a\,b^2x^6-6\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)b^3x^9-6\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a\,b^2x^6-6\log\left(\right.}{6a^4x^6\left(b\,x^3+a\right)}$$

input
```
int((B*x^3+A)/x^7/(b*x^3+a)^3,x)
```

output
```
( - 6*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**2*x**6 - 6*
log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**3*x**9 - 6*log(a**(
1/3) + b**(1/3)*x)*a*b**2*x**6 - 6*log(a**(1/3) + b**(1/3)*x)*b**3*x**9 +
18*log(x)*a*b**2*x**6 + 18*log(x)*b**3*x**9 - a**3 + 3*a**2*b*x**3 - 6*b**
3*x**9)/(6*a**4*x**6*(a + b*x**3))
```

3.98 $\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 232

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ab-3aB)x^2}{2b^4}+\frac{Bx^5}{5b^3}-\frac{a^2(Ab-aB)x^2}{6b^4(a+bx^3)^2}+\frac{a(7Ab-10aB)x^2}{9b^4(a+bx^3)} + \frac{4a^{2/3}(5Ab-11aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{14/3}} + \frac{4a^{2/3}(5Ab-11aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{14/3}} - \frac{2a^{2/3}(5Ab-11aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27b^{14/3}}$$

output
```
1/2*(A*b-3*B*a)*x^2/b^4+1/5*B*x^5/b^3-1/6*a^2*(A*b-B*a)*x^2/b^4/(b*x^3+a)^
2+1/9*a*(7*A*b-10*B*a)*x^2/b^4/(b*x^3+a)+4/27*a^(2/3)*(5*A*b-11*B*a)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/b^(14/3)+4/27*a^(2/3)
*(5*A*b-11*B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(14/3)-2/27*a^(2/3)*(5*A*b-11*B*a)
*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(14/3)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.93

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \frac{135b^{2/3}(Ab-3aB)x^2+54b^{5/3}Bx^5+\frac{45a^2b^{2/3}(-Ab+aB)x^2}{(a+bx^3)^2}+\frac{30ab^{2/3}(7Ab-10aB)x^2}{a+bx^3}-40\sqrt{3}a^{2/3}(-5Ab+11aB)}{}$$

input
```
Integrate[(x^10*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
(135*b^(2/3)*(A*b - 3*a*B)*x^2 + 54*b^(5/3)*B*x^5 + (45*a^2*b^(2/3)*(-(A*b
) + a*B)*x^2)/(a + b*x^3)^2 + (30*a*b^(2/3)*(7*A*b - 10*a*B)*x^2)/(a + b*x
^3) - 40*Sqrt[3]*a^(2/3)*(-5*A*b + 11*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]] - 40*a^(2/3)*(-5*A*b + 11*a*B)*Log[a^(1/3) + b^(1/3)*x] + 20*
a^(2/3)*(-5*A*b + 11*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(270*b^(14/3))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {957, 817, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \; 957$$

$$\frac{x^{11}(Ab-aB)}{6ab\,(a+bx^3)^2}-\frac{(5Ab-11aB)\int \frac{x^{10}}{(bx^3+a)^2}dx}{6ab}$$

$$\Big\downarrow \; 817$$

$$\frac{x^{11}(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(5Ab-11aB)\left(\frac{8\int\frac{x^7}{bx^3+a}dx}{3b}-\frac{x^8}{3b(a+bx^3)}\right)}{6ab}$$

$$\Big\downarrow \text{831}$$

$$\frac{x^{11}(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(5Ab-11aB)\left(\frac{8\int\left(\frac{x^4}{b}+\frac{a^2x}{b^2\left(bx^3+a\right)}-\frac{ax}{b^2}\right)dx}{3b}-\frac{x^8}{3b(a+bx^3)}\right)}{6ab}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{11}(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(5Ab-11aB)\left(\frac{8\left(-\frac{a^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}}+\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}}-\frac{ax^2}{2b^2}+\frac{x^5}{5b}\right)}{3b}-\frac{x^8}{3b(a+bx^3)}\right)}{6ab}$$

input
```
Int[(x^10*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^11)/(6*a*b*(a + b*x^3)^2) - ((5*A*b - 11*a*B)*(-1/3*x^8/(b*
(a + b*x^3)) + (8*(-1/2*(a*x^2)/b^2 + x^5/(5*b) - (a^(5/3)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) - (a^(5/3)*Log[a^(1/
3) + b^(1/3)*x])/(3*b^(8/3)) + (a^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(6*b^(8/3))))/(3*b)))/(6*a*b)
```

Defintions of rubi rules used

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.97 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.50

method	result
risch	$\frac{B\,x^{5}}{5b^{3}}+\frac{A\,x^{2}}{2b^{3}}-\frac{3Ba\,x^{2}}{2b^{4}}+\frac{\left(\frac{7}{9}a\,b^{2}A-\frac{10}{9}a^{2}bB\right)x^{5}+\frac{a^{2}(11Ab-17Ba)x^{2}}{18}}{b^{4}\left(b\,x^{3}+a\right)^{2}}-\frac{4a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(5Ab-11Ba)\ln\left(x-\_R\right)}{\_R}\right)}{27b^{5}}$
default	$\frac{\frac{bB\,x^{5}}{5}+\frac{(Ab-3Ba)x^{2}}{2}}{b^{4}}-\frac{a\left(\frac{\left(-\frac{7}{9}b^{2}A+\frac{10}{9}abB\right)x^{5}-\frac{a(11Ab-17Ba)x^{2}}{18}}{\left(b\,x^{3}+a\right)^{2}}+\left(\frac{20Ab}{9}-\frac{44Ba}{9}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\right.\right.}{b^{4}}$

input
```
int(x^10*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*B*x^5/b^3+1/2/b^3*A*x^2-3/2/b^4*B*a*x^2+((7/9*a*b^2*A-10/9*a^2*b*B)*x^
5+1/18*a^2*(11*A*b-17*B*a)*x^2)/b^4/(b*x^3+a)^2-4/27/b^5*a*sum((5*A*b-11*B
*a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.57

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \frac{54\,Bb^{3}x^{11}-27\,(11\,Bab^{2}-5\,Ab^{3})x^{8}-96\,(11\,Ba^{2}b-5\,Aab^{2})x^{5}-60\,(11\,Ba^{3}-5\,Aa^{2}b)x^{2}+40\,\sqrt{3}((11\,B}{}$$

input
```
integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/270*(54*B*b^3*x^11 - 27*(11*B*a*b^2 - 5*A*b^3)*x^8 - 96*(11*B*a^2*b - 5*
A*a*b^2)*x^5 - 60*(11*B*a^3 - 5*A*a^2*b)*x^2 + 40*sqrt(3)*((11*B*a*b^2 - 5
*A*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5*A*a*b^2)*x^3)*(a^2/
b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 20*
((11*B*a*b^2 - 5*A*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5*A*a
*b^2)*x^3)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(
1/3)) - 40*((11*B*a*b^2 - 5*A*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^
2*b - 5*A*a*b^2)*x^3)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/(b^6*x
^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.83

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx$$
$$= \frac{Bx^5}{5b^3} + x^2\left(\frac{A}{2b^3} - \frac{3Ba}{2b^4}\right) + \frac{x^5\cdot(14Aab^2 - 20Ba^2b) + x^2\cdot(11Aa^2b - 17Ba^3)}{18a^2b^4 + 36ab^5x^3 + 18b^6x^6}$$
$$+ \mathrm{RootSum}\left(19683t^3b^{14} - 8000A^3a^2b^3 + 52800A^2Ba^3b^2 - 116160AB^2a^4b + 85184B^3a^5, \left(t \mapsto t\log\left(\frac{}{4}\right.\right.\right.$$

input
```
integrate(x**10*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
B*x**5/(5*b**3) + x**2*(A/(2*b**3) - 3*B*a/(2*b**4)) + (x**5*(14*A*a*b**2
- 20*B*a**2*b) + x**2*(11*A*a**2*b - 17*B*a**3))/(18*a**2*b**4 + 36*a*b**5
*x**3 + 18*b**6*x**6) + RootSum(19683*_t**3*b**14 - 8000*A**3*a**2*b**3 +
52800*A**2*B*a**3*b**2 - 116160*A*B**2*a**4*b + 85184*B**3*a**5, Lambda(_t
, _t*log(729*_t**2*b**9/(400*A**2*a*b**2 - 1760*A*B*a**2*b + 1936*B**2*a**
3) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.98

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{2\,(10\,Ba^2b-7\,Aab^2)x^5+(17\,Ba^3-11\,Aa^2b)x^2}{18\,(b^6x^6+2\,ab^5x^3+a^2b^4)} + \frac{4\,\sqrt{3}(11\,Ba^2-5\,Aab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{2\,Bbx^5-5\,(3\,Ba-Ab)x^2}{10\,b^4} + \frac{2\,(11\,Ba^2-5\,Aab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{4\,(11\,Ba^2-5\,Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*(2*(10*B*a^2*b - 7*A*a*b^2)*x^5 + (17*B*a^3 - 11*A*a^2*b)*x^2)/(b^6*
x^6 + 2*a*b^5*x^3 + a^2*b^4) + 4/27*sqrt(3)*(11*B*a^2 - 5*A*a*b)*arctan(1/
3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(1/3)) + 1/10*(2*B*b
*x^5 - 5*(3*B*a - A*b)*x^2)/b^4 + 2/27*(11*B*a^2 - 5*A*a*b)*log(x^2 - x*(a
/b)^(1/3) + (a/b)^(2/3))/(b^5*(a/b)^(1/3)) - 4/27*(11*B*a^2 - 5*A*a*b)*log
(x + (a/b)^(1/3))/(b^5*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.12

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx$$
$$= -\frac{4\left(11\,Ba^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,Aab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^4}$$
$$-\frac{4\sqrt{3}\left(11\left(-ab^2\right)^{\frac{2}{3}}Ba-5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^6}$$
$$+\frac{2\left(11\left(-ab^2\right)^{\frac{2}{3}}Ba-5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,b^6}$$
$$-\frac{20\,Ba^2bx^5-14\,Aab^2x^5+17\,Ba^3x^2-11\,Aa^2bx^2}{18\left(bx^3+a\right)^2b^4}$$
$$+\frac{2\,Bb^{12}x^5-15\,Bab^{11}x^2+5\,Ab^{12}x^2}{10\,b^{15}}$$

input
```
integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-4/27*(11*B*a^2*(-a/b)^(1/3) - 5*A*a*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(
x - (-a/b)^(1/3)))/(a*b^4) - 4/27*sqrt(3)*(11*(-a*b^2)^(2/3)*B*a - 5*(-a*b
^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^6 +
 2/27*(11*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/b^6 - 1/18*(20*B*a^2*b*x^5 - 14*A*a*b^2*x^5 + 17*B*a^3
*x^2 - 11*A*a^2*b*x^2)/((b*x^3 + a)^2*b^4) + 1/10*(2*B*b^12*x^5 - 15*B*a*b
^11*x^2 + 5*A*b^12*x^2)/b^15
```

Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.92

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^5\left(\frac{7\,A\,a\,b^2}{9}-\frac{10\,B\,a^2\,b}{9}\right)-x^2\left(\frac{17\,B\,a^3}{18}-\frac{11\,A\,a^2\,b}{18}\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+x^2\left(\frac{A}{2\,b^3}-\frac{3\,B\,a}{2\,b^4}\right) + \frac{B\,x^5}{5\,b^3}+\frac{4\,a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(5\,A\,b-11\,B\,a)}{27\,b^{14/3}} + \frac{4\,a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-11\,B\,a)}{27\,b^{14/3}} - \frac{4\,a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-11\,B\,a)}{27\,b^{14/3}}$$

input
```
int((x^10*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
(x^5*((7*A*a*b^2)/9 - (10*B*a^2*b)/9) - x^2*((17*B*a^3)/18 - (11*A*a^2*b)/
18))/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + x^2*(A/(2*b^3) - (3*B*a)/(2*b^4))
 + (B*x^5)/(5*b^3) + (4*a^(2/3)*log(b^(1/3)*x + a^(1/3))*(5*A*b - 11*B*a))
/(27*b^(14/3)) + (4*a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(5*A*b - 11*B*a))/(27*b^(14/3)) - (4*a^(2/3)*log(
3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b
- 11*B*a))/(27*b^(14/3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.85

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{-40\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a^3-40\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a^2b\,x^3-60b^{\frac{2}{3}}a^{\frac{7}{3}}x^2-36b^{\frac{5}{3}}a^{\frac{4}{3}}x^5+9b^{\frac{8}{3}}a^{\frac{1}{3}}x^8+20\log\Big(a}{45b^{\frac{11}{3}}a}$$

input
```
int(x^10*(B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
( - 40*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**3 - 4
0*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2*b*x**3 -
 60*b**(2/3)*a**(1/3)*a**2*x**2 - 36*b**(2/3)*a**(1/3)*a*b*x**5 + 9*b**(2/
3)*a**(1/3)*b**2*x**8 + 20*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x
**2)*a**3 + 20*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b*
x**3 - 40*log(a**(1/3) + b**(1/3)*x)*a**3 - 40*log(a**(1/3) + b**(1/3)*x)*
a**2*b*x**3)/(45*b**(2/3)*a**(1/3)*b**3*(a + b*x**3))
```

3.99 $\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 223

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ab-3aB)x}{b^4}+\frac{Bx^4}{4b^3}-\frac{a^2(Ab-aB)x}{6b^4(a+bx^3)^2}+\frac{a(13Ab-19aB)x}{18b^4(a+bx^3)} + \frac{7\sqrt[3]{a}(2Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{13/3}} - \frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{13/3}} + \frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{13/3}}$$

output
```
(A*b-3*B*a)*x/b^4+1/4*B*x^4/b^3-1/6*a^2*(A*b-B*a)*x/b^4/(b*x^3+a)^2+1/18*a
*(13*A*b-19*B*a)*x/b^4/(b*x^3+a)+7/27*a^(1/3)*(2*A*b-5*B*a)*arctan(1/3*(a^
(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/b^(13/3)-7/27*a^(1/3)*(2*A*b-5
*B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(13/3)+7/54*a^(1/3)*(2*A*b-5*B*a)*ln(a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(13/3)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.94

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \frac{108\sqrt[3]{b}(Ab-3aB)x+27b^{4/3}Bx^4+\frac{18a^2\sqrt[3]{b}(-Ab+aB)x}{(a+bx^3)^2}+\frac{6a\sqrt[3]{b}(13Ab-19aB)x}{a+bx^3}-28\sqrt{3}\sqrt[3]{a}(-2Ab+5aB)\,\mathrm{arcta}}{1}$$

input
```
Integrate[(x^9*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
(108*b^(1/3)*(A*b - 3*a*B)*x + 27*b^(4/3)*B*x^4 + (18*a^2*b^(1/3)*(-(A*b)
+ a*B)*x)/(a + b*x^3)^2 + (6*a*b^(1/3)*(13*A*b - 19*a*B)*x)/(a + b*x^3) -
28*Sqrt[3]*a^(1/3)*(-2*A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]] + 28*a^(1/3)*(-2*A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x] - 14*a^(1/3)*
(-2*A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(108*b^(1
3/3))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {957, 817, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \; 957$$

$$\frac{x^{10}(Ab-aB)}{6ab\,(a+bx^3)^2}-\frac{(2Ab-5aB)\int \frac{x^9}{(bx^3+a)^2}dx}{3ab}$$

$$\Big\downarrow \; 817$$

$$\frac{x^{10}(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(2Ab-5aB)\left(\frac{7\int\frac{x^6}{bx^3+a}dx}{3b}-\frac{x^7}{3b(a+bx^3)}\right)}{3ab}$$

$\Big\downarrow$ 831

$$\frac{x^{10}(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(2Ab-5aB)\left(\frac{7\int\left(\frac{x^3}{b}+\frac{a^2}{b^2\left(bx^3+a\right)}-\frac{a}{b^2}\right)dx}{3b}-\frac{x^7}{3b(a+bx^3)}\right)}{3ab}$$

$\Big\downarrow$ 2009

$$\frac{x^{10}(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(2Ab-5aB)\left(\frac{7\left(-\frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{7/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}}-\frac{ax}{b^2}+\frac{x^4}{4b}\right)}{3b}-\frac{x^7}{3b(a+bx^3)}\right)}{3ab}$$

input
```
Int[(x^9*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^10)/(6*a*b*(a + b*x^3)^2) - ((2*A*b - 5*a*B)*(-1/3*x^7/(b*(
a + b*x^3)) + (7*(-((a*x)/b^2) + x^4/(4*b) - (a^(4/3)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) + (a^(4/3)*Log[a^(1/3) +
b^(1/3)*x])/(3*b^(7/3)) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(6*b^(7/3))))/(3*b)))/(3*a*b)
```

Defintions of rubi rules used

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.83 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.49

method	result
risch	$\frac{B\,x^{4}}{4b^{3}}+\frac{Ax}{b^{3}}-\frac{3Bax}{b^{4}}+\frac{\left(\frac{13}{18}a\,b^{2}A-\frac{19}{18}a^{2}bB\right)x^{4}+\frac{a^{2}(5Ab-8Ba)x}{9}}{b^{4}(b\,x^{3}+a)^{2}}-\frac{7a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(2Ab-5Ba)\ln\left(x-\_R\right)}{\_R^{2}}\right)}{27b^{5}}$
default	$\frac{\frac{1}{4}bB\,x^{4}+Abx-3Bax}{b^{4}}-\frac{a\left(\frac{\left(-\frac{13}{18}b^{2}A+\frac{19}{18}abB\right)x^{4}-\frac{a(5Ab-8Ba)x}{9}}{\left(b\,x^{3}+a\right)^{2}}+\frac{7(2Ab-5Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\mathrm{arctan}}{}\right.}{9}\right.}{b^{4}}$

input
```
int(x^9*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*B*x^4/b^3+1/b^3*A*x-3/b^4*B*a*x+((13/18*a*b^2*A-19/18*a^2*b*B)*x^4+1/9
*a^2*(5*A*b-8*B*a)*x)/b^4/(b*x^3+a)^2-7/27/b^5*a*sum((2*A*b-5*B*a)/_R^2*ln
(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.56

$$\int\frac{x^{9}(A+Bx^{3})}{(a+bx^{3})^{3}}\,dx$$

$$=\frac{27\,Bb^{3}x^{10}-54\,(5\,Bab^{2}-2\,Ab^{3})x^{7}-147\,(5\,Ba^{2}b-2\,Aab^{2})x^{4}-28\,\sqrt{3}((5\,Bab^{2}-2\,Ab^{3})x^{6}+5\,Ba^{3}-2}{}$$

input
```
integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/108*(27*B*b^3*x^10 - 54*(5*B*a*b^2 - 2*A*b^3)*x^7 - 147*(5*B*a^2*b - 2*A
*a*b^2)*x^4 - 28*sqrt(3)*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2*b
+ 2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-
a/b)^(2/3) - sqrt(3)*a)/a) + 14*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A
*a^2*b + 2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3)) - 28*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2*b
+ 2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) - 84*(
5*B*a^3 - 2*A*a^2*b)*x)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.73

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx$$
$$= \frac{Bx^4}{4b^3} + x\left(\frac{A}{b^3} - \frac{3Ba}{b^4}\right) + \frac{x^4\cdot(13Aab^2 - 19Ba^2b) + x(10Aa^2b - 16Ba^3)}{18a^2b^4 + 36ab^5x^3 + 18b^6x^6}$$
$$+\text{RootSum}\left(19683t^3b^{13} + 2744A^3ab^3 - 20580A^2Ba^2b^2 + 51450AB^2a^3b - 42875B^3a^4, \left(t \mapsto t\log\left(\frac{\quad}{-1\ldots}\right.\right.\right.$$

input
```
integrate(x**9*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
B*x**4/(4*b**3) + x*(A/b**3 - 3*B*a/b**4) + (x**4*(13*A*a*b**2 - 19*B*a**2
*b) + x*(10*A*a**2*b - 16*B*a**3))/(18*a**2*b**4 + 36*a*b**5*x**3 + 18*b**
6*x**6) + RootSum(19683*_t**3*b**13 + 2744*A**3*a*b**3 - 20580*A**2*B*a**2
*b**2 + 51450*A*B**2*a**3*b - 42875*B**3*a**4, Lambda(_t, _t*log(27*_t*b**
4/(-14*A*b + 35*B*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{(19\,Ba^2b-13\,Aab^2)x^4+2\,(8\,Ba^3-5\,Aa^2b)x}{18\,(b^6x^6+2\,ab^5x^3+a^2b^4)} + \frac{Bbx^4-4\,(3\,Ba-Ab)x}{4\,b^4} + \frac{7\,\sqrt{3}(5\,Ba^2-2\,Aab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{7\,(5\,Ba^2-2\,Aab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{7\,(5\,Ba^2-2\,Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*((19*B*a^2*b - 13*A*a*b^2)*x^4 + 2*(8*B*a^3 - 5*A*a^2*b)*x)/(b^6*x^6
 + 2*a*b^5*x^3 + a^2*b^4) + 1/4*(B*b*x^4 - 4*(3*B*a - A*b)*x)/b^4 + 7/27*s
qrt(3)*(5*B*a^2 - 2*A*a*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/(b^5*(a/b)^(2/3)) - 7/54*(5*B*a^2 - 2*A*a*b)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b^5*(a/b)^(2/3)) + 7/27*(5*B*a^2 - 2*A*a*b)*log(x + (a/b)^
(1/3))/(b^5*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.05

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{7\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba-2\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27b^5} - \frac{7\left(5Ba^2-2Aab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27ab^4} + \frac{7\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba-2\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54b^5} - \frac{19Ba^2bx^4-13Aab^2x^4+16Ba^3x-10Aa^2bx}{18\left(bx^3+a\right)^2b^4} + \frac{Bb^9x^4-12Bab^8x+4Ab^9x}{4b^{12}}$$

input
```
integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
7/27*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 2*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt
(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 - 7/27*(5*B*a^2 - 2*A*a*b)*(-a/
b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) + 7/54*(5*(-a*b^2)^(1/3)*B*a -
 2*(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^5 - 1/18
*(19*B*a^2*b*x^4 - 13*A*a*b^2*x^4 + 16*B*a^3*x - 10*A*a^2*b*x)/((b*x^3 + a
)^2*b^4) + 1/4*(B*b^9*x^4 - 12*B*a*b^8*x + 4*A*b^9*x)/b^12
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.02

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^4\left(\frac{13\,A\,a\,b^2}{18}-\frac{19\,B\,a^2\,b}{18}\right)-x\left(\frac{8\,B\,a^3}{9}-\frac{5\,A\,a^2\,b}{9}\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+x\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)+\frac{B\,x^4}{4\,b^3}+\frac{7\,(-a)^{1/3}\,\ln\left((-a)^{4/3}+a\,b^{1/3}\,x\right)\,(2\,A\,b-5\,B\,a)}{27\,b^{13/3}}-\frac{7\,(-a)^{1/3}\,\ln\left((-a)^{4/3}-2\,a\,b^{1/3}\,x+\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b-5\,B\,a)}{27\,b^{13/3}}+\frac{7\,(-a)^{1/3}\,\ln\left(2\,a\,b^{1/3}\,x-(-a)^{4/3}+\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b-5\,B\,a)}{27\,b^{13/3}}$$

input
```
int((x^9*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
(x^4*((13*A*a*b^2)/18 - (19*B*a^2*b)/18) - x*((8*B*a^3)/9 - (5*A*a^2*b)/9)
)/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + x*(A/b^3 - (3*B*a)/b^4) + (B*x^4)/(4
*b^3) + (7*(-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*x)*(2*A*b - 5*B*a))/(27*b
^(13/3)) - (7*(-a)^(1/3)*log((-a)^(4/3) + 3^(1/2)*(-a)^(4/3)*1i - 2*a*b^(1
/3)*x)*((3^(1/2)*1i)/2 + 1/2)*(2*A*b - 5*B*a))/(27*b^(13/3)) + (7*(-a)^(1/
3)*log(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2
 - 1/2)*(2*A*b - 5*B*a))/(27*b^(13/3))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.84

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{-28a^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)-28a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^3-14a^{\frac{7}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-14a^{\frac{4}{3}}\log\left(a\right.}{36b^{\frac{10}{3}}\,(b}$$

input
```
int(x^9*(B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
( - 28*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))
*a**2 - 28*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(
3)))*a*b*x**3 - 14*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*
x**2)*a**2 - 14*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**
2)*a*b*x**3 + 28*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**2 + 28*a**(1/3)*lo
g(a**(1/3) + b**(1/3)*x)*a*b*x**3 - 84*b**(1/3)*a**2*x - 63*b**(1/3)*a*b*x
**4 + 9*b**(1/3)*b**2*x**7)/(36*b**(1/3)*b**3*(a + b*x**3))
```

3.100 $\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 208

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^2}{2b^3}+\frac{a(Ab-aB)x^2}{6b^3(a+bx^3)^2}-\frac{(4Ab-7aB)x^2}{9b^3(a+bx^3)} - \frac{5(Ab-4aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\sqrt[3]{a}b^{11/3}} - \frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}b^{11/3}} + \frac{5(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54\sqrt[3]{a}b^{11/3}}$$

output
```
1/2*B*x^2/b^3+1/6*a*(A*b-B*a)*x^2/b^3/(b*x^3+a)^2-1/9*(4*A*b-7*B*a)*x^2/b^
3/(b*x^3+a)-5/27*(A*b-4*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1
/3))*3^(1/2)/a^(1/3)/b^(11/3)-5/27*(A*b-4*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/
3)/b^(11/3)+5/54*(A*b-4*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(
1/3)/b^(11/3)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.93

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \frac{27b^{2/3}Bx^2 + \frac{9ab^{2/3}(Ab-aB)x^2}{(a+bx^3)^2} - \frac{6b^{2/3}(4Ab-7aB)x^2}{a+bx^3} + \frac{10\sqrt{3}(-Ab+4aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{10(-Ab+4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{\sqrt[3]{a}}}{54b^{11/3}}$$

input
```
Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
(27*b^(2/3)*B*x^2 + (9*a*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3)^2 - (6*b^(2/
3)*(4*A*b - 7*a*B)*x^2)/(a + b*x^3) + (10*Sqrt[3]*(-(A*b) + 4*a*B)*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (10*(-(A*b) + 4*a*B)*Log[a
^(1/3) + b^(1/3)*x])/a^(1/3) + (5*(A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/a^(1/3))/(54*b^(11/3))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {957, 817, 843, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^8(Ab-aB)}{6ab\,(a+bx^3)^2} - \frac{(Ab-4aB)\int \frac{x^7}{(bx^3+a)^2}dx}{3ab}$$

$$\Big\downarrow \text{817}$$

$$\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-4aB)\left(\frac{5\int\frac{x^4}{bx^3+a}dx}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}$$

$$\Big\downarrow \textcolor{blue}{843}$$

$$\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-4aB)\left(\frac{5\left(\frac{x^2}{2b}-\frac{a\int\frac{x}{bx^3+a}dx}{b}\right)}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}$$

$$\Big\downarrow \textcolor{blue}{821}$$

$$\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-4aB)\left(\frac{5\left(\frac{x^2}{2b}-\frac{a\left(\frac{\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}$$

$$\Big\downarrow \textcolor{blue}{16}$$

$$\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-4aB)\left(\frac{5\left(\frac{x^2}{2b}-\frac{a\left(\frac{\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}$$

$\Big\downarrow$ 1142

$$\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-4aB)\left(\frac{5\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}$$

$\downarrow$ 25

$$\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-4aB)\left(\frac{5\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}$$

$\downarrow$ 27

$$\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-4aB)\left(\frac{5\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}$$

$$\Big\downarrow \; 1082$$

$$\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-4aB)\left(\frac{5\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{x^8(Ab-aB)}{6ab\,(a+bx^3)^2}-(Ab-4aB)\left(\frac{5\left(\frac{x^2}{2b}-\frac{a\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}$$

$\downarrow$ 1103

$$\frac{\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-4aB)\left(\frac{5\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}}{}$$

input
```
Int[(x^7*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^8)/(6*a*b*(a + b*x^3)^2) - ((A*b - 4*a*B)*(-1/3*x^5/(b*(a +
 b*x^3)) + (5*(x^2/(2*b) - (a*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2
/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) +
 Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^
(1/3))))/b))/(3*b)))/(3*a*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.86 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.43

method	result
risch	$\frac{Bx^2}{2b^3}+\frac{\left(-\frac{4}{9}b^2A+\frac{7}{9}abB\right)x^5-\frac{a(5Ab-11Ba)x^2}{18}}{b^3(bx^3+a)^2}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab-4Ba)\ln\left(x-\_R\right)}{\_R}\right)}{27b^4}$
default	$\frac{Bx^2}{2b^3}+\frac{\frac{\left(-\frac{4}{9}b^2A+\frac{7}{9}abB\right)x^5-\frac{a(5Ab-11Ba)x^2}{18}}{\left(bx^3+a\right)^2}+\left(\frac{5Ab}{9}-\frac{20Ba}{9}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{3}\right.}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{b^3}$

input
```
int(x^7*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x^2/b^3+((-4/9*b^2*A+7/9*a*b*B)*x^5-1/18*a*(5*A*b-11*B*a)*x^2)/b^3/(
b*x^3+a)^2+5/27/b^4*sum((A*b-4*B*a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(168) = 336$.

Time = 0.13 (sec) , antiderivative size = 792, normalized size of antiderivative = 3.81

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/54*(27*B*a*b^4*x^8 + 24*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 15*(4*B*a^3*b^2 -
 A*a^2*b^3)*x^2 - 15*sqrt(1/3)*((4*B*a^2*b^3 - A*a*b^4)*x^6 + 4*B*a^4*b -
A*a^3*b^2 + 2*(4*B*a^3*b^2 - A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2
*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3
)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 5*((4*B*a
*b^2 - A*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b
^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*((4*B*a*
b^2 - A*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^
2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^7*x^6 + 2*a^2*b^6*x^3 + a^3*b^5),
 1/54*(27*B*a*b^4*x^8 + 24*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 15*(4*B*a^3*b^2 -
 A*a^2*b^3)*x^2 - 30*sqrt(1/3)*((4*B*a^2*b^3 - A*a*b^4)*x^6 + 4*B*a^4*b -
A*a^3*b^2 + 2*(4*B*a^3*b^2 - A*a^2*b^3)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arcta
n(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 5*((4*B*
a*b^2 - A*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*
b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*((4*B*a
*b^2 - A*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b
^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^7*x^6 + 2*a^2*b^6*x^3 + a^3*b^5)
]
```

Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.78

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^2}{2b^3} + \frac{x^5(-8Ab^2+14Bab)+x^2(-5Aab+11Ba^2)}{18a^2b^3+36ab^4x^3+18b^5x^6}$$
$$+\mathrm{RootSum}\left(19683t^3ab^{11}+125A^3b^3-1500A^2Bab^2+6000AB^2a^2b-8000B^3a^3, \left(t\mapsto t\log\left(\frac{}{25A^2b^2-}\right.\right.\right.$$

input
```
integrate(x**7*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
B*x**2/(2*b**3) + (x**5*(-8*A*b**2 + 14*B*a*b) + x**2*(-5*A*a*b + 11*B*a**
2))/(18*a**2*b**3 + 36*a*b**4*x**3 + 18*b**5*x**6) + RootSum(19683*_t**3*a
*b**11 + 125*A**3*b**3 - 1500*A**2*B*a*b**2 + 6000*A*B**2*a**2*b - 8000*B*
*3*a**3, Lambda(_t, _t*log(729*_t**2*a*b**7/(25*A**2*b**2 - 200*A*B*a*b +
400*B**2*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.94

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2\,(7\,Bab-4\,Ab^2)x^5+(11\,Ba^2-5\,Aab)x^2}{18\,(b^5x^6+2\,ab^4x^3+a^2b^3)}+\frac{Bx^2}{2\,b^3} - \frac{5\,\sqrt{3}(4\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^4\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{5\,(4\,Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^4\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{5\,(4\,Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^4\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(2*(7*B*a*b - 4*A*b^2)*x^5 + (11*B*a^2 - 5*A*a*b)*x^2)/(b^5*x^6 + 2*a
*b^4*x^3 + a^2*b^3) + 1/2*B*x^2/b^3 - 5/27*sqrt(3)*(4*B*a - A*b)*arctan(1/
3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(1/3)) - 5/54*(4*B*a
 - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(1/3)) + 5/27*(4
*B*a - A*b)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.01

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^2}{2\,b^3} - \frac{5\,\sqrt{3}(4\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{1}{3}}\,b^3} + \frac{5\,(4\,Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{1}{3}}\,b^3} + \frac{5\left(4\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^3} + \frac{14\,Babx^5-8\,Ab^2x^5+11\,Ba^2x^2-5\,Aabx^2}{18\,(bx^3+a)^2b^3}$$

input
```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/2*B*x^2/b^3 - 5/27*sqrt(3)*(4*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^3) + 5/54*(4*B*a - A*b)*log(x^2 +
 x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b^3) + 5/27*(4*B*a*(-a/b)^
(1/3) - A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3)
+ 1/18*(14*B*a*b*x^5 - 8*A*b^2*x^5 + 11*B*a^2*x^2 - 5*A*a*b*x^2)/((b*x^3 +
 a)^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.90

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^2\left(\frac{11\,B\,a^2}{18}-\frac{5\,A\,a\,b}{18}\right)-x^5\left(\frac{4\,A\,b^2}{9}-\frac{7\,B\,a\,b}{9}\right)}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6} + \frac{B\,x^2}{2\,b^3} - \frac{5\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-4\,B\,a\right)}{27\,a^{1/3}\,b^{11/3}} - \frac{5\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-4\,B\,a\right)}{27\,a^{1/3}\,b^{11/3}} + \frac{5\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-4\,B\,a\right)}{27\,a^{1/3}\,b^{11/3}}$$

input
```
int((x^7*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
(x^2*((11*B*a^2)/18 - (5*A*a*b)/18) - x^5*((4*A*b^2)/9 - (7*B*a*b)/9))/(a^
2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (B*x^2)/(2*b^3) - (5*log(b^(1/3)*x + a^(1
/3))*(A*b - 4*B*a))/(27*a^(1/3)*b^(11/3)) - (5*log(3^(1/2)*a^(1/3)*1i - 2*
b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - 4*B*a))/(27*a^(1/3)*b^(
11/3)) + (5*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/
2 + 1/2)*(A*b - 4*B*a))/(27*a^(1/3)*b^(11/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.87

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{10\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a^2+10\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)ab\,x^3+15b^{\frac{2}{3}}a^{\frac{4}{3}}x^2+9b^{\frac{5}{3}}a^{\frac{1}{3}}x^5-5\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right.}{18b^{\frac{8}{3}}a^{\frac{1}{3}}\left(b\,x^3+\right.}$$

input
```
int(x^7*(B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
(10*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2 + 10*s
qrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b*x**3 + 15*b*
*(2/3)*a**(1/3)*a*x**2 + 9*b**(2/3)*a**(1/3)*b*x**5 - 5*log(a**(2/3) - b**
(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2 - 5*log(a**(2/3) - b**(1/3)*a**(1/3
)*x + b**(2/3)*x**2)*a*b*x**3 + 10*log(a**(1/3) + b**(1/3)*x)*a**2 + 10*lo
g(a**(1/3) + b**(1/3)*x)*a*b*x**3)/(18*b**(2/3)*a**(1/3)*b**2*(a + b*x**3)
)
```

3.101 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 199

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx}{b^3}+\frac{a(Ab-aB)x}{6b^3(a+bx^3)^2}-\frac{(7Ab-13aB)x}{18b^3(a+bx^3)} - \frac{2(Ab-7aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{10/3}} + \frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}} - \frac{(Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{2/3}b^{10/3}}$$

output
```
B*x/b^3+1/6*a*(A*b-B*a)*x/b^3/(b*x^3+a)^2-1/18*(7*A*b-13*B*a)*x/b^3/(b*x^3
+a)-2/27*(A*b-7*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(
1/2)/a^(2/3)/b^(10/3)+2/27*(A*b-7*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(10
/3)-1/27*(A*b-7*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(
10/3)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.94

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx$$
$$= \frac{54\sqrt[3]{b}Bx + \frac{9a\sqrt[3]{b}(Ab-aB)x}{(a+bx^3)^2} - \frac{3\sqrt[3]{b}(7Ab-13aB)x}{a+bx^3} + \frac{4\sqrt{3}(-Ab+7aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{4(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \cdots}{54b^{10/3}}$$

input
```
Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
(54*b^(1/3)*B*x + (9*a*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3)^2 - (3*b^(1/3)*(
7*A*b - 13*a*B)*x)/(a + b*x^3) + (4*Sqrt[3]*(-(A*b) + 7*a*B)*ArcTan[(1 - (
2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (4*(A*b - 7*a*B)*Log[a^(1/3) + b
^(1/3)*x])/a^(2/3) + (2*(-(A*b) + 7*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/a^(2/3))/(54*b^(10/3))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {957, 817, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \ 957$$

$$\frac{x^7(Ab-aB)}{6ab(a+bx^3)^2} - \frac{(Ab-7aB)\int \frac{x^6}{(bx^3+a)^2}dx}{6ab}$$

$$\Big\downarrow \ 817$$

$$\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-7aB)\left(\frac{4\int \frac{x^3}{bx^3+a}dx}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}$$

$\big\downarrow$ 843

$$\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-7aB)\left(\frac{4\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^3+a}dx}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}$$

$\big\downarrow$ 750

$$\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-7aB)\left(\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}$$

$\big\downarrow$ 16

$$\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-7aB)\left(\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}$$

$\downarrow$ 1142

$$\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-7aB)\left(\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}$$

$\Big\downarrow$ 25

$$\frac{\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-(Ab-7aB)\left(\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-(Ab-7aB)\left(\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}$$

$\downarrow$ 1082

$$\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-7aB)\left(\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}$$

$\downarrow$ 217

$$\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-7aB)\left(\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}$$

↓ 1103

$$\frac{\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-(Ab-7aB)\left(\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}$$

input `Int[(x^6*(A + B*x^3))/(a + b*x^3)^3,x]`

output
```
((A*b - a*B)*x^7)/(6*a*b*(a + b*x^3)^2) - ((A*b - 7*a*B)*(-1/3*x^4/(b*(a +
 b*x^3)) + (4*(x/b - (a*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-
((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/b))/(3*b
)))/(6*a*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.87 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.43

method	result
risch	$\frac{Bx}{b^3}+\frac{\left(-\frac{7}{18}b^2A+\frac{13}{18}abB\right)x^4-\frac{a(2Ab-5Ba)x}{9}}{b^3(bx^3+a)^2}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(Ab-7Ba)\ln(x-\_R)}{\_R^2}\right)}{27b^4}$
default	$\frac{Bx}{b^3}+\frac{\frac{\left(-\frac{7}{18}b^2A+\frac{13}{18}abB\right)x^4-\frac{a(2Ab-5Ba)x}{9}}{(bx^3+a)^2}+\frac{2(Ab-7Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{9}}{b^3}$

input
```
int(x^6*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
B*x/b^3+((-7/18*b^2*A+13/18*a*b*B)*x^4-1/9*a*(2*A*b-5*B*a)*x)/b^3/(b*x^3+a
)^2+2/27/b^4*sum((A*b-7*B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(161) = 322$.

Time = 0.14 (sec) , antiderivative size = 789, normalized size of antiderivative = 3.96

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/54*(54*B*a^2*b^3*x^7 + 21*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 6*sqrt(1/3)*(
(7*B*a^2*b^3 - A*a*b^4)*x^6 + 7*B*a^4*b - A*a^3*b^2 + 2*(7*B*a^3*b^2 - A*a
^2*b^3)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x -
 a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(
a^2*b)^(1/3)/b))/(b*x^3 + a)) + 2*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a
^2*b + 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2
/3)*x + (a^2*b)^(1/3)*a) - 4*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b
+ 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) +
12*(7*B*a^4*b - A*a^3*b^2)*x)/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4), 1/5
4*(54*B*a^2*b^3*x^7 + 21*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 12*sqrt(1/3)*((7*
B*a^2*b^3 - A*a*b^4)*x^6 + 7*B*a^4*b - A*a^3*b^2 + 2*(7*B*a^3*b^2 - A*a^2*
b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2
*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + 2*((7*B*a*b^2 - A*b^3)*x^6 + 7*B
*a^3 - A*a^2*b + 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 -
(a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 4*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3
- A*a^2*b + 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)
^(2/3)) + 12*(7*B*a^4*b - A*a^3*b^2)*x)/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4
*b^4)]
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.71

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx}{b^3} + \frac{x^4(-7Ab^2+13Bab)+x(-4Aab+10Ba^2)}{18a^2b^3+36ab^4x^3+18b^5x^6} + \mathrm{RootSum}\left(19683t^3a^2b^{10} - 8A^3b^3 + 168A^2Bab^2 - 1176AB^2a^2b + 2744B^3a^3, \left(t \mapsto t\log\left(-\frac{27tab^3}{-2Ab+1\ldots}\right.\right.\right.$$

input
```
integrate(x**6*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
B*x/b**3 + (x**4*(-7*A*b**2 + 13*B*a*b) + x*(-4*A*a*b + 10*B*a**2))/(18*a*
*2*b**3 + 36*a*b**4*x**3 + 18*b**5*x**6) + RootSum(19683*_t**3*a**2*b**10
- 8*A**3*b**3 + 168*A**2*B*a*b**2 - 1176*A*B**2*a**2*b + 2744*B**3*a**3, L
ambda(_t, _t*log(-27*_t*a*b**3/(-2*A*b + 14*B*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.96

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(13\,Bab-7\,Ab^2)x^4+2\,(5\,Ba^2-2\,Aab)x}{18\,(b^5x^6+2\,ab^4x^3+a^2b^3)}+\frac{Bx}{b^3} - \frac{2\,\sqrt{3}(7\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(7\,Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,b^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{2\,(7\,Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*((13*B*a*b - 7*A*b^2)*x^4 + 2*(5*B*a^2 - 2*A*a*b)*x)/(b^5*x^6 + 2*a*b
^4*x^3 + a^2*b^3) + B*x/b^3 - 2/27*sqrt(3)*(7*B*a - A*b)*arctan(1/3*sqrt(3
)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(2/3)) + 1/27*(7*B*a - A*b)*
log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) - 2/27*(7*B*a - A
*b)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.94

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2\sqrt{3}(7\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{2}{3}}b^2} + \frac{(7\,Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,\left(-ab^2\right)^{\frac{2}{3}}b^2} + \frac{Bx}{b^3} + \frac{2\,(7\,Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^3} + \frac{13\,Babx^4-7\,Ab^2x^4+10\,Ba^2x-4\,Aabx}{18\,(bx^3+a)^2b^3}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
2/27*sqrt(3)*(7*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(2/3)*b^2) + 1/27*(7*B*a - A*b)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) + B*x/b^3 + 2/27*(7*B*a - A*b)*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3) + 1/18*(13*B*a*b*x^4 - 7*A*b^2*x
^4 + 10*B*a^2*x - 4*A*a*b*x)/((b*x^3 + a)^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.92

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{B\,x}{b^3} - \frac{x^4\left(\frac{7\,A\,b^2}{18}-\frac{13\,B\,a\,b}{18}\right)-x\left(\frac{5\,B\,a^2}{9}-\frac{2\,A\,a\,b}{9}\right)}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6} + \frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(A\,b-7\,B\,a)}{27\,a^{2/3}\,b^{10/3}} - \frac{2\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-7\,B\,a)}{27\,a^{2/3}\,b^{10/3}} + \frac{2\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-7\,B\,a)}{27\,a^{2/3}\,b^{10/3}}$$

input
```
int((x^6*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
(B*x)/b^3 - (x^4*((7*A*b^2)/18 - (13*B*a*b)/18) - x*((5*B*a^2)/9 - (2*A*a*
b)/9))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (2*log(b^(1/3)*x + a^(1/3))*(A*
b - 7*B*a))/(27*a^(2/3)*b^(10/3)) - (2*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*
x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - 7*B*a))/(27*a^(2/3)*b^(10/3)) +
 (2*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)
*(A*b - 7*B*a))/(27*a^(2/3)*b^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.88

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{4a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)+4a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^3+2a^{\frac{4}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)+2a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}\right.}{9b^{\frac{7}{3}}\left(b\,x^3+a\right)}$$

input
```
int(x^6*(B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
(4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a +
 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b*x
**3 + 2*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a + 2
*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**3 - 4*a
**(1/3)*log(a**(1/3) + b**(1/3)*x)*a - 4*a**(1/3)*log(a**(1/3) + b**(1/3)*
x)*b*x**3 + 12*b**(1/3)*a*x + 9*b**(1/3)*b*x**4)/(9*b**(1/3)*b**2*(a + b*x
**3))
```

3.102 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 198

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{(Ab-aB)x^2}{6b^2(a+bx^3)^2}+\frac{(Ab-4aB)x^2}{9ab^2(a+bx^3)} - \frac{(Ab+5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{8/3}} - \frac{(Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{8/3}} + \frac{(Ab+5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{8/3}}$$

output
```
-1/6*(A*b-B*a)*x^2/b^2/(b*x^3+a)^2+1/9*(A*b-4*B*a)*x^2/a/b^2/(b*x^3+a)-1/2
7*(A*b+5*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^
(4/3)/b^(8/3)-1/27*(A*b+5*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(8/3)+1/54*
(A*b+5*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(8/3)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.91

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{9b^{2/3}(Ab-aB)x^2}{(a+bx^3)^2}+\frac{6b^{2/3}(Ab-4aB)x^2}{a(a+bx^3)}-\frac{2\sqrt{3}(Ab+5aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}-\frac{2(Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{4/3}}+\frac{(Ab+5aB)\log\left(a\right.}{}}{54b^{8/3}}$$

input
```
Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((-9*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3)^2 + (6*b^(2/3)*(A*b - 4*a*B)*x^2
)/(a*(a + b*x^3)) - (2*Sqrt[3]*(A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]])/a^(4/3) - (2*(A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(4/
3) + ((A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3
))/(54*b^(8/3))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {957, 817, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(5aB+Ab)\int \frac{x^4}{(bx^3+a)^2}dx}{6ab}+\frac{x^5(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$$\Big\downarrow \text{817}$$

$$\frac{(5aB+Ab)\left(\frac{2\int \frac{x}{bx^3+a}dx}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6ab}+\frac{x^5(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$\Big\downarrow$ 821

$$\frac{(5aB+Ab)\left(\frac{2\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6ab}+\frac{x^5(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$\Big\downarrow$ 16

$$\frac{(5aB+Ab)\left(\frac{2\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6ab}+\frac{x^5(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$\Big\downarrow$ 1142

$$\frac{(5aB+Ab)\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6ab}+\frac{x^5(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(5aB+Ab)\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6ab}+\frac{x^5(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(5aB+Ab)\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6ab}+\frac{x^5(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(5aB+Ab)\left(\frac{2\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6ab}+\frac{x^5(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 217

$$\frac{(5aB+Ab)\left(\frac{2\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6ab}+\frac{x^5(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{(5aB+Ab)\left(\frac{2\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6ab}+\frac{x^5(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input
```
Int[(x^4*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^5)/(6*a*b*(a + b*x^3)^2) + ((A*b + 5*a*B)*(-1/3*x^2/(b*(a +
 b*x^3)) + (2*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[
3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/(3*b)))
/(6*a*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.79 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\frac{(Ab-4Ba)x^5}{9ab}-\frac{(Ab+5Ba)x^2}{18b^2}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab+5Ba)\ln\left(x-\_R\right)}{\_R}}{27a\,b^3}$	85
default	$\frac{\frac{(Ab-4Ba)x^5}{9ab}-\frac{(Ab+5Ba)x^2}{18b^2}}{(b\,x^3+a)^2}+\frac{(Ab+5Ba)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^2a}$	154

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/9*(A*b-4*B*a)/a/b*x^5-1/18*(A*b+5*B*a)/b^2*x^2)/(b*x^3+a)^2+1/27/a/b^3*
sum((A*b+5*B*a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(158) = 316$.

Time = 0.14 (sec) , antiderivative size = 756, normalized size of antiderivative = 3.82

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[-1/54*(6*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 3*(5*B*a^3*b^2 + A*a^2*b^3)*x^2 -
3*sqrt(1/3)*((5*B*a^2*b^3 + A*a*b^4)*x^6 + 5*B*a^4*b + A*a^3*b^2 + 2*(5*B*
a^3*b^2 + A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*
sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^
(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((5*B*a*b^2 + A*b^3)*x^6 + 5
*B*a^3 + A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2
 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b^2 + A*b^3)*x^6 + 5*B
*a^3 + A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a
*b^2)^(1/3)))/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4), -1/54*(6*(4*B*a^2*b
^3 - A*a*b^4)*x^5 + 3*(5*B*a^3*b^2 + A*a^2*b^3)*x^2 - 6*sqrt(1/3)*((5*B*a^
2*b^3 + A*a*b^4)*x^6 + 5*B*a^4*b + A*a^3*b^2 + 2*(5*B*a^3*b^2 + A*a^2*b^3)
*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sq
rt(-(-a*b^2)^(1/3)/a)/b) - ((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 + A*a^2*b +
2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b
*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 + A*a^2*b + 2*
(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*
b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4)]
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.78

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^5\cdot(2Ab^2-8Bab)+x^2(-Aab-5Ba^2)}{18a^3b^2+36a^2b^3x^3+18ab^4x^6}$$
$$+\operatorname{RootSum}\left(19683t^3a^4b^8+A^3b^3+15A^2Bab^2+75AB^2a^2b+125B^3a^3,\left(t\mapsto t\log\left(\frac{729t^2a^3b^5}{A^2b^2+10ABab+\right.\right.\right.$$

input
```
integrate(x**4*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
(x**5*(2*A*b**2 - 8*B*a*b) + x**2*(-A*a*b - 5*B*a**2))/(18*a**3*b**2 + 36*
a**2*b**3*x**3 + 18*a*b**4*x**6) + RootSum(19683*_t**3*a**4*b**8 + A**3*b*
*3 + 15*A**2*B*a*b**2 + 75*A*B**2*a**2*b + 125*B**3*a**3, Lambda(_t, _t*lo
g(729*_t**2*a**3*b**5/(A**2*b**2 + 10*A*B*a*b + 25*B**2*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.98

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{2\,(4\,Bab-Ab^2)x^5+(5\,Ba^2+Aab)x^2}{18\,(ab^4x^6+2\,a^2b^3x^3+a^3b^2)} + \frac{\sqrt{3}(5\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(5\,Ba+Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(5\,Ba+Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*(2*(4*B*a*b - A*b^2)*x^5 + (5*B*a^2 + A*a*b)*x^2)/(a*b^4*x^6 + 2*a^2
*b^3*x^3 + a^3*b^2) + 1/27*sqrt(3)*(5*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x -
 (a/b)^(1/3))/(a/b)^(1/3))/(a*b^3*(a/b)^(1/3)) + 1/54*(5*B*a + A*b)*log(x^
2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(1/3)) - 1/27*(5*B*a + A*b)*
log(x + (a/b)^(1/3))/(a*b^3*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.04

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\sqrt{3}(5\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}ab^2} - \frac{(5\,Ba+Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}ab^2} - \frac{\left(5\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}+Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^2} - \frac{8\,Babx^5-2\,Ab^2x^5+5\,Ba^2x^2+Aabx^2}{18\,\left(bx^3+a\right)^2ab^2}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/27*sqrt(3)*(5*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(1/3)*a*b^2) - 1/54*(5*B*a + A*b)*log(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a*b^2) - 1/27*(5*B*a*(-a/b)^(1/3) + A*b*
(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^2) - 1/18*(8*
B*a*b*x^5 - 2*A*b^2*x^5 + 5*B*a^2*x^2 + A*a*b*x^2)/((b*x^3 + a)^2*a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.88

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{\frac{x^2\,(A\,b+5\,B\,a)}{18\,b^2}-\frac{x^5\,(A\,b-4\,B\,a)}{9\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b+5\,B\,a\right)}{27\,a^{4/3}\,b^{8/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b+5\,B\,a\right)}{27\,a^{4/3}\,b^{8/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b+5\,B\,a\right)}{27\,a^{4/3}\,b^{8/3}}$$

input
```
int((x^4*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
(log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A
*b + 5*B*a))/(27*a^(4/3)*b^(8/3)) - (log(b^(1/3)*x + a^(1/3))*(A*b + 5*B*a
))/(27*a^(4/3)*b^(8/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))
*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a))/(27*a^(4/3)*b^(8/3)) - ((x^2*(A*b +
 5*B*a))/(18*b^2) - (x^5*(A*b - 4*B*a))/(9*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^
3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.80

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^3-3b^{\frac{2}{3}}a^{\frac{1}{3}}x^2+\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a+\log\left(a^{\frac{2}{3}}-\right.}{9b^{\frac{5}{3}}a^{\frac{1}{3}}\left(b\,x^3+a\right)}$$

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a - 2*sqr
t(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b*x**3 - 3*b**(2/3
)*a**(1/3)*x**2 + log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a +
log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**3 - 2*log(a**(1/3
) + b**(1/3)*x)*a - 2*log(a**(1/3) + b**(1/3)*x)*b*x**3)/(9*b**(2/3)*a**(1
/3)*b*(a + b*x**3))
```

3.103 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 194

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3} dx = -\frac{(Ab-aB)x}{6b^2(a+bx^3)^2} + \frac{(Ab-7aB)x}{18ab^2(a+bx^3)} - \frac{(Ab+2aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{7/3}} + \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}} - \frac{(Ab+2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{7/3}}$$

output
```
-1/6*(A*b-B*a)*x/b^2/(b*x^3+a)^2+1/18*(A*b-7*B*a)*x/a/b^2/(b*x^3+a)-1/27*(
A*b+2*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/
3)/b^(7/3)+1/27*(A*b+2*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(7/3)-1/54*(A*
b+2*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(7/3)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.92

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{9\sqrt[3]{b}(Ab-aB)x}{(a+bx^3)^2}+\frac{3\sqrt[3]{b}(Ab-7aB)x}{a(a+bx^3)}-\frac{2\sqrt{3}(Ab+2aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}+\frac{2(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}-\frac{(Ab+2aB)\log\left(a^{2/3}\right.}{}}{54b^{7/3}}$$

input
```
Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((-9*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3)^2 + (3*b^(1/3)*(A*b - 7*a*B)*x)/(a
*(a + b*x^3)) - (2*Sqrt[3]*(A*b + 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]])/a^(5/3) + (2*(A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) -
 ((A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(
54*b^(7/3))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {957, 817, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(2aB+Ab)\int \frac{x^3}{(bx^3+a)^2}dx}{3ab}+\frac{x^4(Ab-aB)}{6ab(a+bx^3)^2}$$

$$\Big\downarrow \text{817}$$

$$\frac{(2aB+Ab)\left(\frac{\int \frac{1}{bx^3+a}dx}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 750

$$\frac{(2aB+Ab)\left(\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 16

$$\frac{(2aB+Ab)\left(\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 1142

$$\frac{(2aB+Ab)\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 25

$$\frac{(2aB+Ab)\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(2aB+Ab)\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{(2aB+Ab)\left(\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{(2aB+Ab)\left(\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{(2aB+Ab)\left(\frac{\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input
```
Int[(x^3*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^4)/(6*a*b*(a + b*x^3)^2) + ((A*b + 2*a*B)*(-1/3*x/(b*(a + b
*x^3)) + (Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3)))/(3*b)))/(3*a*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.81 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\frac{(Ab-7Ba)x^4}{18ab}-\frac{(Ab+2Ba)x}{9b^2}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab+2Ba)\ln\left(x-\_R\right)}{\_R^2}}{27a\,b^3}$	83
default	$\frac{\frac{(Ab-7Ba)x^4}{18ab}-\frac{(Ab+2Ba)x}{9b^2}}{(b\,x^3+a)^2}+\frac{(Ab+2Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{9b^2a}$	152

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/18*(A*b-7*B*a)/a/b*x^4-1/9*(A*b+2*B*a)/b^2*x)/(b*x^3+a)^2+1/27/a/b^3*su
m((A*b+2*B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(154) = 308$.

Time = 0.15 (sec) , antiderivative size = 743, normalized size of antiderivative = 3.83

$$\int \frac{x^3(A + Bx^3)}{(a + bx^3)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[-1/54*(3*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 3*sqrt(1/3)*((2*B*a^2*b^3 + A*a*
b^4)*x^6 + 2*B*a^4*b + A*a^3*b^2 + 2*(2*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt(-
(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*
(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b
*x^3 + a)) + ((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b +
 A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)
*a) - 2*((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*
b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(2*B*a^4*b + A*a^3*
b^2)*x)/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3), -1/54*(3*(7*B*a^3*b^2 - A
*a^2*b^3)*x^4 - 6*sqrt(1/3)*((2*B*a^2*b^3 + A*a*b^4)*x^6 + 2*B*a^4*b + A*a
^3*b^2 + 2*(2*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqr
t(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) +
((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^3
)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((2*B
*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^3)*(a^
2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(2*B*a^4*b + A*a^3*b^2)*x)/(a^3*
b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.70

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^4(Ab^2-7Bab)+x(-2Aab-4Ba^2)}{18a^3b^2+36a^2b^3x^3+18ab^4x^6} + \mathrm{RootSum}\left(19683t^3a^5b^7-A^3b^3-6A^2Bab^2-12AB^2a^2b-8B^3a^3,\left(t\mapsto t\log\left(\frac{27ta^2b^2}{Ab+2Ba}+x\right)\right)\right)$$

input
```
integrate(x**3*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
(x**4*(A*b**2 - 7*B*a*b) + x*(-2*A*a*b - 4*B*a**2))/(18*a**3*b**2 + 36*a**
2*b**3*x**3 + 18*a*b**4*x**6) + RootSum(19683*_t**3*a**5*b**7 - A**3*b**3
- 6*A**2*B*a*b**2 - 12*A*B**2*a**2*b - 8*B**3*a**3, Lambda(_t, _t*log(27*_
t*a**2*b**2/(A*b + 2*B*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.99

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{(7\,Bab-Ab^2)x^4+2\,(2\,Ba^2+Aab)x}{18\,(ab^4x^6+2\,a^2b^3x^3+a^3b^2)} + \frac{\sqrt{3}(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(2\,Ba+Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(2\,Ba+Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*((7*B*a*b - A*b^2)*x^4 + 2*(2*B*a^2 + A*a*b)*x)/(a*b^4*x^6 + 2*a^2*b
^3*x^3 + a^3*b^2) + 1/27*sqrt(3)*(2*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(a*b^3*(a/b)^(2/3)) - 1/54*(2*B*a + A*b)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 1/27*(2*B*a + A*b)*lo
g(x + (a/b)^(1/3))/(a*b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.96

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{\sqrt{3}(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}ab} - \frac{(2\,Ba+Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}ab} - \frac{(2\,Ba+Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^2} - \frac{7\,Babx^4-Ab^2x^4+4\,Ba^2x+2\,Aabx}{18\left(bx^3+a\right)^2ab^2}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(2*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/54*(2*B*a + A*b)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b) - 1/27*(2*B*a + A*b)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/(a^2*b^2) - 1/18*(7*B*a*b*x^4 - A*b^2*x^4 + 4*B*a
^2*x + 2*A*a*b*x)/((b*x^3 + a)^2*a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.89

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b+2\,B\,a\right)}{27\,a^{5/3}\,b^{7/3}} - \frac{\frac{x\,(A\,b+2\,B\,a)}{9\,b^2}-\frac{x^4\,(A\,b-7\,B\,a)}{18\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b+2\,B\,a\right)}{27\,a^{5/3}\,b^{7/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b+2\,B\,a\right)}{27\,a^{5/3}\,b^{7/3}}$$

input
```
int((x^3*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
(log(b^(1/3)*x + a^(1/3))*(A*b + 2*B*a))/(27*a^(5/3)*b^(7/3)) - ((x*(A*b +
 2*B*a))/(9*b^2) - (x^4*(A*b - 7*B*a))/(18*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^
3) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/
2)*(A*b + 2*B*a))/(27*a^(5/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/
3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b + 2*B*a))/(27*a^(5/3)*b^(7/3))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.88

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{-2a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)-2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^3-a^{\frac{4}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a\right.}{18b^{\frac{4}{3}}a\left(b\,x^3+a\right)}$$

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
( - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
a - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
b*x**3 - a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a -
a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**3 + 2*a*
*(1/3)*log(a**(1/3) + b**(1/3)*x)*a + 2*a**(1/3)*log(a**(1/3) + b**(1/3)*x
)*b*x**3 - 6*b**(1/3)*a*x)/(18*b**(1/3)*a*b*(a + b*x**3))
```

3.104 $\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 201

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx=\frac{(Ab-aB)x^2}{6ab\,(a+bx^3)^2}+\frac{(2Ab+aB)x^2}{9a^2b\,(a+bx^3)}-\frac{(2Ab+aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{5/3}}-\frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{5/3}}+\frac{(2Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{5/3}}$$

output
```
1/6*(A*b-B*a)*x^2/a/b/(b*x^3+a)^2+1/9*(2*A*b+B*a)*x^2/a^2/b/(b*x^3+a)-1/27
*(2*A*b+B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(
7/3)/b^(5/3)-1/27*(2*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(5/3)+1/54*(
2*A*b+B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(5/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.89

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx$$
$$= \frac{-\frac{9a^{4/3}b^{2/3}(-Ab+aB)x^2}{(a+bx^3)^2}+\frac{6\sqrt[3]{a}b^{2/3}(2Ab+aB)x^2}{a+bx^3}-2\sqrt{3}(2Ab+aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2(2Ab+aB)\log\left(\sqrt[3]{a}\right.}{54a^{7/3}b^{5/3}}$$

input
```
Integrate[(x*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((-9*a^(4/3)*b^(2/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3)^2 + (6*a^(1/3)*b^(2/3
)*(2*A*b + a*B)*x^2)/(a + b*x^3) - 2*Sqrt[3]*(2*A*b + a*B)*ArcTan[(1 - (2*
b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(2*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x] +
(2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*
b^(5/3))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {957, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx$$
$$\Big\downarrow \text{957}$$
$$\frac{(aB+2Ab)\int \frac{x}{(bx^3+a)^2}dx}{3ab}+\frac{x^2(Ab-aB)}{6ab(a+bx^3)^2}$$
$$\Big\downarrow \text{819}$$

$$\frac{(aB+2Ab)\left(\frac{\int \frac{x}{bx^3+a}dx}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3ab}+\frac{x^2(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 821

$$\frac{(aB+2Ab)\left(\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3ab}+\frac{x^2(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 16

$$\frac{(aB+2Ab)\left(\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3ab}+\frac{x^2(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1142

$$\frac{(aB+2Ab)\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3ab}+\frac{x^2(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 25

$$\frac{(aB+2Ab)\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3ab}+\frac{x^2(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 27

$$\frac{(aB+2Ab)\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3ab}+\frac{x^2(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{(aB+2Ab)\left(\frac{\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3ab}+\frac{x^2(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 217

$$\frac{(aB+2Ab)\left(\frac{\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3ab}+\frac{x^2(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\big\downarrow$ 1103

$$\frac{(aB+2Ab)\left(\frac{\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3ab}+\frac{x^2(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input
```
Int[(x*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^2)/(6*a*b*(a + b*x^3)^2) + ((2*A*b + a*B)*(x^2/(3*a*(a + b*
x^3)) + (-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*Arc
Tan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3)))/(3*a)))/(3*a*b
)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.83 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\frac{(2Ab+Ba)x^5}{9a^2}+\frac{(7Ab-Ba)x^2}{18ab}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(2Ab+Ba)\ln\left(x-\_R\right)}{\_R}}{27a^2b^2}$	86
default	$\frac{\frac{(2Ab+Ba)x^5}{9a^2}+\frac{(7Ab-Ba)x^2}{18ab}}{(b\,x^3+a)^2}+\frac{(2Ab+Ba)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b}$	155

input
```
int(x*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/9*(2*A*b+B*a)/a^2*x^5+1/18*(7*A*b-B*a)/a/b*x^2)/(b*x^3+a)^2+1/27/a^2/b^
2*sum((2*A*b+B*a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(160) = 320$.

Time = 0.13 (sec) , antiderivative size = 752, normalized size of antiderivative = 3.74

$$\int \frac{x(A + Bx^3)}{(a + bx^3)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/54*(6*(B*a^2*b^3 + 2*A*a*b^4)*x^5 - 3*(B*a^3*b^2 - 7*A*a^2*b^3)*x^2 + 3
*sqrt(1/3)*((B*a^2*b^3 + 2*A*a*b^4)*x^6 + B*a^4*b + 2*A*a^3*b^2 + 2*(B*a^3
*b^2 + 2*A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*s
qrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(
1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + ((B*a*b^2 + 2*A*b^3)*x^6 + B*
a^3 + 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2
+ (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*((B*a*b^2 + 2*A*b^3)*x^6 + B*a^
3 + 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*
b^2)^(1/3)))/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3), 1/54*(6*(B*a^2*b^3 +
 2*A*a*b^4)*x^5 - 3*(B*a^3*b^2 - 7*A*a^2*b^3)*x^2 + 6*sqrt(1/3)*((B*a^2*b^
3 + 2*A*a*b^4)*x^6 + B*a^4*b + 2*A*a^3*b^2 + 2*(B*a^3*b^2 + 2*A*a^2*b^3)*x
^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt
(-(-a*b^2)^(1/3)/a)/b) + ((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A*a^2*b + 2*
(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x
 + (-a*b^2)^(2/3)) - 2*((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A*a^2*b + 2*(B
*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^
5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.76

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^5\cdot(4Ab^2+2Bab)+x^2\cdot(7Aab-Ba^2)}{18a^4b+36a^3b^2x^3+18a^2b^3x^6}$$
$$+\mathrm{RootSum}\left(19683t^3a^7b^5+8A^3b^3+12A^2Bab^2+6AB^2a^2b+B^3a^3,\left(t\mapsto t\log\left(\frac{729t^2a^5b^3}{4A^2b^2+4ABab+B^2a^2}\right.\right.\right.$$

input
```
integrate(x*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
(x**5*(4*A*b**2 + 2*B*a*b) + x**2*(7*A*a*b - B*a**2))/(18*a**4*b + 36*a**3
*b**2*x**3 + 18*a**2*b**3*x**6) + RootSum(19683*_t**3*a**7*b**5 + 8*A**3*b
**3 + 12*A**2*B*a*b**2 + 6*A*B**2*a**2*b + B**3*a**3, Lambda(_t, _t*log(72
9*_t**2*a**5*b**3/(4*A**2*b**2 + 4*A*B*a*b + B**2*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.97

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2\,(Bab+2\,Ab^2)x^5-(Ba^2-7\,Aab)x^2}{18\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)} + \frac{\sqrt{3}(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba+2\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba+2\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(2*(B*a*b + 2*A*b^2)*x^5 - (B*a^2 - 7*A*a*b)*x^2)/(a^2*b^3*x^6 + 2*a^
3*b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(1/3)) + 1/54*(B*a + 2*A*b)*log(x
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(1/3)) - 1/27*(B*a + 2*A*
b)*log(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.03

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\sqrt{3}(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}a^2b} - \frac{(Ba+2\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{1}{3}}a^2b} - \frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b} + \frac{2\,Babx^5+4\,Ab^2x^5-Ba^2x^2+7\,Aabx^2}{18\left(bx^3+a\right)^2a^2b}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/27*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(1/3)*a^2*b) - 1/54*(B*a + 2*A*b)*log(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2*b) - 1/27*(B*a*(-a/b)^(1/3) + 2*A*b*
(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) + 1/18*(2*B*
a*b*x^5 + 4*A*b^2*x^5 - B*a^2*x^2 + 7*A*a*b*x^2)/((b*x^3 + a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\frac{x^5\,(2\,A\,b+B\,a)}{9\,a^2}+\frac{x^2\,(7\,A\,b-B\,a)}{18\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(2\,A\,b+B\,a)}{27\,a^{7/3}\,b^{5/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b+B\,a)}{27\,a^{7/3}\,b^{5/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b+B\,a)}{27\,a^{7/3}\,b^{5/3}}$$

input
```
int((x*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
((x^5*(2*A*b + B*a))/(9*a^2) + (x^2*(7*A*b - B*a))/(18*a*b))/(a^2 + b^2*x^
6 + 2*a*b*x^3) - (log(b^(1/3)*x + a^(1/3))*(2*A*b + B*a))/(27*a^(7/3)*b^(5
/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 -
1/2)*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(
1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3
))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.79

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx$$
$$= \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^3+6b^{\frac{2}{3}}a^{\frac{1}{3}}x^2+\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a+\log\left(a^{\frac{2}{3}}-\right.}{18b^{\frac{2}{3}}a^{\frac{4}{3}}\left(b\,x^3+a\right)}$$

input
```
int(x*(B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a - 2*sqr
t(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b*x**3 + 6*b**(2/3
)*a**(1/3)*x**2 + log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a +
log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**3 - 2*log(a**(1/3
) + b**(1/3)*x)*a - 2*log(a**(1/3) + b**(1/3)*x)*b*x**3)/(18*b**(2/3)*a**(
1/3)*a*(a + b*x**3))
```

3.105 $\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 197

$$\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx = \frac{(Ab-aB)x}{6ab\,(a+bx^3)^2}+\frac{(5Ab+aB)x}{18a^2b\,(a+bx^3)} - \frac{(5Ab+aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{4/3}}+\frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}} - \frac{(5Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{4/3}}$$

output
```
1/6*(A*b-B*a)*x/a/b/(b*x^3+a)^2+1/18*(5*A*b+B*a)*x/a^2/b/(b*x^3+a)-1/27*(5
*A*b+B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3
)/b^(4/3)+1/27*(5*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(4/3)-1/54*(5*A
*b+B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx$$
$$= \frac{-\frac{9a^{5/3}\sqrt[3]{b}(-Ab+aB)x}{(a+bx^3)^2}+\frac{3a^{2/3}\sqrt[3]{b}(5Ab+aB)x}{a+bx^3}-2\sqrt{3}(5Ab+aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2(5Ab+aB)\log\left(\sqrt[3]{a}+\right.}{54a^{8/3}b^{4/3}}$$

input
```
Integrate[(A + B*x^3)/(a + b*x^3)^3,x]
```

output
```
((-9*a^(5/3)*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3)^2 + (3*a^(2/3)*b^(1/3)*
(5*A*b + a*B)*x)/(a + b*x^3) - 2*Sqrt[3]*(5*A*b + a*B)*ArcTan[(1 - (2*b^(1
/3)*x)/a^(1/3))/Sqrt[3]] + 2*(5*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x] - (5*A
*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(4
/3))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {910, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx$$
$$\Big\downarrow \text{ 910}$$
$$\frac{(aB+5Ab)\int \frac{1}{(bx^3+a)^2}dx}{6ab}+\frac{x(Ab-aB)}{6ab\,(a+bx^3)^2}$$
$$\Big\downarrow \text{ 749}$$

$$\frac{(aB+5Ab)\left(\frac{2\int\frac{1}{bx^3+a}dx}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 750

$$\frac{(aB+5Ab)\left(\frac{2\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 16

$$\frac{(aB+5Ab)\left(\frac{2\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 1142

$$\frac{(aB+5Ab)\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{(aB+5Ab)\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(aB+5Ab)\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{(aB+5Ab)\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 217

$$\frac{(aB+5Ab)\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{(aB+5Ab)\left(\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input
```
Int[(A + B*x^3)/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x)/(6*a*b*(a + b*x^3)^2) + ((5*A*b + a*B)*(x/(3*a*(a + b*x^3)
) + (2*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/(3*a)))/(6*a*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.81 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\frac{(5Ab+Ba)x^4}{18a^2}+\frac{(4Ab-Ba)x}{9ab}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(5Ab+Ba)\ln\left(x-\_R\right)}{\_R^2}}{27b^2a^2}$	84
default	$\frac{\frac{(5Ab+Ba)x^4}{18a^2}+\frac{(4Ab-Ba)x}{9ab}}{(b\,x^3+a)^2}+\frac{(5Ab+Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{9b\,a^2}$	153

input
```
int((B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/18*(5*A*b+B*a)/a^2*x^4+1/9*(4*A*b-B*a)/a/b*x)/(b*x^3+a)^2+1/27/b^2/a^2*
sum((5*A*b+B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(156) = 312$.

Time = 0.10 (sec) , antiderivative size = 743, normalized size of antiderivative = 3.77

$$\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/54*(3*(B*a^3*b^2 + 5*A*a^2*b^3)*x^4 + 3*sqrt(1/3)*((B*a^2*b^3 + 5*A*a*b
^4)*x^6 + B*a^4*b + 5*A*a^3*b^2 + 2*(B*a^3*b^2 + 5*A*a^2*b^3)*x^3)*sqrt(-(
a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(
2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*
x^3 + a)) - ((B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*
A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*
a) + 2*((B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b
^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^4*b - 4*A*a^3*b
^2)*x)/(a^4*b^4*x^6 + 2*a^5*b^3*x^3 + a^6*b^2), 1/54*(3*(B*a^3*b^2 + 5*A*a
^2*b^3)*x^4 + 6*sqrt(1/3)*((B*a^2*b^3 + 5*A*a*b^4)*x^6 + B*a^4*b + 5*A*a^3
*b^2 + 2*(B*a^3*b^2 + 5*A*a^2*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(
1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((
B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3)*
(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b
^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3)*(a^2*
b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^4*b - 4*A*a^3*b^2)*x)/(a^4*b^
4*x^6 + 2*a^5*b^3*x^3 + a^6*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx = \frac{x^4\cdot(5Ab^2+Bab)+x(8Aab-2Ba^2)}{18a^4b+36a^3b^2x^3+18a^2b^3x^6} + \operatorname{RootSum}\left(19683t^3a^8b^4-125A^3b^3-75A^2Bab^2-15AB^2a^2b-B^3a^3, \left(t\mapsto t\log\left(\frac{27ta^3b}{5Ab+Ba}+x\right)\right)\right)$$

input
```
integrate((B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
(x**4*(5*A*b**2 + B*a*b) + x*(8*A*a*b - 2*B*a**2))/(18*a**4*b + 36*a**3*b*
*2*x**3 + 18*a**2*b**3*x**6) + RootSum(19683*_t**3*a**8*b**4 - 125*A**3*b*
*3 - 75*A**2*B*a*b**2 - 15*A*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(27
*_t*a**3*b/(5*A*b + B*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx = \frac{(Bab+5\,Ab^2)x^4-2\,(Ba^2-4\,Aab)x}{18\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)} + \frac{\sqrt{3}(Ba+5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba+5\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba+5\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*((B*a*b + 5*A*b^2)*x^4 - 2*(B*a^2 - 4*A*a*b)*x)/(a^2*b^3*x^6 + 2*a^3*
b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(B*a + 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a
/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) - 1/54*(B*a + 5*A*b)*log(x^2
 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(2/3)) + 1/27*(B*a + 5*A*b)
*log(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^3}\,dx = -\frac{\sqrt{3}(Ba+5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^2} - \frac{(Ba+5\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^2} - \frac{(Ba+5\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b} + \frac{Babx^4+5\,Ab^2x^4-2\,Ba^2x+8\,Aabx}{18\left(bx^3+a\right)^2a^2b}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(B*a + 5*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(B*a + 5*A*b)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27*(B*a + 5*A*b)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/(a^3*b) + 1/18*(B*a*b*x^4 + 5*A*b^2*x^4 - 2*B*a^2
*x + 8*A*a*b*x)/((b*x^3 + a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^3}\,dx = \frac{\frac{x^4\,(5\,A\,b+B\,a)}{18\,a^2}+\frac{x\,(4\,A\,b-B\,a)}{9\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6} + \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(5\,A\,b+B\,a)}{27\,a^{8/3}\,b^{4/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b+B\,a)}{27\,a^{8/3}\,b^{4/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b+B\,a)}{27\,a^{8/3}\,b^{4/3}}$$

input
```
int((A + B*x^3)/(a + b*x^3)^3,x)
```

output
```
((x^4*(5*A*b + B*a))/(18*a^2) + (x*(4*A*b - B*a))/(9*a*b))/(a^2 + b^2*x^6
+ 2*a*b*x^3) + (log(b^(1/3)*x + a^(1/3))*(5*A*b + B*a))/(27*a^(8/3)*b^(4/3
)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/
2)*(5*A*b + B*a))/(27*a^(8/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/
3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a))/(27*a^(8/3)*b^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx$$
$$= \frac{-2a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)-2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^3-a^{\frac{4}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a\right.}{9b^{\frac{1}{3}}a^2\left(b\,x^3+a\right)}$$

input
```
int((B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
( - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
a - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
b*x**3 - a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a -
a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**3 + 2*a*
*(1/3)*log(a**(1/3) + b**(1/3)*x)*a + 2*a**(1/3)*log(a**(1/3) + b**(1/3)*x
)*b*x**3 + 3*b**(1/3)*a*x)/(9*b**(1/3)*a**2*(a + b*x**3))
```

3.106 $\int \frac{A+Bx^3}{x^2(a+bx^3)^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 208

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^3} \, dx = -\frac{A}{a^3x} - \frac{(Ab-aB)x^2}{6a^2(a+bx^3)^2} - \frac{(5Ab-2aB)x^2}{9a^3(a+bx^3)} + \frac{2(7Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{2/3}} + \frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{2/3}} - \frac{(7Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}b^{2/3}}$$

output
```
-A/a^3/x-1/6*(A*b-B*a)*x^2/a^2/(b*x^3+a)^2-1/9*(5*A*b-2*B*a)*x^2/a^3/(b*x^
3+a)+2/27*(7*A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^
(1/2)/a^(10/3)/b^(2/3)+2/27*(7*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(
2/3)-1/27*(7*A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b
^(2/3)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx$$

$$= \frac{-\frac{54\sqrt[3]{a}A}{x}+\frac{9a^{4/3}(-Ab+aB)x^2}{(a+bx^3)^2}+\frac{6\sqrt[3]{a}(-5Ab+2aB)x^2}{a+bx^3}+\frac{4\sqrt{3}(7Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{4(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} -}{54a^{10/3}}$$

input
```
Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^3),x]
```

output
```
((-54*a^(1/3)*A)/x + (9*a^(4/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3)^2 + (6*a^(
1/3)*(-5*A*b + 2*a*B)*x^2)/(a + b*x^3) + (4*Sqrt[3]*(7*A*b - a*B)*ArcTan[(
1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (4*(7*A*b - a*B)*Log[a^(1/3
) + b^(1/3)*x])/b^(2/3) + (2*(-7*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/b^(2/3))/(54*a^(10/3))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {957, 819, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{(7Ab-aB)\int \frac{1}{x^2(bx^3+a)^2}dx}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{(7Ab-aB)\left(\frac{4\int \frac{1}{x^2\left(bx^3+a\right)}dx}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 847$$

$$\frac{(7Ab-aB)\left(\frac{4\left(-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 821$$

$$\frac{(7Ab-aB)\left(\frac{4\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 16$$

$$\frac{(7Ab-aB)\left(\frac{4\left(-\frac{b\left(\frac{\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1142

$$\frac{(7Ab-aB)\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{(7Ab-aB)\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$\big\downarrow$ 27

$$\frac{(7Ab-aB)\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$\downarrow$ 1082

$$\frac{(7Ab-aB)\left(\frac{4\left(-\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}$$

$$\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 217

$$\frac{(7Ab-aB)\left(\frac{4\left(-\frac{b\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$$\downarrow 1103$$

$$\frac{(7Ab-aB)\left(\frac{4\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

input
```
Int[(A + B*x^3)/(x^2*(a + b*x^3)^3),x]
```

output
```
(A*b - a*B)/(6*a*b*x*(a + b*x^3)^2) + ((7*A*b - a*B)*(1/(3*a*x*(a + b*x^3)
) + (4*(-(1/(a*x)) - (b*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) +
 (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))
))/a))/(3*a)))/(6*a*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.76

method	result
default	$-\frac{A}{a^3 x}-\frac{\frac{\left(\frac{5}{9}b^2A-\frac{2}{9}abB\right)x^5+\frac{a(13Ab-7Ba)x^2}{18}}{\left(b\,x^3+a\right)^2}+\left(\frac{14Ab}{9}-\frac{2Ba}{9}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{3}\right.}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{a^3}$
risch	$\frac{-\frac{2b(7Ab-Ba)x^6}{9a^3}-\frac{7(7Ab-Ba)x^3}{18a^2}-\frac{A}{a}}{x(b\,x^3+a)^2}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}b^2\_Z^3-343A^3b^3+147A^2Ba\,b^2-21A\,B^2a^2b+B^3a^3\right)}\_R\ln\left(\left(-4\_R^3a^{10}\right.\right.\right.}{27}$

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-A/a^3/x-1/a^3*(((5/9*b^2*A-2/9*a*b*B)*x^5+1/18*a*(13*A*b-7*B*a)*x^2)/(b*x
^3+a)^2+(14/9*A*b-2/9*B*a)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/
b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(164) = 328$.

Time = 0.17 (sec) , antiderivative size = 776, normalized size of antiderivative = 3.73

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/54*(12*(B*a^2*b^3 - 7*A*a*b^4)*x^6 - 54*A*a^3*b^2 + 21*(B*a^3*b^2 - 7*A
*a^2*b^3)*x^3 - 6*sqrt(1/3)*((B*a^2*b^3 - 7*A*a*b^4)*x^7 + 2*(B*a^3*b^2 -
7*A*a^2*b^3)*x^4 + (B*a^4*b - 7*A*a^3*b^2)*x)*sqrt(-(a*b^2)^(1/3)/a)*log((
2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)
*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*((B*a*b^2
 - 7*A*b^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*
b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*((B*a*b^2
- 7*A*b^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b
^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^4*b^4*x^7 + 2*a^5*b^3*x^4 + a^6*b^2
*x), 1/54*(12*(B*a^2*b^3 - 7*A*a*b^4)*x^6 - 54*A*a^3*b^2 + 21*(B*a^3*b^2 -
 7*A*a^2*b^3)*x^3 - 12*sqrt(1/3)*((B*a^2*b^3 - 7*A*a*b^4)*x^7 + 2*(B*a^3*b
^2 - 7*A*a^2*b^3)*x^4 + (B*a^4*b - 7*A*a^3*b^2)*x)*sqrt((a*b^2)^(1/3)/a)*a
rctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) + 2*((B*
a*b^2 - 7*A*b^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x
)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*((B*a
*b^2 - 7*A*b^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)
*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^4*b^4*x^7 + 2*a^5*b^3*x^4 + a^
6*b^2*x)]
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^3}\,dx = \frac{-18Aa^2+x^6(-28Ab^2+4Bab)+x^3(-49Aab+7Ba^2)}{18a^5x+36a^4bx^4+18a^3b^2x^7}$$
$$+\text{RootSum}\left(19683t^3a^{10}b^2-2744A^3b^3+1176A^2Bab^2-168AB^2a^2b+8B^3a^3,\left(t\mapsto t\log\left(\frac{7}{196A^2b^2-}\right.\right.\right.$$

input
```
integrate((B*x**3+A)/x**2/(b*x**3+a)**3,x)
```

output
```
(-18*A*a**2 + x**6*(-28*A*b**2 + 4*B*a*b) + x**3*(-49*A*a*b + 7*B*a**2))/(
18*a**5*x + 36*a**4*b*x**4 + 18*a**3*b**2*x**7) + RootSum(19683*_t**3*a**1
0*b**2 - 2744*A**3*b**3 + 1176*A**2*B*a*b**2 - 168*A*B**2*a**2*b + 8*B**3*
a**3, Lambda(_t, _t*log(729*_t**2*a**7*b/(196*A**2*b**2 - 56*A*B*a*b + 4*B
**2*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^3}\,dx = \frac{4\,(Bab-7\,Ab^2)x^6+7\,(Ba^2-7\,Aab)x^3-18\,Aa^2}{18\,(a^3b^2x^7+2\,a^4bx^4+a^5x)} + \frac{2\,\sqrt{3}(Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba-7\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^3b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2\,(Ba-7\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(4*(B*a*b - 7*A*b^2)*x^6 + 7*(B*a^2 - 7*A*a*b)*x^3 - 18*A*a^2)/(a^3*b
^2*x^7 + 2*a^4*b*x^4 + a^5*x) + 2/27*sqrt(3)*(B*a - 7*A*b)*arctan(1/3*sqrt
(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(1/3)) + 1/27*(B*a - 7*A
*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(1/3)) - 2/27*(B*a
 - 7*A*b)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx = \frac{2\sqrt{3}(Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}a^3} - \frac{(Ba-7\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}a^3} - \frac{2\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4} - \frac{A}{a^3x} + \frac{4\,Babx^5-10\,Ab^2x^5+7\,Ba^2x^2-13\,Aabx^2}{18\left(bx^3+a\right)^2a^3}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
2/27*sqrt(3)*(B*a - 7*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(1/3)*a^3) - 1/27*(B*a - 7*A*b)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^3) - 2/27*(B*a*(-a/b)^(1/3) - 7*A*b*(-a/
b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 - A/(a^3*x) + 1/18*(
4*B*a*b*x^5 - 10*A*b^2*x^5 + 7*B*a^2*x^2 - 13*A*a*b*x^2)/((b*x^3 + a)^2*a^
3)
```

Mupad [B] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx = \frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(7\,A\,b-B\,a\right)}{27\,a^{10/3}\,b^{2/3}} - \frac{\frac{A}{a}+\frac{7\,x^3\,\left(7\,A\,b-B\,a\right)}{18\,a^2}+\frac{2\,b\,x^6\,\left(7\,A\,b-B\,a\right)}{9\,a^3}}{a^2\,x+2\,a\,b\,x^4+b^2\,x^7} + \frac{2\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(7\,A\,b-B\,a\right)}{27\,a^{10/3}\,b^{2/3}} - \frac{2\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(7\,A\,b-B\,a\right)}{27\,a^{10/3}\,b^{2/3}}$$

input
```
int((A + B*x^3)/(x^2*(a + b*x^3)^3),x)
```

output
```
(2*log(b^(1/3)*x + a^(1/3))*(7*A*b - B*a))/(27*a^(10/3)*b^(2/3)) - (A/a +
(7*x^3*(7*A*b - B*a))/(18*a^2) + (2*b*x^6*(7*A*b - B*a))/(9*a^3))/(a^2*x +
 b^2*x^7 + 2*a*b*x^4) + (2*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))
*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - B*a))/(27*a^(10/3)*b^(2/3)) - (2*log(3^(1
/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - B*
a))/(27*a^(10/3)*b^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx = \frac{4\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)abx+4\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2x^4-9b^{\frac{2}{3}}a^{\frac{4}{3}}-12b^{\frac{5}{3}}a^{\frac{1}{3}}x^3-2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)}{9b^{\frac{2}{3}}a^{\frac{7}{3}}x\left(b\,x^3+a\right)}$$

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^3,x)
```

output
```
(4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b*x + 4*sq
rt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**2*x**4 - 9*b**
(2/3)*a**(1/3)*a - 12*b**(2/3)*a**(1/3)*b*x**3 - 2*log(a**(2/3) - b**(1/3)
*a**(1/3)*x + b**(2/3)*x**2)*a*b*x - 2*log(a**(2/3) - b**(1/3)*a**(1/3)*x
+ b**(2/3)*x**2)*b**2*x**4 + 4*log(a**(1/3) + b**(1/3)*x)*a*b*x + 4*log(a*
*(1/3) + b**(1/3)*x)*b**2*x**4)/(9*b**(2/3)*a**(1/3)*a**2*x*(a + b*x**3))
```

3.107 $\int \frac{A+Bx^3}{x^3(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 206

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^3}\,dx = -\frac{A}{2a^3x^2} - \frac{(Ab-aB)x}{6a^2(a+bx^3)^2} - \frac{(11Ab-5aB)x}{18a^3(a+bx^3)} + \frac{5(4Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt[3]{b}} - \frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}} + \frac{5(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}\sqrt[3]{b}}$$

output
```
-1/2*A/a^3/x^2-1/6*(A*b-B*a)*x/a^2/(b*x^3+a)^2-1/18*(11*A*b-5*B*a)*x/a^3/(
b*x^3+a)+5/27*(4*A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3)
)*3^(1/2)/a^(11/3)/b^(1/3)-5/27*(4*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)
/b^(1/3)+5/54*(4*A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/
3)/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{27a^{2/3}A}{x^2}+\frac{9a^{5/3}(-Ab+aB)x}{(a+bx^3)^2}+\frac{3a^{2/3}(-11Ab+5aB)x}{a+bx^3}+\frac{10\sqrt{3}(4Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{10(-4Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}}{54a^{11/3}}$$

input
```
Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^3),x]
```

output
```
((-27*a^(2/3)*A)/x^2 + (9*a^(5/3)*(-(A*b) + a*B)*x)/(a + b*x^3)^2 + (3*a^(
2/3)*(-11*A*b + 5*a*B)*x)/(a + b*x^3) + (10*Sqrt[3]*(4*A*b - a*B)*ArcTan[(
1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (10*(-4*A*b + a*B)*Log[a^(1
/3) + b^(1/3)*x])/b^(1/3) + (5*(4*A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/b^(1/3))/(54*a^(11/3))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {957, 819, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(4Ab-aB)\int \frac{1}{x^3(bx^3+a)^2}dx}{3ab}+\frac{Ab-aB}{6abx^2(a+bx^3)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{(4Ab-aB)\left(\frac{5\int \frac{1}{x^3\left(bx^3+a\right)}dx}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 847

$$\frac{(4Ab-aB)\left(\frac{5\left(-\frac{b\int \frac{1}{bx^3+a}dx}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 750

$$\frac{(4Ab-aB)\left(\frac{5\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 16

$$\frac{(4Ab-aB)\left(\frac{5\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1142$$

$$\frac{(4Ab-aB)\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}$$

$$\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow 25$$

$$\frac{(4Ab-aB)\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}$$

$$\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$\downarrow$ 27

$$\frac{(4Ab-aB)\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}$$

$$\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$\downarrow$ 1082

$$\frac{(4Ab-aB)\left(\frac{5\left(-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}$$

$$\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 217$$

$$\frac{(4Ab-aB)\left(\frac{5\left(-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 1103$$

$$\frac{(4Ab-aB)\left(\frac{5\left(-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

input

```
Int[(A + B*x^3)/(x^3*(a + b*x^3)^3),x]
```

output
```
(A*b - a*B)/(6*a*b*x^2*(a + b*x^3)^2) + ((4*A*b - a*B)*(1/(3*a*x^2*(a + b*
x^3)) + (5*(-1/2*1/(a*x^2) - (b*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/
3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) -
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/
a))/(3*a)))/(3*a*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.77

method	result
default	$-\frac{A}{2a^{3}x^{2}}-\frac{\frac{\left(\frac{11}{18}b^{2}A-\frac{5}{18}abB\right)x^{4}+\frac{a(7Ab-4Ba)x}{9}}{\left(b\,x^{3}+a\right)^{2}}+\frac{5(4Ab-Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{9}}{a^{3}}$
risch	$\frac{-\frac{5b(4Ab-Ba)x^{6}}{18a^{3}}-\frac{4(4Ab-Ba)x^{3}}{9a^{2}}-\frac{A}{2a}}{x^{2}\left(b\,x^{3}+a\right)^{2}}+\frac{5\left(\sum_{\_R=\operatorname{RootOf}\left(a^{11}b\_Z^{3}+64A^{3}b^{3}-48A^{2}Ba\,b^{2}+12A\,B^{2}a^{2}b-B^{3}a^{3}\right)}\_R\ln\left(\left(-4\_R^{3}a^{11}b-\right.\right.\right.}{27}$

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/a^3/x^2-1/a^3*((((11/18*b^2*A-5/18*a*b*B)*x^4+1/9*a*(7*A*b-4*B*a)*x)
/(b*x^3+a)^2+5/9*(4*A*b-B*a)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a
/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(160) = 320.

Time = 0.12 (sec) , antiderivative size = 812, normalized size of antiderivative = 3.94

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/54*(15*(B*a^3*b^2 - 4*A*a^2*b^3)*x^6 - 27*A*a^4*b + 24*(B*a^4*b - 4*A*a
^3*b^2)*x^3 - 15*sqrt(1/3)*((B*a^2*b^3 - 4*A*a*b^4)*x^8 + 2*(B*a^3*b^2 - 4
*A*a^2*b^3)*x^5 + (B*a^4*b - 4*A*a^3*b^2)*x^2)*sqrt((-a^2*b)^(1/3)/b)*log(
(2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b
)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 5*((B
*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*
x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 1
0*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^
2*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^5*b^3*x^8 + 2*a^6
*b^2*x^5 + a^7*b*x^2), 1/54*(15*(B*a^3*b^2 - 4*A*a^2*b^3)*x^6 - 27*A*a^4*b
 + 24*(B*a^4*b - 4*A*a^3*b^2)*x^3 + 30*sqrt(1/3)*((B*a^2*b^3 - 4*A*a*b^4)*
x^8 + 2*(B*a^3*b^2 - 4*A*a^2*b^3)*x^5 + (B*a^4*b - 4*A*a^3*b^2)*x^2)*sqrt(
-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a
)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 5*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b -
 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-
a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^
2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x +
 (-a^2*b)^(2/3)))/(a^5*b^3*x^8 + 2*a^6*b^2*x^5 + a^7*b*x^2)]
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^3}{x^3\,(a+bx^3)^3}\,dx = \frac{-9Aa^2+x^6(-20Ab^2+5Bab)+x^3(-32Aab+8Ba^2)}{18a^5x^2+36a^4bx^5+18a^3b^2x^8} + \mathrm{RootSum}\left(19683t^3a^{11}b+8000A^3b^3-6000A^2Bab^2+1500AB^2a^2b-125B^3a^3, \left(t\mapsto t\log\left(\frac{27ta}{-20Ab+}\right.\right.\right.$$

input
```
integrate((B*x**3+A)/x**3/(b*x**3+a)**3,x)
```

output
```
(-9*A*a**2 + x**6*(-20*A*b**2 + 5*B*a*b) + x**3*(-32*A*a*b + 8*B*a**2))/(1
8*a**5*x**2 + 36*a**4*b*x**5 + 18*a**3*b**2*x**8) + RootSum(19683*_t**3*a*
*11*b + 8000*A**3*b**3 - 6000*A**2*B*a*b**2 + 1500*A*B**2*a**2*b - 125*B**
3*a**3, Lambda(_t, _t*log(27*_t*a**4/(-20*A*b + 5*B*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx = \frac{5\,(Bab-4\,Ab^2)x^6+8\,(Ba^2-4\,Aab)x^3-9\,Aa^2}{18\,(a^3b^2x^8+2\,a^4bx^5+a^5x^2)} + \frac{5\,\sqrt{3}(Ba-4\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5\,(Ba-4\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{5\,(Ba-4\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(5*(B*a*b - 4*A*b^2)*x^6 + 8*(B*a^2 - 4*A*a*b)*x^3 - 9*A*a^2)/(a^3*b^
2*x^8 + 2*a^4*b*x^5 + a^5*x^2) + 5/27*sqrt(3)*(B*a - 4*A*b)*arctan(1/3*sqr
t(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) - 5/54*(B*a - 4*
A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) + 5/27*(B*
a - 4*A*b)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx = -\frac{5\,(Ba-4\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4} + \frac{5\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-4\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b} + \frac{5\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-4\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b} + \frac{5\,Babx^6-20\,Ab^2x^6+8\,Ba^2x^3-32\,Aabx^3-9\,Aa^2}{18\left(bx^4+ax\right)^2a^3}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-5/27*(B*a - 4*A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 + 5/27*sqr
t(3)*((-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b) + 5/54*((-a*b^2)^(1/3)*B*a - 4*(-a*b^
2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) + 1/18*(5*B
*a*b*x^6 - 20*A*b^2*x^6 + 8*B*a^2*x^3 - 32*A*a*b*x^3 - 9*A*a^2)/((b*x^4 +
a*x)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx = -\frac{\frac{A}{2\,a}+\frac{4\,x^3\,(4\,A\,b-B\,a)}{9\,a^2}+\frac{5\,b\,x^6\,(4\,A\,b-B\,a)}{18\,a^3}}{a^2\,x^2+2\,a\,b\,x^5+b^2\,x^8} - \frac{5\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(4\,A\,b-B\,a)}{27\,a^{11/3}\,b^{1/3}} + \frac{5\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(4\,A\,b-B\,a)}{27\,a^{11/3}\,b^{1/3}} - \frac{5\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(4\,A\,b-B\,a)}{27\,a^{11/3}\,b^{1/3}}$$

input
```
int((A + B*x^3)/(x^3*(a + b*x^3)^3),x)
```

output
```
(5*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(4*A*b - B*a))/(27*a^(11/3)*b^(1/3)) - (5*log(b^(1/3)*x + a^(1/3))*(4*A*b
- B*a))/(27*a^(11/3)*b^(1/3)) - (A/(2*a) + (4*x^3*(4*A*b - B*a))/(9*a^2) +
 (5*b*x^6*(4*A*b - B*a))/(18*a^3))/(a^2*x^2 + b^2*x^8 + 2*a*b*x^5) - (5*lo
g(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*A*
b - B*a))/(27*a^(11/3)*b^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx$$
$$= \frac{10a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^2+10a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2x^5+5a^{\frac{4}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)b\,x^2+5a^{\frac{1}{3}}\mathrm{lo}}{18b^{\frac{1}{3}}a^3x^2}$$

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^3,x)
```

output
```
(10*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*
b*x**2 + 10*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt
(3)))*b**2*x**5 + 5*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)
*x**2)*a*b*x**2 + 5*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)
*x**2)*b**2*x**5 - 10*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a*b*x**2 - 10*a*
*(1/3)*log(a**(1/3) + b**(1/3)*x)*b**2*x**5 - 9*b**(1/3)*a**2 - 15*b**(1/3
)*a*b*x**3)/(18*b**(1/3)*a**3*x**2*(a + b*x**3))
```

3.108 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 228

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx = -\frac{A}{4a^3x^4}+\frac{3Ab-aB}{a^4x}+\frac{b(Ab-aB)x^2}{6a^3\left(a+bx^3\right)^2}+\frac{b(8Ab-5aB)x^2}{9a^4\left(a+bx^3\right)} - \frac{7\sqrt[3]{b}(5Ab-2aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{13/3}} - \frac{7\sqrt[3]{b}(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{13/3}} + \frac{7\sqrt[3]{b}(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{13/3}}$$

output
```
-1/4*A/a^3/x^4+(3*A*b-B*a)/a^4/x+1/6*b*(A*b-B*a)*x^2/a^3/(b*x^3+a)^2+1/9*b
*(8*A*b-5*B*a)*x^2/a^4/(b*x^3+a)-7/27*b^(1/3)*(5*A*b-2*B*a)*arctan(1/3*(a^
(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(13/3)-7/27*b^(1/3)*(5*A*b-2
*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(13/3)+7/54*b^(1/3)*(5*A*b-2*B*a)*ln(a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(13/3)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$$

$$= \frac{-\frac{27a^{4/3}A}{x^4}-\frac{108\sqrt[3]{a}(-3Ab+aB)}{x}-\frac{18a^{4/3}b(-Ab+aB)x^2}{(a+bx^3)^2}-\frac{12\sqrt[3]{a}b(-8Ab+5aB)x^2}{a+bx^3}-28\sqrt{3}\sqrt[3]{b}(5Ab-2aB)\arctan\left(\frac{1-\cdots}{\cdots}\right)\cdots}{108a^{1\cdots}}$$

input

```
Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^3),x]
```

output

```
((-27*a^(4/3)*A)/x^4 - (108*a^(1/3)*(-3*A*b + a*B))/x - (18*a^(4/3)*b*(-(A
*b) + a*B)*x^2)/(a + b*x^3)^2 - (12*a^(1/3)*b*(-8*A*b + 5*a*B)*x^2)/(a + b
*x^3) - 28*Sqrt[3]*b^(1/3)*(5*A*b - 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]] + 28*b^(1/3)*(-5*A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*b
^(1/3)*(5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(10
8*a^(13/3))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {957, 819, 847, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{(5Ab-2aB)\int \frac{1}{x^5(bx^3+a)^2}dx}{3ab}+\frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{(5Ab-2aB)\left(\frac{7\int \frac{1}{x^5\left(bx^3+a\right)}dx}{3a}+\frac{1}{3ax^4(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 847

$$\frac{(5Ab-2aB)\left(\frac{7\left(-\frac{b\int \frac{1}{x^2\left(bx^3+a\right)}dx}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{1}{3ax^4(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 847

$$\frac{(5Ab-2aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{1}{3ax^4(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 821

$$\frac{(5Ab-2aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{1}{3ax^4(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 16$$

$$\frac{(5Ab-2aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{1}{3ax^4(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1142$$

$$(5Ab-2aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{}{3a}\right.$$

$\downarrow$ 25

$$(5Ab-2aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{\cdots}{3ax\cdots}\right.$$

$\big\downarrow$ 27

$$\frac{(5Ab-2aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{}{3a}\right.}{3ab}$$

$$\frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2}$$

$\big\downarrow$ 1082

$$(5Ab-2aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3a}+\right.$$

↓ 217

$$(5Ab-2aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{1}{3ax^4(a+bx^3)}\right.$$

↓ 1103

$$(5Ab-2aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{1}{3ax^4(a+bx^3)}$$

input
```
Int[(A + B*x^3)/(x^5*(a + b*x^3)^3),x]
```

output
```
(A*b - a*B)/(6*a*b*x^4*(a + b*x^3)^2) + ((5*A*b - 2*a*B)*(1/(3*a*x^4*(a +
b*x^3)) + (7*(-1/4*1/(a*x^4) - (b*(-(1/(a*x)) - (b*(-1/3*Log[a^(1/3) + b^(
1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^
(1/3)))/(3*a^(1/3)*b^(1/3))))/a))/a))/(3*a)))/(3*a*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.77

method	result
default	$-\frac{A}{4a^3x^4}-\frac{-3Ab+Ba}{a^4x}+\frac{b\left(\frac{\left(\frac{8}{9}b^2A-\frac{5}{9}abB\right)x^5+\frac{a(19Ab-13Ba)x^2}{18}}{\left(b\,x^3+a\right)^2}+\left(\frac{35Ab}{9}-\frac{14Ba}{9}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\right.\right.}{a^4}$
risch	$\frac{\frac{7b^2(5Ab-2Ba)x^9}{9a^4}+\frac{49b(5Ab-2Ba)x^6}{36a^3}+\frac{(5Ab-2Ba)x^3}{2a^2}-\frac{A}{4a}}{x^4(b\,x^3+a)^2}+\frac{7\left(\sum_{\_R=\mathrm{RootOf}\left(a^{13}\_Z^3+125A^3b^4-150A^2Ba\,b^3+60A\,B^2a^2b^2-8B^3a^3b\right)}\right.}{}$

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/a^3/x^4-(-3*A*b+B*a)/a^4/x+1/a^4*b*(((8/9*b^2*A-5/9*a*b*B)*x^5+1/18
*a*(19*A*b-13*B*a)*x^2)/(b*x^3+a)^2+(35/9*A*b-14/9*B*a)*(-1/3/b/(a/b)^(1/3
)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/
3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.61

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx =$$

$$-\frac{84\,(2\,Bab^2-5\,Ab^3)x^9+147\,(2\,Ba^2b-5\,Aab^2)x^6+27\,Aa^3+54\,(2\,Ba^3-5\,Aa^2b)x^3+28\,\sqrt{3}((2\,Bab^2}{}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/108*(84*(2*B*a*b^2 - 5*A*b^3)*x^9 + 147*(2*B*a^2*b - 5*A*a*b^2)*x^6 + 2
7*A*a^3 + 54*(2*B*a^3 - 5*A*a^2*b)*x^3 + 28*sqrt(3)*((2*B*a*b^2 - 5*A*b^3)
*x^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)*(-b/a)^
(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 14*((2*B*a*b^2 -
5*A*b^3)*x^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)
*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1/3)) + 28*((2*B*a*
b^2 - 5*A*b^3)*x^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b
)*x^4)*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^4*b^2*x^10 + 2*a^5*b*x^7
 + a^6*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$$

$$= \text{RootSum}\left(19683t^3a^{13}+42875A^3b^4-51450A^2Bab^3+20580AB^2a^2b^2-2744B^3a^3b,\left(t\mapsto t\log\left(\frac{}{1225A}\right.\right.\right.$$

$$+\frac{-9Aa^3+x^9\cdot(140Ab^3-56Bab^2)+x^6\cdot(245Aab^2-98Ba^2b)+x^3\cdot(90Aa^2b-36Ba^3)}{36a^6x^4+72a^5bx^7+36a^4b^2x^{10}}$$

input
```
integrate((B*x**3+A)/x**5/(b*x**3+a)**3,x)
```

output
```
RootSum(19683*_t**3*a**13 + 42875*A**3*b**4 - 51450*A**2*B*a*b**3 + 20580*
A*B**2*a**2*b**2 - 2744*B**3*a**3*b, Lambda(_t, _t*log(729*_t**2*a**9/(122
5*A**2*b**3 - 980*A*B*a*b**2 + 196*B**2*a**2*b) + x))) + (-9*A*a**3 + x**9
*(140*A*b**3 - 56*B*a*b**2) + x**6*(245*A*a*b**2 - 98*B*a**2*b) + x**3*(90
*A*a**2*b - 36*B*a**3))/(36*a**6*x**4 + 72*a**5*b*x**7 + 36*a**4*b**2*x**1
0)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^3}{x^5 (a+bx^3)^3}\,dx$$
$$= -\frac{28\,(2\,Bab^2-5\,Ab^3)x^9+49\,(2\,Ba^2b-5\,Aab^2)x^6+9\,Aa^3+18\,(2\,Ba^3-5\,Aa^2b)x^3}{36\,(a^4b^2x^{10}+2\,a^5bx^7+a^6x^4)}$$
$$-\frac{7\sqrt{3}(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$-\frac{7\,(2\,Ba-5\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{7\,(2\,Ba-5\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/36*(28*(2*B*a*b^2 - 5*A*b^3)*x^9 + 49*(2*B*a^2*b - 5*A*a*b^2)*x^6 + 9*A
*a^3 + 18*(2*B*a^3 - 5*A*a^2*b)*x^3)/(a^4*b^2*x^10 + 2*a^5*b*x^7 + a^6*x^4
) - 7/27*sqrt(3)*(2*B*a - 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a
/b)^(1/3))/(a^4*(a/b)^(1/3)) - 7/54*(2*B*a - 5*A*b)*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(a^4*(a/b)^(1/3)) + 7/27*(2*B*a - 5*A*b)*log(x + (a/b)^(1
/3))/(a^4*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx = \frac{7\left(2\,Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^5} + \frac{7\sqrt{3}\left(2\left(-ab^2\right)^{\frac{2}{3}}Ba-5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5b} - \frac{7\left(2\left(-ab^2\right)^{\frac{2}{3}}Ba-5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^5b} - \frac{10\,Bab^2x^5-16\,Ab^3x^5+13\,Ba^2bx^2-19\,Aab^2x^2}{18\left(bx^3+a\right)^2a^4} - \frac{4\,Bax^3-12\,Abx^3+Aa}{4\,a^4x^4}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
7/27*(2*B*a*b*(-a/b)^(1/3) - 5*A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/a^5 + 7/27*sqrt(3)*(2*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/
3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b) - 7/
54*(2*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/(a^5*b) - 1/18*(10*B*a*b^2*x^5 - 16*A*b^3*x^5 + 13*B*a^2*b
*x^2 - 19*A*a*b^2*x^2)/((b*x^3 + a)^2*a^4) - 1/4*(4*B*a*x^3 - 12*A*b*x^3 +
 A*a)/(a^4*x^4)
```

Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx=\frac{\frac{x^3\,(5\,A\,b-2\,B\,a)}{2\,a^2}-\frac{A}{4\,a}+\frac{7\,b^2\,x^9\,(5\,A\,b-2\,B\,a)}{9\,a^4}+\frac{49\,b\,x^6\,(5\,A\,b-2\,B\,a)}{36\,a^3}}{a^2\,x^4+2\,a\,b\,x^7+b^2\,x^{10}}$$
$$+\frac{7\,(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3}+b^3\,x\right)\,(5\,A\,b-2\,B\,a)}{27\,a^{13/3}}$$
$$+\frac{7\,(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3}-2\,b^3\,x+\sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-2\,B\,a)}{27\,a^{13/3}}$$
$$-\frac{7\,(-b)^{1/3}\,\ln\left(2\,b^3\,x-a^{1/3}\,(-b)^{8/3}+\sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-2\,B\,a)}{27\,a^{13/3}}$$

input
```
int((A + B*x^3)/(x^5*(a + b*x^3)^3),x)
```

output
```
((x^3*(5*A*b - 2*B*a))/(2*a^2) - A/(4*a) + (7*b^2*x^9*(5*A*b - 2*B*a))/(9*
a^4) + (49*b*x^6*(5*A*b - 2*B*a))/(36*a^3))/(a^2*x^4 + b^2*x^10 + 2*a*b*x^
7) + (7*(-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) + b^3*x)*(5*A*b - 2*B*a))/(27*a^
(13/3)) + (7*(-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) - 2*b^3*x + 3^(1/2)*a^(1/3)
*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(5*A*b - 2*B*a))/(27*a^(13/3)) - (7
*(-b)^(1/3)*log(2*b^3*x - a^(1/3)*(-b)^(8/3) + 3^(1/2)*a^(1/3)*(-b)^(8/3)*
1i)*((3^(1/2)*1i)/2 + 1/2)*(5*A*b - 2*B*a))/(27*a^(13/3))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$$
$$=\frac{-28\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a\,b^2x^4-28\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b^3x^7-9b^{\frac{2}{3}}a^{\frac{7}{3}}+63b^{\frac{5}{3}}a^{\frac{4}{3}}x^3+84b^{\frac{8}{3}}a^{\frac{1}{3}}x^6+14\log\left(a\right.}{36b^{\frac{2}{3}}}$$

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^3,x)
```

output
```
( - 28*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**2*x
**4 - 28*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**3*x
**7 - 9*b**(2/3)*a**(1/3)*a**2 + 63*b**(2/3)*a**(1/3)*a*b*x**3 + 84*b**(2/
3)*a**(1/3)*b**2*x**6 + 14*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x
**2)*a*b**2*x**4 + 14*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*
b**3*x**7 - 28*log(a**(1/3) + b**(1/3)*x)*a*b**2*x**4 - 28*log(a**(1/3) +
b**(1/3)*x)*b**3*x**7)/(36*b**(2/3)*a**(1/3)*a**3*x**4*(a + b*x**3))
```

3.109 $\int \frac{A+Bx^3}{x^6(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 227

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx = -\frac{A}{5a^3x^5}+\frac{3Ab-aB}{2a^4x^2}+\frac{b(Ab-aB)x}{6a^3\left(a+bx^3\right)^2}+\frac{b(17Ab-11aB)x}{18a^4\left(a+bx^3\right)} - \frac{4b^{2/3}(11Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{14/3}} + \frac{4b^{2/3}(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{14/3}} - \frac{2b^{2/3}(11Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{14/3}}$$

output
```
-1/5*A/a^3/x^5+1/2*(3*A*b-B*a)/a^4/x^2+1/6*b*(A*b-B*a)*x/a^3/(b*x^3+a)^2+1
/18*b*(17*A*b-11*B*a)*x/a^4/(b*x^3+a)-4/27*b^(2/3)*(11*A*b-5*B*a)*arctan(1
/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(14/3)+4/27*b^(2/3)*(1
1*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)-2/27*b^(2/3)*(11*A*b-5*B*a)*ln
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14/3)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^3}\,dx$$

$$= \frac{-\frac{54a^{5/3}A}{x^5} - \frac{135a^{2/3}(-3Ab+aB)}{x^2} - \frac{45a^{5/3}b(-Ab+aB)x}{(a+bx^3)^2} - \frac{15a^{2/3}b(-17Ab+11aB)x}{a+bx^3} - 40\sqrt{3}b^{2/3}(11Ab-5aB)\arctan\Bigg(}{270}$$

input
```
Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^3),x]
```

output
```
((-54*a^(5/3)*A)/x^5 - (135*a^(2/3)*(-3*A*b + a*B))/x^2 - (45*a^(5/3)*b*(-
(A*b) + a*B)*x)/(a + b*x^3)^2 - (15*a^(2/3)*b*(-17*A*b + 11*a*B)*x)/(a + b
*x^3) - 40*Sqrt[3]*b^(2/3)*(11*A*b - 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]] + 40*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(1/3) + b^(1/3)*x] + 20*
b^(2/3)*(-11*A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(270*a^(14/3))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {957, 819, 847, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^3}\,dx$$

$$\Big\downarrow \; 957$$

$$\frac{(11Ab-5aB)\int \frac{1}{x^6(bx^3+a)^2}dx}{6ab} + \frac{Ab-aB}{6abx^5 (a+bx^3)^2}$$

$$\Big\downarrow \; 819$$

$$\frac{(11Ab-5aB)\left(\frac{8\int \frac{1}{x^6\left(bx^3+a\right)}dx}{3a}+\frac{1}{3ax^5(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 847

$$\frac{(11Ab-5aB)\left(\frac{8\left(-\frac{b\int \frac{1}{x^3\left(bx^3+a\right)}dx}{a}-\frac{1}{5ax^5}\right)}{3a}+\frac{1}{3ax^5(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 847

$$\frac{(11Ab-5aB)\left(\frac{8\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^3+a}dx}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3a}+\frac{1}{3ax^5(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 750

$$\frac{(11Ab-5aB)\left(\frac{8\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3a}+\frac{1}{3ax^5(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}$$

$\downarrow$ 16

$$\frac{(11Ab-5aB)\left(\frac{8\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3a}+\frac{1}{3ax^5(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1142

$$(11Ab-5aB)\left(\frac{8\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3a}+\right.$$

↓ 25

$$(11Ab-5aB)\left(\frac{8\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3a}+\right.$$

$\downarrow$ 27

$$\frac{(11Ab-5aB)\left(\frac{8\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3a}+\right.}{6ab}$$

$$\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}$$

$\downarrow$ 1082

$$(11Ab-5aB)\left(\frac{8\left(-\frac{b\left(-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3a}\right.$$

↓ 217

$$(11Ab-5aB)\left(\frac{8\left(-\frac{b\left(-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3a}+\frac{1}{3ax^5(a+bx}\right.$$

↓ 1103

$$(11Ab-5aB)\left(\frac{8\left(-\frac{b\left(-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3a}+\frac{1}{3ax^5(a+b$$

input
```
Int[(A + B*x^3)/(x^6*(a + b*x^3)^3),x]
```

output
```
(A*b - a*B)/(6*a*b*x^5*(a + b*x^3)^2) + ((11*A*b - 5*a*B)*(1/(3*a*x^5*(a +
 b*x^3)) + (8*(-1/5*1/(a*x^5) - (b*(-1/2*1/(a*x^2) - (b*(Log[a^(1/3) + b^(
1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3
))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*
b^(1/3)))/(3*a^(2/3))))/a))/a))/(3*a)))/(6*a*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.77

method	result
default	$-\frac{A}{5a^3x^5}-\frac{-3Ab+Ba}{2x^2a^4}+\frac{b\left(\frac{\left(\frac{17}{18}b^2A-\frac{11}{18}abB\right)x^4+\frac{a(10Ab-7Ba)x}{9}}{\left(b\,x^3+a\right)^2}+\frac{4(11Ab-5Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\ \text{ar}}{}\right.}{9}\right.}{a^4}$
risch	$\frac{\frac{2b^2(11Ab-5Ba)x^9}{9a^4}+\frac{16b(11Ab-5Ba)x^6}{45a^3}+\frac{(11Ab-5Ba)x^3}{10a^2}-\frac{A}{5a}}{x^5\left(b\,x^3+a\right)^2}+\frac{4\left(\sum_{\_R=\text{RootOf}\left(a^{14}\_Z^3-1331A^3b^5+1815A^2Ba\,b^4-825A\,B^2a^2b^3+125B\right.}\right.}{}$

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/5*A/a^3/x^5-1/2*(-3*A*b+B*a)/x^2/a^4+1/a^4*b*(((17/18*b^2*A-11/18*a*b*B
)*x^4+1/9*a*(10*A*b-7*B*a)*x)/(b*x^3+a)^2+4/9*(11*A*b-5*B*a)*(1/3/b/(a/b)^
(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(181) = 362$.

Time = 0.10 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.69

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx =$$

$$-\frac{60\,(5\,Bab^2-11\,Ab^3)x^9+96\,(5\,Ba^2b-11\,Aab^2)x^6+54\,Aa^3+27\,(5\,Ba^3-11\,Aa^2b)x^3+40\,\sqrt{3}((5\,Ba}{}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/270*(60*(5*B*a*b^2 - 11*A*b^3)*x^9 + 96*(5*B*a^2*b - 11*A*a*b^2)*x^6 +
54*A*a^3 + 27*(5*B*a^3 - 11*A*a^2*b)*x^3 + 40*sqrt(3)*((5*B*a*b^2 - 11*A*b
^3)*x^11 + 2*(5*B*a^2*b - 11*A*a*b^2)*x^8 + (5*B*a^3 - 11*A*a^2*b)*x^5)*(b
^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) -
20*((5*B*a*b^2 - 11*A*b^3)*x^11 + 2*(5*B*a^2*b - 11*A*a*b^2)*x^8 + (5*B*a^
3 - 11*A*a^2*b)*x^5)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) +
 a^2*(b^2/a^2)^(2/3)) + 40*((5*B*a*b^2 - 11*A*b^3)*x^11 + 2*(5*B*a^2*b - 1
1*A*a*b^2)*x^8 + (5*B*a^3 - 11*A*a^2*b)*x^5)*(b^2/a^2)^(1/3)*log(b*x + a*(
b^2/a^2)^(1/3)))/(a^4*b^2*x^11 + 2*a^5*b*x^8 + a^6*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx$$

$$= \operatorname{RootSum}\left(19683t^3a^{14}-85184A^3b^5+116160A^2Bab^4-52800AB^2a^2b^3+8000B^3a^3b^2,\left(t\mapsto t\log\left(-\right.\right.\right.$$

$$+\frac{-18Aa^3+x^9\cdot(220Ab^3-100Bab^2)+x^6\cdot(352Aab^2-160Ba^2b)+x^3\cdot(99Aa^2b-45Ba^3)}{90a^6x^5+180a^5bx^8+90a^4b^2x^{11}}$$

input
```
integrate((B*x**3+A)/x**6/(b*x**3+a)**3,x)
```

output
```
RootSum(19683*_t**3*a**14 - 85184*A**3*b**5 + 116160*A**2*B*a*b**4 - 52800
*A*B**2*a**2*b**3 + 8000*B**3*a**3*b**2, Lambda(_t, _t*log(-27*_t*a**5/(-4
4*A*b**2 + 20*B*a*b) + x))) + (-18*A*a**3 + x**9*(220*A*b**3 - 100*B*a*b**
2) + x**6*(352*A*a*b**2 - 160*B*a**2*b) + x**3*(99*A*a**2*b - 45*B*a**3))/
(90*a**6*x**5 + 180*a**5*b*x**8 + 90*a**4*b**2*x**11)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^3}{x^6(a+bx^3)^3}\,dx = -\frac{20\,(5\,Bab^2-11\,Ab^3)x^9+32\,(5\,Ba^2b-11\,Aab^2)x^6+18\,Aa^3+9\,(5\,Ba^3-11\,Aa^2b)x^3}{90\,(a^4b^2x^{11}+2\,a^5bx^8+a^6x^5)} - \frac{4\sqrt{3}(5\,Ba-11\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2\,(5\,Ba-11\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{4\,(5\,Ba-11\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/90*(20*(5*B*a*b^2 - 11*A*b^3)*x^9 + 32*(5*B*a^2*b - 11*A*a*b^2)*x^6 + 1
8*A*a^3 + 9*(5*B*a^3 - 11*A*a^2*b)*x^3)/(a^4*b^2*x^11 + 2*a^5*b*x^8 + a^6*
x^5) - 4/27*sqrt(3)*(5*B*a - 11*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3)
)/(a/b)^(1/3))/(a^4*(a/b)^(2/3)) + 2/27*(5*B*a - 11*A*b)*log(x^2 - x*(a/b)
^(1/3) + (a/b)^(2/3))/(a^4*(a/b)^(2/3)) - 4/27*(5*B*a - 11*A*b)*log(x + (a
/b)^(1/3))/(a^4*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^3}{x^6\,(a+bx^3)^3}\,dx$$

$$= -\frac{4\,\sqrt{3}\left(5\,(-ab^2)^{\frac{1}{3}}\,Ba - 11\,(-ab^2)^{\frac{1}{3}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5}$$
$$+ \frac{4\,(5\,Bab-11\,Ab^2)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^5}$$
$$- \frac{2\left(5\,(-ab^2)^{\frac{1}{3}}\,Ba - 11\,(-ab^2)^{\frac{1}{3}}\,Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^5}$$
$$- \frac{11\,Bab^2x^4 - 17\,Ab^3x^4 + 14\,Ba^2bx - 20\,Aab^2x}{18\,(bx^3+a)^2a^4} - \frac{5\,Bax^3 - 15\,Abx^3 + 2\,Aa}{10\,a^4x^5}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-4/27*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 11*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sq
rt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^5 + 4/27*(5*B*a*b - 11*A*b^2)*(
-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^5 - 2/27*(5*(-a*b^2)^(1/3)*B*a -
11*(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^5 - 1/18
*(11*B*a*b^2*x^4 - 17*A*b^3*x^4 + 14*B*a^2*b*x - 20*A*a*b^2*x)/((b*x^3 + a
)^2*a^4) - 1/10*(5*B*a*x^3 - 15*A*b*x^3 + 2*A*a)/(a^4*x^5)
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^3}\,dx$$
$$= \frac{\frac{x^3\,(11\,A\,b-5\,B\,a)}{10\,a^2}-\frac{A}{5\,a}+\frac{2\,b^2\,x^9\,(11\,A\,b-5\,B\,a)}{9\,a^4}+\frac{16\,b\,x^6\,(11\,A\,b-5\,B\,a)}{45\,a^3}}{a^2\,x^5+2\,a\,b\,x^8+b^2\,x^{11}}$$
$$+\frac{4\,b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(11\,A\,b-5\,B\,a)}{27\,a^{14/3}}$$
$$-\frac{4\,b^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(11\,A\,b-5\,B\,a)}{27\,a^{14/3}}$$
$$+\frac{4\,b^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(11\,A\,b-5\,B\,a)}{27\,a^{14/3}}$$

input
```
int((A + B*x^3)/(x^6*(a + b*x^3)^3),x)
```

output
```
((x^3*(11*A*b - 5*B*a))/(10*a^2) - A/(5*a) + (2*b^2*x^9*(11*A*b - 5*B*a))/
(9*a^4) + (16*b*x^6*(11*A*b - 5*B*a))/(45*a^3))/(a^2*x^5 + b^2*x^11 + 2*a*
b*x^8) + (4*b^(2/3)*log(b^(1/3)*x + a^(1/3))*(11*A*b - 5*B*a))/(27*a^(14/3
)) - (4*b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*
1i)/2 + 1/2)*(11*A*b - 5*B*a))/(27*a^(14/3)) + (4*b^(2/3)*log(3^(1/2)*a^(1
/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a))/(
27*a^(14/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^3}\,dx$$
$$= \frac{-40a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2x^5-40a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b^3x^8-20a^{\frac{4}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)b^2x^5-2}{4}$$

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^3,x)
```

output
```
( - 40*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))
*a*b**2*x**5 - 40*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3
)*sqrt(3)))*b**3*x**8 - 20*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b
**(2/3)*x**2)*a*b**2*x**5 - 20*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x
 + b**(2/3)*x**2)*b**3*x**8 + 40*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a*b**
2*x**5 + 40*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b**3*x**8 - 9*b**(1/3)*a**
3 + 36*b**(1/3)*a**2*b*x**3 + 60*b**(1/3)*a*b**2*x**6)/(45*b**(1/3)*a**4*x
**5*(a + b*x**3))
```

3.110 $\int x^{5/2}(a+bx^3)(A+Bx^3)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 39

$$\int x^{5/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{7}aAx^{7/2}+\frac{2}{13}(Ab+aB)x^{13/2}+\frac{2}{19}bBx^{19/2}$$

output
```
2/7*a*A*x^(7/2)+2/13*(A*b+B*a)*x^(13/2)+2/19*b*B*x^(19/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int x^{5/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2x^{7/2}(247aA+133Abx^3+133aBx^3+91bBx^6)}{1729}$$

input
```
Integrate[x^(5/2)*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(2*x^(7/2)*(247*a*A + 133*A*b*x^3 + 133*a*B*x^3 + 91*b*B*x^6))/1729
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^{11/2}(aB+Ab)+aAx^{5/2}+bBx^{17/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{13}x^{13/2}(aB+Ab)+\frac{2}{7}aAx^{7/2}+\frac{2}{19}bBx^{19/2}$$

input
```
Int[x^(5/2)*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(13/2))/13 + (2*b*B*x^(19/2))/19
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{7}{2}}}{7}+\frac{2(Ab+Ba)x^{\frac{13}{2}}}{13}+\frac{2bB\,x^{\frac{19}{2}}}{19}$	28
default	$\frac{2aA\,x^{\frac{7}{2}}}{7}+\frac{2(Ab+Ba)x^{\frac{13}{2}}}{13}+\frac{2bB\,x^{\frac{19}{2}}}{19}$	28
gosper	$\frac{2x^{\frac{7}{2}}(91bB\,x^{6}+133Ab\,x^{3}+133Ba\,x^{3}+247Aa)}{1729}$	32
trager	$\frac{2x^{\frac{7}{2}}(91bB\,x^{6}+133Ab\,x^{3}+133Ba\,x^{3}+247Aa)}{1729}$	32
risch	$\frac{2x^{\frac{7}{2}}(91bB\,x^{6}+133Ab\,x^{3}+133Ba\,x^{3}+247Aa)}{1729}$	32
orering	$\frac{2x^{\frac{7}{2}}(91bB\,x^{6}+133Ab\,x^{3}+133Ba\,x^{3}+247Aa)}{1729}$	32

input
```
int(x^(5/2)*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/7*a*A*x^(7/2)+2/13*(A*b+B*a)*x^(13/2)+2/19*b*B*x^(19/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{1729}\left(91\,Bbx^9+133\,(Ba+Ab)x^6+247\,Aax^3\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/1729*(91*B*b*x^9 + 133*(B*a + A*b)*x^6 + 247*A*a*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{5/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2Aax^{\frac{7}{2}}}{7}+\frac{2Abx^{\frac{13}{2}}}{13}+\frac{2Bax^{\frac{13}{2}}}{13}+\frac{2Bbx^{\frac{19}{2}}}{19}$$

input
```
integrate(x**(5/2)*(b*x**3+a)*(B*x**3+A),x)
```

output
```
2*A*a*x**(7/2)/7 + 2*A*b*x**(13/2)/13 + 2*B*a*x**(13/2)/13 + 2*B*b*x**(19/
2)/19
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{5/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{19}\,Bbx^{\frac{19}{2}}+\frac{2}{13}\,(Ba+Ab)x^{\frac{13}{2}}+\frac{2}{7}\,Aax^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/19*B*b*x^(19/2) + 2/13*(B*a + A*b)*x^(13/2) + 2/7*A*a*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{5/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{19}\,Bbx^{\frac{19}{2}}+\frac{2}{13}\,Bax^{\frac{13}{2}}+\frac{2}{13}\,Abx^{\frac{13}{2}}+\frac{2}{7}\,Aax^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

output
```
2/19*B*b*x^(19/2) + 2/13*B*a*x^(13/2) + 2/13*A*b*x^(13/2) + 2/7*A*a*x^(7/2
)
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{5/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2\,x^{7/2}\,\left(247\,A\,a+133\,A\,b\,x^3+133\,B\,a\,x^3+91\,B\,b\,x^6\right)}{1729}$$

input
```
int(x^(5/2)*(A + B*x^3)*(a + b*x^3),x)
```

output
```
(2*x^(7/2)*(247*A*a + 133*A*b*x^3 + 133*B*a*x^3 + 91*B*b*x^6))/1729
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int x^{5/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2\sqrt{x}\,x^3(91b^2x^6+266ab\,x^3+247a^2)}{1729}$$

input
```
int(x^(5/2)*(b*x^3+a)*(B*x^3+A),x)
```

output
```
(2*sqrt(x)*x**3*(247*a**2 + 266*a*b*x**3 + 91*b**2*x**6))/1729
```

3.111 $\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 39

$$\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{5}aAx^{5/2}+\frac{2}{11}(Ab+aB)x^{11/2}+\frac{2}{17}bBx^{17/2}$$

output
```
2/5*a*A*x^(5/2)+2/11*(A*b+B*a)*x^(11/2)+2/17*b*B*x^(17/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{935}x^{5/2}\left(187aA+85Abx^3+85aBx^3+55bBx^6\right)$$

input
```
Integrate[x^(3/2)*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(2*x^(5/2)*(187*a*A + 85*A*b*x^3 + 85*a*B*x^3 + 55*b*B*x^6))/935
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^{9/2}(aB+Ab)+aAx^{3/2}+bBx^{15/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{11}x^{11/2}(aB+Ab)+\frac{2}{5}aAx^{5/2}+\frac{2}{17}bBx^{17/2}$$

input
```
Int[x^(3/2)*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(11/2))/11 + (2*b*B*x^(17/2))/17
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{5}{2}}}{5}+\frac{2(Ab+Ba)x^{\frac{11}{2}}}{11}+\frac{2bB\,x^{\frac{17}{2}}}{17}$	28
default	$\frac{2aA\,x^{\frac{5}{2}}}{5}+\frac{2(Ab+Ba)x^{\frac{11}{2}}}{11}+\frac{2bB\,x^{\frac{17}{2}}}{17}$	28
gosper	$\frac{2x^{\frac{5}{2}}(55bB\,x^{6}+85Ab\,x^{3}+85Ba\,x^{3}+187Aa)}{935}$	32
trager	$\frac{2x^{\frac{5}{2}}(55bB\,x^{6}+85Ab\,x^{3}+85Ba\,x^{3}+187Aa)}{935}$	32
risch	$\frac{2x^{\frac{5}{2}}(55bB\,x^{6}+85Ab\,x^{3}+85Ba\,x^{3}+187Aa)}{935}$	32
orering	$\frac{2x^{\frac{5}{2}}(55bB\,x^{6}+85Ab\,x^{3}+85Ba\,x^{3}+187Aa)}{935}$	32

input
```
int(x^(3/2)*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/5*a*A*x^(5/2)+2/11*(A*b+B*a)*x^(11/2)+2/17*b*B*x^(17/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{3/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{935}\left(55\,Bbx^8+85\,(Ba+Ab)x^5+187\,Aax^2\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/935*(55*B*b*x^8 + 85*(B*a + A*b)*x^5 + 187*A*a*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2Aax^{\frac{5}{2}}}{5}+\frac{2Abx^{\frac{11}{2}}}{11}+\frac{2Bax^{\frac{11}{2}}}{11}+\frac{2Bbx^{\frac{17}{2}}}{17}$$

input
```
integrate(x**(3/2)*(b*x**3+a)*(B*x**3+A),x)
```

output
```
2*A*a*x**(5/2)/5 + 2*A*b*x**(11/2)/11 + 2*B*a*x**(11/2)/11 + 2*B*b*x**(17/
2)/17
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{17}\,Bbx^{\frac{17}{2}}+\frac{2}{11}\,(Ba+Ab)x^{\frac{11}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/17*B*b*x^(17/2) + 2/11*(B*a + A*b)*x^(11/2) + 2/5*A*a*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{17}\,Bbx^{\frac{17}{2}}+\frac{2}{11}\,Bax^{\frac{11}{2}}+\frac{2}{11}\,Abx^{\frac{11}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

output
```
2/17*B*b*x^(17/2) + 2/11*B*a*x^(11/2) + 2/11*A*b*x^(11/2) + 2/5*A*a*x^(5/2
)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2\,x^{5/2}\,(187\,A\,a+85\,A\,b\,x^3+85\,B\,a\,x^3+55\,B\,b\,x^6)}{935}$$

input
```
int(x^(3/2)*(A + B*x^3)*(a + b*x^3),x)
```

output
```
(2*x^(5/2)*(187*A*a + 85*A*b*x^3 + 85*B*a*x^3 + 55*B*b*x^6))/935
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2\sqrt{x}\,x^2(55b^2x^6+170ab\,x^3+187a^2)}{935}$$

input
```
int(x^(3/2)*(b*x^3+a)*(B*x^3+A),x)
```

output
```
(2*sqrt(x)*x**2*(187*a**2 + 170*a*b*x**3 + 55*b**2*x**6))/935
```

3.112 $\int \sqrt{x}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \sqrt{x}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{3}aAx^{3/2}+\frac{2}{9}(Ab+aB)x^{9/2}+\frac{2}{15}bBx^{15/2}$$

output
```
2/3*a*A*x^(3/2)+2/9*(A*b+B*a)*x^(9/2)+2/15*b*B*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \sqrt{x}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{45}x^{3/2}\left(15aA+5Abx^3+5aBx^3+3bBx^6\right)$$

input
```
Integrate[Sqrt[x]*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(2*x^(3/2)*(15*a*A + 5*A*b*x^3 + 5*a*B*x^3 + 3*b*B*x^6))/45
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+bx^3)\left(A+Bx^3\right)dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^{7/2}(aB+Ab)+aA\sqrt{x}+bBx^{13/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{15}bBx^{15/2}$$

input
```
Int[Sqrt[x]*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(9/2))/9 + (2*b*B*x^(15/2))/15
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{3}{2}}}{3}+\frac{2(Ab+Ba)x^{\frac{9}{2}}}{9}+\frac{2bB\,x^{\frac{15}{2}}}{15}$	28
default	$\frac{2aA\,x^{\frac{3}{2}}}{3}+\frac{2(Ab+Ba)x^{\frac{9}{2}}}{9}+\frac{2bB\,x^{\frac{15}{2}}}{15}$	28
gosper	$\frac{2x^{\frac{3}{2}}\left(3bB\,x^{6}+5Ab\,x^{3}+5Ba\,x^{3}+15Aa\right)}{45}$	32
trager	$\frac{2x^{\frac{3}{2}}\left(3bB\,x^{6}+5Ab\,x^{3}+5Ba\,x^{3}+15Aa\right)}{45}$	32
risch	$\frac{2x^{\frac{3}{2}}\left(3bB\,x^{6}+5Ab\,x^{3}+5Ba\,x^{3}+15Aa\right)}{45}$	32
orering	$\frac{2x^{\frac{3}{2}}\left(3bB\,x^{6}+5Ab\,x^{3}+5Ba\,x^{3}+15Aa\right)}{45}$	32

input
```
int(x^(1/2)*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/3*a*A*x^(3/2)+2/9*(A*b+B*a)*x^(9/2)+2/15*b*B*x^(15/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \sqrt{x}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{45}\left(3\,Bbx^7+5\,(Ba+Ab)x^4+15\,Aax\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/45*(3*B*b*x^7 + 5*(B*a + A*b)*x^4 + 15*A*a*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \sqrt{x}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2Aax^{\frac{3}{2}}}{3}+\frac{2Abx^{\frac{9}{2}}}{9}+\frac{2Bax^{\frac{9}{2}}}{9}+\frac{2Bbx^{\frac{15}{2}}}{15}$$

input
```
integrate(x**(1/2)*(b*x**3+a)*(B*x**3+A),x)
```

output
```
2*A*a*x**(3/2)/3 + 2*A*b*x**(9/2)/9 + 2*B*a*x**(9/2)/9 + 2*B*b*x**(15/2)/1
5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{15}\,Bbx^{\frac{15}{2}}+\frac{2}{9}\,(Ba+Ab)x^{\frac{9}{2}}+\frac{2}{3}\,Aax^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/15*B*b*x^(15/2) + 2/9*(B*a + A*b)*x^(9/2) + 2/3*A*a*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \sqrt{x}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{15}\,Bbx^{\frac{15}{2}}+\frac{2}{9}\,Bax^{\frac{9}{2}}+\frac{2}{9}\,Abx^{\frac{9}{2}}+\frac{2}{3}\,Aax^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

output
```
2/15*B*b*x^(15/2) + 2/9*B*a*x^(9/2) + 2/9*A*b*x^(9/2) + 2/3*A*a*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \sqrt{x}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2\,x^{3/2}\,\left(15\,A\,a+5\,A\,b\,x^3+5\,B\,a\,x^3+3\,B\,b\,x^6\right)}{45}$$

input
```
int(x^(1/2)*(A + B*x^3)*(a + b*x^3),x)
```

output
```
(2*x^(3/2)*(15*A*a + 5*A*b*x^3 + 5*B*a*x^3 + 3*B*b*x^6))/45
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

$$\int \sqrt{x}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2\sqrt{x}\,x(3b^2x^6+10ab\,x^3+15a^2)}{45}$$

input
```
int(x^(1/2)*(b*x^3+a)*(B*x^3+A),x)
```

output
```
(2*sqrt(x)*x*(15*a**2 + 10*a*b*x**3 + 3*b**2*x**6))/45
```

3.113 $\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}} \, dx = 2aA\sqrt{x} + \frac{2}{7}(Ab+aB)x^{7/2} + \frac{2}{13}bBx^{13/2}$$

output
```
2*a*A*x^(1/2)+2/7*(A*b+B*a)*x^(7/2)+2/13*b*B*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}} \, dx = \frac{2}{91}\sqrt{x}(91aA + 13Abx^3 + 13aBx^3 + 7bBx^6)$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(91*a*A + 13*A*b*x^3 + 13*a*B*x^3 + 7*b*B*x^6))/91
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{ 950}$$

$$\int \left(x^{5/2}(aB+Ab)+\frac{aA}{\sqrt{x}}+bBx^{11/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{7}x^{7/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{13}bBx^{13/2}$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/Sqrt[x],x]
```

output
```
2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(7/2))/7 + (2*b*B*x^(13/2))/13
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$2aA\sqrt{x}+\frac{2(Ab+Ba)x^{\frac{7}{2}}}{7}+\frac{2bB\,x^{\frac{13}{2}}}{13}$	28
default	$2aA\sqrt{x}+\frac{2(Ab+Ba)x^{\frac{7}{2}}}{7}+\frac{2bB\,x^{\frac{13}{2}}}{13}$	28
trager	$\left(\frac{2}{13}bB\,x^{6}+\frac{2}{7}Ab\,x^{3}+\frac{2}{7}Ba\,x^{3}+2Aa\right)\sqrt{x}$	31
gosper	$\frac{2\sqrt{x}\,(7bB\,x^{6}+13Ab\,x^{3}+13Ba\,x^{3}+91Aa)}{91}$	32
risch	$\frac{2\sqrt{x}\,(7bB\,x^{6}+13Ab\,x^{3}+13Ba\,x^{3}+91Aa)}{91}$	32
orering	$\frac{2\sqrt{x}\,(7bB\,x^{6}+13Ab\,x^{3}+13Ba\,x^{3}+91Aa)}{91}$	32

input
```
int((b*x^3+a)*(B*x^3+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a*A*x^(1/2)+2/7*(A*b+B*a)*x^(7/2)+2/13*b*B*x^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)\,(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{91}\left(7\,Bbx^6+13\,(Ba+Ab)x^3+91\,Aa\right)\sqrt{x}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
```

output
```
2/91*(7*B*b*x^6 + 13*(B*a + A*b)*x^3 + 91*A*a)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx = 2Aa\sqrt{x} + \frac{2Abx^{\frac{7}{2}}}{7} + \frac{2Bax^{\frac{7}{2}}}{7} + \frac{2Bbx^{\frac{13}{2}}}{13}$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x**(1/2),x)
```

output
```
2*A*a*sqrt(x) + 2*A*b*x**(7/2)/7 + 2*B*a*x**(7/2)/7 + 2*B*b*x**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{13}\,Bbx^{\frac{13}{2}} + \frac{2}{7}\,(Ba+Ab)x^{\frac{7}{2}} + 2\,Aa\sqrt{x}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
```

output
```
2/13*B*b*x^(13/2) + 2/7*(B*a + A*b)*x^(7/2) + 2*A*a*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{13}\,Bbx^{\frac{13}{2}} + \frac{2}{7}\,Bax^{\frac{7}{2}} + \frac{2}{7}\,Abx^{\frac{7}{2}} + 2\,Aa\sqrt{x}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

output
```
2/13*B*b*x^(13/2) + 2/7*B*a*x^(7/2) + 2/7*A*b*x^(7/2) + 2*A*a*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(91\,A\,a+13\,A\,b\,x^3+13\,B\,a\,x^3+7\,B\,b\,x^6)}{91}$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x^(1/2),x)
```

output
```
(2*x^(1/2)*(91*A*a + 13*A*b*x^3 + 13*B*a*x^3 + 7*B*b*x^6))/91
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(7b^2x^6+26ab\,x^3+91a^2)}{91}$$

input
```
int((b*x^3+a)*(B*x^3+A)/x^(1/2),x)
```

output
```
(2*sqrt(x)*(91*a**2 + 26*a*b*x**3 + 7*b**2*x**6))/91
```

3.114 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}} dx = -\frac{2aA}{\sqrt{x}} + \frac{2}{5}(Ab+aB)x^{5/2} + \frac{2}{11}bBx^{11/2}$$

output
```
-2*a*A/x^(1/2)+2/5*(A*b+B*a)*x^(5/2)+2/11*b*B*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}} dx = -\frac{2(55aA-11Abx^3-11aBx^3-5bBx^6)}{55\sqrt{x}}$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/x^(3/2),x]
```

output
```
(-2*(55*a*A - 11*A*b*x^3 - 11*a*B*x^3 - 5*b*B*x^6))/(55*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^{3/2}(aB+Ab)+\frac{aA}{x^{3/2}}+bBx^{9/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}x^{5/2}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{11}bBx^{11/2}$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/x^(3/2),x]
```

output
```
(-2*a*A)/Sqrt[x] + (2*(A*b + a*B)*x^(5/2))/5 + (2*b*B*x^(11/2))/11
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2bB\,x^{\frac{11}{2}}}{11}+\frac{2Ab\,x^{\frac{5}{2}}}{5}+\frac{2Ba\,x^{\frac{5}{2}}}{5}-\frac{2aA}{\sqrt{x}}$	30
default	$\frac{2bB\,x^{\frac{11}{2}}}{11}+\frac{2Ab\,x^{\frac{5}{2}}}{5}+\frac{2Ba\,x^{\frac{5}{2}}}{5}-\frac{2aA}{\sqrt{x}}$	30
gosper	$-\frac{2(-5bB\,x^{6}-11Ab\,x^{3}-11Ba\,x^{3}+55Aa)}{55\sqrt{x}}$	32
trager	$-\frac{2(-5bB\,x^{6}-11Ab\,x^{3}-11Ba\,x^{3}+55Aa)}{55\sqrt{x}}$	32
risch	$-\frac{2(-5bB\,x^{6}-11Ab\,x^{3}-11Ba\,x^{3}+55Aa)}{55\sqrt{x}}$	32
orering	$-\frac{2(-5bB\,x^{6}-11Ab\,x^{3}-11Ba\,x^{3}+55Aa)}{55\sqrt{x}}$	32

input
```
int((b*x^3+a)*(B*x^3+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/11*b*B*x^(11/2)+2/5*A*b*x^(5/2)+2/5*B*a*x^(5/2)-2*a*A/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx=\frac{2\,(5\,Bbx^6+11\,(Ba+Ab)x^3-55\,Aa)}{55\,\sqrt{x}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
```

output
```
2/55*(5*B*b*x^6 + 11*(B*a + A*b)*x^3 - 55*A*a)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx = -\frac{2Aa}{\sqrt{x}} + \frac{2Abx^{\frac{5}{2}}}{5} + \frac{2Bax^{\frac{5}{2}}}{5} + \frac{2Bbx^{\frac{11}{2}}}{11}$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x**(3/2),x)
```

output
```
-2*A*a/sqrt(x) + 2*A*b*x**(5/2)/5 + 2*B*a*x**(5/2)/5 + 2*B*b*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx = \frac{2}{11}\,Bbx^{\frac{11}{2}} + \frac{2}{5}\,(Ba+Ab)x^{\frac{5}{2}} - \frac{2\,Aa}{\sqrt{x}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
```

output
```
2/11*B*b*x^(11/2) + 2/5*(B*a + A*b)*x^(5/2) - 2*A*a/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx = \frac{2}{11}\,Bbx^{\frac{11}{2}} + \frac{2}{5}\,Bax^{\frac{5}{2}} + \frac{2}{5}\,Abx^{\frac{5}{2}} - \frac{2\,Aa}{\sqrt{x}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="giac")
```

output
```
2/11*B*b*x^(11/2) + 2/5*B*a*x^(5/2) + 2/5*A*b*x^(5/2) - 2*A*a/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx = \frac{22\,A\,b\,x^3-110\,A\,a+22\,B\,a\,x^3+10\,B\,b\,x^6}{55\,\sqrt{x}}$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x^(3/2),x)
```

output
```
(22*A*b*x^3 - 110*A*a + 22*B*a*x^3 + 10*B*b*x^6)/(55*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx = \frac{\frac{2}{11}b^2x^6+\frac{4}{5}ab\,x^3-2a^2}{\sqrt{x}}$$

input
```
int((b*x^3+a)*(B*x^3+A)/x^(3/2),x)
```

output
```
(2*( - 55*a**2 + 22*a*b*x**3 + 5*b**2*x**6))/(55*sqrt(x))
```

3.115 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx = -\frac{2aA}{3x^{3/2}} + \frac{2}{3}(Ab+aB)x^{3/2} + \frac{2}{9}bBx^{9/2}$$

output
```
-2/3*a*A/x^(3/2)+2/3*(A*b+B*a)*x^(3/2)+2/9*b*B*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx = \frac{2(-3aA+3Abx^3+3aBx^3+bBx^6)}{9x^{3/2}}$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/x^(5/2),x]
```

output
```
(2*(-3*a*A + 3*A*b*x^3 + 3*a*B*x^3 + b*B*x^6))/(9*x^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx$$

$$\Big\downarrow \ 950$$

$$\int \left(\sqrt{x}(aB+Ab)+\frac{aA}{x^{5/2}}+bBx^{7/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{3}x^{3/2}(aB+Ab)-\frac{2aA}{3x^{3/2}}+\frac{2}{9}bBx^{9/2}$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/x^(5/2),x]
```

output
```
(-2*a*A)/(3*x^(3/2)) + (2*(A*b + a*B)*x^(3/2))/3 + (2*b*B*x^(9/2))/9
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2bB\,x^{\frac{9}{2}}}{9}+\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2Ba\,x^{\frac{3}{2}}}{3}-\frac{2aA}{3x^{\frac{3}{2}}}$	30
default	$\frac{2bB\,x^{\frac{9}{2}}}{9}+\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2Ba\,x^{\frac{3}{2}}}{3}-\frac{2aA}{3x^{\frac{3}{2}}}$	30
gosper	$-\frac{2(-bB\,x^{6}-3Ab\,x^{3}-3Ba\,x^{3}+3Aa)}{9x^{\frac{3}{2}}}$	32
trager	$-\frac{2(-bB\,x^{6}-3Ab\,x^{3}-3Ba\,x^{3}+3Aa)}{9x^{\frac{3}{2}}}$	32
risch	$-\frac{2(-bB\,x^{6}-3Ab\,x^{3}-3Ba\,x^{3}+3Aa)}{9x^{\frac{3}{2}}}$	32
orering	$-\frac{2(-bB\,x^{6}-3Ab\,x^{3}-3Ba\,x^{3}+3Aa)}{9x^{\frac{3}{2}}}$	32

input
```
int((b*x^3+a)*(B*x^3+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*b*B*x^(9/2)+2/3*A*b*x^(3/2)+2/3*B*a*x^(3/2)-2/3*a*A/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx=\frac{2\,(Bbx^6+3\,(Ba+Ab)x^3-3\,Aa)}{9\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
```

output
```
2/9*(B*b*x^6 + 3*(B*a + A*b)*x^3 - 3*A*a)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx = -\frac{2Aa}{3x^{\frac{3}{2}}} + \frac{2Abx^{\frac{3}{2}}}{3} + \frac{2Bax^{\frac{3}{2}}}{3} + \frac{2Bbx^{\frac{9}{2}}}{9}$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x**(5/2),x)
```

output
```
-2*A*a/(3*x**(3/2)) + 2*A*b*x**(3/2)/3 + 2*B*a*x**(3/2)/3 + 2*B*b*x**(9/2)
/9
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx = \frac{2}{9}\,Bbx^{\frac{9}{2}} + \frac{2}{3}\,(Ba+Ab)x^{\frac{3}{2}} - \frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

output
```
2/9*B*b*x^(9/2) + 2/3*(B*a + A*b)*x^(3/2) - 2/3*A*a/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx = \frac{2}{9}\,Bbx^{\frac{9}{2}} + \frac{2}{3}\,Bax^{\frac{3}{2}} + \frac{2}{3}\,Abx^{\frac{3}{2}} - \frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

output
```
2/9*B*b*x^(9/2) + 2/3*B*a*x^(3/2) + 2/3*A*b*x^(3/2) - 2/3*A*a/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx = \frac{6\,A\,b\,x^3-6\,A\,a+6\,B\,a\,x^3+2\,B\,b\,x^6}{9\,x^{3/2}}$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x^(5/2),x)
```

output
```
(6*A*b*x^3 - 6*A*a + 6*B*a*x^3 + 2*B*b*x^6)/(9*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx = \frac{\frac{2}{9}b^2x^6+\frac{4}{3}ab\,x^3-\frac{2}{3}a^2}{\sqrt{x}\,x}$$

input
```
int((b*x^3+a)*(B*x^3+A)/x^(5/2),x)
```

output
```
(2*( - 3*a**2 + 6*a*b*x**3 + b**2*x**6))/(9*sqrt(x)*x)
```

3.116 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}} dx = -\frac{2aA}{5x^{5/2}} + 2(Ab+aB)\sqrt{x} + \frac{2}{7}bBx^{7/2}$$

output
```
-2/5*a*A/x^(5/2)+2*(A*b+B*a)*x^(1/2)+2/7*b*B*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}} dx = -\frac{2(7aA - 35Abx^3 - 35aBx^3 - 5bBx^6)}{35x^{5/2}}$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/x^(7/2),x]
```

output
```
(-2*(7*a*A - 35*A*b*x^3 - 35*a*B*x^3 - 5*b*B*x^6))/(35*x^(5/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx$$

$$\Big\downarrow \ 950$$

$$\int \left(\frac{aB+Ab}{\sqrt{x}}+\frac{aA}{x^{7/2}}+bBx^{5/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$2\sqrt{x}(aB+Ab)-\frac{2aA}{5x^{5/2}}+\frac{2}{7}bBx^{7/2}$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/x^(7/2),x]
```

output
```
(-2*a*A)/(5*x^(5/2)) + 2*(A*b + a*B)*Sqrt[x] + (2*b*B*x^(7/2))/7
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2bB\,x^{\frac{7}{2}}}{7}+2Ab\sqrt{x}+2Ba\sqrt{x}-\frac{2aA}{5x^{\frac{5}{2}}}$	30
default	$\frac{2bB\,x^{\frac{7}{2}}}{7}+2Ab\sqrt{x}+2Ba\sqrt{x}-\frac{2aA}{5x^{\frac{5}{2}}}$	30
gosper	$-\frac{2(-5bB\,x^{6}-35Ab\,x^{3}-35Ba\,x^{3}+7Aa)}{35x^{\frac{5}{2}}}$	32
trager	$-\frac{2(-5bB\,x^{6}-35Ab\,x^{3}-35Ba\,x^{3}+7Aa)}{35x^{\frac{5}{2}}}$	32
risch	$-\frac{2(-5bB\,x^{6}-35Ab\,x^{3}-35Ba\,x^{3}+7Aa)}{35x^{\frac{5}{2}}}$	32
orering	$-\frac{2(-5bB\,x^{6}-35Ab\,x^{3}-35Ba\,x^{3}+7Aa)}{35x^{\frac{5}{2}}}$	32

input
```
int((b*x^3+a)*(B*x^3+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*b*B*x^(7/2)+2*A*b*x^(1/2)+2*B*a*x^(1/2)-2/5*a*A/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx = \frac{2\,(5\,Bbx^6+35\,(Ba+Ab)x^3-7\,Aa)}{35\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
```

output
```
2/35*(5*B*b*x^6 + 35*(B*a + A*b)*x^3 - 7*A*a)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx = -\frac{2Aa}{5x^{\frac{5}{2}}} + 2Ab\sqrt{x} + 2Ba\sqrt{x} + \frac{2Bbx^{\frac{7}{2}}}{7}$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x**(7/2),x)
```

output
```
-2*A*a/(5*x**(5/2)) + 2*A*b*sqrt(x) + 2*B*a*sqrt(x) + 2*B*b*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx = \frac{2}{7}Bbx^{\frac{7}{2}} + 2(Ba+Ab)\sqrt{x} - \frac{2\,Aa}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
```

output
```
2/7*B*b*x^(7/2) + 2*(B*a + A*b)*sqrt(x) - 2/5*A*a/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx = \frac{2}{7}Bbx^{\frac{7}{2}} + 2\,Ba\sqrt{x} + 2\,Ab\sqrt{x} - \frac{2\,Aa}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="giac")
```

output
```
2/7*B*b*x^(7/2) + 2*B*a*sqrt(x) + 2*A*b*sqrt(x) - 2/5*A*a/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx = \frac{2\,A\,b\,x^3 - \frac{2\,A\,a}{5} + 2\,B\,a\,x^3 + \frac{2\,B\,b\,x^6}{7}}{x^{5/2}}$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x^(7/2),x)
```

output
```
(2*A*b*x^3 - (2*A*a)/5 + 2*B*a*x^3 + (2*B*b*x^6)/7)/x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx = \frac{\frac{2}{7}b^2x^6 + 4ab\,x^3 - \frac{2}{5}a^2}{\sqrt{x}\,x^2}$$

input
```
int((b*x^3+a)*(B*x^3+A)/x^(7/2),x)
```

output
```
(2*( - 7*a**2 + 70*a*b*x**3 + 5*b**2*x**6))/(35*sqrt(x)*x**2)
```

3.117 $\int x^{5/2}(a+bx^3)^2 (A+Bx^3) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int x^{5/2}(a+bx^3)^2 (A+Bx^3) \, dx = \frac{2}{7}a^2Ax^{7/2} + \frac{2}{13}a(2Ab+aB)x^{13/2} + \frac{2}{19}b(Ab+2aB)x^{19/2} + \frac{2}{25}b^2Bx^{25/2}$$

output
```
2/7*a^2*A*x^(7/2)+2/13*a*(2*A*b+B*a)*x^(13/2)+2/19*b*(A*b+2*B*a)*x^(19/2)+
2/25*b^2*B*x^(25/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int x^{5/2}(a+bx^3)^2 (A+Bx^3) \, dx = \frac{2x^{7/2}(475a^2(13A+7Bx^3)+350abx^3(19A+13Bx^3)+91b^2x^6(25A+19Bx^3))}{43225}$$

input
```
Integrate[x^(5/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(2*x^(7/2)*(475*a^2*(13*A + 7*B*x^3) + 350*a*b*x^3*(19*A + 13*B*x^3) + 91*
b^2*x^6*(25*A + 19*B*x^3)))/43225
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^2Ax^{5/2}+bx^{17/2}(2aB+Ab)+ax^{11/2}(aB+2Ab)+b^2Bx^{23/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{19}bx^{19/2}(2aB+Ab)+\frac{2}{13}ax^{13/2}(aB+2Ab)+\frac{2}{25}b^2Bx^{25/2}$$

input
```
Int[x^(5/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b + 2*a*B)
*x^(19/2))/19 + (2*b^2*B*x^(25/2))/25
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2b^2 B\, x^{\frac{25}{2}}}{25}+\frac{2(b^2 A+2abB)x^{\frac{19}{2}}}{19}+\frac{2(2abA+a^2 B)x^{\frac{13}{2}}}{13}+\frac{2a^2 A\, x^{\frac{7}{2}}}{7}$	52
default	$\frac{2b^2 B\, x^{\frac{25}{2}}}{25}+\frac{2(b^2 A+2abB)x^{\frac{19}{2}}}{19}+\frac{2(2abA+a^2 B)x^{\frac{13}{2}}}{13}+\frac{2a^2 A\, x^{\frac{7}{2}}}{7}$	52
gosper	$\frac{2x^{\frac{7}{2}}\left(1729b^2 B\, x^9+2275A\, b^2 x^6+4550Bab\, x^6+6650aAb\, x^3+3325B\, a^2 x^3+6175a^2 A\right)}{43225}$	56
trager	$\frac{2x^{\frac{7}{2}}\left(1729b^2 B\, x^9+2275A\, b^2 x^6+4550Bab\, x^6+6650aAb\, x^3+3325B\, a^2 x^3+6175a^2 A\right)}{43225}$	56
risch	$\frac{2x^{\frac{7}{2}}\left(1729b^2 B\, x^9+2275A\, b^2 x^6+4550Bab\, x^6+6650aAb\, x^3+3325B\, a^2 x^3+6175a^2 A\right)}{43225}$	56
orering	$\frac{2x^{\frac{7}{2}}\left(1729b^2 B\, x^9+2275A\, b^2 x^6+4550Bab\, x^6+6650aAb\, x^3+3325B\, a^2 x^3+6175a^2 A\right)}{43225}$	56

input
```
int(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/25*b^2*B*x^(25/2)+2/19*(A*b^2+2*B*a*b)*x^(19/2)+2/13*(2*A*a*b+B*a^2)*x^(
13/2)+2/7*a^2*A*x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{43225}\left(1729\,Bb^2x^{12}+2275\left(2\,Bab+Ab^2\right)x^9+3325\left(Ba^2+2\,Aab\right)x^6+6175\,Aa^2x^3\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/43225*(1729*B*b^2*x^12 + 2275*(2*B*a*b + A*b^2)*x^9 + 3325*(B*a^2 + 2*A*
a*b)*x^6 + 6175*A*a^2*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2Aa^2x^{\frac{7}{2}}}{7}+\frac{4Aabx^{\frac{13}{2}}}{13}+\frac{2Ab^2x^{\frac{19}{2}}}{19}+\frac{2Ba^2x^{\frac{13}{2}}}{13}+\frac{4Babx^{\frac{19}{2}}}{19}+\frac{2Bb^2x^{\frac{25}{2}}}{25}$$

input
```
integrate(x**(5/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

output
```
2*A*a**2*x**(7/2)/7 + 4*A*a*b*x**(13/2)/13 + 2*A*b**2*x**(19/2)/19 + 2*B*a
**2*x**(13/2)/13 + 4*B*a*b*x**(19/2)/19 + 2*B*b**2*x**(25/2)/25
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{25}\,Bb^2x^{\frac{25}{2}}+\frac{2}{19}\,\left(2\,Bab+Ab^2\right)x^{\frac{19}{2}}+\frac{2}{13}\,\left(Ba^2+2\,Aab\right)x^{\frac{13}{2}}+\frac{2}{7}\,Aa^2x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/25*B*b^2*x^(25/2) + 2/19*(2*B*a*b + A*b^2)*x^(19/2) + 2/13*(B*a^2 + 2*A*
a*b)*x^(13/2) + 2/7*A*a^2*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{25}\,Bb^2x^{\frac{25}{2}}+\frac{4}{19}\,Babx^{\frac{19}{2}}+\frac{2}{19}\,Ab^2x^{\frac{19}{2}}+\frac{2}{13}\,Ba^2x^{\frac{13}{2}}+\frac{4}{13}\,Aabx^{\frac{13}{2}}+\frac{2}{7}\,Aa^2x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

output
```
2/25*B*b^2*x^(25/2) + 4/19*B*a*b*x^(19/2) + 2/19*A*b^2*x^(19/2) + 2/13*B*a
^2*x^(13/2) + 4/13*A*a*b*x^(13/2) + 2/7*A*a^2*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=x^{13/2}\left(\frac{2\,B\,a^2}{13}+\frac{4\,A\,b\,a}{13}\right)+x^{19/2}\left(\frac{2\,A\,b^2}{19}+\frac{4\,B\,a\,b}{19}\right)+\frac{2\,A\,a^2\,x^{7/2}}{7}+\frac{2\,B\,b^2\,x^{25/2}}{25}$$

input
```
int(x^(5/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

output
```
x^(13/2)*((2*B*a^2)/13 + (4*A*a*b)/13) + x^(19/2)*((2*A*b^2)/19 + (4*B*a*b
)/19) + (2*A*a^2*x^(7/2))/7 + (2*B*b^2*x^(25/2))/25
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.62

$$\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2\sqrt{x}\,x^3(1729b^3x^9+6825a\,b^2x^6+9975a^2b\,x^3+6175a^3)}{43225}$$

input
```
int(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x)
```

output
```
(2*sqrt(x)*x**3*(6175*a**3 + 9975*a**2*b*x**3 + 6825*a*b**2*x**6 + 1729*b*
*3*x**9))/43225
```

3.118 $\int x^{3/2}(a+bx^3)^2\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{5}a^2Ax^{5/2}+\frac{2}{11}a(2Ab+aB)x^{11/2}+\frac{2}{17}b(Ab+2aB)x^{17/2}+\frac{2}{23}b^2Bx^{23/2}$$

output
```
2/5*a^2*A*x^(5/2)+2/11*a*(2*A*b+B*a)*x^(11/2)+2/17*b*(A*b+2*B*a)*x^(17/2)+
2/23*b^2*B*x^(23/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2x^{5/2}(391a^2(11A+5Bx^3)+230abx^3(17A+11Bx^3)+55b^2x^6(23A+17Bx^3))}{21505}$$

input
```
Integrate[x^(3/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(2*x^(5/2)*(391*a^2*(11*A + 5*B*x^3) + 230*a*b*x^3*(17*A + 11*B*x^3) + 55*
b^2*x^6*(23*A + 17*B*x^3)))/21505
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2} \left(a+b x^3\right)^2 \left(A+B x^3\right) \, dx$$

$$\Big\downarrow \text{ 950}$$

$$\int \left(a^2 A x^{3/2}+b x^{15/2} (2 a B+A b)+a x^{9/2} (a B+2 A b)+b^2 B x^{21/2}\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{5} a^2 A x^{5/2}+\frac{2}{17} b x^{17/2} (2 a B+A b)+\frac{2}{11} a x^{11/2} (a B+2 A b)+\frac{2}{23} b^2 B x^{23/2}$$

input
```
Int[x^(3/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b + 2*a*B)
*x^(17/2))/17 + (2*b^2*B*x^(23/2))/23
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2b^2 B x^{\frac{23}{2}}}{23}+\frac{2(b^2 A+2abB)x^{\frac{17}{2}}}{17}+\frac{2(2abA+a^2 B)x^{\frac{11}{2}}}{11}+\frac{2a^2 A x^{\frac{5}{2}}}{5}$	52
default	$\frac{2b^2 B x^{\frac{23}{2}}}{23}+\frac{2(b^2 A+2abB)x^{\frac{17}{2}}}{17}+\frac{2(2abA+a^2 B)x^{\frac{11}{2}}}{11}+\frac{2a^2 A x^{\frac{5}{2}}}{5}$	52
gosper	$\frac{2x^{\frac{5}{2}}(935b^2 B x^9+1265A b^2 x^6+2530Bab x^6+3910aAb x^3+1955B a^2 x^3+4301a^2 A)}{21505}$	56
trager	$\frac{2x^{\frac{5}{2}}(935b^2 B x^9+1265A b^2 x^6+2530Bab x^6+3910aAb x^3+1955B a^2 x^3+4301a^2 A)}{21505}$	56
risch	$\frac{2x^{\frac{5}{2}}(935b^2 B x^9+1265A b^2 x^6+2530Bab x^6+3910aAb x^3+1955B a^2 x^3+4301a^2 A)}{21505}$	56
orering	$\frac{2x^{\frac{5}{2}}(935b^2 B x^9+1265A b^2 x^6+2530Bab x^6+3910aAb x^3+1955B a^2 x^3+4301a^2 A)}{21505}$	56

input
```
int(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/23*b^2*B*x^(23/2)+2/17*(A*b^2+2*B*a*b)*x^(17/2)+2/11*(2*A*a*b+B*a^2)*x^(
11/2)+2/5*a^2*A*x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{3/2}(a+bx^3)^2(A+Bx^3)\,dx=\frac{2}{21505}\left(935\,Bb^2x^{11}+1265\left(2\,Bab+Ab^2\right)x^8+1955\left(Ba^2+2\,Aab\right)x^5+4301\,Aa^2x^2\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/21505*(935*B*b^2*x^11 + 1265*(2*B*a*b + A*b^2)*x^8 + 1955*(B*a^2 + 2*A*a
*b)*x^5 + 4301*A*a^2*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2Aa^2x^{\frac{5}{2}}}{5}+\frac{4Aabx^{\frac{11}{2}}}{11}+\frac{2Ab^2x^{\frac{17}{2}}}{17}+\frac{2Ba^2x^{\frac{11}{2}}}{11}+\frac{4Babx^{\frac{17}{2}}}{17}+\frac{2Bb^2x^{\frac{23}{2}}}{23}$$

input
```
integrate(x**(3/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

output
```
2*A*a**2*x**(5/2)/5 + 4*A*a*b*x**(11/2)/11 + 2*A*b**2*x**(17/2)/17 + 2*B*a
**2*x**(11/2)/11 + 4*B*a*b*x**(17/2)/17 + 2*B*b**2*x**(23/2)/23
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{23}\,Bb^2x^{\frac{23}{2}}+\frac{2}{17}\,\left(2\,Bab+Ab^2\right)x^{\frac{17}{2}}+\frac{2}{11}\,\left(Ba^2+2\,Aab\right)x^{\frac{11}{2}}+\frac{2}{5}\,Aa^2x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/23*B*b^2*x^(23/2) + 2/17*(2*B*a*b + A*b^2)*x^(17/2) + 2/11*(B*a^2 + 2*A*
a*b)*x^(11/2) + 2/5*A*a^2*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{23}\,Bb^2x^{\frac{23}{2}}+\frac{4}{17}\,Babx^{\frac{17}{2}}+\frac{2}{17}\,Ab^2x^{\frac{17}{2}}+\frac{2}{11}\,Ba^2x^{\frac{11}{2}}+\frac{4}{11}\,Aabx^{\frac{11}{2}}+\frac{2}{5}\,Aa^2x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

output
```
2/23*B*b^2*x^(23/2) + 4/17*B*a*b*x^(17/2) + 2/17*A*b^2*x^(17/2) + 2/11*B*a
^2*x^(11/2) + 4/11*A*a*b*x^(11/2) + 2/5*A*a^2*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=x^{11/2}\left(\frac{2\,B\,a^2}{11}+\frac{4\,A\,b\,a}{11}\right)+x^{17/2}\left(\frac{2\,A\,b^2}{17}+\frac{4\,B\,a\,b}{17}\right)+\frac{2\,A\,a^2\,x^{5/2}}{5}+\frac{2\,B\,b^2\,x^{23/2}}{23}$$

input
```
int(x^(3/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

output
```
x^(11/2)*((2*B*a^2)/11 + (4*A*a*b)/11) + x^(17/2)*((2*A*b^2)/17 + (4*B*a*b
)/17) + (2*A*a^2*x^(5/2))/5 + (2*B*b^2*x^(23/2))/23
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.62

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2\sqrt{x}\,x^2(935b^3x^9+3795a\,b^2x^6+5865a^2b\,x^3+4301a^3)}{21505}$$

input
```
int(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x)
```

output
```
(2*sqrt(x)*x**2*(4301*a**3 + 5865*a**2*b*x**3 + 3795*a*b**2*x**6 + 935*b**
3*x**9))/21505
```

3.119 $\int \sqrt{x}(a+bx^3)^2 (A+Bx^3) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \sqrt{x}(a+bx^3)^2 (A+Bx^3) \, dx = \frac{2}{3}a^2Ax^{3/2} + \frac{2}{9}a(2Ab+aB)x^{9/2} + \frac{2}{15}b(Ab+2aB)x^{15/2} + \frac{2}{21}b^2Bx^{21/2}$$

output
```
2/3*a^2*A*x^(3/2)+2/9*a*(2*A*b+B*a)*x^(9/2)+2/15*b*(A*b+2*B*a)*x^(15/2)+2/
21*b^2*B*x^(21/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \sqrt{x}(a+bx^3)^2 (A+Bx^3) \, dx = \frac{2}{315}x^{3/2}\left(35a^2(3A+Bx^3) + 14abx^3(5A+3Bx^3) + 3b^2x^6(7A+5Bx^3)\right)$$

input
```
Integrate[Sqrt[x]*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(2*x^(3/2)*(35*a^2*(3*A + B*x^3) + 14*a*b*x^3*(5*A + 3*B*x^3) + 3*b^2*x^6*
(7*A + 5*B*x^3)))/315
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^2A\sqrt{x}+bx^{13/2}(2aB+Ab)+ax^{7/2}(aB+2Ab)+b^2Bx^{19/2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{21}b^2Bx^{21/2}$$

input
```
Int[Sqrt[x]*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*b*(A*b + 2*a*B)*x
^(15/2))/15 + (2*b^2*B*x^(21/2))/21
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{21}{2}}}{21}+\frac{2(b^2A+2abB)x^{\frac{15}{2}}}{15}+\frac{2(2abA+a^2B)x^{\frac{9}{2}}}{9}+\frac{2a^2Ax^{\frac{3}{2}}}{3}$	52
default	$\frac{2b^2Bx^{\frac{21}{2}}}{21}+\frac{2(b^2A+2abB)x^{\frac{15}{2}}}{15}+\frac{2(2abA+a^2B)x^{\frac{9}{2}}}{9}+\frac{2a^2Ax^{\frac{3}{2}}}{3}$	52
gosper	$\frac{2x^{\frac{3}{2}}(15b^2Bx^9+21Ab^2x^6+42Babx^6+70aAbx^3+35Ba^2x^3+105a^2A)}{315}$	56
trager	$\frac{2x^{\frac{3}{2}}(15b^2Bx^9+21Ab^2x^6+42Babx^6+70aAbx^3+35Ba^2x^3+105a^2A)}{315}$	56
risch	$\frac{2x^{\frac{3}{2}}(15b^2Bx^9+21Ab^2x^6+42Babx^6+70aAbx^3+35Ba^2x^3+105a^2A)}{315}$	56
orering	$\frac{2x^{\frac{3}{2}}(15b^2Bx^9+21Ab^2x^6+42Babx^6+70aAbx^3+35Ba^2x^3+105a^2A)}{315}$	56

input
```
int(x^(1/2)*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/21*b^2*B*x^(21/2)+2/15*(A*b^2+2*B*a*b)*x^(15/2)+2/9*(2*A*a*b+B*a^2)*x^(9
/2)+2/3*a^2*A*x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \sqrt{x}(a+bx^3)^2\left(A+Bx^3\right)\,dx$$
$$=\frac{2}{315}\left(15\,Bb^2x^{10}+21\left(2\,Bab+Ab^2\right)x^7+35\left(Ba^2+2\,Aab\right)x^4+105\,Aa^2x\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/315*(15*B*b^2*x^10 + 21*(2*B*a*b + A*b^2)*x^7 + 35*(B*a^2 + 2*A*a*b)*x^4
 + 105*A*a^2*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \sqrt{x}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx = \frac{2Aa^2x^{\frac{3}{2}}}{3} + \frac{4Aabx^{\frac{9}{2}}}{9} + \frac{2Ab^2x^{\frac{15}{2}}}{15} + \frac{2Ba^2x^{\frac{9}{2}}}{9} + \frac{4Babx^{\frac{15}{2}}}{15} + \frac{2Bb^2x^{\frac{21}{2}}}{21}$$

input
```
integrate(x**(1/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

output
```
2*A*a**2*x**(3/2)/3 + 4*A*a*b*x**(9/2)/9 + 2*A*b**2*x**(15/2)/15 + 2*B*a**
2*x**(9/2)/9 + 4*B*a*b*x**(15/2)/15 + 2*B*b**2*x**(21/2)/21
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx = \frac{2}{21}\,Bb^2x^{\frac{21}{2}} + \frac{2}{15}\left(2\,Bab + Ab^2\right)x^{\frac{15}{2}} + \frac{2}{9}\left(Ba^2 + 2\,Aab\right)x^{\frac{9}{2}} + \frac{2}{3}\,Aa^2x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/21*B*b^2*x^(21/2) + 2/15*(2*B*a*b + A*b^2)*x^(15/2) + 2/9*(B*a^2 + 2*A*a
*b)*x^(9/2) + 2/3*A*a^2*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \sqrt{x}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{21}Bb^2x^{\frac{21}{2}}+\frac{4}{15}Babx^{\frac{15}{2}}+\frac{2}{15}Ab^2x^{\frac{15}{2}}+\frac{2}{9}Ba^2x^{\frac{9}{2}}+\frac{4}{9}Aabx^{\frac{9}{2}}+\frac{2}{3}Aa^2x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

output
```
2/21*B*b^2*x^(21/2) + 4/15*B*a*b*x^(15/2) + 2/15*A*b^2*x^(15/2) + 2/9*B*a^
2*x^(9/2) + 4/9*A*a*b*x^(9/2) + 2/3*A*a^2*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=x^{9/2}\left(\frac{2\,B\,a^2}{9}+\frac{4\,A\,b\,a}{9}\right)+x^{15/2}\left(\frac{2\,A\,b^2}{15}+\frac{4\,B\,a\,b}{15}\right)+\frac{2\,A\,a^2\,x^{3/2}}{3}+\frac{2\,B\,b^2\,x^{21/2}}{21}$$

input
```
int(x^(1/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

output
```
x^(9/2)*((2*B*a^2)/9 + (4*A*a*b)/9) + x^(15/2)*((2*A*b^2)/15 + (4*B*a*b)/1
5) + (2*A*a^2*x^(3/2))/3 + (2*B*b^2*x^(21/2))/21
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.59

$$\int \sqrt{x}(a+bx^3)^2\,(A+Bx^3)\;dx=\frac{2\sqrt{x}\,x(5b^3x^9+21a\,b^2x^6+35a^2b\,x^3+35a^3)}{105}$$

input
```
int(x^(1/2)*(b*x^3+a)^2*(B*x^3+A),x)
```

output
```
(2*sqrt(x)*x*(35*a**3 + 35*a**2*b*x**3 + 21*a*b**2*x**6 + 5*b**3*x**9))/10
5
```

3.120 $\int \frac{(a+bx^3)^2(A+Bx^3)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{\sqrt{x}}\,dx = 2a^2A\sqrt{x}+\frac{2}{7}a(2Ab+aB)x^{7/2} + \frac{2}{13}b(Ab+2aB)x^{13/2}+\frac{2}{19}b^2Bx^{19/2}$$

output
```
2*a^2*A*x^(1/2)+2/7*a*(2*A*b+B*a)*x^(7/2)+2/13*b*(A*b+2*B*a)*x^(13/2)+2/19
*b^2*B*x^(19/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2\sqrt{x}(247a^2(7A+Bx^3)+38abx^3(13A+7Bx^3)+7b^2x^6(19A+13Bx^3))}{1729}$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(247*a^2*(7*A + B*x^3) + 38*a*b*x^3*(13*A + 7*B*x^3) + 7*b^2*x^
6*(19*A + 13*B*x^3)))/1729
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2A}{\sqrt{x}}+bx^{11/2}(2aB+Ab)+ax^{5/2}(aB+2Ab)+b^2Bx^{17/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2a^2A\sqrt{x}+\frac{2}{13}bx^{13/2}(2aB+Ab)+\frac{2}{7}ax^{7/2}(aB+2Ab)+\frac{2}{19}b^2Bx^{19/2}$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/Sqrt[x],x]
```

output
```
2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(7/2))/7 + (2*b*(A*b + 2*a*B)*x^(13
/2))/13 + (2*b^2*B*x^(19/2))/19
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2b^2 B x^{\frac{19}{2}}}{19}+\frac{2(b^2 A+2abB)x^{\frac{13}{2}}}{13}+\frac{2(2abA+a^2 B)x^{\frac{7}{2}}}{7}+2a^2 A\sqrt{x}$	52
default	$\frac{2b^2 B x^{\frac{19}{2}}}{19}+\frac{2(b^2 A+2abB)x^{\frac{13}{2}}}{13}+\frac{2(2abA+a^2 B)x^{\frac{7}{2}}}{7}+2a^2 A\sqrt{x}$	52
trager	$\left(\frac{2}{19}b^2 B x^9+\frac{2}{13}A b^2 x^6+\frac{4}{13}Bab x^6+\frac{4}{7}aAb x^3+\frac{2}{7}B a^2 x^3+2a^2 A\right)\sqrt{x}$	55
gosper	$\frac{2\sqrt{x}\,(91b^2 B x^9+133A b^2 x^6+266Bab x^6+494aAb x^3+247B a^2 x^3+1729a^2 A)}{1729}$	56
risch	$\frac{2\sqrt{x}\,(91b^2 B x^9+133A b^2 x^6+266Bab x^6+494aAb x^3+247B a^2 x^3+1729a^2 A)}{1729}$	56
orering	$\frac{2\sqrt{x}\,(91b^2 B x^9+133A b^2 x^6+266Bab x^6+494aAb x^3+247B a^2 x^3+1729a^2 A)}{1729}$	56

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/19*b^2*B*x^(19/2)+2/13*(A*b^2+2*B*a*b)*x^(13/2)+2/7*(2*A*a*b+B*a^2)*x^(7
/2)+2*a^2*A*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{\sqrt{x}}\,dx$$
$$=\frac{2}{1729}\left(91\,Bb^2x^9+133\,(2\,Bab+Ab^2)x^6+247\,\left(Ba^2+2\,Aab\right)x^3+1729\,Aa^2\right)\sqrt{x}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
```

output
```
2/1729*(91*B*b^2*x^9 + 133*(2*B*a*b + A*b^2)*x^6 + 247*(B*a^2 + 2*A*a*b)*x
^3 + 1729*A*a^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}} \, dx = 2Aa^2\sqrt{x} + \frac{4Aabx^{\frac{7}{2}}}{7} + \frac{2Ab^2x^{\frac{13}{2}}}{13} + \frac{2Ba^2x^{\frac{7}{2}}}{7} + \frac{4Babx^{\frac{13}{2}}}{13} + \frac{2Bb^2x^{\frac{19}{2}}}{19}$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**(1/2),x)
```

output
```
2*A*a**2*sqrt(x) + 4*A*a*b*x**(7/2)/7 + 2*A*b**2*x**(13/2)/13 + 2*B*a**2*x
**(7/2)/7 + 4*B*a*b*x**(13/2)/13 + 2*B*b**2*x**(19/2)/19
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}} \, dx = \frac{2}{19} Bb^2x^{\frac{19}{2}} + \frac{2}{13} \left(2 Bab + Ab^2\right)x^{\frac{13}{2}} + \frac{2}{7} \left(Ba^2 + 2 Aab\right)x^{\frac{7}{2}} + 2 Aa^2\sqrt{x}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
```

output
```
2/19*B*b^2*x^(19/2) + 2/13*(2*B*a*b + A*b^2)*x^(13/2) + 2/7*(B*a^2 + 2*A*a
*b)*x^(7/2) + 2*A*a^2*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{19}Bb^2x^{\frac{19}{2}} + \frac{4}{13}Babx^{\frac{13}{2}} + \frac{2}{13}Ab^2x^{\frac{13}{2}} + \frac{2}{7}Ba^2x^{\frac{7}{2}} + \frac{4}{7}Aabx^{\frac{7}{2}} + 2\,Aa^2\sqrt{x}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

output
```
2/19*B*b^2*x^(19/2) + 4/13*B*a*b*x^(13/2) + 2/13*A*b^2*x^(13/2) + 2/7*B*a^
2*x^(7/2) + 4/7*A*a*b*x^(7/2) + 2*A*a^2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}}\,dx = x^{7/2}\left(\frac{2\,B\,a^2}{7} + \frac{4\,A\,b\,a}{7}\right) + x^{13/2}\left(\frac{2\,A\,b^2}{13} + \frac{4\,B\,a\,b}{13}\right) + 2\,A\,a^2\,\sqrt{x} + \frac{2\,B\,b^2\,x^{19/2}}{19}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^(1/2),x)
```

output
```
x^(7/2)*((2*B*a^2)/7 + (4*A*a*b)/7) + x^(13/2)*((2*A*b^2)/13 + (4*B*a*b)/1
3) + 2*A*a^2*x^(1/2) + (2*B*b^2*x^(19/2))/19
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}} \, dx = \frac{2\sqrt{x}\,(91b^3x^9+399a\,b^2x^6+741a^2b\,x^3+1729a^3)}{1729}$$

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x)
```

output
```
(2*sqrt(x)*(1729*a**3 + 741*a**2*b*x**3 + 399*a*b**2*x**6 + 91*b**3*x**9))
/1729
```

3.121 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx = -\frac{2a^2A}{\sqrt{x}} + \frac{2}{5}a(2Ab+aB)x^{5/2} + \frac{2}{11}b(Ab+2aB)x^{11/2} + \frac{2}{17}b^2Bx^{17/2}$$

output
```
-2*a^2*A/x^(1/2)+2/5*a*(2*A*b+B*a)*x^(5/2)+2/11*b*(A*b+2*B*a)*x^(11/2)+2/1
7*b^2*B*x^(17/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx = -\frac{2(935a^2A-374aAbx^3-187a^2Bx^3-85Ab^2x^6-170abBx^6-55b^2Bx^9)}{935\sqrt{x}}$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(3/2),x]
```

output
```
(-2*(935*a^2*A - 374*a*A*b*x^3 - 187*a^2*B*x^3 - 85*A*b^2*x^6 - 170*a*b*B*
x^6 - 55*b^2*B*x^9))/(935*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{3/2}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2A}{x^{3/2}}+bx^{9/2}(2aB+Ab)+ax^{3/2}(aB+2Ab)+b^2Bx^{15/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{17}b^2Bx^{17/2}$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x^(3/2),x]
```

output
```
(-2*a^2*A)/Sqrt[x] + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*b*(A*b + 2*a*B)*x^
(11/2))/11 + (2*b^2*B*x^(17/2))/17
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{17}{2}}}{17}+\frac{2Ab^2x^{\frac{11}{2}}}{11}+\frac{4Babx^{\frac{11}{2}}}{11}+\frac{4Aabx^{\frac{5}{2}}}{5}+\frac{2Ba^2x^{\frac{5}{2}}}{5}-\frac{2a^2A}{\sqrt{x}}$	54
default	$\frac{2b^2Bx^{\frac{17}{2}}}{17}+\frac{2Ab^2x^{\frac{11}{2}}}{11}+\frac{4Babx^{\frac{11}{2}}}{11}+\frac{4Aabx^{\frac{5}{2}}}{5}+\frac{2Ba^2x^{\frac{5}{2}}}{5}-\frac{2a^2A}{\sqrt{x}}$	54
gosper	$-\frac{2(-55b^2Bx^9-85Ab^2x^6-170Babx^6-374aAbx^3-187Ba^2x^3+935a^2A)}{935\sqrt{x}}$	56
trager	$-\frac{2(-55b^2Bx^9-85Ab^2x^6-170Babx^6-374aAbx^3-187Ba^2x^3+935a^2A)}{935\sqrt{x}}$	56
risch	$-\frac{2(-55b^2Bx^9-85Ab^2x^6-170Babx^6-374aAbx^3-187Ba^2x^3+935a^2A)}{935\sqrt{x}}$	56
orering	$-\frac{2(-55b^2Bx^9-85Ab^2x^6-170Babx^6-374aAbx^3-187Ba^2x^3+935a^2A)}{935\sqrt{x}}$	56

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/17*b^2*B*x^(17/2)+2/11*A*b^2*x^(11/2)+4/11*B*a*b*x^(11/2)+4/5*A*a*b*x^(5
/2)+2/5*B*a^2*x^(5/2)-2*a^2*A/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{3/2}}\,dx = \frac{2\,(55\,Bb^2x^9+85\,(2\,Bab+Ab^2)x^6+187\,(Ba^2+2\,Aab)x^3-935\,Aa^2)}{935\,\sqrt{x}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
```

output
```
2/935*(55*B*b^2*x^9 + 85*(2*B*a*b + A*b^2)*x^6 + 187*(B*a^2 + 2*A*a*b)*x^3
 - 935*A*a^2)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{3/2}}\,dx = -\frac{2Aa^2}{\sqrt{x}}+\frac{4Aabx^{\frac{5}{2}}}{5}+\frac{2Ab^2x^{\frac{11}{2}}}{11}+\frac{2Ba^2x^{\frac{5}{2}}}{5}+\frac{4Babx^{\frac{11}{2}}}{11}+\frac{2Bb^2x^{\frac{17}{2}}}{17}$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**(3/2),x)
```

output
```
-2*A*a**2/sqrt(x) + 4*A*a*b*x**(5/2)/5 + 2*A*b**2*x**(11/2)/11 + 2*B*a**2*
x**(5/2)/5 + 4*B*a*b*x**(11/2)/11 + 2*B*b**2*x**(17/2)/17
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{3/2}}\,dx = \frac{2}{17}\,Bb^2x^{\frac{17}{2}}+\frac{2}{11}\,(2\,Bab+Ab^2)x^{\frac{11}{2}}+\frac{2}{5}\,(Ba^2+2\,Aab)x^{\frac{5}{2}}-\frac{2\,Aa^2}{\sqrt{x}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
```

output
```
2/17*B*b^2*x^(17/2) + 2/11*(2*B*a*b + A*b^2)*x^(11/2) + 2/5*(B*a^2 + 2*A*a
*b)*x^(5/2) - 2*A*a^2/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx = \frac{2}{17}Bb^2x^{\frac{17}{2}} + \frac{4}{11}Babx^{\frac{11}{2}} + \frac{2}{11}Ab^2x^{\frac{11}{2}} + \frac{2}{5}Ba^2x^{\frac{5}{2}} + \frac{4}{5}Aabx^{\frac{5}{2}} - \frac{2Aa^2}{\sqrt{x}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="giac")
```

output
```
2/17*B*b^2*x^(17/2) + 4/11*B*a*b*x^(11/2) + 2/11*A*b^2*x^(11/2) + 2/5*B*a^
2*x^(5/2) + 4/5*A*a*b*x^(5/2) - 2*A*a^2/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx = x^{5/2}\left(\frac{2\,B\,a^2}{5} + \frac{4\,A\,b\,a}{5}\right) + x^{11/2}\left(\frac{2\,A\,b^2}{11} + \frac{4\,B\,a\,b}{11}\right) - \frac{2\,A\,a^2}{\sqrt{x}} + \frac{2\,B\,b^2\,x^{17/2}}{17}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^(3/2),x)
```

output
```
x^(5/2)*((2*B*a^2)/5 + (4*A*a*b)/5) + x^(11/2)*((2*A*b^2)/11 + (4*B*a*b)/1
1) - (2*A*a^2)/x^(1/2) + (2*B*b^2*x^(17/2))/17
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{3/2}}\,dx = \frac{\frac{2}{17}b^3x^9+\frac{6}{11}a\,b^2x^6+\frac{6}{5}a^2b\,x^3-2a^3}{\sqrt{x}}$$

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x)
```

output
```
(2*( - 935*a**3 + 561*a**2*b*x**3 + 255*a*b**2*x**6 + 55*b**3*x**9))/(935*
sqrt(x))
```

3.122 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{5/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{5/2}} dx = -\frac{2a^2A}{3x^{3/2}} + \frac{2}{3}a(2Ab+aB)x^{3/2} + \frac{2}{9}b(Ab+2aB)x^{9/2} + \frac{2}{15}b^2Bx^{15/2}$$

output
```
-2/3*a^2*A/x^(3/2)+2/3*a*(2*A*b+B*a)*x^(3/2)+2/9*b*(A*b+2*B*a)*x^(9/2)+2/1
5*b^2*B*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{5/2}} dx = \frac{2(-15a^2A+30aAbx^3+15a^2Bx^3+5Ab^2x^6+10abBx^6+3b^2Bx^9)}{45x^{3/2}}$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(5/2),x]
```

output
```
(2*(-15*a^2*A + 30*a*A*b*x^3 + 15*a^2*B*x^3 + 5*A*b^2*x^6 + 10*a*b*B*x^6 +
 3*b^2*B*x^9))/(45*x^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{5/2}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2A}{x^{5/2}} + bx^{7/2}(2aB+Ab) + a\sqrt{x}(aB+2Ab) + b^2Bx^{13/2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2A}{3x^{3/2}} + \frac{2}{9}bx^{9/2}(2aB+Ab) + \frac{2}{3}ax^{3/2}(aB+2Ab) + \frac{2}{15}b^2Bx^{15/2}$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x^(5/2),x]
```

output
```
(-2*a^2*A)/(3*x^(3/2)) + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*b*(A*b + 2*a*B
)*x^(9/2))/9 + (2*b^2*B*x^(15/2))/15
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2b^2 B x^{\frac{15}{2}}}{15}+\frac{2A b^2 x^{\frac{9}{2}}}{9}+\frac{4Bab x^{\frac{9}{2}}}{9}+\frac{4Aab x^{\frac{3}{2}}}{3}+\frac{2B a^2 x^{\frac{3}{2}}}{3}-\frac{2a^2 A}{3x^{\frac{3}{2}}}$	54
default	$\frac{2b^2 B x^{\frac{15}{2}}}{15}+\frac{2A b^2 x^{\frac{9}{2}}}{9}+\frac{4Bab x^{\frac{9}{2}}}{9}+\frac{4Aab x^{\frac{3}{2}}}{3}+\frac{2B a^2 x^{\frac{3}{2}}}{3}-\frac{2a^2 A}{3x^{\frac{3}{2}}}$	54
gosper	$-\frac{2(-3b^2 B x^9-5A b^2 x^6-10Bab x^6-30aAb x^3-15B a^2 x^3+15a^2 A)}{45x^{\frac{3}{2}}}$	56
trager	$-\frac{2(-3b^2 B x^9-5A b^2 x^6-10Bab x^6-30aAb x^3-15B a^2 x^3+15a^2 A)}{45x^{\frac{3}{2}}}$	56
risch	$-\frac{2(-3b^2 B x^9-5A b^2 x^6-10Bab x^6-30aAb x^3-15B a^2 x^3+15a^2 A)}{45x^{\frac{3}{2}}}$	56
orering	$-\frac{2(-3b^2 B x^9-5A b^2 x^6-10Bab x^6-30aAb x^3-15B a^2 x^3+15a^2 A)}{45x^{\frac{3}{2}}}$	56

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*b^2*B*x^(15/2)+2/9*A*b^2*x^(9/2)+4/9*B*a*b*x^(9/2)+4/3*A*a*b*x^(3/2)+
2/3*B*a^2*x^(3/2)-2/3*a^2*A/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{5/2}}\,dx = \frac{2\,(3\,Bb^2x^9+5\,(2\,Bab+Ab^2)x^6+15\,(Ba^2+2\,Aab)x^3-15\,Aa^2)}{45\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
```

output
```
2/45*(3*B*b^2*x^9 + 5*(2*B*a*b + A*b^2)*x^6 + 15*(B*a^2 + 2*A*a*b)*x^3 - 1
5*A*a^2)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{5/2}}\,dx = -\frac{2Aa^2}{3x^{\frac{3}{2}}}+\frac{4Aabx^{\frac{3}{2}}}{3}+\frac{2Ab^2x^{\frac{9}{2}}}{9}+\frac{2Ba^2x^{\frac{3}{2}}}{3}+\frac{4Babx^{\frac{9}{2}}}{9}+\frac{2Bb^2x^{\frac{15}{2}}}{15}$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**(5/2),x)
```

output
```
-2*A*a**2/(3*x**(3/2)) + 4*A*a*b*x**(3/2)/3 + 2*A*b**2*x**(9/2)/9 + 2*B*a*
*2*x**(3/2)/3 + 4*B*a*b*x**(9/2)/9 + 2*B*b**2*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{5/2}}\,dx = \frac{2}{15}\,Bb^2x^{\frac{15}{2}}+\frac{2}{9}\,(2\,Bab+Ab^2)x^{\frac{9}{2}}+\frac{2}{3}\,(Ba^2+2\,Aab)x^{\frac{3}{2}}-\frac{2\,Aa^2}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

output
```
2/15*B*b^2*x^(15/2) + 2/9*(2*B*a*b + A*b^2)*x^(9/2) + 2/3*(B*a^2 + 2*A*a*b
)*x^(3/2) - 2/3*A*a^2/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{5/2}}\,dx = \frac{2}{15}\,Bb^2x^{\frac{15}{2}} + \frac{4}{9}\,Babx^{\frac{9}{2}} + \frac{2}{9}\,Ab^2x^{\frac{9}{2}} + \frac{2}{3}\,Ba^2x^{\frac{3}{2}} + \frac{4}{3}\,Aabx^{\frac{3}{2}} - \frac{2\,Aa^2}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

output
```
2/15*B*b^2*x^(15/2) + 4/9*B*a*b*x^(9/2) + 2/9*A*b^2*x^(9/2) + 2/3*B*a^2*x^
(3/2) + 4/3*A*a*b*x^(3/2) - 2/3*A*a^2/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{5/2}}\,dx = x^{3/2}\left(\frac{2\,B\,a^2}{3} + \frac{4\,A\,b\,a}{3}\right) + x^{9/2}\left(\frac{2\,A\,b^2}{9} + \frac{4\,B\,a\,b}{9}\right) - \frac{2\,A\,a^2}{3\,x^{3/2}} + \frac{2\,B\,b^2\,x^{15/2}}{15}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^(5/2),x)
```

output
```
x^(3/2)*((2*B*a^2)/3 + (4*A*a*b)/3) + x^(9/2)*((2*A*b^2)/9 + (4*B*a*b)/9)
- (2*A*a^2)/(3*x^(3/2)) + (2*B*b^2*x^(15/2))/15
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{5/2}} \, dx = \frac{\frac{2}{15} b^3 x^9 + \frac{2}{3} a \, b^2 x^6 + 2a^2 b \, x^3 - \frac{2}{3} a^3}{\sqrt{x} \, x}$$

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x)
```

output
```
(2*( - 5*a**3 + 15*a**2*b*x**3 + 5*a*b**2*x**6 + b**3*x**9))/(15*sqrt(x)*x
)
```

3.123 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}} dx = -\frac{2a^2A}{5x^{5/2}} + 2a(2Ab+aB)\sqrt{x} + \frac{2}{7}b(Ab+2aB)x^{7/2} + \frac{2}{13}b^2Bx^{13/2}$$

output
```
-2/5*a^2*A/x^(5/2)+2*a*(2*A*b+B*a)*x^(1/2)+2/7*b*(A*b+2*B*a)*x^(7/2)+2/13*
b^2*B*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}} dx = -\frac{2(91a^2A - 910aAbx^3 - 455a^2Bx^3 - 65Ab^2x^6 - 130abBx^6 - 35b^2Bx^9)}{455x^{5/2}}$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(7/2),x]
```

output
```
(-2*(91*a^2*A - 910*a*A*b*x^3 - 455*a^2*B*x^3 - 65*A*b^2*x^6 - 130*a*b*B*x
^6 - 35*b^2*B*x^9))/(455*x^(5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{7/2}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2 A}{x^{7/2}}+bx^{5/2}(2aB+Ab)+\frac{a(aB+2Ab)}{\sqrt{x}}+b^2Bx^{11/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2A}{5x^{5/2}}+\frac{2}{7}bx^{7/2}(2aB+Ab)+2a\sqrt{x}(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x^(7/2),x]
```

output
```
(-2*a^2*A)/(5*x^(5/2)) + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*b*(A*b + 2*a*B)*x^
(7/2))/7 + (2*b^2*B*x^(13/2))/13
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2b^2 B x^{\frac{13}{2}}}{13}+\frac{2A b^2 x^{\frac{7}{2}}}{7}+\frac{4Bab x^{\frac{7}{2}}}{7}+4abA\sqrt{x}+2a^2 B\sqrt{x}-\frac{2a^2 A}{5x^{\frac{5}{2}}}$	54
default	$\frac{2b^2 B x^{\frac{13}{2}}}{13}+\frac{2A b^2 x^{\frac{7}{2}}}{7}+\frac{4Bab x^{\frac{7}{2}}}{7}+4abA\sqrt{x}+2a^2 B\sqrt{x}-\frac{2a^2 A}{5x^{\frac{5}{2}}}$	54
gosper	$-\frac{2\left(-35b^2 B x^9-65A b^2 x^6-130Bab x^6-910aAb x^3-455B a^2 x^3+91a^2 A\right)}{455x^{\frac{5}{2}}}$	56
trager	$-\frac{2\left(-35b^2 B x^9-65A b^2 x^6-130Bab x^6-910aAb x^3-455B a^2 x^3+91a^2 A\right)}{455x^{\frac{5}{2}}}$	56
risch	$-\frac{2\left(-35b^2 B x^9-65A b^2 x^6-130Bab x^6-910aAb x^3-455B a^2 x^3+91a^2 A\right)}{455x^{\frac{5}{2}}}$	56
orering	$-\frac{2\left(-35b^2 B x^9-65A b^2 x^6-130Bab x^6-910aAb x^3-455B a^2 x^3+91a^2 A\right)}{455x^{\frac{5}{2}}}$	56

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/13*b^2*B*x^(13/2)+2/7*A*b^2*x^(7/2)+4/7*B*a*b*x^(7/2)+4*a*b*A*x^(1/2)+2*
a^2*B*x^(1/2)-2/5*a^2*A/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{7/2}}\,dx = \frac{2\,(35\,Bb^2x^9+65\,(2\,Bab+Ab^2)x^6+455\,(Ba^2+2\,Aab)x^3-91\,Aa^2)}{455\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
```

output
```
2/455*(35*B*b^2*x^9 + 65*(2*B*a*b + A*b^2)*x^6 + 455*(B*a^2 + 2*A*a*b)*x^3
 - 91*A*a^2)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{7/2}}\,dx = -\frac{2Aa^2}{5x^{\frac{5}{2}}}+4Aab\sqrt{x} + \frac{2Ab^2x^{\frac{7}{2}}}{7}+2Ba^2\sqrt{x}+\frac{4Babx^{\frac{7}{2}}}{7}+\frac{2Bb^2x^{\frac{13}{2}}}{13}$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**(7/2),x)
```

output
```
-2*A*a**2/(5*x**(5/2)) + 4*A*a*b*sqrt(x) + 2*A*b**2*x**(7/2)/7 + 2*B*a**2*
sqrt(x) + 4*B*a*b*x**(7/2)/7 + 2*B*b**2*x**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{7/2}}\,dx = \frac{2}{13}\,Bb^2x^{\frac{13}{2}} + \frac{2}{7}\left(2\,Bab+Ab^2\right)x^{\frac{7}{2}}+2\left(Ba^2+2\,Aab\right)\sqrt{x}-\frac{2\,Aa^2}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
```

output
```
2/13*B*b^2*x^(13/2) + 2/7*(2*B*a*b + A*b^2)*x^(7/2) + 2*(B*a^2 + 2*A*a*b)*
sqrt(x) - 2/5*A*a^2/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}}\,dx = \frac{2}{13}\,Bb^2x^{\frac{13}{2}} + \frac{4}{7}\,Babx^{\frac{7}{2}} + \frac{2}{7}\,Ab^2x^{\frac{7}{2}} + 2\,Ba^2\sqrt{x} + 4\,Aab\sqrt{x} - \frac{2\,Aa^2}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="giac")
```

output
```
2/13*B*b^2*x^(13/2) + 4/7*B*a*b*x^(7/2) + 2/7*A*b^2*x^(7/2) + 2*B*a^2*sqrt
(x) + 4*A*a*b*sqrt(x) - 2/5*A*a^2/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}}\,dx = \sqrt{x}\left(2\,B\,a^2+4\,A\,b\,a\right) + x^{7/2}\left(\frac{2\,A\,b^2}{7}+\frac{4\,B\,a\,b}{7}\right) - \frac{2\,A\,a^2}{5\,x^{5/2}} + \frac{2\,B\,b^2\,x^{13/2}}{13}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^(7/2),x)
```

output
```
x^(1/2)*(2*B*a^2 + 4*A*a*b) + x^(7/2)*((2*A*b^2)/7 + (4*B*a*b)/7) - (2*A*a
^2)/(5*x^(5/2)) + (2*B*b^2*x^(13/2))/13
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{7/2}}\, dx = \frac{\frac{2}{13}b^3x^9 + \frac{6}{7}a\, b^2x^6 + 6a^2b\, x^3 - \frac{2}{5}a^3}{\sqrt{x}\, x^2}$$

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x)
```

output
```
(2*( - 91*a**3 + 1365*a**2*b*x**3 + 195*a*b**2*x**6 + 35*b**3*x**9))/(455*
sqrt(x)*x**2)
```

3.124 $\int x^{5/2}(a+bx^3)^3 (A+Bx^3) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int x^{5/2}(a+bx^3)^3 (A+Bx^3) \, dx = \frac{2}{7}a^3Ax^{7/2} + \frac{2}{13}a^2(3Ab+aB)x^{13/2} + \frac{6}{19}ab(Ab+aB)x^{19/2} + \frac{2}{25}b^2(Ab+3aB)x^{25/2} + \frac{2}{31}b^3Bx^{31/2}$$

output
```
2/7*a^3*A*x^(7/2)+2/13*a^2*(3*A*b+B*a)*x^(13/2)+6/19*a*b*(A*b+B*a)*x^(19/2
)+2/25*b^2*(A*b+3*B*a)*x^(25/2)+2/31*b^3*B*x^(31/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07

$$\int x^{5/2}(a+bx^3)^3 (A+Bx^3) \, dx = \frac{2}{91}a^3x^{7/2}(13A+7Bx^3) + \frac{6}{247}a^2bx^{13/2}(19A+13Bx^3) + \frac{6}{475}ab^2x^{19/2}(25A+19Bx^3) + \frac{2}{775}b^3x^{25/2}(31A+25Bx^3)$$

input
```
Integrate[x^(5/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

output
```
(2*a^3*x^(7/2)*(13*A + 7*B*x^3))/91 + (6*a^2*b*x^(13/2)*(19*A + 13*B*x^3))
/247 + (6*a*b^2*x^(19/2)*(25*A + 19*B*x^3))/475 + (2*b^3*x^(25/2)*(31*A +
25*B*x^3))/775
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^3Ax^{5/2}+a^2x^{11/2}(aB+3Ab)+b^2x^{23/2}(3aB+Ab)+3abx^{17/2}(aB+Ab)+b^3Bx^{29/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}a^3Ax^{7/2}+\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{25}b^2x^{25/2}(3aB+Ab)+\frac{6}{19}abx^{19/2}(aB+Ab)+\frac{2}{31}b^3Bx^{31/2}$$

input
```
Int[x^(5/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

output
```
(2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*
B)*x^(19/2))/19 + (2*b^2*(A*b + 3*a*B)*x^(25/2))/25 + (2*b^3*B*x^(31/2))/3
1
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2b^3 B x^{\frac{31}{2}}}{31}+\frac{2(b^3 A+3a b^2 B) x^{\frac{25}{2}}}{25}+\frac{2(3a b^2 A+3a^2 bB) x^{\frac{19}{2}}}{19}+\frac{2(3a^2 bA+a^3 B) x^{\frac{13}{2}}}{13}+\frac{2a^3 A x^{\frac{7}{2}}}{7}$
default	$\frac{2b^3 B x^{\frac{31}{2}}}{31}+\frac{2(b^3 A+3a b^2 B) x^{\frac{25}{2}}}{25}+\frac{2(3a b^2 A+3a^2 bB) x^{\frac{19}{2}}}{19}+\frac{2(3a^2 bA+a^3 B) x^{\frac{13}{2}}}{13}+\frac{2a^3 A x^{\frac{7}{2}}}{7}$
gosper	$\frac{2x^{\frac{7}{2}}(43225B b^3 x^{12}+53599A x^9 b^3+160797B x^9 a b^2+211575A x^6 a b^2+211575B x^6 a^2 b+309225a^2 Ab x^3+103075B x^3 a}{1339975}$
trager	$\frac{2x^{\frac{7}{2}}(43225B b^3 x^{12}+53599A x^9 b^3+160797B x^9 a b^2+211575A x^6 a b^2+211575B x^6 a^2 b+309225a^2 Ab x^3+103075B x^3 a}{1339975}$
risch	$\frac{2x^{\frac{7}{2}}(43225B b^3 x^{12}+53599A x^9 b^3+160797B x^9 a b^2+211575A x^6 a b^2+211575B x^6 a^2 b+309225a^2 Ab x^3+103075B x^3 a}{1339975}$
orering	$\frac{2x^{\frac{7}{2}}(43225B b^3 x^{12}+53599A x^9 b^3+160797B x^9 a b^2+211575A x^6 a b^2+211575B x^6 a^2 b+309225a^2 Ab x^3+103075B x^3 a}{1339975}$

input
```
int(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/31*b^3*B*x^(31/2)+2/25*(A*b^3+3*B*a*b^2)*x^(25/2)+2/19*(3*A*a*b^2+3*B*a^
2*b)*x^(19/2)+2/13*(3*A*a^2*b+B*a^3)*x^(13/2)+2/7*a^3*A*x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{5/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2}{1339975}\left(43225\,Bb^3x^{15}+53599\left(3\,Bab^2+Ab^3\right)x^{12}+211575\left(Ba^2b+Aab^2\right)x^9+191425\,Aa^3\right.$$

input
```
integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/1339975*(43225*B*b^3*x^15 + 53599*(3*B*a*b^2 + A*b^3)*x^12 + 211575*(B*a
^2*b + A*a*b^2)*x^9 + 191425*A*a^3*x^3 + 103075*(B*a^3 + 3*A*a^2*b)*x^6)*s
qrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{5/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2Aa^3x^{\frac{7}{2}}}{7}+\frac{6Aa^2bx^{\frac{13}{2}}}{13}+\frac{6Aab^2x^{\frac{19}{2}}}{19}+\frac{2Ab^3x^{\frac{25}{2}}}{25}+\frac{2Ba^3x^{\frac{13}{2}}}{13}+\frac{6Ba^2bx^{\frac{19}{2}}}{19}+\frac{6Bab^2x^{\frac{25}{2}}}{25}+\frac{2Bb^3x^{\frac{31}{2}}}{31}$$

input
```
integrate(x**(5/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

output
```
2*A*a**3*x**(7/2)/7 + 6*A*a**2*b*x**(13/2)/13 + 6*A*a*b**2*x**(19/2)/19 +
2*A*b**3*x**(25/2)/25 + 2*B*a**3*x**(13/2)/13 + 6*B*a**2*b*x**(19/2)/19 +
6*B*a*b**2*x**(25/2)/25 + 2*B*b**3*x**(31/2)/31
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{5/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2}{31}\,Bb^3x^{\frac{31}{2}}+\frac{2}{25}\left(3\,Bab^2+Ab^3\right)x^{\frac{25}{2}}+\frac{6}{19}\left(Ba^2b+Aab^2\right)x^{\frac{19}{2}}+\frac{2}{7}\,Aa^3x^{\frac{7}{2}}+\frac{2}{13}\left(Ba^3+3\,Aa^2b\right)x^{\frac{13}{2}}$$

input
```
integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/31*B*b^3*x^(31/2) + 2/25*(3*B*a*b^2 + A*b^3)*x^(25/2) + 6/19*(B*a^2*b +
A*a*b^2)*x^(19/2) + 2/7*A*a^3*x^(7/2) + 2/13*(B*a^3 + 3*A*a^2*b)*x^(13/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{5/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2}{31}\,Bb^3x^{\frac{31}{2}}+\frac{6}{25}\,Bab^2x^{\frac{25}{2}}+\frac{2}{25}\,Ab^3x^{\frac{25}{2}}+\frac{6}{19}\,Ba^2bx^{\frac{19}{2}}+\frac{6}{19}\,Aab^2x^{\frac{19}{2}}+\frac{2}{13}\,Ba^3x^{\frac{13}{2}}+\frac{6}{13}\,Aa^2bx^{\frac{13}{2}}+\frac{2}{7}\,Aa^3x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

output
```
2/31*B*b^3*x^(31/2) + 6/25*B*a*b^2*x^(25/2) + 2/25*A*b^3*x^(25/2) + 6/19*B
*a^2*b*x^(19/2) + 6/19*A*a*b^2*x^(19/2) + 2/13*B*a^3*x^(13/2) + 6/13*A*a^2
*b*x^(13/2) + 2/7*A*a^3*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{5/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=x^{13/2}\left(\frac{2\,B\,a^3}{13}+\frac{6\,A\,b\,a^2}{13}\right)+x^{25/2}\left(\frac{2\,A\,b^3}{25}+\frac{6\,B\,a\,b^2}{25}\right)+\frac{2\,A\,a^3\,x^{7/2}}{7}+\frac{2\,B\,b^3\,x^{31/2}}{31}+\frac{6\,a\,b\,x^{19/2}\,(A\,b+B\,a)}{19}$$

input
```
int(x^(5/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

output
```
x^(13/2)*((2*B*a^3)/13 + (6*A*a^2*b)/13) + x^(25/2)*((2*A*b^3)/25 + (6*B*a
*b^2)/25) + (2*A*a^3*x^(7/2))/7 + (2*B*b^3*x^(31/2))/31 + (6*a*b*x^(19/2)*
(A*b + B*a))/19
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.59

$$\int x^{5/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2\sqrt{x}\,x^3(43225b^4x^{12}+214396a\,b^3x^9+423150a^2b^2x^6+412300a^3b\,x^3+191425a^4)}{1339975}$$

input
```
int(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x)
```

output
```
(2*sqrt(x)*x**3*(191425*a**4 + 412300*a**3*b*x**3 + 423150*a**2*b**2*x**6
+ 214396*a*b**3*x**9 + 43225*b**4*x**12))/1339975
```

3.125 $\int x^{3/2}(a+bx^3)^3\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2}{5}a^3Ax^{5/2}+\frac{2}{11}a^2(3Ab+aB)x^{11/2}+\frac{6}{17}ab(Ab+aB)x^{17/2}+\frac{2}{23}b^2(Ab+3aB)x^{23/2}+\frac{2}{29}b^3Bx^{29/2}$$

output
```
2/5*a^3*A*x^(5/2)+2/11*a^2*(3*A*b+B*a)*x^(11/2)+6/17*a*b*(A*b+B*a)*x^(17/2
)+2/23*b^2*(A*b+3*B*a)*x^(23/2)+2/29*b^3*B*x^(29/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2x^{5/2}(11339a^3(11A+5Bx^3)+10005a^2bx^3(17A+11Bx^3)+4785ab^2x^6(23A+17Bx^3)+935}{623645}$$

input
```
Integrate[x^(3/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

output
```
(2*x^(5/2)*(11339*a^3*(11*A + 5*B*x^3) + 10005*a^2*b*x^3*(17*A + 11*B*x^3)
 + 4785*a*b^2*x^6*(23*A + 17*B*x^3) + 935*b^3*x^9*(29*A + 23*B*x^3)))/6236
45
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{ 950}$$

$$\int \left(a^3Ax^{3/2}+a^2x^{9/2}(aB+3Ab)+b^2x^{21/2}(3aB+Ab)+3abx^{15/2}(aB+Ab)+b^3Bx^{27/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{5}a^3Ax^{5/2}+\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{23}b^2x^{23/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{29}b^3Bx^{29/2}$$

input
```
Int[x^(3/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

output
```
(2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (6*a*b*(A*b + a*
B)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(23/2))/23 + (2*b^3*B*x^(29/2))/2
9
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2b^3Bx^{\frac{29}{2}}}{29}+\frac{2(b^3A+3a b^2B)x^{\frac{23}{2}}}{23}+\frac{2(3a b^2A+3a^2bB)x^{\frac{17}{2}}}{17}+\frac{2(3a^2bA+a^3B)x^{\frac{11}{2}}}{11}+\frac{2a^3Ax^{\frac{5}{2}}}{5}$
default	$\frac{2b^3Bx^{\frac{29}{2}}}{29}+\frac{2(b^3A+3a b^2B)x^{\frac{23}{2}}}{23}+\frac{2(3a b^2A+3a^2bB)x^{\frac{17}{2}}}{17}+\frac{2(3a^2bA+a^3B)x^{\frac{11}{2}}}{11}+\frac{2a^3Ax^{\frac{5}{2}}}{5}$
gosper	$\frac{2x^{\frac{5}{2}}(21505Bb^3x^{12}+27115Ax^9b^3+81345Bx^9ab^2+110055Ax^6ab^2+110055Bx^6a^2b+170085a^2Abx^3+56695Bx^3a^3}{623645}$
trager	$\frac{2x^{\frac{5}{2}}(21505Bb^3x^{12}+27115Ax^9b^3+81345Bx^9ab^2+110055Ax^6ab^2+110055Bx^6a^2b+170085a^2Abx^3+56695Bx^3a^3}{623645}$
risch	$\frac{2x^{\frac{5}{2}}(21505Bb^3x^{12}+27115Ax^9b^3+81345Bx^9ab^2+110055Ax^6ab^2+110055Bx^6a^2b+170085a^2Abx^3+56695Bx^3a^3}{623645}$
orering	$\frac{2x^{\frac{5}{2}}(21505Bb^3x^{12}+27115Ax^9b^3+81345Bx^9ab^2+110055Ax^6ab^2+110055Bx^6a^2b+170085a^2Abx^3+56695Bx^3a^3}{623645}$

input
```
int(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/29*b^3*B*x^(29/2)+2/23*(A*b^3+3*B*a*b^2)*x^(23/2)+2/17*(3*A*a*b^2+3*B*a^
2*b)*x^(17/2)+2/11*(3*A*a^2*b+B*a^3)*x^(11/2)+2/5*a^3*A*x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{3/2}(a+bx^3)^3(A+Bx^3)\,dx=\frac{2}{623645}\left(21505\,Bb^3x^{14}+27115\left(3\,Bab^2+Ab^3\right)x^{11}+110055\left(Ba^2b+Aab^2\right)x^8+124729\,Aa^3x$$

input
```
integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/623645*(21505*B*b^3*x^14 + 27115*(3*B*a*b^2 + A*b^3)*x^11 + 110055*(B*a^
2*b + A*a*b^2)*x^8 + 124729*A*a^3*x^2 + 56695*(B*a^3 + 3*A*a^2*b)*x^5)*sqr
t(x)
```

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{3/2}(a+bx^3)^3(A+Bx^3)\,dx=\frac{2Aa^3x^{\frac{5}{2}}}{5}+\frac{6Aa^2bx^{\frac{11}{2}}}{11}+\frac{6Aab^2x^{\frac{17}{2}}}{17}+\frac{2Ab^3x^{\frac{23}{2}}}{23}+\frac{2Ba^3x^{\frac{11}{2}}}{11}+\frac{6Ba^2bx^{\frac{17}{2}}}{17}+\frac{6Bab^2x^{\frac{23}{2}}}{23}+\frac{2Bb^3x^{\frac{29}{2}}}{29}$$

input
```
integrate(x**(3/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

output
```
2*A*a**3*x**(5/2)/5 + 6*A*a**2*b*x**(11/2)/11 + 6*A*a*b**2*x**(17/2)/17 +
2*A*b**3*x**(23/2)/23 + 2*B*a**3*x**(11/2)/11 + 6*B*a**2*b*x**(17/2)/17 +
6*B*a*b**2*x**(23/2)/23 + 2*B*b**3*x**(29/2)/29
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2}{29}\,Bb^3x^{\frac{29}{2}}+\frac{2}{23}\,\left(3\,Bab^2+Ab^3\right)x^{\frac{23}{2}}+\frac{6}{17}\,\left(Ba^2b+Aab^2\right)x^{\frac{17}{2}}+\frac{2}{5}\,Aa^3x^{\frac{5}{2}}+\frac{2}{11}\,\left(Ba^3+3\,Aa^2b\right)x^{\frac{11}{2}}$$

input
```
integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/29*B*b^3*x^(29/2) + 2/23*(3*B*a*b^2 + A*b^3)*x^(23/2) + 6/17*(B*a^2*b +
A*a*b^2)*x^(17/2) + 2/5*A*a^3*x^(5/2) + 2/11*(B*a^3 + 3*A*a^2*b)*x^(11/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2}{29}\,Bb^3x^{\frac{29}{2}}+\frac{6}{23}\,Bab^2x^{\frac{23}{2}}+\frac{2}{23}\,Ab^3x^{\frac{23}{2}}+\frac{6}{17}\,Ba^2bx^{\frac{17}{2}}+\frac{6}{17}\,Aab^2x^{\frac{17}{2}}+\frac{2}{11}\,Ba^3x^{\frac{11}{2}}+\frac{6}{11}\,Aa^2bx^{\frac{11}{2}}+\frac{2}{5}\,Aa^3x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

output
```
2/29*B*b^3*x^(29/2) + 6/23*B*a*b^2*x^(23/2) + 2/23*A*b^3*x^(23/2) + 6/17*B
*a^2*b*x^(17/2) + 6/17*A*a*b^2*x^(17/2) + 2/11*B*a^3*x^(11/2) + 6/11*A*a^2
*b*x^(11/2) + 2/5*A*a^3*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=x^{11/2}\left(\frac{2\,B\,a^3}{11}+\frac{6\,A\,b\,a^2}{11}\right)+x^{23/2}\left(\frac{2\,A\,b^3}{23}+\frac{6\,B\,a\,b^2}{23}\right)+\frac{2\,A\,a^3\,x^{5/2}}{5}+\frac{2\,B\,b^3\,x^{29/2}}{29}+\frac{6\,a\,b\,x^{17/2}\,(A\,b+B\,a)}{17}$$

input
```
int(x^(3/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

output
```
x^(11/2)*((2*B*a^3)/11 + (6*A*a^2*b)/11) + x^(23/2)*((2*A*b^3)/23 + (6*B*a
*b^2)/23) + (2*A*a^3*x^(5/2))/5 + (2*B*b^3*x^(29/2))/29 + (6*a*b*x^(17/2)*
(A*b + B*a))/17
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.59

$$\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2\sqrt{x}\,x^2(21505b^4x^{12}+108460a\,b^3x^9+220110a^2b^2x^6+226780a^3b\,x^3+124729a^4)}{623645}$$

input
```
int(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x)
```

output
```
(2*sqrt(x)*x**2*(124729*a**4 + 226780*a**3*b*x**3 + 220110*a**2*b**2*x**6
+ 108460*a*b**3*x**9 + 21505*b**4*x**12))/623645
```

3.126 $\int \sqrt{x}(a+bx^3)^3 (A+Bx^3)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \sqrt{x}(a+bx^3)^3 (A+Bx^3)\, dx = \frac{2}{3}a^3Ax^{3/2} + \frac{2}{9}a^2(3Ab+aB)x^{9/2} + \frac{2}{5}ab(Ab+aB)x^{15/2} + \frac{2}{21}b^2(Ab+3aB)x^{21/2} + \frac{2}{27}b^3Bx^{27/2}$$

output
```
2/3*a^3*A*x^(3/2)+2/9*a^2*(3*A*b+B*a)*x^(9/2)+2/5*a*b*(A*b+B*a)*x^(15/2)+2
/21*b^2*(A*b+3*B*a)*x^(21/2)+2/27*b^3*B*x^(27/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \sqrt{x}(a+bx^3)^3 (A+Bx^3)\, dx = \frac{2}{945}x^{3/2}\left(105a^3(3A+Bx^3) + 63a^2bx^3(5A+3Bx^3) + 27ab^2x^6(7A+5Bx^3) + 5b^3x^9(9A+7Bx^3)\right)$$

input
```
Integrate[Sqrt[x]*(a + b*x^3)^3*(A + B*x^3),x]
```

output
```
(2*x^(3/2)*(105*a^3*(3*A + B*x^3) + 63*a^2*b*x^3*(5*A + 3*B*x^3) + 27*a*b^
2*x^6*(7*A + 5*B*x^3) + 5*b^3*x^9*(9*A + 7*B*x^3)))/945
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^3A\sqrt{x}+a^2x^{7/2}(aB+3Ab)+b^2x^{19/2}(3aB+Ab)+3abx^{13/2}(aB+Ab)+b^3Bx^{25/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^3Ax^{3/2}+\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{27}b^3Bx^{27/2}$$

input
```
Int[Sqrt[x]*(a + b*x^3)^3*(A + B*x^3),x]
```

output
```
(2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (2*a*b*(A*b + a*B)
*x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2*b^3*B*x^(27/2))/27
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2b^{3}Bx^{\frac{27}{2}}}{27}+\frac{2(b^{3}A+3a\,b^{2}B)x^{\frac{21}{2}}}{21}+\frac{2(3a\,b^{2}A+3a^{2}bB)x^{\frac{15}{2}}}{15}+\frac{2(3a^{2}bA+a^{3}B)x^{\frac{9}{2}}}{9}+\frac{2a^{3}Ax^{\frac{3}{2}}}{3}$	76
default	$\frac{2b^{3}Bx^{\frac{27}{2}}}{27}+\frac{2(b^{3}A+3a\,b^{2}B)x^{\frac{21}{2}}}{21}+\frac{2(3a\,b^{2}A+3a^{2}bB)x^{\frac{15}{2}}}{15}+\frac{2(3a^{2}bA+a^{3}B)x^{\frac{9}{2}}}{9}+\frac{2a^{3}Ax^{\frac{3}{2}}}{3}$	76
gosper	$\frac{2x^{\frac{3}{2}}(35B\,b^{3}x^{12}+45A\,x^{9}b^{3}+135B\,x^{9}a\,b^{2}+189A\,x^{6}a\,b^{2}+189B\,x^{6}a^{2}b+315a^{2}Ab\,x^{3}+105B\,x^{3}a^{3}+315a^{3}A)}{945}$	80
trager	$\frac{2x^{\frac{3}{2}}(35B\,b^{3}x^{12}+45A\,x^{9}b^{3}+135B\,x^{9}a\,b^{2}+189A\,x^{6}a\,b^{2}+189B\,x^{6}a^{2}b+315a^{2}Ab\,x^{3}+105B\,x^{3}a^{3}+315a^{3}A)}{945}$	80
risch	$\frac{2x^{\frac{3}{2}}(35B\,b^{3}x^{12}+45A\,x^{9}b^{3}+135B\,x^{9}a\,b^{2}+189A\,x^{6}a\,b^{2}+189B\,x^{6}a^{2}b+315a^{2}Ab\,x^{3}+105B\,x^{3}a^{3}+315a^{3}A)}{945}$	80
orering	$\frac{2x^{\frac{3}{2}}(35B\,b^{3}x^{12}+45A\,x^{9}b^{3}+135B\,x^{9}a\,b^{2}+189A\,x^{6}a\,b^{2}+189B\,x^{6}a^{2}b+315a^{2}Ab\,x^{3}+105B\,x^{3}a^{3}+315a^{3}A)}{945}$	80

input
```
int(x^(1/2)*(b*x^3+a)^3*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/27*b^3*B*x^(27/2)+2/21*(A*b^3+3*B*a*b^2)*x^(21/2)+2/15*(3*A*a*b^2+3*B*a^
2*b)*x^(15/2)+2/9*(3*A*a^2*b+B*a^3)*x^(9/2)+2/3*a^3*A*x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \sqrt{x}(a+bx^3)^3\left(A+Bx^3\right)\,dx$$
$$=\frac{2}{945}\left(35\,Bb^3x^{13}+45\left(3\,Bab^2+Ab^3\right)x^{10}+189\left(Ba^2b+Aab^2\right)x^7+315\,Aa^3x+105\left(Ba^3+3\,Aa^2b\right)x^4\right)\sqrt{}$$

input
```
integrate(x^(1/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/945*(35*B*b^3*x^13 + 45*(3*B*a*b^2 + A*b^3)*x^10 + 189*(B*a^2*b + A*a*b^
2)*x^7 + 315*A*a^3*x + 105*(B*a^3 + 3*A*a^2*b)*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \sqrt{x}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2Aa^3x^{\frac{3}{2}}}{3}+\frac{2Aa^2bx^{\frac{9}{2}}}{3}+\frac{2Aab^2x^{\frac{15}{2}}}{5}+\frac{2Ab^3x^{\frac{21}{2}}}{21}+\frac{2Ba^3x^{\frac{9}{2}}}{9}+\frac{2Ba^2bx^{\frac{15}{2}}}{5}+\frac{2Bab^2x^{\frac{21}{2}}}{7}+\frac{2Bb^3x^{\frac{27}{2}}}{27}$$

input
```
integrate(x**(1/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

output
```
2*A*a**3*x**(3/2)/3 + 2*A*a**2*b*x**(9/2)/3 + 2*A*a*b**2*x**(15/2)/5 + 2*A
*b**3*x**(21/2)/21 + 2*B*a**3*x**(9/2)/9 + 2*B*a**2*b*x**(15/2)/5 + 2*B*a*
b**2*x**(21/2)/7 + 2*B*b**3*x**(27/2)/27
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \sqrt{x}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2}{27}\,Bb^3x^{\frac{27}{2}}+\frac{2}{21}\,\left(3\,Bab^2+Ab^3\right)x^{\frac{21}{2}}+\frac{2}{5}\,\left(Ba^2b+Aab^2\right)x^{\frac{15}{2}}+\frac{2}{3}\,Aa^3x^{\frac{3}{2}}+\frac{2}{9}\,\left(Ba^3+3\,Aa^2b\right)x^{\frac{9}{2}}$$

input
```
integrate(x^(1/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/27*B*b^3*x^(27/2) + 2/21*(3*B*a*b^2 + A*b^3)*x^(21/2) + 2/5*(B*a^2*b + A
*a*b^2)*x^(15/2) + 2/3*A*a^3*x^(3/2) + 2/9*(B*a^3 + 3*A*a^2*b)*x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \sqrt{x}(a+bx^3)^3 (A+Bx^3)\ dx = \frac{2}{27}Bb^3x^{\frac{27}{2}} + \frac{2}{7}Bab^2x^{\frac{21}{2}} + \frac{2}{21}Ab^3x^{\frac{21}{2}} + \frac{2}{5}Ba^2bx^{\frac{15}{2}} + \frac{2}{5}Aab^2x^{\frac{15}{2}} + \frac{2}{9}Ba^3x^{\frac{9}{2}} + \frac{2}{3}Aa^2bx^{\frac{9}{2}} + \frac{2}{3}Aa^3x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

output
```
2/27*B*b^3*x^(27/2) + 2/7*B*a*b^2*x^(21/2) + 2/21*A*b^3*x^(21/2) + 2/5*B*a
^2*b*x^(15/2) + 2/5*A*a*b^2*x^(15/2) + 2/9*B*a^3*x^(9/2) + 2/3*A*a^2*b*x^(
9/2) + 2/3*A*a^3*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(a+bx^3)^3 (A+Bx^3)\ dx = x^{9/2}\left(\frac{2\,B\,a^3}{9}+\frac{2\,A\,b\,a^2}{3}\right)+x^{21/2}\left(\frac{2\,A\,b^3}{21}+\frac{2\,B\,a\,b^2}{7}\right) + \frac{2\,A\,a^3\,x^{3/2}}{3}+\frac{2\,B\,b^3\,x^{27/2}}{27}+\frac{2\,a\,b\,x^{15/2}\,(A\,b+B\,a)}{5}$$

input
```
int(x^(1/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

output
```
x^(9/2)*((2*B*a^3)/9 + (2*A*a^2*b)/3) + x^(21/2)*((2*A*b^3)/21 + (2*B*a*b^
2)/7) + (2*A*a^3*x^(3/2))/3 + (2*B*b^3*x^(27/2))/27 + (2*a*b*x^(15/2)*(A*b
 + B*a))/5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

$$\int \sqrt{x}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx$$
$$=\frac{2\sqrt{x}\,x(35b^4x^{12}+180a\,b^3x^9+378a^2b^2x^6+420a^3b\,x^3+315a^4)}{945}$$

input
```
int(x^(1/2)*(b*x^3+a)^3*(B*x^3+A),x)
```

output
```
(2*sqrt(x)*x*(315*a**4 + 420*a**3*b*x**3 + 378*a**2*b**2*x**6 + 180*a*b**3
*x**9 + 35*b**4*x**12))/945
```

3.127 $\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx = 2a^3A\sqrt{x}+\frac{2}{7}a^2(3Ab+aB)x^{7/2}+\frac{6}{13}ab(Ab+aB)x^{13/2} + \frac{2}{19}b^2(Ab+3aB)x^{19/2}+\frac{2}{25}b^3Bx^{25/2}$$

output
```
2*a^3*A*x^(1/2)+2/7*a^2*(3*A*b+B*a)*x^(7/2)+6/13*a*b*(A*b+B*a)*x^(13/2)+2/
19*b^2*(A*b+3*B*a)*x^(19/2)+2/25*b^3*B*x^(25/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2\sqrt{x}(6175a^3(7A+Bx^3)+1425a^2bx^3(13A+7Bx^3)+525ab^2x^6(19A+13Bx^3)+91b^3x^9(25A+19Bx^3)}{43225}$$

input
```
Integrate[((a + b*x^3)^3*(A + B*x^3))/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(6175*a^3*(7*A + B*x^3) + 1425*a^2*b*x^3*(13*A + 7*B*x^3) + 525
*a*b^2*x^6*(19*A + 13*B*x^3) + 91*b^3*x^9*(25*A + 19*B*x^3)))/43225
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx$$
$$\Big\downarrow \text{950}$$
$$\int \left(\frac{a^3A}{\sqrt{x}}+a^2x^{5/2}(aB+3Ab)+b^2x^{17/2}(3aB+Ab)+3abx^{11/2}(aB+Ab)+b^3Bx^{23/2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$2a^3A\sqrt{x}+\frac{2}{7}a^2x^{7/2}(aB+3Ab)+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

input
```
Int[((a + b*x^3)^3*(A + B*x^3))/Sqrt[x],x]
```

output
```
2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A*b + a*B)*x^(
13/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(25/2))/25
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{2b^3 B x^{\frac{25}{2}}}{25}+\frac{2(b^3 A+3a b^2 B)x^{\frac{19}{2}}}{19}+\frac{2(3a b^2 A+3a^2 bB)x^{\frac{13}{2}}}{13}+\frac{2(3a^2 bA+a^3 B)x^{\frac{7}{2}}}{7}+2a^3 A\sqrt{x}$
default	$\frac{2b^3 B x^{\frac{25}{2}}}{25}+\frac{2(b^3 A+3a b^2 B)x^{\frac{19}{2}}}{19}+\frac{2(3a b^2 A+3a^2 bB)x^{\frac{13}{2}}}{13}+\frac{2(3a^2 bA+a^3 B)x^{\frac{7}{2}}}{7}+2a^3 A\sqrt{x}$
trager	$\left(\frac{2}{25}B\,b^3x^{12}+\frac{2}{19}A\,x^9b^3+\frac{6}{19}B\,x^9a\,b^2+\frac{6}{13}A\,x^6a\,b^2+\frac{6}{13}B\,x^6a^2b+\frac{6}{7}a^2Ab\,x^3+\frac{2}{7}B\,x^3a^3$
gosper	$\frac{2\sqrt{x}\,(1729B\,b^3x^{12}+2275A\,x^9b^3+6825B\,x^9a\,b^2+9975A\,x^6a\,b^2+9975B\,x^6a^2b+18525a^2Ab\,x^3+6175B\,x^3a^3+43225a^3}{43225}$
risch	$\frac{2\sqrt{x}\,(1729B\,b^3x^{12}+2275A\,x^9b^3+6825B\,x^9a\,b^2+9975A\,x^6a\,b^2+9975B\,x^6a^2b+18525a^2Ab\,x^3+6175B\,x^3a^3+43225a^3}{43225}$
orering	$\frac{2\sqrt{x}\,(1729B\,b^3x^{12}+2275A\,x^9b^3+6825B\,x^9a\,b^2+9975A\,x^6a\,b^2+9975B\,x^6a^2b+18525a^2Ab\,x^3+6175B\,x^3a^3+43225a^3}{43225}$

input
```
int((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/25*b^3*B*x^(25/2)+2/19*(A*b^3+3*B*a*b^2)*x^(19/2)+2/13*(3*A*a*b^2+3*B*a^
2*b)*x^(13/2)+2/7*(3*A*a^2*b+B*a^3)*x^(7/2)+2*a^3*A*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx$$
$$=\frac{2}{43225}\left(1729\,Bb^3x^{12}+2275\left(3\,Bab^2+Ab^3\right)x^9+9975\left(Ba^2b+Aab^2\right)x^6+43225\,Aa^3+6175\left(Ba^3+3\,A\right.\right.$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
```

output
```
2/43225*(1729*B*b^3*x^12 + 2275*(3*B*a*b^2 + A*b^3)*x^9 + 9975*(B*a^2*b +
A*a*b^2)*x^6 + 43225*A*a^3 + 6175*(B*a^3 + 3*A*a^2*b)*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{\sqrt{x}}\,dx = 2Aa^3\sqrt{x} + \frac{6Aa^2bx^{\frac{7}{2}}}{7} + \frac{6Aab^2x^{\frac{13}{2}}}{13} + \frac{2Ab^3x^{\frac{19}{2}}}{19} + \frac{2Ba^3x^{\frac{7}{2}}}{7} + \frac{6Ba^2bx^{\frac{13}{2}}}{13} + \frac{6Bab^2x^{\frac{19}{2}}}{19} + \frac{2Bb^3x^{\frac{25}{2}}}{25}$$

input
```
integrate((b*x**3+a)**3*(B*x**3+A)/x**(1/2),x)
```

output
```
2*A*a**3*sqrt(x) + 6*A*a**2*b*x**(7/2)/7 + 6*A*a*b**2*x**(13/2)/13 + 2*A*b
**3*x**(19/2)/19 + 2*B*a**3*x**(7/2)/7 + 6*B*a**2*b*x**(13/2)/13 + 6*B*a*b
**2*x**(19/2)/19 + 2*B*b**3*x**(25/2)/25
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{25}\,Bb^3x^{\frac{25}{2}} + \frac{2}{19}\,\left(3\,Bab^2 + Ab^3\right)x^{\frac{19}{2}} + \frac{6}{13}\,\left(Ba^2b + Aab^2\right)x^{\frac{13}{2}} + 2\,Aa^3\sqrt{x} + \frac{2}{7}\,\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{7}{2}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
```

output
```
2/25*B*b^3*x^(25/2) + 2/19*(3*B*a*b^2 + A*b^3)*x^(19/2) + 6/13*(B*a^2*b +
A*a*b^2)*x^(13/2) + 2*A*a^3*sqrt(x) + 2/7*(B*a^3 + 3*A*a^2*b)*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{25}Bb^3x^{\frac{25}{2}} + \frac{6}{19}Bab^2x^{\frac{19}{2}} + \frac{2}{19}Ab^3x^{\frac{19}{2}} + \frac{6}{13}Ba^2bx^{\frac{13}{2}} + \frac{6}{13}Aab^2x^{\frac{13}{2}} + \frac{2}{7}Ba^3x^{\frac{7}{2}} + \frac{6}{7}Aa^2bx^{\frac{7}{2}} + 2\,Aa^3\sqrt{x}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

output
```
2/25*B*b^3*x^(25/2) + 6/19*B*a*b^2*x^(19/2) + 2/19*A*b^3*x^(19/2) + 6/13*B
*a^2*b*x^(13/2) + 6/13*A*a*b^2*x^(13/2) + 2/7*B*a^3*x^(7/2) + 6/7*A*a^2*b*
x^(7/2) + 2*A*a^3*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx = x^{7/2}\left(\frac{2\,B\,a^3}{7}+\frac{6\,A\,b\,a^2}{7}\right) + x^{19/2}\left(\frac{2\,A\,b^3}{19}+\frac{6\,B\,a\,b^2}{19}\right) + 2\,A\,a^3\,\sqrt{x} + \frac{2\,B\,b^3\,x^{25/2}}{25} + \frac{6\,a\,b\,x^{13/2}\,(A\,b+B\,a)}{13}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^3)/x^(1/2),x)
```

output
```
x^(7/2)*((2*B*a^3)/7 + (6*A*a^2*b)/7) + x^(19/2)*((2*A*b^3)/19 + (6*B*a*b^
2)/19) + 2*A*a^3*x^(1/2) + (2*B*b^3*x^(25/2))/25 + (6*a*b*x^(13/2)*(A*b +
B*a))/13
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{\sqrt{x}}\,dx$$
$$= \frac{2\sqrt{x}\,(1729b^4x^{12}+9100a\,b^3x^9+19950a^2b^2x^6+24700a^3b\,x^3+43225a^4)}{43225}$$

input
```
int((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x)
```

output
```
(2*sqrt(x)*(43225*a**4 + 24700*a**3*b*x**3 + 19950*a**2*b**2*x**6 + 9100*a
*b**3*x**9 + 1729*b**4*x**12))/43225
```

3.128 $\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{3/2}}\,dx = -\frac{2a^3A}{\sqrt{x}} + \frac{2}{5}a^2(3Ab+aB)x^{5/2} + \frac{6}{11}ab(Ab+aB)x^{11/2} + \frac{2}{17}b^2(Ab+3aB)x^{17/2} + \frac{2}{23}b^3Bx^{23/2}$$

output
```
-2*a^3*A/x^(1/2)+2/5*a^2*(3*A*b+B*a)*x^(5/2)+6/11*a*b*(A*b+B*a)*x^(11/2)+2
/17*b^2*(A*b+3*B*a)*x^(17/2)+2/23*b^3*B*x^(23/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{3/2}}\,dx = -\frac{2(21505a^3A - 12903a^2Abx^3 - 4301a^3Bx^3 - 5865aAb^2x^6 - 5865a^2bBx^6 - 1265Ab^3x^9 - 3795ab^2Bx^9 -}{21505\sqrt{x}}$$

input
```
Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(3/2),x]
```

output
```
(-2*(21505*a^3*A - 12903*a^2*A*b*x^3 - 4301*a^3*B*x^3 - 5865*a*A*b^2*x^6 -
 5865*a^2*b*B*x^6 - 1265*A*b^3*x^9 - 3795*a*b^2*B*x^9 - 935*b^3*B*x^12))/(
21505*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^3A}{x^{3/2}}+a^2x^{3/2}(aB+3Ab)+b^2x^{15/2}(3aB+Ab)+3abx^{9/2}(aB+Ab)+b^3Bx^{21/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3A}{\sqrt{x}}+\frac{2}{5}a^2x^{5/2}(aB+3Ab)+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

input
```
Int[((a + b*x^3)^3*(A + B*x^3))/x^(3/2),x]
```

output
```
(-2*a^3*A)/Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(5/2))/5 + (6*a*b*(A*b + a*B)*
x^(11/2))/11 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b^3*B*x^(23/2))/23
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{2b^3Bx^{\frac{23}{2}}}{23}+\frac{2Ab^3x^{\frac{17}{2}}}{17}+\frac{6Bab^2x^{\frac{17}{2}}}{17}+\frac{6Aab^2x^{\frac{11}{2}}}{11}+\frac{6Ba^2bx^{\frac{11}{2}}}{11}+\frac{6Aa^2bx^{\frac{5}{2}}}{5}+\frac{2Ba^3x^{\frac{5}{2}}}{5}-\frac{2a^3A}{\sqrt{x}}$
default	$\frac{2b^3Bx^{\frac{23}{2}}}{23}+\frac{2Ab^3x^{\frac{17}{2}}}{17}+\frac{6Bab^2x^{\frac{17}{2}}}{17}+\frac{6Aab^2x^{\frac{11}{2}}}{11}+\frac{6Ba^2bx^{\frac{11}{2}}}{11}+\frac{6Aa^2bx^{\frac{5}{2}}}{5}+\frac{2Ba^3x^{\frac{5}{2}}}{5}-\frac{2a^3A}{\sqrt{x}}$
gosper	$-\frac{2(-935Bb^3x^{12}-1265Ax^9b^3-3795Bx^9ab^2-5865Ax^6ab^2-5865Bx^6a^2b-12903a^2Abx^3-4301Bx^3a^3+21505a^3A}{21505\sqrt{x}}$
trager	$-\frac{2(-935Bb^3x^{12}-1265Ax^9b^3-3795Bx^9ab^2-5865Ax^6ab^2-5865Bx^6a^2b-12903a^2Abx^3-4301Bx^3a^3+21505a^3A}{21505\sqrt{x}}$
risch	$-\frac{2(-935Bb^3x^{12}-1265Ax^9b^3-3795Bx^9ab^2-5865Ax^6ab^2-5865Bx^6a^2b-12903a^2Abx^3-4301Bx^3a^3+21505a^3A}{21505\sqrt{x}}$
orering	$-\frac{2(-935Bb^3x^{12}-1265Ax^9b^3-3795Bx^9ab^2-5865Ax^6ab^2-5865Bx^6a^2b-12903a^2Abx^3-4301Bx^3a^3+21505a^3A}{21505\sqrt{x}}$

input
```
int((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/23*b^3*B*x^(23/2)+2/17*A*b^3*x^(17/2)+6/17*B*a*b^2*x^(17/2)+6/11*A*a*b^2
*x^(11/2)+6/11*B*a^2*b*x^(11/2)+6/5*A*a^2*b*x^(5/2)+2/5*B*a^3*x^(5/2)-2*a^
3*A/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{3/2}}\,dx = \frac{2\,(935\,Bb^3x^{12} + 1265\,(3\,Bab^2 + Ab^3)x^9 + 5865\,(Ba^2b + Aab^2)x^6 - 21505\,A}{21505\,\sqrt{x}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
```

output
```
2/21505*(935*B*b^3*x^12 + 1265*(3*B*a*b^2 + A*b^3)*x^9 + 5865*(B*a^2*b + A
*a*b^2)*x^6 - 21505*A*a^3 + 4301*(B*a^3 + 3*A*a^2*b)*x^3)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{3/2}}\,dx = -\frac{2Aa^3}{\sqrt{x}} + \frac{6Aa^2bx^{\frac{5}{2}}}{5} + \frac{6Aab^2x^{\frac{11}{2}}}{11} + \frac{2Ab^3x^{\frac{17}{2}}}{17} + \frac{2Ba^3x^{\frac{5}{2}}}{5} + \frac{6Ba^2bx^{\frac{11}{2}}}{11} + \frac{6Bab^2x^{\frac{17}{2}}}{17} + \frac{2Bb^3x^{\frac{23}{2}}}{23}$$

input
```
integrate((b*x**3+a)**3*(B*x**3+A)/x**(3/2),x)
```

output
```
-2*A*a**3/sqrt(x) + 6*A*a**2*b*x**(5/2)/5 + 6*A*a*b**2*x**(11/2)/11 + 2*A*
b**3*x**(17/2)/17 + 2*B*a**3*x**(5/2)/5 + 6*B*a**2*b*x**(11/2)/11 + 6*B*a*
b**2*x**(17/2)/17 + 2*B*b**3*x**(23/2)/23
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{3/2}}\,dx = \frac{2}{23}\,Bb^3x^{\frac{23}{2}} + \frac{2}{17}\,(3\,Bab^2 + Ab^3)x^{\frac{17}{2}} + \frac{6}{11}\,(Ba^2b + Aab^2)x^{\frac{11}{2}} - \frac{2\,Aa^3}{\sqrt{x}} + \frac{2}{5}\,(Ba^3 + 3\,Aa^2b)x^{\frac{5}{2}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
```

output
```
2/23*B*b^3*x^(23/2) + 2/17*(3*B*a*b^2 + A*b^3)*x^(17/2) + 6/11*(B*a^2*b +
A*a*b^2)*x^(11/2) - 2*A*a^3/sqrt(x) + 2/5*(B*a^3 + 3*A*a^2*b)*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^3\,(A+Bx^3)}{x^{3/2}}\,dx = \frac{2}{23}\,Bb^3x^{\frac{23}{2}} + \frac{6}{17}\,Bab^2x^{\frac{17}{2}} + \frac{2}{17}\,Ab^3x^{\frac{17}{2}} + \frac{6}{11}\,Ba^2bx^{\frac{11}{2}} + \frac{6}{11}\,Aab^2x^{\frac{11}{2}} + \frac{2}{5}\,Ba^3x^{\frac{5}{2}} + \frac{6}{5}\,Aa^2bx^{\frac{5}{2}} - \frac{2\,Aa^3}{\sqrt{x}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="giac")
```

output
```
2/23*B*b^3*x^(23/2) + 6/17*B*a*b^2*x^(17/2) + 2/17*A*b^3*x^(17/2) + 6/11*B
*a^2*b*x^(11/2) + 6/11*A*a*b^2*x^(11/2) + 2/5*B*a^3*x^(5/2) + 6/5*A*a^2*b*
x^(5/2) - 2*A*a^3/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^3)^3\,(A+Bx^3)}{x^{3/2}}\,dx = x^{5/2}\left(\frac{2\,B\,a^3}{5}+\frac{6\,A\,b\,a^2}{5}\right) + x^{17/2}\left(\frac{2\,A\,b^3}{17}+\frac{6\,B\,a\,b^2}{17}\right) - \frac{2\,A\,a^3}{\sqrt{x}} + \frac{2\,B\,b^3\,x^{23/2}}{23} + \frac{6\,a\,b\,x^{11/2}\,(A\,b+B\,a)}{11}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^3)/x^(3/2),x)
```

output
```
x^(5/2)*((2*B*a^3)/5 + (6*A*a^2*b)/5) + x^(17/2)*((2*A*b^3)/17 + (6*B*a*b^
2)/17) - (2*A*a^3)/x^(1/2) + (2*B*b^3*x^(23/2))/23 + (6*a*b*x^(11/2)*(A*b
+ B*a))/11
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{3/2}}\,dx = \frac{\frac{2}{23}b^4x^{12}+\frac{8}{17}a\,b^3x^9+\frac{12}{11}a^2b^2x^6+\frac{8}{5}a^3b\,x^3-2a^4}{\sqrt{x}}$$

input
```
int((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x)
```

output
```
(2*( - 21505*a**4 + 17204*a**3*b*x**3 + 11730*a**2*b**2*x**6 + 5060*a*b**3
*x**9 + 935*b**4*x**12))/(21505*sqrt(x))
```

3.129 $\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{5/2}}\,dx = -\frac{2a^3A}{3x^{3/2}} + \frac{2}{3}a^2(3Ab+aB)x^{3/2} + \frac{2}{3}ab(Ab+aB)x^{9/2} + \frac{2}{15}b^2(Ab+3aB)x^{15/2} + \frac{2}{21}b^3Bx^{21/2}$$

output
```
-2/3*a^3*A/x^(3/2)+2/3*a^2*(3*A*b+B*a)*x^(3/2)+2/3*a*b*(A*b+B*a)*x^(9/2)+2
/15*b^2*(A*b+3*B*a)*x^(15/2)+2/21*b^3*B*x^(21/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{5/2}}\,dx = \frac{2(-35a^3(A-Bx^3)+35a^2bx^3(3A+Bx^3)+7ab^2x^6(5A+3Bx^3)+b^3x^9(7A}{105x^{3/2}}$$

input
```
Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(5/2),x]
```

output
```
(2*(-35*a^3*(A - B*x^3) + 35*a^2*b*x^3*(3*A + B*x^3) + 7*a*b^2*x^6*(5*A +
3*B*x^3) + b^3*x^9*(7*A + 5*B*x^3)))/(105*x^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{5/2}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^3A}{x^{5/2}}+a^2\sqrt{x}(aB+3Ab)+b^2x^{13/2}(3aB+Ab)+3abx^{7/2}(aB+Ab)+b^3Bx^{19/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3A}{3x^{3/2}}+\frac{2}{3}a^2x^{3/2}(aB+3Ab)+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

input
```
Int[((a + b*x^3)^3*(A + B*x^3))/x^(5/2),x]
```

output
```
(-2*a^3*A)/(3*x^(3/2)) + (2*a^2*(3*A*b + a*B)*x^(3/2))/3 + (2*a*b*(A*b + a
*B)*x^(9/2))/3 + (2*b^2*(A*b + 3*a*B)*x^(15/2))/15 + (2*b^3*B*x^(21/2))/21
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

method	result	siz
derivativedivides	$\frac{2b^3Bx^{\frac{21}{2}}}{21}+\frac{2Ab^3x^{\frac{15}{2}}}{15}+\frac{2Bab^2x^{\frac{15}{2}}}{5}+\frac{2Aab^2x^{\frac{9}{2}}}{3}+\frac{2Ba^2bx^{\frac{9}{2}}}{3}+2Aa^2bx^{\frac{3}{2}}+\frac{2Ba^3x^{\frac{3}{2}}}{3}-\frac{2a^3A}{3x^{\frac{3}{2}}}$	78
default	$\frac{2b^3Bx^{\frac{21}{2}}}{21}+\frac{2Ab^3x^{\frac{15}{2}}}{15}+\frac{2Bab^2x^{\frac{15}{2}}}{5}+\frac{2Aab^2x^{\frac{9}{2}}}{3}+\frac{2Ba^2bx^{\frac{9}{2}}}{3}+2Aa^2bx^{\frac{3}{2}}+\frac{2Ba^3x^{\frac{3}{2}}}{3}-\frac{2a^3A}{3x^{\frac{3}{2}}}$	78
gosper	$-\frac{2(-5Bb^3x^{12}-7Ax^9b^3-21Bx^9ab^2-35Ax^6ab^2-35Bx^6a^2b-105a^2Abx^3-35Bx^3a^3+35a^3A)}{105x^{\frac{3}{2}}}$	80
trager	$-\frac{2(-5Bb^3x^{12}-7Ax^9b^3-21Bx^9ab^2-35Ax^6ab^2-35Bx^6a^2b-105a^2Abx^3-35Bx^3a^3+35a^3A)}{105x^{\frac{3}{2}}}$	80
risch	$-\frac{2(-5Bb^3x^{12}-7Ax^9b^3-21Bx^9ab^2-35Ax^6ab^2-35Bx^6a^2b-105a^2Abx^3-35Bx^3a^3+35a^3A)}{105x^{\frac{3}{2}}}$	80
orering	$-\frac{2(-5Bb^3x^{12}-7Ax^9b^3-21Bx^9ab^2-35Ax^6ab^2-35Bx^6a^2b-105a^2Abx^3-35Bx^3a^3+35a^3A)}{105x^{\frac{3}{2}}}$	80

input
```
int((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*b^3*B*x^(21/2)+2/15*A*b^3*x^(15/2)+2/5*B*a*b^2*x^(15/2)+2/3*A*a*b^2*x
^(9/2)+2/3*B*a^2*b*x^(9/2)+2*A*a^2*b*x^(3/2)+2/3*B*a^3*x^(3/2)-2/3*a^3*A/x
^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int\frac{(a+bx^3)^3(A+Bx^3)}{x^{5/2}}\,dx=\frac{2\left(5\,Bb^3x^{12}+7\,(3\,Bab^2+Ab^3)x^9+35\,(Ba^2b+Aab^2)x^6-35\,Aa^3+35\,(Ba^{}\right.}{105\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
```

output
```
2/105*(5*B*b^3*x^12 + 7*(3*B*a*b^2 + A*b^3)*x^9 + 35*(B*a^2*b + A*a*b^2)*x
^6 - 35*A*a^3 + 35*(B*a^3 + 3*A*a^2*b)*x^3)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{5/2}}\,dx = -\frac{2Aa^3}{3x^{\frac{3}{2}}} + 2Aa^2bx^{\frac{3}{2}} + \frac{2Aab^2x^{\frac{9}{2}}}{3} + \frac{2Ab^3x^{\frac{15}{2}}}{15} + \frac{2Ba^3x^{\frac{3}{2}}}{3} + \frac{2Ba^2bx^{\frac{9}{2}}}{3} + \frac{2Bab^2x^{\frac{15}{2}}}{5} + \frac{2Bb^3x^{\frac{21}{2}}}{21}$$

input
```
integrate((b*x**3+a)**3*(B*x**3+A)/x**(5/2),x)
```

output
```
-2*A*a**3/(3*x**(3/2)) + 2*A*a**2*b*x**(3/2) + 2*A*a*b**2*x**(9/2)/3 + 2*A
*b**3*x**(15/2)/15 + 2*B*a**3*x**(3/2)/3 + 2*B*a**2*b*x**(9/2)/3 + 2*B*a*b
**2*x**(15/2)/5 + 2*B*b**3*x**(21/2)/21
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{5/2}}\,dx = \frac{2}{21}\,Bb^3x^{\frac{21}{2}} + \frac{2}{15}\left(3\,Bab^2 + Ab^3\right)x^{\frac{15}{2}} + \frac{2}{3}\left(Ba^2b + Aab^2\right)x^{\frac{9}{2}} - \frac{2\,Aa^3}{3\,x^{\frac{3}{2}}} + \frac{2}{3}\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{3}{2}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

output
```
2/21*B*b^3*x^(21/2) + 2/15*(3*B*a*b^2 + A*b^3)*x^(15/2) + 2/3*(B*a^2*b + A
*a*b^2)*x^(9/2) - 2/3*A*a^3/x^(3/2) + 2/3*(B*a^3 + 3*A*a^2*b)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{5/2}}\,dx = \frac{2}{21}\,Bb^3x^{\frac{21}{2}} + \frac{2}{5}\,Bab^2x^{\frac{15}{2}} + \frac{2}{15}\,Ab^3x^{\frac{15}{2}} + \frac{2}{3}\,Ba^2bx^{\frac{9}{2}} + \frac{2}{3}\,Aab^2x^{\frac{9}{2}} + \frac{2}{3}\,Ba^3x^{\frac{3}{2}} + 2\,Aa^2bx^{\frac{3}{2}} - \frac{2\,Aa^3}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

output
```
2/21*B*b^3*x^(21/2) + 2/5*B*a*b^2*x^(15/2) + 2/15*A*b^3*x^(15/2) + 2/3*B*a
^2*b*x^(9/2) + 2/3*A*a*b^2*x^(9/2) + 2/3*B*a^3*x^(3/2) + 2*A*a^2*b*x^(3/2)
 - 2/3*A*a^3/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{5/2}}\,dx = x^{3/2}\left(\frac{2\,B\,a^3}{3} + 2\,A\,b\,a^2\right) + x^{15/2}\left(\frac{2\,A\,b^3}{15} + \frac{2\,B\,a\,b^2}{5}\right) - \frac{2\,A\,a^3}{3\,x^{3/2}} + \frac{2\,B\,b^3\,x^{21/2}}{21} + \frac{2\,a\,b\,x^{9/2}\,(A\,b+B\,a)}{3}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^3)/x^(5/2),x)
```

output
```
x^(3/2)*((2*B*a^3)/3 + 2*A*a^2*b) + x^(15/2)*((2*A*b^3)/15 + (2*B*a*b^2)/5
) - (2*A*a^3)/(3*x^(3/2)) + (2*B*b^3*x^(21/2))/21 + (2*a*b*x^(9/2)*(A*b +
B*a))/3
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{5/2}}\,dx = \frac{\frac{2}{21}b^4x^{12}+\frac{8}{15}a\,b^3x^9+\frac{4}{3}a^2b^2x^6+\frac{8}{3}a^3b\,x^3-\frac{2}{3}a^4}{\sqrt{x}\,x}$$

input
```
int((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x)
```

output
```
(2*( - 35*a**4 + 140*a**3*b*x**3 + 70*a**2*b**2*x**6 + 28*a*b**3*x**9 + 5*
b**4*x**12))/(105*sqrt(x)*x)
```

3.130 $\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}}\,dx = -\frac{2a^3A}{5x^{5/2}} + 2a^2(3Ab+aB)\sqrt{x} + \frac{6}{7}ab(Ab+aB)x^{7/2} + \frac{2}{13}b^2(Ab+3aB)x^{13/2} + \frac{2}{19}b^3Bx^{19/2}$$

output
```
-2/5*a^3*A/x^(5/2)+2*a^2*(3*A*b+B*a)*x^(1/2)+6/7*a*b*(A*b+B*a)*x^(7/2)+2/1
3*b^2*(A*b+3*B*a)*x^(13/2)+2/19*b^3*B*x^(19/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}}\,dx = \frac{-3458a^3(A-5Bx^3)+7410a^2bx^3(7A+Bx^3)+570ab^2x^6(13A+7Bx^3)+7}{8645x^{5/2}}$$

input
```
Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(7/2),x]
```

output
```
(-3458*a^3*(A - 5*B*x^3) + 7410*a^2*b*x^3*(7*A + B*x^3) + 570*a*b^2*x^6*(1
3*A + 7*B*x^3) + 70*b^3*x^9*(19*A + 13*B*x^3))/(8645*x^(5/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{7/2}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^3 A}{x^{7/2}}+\frac{a^2(aB+3Ab)}{\sqrt{x}}+b^2x^{11/2}(3aB+Ab)+3abx^{5/2}(aB+Ab)+b^3Bx^{17/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3A}{5x^{5/2}}+2a^2\sqrt{x}(aB+3Ab)+\frac{2}{13}b^2x^{13/2}(3aB+Ab)+\frac{6}{7}abx^{7/2}(aB+Ab)+\frac{2}{19}b^3Bx^{19/2}$$

input
```
Int[((a + b*x^3)^3*(A + B*x^3))/x^(7/2),x]
```

output
```
(-2*a^3*A)/(5*x^(5/2)) + 2*a^2*(3*A*b + a*B)*Sqrt[x] + (6*a*b*(A*b + a*B)*
x^(7/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(13/2))/13 + (2*b^3*B*x^(19/2))/19
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{2b^3 B\, x^{\frac{19}{2}}}{19}+\frac{2A\, b^3 x^{\frac{13}{2}}}{13}+\frac{6Ba\, b^2 x^{\frac{13}{2}}}{13}+\frac{6Aa\, b^2 x^{\frac{7}{2}}}{7}+\frac{6B\, a^2 b\, x^{\frac{7}{2}}}{7}+6A\, a^2 b\sqrt{x}+2B\, a^3\sqrt{x}-\frac{2a^3 A}{5x^{\frac{5}{2}}}$
default	$\frac{2b^3 B\, x^{\frac{19}{2}}}{19}+\frac{2A\, b^3 x^{\frac{13}{2}}}{13}+\frac{6Ba\, b^2 x^{\frac{13}{2}}}{13}+\frac{6Aa\, b^2 x^{\frac{7}{2}}}{7}+\frac{6B\, a^2 b\, x^{\frac{7}{2}}}{7}+6A\, a^2 b\sqrt{x}+2B\, a^3\sqrt{x}-\frac{2a^3 A}{5x^{\frac{5}{2}}}$
gosper	$-\frac{2(-455B\, b^3 x^{12}-665A\, x^9 b^3-1995B\, x^9 a\, b^2-3705A\, x^6 a\, b^2-3705B\, x^6 a^2 b-25935a^2 Ab\, x^3-8645B\, x^3 a^3+1729a^3 A)}{8645x^{\frac{5}{2}}}$
trager	$-\frac{2(-455B\, b^3 x^{12}-665A\, x^9 b^3-1995B\, x^9 a\, b^2-3705A\, x^6 a\, b^2-3705B\, x^6 a^2 b-25935a^2 Ab\, x^3-8645B\, x^3 a^3+1729a^3 A)}{8645x^{\frac{5}{2}}}$
risch	$-\frac{2(-455B\, b^3 x^{12}-665A\, x^9 b^3-1995B\, x^9 a\, b^2-3705A\, x^6 a\, b^2-3705B\, x^6 a^2 b-25935a^2 Ab\, x^3-8645B\, x^3 a^3+1729a^3 A)}{8645x^{\frac{5}{2}}}$
orering	$-\frac{2(-455B\, b^3 x^{12}-665A\, x^9 b^3-1995B\, x^9 a\, b^2-3705A\, x^6 a\, b^2-3705B\, x^6 a^2 b-25935a^2 Ab\, x^3-8645B\, x^3 a^3+1729a^3 A)}{8645x^{\frac{5}{2}}}$

input
```
int((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/19*b^3*B*x^(19/2)+2/13*A*b^3*x^(13/2)+6/13*B*a*b^2*x^(13/2)+6/7*A*a*b^2*
x^(7/2)+6/7*B*a^2*b*x^(7/2)+6*A*a^2*b*x^(1/2)+2*B*a^3*x^(1/2)-2/5*a^3*A/x^
(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^3\,(A+Bx^3)}{x^{7/2}}\,dx=\frac{2\,(455\,Bb^3x^{12}+665\,(3\,Bab^2+Ab^3)x^9+3705\,(Ba^2b+Aab^2)x^6-1729\,Aa^3}{8645\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
```

output
```
2/8645*(455*B*b^3*x^12 + 665*(3*B*a*b^2 + A*b^3)*x^9 + 3705*(B*a^2*b + A*a
*b^2)*x^6 - 1729*A*a^3 + 8645*(B*a^3 + 3*A*a^2*b)*x^3)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{7/2}} dx = -\frac{2Aa^3}{5x^{\frac{5}{2}}} + 6Aa^2b\sqrt{x} + \frac{6Aab^2x^{\frac{7}{2}}}{7} + \frac{2Ab^3x^{\frac{13}{2}}}{13} + 2Ba^3\sqrt{x} + \frac{6Ba^2bx^{\frac{7}{2}}}{7} + \frac{6Bab^2x^{\frac{13}{2}}}{13} + \frac{2Bb^3x^{\frac{19}{2}}}{19}$$

input
```
integrate((b*x**3+a)**3*(B*x**3+A)/x**(7/2),x)
```

output
```
-2*A*a**3/(5*x**(5/2)) + 6*A*a**2*b*sqrt(x) + 6*A*a*b**2*x**(7/2)/7 + 2*A*
b**3*x**(13/2)/13 + 2*B*a**3*sqrt(x) + 6*B*a**2*b*x**(7/2)/7 + 6*B*a*b**2*
x**(13/2)/13 + 2*B*b**3*x**(19/2)/19
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{7/2}} dx = \frac{2}{19} Bb^3x^{\frac{19}{2}} + \frac{2}{13} (3 Bab^2 + Ab^3)x^{\frac{13}{2}} + \frac{6}{7} (Ba^2b + Aab^2)x^{\frac{7}{2}} - \frac{2 Aa^3}{5 x^{\frac{5}{2}}} + 2 (Ba^3 + 3 Aa^2b)\sqrt{x}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
```

output
```
2/19*B*b^3*x^(19/2) + 2/13*(3*B*a*b^2 + A*b^3)*x^(13/2) + 6/7*(B*a^2*b + A
*a*b^2)*x^(7/2) - 2/5*A*a^3/x^(5/2) + 2*(B*a^3 + 3*A*a^2*b)*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{7/2}}\,dx = \frac{2}{19}Bb^3x^{\frac{19}{2}} + \frac{6}{13}Bab^2x^{\frac{13}{2}} + \frac{2}{13}Ab^3x^{\frac{13}{2}} + \frac{6}{7}Ba^2bx^{\frac{7}{2}} + \frac{6}{7}Aab^2x^{\frac{7}{2}} + 2Ba^3\sqrt{x} + 6Aa^2b\sqrt{x} - \frac{2Aa^3}{5x^{\frac{5}{2}}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="giac")
```

output
```
2/19*B*b^3*x^(19/2) + 6/13*B*a*b^2*x^(13/2) + 2/13*A*b^3*x^(13/2) + 6/7*B*
a^2*b*x^(7/2) + 6/7*A*a*b^2*x^(7/2) + 2*B*a^3*sqrt(x) + 6*A*a^2*b*sqrt(x)
- 2/5*A*a^3/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{7/2}}\,dx = \sqrt{x}\left(2Ba^3+6Aba^2\right) + x^{13/2}\left(\frac{2Ab^3}{13}+\frac{6Bab^2}{13}\right) - \frac{2Aa^3}{5x^{5/2}} + \frac{2Bb^3x^{19/2}}{19} + \frac{6abx^{7/2}(Ab+Ba)}{7}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^3)/x^(7/2),x)
```

output
```
x^(1/2)*(2*B*a^3 + 6*A*a^2*b) + x^(13/2)*((2*A*b^3)/13 + (6*B*a*b^2)/13) -
 (2*A*a^3)/(5*x^(5/2)) + (2*B*b^3*x^(19/2))/19 + (6*a*b*x^(7/2)*(A*b + B*a
))/7
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{7/2}} \, dx = \frac{\frac{2}{19}b^4x^{12} + \frac{8}{13}a\,b^3x^9 + \frac{12}{7}a^2b^2x^6 + 8a^3b\,x^3 - \frac{2}{5}a^4}{\sqrt{x}\,x^2}$$

input
```
int((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x)
```

output
```
(2*( - 1729*a**4 + 34580*a**3*b*x**3 + 7410*a**2*b**2*x**6 + 2660*a*b**3*x
**9 + 455*b**4*x**12))/(8645*sqrt(x)*x**2)
```

3.131 $\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3} dx = \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{9/2}}{9b} - \frac{2\sqrt{a}(Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}$$

output
```
2/3*(A*b-B*a)*x^(3/2)/b^2+2/9*B*x^(9/2)/b-2/3*a^(1/2)*(A*b-B*a)*arctan(b^(
1/2)*x^(3/2)/a^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3} dx = \frac{2x^{3/2}(3Ab-3aB+bBx^3)}{9b^2} + \frac{2\sqrt{a}(-Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}$$

input
```
Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(2*x^(3/2)*(3*A*b - 3*a*B + b*B*x^3))/(9*b^2) + (2*Sqrt[a]*(-(A*b) + a*B)*
ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(5/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {959, 843, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}dx$$

$$\Big\downarrow \text{ 959}$$

$$\frac{(Ab-aB)\int \frac{x^{7/2}}{bx^3+a}dx}{b}+\frac{2Bx^{9/2}}{9b}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{bx^3+a}dx}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\int \frac{x}{bx^3+a}d\sqrt{x}}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\int \frac{1}{a+bx}dx^{3/2}}{3b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{3/2}}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

input
```
Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(2*B*x^(9/2))/(9*b) + ((A*b - a*B)*((2*x^(3/2))/(3*b) - (2*Sqrt[a]*ArcTan[
(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(3/2))))/b
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{2x^{\frac{3}{2}}(bB\,x^{3}+3Ab-3Ba)}{9b^{2}}-\frac{2a(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b^{2}\sqrt{ab}}$	55
derivativedivides	$\frac{\frac{2bB\,x^{\frac{9}{2}}}{9}+\frac{2Ab\,x^{\frac{3}{2}}}{3}-\frac{2Ba\,x^{\frac{3}{2}}}{3}}{b^{2}}-\frac{2a(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b^{2}\sqrt{ab}}$	58
default	$\frac{\frac{2bB\,x^{\frac{9}{2}}}{9}+\frac{2Ab\,x^{\frac{3}{2}}}{3}-\frac{2Ba\,x^{\frac{3}{2}}}{3}}{b^{2}}-\frac{2a(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b^{2}\sqrt{ab}}$	58

input
```
int(x^(7/2)*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/9*x^(3/2)*(B*b*x^3+3*A*b-3*B*a)/b^2-2/3*a*(A*b-B*a)/b^2/(a*b)^(1/2)*arct
an(b*x^(3/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.96

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx=\left[-\frac{3\,(Ba-Ab)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^3-2\,bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}-a}{bx^3+a}\right)-2\,(Bbx^4-3\,(Ba-Ab)x)\sqrt{x}}{9\,b^2},\ \frac{2\left(3\right.}{}\right.$$

input
```
integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/9*(3*(B*a - A*b)*sqrt(-a/b)*log((b*x^3 - 2*b*x^(3/2)*sqrt(-a/b) - a)/(
b*x^3 + a)) - 2*(B*b*x^4 - 3*(B*a - A*b)*x)*sqrt(x))/b^2, 2/9*(3*(B*a - A*
b)*sqrt(a/b)*arctan(b*x^(3/2)*sqrt(a/b)/a) + (B*b*x^4 - 3*(B*a - A*b)*x)*s
qrt(x))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(70) = 140$.

Time = 66.77 (sec) , antiderivative size = 428, normalized size of antiderivative = 5.86

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx = \begin{cases} \tilde{\infty}\left(\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{9}{2}}}{9}\right) \\ \frac{\frac{2Ax^{\frac{9}{2}}}{9}+\frac{2Bx^{\frac{15}{2}}}{15}}{a} \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{9}{2}}}{9}}{b} \\ -\frac{Aa\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}}+\frac{Aa\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}}+\frac{Aa\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}}-\frac{Aa\log}{} \end{cases}$$

input
```
integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a),x)
```

output
```
Piecewise((zoo*(2*A*x**(3/2)/3 + 2*B*x**(9/2)/9), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(9/2)/9 + 2*B*x**(15/2)/15)/a, Eq(b, 0)), ((2*A*x**(3/2)/3 + 2*B*x*
*(9/2)/9)/b, Eq(a, 0)), (-A*a*log(sqrt(x) - (-a/b)**(1/6))/(3*b**2*sqrt(-a
/b)) + A*a*log(sqrt(x) + (-a/b)**(1/6))/(3*b**2*sqrt(-a/b)) + A*a*log(-4*s
qrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b**2*sqrt(-a/b)) - A*a*lo
g(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b**2*sqrt(-a/b)) + 2
*A*x**(3/2)/(3*b) + B*a**2*log(sqrt(x) - (-a/b)**(1/6))/(3*b**3*sqrt(-a/b)
) - B*a**2*log(sqrt(x) + (-a/b)**(1/6))/(3*b**3*sqrt(-a/b)) - B*a**2*log(-
4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b**3*sqrt(-a/b)) + B*a
**2*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b**3*sqrt(-a/b
)) - 2*B*a*x**(3/2)/(3*b**2) + 2*B*x**(9/2)/(9*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,(Ba^2-Aab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b^2}+\frac{2\left(Bbx^{\frac{9}{2}}-3\,(Ba-Ab)x^{\frac{3}{2}}\right)}{9\,b^2}$$

input
```
integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
2/3*(B*a^2 - A*a*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/9*(B*b
*x^(9/2) - 3*(B*a - A*b)*x^(3/2))/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,(Ba^2-Aab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b^2} + \frac{2\left(Bb^2x^{\frac{9}{2}}-3\,Babx^{\frac{3}{2}}+3\,Ab^2x^{\frac{3}{2}}\right)}{9\,b^3}$$

input
```
integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*(B*a^2 - A*a*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/9*(B*b
^2*x^(9/2) - 3*B*a*b*x^(3/2) + 3*A*b^2*x^(3/2))/b^3
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.52

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx = x^{3/2}\left(\frac{2\,A}{3\,b}-\frac{2\,B\,a}{3\,b^2}\right)+\frac{2\,B\,x^{9/2}}{9\,b} - \frac{2\,\sqrt{a}\,\mathrm{atan}\left(\frac{72\,b^{3/2}\,x^{3/2}\,\left(A^2\,a^2\,b^2-2\,A\,B\,a^3\,b+B^2\,a^4\right)}{\sqrt{a}\,\left(72\,A\,a^2\,b^2-72\,B\,a^3\,b\right)\,\left(A\,b-B\,a\right)}\right)\,\left(A\,b-B\,a\right)}{3\,b^{5/2}}$$

input
```
int((x^(7/2)*(A + B*x^3))/(a + b*x^3),x)
```

output
```
x^(3/2)*((2*A)/(3*b) - (2*B*a)/(3*b^2)) + (2*B*x^(9/2))/(9*b) - (2*a^(1/2)
*atan((72*b^(3/2)*x^(3/2)*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/(a^(1/2)*
(72*A*a^2*b^2 - 72*B*a^3*b)*(A*b - B*a)))*(A*b - B*a))/(3*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.10

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\sqrt{x}\,x^4}{9}$$

input
```
int(x^(7/2)*(B*x^3+A)/(b*x^3+a),x)
```

output
```
(2*sqrt(x)*x**4)/9
```

3.132 $\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx = \frac{2Bx^{3/2}}{3b} + \frac{2(Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}}$$

output
```
2/3*B*x^(3/2)/b+2/3*(A*b-B*a)*arctan(b^(1/2)*x^(3/2)/a^(1/2))/a^(1/2)/b^(3
/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx = \frac{2Bx^{3/2}}{3b} - \frac{2(-Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(2*B*x^(3/2))/(3*b) - (2*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])
/(3*Sqrt[a]*b^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {959, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(Ab-aB)\int \frac{\sqrt{x}}{bx^3+a}dx}{b}+\frac{2Bx^{3/2}}{3b}$$

$$\Big\downarrow \text{851}$$

$$\frac{2(Ab-aB)\int \frac{x}{bx^3+a}d\sqrt{x}}{b}+\frac{2Bx^{3/2}}{3b}$$

$$\Big\downarrow \text{807}$$

$$\frac{2(Ab-aB)\int \frac{1}{a+bx}dx^{3/2}}{3b}+\frac{2Bx^{3/2}}{3b}$$

$$\Big\downarrow \text{218}$$

$$\frac{2(Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}}+\frac{2Bx^{3/2}}{3b}$$

input
```
Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(2*B*x^(3/2))/(3*b) + (2*(A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3
*Sqrt[a]*b^(3/2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2B\,x^{\frac{3}{2}}}{3b}+\frac{2(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b\sqrt{ab}}$	40
default	$\frac{2B\,x^{\frac{3}{2}}}{3b}+\frac{2(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b\sqrt{ab}}$	40
risch	$\frac{2B\,x^{\frac{3}{2}}}{3b}+\frac{2(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b\sqrt{ab}}$	40

input
```
int(x^(1/2)*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*B*x^(3/2)/b+2/3*(A*b-B*a)/b/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx = \left[\frac{2\,Babx^{\frac{3}{2}} + (Ba-Ab)\sqrt{-ab}\log\left(\frac{bx^3-2\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)}{3\,ab^2}, \frac{2\left(Babx^{\frac{3}{2}} - (Ba-Ab)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x^{\frac{3}{2}}}{a}\right)\right)}{3\,ab^2}\right]$$

input
```
integrate(x^(1/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/3*(2*B*a*b*x^(3/2) + (B*a - A*b)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x
^(3/2) - a)/(b*x^3 + a)))/(a*b^2), 2/3*(B*a*b*x^(3/2) - (B*a - A*b)*sqrt(a
*b)*arctan(sqrt(a*b)*x^(3/2)/a))/(a*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(51) = 102.

Time = 4.92 (sec) , antiderivative size = 381, normalized size of antiderivative = 7.19

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}} + \frac{2Bx^{\frac{3}{2}}}{3}\right) \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{9}{2}}}{9}}{a} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+\frac{2Bx^{\frac{3}{2}}}{3}}{b} \\ \frac{A\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}} - \frac{A\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}} - \frac{A\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}} + \frac{A\log\left(4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}} - \end{cases}$$

input
```
integrate(x**(1/2)*(B*x**3+A)/(b*x**3+a),x)
```

output
```
Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*x**(3/2)/3), Eq(a, 0) & Eq(b, 0)),
 ((2*A*x**(3/2)/3 + 2*B*x**(9/2)/9)/a, Eq(b, 0)), ((-2*A/(3*x**(3/2)) + 2*
B*x**(3/2)/3)/b, Eq(a, 0)), (A*log(sqrt(x) - (-a/b)**(1/6))/(3*b*sqrt(-a/b
)) - A*log(sqrt(x) + (-a/b)**(1/6))/(3*b*sqrt(-a/b)) - A*log(-4*sqrt(x)*(-
a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b*sqrt(-a/b)) + A*log(4*sqrt(x)*(-
a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b*sqrt(-a/b)) - B*a*log(sqrt(x) -
(-a/b)**(1/6))/(3*b**2*sqrt(-a/b)) + B*a*log(sqrt(x) + (-a/b)**(1/6))/(3*b
**2*sqrt(-a/b)) + B*a*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3)
)/(3*b**2*sqrt(-a/b)) - B*a*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**
(1/3))/(3*b**2*sqrt(-a/b)) + 2*B*x**(3/2)/(3*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,b} - \frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b}$$

input
```
integrate(x^(1/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
2/3*B*x^(3/2)/b - 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b
)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,b} - \frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b}$$

input
```
integrate(x^(1/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*B*x^(3/2)/b - 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b
)
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,B\,x^{3/2}}{3\,b} - \frac{2\,\mathrm{atan}\left(\frac{3\,\sqrt{a}\,b^{3/2}\,x^{3/2}\,\left(24\,A^2\,b^3-48\,A\,B\,a\,b^2+24\,B^2\,a^2\,b\right)}{\left(72\,B\,a^2\,b^2-72\,A\,a\,b^3\right)\,\left(A\,b-B\,a\right)}\right)\,\left(A\,b-B\,a\right)}{3\,\sqrt{a}\,b^{3/2}}$$

input
```
int((x^(1/2)*(A + B*x^3))/(a + b*x^3),x)
```

output
```
(2*B*x^(3/2))/(3*b) - (2*atan((3*a^(1/2)*b^(3/2)*x^(3/2)*(24*A^2*b^3 + 24*
B^2*a^2*b - 48*A*B*a*b^2))/((72*B*a^2*b^2 - 72*A*a*b^3)*(A*b - B*a)))*(A*b
 - B*a))/(3*a^(1/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\sqrt{x}\,x}{3}$$

input
```
int(x^(1/2)*(B*x^3+A)/(b*x^3+a),x)
```

output
```
(2*sqrt(x)*x)/3
```

3.133 $\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}\,dx = -\frac{2A}{3ax^{3/2}} - \frac{2(Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}$$

output
```
-2/3*A/a/x^(3/2)-2/3*(A*b-B*a)*arctan(b^(1/2)*x^(3/2)/a^(1/2))/a^(3/2)/b^(
1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}\,dx = -\frac{2A}{3ax^{3/2}} + \frac{2(-Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}$$

input
```
Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)),x]
```

output
```
(-2*A)/(3*a*x^(3/2)) + (2*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]]
)/(3*a^(3/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)}dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(Ab-aB)\int \frac{\sqrt{x}}{bx^3+a}dx}{a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \; 851$$

$$-\frac{2(Ab-aB)\int \frac{x}{bx^3+a}d\sqrt{x}}{a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \; 807$$

$$-\frac{2(Ab-aB)\int \frac{1}{a+bx}dx^{3/2}}{3a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \; 218$$

$$-\frac{2(Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}-\frac{2A}{3ax^{3/2}}$$

input
```
Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)),x]
```

output
```
(-2*A)/(3*a*x^(3/2)) - (2*(A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(
3*a^(3/2)*Sqrt[b])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{2A}{3a\,x^{\frac{3}{2}}}-\frac{2(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3a\sqrt{ab}}$	40
default	$-\frac{2A}{3a\,x^{\frac{3}{2}}}-\frac{2(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3a\sqrt{ab}}$	40
risch	$-\frac{2A}{3a\,x^{\frac{3}{2}}}-\frac{2(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3a\sqrt{ab}}$	40

input
```
int((B*x^3+A)/x^(5/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*A/a/x^(3/2)-2/3/a*(A*b-B*a)/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.26

$$\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)}\,dx = \left[\frac{(Ba-Ab)\sqrt{-ab}x^2\log\left(\frac{bx^3+2\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)-2\,Aab\sqrt{x}}{3\,a^2bx^2}, \frac{2\left((Ba-Ab)\sqrt{ab}x^2\arctan\right.}{3\,a^2bx^2}\right.$$

input
```
integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/3*((B*a - A*b)*sqrt(-a*b)*x^2*log((b*x^3 + 2*sqrt(-a*b)*x^(3/2) - a)/(b
*x^3 + a)) - 2*A*a*b*sqrt(x))/(a^2*b*x^2), 2/3*((B*a - A*b)*sqrt(a*b)*x^2*
arctan(sqrt(a*b)*x^(3/2)/a) - A*a*b*sqrt(x))/(a^2*b*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(53) = 106$.

Time = 147.71 (sec) , antiderivative size = 371, normalized size of antiderivative = 7.00

$$\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) \\ \frac{-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+\frac{2Bx^{\frac{3}{2}}}{3}}{a} \\ -\frac{A\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3a\sqrt{-\frac{a}{b}}}+\frac{A\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3a\sqrt{-\frac{a}{b}}}+\frac{A\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3a\sqrt{-\frac{a}{b}}}-\frac{A\log\left(4\sqrt{x}\right.}{} \end{cases}$$

input
```
integrate((B*x**3+A)/x**(5/2)/(b*x**3+a),x)
```

output
```
Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0)
), ((-2*A/(9*x**(9/2)) - 2*B/(3*x**(3/2)))/b, Eq(a, 0)), ((-2*A/(3*x**(3/2
)) + 2*B*x**(3/2)/3)/a, Eq(b, 0)), (-A*log(sqrt(x) - (-a/b)**(1/6))/(3*a*s
qrt(-a/b)) + A*log(sqrt(x) + (-a/b)**(1/6))/(3*a*sqrt(-a/b)) + A*log(-4*sq
rt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*a*sqrt(-a/b)) - A*log(4*sq
rt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*a*sqrt(-a/b)) - 2*A/(3*a*x
**(3/2)) + B*log(sqrt(x) - (-a/b)**(1/6))/(3*b*sqrt(-a/b)) - B*log(sqrt(x)
 + (-a/b)**(1/6))/(3*b*sqrt(-a/b)) - B*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x
+ 4*(-a/b)**(1/3))/(3*b*sqrt(-a/b)) + B*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x
+ 4*(-a/b)**(1/3))/(3*b*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^3}{x^{5/2}\,(a+bx^3)}\,dx = \frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a} - \frac{2\,A}{3\,ax^{\frac{3}{2}}}$$

input
```
integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a) - 2/3*A/(a*x^(3/
2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^3}{x^{5/2}\,(a+bx^3)}\,dx = \frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a} - \frac{2\,A}{3\,ax^{\frac{3}{2}}}$$

input
```
integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a) - 2/3*A/(a*x^(3/
2))
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.92

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}\,dx=-\frac{2\,A}{3\,a\,x^{3/2}}-\frac{2\,\mathrm{atan}\left(\frac{3\,a^{3/2}\,\sqrt{b}\,x^{3/2}\,\left(24\,A^2\,a^3\,b^5-48\,A\,B\,a^4\,b^4+24\,B^2\,a^5\,b^3\right)}{\left(A\,b-B\,a\right)\,\left(72\,A\,a^5\,b^4-72\,B\,a^6\,b^3\right)}\right)\,\left(A\,b-B\,a\right)}{3\,a^{3/2}\,\sqrt{b}}$$

input
```
int((A + B*x^3)/(x^(5/2)*(a + b*x^3)),x)
```

output
```
- (2*A)/(3*a*x^(3/2)) - (2*atan((3*a^(3/2)*b^(1/2)*x^(3/2)*(24*A^2*a^3*b^5
 + 24*B^2*a^5*b^3 - 48*A*B*a^4*b^4))/((A*b - B*a)*(72*A*a^5*b^4 - 72*B*a^6
*b^3)))*(A*b - B*a))/(3*a^(3/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.17

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}\,dx=-\frac{2}{3\sqrt{x}\,x}$$

input
```
int((B*x^3+A)/x^(5/2)/(b*x^3+a),x)
```

output
```
( - 2)/(3*sqrt(x)*x)
```

3.134 $\int \frac{A+Bx^3}{x^{11/2}(a+bx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{A+Bx^3}{x^{11/2}\left(a+bx^3\right)}\,dx=-\frac{2A}{9ax^{9/2}}+\frac{2(Ab-aB)}{3a^2x^{3/2}}+\frac{2\sqrt{b}(Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}}$$

output
```
-2/9*A/a/x^(9/2)+2/3*(A*b-B*a)/a^2/x^(3/2)+2/3*b^(1/2)*(A*b-B*a)*arctan(b^
(1/2)*x^(3/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^3}{x^{11/2}\left(a+bx^3\right)}\,dx=-\frac{2\left(aA-3Abx^3+3aBx^3\right)}{9a^2x^{9/2}}-\frac{2\sqrt{b}(-Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}}$$

input
```
Integrate[(A + B*x^3)/(x^(11/2)*(a + b*x^3)),x]
```

output
```
(-2*(a*A - 3*A*b*x^3 + 3*a*B*x^3))/(9*a^2*x^(9/2)) - (2*Sqrt[b]*(-(A*b) +
a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(5/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {955, 847, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{11/2}\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(Ab-aB)\int \frac{1}{x^{5/2}(bx^3+a)}dx}{a}-\frac{2A}{9ax^{9/2}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(Ab-aB)\left(-\frac{b\int \frac{\sqrt{x}}{bx^3+a}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2A}{9ax^{9/2}}$$

$$\Big\downarrow \text{851}$$

$$-\frac{(Ab-aB)\left(-\frac{2b\int \frac{x}{bx^3+a}d\sqrt{x}}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2A}{9ax^{9/2}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{(Ab-aB)\left(-\frac{2b\int \frac{1}{a+bx}dx^{3/2}}{3a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2A}{9ax^{9/2}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{(Ab-aB)\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2A}{9ax^{9/2}}$$

input
```
Int[(A + B*x^3)/(x^(11/2)*(a + b*x^3)),x]
```

output
```
(-2*A)/(9*a*x^(9/2)) - ((A*b - a*B)*(-2/(3*a*x^(3/2)) - (2*Sqrt[b]*ArcTan[
(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(3/2))))/a
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{2(Ab-Ba)b\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3a^{2}\sqrt{ab}}-\frac{2A}{9a\,x^{\frac{9}{2}}}-\frac{2(-Ab+Ba)}{3a^{2}x^{\frac{3}{2}}}$	57
default	$\frac{2(Ab-Ba)b\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3a^{2}\sqrt{ab}}-\frac{2A}{9a\,x^{\frac{9}{2}}}-\frac{2(-Ab+Ba)}{3a^{2}x^{\frac{3}{2}}}$	57
risch	$-\frac{2(-3Ab\,x^{3}+3Ba\,x^{3}+Aa)}{9a^{2}x^{\frac{9}{2}}}+\frac{2(Ab-Ba)b\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3a^{2}\sqrt{ab}}$	58

input
```
int((B*x^3+A)/x^(11/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*(A*b-B*a)/a^2*b/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))-2/9*A/a/x^(9
/2)-2/3/a^2*(-A*b+B*a)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.04

$$\int \frac{A+Bx^3}{x^{11/2}\,(a+bx^3)}\,dx = \left[-\frac{3\,(Ba-Ab)x^5\sqrt{-\frac{b}{a}}\log\left(\frac{bx^3+2\,ax^{\frac{3}{2}}\sqrt{-\frac{b}{a}}-a}{bx^3+a}\right)+2\,(3\,(Ba-Ab)x^3+Aa)\sqrt{x}}{9\,a^2x^5},\; -\frac{2\left(3\,(Ba-Ab)x^5\sqrt{\frac{b}{a}}\arctan\left(x^{\frac{3}{2}}\sqrt{\frac{b}{a}}\right)+(3\,(Ba-Ab)x^3+Aa)\sqrt{x}\right)}{9\,a^2x^5}\right]$$

input
```
integrate((B*x^3+A)/x^(11/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/9*(3*(B*a - A*b)*x^5*sqrt(-b/a)*log((b*x^3 + 2*a*x^(3/2)*sqrt(-b/a) -
a)/(b*x^3 + a)) + 2*(3*(B*a - A*b)*x^3 + A*a)*sqrt(x))/(a^2*x^5), -2/9*(3*
(B*a - A*b)*x^5*sqrt(b/a)*arctan(x^(3/2)*sqrt(b/a)) + (3*(B*a - A*b)*x^3 +
 A*a)*sqrt(x))/(a^2*x^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^{11/2}\left(a+bx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((B*x**3+A)/x**(11/2)/(b*x**3+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^3}{x^{11/2}\left(a+bx^3\right)}\,dx=-\frac{2\,(Bab-Ab^2)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a^2}-\frac{2\,(3\,(Ba-Ab)x^3+Aa)}{9\,a^2x^{\frac{9}{2}}}$$

input
```
integrate((B*x^3+A)/x^(11/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-2/3*(B*a*b - A*b^2)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2) - 2/9*(3*
(B*a - A*b)*x^3 + A*a)/(a^2*x^(9/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^3}{x^{11/2}(a+bx^3)}\,dx = -\frac{2\,(Bab-Ab^2)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a^2} - \frac{2\,(3\,Bax^3-3\,Abx^3+Aa)}{9\,a^2x^{\frac{9}{2}}}$$

input
```
integrate((B*x^3+A)/x^(11/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
-2/3*(B*a*b - A*b^2)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2) - 2/9*(3*
B*a*x^3 - 3*A*b*x^3 + A*a)/(a^2*x^(9/2))
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^3}{x^{11/2}(a+bx^3)}\,dx = \frac{2\,A\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\,a^{5/2}} - \frac{2\,B}{3\,a\,x^{3/2}} - \frac{2\,A}{9\,a\,x^{9/2}} - \frac{2\,B\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\,a^{3/2}} + \frac{2\,A\,b}{3\,a^2\,x^{3/2}}$$

input
```
int((A + B*x^3)/(x^(11/2)*(a + b*x^3)),x)
```

output
```
(2*A*b^(3/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)))/(3*a^(5/2)) - (2*B)/(3*a*x^(
3/2)) - (2*A)/(9*a*x^(9/2)) - (2*B*b^(1/2)*atan((b^(1/2)*x^(3/2))/a^(1/2))
)/(3*a^(3/2)) + (2*A*b)/(3*a^2*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.12

$$\int \frac{A + Bx^3}{x^{11/2}\,(a + bx^3)}\,dx = -\frac{2}{9\sqrt{x}\,x^4}$$

input
```
int((B*x^3+A)/x^(11/2)/(b*x^3+a),x)
```

output
```
( - 2)/(9*sqrt(x)*x**4)
```

3.135 $\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 225

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2(Ab-aB)\sqrt{x}}{b^2}+\frac{2Bx^{7/2}}{7b} + \frac{\sqrt[6]{a}(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} - \frac{\sqrt[6]{a}(Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} - \frac{2\sqrt[6]{a}(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} - \frac{\sqrt[6]{a}(Ab-aB)\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{\sqrt{3}b^{13/6}}$$

output
```
2*(A*b-B*a)*x^(1/2)/b^2+2/7*B*x^(7/2)/b-1/3*a^(1/6)*(A*b-B*a)*arctan(-3^(1
/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/b^(13/6)-1/3*a^(1/6)*(A*b-B*a)*arctan(3^(1/
2)+2*b^(1/6)*x^(1/2)/a^(1/6))/b^(13/6)-2/3*a^(1/6)*(A*b-B*a)*arctan(b^(1/6
)*x^(1/2)/a^(1/6))/b^(13/6)-1/3*a^(1/6)*(A*b-B*a)*arctanh(3^(1/2)*a^(1/6)*
b^(1/6)*x^(1/2)/(a^(1/3)+b^(1/3)*x))*3^(1/2)/b^(13/6)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.76

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{6\sqrt[6]{b}\sqrt{x}(7Ab-7aB+bBx^3)+14\sqrt[6]{a}(-Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-7\sqrt[6]{a}(-Ab+}{21b^{13/6}}$$

input
```
Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(6*b^(1/6)*Sqrt[x]*(7*A*b - 7*a*B + b*B*x^3) + 14*a^(1/6)*(-(A*b) + a*B)*A
rcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - 7*a^(1/6)*(-(A*b) + a*B)*ArcTan[(a^(1/3
) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] + 7*Sqrt[3]*a^(1/6)*(-(A*b) + a*
B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(21*b
^(13/6))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {959, 843, 851, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}dx$$

$$\Big\downarrow \text{ 959}$$

$$\frac{(Ab-aB)\int \frac{x^{5/2}}{bx^3+a}dx}{b}+\frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}\left(bx^3+a\right)}dx}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\int \frac{1}{bx^3+a}d\sqrt{x}}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$\Big\downarrow$ 753

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}\right)}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$\Big\downarrow$ 27

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}\right)}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$\Big\downarrow$ 218

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}\right)}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$\downarrow$ 1142

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int-\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}}{}\right.}{b}\right.}{b}$$

$$\frac{2Bx^{7/2}}{7b}$$

$\downarrow$ 25

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\sqrt{3}\int\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+}}{}\right.}{b}\right.}{b}$$

$$\frac{2Bx^{7/2}}{7b}$$

$\downarrow$ 27

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\cdots}\cdots}{\cdots}\cdots}{b}\cdots}{b}\cdots$$

$$\frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow \; 1082$$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int \frac{1}{-x-\cdots}\cdots}{\cdots}}{6a^{5/6}}\cdots}{b}\cdots}{b}\cdots$$

$$\frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow \; 217$$

$$(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+ \cdots}{6a^{5/6}}\right)}{b}\right.$$

$$b$$

$$\frac{2Bx^{7/2}}{7b}$$

$\downarrow$ 1103

$$(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}+\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1 \cdots\right.\right.}{\sqrt[6]{b}}\cdots}{\cdots}\right)}{b}\right.$$

$$b$$

$$\frac{2Bx^{7/2}}{7b}$$

input
```
Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(2*B*x^(7/2))/(7*b) + ((A*b - a*B)*((2*Sqrt[x])/b - (2*a*(ArcTan[(b^(1/6)*
Sqrt[x])/a^(1/6)]/(3*a^(5/6)*b^(1/6)) + (-(ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*
Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6)) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(
1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6)) + (ArcTan[Sqrt
[3]*(1 + (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6) + (Sqrt[3]*Log[a^
(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5
/6))))/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.90

method	result
risch	$\frac{2(bB\,x^3+7Ab-7Ba)\sqrt{x}}{7b^2}-\frac{a(Ab-Ba)\left(\frac{2\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x-\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\text{arcta}}{}\right.}{b^2}$
derivativedivides	$\frac{\frac{2bB\,x^{\frac{7}{2}}}{7}+2Ab\sqrt{x}-2Ba\sqrt{x}}{b^2}-\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right.}{6a}\right.}{b}$
default	$\frac{\frac{2bB\,x^{\frac{7}{2}}}{7}+2Ab\sqrt{x}-2Ba\sqrt{x}}{b^2}-\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right.}{6a}\right.}{b}$

input
```
int(x^(5/2)*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/7*(B*b*x^3+7*A*b-7*B*a)*x^(1/2)/b^2-a*(A*b-B*a)/b^2*(2/3/a*(a/b)^(1/6)*a
rctan(x^(1/2)/(a/b)^(1/6))-1/6/a*3^(1/2)*(a/b)^(1/6)*ln(x-3^(1/2)*(a/b)^(1
/6)*x^(1/2)+(a/b)^(1/3))+1/3/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^
(1/2))+1/6/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1
/3))+1/3/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1289 vs. $2(163) = 326$.

Time = 0.13 (sec) , antiderivative size = 1289, normalized size of antiderivative = 5.73

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/42*(14*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3
*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*l
og(b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^
3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6) - (B*a -
 A*b)*sqrt(x)) - 14*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 -
20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^1
3)^(1/6)*log(-b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3
*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/
6) - (B*a - A*b)*sqrt(x)) + 7*(sqrt(-3)*b^2 + b^2)*(-(B^6*a^7 - 6*A*B^5*a^
6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5
*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log(-2*(B*a - A*b)*sqrt(x) + (sqrt(-3)
*b^2 + b^2)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a
^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)) -
7*(sqrt(-3)*b^2 + b^2)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 2
0*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13
)^(1/6)*log(-2*(B*a - A*b)*sqrt(x) - (sqrt(-3)*b^2 + b^2)*(-(B^6*a^7 - 6*A
*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4
- 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)) + 7*(sqrt(-3)*b^2 - b^2)*(-(B^
6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B
^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log(-2*(B*a - A*b...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(209) = 418$.

Time = 30.32 (sec) , antiderivative size = 605, normalized size of antiderivative = 2.69

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \begin{cases} \tilde{\infty}\left(2A\sqrt{x}+\frac{2Bx^{\frac{7}{2}}}{7}\right) \\ \frac{\frac{2Ax^{\frac{7}{2}}}{7}+\frac{2Bx^{\frac{13}{2}}}{13}}{a} \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{7}{2}}}{7}}{b} \\ \frac{2A\sqrt{x}}{b}+\frac{A\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3b}-\frac{A\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3b}+\frac{A\sqrt[6]{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}\right.}{6b} \end{cases}$$

input
```
integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a),x)
```

output
```
Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(7/2)/7), Eq(a, 0) & Eq(b, 0)), ((2*A
*x**(7/2)/7 + 2*B*x**(13/2)/13)/a, Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(7/2)
/7)/b, Eq(a, 0)), (2*A*sqrt(x)/b + A*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(
1/6))/(3*b) - A*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(3*b) + A*(-a/b
)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b) - A*(
-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b) -
sqrt(3)*A*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)
/3)/(3*b) - sqrt(3)*A*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6
)) + sqrt(3)/3)/(3*b) - 2*B*a*sqrt(x)/b**2 - B*a*(-a/b)**(1/6)*log(sqrt(x)
 - (-a/b)**(1/6))/(3*b**2) + B*a*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6)
)/(3*b**2) - B*a*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/
b)**(1/3))/(6*b**2) + B*a*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x
+ 4*(-a/b)**(1/3))/(6*b**2) + sqrt(3)*B*a*(-a/b)**(1/6)*atan(2*sqrt(3)*sqr
t(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*b**2) + sqrt(3)*B*a*(-a/b)**(1/6)*a
tan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*b**2) + 2*B*x**(7/
2)/(7*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.31

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{\left(\frac{\sqrt{3}(Ba-Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(Ba-Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\cdots\right.}{a^{\frac{2}{3}}b\cdots}\right.}{\cdots} + \frac{2\left(Bbx^{\frac{7}{2}}-7\,(Ba-Ab)\sqrt{x}\right)}{7\,b^2}$$

input
```
integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/6*(sqrt(3)*(B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
 a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*a - A*b)*log(-sqrt(3)*a^(1/6)*b^(
1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) - A
*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*s
qrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan((
sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(
1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*ar
ctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))
/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))*a/b^2 + 2/7*(B*b*x^(7/2) - 7*(B*a - A*
b)*sqrt(x))/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.28

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{\sqrt{3}\left((ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,b^3} - \frac{\sqrt{3}\left((ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,b^3} + \frac{\left((ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,b^3} + \frac{\left((ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,b^3} + \frac{2\left((ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,b^3} + \frac{2\left(Bb^6x^{\frac{7}{2}}-7\,Bab^5\sqrt{x}+7\,Ab^6\sqrt{x}\right)}{7\,b^7}$$

input
```
integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a
/b)^(1/6) + x + (a/b)^(1/3))/b^3 - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5
)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/b^3 + 1/3
*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*s
qrt(x))/(a/b)^(1/6))/b^3 + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arc
tan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/b^3 + 2/3*((a*b^5)^(1/
6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/b^3 + 2/7*(B*b^6*x
^(7/2) - 7*B*a*b^5*sqrt(x) + 7*A*b^6*sqrt(x))/b^7
```

Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 1933, normalized size of antiderivative = 8.59

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
int((x^(5/2)*(A + B*x^3))/(a + b*x^3),x)
```

output
```
x^(1/2)*((2*A)/b - (2*B*a)/b^2) + (2*B*x^(7/2))/(7*b) + ((-a)^(1/6)*atan((
((-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^
6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*(A*b - B*a)
*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/
(3*b^(13/6)) + ((-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4
 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^(1
/6)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))
/b^(19/6))*1i)/(3*b^(13/6)))/(((-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^
8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^
3 - (96*(-a)^(1/6)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*
A^2*B*a^5*b^2))/b^(19/6)))/(3*b^(13/6)) - ((-a)^(1/6)*(A*b - B*a)*((96*x^(
1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*
a^5*b^3))/b^3 + (96*(-a)^(1/6)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^
2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6)))/(3*b^(13/6))))*(A*b - B*a)*2i)/(3*b
^(13/6)) + ((-a)^(1/6)*atan(((((-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a
)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b
- 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a
)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)
/(3*b^(13/6)) + ((-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*((96*x^(1/2
)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.03

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\sqrt{x}\,x^3}{7}$$

input

```
int(x^(5/2)*(B*x^3+A)/(b*x^3+a),x)
```

output

```
(2*sqrt(x)*x**3)/7
```

3.136 $\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 207

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2Bx^{5/2}}{5b} - \frac{(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} + \frac{(Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} + \frac{2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} - \frac{(Ab-aB)\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{\sqrt{3}\sqrt[6]{a}b^{11/6}}$$

output
```
2/5*B*x^(5/2)/b+1/3*(A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a
^(1/6)/b^(11/6)+1/3*(A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^
(1/6)/b^(11/6)+2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(1/6)/b^(11
/6)-1/3*(A*b-B*a)*arctanh(3^(1/2)*a^(1/6)*b^(1/6)*x^(1/2)/(a^(1/3)+b^(1/3)
*x))*3^(1/2)/a^(1/6)/b^(11/6)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.74

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{6\sqrt[6]{a}b^{5/6}Bx^{5/2}+10(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-5(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{15\sqrt[6]{a}b^{11/6}}$$

input
```
Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(6*a^(1/6)*b^(5/6)*B*x^(5/2) + 10*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(
1/6)] - 5*(A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x
])] - 5*Sqrt[3]*(A*b - a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(
1/3) + b^(1/3)*x)])/(15*a^(1/6)*b^(11/6))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.27, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {959, 851, 824, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(Ab-aB)\int \frac{x^{3/2}}{bx^3+a}dx}{b}+\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \text{851}$$

$$\frac{2(Ab-aB)\int \frac{x^2}{bx^3+a}d\sqrt{x}}{b}+\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \text{824}$$

$$\frac{2(Ab-aB)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}+\frac{\int -\frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}+\frac{\int -\frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}\right)}{b}+\frac{2Bx^{5/2}}{5b}$$

$\big\downarrow$ 27

$$\frac{2(Ab-aB)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}\right)}{b}+\frac{2Bx^{5/2}}{5b}$$

$\big\downarrow$ 218

$$\frac{2(Ab-aB)\left(-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}\right)}{b}+\frac{2Bx^{5/2}}{5b}$$

$\big\downarrow$ 1142

$$\frac{2(Ab-aB)\left(-\frac{-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}$$

$$\frac{2Bx^{5/2}}{5b}$$

$\big\downarrow$ 25

$$\frac{2(Ab-aB)\left(-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-\cdots}{6\sqrt[6]{\cdots}}\cdots\right.}{b}$$

$$\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \; 27$$

$$\frac{2(Ab-aB)\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}\cdots}{\cdots}\cdots\right.}{b}$$

$$\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \; 1082$$

$$\frac{2(Ab-aB)\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\cdots}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\cdots}\cdots}{6\sqrt[6]{a}b^{2/3}}\cdots\right.}{b}$$

$$\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \; 217$$

$$\frac{2(Ab-aB)\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\cdots\right)}{\cdots}}{6\sqrt[6]{a}b^{2/3}}\cdots\right.}{b}$$

$$\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \; 1103$$

$$\frac{2(Ab-aB)\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{}\cdots\right.}{2\sqrt[6]{b}}}{6\sqrt[6]{}\cdots}\right.}{b}$$

$$\frac{2Bx^{5/2}}{5b}$$

input
```
Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(2*B*x^(5/2))/(5*b) + (2*(A*b - a*B)*(ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3
*a^(1/6)*b^(5/6)) - (ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1
/6)))]/b^(1/6) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] +
b^(1/3)*x])/(2*b^(1/6)))/(6*a^(1/6)*b^(2/3)) - (-(ArcTan[Sqrt[3]*(1 + (2*b
^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6)) + (Sqrt[3]*Log[a^(1/3) + Sqrt
[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(1/6)*b^(2/3))
))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.92

method	result
risch	$\frac{2Bx^{\frac{5}{2}}}{5b}+\frac{(Ab-Ba)\left(\frac{2\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x\right.}{}\right.}{b}$
derivativedivides	$\frac{2Bx^{\frac{5}{2}}}{5b}+\frac{2\left(\frac{\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\right.}{12a}\right.}{b}$
default	$\frac{2Bx^{\frac{5}{2}}}{5b}+\frac{2\left(\frac{\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\right.}{12a}\right.}{b}$

input
```
int(x^(3/2)*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/5*B*x^(5/2)/b+(A*b-B*a)/b*(2/3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))
+1/6/a*3^(1/2)*(a/b)^(5/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1
/3/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))-1/6/a*3^(1/2)*(a/b)
^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/3/b/(a/b)^(1/6)*arc
tan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1603 vs. $2(147) = 294$.

Time = 0.12 (sec) , antiderivative size = 1603, normalized size of antiderivative = 7.74

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/30*(12*B*x^(5/2) + 10*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2
- 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^
11))^(1/6)*log(a*b^9*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*
A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^
(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3
 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) - 10*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 1
5*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^
5 + A^6*b^6)/(a*b^11))^(1/6)*log(-a*b^9*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^
2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 +
A^6*b^6)/(a*b^11))^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 -
 10*A^3*B^2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) - 5*(sqrt(-3)*b -
b)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6)*log((sqrt(-3
)*a*b^9 + a*b^9)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*
B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6
) - 2*(B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3 +
 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) + 5*(sqrt(-3)*b - b)*(-(B^6*a^6 - 6*A*B
^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 -
6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6)*log(-(sqrt(-3)*a*b^9 + a*b^9)*(-(
B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(199) = 398$.

Time = 12.62 (sec) , antiderivative size = 581, normalized size of antiderivative = 2.81

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{5}{2}}}{5}\right) \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{11}{2}}}{11}}{a} \\ \frac{-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{5}{2}}}{5}}{b} \\ \frac{A\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3b\sqrt[6]{-\frac{a}{b}}} - \frac{A\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3b\sqrt[6]{-\frac{a}{b}}} + \frac{A\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6b\sqrt[6]{-\frac{a}{b}}} - \frac{A\log\left(4\sqrt{x}\right.}{} \end{cases}$$

input
```
integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a),x)
```

output
```
Piecewise((zoo*(-2*A/sqrt(x) + 2*B*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((2*
A*x**(5/2)/5 + 2*B*x**(11/2)/11)/a, Eq(b, 0)), ((-2*A/sqrt(x) + 2*B*x**(5/
2)/5)/b, Eq(a, 0)), (A*log(sqrt(x) - (-a/b)**(1/6))/(3*b*(-a/b)**(1/6)) -
A*log(sqrt(x) + (-a/b)**(1/6))/(3*b*(-a/b)**(1/6)) + A*log(-4*sqrt(x)*(-a/
b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b*(-a/b)**(1/6)) - A*log(4*sqrt(x)*(
-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b*(-a/b)**(1/6)) + sqrt(3)*A*atan
(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*b*(-a/b)**(1/6)) + sq
rt(3)*A*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*b*(-a/b)*
*(1/6)) - B*a*log(sqrt(x) - (-a/b)**(1/6))/(3*b**2*(-a/b)**(1/6)) + B*a*lo
g(sqrt(x) + (-a/b)**(1/6))/(3*b**2*(-a/b)**(1/6)) - B*a*log(-4*sqrt(x)*(-a
/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b**2*(-a/b)**(1/6)) + B*a*log(4*sqr
t(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b**2*(-a/b)**(1/6)) - sqrt(
3)*B*a*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*b**2*(-a/b
)**(1/6)) - sqrt(3)*B*a*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)
/3)/(3*b**2*(-a/b)**(1/6)) + 2*B*x**(5/2)/(5*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.02

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,Bx^{\frac{5}{2}}}{5\,b} + \frac{(Ba-Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}} - \frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}} - \frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} - \frac{2\arctan\left(-\frac{\sqrt{3}\cdots}{\cdots}\right)}{b^{\frac{2}{3}}\sqrt{\cdots}}\cdots\right)}{6\,b}$$

input
```
integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
2/5*B*x^(5/2)/b + 1/6*(B*a - A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqr
t(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/
6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sq
rt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)
*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*s
qrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(
b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/b
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.26

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,Bx^{\frac{5}{2}}}{5\,b} - \frac{(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,(ab^5)^{\frac{1}{6}}\,b} - \frac{(Ba-Ab)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,(ab^5)^{\frac{1}{6}}\,b} - \frac{2\left(Ba\left(\frac{a}{b}\right)^{\frac{5}{6}} - Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab} + \frac{\sqrt{3}\left((ab^5)^{\frac{5}{6}}\,Ba - (ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,ab^6} - \frac{\sqrt{3}\left((ab^5)^{\frac{5}{6}}\,Ba - (ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,ab^6}$$

input
```
integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/5*B*x^(5/2)/b - 1/3*(B*a - A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))
/(a/b)^(1/6))/((a*b^5)^(1/6)*b) - 1/3*(B*a - A*b)*arctan(-(sqrt(3)*(a/b)^(
1/6) - 2*sqrt(x))/(a/b)^(1/6))/((a*b^5)^(1/6)*b) - 2/3*(B*a*(a/b)^(5/6) -
A*b*(a/b)^(5/6))*arctan(sqrt(x)/(a/b)^(1/6))/(a*b) + 1/6*sqrt(3)*((a*b^5)^
(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)
^(1/3))/(a*b^6) - 1/6*sqrt(3)*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(
-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^6)
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 1640, normalized size of antiderivative = 7.92

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
int((x^(3/2)*(A + B*x^3))/(a + b*x^3),x)
```

output
```
(2*B*x^(5/2))/(5*b) + (atan((((A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a^6 +
 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4
 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a
)^(1/3)*b^(11/3)) + ((A*b - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2
*a^5*b + 96*A^2*B*a^4*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^
2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b
^(11/3)))/(((A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a^6 + 96*A*B^2*a^5*b -
96*A^2*B*a^4*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2
 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3)) - (
(A*b - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96*A^2*B*a^4
*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*
a^4*b^3))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3))))*(A*b - B*a)*2
i)/(3*(-a)^(1/6)*b^(11/6)) + (atan(((((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^
2*(32*A^3*a^3*b^3 - 32*B^3*a^6 + 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1
/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2
- 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/3)) +
 (((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96
*A*B^2*a^5*b + 96*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B
*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)
*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/3)))/((((3^(1/2)*1i)/2 - 1/2)^2*(A*b ...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.03

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\sqrt{x}\,x^2}{5}$$

input
```
int(x^(3/2)*(B*x^3+A)/(b*x^3+a),x)
```

output
```
(2*sqrt(x)*x**2)/5
```

3.137 $\int \frac{A+Bx^3}{\sqrt{x}(a+bx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 204

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx = \frac{2B\sqrt{x}}{b} - \frac{(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} + \frac{(Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} + \frac{2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} + \frac{(Ab-aB)\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{\sqrt{3}a^{5/6}b^{7/6}}$$

output
```
2*B*x^(1/2)/b+1/3*(A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(
5/6)/b^(7/6)+1/3*(A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(5/
6)/b^(7/6)+2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(5/6)/b^(7/6)+1
/3*(A*b-B*a)*arctanh(3^(1/2)*a^(1/6)*b^(1/6)*x^(1/2)/(a^(1/3)+b^(1/3)*x))*
3^(1/2)/a^(5/6)/b^(7/6)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx$$
$$= \frac{6a^{5/6}\sqrt[6]{b}B\sqrt{x}+2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)+\sqrt{3}(Ab-aB)\text{arctanh}}{3a^{5/6}b^{7/6}}$$

input
```
Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)),x]
```

output
```
(6*a^(5/6)*b^(1/6)*B*Sqrt[x] + 2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1
/6)] - (A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])]
 + Sqrt[3]*(A*b - a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3)
+ b^(1/3)*x)])/(3*a^(5/6)*b^(7/6))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.23, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {959, 851, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx$$
$$\Big\downarrow \; 959$$
$$\frac{(Ab-aB)\int \frac{1}{\sqrt{x}(bx^3+a)}dx}{b}+\frac{2B\sqrt{x}}{b}$$
$$\Big\downarrow \; 851$$
$$\frac{2(Ab-aB)\int \frac{1}{bx^3+a}d\sqrt{x}}{b}+\frac{2B\sqrt{x}}{b}$$
$$\Big\downarrow \; 753$$

$$\frac{2(Ab-aB)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}\right)}{b}+\frac{2B\sqrt{x}}{b}$$

$\downarrow$ 27

$$\frac{2(Ab-aB)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}\right)}{b}+\frac{2B\sqrt{x}}{b}$$

$\downarrow$ 218

$$\frac{2(Ab-aB)\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}\right)}{b}+\frac{2B\sqrt{x}}{b}$$

$\downarrow$ 1142

$$\frac{2(Ab-aB)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{\ldots}}{6a\ldots}\ldots\right)}{b}+\frac{2B\sqrt{x}}{b}$$

$\downarrow$ 25

$$\frac{2(Ab-aB)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}\cdots}{6a^{5}\cdots}\cdots\right)}{b}\cdots$$

$$\frac{2B\sqrt{x}}{b}$$

$\Big\downarrow$ 27

$$\frac{2(Ab-aB)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\cdots}{\cdots}\cdots\right)}{b}\cdots$$

$$\frac{2B\sqrt{x}}{b}$$

$\Big\downarrow$ 1082

$$\frac{2(Ab-aB)\left(\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(2\cdots\right)}{\sqrt{3}\sqrt[6]{\cdots}}}{6a^{5/6}}\cdots\right)}{b}\cdots$$

$$\frac{2B\sqrt{x}}{b}$$

$\Big\downarrow$ 217

$$\frac{2(Ab-aB)\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\cdots\right)}{\cdots}}{6a^{5/6}}\cdots\right)}{b}\cdots$$

$$\frac{2B\sqrt{x}}{b}$$

$\Big\downarrow$ 1103

$$\frac{2(Ab-aB)\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}+\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)}{\sqrt[6]{b}}+\cdots}{\cdots}\right.}{b}$$

$$\frac{2B\sqrt{x}}{b}$$

input
```
Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)),x]
```

output
```
(2*B*Sqrt[x])/b + (2*(A*b - a*B)*(ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3*a^(
5/6)*b^(1/6)) + (-(ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6
)))]/b^(1/6)) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b
^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6)) + (ArcTan[Sqrt[3]*(1 + (2*b^(1/6)*Sqrt
[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6) + (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*
b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6))))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.92

method	result
risch	$\frac{2B\sqrt{x}}{b}+\frac{(Ab-Ba)\left(\frac{2\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x-\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{3a}+\ldots}{b}$
derivativedivides	$\frac{2B\sqrt{x}}{b}+\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6a}-\ldots\right)}{b}$
default	$\frac{2B\sqrt{x}}{b}+\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6a}-\ldots\right)}{b}$

input
```
int((B*x^3+A)/x^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2*B*x^(1/2)/b+(A*b-B*a)/b*(2/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-1
/6/a*3^(1/2)*(a/b)^(1/6)*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/3
/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))+1/6/a*3^(1/2)*(a/b)^(
1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/3/a*(a/b)^(1/6)*arcta
n(2*x^(1/2)/(a/b)^(1/6)+3^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1245 vs. $2(147) = 294$.

Time = 0.14 (sec) , antiderivative size = 1245, normalized size of antiderivative = 6.10

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(2*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(a
*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6) - (B*a - A*
b)*sqrt(x)) - 2*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3
*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1
/6)*log(-a*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*
a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6) -
 (B*a - A*b)*sqrt(x)) + (sqrt(-3)*b + b)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A
^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 +
 A^6*b^6)/(a^5*b^7))^(1/6)*log(-2*(B*a - A*b)*sqrt(x) + (sqrt(-3)*a*b + a*
b)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)) - (sqrt(-3
)*b + b)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(-
2*(B*a - A*b)*sqrt(x) - (sqrt(-3)*a*b + a*b)*(-(B^6*a^6 - 6*A*B^5*a^5*b +
15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b
^5 + A^6*b^6)/(a^5*b^7))^(1/6)) + (sqrt(-3)*b - b)*(-(B^6*a^6 - 6*A*B^5*a^
5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5
*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(-2*(B*a - A*b)*sqrt(x) + (sqrt...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(197) = 394$.

Time = 5.61 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.74

$$\int \frac{A + Bx^3}{\sqrt{x}\,(a + bx^3)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{5x^{\frac{5}{2}}} + 2B\sqrt{x}\right) \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}} + 2B\sqrt{x}}{b} \\ \frac{2A\sqrt{x} + \frac{2Bx^{\frac{7}{2}}}{7}}{a} \\ -\frac{A\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[6]{-\frac{a}{b}}\right)}{3a} + \frac{A\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[6]{-\frac{a}{b}}\right)}{3a} - \frac{A\sqrt[6]{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{6a} + \frac{A\sqrt[6]{-\frac{a}{b}}\log\left(\right.}{} \end{cases}$$

input
```
integrate((B*x**3+A)/x**(1/2)/(b*x**3+a),x)
```

output
```
Piecewise((zoo*(-2*A/(5*x**(5/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((
-2*A/(5*x**(5/2)) + 2*B*sqrt(x))/b, Eq(a, 0)), ((2*A*sqrt(x) + 2*B*x**(7/2
)/7)/a, Eq(b, 0)), (-A*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(3*a) +
A*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(3*a) - A*(-a/b)**(1/6)*log(-
4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a) + A*(-a/b)**(1/6)*l
og(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a) + sqrt(3)*A*(-a/
b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*a) + sq
rt(3)*A*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3
)/(3*a) + 2*B*sqrt(x)/b + B*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(3*
b) - B*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(3*b) + B*(-a/b)**(1/6)*
log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b) - B*(-a/b)**(1
/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b) - sqrt(3)*B
*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*b)
 - sqrt(3)*B*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt
(3)/3)/(3*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx = \frac{2\,B\sqrt{x}}{b}$$

$$-\frac{\frac{\sqrt{3}(Ba-Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(Ba-Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + \frac{2\,(}{}}{6\,b}$$

input
```
integrate((B*x^3+A)/x^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
2*B*sqrt(x)/b - 1/6*(sqrt(3)*(B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(
x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*a - A*b)*log(-sqr
t(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*
(B*a*b^(1/3) - A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a
^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b
^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*
b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(
1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a
^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx = \frac{2\,B\sqrt{x}}{b} - \frac{\sqrt{3}\Big(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\Big)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,ab^2} + \frac{\sqrt{3}\Big(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\Big)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,ab^2} - \frac{\Big(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\Big)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab^2} - \frac{\Big(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\Big)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab^2} - \frac{2\left(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab^2}$$

input
```
integrate((B*x^3+A)/x^(1/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
2*B*sqrt(x)/b - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sq
rt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^2) + 1/6*sqrt(3)*((a*b^5
)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a
/b)^(1/3))/(a*b^2) - 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((s
qrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a*b^2) - 1/3*((a*b^5)^(1/6)*
B*a - (a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(
1/6))/(a*b^2) - 2/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)
/(a/b)^(1/6))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 1915, normalized size of antiderivative = 9.39

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^3)/(x^(1/2)*(a + b*x^3)),x)
```

output
```
(2*B*x^(1/2))/b + (atan((((x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^
2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((A*b - B*a)*(288*A^3*a
*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(
5/6)*b^(7/6)))*(A*b - B*a)*1i)/(3*(-a)^(5/6)*b^(7/6)) + ((x^(1/2)*(96*A^4*
b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3
*b^2) + ((A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3
- 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*(A*b - B*a)*1i)/(3*(-a)^(5/6
)*b^(7/6)))/(((x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 -
384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((A*b - B*a)*(288*A^3*a*b^5 - 288*B
^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)
))*(A*b - B*a))/(3*(-a)^(5/6)*b^(7/6)) - ((x^(1/2)*(96*A^4*b^5 + 96*B^4*a^
4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + ((A*b -
 B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2
*b^4))/(3*(-a)^(5/6)*b^(7/6)))*(A*b - B*a))/(3*(-a)^(5/6)*b^(7/6))))*(A*b
- B*a)*2i)/(3*(-a)^(5/6)*b^(7/6)) + (atan(((((3^(1/2)*1i)/2 - 1/2)*(A*b -
B*a)*(x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B
*a*b^4 - 384*A*B^3*a^3*b^2) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3
*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)
^(5/6)*b^(7/6)))*1i)/(3*(-a)^(5/6)*b^(7/6)) + (((3^(1/2)*1i)/2 - 1/2)*(A*b
 - B*a)*(x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.02

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx=2\sqrt{x}$$

input
```
int((B*x^3+A)/x^(1/2)/(b*x^3+a),x)
```

output
```
2*sqrt(x)
```

3.138 $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 204

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx = -\frac{2A}{a\sqrt{x}}+\frac{(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{(Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} + \frac{(Ab-aB)\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{\sqrt{3}a^{7/6}b^{5/6}}$$

output
```
-2*A/a/x^(1/2)-1/3*(A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^
(7/6)/b^(5/6)-1/3*(A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(7
/6)/b^(5/6)-2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(7/6)/b^(5/6)+
1/3*(A*b-B*a)*arctanh(3^(1/2)*a^(1/6)*b^(1/6)*x^(1/2)/(a^(1/3)+b^(1/3)*x))
*3^(1/2)/a^(7/6)/b^(5/6)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx = \frac{-\frac{6\sqrt[6]{a}A}{\sqrt{x}} + \frac{2(-Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{b^{5/6}} + \frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{b^{5/6}} + \frac{\sqrt{3}(Ab-aB)\operatorname{arctanh}(}{b^{5/6}}}{3a^{7/6}}$$

input
```
Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)),x]
```

output
```
((-6*a^(1/6)*A)/Sqrt[x] + (2*(-(A*b) + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/
6)])/b^(5/6) + ((A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*
Sqrt[x])])/b^(5/6) + (Sqrt[3]*(A*b - a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)
*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/b^(5/6))/(3*a^(7/6))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.28, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {955, 851, 824, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(Ab-aB)\int \frac{x^{3/2}}{bx^3+a}dx}{a}-\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \text{ 851}$$

$$-\frac{2(Ab-aB)\int \frac{x^2}{bx^3+a}d\sqrt{x}}{a}-\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \text{ 824}$$

$$-\frac{2(Ab-aB)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}+\frac{\int -\frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}+\frac{\int -\frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}\right)}{a}-\frac{2A}{a\sqrt{x}}$$

$\big\downarrow$ 27

$$-\frac{2(Ab-aB)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}\right)}{a}-\frac{2A}{a\sqrt{x}}$$

$\big\downarrow$ 218

$$-\frac{2(Ab-aB)\left(-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}\right)}{a}-\frac{2A}{a\sqrt{x}}$$

$\big\downarrow$ 1142

$$-\frac{2(Ab-aB)\left(-\frac{-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}}{2\sqrt[6]{b}}}{}\right.}{a}$$

$$\frac{2A}{a\sqrt{x}}$$

$\big\downarrow$ 25

$$-\frac{2(Ab-aB)\left(-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{\cdots}}{2\sqrt[6]{b}}\cdots}{6\cdots}\cdots\right)}{a}$$

$$\frac{2A}{a\sqrt{x}}$$

$\downarrow$ 27

$$-\frac{2(Ab-aB)\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{\cdots}}\cdots}{\cdots}\cdots\right)}{a}$$

$$\frac{2A}{a\sqrt{x}}$$

$\downarrow$ 1082

$$-\frac{2(Ab-aB)\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\cdots}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\cdots}\cdots}{6\sqrt[6]{a}b^{2/3}}\cdots\right)}{a}$$

$$\frac{2A}{a\sqrt{x}}$$

$\downarrow$ 217

$$-\frac{2(Ab-aB)\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\cdots}{\cdots}}{6\sqrt[6]{a}b^{2/3}}\cdots\right)}{a}$$

$$\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \; 1103$$

$$-\frac{2(Ab-aB)\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\cdots\right.}{2\sqrt[6]{b}}\cdots}{6\cdots}\right.}{a}\cdots$$

$$\frac{2A}{a\sqrt{x}}$$

input
```
Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)),x]
```

output
```
(-2*A)/(a*Sqrt[x]) - (2*(A*b - a*B)*(ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3*
a^(1/6)*b^(5/6)) - (ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/
6)))]/b^(1/6) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b
^(1/3)*x])/(2*b^(1/6)))/(6*a^(1/6)*b^(2/3)) - (-(ArcTan[Sqrt[3]*(1 + (2*b^
(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6)) + (Sqrt[3]*Log[a^(1/3) + Sqrt[
3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(1/6)*b^(2/3)))
)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{2\left(\frac{\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right.\right.}{12a}\cdots\right.}{a}$
default	$-\frac{2\left(\frac{\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right.\right.}{12a}\cdots\right.}{a}$
risch	$-\frac{2A}{a\sqrt{x}}-\frac{(Ab-Ba)\left(\frac{2\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\cdots\right.}{\cdots}\right.}{a}$

input
```
int((B*x^3+A)/x^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-2*(1/3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(5/
6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(
2*x^(1/2)/(a/b)^(1/6)-3^(1/2))-1/12/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/
b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/
6)+3^(1/2)))/a*(A*b-B*a)-2*A/a/x^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1636 vs. $2(147) = 294$.

Time = 0.13 (sec) , antiderivative size = 1636, normalized size of antiderivative = 8.02

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*(2*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a
^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6)*lo
g(a^6*b^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3
*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(5/6) - (B
^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3 + 5*A^4*B
*a*b^4 - A^5*b^5)*sqrt(x)) - 2*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4
*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b
^6)/(a^7*b^5))^(1/6)*log(-a^6*b^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*
a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^
6)/(a^7*b^5))^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A
^3*B^2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) - (sqrt(-3)*a*x - a*x)*
(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*
A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6)*log((sqrt(-3)*
a^6*b^4 + a^6*b^4)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^
3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(
5/6) - 2*(B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^
3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) + (sqrt(-3)*a*x - a*x)*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b
^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6)*log(-(sqrt(-3)*a^6*b^4 + a^
6*b^4)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(197) = 394$.

Time = 9.88 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.75

$$\int \frac{A+Bx^3}{x^{3/2}\,(a+bx^3)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}} - \frac{2B}{\sqrt{x}}\right) \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}} - \frac{2B}{\sqrt{x}}}{b} \\ \frac{-\frac{2A}{\sqrt{x}} + \frac{2Bx^{\frac{5}{2}}}{5}}{a} \\ -\frac{A\log\left(\sqrt{x} - \sqrt[6]{-\frac{a}{b}}\right)}{3a\sqrt[6]{-\frac{a}{b}}} + \frac{A\log\left(\sqrt{x} + \sqrt[6]{-\frac{a}{b}}\right)}{3a\sqrt[6]{-\frac{a}{b}}} - \frac{A\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{6a\sqrt[6]{-\frac{a}{b}}} + \frac{A\log\left(4\sqrt{x}\right.}{} \end{cases}$$

input
```
integrate((B*x**3+A)/x**(3/2)/(b*x**3+a),x)
```

output
```
Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((
-2*A/(7*x**(7/2)) - 2*B/sqrt(x))/b, Eq(a, 0)), ((-2*A/sqrt(x) + 2*B*x**(5/
2)/5)/a, Eq(b, 0)), (-A*log(sqrt(x) - (-a/b)**(1/6))/(3*a*(-a/b)**(1/6)) +
 A*log(sqrt(x) + (-a/b)**(1/6))/(3*a*(-a/b)**(1/6)) - A*log(-4*sqrt(x)*(-a
/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a*(-a/b)**(1/6)) + A*log(4*sqrt(x)*
(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a*(-a/b)**(1/6)) - sqrt(3)*A*ata
n(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*a*(-a/b)**(1/6)) - s
qrt(3)*A*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*a*(-a/b)
**(1/6)) - 2*A/(a*sqrt(x)) + B*log(sqrt(x) - (-a/b)**(1/6))/(3*b*(-a/b)**(
1/6)) - B*log(sqrt(x) + (-a/b)**(1/6))/(3*b*(-a/b)**(1/6)) + B*log(-4*sqrt
(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b*(-a/b)**(1/6)) - B*log(4*s
qrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b*(-a/b)**(1/6)) + sqrt(3
)*B*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*b*(-a/b)**(1/
6)) + sqrt(3)*B*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*b
*(-a/b)**(1/6)), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx = -\frac{(Ba-Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(-\sqrt{\ldots}\right.}{b^{\frac{2}{3}}\sqrt{\ldots}}\right.}{6a} - \frac{2A}{a\sqrt{x}}$$

input
```
integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/6*(B*a - A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x
+ a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x
) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(
1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(
1/3))) - 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1
/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/s
qrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/a - 2*A/(a*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)}\,dx = \frac{(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,(ab^5)^{\frac{1}{6}}\,a} + \frac{(Ba-Ab)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,(ab^5)^{\frac{1}{6}}\,a} + \frac{2\,(Ba-Ab)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,(ab^5)^{\frac{1}{6}}\,a} - \frac{2\,A}{a\sqrt{x}} - \frac{\sqrt{3}\left((ab^5)^{\frac{5}{6}}\,Ba-(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^2b^5} + \frac{\sqrt{3}\left((ab^5)^{\frac{5}{6}}\,Ba-(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^2b^5}$$

input
```
integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*(B*a - A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/((a*
b^5)^(1/6)*a) + 1/3*(B*a - A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/
(a/b)^(1/6))/((a*b^5)^(1/6)*a) + 2/3*(B*a - A*b)*arctan(sqrt(x)/(a/b)^(1/6
))/((a*b^5)^(1/6)*a) - 2*A/(a*sqrt(x)) - 1/6*sqrt(3)*((a*b^5)^(5/6)*B*a -
(a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2
*b^5) + 1/6*sqrt(3)*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(-sqrt(3)*s
qrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^5)
```

Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 1700, normalized size of antiderivative = 8.33

$$\int \frac{A+Bx^3}{x^{3/2}\,(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^3)/(x^(3/2)*(a + b*x^3)),x)
```

output
```
(atan((((A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*b^
4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a
^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(
5/3)) + ((A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b
^4 - 96*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*
a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^
(5/3)))/(((A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*
b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2
*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5
/3)) - ((A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^
4 - 96*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a
^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3
))))*(A*b - B*a)*2i)/(3*(-a)^(7/6)*b^(5/6)) - (2*A)/(a*x^(1/2)) + (atan(((
((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 -
 96*A*B^2*a^11*b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(
A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*
(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3)) + (((3^(1/2)*1i)/2 - 1/2)^2*
(A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 - 96*A
^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^10*
b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.03

$$\int \frac{A+Bx^3}{x^{3/2}\,(a+bx^3)}\,dx = -\frac{2}{\sqrt{x}}$$

input
```
int((B*x^3+A)/x^(3/2)/(b*x^3+a),x)
```

output
```
( - 2)/sqrt(x)
```

3.139 $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 207

$$\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)}\,dx = -\frac{2A}{5ax^{5/2}} + \frac{(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} - \frac{(Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} - \frac{2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} - \frac{(Ab-aB)\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{\sqrt{3}a^{11/6}\sqrt[6]{b}}$$

output
```
-2/5*A/a/x^(5/2)-1/3*(A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/
a^(11/6)/b^(1/6)-1/3*(A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a
^(11/6)/b^(1/6)-2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(
1/6)-1/3*(A*b-B*a)*arctanh(3^(1/2)*a^(1/6)*b^(1/6)*x^(1/2)/(a^(1/3)+b^(1/3
)*x))*3^(1/2)/a^(11/6)/b^(1/6)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx = \frac{-\frac{6a^{5/6}A}{x^{5/2}}+\frac{10(-Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{\sqrt[6]{b}}+\frac{5(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{\sqrt[6]{b}}+\frac{5\sqrt{3}(-Ab+aB)\text{arcta}\ldots}{\ldots}}{15a^{11/6}}$$

input
```
Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)),x]
```

output
```
((-6*a^(5/6)*A)/x^(5/2) + (10*(-(A*b) + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1
/6)])/b^(1/6) + (5*(A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/
6)*Sqrt[x])])/b^(1/6) + (5*Sqrt[3]*(-(A*b) + a*B)*ArcTanh[(Sqrt[3]*a^(1/6)
*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/b^(1/6))/(15*a^(11/6))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.22, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {955, 851, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}dx$$

$$\Big\downarrow \ 955$$

$$-\frac{(Ab-aB)\int \frac{1}{\sqrt{x}\left(bx^3+a\right)}dx}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \ 851$$

$$-\frac{2(Ab-aB)\int \frac{1}{bx^3+a}d\sqrt{x}}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \ 753$$

$$-\frac{2(Ab-aB)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}\right)}{a}-\frac{2A}{5ax^{5/2}}$$

$\downarrow$ 27

$$-\frac{2(Ab-aB)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}\right)}{a}-\frac{2A}{5ax^{5/2}}$$

$\downarrow$ 218

$$-\frac{2(Ab-aB)\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}\right)}{a}-\frac{2A}{5ax^{5/2}}$$

$\downarrow$ 1142

$$-\frac{2(Ab-aB)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}\cdots}{\cdots}\right)}{a}-\frac{2A}{5ax^{5/2}}$$

$\downarrow$ 25

$$-\frac{2(Ab-aB)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\ldots}{6\ldots}\right.\ldots}{a}$$

$$\frac{2A}{5ax^{5/2}}$$

$\Big\downarrow$ 27

$$-\frac{2(Ab-aB)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\ldots}\ldots}{\ldots}\right.\ldots}{a}$$

$$\frac{2A}{5ax^{5/2}}$$

$\Big\downarrow$ 1082

$$-\frac{2(Ab-aB)\left(\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(\ldots\right.}{\sqrt{\ldots}}}{6a^{5/6}}\right.\ldots}{a}$$

$$\frac{2A}{5ax^{5/2}}$$

$\Big\downarrow$ 217

$$-\frac{2(Ab-aB)\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\ldots\right.}{\ldots}}{6a^{5/6}}\right.\ldots}{a}$$

$$\frac{2A}{5ax^{5/2}}$$

$\Big\downarrow$ 1103

$$-\frac{2(Ab-aB)\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}+\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)}{\sqrt[6]{b}}}{\ldots}\right)}{a}$$

$$\frac{2A}{5ax^{5/2}}$$

input
```
Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)),x]
```

output
```
(-2*A)/(5*a*x^(5/2)) - (2*(A*b - a*B)*(ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(
3*a^(5/6)*b^(1/6)) + (-(ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a
^(1/6)))]/b^(1/6)) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x
] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6)) + (ArcTan[Sqrt[3]*(1 + (2*b^(1/6)
*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6) + (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(
1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6))))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.91

method	result
derivativedivides	$-\frac{2A}{5a\,x^{\frac{5}{2}}}+\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}}{}\right.}{a}$
default	$-\frac{2A}{5a\,x^{\frac{5}{2}}}+\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}}{}\right.}{a}$
risch	$-\frac{2A}{5a\,x^{\frac{5}{2}}}-\frac{(Ab-Ba)\left(\frac{2\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x-\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{3a}+\right.}{a}$

input
```
int((B*x^3+A)/x^(7/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-2/5*A/a/x^(5/2)+2*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))+1/12/a*3
^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/
b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))-1/12/a*3^(1/2)*(a/b)^(1/6)*
ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x
^(1/2)/(a/b)^(1/6)-3^(1/2)))*(-A*b+B*a)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1290 vs. 2(147) = 294.

Time = 0.11 (sec) , antiderivative size = 1290, normalized size of antiderivative = 6.23

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/30*(10*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B
^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)
*log(a^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6) - (B*a
 - A*b)*sqrt(x)) - 10*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^
2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^
11*b))^(1/6)*log(-a^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20
*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))
^(1/6) - (B*a - A*b)*sqrt(x)) + 5*(sqrt(-3)*a*x^3 + a*x^3)*(-(B^6*a^6 - 6*
A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4
 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*log(-2*(B*a - A*b)*sqrt(x) + (
sqrt(-3)*a^2 + a^2)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A
^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(
1/6)) - 5*(sqrt(-3)*a*x^3 + a*x^3)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4
*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b
^6)/(a^11*b))^(1/6)*log(-2*(B*a - A*b)*sqrt(x) - (sqrt(-3)*a^2 + a^2)*(-(B
^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*
B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)) + 5*(sqrt(-3)*a*x^
3 - a*x^3)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^
3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*l...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(199) = 398$.

Time = 51.01 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.83

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{11x^{\frac{11}{2}}}-\frac{2B}{5x^{\frac{5}{2}}}\right) \\ \frac{-\frac{2A}{11x^{\frac{11}{2}}}-\frac{2B}{5x^{\frac{5}{2}}}}{b} \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}}+2B\sqrt{x}}{a} \\ -\frac{2A}{5ax^{\frac{5}{2}}}+\frac{Ab\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3a^2}-\frac{Ab\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3a^2}+\frac{Ab\sqrt[6]{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{\right.}}{6a^2} \end{cases}$$

input
```
integrate((B*x**3+A)/x**(7/2)/(b*x**3+a),x)
```

output
```
Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(5*x**(5/2))), Eq(a, 0) & Eq(b,
0)), ((-2*A/(11*x**(11/2)) - 2*B/(5*x**(5/2)))/b, Eq(a, 0)), ((-2*A/(5*x**
(5/2)) + 2*B*sqrt(x))/a, Eq(b, 0)), (-2*A/(5*a*x**(5/2)) + A*b*(-a/b)**(1/
6)*log(sqrt(x) - (-a/b)**(1/6))/(3*a**2) - A*b*(-a/b)**(1/6)*log(sqrt(x) +
 (-a/b)**(1/6))/(3*a**2) + A*b*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6)
+ 4*x + 4*(-a/b)**(1/3))/(6*a**2) - A*b*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)
**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a**2) - sqrt(3)*A*b*(-a/b)**(1/6)*atan
(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*a**2) - sqrt(3)*A*b*(
-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*a**2
) - B*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(3*a) + B*(-a/b)**(1/6)*l
og(sqrt(x) + (-a/b)**(1/6))/(3*a) - B*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)*
*(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a) + B*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/
b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a) + sqrt(3)*B*(-a/b)**(1/6)*atan(2*
sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*a) + sqrt(3)*B*(-a/b)**(
1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*a), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx = \frac{\frac{\sqrt{3}(Ba-Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(Ba-Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\text{ar}}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{}}}{} - \frac{2\,A}{5\,ax^{\frac{5}{2}}}$$

input
```
integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/6*(sqrt(3)*(B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
 a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*a - A*b)*log(-sqrt(3)*a^(1/6)*b^(
1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) - A
*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*s
qrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan((
sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(
1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*ar
ctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))
/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/a - 2/5*A/(a*x^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx = \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^2b} - \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^2b} + \frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^2b} + \frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^2b} + \frac{2\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^2b} - \frac{2\,A}{5\,ax^{\frac{5}{2}}}$$

input
```
integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a
/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b) - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a
*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*
b) + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/
6) + 2*sqrt(x))/(a/b)^(1/6))/(a^2*b) + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1
/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b) +
2/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a
^2*b) - 2/5*A/(a*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 2023, normalized size of antiderivative = 9.77

$$\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^3)/(x^(7/2)*(a + b*x^3)),x)
```

output
```
- (2*A)/(5*a*x^(5/2)) - (atan((((x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5
+ 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - ((A*b - B
*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^
8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)) +
 ((x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*
B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + ((A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3
*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)
))*(A*b - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)))/(((x^(1/2)*(96*A^4*a^5*b^9 + 9
6*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^
8) - ((A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6
- 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)^(11/6)
*b^(1/6)) - ((x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b
^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + ((A*b - B*a)*(288*A^3*a^7*b^
8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11
/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)^(11/6)*b^(1/6))))*(A*b - B*a)*2i)/(3*(-
a)^(11/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(96*A^4*a^5*b
^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*
a^6*b^8) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*
a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6))
)*(A*b - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)) + (((3^(1/2)*1i)/2 - 1/2)*(x^...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.04

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx=-\frac{2}{5\sqrt{x}\,x^2}$$

input
```
int((B*x^3+A)/x^(7/2)/(b*x^3+a),x)
```

output
```
( - 2)/(5*sqrt(x)*x**2)
```

3.140 $\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(3Ab-5aB)x^{3/2}}{3b^3}+\frac{2Bx^{9/2}}{9b^2} - \frac{(Ab-aB)x^{9/2}}{3b^2(a+bx^3)}-\frac{\sqrt{a}(3Ab-5aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{7/2}}$$

output
```
1/3*(3*A*b-5*B*a)*x^(3/2)/b^3+2/9*B*x^(9/2)/b^2-1/3*(A*b-B*a)*x^(9/2)/b^2/
(b*x^3+a)-1/3*a^(1/2)*(3*A*b-5*B*a)*arctan(b^(1/2)*x^(3/2)/a^(1/2))/b^(7/2
)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{x^{3/2}(9aAb-15a^2B+6Ab^2x^3-10abBx^3+2b^2Bx^6)}{9b^3(a+bx^3)} + \frac{\sqrt{a}(-3Ab+5aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{7/2}}$$

input
```
Integrate[(x^(13/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(x^(3/2)*(9*a*A*b - 15*a^2*B + 6*A*b^2*x^3 - 10*a*b*B*x^3 + 2*b^2*B*x^6))/
(9*b^3*(a + b*x^3)) + (Sqrt[a]*(-3*A*b + 5*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/S
qrt[a]])/(3*b^(7/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {957, 843, 843, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{x^{15/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(3Ab-5aB)\int \frac{x^{13/2}}{bx^3+a}dx}{2ab}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x^{15/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(3Ab-5aB)\left(\frac{2x^{9/2}}{9b}-\frac{a\int \frac{x^{7/2}}{bx^3+a}dx}{b}\right)}{2ab}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x^{15/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(3Ab-5aB)\left(\frac{2x^{9/2}}{9b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{bx^3+a}dx}{b}\right)}{b}\right)}{2ab}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{x^{15/2}(Ab-aB)}{3ab(a+bx^3)}-\frac{(3Ab-5aB)\left(\frac{2x^{9/2}}{9b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{2a\int\frac{x}{bx^3+a}d\sqrt{x}}{b}\right)}{b}\right)}{2ab}$$

$\Big\downarrow$ 807

$$\frac{x^{15/2}(Ab-aB)}{3ab(a+bx^3)}-\frac{(3Ab-5aB)\left(\frac{2x^{9/2}}{9b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{2a\int\frac{1}{a+bx}dx^{3/2}}{3b}\right)}{b}\right)}{2ab}$$

$\Big\downarrow$ 218

$$\frac{x^{15/2}(Ab-aB)}{3ab(a+bx^3)}-\frac{(3Ab-5aB)\left(\frac{2x^{9/2}}{9b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{3/2}}\right)}{b}\right)}{2ab}$$

input
```
Int[(x^(13/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^(15/2))/(3*a*b*(a + b*x^3)) - ((3*A*b - 5*a*B)*((2*x^(9/2))
/(9*b) - (a*((2*x^(3/2))/(3*b) - (2*Sqrt[a]*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[
a]])/(3*b^(3/2))))/b))/(2*a*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{2x^{\frac{3}{2}}\left(bB\,x^{3}+3Ab-6Ba\right)}{9b^{3}}-\frac{a\left(\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x^{\frac{3}{2}}}{3\left(b\,x^{3}+a\right)}+\frac{\left(3Ab-5Ba\right)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}}\right)}{b^{3}}$	82
derivativedivides	$\frac{\frac{2bB\,x^{\frac{9}{2}}}{9}+\frac{2\left(Ab-2Ba\right)x^{\frac{3}{2}}}{3}}{b^{3}}-\frac{2a\left(\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x^{\frac{3}{2}}}{b\,x^{3}+a}+\frac{\left(3Ab-5Ba\right)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{3b^{3}}$	84
default	$\frac{\frac{2bB\,x^{\frac{9}{2}}}{9}+\frac{2\left(Ab-2Ba\right)x^{\frac{3}{2}}}{3}}{b^{3}}-\frac{2a\left(\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x^{\frac{3}{2}}}{b\,x^{3}+a}+\frac{\left(3Ab-5Ba\right)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{3b^{3}}$	84

input
```
int(x^(13/2)*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/9*x^(3/2)*(B*b*x^3+3*A*b-6*B*a)/b^3-a/b^3*(2/3*(-1/2*A*b+1/2*B*a)*x^(3/2
)/(b*x^3+a)+1/3*(3*A*b-5*B*a)/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.45

$$\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \left[-\frac{3\left((5\,Bab-3\,Ab^2)x^3+5\,Ba^2-3\,Aab\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^3-2\,bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}-a}{bx^3+a}\right)-2\,(2\,Bb^2}{18\,(b^4x^3+ab^3)}\right.$$

input
```
integrate(x^(13/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/18*(3*((5*B*a*b - 3*A*b^2)*x^3 + 5*B*a^2 - 3*A*a*b)*sqrt(-a/b)*log((b*
x^3 - 2*b*x^(3/2)*sqrt(-a/b) - a)/(b*x^3 + a)) - 2*(2*B*b^2*x^7 - 2*(5*B*a
*b - 3*A*b^2)*x^4 - 3*(5*B*a^2 - 3*A*a*b)*x)*sqrt(x))/(b^4*x^3 + a*b^3), 1
/9*(3*((5*B*a*b - 3*A*b^2)*x^3 + 5*B*a^2 - 3*A*a*b)*sqrt(a/b)*arctan(b*x^(
3/2)*sqrt(a/b)/a) + (2*B*b^2*x^7 - 2*(5*B*a*b - 3*A*b^2)*x^4 - 3*(5*B*a^2
- 3*A*a*b)*x)*sqrt(x))/(b^4*x^3 + a*b^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(13/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(Ba^2-Aab)x^{\frac{3}{2}}}{3\,(b^4x^3+ab^3)} + \frac{(5\,Ba^2-3\,Aab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b^3} + \frac{2\left(Bbx^{\frac{9}{2}}-3\,(2\,Ba-Ab)x^{\frac{3}{2}}\right)}{9\,b^3}$$

input
```
integrate(x^(13/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(B*a^2 - A*a*b)*x^(3/2)/(b^4*x^3 + a*b^3) + 1/3*(5*B*a^2 - 3*A*a*b)*a
rctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/9*(B*b*x^(9/2) - 3*(2*B*a -
 A*b)*x^(3/2))/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.94

$$\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(5\,Ba^2-3\,Aab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b^3} - \frac{Ba^2x^{\frac{3}{2}}-Aabx^{\frac{3}{2}}}{3\,(bx^3+a)b^3} + \frac{2\left(Bb^4x^{\frac{9}{2}}-6\,Bab^3x^{\frac{3}{2}}+3\,Ab^4x^{\frac{3}{2}}\right)}{9\,b^6}$$

input
```
integrate(x^(13/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*(5*B*a^2 - 3*A*a*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/3*
(B*a^2*x^(3/2) - A*a*b*x^(3/2))/((b*x^3 + a)*b^3) + 2/9*(B*b^4*x^(9/2) - 6
*B*a*b^3*x^(3/2) + 3*A*b^4*x^(3/2))/b^6
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.38

$$\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = x^{3/2}\left(\frac{2\,A}{3\,b^2}-\frac{4\,B\,a}{3\,b^3}\right)+\frac{2\,B\,x^{9/2}}{9\,b^2}-\frac{x^{3/2}\left(\frac{B\,a^2}{3}-\frac{A\,a\,b}{3}\right)}{b^4\,x^3+a\,b^3} + \frac{\sqrt{a}\,\mathrm{atan}\left(\frac{36\,\sqrt{b}\,x^{3/2}\,\left(9\,A^2\,a^2\,b^2-30\,A\,B\,a^3\,b+25\,B^2\,a^4\right)}{\sqrt{a}\,\left(3\,A\,b-5\,B\,a\right)\,\left(180\,B\,a^3-108\,A\,a^2\,b\right)}\right)\,\left(3\,A\,b-5\,B\,a\right)}{3\,b^{7/2}}$$

input
```
int((x^(13/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
x^(3/2)*((2*A)/(3*b^2) - (4*B*a)/(3*b^3)) + (2*B*x^(9/2))/(9*b^2) - (x^(3/
2)*((B*a^2)/3 - (A*a*b)/3))/(a*b^3 + b^4*x^3) + (a^(1/2)*atan((36*b^(1/2)*
x^(3/2)*(25*B^2*a^4 + 9*A^2*a^2*b^2 - 30*A*B*a^3*b))/(a^(1/2)*(3*A*b - 5*B
*a)*(180*B*a^3 - 108*A*a^2*b)))*(3*A*b - 5*B*a))/(3*b^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12

$$\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{-\frac{2b^{\frac{1}{6}}a^{\frac{13}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)}{3}+\frac{2b^{\frac{1}{6}}a^{\frac{13}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)}{3}-\frac{2b^{\frac{1}{6}}a^{\frac{13}{6}}\,atan\left(\frac{\sqrt{x}\,b^{\frac{1}{6}}}{a^{\frac{1}{6}}}\right)}{3}-\frac{2\sqrt{x}\cdots}{}}{b^{\frac{8}{3}}a^{\frac{2}{3}}}$$

input
```
int(x^(13/2)*(B*x^3+A)/(b*x^3+a)^2,x)
```

output
```
(2*( - 3*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*b**
(1/3))/(b**(1/6)*a**(1/6)))*a**2 + 3*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(
1/6)*sqrt(3) + 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*a**2 - 3*b**(1/6)*
a**(1/6)*atan((sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*a**2 - 3*sqrt(x)*b**
(2/3)*a**(2/3)*a*x + sqrt(x)*b**(2/3)*a**(2/3)*b*x**4))/(9*b**(2/3)*a**(2/
3)*b**2)
```

3.141 $\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{2Bx^{3/2}}{3b^2} - \frac{(Ab-aB)x^{3/2}}{3b^2(a+bx^3)} + \frac{(Ab-3aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{5/2}}$$

output
```
2/3*B*x^(3/2)/b^2-1/3*(A*b-B*a)*x^(3/2)/b^2/(b*x^3+a)+1/3*(A*b-3*B*a)*arct
an(b^(1/2)*x^(3/2)/a^(1/2))/a^(1/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{x^{3/2}(-Ab+3aB+2bBx^3)}{3b^2(a+bx^3)} + \frac{(Ab-3aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{5/2}}$$

input
```
Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(x^(3/2)*(-(A*b) + 3*a*B + 2*b*B*x^3))/(3*b^2*(a + b*x^3)) + ((A*b - 3*a*B
)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*Sqrt[a]*b^(5/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {957, 843, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^{9/2}(Ab-aB)}{3ab(a+bx^3)}-\frac{(Ab-3aB)\int \frac{x^{7/2}}{bx^3+a}dx}{2ab}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^{9/2}(Ab-aB)}{3ab(a+bx^3)}-\frac{(Ab-3aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{bx^3+a}dx}{b}\right)}{2ab}$$

$$\Big\downarrow \text{851}$$

$$\frac{x^{9/2}(Ab-aB)}{3ab(a+bx^3)}-\frac{(Ab-3aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\int \frac{x}{bx^3+a}d\sqrt{x}}{b}\right)}{2ab}$$

$$\Big\downarrow \text{807}$$

$$\frac{x^{9/2}(Ab-aB)}{3ab(a+bx^3)}-\frac{(Ab-3aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\int \frac{1}{a+bx}dx^{3/2}}{3b}\right)}{2ab}$$

$$\Big\downarrow \text{218}$$

$$\frac{x^{9/2}(Ab-aB)}{3ab(a+bx^3)}-\frac{(Ab-3aB)\left(\frac{2x^{3/2}}{3b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{3/2}}\right)}{2ab}$$

input
```
Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^(9/2))/(3*a*b*(a + b*x^3)) - ((A*b - 3*a*B)*((2*x^(3/2))/(3
*b) - (2*Sqrt[a]*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(3/2))))/(2*a*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{2Bx^{\frac{3}{2}}}{3b^2}+\frac{\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x^{\frac{3}{2}}}{3\left(b\,x^3+a\right)}+\frac{(Ab-3Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}}}{b^2}$	65
default	$\frac{2Bx^{\frac{3}{2}}}{3b^2}+\frac{\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x^{\frac{3}{2}}}{3\left(b\,x^3+a\right)}+\frac{(Ab-3Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}}}{b^2}$	65
risch	$\frac{2Bx^{\frac{3}{2}}}{3b^2}+\frac{\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x^{\frac{3}{2}}}{3\left(b\,x^3+a\right)}+\frac{(Ab-3Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}}}{b^2}$	65

input
```
int(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*B*x^(3/2)/b^2+2/3/b^2*((-1/2*A*b+1/2*B*a)*x^(3/2)/(b*x^3+a)+1/2*(A*b-3
*B*a)/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.71

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \left[\frac{((3\,Bab-Ab^2)x^3+3\,Ba^2-Aab)\sqrt{-ab}\log\left(\frac{bx^3-2\,\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)+2\,(2\,Bab^2x^4+(3\right.$$

$$\left.-\frac{((3\,Bab-Ab^2)x^3+3\,Ba^2-Aab)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x^{\frac{3}{2}}}{a}\right)-(2\,Bab^2x^4+(3\,Ba^2b-Aab^2)x)\sqrt{x}}{3\,(ab^4x^3+a^2b^3)}\right]$$

input
```
integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/6*(((3*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b)*sqrt(-a*b)*log((b*x^3 - 2*
sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) + 2*(2*B*a*b^2*x^4 + (3*B*a^2*b - A*a
*b^2)*x)*sqrt(x))/(a*b^4*x^3 + a^2*b^3), -1/3*(((3*B*a*b - A*b^2)*x^3 + 3*
B*a^2 - A*a*b)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) - (2*B*a*b^2*x^4 + (3
*B*a^2*b - A*a*b^2)*x)*sqrt(x))/(a*b^4*x^3 + a^2*b^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ba-Ab)x^{\frac{3}{2}}}{3\,(b^3x^3+ab^2)} + \frac{2\,Bx^{\frac{3}{2}}}{3\,b^2} - \frac{(3\,Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b^2}$$

input
```
integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(B*a - A*b)*x^(3/2)/(b^3*x^3 + a*b^2) + 2/3*B*x^(3/2)/b^2 - 1/3*(3*B*a
 - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,b^2} - \frac{(3\,Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b^2} + \frac{Bax^{\frac{3}{2}}-Abx^{\frac{3}{2}}}{3\,(bx^3+a)b^2}$$

input
```
integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
2/3*B*x^(3/2)/b^2 - 1/3*(3*B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*
b)*b^2) + 1/3*(B*a*x^(3/2) - A*b*x^(3/2))/((b*x^3 + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.41

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{2\,B\,x^{3/2}}{3\,b^2} - \frac{x^{3/2}\left(\frac{A\,b}{3}-\frac{B\,a}{3}\right)}{b^3\,x^3+a\,b^2} + \frac{\operatorname{atan}\left(\frac{36\,\sqrt{a}\,b^{3/2}\,x^{3/2}\,(A^2\,b^2-6\,A\,B\,a\,b+9\,B^2\,a^2)}{(A\,b-3\,B\,a)\,(36\,A\,a\,b^2-108\,B\,a^2\,b)}\right)(A\,b-3\,B\,a)}{3\,\sqrt{a}\,b^{5/2}}$$

input
```
int((x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
(2*B*x^(3/2))/(3*b^2) - (x^(3/2)*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) +
(atan((36*a^(1/2)*b^(3/2)*x^(3/2)*(A^2*b^2 + 9*B^2*a^2 - 6*A*B*a*b))/((A*b
 - 3*B*a)*(36*A*a*b^2 - 108*B*a^2*b)))*(A*b - 3*B*a))/(3*a^(1/2)*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.23

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{\frac{2b^{\frac{1}{6}}a^{\frac{7}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)}{3} - \frac{2b^{\frac{1}{6}}a^{\frac{7}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)}{3} + \frac{2b^{\frac{1}{6}}a^{\frac{7}{6}}\,atan\left(\frac{\sqrt{x}\,b^{\frac{1}{6}}}{a^{\frac{1}{6}}}\right)}{3} + \frac{2\sqrt{x}\,b^{\frac{2}{3}}a^{\frac{2}{3}}}{3}}{b^{\frac{5}{3}}a^{\frac{2}{3}}}$$

input
```
int(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x)
```

output
```
(2*(b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*b**(1/3)
)/(b**(1/6)*a**(1/6)))*a - b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(
3) + 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*a + b**(1/6)*a**(1/6)*atan((
sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*a + sqrt(x)*b**(2/3)*a**(2/3)*x))/(
3*b**(2/3)*a**(2/3)*b)
```

3.142 $\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2} dx = \frac{(Ab-aB)x^{3/2}}{3ab(a+bx^3)} + \frac{(Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}$$

output
```
1/3*(A*b-B*a)*x^(3/2)/a/b/(b*x^3+a)+1/3*(A*b+B*a)*arctan(b^(1/2)*x^(3/2)/a
^(1/2))/a^(3/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2} dx = -\frac{(-Ab+aB)x^{3/2}}{3ab(a+bx^3)} + \frac{(Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
-1/3*((-(A*b) + a*B)*x^(3/2))/(a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqr
t[b]*x^(3/2))/Sqrt[a]])/(3*a^(3/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {957, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{(aB+Ab)\int \frac{\sqrt{x}}{bx^3+a}dx}{2ab}+\frac{x^{3/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{(aB+Ab)\int \frac{x}{bx^3+a}d\sqrt{x}}{ab}+\frac{x^{3/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{(aB+Ab)\int \frac{1}{a+bx}dx^{3/2}}{3ab}+\frac{x^{3/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(aB+Ab)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}+\frac{x^{3/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

input
```
Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^(3/2))/(3*a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x
^(3/2))/Sqrt[a]])/(3*a^(3/2)*b^(3/2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{(Ab-Ba)x^{\frac{3}{2}}}{3ab(b\,x^{3}+a)}+\frac{(Ab+Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3ab\sqrt{ab}}$	61
default	$\frac{(Ab-Ba)x^{\frac{3}{2}}}{3ab(b\,x^{3}+a)}+\frac{(Ab+Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3ab\sqrt{ab}}$	61

input
```
int(x^(1/2)*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(A*b-B*a)*x^(3/2)/a/b/(b*x^3+a)+1/3*(A*b+B*a)/a/b/(a*b)^(1/2)*arctan(b
*x^(3/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.68

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2}\,dx = \left[-\frac{2\,(Ba^2b-Aab^2)x^{\frac{3}{2}}+((Bab+Ab^2)x^3+Ba^2+Aab)\sqrt{-ab}\log\left(\frac{bx^3-2\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)}{6\,(a^2b^3x^3+a^3b^2)}, -\frac{(Ba^2b-Aab^2)x^{\frac{3}{2}}-((Bab+Ab^2)x^3+Ba^2+Aab)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x^{\frac{3}{2}}}{a}\right)}{3\,(a^2b^3x^3+a^3b^2)}\right]$$

input
```
integrate(x^(1/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/6*(2*(B*a^2*b - A*a*b^2)*x^(3/2) + ((B*a*b + A*b^2)*x^3 + B*a^2 + A*a*
b)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)))/(a^2*b^
3*x^3 + a^3*b^2), -1/3*((B*a^2*b - A*a*b^2)*x^(3/2) - ((B*a*b + A*b^2)*x^3
 + B*a^2 + A*a*b)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a))/(a^2*b^3*x^3 + a^
3*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. 2(61) = 122.

Time = 74.23 (sec) , antiderivative size = 1042, normalized size of antiderivative = 14.68

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(1/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0)
), ((2*A*x**(3/2)/3 + 2*B*x**(9/2)/9)/a**2, Eq(b, 0)), ((-2*A/(9*x**(9/2))
 - 2*B/(3*x**(3/2)))/b**2, Eq(a, 0)), (2*A*a*b*x**(3/2)/(6*a**3*b + 6*a**2
*b**2*x**3) - A*a*b*sqrt(-a/b)*log(sqrt(x) - (-a/b)**(1/6))/(6*a**3*b + 6*
a**2*b**2*x**3) + A*a*b*sqrt(-a/b)*log(sqrt(x) + (-a/b)**(1/6))/(6*a**3*b
+ 6*a**2*b**2*x**3) + A*a*b*sqrt(-a/b)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x
+ 4*(-a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x**3) - A*a*b*sqrt(-a/b)*log(4*
sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x**
3) - A*b**2*x**3*sqrt(-a/b)*log(sqrt(x) - (-a/b)**(1/6))/(6*a**3*b + 6*a**
2*b**2*x**3) + A*b**2*x**3*sqrt(-a/b)*log(sqrt(x) + (-a/b)**(1/6))/(6*a**3
*b + 6*a**2*b**2*x**3) + A*b**2*x**3*sqrt(-a/b)*log(-4*sqrt(x)*(-a/b)**(1/
6) + 4*x + 4*(-a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x**3) - A*b**2*x**3*sq
rt(-a/b)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a**3*b +
6*a**2*b**2*x**3) - 2*B*a**2*x**(3/2)/(6*a**3*b + 6*a**2*b**2*x**3) - B*a*
*2*sqrt(-a/b)*log(sqrt(x) - (-a/b)**(1/6))/(6*a**3*b + 6*a**2*b**2*x**3) +
 B*a**2*sqrt(-a/b)*log(sqrt(x) + (-a/b)**(1/6))/(6*a**3*b + 6*a**2*b**2*x*
*3) + B*a**2*sqrt(-a/b)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/
3))/(6*a**3*b + 6*a**2*b**2*x**3) - B*a**2*sqrt(-a/b)*log(4*sqrt(x)*(-a/b)
**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x**3) - B*a*b*x**
3*sqrt(-a/b)*log(sqrt(x) - (-a/b)**(1/6))/(6*a**3*b + 6*a**2*b**2*x**3)...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(Ba-Ab)x^{\frac{3}{2}}}{3\,(ab^2x^3+a^2b)} + \frac{(Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}ab}$$

input
```
integrate(x^(1/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(B*a - A*b)*x^(3/2)/(a*b^2*x^3 + a^2*b) + 1/3*(B*a + A*b)*arctan(b*x^
(3/2)/sqrt(a*b))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}ab} - \frac{Bax^{\frac{3}{2}} - Abx^{\frac{3}{2}}}{3\,(bx^3+a)ab}$$

input
```
integrate(x^(1/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*(B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/3*(B*a*x^(
3/2) - A*b*x^(3/2))/((b*x^3 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{B\,a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right) + A\,b^2\,x^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right) + A\,a\,b\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right) + A\,\sqrt{a}\,b^{3/2}\,x^{3/2} - B\,a^{3/2}\,\sqrt{b}\,x^{3/2} + B\,a}{3\,a^{5/2}\,b^{3/2} + 3\,a^{3/2}\,b^{5/2}\,x^3}$$

input
```
int((x^(1/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
(B*a^2*atan((b^(1/2)*x^(3/2))/a^(1/2)) + A*b^2*x^3*atan((b^(1/2)*x^(3/2))/
a^(1/2)) + A*a*b*atan((b^(1/2)*x^(3/2))/a^(1/2)) + A*a^(1/2)*b^(3/2)*x^(3/
2) - B*a^(3/2)*b^(1/2)*x^(3/2) + B*a*b*x^3*atan((b^(1/2)*x^(3/2))/a^(1/2))
)/(3*a^(5/2)*b^(3/2) + 3*a^(3/2)*b^(5/2)*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{-\frac{2atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)}{3}+\frac{2atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)}{3}-\frac{2atan\left(\frac{\sqrt{x}\,b^{\frac{1}{6}}}{a^{\frac{1}{6}}}\right)}{3}}{\sqrt{b}\,\sqrt{a}}$$

input
```
int(x^(1/2)*(B*x^3+A)/(b*x^3+a)^2,x)
```

output
```
(2*b**(1/6)*a**(1/6)*( - atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*b**(1
/3))/(b**(1/6)*a**(1/6))) + atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*b*
*(1/3))/(b**(1/6)*a**(1/6))) - atan((sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6))
)))/(3*b**(2/3)*a**(2/3))
```

3.143 $\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)^2} dx = -\frac{2A}{3a^2x^{3/2}} - \frac{(Ab-aB)x^{3/2}}{3a^2(a+bx^3)} - \frac{(3Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}$$

output
```
-2/3*A/a^2/x^(3/2)-1/3*(A*b-B*a)*x^(3/2)/a^2/(b*x^3+a)-1/3*(3*A*b-B*a)*arc
tan(b^(1/2)*x^(3/2)/a^(1/2))/a^(5/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)^2} dx = \frac{-2aA-3Abx^3+aBx^3}{3a^2x^{3/2}(a+bx^3)} + \frac{(-3Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}$$

input
```
Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x]
```

output
```
(-2*a*A - 3*A*b*x^3 + a*B*x^3)/(3*a^2*x^(3/2)*(a + b*x^3)) + ((-3*A*b + a*
B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(5/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {957, 847, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{(3Ab-aB)\int \frac{1}{x^{5/2}(bx^3+a)}dx}{2ab}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{(3Ab-aB)\left(-\frac{b\int \frac{\sqrt{x}}{bx^3+a}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{2ab}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{(3Ab-aB)\left(-\frac{2b\int \frac{x}{bx^3+a}d\sqrt{x}}{a}-\frac{2}{3ax^{3/2}}\right)}{2ab}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{(3Ab-aB)\left(-\frac{2b\int \frac{1}{a+bx}dx^{3/2}}{3a}-\frac{2}{3ax^{3/2}}\right)}{2ab}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(3Ab-aB)\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{2}{3ax^{3/2}}\right)}{2ab}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}$$

input
```
Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x]
```

output
```
(A*b - a*B)/(3*a*b*x^(3/2)*(a + b*x^3)) + ((3*A*b - a*B)*(-2/(3*a*x^(3/2))
 - (2*Sqrt[b]*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(3/2))))/(2*a*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{2A}{3a^2x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)x^{\frac{3}{2}}}{b\,x^3+a}+\frac{(3Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{3a^2}$	66
default	$-\frac{2A}{3a^2x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)x^{\frac{3}{2}}}{b\,x^3+a}+\frac{(3Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{3a^2}$	66
risch	$-\frac{2A}{3a^2x^{\frac{3}{2}}}-\frac{\frac{2\left(\frac{Ab}{2}-\frac{Ba}{2}\right)x^{\frac{3}{2}}}{3\left(b\,x^3+a\right)}+\frac{(3Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}}}{a^2}$	67

input
```
int((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*A/a^2/x^(3/2)-2/3/a^2*((1/2*A*b-1/2*B*a)*x^(3/2)/(b*x^3+a)+1/2*(3*A*b
-B*a)/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.80

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx = \left[\frac{((Bab-3\,Ab^2)x^5+(Ba^2-3\,Aab)x^2)\sqrt{-ab}\log\left(\frac{bx^3+2\,\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)-2\,(2\,Aa^2b-(}{6\,(a^3b^2x^5+a^4bx^2)}\right.$$

input
```
integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/6*(((B*a*b - 3*A*b^2)*x^5 + (B*a^2 - 3*A*a*b)*x^2)*sqrt(-a*b)*log((b*x^
3 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) - 2*(2*A*a^2*b - (B*a^2*b - 3*A
*a*b^2)*x^3)*sqrt(x))/(a^3*b^2*x^5 + a^4*b*x^2), 1/3*(((B*a*b - 3*A*b^2)*x
^5 + (B*a^2 - 3*A*a*b)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) - (2*A*a
^2*b - (B*a^2*b - 3*A*a*b^2)*x^3)*sqrt(x))/(a^3*b^2*x^5 + a^4*b*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)/x**(5/2)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)^2}\,dx = \frac{(Ba-3\,Ab)x^3-2\,Aa}{3\left(a^2bx^{\frac{9}{2}}+a^3x^{\frac{3}{2}}\right)} + \frac{(Ba-3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a^2}$$

input
```
integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*((B*a - 3*A*b)*x^3 - 2*A*a)/(a^2*b*x^(9/2) + a^3*x^(3/2)) + 1/3*(B*a -
 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx=\frac{(Ba-3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a^2}+\frac{Bax^3-3\,Abx^3-2\,Aa}{3\left(bx^{\frac{9}{2}}+ax^{\frac{3}{2}}\right)a^2}$$

input
```
integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*(B*a - 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(B*a*x
^3 - 3*A*b*x^3 - 2*A*a)/((b*x^(9/2) + a*x^(3/2))*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.67

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx=$$

$$-\frac{2\,A\,a^{3/2}\,\sqrt{b}-B\,a^2\,x^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+3\,A\,b^2\,x^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+3\,A\,\sqrt{a}\,b^{3/2}\,x^3-B\,a^{3/2}\,\sqrt{b}\,x^3+3\,A\,a}{3\,a^{7/2}\,\sqrt{b}\,x^{3/2}+3\,a^{5/2}\,b^{3/2}\,x^{9/2}}$$

input
```
int((A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x)
```

output
```
-(2*A*a^(3/2)*b^(1/2) - B*a^2*x^(3/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)) + 3*
A*b^2*x^(9/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)) + 3*A*a^(1/2)*b^(3/2)*x^3 -
B*a^(3/2)*b^(1/2)*x^3 + 3*A*a*b*x^(3/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)) -
B*a*b*x^(9/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)))/(3*a^(7/2)*b^(1/2)*x^(3/2)
+ 3*a^(5/2)*b^(3/2)*x^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.39

$$\int \frac{A+Bx^3}{x^{5/2}\,(a+bx^3)^2}\,dx = \frac{\frac{2\sqrt{x}\,b^{\frac{7}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)x}{3}-\frac{2\sqrt{x}\,b^{\frac{7}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)x}{3}+\frac{2\sqrt{x}\,b^{\frac{7}{6}}a^{\frac{1}{6}}\,atan\left(\frac{\sqrt{x}\,b^{\frac{1}{6}}}{a^{\frac{1}{6}}}\right)x}{3}}{\sqrt{x}\,b^{\frac{2}{3}}a^{\frac{5}{3}}x}$$

input
```
int((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x)
```

output
```
(2*(sqrt(x)*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*
b**(1/3))/(b**(1/6)*a**(1/6)))*b*x - sqrt(x)*b**(1/6)*a**(1/6)*atan((b**(1
/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*b*x + sqrt
(x)*b**(1/6)*a**(1/6)*atan((sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*b*x - b
**(2/3)*a**(2/3)))/(3*sqrt(x)*b**(2/3)*a**(2/3)*a*x)
```

3.144 $\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 236

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{2B\sqrt{x}}{b^2} - \frac{(Ab-aB)\sqrt{x}}{3b^2(a+bx^3)} - \frac{(Ab-7aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}} + \frac{(Ab-7aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}} + \frac{(Ab-7aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}} + \frac{(Ab-7aB)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{6\sqrt{3}a^{5/6}b^{13/6}}$$

output
```
2*B*x^(1/2)/b^2-1/3*(A*b-B*a)*x^(1/2)/b^2/(b*x^3+a)+1/18*(A*b-7*B*a)*arcta
n(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(5/6)/b^(13/6)+1/18*(A*b-7*B*a)*ar
ctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(5/6)/b^(13/6)+1/9*(A*b-7*B*a)*a
rctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(5/6)/b^(13/6)+1/18*(A*b-7*B*a)*arctanh(3
^(1/2)*a^(1/6)*b^(1/6)*x^(1/2)/(a^(1/3)+b^(1/3)*x))*3^(1/2)/a^(5/6)/b^(13/
6)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.77

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{\frac{6\sqrt[6]{b}\sqrt{x}(-Ab+7aB+6bBx^3)}{a+bx^3}+\frac{2(Ab-7aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{a^{5/6}}+\frac{(-Ab+7aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{a^{5/6}}+\cdots}{18b^{13/6}}$$

input
```
Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((6*b^(1/6)*Sqrt[x]*(-(A*b) + 7*a*B + 6*b*B*x^3))/(a + b*x^3) + (2*(A*b -
7*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/a^(5/6) + ((-(A*b) + 7*a*B)*ArcT
an[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/a^(5/6) + (Sqrt[3]*(A
*b - 7*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x
)])/a^(5/6))/(18*b^(13/6))
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {957, 843, 851, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^{7/2}(Ab-aB)}{3ab(a+bx^3)}-\frac{(Ab-7aB)\int \frac{x^{5/2}}{bx^3+a}dx}{6ab}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^{7/2}(Ab-aB)}{3ab(a+bx^3)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(bx^3+a)}dx}{b}\right)}{6ab}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{x^{7/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{bx^3+a}d\sqrt{x}}{b}\right)}{6ab}$$

$$\Big\downarrow \text{ 753}$$

$$\frac{x^{7/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int\frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}+\frac{\int\frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}\right)}{b}\right)}{6ab}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^{7/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int\frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int\frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}\right)}{b}\right)}{6ab}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{x^{7/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int\frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}\right)}{b}\right)}{6ab}$$

$\downarrow$ 1142

$$\frac{x^{7/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int-\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{}}\cdots}{\cdots}\right.}{b}\cdots\right.}{6ab}$$

$\downarrow$ 25

$$\frac{x^{7/2}(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\sqrt{3}\int\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}\cdots}\cdots}{\cdots}\cdots\right)}{b}\cdots\right)}{6ab}$$

$\downarrow$ 27

$$\frac{x^{7/2}(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}\cdots}\cdots}{\cdots}\cdots\right)}{b}\cdots\right)}{6ab}$$

$\downarrow$ 1082

$$\frac{x^{7/2}(Ab-aB)}{3ab\,(a+bx^3)} - \frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b} - \frac{2a\left(\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}\,d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}} + \frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}\,d\sqrt{x}}{6a^{5/6}} + \frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}\,d\sqrt{x} - \cdots}{6a^{5/6}} \cdots\right)}{b}\right)}{6ab}$$

$\downarrow$ 217

$$\frac{x^{7/2}(Ab-aB)}{3ab\,(a+bx^3)} - \frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b} - \frac{2a\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}\,d\sqrt{x} - \frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6a^{5/6}} + \frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}\,d\sqrt{x} + \cdots}{6a^{5/6}} \cdots\right)}{b}\right)}{6ab}$$

$\downarrow$ 1103

$$\frac{x^{7/2}(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}+\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+\cdots\right)\right)}{\sqrt[6]{b}}\cdots\right)}{b}\right)}{6ab}$$

input
```
Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^(7/2))/(3*a*b*(a + b*x^3)) - ((A*b - 7*a*B)*((2*Sqrt[x])/b
- (2*a*(ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3*a^(5/6)*b^(1/6)) + (-(ArcTan[
Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6)) - (Sqrt[3]*L
og[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6
*a^(5/6)) + (ArcTan[Sqrt[3]*(1 + (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b
^(1/6) + (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*
x])/(2*b^(1/6)))/(6*a^(5/6))))/b))/(6*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{6}+\frac{Ba}{6}\right)\sqrt{x}}{b\,x^3+a}+\frac{(Ab-7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\cdots\right.}{6a}\cdots\right)}{3}}{b^2}$
default	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{6}+\frac{Ba}{6}\right)\sqrt{x}}{b\,x^3+a}+\frac{(Ab-7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\cdots\right.}{6a}\cdots\right)}{3}}{b^2}$
risch	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{6}+\frac{Ba}{6}\right)\sqrt{x}}{b\,x^3+a}+\frac{(Ab-7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\cdots\right.}{6a}\cdots\right)}{3}}{b^2}$

input
```
int(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*B*x^(1/2)/b^2+2/b^2*((-1/6*A*b+1/6*B*a)*x^(1/2)/(b*x^3+a)+1/6*(A*b-7*B*a
)*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(1/6
)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2
*x^(1/2)/(a/b)^(1/6)+3^(1/2))-1/12/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^
(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6
)-3^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1426 vs. $2(176) = 352$.

Time = 0.15 (sec) , antiderivative size = 1426, normalized size of antiderivative = 6.04

$$\int \frac{x^{5/2}(A + Bx^3)}{(a + bx^3)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/36*(2*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A
^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b
^5 + A^6*b^6)/(a^5*b^13))^(1/6)*log(a*b^2*(-(117649*B^6*a^6 - 100842*A*B^5
*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^
4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6) - (7*B*a - A*b)*sqrt(x)) -
 2*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^
4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 +
A^6*b^6)/(a^5*b^13))^(1/6)*log(-a*b^2*(-(117649*B^6*a^6 - 100842*A*B^5*a^5
*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 -
42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6) - (7*B*a - A*b)*sqrt(x)) + (b^
3*x^3 + a*b^2 + sqrt(-3)*(b^3*x^3 + a*b^2))*(-(117649*B^6*a^6 - 100842*A*B
^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*
b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*log(-(7*B*a - A*b)*sqrt(
x) + 1/2*(sqrt(-3)*a*b^2 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b +
 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A
^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)) - (b^3*x^3 + a*b^2 + sqrt(-3)*(b^
3*x^3 + a*b^2))*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4
*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b
^6)/(a^5*b^13))^(1/6)*log(-(7*B*a - A*b)*sqrt(x) - 1/2*(sqrt(-3)*a*b^2 + a
*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1658 vs. $2(228) = 456$.

Time = 170.94 (sec) , antiderivative size = 1658, normalized size of antiderivative = 7.03

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(5*x**(5/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(7/2)/7 + 2*B*x**(13/2)/13)/a**2, Eq(b, 0)), ((-2*A/(5*x**(5/2)) +
2*B*sqrt(x))/b**2, Eq(a, 0)), (-12*A*a*b*sqrt(x)/(36*a**2*b**2 + 36*a*b**3
*x**3) - 2*A*a*b*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(36*a**2*b**2
+ 36*a*b**3*x**3) + 2*A*a*b*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(36
*a**2*b**2 + 36*a*b**3*x**3) - A*a*b*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**
(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2 + 36*a*b**3*x**3) + A*a*b*(-a
/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b
**2 + 36*a*b**3*x**3) + 2*sqrt(3)*A*a*b*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(
x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**2*b**2 + 36*a*b**3*x**3) + 2*sqrt
(3)*A*a*b*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)
/3)/(36*a**2*b**2 + 36*a*b**3*x**3) - 2*A*b**2*x**3*(-a/b)**(1/6)*log(sqrt
(x) - (-a/b)**(1/6))/(36*a**2*b**2 + 36*a*b**3*x**3) + 2*A*b**2*x**3*(-a/b
)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**2*b**2 + 36*a*b**3*x**3) - A*
b**2*x**3*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/
3))/(36*a**2*b**2 + 36*a*b**3*x**3) + A*b**2*x**3*(-a/b)**(1/6)*log(4*sqrt
(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2 + 36*a*b**3*x**3)
 + 2*sqrt(3)*A*b**2*x**3*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(
1/6)) - sqrt(3)/3)/(36*a**2*b**2 + 36*a*b**3*x**3) + 2*sqrt(3)*A*b**2*x**3
*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.32

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ba-Ab)\sqrt{x}}{3\,(b^3x^3+ab^2)} + \frac{2\,B\sqrt{x}}{b^2}$$

$$-\frac{\frac{\sqrt{3}(7\,Ba-Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(7\,Ba-Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(7\,Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + \frac{2\,(\cdots}{\cdots}}{36\,b^2}$$

input
```
integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(B*a - A*b)*sqrt(x)/(b^3*x^3 + a*b^2) + 2*B*sqrt(x)/b^2 - 1/36*(sqrt(3
)*(7*B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))
/(a^(5/6)*b^(1/6)) - sqrt(3)*(7*B*a - A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sq
rt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(7*B*a*b^(1/3) - A*b^(4
/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a
^(1/3)*b^(1/3))) + 2*(7*B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan((sqr
t(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3
)*sqrt(a^(1/3)*b^(1/3))) + 2*(7*B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arc
tan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/
(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/b^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.33

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{2\,B\sqrt{x}}{b^2} - \frac{\sqrt{3}\left(7\,(ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,ab^3} + \frac{\sqrt{3}\left(7\,(ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,ab^3} + \frac{Ba\sqrt{x}-Ab\sqrt{x}}{3\,(bx^3+a)b^2} - \frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,ab^3} - \frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,ab^3} - \frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,ab^3}$$

input
```
integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
2*B*sqrt(x)/b^2 - 1/36*sqrt(3)*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*l
og(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^3) + 1/36*sqrt(3)*(
7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6)
+ x + (a/b)^(1/3))/(a*b^3) + 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*
b^2) - 1/18*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b
)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a*b^3) - 1/18*(7*(a*b^5)^(1/6)*B*a - (a
*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a
*b^3) - 1/9*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)
^(1/6))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 1884, normalized size of antiderivative = 7.98

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int((x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
(2*B*x^(1/2))/b^2 - (x^(1/2)*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) - (ata
n(((((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3
*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) - (2*(A*b - 7*B*a)*(343*B^3*a^4 - A^3*a
*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A*b
 - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)) + (((2*x^(1/2)*(A^4*b^4 + 2401*B^4*
a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) +
 (2*(A*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^
2*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6
)))/((((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B
^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) - (2*(A*b - 7*B*a)*(343*B^3*a^4 - A^3
*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A
*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6)) - (((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a
^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) +
(2*(A*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2
*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6))))
*(A*b - 7*B*a)*1i)/(9*(-a)^(5/6)*b^(13/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2
)*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a
^3*b - 28*A^3*B*a*b^3))/(27*b^3) - (2*((3^(1/2)*1i)/2 - 1/2)*(A*b - 7*B*a)
*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^
(5/6)*b^(19/6)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)) + (((3^(1/2...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.67

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{2b^{\frac{1}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-2b^{\frac{1}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-4b^{\frac{1}{6}}a^{\frac{1}{6}}\,atan\left(\frac{\sqrt{x}\,b^{\frac{1}{6}}}{a^{\frac{1}{6}}}\right)}{}$$

input
```
int(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x)
```

output
```
(2*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*b**(1/3))
/(b**(1/6)*a**(1/6))) - 2*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3
) + 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6))) - 4*b**(1/6)*a**(1/6)*atan((s
qrt(x)*b**(1/3))/(b**(1/6)*a**(1/6))) + b**(1/6)*a**(1/6)*sqrt(3)*log( - s
qrt(x)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*x) - b**(1/6)*a**(1
/6)*sqrt(3)*log(sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*x)
 + 12*sqrt(x)*b**(1/3))/(6*b**(1/3)*b)
```

3.145 $\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 228

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx=\frac{(Ab-aB)x^{5/2}}{3ab\,(a+bx^3)}-\frac{(Ab+5aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}}+\frac{(Ab+5aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}}+\frac{(Ab+5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}}-\frac{(Ab+5aB)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{6\sqrt{3}a^{7/6}b^{11/6}}$$

output
```
1/3*(A*b-B*a)*x^(5/2)/a/b/(b*x^3+a)+1/18*(A*b+5*B*a)*arctan(-3^(1/2)+2*b^(
1/6)*x^(1/2)/a^(1/6))/a^(7/6)/b^(11/6)+1/18*(A*b+5*B*a)*arctan(3^(1/2)+2*b
^(1/6)*x^(1/2)/a^(1/6))/a^(7/6)/b^(11/6)+1/9*(A*b+5*B*a)*arctan(b^(1/6)*x^
(1/2)/a^(1/6))/a^(7/6)/b^(11/6)-1/18*(A*b+5*B*a)*arctanh(3^(1/2)*a^(1/6)*b
^(1/6)*x^(1/2)/(a^(1/3)+b^(1/3)*x))*3^(1/2)/a^(7/6)/b^(11/6)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.74

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{-\frac{6\sqrt[6]{a}b^{5/6}(-Ab+aB)x^{5/2}}{a+bx^3}+2(Ab+5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-(Ab+5aB)\arctan\left(\frac{\sqrt[3]{a}}{\sqrt[6]{a}}\right.}{18a^{7/6}b^{11/6}}$$

input
```
Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((-6*a^(1/6)*b^(5/6)*(-(A*b) + a*B)*x^(5/2))/(a + b*x^3) + 2*(A*b + 5*a*B)
*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - (A*b + 5*a*B)*ArcTan[(a^(1/3) - b^(1/
3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] - Sqrt[3]*(A*b + 5*a*B)*ArcTanh[(Sqrt[3]*
a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(18*a^(7/6)*b^(11/6))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.26, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {957, 851, 824, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{(5aB+Ab)\int \frac{x^{3/2}}{bx^3+a}dx}{6ab}+\frac{x^{5/2}(Ab-aB)}{3ab\,(a+bx^3)}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{(5aB+Ab)\int \frac{x^2}{bx^3+a}d\sqrt{x}}{3ab}+\frac{x^{5/2}(Ab-aB)}{3ab\,(a+bx^3)}$$

$$\Big\downarrow \text{ 824}$$

$$\frac{(5aB+Ab)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}+\frac{\int -\frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}+\frac{\int -\frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}\right)}{3ab}+\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$\big\downarrow$ 27

$$\frac{(5aB+Ab)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}\right)}{3ab}+\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$\big\downarrow$ 218

$$\frac{(5aB+Ab)\left(-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}\right)}{3ab}+\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$\big\downarrow$ 1142

$$\frac{(5aB+Ab)\left(-\frac{-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}\right.}{3ab}$$

$$\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$\big\downarrow$ 25

$$\frac{(5aB+Ab)\left(-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-}{6\sqrt[6]{}}\right.}{3ab}$$

$$\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$\big\downarrow$ 27

$$\frac{(5aB+Ab)\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d}{}\right.}{3ab}$$

$$\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$\big\downarrow$ 1082

$$\frac{(5aB+Ab)\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}\right.}{3ab}$$

$$\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$\big\downarrow$ 217

$$\frac{(5aB+Ab)\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{}\right.}{}}{6\sqrt[6]{a}b^{2/3}}\right.}{3ab}$$

$$\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{(5aB+Ab)\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{\cdots}\right)}{2\sqrt[6]{b}}\cdots}{6\sqrt[6]{\cdots}}\right.}{3ab}$$

$$\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

input
```
Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^(5/2))/(3*a*b*(a + b*x^3)) + ((A*b + 5*a*B)*(ArcTan[(b^(1/6
)*Sqrt[x])/a^(1/6)]/(3*a^(1/6)*b^(5/6)) - (ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*
Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1
/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(1/6)*b^(2/3)) - (-(Ar
cTan[Sqrt[3]*(1 + (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6)) + (Sqrt
[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)
))/(6*a^(1/6)*b^(2/3))))/(3*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{(Ab-Ba)x^{\frac{5}{2}}}{3ab(b\,x^3+a)}+\frac{(Ab+5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{5}{6}}\sqrt{3}\,\ln\left(x-\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{\ldots}\right.}{12a}\ldots\right.}{3ab}\ldots$
default	$\frac{(Ab-Ba)x^{\frac{5}{2}}}{3ab(b\,x^3+a)}+\frac{(Ab+5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{5}{6}}\sqrt{3}\,\ln\left(x-\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{\ldots}\right.}{12a}\ldots\right.}{3ab}\ldots$

input
```
int(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(A*b-B*a)*x^(5/2)/a/b/(b*x^3+a)+1/3*(A*b+5*B*a)/a/b*(1/12/a*(a/b)^(5/6
)*3^(1/2)*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*
arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))-1/12/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1
/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a
/b)^(1/6)+3^(1/2))+1/3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1773 vs. 2(166) = 332.

Time = 0.15 (sec) , antiderivative size = 1773, normalized size of antiderivative = 7.78

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/36*(12*(B*a - A*b)*x^(5/2) - 2*(a*b^2*x^3 + a^2*b)*(-(15625*B^6*a^6 + 1
8750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B
^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(a^6*b^9*(-(15
625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*
b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6) +
(3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*A^3*B^2*a^2*
b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) + 2*(a*b^2*x^3 + a^2*b)*(-(15625*
B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3
+ 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(-a
^6*b^9*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*
A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^1
1))^(5/6) + (3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*
A^3*B^2*a^2*b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) - (a*b^2*x^3 + a^2*b
- sqrt(-3)*(a*b^2*x^3 + a^2*b))*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 937
5*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*
a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(1/2*(sqrt(-3)*a^6*b^9 + a^6*b^9)*(-
(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a
^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6)
 + (3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*A^3*B^2*a
^2*b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) + (a*b^2*x^3 + a^2*b - sqrt...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1885 vs. $2(216) = 432$.

Time = 120.59 (sec) , antiderivative size = 1885, normalized size of antiderivative = 8.27

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(5/2)/5 + 2*B*x**(11/2)/11)/a**2, Eq(b, 0)), ((-2*A/(7*x**(7/2)) -
2*B/sqrt(x))/b**2, Eq(a, 0)), (2*A*a*b*log(sqrt(x) - (-a/b)**(1/6))/(36*a*
*2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) - 2*A*a*b*log(sqrt(x
) + (-a/b)**(1/6))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1
/6)) + A*a*b*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**
2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) - A*a*b*log(4*sqrt(x)
*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a
*b**3*x**3*(-a/b)**(1/6)) + 2*sqrt(3)*A*a*b*atan(2*sqrt(3)*sqrt(x)/(3*(-a/
b)**(1/6)) - sqrt(3)/3)/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b
)**(1/6)) + 2*sqrt(3)*A*a*b*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqr
t(3)/3)/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 12*A
*b**2*x**(5/2)*(-a/b)**(1/6)/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*
(-a/b)**(1/6)) + 2*A*b**2*x**3*log(sqrt(x) - (-a/b)**(1/6))/(36*a**2*b**2*
(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) - 2*A*b**2*x**3*log(sqrt(x)
+ (-a/b)**(1/6))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6
)) + A*b**2*x**3*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36
*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) - A*b**2*x**3*log
(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2*(-a/b)**(1
/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 2*sqrt(3)*A*b**2*x**3*atan(2*sqrt...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.03

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(Ba-Ab)x^{\frac{5}{2}}}{3\,(ab^2x^3+a^2b)} - \frac{(5\,Ba+Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}} - \frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}} - \frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} - \frac{2\arctan\left(-\right.}{b^{\frac{2}{3}}}\right.}{36\,ab}$$

input
```
integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(B*a - A*b)*x^(5/2)/(a*b^2*x^3 + a^2*b) - 1/36*(5*B*a + A*b)*(sqrt(3)
*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/
6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/
(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))
/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(
3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sq
rt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^
(2/3)*sqrt(a^(1/3)*b^(1/3))))/(a*b)
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.26

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(5\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,(ab^5)^{\frac{1}{6}}\,ab} + \frac{(5\,Ba+Ab)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,(ab^5)^{\frac{1}{6}}\,ab} + \frac{\left(5\,Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}+Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^2b} - \frac{Bax^{\frac{5}{2}}-Abx^{\frac{5}{2}}}{3\,(bx^3+a)ab} - \frac{\sqrt{3}\left(5\,(ab^5)^{\frac{5}{6}}\,Ba+(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^2b^6} + \frac{\sqrt{3}\left(5\,(ab^5)^{\frac{5}{6}}\,Ba+(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^2b^6}$$

input
```
integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/18*(5*B*a + A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(
(a*b^5)^(1/6)*a*b) + 1/18*(5*B*a + A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*s
qrt(x))/(a/b)^(1/6))/((a*b^5)^(1/6)*a*b) + 1/9*(5*B*a*(a/b)^(5/6) + A*b*(a
/b)^(5/6))*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b) - 1/3*(B*a*x^(5/2) - A*b*x^
(5/2))/((b*x^3 + a)*a*b) - 1/36*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + (a*b^5)^(5/
6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^6) + 1/3
6*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + (a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(
a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^6)
```

Mupad [B] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 1578, normalized size of antiderivative = 6.92

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int((x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
(x^(5/2)*(A*b - B*a))/(3*a*b*(a + b*x^3)) - (atan(((((3^(1/2)*1i)/2 - 1/2)
^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20
*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4
 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*
(-a)^(7/3)*b^(11/3)) - (((3^(1/2)*1i)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b
^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 + (x^(1/2)*((3^
(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B
*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)))/((((3^
(1/2)*1i)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 10
0*A*B^2*a^2*b + 20*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*
B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^
(11/6))))/(324*(-a)^(7/3)*b^(11/3)) + (((3^(1/2)*1i)/2 - 1/2)^2*(A*b + 5*B
*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2
+ (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^
3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11
/3))))*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*1i)/(9*(-a)^(7/6)*b^(11/6)) -
(atan(((((3^(1/2)*1i)/2 + 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3
*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 + 1/
2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(
-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)) - (((3^(1/2)*1i)/2 +...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.55

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{-2atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)+2atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)+4atan\left(\frac{\sqrt{x}\,b^{\frac{1}{6}}}{a^{\frac{1}{6}}}\right)+\sqrt{3}\log\left(-\cdots\right.}{6b^{\frac{5}{6}}a^{\frac{1}{6}}}$$

input
```
int(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x)
```

output
```
(b**(1/6)*a**(1/6)*( - 2*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*b**(1
/3))/(b**(1/6)*a**(1/6))) + 2*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*
b**(1/3))/(b**(1/6)*a**(1/6))) + 4*atan((sqrt(x)*b**(1/3))/(b**(1/6)*a**(1
/6))) + sqrt(3)*log( - sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + b**(
1/3)*x) - sqrt(3)*log(sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + b**(1
/3)*x)))/(6*a**(1/3)*b)
```

3.146 $\int \frac{A+Bx^3}{\sqrt{x}(a+bx^3)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 228

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^2}\,dx = \frac{(Ab-aB)\sqrt{x}}{3ab\,(a+bx^3)} - \frac{(5Ab+aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}} + \frac{(5Ab+aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}} + \frac{(5Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}} + \frac{(5Ab+aB)\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{6\sqrt{3}a^{11/6}b^{7/6}}$$

output
```
1/3*(A*b-B*a)*x^(1/2)/a/b/(b*x^3+a)+1/18*(5*A*b+B*a)*arctan(-3^(1/2)+2*b^(
1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(7/6)+1/18*(5*A*b+B*a)*arctan(3^(1/2)+2*b
^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(7/6)+1/9*(5*A*b+B*a)*arctan(b^(1/6)*x^
(1/2)/a^(1/6))/a^(11/6)/b^(7/6)+1/18*(5*A*b+B*a)*arctanh(3^(1/2)*a^(1/6)*b
^(1/6)*x^(1/2)/(a^(1/3)+b^(1/3)*x))*3^(1/2)/a^(11/6)/b^(7/6)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^2}\,dx = \frac{-\frac{6a^{5/6}\sqrt[6]{b}(-Ab+aB)\sqrt{x}}{a+bx^3}+2(5Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-(5Ab+aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)+\sqrt{3}(5Ab+a}{18a^{11/6}b^{7/6}}$$

input
```
Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^2),x]
```

output
```
((-6*a^(5/6)*b^(1/6)*(-(A*b) + a*B)*Sqrt[x])/(a + b*x^3) + 2*(5*A*b + a*B)
*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - (5*A*b + a*B)*ArcTan[(a^(1/3) - b^(1/
3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] + Sqrt[3]*(5*A*b + a*B)*ArcTanh[(Sqrt[3]*
a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(18*a^(11/6)*b^(7/6))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {957, 851, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(aB+5Ab)\int \frac{1}{\sqrt{x}(bx^3+a)}dx}{6ab}+\frac{\sqrt{x}(Ab-aB)}{3ab\,(a+bx^3)}$$

$$\Big\downarrow \text{851}$$

$$\frac{(aB+5Ab)\int \frac{1}{bx^3+a}d\sqrt{x}}{3ab}+\frac{\sqrt{x}(Ab-aB)}{3ab\,(a+bx^3)}$$

$$\Big\downarrow \text{753}$$

$$\frac{(aB+5Ab)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}\right)}{3ab}+\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{(aB+5Ab)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}\right)}{3ab}+\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{(aB+5Ab)\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}\right)}{3ab}+\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{(aB+5Ab)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}\cdots}{6a\cdots}\cdots\right)}{3ab}+\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{(aB+5Ab)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5}}\right.}{3ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{(aB+5Ab)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d}{}\right.}{3ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{(aB+5Ab)\left(\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(2\right.}{\sqrt{3}}}{6a^{5/6}}\right.}{3ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{(aB+5Ab)\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\right.}{}}{6a^{5/6}}\right.}{3ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{(aB+5Ab)\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}+\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)}{\sqrt[6]{b}}+\cdots}{\cdots}\right.}{3ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

input
```
Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^2),x]
```

output
```
((A*b - a*B)*Sqrt[x])/(3*a*b*(a + b*x^3)) + ((5*A*b + a*B)*(ArcTan[(b^(1/6
)*Sqrt[x])/a^(1/6)]/(3*a^(5/6)*b^(1/6)) + (-(ArcTan[Sqrt[3]*(1 - (2*b^(1/6
)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6)) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a
^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6)) + (ArcTan[Sq
rt[3]*(1 + (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6) + (Sqrt[3]*Log[
a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^
(5/6))))/(3*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{(Ab-Ba)\sqrt{x}}{3ab(b\,x^3+a)}+\frac{(5Ab+Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{\right.}}{6a}\right.}{3ab}$
default	$\frac{(Ab-Ba)\sqrt{x}}{3ab(b\,x^3+a)}+\frac{(5Ab+Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{\right.}}{6a}\right.}{3ab}$

input
```
int((B*x^3+A)/x^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(A*b-B*a)*x^(1/2)/a/b/(b*x^3+a)+1/3*(5*A*b+B*a)/a/b*(1/3/a*(a/b)^(1/6)
*arctan(x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)
^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)
+3^(1/2))-1/12/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b
)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1417 vs. $2(166) = 332$.

Time = 0.12 (sec) , antiderivative size = 1417, normalized size of antiderivative = 6.21

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/36*(2*(a*b^2*x^3 + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*
b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15
625*A^6*b^6)/(a^11*b^7))^(1/6)*log(a^2*b*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375
*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5
*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6) + (B*a + 5*A*b)*sqrt(x)) - 2*(
a*b^2*x^3 + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 250
0*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b
^6)/(a^11*b^7))^(1/6)*log(-a^2*b*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4
*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5
 + 15625*A^6*b^6)/(a^11*b^7))^(1/6) + (B*a + 5*A*b)*sqrt(x)) + (a*b^2*x^3
+ a^2*b + sqrt(-3)*(a*b^2*x^3 + a^2*b))*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*
A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*
B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*log((B*a + 5*A*b)*sqrt(x) + 1/2
*(sqrt(-3)*a^2*b + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^
2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 1562
5*A^6*b^6)/(a^11*b^7))^(1/6)) - (a*b^2*x^3 + a^2*b + sqrt(-3)*(a*b^2*x^3 +
 a^2*b))*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*
a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*
b^7))^(1/6)*log((B*a + 5*A*b)*sqrt(x) - 1/2*(sqrt(-3)*a^2*b + a^2*b)*(-(B^
6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1632 vs. $2(216) = 432$.

Time = 95.63 (sec) , antiderivative size = 1632, normalized size of antiderivative = 7.16

$$\int \frac{A + Bx^3}{\sqrt{x}\,(a + bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x**3+A)/x**(1/2)/(b*x**3+a)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(5*x**(5/2))), Eq(a, 0) & Eq(b,
0)), ((2*A*sqrt(x) + 2*B*x**(7/2)/7)/a**2, Eq(b, 0)), ((-2*A/(11*x**(11/2)
) - 2*B/(5*x**(5/2)))/b**2, Eq(a, 0)), (12*A*a*b*sqrt(x)/(36*a**3*b + 36*a
**2*b**2*x**3) - 10*A*a*b*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(36*a
**3*b + 36*a**2*b**2*x**3) + 10*A*a*b*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**
(1/6))/(36*a**3*b + 36*a**2*b**2*x**3) - 5*A*a*b*(-a/b)**(1/6)*log(-4*sqrt
(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b + 36*a**2*b**2*x**3)
 + 5*A*a*b*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/
3))/(36*a**3*b + 36*a**2*b**2*x**3) + 10*sqrt(3)*A*a*b*(-a/b)**(1/6)*atan(
2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**3*b + 36*a**2*b**2
*x**3) + 10*sqrt(3)*A*a*b*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**
(1/6)) + sqrt(3)/3)/(36*a**3*b + 36*a**2*b**2*x**3) - 10*A*b**2*x**3*(-a/b
)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(36*a**3*b + 36*a**2*b**2*x**3) + 10
*A*b**2*x**3*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**3*b + 36*a*
*2*b**2*x**3) - 5*A*b**2*x**3*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) +
 4*x + 4*(-a/b)**(1/3))/(36*a**3*b + 36*a**2*b**2*x**3) + 5*A*b**2*x**3*(-
a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*
b + 36*a**2*b**2*x**3) + 10*sqrt(3)*A*b**2*x**3*(-a/b)**(1/6)*atan(2*sqrt(
3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**3*b + 36*a**2*b**2*x**3)
+ 10*sqrt(3)*A*b**2*x**3*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^2}\,dx = -\frac{(Ba-Ab)\sqrt{x}}{3\,(ab^2x^3+a^2b)} + \frac{\frac{\sqrt{3}(Ba+5\,Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(Ba+5\,Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(Bab^{\frac{1}{3}}+5\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + }{36\,ab}$$

input
```
integrate((B*x^3+A)/x^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(B*a - A*b)*sqrt(x)/(a*b^2*x^3 + a^2*b) + 1/36*(sqrt(3)*(B*a + 5*A*b)
*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/
6)) - sqrt(3)*(B*a + 5*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)
*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) + 5*A*b^(4/3))*arctan(b^(
1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))
) + 2*(B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^
(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*
b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a
^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt
(a^(1/3)*b^(1/3))))/(a*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^2}\,dx = \frac{\sqrt{3}\left((ab^5)^{\frac{1}{6}}\,Ba+5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^2b^2} - \frac{\sqrt{3}\left((ab^5)^{\frac{1}{6}}\,Ba+5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^2b^2} - \frac{Ba\sqrt{x}-Ab\sqrt{x}}{3\,(bx^3+a)ab} + \frac{\left((ab^5)^{\frac{1}{6}}\,Ba+5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^2b^2} + \frac{\left((ab^5)^{\frac{1}{6}}\,Ba+5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^2b^2} + \frac{\left((ab^5)^{\frac{1}{6}}\,Ba+5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^2b^2}$$

input
```
integrate((B*x^3+A)/x^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/36*sqrt(3)*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)
*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^2) - 1/36*sqrt(3)*((a*b^5)^(1/6)*B*
a + 5*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3
))/(a^2*b^2) - 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*a*b) + 1/18*((
a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sq
rt(x))/(a/b)^(1/6))/(a^2*b^2) + 1/18*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*
A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^2) + 1/
9*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a
^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 1922, normalized size of antiderivative = 8.43

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^3)/(x^(1/2)*(a + b*x^3)^2),x)
```

output
```
(atan(((((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A
^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*(5*A*b + B*a)*(125*A^3*b^5 +
 B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)
))*(5*A*b + B*a)*1i)/(18*(-a)^(11/6)*b^(7/6)) + (((2*x^(1/2)*(625*A^4*b^5
+ B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(
27*a^4) + (2*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 1
5*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*(5*A*b + B*a)*1i)/(18*(-a)^(11
/6)*b^(7/6)))/((((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3
 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*(5*A*b + B*a)*(125*A
^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)
*b^(7/6)))*(5*A*b + B*a))/(18*(-a)^(11/6)*b^(7/6)) - (((2*x^(1/2)*(625*A^4
*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^
2))/(27*a^4) + (2*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^
4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*(5*A*b + B*a))/(18*(-a)^(
11/6)*b^(7/6))))*(5*A*b + B*a)*1i)/(9*(-a)^(11/6)*b^(7/6)) + (atan(((((3^(
1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150
*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*((3^
(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b
^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*1i)/(18*(-a)^(11/6)*b^(7
/6)) + (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 +...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.55

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^2}\,dx$$
$$= \frac{-2atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)+2atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)+4atan\left(\frac{\sqrt{x}\,b^{\frac{1}{6}}}{a^{\frac{1}{6}}}\right)-\sqrt{3}\log\left(-\sqrt{x}\,b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+a^{\frac{1}{3}}+b^{\frac{1}{3}}\cdots}{6b^{\frac{1}{6}}a^{\frac{5}{6}}}$$

input
```
int((B*x^3+A)/x^(1/2)/(b*x^3+a)^2,x)
```

output
```
(b**(1/6)*a**(1/6)*( - 2*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*b**(1
/3))/(b**(1/6)*a**(1/6))) + 2*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*
b**(1/3))/(b**(1/6)*a**(1/6))) + 4*atan((sqrt(x)*b**(1/3))/(b**(1/6)*a**(1
/6))) - sqrt(3)*log( - sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + b**(
1/3)*x) + sqrt(3)*log(sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + b**(1
/3)*x)))/(6*b**(1/3)*a)
```

3.147 $\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 240

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2} \, dx = -\frac{2A}{a^2\sqrt{x}} - \frac{(Ab-aB)x^{5/2}}{3a^2\left(a+bx^3\right)} + \frac{(7Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{13/6}b^{5/6}} - \frac{(7Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{13/6}b^{5/6}} - \frac{(7Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}} + \frac{(7Ab-aB)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{6\sqrt{3}a^{13/6}b^{5/6}}$$

output
```
-2*A/a^2/x^(1/2)-1/3*(A*b-B*a)*x^(5/2)/a^2/(b*x^3+a)-1/18*(7*A*b-B*a)*arct
an(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(5/6)-1/18*(7*A*b-B*a)*a
rctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(5/6)-1/9*(7*A*b-B*a)*
arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(5/6)+1/18*(7*A*b-B*a)*arctanh(
3^(1/2)*a^(1/6)*b^(1/6)*x^(1/2)/(a^(1/3)+b^(1/3)*x))*3^(1/2)/a^(13/6)/b^(5
/6)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx = \frac{\frac{6\sqrt[6]{a}(-6aA-7Abx^3+aBx^3)}{\sqrt{x}(a+bx^3)} + \frac{2(-7Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{b^{5/6}} + \frac{(7Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{b^{5/6}} + \cdots}{18a^{13/6}}$$

input
```
Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x]
```

output
```
((6*a^(1/6)*(-6*a*A - 7*A*b*x^3 + a*B*x^3))/(Sqrt[x]*(a + b*x^3)) + (2*(-7
*A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/b^(5/6) + ((7*A*b - a*B)*Ar
cTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/b^(5/6) + (Sqrt[3]*
(7*A*b - a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)
*x)])/b^(5/6))/(18*a^(13/6))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {957, 847, 851, 824, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \; 957$$

$$\frac{(7Ab-aB)\int \frac{1}{x^{3/2}(bx^3+a)}dx}{6ab}+\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

$$\Big\downarrow \; 847$$

$$\frac{(7Ab-aB)\left(-\frac{b\int \frac{x^{3/2}}{bx^3+a}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{6ab}+\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

$\Big\downarrow$ 851

$$\frac{(7Ab-aB)\left(-\frac{2b\int \frac{x^2}{bx^3+a}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{6ab}+\frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)}$$

$\Big\downarrow$ 824

$$\frac{(7Ab-aB)\left(-\frac{2b\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}+\frac{\int -\frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}+\frac{\int -\frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{6ab}+\frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)}$$

$\Big\downarrow$ 27

$$\frac{(7Ab-aB)\left(-\frac{2b\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{6ab}+\frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)}$$

$\Big\downarrow$ 218

$$\frac{(7Ab-aB)\left(-\frac{2b\left(-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{6ab}+\frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)}$$

$\big\downarrow$ 1142

$$\frac{(7Ab-aB)\left(-\frac{2b\left(-\frac{-\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\sqrt{3}\int\frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}\cdots}{2\sqrt[6]{b}}\cdots\right)}{a}\cdots\right)}{6ab}$$

$$\frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)}$$

$\big\downarrow$ 25

$$\frac{(7Ab-aB)\left(-\frac{2b\left(-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{\cdots}}{2\sqrt[6]{b}}\cdots}{\cdots}\cdots\right)}{a}\cdots\right)}{6ab\cdots}\cdots$$

$$\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

$\Big\downarrow$ 27

$$\frac{(7Ab-aB)\left(-\frac{2b\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\cdots}\cdots}{\cdots}\cdots\right)}{a}\cdots\right)}{6ab\cdots}\cdots$$

$$\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

$\Big\downarrow$ 1082

$$\frac{(7Ab-aB)\left(-\frac{2b\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int\frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\int\frac{1}{-x-\frac{1}{3}}d\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{\ldots}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{\ldots}}}{6\sqrt[6]{a}b^{2/3}}\ldots}{a}\ldots\right.}{6ab}\ldots$$

$$\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

$\downarrow$ 217

$$\frac{(7Ab-aB)\left(-\frac{2b\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan(\ldots}{\ldots}}{6\sqrt[6]{a}b^{2/3}}\ldots}{a}\ldots\right.}{6ab}\ldots$$

$$\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

$\downarrow$ 1103

$$\frac{(7Ab-aB)\left(-\frac{2b\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\cdots\right)}{2\sqrt[6]{b}}\cdots}{6\cdots}\right)}{a}\cdots\right)}{6ab}$$

$$\frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)}$$

input
```
Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x]
```

output
```
(A*b - a*B)/(3*a*b*Sqrt[x]*(a + b*x^3)) + ((7*A*b - a*B)*(-2/(a*Sqrt[x]) -
 (2*b*(ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3*a^(1/6)*b^(5/6)) - (ArcTan[Sqr
t[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6) - (Sqrt[3]*Log[a
^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(
1/6)*b^(2/3)) - (-(ArcTan[Sqrt[3]*(1 + (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6
)))]/b^(1/6)) + (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b
^(1/3)*x])/(2*b^(1/6)))/(6*a^(1/6)*b^(2/3))))/a))/(6*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{2A}{a^2\sqrt{x}}-\frac{2\left(\frac{\left(\frac{Ab}{6}-\frac{Ba}{6}\right)x^{\frac{5}{2}}}{b\,x^3+a}+\left(\frac{7Ab}{6}-\frac{Ba}{6}\right)\left(\frac{\left(\frac{a}{b}\right)^{\frac{5}{6}}\sqrt{3}\,\ln\left(x-\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\cdots}{}\right.\right.}{a^2}$
default	$-\frac{2A}{a^2\sqrt{x}}-\frac{2\left(\frac{\left(\frac{Ab}{6}-\frac{Ba}{6}\right)x^{\frac{5}{2}}}{b\,x^3+a}+\left(\frac{7Ab}{6}-\frac{Ba}{6}\right)\left(\frac{\left(\frac{a}{b}\right)^{\frac{5}{6}}\sqrt{3}\,\ln\left(x-\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\cdots}{}\right.\right.}{a^2}$
risch	$-\frac{2A}{a^2\sqrt{x}}-\frac{\frac{2\left(\frac{Ab}{6}-\frac{Ba}{6}\right)x^{\frac{5}{2}}}{b\,x^3+a}+2\left(\frac{7Ab}{6}-\frac{Ba}{6}\right)\left(\frac{\left(\frac{a}{b}\right)^{\frac{5}{6}}\sqrt{3}\,\ln\left(x-\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\cdots}{}\right.}{a^2}$

input
```
int((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*A/a^2/x^(1/2)-2/a^2*((1/6*A*b-1/6*B*a)*x^(5/2)/(b*x^3+a)+(7/6*A*b-1/6*B
*a)*(1/12/a*(a/b)^(5/6)*3^(1/2)*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/
3))+1/6/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))-1/12/a*3^(1/2)
*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/b/(a/b)^(1/
6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))+1/3/b/(a/b)^(1/6)*arctan(x^(1/2)/
(a/b)^(1/6))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1788 vs. $2(172) = 344$.

Time = 0.12 (sec) , antiderivative size = 1788, normalized size of antiderivative = 7.45

$$\int \frac{A + Bx^3}{x^{3/2} (a + bx^3)^2} \, dx = \text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/36*(2*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4
*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 +
 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log(a^11*b^4*(-(B^6*a^6 - 42*A*B^5*a^5*
b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 1
00842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) - (B^5*a^5 - 35*A*B^
4*a^4*b + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3 + 12005*A^4*B*a*b^4 -
 16807*A^5*b^5)*sqrt(x)) - 2*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5
*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 -
100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log(-a^11*b^4*(-(B^
6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 3601
5*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6)
 - (B^5*a^5 - 35*A*B^4*a^4*b + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3
+ 12005*A^4*B*a*b^4 - 16807*A^5*b^5)*sqrt(x)) + (a^2*b*x^4 + a^3*x - sqrt(
-3)*(a^2*b*x^4 + a^3*x))*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2
 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117
649*A^6*b^6)/(a^13*b^5))^(1/6)*log(1/2*(sqrt(-3)*a^11*b^4 + a^11*b^4)*(-(B
^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 360
15*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6
) - (B^5*a^5 - 35*A*B^4*a^4*b + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3
 + 12005*A^4*B*a*b^4 - 16807*A^5*b^5)*sqrt(x)) - (a^2*b*x^4 + a^3*x - s...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2200 vs. $2(228) = 456$.

Time = 165.90 (sec) , antiderivative size = 2200, normalized size of antiderivative = 9.17

$$\int \frac{A + Bx^3}{x^{3/2} (a + bx^3)^2} \, dx = \text{Too large to display}$$

input
```
integrate((B*x**3+A)/x**(3/2)/(b*x**3+a)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(13*x**(13/2)) - 2*B/(7*x**(7/2))), Eq(a, 0) & Eq(b,
0)), ((-2*A/sqrt(x) + 2*B*x**(5/2)/5)/a**2, Eq(b, 0)), ((-2*A/(13*x**(13/2
)) - 2*B/(7*x**(7/2)))/b**2, Eq(a, 0)), (-14*A*a*b*sqrt(x)*log(sqrt(x) - (
-a/b)**(1/6))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a
/b)**(1/6)) + 14*A*a*b*sqrt(x)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**3*b*sqr
t(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - 7*A*a*b*sqrt(x
)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b*sqrt(x)
*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 7*A*a*b*sqrt(x)*lo
g(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b*sqrt(x)*(-a/
b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - 14*sqrt(3)*A*a*b*sqrt(x
)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**3*b*sqrt(x)
*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - 14*sqrt(3)*A*a*b*s
qrt(x)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**3*b*sq
rt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - 72*A*a*b*(-a/
b)**(1/6)/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)*
*(1/6)) - 14*A*b**2*x**(7/2)*log(sqrt(x) - (-a/b)**(1/6))/(36*a**3*b*sqrt(
x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 14*A*b**2*x**(7/
2)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2
*b**2*x**(7/2)*(-a/b)**(1/6)) - 7*A*b**2*x**(7/2)*log(-4*sqrt(x)*(-a/b)**(
1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**...
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx = \frac{(Ba-7\,Ab)x^3-6\,Aa}{3\left(a^2bx^{\frac{7}{2}}+a^3\sqrt{x}\right)}$$
$$-\frac{(Ba-7\,Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(-\cdots\right.}{b^{\cdots}}\cdots\right.}{36\,a^2}$$

input
```
integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*((B*a - 7*A*b)*x^3 - 6*A*a)/(a^2*b*x^(7/2) + a^3*sqrt(x)) - 1/36*(B*a
- 7*A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3
))/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1
/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2
*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) -
 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1
/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1
/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/a^2
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx = \frac{(Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,\left(ab^5\right)^{\frac{1}{6}}\,a^2} + \frac{(Ba-7\,Ab)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,\left(ab^5\right)^{\frac{1}{6}}\,a^2} + \frac{(Ba-7\,Ab)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,\left(ab^5\right)^{\frac{1}{6}}\,a^2} + \frac{Bax^3-7\,Abx^3-6\,Aa}{3\left(bx^{\frac{7}{2}}+a\sqrt{x}\right)a^2} - \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba-7\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^3b^5} + \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba-7\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^3b^5}$$

input

```
integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output

```
1/18*(B*a - 7*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(
(a*b^5)^(1/6)*a^2) + 1/18*(B*a - 7*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*s
qrt(x))/(a/b)^(1/6))/((a*b^5)^(1/6)*a^2) + 1/9*(B*a - 7*A*b)*arctan(sqrt(x
)/(a/b)^(1/6))/((a*b^5)^(1/6)*a^2) + 1/3*(B*a*x^3 - 7*A*b*x^3 - 6*A*a)/((b
*x^(7/2) + a*sqrt(x))*a^2) - 1/36*sqrt(3)*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(
5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^5) + 1
/36*sqrt(3)*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)
*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^5)
```

Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 1757, normalized size of antiderivative = 7.32

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x)
```

output
```
(atan((((7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^2
*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147208*A^2*a^
17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^
(5/6)))*1i)/((-a)^(13/3)*b^(5/3)) + ((7*A*b - B*a)^2*(27783*A^3*a^15*b^6 -
 81*B^3*a^18*b^3 + 1701*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*(
7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^
18*b^5))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)))/((((7*A*b -
 B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^2*a^17*b^4 + 1190
7*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*
B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(
13/3)*b^(5/3)) - ((7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 +
1701*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147
208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)
^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3))))*(7*A*b - B*a)*1i)/(9*(-a)^(13/6
)*b^(5/6)) - ((2*A)/a + (x^3*(7*A*b - B*a))/(3*a^2))/(a*x^(1/2) + b*x^(7/2
)) + (atan(((((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 2
7783*A^3*a^15*b^6 - 1701*A*B^2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*
((3^(1/2)*1i)/2 - 1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a
^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13
/3)*b^(5/3)) + ((((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(27783*A^3*a^1...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx = \frac{2\sqrt{x}\,b^{\frac{1}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-2\sqrt{x}\,b^{\frac{1}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-4\sqrt{x}\,b^{\frac{1}{6}}a^{\frac{1}{6}}\,ata\cdots}{\cdots}$$

input
```
int((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x)
```

output
```
(2*sqrt(x)*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*b
**(1/3))/(b**(1/6)*a**(1/6))) - 2*sqrt(x)*b**(1/6)*a**(1/6)*atan((b**(1/6)
*a**(1/6)*sqrt(3) + 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6))) - 4*sqrt(x)*b
**(1/6)*a**(1/6)*atan((sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6))) - sqrt(x)*b*
*(1/6)*a**(1/6)*sqrt(3)*log( - sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3
) + b**(1/3)*x) + sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3)*log(sqrt(x)*b**(1/6)*a
**(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*x) - 12*a**(1/3))/(6*sqrt(x)*a**(1/3
)*a)
```

3.148 $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 242

$$\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^2}\,dx = -\frac{2A}{5a^2x^{5/2}} - \frac{(Ab-aB)\sqrt{x}}{3a^2(a+bx^3)} + \frac{(11Ab-5aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{6\sqrt{3}a^{17/6}\sqrt[6]{b}}$$

output
```
-2/5*A/a^2/x^(5/2)-1/3*(A*b-B*a)*x^(1/2)/a^2/(b*x^3+a)-1/18*(11*A*b-5*B*a)
*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(1/6)-1/18*(11*A*b-
5*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(1/6)-1/9*(11*
A*b-5*B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(1/6)-1/18*(11*A*b-5
*B*a)*arctanh(3^(1/2)*a^(1/6)*b^(1/6)*x^(1/2)/(a^(1/3)+b^(1/3)*x))*3^(1/2)
/a^(17/6)/b^(1/6)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx = \frac{\frac{6a^{5/6}(-6aA-11Abx^3+5aBx^3)}{x^{5/2}(a+bx^3)} + \frac{10(-11Ab+5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{\sqrt[6]{b}} + \frac{5(11Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{\sqrt[6]{b}}}{90a^{17/6}}$$

input
```
Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x]
```

output
```
((6*a^(5/6)*(-6*a*A - 11*A*b*x^3 + 5*a*B*x^3))/(x^(5/2)*(a + b*x^3)) + (10
*(-11*A*b + 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/b^(1/6) + (5*(11*A*b
 - 5*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/b^(1/6)
 + (5*Sqrt[3]*(-11*A*b + 5*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/
(a^(1/3) + b^(1/3)*x)])/b^(1/6))/(90*a^(17/6))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.23, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {957, 847, 851, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \; 957$$

$$\frac{(11Ab-5aB)\int \frac{1}{x^{7/2}(bx^3+a)}dx}{6ab}+\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \; 847$$

$$\frac{(11Ab-5aB)\left(-\frac{b\int \frac{1}{\sqrt{x}\left(bx^3+a\right)}dx}{a}-\frac{2}{5ax^{5/2}}\right)}{6ab}+\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{851}$$

$$\frac{(11Ab-5aB)\left(-\frac{2b\int \frac{1}{bx^3+a}d\sqrt{x}}{a}-\frac{2}{5ax^{5/2}}\right)}{6ab}+\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{753}$$

$$\frac{(11Ab-5aB)\left(-\frac{2b\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{6ab}+\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(11Ab-5aB)\left(-\frac{2b\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{6ab}+\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{(11Ab-5aB)\left(-\frac{2b\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{6ab}+\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$\big\downarrow$ 1142

$$\frac{(11Ab-5aB)\left(-\frac{2b\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\cdots}\cdots}{\cdots}\cdots\right)}{a}\cdots\right)}{6ab}$$

$$\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$\big\downarrow$ 25

$$\frac{(11Ab-5aB)\left(-\frac{2b\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\sqrt{3}\int\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{\cdots}}\cdots}{\cdots}\right.}{a}\cdots\right)}{6ab\cdots}$$

$$\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$\downarrow$ 27

$$\frac{(11Ab-5aB)\left(-\frac{2b\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{\cdots}}\cdots}{\cdots}\right.}{a}\cdots\right)}{6ab\cdots}$$

$$\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$\downarrow$ 1082

$$\frac{(11Ab-5aB)\left(-\frac{2b\left(\frac{\frac{\int\frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int\frac{1}{-x-\frac{1}{3}}}{}}{6a^{5/6}}}{a}}{6ab}$$

$$\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$\downarrow$ 217

$$\frac{(11Ab-5aB)\left(-\frac{2b\left(\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan}{}}{6a^{5/6}}}{a}}{6ab}$$

$$\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$\downarrow$ 1103

$$\frac{(11Ab-5aB)\left(-\frac{2b\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}+\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)}{\sqrt[6]{b}}\cdots\right)}{a}\cdots\right)}{6ab}$$

$$\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

input
```
Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x]
```

output
```
(A*b - a*B)/(3*a*b*x^(5/2)*(a + b*x^3)) + ((11*A*b - 5*a*B)*(-2/(5*a*x^(5/
2)) - (2*b*(ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3*a^(5/6)*b^(1/6)) + (-(Arc
Tan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6)) - (Sqrt[
3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6))
)/(6*a^(5/6)) + (ArcTan[Sqrt[3]*(1 + (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6))
)]/b^(1/6) + (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1
/3)*x])/(2*b^(1/6)))/(6*a^(5/6))))/a))/(6*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.88

method	result
derivativedivides	$-\frac{2A}{5a^2x^{\frac{5}{2}}}-\frac{2\left(\frac{\left(\frac{Ab}{6}-\frac{Ba}{6}\right)\sqrt{x}}{b\,x^3+a}+\frac{(11Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\text{ar}\cdots}{\cdots}\right)}{\cdots}\right)}{a^2}$
default	$-\frac{2A}{5a^2x^{\frac{5}{2}}}-\frac{2\left(\frac{\left(\frac{Ab}{6}-\frac{Ba}{6}\right)\sqrt{x}}{b\,x^3+a}+\frac{(11Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\text{ar}\cdots}{\cdots}\right)}{\cdots}\right)}{a^2}$
risch	$-\frac{2A}{5a^2x^{\frac{5}{2}}}-\frac{\frac{2\left(\frac{Ab}{6}-\frac{Ba}{6}\right)\sqrt{x}}{b\,x^3+a}+\frac{(11Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\text{arcta}\cdots}{\cdots}\right)}{3\cdots}}{a^2}$

input
```
int((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/5*A/a^2/x^(5/2)-2/a^2*((1/6*A*b-1/6*B*a)*x^(1/2)/(b*x^3+a)+1/6*(11*A*b-
5*B*a)*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)
^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arc
tan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))-1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x-3^(1/2)
*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)
^(1/6)-3^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1463 vs. $2(176) = 352$.

Time = 0.10 (sec) , antiderivative size = 1463, normalized size of antiderivative = 6.05

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/180*(10*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1
134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4
 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*log(a^3*(-(15625
*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*
a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)
/(a^17*b))^(1/6) - (5*B*a - 11*A*b)*sqrt(x)) - 10*(a^2*b*x^6 + a^3*x^3)*(-
(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^
3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^
6*b^6)/(a^17*b))^(1/6)*log(-a^3*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 11
34375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4
- 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6) - (5*B*a - 11*A*b
)*sqrt(x)) + 5*(a^2*b*x^6 + a^3*x^3 + sqrt(-3)*(a^2*b*x^6 + a^3*x^3))*(-(1
5625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*
B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*
b^6)/(a^17*b))^(1/6)*log(-(5*B*a - 11*A*b)*sqrt(x) + 1/2*(sqrt(-3)*a^3 + a
^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327
500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771
561*A^6*b^6)/(a^17*b))^(1/6)) - 5*(a^2*b*x^6 + a^3*x^3 + sqrt(-3)*(a^2*b*x
^6 + a^3*x^3))*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4
*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^3}{x^{7/2}\left(a + bx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)/x**(7/2)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx = \frac{(5\,Ba-11\,Ab)x^3-6\,Aa}{15\left(a^2bx^{\frac{11}{2}}+a^3x^{\frac{5}{2}}\right)} + \frac{\frac{\sqrt{3}(5\,Ba-11\,Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(5\,Ba-11\,Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(5\,Bab^{\frac{1}{3}}-11\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}{36\,a^2}$$

input
```
integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/15*((5*B*a - 11*A*b)*x^3 - 6*A*a)/(a^2*b*x^(11/2) + a^3*x^(5/2)) + 1/36*
(sqrt(3)*(5*B*a - 11*A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x
+ a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(5*B*a - 11*A*b)*log(-sqrt(3)*a^(1/
6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(5*B*a*b^(
1/3) - 11*A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3
)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 11*A*a^(1/3)*b
^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*
b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 11*
A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/s
qrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/a^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx = \frac{\sqrt{3}\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^3b} - \frac{\sqrt{3}\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^3b} + \frac{Ba\sqrt{x}-Ab\sqrt{x}}{3\left(bx^3+a\right)a^2} + \frac{\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^3b} + \frac{\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^3b} + \frac{\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^3b} - \frac{2\,A}{5\,a^2x^{\frac{5}{2}}}$$

input
```
integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/36*sqrt(3)*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt
(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b) - 1/36*sqrt(3)*(5*(a*b^5)^(1/6)
*B*a - 11*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^
(1/3))/(a^3*b) + 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*a^2) + 1/18*
(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) +
 2*sqrt(x))/(a/b)^(1/6))/(a^3*b) + 1/18*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^
(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^3*b)
+ 1/9*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1
/6))/(a^3*b) - 2/5*A/(a^2*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 2080, normalized size of antiderivative = 8.60

$$\int \frac{A+Bx^3}{x^{7/2} (a+bx^3)^2} \, dx = \text{Too large to display}$$

input
```
int((A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x)
```

output
```
- ((2*A)/(5*a) + (x^3*(11*A*b - 5*B*a))/(15*a^2))/(a*x^(5/2) + b*x^(11/2))
 - (atan((((x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 2646270
0*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) - (
(11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*
A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A
*b - 5*B*a)*1i)/(18*(-a)^(17/6)*b^(1/6)) + ((x^(1/2)*(21346578*A^4*a^10*b^
9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b
^6 - 38811960*A^3*B*a^11*b^8) + ((11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 -
 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7)
)/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*(-a)^(17/6)*b^(1/6)))
/(((x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^
2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) - ((11*A*b
- 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^
15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B
*a))/(18*(-a)^(17/6)*b^(1/6)) - ((x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*
B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 388119
60*A^3*B*a^11*b^8) + ((11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^
3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^
(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^(17/6)*b^(1/6))))*(11*A*b - 5*
B*a)*1i)/(9*(-a)^(17/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(x^(1...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx^3}{x^{7/2} (a+bx^3)^2} \, dx = \frac{10\sqrt{x}\, b^{\frac{7}{6}} a^{\frac{1}{6}}\, atan\left(\frac{b^{\frac{1}{6}} a^{\frac{1}{6}} \sqrt{3}-2\sqrt{x}\, b^{\frac{1}{3}}}{b^{\frac{1}{6}} a^{\frac{1}{6}}}\right) x^2 - 10\sqrt{x}\, b^{\frac{7}{6}} a^{\frac{1}{6}}\, atan\left(\frac{b^{\frac{1}{6}} a^{\frac{1}{6}} \sqrt{3}+2\sqrt{x}\, b^{\frac{1}{3}}}{b^{\frac{1}{6}} a^{\frac{1}{6}}}\right) x^2 - 20\sqrt{3}}{}$$

input
```
int((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x)
```

output
```
(10*sqrt(x)*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*
b**(1/3))/(b**(1/6)*a**(1/6)))*b*x**2 - 10*sqrt(x)*b**(1/6)*a**(1/6)*atan(
(b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*b*x*
*2 - 20*sqrt(x)*b**(1/6)*a**(1/6)*atan((sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/
6)))*b*x**2 + 5*sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3)*log( - sqrt(x)*b**(1/6)*
a**(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*x)*b*x**2 - 5*sqrt(x)*b**(1/6)*a**(
1/6)*sqrt(3)*log(sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*x
)*b*x**2 - 12*b**(1/3)*a)/(30*sqrt(x)*b**(1/3)*a**2*x**2)
```

3.149 $\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2Bx^{3/2}}{3b^3} - \frac{(Ab-aB)x^{9/2}}{6b^2(a+bx^3)^2} - \frac{(3Ab-7aB)x^{3/2}}{12b^3(a+bx^3)} + \frac{(Ab-5aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4\sqrt{a}b^{7/2}}$$

output
```
2/3*B*x^(3/2)/b^3-1/6*(A*b-B*a)*x^(9/2)/b^2/(b*x^3+a)^2-1/12*(3*A*b-7*B*a)
*x^(3/2)/b^3/(b*x^3+a)+1/4*(A*b-5*B*a)*arctan(b^(1/2)*x^(3/2)/a^(1/2))/a^(
1/2)/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^{3/2}(-3aAb+15a^2B-5Ab^2x^3+25abBx^3+8b^2Bx^6)}{12b^3(a+bx^3)^2} + \frac{(Ab-5aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4\sqrt{a}b^{7/2}}$$

input
```
Integrate[(x^(13/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
(x^(3/2)*(-3*a*A*b + 15*a^2*B - 5*A*b^2*x^3 + 25*a*b*B*x^3 + 8*b^2*B*x^6))
/(12*b^3*(a + b*x^3)^2) + ((A*b - 5*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]]
)/(4*Sqrt[a]*b^(7/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {957, 817, 843, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^{15/2}(Ab-aB)}{6ab\,(a+bx^3)^2}-\frac{(Ab-5aB)\int \frac{x^{13/2}}{(bx^3+a)^2}dx}{4ab}$$

$$\Big\downarrow \text{817}$$

$$\frac{x^{15/2}(Ab-aB)}{6ab\,(a+bx^3)^2}-\frac{(Ab-5aB)\left(\frac{3\int \frac{x^{7/2}}{bx^3+a}dx}{2b}-\frac{x^{9/2}}{3b(a+bx^3)}\right)}{4ab}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^{15/2}(Ab-aB)}{6ab\,(a+bx^3)^2}-\frac{(Ab-5aB)\left(\frac{3\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{bx^3+a}dx}{b}\right)}{2b}-\frac{x^{9/2}}{3b(a+bx^3)}\right)}{4ab}$$

$$\Big\downarrow \text{851}$$

$$\frac{x^{15/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-5aB)\left(\frac{3\left(\frac{2x^{3/2}}{3b}-\frac{2a\int\frac{x}{bx^3+a}d\sqrt{x}}{b}\right)}{2b}-\frac{x^{9/2}}{3b(a+bx^3)}\right)}{4ab}$$

$\Big\downarrow$ 807

$$\frac{x^{15/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-5aB)\left(\frac{3\left(\frac{2x^{3/2}}{3b}-\frac{2a\int\frac{1}{a+bx}dx^{3/2}}{3b}\right)}{2b}-\frac{x^{9/2}}{3b(a+bx^3)}\right)}{4ab}$$

$\Big\downarrow$ 218

$$\frac{x^{15/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-5aB)\left(\frac{3\left(\frac{2x^{3/2}}{3b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{3/2}}\right)}{2b}-\frac{x^{9/2}}{3b(a+bx^3)}\right)}{4ab}$$

input
```
Int[(x^(13/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^(15/2))/(6*a*b*(a + b*x^3)^2) - ((A*b - 5*a*B)*(-1/3*x^(9/2
)/(b*(a + b*x^3)) + (3*((2*x^(3/2))/(3*b) - (2*Sqrt[a]*ArcTan[(Sqrt[b]*x^(
3/2))/Sqrt[a]])/(3*b^(3/2))))/(2*b)))/(4*a*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{2Bx^{\frac{3}{2}}}{3b^3}+\frac{\frac{2\left(\left(-\frac{5}{8}b^2A+\frac{9}{8}abB\right)x^{\frac{9}{2}}-\frac{a(3Ab-7Ba)x^{\frac{3}{2}}}{8}\right)}{3\left(bx^3+a\right)^2}+\frac{(Ab-5Ba)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\sqrt{ab}}}{b^3}$	85
default	$\frac{2Bx^{\frac{3}{2}}}{3b^3}+\frac{\frac{2\left(\left(-\frac{5}{8}b^2A+\frac{9}{8}abB\right)x^{\frac{9}{2}}-\frac{a(3Ab-7Ba)x^{\frac{3}{2}}}{8}\right)}{3\left(bx^3+a\right)^2}+\frac{(Ab-5Ba)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\sqrt{ab}}}{b^3}$	85
risch	$\frac{2Bx^{\frac{3}{2}}}{3b^3}+\frac{\frac{\frac{2\left(-\frac{5}{8}b^2A+\frac{9}{8}abB\right)x^{\frac{9}{2}}}{3}-\frac{a(3Ab-7Ba)x^{\frac{3}{2}}}{12}}{\left(bx^3+a\right)^2}+\frac{(Ab-5Ba)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\sqrt{ab}}}{b^3}$	85

input
```
int(x^(13/2)*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*B*x^(3/2)/b^3+2/3/b^3*(((-5/8*b^2*A+9/8*a*b*B)*x^(9/2)-1/8*a*(3*A*b-7*
B*a)*x^(3/2))/(b*x^3+a)^2+3/8*(A*b-5*B*a)/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*
b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 344, normalized size of antiderivative = 3.07

$$\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \left[\frac{3\left((5\,Bab^2-Ab^3)x^6+5\,Ba^3-Aa^2b+2\,(5\,Ba^2b-Aab^2)x^3\right)\sqrt{-ab}\log\left(\frac{bx^3-2\sqrt{-}}{bx^3}\right.}{24\,(ab^6x^6+2\,a^2b}\right.$$

$$-\frac{3\left((5\,Bab^2-Ab^3)x^6+5\,Ba^3-Aa^2b+2\,(5\,Ba^2b-Aab^2)x^3\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x^{\frac{3}{2}}}{a}\right)-(8\,Bab^3x^7+5\,(5\,Ba}{12\,(ab^6x^6+2\,a^2b^5x^3+a^3b^4)}$$

input
```
integrate(x^(13/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/24*(3*((5*B*a*b^2 - A*b^3)*x^6 + 5*B*a^3 - A*a^2*b + 2*(5*B*a^2*b - A*a
*b^2)*x^3)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a))
+ 2*(8*B*a*b^3*x^7 + 5*(5*B*a^2*b^2 - A*a*b^3)*x^4 + 3*(5*B*a^3*b - A*a^2*
b^2)*x)*sqrt(x))/(a*b^6*x^6 + 2*a^2*b^5*x^3 + a^3*b^4), -1/12*(3*((5*B*a*b
^2 - A*b^3)*x^6 + 5*B*a^3 - A*a^2*b + 2*(5*B*a^2*b - A*a*b^2)*x^3)*sqrt(a*
b)*arctan(sqrt(a*b)*x^(3/2)/a) - (8*B*a*b^3*x^7 + 5*(5*B*a^2*b^2 - A*a*b^3
)*x^4 + 3*(5*B*a^3*b - A*a^2*b^2)*x)*sqrt(x))/(a*b^6*x^6 + 2*a^2*b^5*x^3 +
 a^3*b^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(13/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.90

$$\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(9\,Bab-5\,Ab^2)x^{\frac{9}{2}}+(7\,Ba^2-3\,Aab)x^{\frac{3}{2}}}{12\,(b^5x^6+2\,ab^4x^3+a^2b^3)} + \frac{2\,Bx^{\frac{3}{2}}}{3\,b^3} - \frac{(5\,Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^3}$$

input
```
integrate(x^(13/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/12*((9*B*a*b - 5*A*b^2)*x^(9/2) + (7*B*a^2 - 3*A*a*b)*x^(3/2))/(b^5*x^6
+ 2*a*b^4*x^3 + a^2*b^3) + 2/3*B*x^(3/2)/b^3 - 1/4*(5*B*a - A*b)*arctan(b*
x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

$$\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,b^3} - \frac{(5\,Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^3} + \frac{9\,Babx^{\frac{9}{2}}-5\,Ab^2x^{\frac{9}{2}}+7\,Ba^2x^{\frac{3}{2}}-3\,Aabx^{\frac{3}{2}}}{12\,(bx^3+a)^2b^3}$$

input
```
integrate(x^(13/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
2/3*B*x^(3/2)/b^3 - 1/4*(5*B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*
b)*b^3) + 1/12*(9*B*a*b*x^(9/2) - 5*A*b^2*x^(9/2) + 7*B*a^2*x^(3/2) - 3*A*
a*b*x^(3/2))/((b*x^3 + a)^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.29

$$\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^{3/2}\left(\frac{7\,B\,a^2}{12}-\frac{A\,a\,b}{4}\right)-x^{9/2}\left(\frac{5\,A\,b^2}{12}-\frac{3\,B\,a\,b}{4}\right)}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6} + \frac{2\,B\,x^{3/2}}{3\,b^3} - \frac{\mathrm{atan}\left(\frac{27\,\sqrt{a}\,\sqrt{b}\,x^{3/2}\,(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2)}{(135\,B\,a^2-27\,A\,a\,b)\,(A\,b-5\,B\,a)}\right)\,(A\,b-5\,B\,a)}{4\,\sqrt{a}\,b^{7/2}}$$

input
```
int((x^(13/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
(x^(3/2)*((7*B*a^2)/12 - (A*a*b)/4) - x^(9/2)*((5*A*b^2)/12 - (3*B*a*b)/4)
)/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (2*B*x^(3/2))/(3*b^3) - (atan((27*a^
(1/2)*b^(1/2)*x^(3/2)*(A^2*b^2 + 25*B^2*a^2 - 10*A*B*a*b))/((135*B*a^2 - 2
7*A*a*b)*(A*b - 5*B*a)))*(A*b - 5*B*a))/(4*a^(1/2)*b^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.96

$$\int \frac{x^{13/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{3b^{\frac{1}{6}}a^{\frac{13}{6}}atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)+3b^{\frac{7}{6}}a^{\frac{7}{6}}atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)x^3-3b^{\frac{1}{6}}a^{\frac{13}{6}}atan\left(\frac{b^{\frac{1}{6}}\cdots}{}\right.}{}$$

input
```
int(x^(13/2)*(B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
(3*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*b**(1/3))
/(b**(1/6)*a**(1/6)))*a**2 + 3*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*s
qrt(3) - 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*a*b*x**3 - 3*b**(1/6)*a*
*(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**
(1/6)))*a**2 - 3*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqr
t(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*a*b*x**3 + 3*b**(1/6)*a**(1/6)*atan((s
qrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*a**2 + 3*b**(1/6)*a**(1/6)*atan((sqr
t(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*a*b*x**3 + 3*sqrt(x)*b**(2/3)*a**(2/3)
*a*x + 2*sqrt(x)*b**(2/3)*a**(2/3)*b*x**4)/(3*b**(2/3)*a**(2/3)*b**2*(a +
b*x**3))
```

3.150 $\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 101

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx=-\frac{(Ab-aB)x^{3/2}}{6b^2(a+bx^3)^2}+\frac{(Ab-5aB)x^{3/2}}{12ab^2(a+bx^3)}+\frac{(Ab+3aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}$$

output
```
-1/6*(A*b-B*a)*x^(3/2)/b^2/(b*x^3+a)^2+1/12*(A*b-5*B*a)*x^(3/2)/a/b^2/(b*x
^3+a)+1/12*(A*b+3*B*a)*arctan(b^(1/2)*x^(3/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx=-\frac{x^{3/2}(aAb+3a^2B-Ab^2x^3+5abBx^3)}{12ab^2(a+bx^3)^2}+\frac{(Ab+3aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}$$

input
```
Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
-1/12*(x^(3/2)*(a*A*b + 3*a^2*B - A*b^2*x^3 + 5*a*b*B*x^3))/(a*b^2*(a + b*
x^3)^2) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(3/2)*b^
(5/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {957, 817, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(3aB+Ab)\int \frac{x^{7/2}}{(bx^3+a)^2}dx}{4ab}+\frac{x^{9/2}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$$\Big\downarrow \text{817}$$

$$\frac{(3aB+Ab)\left(\frac{\int \frac{\sqrt{x}}{bx^3+a}dx}{2b}-\frac{x^{3/2}}{3b(a+bx^3)}\right)}{4ab}+\frac{x^{9/2}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$$\Big\downarrow \text{851}$$

$$\frac{(3aB+Ab)\left(\frac{\int \frac{x}{bx^3+a}d\sqrt{x}}{b}-\frac{x^{3/2}}{3b(a+bx^3)}\right)}{4ab}+\frac{x^{9/2}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$$\Big\downarrow \text{807}$$

$$\frac{(3aB+Ab)\left(\frac{\int \frac{1}{a+bx}dx^{3/2}}{3b}-\frac{x^{3/2}}{3b(a+bx^3)}\right)}{4ab}+\frac{x^{9/2}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{(3aB+Ab)\left(\frac{\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}}-\frac{x^{3/2}}{3b(a+bx^3)}\right)}{4ab}+\frac{x^{9/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input
```
Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^(9/2))/(6*a*b*(a + b*x^3)^2) + ((A*b + 3*a*B)*(-1/3*x^(3/2)
/(b*(a + b*x^3)) + ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]]/(3*Sqrt[a]*b^(3/2))))
/(4*a*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\frac{(Ab-5Ba)x^{\frac{9}{2}}}{12ab}-\frac{(Ab+3Ba)x^{\frac{3}{2}}}{12b^2}}{(b\,x^3+a)^2}+\frac{(Ab+3Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12b^2a\sqrt{ab}}$	81
default	$\frac{\frac{(Ab-5Ba)x^{\frac{9}{2}}}{12ab}-\frac{(Ab+3Ba)x^{\frac{3}{2}}}{12b^2}}{(b\,x^3+a)^2}+\frac{(Ab+3Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12b^2a\sqrt{ab}}$	81

input
```
int(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*(1/8*(A*b-5*B*a)/a/b*x^(9/2)-1/8*(A*b+3*B*a)/b^2*x^(3/2))/(b*x^3+a)^2+
1/12*(A*b+3*B*a)/b^2/a/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.11

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \left[-\frac{((3\,Bab^2+Ab^3)x^6+3\,Ba^3+Aa^2b+2\,(3\,Ba^2b+Aab^2)x^3)\sqrt{-ab}\log\left(\frac{bx^3-2\sqrt{-a}}{bx^3+}\right.}{24\,(a^2b^5x^6+2\,a^3b^4x^3+a}\right.$$

input
```
integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[-1/24*(((3*B*a*b^2 + A*b^3)*x^6 + 3*B*a^3 + A*a^2*b + 2*(3*B*a^2*b + A*a*
b^2)*x^3)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) +
 2*((5*B*a^2*b^2 - A*a*b^3)*x^4 + (3*B*a^3*b + A*a^2*b^2)*x)*sqrt(x))/(a^2
*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3), 1/12*(((3*B*a*b^2 + A*b^3)*x^6 + 3*B*
a^3 + A*a^2*b + 2*(3*B*a^2*b + A*a*b^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x^
(3/2)/a) - ((5*B*a^2*b^2 - A*a*b^3)*x^4 + (3*B*a^3*b + A*a^2*b^2)*x)*sqrt(
x))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.95

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{(5\,Bab-Ab^2)x^{\frac{9}{2}}+(3\,Ba^2+Aab)x^{\frac{3}{2}}}{12\,(ab^4x^6+2\,a^2b^3x^3+a^3b^2)}+\frac{(3\,Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}ab^2}$$

input
```
integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/12*((5*B*a*b - A*b^2)*x^(9/2) + (3*B*a^2 + A*a*b)*x^(3/2))/(a*b^4*x^6 +
 2*a^2*b^3*x^3 + a^3*b^2) + 1/12*(3*B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))
/(sqrt(a*b)*a*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(3\,Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}ab^2} - \frac{5\,Babx^{\frac{9}{2}} - Ab^2x^{\frac{9}{2}} + 3\,Ba^2x^{\frac{3}{2}} + Aabx^{\frac{3}{2}}}{12\,(bx^3+a)^2ab^2}$$

input
```
integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/12*(3*B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/12*(5
*B*a*b*x^(9/2) - A*b^2*x^(9/2) + 3*B*a^2*x^(3/2) + A*a*b*x^(3/2))/((b*x^3
+ a)^2*a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.32

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\mathrm{atan}\left(\frac{9\,b^{3/2}\,x^{3/2}\,(A^2\,b^2+6\,A\,B\,a\,b+9\,B^2\,a^2)}{\sqrt{a}\,(9\,A\,b^2+27\,B\,a\,b)\,(A\,b+3\,B\,a)}\right)\,(A\,b+3\,B\,a)}{12\,a^{3/2}\,b^{5/2}} - \frac{\frac{x^{3/2}\,(A\,b+3\,B\,a)}{12\,b^2} - \frac{x^{9/2}\,(A\,b-5\,B\,a)}{12\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

input
```
int((x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
(atan((9*b^(3/2)*x^(3/2)*(A^2*b^2 + 9*B^2*a^2 + 6*A*B*a*b))/(a^(1/2)*(9*A*
b^2 + 27*B*a*b)*(A*b + 3*B*a)))*(A*b + 3*B*a))/(12*a^(3/2)*b^(5/2)) - ((x^
(3/2)*(A*b + 3*B*a))/(12*b^2) - (x^(9/2)*(A*b - 5*B*a))/(12*a*b))/(a^2 + b
^2*x^6 + 2*a*b*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.02

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{-b^{\frac{1}{6}}a^{\frac{7}{6}}atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-b^{\frac{7}{6}}a^{\frac{1}{6}}atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)x^3+b^{\frac{1}{6}}a^{\frac{7}{6}}atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}\cdots}{b^{\frac{1}{6}}\cdots}\right.}{}$$

input
```
int(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
( - b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*b**(1/3)
)/(b**(1/6)*a**(1/6)))*a - b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(
3) - 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*b*x**3 + b**(1/6)*a**(1/6)*a
tan((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*
a + b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*b**(1/3)
)/(b**(1/6)*a**(1/6)))*b*x**3 - b**(1/6)*a**(1/6)*atan((sqrt(x)*b**(1/3))/
(b**(1/6)*a**(1/6)))*a - b**(1/6)*a**(1/6)*atan((sqrt(x)*b**(1/3))/(b**(1/
6)*a**(1/6)))*b*x**3 - sqrt(x)*b**(2/3)*a**(2/3)*x)/(3*b**(2/3)*a**(2/3)*b
*(a + b*x**3))
```

3.151 $\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx=\frac{(Ab-aB)x^{3/2}}{6ab\,(a+bx^3)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\,(a+bx^3)}+\frac{(3Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}$$

output
```
1/6*(A*b-B*a)*x^(3/2)/a/b/(b*x^3+a)^2+1/12*(3*A*b+B*a)*x^(3/2)/a^2/b/(b*x^
3+a)+1/12*(3*A*b+B*a)*arctan(b^(1/2)*x^(3/2)/a^(1/2))/a^(5/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx=-\frac{x^{3/2}(-5aAb+a^2B-3Ab^2x^3-abBx^3)}{12a^2b\,(a+bx^3)^2}+\frac{(3Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
-1/12*(x^(3/2)*(-5*a*A*b + a^2*B - 3*A*b^2*x^3 - a*b*B*x^3))/(a^2*b*(a + b
*x^3)^2) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(5/2)*b
^(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {957, 819, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(aB+3Ab)\int \frac{\sqrt{x}}{(bx^3+a)^2}dx}{4ab}+\frac{x^{3/2}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{(aB+3Ab)\left(\frac{\int \frac{\sqrt{x}}{bx^3+a}dx}{2a}+\frac{x^{3/2}}{3a(a+bx^3)}\right)}{4ab}+\frac{x^{3/2}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$$\Big\downarrow \text{851}$$

$$\frac{(aB+3Ab)\left(\frac{\int \frac{x}{bx^3+a}d\sqrt{x}}{a}+\frac{x^{3/2}}{3a(a+bx^3)}\right)}{4ab}+\frac{x^{3/2}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$$\Big\downarrow \text{807}$$

$$\frac{(aB+3Ab)\left(\frac{\int \frac{1}{a+bx}dx^{3/2}}{3a}+\frac{x^{3/2}}{3a(a+bx^3)}\right)}{4ab}+\frac{x^{3/2}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{(aB+3Ab)\left(\frac{\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}+\frac{x^{3/2}}{3a(a+bx^3)}\right)}{4ab}+\frac{x^{3/2}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

input
```
Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^(3/2))/(6*a*b*(a + b*x^3)^2) + ((3*A*b + a*B)*(x^(3/2)/(3*a
*(a + b*x^3)) + ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]]/(3*a^(3/2)*Sqrt[b])))/(4
*a*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{(3Ab+Ba)x^{\frac{9}{2}}}{12a^2}+\frac{(5Ab-Ba)x^{\frac{3}{2}}}{12ab}}{(b\,x^3+a)^2}+\frac{(3Ab+Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12a^2b\sqrt{ab}}$	82
default	$\frac{\frac{(3Ab+Ba)x^{\frac{9}{2}}}{12a^2}+\frac{(5Ab-Ba)x^{\frac{3}{2}}}{12ab}}{(b\,x^3+a)^2}+\frac{(3Ab+Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12a^2b\sqrt{ab}}$	82

input
```
int(x^(1/2)*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*(1/8*(3*A*b+B*a)/a^2*x^(9/2)+1/8*(5*A*b-B*a)/a/b*x^(3/2))/(b*x^3+a)^2+
1/12*(3*A*b+B*a)/a^2/b/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.01

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx$$
$$= \left[-\frac{((Bab^2+3\,Ab^3)x^6+Ba^3+3\,Aa^2b+2\,(Ba^2b+3\,Aab^2)x^3)\sqrt{-ab}\log\left(\frac{bx^3-2\,\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)-2\,((Ba^2b^2+}{24\,(a^3b^4x^6+2\,a^4b^3x^3+a^5b^2)}\right.$$

input
```
integrate(x^(1/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[-1/24*(((B*a*b^2 + 3*A*b^3)*x^6 + B*a^3 + 3*A*a^2*b + 2*(B*a^2*b + 3*A*a*
b^2)*x^3)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) -
 2*((B*a^2*b^2 + 3*A*a*b^3)*x^4 - (B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(x))/(a^3
*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2), 1/12*(((B*a*b^2 + 3*A*b^3)*x^6 + B*a^
3 + 3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x^
(3/2)/a) + ((B*a^2*b^2 + 3*A*a*b^3)*x^4 - (B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(
x))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Bab+3\,Ab^2)x^{\frac{9}{2}}-(Ba^2-5\,Aab)x^{\frac{3}{2}}}{12\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)}+\frac{(Ba+3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}a^2b}$$

input
```
integrate(x^(1/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/12*((B*a*b + 3*A*b^2)*x^(9/2) - (B*a^2 - 5*A*a*b)*x^(3/2))/(a^2*b^3*x^6
+ 2*a^3*b^2*x^3 + a^4*b) + 1/12*(B*a + 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/
(sqrt(a*b)*a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ba+3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}a^2b} + \frac{Babx^{\frac{9}{2}}+3\,Ab^2x^{\frac{9}{2}}-Ba^2x^{\frac{3}{2}}+5\,Aabx^{\frac{3}{2}}}{12\,(bx^3+a)^2a^2b}$$

input
```
integrate(x^(1/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/12*(B*a + 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/12*(B
*a*b*x^(9/2) + 3*A*b^2*x^(9/2) - B*a^2*x^(3/2) + 5*A*a*b*x^(3/2))/((b*x^3
+ a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\frac{x^{9/2}\,(3\,A\,b+B\,a)}{12\,a^2}+\frac{x^{3/2}\,(5\,A\,b-B\,a)}{12\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6} + \frac{\mathrm{atan}\left(\frac{b^{3/2}\,x^{3/2}\,(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b)}{\sqrt{a}\,(3\,A\,b+B\,a)\,(3\,A\,b^3+B\,a\,b^2)}\right)\,(3\,A\,b+B\,a)}{12\,a^{5/2}\,b^{3/2}}$$

input
```
int((x^(1/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
((x^(9/2)*(3*A*b + B*a))/(12*a^2) + (x^(3/2)*(5*A*b - B*a))/(12*a*b))/(a^2
 + b^2*x^6 + 2*a*b*x^3) + (atan((b^(3/2)*x^(3/2)*(9*A^2*b^3 + B^2*a^2*b +
6*A*B*a*b^2))/(a^(1/2)*(3*A*b + B*a)*(3*A*b^3 + B*a*b^2)))*(3*A*b + B*a))/
(12*a^(5/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx$$
$$= \frac{-b^{\frac{1}{6}}a^{\frac{7}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-b^{\frac{7}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)x^3+b^{\frac{1}{6}}a^{\frac{7}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)+b^{\frac{7}{6}}a^{\frac{1}{6}}\,atan}{3b^{\frac{2}{3}}a^{\frac{5}{3}}\,(b\,x^3+a)}$$

input
```
int(x^(1/2)*(B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
( - b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*b**(1/3)
)/(b**(1/6)*a**(1/6)))*a - b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(
3) - 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*b*x**3 + b**(1/6)*a**(1/6)*a
tan((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*
a + b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*b**(1/3)
)/(b**(1/6)*a**(1/6)))*b*x**3 - b**(1/6)*a**(1/6)*atan((sqrt(x)*b**(1/3))/
(b**(1/6)*a**(1/6)))*a - b**(1/6)*a**(1/6)*atan((sqrt(x)*b**(1/3))/(b**(1/
6)*a**(1/6)))*b*x**3 + sqrt(x)*b**(2/3)*a**(2/3)*x)/(3*b**(2/3)*a**(2/3)*a
*(a + b*x**3))
```

3.152 $\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx=-\frac{2A}{3a^3x^{3/2}}-\frac{(Ab-aB)x^{3/2}}{6a^2\left(a+bx^3\right)^2}-\frac{(7Ab-3aB)x^{3/2}}{12a^3\left(a+bx^3\right)}-\frac{(5Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}$$

output
```
-2/3*A/a^3/x^(3/2)-1/6*(A*b-B*a)*x^(3/2)/a^2/(b*x^3+a)^2-1/12*(7*A*b-3*B*a
)*x^(3/2)/a^3/(b*x^3+a)-1/4*(5*A*b-B*a)*arctan(b^(1/2)*x^(3/2)/a^(1/2))/a^
(7/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx=\frac{-8a^2A-25aAbx^3+5a^2Bx^3-15Ab^2x^6+3abBx^6}{12a^3x^{3/2}\left(a+bx^3\right)^2}+\frac{(-5Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}$$

input
```
Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x]
```

output
```
(-8*a^2*A - 25*a*A*b*x^3 + 5*a^2*B*x^3 - 15*A*b^2*x^6 + 3*a*b*B*x^6)/(12*a
^3*x^(3/2)*(a + b*x^3)^2) + ((-5*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[
a]])/(4*a^(7/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {957, 819, 847, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(5Ab-aB)\int \frac{1}{x^{5/2}\left(bx^3+a\right)^2}dx}{4ab}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{(5Ab-aB)\left(\frac{3\int \frac{1}{x^{5/2}\left(bx^3+a\right)}dx}{2a}+\frac{1}{3ax^{3/2}\left(a+bx^3\right)}\right)}{4ab}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{847}$$

$$\frac{(5Ab-aB)\left(\frac{3\left(-\frac{b\int \frac{\sqrt{x}}{bx^3+a}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{3ax^{3/2}\left(a+bx^3\right)}\right)}{4ab}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{851}$$

$$\frac{(5Ab-aB)\left(\frac{3\left(-\frac{2b\int\frac{x}{bx^3+a}d\sqrt{x}}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{3ax^{3/2}(a+bx^3)}\right)}{4ab}+\frac{Ab-aB}{6abx^{3/2}(a+bx^3)^2}$$

$$\Big\downarrow \text{807}$$

$$\frac{(5Ab-aB)\left(\frac{3\left(-\frac{2b\int\frac{1}{a+bx}dx^{3/2}}{3a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{3ax^{3/2}(a+bx^3)}\right)}{4ab}+\frac{Ab-aB}{6abx^{3/2}(a+bx^3)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{(5Ab-aB)\left(\frac{3\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{3ax^{3/2}(a+bx^3)}\right)}{4ab}+\frac{Ab-aB}{6abx^{3/2}(a+bx^3)^2}$$

input
```
Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x]
```

output
```
(A*b - a*B)/(6*a*b*x^(3/2)*(a + b*x^3)^2) + ((5*A*b - a*B)*(1/(3*a*x^(3/2)
*(a + b*x^3)) + (3*(-2/(3*a*x^(3/2)) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*x^(3/2))
/Sqrt[a]])/(3*a^(3/2))))/(2*a)))/(4*a*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$-\frac{2A}{3a^3x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(\frac{7}{8}b^2A-\frac{3}{8}abB\right)x^{\frac{9}{2}}+\frac{a(9Ab-5Ba)x^{\frac{3}{2}}}{8}}{\left(bx^3+a\right)^2}+\frac{3(5Ab-Ba)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{3a^3}$	86
default	$-\frac{2A}{3a^3x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(\frac{7}{8}b^2A-\frac{3}{8}abB\right)x^{\frac{9}{2}}+\frac{a(9Ab-5Ba)x^{\frac{3}{2}}}{8}}{\left(bx^3+a\right)^2}+\frac{3(5Ab-Ba)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{3a^3}$	86
risch	$-\frac{2A}{3a^3x^{\frac{3}{2}}}-\frac{\frac{\frac{2\left(\frac{7}{8}b^2A-\frac{3}{8}abB\right)x^{\frac{9}{2}}}{3}+\frac{a(9Ab-5Ba)x^{\frac{3}{2}}}{12}}{\left(bx^3+a\right)^2}+\frac{(5Ab-Ba)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\sqrt{ab}}}{a^3}$	87

input
```
int((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*A/a^3/x^(3/2)-2/3/a^3*(((7/8*b^2*A-3/8*a*b*B)*x^(9/2)+1/8*a*(9*A*b-5*
B*a)*x^(3/2))/(b*x^3+a)^2+3/8*(5*A*b-B*a)/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*
b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.07

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx=\left[\frac{3\left((Bab^2-5\,Ab^3)x^8+2\,(Ba^2b-5\,Aab^2)x^5+(Ba^3-5\,Aa^2b)x^2\right)\sqrt{-ab}\log\left(\frac{bx^3+2}{}\right.}{24\left(a^4b^3x^8+2\,a^5b\right.}\right.$$

input
```
integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/24*(3*((B*a*b^2 - 5*A*b^3)*x^8 + 2*(B*a^2*b - 5*A*a*b^2)*x^5 + (B*a^3 -
 5*A*a^2*b)*x^2)*sqrt(-a*b)*log((b*x^3 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3
+ a)) + 2*(3*(B*a^2*b^2 - 5*A*a*b^3)*x^6 - 8*A*a^3*b + 5*(B*a^3*b - 5*A*a^
2*b^2)*x^3)*sqrt(x))/(a^4*b^3*x^8 + 2*a^5*b^2*x^5 + a^6*b*x^2), 1/12*(3*((
B*a*b^2 - 5*A*b^3)*x^8 + 2*(B*a^2*b - 5*A*a*b^2)*x^5 + (B*a^3 - 5*A*a^2*b)
*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) + (3*(B*a^2*b^2 - 5*A*a*b^3)*x
^6 - 8*A*a^3*b + 5*(B*a^3*b - 5*A*a^2*b^2)*x^3)*sqrt(x))/(a^4*b^3*x^8 + 2*
a^5*b^2*x^5 + a^6*b*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)/x**(5/2)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx = \frac{3\,(Bab-5\,Ab^2)x^6+5\,(Ba^2-5\,Aab)x^3-8\,Aa^2}{12\,\left(a^3b^2x^{\frac{15}{2}}+2\,a^4bx^{\frac{9}{2}}+a^5x^{\frac{3}{2}}\right)} + \frac{(Ba-5\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^3}$$

input
```
integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/12*(3*(B*a*b - 5*A*b^2)*x^6 + 5*(B*a^2 - 5*A*a*b)*x^3 - 8*A*a^2)/(a^3*b^
2*x^(15/2) + 2*a^4*b*x^(9/2) + a^5*x^(3/2)) + 1/4*(B*a - 5*A*b)*arctan(b*x
^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx=\frac{(Ba-5\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^3}-\frac{2\,A}{3\,a^3x^{\frac{3}{2}}}+\frac{3\,Babx^{\frac{9}{2}}-7\,Ab^2x^{\frac{9}{2}}+5\,Ba^2x^{\frac{3}{2}}-9\,Aabx^{\frac{3}{2}}}{12\,(bx^3+a)^2a^3}$$

input
```
integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/4*(B*a - 5*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/3*A/(a^3
*x^(3/2)) + 1/12*(3*B*a*b*x^(9/2) - 7*A*b^2*x^(9/2) + 5*B*a^2*x^(3/2) - 9*
A*a*b*x^(3/2))/((b*x^3 + a)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx=-\frac{\frac{2\,A}{3\,a}+\frac{5\,x^3\,(5\,A\,b-B\,a)}{12\,a^2}+\frac{b\,x^6\,(5\,A\,b-B\,a)}{4\,a^3}}{a^2\,x^{3/2}+b^2\,x^{15/2}+2\,a\,b\,x^{9/2}}-\frac{\mathrm{atan}\left(\frac{8\,a^{7/2}\,\sqrt{b}\,x^{3/2}\,(86400\,A^2\,a^9\,b^5-34560\,A\,B\,a^{10}\,b^4+3456\,B^2\,a^{11}\,b^3)}{(5\,A\,b-B\,a)\,(138240\,A\,a^{13}\,b^4-27648\,B\,a^{14}\,b^3)}\right)\,(5\,A\,b-B\,a)}{4\,a^{7/2}\,\sqrt{b}}$$

input
```
int((A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x)
```

output
```
- ((2*A)/(3*a) + (5*x^3*(5*A*b - B*a))/(12*a^2) + (b*x^6*(5*A*b - B*a))/(4
*a^3))/(a^2*x^(3/2) + b^2*x^(15/2) + 2*a*b*x^(9/2)) - (atan((8*a^(7/2)*b^(
1/2)*x^(3/2)*(86400*A^2*a^9*b^5 + 3456*B^2*a^11*b^3 - 34560*A*B*a^10*b^4))
/((5*A*b - B*a)*(138240*A*a^13*b^4 - 27648*B*a^14*b^3)))*(5*A*b - B*a))/(4
*a^(7/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.09

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx = \frac{3\sqrt{x}\,b^{\frac{7}{6}}a^{\frac{7}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)x+3\sqrt{x}\,b^{\frac{13}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)x^4-3\sqrt{x}\,b^{\frac{7}{6}}}{}$$

input
```
int((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x)
```

output
```
(3*sqrt(x)*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*b
**(1/3))/(b**(1/6)*a**(1/6)))*a*b*x + 3*sqrt(x)*b**(1/6)*a**(1/6)*atan((b*
*(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*b**2*x*
*4 - 3*sqrt(x)*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt(
x)*b**(1/3))/(b**(1/6)*a**(1/6)))*a*b*x - 3*sqrt(x)*b**(1/6)*a**(1/6)*atan
((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*b**
2*x**4 + 3*sqrt(x)*b**(1/6)*a**(1/6)*atan((sqrt(x)*b**(1/3))/(b**(1/6)*a**
(1/6)))*a*b*x + 3*sqrt(x)*b**(1/6)*a**(1/6)*atan((sqrt(x)*b**(1/3))/(b**(1
/6)*a**(1/6)))*b**2*x**4 - 2*b**(2/3)*a**(2/3)*a - 3*b**(2/3)*a**(2/3)*b*x
**3)/(3*sqrt(x)*b**(2/3)*a**(2/3)*a**2*x*(a + b*x**3))
```

3.153 $\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 261

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3} dx = -\frac{(Ab-aB)\sqrt{x}}{6b^2(a+bx^3)^2} + \frac{(Ab-13aB)\sqrt{x}}{36ab^2(a+bx^3)} - \frac{(5Ab+7aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{11/6}b^{13/6}} + \frac{(5Ab+7aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{11/6}b^{13/6}} + \frac{(5Ab+7aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}} + \frac{(5Ab+7aB)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{72\sqrt{3}a^{11/6}b^{13/6}}$$

output
```
-1/6*(A*b-B*a)*x^(1/2)/b^2/(b*x^3+a)^2+1/36*(A*b-13*B*a)*x^(1/2)/a/b^2/(b*
x^3+a)+1/216*(5*A*b+7*B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(1
1/6)/b^(13/6)+1/216*(5*A*b+7*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6)
)/a^(11/6)/b^(13/6)+1/108*(5*A*b+7*B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^
(11/6)/b^(13/6)+1/216*(5*A*b+7*B*a)*arctanh(3^(1/2)*a^(1/6)*b^(1/6)*x^(1/2
)/(a^(1/3)+b^(1/3)*x))*3^(1/2)/a^(11/6)/b^(13/6)
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.74

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{-\frac{6a^{5/6}\sqrt[6]{b}\sqrt{x}(7a^2B-Ab^2x^3+ab(5A+13Bx^3))}{(a+bx^3)^2}+2(5Ab+7aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-(5Ab+7}{216a^{11/6}b^{13/6}}$$

input
```
Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((-6*a^(5/6)*b^(1/6)*Sqrt[x]*(7*a^2*B - A*b^2*x^3 + a*b*(5*A + 13*B*x^3)))
/(a + b*x^3)^2 + 2*(5*A*b + 7*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - (5*
A*b + 7*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] + Sqr
t[3]*(5*A*b + 7*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) +
b^(1/3)*x)])/(216*a^(11/6)*b^(13/6))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {957, 817, 851, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}dx$$

$$\Big\downarrow \; 957$$

$$\frac{(7aB+5Ab)\int \frac{x^{5/2}}{(bx^3+a)^2}dx}{12ab}+\frac{x^{7/2}(Ab-aB)}{6ab(a+bx^3)^2}$$

$$\Big\downarrow \; 817$$

$$\frac{(7aB+5Ab)\left(\frac{\int \frac{1}{\sqrt{x}(bx^3+a)}dx}{6b}-\frac{\sqrt{x}}{3b(a+bx^3)}\right)}{12ab}+\frac{x^{7/2}(Ab-aB)}{6ab(a+bx^3)^2}$$

$$\Big\downarrow \text{851}$$

$$\frac{(7aB+5Ab)\left(\frac{\int \frac{1}{bx^3+a}d\sqrt{x}}{3b}-\frac{\sqrt{x}}{3b(a+bx^3)}\right)}{12ab}+\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{753}$$

$$\frac{(7aB+5Ab)\left(\frac{\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}}{3b}-\frac{\sqrt{x}}{3b(a+bx^3)}\right)}{12ab}+\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(7aB+5Ab)\left(\frac{\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}}{3b}-\frac{\sqrt{x}}{3b(a+bx^3)}\right)}{12ab}+\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{(7aB+5Ab)\left(\frac{\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}}{3b}-\frac{\sqrt{x}}{3b(a+bx^3)}\right)}{12ab}+\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1142}$$

$$\frac{(7aB+5Ab)\left(\frac{\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}\cdots}{6a^{5/6}\cdots}}{3b}\cdots\right)}{12ab}\cdots$$

$$\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 25

$$\frac{(7aB+5Ab)\left(\frac{\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\cdots}{6a^{5/6}\cdots}}{3b}\cdots\right)}{12ab}\cdots$$

$$\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 27

$$\frac{(7aB+5Ab)\left(\frac{\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\cdots}{6a^{5/6}\cdots}}{3b}\cdots\right)}{12ab}\cdots$$

$$\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{(7aB+5Ab)\left(\frac{\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(\frac{2\sqrt[6]{b}\cdots}{\sqrt{3}\sqrt[6]{\cdots}}\right)}{\sqrt{3}\sqrt[6]{b}\cdots}}{6a^{5/6}}\cdots}{3b}\cdots\right)}{12ab\cdots}$$

$$\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{(7aB+5Ab)\left(\frac{\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\left(\frac{2\cdots}{\sqrt{\cdots}}\cdots\right)\right)}{\sqrt[6]{\cdots}}}{6a^{5/6}}\cdots}{3b}\cdots\right)}{12ab\cdots}$$

$$\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{(7aB+5Ab)\left(\frac{\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}+\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)}{\sqrt[6]{b}}+\frac{\sqrt{3}\log\cdots}{\cdots}}{6a^{5/6}\cdots}\cdots}{3b}\cdots\right)}{12ab\cdots}$$

$$\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input

```
Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^(7/2))/(6*a*b*(a + b*x^3)^2) + ((5*A*b + 7*a*B)*(-1/3*Sqrt[
x]/(b*(a + b*x^3)) + (ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3*a^(5/6)*b^(1/6)
) + (-(ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6)
) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(
2*b^(1/6)))/(6*a^(5/6)) + (ArcTan[Sqrt[3]*(1 + (2*b^(1/6)*Sqrt[x])/(Sqrt[3
]*a^(1/6)))]/b^(1/6) + (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6)))/(3*b)))/(12*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\frac{(Ab-13Ba)x^{\frac{7}{2}}}{36ab}-\frac{(5Ab+7Ba)\sqrt{x}}{36b^2}}{(b\,x^3+a)^2}+\frac{(5Ab+7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}}{}\right.}{}$
default	$\frac{\frac{(Ab-13Ba)x^{\frac{7}{2}}}{36ab}-\frac{(5Ab+7Ba)\sqrt{x}}{36b^2}}{(b\,x^3+a)^2}+\frac{(5Ab+7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}}{}\right.}{}$

input
```
int(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/72*(A*b-13*B*a)/a/b*x^(7/2)-1/72*(5*A*b+7*B*a)/b^2*x^(1/2))/(b*x^3+a)
^2+1/36*(5*A*b+7*B*a)/b^2/a*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))
+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+
1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))-1/12/a*3^(1/2)*(a/
b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*a
rctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. 2(196) = 392.

Time = 0.10 (sec) , antiderivative size = 1618, normalized size of antiderivative = 6.20

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/432*(2*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504210*
A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4
*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*log(
a^2*b^2*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 +
857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 156
25*A^6*b^6)/(a^11*b^13))^(1/6) + (7*B*a + 5*A*b)*sqrt(x)) - 2*(a*b^4*x^6 +
 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*
A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250
*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*log(-a^2*b^2*(-(117649*B^
6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b
^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^
13))^(1/6) + (7*B*a + 5*A*b)*sqrt(x)) + (a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b
^2 + sqrt(-3)*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2))*(-(117649*B^6*a^6 + 5
04210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 4593
75*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6
)*log((7*B*a + 5*A*b)*sqrt(x) + 1/2*(sqrt(-3)*a^2*b^2 + a^2*b^2)*(-(117649
*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^
3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11
*b^13))^(1/6)) - (a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2 + sqrt(-3)*(a*b^4*x^
6 + 2*a^2*b^3*x^3 + a^3*b^2))*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.31

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{(13\,Bab-Ab^2)x^{\frac{7}{2}}+(7\,Ba^2+5\,Aab)\sqrt{x}}{36\,(ab^4x^6+2\,a^2b^3x^3+a^3b^2)} + \frac{\frac{\sqrt{3}(7\,Ba+5\,Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(7\,Ba+5\,Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(7\,Bab^{\frac{1}{3}}+5\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + }{432\,ab^2}$$

input
```
integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/36*((13*B*a*b - A*b^2)*x^(7/2) + (7*B*a^2 + 5*A*a*b)*sqrt(x))/(a*b^4*x^
6 + 2*a^2*b^3*x^3 + a^3*b^2) + 1/432*(sqrt(3)*(7*B*a + 5*A*b)*log(sqrt(3)*
a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)
*(7*B*a + 5*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3
))/(a^(5/6)*b^(1/6)) + 4*(7*B*a*b^(1/3) + 5*A*b^(4/3))*arctan(b^(1/3)*sqrt
(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(7*
B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6) +
 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3)
)) + 2*(7*B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6
)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1
/3)*b^(1/3))))/(a*b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.26

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\sqrt{3}\left(7\,(ab^5)^{\frac{1}{6}}\,Ba+5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^2b^3}$$
$$-\frac{\sqrt{3}\left(7\,(ab^5)^{\frac{1}{6}}\,Ba+5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^2b^3}$$
$$+\frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba+5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^2b^3}$$
$$+\frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba+5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^2b^3}$$
$$+\frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba+5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^2b^3}$$
$$-\frac{13\,Babx^{\frac{7}{2}}-Ab^2x^{\frac{7}{2}}+7\,Ba^2\sqrt{x}+5\,Aab\sqrt{x}}{36\,(bx^3+a)^2ab^2}$$

input
```
integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/432*sqrt(3)*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt
(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^3) - 1/432*sqrt(3)*(7*(a*b^5)^(1
/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b
)^(1/3))/(a^2*b^3) + 1/216*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arc
tan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^3) + 1/216*(7*(a
*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sq
rt(x))/(a/b)^(1/6))/(a^2*b^3) + 1/108*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/
6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b^3) - 1/36*(13*B*a*b*x^(7/2) - A
*b^2*x^(7/2) + 7*B*a^2*sqrt(x) + 5*A*a*b*sqrt(x))/((b*x^3 + a)^2*a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 1944, normalized size of antiderivative = 7.45

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
int((x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
(atan((((((5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 5
25*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)) - (x^(1/2)*(625*A^4*b^4 + 2
401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))
/(279936*a^4*b^3))*(5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)) - ((((5*
A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^
2))/(279936*(-a)^(23/6)*b^(19/6)) + (x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 +
 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*
b^3))*(5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)))/(((((5*A*b + 7*B*a)*
(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(
-a)^(23/6)*b^(19/6)) - (x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2
*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3))*(5*A*b
+ 7*B*a))/(216*(-a)^(11/6)*b^(13/6)) + ((((5*A*b + 7*B*a)*(125*A^3*b^3 + 3
43*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19
/6)) + (x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*
A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3))*(5*A*b + 7*B*a))/(216*(
-a)^(11/6)*b^(13/6))))*(5*A*b + 7*B*a)*1i)/(108*(-a)^(11/6)*b^(13/6)) - ((
x^(1/2)*(5*A*b + 7*B*a))/(36*b^2) - (x^(7/2)*(A*b - 13*B*a))/(36*a*b))/(a^
2 + b^2*x^6 + 2*a*b*x^3) + (atan(((((3^(1/2)*1i)/2 - 1/2)*((x^(1/2)*(625*A
^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3
*B*a*b^3))/(279936*a^4*b^3) - (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + 7*B*a)*(...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.27

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{-2b^{\frac{5}{6}}a^{\frac{7}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-2b^{\frac{11}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)x^3+2b^{\frac{5}{6}}a^{\frac{7}{6}}\,atan\left(b^{\frac{1}{6}}\right.}{}$$

input
```
int(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
( - 2*b**(5/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*b**(1/
3))/(b**(1/6)*a**(1/6)))*a - 2*b**(5/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*s
qrt(3) - 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*b*x**3 + 2*b**(5/6)*a**(
1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1
/6)))*a + 2*b**(5/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*
b**(1/3))/(b**(1/6)*a**(1/6)))*b*x**3 + 4*b**(5/6)*a**(1/6)*atan((sqrt(x)*
b**(1/3))/(b**(1/6)*a**(1/6)))*a + 4*b**(5/6)*a**(1/6)*atan((sqrt(x)*b**(1
/3))/(b**(1/6)*a**(1/6)))*b*x**3 - b**(5/6)*a**(1/6)*sqrt(3)*log( - sqrt(x
)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*x)*a - b**(5/6)*a**(1/6)
*sqrt(3)*log( - sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*x)
*b*x**3 + b**(5/6)*a**(1/6)*sqrt(3)*log(sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3)
+ a**(1/3) + b**(1/3)*x)*a + b**(5/6)*a**(1/6)*sqrt(3)*log(sqrt(x)*b**(1/6
)*a**(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*x)*b*x**3 - 12*sqrt(x)*a*b)/(36*a
*b**2*(a + b*x**3))
```

3.154 $\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 265

$$\begin{aligned}\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx &= \frac{(Ab-aB)x^{5/2}}{6ab\,(a+bx^3)^2}+\frac{(7Ab+5aB)x^{5/2}}{36a^2b\,(a+bx^3)} \\ &- \frac{(7Ab+5aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{13/6}b^{11/6}}+\frac{(7Ab+5aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{13/6}b^{11/6}} \\ &+ \frac{(7Ab+5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}}-\frac{(7Ab+5aB)\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{72\sqrt{3}a^{13/6}b^{11/6}}\end{aligned}$$

output
```
1/6*(A*b-B*a)*x^(5/2)/a/b/(b*x^3+a)^2+1/36*(7*A*b+5*B*a)*x^(5/2)/a^2/b/(b*
x^3+a)+1/216*(7*A*b+5*B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(1
3/6)/b^(11/6)+1/216*(7*A*b+5*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6)
)/a^(13/6)/b^(11/6)+1/108*(7*A*b+5*B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^
(13/6)/b^(11/6)-1/216*(7*A*b+5*B*a)*arctanh(3^(1/2)*a^(1/6)*b^(1/6)*x^(1/2
)/(a^(1/3)+b^(1/3)*x))*3^(1/2)/a^(13/6)/b^(11/6)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.73

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\frac{6\sqrt[6]{a}b^{5/6}x^{5/2}(-a^2B+7Ab^2x^3+ab(13A+5Bx^3))}{(a+bx^3)^2}+2(7Ab+5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-(7Ab+5}{216a^{13/6}b^{11/6}}$$

input
```
Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((6*a^(1/6)*b^(5/6)*x^(5/2)*(-(a^2*B) + 7*A*b^2*x^3 + a*b*(13*A + 5*B*x^3)
))/(a + b*x^3)^2 + 2*(7*A*b + 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - (
7*A*b + 5*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] - S
qrt[3]*(7*A*b + 5*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3)
+ b^(1/3)*x)])/(216*a^(13/6)*b^(11/6))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {957, 819, 851, 824, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{(5aB+7Ab)\int \frac{x^{3/2}}{(bx^3+a)^2}dx}{12ab}+\frac{x^{5/2}(Ab-aB)}{6ab(a+bx^3)^2}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{(5aB+7Ab)\left(\frac{\int \frac{x^{3/2}}{bx^3+a}dx}{6a}+\frac{x^{5/2}}{3a(a+bx^3)}\right)}{12ab}+\frac{x^{5/2}(Ab-aB)}{6ab(a+bx^3)^2}$$

$$\Big\downarrow \; 851$$

$$\frac{(5aB+7Ab)\left(\frac{\int \frac{x^2}{bx^3+a}d\sqrt{x}}{3a}+\frac{x^{5/2}}{3a(a+bx^3)}\right)}{12ab}+\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 824$$

$$\frac{(5aB+7Ab)\left(\frac{\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}+\frac{\int -\frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}+\frac{\int -\frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}}{3a}+\frac{x^{5/2}}{3a(a+bx^3)}\right)}{12ab}+\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(5aB+7Ab)\left(\frac{\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}}{3a}+\frac{x^{5/2}}{3a(a+bx^3)}\right)}{12ab}+\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{(5aB+7Ab)\left(\frac{-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}}{3a}+\frac{x^{5/2}}{3a(a+bx^3)}\right)}{12ab}+\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1142$$

$$\frac{(5aB+7Ab)\left(-\frac{\frac{-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}\cdots}{6\sqrt[6]{\cdots}}}{3a}\cdots\right.}{12ab}$$

$$\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 25

$$\frac{(5aB+7Ab)\left(-\frac{\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-\frac{1}{2}\cdots}{6\sqrt[6]{a}\cdots}}{3a}\cdots\right.}{12ab}$$

$$\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 27

$$\frac{(5aB+7Ab)\left(-\frac{\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\cdots}{6\sqrt[6]{\cdots}}}{3a}\cdots\right.}{12ab}$$

$$\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 1082

$$\frac{(5aB+7Ab)\left(\frac{-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\cdots}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}\cdots}\cdots}{6\sqrt[6]{a}b^{2/3}}\cdots}{3a}\cdots\right)}{12ab\cdots}$$

$$\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 217

$$\frac{(5aB+7Ab)\left(\frac{-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\left(\cdots\right)\right)}{\sqrt[6]{\cdots}}}{6\sqrt[6]{a}b^{2/3}}\cdots}{3a}\cdots\right)}{12ab\cdots}$$

$$\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{(5aB+7Ab)\left(\frac{\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\cdots\right)}{2\sqrt[6]{b}}\cdots}{6\sqrt[6]{a}b^{2\cdots}}\cdots}{3a}\cdots\right)}{12ab\cdots}$$

$$\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input

```
Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^(5/2))/(6*a*b*(a + b*x^3)^2) + ((7*A*b + 5*a*B)*(x^(5/2)/(3
*a*(a + b*x^3)) + (ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3*a^(1/6)*b^(5/6)) -
 (ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6) - (S
qrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1
/6)))/(6*a^(1/6)*b^(2/3)) - (-(ArcTan[Sqrt[3]*(1 + (2*b^(1/6)*Sqrt[x])/(Sq
rt[3]*a^(1/6)))]/b^(1/6)) + (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)
*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(1/6)*b^(2/3)))/(3*a)))/(12*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{(7Ab+5Ba)x^{\frac{11}{2}}}{36a^2}+\frac{(13Ab-Ba)x^{\frac{5}{2}}}{36ab}}{(b\,x^3+a)^2}+\frac{(7Ab+5Ba)\left(\frac{\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{}}{\left(\frac{a}{b}\right.}\right.}{6b\left(\frac{a}{b}\right.}\right.}{36a^2b}$
default	$\frac{\frac{(7Ab+5Ba)x^{\frac{11}{2}}}{36a^2}+\frac{(13Ab-Ba)x^{\frac{5}{2}}}{36ab}}{(b\,x^3+a)^2}+\frac{(7Ab+5Ba)\left(\frac{\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{}}{\left(\frac{a}{b}\right.}\right.}{6b\left(\frac{a}{b}\right.}\right.}{36a^2b}$

input
```
int(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/72*(7*A*b+5*B*a)/a^2*x^(11/2)+1/72*(13*A*b-B*a)/a/b*x^(5/2))/(b*x^3+a
)^2+1/36*(7*A*b+5*B*a)/a^2/b*(1/3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6)
)+1/12/a*3^(1/2)*(a/b)^(5/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))
+1/6/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))-1/12/a*3^(1/2)*(a
/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*
arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1959 vs. $2(199) = 398$.

Time = 0.13 (sec) , antiderivative size = 1959, normalized size of antiderivative = 7.39

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/432*(2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 + 131250*A
*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*
B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)*log(
a^11*b^9*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 +
857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117
649*A^6*b^6)/(a^13*b^11))^(5/6) + (3125*B^5*a^5 + 21875*A*B^4*a^4*b + 6125
0*A^2*B^3*a^3*b^2 + 85750*A^3*B^2*a^2*b^3 + 60025*A^4*B*a*b^4 + 16807*A^5*
b^5)*sqrt(x)) - 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 +
 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 90
0375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(
1/6)*log(-a^11*b^9*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*
a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a
*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(5/6) + (3125*B^5*a^5 + 21875*A*B^4*a^
4*b + 61250*A^2*B^3*a^3*b^2 + 85750*A^3*B^2*a^2*b^3 + 60025*A^4*B*a*b^4 +
16807*A^5*b^5)*sqrt(x)) + (a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b - sqrt(-3)*
(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b))*(-(15625*B^6*a^6 + 131250*A*B^5*a^5
*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*
b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)*log(1/2*(sqr
t(-3)*a^11*b^9 + a^11*b^9)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*
A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.02

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(5\,Bab+7\,Ab^2)x^{\frac{11}{2}}-(Ba^2-13\,Aab)x^{\frac{5}{2}}}{36\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)} - \frac{(5\,Ba+7\,Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}} - \frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}} - \frac{2\,\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} - \frac{2\,\arctan\left(\right.}{}\right.}{432\,a^2b}$$

input
```
integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/36*((5*B*a*b + 7*A*b^2)*x^(11/2) - (B*a^2 - 13*A*a*b)*x^(5/2))/(a^2*b^3*
x^6 + 2*a^3*b^2*x^3 + a^4*b) - 1/432*(5*B*a + 7*A*b)*(sqrt(3)*log(sqrt(3)*
a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)
*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5
/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)
*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3)*a^(1/6)*b^
(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^
(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^
(1/3)*b^(1/3))))/(a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.18

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(5\,Ba+7\,Ab)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,(ab^5)^{\frac{1}{6}}\,a^2b} + \frac{(5\,Ba+7\,Ab)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,(ab^5)^{\frac{1}{6}}\,a^2b} + \frac{\left(5\,Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}+7\,Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^3b} + \frac{5\,Babx^{\frac{11}{2}}+7\,Ab^2x^{\frac{11}{2}}-Ba^2x^{\frac{5}{2}}+13\,Aabx^{\frac{5}{2}}}{36\,(bx^3+a)^2a^2b} - \frac{\sqrt{3}\left(5\,(ab^5)^{\frac{5}{6}}\,Ba+7\,(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^6} + \frac{\sqrt{3}\left(5\,(ab^5)^{\frac{5}{6}}\,Ba+7\,(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^6}$$

input
```
integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/216*(5*B*a + 7*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6)
)/((a*b^5)^(1/6)*a^2*b) + 1/216*(5*B*a + 7*A*b)*arctan(-(sqrt(3)*(a/b)^(1/
6) - 2*sqrt(x))/(a/b)^(1/6))/((a*b^5)^(1/6)*a^2*b) + 1/108*(5*B*a*(a/b)^(5
/6) + 7*A*b*(a/b)^(5/6))*arctan(sqrt(x)/(a/b)^(1/6))/(a^3*b) + 1/36*(5*B*a
*b*x^(11/2) + 7*A*b^2*x^(11/2) - B*a^2*x^(5/2) + 13*A*a*b*x^(5/2))/((b*x^3
 + a)^2*a^2*b) - 1/432*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + 7*(a*b^5)^(5/6)*A*b)
*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^6) + 1/432*sqrt
(3)*(5*(a*b^5)^(5/6)*B*a + 7*(a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)
^(1/6) + x + (a/b)^(1/3))/(a^3*b^6)
```

Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 1672, normalized size of antiderivative = 6.31

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
int((x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
((x^(11/2)*(7*A*b + 5*B*a))/(36*a^2) + (x^(5/2)*(13*A*b - B*a))/(36*a*b))/
(a^2 + b^2*x^6 + 2*a*b*x^3) + (atan((((343*A^3*b^3 + 125*B^3*a^3 + 525*A*
B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*(7*A*b + 5*B*a)*(49*A^2
*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*(7*A*b
 + 5*B*a)^2*1i)/(46656*(-a)^(13/3)*b^(11/3)) - (((343*A^3*b^3 + 125*B^3*a^
3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1/2)*(7*A*b + 5*B*
a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6
)))*(7*A*b + 5*B*a)^2*1i)/(46656*(-a)^(13/3)*b^(11/3)))/((((343*A^3*b^3 +
125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*(7*
A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/
6)*b^(11/6)))*(7*A*b + 5*B*a)^2)/(46656*(-a)^(13/3)*b^(11/3)) + (((343*A^3
*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1
/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-
a)^(19/6)*b^(11/6)))*(7*A*b + 5*B*a)^2)/(46656*(-a)^(13/3)*b^(11/3))))*(7*
A*b + 5*B*a)*1i)/(108*(-a)^(13/6)*b^(11/6)) + (atan(((((3^(1/2)*1i)/2 - 1/
2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735
*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b + 5*B*a)
*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6))
)*1i)/(46656*(-a)^(13/3)*b^(11/3)) - ((((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b + 5*
B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)...
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.27

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{-2b^{\frac{1}{6}}a^{\frac{7}{6}}atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-2b^{\frac{7}{6}}a^{\frac{1}{6}}atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)x^3+2b^{\frac{1}{6}}a^{\frac{7}{6}}atan\left(\frac{b^{\frac{1}{6}}a\cdots}{\cdots}\right.}{\cdots}$$

input

```
int(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x)
```

output

```
( - 2*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*b**(1/
3))/(b**(1/6)*a**(1/6)))*a - 2*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*s
qrt(3) - 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*b*x**3 + 2*b**(1/6)*a**(
1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1
/6)))*a + 2*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*
b**(1/3))/(b**(1/6)*a**(1/6)))*b*x**3 + 4*b**(1/6)*a**(1/6)*atan((sqrt(x)*
b**(1/3))/(b**(1/6)*a**(1/6)))*a + 4*b**(1/6)*a**(1/6)*atan((sqrt(x)*b**(1
/3))/(b**(1/6)*a**(1/6)))*b*x**3 + b**(1/6)*a**(1/6)*sqrt(3)*log( - sqrt(x
)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*x)*a + b**(1/6)*a**(1/6)
*sqrt(3)*log( - sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*x)
*b*x**3 - b**(1/6)*a**(1/6)*sqrt(3)*log(sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3)
+ a**(1/3) + b**(1/3)*x)*a - b**(1/6)*a**(1/6)*sqrt(3)*log(sqrt(x)*b**(1/6
)*a**(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*x)*b*x**3 + 12*sqrt(x)*a**(1/3)*b
*x**2)/(36*a**(1/3)*a*b*(a + b*x**3))
```

3.155 $\int \frac{A+Bx^3}{\sqrt{x}(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 260

$$\begin{aligned}\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx &= \frac{(Ab-aB)\sqrt{x}}{6ab\,(a+bx^3)^2}+\frac{(11Ab+aB)\sqrt{x}}{36a^2b\,(a+bx^3)} \\ &- \frac{5(11Ab+aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{17/6}b^{7/6}} \\ &+ \frac{5(11Ab+aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{17/6}b^{7/6}} \\ &+ \frac{5(11Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}} \\ &+ \frac{5(11Ab+aB)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{72\sqrt{3}a^{17/6}b^{7/6}}\end{aligned}$$

output
```
1/6*(A*b-B*a)*x^(1/2)/a/b/(b*x^3+a)^2+1/36*(11*A*b+B*a)*x^(1/2)/a^2/b/(b*x
^3+a)+5/216*(11*A*b+B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/
6)/b^(7/6)+5/216*(11*A*b+B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^
(17/6)/b^(7/6)+5/108*(11*A*b+B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)
/b^(7/6)+5/216*(11*A*b+B*a)*arctanh(3^(1/2)*a^(1/6)*b^(1/6)*x^(1/2)/(a^(1/
3)+b^(1/3)*x))*3^(1/2)/a^(17/6)/b^(7/6)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx = \frac{\frac{6a^{5/6}\sqrt[6]{b}\sqrt{x}(-5a^2B+11Ab^2x^3+ab(17A+Bx^3))}{(a+bx^3)^2}+10(11Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-5(11Ab+aB)\arctan\left(\frac{\sqrt[3]{a}-}{\sqrt[6]{a}\sqrt[6]{}}\right.}{216a^{17/6}b^{7/6}}$$

input
```
Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^3),x]
```

output
```
((6*a^(5/6)*b^(1/6)*Sqrt[x]*(-5*a^2*B + 11*A*b^2*x^3 + a*b*(17*A + B*x^3))
)/(a + b*x^3)^2 + 10*(11*A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - 5*
(11*A*b + a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] + 5
*Sqrt[3]*(11*A*b + a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3)
 + b^(1/3)*x)])/(216*a^(17/6)*b^(7/6))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {957, 819, 851, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^3}dx$$

$\Big\downarrow$ 957

$$\frac{(aB+11Ab)\int \frac{1}{\sqrt{x}\left(bx^3+a\right)^2}dx}{12ab}+\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 819

$$\frac{(aB+11Ab)\left(\frac{5\int \frac{1}{\sqrt{x}\left(bx^3+a\right)}dx}{6a}+\frac{\sqrt{x}}{3a(a+bx^3)}\right)}{12ab}+\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 851

$$\frac{(aB+11Ab)\left(\frac{5\int \frac{1}{bx^3+a}d\sqrt{x}}{3a}+\frac{\sqrt{x}}{3a(a+bx^3)}\right)}{12ab}+\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 753

$$\frac{(aB+11Ab)\left(\frac{5\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}\right)}{3a}+\frac{\sqrt{x}}{3a(a+bx^3)}\right)}{12ab}+\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 27

$$\frac{(aB+11Ab)\left(\frac{5\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}\right)}{3a}+\frac{\sqrt{x}}{3a(a+bx^3)}\right)}{12ab}+\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 218

$$\frac{(aB+11Ab)\left(\frac{5\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}\right)}{3a}+\frac{\sqrt{x}}{3a(a+bx^3)}\right)}{12ab}+\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 1142

$$\frac{(aB+11Ab)\left(\frac{5\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d}{}\right.}{3a}\right.}{12ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 25

$$\frac{(aB+11Ab)\left(\frac{5\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{}}{6a}\right)}{3a}}{12ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 27

$$\frac{(aB+11Ab)\left(\frac{5\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d}{6}\right)}{3a}}{12ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{(aB+11Ab)\left(\frac{5\left(\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(\cdots\right)}{\sqrt{3}\sqrt[6]{\cdots}}\cdots}{6a^{5/6}}\cdots\right)}{3a}\cdots\right)}{12ab}\cdots$$

$$\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 217

$$\frac{(aB+11Ab)\left(\frac{5\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\left(\cdots\right)\right)}{\cdots}}{6a^{5/6}}\cdots\right)}{3a}\cdots\right)}{12ab}\cdots$$

$$\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{(aB+11Ab)\left(\frac{5\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}+\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)}{\sqrt[6]{b}}+\cdots}{\cdots}\right)}{3a}\cdots\right)}{12ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input
```
Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^3),x]
```

output
```
((A*b - a*B)*Sqrt[x])/(6*a*b*(a + b*x^3)^2) + ((11*A*b + a*B)*(Sqrt[x]/(3*
a*(a + b*x^3)) + (5*(ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3*a^(5/6)*b^(1/6))
 + (-(ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6))
 - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2
*b^(1/6)))/(6*a^(5/6)) + (ArcTan[Sqrt[3]*(1 + (2*b^(1/6)*Sqrt[x])/(Sqrt[3]
*a^(1/6)))]/b^(1/6) + (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[
x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6))))/(3*a)))/(12*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{(11Ab+Ba)x^{\frac{7}{2}}}{36a^2}+\frac{(17Ab-5Ba)\sqrt{x}}{36ab}}{(bx^3+a)^2}+\frac{5(11Ab+Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\right.}{}$
default	$\frac{\frac{(11Ab+Ba)x^{\frac{7}{2}}}{36a^2}+\frac{(17Ab-5Ba)\sqrt{x}}{36ab}}{(bx^3+a)^2}+\frac{5(11Ab+Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\right.}{}$

input
```
int((B*x^3+A)/x^(1/2)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/72*(11*A*b+B*a)/a^2*x^(7/2)+1/72*(17*A*b-5*B*a)/a/b*x^(1/2))/(b*x^3+a
)^2+5/36*(11*A*b+B*a)/a^2/b*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))
+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+
1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))-1/12/a*3^(1/2)*(a/
b)^(1/6)*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*a
rctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1588 vs. $2(194) = 388$.

Time = 0.13 (sec) , antiderivative size = 1588, normalized size of antiderivative = 6.11

$$\int \frac{A + Bx^3}{\sqrt{x}\,(a + bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^(1/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/432*(10*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*
b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4
+ 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)*log(5*a^3*b*(-(B
^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 2
19615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^
(1/6) + 5*(B*a + 11*A*b)*sqrt(x)) - 10*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*
b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*
b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17
*b^7))^(1/6)*log(-5*a^3*b*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b
^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 +
 1771561*A^6*b^6)/(a^17*b^7))^(1/6) + 5*(B*a + 11*A*b)*sqrt(x)) + 5*(a^2*b
^3*x^6 + 2*a^3*b^2*x^3 + a^4*b + sqrt(-3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a
^4*b))*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*
a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(
a^17*b^7))^(1/6)*log(5*(B*a + 11*A*b)*sqrt(x) + 5/2*(sqrt(-3)*a^3*b + a^3*
b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*
b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17
*b^7))^(1/6)) - 5*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b + sqrt(-3)*(a^2*b^3
*x^6 + 2*a^3*b^2*x^3 + a^4*b))*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*
a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)/x**(1/2)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx = \frac{(Bab+11\,Ab^2)x^{\frac{7}{2}}-(5\,Ba^2-17\,Aab)\sqrt{x}}{36\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)} + \frac{5\left(\frac{\sqrt{3}(Ba+11\,Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(Ba+11\,Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(Bab^{\frac{1}{3}}+11\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\cdots}{\sqrt{a^{\frac{1}{3}}\cdots}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\cdots\right)}{432\,a^2b}$$

input
```
integrate((B*x^3+A)/x^(1/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/36*((B*a*b + 11*A*b^2)*x^(7/2) - (5*B*a^2 - 17*A*a*b)*sqrt(x))/(a^2*b^3*
x^6 + 2*a^3*b^2*x^3 + a^4*b) + 5/432*(sqrt(3)*(B*a + 11*A*b)*log(sqrt(3)*a
^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*
(B*a + 11*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))
/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) + 11*A*b^(4/3))*arctan(b^(1/3)*sqrt(x)
/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(
4/3)*b^(1/3) + 11*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b
^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) +
 2*(B*a^(4/3)*b^(1/3) + 11*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(
1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b
^(1/3))))/(a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.24

$$
\begin{aligned}
&\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx \\
&= \frac{5\sqrt{3}\left((ab^5)^{\frac{1}{6}}Ba+11\,(ab^5)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^2} \\
&\quad-\frac{5\sqrt{3}\left((ab^5)^{\frac{1}{6}}Ba+11\,(ab^5)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^2} \\
&\quad+\frac{5\left((ab^5)^{\frac{1}{6}}Ba+11\,(ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^3b^2} \\
&\quad+\frac{5\left((ab^5)^{\frac{1}{6}}Ba+11\,(ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^3b^2} \\
&\quad+\frac{5\left((ab^5)^{\frac{1}{6}}Ba+11\,(ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^3b^2} \\
&\quad+\frac{Babx^{\frac{7}{2}}+11\,Ab^2x^{\frac{7}{2}}-5\,Ba^2\sqrt{x}+17\,Aab\sqrt{x}}{36\,(bx^3+a)^2a^2b}
\end{aligned}
$$

input
```
integrate((B*x^3+A)/x^(1/2)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
5/432*sqrt(3)*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(
x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^2) - 5/432*sqrt(3)*((a*b^5)^(1/6)
*B*a + 11*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^
(1/3))/(a^3*b^2) + 5/216*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*arctan
((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^2) + 5/216*((a*b^5)
^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x
))/(a/b)^(1/6))/(a^3*b^2) + 5/108*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*
b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^3*b^2) + 1/36*(B*a*b*x^(7/2) + 11*A*b^2*
x^(7/2) - 5*B*a^2*sqrt(x) + 17*A*a*b*sqrt(x))/((b*x^3 + a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 1952, normalized size of antiderivative = 7.51

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^3)/(x^(1/2)*(a + b*x^3)^3),x)
```

output
```
((x^(7/2)*(11*A*b + B*a))/(36*a^2) + (x^(1/2)*(17*A*b - 5*B*a))/(36*a*b))/
(a^2 + b^2*x^6 + 2*a*b*x^3) - (atan(((((625*x^(1/2)*(14641*A^4*b^5 + B^4*a
^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936
*a^8) - (625*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4
+ 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*(11*A*b + B*a)*5i)/(216
*(-a)^(17/6)*b^(7/6)) + (((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^
2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*
(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^
2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*(11*A*b + B*a)*5i)/(216*(-a)^(17/6)*
b^(7/6)))/((5*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b
^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*(11*A*b + B
*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(27
9936*(-a)^(47/6)*b^(7/6)))*(11*A*b + B*a))/(216*(-a)^(17/6)*b^(7/6)) - (5*
((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*
B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*(11*A*b + B*a)*(1331*A^3*
b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/
6)*b^(7/6)))*(11*A*b + B*a))/(216*(-a)^(17/6)*b^(7/6))))*(11*A*b + B*a)*5i
)/(108*(-a)^(17/6)*b^(7/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a
)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^
3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*((3^(1/2)*1i)/2 - 1/...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx = \frac{-10b^{\frac{5}{6}}a^{\frac{7}{6}}atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-10b^{\frac{11}{6}}a^{\frac{1}{6}}atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)x^3+10b^{\frac{5}{6}}a^{\frac{7}{6}}atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)+1\cdots}{}$$

input

```
int((B*x^3+A)/x^(1/2)/(b*x^3+a)^3,x)
```

output

```
( - 10*b**(5/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*b**(1
/3))/(b**(1/6)*a**(1/6)))*a - 10*b**(5/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)
*sqrt(3) - 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*b*x**3 + 10*b**(5/6)*a
**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*b**(1/3))/(b**(1/6)*a*
*(1/6)))*a + 10*b**(5/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt
(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*b*x**3 + 20*b**(5/6)*a**(1/6)*atan((sqr
t(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*a + 20*b**(5/6)*a**(1/6)*atan((sqrt(x)
*b**(1/3))/(b**(1/6)*a**(1/6)))*b*x**3 - 5*b**(5/6)*a**(1/6)*sqrt(3)*log(
- sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*x)*a - 5*b**(5/6
)*a**(1/6)*sqrt(3)*log( - sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + b
**(1/3)*x)*b*x**3 + 5*b**(5/6)*a**(1/6)*sqrt(3)*log(sqrt(x)*b**(1/6)*a**(1
/6)*sqrt(3) + a**(1/3) + b**(1/3)*x)*a + 5*b**(5/6)*a**(1/6)*sqrt(3)*log(s
qrt(x)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*x)*b*x**3 + 12*sqrt
(x)*a*b)/(36*a**2*b*(a + b*x**3))
```

3.156 $\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 270

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx=-\frac{2A}{a^3\sqrt{x}}-\frac{(Ab-aB)x^{5/2}}{6a^2\left(a+bx^3\right)^2}-\frac{(19Ab-7aB)x^{5/2}}{36a^3\left(a+bx^3\right)}+\frac{7(13Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{19/6}b^{5/6}}-\frac{7(13Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{19/6}b^{5/6}}-\frac{7(13Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{19/6}b^{5/6}}+\frac{7(13Ab-aB)\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{72\sqrt{3}a^{19/6}b^{5/6}}$$

output
```
-2*A/a^3/x^(1/2)-1/6*(A*b-B*a)*x^(5/2)/a^2/(b*x^3+a)^2-1/36*(19*A*b-7*B*a)
*x^(5/2)/a^3/(b*x^3+a)-7/216*(13*A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2
)/a^(1/6))/a^(19/6)/b^(5/6)-7/216*(13*A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^
(1/2)/a^(1/6))/a^(19/6)/b^(5/6)-7/108*(13*A*b-B*a)*arctan(b^(1/6)*x^(1/2)/
a^(1/6))/a^(19/6)/b^(5/6)+7/216*(13*A*b-B*a)*arctanh(3^(1/2)*a^(1/6)*b^(1/
6)*x^(1/2)/(a^(1/3)+b^(1/3)*x))*3^(1/2)/a^(19/6)/b^(5/6)
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx=\frac{-\frac{6\sqrt[6]{a}\left(91Ab^2x^6+a^2\left(72A-13Bx^3\right)+abx^3\left(169A-7Bx^3\right)\right)}{\sqrt{x}\left(a+bx^3\right)^2}+\frac{14(-13Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{b^{5/6}}+\frac{7(13Ab-}{}}{216a^{19/6}}$$

input
```
Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x]
```

output
```
((-6*a^(1/6)*(91*A*b^2*x^6 + a^2*(72*A - 13*B*x^3) + a*b*x^3*(169*A - 7*B*
x^3)))/(Sqrt[x]*(a + b*x^3)^2) + (14*(-13*A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[
x])/a^(1/6)])/b^(5/6) + (7*(13*A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^
(1/6)*b^(1/6)*Sqrt[x])])/b^(5/6) + (7*Sqrt[3]*(13*A*b - a*B)*ArcTanh[(Sqrt
[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/b^(5/6))/(216*a^(19/6
))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.24, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {957, 819, 847, 851, 824, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(13Ab-aB)\int \frac{1}{x^{3/2}\left(bx^3+a\right)^2}dx}{12ab}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{(13Ab-aB)\left(\frac{7\int \frac{1}{x^{3/2}\left(bx^3+a\right)}dx}{6a}+\frac{1}{3a\sqrt{x}(a+bx^3)}\right)}{12ab}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{847}$$

$$\frac{(13Ab-aB)\left(\frac{7\left(-\frac{b\int \frac{x^{3/2}}{bx^3+a}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{6a}+\frac{1}{3a\sqrt{x}(a+bx^3)}\right)}{12ab}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{851}$$

$$\frac{(13Ab-aB)\left(\frac{7\left(-\frac{2b\int \frac{x^2}{bx^3+a}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{6a}+\frac{1}{3a\sqrt{x}(a+bx^3)}\right)}{12ab}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{824}$$

$$\frac{(13Ab-aB)\left(\frac{7\left(-\frac{2b\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}+\frac{\int -\frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}+\frac{\int -\frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{6a}+\right)}{12ab}$$

$$\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$\downarrow$ 27

$$\frac{(13Ab-aB)\left(\frac{7\left(-\frac{2b\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{6a}+\frac{1}{3a\sqrt{x}(a+}\right.}{12ab}$$

$$\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$\downarrow$ 218

$$\frac{(13Ab-aB)\left(\frac{7\left(-\frac{2b\left(-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{6a}+\frac{1}{3a\sqrt{x}(a+}\right.}{12ab}$$

$$\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$\downarrow$ 1142

$$\frac{(13Ab-aB)\left(\frac{7\left(-\frac{2b\left(-\frac{-\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int-\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\sqrt{3}\int\frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right.}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+}}{2\sqrt[6]{b}}\right.}{a}\right.}{6a}\right.}{12ab}$$

$$\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$\downarrow$ 25

$$\frac{(13Ab-aB)\left(7\left(-\frac{2b\left(-\frac{\frac{\sqrt{3}\int\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\sqrt{3}\int\frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{\cdots}}}{2\sqrt[6]{b}}\cdots\right)}{a}\cdots\right)}{6a}\cdots\right)}{12ab}$$

$$\frac{Ab-aB}{6ab\sqrt{x}\,(a+bx^3)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(13Ab-aB)\left(\frac{7\left(-\frac{2b\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}\cdots}{\cdots}\cdots\right)}{a}\cdots\right)}{6a}\cdots\right)}{12ab}$$

$$\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$\big\downarrow$ 1082

$$\frac{(13Ab-aB)\left(\frac{7\left(-\frac{2b\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{}}}{6\sqrt[6]{a}b^{2/3}}}{a}}{6a}}{12ab}$$

$$\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 217

$$\frac{(13Ab-aB)\left(\frac{7\left(-\frac{2b\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\text{arct}\cdots}{6\sqrt[6]{a}b^{2/3}}\cdots\right)}{a}\cdots\right)}{6a}\cdots\right)}{12ab}\cdots$$

$$\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$\downarrow$ 1103

$$\frac{(13Ab-aB)\left(\frac{7\left(-\frac{2b\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{\cdots}\right.}{2\sqrt[6]{b}}\cdots\right)}{a}\cdots\right)}{6a}\cdots\right)}{12ab}$$

$$\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

input

```
Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x]
```

output

```
(A*b - a*B)/(6*a*b*Sqrt[x]*(a + b*x^3)^2) + ((13*A*b - a*B)*(1/(3*a*Sqrt[x
]*(a + b*x^3)) + (7*(-2/(a*Sqrt[x]) - (2*b*(ArcTan[(b^(1/6)*Sqrt[x])/a^(1/
6)]/(3*a^(1/6)*b^(5/6)) - (ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3
]*a^(1/6)))]/b^(1/6) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(1/6)*b^(2/3)) - (-(ArcTan[Sqrt[3]*(1
+ (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6)) + (Sqrt[3]*Log[a^(1/3)
+ Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(1/6)*b^
(2/3))))/a))/(6*a)))/(12*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.87

method	result
derivativedivides	$-\frac{2A}{a^3\sqrt{x}}-\frac{2\left(\frac{\left(\frac{19}{72}b^2A-\frac{7}{72}abB\right)x^{\frac{11}{2}}+\frac{a(25Ab-13Ba)x^{\frac{5}{2}}}{72}}{\left(bx^3+a\right)^2}+\left(\frac{91Ab}{72}-\frac{7Ba}{72}\right)\left(\frac{\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}\right.}{12a}\right.\right.}{a^3}$
default	$-\frac{2A}{a^3\sqrt{x}}-\frac{2\left(\frac{\left(\frac{19}{72}b^2A-\frac{7}{72}abB\right)x^{\frac{11}{2}}+\frac{a(25Ab-13Ba)x^{\frac{5}{2}}}{72}}{\left(bx^3+a\right)^2}+\left(\frac{91Ab}{72}-\frac{7Ba}{72}\right)\left(\frac{\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}\right.}{12a}\right.\right.}{a^3}$
risch	$-\frac{2A}{a^3\sqrt{x}}-\frac{\frac{2\left(\frac{19}{72}b^2A-\frac{7}{72}abB\right)x^{\frac{11}{2}}+\frac{a(25Ab-13Ba)x^{\frac{5}{2}}}{36}}{\left(bx^3+a\right)^2}+2\left(\frac{91Ab}{72}-\frac{7Ba}{72}\right)\left(\frac{\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\right.}{12a}\right.}{a^3}$

input
```
int((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*A/a^3/x^(1/2)-2/a^3*(((19/72*b^2*A-7/72*a*b*B)*x^(11/2)+1/72*a*(25*A*b-
13*B*a)*x^(5/2))/(b*x^3+a)^2+(91/72*A*b-7/72*B*a)*(1/3/b/(a/b)^(1/6)*arcta
n(x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(5/6)*ln(3^(1/2)*(a/b)^(1/6)*x
^(1/2)-x-(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/
2))-1/12/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3
))+1/6/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1934 vs. 2(198) = 396.

Time = 0.13 (sec) , antiderivative size = 1934, normalized size of antiderivative = 7.16

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/432*(14*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5*a^5*b
 + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 -
 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*log(16807*a^16*b
^4*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*
b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^1
9*b^5))^(5/6) - 16807*(B^5*a^5 - 65*A*B^4*a^4*b + 1690*A^2*B^3*a^3*b^2 - 2
1970*A^3*B^2*a^2*b^3 + 142805*A^4*B*a*b^4 - 371293*A^5*b^5)*sqrt(x)) - 14*
(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2
*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^
5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*log(-16807*a^16*b^4*(-(B^6*
a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 4284
15*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(5
/6) - 16807*(B^5*a^5 - 65*A*B^4*a^4*b + 1690*A^2*B^3*a^3*b^2 - 21970*A^3*B
^2*a^2*b^3 + 142805*A^4*B*a*b^4 - 371293*A^5*b^5)*sqrt(x)) + 7*(a^3*b^2*x^
7 + 2*a^4*b*x^4 + a^5*x - sqrt(-3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x))*(-
(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 +
 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5
))^(1/6)*log(16807/2*(sqrt(-3)*a^16*b^4 + a^16*b^4)*(-(B^6*a^6 - 78*A*B^5*
a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*
b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(5/6) - 16807*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)/x**(3/2)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx = \frac{7\,(Bab-13\,Ab^2)x^6+13\,(Ba^2-13\,Aab)x^3-72\,Aa^2}{36\left(a^3b^2x^{\frac{13}{2}}+2\,a^4bx^{\frac{7}{2}}+a^5\sqrt{x}\right)}$$

$$-\frac{7\,(Ba-13\,Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan \cdots}{\cdots}\right.}{432\,a^3}$$

input
```
integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/36*(7*(B*a*b - 13*A*b^2)*x^6 + 13*(B*a^2 - 13*A*a*b)*x^3 - 72*A*a^2)/(a^
3*b^2*x^(13/2) + 2*a^4*b*x^(7/2) + a^5*sqrt(x)) - 7/432*(B*a - 13*A*b)*(sq
rt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*
b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1
/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqr
t(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(
sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/
3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3))
)/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/a^3
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)^3}\,dx = \frac{7\,(Ba-13\,Ab)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,(ab^5)^{\frac{1}{6}}\,a^3} + \frac{7\,(Ba-13\,Ab)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,(ab^5)^{\frac{1}{6}}\,a^3} + \frac{7\,(Ba-13\,Ab)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,(ab^5)^{\frac{1}{6}}\,a^3} - \frac{2\,A}{a^3\sqrt{x}} + \frac{7\,Babx^{\frac{11}{2}}-19\,Ab^2x^{\frac{11}{2}}+13\,Ba^2x^{\frac{5}{2}}-25\,Aabx^{\frac{5}{2}}}{36\,(bx^3+a)^2a^3} - \frac{7\,\sqrt{3}\left((ab^5)^{\frac{5}{6}}\,Ba-13\,(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^4b^5} + \frac{7\,\sqrt{3}\left((ab^5)^{\frac{5}{6}}\,Ba-13\,(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^4b^5}$$

input
```
integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
7/216*(B*a - 13*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))
/((a*b^5)^(1/6)*a^3) + 7/216*(B*a - 13*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) -
 2*sqrt(x))/(a/b)^(1/6))/((a*b^5)^(1/6)*a^3) + 7/108*(B*a - 13*A*b)*arctan
(sqrt(x)/(a/b)^(1/6))/((a*b^5)^(1/6)*a^3) - 2*A/(a^3*sqrt(x)) + 1/36*(7*B*
a*b*x^(11/2) - 19*A*b^2*x^(11/2) + 13*B*a^2*x^(5/2) - 25*A*a*b*x^(5/2))/((
b*x^3 + a)^2*a^3) - 7/432*sqrt(3)*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*A*
b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^4*b^5) + 7/432*sq
rt(3)*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b
)^(1/6) + x + (a/b)^(1/3))/(a^4*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 1786, normalized size of antiderivative = 6.61

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x)
```

output
```
(atan((((13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^
21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (3
43*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 8294063127920
64*B^2*a^26*b^4 - 2156456413259366 4*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b
^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)) + ((13*A*b - B*a)^2*(62019785528064*A^3
*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 1431
2258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(14016966686185881
6*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 2156456413259366 4*A*B*a^25
*b^5))/(10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)))/(((13*A*
b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 11009
42938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13
*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^
4 - 2156456413259366 4*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6))))/((-a
)^(19/3)*b^(5/3)) - ((13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229
306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*
a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 +
829406312792064*B^2*a^26*b^4 - 2156456413259366 4*A*B*a^25*b^5))/(10077696*
(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3))))*(13*A*b - B*a)*7i)/(108*(-a
)^(19/6)*b^(5/6)) - ((2*A)/a + (13*x^3*(13*A*b - B*a))/(36*a^2) + (7*b*x^6
*(13*A*b - B*a))/(36*a^3))/(a^2*x^(1/2) + b^2*x^(13/2) + 2*a*b*x^(7/2))...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx = \frac{14\sqrt{x}\,b^{\frac{1}{6}}a^{\frac{7}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)+14\sqrt{x}\,b^{\frac{7}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)x^3-14\sqrt{x}\,b}{}$$

input
```
int((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x)
```

output
```
(14*sqrt(x)*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*
b**(1/3))/(b**(1/6)*a**(1/6)))*a + 14*sqrt(x)*b**(1/6)*a**(1/6)*atan((b**(
1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*b*x**3 -
14*sqrt(x)*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*b
**(1/3))/(b**(1/6)*a**(1/6)))*a - 14*sqrt(x)*b**(1/6)*a**(1/6)*atan((b**(1
/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*b*x**3 - 2
8*sqrt(x)*b**(1/6)*a**(1/6)*atan((sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*a
 - 28*sqrt(x)*b**(1/6)*a**(1/6)*atan((sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)
))*b*x**3 - 7*sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3)*log( - sqrt(x)*b**(1/6)*a*
*(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*x)*a - 7*sqrt(x)*b**(1/6)*a**(1/6)*sq
rt(3)*log( - sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*x)*b*
x**3 + 7*sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3)*log(sqrt(x)*b**(1/6)*a**(1/6)*s
qrt(3) + a**(1/3) + b**(1/3)*x)*a + 7*sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3)*lo
g(sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*x)*b*x**3 - 72*a
**(1/3)*a - 84*a**(1/3)*b*x**3)/(36*sqrt(x)*a**(1/3)*a**2*(a + b*x**3))
```

3.157 $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 272

$$\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^3}\,dx = -\frac{2A}{5a^3x^{5/2}} - \frac{(Ab-aB)\sqrt{x}}{6a^2(a+bx^3)^2} - \frac{(23Ab-11aB)\sqrt{x}}{36a^3(a+bx^3)} + \frac{11(17Ab-5aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{23/6}\sqrt[6]{b}} - \frac{11(17Ab-5aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{23/6}\sqrt[6]{b}} - \frac{11(17Ab-5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{23/6}\sqrt[6]{b}} - \frac{11(17Ab-5aB)\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{72\sqrt{3}a^{23/6}\sqrt[6]{b}}$$

output
```
-2/5*A/a^3/x^(5/2)-1/6*(A*b-B*a)*x^(1/2)/a^2/(b*x^3+a)^2-1/36*(23*A*b-11*B
*a)*x^(1/2)/a^3/(b*x^3+a)-11/216*(17*A*b-5*B*a)*arctan(-3^(1/2)+2*b^(1/6)*
x^(1/2)/a^(1/6))/a^(23/6)/b^(1/6)-11/216*(17*A*b-5*B*a)*arctan(3^(1/2)+2*b
^(1/6)*x^(1/2)/a^(1/6))/a^(23/6)/b^(1/6)-11/108*(17*A*b-5*B*a)*arctan(b^(1
/6)*x^(1/2)/a^(1/6))/a^(23/6)/b^(1/6)-11/216*(17*A*b-5*B*a)*arctanh(3^(1/2
)*a^(1/6)*b^(1/6)*x^(1/2)/(a^(1/3)+b^(1/3)*x))*3^(1/2)/a^(23/6)/b^(1/6)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx = \frac{-\frac{6a^{5/6}\left(187Ab^2x^6+a^2\left(72A-85Bx^3\right)+abx^3\left(289A-55Bx^3\right)\right)}{x^{5/2}\left(a+bx^3\right)^2}+\frac{110(-17Ab+5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{\sqrt[6]{b}}+\frac{55(1\ldots}{\ldots}}{1080a^{23/6}}$$

input
```
Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x]
```

output
```
((-6*a^(5/6)*(187*A*b^2*x^6 + a^2*(72*A - 85*B*x^3) + a*b*x^3*(289*A - 55*
B*x^3)))/(x^(5/2)*(a + b*x^3)^2) + (110*(-17*A*b + 5*a*B)*ArcTan[(b^(1/6)*
Sqrt[x])/a^(1/6)])/b^(1/6) + (55*(17*A*b - 5*a*B)*ArcTan[(a^(1/3) - b^(1/3
)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/b^(1/6) + (55*Sqrt[3]*(-17*A*b + 5*a*B)*A
rcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/b^(1/6))/
(1080*a^(23/6))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.20, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {957, 819, 847, 851, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{(17Ab-5aB)\int \frac{1}{x^{7/2}\left(bx^3+a\right)^2}dx}{12ab}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{(17Ab-5aB)\left(\frac{11\int \frac{1}{x^{7/2}\left(bx^3+a\right)}dx}{6a}+\frac{1}{3ax^{5/2}(a+bx^3)}\right)}{12ab}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$\downarrow$ 847

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{b\int \frac{1}{\sqrt{x}\left(bx^3+a\right)}dx}{a}-\frac{2}{5ax^{5/2}}\right)}{6a}+\frac{1}{3ax^{5/2}(a+bx^3)}\right)}{12ab}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$\downarrow$ 851

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{2b\int \frac{1}{bx^3+a}d\sqrt{x}}{a}-\frac{2}{5ax^{5/2}}\right)}{6a}+\frac{1}{3ax^{5/2}(a+bx^3)}\right)}{12ab}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$\downarrow$ 753

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{2b\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{6a}\right)}{12ab}$$

$$\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$\downarrow$ 27

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{2b\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{6a}+\frac{}{3ax^{5}}\right.}{12ab}$$

$$\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 218

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{2b\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{6a}+\frac{}{3ax^{5/}}\right.}{12ab}$$

$$\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1142

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{2b\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int-\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{\cdots}}}{\cdots}\right)}{a}\cdots\right)}{6a}\cdots\right)}{12ab}$$

$$\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$\downarrow$ 25

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{2b\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+ \cdots}}{\cdots}\cdots\right)}{a}\cdots\right)}{6a}\cdots\right)}{12ab}\cdots$$

$$\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$(17Ab-5aB)\left(\frac{11\left(-\frac{2b\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}}{}\right.}{a}\right.}{6a}\right.}{12ab}$$

$$\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$\downarrow$ 1082

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int \frac{\cdots}{-x \cdots}}{\cdots}}{6a^{5/6}} \cdots\right)}{a} \cdots\right)}{6a} \cdots\right)}{12ab} \cdots$$

$$\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$\downarrow$ 217

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{2b\left(\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\text{arc}}{}}{6a^{5/6}}}{a}\right)}{6a}\right)}{12ab}$$

$$\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$\downarrow$ 1103

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{2b\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}+\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+\cdots\right)\right)}{\sqrt[6]{b}}\cdots}{\cdots}\right)}{a}\cdots\right)}{6a}\cdots\right)}{12ab}$$

$$\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

input

```
Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x]
```

output

```
(A*b - a*B)/(6*a*b*x^(5/2)*(a + b*x^3)^2) + ((17*A*b - 5*a*B)*(1/(3*a*x^(5
/2)*(a + b*x^3)) + (11*(-2/(5*a*x^(5/2)) - (2*b*(ArcTan[(b^(1/6)*Sqrt[x])/
a^(1/6)]/(3*a^(5/6)*b^(1/6)) + (-(ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/
(Sqrt[3]*a^(1/6)))]/b^(1/6)) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1
/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6)) + (ArcTan[Sqrt[3]*(1 +
(2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6) + (Sqrt[3]*Log[a^(1/3) + S
qrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6))))/a)
)/(6*a)))/(12*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.87

method	result
derivativedivides	$-\frac{2A}{5a^3x^{\frac{5}{2}}}-\frac{2\left(\frac{\left(\frac{23}{72}b^2A-\frac{11}{72}abB\right)x^{\frac{7}{2}}+\frac{a(29Ab-17Ba)\sqrt{x}}{72}}{\left(bx^3+a\right)^2}+\frac{11(17Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\cdots\right.\right.}{12a}\cdots\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{2A}{5a^3x^{\frac{5}{2}}}-\frac{2\left(\frac{\left(\frac{23}{72}b^2A-\frac{11}{72}abB\right)x^{\frac{7}{2}}+\frac{a(29Ab-17Ba)\sqrt{x}}{72}}{\left(bx^3+a\right)^2}+\frac{11(17Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\cdots\right.\right.}{12a}\cdots\right.}{\cdots}\right.}{\cdots}$
risch	$-\frac{2A}{5a^3x^{\frac{5}{2}}}-\frac{\frac{2\left(\frac{23}{72}b^2A-\frac{11}{72}abB\right)x^{\frac{7}{2}}+\frac{a(29Ab-17Ba)\sqrt{x}}{36}}{\left(bx^3+a\right)^2}+\frac{11(17Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\cdots\right.\right.}{12a}\cdots\right.}{\cdots}}{\cdots}$

input
```
int((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/5*A/a^3/x^(5/2)-2/a^3*(((23/72*b^2*A-11/72*a*b*B)*x^(7/2)+1/72*a*(29*A*
b-17*B*a)*x^(1/2))/(b*x^3+a)^2+11/72*(17*A*b-5*B*a)*(1/3/a*(a/b)^(1/6)*arc
tan(x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/
6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(
1/2))-1/12/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1
/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. $2(202) = 404$.

Time = 0.11 (sec) , antiderivative size = 1608, normalized size of antiderivative = 5.91

$$\int \frac{A + Bx^3}{x^{7/2} (a + bx^3)^3} \, dx = \text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/2160*(110*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6*a^6 - 3187
50*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 3132
0375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^
(1/6)*log(11*a^4*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a
^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^
5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6) - 11*(5*B*a - 17*A*b)*sqrt(x
)) - 110*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6*a^6 - 318750*A
*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375
*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6
)*log(-11*a^4*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*
b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B
*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6) - 11*(5*B*a - 17*A*b)*sqrt(x))
+ 55*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3 + sqrt(-3)*(a^3*b^2*x^9 + 2*a^4*
b*x^6 + a^5*x^3))*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*
a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A
^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6)*log(-11*(5*B*a - 17*A*b)*sq
rt(x) + 11/2*(sqrt(-3)*a^4 + a^4)*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b +
2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*
b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6)) - 55*(a^3*
b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3 + sqrt(-3)*(a^3*b^2*x^9 + 2*a^4*b*x^6 +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)/x**(7/2)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx = \frac{11\,(5\,Bab-17\,Ab^2)x^6+17\,(5\,Ba^2-17\,Aab)x^3-72\,Aa^2}{180\left(a^3b^2x^{\frac{17}{2}}+2\,a^4bx^{\frac{11}{2}}+a^5x^{\frac{5}{2}}\right)} + \frac{11\left(\frac{\sqrt{3}(5\,Ba-17\,Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}(5\,Ba-17\,Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(5\,Bab^{\frac{1}{3}}-17\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b\cdots}{\sqrt{\cdots}}\right.}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\cdots\right.}{432\,a^3}$$

input
```
integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/180*(11*(5*B*a*b - 17*A*b^2)*x^6 + 17*(5*B*a^2 - 17*A*a*b)*x^3 - 72*A*a^
2)/(a^3*b^2*x^(17/2) + 2*a^4*b*x^(11/2) + a^5*x^(5/2)) + 11/432*(sqrt(3)*(
5*B*a - 17*A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))
/(a^(5/6)*b^(1/6)) - sqrt(3)*(5*B*a - 17*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)
*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(5*B*a*b^(1/3) - 17*
A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*
sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 17*A*a^(1/3)*b^(4/3))*ar
ctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/
(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 17*A*a^(1/3)*
b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3
)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/a^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^3}\,dx = \frac{11\sqrt{3}\left(5\,(ab^5)^{\frac{1}{6}}Ba-17\,(ab^5)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^4b} - \frac{11\sqrt{3}\left(5\,(ab^5)^{\frac{1}{6}}Ba-17\,(ab^5)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^4b} + \frac{11\left(5\,(ab^5)^{\frac{1}{6}}Ba-17\,(ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^4b} + \frac{11\left(5\,(ab^5)^{\frac{1}{6}}Ba-17\,(ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^4b} + \frac{11\left(5\,(ab^5)^{\frac{1}{6}}Ba-17\,(ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^4b} + \frac{11\,Babx^{\frac{7}{2}}-23\,Ab^2x^{\frac{7}{2}}+17\,Ba^2\sqrt{x}-29\,Aab\sqrt{x}}{36\,(bx^3+a)^2a^3} - \frac{2\,A}{5\,a^3x^{\frac{5}{2}}}$$

input
```
integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
11/432*sqrt(3)*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sq
rt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^4*b) - 11/432*sqrt(3)*(5*(a*b^5)^(
1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a
/b)^(1/3))/(a^4*b) + 11/216*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*a
rctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^4*b) + 11/216*(5*(
a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*
sqrt(x))/(a/b)^(1/6))/(a^4*b) + 11/108*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(
1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^4*b) + 1/36*(11*B*a*b*x^(7/2) - 2
3*A*b^2*x^(7/2) + 17*B*a^2*sqrt(x) - 29*A*a*b*sqrt(x))/((b*x^3 + a)^2*a^3)
 - 2/5*A/(a^3*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 2109, normalized size of antiderivative = 7.75

$$\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x)
```

output
```
- ((2*A)/(5*a) + (17*x^3*(17*A*b - 5*B*a))/(180*a^2) + (11*b*x^6*(17*A*b -
 5*B*a))/(180*a^3))/(a^2*x^(5/2) + b^2*x^(17/2) + 2*a*b*x^(11/2)) - (atan(
(((x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^
5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6
 - 521928791337000960*A^3*B*a^16*b^8) - (11*(17*A*b - 5*B*a)*(512439176949
055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*
A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1
/6)))*(17*A*b - 5*B*a)*11i)/(216*(-a)^(23/6)*b^(1/6)) + ((x^(1/2)*(4436394
72636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441
600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 5219287913370009
60*A^3*B*a^16*b^8) + (11*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8
 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 45
2152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*(17*A*b - 5*B
*a)*11i)/(216*(-a)^(23/6)*b^(1/6)))/((11*(x^(1/2)*(443639472636450816*A^4*
a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17
*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^
8) - (11*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 1303783780147
2000*B^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 452152214955048960
*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*(17*A*b - 5*B*a))/(216*(-a)^(
23/6)*b^(1/6)) - (11*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 331981...
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^3}\,dx = \frac{110\sqrt{x}\,b^{\frac{7}{6}}a^{\frac{7}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)x^2+110\sqrt{x}\,b^{\frac{13}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2\sqrt{x}\,b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)x^5-11}{}$$

input
```
int((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x)
```

output
```
(110*sqrt(x)*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)
*b**(1/3))/(b**(1/6)*a**(1/6)))*a*b*x**2 + 110*sqrt(x)*b**(1/6)*a**(1/6)*a
tan((b**(1/6)*a**(1/6)*sqrt(3) - 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*
b**2*x**5 - 110*sqrt(x)*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3)
+ 2*sqrt(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*a*b*x**2 - 110*sqrt(x)*b**(1/6)
*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*sqrt(x)*b**(1/3))/(b**(1/6)*
a**(1/6)))*b**2*x**5 - 220*sqrt(x)*b**(1/6)*a**(1/6)*atan((sqrt(x)*b**(1/3
))/(b**(1/6)*a**(1/6)))*a*b*x**2 - 220*sqrt(x)*b**(1/6)*a**(1/6)*atan((sqr
t(x)*b**(1/3))/(b**(1/6)*a**(1/6)))*b**2*x**5 + 55*sqrt(x)*b**(1/6)*a**(1/
6)*sqrt(3)*log( - sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*
x)*a*b*x**2 + 55*sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3)*log( - sqrt(x)*b**(1/6)
*a**(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*x)*b**2*x**5 - 55*sqrt(x)*b**(1/6)
*a**(1/6)*sqrt(3)*log(sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + b**(1
/3)*x)*a*b*x**2 - 55*sqrt(x)*b**(1/6)*a**(1/6)*sqrt(3)*log(sqrt(x)*b**(1/6
)*a**(1/6)*sqrt(3) + a**(1/3) + b**(1/3)*x)*b**2*x**5 - 72*b**(1/3)*a**2 -
 132*b**(1/3)*a*b*x**3)/(180*sqrt(x)*b**(1/3)*a**3*x**2*(a + b*x**3))
```

3.158 $\int x^8\sqrt{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int x^8\sqrt{a+bx^3}(A+Bx^3)\,dx=\frac{2a^2(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^4}-\frac{2a(2Ab-3aB)\left(a+bx^3\right)^{5/2}}{15b^4}+\frac{2(Ab-3aB)\left(a+bx^3\right)^{7/2}}{21b^4}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^4}$$

output
```
2/9*a^2*(A*b-B*a)*(b*x^3+a)^(3/2)/b^4-2/15*a*(2*A*b-3*B*a)*(b*x^3+a)^(5/2)
/b^4+2/21*(A*b-3*B*a)*(b*x^3+a)^(7/2)/b^4+2/27*B*(b*x^3+a)^(9/2)/b^4
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.73

$$\int x^8\sqrt{a+bx^3}(A+Bx^3)\,dx=\frac{2\left(a+bx^3\right)^{3/2}\left(-16a^3B+24a^2b(A+Bx^3)-6ab^2x^3(6A+5Bx^3)+5b^3x^6(9A+7Bx^3)\right)}{945b^4}$$

input
```
Integrate[x^8*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*(a + b*x^3)^(3/2)*(-16*a^3*B + 24*a^2*b*(A + B*x^3) - 6*a*b^2*x^3*(6*A
+ 5*B*x^3) + 5*b^3*x^6*(9*A + 7*B*x^3)))/(945*b^4)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int x^6\sqrt{bx^3+a}\left(Bx^3+A\right)dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int\left(\frac{B\left(bx^3+a\right)^{7/2}}{b^3}+\frac{(Ab-3aB)\left(bx^3+a\right)^{5/2}}{b^3}+\frac{a(3aB-2Ab)\left(bx^3+a\right)^{3/2}}{b^3}-\frac{a^2(aB-Ab)\sqrt{bx^3+a}}{b^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2a^2\left(a+bx^3\right)^{3/2}(Ab-aB)}{3b^4}+\frac{2\left(a+bx^3\right)^{7/2}(Ab-3aB)}{7b^4}-\frac{2a\left(a+bx^3\right)^{5/2}(2Ab-3aB)}{5b^4}+\frac{2B\left(a+bx^3\right)^{9/2}}{9b^4}\right)$$

input
```
Int[x^8*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
((2*a^2*(A*b - a*B)*(a + b*x^3)^(3/2))/(3*b^4) - (2*a*(2*A*b - 3*a*B)*(a +
 b*x^3)^(5/2))/(5*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(7/2))/(7*b^4) + (2*
B*(a + b*x^3)^(9/2))/(9*b^4))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{16\left(\frac{15\left(\frac{7Bx^3}{9}+A\right)x^6b^3}{8}-\frac{3a\left(\frac{5Bx^3}{6}+A\right)x^3b^2}{2}+a^2\left(Bx^3+A\right)b-\frac{2a^3B}{3}\right)\left(bx^3+a\right)^{\frac{3}{2}}}{315b^4}$
gosper	$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}\left(35b^3Bx^9+45Ab^3x^6-30Bab^2x^6-36aAb^2x^3+24Ba^2bx^3+24a^2bA-16a^3B\right)}{945b^4}$
orering	$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}\left(35b^3Bx^9+45Ab^3x^6-30Bab^2x^6-36aAb^2x^3+24Ba^2bx^3+24a^2bA-16a^3B\right)}{945b^4}$
trager	$\frac{2\left(35Bx^{12}b^4+45Ab^4x^9+5Bx^9ab^3+9Ax^6ab^3-6Bx^6a^2b^2-12Aa^2b^2x^3+8Ba^3bx^3+24Aa^3b-16Ba^4\right)\sqrt{bx^3+a}}{945b^4}$
risch	$\frac{2\left(35Bx^{12}b^4+45Ab^4x^9+5Bx^9ab^3+9Ax^6ab^3-6Bx^6a^2b^2-12Aa^2b^2x^3+8Ba^3bx^3+24Aa^3b-16Ba^4\right)\sqrt{bx^3+a}}{945b^4}$
elliptic	$\frac{2Bx^{12}\sqrt{bx^3+a}}{27}+\frac{2\left(Ab+\frac{Ba}{9}\right)x^9\sqrt{bx^3+a}}{21b}+\frac{2\left(Aa-\frac{6a\left(Ab+\frac{Ba}{9}\right)}{7b}\right)x^6\sqrt{bx^3+a}}{15b}-\frac{8a\left(Aa-\frac{6a\left(Ab+\frac{Ba}{9}\right)}{7b}\right)x^3\sqrt{bx^3+a}}{45b^2}$
default	$A\left(\frac{2x^9\sqrt{bx^3+a}}{21}+\frac{2ax^6\sqrt{bx^3+a}}{105b}-\frac{8a^2x^3\sqrt{bx^3+a}}{315b^2}+\frac{16a^3\sqrt{bx^3+a}}{315b^3}\right)+B\left(\frac{2x^{12}\sqrt{bx^3+a}}{27}+\frac{2ax^9\sqrt{bx^3+a}}{189b}-\right.$

input
```
int(x^8*(b*x^3+a)^(1/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
16/315*(15/8*(7/9*B*x^3+A)*x^6*b^3-3/2*a*(5/6*B*x^3+A)*x^3*b^2+a^2*(B*x^3+
A)*b-2/3*a^3*B)*(b*x^3+a)^(3/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.96

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{2\,(35\,Bb^4x^{12}+5\,(Bab^3+9\,Ab^4)x^9-3\,(2\,Ba^2b^2-3\,Aab^3)x^6-16\,Ba^4+24\,Aa^3b+4\,(2\,Ba^3b-3\,Aa^2b^2)}{945\,b^4}$$

input
```
integrate(x^8*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/945*(35*B*b^4*x^12 + 5*(B*a*b^3 + 9*A*b^4)*x^9 - 3*(2*B*a^2*b^2 - 3*A*a*
b^3)*x^6 - 16*B*a^4 + 24*A*a^3*b + 4*(2*B*a^3*b - 3*A*a^2*b^2)*x^3)*sqrt(b
*x^3 + a)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(100) = 200.

Time = 0.42 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.13

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\begin{cases}\frac{16Aa^3\sqrt{a+bx^3}}{315b^3}-\frac{8Aa^2x^3\sqrt{a+bx^3}}{315b^2}+\frac{2Aax^6\sqrt{a+bx^3}}{105b}+\frac{2Ax^9\sqrt{a+bx^3}}{21}-\frac{32Ba^4\sqrt{a+bx^3}}{945b^4}+\frac{16Ba^3x^3\sqrt{a+bx^3}}{945b^3}-\frac{4Ba^2x^6\sqrt{a+bx^3}}{315b^2} \\ \sqrt{a}\left(\frac{Ax^9}{9}+\frac{Bx^{12}}{12}\right)\end{cases}$$

input
```
integrate(x**8*(b*x**3+a)**(1/2)*(B*x**3+A),x)
```

output
```
Piecewise((16*A*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*A*a**2*x**3*sqrt(a +
b*x**3)/(315*b**2) + 2*A*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*A*x**9*sqrt(a
 + b*x**3)/21 - 32*B*a**4*sqrt(a + b*x**3)/(945*b**4) + 16*B*a**3*x**3*sqr
t(a + b*x**3)/(945*b**3) - 4*B*a**2*x**6*sqrt(a + b*x**3)/(315*b**2) + 2*B
*a*x**9*sqrt(a + b*x**3)/(189*b) + 2*B*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)
), (sqrt(a)*(A*x**9/9 + B*x**12/12), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{2}{945}\,B\left(\frac{35\,(bx^3+a)^{\frac{9}{2}}}{b^4}-\frac{135\,(bx^3+a)^{\frac{7}{2}}a}{b^4}+\frac{189\,(bx^3+a)^{\frac{5}{2}}a^2}{b^4}-\frac{105\,(bx^3+a)^{\frac{3}{2}}a^3}{b^4}\right)+\frac{2}{315}\,A\left(\frac{15\,(bx^3+a)^{\frac{7}{2}}}{b^3}-\frac{42\,(bx^3+a)^{\frac{5}{2}}a}{b^3}+\frac{35\,(bx^3+a)^{\frac{3}{2}}a^2}{b^3}\right)$$

input
```
integrate(x^8*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/945*B*(35*(b*x^3 + a)^(9/2)/b^4 - 135*(b*x^3 + a)^(7/2)*a/b^4 + 189*(b*x
^3 + a)^(5/2)*a^2/b^4 - 105*(b*x^3 + a)^(3/2)*a^3/b^4) + 2/315*A*(15*(b*x^
3 + a)^(7/2)/b^3 - 42*(b*x^3 + a)^(5/2)*a/b^3 + 35*(b*x^3 + a)^(3/2)*a^2/b
^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{2\left(35\,(bx^3+a)^{\frac{9}{2}}B-135\,(bx^3+a)^{\frac{7}{2}}Ba+189\,(bx^3+a)^{\frac{5}{2}}Ba^2-105\,(bx^3+a)^{\frac{3}{2}}Ba^3+45\,(bx^3+a)^{\frac{7}{2}}Ab-\right.}{945\,b^4}$$

input
```
integrate(x^8*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
2/945*(35*(b*x^3 + a)^(9/2)*B - 135*(b*x^3 + a)^(7/2)*B*a + 189*(b*x^3 + a
)^(5/2)*B*a^2 - 105*(b*x^3 + a)^(3/2)*B*a^3 + 45*(b*x^3 + a)^(7/2)*A*b - 1
26*(b*x^3 + a)^(5/2)*A*a*b + 105*(b*x^3 + a)^(3/2)*A*a^2*b)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\,B\,x^{12}\,\sqrt{b\,x^3+a}}{27}+\frac{x^9\,\sqrt{b\,x^3+a}\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{21\,b}+\frac{8\,a^2\,\left(2\,A\,a-\frac{6\,a\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{7\,b}\right)\,\sqrt{b\,x^3+a}}{45\,b^3}+\frac{x^6\,\left(2\,A\,a-\frac{6\,a\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{7\,b}\right)\,\sqrt{b\,x^3+a}}{15\,b}-\frac{4\,a\,x^3\,\left(2\,A\,a-\frac{6\,a\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{7\,b}\right)\,\sqrt{b\,x^3+a}}{45\,b^2}$$

input
```
int(x^8*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

output
```
(2*B*x^12*(a + b*x^3)^(1/2))/27 + (x^9*(a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/
9))/(21*b) + (8*a^2*(2*A*a - (6*a*(2*A*b + (2*B*a)/9))/(7*b))*(a + b*x^3)^
(1/2))/(45*b^3) + (x^6*(2*A*a - (6*a*(2*A*b + (2*B*a)/9))/(7*b))*(a + b*x^
3)^(1/2))/(15*b) - (4*a*x^3*(2*A*a - (6*a*(2*A*b + (2*B*a)/9))/(7*b))*(a +
 b*x^3)^(1/2))/(45*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.54

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\sqrt{b\,x^3+a}\,(35b^4x^{12}+50a\,b^3x^9+3a^2b^2x^6-4a^3b\,x^3+8a^4)}{945b^3}$$

input
```
int(x^8*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

output
```
(2*sqrt(a + b*x**3)*(8*a**4 - 4*a**3*b*x**3 + 3*a**2*b**2*x**6 + 50*a*b**3
*x**9 + 35*b**4*x**12))/(945*b**3)
```

3.159 $\int x^5\sqrt{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int x^5\sqrt{a+bx^3}(A+Bx^3)\,dx = -\frac{2a(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^3} + \frac{2(Ab-2aB)\left(a+bx^3\right)^{5/2}}{15b^3} + \frac{2B\left(a+bx^3\right)^{7/2}}{21b^3}$$

output
```
-2/9*a*(A*b-B*a)*(b*x^3+a)^(3/2)/b^3+2/15*(A*b-2*B*a)*(b*x^3+a)^(5/2)/b^3+
2/21*B*(b*x^3+a)^(7/2)/b^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int x^5\sqrt{a+bx^3}(A+Bx^3)\,dx = \frac{2\left(a+bx^3\right)^{3/2}\left(-14aAb+8a^2B+21Ab^2x^3-12abBx^3+15b^2Bx^6\right)}{315b^3}$$

input
```
Integrate[x^5*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*(a + b*x^3)^(3/2)*(-14*a*A*b + 8*a^2*B + 21*A*b^2*x^3 - 12*a*b*B*x^3 +
15*b^2*B*x^6))/(315*b^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int x^3\sqrt{bx^3+a}\left(Bx^3+A\right)dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int\left(\frac{B\left(bx^3+a\right)^{5/2}}{b^2}+\frac{(Ab-2aB)\left(bx^3+a\right)^{3/2}}{b^2}+\frac{a(aB-Ab)\sqrt{bx^3+a}}{b^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2\left(a+bx^3\right)^{5/2}(Ab-2aB)}{5b^3}-\frac{2a\left(a+bx^3\right)^{3/2}(Ab-aB)}{3b^3}+\frac{2B\left(a+bx^3\right)^{7/2}}{7b^3}\right)$$

input
```
Int[x^5*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
((-2*a*(A*b - a*B)*(a + b*x^3)^(3/2))/(3*b^3) + (2*(A*b - 2*a*B)*(a + b*x^
3)^(5/2))/(5*b^3) + (2*B*(a + b*x^3)^(7/2))/(7*b^3))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$-\frac{4\left(-\frac{3\left(\frac{5 B x^{3}}{7}+A\right) x^{3} b^{2}}{2}+a\left(\frac{6 B x^{3}}{7}+A\right) b-\frac{4 a^{2} B}{7}\right)\left(b x^{3}+a\right)^{\frac{3}{2}}}{45 b^{3}}$
gosper	$-\frac{2\left(b x^{3}+a\right)^{\frac{3}{2}}\left(-15 b^{2} B x^{6}-21 A b^{2} x^{3}+12 B a b x^{3}+14 a b A-8 a^{2} B\right)}{315 b^{3}}$
orering	$-\frac{2\left(b x^{3}+a\right)^{\frac{3}{2}}\left(-15 b^{2} B x^{6}-21 A b^{2} x^{3}+12 B a b x^{3}+14 a b A-8 a^{2} B\right)}{315 b^{3}}$
trager	$-\frac{2\left(-15 b^{3} B x^{9}-21 A b^{3} x^{6}-3 B a b^{2} x^{6}-7 a A b^{2} x^{3}+4 B a^{2} b x^{3}+14 a^{2} b A-8 a^{3} B\right)\sqrt{b x^{3}+a}}{315 b^{3}}$
risch	$-\frac{2\left(-15 b^{3} B x^{9}-21 A b^{3} x^{6}-3 B a b^{2} x^{6}-7 a A b^{2} x^{3}+4 B a^{2} b x^{3}+14 a^{2} b A-8 a^{3} B\right)\sqrt{b x^{3}+a}}{315 b^{3}}$
elliptic	$\frac{2 B x^{9}\sqrt{b x^{3}+a}}{21}+\frac{2\left(A b+\frac{B a}{7}\right) x^{6}\sqrt{b x^{3}+a}}{15 b}+\frac{2\left(A a-\frac{4 a\left(A b+\frac{B a}{7}\right)}{5 b}\right) x^{3}\sqrt{b x^{3}+a}}{9 b}-\frac{4 a\left(A a-\frac{4 a\left(A b+\frac{B a}{7}\right)}{5 b}\right)\sqrt{b x^{3}+a}}{9 b^{2}}$
default	$A\left(\frac{2 x^{6}\sqrt{b x^{3}+a}}{15}+\frac{2 a x^{3}\sqrt{b x^{3}+a}}{45 b}-\frac{4 a^{2}\sqrt{b x^{3}+a}}{45 b^{2}}\right)+B\left(\frac{2 x^{9}\sqrt{b x^{3}+a}}{21}+\frac{2 a x^{6}\sqrt{b x^{3}+a}}{105 b}-\frac{8 a^{2} x^{3}\sqrt{b x^{3}+a}}{315 b^{2}}+\right.$

input
```
int(x^5*(b*x^3+a)^(1/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
-4/45*(-3/2*(5/7*B*x^3+A)*x^3*b^2+a*(6/7*B*x^3+A)*b-4/7*a^2*B)*(b*x^3+a)^(
3/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2\,(15\,Bb^3x^9+3\,(Bab^2+7\,Ab^3)x^6+8\,Ba^3-14\,Aa^2b-(4\,Ba^2b-7\,Aab^2)x^3)\sqrt{bx^3+a}}{315\,b^3}$$

input
```
integrate(x^5*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/315*(15*B*b^3*x^9 + 3*(B*a*b^2 + 7*A*a*b^3)*x^6 + 8*B*a^3 - 14*A*a^2*b - (
4*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(70) = 140.

Time = 0.30 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.30

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \begin{cases} -\frac{4Aa^2\sqrt{a+bx^3}}{45b^2}+\frac{2Aax^3\sqrt{a+bx^3}}{45b}+\frac{2Ax^6\sqrt{a+bx^3}}{15}+\frac{16Ba^3\sqrt{a+bx^3}}{315b^3}-\frac{8Ba^2x^3\sqrt{a+bx^3}}{315b^2}+\frac{2Bax^6\sqrt{a+bx^3}}{105b}+\frac{2Bx^9\sqrt{a+bx^3}}{21} & \text{fo} \\ \sqrt{a}\left(\frac{Ax^6}{6}+\frac{Bx^9}{9}\right) & \text{o} \end{cases}$$

input
```
integrate(x**5*(b*x**3+a)**(1/2)*(B*x**3+A),x)
```

output
```
Piecewise((-4*A*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*A*a*x**3*sqrt(a + b*x*
*3)/(45*b) + 2*A*x**6*sqrt(a + b*x**3)/15 + 16*B*a**3*sqrt(a + b*x**3)/(31
5*b**3) - 8*B*a**2*x**3*sqrt(a + b*x**3)/(315*b**2) + 2*B*a*x**6*sqrt(a +
b*x**3)/(105*b) + 2*B*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (sqrt(a)*(A*x**
6/6 + B*x**9/9), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2}{315}\,B\left(\frac{15\,(bx^3+a)^{\frac{7}{2}}}{b^3}-\frac{42\,(bx^3+a)^{\frac{5}{2}}a}{b^3}+\frac{35\,(bx^3+a)^{\frac{3}{2}}a^2}{b^3}\right)+\frac{2}{45}\,A\left(\frac{3\,(bx^3+a)^{\frac{5}{2}}}{b^2}-\frac{5\,(bx^3+a)^{\frac{3}{2}}a}{b^2}\right)$$

input
```
integrate(x^5*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/315*B*(15*(b*x^3 + a)^(7/2)/b^3 - 42*(b*x^3 + a)^(5/2)*a/b^3 + 35*(b*x^3
 + a)^(3/2)*a^2/b^3) + 2/45*A*(3*(b*x^3 + a)^(5/2)/b^2 - 5*(b*x^3 + a)^(3/
2)*a/b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2\left(15\,(bx^3+a)^{\frac{7}{2}}B-42\,(bx^3+a)^{\frac{5}{2}}Ba+35\,(bx^3+a)^{\frac{3}{2}}Ba^2+21\,(bx^3+a)^{\frac{5}{2}}Ab-35\,(bx^3+a)^{\frac{3}{2}}Aab\right)}{315\,b^3}$$

input
```
integrate(x^5*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
2/315*(15*(b*x^3 + a)^(7/2)*B - 42*(b*x^3 + a)^(5/2)*B*a + 35*(b*x^3 + a)^
(3/2)*B*a^2 + 21*(b*x^3 + a)^(5/2)*A*b - 35*(b*x^3 + a)^(3/2)*A*a*b)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.56

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\,B\,x^9\,\sqrt{b\,x^3+a}}{21}+\frac{x^6\,\sqrt{b\,x^3+a}\,\left(2\,A\,b+\frac{2\,B\,a}{7}\right)}{15\,b}-\frac{2\,a\,\left(2\,A\,a-\frac{4\,a\,\left(2\,A\,b+\frac{2\,B\,a}{7}\right)}{5\,b}\right)\,\sqrt{b\,x^3+a}}{9\,b^2}+\frac{x^3\,\left(2\,A\,a-\frac{4\,a\,\left(2\,A\,b+\frac{2\,B\,a}{7}\right)}{5\,b}\right)\,\sqrt{b\,x^3+a}}{9\,b}$$

input
```
int(x^5*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

output
```
(2*B*x^9*(a + b*x^3)^(1/2))/21 + (x^6*(a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/7
))/(15*b) - (2*a*(2*A*a - (4*a*(2*A*b + (2*B*a)/7))/(5*b))*(a + b*x^3)^(1/
2))/(9*b^2) + (x^3*(2*A*a - (4*a*(2*A*b + (2*B*a)/7))/(5*b))*(a + b*x^3)^(
1/2))/(9*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\sqrt{b\,x^3+a}\,\left(5b^3x^9+8a\,b^2x^6+a^2b\,x^3-2a^3\right)}{105b^2}$$

input
```
int(x^5*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

output
```
(2*sqrt(a + b*x**3)*( - 2*a**3 + a**2*b*x**3 + 8*a*b**2*x**6 + 5*b**3*x**9
))/(105*b**2)
```

3.160 $\int x^2\sqrt{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^2}$$

output
```
2/9*(A*b-B*a)*(b*x^3+a)^(3/2)/b^2+2/15*B*(b*x^3+a)^(5/2)/b^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\left(a+bx^3\right)^{3/2}\left(5Ab-2aB+3bBx^3\right)}{45b^2}$$

input
```
Integrate[x^2*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*(a + b*x^3)^(3/2)*(5*A*b - 2*a*B + 3*b*B*x^3))/(45*b^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \sqrt{bx^3+a}\left(Bx^3+A\right)dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\frac{B\left(bx^3+a\right)^{3/2}}{b}+\frac{(Ab-aB)\sqrt{bx^3+a}}{b}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2\left(a+bx^3\right)^{3/2}(Ab-aB)}{3b^2}+\frac{2B\left(a+bx^3\right)^{5/2}}{5b^2}\right)$$

input
```
Int[x^2*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
((2*(A*b - a*B)*(a + b*x^3)^(3/2))/(3*b^2) + (2*B*(a + b*x^3)^(5/2))/(5*b^
2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{2(b\,x^3+a)^{\frac{3}{2}}(3bB\,x^3+5Ab-2Ba)}{45b^2}$	31
orering	$\frac{2(b\,x^3+a)^{\frac{3}{2}}(3bB\,x^3+5Ab-2Ba)}{45b^2}$	31
pseudoelliptic	$\frac{2((3B\,x^3+5A)b-2Ba)(b\,x^3+a)^{\frac{3}{2}}}{45b^2}$	32
trager	$\frac{2(3b^2B\,x^6+5A\,b^2x^3+Bab\,x^3+5abA-2a^2B)\sqrt{b\,x^3+a}}{45b^2}$	52
risch	$\frac{2(3b^2B\,x^6+5A\,b^2x^3+Bab\,x^3+5abA-2a^2B)\sqrt{b\,x^3+a}}{45b^2}$	52
default	$\frac{2A(b\,x^3+a)^{\frac{3}{2}}}{9b}+B\left(\frac{2x^6\sqrt{b\,x^3+a}}{15}+\frac{2a\,x^3\sqrt{b\,x^3+a}}{45b}-\frac{4a^2\sqrt{b\,x^3+a}}{45b^2}\right)$	69
elliptic	$\frac{2B\,x^6\sqrt{b\,x^3+a}}{15}+\frac{2\left(Ab+\frac{Ba}{5}\right)x^3\sqrt{b\,x^3+a}}{9b}+\frac{2\left(Aa-\frac{2a\left(Ab+\frac{Ba}{5}\right)}{3b}\right)\sqrt{b\,x^3+a}}{3b}$	74

input
```
int(x^2*(b*x^3+a)^(1/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/45*(b*x^3+a)^(3/2)*(3*B*b*x^3+5*A*b-2*B*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\,(3\,Bb^2x^6+(Bab+5\,Ab^2)x^3-2\,Ba^2+5\,Aab)\sqrt{bx^3+a}}{45\,b^2}$$

input
```
integrate(x^2*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/45*(3*B*b^2*x^6 + (B*a*b + 5*A*b^2)*x^3 - 2*B*a^2 + 5*A*a*b)*sqrt(b*x^3
+ a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(44) = 88.

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.54

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\begin{cases}\frac{2Aa\sqrt{a+bx^3}}{9b}+\frac{2Ax^3\sqrt{a+bx^3}}{9}-\frac{4Ba^2\sqrt{a+bx^3}}{45b^2}+\frac{2Bax^3\sqrt{a+bx^3}}{45b}+\frac{2Bx^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \sqrt{a}\left(\frac{Ax^3}{3}+\frac{Bx^6}{6}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(b*x**3+a)**(1/2)*(B*x**3+A),x)
```

output
```
Piecewise((2*A*a*sqrt(a + b*x**3)/(9*b) + 2*A*x**3*sqrt(a + b*x**3)/9 - 4*
B*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*B*a*x**3*sqrt(a + b*x**3)/(45*b) + 2
*B*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*(A*x**3/3 + B*x**6/6), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2}{45}\,B\left(\frac{3\,(bx^3+a)^{\frac{5}{2}}}{b^2}-\frac{5\,(bx^3+a)^{\frac{3}{2}}a}{b^2}\right)+\frac{2\,(bx^3+a)^{\frac{3}{2}}A}{9\,b}$$

input
```
integrate(x^2*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/45*B*(3*(b*x^3 + a)^(5/2)/b^2 - 5*(b*x^3 + a)^(3/2)*a/b^2) + 2/9*(b*x^3
+ a)^(3/2)*A/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\left(3\,(bx^3+a)^{\frac{5}{2}}B-5\,(bx^3+a)^{\frac{3}{2}}Ba+5\,(bx^3+a)^{\frac{3}{2}}Ab\right)}{45\,b^2}$$

input
```
integrate(x^2*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
2/45*(3*(b*x^3 + a)^(5/2)*B - 5*(b*x^3 + a)^(3/2)*B*a + 5*(b*x^3 + a)^(3/2
)*A*b)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{6\,B\,{\left(b\,x^3+a\right)}^{5/2}+10\,A\,b\,{\left(b\,x^3+a\right)}^{3/2}-10\,B\,a\,{\left(b\,x^3+a\right)}^{3/2}}{45\,b^2}$$

input
```
int(x^2*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

output
```
(6*B*(a + b*x^3)^(5/2) + 10*A*b*(a + b*x^3)^(3/2) - 10*B*a*(a + b*x^3)^(3/
2))/(45*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\sqrt{b\,x^3+a}\,(b^2x^6+2ab\,x^3+a^2)}{15b}$$

input
```
int(x^2*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

output
```
(2*sqrt(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6))/(15*b)
```

3.161 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{2}{3}A\sqrt{a+bx^3} + \frac{2B(a+bx^3)^{3/2}}{9b} - \frac{2}{3}\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

output
```
2/3*A*(b*x^3+a)^(1/2)+2/9*B*(b*x^3+a)^(3/2)/b-2/3*a^(1/2)*A*arctanh((b*x^3
+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{2\sqrt{a+bx^3}(3Ab+aB+bBx^3)}{9b} - \frac{2}{3}\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x,x]
```

output
```
(2*Sqrt[a + b*x^3]*(3*A*b + a*B + b*B*x^3))/(9*b) - (2*Sqrt[a]*A*ArcTanh[S
qrt[a + b*x^3]/Sqrt[a]])/3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {948, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{bx^3+a}(Bx^3+A)}{x^3}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(A\int \frac{\sqrt{bx^3+a}}{x^3}dx^3+\frac{2B(a+bx^3)^{3/2}}{3b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(A\left(a\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+2\sqrt{a+bx^3}\right)+\frac{2B(a+bx^3)^{3/2}}{3b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(A\left(\frac{2a\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{b}+2\sqrt{a+bx^3}\right)+\frac{2B(a+bx^3)^{3/2}}{3b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(A\left(2\sqrt{a+bx^3}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)+\frac{2B(a+bx^3)^{3/2}}{3b}\right)$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x,x]
```

output
```
((2*B*(a + b*x^3)^(3/2))/(3*b) + A*(2*Sqrt[a + b*x^3] - 2*Sqrt[a]*ArcTanh[
Sqrt[a + b*x^3]/Sqrt[a]]))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

method	result	size
default	$A\left(\frac{2\sqrt{b\,x^{3}+a}}{3}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3}\right)+\frac{2B\left(b\,x^{3}+a\right)^{\frac{3}{2}}}{9b}$	50
elliptic	$\frac{2B\,x^{3}\sqrt{b\,x^{3}+a}}{9}+\frac{2\left(Ab+\frac{Ba}{3}\right)\sqrt{b\,x^{3}+a}}{3b}-\frac{2\sqrt{a}\,A\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3}$	59
pseudoelliptic	$\frac{2Bb\,x^{3}\sqrt{b\,x^{3}+a}-6\sqrt{a}\,bA\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)+6Ab\sqrt{b\,x^{3}+a}+2Ba\sqrt{b\,x^{3}+a}}{9b}$	70

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
```

output
```
A*(2/3*(b*x^3+a)^(1/2)-2/3*a^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+2/9*B
*(b*x^3+a)^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx$$
$$=\left[\frac{3\,A\sqrt{a}b\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(Bbx^3+Ba+3\,Ab)\sqrt{bx^3+a}}{9\,b},\frac{2\left(3\,A\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^3+a}}\right)+(B\right.}{9\,b}\right.$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x,x, algorithm="fricas")
```

output
```
[1/9*(3*A*sqrt(a)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2
*(B*b*x^3 + B*a + 3*A*b)*sqrt(b*x^3 + a))/b, 2/9*(3*A*sqrt(-a)*b*arctan(sq
rt(-a)/sqrt(b*x^3 + a)) + (B*b*x^3 + B*a + 3*A*b)*sqrt(b*x^3 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{A\left(\begin{cases} \frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{a+bx^3} & \text{for}\ b\neq 0 \\ -\sqrt{a}\log\left(\frac{1}{x^3}\right) & \text{otherwise} \end{cases}\right)}{3} - \frac{B\left(\begin{cases} -\sqrt{a}x^3 & \text{for}\ b=0 \\ -\frac{2(a+bx^3)^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}\right)}{3}$$

input
```
integrate((b*x**3+a)**(1/2)*(B*x**3+A)/x,x)
```

output
```
A*Piecewise((2*a*atan(sqrt(a + b*x**3)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b*x
**3), Ne(b, 0)), (-sqrt(a)*log(x**(-3)), True))/3 - B*Piecewise((-sqrt(a)*
x**3, Eq(b, 0)), (-2*(a + b*x**3)**(3/2)/(3*b), True))/3
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{1}{3}\left(\sqrt{a}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right) + 2\sqrt{bx^3+a}\right)A + \frac{2\,(bx^3+a)^{\frac{3}{2}}B}{9\,b}$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x,x, algorithm="maxima")
```

output
```
1/3*(sqrt(a)*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))
+ 2*sqrt(b*x^3 + a))*A + 2/9*(b*x^3 + a)^(3/2)*B/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{2\,Aa\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}} + \frac{2\left((bx^3+a)^{\frac{3}{2}}Bb^2 + 3\,\sqrt{bx^3+a}Ab^3\right)}{9\,b^3}$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x,x, algorithm="giac")
```

output
```
2/3*A*a*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/9*((b*x^3 + a)^(3/2)
*B*b^2 + 3*sqrt(b*x^3 + a)*A*b^3)/b^3
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{2\,B\,x^3\,\sqrt{b\,x^3+a}}{9} + \frac{\sqrt{b\,x^3+a}\,\left(2\,A\,b+\frac{2\,B\,a}{3}\right)}{3\,b} + \frac{A\,\sqrt{a}\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x,x)
```

output
```
(2*B*x^3*(a + b*x^3)^(1/2))/9 + ((a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/3))/(3
*b) + (A*a^(1/2)*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) +
 a^(1/2)))/x^6))/3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{8\sqrt{b\,x^3+a}\,a}{9}+\frac{2\sqrt{b\,x^3+a}\,b\,x^3}{9} + \frac{\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)a}{3}-\frac{\sqrt{a}\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)a}{3}$$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x,x)
```

output
```
(8*sqrt(a + b*x**3)*a + 2*sqrt(a + b*x**3)*b*x**3 + 3*sqrt(a)*log(sqrt(a +
 b*x**3) - sqrt(a))*a - 3*sqrt(a)*log(sqrt(a + b*x**3) + sqrt(a))*a)/9
```

3.162 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx = \frac{2}{3}B\sqrt{a+bx^3} - \frac{A\sqrt{a+bx^3}}{3x^3} - \frac{(Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

output
```
2/3*B*(b*x^3+a)^(1/2)-1/3*A*(b*x^3+a)^(1/2)/x^3-1/3*(A*b+2*B*a)*arctanh((b
*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx = \frac{\sqrt{a+bx^3}(-A+2Bx^3)}{3x^3} + \frac{(-Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^4,x]
```

output
```
(Sqrt[a + b*x^3]*(-A + 2*B*x^3))/(3*x^3) + ((-(A*b) - 2*a*B)*ArcTanh[Sqrt[
a + b*x^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {948, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{bx^3+a}(Bx^3+A)}{x^6}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{(2aB+Ab)\int \frac{\sqrt{bx^3+a}}{x^3}dx^3}{2a}-\frac{A(a+bx^3)^{3/2}}{ax^3}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{(2aB+Ab)\left(a\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+2\sqrt{a+bx^3}\right)}{2a}-\frac{A(a+bx^3)^{3/2}}{ax^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{(2aB+Ab)\left(\frac{2a\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{b}+2\sqrt{a+bx^3}\right)}{2a}-\frac{A(a+bx^3)^{3/2}}{ax^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{(2aB+Ab)\left(2\sqrt{a+bx^3}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)}{2a}-\frac{A(a+bx^3)^{3/2}}{ax^3}\right)$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^4,x]
```

output
```
(-((A*(a + b*x^3)^(3/2))/(a*x^3)) + ((A*b + 2*a*B)*(2*Sqrt[a + b*x^3] - 2*
Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]))/(2*a))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{2B\sqrt{b\,x^{3}+a}}{3}-\frac{A\sqrt{b\,x^{3}+a}}{3x^{3}}-\frac{(Ab+2Ba)\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$	56
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{3x^{3}}+\frac{2B\sqrt{b\,x^{3}+a}}{3}-\frac{2\left(\frac{Ab}{2}+Ba\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$	56
pseudoelliptic	$-\frac{(Ab+2Ba)\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)x^{3}+\sqrt{b\,x^{3}+a}\,(-2B\,x^{3}+A)\sqrt{a}}{3\sqrt{a}\,x^{3}}$	57
default	$A\left(-\frac{\sqrt{b\,x^{3}+a}}{3x^{3}}-\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}\right)+B\left(\frac{2\sqrt{b\,x^{3}+a}}{3}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3}\right)$	72

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2/3*B*(b*x^3+a)^(1/2)-1/3*A*(b*x^3+a)^(1/2)/x^3-1/3*(A*b+2*B*a)*arctanh((b
*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx = \left[\frac{(2\,Ba+Ab)\sqrt{a}x^3\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(2\,Bax^3-Aa)\sqrt{bx^3+a}}{6\,ax^3}, \frac{(2\,Ba+Ab)\sqrt{-a}x^3\arctan\left(\frac{\vphantom{x}}{\sqrt{\vphantom{x}}}\right.}{3}\right.$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^4,x, algorithm="fricas")
```

output
```
[1/6*((2*B*a + A*b)*sqrt(a)*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2
*a)/x^3) + 2*(2*B*a*x^3 - A*a)*sqrt(b*x^3 + a))/(a*x^3), 1/3*((2*B*a + A*b
)*sqrt(-a)*x^3*arctan(sqrt(-a)/sqrt(b*x^3 + a)) + (2*B*a*x^3 - A*a)*sqrt(b
*x^3 + a))/(a*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(65) = 130$.

Time = 14.75 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{Ab\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}} - \frac{2B\sqrt{a}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{2Ba}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2B\sqrt{b}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

input
```
integrate((b*x**3+a)**(1/2)*(B*x**3+A)/x**4,x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - A*b*asinh(sqrt(a)/(sqrt(b)*
x**(3/2)))/(3*sqrt(a)) - 2*B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 +
 2*B*a/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*B*sqrt(b)*x**(3/2)/(3
*sqrt(a/(b*x**3) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx = \frac{1}{6}\left(\frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\sqrt{bx^3+a}}{x^3}\right)A + \frac{1}{3}\left(\sqrt{a}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right) + 2\sqrt{bx^3+a}\right)B$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^4,x, algorithm="maxima")
```

output
```
1/6*(b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a
) - 2*sqrt(b*x^3 + a)/x^3)*A + 1/3*(sqrt(a)*log((sqrt(b*x^3 + a) - sqrt(a)
)/(sqrt(b*x^3 + a) + sqrt(a))) + 2*sqrt(b*x^3 + a))*B
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx$$
$$=\frac{1}{3}\,b\left(\frac{2\sqrt{bx^3+a}B}{b}+\frac{(2\,Ba+Ab)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}b}-\frac{\sqrt{bx^3+a}A}{bx^3}\right)$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^4,x, algorithm="giac")
```

output
```
1/3*b*(2*sqrt(b*x^3 + a)*B/b + (2*B*a + A*b)*arctan(sqrt(b*x^3 + a)/sqrt(-
a))/(sqrt(-a)*b) - sqrt(b*x^3 + a)*A/(b*x^3))
```

Mupad [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx=\frac{2\,B\,\sqrt{b\,x^3+a}}{3}-\frac{A\,\sqrt{b\,x^3+a}}{3\,x^3}$$
$$+\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\,\left(\frac{A\,b}{2}+B\,a\right)}{3\,\sqrt{a}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^4,x)
```

output
```
(2*B*(a + b*x^3)^(1/2))/3 - (A*(a + b*x^3)^(1/2))/(3*x^3) + (log((((a + b*
x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*((A*b)/2 + B*a
))/(3*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx$$
$$= \frac{-2\sqrt{b\,x^3+a}\,a+4\sqrt{b\,x^3+a}\,b\,x^3+3\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)b\,x^3-3\sqrt{a}\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)b\,x^3}{6x^3}$$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^4,x)
```

output
```
( - 2*sqrt(a + b*x**3)*a + 4*sqrt(a + b*x**3)*b*x**3 + 3*sqrt(a)*log(sqrt(
a + b*x**3) - sqrt(a))*b*x**3 - 3*sqrt(a)*log(sqrt(a + b*x**3) + sqrt(a))*
b*x**3)/(6*x**3)
```

3.163 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx = -\frac{A\sqrt{a+bx^3}}{6x^6} - \frac{(Ab+4aB)\sqrt{a+bx^3}}{12ax^3} + \frac{b(Ab-4aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$

output
```
-1/6*A*(b*x^3+a)^(1/2)/x^6-1/12*(A*b+4*B*a)*(b*x^3+a)^(1/2)/a/x^3+1/12*b*(
A*b-4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx = \frac{\sqrt{a+bx^3}(-2aA-Abx^3-4aBx^3)}{12ax^6} - \frac{b(-Ab+4aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^7,x]
```

output
```
(Sqrt[a + b*x^3]*(-2*a*A - A*b*x^3 - 4*a*B*x^3))/(12*a*x^6) - (b*(-(A*b) +
 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {948, 87, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{bx^3+a}(Bx^3+A)}{x^9}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(-\frac{(Ab-4aB)\int \frac{\sqrt{bx^3+a}}{x^6}dx^3}{4a}-\frac{A(a+bx^3)^{3/2}}{2ax^6}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}\left(-\frac{(Ab-4aB)\left(\frac{1}{2}b\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3-\frac{\sqrt{a+bx^3}}{x^3}\right)}{4a}-\frac{A(a+bx^3)^{3/2}}{2ax^6}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{(Ab-4aB)\left(\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}-\frac{\sqrt{a+bx^3}}{x^3}\right)}{4a}-\frac{A(a+bx^3)^{3/2}}{2ax^6}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{(Ab-4aB)\left(-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^3}}{x^3}\right)}{4a}-\frac{A(a+bx^3)^{3/2}}{2ax^6}\right)$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^7,x]
```

output
```
(-1/2*(A*(a + b*x^3)^(3/2))/(a*x^6) - ((A*b - 4*a*B)*(-(Sqrt[a + b*x^3]/x^
3) - (b*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/Sqrt[a]))/(4*a))/3
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

method	result	size
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(A b\,x^{3}+4 B a\,x^{3}+2 A a\right)}{12 x^{6} a}+\frac{b \left(A b-4 B a\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{12 a^{\frac{3}{2}}}$	65
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{6 x^{6}}-\frac{\left(A b+4 B a\right)\sqrt{b\,x^{3}+a}}{12 a\,x^{3}}+\frac{b \left(A b-4 B a\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{12 a^{\frac{3}{2}}}$	70
pseudoelliptic	$-\frac{-b\,x^{6} \left(A b-4 B a\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)+\sqrt{b\,x^{3}+a}\,\left(\left(4 B\,x^{3}+2 A\right) a^{\frac{3}{2}}+A\sqrt{a}\,b\,x^{3}\right)}{12 a^{\frac{3}{2}} x^{6}}$	72
default	$A \left(-\frac{\sqrt{b\,x^{3}+a}}{6 x^{6}}-\frac{b\sqrt{b\,x^{3}+a}}{12 a\,x^{3}}+\frac{b^{2} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{12 a^{\frac{3}{2}}}\right)+B \left(-\frac{\sqrt{b\,x^{3}+a}}{3 x^{3}}-\frac{b \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}\right)$	96

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b*x^3+a)^(1/2)*(A*b*x^3+4*B*a*x^3+2*A*a)/x^6/a+1/12*b*(A*b-4*B*a)*a
rctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.99

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx$$
$$= \left[-\frac{(4\,Bab-Ab^2)\sqrt{a}x^6\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,((4\,Ba^2+Aab)x^3+2\,Aa^2)\sqrt{bx^3+a}}{24\,a^2x^6}, \frac{(4\,Bab-Ab^2)}{}\right.$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^7,x, algorithm="fricas")
```

output
```
[-1/24*((4*B*a*b - A*b^2)*sqrt(a)*x^6*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(
a) + 2*a)/x^3) + 2*((4*B*a^2 + A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^2
*x^6), 1/12*((4*B*a*b - A*b^2)*sqrt(-a)*x^6*arctan(sqrt(-a)/sqrt(b*x^3 + a
)) - ((4*B*a^2 + A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^2*x^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(75) = 150$.

Time = 39.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx = -\frac{Aa}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{A\sqrt{b}}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{Ab^{\frac{3}{2}}}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

input
```
integrate((b*x**3+a)**(1/2)*(B*x**3+A)/x**7,x)
```

output
```
-A*a/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - A*sqrt(b)/(4*x**(9/2)*sq
rt(a/(b*x**3) + 1)) - A*b**(3/2)/(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) + A*
b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2)) - B*sqrt(b)*sqrt(a/(b
*x**3) + 1)/(3*x**(3/2)) - B*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a
))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(70) = 140$.

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx$$
$$= -\frac{1}{24}\left(\frac{b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\left((bx^3+a)^{\frac{3}{2}}b^2+\sqrt{bx^3+a}ab^2\right)}{(bx^3+a)^2a-2\,(bx^3+a)a^2+a^3}\right)A$$
$$+\frac{1}{6}\left(\frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{bx^3+a}}{x^3}\right)B$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^7,x, algorithm="maxima")
```

output
```
-1/24*(b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^
(3/2) + 2*((b*x^3 + a)^(3/2)*b^2 + sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2*a
 - 2*(b*x^3 + a)*a^2 + a^3))*A + 1/6*(b*log((sqrt(b*x^3 + a) - sqrt(a))/(s
qrt(b*x^3 + a) + sqrt(a)))/sqrt(a) - 2*sqrt(b*x^3 + a)/x^3)*B
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx$$
$$= \frac{\frac{(4\,Bab^2-Ab^3)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{4\,(bx^3+a)^{\frac{3}{2}}Bab^2-4\sqrt{bx^3+a}Ba^2b^2+(bx^3+a)^{\frac{3}{2}}Ab^3+\sqrt{bx^3+a}Aab^3}{ab^2x^6}}{12\,b}$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^7,x, algorithm="giac")
```

output
```
1/12*((4*B*a*b^2 - A*b^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) -
(4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a^2*b^2 + (b*x^3 + a)^(
3/2)*A*b^3 + sqrt(b*x^3 + a)*A*a*b^3)/(a*b^2*x^6))/b
```

Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx = \frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)\,\left(A\,b-4\,B\,a\right)}{24\,a^{3/2}} - \frac{\left(4\,B\,a^2+A\,b\,a\right)\,\sqrt{b\,x^3+a}}{12\,a^2\,x^3} - \frac{A\,\sqrt{b\,x^3+a}}{6\,x^6}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^7,x)
```

output
```
(b*log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6
)*(A*b - 4*B*a))/(24*a^(3/2)) - ((4*B*a^2 + A*a*b)*(a + b*x^3)^(1/2))/(12*
a^2*x^3) - (A*(a + b*x^3)^(1/2))/(6*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx = \frac{-4\sqrt{b\,x^3+a}\,a^2-10\sqrt{b\,x^3+a}\,ab\,x^3+3\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)b^2x^6-3\sqrt{a}\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)b^2x^6}{24a\,x^6}$$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^7,x)
```

output
```
( - 4*sqrt(a + b*x**3)*a**2 - 10*sqrt(a + b*x**3)*a*b*x**3 + 3*sqrt(a)*log
(sqrt(a + b*x**3) - sqrt(a))*b**2*x**6 - 3*sqrt(a)*log(sqrt(a + b*x**3) +
sqrt(a))*b**2*x**6)/(24*a*x**6)
```

3.164 $\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 303

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{6a(17Ab-8aB)x\sqrt{a+bx^3}}{935b^2}+\frac{2(17Ab-8aB)x^4\sqrt{a+bx^3}}{187b}+\frac{2Bx^4\left(a+bx^3\right)^{3/2}}{17b}$$
$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2(17Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}\right.\right.}{935b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
6/935*a*(17*A*b-8*B*a)*x*(b*x^3+a)^(1/2)/b^2+2/187*(17*A*b-8*B*a)*x^4*(b*x
^3+a)^(1/2)/b+2/17*B*x^4*(b*x^3+a)^(3/2)/b-4/935*3^(3/4)*(1/2*6^(1/2)+1/2*
2^(1/2))*a^2*(17*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1
/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(
7/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)
/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.76 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.29

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{2x\sqrt{a+bx^3}\left(-\left(\left(a+bx^3\right)\left(-17Ab+8aB-11bBx^3\right)\right)+\frac{a(-17Ab+8aB)\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{187b^2}$$

input
```
Integrate[x^3*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(-((a + b*x^3)*(-17*A*b + 8*a*B - 11*b*B*x^3)) + (a*(
-17*A*b + 8*a*B)*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 +
 (b*x^3)/a]))/(187*b^2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {959, 811, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$\Big\downarrow \text{959}$$
$$\frac{(17Ab-8aB)\int x^3\sqrt{bx^3+a}dx}{17b}+\frac{2Bx^4\left(a+bx^3\right)^{3/2}}{17b}$$
$$\Big\downarrow \text{811}$$
$$\frac{(17Ab-8aB)\left(\frac{3}{11}a\int\frac{x^3}{\sqrt{bx^3+a}}dx+\frac{2}{11}x^4\sqrt{a+bx^3}\right)}{17b}+\frac{2Bx^4\left(a+bx^3\right)^{3/2}}{17b}$$
$$\Big\downarrow \text{843}$$

$$\frac{(17Ab-8aB)\left(\frac{3}{11}a\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{2a\int\frac{1}{\sqrt{bx^3+a}}dx}{5b}\right)+\frac{2}{11}x^4\sqrt{a+bx^3}\right)}{17b}+\frac{2Bx^4(a+bx^3)^{3/2}}{17b}$$

$\big\downarrow$ 759

$$\frac{(17Ab-8aB)\left(\frac{3}{11}a\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-\right.}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.}{17b}$$

$$\frac{2Bx^4(a+bx^3)^{3/2}}{17b}$$

input
```
Int[x^3*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*B*x^4*(a + b*x^3)^(3/2))/(17*b) + ((17*A*b - 8*a*B)*((2*x^4*Sqrt[a + b*
x^3])/11 + (3*a*((2*x*Sqrt[a + b*x^3])/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/
4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3])))/11))/(17*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.15

method	result
risch	$\frac{2x(55b^2Bx^6+85Ab^2x^3+15Babx^3+51abA-24a^2B)\sqrt{bx^3+a}}{935b^2}+\frac{4ia^2(17Ab-8Ba)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{}$
elliptic	$\frac{2x^7\sqrt{bx^3+a}\,B}{17}+\frac{2\left(Ab+\frac{3Ba}{17}\right)x^4\sqrt{bx^3+a}}{11b}+\frac{2\left(Aa-\frac{8a\left(Ab+\frac{3Ba}{17}\right)}{11b}\right)x\sqrt{bx^3+a}}{5b}+\frac{4ia\left(Aa-\frac{8a\left(Ab+\frac{3Ba}{17}\right)}{11b}\right)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i(}{}}}{}$
default	$A\left(\frac{2x^4\sqrt{bx^3+a}}{11}+\frac{6ax\sqrt{bx^3+a}}{55b}+\frac{4ia^2\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}}{}\right.$

input
```
int(x^3*(b*x^3+a)^(1/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/935*x*(55*B*b^2*x^6+85*A*b^2*x^3+15*B*a*b*x^3+51*A*a*b-24*B*a^2)/b^2*(b*
x^3+a)^(1/2)+4/935*I*a^2*(17*A*b-8*B*a)/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.30

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2\left(6\left(8\,Ba^3-17\,Aa^2b\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(55\,Bb^3x^7+5\left(3\,Bab^2+17\,Ab^3\right)x^4-3\left(8\,B\right.\right.\right.}{935\,b^3}$$

input
```
integrate(x^3*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/935*(6*(8*B*a^3 - 17*A*a^2*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x)
+ (55*B*b^3*x^7 + 5*(3*B*a*b^2 + 17*A*b^3)*x^4 - 3*(8*B*a^2*b - 17*A*a*b^2
)*x)*sqrt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.27

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{A\sqrt{a}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{B\sqrt{a}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**3*(b*x**3+a)**(1/2)*(B*x**3+A),x)
```

output
```
A*sqrt(a)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(7/3)) + B*sqrt(a)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Maxima [F]

$$\int x^3\sqrt{a+bx^3}(A+Bx^3)\ dx=\int (Bx^3+A)\sqrt{bx^3+a}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{a+bx^3}(A+Bx^3)\ dx=\int (Bx^3+A)\sqrt{bx^3+a}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int x^3\,\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}\,dx$$

input
```
int(x^3*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

output
```
int(x^3*(A + B*x^3)*(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{\frac{54\sqrt{b\,x^3+a}\,a^2x}{935}+\frac{40\sqrt{b\,x^3+a}\,ab\,x^4}{187}+\frac{2\sqrt{b\,x^3+a}\,b^2x^7}{17}-\frac{54\left(\int\frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a^3}{935}}{b}$$

input
```
int(x^3*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

output
```
(2*(27*sqrt(a + b*x**3)*a**2*x + 100*sqrt(a + b*x**3)*a*b*x**4 + 55*sqrt(a
 + b*x**3)*b**2*x**7 - 27*int(sqrt(a + b*x**3)/(a + b*x**3),x)*a**3))/(935
*b)
```

3.165 $\int \sqrt{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 268

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2(11Ab-2aB)x\sqrt{a+bx^3}}{55b} + \frac{2Bx\left(a+bx^3\right)^{3/2}}{11b} + \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a(11Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}}\right.\right.}{55b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/55*(11*A*b-2*B*a)*x*(b*x^3+a)^(1/2)/b+2/11*B*x*(b*x^3+a)^(3/2)/b+2/55*3^
(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a*(11*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/
2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*
x),I*3^(1/2)+2*I)/b^(4/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3
)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.76 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.28

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2x\sqrt{a+bx^3}\left(B(a+bx^3)+\frac{(11Ab-2aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{11b}$$

input
```
Integrate[Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(B*(a + b*x^3) + ((11*A*b - 2*a*B)*Hypergeometric2F1[
-1/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(11*b)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {913, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(11Ab-2aB)\int \sqrt{bx^3+a}dx}{11b}+\frac{2Bx\left(a+bx^3\right)^{3/2}}{11b}$$

$$\Big\downarrow \text{748}$$

$$\frac{(11Ab-2aB)\left(\frac{3}{5}a\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{2}{5}x\sqrt{a+bx^3}\right)}{11b}+\frac{2Bx\left(a+bx^3\right)^{3/2}}{11b}$$

$$\Big\downarrow \text{759}$$

$$\frac{(11Ab-2aB)\left(\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2}{5}x\sqrt{\cdots}\right.}{11b}$$

$$\frac{2Bx\left(a+bx^3\right)^{3/2}}{11b}$$

input
```
Int[Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*B*x*(a + b*x^3)^(3/2))/(11*b) + ((11*A*b - 2*a*B)*((2*x*Sqrt[a + b*x^3]
)/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(11*b)
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.21

method	result
risch	$\frac{2x(5bB\,x^3+11Ab+3Ba)\sqrt{b\,x^3+a}}{55b}-\frac{2ia(11Ab-2Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(\cdots}{\cdots}}}\cdots}{\cdots}$
elliptic	$\frac{2B\,x^4\sqrt{b\,x^3+a}}{11}+\frac{2\left(Ab+\frac{3Ba}{11}\right)x\sqrt{b\,x^3+a}}{5b}-\frac{2i\left(Aa-\frac{2a\left(Ab+\frac{3Ba}{11}\right)}{5b}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-\frac{3\cdots}{\cdots}}}\cdots}{\cdots}$
default	$A\left(\frac{2x\sqrt{b\,x^3+a}}{5}-\frac{2ia\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\cdots\right)}{(-\cdots}}\cdots}{5b\sqrt{\cdots}}\cdots\right.$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/55*x*(5*B*b*x^3+11*A*b+3*B*a)/b*(b*x^3+a)^(1/2)-2/55*I*a*(11*A*b-2*B*a)/
b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.25

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = -\frac{2\left(3\left(2\,Ba^2-11\,Aab\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\left(5\,Bb^2x^4+\left(3\,Bab+11\,Ab^2\right)x\right)\sqrt{bx^3+a}\right)}{55\,b^2}$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
-2/55*(3*(2*B*a^2 - 11*A*a*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) -
(5*B*b^2*x^4 + (3*B*a*b + 11*A*b^2)*x)*sqrt(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.31

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{A\sqrt{a}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{B\sqrt{a}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)*(B*x**3+A),x)
```

output
```
A*sqrt(a)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + B*sqrt(a)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Maxima [F]

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\sqrt{bx^3+a}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}\,dx$$

input
```
int((A + B*x^3)*(a + b*x^3)^(1/2),x)
```

output
```
int((A + B*x^3)*(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{28\sqrt{b\,x^3+a}\,ax}{55}+\frac{2\sqrt{b\,x^3+a}\,b\,x^4}{11}+\frac{27\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a^2}{55}$$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A),x)
```

output
```
(28*sqrt(a + b*x**3)*a*x + 10*sqrt(a + b*x**3)*b*x**4 + 27*int(sqrt(a + b*
x**3)/(a + b*x**3),x)*a**2)/55
```

3.166 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 269

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \frac{(5Ab+4aB)x\sqrt{a+bx^3}}{10a} - \frac{A(a+bx^3)^{3/2}}{2ax^2} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}(5Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/10*(5*A*b+4*B*a)*x*(b*x^3+a)^(1/2)/a-1/2*A*(b*x^3+a)^(3/2)/a/x^2+1/10*3^
(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(5*A*b+4*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*
EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),
I*3^(1/2)+2*I)/b^(1/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b
^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \frac{\sqrt{a+bx^3}\left(-A(a+bx^3)+\frac{(5Ab+4aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{2ax^2}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^3,x]
```

output
```
(Sqrt[a + b*x^3]*(-(A*(a + b*x^3)) + ((5*A*b + 4*a*B)*x^3*Hypergeometric2F
1[-1/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(2*a*x^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(4aB+5Ab)\int \sqrt{bx^3+a}dx}{4a}-\frac{A(a+bx^3)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{748}$$

$$\frac{(4aB+5Ab)\left(\frac{3}{5}a\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{2}{5}x\sqrt{a+bx^3}\right)}{4a}-\frac{A(a+bx^3)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{759}$$

$$\frac{(4aB+5Ab)\left(\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2}{5}x\sqrt{a\right.}}{4a}$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{2ax^2}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^3,x]
```

output
```
-1/2*(A*(a + b*x^3)^(3/2))/(a*x^2) + ((5*A*b + 4*a*B)*((2*x*Sqrt[a + b*x^3
])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(4*a)
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
 + 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(-4B\,x^3+5A)}{10x^2}-\frac{2i\left(\frac{3Ab}{4}+\frac{3Ba}{5}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}}{}$
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{2x^2}+\frac{2\sqrt{b\,x^3+a}\,xB}{5}-\frac{2i\left(\frac{3Ab}{4}+\frac{3Ba}{5}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a}{2b}}}}{}$
default	$B\left(\frac{2x\sqrt{b\,x^3+a}}{5}-\frac{2ia\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\right.}{(-}}}{5b\sqrt{}}\right.$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/10*(b*x^3+a)^(1/2)*(-4*B*x^3+5*A)/x^2-2/3*I*(3/4*A*b+3/5*B*a)*3^(1/2)/b
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \frac{3\,(4\,Ba+5\,Ab)\sqrt{b}x^2\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(4\,Bbx^3-5\,Ab)\sqrt{bx^3+a}}{10\,bx^2}$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^3,x, algorithm="fricas")
```

output
```
1/10*(3*(4*B*a + 5*A*b)*sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x) + (4
*B*b*x^3 - 5*A*b)*sqrt(b*x^3 + a))/(b*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{B\sqrt{a}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)*(B*x**3+A)/x**3,x)
```

output
```
A*sqrt(a)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**2*gamma(1/3)) + B*sqrt(a)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^3,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^3,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^3}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^3,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \frac{-16\sqrt{b\,x^3+a}\,a+2\sqrt{b\,x^3+a}\,b\,x^3-27\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^6+a\,x^3}dx\right)a^2x^2}{5x^2}$$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^3,x)
```

output
```
( - 16*sqrt(a + b*x**3)*a + 2*sqrt(a + b*x**3)*b*x**3 - 27*int(sqrt(a + b*
x**3)/(a*x**3 + b*x**6),x)*a**2*x**2)/(5*x**2)
```

3.167 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 272

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx = \frac{(Ab-10aB)\sqrt{a+bx^3}}{20ax^2} - \frac{A(a+bx^3)^{3/2}}{5ax^5} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}(Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\ldots\right.}{20a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/20*(A*b-10*B*a)*(b*x^3+a)^(1/2)/a/x^2-1/5*A*(b*x^3+a)^(3/2)/a/x^5-1/20*3
^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*b^(2/3)*(A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*
((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2
)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(
1/3)*x),I*3^(1/2)+2*I)/a/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)
+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.29

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx = \frac{\sqrt{a+bx^3}\left(-2A(a+bx^3)+\frac{\left(\frac{Ab}{2}-5aB\right)x^3\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{10ax^5}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^6,x]
```

output
```
(Sqrt[a + b*x^3]*(-2*A*(a + b*x^3) + (((A*b)/2 - 5*a*B)*x^3*Hypergeometric
2F1[-2/3, -1/2, 1/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(10*a*x^5)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 809, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(Ab-10aB)\int \frac{\sqrt{bx^3+a}}{x^3}dx}{10a}-\frac{A(a+bx^3)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{ 809}$$

$$-\frac{(Ab-10aB)\left(\frac{3}{4}b\int \frac{1}{\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{2x^2}\right)}{10a}-\frac{A(a+bx^3)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{ 759}$$

$$-\frac{(Ab-10aB)\left(\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{a+}}{2x}\right.}{10a}$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{5ax^5}$$

input

```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^6,x]
```

output

```
-1/5*(A*(a + b*x^3)^(3/2))/(a*x^5) - ((A*b - 10*a*B)*(-1/2*Sqrt[a + b*x^3]
/x^2 + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*
x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(10*a)
```

Defintions of rubi rules used

rule 759

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 955

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(3 A b\,x^{3}+10 B a\,x^{3}+4 A a\right)}{20 x^{5} a}+\frac{i (A b-10 B a) \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(\cdots\right.}{}}}}{}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{5 x^{5}}-\frac{(3 A b+10 B a)\sqrt{b\,x^{3}+a}}{20 a\,x^{2}}-\frac{2 i \left(B b-\frac{b (3 A b+10 B a)}{40 a}\right) \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{}{-\frac{3\left(-a\cdots\right.}{2\cdots}}}}{}$
default	$A\left(-\frac{\sqrt{b\,x^{3}+a}}{5 x^{5}}-\frac{3 b\sqrt{b\,x^{3}+a}}{20 a\,x^{2}}+\frac{i b \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}}}\,\sqrt{-\cdots}}{}\right.$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/20*(b*x^3+a)^(1/2)*(3*A*b*x^3+10*B*a*x^3+4*A*a)/x^5/a+1/20*I*(A*b-10*B*
a)/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx = \frac{3\,(10\,Ba-Ab)\sqrt{b}x^5\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-((10\,Ba+3\,Ab)x^3+4\,Aa)\sqrt{bx^3+a}}{20\,ax^5}$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^6,x, algorithm="fricas")
```

output
```
1/20*(3*(10*B*a - A*b)*sqrt(b)*x^5*weierstrassPInverse(0, -4*a/b, x) - ((1
0*B*a + 3*A*b)*x^3 + 4*A*a)*sqrt(b*x^3 + a))/(a*x^5)
```

Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)*(B*x**3+A)/x**6,x)
```

output
```
A*sqrt(a)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**5*gamma(-2/3)) + B*sqrt(a)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^6,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^6,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^6}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^6,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx = \frac{4\sqrt{b\,x^3+a}\,a-14\sqrt{b\,x^3+a}\,b\,x^3+27\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^9+a\,x^6}dx\right)a^2x^5}{7x^5}$$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^6,x)
```

output
```
(4*sqrt(a + b*x**3)*a - 14*sqrt(a + b*x**3)*b*x**3 + 27*int(sqrt(a + b*x**
3)/(a*x**6 + b*x**9),x)*a**2*x**5)/(7*x**5)
```

3.168 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 305

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = \frac{(7Ab-16aB)\sqrt{a+bx^3}}{80ax^5}+\frac{3b(7Ab-16aB)\sqrt{a+bx^3}}{320a^2x^2}-\frac{A(a+bx^3)^{3/2}}{8ax^8} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}(7Ab-16aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{320a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
1/80*(7*A*b-16*B*a)*(b*x^3+a)^(1/2)/a/x^5+3/320*b*(7*A*b-16*B*a)*(b*x^3+a)
^(1/2)/a^2/x^2-1/8*A*(b*x^3+a)^(3/2)/a/x^8+1/320*3^(3/4)*(1/2*6^(1/2)+1/2*
2^(1/2))*b^(5/3)*(7*A*b-16*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-
3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)
/a^2/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)
/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.26

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = \frac{\sqrt{a+bx^3}\left(-5A(a+bx^3)+\frac{\left(\frac{7Ab}{2}-8aB\right)x^3\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{1}{2},-\frac{2}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{40ax^8}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^9,x]
```

output
```
(Sqrt[a + b*x^3]*(-5*A*(a + b*x^3) + (((7*A*b)/2 - 8*a*B)*x^3*Hypergeometr
ic2F1[-5/3, -1/2, -2/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(40*a*x^8)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 809, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(7Ab-16aB)\int \frac{\sqrt{bx^3+a}}{x^6}dx}{16a}-\frac{A\left(a+bx^3\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \text{809}$$

$$-\frac{(7Ab-16aB)\left(\frac{3}{10}b\int \frac{1}{x^3\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{5x^5}\right)}{16a}-\frac{A\left(a+bx^3\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(7Ab-16aB)\left(\frac{3}{10}b\left(-\frac{b\int\frac{1}{\sqrt{bx^3+a}}dx}{4a}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)-\frac{\sqrt{a+bx^3}}{5x^5}\right)}{16a}-\frac{A(a+bx^3)^{3/2}}{8ax^8}$$

$\big\downarrow$ 759

$$-\frac{(7Ab-16aB)\left(\frac{3}{10}b\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \right.\right.}{16a}$$

$$\frac{A(a+bx^3)^{3/2}}{8ax^8}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^9,x]
```

output
```
-1/8*(A*(a + b*x^3)^(3/2))/(a*x^8) - ((7*A*b - 16*a*B)*(-1/5*Sqrt[a + b*x^
3]/x^5 + (3*b*(-1/2*Sqrt[a + b*x^3]/(a*x^2) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
 + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^
(1/4)*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*Sqrt[a + b*x^3])))/10))/(16*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-21A\,b^{2}x^{6}+48Bab\,x^{6}+12aAb\,x^{3}+64B\,a^{2}x^{3}+40a^{2}A\right)}{320x^{8}a^{2}}-\frac{ib(7Ab-16Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{8x^{8}}-\frac{(3Ab+16Ba)\sqrt{b\,x^{3}+a}}{80a\,x^{5}}+\frac{3b(7Ab-16Ba)\sqrt{b\,x^{3}+a}}{320a^{2}x^{2}}-\frac{ib(7Ab-16Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$
default	$A\left(-\frac{\sqrt{b\,x^{3}+a}}{8x^{8}}-\frac{3b\sqrt{b\,x^{3}+a}}{80a\,x^{5}}+\frac{21b^{2}\sqrt{b\,x^{3}+a}}{320a^{2}x^{2}}-\frac{7ib^{2}\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\cdots}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\cdots}}}{}\right.$

input

```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/320*(b*x^3+a)^(1/2)*(-21*A*b^2*x^6+48*B*a*b*x^6+12*A*a*b*x^3+64*B*a^2*x
^3+40*A*a^2)/x^8/a^2-1/320*I*b*(7*A*b-16*B*a)/a^2*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.29

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = -\frac{3\,(16\,Bab-7\,Ab^2)\sqrt{b}x^8\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(3\,(16\,Bab-7\,Ab^2)x^6+4\,(16\,Ba^2+3\,Aab}{320\,a^2x^8}$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^9,x, algorithm="fricas")
```

output
```
-1/320*(3*(16*B*a*b - 7*A*b^2)*sqrt(b)*x^8*weierstrassPInverse(0, -4*a/b,
x) + (3*(16*B*a*b - 7*A*b^2)*x^6 + 4*(16*B*a^2 + 3*A*a*b)*x^3 + 40*A*a^2)*
sqrt(b*x^3 + a))/(a^2*x^8)
```

Sympy [A] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{8}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)*(B*x**3+A)/x**9,x)
```

output
```
A*sqrt(a)*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**8*gamma(-5/3)) + B*sqrt(a)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^9}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^9,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^9, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^9}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^9,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx=\int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^9}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^9,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^9, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx=\frac{-8\sqrt{b\,x^3+a}\,a-26\sqrt{b\,x^3+a}\,b\,x^3+27\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^{12}+a\,x^9}dx\right)a^2x^8}{91x^8}$$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^9,x)
```

output
```
( - 8*sqrt(a + b*x**3)*a - 26*sqrt(a + b*x**3)*b*x**3 + 27*int(sqrt(a + b*
x**3)/(a*x**9 + b*x**12),x)*a**2*x**8)/(91*x**8)
```

3.169 $\int x^4\sqrt{a+bx^3}(A+Bx^3)\ dx$

Optimal result

Integrand size = 22, antiderivative size = 581

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{6a(19Ab-10aB)x^2\sqrt{a+bx^3}}{1729b^2}$$

$$+\frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b}-\frac{24a^2(19Ab-10aB)\sqrt{a+bx^3}}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}$$

$$+\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}(19Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{8\sqrt{2}3^{3/4}a^{7/3}(19Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
6/1729*a*(19*A*b-10*B*a)*x^2*(b*x^3+a)^(1/2)/b^2+2/247*(19*A*b-10*B*a)*x^5
*(b*x^3+a)^(1/2)/b-24/1729*a^2*(19*A*b-10*B*a)*(b*x^3+a)^(1/2)/b^(8/3)/((1
+3^(1/2))*a^(1/3)+b^(1/3)*x)+2/19*B*x^5*(b*x^3+a)^(3/2)/b+12/1729*3^(1/4)*
(1/2*6^(1/2)-1/2*2^(1/2))*a^(7/3)*(19*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*((a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1
/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)
*x),I*3^(1/2)+2*I)/b^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/
3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-8/1729*2^(1/2)*3^(3/4)*a^(7/3)*(19*
A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
(1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(
1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(8/3)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.00 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.16

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2x^2\sqrt{a+bx^3}\left(-\left(\left(a+bx^3\right)\left(-19Ab+10aB-13bBx^3\right)\right)+\frac{a(-19Ab+10aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{247b^2}$$

input
```
Integrate[x^4*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*x^2*Sqrt[a + b*x^3]*(-((a + b*x^3)*(-19*A*b + 10*a*B - 13*b*B*x^3)) + (
a*(-19*A*b + 10*a*B)*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt
[1 + (b*x^3)/a]))/(247*b^2)
```

Rubi [A] (warning: unable to verify)

Time = 1.00 (sec) , antiderivative size = 577, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {959, 811, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(19Ab-10aB)\int x^4\sqrt{bx^3+a}dx}{19b}+\frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}$$

$$\Big\downarrow \text{811}$$

$$\frac{(19Ab-10aB)\left(\frac{3}{13}a\int \frac{x^4}{\sqrt{bx^3+a}}dx+\frac{2}{13}x^5\sqrt{a+bx^3}\right)}{19b}+\frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}$$

$$\Big\downarrow \text{843}$$

$$\frac{(19Ab-10aB)\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\int \frac{x}{\sqrt{bx^3+a}}dx}{7b}\right)+\frac{2}{13}x^5\sqrt{a+bx^3}\right)}{19b}+\frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}$$

$$\Big\downarrow \text{832}$$

$$\frac{(19Ab-10aB)\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{7b}\right)+\frac{2}{13}x^5\sqrt{a+bx^3}\right)}{19b}+\frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}$$

$$\Big\downarrow \text{759}$$

$$\frac{(19Ab-10aB)\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{Ellip}}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}}}\right)}{7b}\right.\right.}{19b}$$

$$\frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}$$

$$\Big\downarrow \; 2416$$

$$\frac{(19Ab-10aB)\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\text{arc}\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a}}}{\sqrt[3]{b}}\right.}{}\right.\right.}{}$$

$$\frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}$$

input
```
Int[x^4*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*B*x^5*(a + b*x^3)^(3/2))/(19*b) + ((19*A*b - 10*a*B)*((2*x^5*Sqrt[a + b
*x^3])/13 + (3*a*((2*x^2*Sqrt[a + b*x^3])/(7*b) - (4*a*(((2*Sqrt[a + b*x^3
])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[
3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + S
qrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(7*b)))/13))/(19*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.87

method	result
risch	$\frac{2x^2\left(91b^2B\,x^6+133A\,b^2x^3+21Bab\,x^3+57abA-30a^2B\right)\sqrt{b\,x^3+a}}{1729b^2}+\frac{8ia^2(19Ab-10Ba)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2 \cdots\right.}{2b}\cdots\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}\cdots}}\cdots}{\cdots}$
elliptic	$\frac{2B\,x^8\sqrt{b\,x^3+a}}{19}+\frac{2\left(Ab+\frac{3Ba}{19}\right)x^5\sqrt{b\,x^3+a}}{13b}+\frac{2\left(Aa-\frac{10a\left(Ab+\frac{3Ba}{19}\right)}{13b}\right)x^2\sqrt{b\,x^3+a}}{7b}+\frac{8ia\left(Aa-\frac{10a\left(Ab+\frac{3Ba}{19}\right)}{13b}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\cdots}{\cdots}$
default	Expression too large to display

input
```
int(x^4*(b*x^3+a)^(1/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/1729*x^2*(91*B*b^2*x^6+133*A*b^2*x^3+21*B*a*b*x^3+57*A*a*b-30*B*a^2)/b^2
*(b*x^3+a)^(1/2)+8/1729*I*a^2*(19*A*b-10*B*a)/b^3*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.18

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = -\frac{2\left(12\left(10\,Ba^3-19\,Aa^2b\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-\left(91\,Bb^3x^8+\right.\right.}{1729\,b^3}$$

input
```
integrate(x^4*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
-2/1729*(12*(10*B*a^3 - 19*A*a^2*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, wei
erstrassPInverse(0, -4*a/b, x)) - (91*B*b^3*x^8 + 7*(3*B*a*b^2 + 19*A*b^3)
*x^5 - 3*(10*B*a^2*b - 19*A*a*b^2)*x^2)*sqrt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.14

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{A\sqrt{a}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{B\sqrt{a}x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**4*(b*x**3+a)**(1/2)*(B*x**3+A),x)
```

output
```
A*sqrt(a)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(8/3)) + B*sqrt(a)*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3))
```

Maxima [F]

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\sqrt{bx^3+a}x^4\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^4, x)
```

Giac [F]

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}x^4\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int x^4\,\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}\,dx$$

input
```
int(x^4*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

output
```
int(x^4*(A + B*x^3)*(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{\frac{54\sqrt{b\,x^3+a}\,a^2x^2}{1729}+\frac{44\sqrt{b\,x^3+a}\,ab\,x^5}{247}+\frac{2\sqrt{b\,x^3+a}\,b^2x^8}{19}-\frac{108\left(\int\frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a^3}{1729}}{b}$$

input
```
int(x^4*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

output
```
(2*(27*sqrt(a + b*x**3)*a**2*x**2 + 154*sqrt(a + b*x**3)*a*b*x**5 + 91*sqr
t(a + b*x**3)*b**2*x**8 - 54*int((sqrt(a + b*x**3)*x)/(a + b*x**3),x)*a**3
))/(1729*b)
```

3.170 $\int x\sqrt{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 548

$$\begin{aligned}\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx &= \frac{2(13Ab-4aB)x^2\sqrt{a+bx^3}}{91b}+\frac{6a(13Ab-4aB)\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b}\\ &-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-\right.}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\\ &+\frac{2\sqrt{2}3^{3/4}a^{4/3}(13Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.\right.}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\end{aligned}$$

output
```
2/91*(13*A*b-4*B*a)*x^2*(b*x^3+a)^(1/2)/b+6/91*a*(13*A*b-4*B*a)*(b*x^3+a)^
(1/2)/b^(5/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+2/13*B*x^2*(b*x^3+a)^(3/2)/b
-3/91*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(4/3)*(13*A*b-4*B*a)*(a^(1/3)+b^
(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(
1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^
(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3
^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+2/91*2^(1/2)*3^(3/4)*a
^(4/3)*(13*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a
^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(5/3)/(
a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^
3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.83 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.14

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{x^2\sqrt{a+bx^3}\left(4B(a+bx^3)+\frac{(13Ab-4aB)\,\mathrm{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{26b}$$

input
```
Integrate[x*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(x^2*Sqrt[a + b*x^3]*(4*B*(a + b*x^3) + ((13*A*b - 4*a*B)*Hypergeometric2F
1[-1/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(26*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.95 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {959, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(13Ab-4aB)\int x\sqrt{bx^3+a}dx}{13b}+\frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b}$$

$$\Big\downarrow \; 811$$

$$\frac{(13Ab-4aB)\left(\frac{3}{7}a\int \frac{x}{\sqrt{bx^3+a}}dx+\frac{2}{7}x^2\sqrt{a+bx^3}\right)}{13b}+\frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b}$$

$$\Big\downarrow \; 832$$

$$\frac{(13Ab-4aB)\left(\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{7}x^2\sqrt{a+bx^3}\right)}{13b}+\frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b}$$

$$\Big\downarrow \; 759$$

$$\frac{(13Ab-4aB)\left(\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots\right.\right.}{13b}+\frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b}$$

$$\Big\downarrow \; 2416$$

$$\frac{(13Ab-4aB)\left(\frac{3}{7}a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.}{}$$

$$\frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b}$$

input
```
Int[x*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*B*x^2*(a + b*x^3)^(3/2))/(13*b) + ((13*A*b - 4*a*B)*((2*x^2*Sqrt[a + b*
x^3])/7 + (3*a*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(
1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/
3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]))/7))/(13*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 479, normalized size of antiderivative = 0.87

method	result
risch	$\frac{2x^{2}\left(7bB\,x^{3}+13Ab+3Ba\right)\sqrt{b\,x^{3}+a}}{91b}-\frac{2ia(13Ab-4Ba)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\cdots}{\cdots}}}\cdots}{\cdots}$
elliptic	$\frac{2x^{5}\sqrt{b\,x^{3}+a}\,B}{13}+\frac{2\left(Ab+\frac{3Ba}{13}\right)x^{2}\sqrt{b\,x^{3}+a}}{7b}-\frac{2i\left(Aa-\frac{4a\left(Ab+\frac{3Ba}{13}\right)}{7b}\right)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-\cdots}}\cdots}{\cdots}$
default	Expression too large to display

input
```
int(x*(b*x^3+a)^(1/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/91*x^2*(7*B*b*x^3+13*A*b+3*B*a)/b*(b*x^3+a)^(1/2)-2/91*I*a*(13*A*b-4*B*a
)/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.14

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{2\left(3\left(4\,Ba^2-13\,Aab\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(7\,Bb^2x^5+\left(3\,Bab\right.\right.\right.}{91\,b^2}$$

input
```
integrate(x*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/91*(3*(4*B*a^2 - 13*A*a*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstras
sPInverse(0, -4*a/b, x)) + (7*B*b^2*x^5 + (3*B*a*b + 13*A*b^2)*x^2)*sqrt(b
*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.15

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{A\sqrt{a}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{B\sqrt{a}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x*(b*x**3+a)**(1/2)*(B*x**3+A),x)
```

output
```
A*sqrt(a)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(5/3)) + B*sqrt(a)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3))
```

Maxima [F]

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\sqrt{bx^3+a}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x, x)
```

Giac [F]

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int x\,\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}\,dx$$

input
```
int(x*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

output
```
int(x*(A + B*x^3)*(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{32\sqrt{b\,x^3+a}\,a\,x^2}{91}+\frac{2\sqrt{b\,x^3+a}\,b\,x^5}{13}+\frac{27\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a^2}{91}$$

input
```
int(x*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

output
```
(32*sqrt(a + b*x**3)*a*x**2 + 14*sqrt(a + b*x**3)*b*x**5 + 27*int((sqrt(a
+ b*x**3)*x)/(a + b*x**3),x)*a**2)/91
```

3.171 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 545

$$
\begin{aligned}
&\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx \\
&= \frac{(7Ab+2aB)x^2\sqrt{a+bx^3}}{7a} + \frac{3(7Ab+2aB)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A\left(a+bx^3\right)^{3/2}}{ax} \\
&\quad - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7\ldots\right.}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} \\
&\quad + \frac{\sqrt{2}\,3^{3/4}\sqrt[3]{a}(7Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-\ldots\right.}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}
$$

output
```
1/7*(7*A*b+2*B*a)*x^2*(b*x^3+a)^(1/2)/a+3/7*(7*A*b+2*B*a)*(b*x^3+a)^(1/2)/
b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)-A*(b*x^3+a)^(3/2)/a/x-3/14*3^(1/4)
*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/3)*(7*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/
2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*
x),I*3^(1/2)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3
)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+1/7*2^(1/2)*3^(3/4)*a^(1/3)*(7*A*b+2
*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(
1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x
)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)
+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = \frac{\sqrt{a+bx^3}\left(-2A(a+bx^3)+\frac{(7Ab+2aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{2ax}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^2,x]
```

output
```
(Sqrt[a + b*x^3]*(-2*A*(a + b*x^3) + ((7*A*b + 2*a*B)*x^3*Hypergeometric2F
1[-1/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(2*a*x)
```

Rubi [A] (warning: unable to verify)

Time = 0.93 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {955, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(2aB+7Ab)\int x\sqrt{bx^3+a}dx}{2a}-\frac{A(a+bx^3)^{3/2}}{ax}$$

$$\Big\downarrow \text{811}$$

$$\frac{(2aB+7Ab)\left(\frac{3}{7}a\int \frac{x}{\sqrt{bx^3+a}}dx+\frac{2}{7}x^2\sqrt{a+bx^3}\right)}{2a}-\frac{A(a+bx^3)^{3/2}}{ax}$$

$$\Big\downarrow \text{832}$$

$$\frac{(2aB+7Ab)\left(\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{7}x^2\sqrt{a+bx^3}\right)}{2a}-\frac{A(a+bx^3)^{3/2}}{ax}$$

$$\Big\downarrow \text{759}$$

$$\frac{(2aB+7Ab)\left(\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}\ldots}{\sqrt[3]{b}\ldots}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\ldots\right.\right.}{2a}$$

$$\frac{A(a+bx^3)^{3/2}}{ax}$$

$$\Big\downarrow \text{2416}$$

$$\frac{(2aB+7Ab)\left(\frac{3}{7}a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.}{}$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{ax}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^2,x]
```

output
```
-((A*(a + b*x^3)^(3/2))/(a*x)) + ((7*A*b + 2*a*B)*((2*x^2*Sqrt[a + b*x^3])
/7 + (3*a*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3)
- (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
])))/7))/(2*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 469, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(-2B\,x^3+7A)}{7x}-\frac{2i\left(\frac{3Ab}{2}+\frac{3Ba}{7}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\cdots}}\cdots}{\cdots}$
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{x}+\frac{2\sqrt{b\,x^3+a}\,x^2B}{7}-\frac{2i\left(\frac{3Ab}{2}+\frac{3Ba}{7}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\cdots}{2b\cdots}}}\cdots}{\cdots}$
default	Expression too large to display

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/7*(b*x^3+a)^(1/2)*(-2*B*x^3+7*A)/x-2/3*I*(3/2*A*b+3/7*B*a)*3^(1/2)/b*(-
a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2
)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = -\frac{3\,(2\,Ba+7\,Ab)\sqrt{b}x\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-(2\,Bbx^3-7\,Ab)\sqrt{bx^3}}{7\,bx}$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^2,x, algorithm="fricas")
```

output
```
-1/7*(3*(2*B*a + 7*A*b)*sqrt(b)*x*weierstrassZeta(0, -4*a/b, weierstrassPI
nverse(0, -4*a/b, x)) - (2*B*b*x^3 - 7*A*b)*sqrt(b*x^3 + a))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{B\sqrt{a}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)*(B*x**3+A)/x**2,x)
```

output
```
A*sqrt(a)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x*gamma(2/3)) + B*sqrt(a)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^2,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^2,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^2}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^2,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = \frac{20\sqrt{b\,x^3+a}\,a+2\sqrt{b\,x^3+a}\,b\,x^3+27\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^5+a\,x^2}dx\right)a^2x}{7x}$$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^2,x)
```

output
```
(20*sqrt(a + b*x**3)*a + 2*sqrt(a + b*x**3)*b*x**3 + 27*int(sqrt(a + b*x**
3)/(a*x**2 + b*x**5),x)*a**2*x)/(7*x)
```

3.172 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 546

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx$$

$$= -\frac{(Ab+8aB)\sqrt{a+bx^3}}{8ax} + \frac{3\sqrt[3]{b}(Ab+8aB)\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A(a+bx^3)^{3/2}}{4ax^4}$$

$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{3^{3/4}\sqrt[3]{b}(Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4}{4\sqrt{2}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/8*(A*b+8*B*a)*(b*x^3+a)^(1/2)/a/x+3/8*b^(1/3)*(A*b+8*B*a)*(b*x^3+a)^(1/
2)/a/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)-1/4*A*(b*x^3+a)^(3/2)/a/x^4-3/16*3^(1
/4)*(1/2*6^(1/2)-1/2*2^(1/2))*b^(1/3)*(A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*((a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1
/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)
*x),I*3^(1/2)+2*I)/a^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/
3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+1/8*3^(3/4)*b^(1/3)*(A*b+8*B*a)*(a^
(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(
1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(
1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)/a^(2/3)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx = \frac{\sqrt{a+bx^3}\left(-A(a+bx^3)-\frac{(Ab+8aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{4ax^4}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^5,x]
```

output
```
(Sqrt[a + b*x^3]*(-(A*(a + b*x^3)) - ((A*b + 8*a*B)*x^3*Hypergeometric2F1[
-1/2, -1/3, 2/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(4*a*x^4)
```

Rubi [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {955, 809, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^5}dx$$

$$\Big\downarrow \text{ 955}$$

$$\frac{(8aB+Ab)\int \frac{\sqrt{bx^3+a}}{x^2}dx}{8a}-\frac{A\left(a+bx^3\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{ 809}$$

$$\frac{(8aB+Ab)\left(\frac{3}{2}b\int \frac{x}{\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{x}\right)}{8a}-\frac{A\left(a+bx^3\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{ 832}$$

$$\frac{(8aB+Ab)\left(\frac{3}{2}b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)-\frac{\sqrt{a+bx^3}}{x}\right)}{8a}-\frac{A\left(a+bx^3\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{(8aB+Ab)\left(\frac{3}{2}b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x \cdots}{\sqrt[3]{b}x \cdots}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} \cdots\right.\right.}{8a}$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{ 2416}$$

$$(8aB+Ab)\left(\frac{3}{2}b\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{4ax^4}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^5,x]
```

output
```
-1/4*(A*(a + b*x^3)^(3/2))/(a*x^4) + ((A*b + 8*a*B)*(-(Sqrt[a + b*x^3]/x)
+ (3*b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (
2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))
)/2))/(8*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.88

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(3Ab\,x^{3}+8Ba\,x^{3}+2Aa\right)}{8x^{4}a}-\frac{i(Ab+8Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a \right.}{2b}}}}{}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{4x^{4}}-\frac{(3Ab+8Ba)\sqrt{b\,x^{3}+a}}{8ax}-\frac{2i\left(Bb+\frac{b(3Ab+8Ba)}{16a}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-}{-\frac{3\left(-a\,b^{2}\right.}{2b}}}}{}$
default	Expression too large to display

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^3+a)^(1/2)*(3*A*b*x^3+8*B*a*x^3+2*A*a)/x^4/a-1/8*I*(A*b+8*B*a)/a
*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx =$$
$$-\frac{3\,(8\,Ba+Ab)\sqrt{b}x^4\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left((8\,Ba+3\,Ab)x^3+2\,A\right.}{8\,ax^4}$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^5,x, algorithm="fricas")
```

output
```
-1/8*(3*(8*B*a + A*b)*sqrt(b)*x^4*weierstrassZeta(0, -4*a/b, weierstrassPI
nverse(0, -4*a/b, x)) + ((8*B*a + 3*A*b)*x^3 + 2*A*a)*sqrt(b*x^3 + a))/(a*
x^4)
```

Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)*(B*x**3+A)/x**5,x)
```

output
```
A*sqrt(a)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**4*gamma(-1/3)) + B*sqrt(a)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^5,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^5, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx=\int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^5,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx=\int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^5}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^5,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx=\frac{-8\sqrt{b\,x^3+a}\,a+10\sqrt{b\,x^3+a}\,b\,x^3-27\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^8+a\,x^5}dx\right)a^2x^4}{5x^4}$$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^5,x)
```

output
```
( - 8*sqrt(a + b*x**3)*a + 10*sqrt(a + b*x**3)*b*x**3 - 27*int(sqrt(a + b*
x**3)/(a*x**5 + b*x**8),x)*a**2*x**4)/(5*x**4)
```

3.173 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 581

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx = \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4} + \frac{3b(5Ab-14aB)\sqrt{a+bx^3}}{112a^2x} - \frac{3b^{4/3}(5Ab-14aB)\sqrt{a+bx^3}}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A(a+bx^3)^{3/2}}{7ax^7}$$
$$+ \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}(5Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-\right.}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$- \frac{3^{3/4}b^{4/3}(5Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7\right.}{56\sqrt{2}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/56*(5*A*b-14*B*a)*(b*x^3+a)^(1/2)/a/x^4+3/112*b*(5*A*b-14*B*a)*(b*x^3+a)
^(1/2)/a^2/x-3/112*b^(4/3)*(5*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^2/((1+3^(1/2))
*a^(1/3)+b^(1/3)*x)-1/7*A*(b*x^3+a)^(3/2)/a/x^7+3/224*3^(1/4)*(1/2*6^(1/2)
-1/2*2^(1/2))*b^(4/3)*(5*A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE
(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)
+2*I)/a^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)
^2)^(1/2)/(b*x^3+a)^(1/2)-1/112*3^(3/4)*b^(4/3)*(5*A*b-14*B*a)*(a^(1/3)+b^
(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(
1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^
(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)/a^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*
x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx = \frac{\sqrt{a+bx^3}\left(-4A(a+bx^3)+\frac{\left(\frac{5Ab}{2}-7aB\right)x^3\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{1}{2},-\frac{1}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{28ax^7}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^8,x]
```

output
```
(Sqrt[a + b*x^3]*(-4*A*(a + b*x^3) + (((5*A*b)/2 - 7*a*B)*x^3*Hypergeometr
ic2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(28*a*x^7)
```

Rubi [A] (warning: unable to verify)

Time = 1.02 (sec) , antiderivative size = 575, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 809, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^8}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(5Ab-14aB)\int \frac{\sqrt{bx^3+a}}{x^5}dx}{14a}-\frac{A\left(a+bx^3\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{809}$$

$$-\frac{(5Ab-14aB)\left(\frac{3}{8}b\int \frac{1}{x^2\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{4x^4}\right)}{14a}-\frac{A\left(a+bx^3\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(5Ab-14aB)\left(\frac{3}{8}b\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)-\frac{\sqrt{a+bx^3}}{4x^4}\right)}{14a}-\frac{A\left(a+bx^3\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{832}$$

$$-\frac{(5Ab-14aB)\left(\frac{3}{8}b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)-\frac{\sqrt{a+bx^3}}{4x^4}\right)}{14a}-\frac{A\left(a+bx^3\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{759}$$

$$-\frac{(5Ab-14aB)\left(\frac{3}{8}b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+}{\sqrt[3]{b}x+}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{2a}\right.\right.}{14a}$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{7ax^7}$$

$\downarrow$ 2416

$$-\frac{(5Ab-14aB)\left(\frac{3}{8}b\left(\frac{b\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}\right.\right.}{}$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{7ax^7}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^8,x]
```

output
```
-1/7*(A*(a + b*x^3)^(3/2))/(a*x^7) - ((5*A*b - 14*a*B)*(-1/4*Sqrt[a + b*x^
3]/x^4 + (3*b*(-(Sqrt[a + b*x^3]/(a*x)) + (b*(((2*Sqrt[a + b*x^3])/(b^(1/3
)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b
^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^
(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]
)/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(2*a)))/8))/(14*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,\left(-15A\,b^2x^6+42Bab\,x^6+6aAb\,x^3+28B\,a^2x^3+16a^2A\right)}{112x^7a^2}+\frac{ib(5Ab-14Ba)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)}{2b}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{}$
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{7x^7}-\frac{(3Ab+14Ba)\sqrt{b\,x^3+a}}{56a\,x^4}+\frac{3b(5Ab-14Ba)\sqrt{b\,x^3+a}}{112a^2x}+\frac{ib(5Ab-14Ba)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b\right.}{2b}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{}$
default	Expression too large to display

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/112*(b*x^3+a)^(1/2)*(-15*A*b^2*x^6+42*B*a*b*x^6+6*A*a*b*x^3+28*B*a^2*x^
3+16*A*a^2)/x^7/a^2+1/112*I*b*(5*A*b-14*B*a)/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx = -\frac{3\,(14\,Bab-5\,Ab^2)\sqrt{b}x^7\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+(3\,(14\,Bab-5\,Ab}{112\,a^2x^7}$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^8,x, algorithm="fricas")
```

output
```
-1/112*(3*(14*B*a*b - 5*A*b^2)*sqrt(b)*x^7*weierstrassZeta(0, -4*a/b, weie
rstrassPInverse(0, -4*a/b, x)) + (3*(14*B*a*b - 5*A*b^2)*x^6 + 2*(14*B*a^2
 + 3*A*a*b)*x^3 + 16*A*a^2)*sqrt(b*x^3 + a))/(a^2*x^7)
```

Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)*(B*x**3+A)/x**8,x)
```

output
```
A*sqrt(a)*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**7*gamma(-4/3)) + B*sqrt(a)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^8}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^8,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx=\int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^8}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^8,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx=\int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^8}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^8,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx=\frac{-4\sqrt{b\,x^3+a}\,a-22\sqrt{b\,x^3+a}\,b\,x^3+27\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^{11}+a\,x^8}dx\right)a^2x^7}{55x^7}$$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^8,x)
```

output
```
( - 4*sqrt(a + b*x**3)*a - 22*sqrt(a + b*x**3)*b*x**3 + 27*int(sqrt(a + b*
x**3)/(a*x**8 + b*x**11),x)*a**2*x**7)/(55*x**7)
```

3.174 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 614

$$\begin{aligned}\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx &= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7}+\frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4}\\ &-\frac{3b^2(11Ab-20aB)\sqrt{a+bx^3}}{448a^3x}+\frac{3b^{7/3}(11Ab-20aB)\sqrt{a+bx^3}}{448a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{A(a+bx^3)^{3/2}}{10ax^{10}}\\ &-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{7/3}(11Ab-20aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-\right.}{896a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\\ &+\frac{3^{3/4}b^{7/3}(11Ab-20aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7\right.}{224\sqrt{2}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\end{aligned}$$

output
```
1/140*(11*A*b-20*B*a)*(b*x^3+a)^(1/2)/a/x^7+3/1120*b*(11*A*b-20*B*a)*(b*x^
3+a)^(1/2)/a^2/x^4-3/448*b^2*(11*A*b-20*B*a)*(b*x^3+a)^(1/2)/a^3/x+3/448*b
^(7/3)*(11*A*b-20*B*a)*(b*x^3+a)^(1/2)/a^3/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)
-1/10*A*(b*x^3+a)^(3/2)/a/x^10-3/896*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*b^(
7/3)*(11*A*b-20*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^
(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/a^(8/3)/(a
^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3
+a)^(1/2)+1/448*3^(3/4)*b^(7/3)*(11*A*b-20*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2
)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x
),I*3^(1/2)+2*I)*2^(1/2)/a^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))
*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \frac{\sqrt{a+bx^3}\left(-7A(a+bx^3)+\frac{\left(\frac{11Ab}{2}-10aB\right)x^3\,\text{Hypergeometric2F1}\left(-\frac{7}{3},-\frac{1}{2},-\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{70ax^{10}}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^11,x]
```

output
```
(Sqrt[a + b*x^3]*(-7*A*(a + b*x^3) + (((11*A*b)/2 - 10*a*B)*x^3*Hypergeome
tric2F1[-7/3, -1/2, -4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(70*a*x^10)
```

Rubi [A] (warning: unable to verify)

Time = 1.04 (sec) , antiderivative size = 605, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {955, 809, 847, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(11Ab-20aB)\int \frac{\sqrt{bx^3+a}}{x^8}dx}{20a}-\frac{A(a+bx^3)^{3/2}}{10ax^{10}}$$

$$\Big\downarrow \text{809}$$

$$-\frac{(11Ab-20aB)\left(\frac{3}{14}b\int \frac{1}{x^5\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{7x^7}\right)}{20a}-\frac{A(a+bx^3)^{3/2}}{10ax^{10}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(11Ab-20aB)\left(\frac{3}{14}b\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+a}}dx}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)-\frac{\sqrt{a+bx^3}}{7x^7}\right)}{20a}-\frac{A(a+bx^3)^{3/2}}{10ax^{10}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(11Ab-20aB)\left(\frac{3}{14}b\left(-\frac{5b\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)-\frac{\sqrt{a+bx^3}}{7x^7}\right)}{20a}-\frac{A(a+bx^3)^{3/2}}{10ax^{10}}$$

$$\Big\downarrow \text{832}$$

$$-\frac{(11Ab-20aB)\left(\frac{3}{14}b\left(-\frac{5b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)-\frac{\sqrt{a+bx^3}}{7x^7}\right)}{20a}-\frac{A\left(a+bx^3\right)^{3/2}}{10ax^{10}}$$

$\Big\downarrow$ 759

$$-\frac{(11Ab-20aB)\left(\frac{3}{14}b\left(-\frac{5b\left(\frac{b\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{2a}\right)}{8a}\right)\right)}{20a}$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{10ax^{10}}$$

$\downarrow$ 2416

$$-\frac{(11Ab-20aB)\left(\frac{3}{14}b\left(-\frac{5b\left(\frac{b\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x\cdots}{\sqrt[3]{b}x\cdots}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots\right)}{\sqrt[3]{b}}\cdots\right)}{\cdots}\cdots\right)\cdots\right)}{\cdots}$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{10ax^{10}}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^11,x]
```

output
```
-1/10*(A*(a + b*x^3)^(3/2))/(a*x^10) - ((11*A*b - 20*a*B)*(-1/7*Sqrt[a + b
*x^3]/x^7 + (3*b*(-1/4*Sqrt[a + b*x^3]/(a*x^4) - (5*b*(-(Sqrt[a + b*x^3]/(
a*x)) + (b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3)
 - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])))/(2*a)))/(8*a)))/14))/(20*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 530, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(165A\,x^{9}b^{3}-300B\,x^{9}a\,b^{2}-66A\,x^{6}a\,b^{2}+120B\,x^{6}a^{2}b+48a^{2}Ab\,x^{3}+320B\,x^{3}a^{3}+224a^{3}A\right)}{2240x^{10}a^{3}}-\frac{ib^{2}(11Ab-20Ba)\sqrt{3}\,(-a\,b \cdots}{\cdots} \cdots ib^{2}(11Ab-20Ba)\sqrt{3}\,(- \cdots$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{10x^{10}}-\frac{(3Ab+20Ba)\sqrt{b\,x^{3}+a}}{140a\,x^{7}}+\frac{3b(11Ab-20Ba)\sqrt{b\,x^{3}+a}}{1120a^{2}x^{4}}-\frac{3b^{2}(11Ab-20Ba)\sqrt{b\,x^{3}+a}}{448a^{3}x}-\cdots$
default	Expression too large to display

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/2240*(b*x^3+a)^(1/2)*(165*A*b^3*x^9-300*B*a*b^2*x^9-66*A*a*b^2*x^6+120*
B*a^2*b*x^6+48*A*a^2*b*x^3+320*B*a^3*x^3+224*A*a^3)/x^10/a^3-1/448*I*b^2*(
11*A*b-20*B*a)/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2
)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I
*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(
-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx$$
$$= \frac{15\,(20\,Bab^2-11\,Ab^3)\sqrt{b}x^{10}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+(15\,(20\,Bab^2-\cdots}{2240\,a^3x^{10}}$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^11,x, algorithm="fricas")
```

output
```
1/2240*(15*(20*B*a*b^2 - 11*A*b^3)*sqrt(b)*x^10*weierstrassZeta(0, -4*a/b,
 weierstrassPInverse(0, -4*a/b, x)) + (15*(20*B*a*b^2 - 11*A*b^3)*x^9 - 6*
(20*B*a^2*b - 11*A*a*b^2)*x^6 - 224*A*a^3 - 16*(20*B*a^3 + 3*A*a^2*b)*x^3)
*sqrt(b*x^3 + a))/(a^3*x^10)
```

Sympy [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{10}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{10}{3}, -\frac{1}{2} \\ -\frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^{10}\Gamma\left(-\frac{7}{3}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{3}, -\frac{1}{2} \\ -\frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)*(B*x**3+A)/x**11,x)
```

output
```
A*sqrt(a)*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*x**10*gamma(-7/3)) + B*sqrt(a)*gamma(-7/3)*hyper((-7/3, -1/2), (-4
/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^11,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^11, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^11,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^11, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^{11}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^11,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^11, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \frac{-16\sqrt{b\,x^3+a}\,a-34\sqrt{b\,x^3+a}\,b\,x^3+27\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^{14}+a\,x^{11}}dx\right)a^2x^{10}}{187x^{10}}$$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^11,x)
```

output
```
( - 16*sqrt(a + b*x**3)*a - 34*sqrt(a + b*x**3)*b*x**3 + 27*int(sqrt(a + b
*x**3)/(a*x**11 + b*x**14),x)*a**2*x**10)/(187*x**10)
```

3.175 $\int x^8(a+bx^3)^{3/2}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int x^8(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2a^2(Ab-aB)(a+bx^3)^{5/2}}{15b^4} - \frac{2a(2Ab-3aB)(a+bx^3)^{7/2}}{21b^4} + \frac{2(Ab-3aB)(a+bx^3)^{9/2}}{27b^4} + \frac{2B(a+bx^3)^{11/2}}{33b^4}$$

output
```
2/15*a^2*(A*b-B*a)*(b*x^3+a)^(5/2)/b^4-2/21*a*(2*A*b-3*B*a)*(b*x^3+a)^(7/2
)/b^4+2/27*(A*b-3*B*a)*(b*x^3+a)^(9/2)/b^4+2/33*B*(b*x^3+a)^(11/2)/b^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int x^8(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2(a+bx^3)^{5/2}(88a^2Ab-48a^3B-220aAb^2x^3+120a^2bBx^3+385Ab^3x^6-210ab^2Bx^6+315b^3}{10395b^4}$$

input
```
Integrate[x^8*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*(a + b*x^3)^(5/2)*(88*a^2*A*b - 48*a^3*B - 220*a*A*b^2*x^3 + 120*a^2*b*
B*x^3 + 385*A*b^3*x^6 - 210*a*b^2*B*x^6 + 315*b^3*B*x^9))/(10395*b^4)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int x^6 \left(bx^3+a\right)^{3/2} \left(Bx^3+A\right) dx^3$$

$$\Big\downarrow \; 86$$

$$\frac{1}{3}\int \left(\frac{B\left(bx^3+a\right)^{9/2}}{b^3}+\frac{(Ab-3aB)\left(bx^3+a\right)^{7/2}}{b^3}+\frac{a(3aB-2Ab)\left(bx^3+a\right)^{5/2}}{b^3}-\frac{a^2(aB-Ab)\left(bx^3+a\right)^{3/2}}{b^3}\right) d$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(\frac{2a^2\left(a+bx^3\right)^{5/2}(Ab-aB)}{5b^4}+\frac{2\left(a+bx^3\right)^{9/2}(Ab-3aB)}{9b^4}-\frac{2a\left(a+bx^3\right)^{7/2}(2Ab-3aB)}{7b^4}+\frac{2B\left(a+bx^3\right)^{11/2}}{11b^4}\right)$$

input
```
Int[x^8*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
((2*a^2*(A*b - a*B)*(a + b*x^3)^(5/2))/(5*b^4) - (2*a*(2*A*b - 3*a*B)*(a +
 b*x^3)^(7/2))/(7*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(9/2))/(9*b^4) + (2*
B*(a + b*x^3)^(11/2))/(11*b^4))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{16\left(\frac{35\left(\frac{9Bx^3}{11}+A\right)x^6b^3}{8}-\frac{5a\left(\frac{21Bx^3}{22}+A\right)x^3b^2}{2}+a^2\left(\frac{15Bx^3}{11}+A\right)b-\frac{6a^3B}{11}\right)\left(bx^3+a\right)^{\frac{5}{2}}}{945b^4}$
gosper	$\frac{2\left(bx^3+a\right)^{\frac{5}{2}}\left(315b^3Bx^9+385Ab^3x^6-210Bab^2x^6-220aAb^2x^3+120Ba^2bx^3+88a^2bA-48a^3B\right)}{10395b^4}$
orering	$\frac{2\left(bx^3+a\right)^{\frac{5}{2}}\left(315b^3Bx^9+385Ab^3x^6-210Bab^2x^6-220aAb^2x^3+120Ba^2bx^3+88a^2bA-48a^3B\right)}{10395b^4}$
trager	$\frac{2(315b^5Bx^{15}+385b^5Ax^{12}+420ab^4Bx^{12}+550ab^4Ax^9+15a^2b^3Bx^9+33a^2Ab^3x^6-18Ba^3b^2x^6-44a^3Ab^2x^3+24Ba^4bx}{10395b^4}$
risch	$\frac{2(315b^5Bx^{15}+385b^5Ax^{12}+420ab^4Bx^{12}+550ab^4Ax^9+15a^2b^3Bx^9+33a^2Ab^3x^6-18Ba^3b^2x^6-44a^3Ab^2x^3+24Ba^4bx}{10395b^4}$
default	$A\left(\frac{2bx^{12}\sqrt{bx^3+a}}{27}+\frac{20ax^9\sqrt{bx^3+a}}{189}+\frac{2a^2x^6\sqrt{bx^3+a}}{315b}-\frac{8a^3x^3\sqrt{bx^3+a}}{945b^2}+\frac{16a^4\sqrt{bx^3+a}}{945b^3}\right)+B\left(\frac{2bx^{15}\sqrt{bx^3}}{33}\right.$
elliptic	$\frac{2Bbx^{15}\sqrt{bx^3+a}}{33}+\frac{2(b^2A+\frac{12}{11}abB)x^{12}\sqrt{bx^3+a}}{27b}+\frac{2\left(2abA+a^2B-\frac{8a\left(b^2A+\frac{12}{11}abB\right)}{9b}\right)x^9\sqrt{bx^3+a}}{21b}+\frac{2\left(a^2A-\frac{6a\left(2a\right.}{}\right.}{}$

input
```
int(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
16/945*(35/8*(9/11*B*x^3+A)*x^6*b^3-5/2*a*(21/22*B*x^3+A)*x^3*b^2+a^2*(15/
11*B*x^3+A)*b-6/11*a^3*B)*(b*x^3+a)^(5/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.20

$$\int x^8(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2\,(315\,Bb^5x^{15}+35\,(12\,Bab^4+11\,Ab^5)x^{12}+5\,(3\,Ba^2b^3+110\,Aab^4)x^9-3\,(6\,Ba^3b^2-11\,Aa^2b}{10395\,b^4}$$

input
```
integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/10395*(315*B*b^5*x^15 + 35*(12*B*a*b^4 + 11*A*b^5)*x^12 + 5*(3*B*a^2*b^3
 + 110*A*a*b^4)*x^9 - 3*(6*B*a^3*b^2 - 11*A*a^2*b^3)*x^6 - 48*B*a^5 + 88*A
*a^4*b + 4*(6*B*a^4*b - 11*A*a^3*b^2)*x^3)*sqrt(b*x^3 + a)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(100) = 200.

Time = 0.68 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.59

$$\int x^8(a+bx^3)^{3/2}(A+Bx^3)\,dx = \begin{cases} \frac{16Aa^4\sqrt{a+bx^3}}{945b^3}-\frac{8Aa^3x^3\sqrt{a+bx^3}}{945b^2}+\frac{2Aa^2x^6\sqrt{a+bx^3}}{315b}+\frac{20Aax^9\sqrt{a+bx^3}}{189}+\frac{2Abx^{12}\sqrt{a+bx^3}}{27}-\frac{32Ba^5\sqrt{a+bx^3}}{3465b^4}+\frac{16}{} \\ a^{\frac{3}{2}}\left(\frac{Ax^9}{9}+\frac{Bx^{12}}{12}\right) \end{cases}$$

input
```
integrate(x**8*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
Piecewise((16*A*a**4*sqrt(a + b*x**3)/(945*b**3) - 8*A*a**3*x**3*sqrt(a +
b*x**3)/(945*b**2) + 2*A*a**2*x**6*sqrt(a + b*x**3)/(315*b) + 20*A*a*x**9*
sqrt(a + b*x**3)/189 + 2*A*b*x**12*sqrt(a + b*x**3)/27 - 32*B*a**5*sqrt(a
+ b*x**3)/(3465*b**4) + 16*B*a**4*x**3*sqrt(a + b*x**3)/(3465*b**3) - 4*B*
a**3*x**6*sqrt(a + b*x**3)/(1155*b**2) + 2*B*a**2*x**9*sqrt(a + b*x**3)/(6
93*b) + 8*B*a*x**12*sqrt(a + b*x**3)/99 + 2*B*b*x**15*sqrt(a + b*x**3)/33,
 Ne(b, 0)), (a**(3/2)*(A*x**9/9 + B*x**12/12), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int x^8(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2}{945}\left(\frac{35\,(bx^3+a)^{\frac{9}{2}}}{b^3} - \frac{90\,(bx^3+a)^{\frac{7}{2}}a}{b^3} + \frac{63\,(bx^3+a)^{\frac{5}{2}}a^2}{b^3}\right)A + \frac{2}{3465}\left(\frac{105\,(bx^3+a)^{\frac{11}{2}}}{b^4} - \frac{385\,(bx^3+a)^{\frac{9}{2}}a}{b^4} + \frac{495\,(bx^3+a)^{\frac{7}{2}}a^2}{b^4} - \frac{231\,(bx^3+a)^{\frac{5}{2}}a^3}{b^4}\right)B$$

input
```
integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/945*(35*(b*x^3 + a)^(9/2)/b^3 - 90*(b*x^3 + a)^(7/2)*a/b^3 + 63*(b*x^3 +
 a)^(5/2)*a^2/b^3)*A + 2/3465*(105*(b*x^3 + a)^(11/2)/b^4 - 385*(b*x^3 + a
)^(9/2)*a/b^4 + 495*(b*x^3 + a)^(7/2)*a^2/b^4 - 231*(b*x^3 + a)^(5/2)*a^3/
b^4)*B
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int x^8(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2\left(315\,(bx^3+a)^{\frac{11}{2}}B - 1155\,(bx^3+a)^{\frac{9}{2}}Ba + 1485\,(bx^3+a)^{\frac{7}{2}}Ba^2 - 693\,(bx^3+a)^{\frac{5}{2}}Ba^3 + 385\right.}{10395\,b^4}$$

input
```
integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
2/10395*(315*(b*x^3 + a)^(11/2)*B - 1155*(b*x^3 + a)^(9/2)*B*a + 1485*(b*x
^3 + a)^(7/2)*B*a^2 - 693*(b*x^3 + a)^(5/2)*B*a^3 + 385*(b*x^3 + a)^(9/2)*
A*b - 990*(b*x^3 + a)^(7/2)*A*a*b + 693*(b*x^3 + a)^(5/2)*A*a^2*b)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.00

$$\int x^8(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx = \frac{20\,A\,a\,x^9\,\sqrt{b\,x^3+a}}{189}+\frac{2\,A\,b\,x^{12}\,\sqrt{b\,x^3+a}}{27}+\frac{8\,B\,a\,x^{12}\,\sqrt{b\,x^3+a}}{99}+\frac{2\,B\,b\,x^{15}\,\sqrt{b\,x^3+a}}{33}+\frac{16\,A\,a^4\,\sqrt{b\,x^3+a}}{945\,b^3}-\frac{32\,B\,a^5\,\sqrt{b\,x^3+a}}{3465\,b^4}-\frac{8\,A\,a^3\,x^3\,\sqrt{b\,x^3+a}}{945\,b^2}+\frac{2\,A\,a^2\,x^6\,\sqrt{b\,x^3+a}}{315\,b}+\frac{16\,B\,a^4\,x^3\,\sqrt{b\,x^3+a}}{3465\,b^3}-\frac{4\,B\,a^3\,x^6\,\sqrt{b\,x^3+a}}{1155\,b^2}+\frac{2\,B\,a^2\,x^9\,\sqrt{b\,x^3+a}}{693\,b}$$

input
```
int(x^8*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

output
```
(20*A*a*x^9*(a + b*x^3)^(1/2))/189 + (2*A*b*x^12*(a + b*x^3)^(1/2))/27 + (
8*B*a*x^12*(a + b*x^3)^(1/2))/99 + (2*B*b*x^15*(a + b*x^3)^(1/2))/33 + (16
*A*a^4*(a + b*x^3)^(1/2))/(945*b^3) - (32*B*a^5*(a + b*x^3)^(1/2))/(3465*b
^4) - (8*A*a^3*x^3*(a + b*x^3)^(1/2))/(945*b^2) + (2*A*a^2*x^6*(a + b*x^3)
^(1/2))/(315*b) + (16*B*a^4*x^3*(a + b*x^3)^(1/2))/(3465*b^3) - (4*B*a^3*x
^6*(a + b*x^3)^(1/2))/(1155*b^2) + (2*B*a^2*x^9*(a + b*x^3)^(1/2))/(693*b)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.65

$$\int x^8(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2\sqrt{b\,x^3+a}\,(63b^5x^{15}+161a\,b^4x^{12}+113a^2b^3x^9+3a^3b^2x^6-4a^4b\,x^3+8a^5)}{2079b^3}$$

input
```
int(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

output
```
(2*sqrt(a + b*x**3)*(8*a**5 - 4*a**4*b*x**3 + 3*a**3*b**2*x**6 + 113*a**2*
b**3*x**9 + 161*a*b**4*x**12 + 63*b**5*x**15))/(2079*b**3)
```

3.176 $\int x^5(a+bx^3)^{3/2}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int x^5\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=-\frac{2a(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{7/2}}{21b^3}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^3}$$

output
```
-2/15*a*(A*b-B*a)*(b*x^3+a)^(5/2)/b^3+2/21*(A*b-2*B*a)*(b*x^3+a)^(7/2)/b^3
+2/27*B*(b*x^3+a)^(9/2)/b^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int x^5\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2\left(a+bx^3\right)^{5/2}\left(-18aAb+8a^2B+45Ab^2x^3-20abBx^3+35b^2Bx^6\right)}{945b^3}$$

input
```
Integrate[x^5*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*(a + b*x^3)^(5/2)*(-18*a*A*b + 8*a^2*B + 45*A*b^2*x^3 - 20*a*b*B*x^3 +
35*b^2*B*x^6))/(945*b^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3} \int x^3 \left(bx^3+a\right)^{3/2} \left(Bx^3+A\right) dx^3$$

$$\Big\downarrow \; 86$$

$$\frac{1}{3} \int \left(\frac{B\left(bx^3+a\right)^{7/2}}{b^2}+\frac{(Ab-2aB)\left(bx^3+a\right)^{5/2}}{b^2}+\frac{a(aB-Ab)\left(bx^3+a\right)^{3/2}}{b^2}\right) dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3} \left(\frac{2\left(a+bx^3\right)^{7/2} (Ab-2aB)}{7b^3}-\frac{2a\left(a+bx^3\right)^{5/2} (Ab-aB)}{5b^3}+\frac{2B\left(a+bx^3\right)^{9/2}}{9b^3}\right)$$

input
```
Int[x^5*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
((-2*a*(A*b - a*B)*(a + b*x^3)^(5/2))/(5*b^3) + (2*(A*b - 2*a*B)*(a + b*x^
3)^(7/2))/(7*b^3) + (2*B*(a + b*x^3)^(9/2))/(9*b^3))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$-\frac{4\left(-\frac{5\left(\frac{7Bx^3}{9}+A\right)x^3b^2}{2}+a\left(\frac{10Bx^3}{9}+A\right)b-\frac{4a^2B}{9}\right)\left(bx^3+a\right)^{\frac{5}{2}}}{105b^3}$
gosper	$-\frac{2\left(bx^3+a\right)^{\frac{5}{2}}\left(-35b^2Bx^6-45Ab^2x^3+20Babx^3+18abA-8a^2B\right)}{945b^3}$
orering	$-\frac{2\left(bx^3+a\right)^{\frac{5}{2}}\left(-35b^2Bx^6-45Ab^2x^3+20Babx^3+18abA-8a^2B\right)}{945b^3}$
trager	$-\frac{2\left(-35Bx^{12}b^4-45Ab^4x^9-50Bx^9ab^3-72Ax^6ab^3-3Bx^6a^2b^2-9Aa^2b^2x^3+4Ba^3bx^3+18Aa^3b-8Ba^4\right)\sqrt{bx^3+a}}{945b^3}$
risch	$-\frac{2\left(-35Bx^{12}b^4-45Ab^4x^9-50Bx^9ab^3-72Ax^6ab^3-3Bx^6a^2b^2-9Aa^2b^2x^3+4Ba^3bx^3+18Aa^3b-8Ba^4\right)\sqrt{bx^3+a}}{945b^3}$
default	$A\left(\frac{2bx^9\sqrt{bx^3+a}}{21}+\frac{16ax^6\sqrt{bx^3+a}}{105}+\frac{2a^2x^3\sqrt{bx^3+a}}{105b}-\frac{4a^3\sqrt{bx^3+a}}{105b^2}\right)+B\left(\frac{2bx^{12}\sqrt{bx^3+a}}{27}+\frac{20ax^9\sqrt{bx^3+a}}{189}\right.$
elliptic	$\frac{2Bbx^{12}\sqrt{bx^3+a}}{27}+\frac{2\left(b^2A+\frac{10}{9}abB\right)x^9\sqrt{bx^3+a}}{21b}+\frac{2\left(2abA+a^2B-\frac{6a\left(b^2A+\frac{10}{9}abB\right)}{7b}\right)x^6\sqrt{bx^3+a}}{15b}+\frac{2\left(a^2A-\frac{4a\left(2ab\right.}{}\right.}{}$

input
```
int(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
-4/105*(-5/2*(7/9*B*x^3+A)*x^3*b^2+a*(10/9*B*x^3+A)*b-4/9*a^2*B)*(b*x^3+a)
^(5/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int x^5 (a+bx^3)^{3/2} (A+Bx^3) \, dx = \frac{2\,(35\,Bb^4x^{12} + 5\,(10\,Bab^3 + 9\,Ab^4)x^9 + 3\,(Ba^2b^2 + 24\,Aab^3)x^6 + 8\,Ba^4 - 18\,Aa^3b - (4\,Ba^3b}{945\,b^3}$$

input
```
integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/945*(35*B*b^4*x^12 + 5*(10*B*a*b^3 + 9*A*b^4)*x^9 + 3*(B*a^2*b^2 + 24*A*
a*b^3)*x^6 + 8*B*a^4 - 18*A*a^3*b - (4*B*a^3*b - 9*A*a^2*b^2)*x^3)*sqrt(b*
x^3 + a)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(70) = 140.

Time = 0.45 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.96

$$\int x^5 (a+bx^3)^{3/2} (A+Bx^3) \, dx = \begin{cases} -\frac{4Aa^3\sqrt{a+bx^3}}{105b^2} + \frac{2Aa^2x^3\sqrt{a+bx^3}}{105b} + \frac{16Aax^6\sqrt{a+bx^3}}{105} + \frac{2Abx^9\sqrt{a+bx^3}}{21} + \frac{16Ba^4\sqrt{a+bx^3}}{945b^3} - \frac{8Ba^3x^3\sqrt{a+bx^3}}{945b^2} + \frac{2}{} \\ a^{\frac{3}{2}}\left(\frac{Ax^6}{6} + \frac{Bx^9}{9}\right) \end{cases}$$

input
```
integrate(x**5*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
Piecewise((-4*A*a**3*sqrt(a + b*x**3)/(105*b**2) + 2*A*a**2*x**3*sqrt(a +
b*x**3)/(105*b) + 16*A*a*x**6*sqrt(a + b*x**3)/105 + 2*A*b*x**9*sqrt(a + b
*x**3)/21 + 16*B*a**4*sqrt(a + b*x**3)/(945*b**3) - 8*B*a**3*x**3*sqrt(a +
 b*x**3)/(945*b**2) + 2*B*a**2*x**6*sqrt(a + b*x**3)/(315*b) + 20*B*a*x**9
*sqrt(a + b*x**3)/189 + 2*B*b*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)), (a**(3
/2)*(A*x**6/6 + B*x**9/9), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int x^5 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{2}{105} \left(\frac{5 \left(bx^3+a\right)^{\frac{7}{2}}}{b^2} - \frac{7 \left(bx^3+a\right)^{\frac{5}{2}} a}{b^2}\right) A + \frac{2}{945} \left(\frac{35 \left(bx^3+a\right)^{\frac{9}{2}}}{b^3} - \frac{90 \left(bx^3+a\right)^{\frac{7}{2}} a}{b^3} + \frac{63 \left(bx^3+a\right)^{\frac{5}{2}} a^2}{b^3}\right) B$$

input
```
integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/105*(5*(b*x^3 + a)^(7/2)/b^2 - 7*(b*x^3 + a)^(5/2)*a/b^2)*A + 2/945*(35*
(b*x^3 + a)^(9/2)/b^3 - 90*(b*x^3 + a)^(7/2)*a/b^3 + 63*(b*x^3 + a)^(5/2)*
a^2/b^3)*B
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int x^5 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{2 \left(35 \left(bx^3+a\right)^{\frac{9}{2}} B - 90 \left(bx^3+a\right)^{\frac{7}{2}} Ba + 63 \left(bx^3+a\right)^{\frac{5}{2}} Ba^2 + 45 \left(bx^3+a\right)^{\frac{7}{2}} Ab - 63 \left(bx^3+a\right)^{\frac{5}{2}}}{945 \, b^3}$$

input
```
integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
2/945*(35*(b*x^3 + a)^(9/2)*B - 90*(b*x^3 + a)^(7/2)*B*a + 63*(b*x^3 + a)^
(5/2)*B*a^2 + 45*(b*x^3 + a)^(7/2)*A*b - 63*(b*x^3 + a)^(5/2)*A*a*b)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.89

$$\int x^5(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{x^6\sqrt{b\,x^3+a}\left(2\,B\,a^2+4\,A\,a\,b-\frac{6\,a\,\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)}{7\,b}\right)}{15\,b} - \frac{2\,a\left(2\,A\,a^2-\frac{4\,a\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{6\,a\,\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)}{7\,b}\right)}{5\,b}\right)\sqrt{b\,x^3+a}}{9\,b^2} + \frac{2\,B\,b\,x^{12}\sqrt{b\,x^3+a}}{27} + \frac{x^3\left(2\,A\,a^2-\frac{4\,a\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{6\,a\,\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)}{7\,b}\right)}{5\,b}\right)\sqrt{b\,x^3+a}}{9\,b} + \frac{x^9\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)\sqrt{b\,x^3+a}}{21\,b}$$

input
```
int(x^5*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

output
```
(x^6*(a + b*x^3)^(1/2)*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2 + (20*B*a*b)/9))
/(7*b)))/(15*b) - (2*a*(2*A*a^2 - (4*a*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2
+ (20*B*a*b)/9))/(7*b)))/(5*b))*(a + b*x^3)^(1/2))/(9*b^2) + (2*B*b*x^12*(
a + b*x^3)^(1/2))/27 + (x^3*(2*A*a^2 - (4*a*(2*B*a^2 + 4*A*a*b - (6*a*(2*A
*b^2 + (20*B*a*b)/9))/(7*b)))/(5*b))*(a + b*x^3)^(1/2))/(9*b) + (x^9*(2*A*
b^2 + (20*B*a*b)/9)*(a + b*x^3)^(1/2))/(21*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int x^5(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2\sqrt{b\,x^3+a}\,(7b^4x^{12}+19a\,b^3x^9+15a^2b^2x^6+a^3b\,x^3-2a^4)}{189b^2}$$

input
```
int(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

output
```
(2*sqrt(a + b*x**3)*( - 2*a**4 + a**3*b*x**3 + 15*a**2*b**2*x***6 + 19*a*b*
*3*x**9 + 7*b**4*x**12))/(189*b**2)
```

3.177 $\int x^2(a+bx^3)^{3/2}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int x^2\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^2}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^2}$$

output
```
2/15*(A*b-B*a)*(b*x^3+a)^(5/2)/b^2+2/21*B*(b*x^3+a)^(7/2)/b^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int x^2\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2\left(a+bx^3\right)^{5/2}\left(7Ab-2aB+5bBx^3\right)}{105b^2}$$

input
```
Integrate[x^2*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*(a + b*x^3)^(5/2)*(7*A*b - 2*a*B + 5*b*B*x^3))/(105*b^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3} \int \left(bx^3+a\right)^{3/2} \left(Bx^3+A\right)dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3} \int \left(\frac{B\left(bx^3+a\right)^{5/2}}{b}+\frac{(Ab-aB)\left(bx^3+a\right)^{3/2}}{b}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{2\left(a+bx^3\right)^{5/2} (Ab-aB)}{5b^2}+\frac{2B\left(a+bx^3\right)^{7/2}}{7b^2}\right)$$

input
```
Int[x^2*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
((2*(A*b - a*B)*(a + b*x^3)^(5/2))/(5*b^2) + (2*B*(a + b*x^3)^(7/2))/(7*b^
2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result
gosper	$\frac{2\left(b\,x^{3}+a\right)^{\frac{5}{2}}\left(5bB\,x^{3}+7Ab-2Ba\right)}{105b^{2}}$
orering	$\frac{2\left(b\,x^{3}+a\right)^{\frac{5}{2}}\left(5bB\,x^{3}+7Ab-2Ba\right)}{105b^{2}}$
pseudoelliptic	$\frac{2\left(\left(5B\,x^{3}+7A\right)b-2Ba\right)\left(b\,x^{3}+a\right)^{\frac{5}{2}}}{105b^{2}}$
trager	$\frac{2\left(5b^{3}B\,x^{9}+7A\,b^{3}x^{6}+8Ba\,b^{2}x^{6}+14aA\,b^{2}x^{3}+B\,a^{2}b\,x^{3}+7a^{2}bA-2a^{3}B\right)\sqrt{b\,x^{3}+a}}{105b^{2}}$
risch	$\frac{2\left(5b^{3}B\,x^{9}+7A\,b^{3}x^{6}+8Ba\,b^{2}x^{6}+14aA\,b^{2}x^{3}+B\,a^{2}b\,x^{3}+7a^{2}bA-2a^{3}B\right)\sqrt{b\,x^{3}+a}}{105b^{2}}$
default	$\frac{2A\left(b\,x^{3}+a\right)^{\frac{5}{2}}}{15b}+B\left(\frac{2b\,x^{9}\sqrt{b\,x^{3}+a}}{21}+\frac{16a\,x^{6}\sqrt{b\,x^{3}+a}}{105}+\frac{2a^{2}x^{3}\sqrt{b\,x^{3}+a}}{105b}-\frac{4a^{3}\sqrt{b\,x^{3}+a}}{105b^{2}}\right)$
elliptic	$\frac{2Bb\,x^{9}\sqrt{b\,x^{3}+a}}{21}+\frac{2\left(b^{2}A+\frac{8}{7}abB\right)x^{6}\sqrt{b\,x^{3}+a}}{15b}+\frac{2\left(2abA+a^{2}B-\frac{4a\left(b^{2}A+\frac{8}{7}abB\right)}{5b}\right)x^{3}\sqrt{b\,x^{3}+a}}{9b}+\frac{2\left(a^{2}A-\frac{2a\left(2abA+\cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/105*(b*x^3+a)^(5/2)*(5*B*b*x^3+7*A*b-2*B*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

$$\int x^2(a+bx^3)^{3/2}(A+Bx^3)\,dx=\frac{2\,(5\,Bb^3x^9+(8\,Bab^2+7\,Ab^3)x^6-2\,Ba^3+7\,Aa^2b+(Ba^2b+14\,Aab^2)x^3)\sqrt{bx^3+a}}{105\,b^2}$$

input
```
integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/105*(5*B*b^3*x^9 + (8*B*a*b^2 + 7*A*b^3)*x^6 - 2*B*a^3 + 7*A*a^2*b + (B*
a^2*b + 14*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(44) = 88.

Time = 0.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.59

$$\int x^2(a+bx^3)^{3/2}(A+Bx^3)\,dx=\begin{cases}\frac{2Aa^2\sqrt{a+bx^3}}{15b}+\frac{4Aax^3\sqrt{a+bx^3}}{15}+\frac{2Abx^6\sqrt{a+bx^3}}{15}-\frac{4Ba^3\sqrt{a+bx^3}}{105b^2}+\frac{2Ba^2x^3\sqrt{a+bx^3}}{105b}+\frac{16Bax^6\sqrt{a+bx^3}}{105}+\frac{2Bbx^9}{} \\ a^{\frac{3}{2}}\left(\frac{Ax^3}{3}+\frac{Bx^6}{6}\right)\end{cases}$$

input
```
integrate(x**2*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
Piecewise((2*A*a**2*sqrt(a + b*x**3)/(15*b) + 4*A*a*x**3*sqrt(a + b*x**3)/
15 + 2*A*b*x**6*sqrt(a + b*x**3)/15 - 4*B*a**3*sqrt(a + b*x**3)/(105*b**2)
 + 2*B*a**2*x**3*sqrt(a + b*x**3)/(105*b) + 16*B*a*x**6*sqrt(a + b*x**3)/1
05 + 2*B*b*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (a**(3/2)*(A*x**3/3 + B*x*
*6/6), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int x^2(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2\,(bx^3+a)^{\frac{5}{2}}A}{15\,b} + \frac{2}{105}\left(\frac{5\,(bx^3+a)^{\frac{7}{2}}}{b^2} - \frac{7\,(bx^3+a)^{\frac{5}{2}}a}{b^2}\right)B$$

input
```
integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/15*(b*x^3 + a)^(5/2)*A/b + 2/105*(5*(b*x^3 + a)^(7/2)/b^2 - 7*(b*x^3 + a
)^(5/2)*a/b^2)*B
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x^2(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2\left(5\,(bx^3+a)^{\frac{7}{2}}B - 7\,(bx^3+a)^{\frac{5}{2}}Ba + 7\,(bx^3+a)^{\frac{5}{2}}Ab\right)}{105\,b^2}$$

input
```
integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
2/105*(5*(b*x^3 + a)^(7/2)*B - 7*(b*x^3 + a)^(5/2)*B*a + 7*(b*x^3 + a)^(5/
2)*A*b)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 150, normalized size of antiderivative = 3.26

$$\int x^2 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\,dx=\frac{\left(2\,A\,a^2-\frac{2\,a\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{4\,a\,\left(2\,A\,b^2+\frac{16\,B\,a\,b}{7}\right)}{5\,b}\right)}{3\,b}\right)\,\sqrt{b\,x^3+a}}{3\,b}+\frac{x^3\,\sqrt{b\,x^3+a}\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{4\,a\,\left(2\,A\,b^2+\frac{16\,B\,a\,b}{7}\right)}{5\,b}\right)}{9\,b}+\frac{2\,B\,b\,x^9\,\sqrt{b\,x^3+a}}{21}+\frac{x^6\,\left(2\,A\,b^2+\frac{16\,B\,a\,b}{7}\right)\,\sqrt{b\,x^3+a}}{15\,b}$$

input
```
int(x^2*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

output
```
((2*A*a^2 - (2*a*(2*B*a^2 + 4*A*a*b - (4*a*(2*A*b^2 + (16*B*a*b)/7))/(5*b)
))/(3*b))*(a + b*x^3)^(1/2))/(3*b) + (x^3*(a + b*x^3)^(1/2)*(2*B*a^2 + 4*A
*a*b - (4*a*(2*A*b^2 + (16*B*a*b)/7))/(5*b)))/(9*b) + (2*B*b*x^9*(a + b*x^
3)^(1/2))/21 + (x^6*(2*A*b^2 + (16*B*a*b)/7)*(a + b*x^3)^(1/2))/(15*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int x^2 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\,dx=\frac{2\sqrt{b\,x^3+a}\,(b^3x^9+3a\,b^2x^6+3a^2b\,x^3+a^3)}{21b}$$

input
```
int(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

output
```
(2*sqrt(a + b*x**3)*(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9))/(2
1*b)
```

3.178 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x} \, dx = \frac{2}{3}aA\sqrt{a+bx^3} + \frac{2}{9}A(a+bx^3)^{3/2} + \frac{2B(a+bx^3)^{5/2}}{15b} - \frac{2}{3}a^{3/2}A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

output
```
2/3*a*A*(b*x^3+a)^(1/2)+2/9*A*(b*x^3+a)^(3/2)+2/15*B*(b*x^3+a)^(5/2)/b-2/3
*a^(3/2)*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x} \, dx = \frac{2\sqrt{a+bx^3}(20aAb+3a^2B+5Ab^2x^3+6abBx^3+3b^2Bx^6)}{45b} - \frac{2}{3}a^{3/2}A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x,x]
```

output
```
(2*Sqrt[a + b*x^3]*(20*a*A*b + 3*a^2*B + 5*A*b^2*x^3 + 6*a*b*B*x^3 + 3*b^2
*B*x^6))/(45*b) - (2*a^(3/2)*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{3/2}(Bx^3+A)}{x^3}dx^3$$

$$\Big\downarrow \ 90$$

$$\frac{1}{3}\left(A\int \frac{(bx^3+a)^{3/2}}{x^3}dx^3+\frac{2B(a+bx^3)^{5/2}}{5b}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{3}\left(A\left(a\int \frac{\sqrt{bx^3+a}}{x^3}dx^3+\frac{2}{3}(a+bx^3)^{3/2}\right)+\frac{2B(a+bx^3)^{5/2}}{5b}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{3}\left(A\left(a\left(a\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+2\sqrt{a+bx^3}\right)+\frac{2}{3}(a+bx^3)^{3/2}\right)+\frac{2B(a+bx^3)^{5/2}}{5b}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{3}\left(A\left(a\left(\frac{2a\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{b}+2\sqrt{a+bx^3}\right)+\frac{2}{3}(a+bx^3)^{3/2}\right)+\frac{2B(a+bx^3)^{5/2}}{5b}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{3}\left(A\left(a\left(2\sqrt{a+bx^3}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+bx^3\right)^{3/2}\right)+\frac{2B\left(a+bx^3\right)^{5/2}}{5b}\right)$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x,x]
```

output
```
((2*B*(a + b*x^3)^(5/2))/(5*b) + A*((2*(a + b*x^3)^(3/2))/3 + a*(2*Sqrt[a
+ b*x^3] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

method	result
default	$A\left(\frac{2x^{3}b\sqrt{b\,x^{3}+a}}{9}+\frac{8a\sqrt{b\,x^{3}+a}}{9}-\frac{2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3}\right)+\frac{2B\left(b\,x^{3}+a\right)^{\frac{5}{2}}}{15b}$
pseudoelliptic	$\frac{-\frac{2a^{\frac{3}{2}}bA\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3}+\frac{8\left(\frac{\left(\frac{3B\,x^{3}}{5}+A\right)x^{3}b^{2}}{4}+a\left(\frac{3B\,x^{3}}{10}+A\right)b+\frac{3a^{2}B}{20}\right)\sqrt{b\,x^{3}+a}}{9}}{b}$
elliptic	$\frac{2Bb\,x^{6}\sqrt{b\,x^{3}+a}}{15}+\frac{2(b^{2}A+\frac{6}{5}abB)x^{3}\sqrt{b\,x^{3}+a}}{9b}+\frac{2\left(2abA+a^{2}B-\frac{2\left(b^{2}A+\frac{6}{5}abB\right)a}{3b}\right)\sqrt{b\,x^{3}+a}}{3b}-\frac{2a^{\frac{3}{2}}A\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}}}{\sqrt{a}}\right.}{3}$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
```

output
```
A*(2/9*x^3*b*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1/2)-2/3*a^(3/2)*arctanh((b*
x^3+a)^(1/2)/a^(1/2)))+2/15*B*(b*x^3+a)^(5/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.09

$$\int\frac{(a+bx^3)^{3/2}\,(A+Bx^3)}{x}\,dx=\left[\frac{15\,Aa^{\frac{3}{2}}b\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(3\,Bb^2x^6+(6\,Bab+5\,Ab^2)x^3+3\,B}{45\,b}\right.$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="fricas")
```

output
```
[1/45*(15*A*a^(3/2)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) +
 2*(3*B*b^2*x^6 + (6*B*a*b + 5*A*b^2)*x^3 + 3*B*a^2 + 20*A*a*b)*sqrt(b*x^3
 + a))/b, 2/45*(15*A*sqrt(-a)*a*b*arctan(sqrt(-a)/sqrt(b*x^3 + a)) + (3*B*
b^2*x^6 + (6*B*a*b + 5*A*b^2)*x^3 + 3*B*a^2 + 20*A*a*b)*sqrt(b*x^3 + a))/b
]
```

Sympy [A] (verification not implemented)

Time = 11.93 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx = \frac{\begin{cases} \frac{2Aa^2\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2Aa\sqrt{a+bx^3} + \frac{2A(a+bx^3)^{\frac{3}{2}}}{3} + \frac{2B(a+bx^3)^{\frac{5}{2}}}{5b} & \text{for } b \neq 0 \\ Aa^{\frac{3}{2}}\log\left(Ba^{\frac{3}{2}}x^3\right) + Ba^{\frac{3}{2}}x^3 & \text{otherwise} \end{cases}}{3}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x,x)
```

output
```
Piecewise((2*A*a**2*atan(sqrt(a + b*x**3)/sqrt(-a))/sqrt(-a) + 2*A*a*sqrt(
a + b*x**3) + 2*A*(a + b*x**3)**(3/2)/3 + 2*B*(a + b*x**3)**(5/2)/(5*b), N
e(b, 0)), (A*a**(3/2)*log(B*a**(3/2)*x**3) + B*a**(3/2)*x**3, True))/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx = \frac{2\,(bx^3+a)^{\frac{5}{2}}B}{15\,b} + \frac{1}{9}\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right) + 2\,(bx^3+a)^{\frac{3}{2}} + 6\,\sqrt{bx^3+a}\,a\right)A$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="maxima")
```

output
```
2/15*(b*x^3 + a)^(5/2)*B/b + 1/9*(3*a^(3/2)*log((sqrt(b*x^3 + a) - sqrt(a)
)/(sqrt(b*x^3 + a) + sqrt(a))) + 2*(b*x^3 + a)^(3/2) + 6*sqrt(b*x^3 + a)*a
)*A
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx = \frac{2\,Aa^2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}} + \frac{2\left(3\,(bx^3+a)^{\frac{5}{2}}Bb^4+5\,(bx^3+a)^{\frac{3}{2}}Ab^5+15\sqrt{bx^3+a}Aab^5\right)}{45\,b^5}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="giac")
```

output
```
2/3*A*a^2*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/45*(3*(b*x^3 + a)^
(5/2)*B*b^4 + 5*(b*x^3 + a)^(3/2)*A*b^5 + 15*sqrt(b*x^3 + a)*A*a*b^5)/b^5
```

Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx = \frac{A\,a^{3/2}\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3} + \frac{\sqrt{b\,x^3+a}\left(2\,B\,a^2+4\,A\,a\,b-\frac{2\,a\left(2\,A\,b^2+\frac{12\,B\,a\,b}{5}\right)}{3\,b}\right)}{3\,b} + \frac{2\,B\,b\,x^6\sqrt{b\,x^3+a}}{15} + \frac{x^3\left(2\,A\,b^2+\frac{12\,B\,a\,b}{5}\right)\sqrt{b\,x^3+a}}{9\,b}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x,x)
```

output
```
(A*a^(3/2)*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/
2)))/x^6))/3 + ((a + b*x^3)^(1/2)*(2*B*a^2 + 4*A*a*b - (2*a*(2*A*b^2 + (12
*B*a*b)/5))/(3*b)))/(3*b) + (2*B*b*x^6*(a + b*x^3)^(1/2))/15 + (x^3*(2*A*b
^2 + (12*B*a*b)/5)*(a + b*x^3)^(1/2))/(9*b)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx = \frac{46\sqrt{b\,x^3+a}\,a^2}{45}+\frac{22\sqrt{b\,x^3+a}\,ab\,x^3}{45}+\frac{2\sqrt{b\,x^3+a}\,b^2x^6}{15}+\frac{\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)a^2}{3}-\frac{\sqrt{a}\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)a^2}{3}$$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x,x)
```

output
```
(46*sqrt(a + b*x**3)*a**2 + 22*sqrt(a + b*x**3)*a*b*x**3 + 6*sqrt(a + b*x*
*3)*b**2*x**6 + 15*sqrt(a)*log(sqrt(a + b*x**3) - sqrt(a))*a**2 - 15*sqrt(
a)*log(sqrt(a + b*x**3) + sqrt(a))*a**2)/45
```

3.179 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4} dx = \frac{2}{3}(Ab+aB)\sqrt{a+bx^3} - \frac{aA\sqrt{a+bx^3}}{3x^3} + \frac{2}{9}B(a+bx^3)^{3/2} - \frac{1}{3}\sqrt{a}(3Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

output
```
2/3*(A*b+B*a)*(b*x^3+a)^(1/2)-1/3*a*A*(b*x^3+a)^(1/2)/x^3+2/9*B*(b*x^3+a)^
(3/2)-1/3*a^(1/2)*(3*A*b+2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4} dx = \frac{\sqrt{a+bx^3}(-3aA+6Abx^3+8aBx^3+2bBx^6)}{9x^3} - \frac{1}{3}\sqrt{a}(3Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^4,x]
```

output
```
(Sqrt[a + b*x^3]*(-3*a*A + 6*A*b*x^3 + 8*a*B*x^3 + 2*b*B*x^6))/(9*x^3) - (
Sqrt[a]*(3*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{3/2}(Bx^3+A)}{x^6}dx^3$$

$$\Big\downarrow \; 87$$

$$\frac{1}{3}\left(\frac{(2aB+3Ab)\int \frac{(bx^3+a)^{3/2}}{x^3}dx^3}{2a}-\frac{A(a+bx^3)^{5/2}}{ax^3}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3}\left(\frac{(2aB+3Ab)\left(a\int \frac{\sqrt{bx^3+a}}{x^3}dx^3+\frac{2}{3}(a+bx^3)^{3/2}\right)}{2a}-\frac{A(a+bx^3)^{5/2}}{ax^3}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3}\left(\frac{(2aB+3Ab)\left(a\left(a\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+2\sqrt{a+bx^3}\right)+\frac{2}{3}(a+bx^3)^{3/2}\right)}{2a}-\frac{A(a+bx^3)^{5/2}}{ax^3}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{(2aB+3Ab)\left(a\left(\frac{2a\int\frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{b}+2\sqrt{a+bx^3}\right)+\frac{2}{3}\left(a+bx^3\right)^{3/2}\right)}{2a}-\frac{A\left(a+bx^3\right)^{5/2}}{ax^3}\right)$$

$\Big\downarrow$ 221

$$\frac{1}{3}\left(\frac{(2aB+3Ab)\left(a\left(2\sqrt{a+bx^3}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+bx^3\right)^{3/2}\right)}{2a}-\frac{A\left(a+bx^3\right)^{5/2}}{ax^3}\right)$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^4,x]
```

output
```
(-((A*(a + b*x^3)^(5/2))/(a*x^3)) + ((3*A*b + 2*a*B)*((2*(a + b*x^3)^(3/2)
)/3 + a*(2*Sqrt[a + b*x^3] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]))
/(2*a))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{a\,x^{3} \left(Ab+\frac{2Ba}{3}\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)-\frac{2\sqrt{b\,x^{3}+a}\,\left(\left(\frac{4B\,x^{3}}{3}-\frac{A}{2}\right) a^{\frac{3}{2}}+b\,x^{3}\sqrt{a}\,\left(\frac{B\,x^{3}}{3}+A\right)\right)}{3}}{\sqrt{a}\,x^{3}}$
elliptic	$-\frac{aA\sqrt{b\,x^{3}+a}}{3x^{3}}+\frac{2Bb\,x^{3}\sqrt{b\,x^{3}+a}}{9}+\frac{2\left(b^{2}A+\frac{4}{3}abB\right)\sqrt{b\,x^{3}+a}}{3b}-\frac{2\left(\frac{3}{2}abA+a^{2}B\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$
default	$A\left(\frac{2b\sqrt{b\,x^{3}+a}}{3}-\frac{a\sqrt{b\,x^{3}+a}}{3x^{3}}-\sqrt{a}\,b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)\right)+B\left(\frac{2x^{3}b\sqrt{b\,x^{3}+a}}{9}+\frac{8a\sqrt{b\,x^{3}+a}}{9}-\frac{2a^{\frac{3}{2}}\,\mathrm{arc}}{}\right.$
risch	$-\frac{aA\sqrt{b\,x^{3}+a}}{3x^{3}}-\frac{\sqrt{a}\,(3Ab+2Ba)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3}+\frac{2Ab\sqrt{b\,x^{3}+a}}{3}+B\,b^{2}\left(\frac{2x^{3}\sqrt{b\,x^{3}+a}}{9b}-\frac{4a\sqrt{b\,x^{3}+a}}{9b^{2}}\right)+$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-(a*x^3*(A*b+2/3*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))-2/3*(b*x^3+a)^(1/2)
*((4/3*B*x^3-1/2*A)*a^(3/2)+b*x^3*a^(1/2)*(1/3*B*x^3+A)))/a^(1/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.75

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx = \left[\frac{3\,(2\,Ba+3\,Ab)\sqrt{a}x^3\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(2\,Bbx^6+2\,(4\,Ba+3\,\ldots}{18\,x^3}\right.$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="fricas")
```

output
```
[1/18*(3*(2*B*a + 3*A*b)*sqrt(a)*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a
) + 2*a)/x^3) + 2*(2*B*b*x^6 + 2*(4*B*a + 3*A*b)*x^3 - 3*A*a)*sqrt(b*x^3 +
 a))/x^3, 1/9*(3*(2*B*a + 3*A*b)*sqrt(-a)*x^3*arctan(sqrt(-a)/sqrt(b*x^3 +
 a)) + (2*B*b*x^6 + 2*(4*B*a + 3*A*b)*x^3 - 3*A*a)*sqrt(b*x^3 + a))/x^3]
```

Sympy [A] (verification not implemented)

Time = 17.34 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.35

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx = -A\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right) - \frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2Aa\sqrt{b}}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2Ab^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} - \frac{2Ba^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{2Ba^2}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2Ba\sqrt{b}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + Bb\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**4,x)
```

output
```
-A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - A*a*sqrt(b)*sqrt(a/(b*x**
3) + 1)/(3*x**(3/2)) + 2*A*a*sqrt(b)/(3*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2
*A*b**(3/2)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) - 2*B*a**(3/2)*asinh(sqrt(a)
/(sqrt(b)*x**(3/2)))/3 + 2*B*a**2/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)
) + 2*B*a*sqrt(b)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + B*b*Piecewise((sqrt(
a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx = \frac{1}{6}\left(3\sqrt{a}b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+4\sqrt{bx^3+a}b-\frac{2\sqrt{bx^3+a}a}{x^3}\right)A + \frac{1}{9}\left(3a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+2(bx^3+a)^{\frac{3}{2}}+6\sqrt{bx^3+a}a\right)B$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="maxima")
```

output
```
1/6*(3*sqrt(a)*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a
))) + 4*sqrt(b*x^3 + a)*b - 2*sqrt(b*x^3 + a)*a/x^3)*A + 1/9*(3*a^(3/2)*lo
g((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a))) + 2*(b*x^3 + a)
^(3/2) + 6*sqrt(b*x^3 + a)*a)*B
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx = \frac{1}{9}b\left(\frac{3(2Ba^2+3Aab)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}b}-\frac{3\sqrt{bx^3+a}Aa}{bx^3}+\frac{2\left((bx^3+a)^{\frac{3}{2}}\right.}{}\right.$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="giac")
```

output
```
1/9*b*(3*(2*B*a^2 + 3*A*a*b)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*b)
 - 3*sqrt(b*x^3 + a)*A*a/(b*x^3) + 2*((b*x^3 + a)^(3/2)*B*b^2 + 3*sqrt(b*x
^3 + a)*B*a*b^2 + 3*sqrt(b*x^3 + a)*A*b^3)/b^3)
```

Mupad [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx = \frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)(3\,A\,b+2\,B\,a)\,\sqrt{\frac{a}{4}}}{3} + \frac{\left(2\,A\,b^2+\frac{8\,B\,a\,b}{3}\right)\sqrt{b\,x^3+a}}{3\,b} - \frac{A\,a\,\sqrt{b\,x^3+a}}{3\,x^3} + \frac{2\,B\,b\,x^3\,\sqrt{b\,x^3+a}}{9}$$

input

```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^4,x)
```

output

```
(log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*
(3*A*b + 2*B*a)*(a/4)^(1/2))/3 + ((2*A*b^2 + (8*B*a*b)/3)*(a + b*x^3)^(1/2
))/(3*b) - (A*a*(a + b*x^3)^(1/2))/(3*x^3) + (2*B*b*x^3*(a + b*x^3)^(1/2))
/9
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx = \frac{-6\sqrt{b\,x^3+a}\,a^2+28\sqrt{b\,x^3+a}\,ab\,x^3+4\sqrt{b\,x^3+a}\,b^2x^6+15\sqrt{a}\log\left(\sqrt{b\,x^3}\right.}{18x^3}$$

input

```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x)
```

output

```
( - 6*sqrt(a + b*x**3)*a**2 + 28*sqrt(a + b*x**3)*a*b*x**3 + 4*sqrt(a + b*
x**3)*b**2*x**6 + 15*sqrt(a)*log(sqrt(a + b*x**3) - sqrt(a))*a*b*x**3 - 15
*sqrt(a)*log(sqrt(a + b*x**3) + sqrt(a))*a*b*x**3)/(18*x**3)
```

3.180 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7} dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7} dx = \frac{2}{3}bB\sqrt{a+bx^3} - \frac{(3Ab+4aB)\sqrt{a+bx^3}}{12x^3} - \frac{A(a+bx^3)^{3/2}}{6x^6} - \frac{b(Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

output
```
2/3*b*B*(b*x^3+a)^(1/2)-1/12*(3*A*b+4*B*a)*(b*x^3+a)^(1/2)/x^3-1/6*A*(b*x^
3+a)^(3/2)/x^6-1/4*b*(A*b+4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7} dx = \frac{\sqrt{a+bx^3}(-2aA-5Abx^3-4aBx^3+8bBx^6)}{12x^6} - \frac{b(Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^7,x]
```

output
```
(Sqrt[a + b*x^3]*(-2*a*A - 5*A*b*x^3 - 4*a*B*x^3 + 8*b*B*x^6))/(12*x^6) -
(b*(A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{3/2}(Bx^3+A)}{x^9}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{(4aB+Ab)\int \frac{(bx^3+a)^{3/2}}{x^6}dx^3}{4a}-\frac{A(a+bx^3)^{5/2}}{2ax^6}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}\left(\frac{(4aB+Ab)\left(\frac{3}{2}b\int \frac{\sqrt{bx^3+a}}{x^3}dx^3-\frac{(a+bx^3)^{3/2}}{x^3}\right)}{4a}-\frac{A(a+bx^3)^{5/2}}{2ax^6}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{(4aB+Ab)\left(\frac{3}{2}b\left(a\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+2\sqrt{a+bx^3}\right)-\frac{(a+bx^3)^{3/2}}{x^3}\right)}{4a}-\frac{A(a+bx^3)^{5/2}}{2ax^6}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{(4aB+Ab)\left(\frac{3}{2}b\left(\frac{2a\int\frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{b}+2\sqrt{a+bx^3}\right)-\frac{(a+bx^3)^{3/2}}{x^3}\right)}{4a}-\frac{A(a+bx^3)^{5/2}}{2ax^6}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{(4aB+Ab)\left(\frac{3}{2}b\left(2\sqrt{a+bx^3}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)-\frac{(a+bx^3)^{3/2}}{x^3}\right)}{4a}-\frac{A(a+bx^3)^{5/2}}{2ax^6}\right)$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^7,x]
```

output
```
(-1/2*(A*(a + b*x^3)^(5/2))/(a*x^6) + ((A*b + 4*a*B)*(-((a + b*x^3)^(3/2)/
x^3) + (3*b*(2*Sqrt[a + b*x^3] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]
]))/2))/(4*a))/3
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$-\frac{b\,x^6(Ab+4Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)+\frac{5\sqrt{b\,x^3+a}\left(\frac{2\left(2B\,x^3+A\right)a^{\frac{3}{2}}}{5}+b\,x^3\sqrt{a}\left(-\frac{8B\,x^3}{5}+A\right)\right)}{3}}{4\sqrt{a}\,x^6}$
risch	$-\frac{\sqrt{b\,x^3+a}\,(5Ab\,x^3+4Ba\,x^3+2Aa)}{12x^6}+\frac{b\left(-\frac{2(3Ab+12Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{16B\sqrt{b\,x^3+a}}{3}\right)}{8}$
elliptic	$-\frac{Aa\sqrt{b\,x^3+a}}{6x^6}-\frac{\left(\frac{5Ab}{4}+Ba\right)\sqrt{b\,x^3+a}}{3x^3}+\frac{2bB\sqrt{b\,x^3+a}}{3}-\frac{2\left(\frac{3}{8}b^2A+\frac{3}{2}abB\right)\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$
default	$A\left(-\frac{a\sqrt{b\,x^3+a}}{6x^6}-\frac{5b\sqrt{b\,x^3+a}}{12x^3}-\frac{b^2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{4\sqrt{a}}\right)+B\left(\frac{2b\sqrt{b\,x^3+a}}{3}-\frac{a\sqrt{b\,x^3+a}}{3x^3}-\sqrt{a}\,b\ \mathrm{arctanh}\right.$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x^6*(A*b+4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))+5/3*(b*x^3+a)^(1/
2)*(2/5*(2*B*x^3+A)*a^(3/2)+b*x^3*a^(1/2)*(-8/5*B*x^3+A)))/a^(1/2)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.88

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx = \left[\frac{3\,(4\,Bab+Ab^2)\sqrt{a}x^6\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(8\,Babx^6-(4\,Ba^2+5\right.}{24\,ax^6}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="fricas")
```

output
```
[1/24*(3*(4*B*a*b + A*b^2)*sqrt(a)*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt
(a) + 2*a)/x^3) + 2*(8*B*a*b*x^6 - (4*B*a^2 + 5*A*a*b)*x^3 - 2*A*a^2)*sqrt
(b*x^3 + a))/(a*x^6), 1/12*(3*(4*B*a*b + A*b^2)*sqrt(-a)*x^6*arctan(sqrt(-
a)/sqrt(b*x^3 + a)) + (8*B*a*b*x^6 - (4*B*a^2 + 5*A*a*b)*x^3 - 2*A*a^2)*sq
rt(b*x^3 + a))/(a*x^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(90) = 180$.

Time = 46.20 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.43

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx = -\frac{Aa^2}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{Aa\sqrt{b}}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{Ab^{\frac{3}{2}}}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4\sqrt{a}} - B\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right) - \frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2Ba\sqrt{b}}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2Bb^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**7,x)
```

output
```
-A*a**2/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - A*a*sqrt(b)/(4*x**(9/
2)*sqrt(a/(b*x**3) + 1)) - A*b**(3/2)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) -
A*b**(3/2)/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - A*b**2*asinh(sqrt(a)/(sqrt
(b)*x**(3/2)))/(4*sqrt(a)) - B*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))
 - B*a*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*B*a*sqrt(b)/(3*x**(3/
2)*sqrt(a/(b*x**3) + 1)) + 2*B*b**(3/2)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(80) = 160$.

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.71

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx = \frac{1}{24}\left(\frac{3\,b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\left(5\,(bx^3+a)^{\frac{3}{2}}b^2 - 3\,\sqrt{bx^3+a}ab^2\right)}{(bx^3+a)^2 - 2\,(bx^3+a)a + a^2}\right)A + \frac{1}{6}\left(3\,\sqrt{a}b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right) + 4\,\sqrt{bx^3+a}b - \frac{2\,\sqrt{bx^3+a}a}{x^3}\right)B$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="maxima")
```

output
```
1/24*(3*b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/s
qrt(a) - 2*(5*(b*x^3 + a)^(3/2)*b^2 - 3*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a
)^2 - 2*(b*x^3 + a)*a + a^2))*A + 1/6*(3*sqrt(a)*b*log((sqrt(b*x^3 + a) -
sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a))) + 4*sqrt(b*x^3 + a)*b - 2*sqrt(b*x^3
 + a)*a/x^3)*B
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx = \frac{8\sqrt{bx^3+a}Bb^2 + \frac{3(4Bab^2+Ab^3)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{4(bx^3+a)^{\frac{3}{2}}Bab^2-4\sqrt{bx^3+a}Ba^2b^2+}{b^2x^6}}{12b}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="giac")
```

output
```
1/12*(8*sqrt(b*x^3 + a)*B*b^2 + 3*(4*B*a*b^2 + A*b^3)*arctan(sqrt(b*x^3 +
a)/sqrt(-a))/sqrt(-a) - (4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B
*a^2*b^2 + 5*(b*x^3 + a)^(3/2)*A*b^3 - 3*sqrt(b*x^3 + a)*A*a*b^3)/(b^2*x^6
))/b
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx = \frac{2\,B\,b\,\sqrt{b\,x^3+a}}{3} - \frac{\sqrt{b\,x^3+a}\,(4\,B\,a^3+5\,A\,b\,a^2)}{12\,a^2\,x^3} - \frac{A\,a\,\sqrt{b\,x^3+a}}{6\,x^6} + \frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)(A\,b+4\,B\,a)}{8\,\sqrt{a}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^7,x)
```

output
```
(2*B*b*(a + b*x^3)^(1/2))/3 - ((a + b*x^3)^(1/2)*(4*B*a^3 + 5*A*a^2*b))/(1
2*a^2*x^3) - (A*a*(a + b*x^3)^(1/2))/(6*x^6) + (b*log((((a + b*x^3)^(1/2)
- a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*(A*b + 4*B*a))/(8*a^(1/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx = \frac{-4\sqrt{b\,x^3+a}\,a^2-18\sqrt{b\,x^3+a}\,ab\,x^3+16\sqrt{b\,x^3+a}\,b^2x^6+15\sqrt{a}\log\left(\sqrt{b\,x^3}\right.}{24x^6}$$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x)
```

output
```
( - 4*sqrt(a + b*x**3)*a**2 - 18*sqrt(a + b*x**3)*a*b*x**3 + 16*sqrt(a + b
*x**3)*b**2*x**6 + 15*sqrt(a)*log(sqrt(a + b*x**3) - sqrt(a))*b**2*x**6 -
15*sqrt(a)*log(sqrt(a + b*x**3) + sqrt(a))*b**2*x**6)/(24*x**6)
```

3.181 $\int x^3(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 336

$$\int x^3\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{54a^2(23Ab-8aB)x\sqrt{a+bx^3}}{21505b^2}+\frac{18a(23Ab-8aB)x^4\sqrt{a+bx^3}}{4301b}+\frac{2(23Ab-8aB)x^4\left(a+bx^3\right)^{3/2}}{391b}+\frac{2Bx^4\left(a+bx^3\right)^{5/2}}{23b}-\frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}a^3(23Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}\right.\right.}{21505b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
54/21505*a^2*(23*A*b-8*B*a)*x*(b*x^3+a)^(1/2)/b^2+18/4301*a*(23*A*b-8*B*a)
*x^4*(b*x^3+a)^(1/2)/b+2/391*(23*A*b-8*B*a)*x^4*(b*x^3+a)^(3/2)/b+2/23*B*x
^4*(b*x^3+a)^(5/2)/b-36/21505*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^3*(23*A*
b-8*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+
3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3
)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(7/3)/(a^(1/3)*(a^(1
/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.80 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.28

$$\int x^3(a+bx^3)^{3/2}(A+Bx^3)\,dx=\frac{2x\sqrt{a+bx^3}\left(-(a+bx^3)^2(-23Ab+8aB-17bBx^3)+\frac{a^2(-23Ab+8aB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},-\frac{b}{}\right.}{\sqrt{1+\frac{bx^3}{a}}}\right.}{391b^2}$$

input
```
Integrate[x^3*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*(-23*A*b + 8*a*B - 17*b*B*x^3)) + (a
^2*(-23*A*b + 8*a*B)*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt
[1 + (b*x^3)/a]))/(391*b^2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {959, 811, 811, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx^3)^{3/2}(A+Bx^3)\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(23Ab-8aB)\int x^3(bx^3+a)^{3/2}dx}{23b}+\frac{2Bx^4(a+bx^3)^{5/2}}{23b}$$

$$\Big\downarrow \text{811}$$

$$\frac{(23Ab-8aB)\left(\frac{9}{17}a\int x^3\sqrt{bx^3+a}dx+\frac{2}{17}x^4(a+bx^3)^{3/2}\right)}{23b}+\frac{2Bx^4(a+bx^3)^{5/2}}{23b}$$

$$\Big\downarrow \text{811}$$

$$\frac{(23Ab-8aB)\left(\frac{9}{17}a\left(\frac{3}{11}a\int \frac{x^3}{\sqrt{bx^3+a}}dx+\frac{2}{11}x^4\sqrt{a+bx^3}\right)+\frac{2}{17}x^4\left(a+bx^3\right)^{3/2}\right)}{23b}+\frac{2Bx^4\left(a+bx^3\right)^{5/2}}{23b}$$

$$\Big\downarrow \; 843$$

$$\frac{(23Ab-8aB)\left(\frac{9}{17}a\left(\frac{3}{11}a\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{2a\int \frac{1}{\sqrt{bx^3+a}}dx}{5b}\right)+\frac{2}{11}x^4\sqrt{a+bx^3}\right)+\frac{2}{17}x^4\left(a+bx^3\right)^{3/2}\right)}{23b}+\frac{2Bx^4\left(a+bx^3\right)^{5/2}}{23b}$$

$$\Big\downarrow \; 759$$

$$\frac{(23Ab-8aB)\left(\frac{9}{17}a\left(\frac{3}{11}a\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.\right.}{23b}$$

$$\frac{2Bx^4\left(a+bx^3\right)^{5/2}}{23b}$$

input
```
Int[x^3*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*B*x^4*(a + b*x^3)^(5/2))/(23*b) + ((23*A*b - 8*a*B)*((2*x^4*(a + b*x^3)
^(3/2))/17 + (9*a*((2*x^4*Sqrt[a + b*x^3]))/11 + (3*a*((2*x*Sqrt[a + b*x^3]
)/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/11))/
17))/(23*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.11

method	result
risch	$\frac{2x\left(935b^{3}B\,x^{9}+1265A\,b^{3}x^{6}+1430Ba\,b^{2}x^{6}+2300aA\,b^{2}x^{3}+135B\,a^{2}b\,x^{3}+621a^{2}bA-216a^{3}B\right)\sqrt{b\,x^{3}+a}}{21505b^{2}}+\frac{36ia^{3}(23Ab-8Ba)\sqrt{3}\,(-$
elliptic	$\frac{2Bb\,x^{10}\sqrt{b\,x^{3}+a}}{23}+\frac{2\left(b^{2}A+\frac{26}{23}abB\right)x^{7}\sqrt{b\,x^{3}+a}}{17b}+\frac{2\left(2abA+a^{2}B-\frac{14a\left(b^{2}A+\frac{26}{23}abB\right)}{17b}\right)x^{4}\sqrt{b\,x^{3}+a}}{11b}+\frac{2\left(a^{2}A-\frac{8a\left(2abA+a^{2}B-\right.}{}\right.}{}$
default	$A\left(\frac{2x^{7}\sqrt{b\,x^{3}+a}\,b}{17}+\frac{40a\,x^{4}\sqrt{b\,x^{3}+a}}{187}+\frac{54a^{2}x\sqrt{b\,x^{3}+a}}{935b}+\frac{36ia^{3}\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{}{-\frac{3(-}{}}}}{}\right.$

input
```
int(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/21505/b^2*x*(935*B*b^3*x^9+1265*A*b^3*x^6+1430*B*a*b^2*x^6+2300*A*a*b^2*
x^3+135*B*a^2*b*x^3+621*A*a^2*b-216*B*a^3)*(b*x^3+a)^(1/2)+36/21505*I*a^3*
(23*A*b-8*B*a)/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2
)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I
*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.34

$$\int x^3(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2\left(54\,(8\,Ba^4-23\,Aa^3b)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(935\,Bb^4x^{10}+55\,(26\,Bab^3+23\right.}{21505\,b^3}$$

input
```
integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/21505*(54*(8*B*a^4 - 23*A*a^3*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b,
x) + (935*B*b^4*x^10 + 55*(26*B*a*b^3 + 23*A*b^4)*x^7 + 5*(27*B*a^2*b^2 +
460*A*a*b^3)*x^4 - 27*(8*B*a^3*b - 23*A*a^2*b^2)*x)*sqrt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.51

$$\int x^3\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{Aa^{\frac{3}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{A\sqrt{a}bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{Ba^{\frac{3}{2}}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{B\sqrt{a}bx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{10}{3}\\\frac{13}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate(x**3*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
A*a**(3/2)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(7/3)) + A*sqrt(a)*b*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + B*a**(3/2)*x**7*gamma(7/3
)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) +
B*sqrt(a)*b*x**10*gamma(10/3)*hyper((-1/2, 10/3), (13/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*gamma(13/3))
```

Maxima [F]

$$\int x^3\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^3, x)
```

Giac [F]

$$\int x^3 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right) \left(bx^3+a\right)^{\frac{3}{2}} x^3 \, dx$$

input
```
integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int x^3 \left(B\,x^3+A\right) \left(b\,x^3+a\right)^{3/2} \,dx$$

input
```
int(x^3*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

output
```
int(x^3*(A + B*x^3)*(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int x^3 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{\frac{162\sqrt{b\,x^3+a}\,a^3x}{4301}+\frac{974\sqrt{b\,x^3+a}\,a^2b\,x^4}{4301}+\frac{98\sqrt{b\,x^3+a}\,a\,b^2x^7}{391}+\frac{2\sqrt{b\,x^3+a}\,b^3x^{10}}{23}-\frac{162\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a^4}{4301}}{b}$$

input
```
int(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

output
```
(2*(81*sqrt(a + b*x**3)*a**3*x + 487*sqrt(a + b*x**3)*a**2*b*x**4 + 539*sq
rt(a + b*x**3)*a*b**2*x**7 + 187*sqrt(a + b*x**3)*b**3*x**10 - 81*int(sqrt
(a + b*x**3)/(a + b*x**3),x)*a**4))/(4301*b)
```

3.182 $\int (a+bx^3)^{3/2} (A+Bx^3)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 299

$$\int (a+bx^3)^{3/2} (A+Bx^3)\, dx = \frac{18a(17Ab-2aB)x\sqrt{a+bx^3}}{935b} + \frac{2(17Ab-2aB)x(a+bx^3)^{3/2}}{187b} + \frac{2Bx(a+bx^3)^{5/2}}{17b} + \frac{18\, 3^{3/4}\sqrt{2+\sqrt{3}}a^2(17Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\ldots}\right.\right.}{935b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
18/935*a*(17*A*b-2*B*a)*x*(b*x^3+a)^(1/2)/b+2/187*(17*A*b-2*B*a)*x*(b*x^3+
a)^(3/2)/b+2/17*B*x*(b*x^3+a)^(5/2)/b+18/935*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1
/2))*a^2*(17*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))
*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(4/3)
/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*
x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.73 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.26

$$\int (a+bx^3)^{3/2} (A+Bx^3) \, dx = \frac{2x\sqrt{a+bx^3}\left(B(a+bx^3)^2 - \frac{a\left(-\frac{17Ab}{2}+aB\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{17b}$$

input
```
Integrate[(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(B*(a + b*x^3)^2 - (a*((-17*A*b)/2 + a*B)*Hypergeomet
ric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(17*b)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {913, 748, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^3)^{3/2} (A+Bx^3) \, dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(17Ab-2aB)\int (bx^3+a)^{3/2}dx}{17b} + \frac{2Bx(a+bx^3)^{5/2}}{17b}$$

$$\Big\downarrow \text{748}$$

$$\frac{(17Ab-2aB)\left(\frac{9}{11}a\int \sqrt{bx^3+a}dx + \frac{2}{11}x(a+bx^3)^{3/2}\right)}{17b} + \frac{2Bx(a+bx^3)^{5/2}}{17b}$$

$$\Big\downarrow \text{748}$$

$$\frac{(17Ab-2aB)\left(\frac{9}{11}a\left(\frac{3}{5}a\int\frac{1}{\sqrt{bx^3+a}}dx+\frac{2}{5}x\sqrt{a+bx^3}\right)+\frac{2}{11}x\left(a+bx^3\right)^{3/2}\right)}{17b}+\frac{2Bx\left(a+bx^3\right)^{5/2}}{17b}$$

$\big\downarrow$ 759

$$\frac{(17Ab-2aB)\left(\frac{9}{11}a\left(\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\cdots\right.\right.}{17b}$$

$$\frac{2Bx\left(a+bx^3\right)^{5/2}}{17b}$$

input
```
Int[(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*B*x*(a + b*x^3)^(5/2))/(17*b) + ((17*A*b - 2*a*B)*((2*x*(a + b*x^3)^(3/
2))/11 + (9*a*((2*x*Sqrt[a + b*x^3])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3])))/11))/(17*b)
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
 + 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.17

method	result
risch	$\frac{2x\left(55b^2Bx^6+85Ab^2x^3+100Babx^3+238abA+27a^2B\right)\sqrt{bx^3+a}}{935b}-\frac{18ia^2(17Ab-2Ba)\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-ab^2\ldots\right.}{2b}\ldots\right)}{\left(-ab^2\right)^{\frac{1}{3}}}}\ldots}{\ldots}$
elliptic	$\frac{2Bbx^7\sqrt{bx^3+a}}{17}+\frac{2\left(b^2A+\frac{20}{17}abB\right)x^4\sqrt{bx^3+a}}{11b}+\frac{2\left(2abA+a^2B-\frac{8a\left(b^2A+\frac{20}{17}abB\right)}{11b}\right)x\sqrt{bx^3+a}}{5b}-\frac{2i\left(a^2A-\frac{2a\left(2abA+a^2B-\frac{8a\ldots}{\ldots}\right)}{5b\ldots}\ldots\right)\ldots}{\ldots}$
default	$A\left(\frac{2bx^4\sqrt{bx^3+a}}{11}+\frac{28ax\sqrt{bx^3+a}}{55}-\frac{18ia^2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}b}{\left(-ab^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\ldots}{2b}}}\ldots}{\ldots}\right.$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/935/b*x*(55*B*b^2*x^6+85*A*b^2*x^3+100*B*a*b*x^3+238*A*a*b+27*B*a^2)*(b*
x^3+a)^(1/2)-18/935*I*a^2*(17*A*b-2*B*a)/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.30

$$\int \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\,dx = -\frac{2\left(27\left(2\,Ba^3-17\,Aa^2b\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\left(55\,Bb^3x^7+5\left(20\,Bab^2+17\,Ab^3\right)x^4+\left(27\,\right.\right.\right.}{935\,b^2}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
-2/935*(27*(2*B*a^3 - 17*A*a^2*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x
) - (55*B*b^3*x^7 + 5*(20*B*a*b^2 + 17*A*b^3)*x^4 + (27*B*a^2*b + 238*A*a*
b^2)*x)*sqrt(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.57

$$\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \frac{Aa^{\frac{3}{2}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{A\sqrt{a}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{Ba^{\frac{3}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{B\sqrt{a}bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
A*a**(3/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + A*sqrt(a)*b*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + B*a**(3/2)*x**4*gamma(4/3)*hyp
er((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + B*sqrt(
a)*b*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(10/3))
```

Maxima [F]

$$\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{3/2}\,dx$$

input
```
int((A + B*x^3)*(a + b*x^3)^(3/2),x)
```

output
```
int((A + B*x^3)*(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{106\sqrt{b\,x^3+a}\,a^2x}{187}+\frac{74\sqrt{b\,x^3+a}\,ab\,x^4}{187}+\frac{2\sqrt{b\,x^3+a}\,b^2x^7}{17}+\frac{81\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a^3}{187}$$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A),x)
```

output
```
(106*sqrt(a + b*x**3)*a**2*x + 74*sqrt(a + b*x**3)*a*b*x**4 + 22*sqrt(a +
b*x**3)*b**2*x**7 + 81*int(sqrt(a + b*x**3)/(a + b*x**3),x)*a**3)/187
```

3.183 $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 295

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^3}\,dx = \frac{9}{110}(11Ab+4aB)x\sqrt{a+bx^3} + \frac{(11Ab+4aB)x(a+bx^3)^{3/2}}{22a} - \frac{A(a+bx^3)^{5/2}}{2ax^2} + \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}a(11Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{110\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
9/110*(11*A*b+4*B*a)*x*(b*x^3+a)^(1/2)+1/22*(11*A*b+4*B*a)*x*(b*x^3+a)^(3/
2)/a-1/2*A*(b*x^3+a)^(5/2)/a/x^2+9/110*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a
*(11*A*b+4*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)
+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(1/3)/(a^(1/3
)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(
1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.00 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.28

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = -\frac{A(a+bx^3)^{5/2}}{2ax^2} - \frac{\left(-\frac{11Ab}{2}-2aB\right)x\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^3,x]
```

output
```
-1/2*(A*(a + b*x^3)^(5/2))/(a*x^2) - (((-11*A*b)/2 - 2*a*B)*x*Sqrt[a + b*x
^3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a
])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 748, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(4aB+11Ab)\int (bx^3+a)^{3/2}\,dx}{4a} - \frac{A(a+bx^3)^{5/2}}{2ax^2}$$

$$\Big\downarrow \text{748}$$

$$\frac{(4aB+11Ab)\left(\frac{9}{11}a\int \sqrt{bx^3+a}dx+\frac{2}{11}x(a+bx^3)^{3/2}\right)}{4a} - \frac{A(a+bx^3)^{5/2}}{2ax^2}$$

$$\Big\downarrow \text{748}$$

$$\frac{(4aB+11Ab)\left(\frac{9}{11}a\left(\frac{3}{5}a\int\frac{1}{\sqrt{bx^3+a}}dx+\frac{2}{5}x\sqrt{a+bx^3}\right)+\frac{2}{11}x\left(a+bx^3\right)^{3/2}\right)}{4a}-\frac{A\left(a+bx^3\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow \text{759}$$

$$\frac{(4aB+11Ab)\left(\frac{9}{11}a\left(\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\right.\right.}{4a}$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{2ax^2}$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^3,x]
```

output
```
-1/2*(A*(a + b*x^3)^(5/2))/(a*x^2) + ((11*A*b + 4*a*B)*((2*x*(a + b*x^3)^(
3/2))/11 + (9*a*((2*x*Sqrt[a + b*x^3])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*b
^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3])))/11))/(4*a)
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
 + 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-20bB\,x^{6}-44Ab\,x^{3}-56Ba\,x^{3}+55Aa\right)}{110x^{2}}-\frac{9ia(11Ab+4Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\cdots}}{\cdots}$
elliptic	$-\frac{Aa\sqrt{b\,x^{3}+a}}{2x^{2}}+\frac{2Bb\,x^{4}\sqrt{b\,x^{3}+a}}{11}+\frac{2(b^{2}A+\frac{14}{11}abB)x\sqrt{b\,x^{3}+a}}{5b}-\frac{2i\left(\frac{7abA}{4}+a^{2}B-\frac{2a\left(b^{2}A+\frac{14}{11}abB\right)}{5b}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-\cdots\right.}{\cdots}\right.}{\cdots}}}{\cdots}$
default	$B\left(\frac{2b\,x^{4}\sqrt{b\,x^{3}+a}}{11}+\frac{28ax\sqrt{b\,x^{3}+a}}{55}-\frac{18ia^{2}\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\cdots}}{2b}}}\cdots}{\cdots}\right.$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/110*(b*x^3+a)^(1/2)*(-20*B*b*x^6-44*A*b*x^3-56*B*a*x^3+55*A*a)/x^2-9/11
0*I*a*(11*A*b+4*B*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),
(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.27

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = \frac{27\,(4\,Ba^2+11\,Aab)\sqrt{b}x^2\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(20\,Bb^2x^6+4}{110\,bx^2}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="fricas")
```

output
```
1/110*(27*(4*B*a^2 + 11*A*a*b)*sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b,
x) + (20*B*b^2*x^6 + 4*(14*B*a*b + 11*A*b^2)*x^3 - 55*A*a*b)*sqrt(b*x^3 +
a))/(b*x^2)
```

Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{A\sqrt{a}bx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{Ba^{\frac{3}{2}}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{B\sqrt{a}bx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**3,x)
```

output
```
A*a**(3/2)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**2*gamma(1/3)) + A*sqrt(a)*b*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + B*a**(3/2)*x*gamma(1/3)*hype
r((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + B*sqrt(a
)*b*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(7/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx=\int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^3}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^3,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx=\frac{-46\sqrt{b\,x^3+a}\,a^2+10\sqrt{b\,x^3+a}\,ab\,x^3+2\sqrt{b\,x^3+a}\,b^2x^6-81\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^6+a\,x^3}dx\right.}{11x^2}$$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x)
```

output
```
( - 46*sqrt(a + b*x**3)*a**2 + 10*sqrt(a + b*x**3)*a*b*x**3 + 2*sqrt(a + b
*x**3)*b**2*x**6 - 81*int(sqrt(a + b*x**3)/(a*x**3 + b*x**6),x)*a**3*x**2)
/(11*x**2)
```

3.184 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 294

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx = -\frac{(Ab+2aB)\sqrt{a+bx^3}}{4x^2} + \frac{b(Ab+2aB)x\sqrt{a+bx^3}}{5a} - \frac{A(a+bx^3)^{5/2}}{5ax^5} + \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{20\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/4*(A*b+2*B*a)*(b*x^3+a)^(1/2)/x^2+1/5*b*(A*b+2*B*a)*x*(b*x^3+a)^(1/2)/a
-1/5*A*(b*x^3+a)^(5/2)/a/x^5+9/20*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*b^(2/3
)*(A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b
^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/(a^(1/3)*(a^(1/3)
+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.28

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx = \frac{\sqrt{a+bx^3}\left(-\frac{2A(a+bx^3)^2}{a}-\frac{5(Ab+2aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{2}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{10x^5}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^6,x]
```

output
```
(Sqrt[a + b*x^3]*((-2*A*(a + b*x^3)^2)/a - (5*(A*b + 2*a*B)*x^3*Hypergeome
tric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(10*x^5)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 809, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(2aB+Ab)\int \frac{(bx^3+a)^{3/2}}{x^3}dx}{2a}-\frac{A(a+bx^3)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{809}$$

$$\frac{(2aB+Ab)\left(\frac{9}{4}b\int \sqrt{bx^3+a}dx-\frac{(a+bx^3)^{3/2}}{2x^2}\right)}{2a}-\frac{A(a+bx^3)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{748}$$

$$\frac{(2aB+Ab)\left(\frac{9}{4}b\left(\frac{3}{5}a\int\frac{1}{\sqrt{bx^3+a}}dx+\frac{2}{5}x\sqrt{a+bx^3}\right)-\frac{(a+bx^3)^{3/2}}{2x^2}\right)}{2a}-\frac{A(a+bx^3)^{5/2}}{5ax^5}$$

$\downarrow$ 759

$$\frac{(2aB+Ab)\left(\frac{9}{4}b\left(\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2}{5}x\right.\right.}{2a}$$

$$\frac{A(a+bx^3)^{5/2}}{5ax^5}$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^6,x]
```

output
```
-1/5*(A*(a + b*x^3)^(5/2))/(a*x^5) + ((A*b + 2*a*B)*(-1/2*(a + b*x^3)^(3/2
)/x^2 + (9*b*((2*x*Sqrt[a + b*x^3])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(a^
(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*b^(1
/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])))/4))/(2*a)
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-8bB\,x^{6}+13Ab\,x^{3}+10Ba\,x^{3}+4Aa\right)}{20x^{5}}-\frac{9i(Ab+2Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\cdots}}\cdots}{\cdots}$
elliptic	$-\frac{Aa\sqrt{b\,x^{3}+a}}{5x^{5}}-\frac{\left(\frac{13Ab}{10}+Ba\right)\sqrt{b\,x^{3}+a}}{2x^{2}}+\frac{2Bbx\sqrt{b\,x^{3}+a}}{5}-\frac{2i\left(b^{2}A+\frac{8abB}{5}-\frac{b(13Ab+10Ba)}{40}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\cdots\right)\cdots}{\left(-a\cdots\right)}}\cdots}{\cdots}$
default	$A\left(-\frac{a\sqrt{b\,x^{3}+a}}{5x^{5}}-\frac{13b\sqrt{b\,x^{3}+a}}{20x^{2}}-\frac{9ib\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{\cdots}}{\cdots}\cdots\right.$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/20*(b*x^3+a)^(1/2)*(-8*B*b*x^6+13*A*b*x^3+10*B*a*x^3+4*A*a)/x^5-9/20*I*
(A*b+2*B*a)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.23

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx = \frac{27\,(2\,Ba+Ab)\sqrt{b}x^5\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(8\,Bbx^6-(10\,Ba+\right.}{20\,x^5}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="fricas")
```

output
```
1/20*(27*(2*B*a + A*b)*sqrt(b)*x^5*weierstrassPInverse(0, -4*a/b, x) + (8*
B*b*x^6 - (10*B*a + 13*A*b)*x^3 - 4*A*a)*sqrt(b*x^3 + a))/x^5
```

Sympy [A] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{3}, -\frac{1}{2} \\ -\frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{A\sqrt{a}b\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{B\sqrt{a}bx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**6,x)
```

output
```
A*a**(3/2)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**5*gamma(-2/3)) + A*sqrt(a)*b*gamma(-2/3)*hyper((-2/3, -1/2), (1/
3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + B*a**(3/2)*gamma(-2/3
)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)
) + B*sqrt(a)*b*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I
*pi)/a)/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^6}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^6,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^6, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx = \frac{74\sqrt{b\,x^3+a}\,a^2 - 182\sqrt{b\,x^3+a}\,ab\,x^3 + 14\sqrt{b\,x^3+a}\,b^2x^6 + 405\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^9+a\,x^6}$$
$$35x^5$$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x)
```

output
```
(74*sqrt(a + b*x**3)*a**2 - 182*sqrt(a + b*x**3)*a*b*x**3 + 14*sqrt(a + b*
x**3)*b**2*x**6 + 405*int(sqrt(a + b*x**3)/(a*x**6 + b*x**9),x)*a**3*x**5)
/(35*x**5)
```

3.185 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 299

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx = \frac{(Ab-16aB)\sqrt{a+bx^3}}{80x^5} + \frac{13b(Ab-16aB)\sqrt{a+bx^3}}{320ax^2} - \frac{A(a+bx^3)^{5/2}}{8ax^8} - \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}(Ab-16aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{320a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/80*(A*b-16*B*a)*(b*x^3+a)^(1/2)/x^5+13/320*b*(A*b-16*B*a)*(b*x^3+a)^(1/2
)/a/x^2-1/8*A*(b*x^3+a)^(5/2)/a/x^8-9/320*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2)
)*b^(5/3)*(A*b-16*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*
a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/a/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^
(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.27

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx=\frac{\sqrt{a+bx^3}\left(-\frac{5A(a+bx^3)^2}{a}+\frac{\left(\frac{Ab}{2}-8aB\right)x^3\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{3}{2},-\frac{2}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{40x^8}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^9,x]
```

output
```
(Sqrt[a + b*x^3]*((-5*A*(a + b*x^3)^2)/a + (((A*b)/2 - 8*a*B)*x^3*Hypergeo
metric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(40*x^8)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 809, 809, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(Ab-16aB)\int \frac{(bx^3+a)^{3/2}}{x^6}dx}{16a}-\frac{A(a+bx^3)^{5/2}}{8ax^8}$$

$$\Big\downarrow \text{809}$$

$$-\frac{(Ab-16aB)\left(\frac{9}{10}b\int \frac{\sqrt{bx^3+a}}{x^3}dx-\frac{(a+bx^3)^{3/2}}{5x^5}\right)}{16a}-\frac{A(a+bx^3)^{5/2}}{8ax^8}$$

$$\Big\downarrow \text{809}$$

$$-\frac{(Ab-16aB)\left(\frac{9}{10}b\left(\frac{3}{4}b\int\frac{1}{\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{2x^2}\right)-\frac{(a+bx^3)^{3/2}}{5x^5}\right)}{16a}-\frac{A(a+bx^3)^{5/2}}{8ax^8}$$

$$\Big\downarrow \text{ 759}$$

$$-\frac{(Ab-16aB)\left(\frac{9}{10}b\left(\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\right.\right.}{16a}$$

$$\frac{A(a+bx^3)^{5/2}}{8ax^8}$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^9,x]
```

output
```
-1/8*(A*(a + b*x^3)^(5/2))/(a*x^8) - ((A*b - 16*a*B)*(-1/5*(a + b*x^3)^(3/
2)/x^5 + (9*b*(-1/2*Sqrt[a + b*x^3]/x^2 + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/
3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3])))/10))/(16*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(27A\,b^{2}x^{6}+208Bab\,x^{6}+76aAb\,x^{3}+64B\,a^{2}x^{3}+40a^{2}A\right)}{320x^{8}a}+\frac{9ib(Ab-16Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right.}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$
elliptic	$-\frac{Aa\sqrt{b\,x^{3}+a}}{8x^{8}}-\frac{\left(\frac{19Ab}{16}+Ba\right)\sqrt{b\,x^{3}+a}}{5x^{5}}-\frac{b(27Ab+208Ba)\sqrt{b\,x^{3}+a}}{320a\,x^{2}}-\frac{2i\left(B\,b^{2}-\frac{b^{2}(27Ab+208Ba)}{640a}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right.}{}}}{}$
default	$A\left(-\frac{a\sqrt{b\,x^{3}+a}}{8x^{8}}-\frac{19b\sqrt{b\,x^{3}+a}}{80x^{5}}-\frac{27b^{2}\sqrt{b\,x^{3}+a}}{320a\,x^{2}}+\frac{9ib^{2}\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(}{}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}}{}\right.$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/320*(b*x^3+a)^(1/2)*(27*A*b^2*x^6+208*B*a*b*x^6+76*A*a*b*x^3+64*B*a^2*x
^3+40*A*a^2)/x^8/a+9/320*I*b*(A*b-16*B*a)/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.30

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx = \frac{27\,(16\,Bab-Ab^2)\sqrt{b}x^8\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-((208\,Bab+27\,A}{320\,ax^8}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="fricas")
```

output
```
1/320*(27*(16*B*a*b - A*b^2)*sqrt(b)*x^8*weierstrassPInverse(0, -4*a/b, x)
 - ((208*B*a*b + 27*A*b^2)*x^6 + 4*(16*B*a^2 + 19*A*a*b)*x^3 + 40*A*a^2)*s
qrt(b*x^3 + a))/(a*x^8)
```

Sympy [A] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)} + \frac{A\sqrt{a}b\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{B\sqrt{a}b\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**9,x)
```

output
```
A*a**(3/2)*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**8*gamma(-5/3)) + A*sqrt(a)*b*gamma(-5/3)*hyper((-5/3, -1/2), (-2
/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + B*a**(3/2)*gamma(-5
/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-
2/3)) + B*sqrt(a)*b*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^9}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^9, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^9}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^9}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^9,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^9, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx = \frac{-62\sqrt{b\,x^3+a}\,a^2+26\sqrt{b\,x^3+a}\,ab\,x^3-182\sqrt{b\,x^3+a}\,b^2x^6-405\left(\int \frac{\sqrt{b\,x^3+}}{b\,x^{12}+a}\right.}{91x^8}$$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x)
```

output
```
( - 62*sqrt(a + b*x**3)*a**2 + 26*sqrt(a + b*x**3)*a*b*x**3 - 182*sqrt(a +
 b*x**3)*b**2*x**6 - 405*int(sqrt(a + b*x**3)/(a*x**9 + b*x**12),x)*a**3*x
**8)/(91*x**8)
```

3.186 $\int x^4(a+bx^3)^{3/2}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 614

$$\int x^4\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{54a^2(5Ab-2aB)x^2\sqrt{a+bx^3}}{8645b^2}$$
$$+\frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b}-\frac{216a^3(5Ab-2aB)\sqrt{a+bx^3}}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{2(5Ab-2aB)x^5\left(a+bx^3\right)^{3/2}}{95b}+\frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b}$$
$$+\frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{72\sqrt{2}3^{3/4}a^{10/3}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
54/8645*a^2*(5*A*b-2*B*a)*x^2*(b*x^3+a)^(1/2)/b^2+18/1235*a*(5*A*b-2*B*a)*
x^5*(b*x^3+a)^(1/2)/b-216/8645*a^3*(5*A*b-2*B*a)*(b*x^3+a)^(1/2)/b^(8/3)/(
(1+3^(1/2))*a^(1/3)+b^(1/3)*x)+2/95*(5*A*b-2*B*a)*x^5*(b*x^3+a)^(3/2)/b+2/
25*B*x^5*(b*x^3+a)^(5/2)/b+108/8645*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1
0/3)*(5*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1
/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(8/3)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a
)^(1/2)-72/8645*2^(1/2)*3^(3/4)*a^(10/3)*(5*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^
2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^
(1/3)*x),I*3^(1/2)+2*I)/b^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*
a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.16

$$\int x^4\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2x^2\sqrt{a+bx^3}\left(-\left(a+bx^3\right)^2\left(-25Ab+10aB-19bBx^3\right)+\frac{5a^2(-5Ab+2aB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},\right.}{\sqrt{1+\frac{bx^3}{a}}}\right.}{475b^2}$$

input
```
Integrate[x^4*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*x^2*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*(-25*A*b + 10*a*B - 19*b*B*x^3)) +
 (5*a^2*(-5*A*b + 2*a*B)*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/
Sqrt[1 + (b*x^3)/a]))/(475*b^2)
```

Rubi [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 601, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {959, 811, 811, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$$

$$\Big\downarrow \ 959$$

$$\frac{(5Ab-2aB)\int x^4 \left(bx^3+a\right)^{3/2}dx}{5b}+\frac{2Bx^5 \left(a+bx^3\right)^{5/2}}{25b}$$

$$\Big\downarrow \ 811$$

$$\frac{(5Ab-2aB)\left(\frac{9}{19}a\int x^4\sqrt{bx^3+a}dx+\frac{2}{19}x^5\left(a+bx^3\right)^{3/2}\right)}{5b}+\frac{2Bx^5 \left(a+bx^3\right)^{5/2}}{25b}$$

$$\Big\downarrow \ 811$$

$$\frac{(5Ab-2aB)\left(\frac{9}{19}a\left(\frac{3}{13}a\int \frac{x^4}{\sqrt{bx^3+a}}dx+\frac{2}{13}x^5\sqrt{a+bx^3}\right)+\frac{2}{19}x^5\left(a+bx^3\right)^{3/2}\right)}{5b}+\frac{2Bx^5 \left(a+bx^3\right)^{5/2}}{25b}$$

$$\Big\downarrow \ 843$$

$$\frac{(5Ab-2aB)\left(\frac{9}{19}a\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\int \frac{x}{\sqrt{bx^3+a}}dx}{7b}\right)+\frac{2}{13}x^5\sqrt{a+bx^3}\right)+\frac{2}{19}x^5\left(a+bx^3\right)^{3/2}\right)}{5b}+\frac{2Bx^5 \left(a+bx^3\right)^{5/2}}{25b}$$

$$\Big\downarrow \ 832$$

$$\frac{(5Ab-2aB)\left(\frac{9}{19}a\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{7b}\right)+\frac{2}{13}x^5\sqrt{a+bx^3}\right)+\frac{2}{19}\right.}{5b}$$

$$\frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b}$$

$\Big\downarrow$ 759

$$\frac{(5Ab-2aB)\left(\frac{9}{19}a\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{\right.}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\right.}}}}\right.}{7b}\right.\right.\right.}{5b}$$

$$\frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b}$$

$\Big\downarrow$ 2416

$$\frac{(5Ab-2aB)\left(\frac{9}{19}a\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}}}}{\sqrt[3]{b}}\right)}{}\right.\right.\right.}{}$$

$$\frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b}$$

input
```
Int[x^4*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*B*x^5*(a + b*x^3)^(5/2))/(25*b) + ((5*A*b - 2*a*B)*((2*x^5*(a + b*x^3)^
(3/2))/19 + (9*a*((2*x^5*Sqrt[a + b*x^3])/13 + (3*a*((2*x^2*Sqrt[a + b*x^3
])/(7*b) - (4*a*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^
(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
 b*x^3])))/(7*b)))/13))/19))/(5*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 527, normalized size of antiderivative = 0.86

method	result
risch	$\frac{2x^{2}\left(1729b^{3}B\,x^{9}+2275A\,b^{3}x^{6}+2548Ba\,b^{2}x^{6}+3850aA\,b^{2}x^{3}+189B\,a^{2}b\,x^{3}+675a^{2}bA-270a^{3}B\right)\sqrt{b\,x^{3}+a}}{43225b^{2}}+\frac{72ia^{3}(5Ab-2Ba)\sqrt{3}\,(}{}$
elliptic	$\frac{2Bb\,x^{11}\sqrt{b\,x^{3}+a}}{25}+\frac{2\left(b^{2}A+\frac{28}{25}abB\right)x^{8}\sqrt{b\,x^{3}+a}}{19b}+\frac{2\left(2abA+a^{2}B-\frac{16a\left(b^{2}A+\frac{28}{25}abB\right)}{19b}\right)x^{5}\sqrt{b\,x^{3}+a}}{13b}+\frac{2\left(a^{2}A-\frac{10a\left(2abA+a^{2}B\right.}{}\right.}{}$
default	Expression too large to display

input
```
int(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/43225/b^2*x^2*(1729*B*b^3*x^9+2275*A*b^3*x^6+2548*B*a*b^2*x^6+3850*A*a*b
^2*x^3+189*B*a^2*b*x^3+675*A*a^2*b-270*B*a^3)*(b*x^3+a)^(1/2)+72/8645*I*a^
3*(5*A*b-2*B*a)/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^
2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-
I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*
(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.21

$$\int x^4 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx =$$

$$-\frac{2 \left(540 \left(2 Ba^4-5 Aa^3 b\right) \sqrt{b} \text{weierstrassZeta}\left(0,-\frac{4 a}{b},\text{weierstrassPInverse}\left(0,-\frac{4 a}{b},x\right)\right)-\left(1729 Bb^4 x^{11}+9\right.\right.}{43225 b^3}$$

input
```
integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
-2/43225*(540*(2*B*a^4 - 5*A*a^3*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, wei
erstrassPInverse(0, -4*a/b, x)) - (1729*B*b^4*x^11 + 91*(28*B*a*b^3 + 25*A
*b^4)*x^8 + 7*(27*B*a^2*b^2 + 550*A*a*b^3)*x^5 - 135*(2*B*a^3*b - 5*A*a^2*
b^2)*x^2)*sqrt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.28

$$\int x^4\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{Aa^{\frac{3}{2}}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{A\sqrt{a}bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}+\frac{Ba^{\frac{3}{2}}x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}+\frac{B\sqrt{a}bx^{11}\Gamma\left(\frac{11}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{3}\\ \frac{14}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{14}{3}\right)}$$

input
```
integrate(x**4*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
A*a**(3/2)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(8/3)) + A*sqrt(a)*b*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + B*a**(3/2)*x**8*gamma(8/3
)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) +
B*sqrt(a)*b*x**11*gamma(11/3)*hyper((-1/2, 11/3), (14/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*gamma(14/3))
```

Maxima [F]

$$\int x^4\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}x^4\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^4, x)
```

Giac [F]

$$\int x^4(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}x^4\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx=\int x^4\left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{3/2}\,dx$$

input
```
int(x^4*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

output
```
int(x^4*(A + B*x^3)*(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int x^4(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx=\frac{\frac{162\sqrt{b\,x^3+a}\,a^3x^2}{8645}+\frac{1154\sqrt{b\,x^3+a}\,a^2b\,x^5}{6175}+\frac{106\sqrt{b\,x^3+a}\,a\,b^2x^8}{475}+\frac{2\sqrt{b\,x^3+a}\,b^3x^{11}}{25}-\frac{324\left(\int\frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a^4}{8645}}{b}$$

input
```
int(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

output
```
(2*(405*sqrt(a + b*x**3)*a**3*x**2 + 4039*sqrt(a + b*x**3)*a**2*b*x**5 + 4
823*sqrt(a + b*x**3)*a*b**2*x**8 + 1729*sqrt(a + b*x**3)*b**3*x**11 - 810*
int((sqrt(a + b*x**3)*x)/(a + b*x**3),x)*a**4))/(43225*b)
```

3.187 $\int x(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 581

$$\int x\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{18a(19Ab-4aB)x^2\sqrt{a+bx^3}}{1729b}$$

$$+\frac{54a^2(19Ab-4aB)\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2(19Ab-4aB)x^2\left(a+bx^3\right)^{3/2}}{247b}+\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}(19Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{18\sqrt{2}3^{3/4}a^{7/3}(19Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
18/1729*a*(19*A*b-4*B*a)*x^2*(b*x^3+a)^(1/2)/b+54/1729*a^2*(19*A*b-4*B*a)*
(b*x^3+a)^(1/2)/b^(5/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+2/247*(19*A*b-4*B*
a)*x^2*(b*x^3+a)^(3/2)/b+2/19*B*x^2*(b*x^3+a)^(5/2)/b-27/1729*3^(1/4)*(1/2
*6^(1/2)-1/2*2^(1/2))*a^(7/3)*(19*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*E
llipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I
*3^(1/2)+2*I)/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^
(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+18/1729*2^(1/2)*3^(3/4)*a^(7/3)*(19*A*b-
4*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^
(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*
x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(5/3)/(a^(1/3)*(a^(1/3
)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.72 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.13

$$\int x\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{x^2\sqrt{a+bx^3}\left(4B(a+bx^3)^2+\frac{a(19Ab-4aB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{38b}$$

input
```
Integrate[x*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(x^2*Sqrt[a + b*x^3]*(4*B*(a + b*x^3)^2 + (a*(19*A*b - 4*a*B)*Hypergeometr
ic2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(38*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 571, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {959, 811, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \ 959$$

$$\frac{(19Ab-4aB)\int x\left(bx^3+a\right)^{3/2}dx}{19b}+\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}$$

$$\Big\downarrow \ 811$$

$$\frac{(19Ab-4aB)\left(\frac{9}{13}a\int x\sqrt{bx^3+a}dx+\frac{2}{13}x^2\left(a+bx^3\right)^{3/2}\right)}{19b}+\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}$$

$$\Big\downarrow \ 811$$

$$\frac{(19Ab-4aB)\left(\frac{9}{13}a\left(\frac{3}{7}a\int\frac{x}{\sqrt{bx^3+a}}dx+\frac{2}{7}x^2\sqrt{a+bx^3}\right)+\frac{2}{13}x^2\left(a+bx^3\right)^{3/2}\right)}{19b}+\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}$$

$$\Big\downarrow \ 832$$

$$\frac{(19Ab-4aB)\left(\frac{9}{13}a\left(\frac{3}{7}a\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{7}x^2\sqrt{a+bx^3}\right)+\frac{2}{13}x^2\left(a+bx^3\right)^{3/2}\right)}{19b}+\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}$$

$$\Big\downarrow \ 759$$

$$\frac{(19Ab-4aB)\left(\frac{9}{13}a\left(\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{arcs}\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.\right.}{19b}$$

$$\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}$$

$\Big\downarrow$ 2416

$$\frac{(19Ab-4aB)\left(\frac{9}{13}a\left(\frac{3}{7}a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{\right.}}{\sqrt[3]{b}x+\left(1+\sqrt{\right.}}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.\right.}{}$$

$$\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}$$

input
```
Int[x*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*B*x^2*(a + b*x^3)^(5/2))/(19*b) + ((19*A*b - 4*a*B)*((2*x^2*(a + b*x^3)
^(3/2))/13 + (9*a*((2*x^2*Sqrt[a + b*x^3])/7 + (3*a*(((2*Sqrt[a + b*x^3])/
(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]
*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[
3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt
[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/7))/13))/(19*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.87

method	result
risch	$\frac{2x^2\left(91b^2Bx^6+133Ab^2x^3+154Babx^3+304abA+27a^2B\right)\sqrt{bx^3+a}}{1729b}-\frac{18ia^2(19Ab-4Ba)\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\right.}{2b}\right.}{\left(-ab^2\right)^{\frac{1}{3}}}}}{}$
elliptic	$\frac{2Bbx^8\sqrt{bx^3+a}}{19}+\frac{2\left(b^2A+\frac{22}{19}abB\right)x^5\sqrt{bx^3+a}}{13b}+\frac{2\left(2abA+a^2B-\frac{10a\left(b^2A+\frac{22}{19}abB\right)}{13b}\right)x^2\sqrt{bx^3+a}}{7b}-\frac{2i\left(a^2A-\frac{4a\left(2abA+a^2B-\right.}{}\right.}{}$
default	Expression too large to display

input
```
int(x*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/1729/b*x^2*(91*B*b^2*x^6+133*A*b^2*x^3+154*B*a*b*x^3+304*A*a*b+27*B*a^2)
*(b*x^3+a)^(1/2)-18/1729*I*a^2*(19*A*b-4*B*a)/b^2*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.17

$$\int x(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2\left(27\,(4\,Ba^3-19\,Aa^2b)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+(91\,Bb\right.}{1729\,b^2}$$

input
```
integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/1729*(27*(4*B*a^3 - 19*A*a^2*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weier
strassPInverse(0, -4*a/b, x)) + (91*B*b^3*x^8 + 7*(22*B*a*b^2 + 19*A*b^3)*
x^5 + (27*B*a^2*b + 304*A*a*b^2)*x^2)*sqrt(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.30

$$\int x(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{Aa^{\frac{3}{2}}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{A\sqrt{a}bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{Ba^{\frac{3}{2}}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{B\sqrt{a}bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
A*a**(3/2)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(5/3)) + A*sqrt(a)*b*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3
,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + B*a**(3/2)*x**5*gamma(5/3)*
hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + B*sq
rt(a)*b*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(11/3))
```

Maxima [F]

$$\int x(a+bx^3)^{3/2}(A+Bx^3)\,dx = \int (Bx^3+A)(bx^3+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x, x)
```

Giac [F]

$$\int x\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int x\left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{3/2}\,dx$$

input
```
int(x*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

output
```
int(x*(A + B*x^3)*(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int x\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{662\sqrt{b\,x^3+a}\,a^2x^2}{1729}+\frac{82\sqrt{b\,x^3+a}\,ab\,x^5}{247}+\frac{2\sqrt{b\,x^3+a}\,b^2x^8}{19}+\frac{405\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a^3}{1729}$$

input
```
int(x*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

output
```
(662*sqrt(a + b*x**3)*a**2*x**2 + 574*sqrt(a + b*x**3)*a*b*x**5 + 182*sqrt
(a + b*x**3)*b**2*x**8 + 405*int((sqrt(a + b*x**3)*x)/(a + b*x**3),x)*a**3
)/1729
```

3.188 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 573

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = \frac{9}{91}(13Ab+2aB)x^2\sqrt{a+bx^3}$$

$$+\frac{27a(13Ab+2aB)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{(13Ab+2aB)x^2(a+bx^3)^{3/2}}{13a}-\frac{A(a+bx^3)^{5/2}}{ax}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{9\sqrt{2}3^{3/4}a^{4/3}(13Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
9/91*(13*A*b+2*B*a)*x^2*(b*x^3+a)^(1/2)+27/91*a*(13*A*b+2*B*a)*(b*x^3+a)^(
1/2)/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+1/13*(13*A*b+2*B*a)*x^2*(b*x^
3+a)^(3/2)/a-A*(b*x^3+a)^(5/2)/a/x-27/182*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2)
)*a^(4/3)*(13*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2)
)*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(2/3
)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b
*x^3+a)^(1/2)+9/91*2^(1/2)*3^(3/4)*a^(4/3)*(13*A*b+2*B*a)*(a^(1/3)+b^(1/3)
*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*
x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)
+b^(1/3)*x),I*3^(1/2)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2
))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.85 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.14

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = -\frac{A(a+bx^3)^{5/2}}{ax} - \frac{\left(-\frac{13Ab}{2}-aB\right)x^2\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^2,x]
```

output
```
-((A*(a + b*x^3)^(5/2))/(a*x)) - (((-13*A*b)/2 - a*B)*x^2*Sqrt[a + b*x^3]*
Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 569, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 811, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx$$

$$\Big\downarrow \ 955$$

$$\frac{(2aB+13Ab)\int x(bx^3+a)^{3/2}dx}{2a}-\frac{A(a+bx^3)^{5/2}}{ax}$$

$$\Big\downarrow \ 811$$

$$\frac{(2aB+13Ab)\left(\frac{9}{13}a\int x\sqrt{bx^3+a}dx+\frac{2}{13}x^2(a+bx^3)^{3/2}\right)}{2a}-\frac{A(a+bx^3)^{5/2}}{ax}$$

$$\Big\downarrow \ 811$$

$$\frac{(2aB+13Ab)\left(\frac{9}{13}a\left(\frac{3}{7}a\int\frac{x}{\sqrt{bx^3+a}}dx+\frac{2}{7}x^2\sqrt{a+bx^3}\right)+\frac{2}{13}x^2(a+bx^3)^{3/2}\right)}{2a}-\frac{A(a+bx^3)^{5/2}}{ax}$$

$$\Big\downarrow \ 832$$

$$\frac{(2aB+13Ab)\left(\frac{9}{13}a\left(\frac{3}{7}a\left(\frac{\int\frac{\sqrt[3]{b}x+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{(1-\sqrt{3})\sqrt[3]{a}\int\frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{7}x^2\sqrt{a+bx^3}\right)+\frac{2}{13}x^2(a+bx^3)^{3/2}\right)}{2a}$$
$$\frac{A(a+bx^3)^{5/2}}{ax}$$

$$\Big\downarrow \ 759$$

$$\frac{(2aB+13Ab)\left(\frac{9}{13}a\left(\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{arcs}\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.\right.}{2a}$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{ax}$$

$\Big\downarrow$ 2416

$$\frac{(2aB+13Ab)\left(\frac{9}{13}a\left(\frac{3}{7}a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{\right.}}{\sqrt[3]{b}x+\left(1+\sqrt{\right.}}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.\right.}{}$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{ax}$$

input

```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^2,x]
```

output
```
-((A*(a + b*x^3)^(5/2))/(a*x)) + ((13*A*b + 2*a*B)*((2*x^2*(a + b*x^3)^(3/
2))/13 + (9*a*((2*x^2*Sqrt[a + b*x^3])/7 + (3*a*(((2*Sqrt[a + b*x^3])/(b^(
1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(
1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]
*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[
3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/7))/13))/(2*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 486, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,\left(-14bB\,x^6-26Ab\,x^3-32Ba\,x^3+91Aa\right)}{91x}-\frac{9ia(13Ab+2Ba)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\quad}{-}}}{\quad}$
elliptic	$-\frac{Aa\sqrt{b\,x^3+a}}{x}+\frac{2Bb\,x^5\sqrt{b\,x^3+a}}{13}+\frac{2\left(b^2A+\frac{16}{13}abB\right)x^2\sqrt{b\,x^3+a}}{7b}-\frac{2i\left(\frac{5abA}{2}+a^2B-\frac{4a\left(b^2A+\frac{16}{13}abB\right)}{7b}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\right.}{}}}{\quad}$
default	Expression too large to display

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/91*(b*x^3+a)^(1/2)*(-14*B*b*x^6-26*A*b*x^3-32*B*a*x^3+91*A*a)/x-9/91*I*
a*(13*A*b+2*B*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2
)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I
*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(
-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.15

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^2}\,dx = -\frac{27\,(2\,Ba^2+13\,Aab)\sqrt{b}x\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-(14\,Bb^2x^6+2\,(16\,B}{91\,bx}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="fricas")
```

output
```
-1/91*(27*(2*B*a^2 + 13*A*a*b)*sqrt(b)*x*weierstrassZeta(0, -4*a/b, weiers
trassPInverse(0, -4*a/b, x)) - (14*B*b^2*x^6 + 2*(16*B*a*b + 13*A*b^2)*x^3
 - 91*A*a*b)*sqrt(b*x^3 + a))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.30

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{A\sqrt{a}bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{Ba^{\frac{3}{2}}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{B\sqrt{a}bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**2,x)
```

output
```
A*a**(3/2)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x*gamma(2/3)) + A*sqrt(a)*b*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + B*a**(3/2)*x**2*gamma(2/3)*h
yper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + B*sqr
t(a)*b*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(8/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^2}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^2,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = \frac{314\sqrt{b\,x^3+a}\,a^2+58\sqrt{b\,x^3+a}\,ab\,x^3+14\sqrt{b\,x^3+a}\,b^2x^6+405\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^5+a\,x^2}\right.}{91x}$$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x)
```

output
```
(314*sqrt(a + b*x**3)*a**2 + 58*sqrt(a + b*x**3)*a*b*x**3 + 14*sqrt(a + b*
x**3)*b**2*x**6 + 405*int(sqrt(a + b*x**3)/(a*x**2 + b*x**5),x)*a**3*x)/(9
1*x)
```

3.189 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 575

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = -\frac{(7Ab+8aB)\sqrt{a+bx^3}}{8x} + \frac{b(7Ab+8aB)x^2\sqrt{a+bx^3}}{28a} + \frac{27\sqrt[3]{b}(7Ab+8aB)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A(a+bx^3)^{5/2}}{4ax^4} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}(7Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-\right.}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{9\,3^{3/4}\sqrt[3]{a}\sqrt[3]{b}(7Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7\right.}{28\sqrt{2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/8*(7*A*b+8*B*a)*(b*x^3+a)^(1/2)/x+1/28*b*(7*A*b+8*B*a)*x^2*(b*x^3+a)^(1
/2)/a+27*b^(1/3)*(7*A*b+8*B*a)*(b*x^3+a)^(1/2)/(56*(1+3^(1/2))*a^(1/3)+56*
b^(1/3)*x)-1/4*A*(b*x^3+a)^(5/2)/a/x^4-27/112*3^(1/4)*(1/2*6^(1/2)-1/2*2^(
1/2))*a^(1/3)*b^(1/3)*(7*A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(
((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+
2*I)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)
/(b*x^3+a)^(1/2)+9/56*3^(3/4)*a^(1/3)*b^(1/3)*(7*A*b+8*B*a)*(a^(1/3)+b^(1/
3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/
3)+b^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1
/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.15

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = -\frac{A(a+bx^3)^{5/2}}{4ax^4} + \frac{\left(-\frac{7Ab}{2}-4aB\right)\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{4x\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^5,x]
```

output
```
-1/4*(A*(a + b*x^3)^(5/2))/(a*x^4) + (((-7*A*b)/2 - 4*a*B)*Sqrt[a + b*x^3]
*Hypergeometric2F1[-3/2, -1/3, 2/3, -((b*x^3)/a)])/(4*x*Sqrt[1 + (b*x^3)/a
])
```

Rubi [A] (warning: unable to verify)

Time = 1.00 (sec) , antiderivative size = 569, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 809, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(8aB+7Ab)\int \frac{(bx^3+a)^{3/2}}{x^2}dx}{8a}-\frac{A(a+bx^3)^{5/2}}{4ax^4}$$

$$\Big\downarrow \text{809}$$

$$\frac{(8aB+7Ab)\left(\frac{9}{2}b\int x\sqrt{bx^3+a}dx-\frac{(a+bx^3)^{3/2}}{x}\right)}{8a}-\frac{A(a+bx^3)^{5/2}}{4ax^4}$$

$$\Big\downarrow \text{811}$$

$$\frac{(8aB+7Ab)\left(\frac{9}{2}b\left(\frac{3}{7}a\int \frac{x}{\sqrt{bx^3+a}}dx+\frac{2}{7}x^2\sqrt{a+bx^3}\right)-\frac{(a+bx^3)^{3/2}}{x}\right)}{8a}-\frac{A(a+bx^3)^{5/2}}{4ax^4}$$

$$\Big\downarrow \text{832}$$

$$\frac{(8aB+7Ab)\left(\frac{9}{2}b\left(\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}x+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{(1-\sqrt{3})\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{7}x^2\sqrt{a+bx^3}\right)-\frac{(a+bx^3)^{3/2}}{x}\right)}{8a}-\frac{A(a+bx^3)^{5/2}}{4ax^4}$$

$$\Big\downarrow \text{759}$$

$$\frac{(8aB+7Ab)\left(\frac{9}{2}b\left(\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.\right.}{8a}$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{4ax^4}$$

$\downarrow$ 2416

$$\frac{(8aB+7Ab)\left(\frac{9}{2}b\left(\frac{3}{7}a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.\right.}{}$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{4ax^4}$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^5,x]
```

output
```
-1/4*(A*(a + b*x^3)^(5/2))/(a*x^4) + ((7*A*b + 8*a*B)*(-((a + b*x^3)^(3/2)
/x) + (9*b*((2*x^2*Sqrt[a + b*x^3])/7 + (3*a*(((2*Sqrt[a + b*x^3])/(b^(1/3
)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b
^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^
(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]
)/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/7))/2))/(8*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 482, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-16bB\,x^{6}+77Ab\,x^{3}+56Ba\,x^{3}+14Aa\right)}{56x^{4}}-\frac{9i(7Ab+8Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{}{-\frac{3(}{}}}}{}$
elliptic	$-\frac{Aa\sqrt{b\,x^{3}+a}}{4x^{4}}-\frac{\left(\frac{11Ab}{8}+Ba\right)\sqrt{b\,x^{3}+a}}{x}+\frac{2Bb\,x^{2}\sqrt{b\,x^{3}+a}}{7}-\frac{2i\left(\frac{27}{16}b^{2}A+\frac{27}{14}abB\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)}{2b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$
default	Expression too large to display

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/56*(b*x^3+a)^(1/2)*(-16*B*b*x^6+77*A*b*x^3+56*B*a*x^3+14*A*a)/x^4-9/56*
I*(7*A*b+8*B*a)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x
+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*
b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.13

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = -\frac{27\,(8\,Ba+7\,Ab)\sqrt{b}x^4\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-(16\,Bbx^6-7\,(8\,Ba+}{56\,x^4}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="fricas")
```

output
```
-1/56*(27*(8*B*a + 7*A*b)*sqrt(b)*x^4*weierstrassZeta(0, -4*a/b, weierstra
ssPInverse(0, -4*a/b, x)) - (16*B*b*x^6 - 7*(8*B*a + 11*A*b)*x^3 - 14*A*a)
*sqrt(b*x^3 + a))/x^4
```

Sympy [A] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.32

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\ -\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{A\sqrt{a}b\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{B\sqrt{a}bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**5,x)
```

output
```
A*a**(3/2)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**4*gamma(-1/3)) + A*sqrt(a)*b*gamma(-1/3)*hyper((-1/2, -1/3), (2/
3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) + B*a**(3/2)*gamma(-1/3)*h
yper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) + B*
sqrt(a)*b*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(5/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^5, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^5}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^5,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^5, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = \frac{-22\sqrt{b\,x^3+a}\,a^2+34\sqrt{b\,x^3+a}\,ab\,x^3+2\sqrt{b\,x^3+a}\,b^2x^6-81\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^8+a\,x^5}dx\right.}{7x^4}$$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x)
```

output
```
( - 22*sqrt(a + b*x**3)*a**2 + 34*sqrt(a + b*x**3)*a*b*x**3 + 2*sqrt(a + b
*x**3)*b**2*x**6 - 81*int(sqrt(a + b*x**3)/(a*x**5 + b*x**8),x)*a**3*x**4)
/(7*x**4)
```

3.190 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 573

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx = -\frac{(Ab+14aB)\sqrt{a+bx^3}}{56x^4} - \frac{11b(Ab+14aB)\sqrt{a+bx^3}}{112ax} + \frac{27b^{4/3}(Ab+14aB)\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A(a+bx^3)^{5/2}}{7ax^7} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(Ab+14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{9\ 3^{3/4}b^{4/3}(Ab+14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-\right.}{56\sqrt{2}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/56*(A*b+14*B*a)*(b*x^3+a)^(1/2)/x^4-11/112*b*(A*b+14*B*a)*(b*x^3+a)^(1/
2)/a/x+27/112*b^(4/3)*(A*b+14*B*a)*(b*x^3+a)^(1/2)/a/((1+3^(1/2))*a^(1/3)+
b^(1/3)*x)-1/7*A*(b*x^3+a)^(5/2)/a/x^7-27/224*3^(1/4)*(1/2*6^(1/2)-1/2*2^(
1/2))*b^(4/3)*(A*b+14*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/
2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/a^(2
/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/
(b*x^3+a)^(1/2)+9/112*3^(3/4)*b^(4/3)*(A*b+14*B*a)*(a^(1/3)+b^(1/3)*x)*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(
1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x),I*3^(1/2)+2*I)*2^(1/2)/a^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/
2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.14

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx=\frac{\sqrt{a+bx^3}\left(-\frac{4A(a+bx^3)^2}{a}-\frac{(Ab+14aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{4}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{28x^7}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^8,x]
```

output
```
(Sqrt[a + b*x^3]*((-4*A*(a + b*x^3)^2)/a - ((A*b + 14*a*B)*x^3*Hypergeomet
ric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(28*x^7)
```

Rubi [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 568, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {955, 809, 809, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^8}\,dx$$

$$\Big\downarrow \ 955$$

$$\frac{(14aB+Ab)\int \frac{\left(bx^3+a\right)^{3/2}}{x^5}dx}{14a}-\frac{A\left(a+bx^3\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \ 809$$

$$\frac{(14aB+Ab)\left(\frac{9}{8}b\int \frac{\sqrt{bx^3+a}}{x^2}dx-\frac{\left(a+bx^3\right)^{3/2}}{4x^4}\right)}{14a}-\frac{A\left(a+bx^3\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \ 809$$

$$\frac{(14aB+Ab)\left(\frac{9}{8}b\left(\frac{3}{2}b\int \frac{x}{\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{x}\right)-\frac{\left(a+bx^3\right)^{3/2}}{4x^4}\right)}{14a}-\frac{A\left(a+bx^3\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \ 832$$

$$\frac{(14aB+Ab)\left(\frac{9}{8}b\left(\frac{3}{2}b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)-\frac{\sqrt{a+bx^3}}{x}\right)-\frac{\left(a+bx^3\right)^{3/2}}{4x^4}\right)}{14a}-\frac{A\left(a+bx^3\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \ 759$$

$$\frac{(14aB+Ab)\left(\frac{9}{8}b\left(\frac{3}{2}b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.\right.}{14a}$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \ 2416$$

$$(14aB+Ab)\left(\frac{9}{8}b\left(\frac{3}{2}b\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.\right.$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{7ax^7}$$

input

```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^8,x]
```

output

```
-1/7*(A*(a + b*x^3)^(5/2))/(a*x^7) + ((A*b + 14*a*B)*(-1/4*(a + b*x^3)^(3/
2)/x^4 + (9*b*(-(Sqrt[a + b*x^3]/x) + (3*b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1
/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/2))/8))/(14*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.88

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(27A\,b^{2}x^{6}+154Bab\,x^{6}+34aAb\,x^{3}+28B\,a^{2}x^{3}+16a^{2}A\right)}{112x^{7}a}-\frac{9ib(Ab+14Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)}{2b}\right.}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$
elliptic	$-\frac{Aa\sqrt{b\,x^{3}+a}}{7x^{7}}-\frac{\left(\frac{17Ab}{14}+Ba\right)\sqrt{b\,x^{3}+a}}{4x^{4}}-\frac{b(27Ab+154Ba)\sqrt{b\,x^{3}+a}}{112ax}-\frac{2i\left(B\,b^{2}+\frac{b^{2}(27Ab+154Ba)}{224a}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\right.}{2}\right.}{}}}{}$
default	Expression too large to display

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/112*(b*x^3+a)^(1/2)*(27*A*b^2*x^6+154*B*a*b*x^6+34*A*a*b*x^3+28*B*a^2*x
^3+16*A*a^2)/x^7/a-9/112*I*b*(A*b+14*B*a)/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.17

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx =$$
$$-\frac{27\,(14\,Bab+Ab^2)\sqrt{b}x^7\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+((154\,Bab+27\,Ab^2)x}{112\,ax^7}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="fricas")
```

output
```
-1/112*(27*(14*B*a*b + A*b^2)*sqrt(b)*x^7*weierstrassZeta(0, -4*a/b, weier
strassPInverse(0, -4*a/b, x)) + ((154*B*a*b + 27*A*b^2)*x^6 + 2*(14*B*a^2
+ 17*A*a*b)*x^3 + 16*A*a^2)*sqrt(b*x^3 + a))/(a*x^7)
```

Sympy [A] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.34

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{A\sqrt{a}b\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{B\sqrt{a}b\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**8,x)
```

output
```
A*a**(3/2)*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**7*gamma(-4/3)) + A*sqrt(a)*b*gamma(-4/3)*hyper((-4/3, -1/2), (-1
/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + B*a**(3/2)*gamma(-4
/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-
1/3)) + B*sqrt(a)*b*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^8, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx=\int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^8}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^8,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^8, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx=\frac{10\sqrt{b\,x^3+a}\,a^2-22\sqrt{b\,x^3+a}\,ab\,x^3+22\sqrt{b\,x^3+a}\,b^2x^6+81\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^{11}+a\,x^8}d}{11x^7}$$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x)
```

output
```
(10*sqrt(a + b*x**3)*a**2 - 22*sqrt(a + b*x**3)*a*b*x**3 + 22*sqrt(a + b*x
**3)*b**2*x**6 + 81*int(sqrt(a + b*x**3)/(a*x**8 + b*x**11),x)*a**3*x**7)/
(11*x**7)
```

3.191 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 605

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx = \frac{(Ab-4aB)\sqrt{a+bx^3}}{28x^7}+\frac{17b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4}$$

$$+\frac{27b^2(Ab-4aB)\sqrt{a+bx^3}}{448a^2x}-\frac{27b^{7/3}(Ab-4aB)\sqrt{a+bx^3}}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{A(a+bx^3)^{5/2}}{10ax^{10}}$$

$$+\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{9\ 3^{3/4}b^{7/3}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-\right.}{224\sqrt{2}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/28*(A*b-4*B*a)*(b*x^3+a)^(1/2)/x^7+17/224*b*(A*b-4*B*a)*(b*x^3+a)^(1/2)/
a/x^4+27/448*b^2*(A*b-4*B*a)*(b*x^3+a)^(1/2)/a^2/x-27/448*b^(7/3)*(A*b-4*B
*a)*(b*x^3+a)^(1/2)/a^2/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)-1/10*A*(b*x^3+a)^(
5/2)/a/x^10+27/896*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*b^(7/3)*(A*b-4*B*a)*(
a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a
^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3
^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/a^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3
)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-9/448*3^(3/4
)*b^(7/3)*(A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a
^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)/a
^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/
2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.14

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx = \frac{\sqrt{a+bx^3}\left(-\frac{7A(a+bx^3)^2}{a}+\frac{5(Ab-4aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{7}{3},-\frac{3}{2},-\frac{4}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{70x^{10}}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^11,x]
```

output
```
(Sqrt[a + b*x^3]*((-7*A*(a + b*x^3)^2)/a + (5*(A*b - 4*a*B)*x^3*Hypergeome
tric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(70*x^1
0)
```

Rubi [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 598, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {955, 809, 809, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(Ab-4aB)\int \frac{(bx^3+a)^{3/2}}{x^8}dx}{4a}-\frac{A(a+bx^3)^{5/2}}{10ax^{10}}$$

$$\Big\downarrow \text{ 809}$$

$$-\frac{(Ab-4aB)\left(\frac{9}{14}b\int \frac{\sqrt{bx^3+a}}{x^5}dx-\frac{(a+bx^3)^{3/2}}{7x^7}\right)}{4a}-\frac{A(a+bx^3)^{5/2}}{10ax^{10}}$$

$$\Big\downarrow \text{ 809}$$

$$-\frac{(Ab-4aB)\left(\frac{9}{14}b\left(\frac{3}{8}b\int \frac{1}{x^2\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{4x^4}\right)-\frac{(a+bx^3)^{3/2}}{7x^7}\right)}{4a}-\frac{A(a+bx^3)^{5/2}}{10ax^{10}}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{(Ab-4aB)\left(\frac{9}{14}b\left(\frac{3}{8}b\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)-\frac{\sqrt{a+bx^3}}{4x^4}\right)-\frac{(a+bx^3)^{3/2}}{7x^7}\right)}{4a}-\frac{A(a+bx^3)^{5/2}}{10ax^{10}}$$

$$\Big\downarrow \text{ 832}$$

$$-\frac{(Ab-4aB)\left(\frac{9}{14}b\left(\frac{3}{8}b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{(1-\sqrt{3})\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)-\frac{\sqrt{a+bx^3}}{4x^4}\right)-\frac{(a+bx^3)^{3/2}}{7x^7}\right)}{4a}-\frac{A(a+bx^3)^{5/2}}{10ax^{10}}$$

$\downarrow$ 759

$$-\frac{(Ab-4aB)\left(\frac{9}{14}b\left(\frac{3}{8}b\left(\frac{b\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{}}{\sqrt[3]{}}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{2a}\right.\right.\right.}{4a}$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{10ax^{10}}$$

$\downarrow$ 2416

$$-\frac{(Ab-4aB)\left(\frac{9}{14}b\left(\frac{3}{8}b\left(\frac{b\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\right.}{\sqrt[3]{b}x+\left(1+\right.}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}\right.\right.\right.}{}$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{10ax^{10}}$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^11,x]
```

output
```
-1/10*(A*(a + b*x^3)^(5/2))/(a*x^10) - ((A*b - 4*a*B)*(-1/7*(a + b*x^3)^(3
/2)/x^7 + (9*b*(-1/4*Sqrt[a + b*x^3]/x^4 + (3*b*(-(Sqrt[a + b*x^3]/(a*x))
+ (b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) -
 (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*
x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*
(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/
(2*a)))/8))/14))/(4*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 529, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-135A\,x^{9}b^{3}+540B\,x^{9}a\,b^{2}+54A\,x^{6}a\,b^{2}+680B\,x^{6}a^{2}b+368a^{2}Ab\,x^{3}+320B\,x^{3}a^{3}+224a^{3}A\right)}{2240x^{10}a^{2}}+\frac{9ib^{2}(Ab-4Ba)\sqrt{3}\,\left(-a\,b\right.}{\ldots}$ $9ib^{2}(Ab-4Ba)\sqrt{3}\,\left(-a\,b^{2}\right.$
elliptic	$-\frac{Aa\sqrt{b\,x^{3}+a}}{10x^{10}}-\frac{\left(\frac{23Ab}{20}+Ba\right)\sqrt{b\,x^{3}+a}}{7x^{7}}-\frac{b(27Ab+340Ba)\sqrt{b\,x^{3}+a}}{1120a\,x^{4}}+\frac{27b^{2}(Ab-4Ba)\sqrt{b\,x^{3}+a}}{448a^{2}x}+$
default	Expression too large to display

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/2240*(b*x^3+a)^(1/2)*(-135*A*b^3*x^9+540*B*a*b^2*x^9+54*A*a*b^2*x^6+680
*B*a^2*b*x^6+368*A*a^2*b*x^3+320*B*a^3*x^3+224*A*a^3)/x^10/a^2+9/448*I*b^2
*(A*b-4*B*a)/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a
*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.20

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx =$$
$$-\frac{135\,(4\,Bab^2-Ab^3)\sqrt{b}x^{10}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+(135\,(4\,Bab^2-Ab^3}{2240\,a^2x^{10}}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="fricas")
```

output
```
-1/2240*(135*(4*B*a*b^2 - A*b^3)*sqrt(b)*x^10*weierstrassZeta(0, -4*a/b, w
eierstrassPInverse(0, -4*a/b, x)) + (135*(4*B*a*b^2 - A*b^3)*x^9 + 2*(340*
B*a^2*b + 27*A*a*b^2)*x^6 + 224*A*a^3 + 16*(20*B*a^3 + 23*A*a^2*b)*x^3)*sq
rt(b*x^3 + a))/(a^2*x^10)
```

Sympy [A] (verification not implemented)

Time = 2.86 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.33

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{10}{3}\right){}_2F_1\left(\begin{matrix} -\frac{10}{3}, -\frac{1}{2} \\ -\frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^{10}\Gamma\left(-\frac{7}{3}\right)} + \frac{A\sqrt{a}b\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix} -\frac{7}{3}, -\frac{1}{2} \\ -\frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix} -\frac{7}{3}, -\frac{1}{2} \\ -\frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{B\sqrt{a}b\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**11,x)
```

output
```
A*a**(3/2)*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*p
i)/a)/(3*x**10*gamma(-7/3)) + A*sqrt(a)*b*gamma(-7/3)*hyper((-7/3, -1/2),
(-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + B*a**(3/2)*gamma
(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamm
a(-4/3)) + B*sqrt(a)*b*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp
_polar(I*pi)/a)/(3*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^11, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^11, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx=\int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^{11}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^11,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^11, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx=\frac{-134\sqrt{b\,x^3+a}\,a^2-238\sqrt{b\,x^3+a}\,ab\,x^3-374\sqrt{b\,x^3+a}\,b^2x^6-405\left(\int \frac{\sqrt{b}}{b\,x^{14}}\right.}{935x^{10}}$$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x)
```

output
```
( - 134*sqrt(a + b*x**3)*a**2 - 238*sqrt(a + b*x**3)*a*b*x**3 - 374*sqrt(a
 + b*x**3)*b**2*x**6 - 405*int(sqrt(a + b*x**3)/(a*x**11 + b*x**14),x)*a**
3*x**10)/(935*x**10)
```

3.192 $\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2a^2(Ab-aB)\sqrt{a+bx^3}}{3b^4} - \frac{2a(2Ab-3aB)\left(a+bx^3\right)^{3/2}}{9b^4} + \frac{2(Ab-3aB)\left(a+bx^3\right)^{5/2}}{15b^4} + \frac{2B\left(a+bx^3\right)^{7/2}}{21b^4}$$

output
```
2/3*a^2*(A*b-B*a)*(b*x^3+a)^(1/2)/b^4-2/9*a*(2*A*b-3*B*a)*(b*x^3+a)^(3/2)/
b^4+2/15*(A*b-3*B*a)*(b*x^3+a)^(5/2)/b^4+2/21*B*(b*x^3+a)^(7/2)/b^4
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}(56a^2Ab-48a^3B-28aAb^2x^3+24a^2bBx^3+21Ab^3x^6-18ab^2Bx^6+15b^3Bx^9)}{315b^4}$$

input
```
Integrate[(x^8*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[a + b*x^3]*(56*a^2*A*b - 48*a^3*B - 28*a*A*b^2*x^3 + 24*a^2*b*B*x^
3 + 21*A*b^3*x^6 - 18*a*b^2*B*x^6 + 15*b^3*B*x^9))/(315*b^4)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (A+Bx^3)}{\sqrt{a+bx^3}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6 (Bx^3+A)}{\sqrt{bx^3+a}}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(\frac{B(bx^3+a)^{5/2}}{b^3}+\frac{(Ab-3aB)(bx^3+a)^{3/2}}{b^3}+\frac{a(3aB-2Ab)\sqrt{bx^3+a}}{b^3}-\frac{a^2(aB-Ab)}{b^3\sqrt{bx^3+a}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2a^2\sqrt{a+bx^3}(Ab-aB)}{b^4}+\frac{2(a+bx^3)^{5/2}(Ab-3aB)}{5b^4}-\frac{2a(a+bx^3)^{3/2}(2Ab-3aB)}{3b^4}+\frac{2B(a+bx^3)^{7/2}}{7b^4}\right)$$

input
```
Int[(x^8*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
((2*a^2*(A*b - a*B)*Sqrt[a + b*x^3])/b^4 - (2*a*(2*A*b - 3*a*B)*(a + b*x^3
)^(3/2))/(3*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(5/2))/(5*b^4) + (2*B*(a +
 b*x^3)^(7/2))/(7*b^4))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{16\left(\frac{3\left(\frac{5B\,x^{3}}{7}+A\right)x^{6}b^{3}}{8}-\frac{a\left(\frac{9B\,x^{3}}{14}+A\right)x^{3}b^{2}}{2}+a^{2}\left(\frac{3B\,x^{3}}{7}+A\right)b-\frac{6a^{3}B}{7}\right)\sqrt{b\,x^{3}+a}}{45b^{4}}$
gosper	$\frac{2\sqrt{b\,x^{3}+a}\,(15b^{3}B\,x^{9}+21A\,b^{3}x^{6}-18Ba\,b^{2}x^{6}-28aA\,b^{2}x^{3}+24B\,a^{2}b\,x^{3}+56a^{2}bA-48a^{3}B)}{315b^{4}}$
trager	$\frac{2\sqrt{b\,x^{3}+a}\,(15b^{3}B\,x^{9}+21A\,b^{3}x^{6}-18Ba\,b^{2}x^{6}-28aA\,b^{2}x^{3}+24B\,a^{2}b\,x^{3}+56a^{2}bA-48a^{3}B)}{315b^{4}}$
risch	$\frac{2\sqrt{b\,x^{3}+a}\,(15b^{3}B\,x^{9}+21A\,b^{3}x^{6}-18Ba\,b^{2}x^{6}-28aA\,b^{2}x^{3}+24B\,a^{2}b\,x^{3}+56a^{2}bA-48a^{3}B)}{315b^{4}}$
orering	$\frac{2\sqrt{b\,x^{3}+a}\,(15b^{3}B\,x^{9}+21A\,b^{3}x^{6}-18Ba\,b^{2}x^{6}-28aA\,b^{2}x^{3}+24B\,a^{2}b\,x^{3}+56a^{2}bA-48a^{3}B)}{315b^{4}}$
elliptic	$\frac{2B\,x^{9}\sqrt{b\,x^{3}+a}}{21b}+\frac{2\left(A-\frac{6Ba}{7b}\right)x^{6}\sqrt{b\,x^{3}+a}}{15b}-\frac{8a\left(A-\frac{6Ba}{7b}\right)x^{3}\sqrt{b\,x^{3}+a}}{45b^{2}}+\frac{16a^{2}\left(A-\frac{6Ba}{7b}\right)\sqrt{b\,x^{3}+a}}{45b^{3}}$
default	$A\left(\frac{2x^{6}\sqrt{b\,x^{3}+a}}{15b}-\frac{8a\,x^{3}\sqrt{b\,x^{3}+a}}{45b^{2}}+\frac{16a^{2}\sqrt{b\,x^{3}+a}}{45b^{3}}\right)+B\left(\frac{2x^{9}\sqrt{b\,x^{3}+a}}{21b}-\frac{4a\,x^{6}\sqrt{b\,x^{3}+a}}{35b^{2}}+\frac{16a^{2}x^{3}\sqrt{b\,x^{3}+a}}{105b^{3}}-\right.$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
16/45*(3/8*(5/7*B*x^3+A)*x^6*b^3-1/2*a*(9/14*B*x^3+A)*x^3*b^2+a^2*(3/7*B*x
^3+A)*b-6/7*a^3*B)*(b*x^3+a)^(1/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\,(15\,Bb^3x^9 - 3\,(6\,Bab^2 - 7\,Ab^3)x^6 - 48\,Ba^3 + 56\,Aa^2b + 4\,(6\,Ba^2b - 7\,Aab^2)x^3)\sqrt{bx^3+a}}{315\,b^4}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(15*B*b^3*x^9 - 3*(6*B*a*b^2 - 7*A*b^3)*x^6 - 48*B*a^3 + 56*A*a^2*b
+ 4*(6*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.70

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \begin{cases} \frac{16Aa^2\sqrt{a+bx^3}}{45b^3} - \frac{8Aax^3\sqrt{a+bx^3}}{45b^2} + \frac{2Ax^6\sqrt{a+bx^3}}{15b} - \frac{32Ba^3\sqrt{a+bx^3}}{105b^4} + \frac{16Ba^2x^3\sqrt{a+bx^3}}{105b^3} - \frac{4Bax^6\sqrt{a+bx^3}}{35b^2} + \frac{2Bx^9\sqrt{a+bx^3}}{21b} & \text{fo} \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{\sqrt{a}} & \text{o} \end{cases}$$

input
```
integrate(x**8*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((16*A*a**2*sqrt(a + b*x**3)/(45*b**3) - 8*A*a*x**3*sqrt(a + b*x*
*3)/(45*b**2) + 2*A*x**6*sqrt(a + b*x**3)/(15*b) - 32*B*a**3*sqrt(a + b*x*
*3)/(105*b**4) + 16*B*a**2*x**3*sqrt(a + b*x**3)/(105*b**3) - 4*B*a*x**6*s
qrt(a + b*x**3)/(35*b**2) + 2*B*x**9*sqrt(a + b*x**3)/(21*b), Ne(b, 0)), (
(A*x**9/9 + B*x**12/12)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2}{105}\,B\left(\frac{5\,(bx^3+a)^{\frac{7}{2}}}{b^4} - \frac{21\,(bx^3+a)^{\frac{5}{2}}a}{b^4} + \frac{35\,(bx^3+a)^{\frac{3}{2}}a^2}{b^4} - \frac{35\,\sqrt{bx^3+a}a^3}{b^4}\right) + \frac{2}{45}\,A\left(\frac{3\,(bx^3+a)^{\frac{5}{2}}}{b^3} - \frac{10\,(bx^3+a)^{\frac{3}{2}}a}{b^3} + \frac{15\,\sqrt{bx^3+a}a^2}{b^3}\right)$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
2/105*B*(5*(b*x^3 + a)^(7/2)/b^4 - 21*(b*x^3 + a)^(5/2)*a/b^4 + 35*(b*x^3
+ a)^(3/2)*a^2/b^4 - 35*sqrt(b*x^3 + a)*a^3/b^4) + 2/45*A*(3*(b*x^3 + a)^(
5/2)/b^3 - 10*(b*x^3 + a)^(3/2)*a/b^3 + 15*sqrt(b*x^3 + a)*a^2/b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = -\frac{2\,(Ba^3-Aa^2b)\sqrt{bx^3+a}}{3\,b^4} + \frac{2\left(15\,(bx^3+a)^{\frac{7}{2}}B - 63\,(bx^3+a)^{\frac{5}{2}}Ba + 105\,(bx^3+a)^{\frac{3}{2}}Ba^2 + 21\,(bx^3+a)^{\frac{5}{2}}Ab - 70\,(bx^3+a)^{\frac{3}{2}}Aab\right)}{315\,b^4}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
-2/3*(B*a^3 - A*a^2*b)*sqrt(b*x^3 + a)/b^4 + 2/315*(15*(b*x^3 + a)^(7/2)*B
 - 63*(b*x^3 + a)^(5/2)*B*a + 105*(b*x^3 + a)^(3/2)*B*a^2 + 21*(b*x^3 + a)
^(5/2)*A*b - 70*(b*x^3 + a)^(3/2)*A*a*b)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{8\,a^2\,\sqrt{b\,x^3+a}\,\left(2\,A-\frac{12\,B\,a}{7\,b}\right)}{45\,b^3}+\frac{x^6\,\sqrt{b\,x^3+a}\,\left(2\,A-\frac{12\,B\,a}{7\,b}\right)}{15\,b}+\frac{2\,B\,x^9\,\sqrt{b\,x^3+a}}{21\,b}-\frac{4\,a\,x^3\,\sqrt{b\,x^3+a}\,\left(2\,A-\frac{12\,B\,a}{7\,b}\right)}{45\,b^2}$$

input
```
int((x^8*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

output
```
(8*a^2*(a + b*x^3)^(1/2)*(2*A - (12*B*a)/(7*b)))/(45*b^3) + (x^6*(a + b*x^
3)^(1/2)*(2*A - (12*B*a)/(7*b)))/(15*b) + (2*B*x^9*(a + b*x^3)^(1/2))/(21*
b) - (4*a*x^3*(a + b*x^3)^(1/2)*(2*A - (12*B*a)/(7*b)))/(45*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.44

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{b\,x^3+a}\,(15b^3x^9+3a\,b^2x^6-4a^2b\,x^3+8a^3)}{315b^3}$$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*(8*a**3 - 4*a**2*b*x**3 + 3*a*b**2*x**6 + 15*b**3*x**9
))/(315*b**3)
```

3.193 $\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = -\frac{2a(Ab-aB)\sqrt{a+bx^3}}{3b^3} + \frac{2(Ab-2aB)\left(a+bx^3\right)^{3/2}}{9b^3} + \frac{2B\left(a+bx^3\right)^{5/2}}{15b^3}$$

output
```
-2/3*a*(A*b-B*a)*(b*x^3+a)^(1/2)/b^3+2/9*(A*b-2*B*a)*(b*x^3+a)^(3/2)/b^3+2
/15*B*(b*x^3+a)^(5/2)/b^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}(-10aAb+8a^2B+5Ab^2x^3-4abBx^3+3b^2Bx^6)}{45b^3}$$

input
```
Integrate[(x^5*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[a + b*x^3]*(-10*a*A*b + 8*a^2*B + 5*A*b^2*x^3 - 4*a*b*B*x^3 + 3*b^
2*B*x^6))/(45*b^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(A+B x^3\right)}{\sqrt{a+b x^3}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^3 \left(B x^3+A\right)}{\sqrt{b x^3+a}}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3} \int \left(\frac{B \left(b x^3+a\right)^{3/2}}{b^2}+\frac{(A b-2 a B) \sqrt{b x^3+a}}{b^2}+\frac{a (a B-A b)}{b^2 \sqrt{b x^3+a}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{2 \left(a+b x^3\right)^{3/2} (A b-2 a B)}{3 b^3}-\frac{2 a \sqrt{a+b x^3} (A b-a B)}{b^3}+\frac{2 B \left(a+b x^3\right)^{5/2}}{5 b^3}\right)$$

input
```
Int[(x^5*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
((-2*a*(A*b - a*B)*Sqrt[a + b*x^3])/b^3 + (2*(A*b - 2*a*B)*(a + b*x^3)^(3/
2))/(3*b^3) + (2*B*(a + b*x^3)^(5/2))/(5*b^3))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$-\frac{4\sqrt{b\,x^{3}+a}\,\left(-\frac{\left(\frac{3B\,x^{3}}{5}+A\right)x^{3}b^{2}}{2}+a\left(\frac{2B\,x^{3}}{5}+A\right)b-\frac{4a^{2}B}{5}\right)}{9b^{3}}$	49
gosper	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-3b^{2}B\,x^{6}-5A\,b^{2}x^{3}+4Bab\,x^{3}+10abA-8a^{2}B\right)}{45b^{3}}$	53
trager	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-3b^{2}B\,x^{6}-5A\,b^{2}x^{3}+4Bab\,x^{3}+10abA-8a^{2}B\right)}{45b^{3}}$	53
risch	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-3b^{2}B\,x^{6}-5A\,b^{2}x^{3}+4Bab\,x^{3}+10abA-8a^{2}B\right)}{45b^{3}}$	53
orering	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-3b^{2}B\,x^{6}-5A\,b^{2}x^{3}+4Bab\,x^{3}+10abA-8a^{2}B\right)}{45b^{3}}$	53
elliptic	$\frac{2B\,x^{6}\sqrt{b\,x^{3}+a}}{15b}+\frac{2\left(A-\frac{4Ba}{5b}\right)x^{3}\sqrt{b\,x^{3}+a}}{9b}-\frac{4a\left(A-\frac{4Ba}{5b}\right)\sqrt{b\,x^{3}+a}}{9b^{2}}$	70
default	$A\left(\frac{2x^{3}\sqrt{b\,x^{3}+a}}{9b}-\frac{4a\sqrt{b\,x^{3}+a}}{9b^{2}}\right)+B\left(\frac{2x^{6}\sqrt{b\,x^{3}+a}}{15b}-\frac{8a\,x^{3}\sqrt{b\,x^{3}+a}}{45b^{2}}+\frac{16a^{2}\sqrt{b\,x^{3}+a}}{45b^{3}}\right)$	92

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/9*(b*x^3+a)^(1/2)*(-1/2*(3/5*B*x^3+A)*x^3*b^2+a*(2/5*B*x^3+A)*b-4/5*a^2
*B)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\,(3\,Bb^2x^6-(4\,Bab-5\,Ab^2)x^3+8\,Ba^2-10\,Aab)\sqrt{bx^3+a}}{45\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(3*B*b^2*x^6 - (4*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 10*A*a*b)*sqrt(b*x
^3 + a)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.70

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \begin{cases} -\frac{4Aa\sqrt{a+bx^3}}{9b^2}+\frac{2Ax^3\sqrt{a+bx^3}}{9b}+\frac{16Ba^2\sqrt{a+bx^3}}{45b^3}-\frac{8Bax^3\sqrt{a+bx^3}}{45b^2}+\frac{2Bx^6\sqrt{a+bx^3}}{15b} & \text{for } b\neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((-4*A*a*sqrt(a + b*x**3)/(9*b**2) + 2*A*x**3*sqrt(a + b*x**3)/(9
*b) + 16*B*a**2*sqrt(a + b*x**3)/(45*b**3) - 8*B*a*x**3*sqrt(a + b*x**3)/(
45*b**2) + 2*B*x**6*sqrt(a + b*x**3)/(15*b), Ne(b, 0)), ((A*x**6/6 + B*x**
9/9)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2}{45}\,B\left(\frac{3\,(bx^3+a)^{\frac{5}{2}}}{b^3} - \frac{10\,(bx^3+a)^{\frac{3}{2}}a}{b^3} + \frac{15\,\sqrt{bx^3+a}a^2}{b^3}\right) + \frac{2}{9}\,A\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^2} - \frac{3\,\sqrt{bx^3+a}a}{b^2}\right)$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
2/45*B*(3*(b*x^3 + a)^(5/2)/b^3 - 10*(b*x^3 + a)^(3/2)*a/b^3 + 15*sqrt(b*x
^3 + a)*a^2/b^3) + 2/9*A*((b*x^3 + a)^(3/2)/b^2 - 3*sqrt(b*x^3 + a)*a/b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\,\sqrt{bx^3+a}(Ba^2-Aab)}{3\,b^3} + \frac{2\left(3\,(bx^3+a)^{\frac{5}{2}}B - 10\,(bx^3+a)^{\frac{3}{2}}Ba + 5\,(bx^3+a)^{\frac{3}{2}}Ab\right)}{45\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(b*x^3 + a)*(B*a^2 - A*a*b)/b^3 + 2/45*(3*(b*x^3 + a)^(5/2)*B - 10
*(b*x^3 + a)^(3/2)*B*a + 5*(b*x^3 + a)^(3/2)*A*b)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{b\,x^3+a}\,(8\,B\,a^2-4\,B\,a\,b\,x^3-10\,A\,a\,b+3\,B\,b^2\,x^6+5\,A\,b^2\,x^3)}{45\,b^3}$$

input
```
int((x^5*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

output
```
(2*(a + b*x^3)^(1/2)*(8*B*a^2 + 5*A*b^2*x^3 + 3*B*b^2*x^6 - 10*A*a*b - 4*B
*a*b*x^3))/(45*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.45

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{b\,x^3+a}\,(3b^2x^6+ab\,x^3-2a^2)}{45b^2}$$

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*( - 2*a**2 + a*b*x**3 + 3*b**2*x**6))/(45*b**2)
```

3.194 $\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}} dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}} dx = \frac{2(Ab-aB)\sqrt{a+bx^3}}{3b^2} + \frac{2B(a+bx^3)^{3/2}}{9b^2}$$

output
```
2/3*(A*b-B*a)*(b*x^3+a)^(1/2)/b^2+2/9*B*(b*x^3+a)^(3/2)/b^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}} dx = \frac{2\sqrt{a+bx^3}(3Ab-2aB+bBx^3)}{9b^2}$$

input
```
Integrate[(x^2*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[a + b*x^3]*(3*A*b - 2*a*B + b*B*x^3))/(9*b^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A + Bx^3)}{\sqrt{a + bx^3}} \, dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{3} \int \frac{Bx^3 + A}{\sqrt{bx^3 + a}} dx^3$$

$$\Big\downarrow \; 53$$

$$\frac{1}{3} \int \left(\frac{\sqrt{bx^3 + a}B}{b} + \frac{Ab - aB}{b\sqrt{bx^3 + a}} \right) dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3} \left(\frac{2\sqrt{a + bx^3}(Ab - aB)}{b^2} + \frac{2B(a + bx^3)^{3/2}}{3b^2} \right)$$

input
```
Int[(x^2*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
((2*(A*b - a*B)*Sqrt[a + b*x^3])/b^2 + (2*B*(a + b*x^3)^(3/2))/(3*b^2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{2\sqrt{b\,x^{3}+a}\,\left(bB\,x^{3}+3Ab-2Ba\right)}{9b^{2}}$	30
trager	$\frac{2\sqrt{b\,x^{3}+a}\,\left(bB\,x^{3}+3Ab-2Ba\right)}{9b^{2}}$	30
risch	$\frac{2\sqrt{b\,x^{3}+a}\,\left(bB\,x^{3}+3Ab-2Ba\right)}{9b^{2}}$	30
pseudoelliptic	$\frac{2\left(\left(\frac{B\,x^{3}}{3}+A\right)b-\frac{2Ba}{3}\right)\sqrt{b\,x^{3}+a}}{3b^{2}}$	30
orering	$\frac{2\sqrt{b\,x^{3}+a}\,\left(bB\,x^{3}+3Ab-2Ba\right)}{9b^{2}}$	30
elliptic	$\frac{2B\,x^{3}\sqrt{b\,x^{3}+a}}{9b}+\frac{2\left(A-\frac{2Ba}{3b}\right)\sqrt{b\,x^{3}+a}}{3b}$	43
default	$\frac{2A\sqrt{b\,x^{3}+a}}{3b}+B\left(\frac{2x^{3}\sqrt{b\,x^{3}+a}}{9b}-\frac{4a\sqrt{b\,x^{3}+a}}{9b^{2}}\right)$	52

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(b*x^3+a)^(1/2)*(B*b*x^3+3*A*b-2*B*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\,(Bbx^3 - 2\,Ba + 3\,Ab)\sqrt{bx^3+a}}{9\,b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(B*b*x^3 - 2*B*a + 3*A*b)*sqrt(b*x^3 + a)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.63

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \begin{cases} \frac{2A\sqrt{a+bx^3}}{3b} - \frac{4Ba\sqrt{a+bx^3}}{9b^2} + \frac{2Bx^3\sqrt{a+bx^3}}{9b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^3}{3}+\frac{Bx^6}{6}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((2*A*sqrt(a + b*x**3)/(3*b) - 4*B*a*sqrt(a + b*x**3)/(9*b**2) +
2*B*x**3*sqrt(a + b*x**3)/(9*b), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/sqrt(a)
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2}{9}\,B\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^2} - \frac{3\,\sqrt{bx^3+a}a}{b^2}\right) + \frac{2\,\sqrt{bx^3+a}A}{3\,b}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
2/9*B*((b*x^3 + a)^(3/2)/b^2 - 3*sqrt(b*x^3 + a)*a/b^2) + 2/3*sqrt(b*x^3 +
 a)*A/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\,(bx^3+a)^{\frac{3}{2}}B}{9\,b^2} - \frac{2\,\sqrt{bx^3+a}(Ba-Ab)}{3\,b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
2/9*(b*x^3 + a)^(3/2)*B/b^2 - 2/3*sqrt(b*x^3 + a)*(B*a - A*b)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\,\sqrt{b\,x^3+a}\,(B\,b\,x^3+3\,A\,b-2\,B\,a)}{9\,b^2}$$

input
```
int((x^2*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

output
```
(2*(a + b*x^3)^(1/2)*(3*A*b - 2*B*a + B*b*x^3))/(9*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.43

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{b\,x^3+a}\,(b\,x^3+a)}{9b}$$

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

output

```
(2*sqrt(a + b*x**3)*(a + b*x**3))/(9*b)
```

3.195 $\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx=\frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

output
```
2/3*B*(b*x^3+a)^(1/2)/b-2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx=\frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Integrate[(A + B*x^3)/(x*Sqrt[a + b*x^3]),x]
```

output
```
(2*B*Sqrt[a + b*x^3])/(3*b) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sq
rt[a])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {948, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^3\sqrt{bx^3+a}}dx^3$$

$$\Big\downarrow \text{ 90}$$

$$\frac{1}{3}\left(A\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+\frac{2B\sqrt{a+bx^3}}{b}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(\frac{2A\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{b}+\frac{2B\sqrt{a+bx^3}}{b}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{3}\left(\frac{2B\sqrt{a+bx^3}}{b}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$$

input
```
Int[(A + B*x^3)/(x*Sqrt[a + b*x^3]),x]
```

output
```
((2*B*Sqrt[a + b*x^3])/b - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/Sqrt[a])
/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{2B\sqrt{b\,x^{3}+a}}{3b}-\frac{2A\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$	37
elliptic	$\frac{2B\sqrt{b\,x^{3}+a}}{3b}-\frac{2A\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$	37
pseudoelliptic	$\frac{\frac{2B\sqrt{b\,x^{3}+a}\,\sqrt{a}}{3}-\frac{2Ab\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3}}{\sqrt{a}\,b}$	42

input
```
int((B*x^3+A)/x/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*B*(b*x^3+a)^(1/2)/b-2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.12

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx = \left[\frac{A\sqrt{a}b\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)+2\sqrt{bx^3+a}Ba}{3ab}, \frac{2\left(A\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^3+a}}\right)+\sqrt{bx^3+a}Ba\right)}{3ab}\right]$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(A*sqrt(a)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*s
qrt(b*x^3 + a)*B*a)/(a*b), 2/3*(A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*x^3 +
a)) + sqrt(b*x^3 + a)*B*a)/(a*b)]
```

Sympy [A] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx = \frac{A\left(\begin{cases}\frac{2\,\mathrm{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for } b\neq 0\\ -\frac{\log\left(\frac{1}{x^3}\right)}{\sqrt{a}} & \text{otherwise}\end{cases}\right)}{3} - \frac{B\left(\begin{cases}-\frac{x^3}{\sqrt{a}} & \text{for } b=0\\ -\frac{2\sqrt{a+bx^3}}{b} & \text{otherwise}\end{cases}\right)}{3}$$

input
```
integrate((B*x**3+A)/x/(b*x**3+a)**(1/2),x)
```

output
```
A*Piecewise((2*atan(sqrt(a + b*x**3)/sqrt(-a))/sqrt(-a), Ne(b, 0)), (-log(
x**(-3))/sqrt(a), True))/3 - B*Piecewise((-x**3/sqrt(a), Eq(b, 0)), (-2*sq
rt(a + b*x**3)/b, True))/3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx = \frac{A\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{3\sqrt{a}} + \frac{2\sqrt{bx^3+a}B}{3b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*A*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
 + 2/3*sqrt(b*x^3 + a)*B/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx = \frac{2A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}} + \frac{2\sqrt{bx^3+a}B}{3b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/3*sqrt(b*x^3 + a)*B/b
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx = \frac{2\,B\,\sqrt{b\,x^3+a}}{3\,b}+\frac{A\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,\sqrt{a}}$$

input
```
int((A + B*x^3)/(x*(a + b*x^3)^(1/2)),x)
```

output
```
(2*B*(a + b*x^3)^(1/2))/(3*b) + (A*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((
a + b*x^3)^(1/2) + a^(1/2)))/x^6))/(3*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{b\,x^3+a}}{3}+\frac{\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{3}-\frac{\sqrt{a}\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{3}$$

input
```
int((B*x^3+A)/x/(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3) + sqrt(a)*log(sqrt(a + b*x**3) - sqrt(a)) - sqrt(a)*lo
g(sqrt(a + b*x**3) + sqrt(a)))/3
```

3.196 $\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx=-\frac{A\sqrt{a+bx^3}}{3ax^3}+\frac{(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

output
```
-1/3*A*(b*x^3+a)^(1/2)/a/x^3+1/3*(A*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/
2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx=-\frac{A\sqrt{a+bx^3}}{3ax^3}+\frac{(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

input
```
Integrate[(A + B*x^3)/(x^4*Sqrt[a + b*x^3]),x]
```

output
```
-1/3*(A*Sqrt[a + b*x^3])/(a*x^3) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/
Sqrt[a]])/(3*a^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^6\sqrt{bx^3+a}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(-\frac{(Ab-2aB)\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{2a}-\frac{A\sqrt{a+bx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{(Ab-2aB)\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{ab}-\frac{A\sqrt{a+bx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{A\sqrt{a+bx^3}}{ax^3}\right)$$

input
```
Int[(A + B*x^3)/(x^4*Sqrt[a + b*x^3]),x]
```

output
```
(-((A*Sqrt[a + b*x^3])/(a*x^3)) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/S
qrt[a]])/a^(3/2))/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{A\sqrt{b\,x^{3}+a}}{3a\,x^{3}}+\frac{(Ab-2Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}$	47
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{3a\,x^{3}}+\frac{(Ab-2Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}$	47
pseudoelliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{3a\,x^{3}}+\frac{(Ab-2Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}$	47
default	$A\left(-\frac{\sqrt{b\,x^{3}+a}}{3a\,x^{3}}+\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}\right)-\frac{2B\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$	62

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*A*(b*x^3+a)^(1/2)/a/x^3+1/3*(A*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/
2))/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.12

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx = \left[-\frac{(2\,Ba-Ab)\sqrt{a}x^3\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,\sqrt{bx^3+a}Aa}{6\,a^2x^3}, \frac{(2\,Ba-Ab)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^3+a}}\right)-}{3\,a^2x^3}\right.$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/6*((2*B*a - A*b)*sqrt(a)*x^3*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) +
2*a)/x^3) + 2*sqrt(b*x^3 + a)*A*a)/(a^2*x^3), 1/3*((2*B*a - A*b)*sqrt(-a)*
x^3*arctan(sqrt(-a)/sqrt(b*x^3 + a)) - sqrt(b*x^3 + a)*A*a)/(a^2*x^3)]
```

Sympy [A] (verification not implemented)

Time = 7.86 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ax^{\frac{3}{2}}} + \frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3a^{\frac{3}{2}}} - \frac{2B\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

input
```
integrate((B*x**3+A)/x**4/(b*x**3+a)**(1/2),x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*x**(3/2)) + A*b*asinh(sqrt(a)/(sqrt(b
)*x**(3/2)))/(3*a**(3/2)) - 2*B*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(
a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(47) = 94.

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.88

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx = -\frac{1}{6}A\left(\frac{2\sqrt{bx^3+a}b}{(bx^3+a)a-a^2} + \frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right) + \frac{B\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*A*(2*sqrt(b*x^3 + a)*b/((b*x^3 + a)*a - a^2) + b*log((sqrt(b*x^3 + a)
 - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)) + 1/3*B*log((sqrt(b*x^3
+ a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx = \frac{1}{3}\,b\left(\frac{(2\,Ba-Ab)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}ab}-\frac{\sqrt{bx^3+a}A}{abx^3}\right)$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/3*b*((2*B*a - A*b)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a*b) - sqr
t(b*x^3 + a)*A/(a*b*x^3))
```

Mupad [B] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)(A\,b-2\,B\,a)}{6\,a^{3/2}}-\frac{A\,\sqrt{b\,x^3+a}}{3\,a\,x^3}$$

input
```
int((A + B*x^3)/(x^4*(a + b*x^3)^(1/2)),x)
```

output
```
(log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*
(A*b - 2*B*a))/(6*a^(3/2)) - (A*(a + b*x^3)^(1/2))/(3*a*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx = \frac{-2\sqrt{b\,x^3+a}\,a+\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)b\,x^3-\sqrt{a}\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)b\,x^3}{6a\,x^3}$$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x**3)*a + sqrt(a)*log(sqrt(a + b*x**3) - sqrt(a))*b*x**3
- sqrt(a)*log(sqrt(a + b*x**3) + sqrt(a))*b*x**3)/(6*a*x**3)
```

3.197 $\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx=-\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3}-\frac{b(3Ab-4aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}$$

output
```
-1/6*A*(b*x^3+a)^(1/2)/a/x^6+1/12*(3*A*b-4*B*a)*(b*x^3+a)^(1/2)/a^2/x^3-1/
12*b*(3*A*b-4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx=\frac{\sqrt{a+bx^3}(-2aA+3Abx^3-4aBx^3)}{12a^2x^6}+\frac{b(-3Ab+4aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}$$

input
```
Integrate[(A + B*x^3)/(x^7*Sqrt[a + b*x^3]),x]
```

output
```
(Sqrt[a + b*x^3]*(-2*a*A + 3*A*b*x^3 - 4*a*B*x^3))/(12*a^2*x^6) + (b*(-3*A
*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(5/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {948, 87, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^9\sqrt{bx^3+a}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(-\frac{(3Ab-4aB)\int \frac{1}{x^6\sqrt{bx^3+a}}dx^3}{4a}-\frac{A\sqrt{a+bx^3}}{2ax^6}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(-\frac{(3Ab-4aB)\left(-\frac{b\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{2a}-\frac{\sqrt{a+bx^3}}{ax^3}\right)}{4a}-\frac{A\sqrt{a+bx^3}}{2ax^6}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{(3Ab-4aB)\left(-\frac{\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{a}-\frac{\sqrt{a+bx^3}}{ax^3}\right)}{4a}-\frac{A\sqrt{a+bx^3}}{2ax^6}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{(3Ab-4aB)\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^3}}{ax^3}\right)}{4a}-\frac{A\sqrt{a+bx^3}}{2ax^6}\right)$$

input
```
Int[(A + B*x^3)/(x^7*Sqrt[a + b*x^3]),x]
```

output
```
(-1/2*(A*Sqrt[a + b*x^3])/(a*x^6) - ((3*A*b - 4*a*B)*(-(Sqrt[a + b*x^3]/(a
*x^3)) + (b*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/a^(3/2)))/(4*a))/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-3 A b\,x^{3}+4 B a\,x^{3}+2 A a \right)}{12 a^{2} x^{6}}-\frac{b \left(3 A b -4 B a \right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{12 a^{\frac{5}{2}}}$	67
pseudoelliptic	$\frac{-b \left(A b -\frac{4 B a}{3}\right) x^{6} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)+\sqrt{b\,x^{3}+a}\,\left(\frac{2 \left(-2 B\,x^{3}-A \right) a^{\frac{3}{2}}}{3}+A \sqrt{a}\, b\,x^{3}\right)}{4 a^{\frac{5}{2}} x^{6}}$	73
elliptic	$-\frac{A \sqrt{b\,x^{3}+a}}{6 a\,x^{6}}+\frac{\left(3 A b -4 B a \right) \sqrt{b\,x^{3}+a}}{12 a^{2} x^{3}}-\frac{b \left(3 A b -4 B a \right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{12 a^{\frac{5}{2}}}$	75
default	$A \left(-\frac{\sqrt{b\,x^{3}+a}}{6 a\,x^{6}}+\frac{b \sqrt{b\,x^{3}+a}}{4 a^{2} x^{3}}-\frac{b^{2} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{4 a^{\frac{5}{2}}}\right)+B \left(-\frac{\sqrt{b\,x^{3}+a}}{3 a\,x^{3}}+\frac{b \,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3 a^{\frac{3}{2}}}\right)$	102

input
```
int((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b*x^3+a)^(1/2)*(-3*A*b*x^3+4*B*a*x^3+2*A*a)/a^2/x^6-1/12*b*(3*A*b-4
*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.89

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx = \left[-\frac{(4\,Bab-3\,Ab^2)\sqrt{a}x^6\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,((4\,Ba^2-3\,Aab)x^3+2\,Aa^2)\sqrt{bx^3+a}}{24\,a^3x^6}, -\frac{(4\,Bab-3\,Ab^2)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^3+a}}\right)+((4\,Ba^2-3\,Aab)x^3+2\,Aa^2)\sqrt{bx^3+a}}{12\,a^3x^6}\right]$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/24*((4*B*a*b - 3*A*b^2)*sqrt(a)*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqr
t(a) + 2*a)/x^3) + 2*((4*B*a^2 - 3*A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/
(a^3*x^6), -1/12*((4*B*a*b - 3*A*b^2)*sqrt(-a)*x^6*arctan(sqrt(-a)/sqrt(b*
x^3 + a)) + ((4*B*a^2 - 3*A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^3*x^6)
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(80) = 160$.

Time = 19.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.81

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx = -\frac{A}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{A\sqrt{b}}{12ax^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{Ab^{\frac{3}{2}}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} -\frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4a^{\frac{5}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ax^{\frac{3}{2}}}+\frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3a^{\frac{3}{2}}}$$

input
```
integrate((B*x**3+A)/x**7/(b*x**3+a)**(1/2),x)
```

output
```
-A/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) + A*sqrt(b)/(12*a*x**(9/2)*s
qrt(a/(b*x**3) + 1)) + A*b**(3/2)/(4*a**2*x**(3/2)*sqrt(a/(b*x**3) + 1)) -
 A*b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*a**(5/2)) - B*sqrt(b)*sqrt(a/
(b*x**3) + 1)/(3*a*x**(3/2)) + B*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*a*
*(3/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(74) = 148$.

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.98

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx$$
$$= -\frac{1}{6}\,B\left(\frac{2\,\sqrt{bx^3+a}b}{(bx^3+a)a-a^2} + \frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right) + \frac{1}{24}\,A\left(\frac{3\,b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{2\left(3\,(bx^3+a)^{\frac{3}{2}}b^2-5\,\sqrt{bx^3+a}ab^2\right)}{(bx^3+a)^2a^2-2\,(bx^3+a)a^3+a^4}\right)$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*B*(2*sqrt(b*x^3 + a)*b/((b*x^3 + a)*a - a^2) + b*log((sqrt(b*x^3 + a)
 - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)) + 1/24*A*(3*b^2*log((sqr
t(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/2) + 2*(3*(b*x^3
 + a)^(3/2)*b^2 - 5*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2*a^2 - 2*(b*x^3 +
 a)*a^3 + a^4))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx = -\frac{\frac{(4\,Bab^2-3\,Ab^3)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{4\,(bx^3+a)^{\frac{3}{2}}Bab^2-4\,\sqrt{bx^3+a}Ba^2b^2-3\,(bx^3+a)^{\frac{3}{2}}Ab^3+5\,\sqrt{bx^3+a}Aab^3}{a^2b^2x^6}}{12\,b}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
-1/12*((4*B*a*b^2 - 3*A*b^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^
2) + (4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a^2*b^2 - 3*(b*x^3
 + a)^(3/2)*A*b^3 + 5*sqrt(b*x^3 + a)*A*a*b^3)/(a^2*b^2*x^6))/b
```

Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx = \frac{\sqrt{b\,x^3+a}\,(3\,A\,b-4\,B\,a)}{12\,a^2\,x^3}-\frac{A\,\sqrt{b\,x^3+a}}{6\,a\,x^6}+\frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\,(3\,A\,b-4\,B\,a)}{24\,a^{5/2}}$$

input
```
int((A + B*x^3)/(x^7*(a + b*x^3)^(1/2)),x)
```

output
```
((a + b*x^3)^(1/2)*(3*A*b - 4*B*a))/(12*a^2*x^3) - (A*(a + b*x^3)^(1/2))/(
6*a*x^6) + (b*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^
(1/2)))/x^6)*(3*A*b - 4*B*a))/(24*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx$$
$$= \frac{-4\sqrt{b\,x^3+a}\,a^2-2\sqrt{b\,x^3+a}\,ab\,x^3-\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)b^2x^6+\sqrt{a}\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)b^2x^6}{24a^2x^6}$$

input
```
int((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x**3)*a**2 - 2*sqrt(a + b*x**3)*a*b*x**3 - sqrt(a)*log(sq
rt(a + b*x**3) - sqrt(a))*b**2*x**6 + sqrt(a)*log(sqrt(a + b*x**3) + sqrt(
a))*b**2*x**6)/(24*a**2*x**6)
```

3.198 $\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 270

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2(11Ab-8aB)x\sqrt{a+bx^3}}{55b^2}+\frac{2Bx^4\sqrt{a+bx^3}}{11b} - \frac{4\sqrt{2+\sqrt{3}}a(11Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right), \ldots\right)}{55\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/55*(11*A*b-8*B*a)*x*(b*x^3+a)^(1/2)/b^2+2/11*B*x^4*(b*x^3+a)^(1/2)/b-4/1
65*(1/2*6^(1/2)+1/2*2^(1/2))*a*(11*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*
EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),
I*3^(1/2)+2*I)*3^(3/4)/b^(7/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a
^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.33

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2x\left(-\left((a+bx^3)\left(-11Ab+8aB-5bBx^3\right)\right)+a(-11Ab+8aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\right.\right.}{55b^2\sqrt{a+bx^3}}$$

input
```
Integrate[(x^3*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(2*x*(-((a + b*x^3)*(-11*A*b + 8*a*B - 5*b*B*x^3)) + a*(-11*A*b + 8*a*B)*S
qrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(55*b^
2*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {959, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(11Ab-8aB)\int \frac{x^3}{\sqrt{bx^3+a}}dx}{11b}+\frac{2Bx^4\sqrt{a+bx^3}}{11b}$$

$$\Big\downarrow \; 843$$

$$\frac{(11Ab-8aB)\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{2a\int \frac{1}{\sqrt{bx^3+a}}dx}{5b}\right)}{11b}+\frac{2Bx^4\sqrt{a+bx^3}}{11b}$$

$$\Big\downarrow \text{759}$$

$$\frac{(11Ab-8aB)\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{11b}$$

$$\frac{2Bx^4\sqrt{a+bx^3}}{11b}$$

input
```
Int[(x^3*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(2*B*x^4*Sqrt[a + b*x^3])/(11*b) + ((11*A*b - 8*a*B)*((2*x*Sqrt[a + b*x^3]
)/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(11*b
)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.20

method	result
risch	$\frac{2x(5bBx^3+11Ab-8Ba)\sqrt{bx^3+a}}{55b^2}+\frac{4i(11Ab-8Ba)a\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(\cdots}{\cdots}}}\cdots}{\cdots}$
elliptic	$\frac{2B\,x^4\sqrt{b\,x^3+a}}{11b}+\frac{2\left(A-\frac{8Ba}{11b}\right)x\sqrt{b\,x^3+a}}{5b}+\frac{4ia\left(A-\frac{8Ba}{11b}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-(\cdots}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\cdots}}\cdots}{\cdots}$
default	$A\left(\frac{2x\sqrt{b\,x^3+a}}{5b}+\frac{4ia\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\cdots\right)}{(-(\cdots}}\cdots}{15b^2\cdots}\cdots\right.$

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/55*x*(5*B*b*x^3+11*A*b-8*B*a)*(b*x^3+a)^(1/2)/b^2+4/165*I*(11*A*b-8*B*a)
*a/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.25

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\left(2\,(8\,Ba^2-11\,Aab)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(5\,Bb^2x^4-(8\,Bab-11\,Ab^2)x)\sqrt{bx^3+a}\right)}{55\,b^3}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/55*(2*(8*B*a^2 - 11*A*a*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (
5*B*b^2*x^4 - (8*B*a*b - 11*A*b^2)*x)*sqrt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.30

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{Ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)}+\frac{Bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**3*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
A*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*s
qrt(a)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*gamma(10/3))
```

Maxima [F]

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)x^3}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^3/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)x^3}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^3/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{x^3\,(B\,x^3+A)}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((x^3*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

output
```
int((x^3*(A + B*x^3))/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{\frac{6\sqrt{b\,x^3+a}\,ax}{55}+\frac{2\sqrt{b\,x^3+a}\,b\,x^4}{11}-\frac{6\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a^2}{55}}{b}$$

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

output
```
(2*(3*sqrt(a + b*x**3)*a*x + 5*sqrt(a + b*x**3)*b*x**4 - 3*int(sqrt(a + b*
x**3)/(a + b*x**3),x)*a**2))/(55*b)
```

3.199 $\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 239

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \frac{2Bx\sqrt{a+bx^3}}{5b} + \frac{2\sqrt{2+\sqrt{3}}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-\right.}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/5*B*x*(b*x^3+a)^(1/2)/b+2/15*(1/2*6^(1/2)+1/2*2^(1/2))*(5*A*b-2*B*a)*(a^
(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(
1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(
1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(4/3)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \frac{2Bx(a+bx^3)+(5Ab-2aB)x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/Sqrt[a + b*x^3],x]
```

output
```
(2*B*x*(a + b*x^3) + (5*A*b - 2*a*B)*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(5*b*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {913, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{ 913}$$

$$\frac{(5Ab-2aB)\int \frac{1}{\sqrt{bx^3+a}}dx}{5b}+\frac{2Bx\sqrt{a+bx^3}}{5b}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-2aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$\frac{2Bx\sqrt{a+bx^3}}{5b}$$

input
```
Int[(A + B*x^3)/Sqrt[a + b*x^3],x]
```

output
```
(2*B*x*Sqrt[a + b*x^3])/(5*b) + (2*Sqrt[2 + Sqrt[3]]*(5*A*b - 2*a*B)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4
)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.29

method	result
risch	$\frac{2Bx\sqrt{b\,x^{3}+a}}{5b}-\frac{2i(5Ab-2Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a \cdots\right.}{\cdots}\right.}{\cdots}}\cdots}{15b\cdots}$
elliptic	$\frac{2Bx\sqrt{b\,x^{3}+a}}{5b}-\frac{2i\left(A-\frac{2Ba}{5b}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b \cdots\right.}{2\cdots}\right.}{\cdots}}\cdots}{3b\cdots}$
default	$-\frac{2iA\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\cdots}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\cdots}{3b\sqrt{b\,x^{3}+a}\cdots}$

input
```
int((B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*B*x*(b*x^3+a)^(1/2)/b-2/15*I*(5*A*b-2*B*a)/b^2*3^(1/2)*(-a*b^2)^(1/3)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.18

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \frac{2\left(\sqrt{bx^3+a}Bbx-(2\,Ba-5\,Ab)\sqrt{b}\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{5\,b^2}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(sqrt(b*x^3 + a)*B*b*x - (2*B*a - 5*A*b)*sqrt(b)*weierstrassPInverse(0
, -4*a/b, x))/b^2
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.33

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
A*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*sqrt(a)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \int \frac{B\,x^3+A}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((A + B*x^3)/(a + b*x^3)^(1/2),x)
```

output
```
int((A + B*x^3)/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx=\frac{2\sqrt{b\,x^3+a}\,x}{5}+\frac{3\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a}{5}$$

input
```
int((B*x^3+A)/(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*x + 3*int(sqrt(a + b*x**3)/(a + b*x**3),x)*a)/5
```

3.200 $\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 243

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{2ax^2} - \frac{\sqrt{2+\sqrt{3}}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7}{2\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-1/2*A*(b*x^3+a)^(1/2)/a/x^2-1/6*(1/2*6^(1/2)+1/2*2^(1/2))*(A*b-4*B*a)*(a^
(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(
1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(
1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a/b^(1/3)/(a^(1/3)*(a^(1/3
)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.32

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx = \frac{-2A(a+bx^3)+(-Ab+4aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)}{4ax^2\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^3*Sqrt[a + b*x^3]),x]
```

output
```
(-2*A*(a + b*x^3) + (-(A*b) + 4*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometri
c2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(4*a*x^2*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {955, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \ 955$$

$$-\frac{(Ab-4aB)\int \frac{1}{\sqrt{bx^3+a}}dx}{4a}-\frac{A\sqrt{a+bx^3}}{2ax^2}$$

$$\Big\downarrow \ 759$$

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-4aB)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$\frac{A\sqrt{a+bx^3}}{2ax^2}$$

input
```
Int[(A + B*x^3)/(x^3*Sqrt[a + b*x^3]),x]
```

output
```
-1/2*(A*Sqrt[a + b*x^3])/(a*x^2) - (Sqrt[2 + Sqrt[3]]*(A*b - 4*a*B)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)
*a*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.87 (sec), antiderivative size = 311, normalized size of antiderivative = 1.28

method	result
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{2a\,x^2}-\frac{2i\left(B-\frac{Ab}{4a}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right.}{2b}\right.}{}}}{3b\sqrt{}}$
risch	$-\frac{A\sqrt{b\,x^3+a}}{2a\,x^2}+\frac{i(Ab-4Ba)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right.}{2b}\right.}{}}}{6ab}$
default	$-\frac{2iB\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{3b\sqrt{b\,x^3+a}}$

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*A*(b*x^3+a)^(1/2)/a/x^2-2/3*I*(B-1/4*A/a*b)*3^(1/2)/b*(-a*b^2)^(1/3)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.21

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx = \frac{(4\,Ba-Ab)\sqrt{b}x^2 \text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\sqrt{bx^3+a}Ab}{2\,abx^2}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((4*B*a - A*b)*sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x) - sqrt(b*
x^3 + a)*A*b)/(a*b*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.34

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx = \frac{A\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^2\Gamma\left(\frac{1}{3}\right)}+\frac{Bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**3/(b*x**3+a)**(1/2),x)
```

output
```
A*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*x**2*gamma(1/3)) + B*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}x^3}\,dx$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^3), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}x^3}\,dx$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{x^3\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int((A + B*x^3)/(x^3*(a + b*x^3)^(1/2)),x)
```

output
```
int((A + B*x^3)/(x^3*(a + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx = \frac{-2\sqrt{b\,x^3+a}-3\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^6+a\,x^3}dx\right)a\,x^2}{x^2}$$

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x**3) - 3*int(sqrt(a + b*x**3)/(a*x**3 + b*x**6),x)*a*x**
2)/x**2
```

3.201 $\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 274

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{5ax^5}+\frac{(7Ab-10aB)\sqrt{a+bx^3}}{20a^2x^2} + \frac{\sqrt{2+\sqrt{3}}b^{2/3}(7Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{20\sqrt[4]{3}a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/5*A*(b*x^3+a)^(1/2)/a/x^5+1/20*(7*A*b-10*B*a)*(b*x^3+a)^(1/2)/a^2/x^2+1
/60*(1/2*6^(1/2)+1/2*2^(1/2))*b^(2/3)*(7*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)
^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1
/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^2/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2)
)*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.28

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx = \frac{-4A(a+bx^3)+(7Ab-10aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{20ax^5\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^6*Sqrt[a + b*x^3]),x]
```

output
```
(-4*A*(a + b*x^3) + (7*A*b - 10*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometri
c2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)])/(20*a*x^5*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(7Ab-10aB)\int \frac{1}{x^3\sqrt{bx^3+a}}dx}{10a}-\frac{A\sqrt{a+bx^3}}{5ax^5}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{(7Ab-10aB)\left(-\frac{b\int \frac{1}{\sqrt{bx^3+a}}dx}{4a}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)}{10a}-\frac{A\sqrt{a+bx^3}}{5ax^5}$$

$$\Big\downarrow \text{ 759}$$

$$-\frac{(7Ab-10aB)\left(-\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)}{10a}$$

$$\frac{A\sqrt{a+bx^3}}{5ax^5}$$

input
```
Int[(A + B*x^3)/(x^6*Sqrt[a + b*x^3]),x]
```

output
```
-1/5*(A*Sqrt[a + b*x^3])/(a*x^5) - ((7*A*b - 10*a*B)*(-1/2*Sqrt[a + b*x^3]
/(a*x^2) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(10*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-7Ab\,x^{3}+10Ba\,x^{3}+4Aa\right)}{20a^{2}x^{5}}-\frac{i(7Ab-10Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{}}{}}}}{}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{5a\,x^{5}}+\frac{(7Ab-10Ba)\sqrt{b\,x^{3}+a}}{20a^{2}x^{2}}-\frac{i(7Ab-10Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b\right)}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i}{}}}}{}$
default	$A\left(-\frac{\sqrt{b\,x^{3}+a}}{5a\,x^{5}}+\frac{7b\sqrt{b\,x^{3}+a}}{20a^{2}x^{2}}-\frac{7ib\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{}}{}\right.$

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(b*x^3+a)^(1/2)*(-7*A*b*x^3+10*B*a*x^3+4*A*a)/a^2/x^5-1/60*I*(7*A*b-
10*B*a)/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.23

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx = -\frac{(10\,Ba-7\,Ab)\sqrt{b}x^5\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+((10\,Ba-7\,Ab)x^3+4\,Aa)\sqrt{bx^3+a}}{20\,a^2x^5}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/20*((10*B*a - 7*A*b)*sqrt(b)*x^5*weierstrassPInverse(0, -4*a/b, x) + ((
10*B*a - 7*A*b)*x^3 + 4*A*a)*sqrt(b*x^3 + a))/(a^2*x^5)
```

Sympy [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.33

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx = \frac{A\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{B\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**6/(b*x**3+a)**(1/2),x)
```

output
```
A*gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqr
t(a)*x**5*gamma(-2/3)) + B*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}x^6}\,dx$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^6), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}x^6}\,dx$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{x^6\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int((A + B*x^3)/(x^6*(a + b*x^3)^(1/2)),x)
```

output
```
int((A + B*x^3)/(x^6*(a + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx=\frac{-2\sqrt{b\,x^3+a}-3\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^9+a\,x^6}dx\right)a\,x^5}{7x^5}$$

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x**3) - 3*int(sqrt(a + b*x**3)/(a*x**6 + b*x**9),x)*a*x**
5)/(7*x**5)
```

3.202 $\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 548

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2(13Ab-10aB)x^2\sqrt{a+bx^3}}{91b^2} + \frac{2Bx^5\sqrt{a+bx^3}}{13b} - \frac{8a(13Ab-10aB)\sqrt{a+bx^3}}{91b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+ \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle| \right.}{91b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{8\sqrt{2}a^{4/3}(13Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right), - \right.}{91\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/91*(13*A*b-10*B*a)*x^2*(b*x^3+a)^(1/2)/b^2+2/13*B*x^5*(b*x^3+a)^(1/2)/b-
8/91*a*(13*A*b-10*B*a)*(b*x^3+a)^(1/2)/b^(8/3)/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x)+4/91*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(4/3)*(13*A*b-10*B*a)*(a^(1/
3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3
)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2
))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/
((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-8/273*2^(1/2)*a^(
4/3)*(13*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^
(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^
(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2
)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.17

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2x^2\left(-\left(\left(a+bx^3\right)\left(-13Ab+10aB-7bBx^3\right)\right)+a(-13Ab+10aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\right.\right.}{91b^2\sqrt{a+bx^3}}$$

input
```
Integrate[(x^4*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(2*x^2*(-((a + b*x^3)*(-13*A*b + 10*a*B - 7*b*B*x^3)) + a*(-13*A*b + 10*a*
B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(9
1*b^2*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.95 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {959, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (A+Bx^3)}{\sqrt{a+bx^3}} \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(13Ab-10aB)\int \frac{x^4}{\sqrt{bx^3+a}}dx}{13b}+\frac{2Bx^5\sqrt{a+bx^3}}{13b}$$

$$\Big\downarrow \text{843}$$

$$\frac{(13Ab-10aB)\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\int \frac{x}{\sqrt{bx^3+a}}dx}{7b}\right)}{13b}+\frac{2Bx^5\sqrt{a+bx^3}}{13b}$$

$$\Big\downarrow \text{832}$$

$$\frac{(13Ab-10aB)\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{7b}\right)}{13b}+\frac{2Bx^5\sqrt{a+bx^3}}{13b}$$

$$\Big\downarrow \text{759}$$

$$\frac{(13Ab-10aB)\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{arc}\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+b}}\right.}{7b}\right.}{13b}$$

$$\frac{2Bx^5\sqrt{a+bx^3}}{13b}$$

$\downarrow$ 2416

$$(13Ab-10aB)\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{}}{\sqrt[3]{}}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.}{}\right.}{1}$$

$$\frac{2Bx^5\sqrt{a+bx^3}}{13b}$$

input
```
Int[(x^4*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(2*B*x^5*Sqrt[a + b*x^3])/(13*b) + ((13*A*b - 10*a*B)*((2*x^2*Sqrt[a + b*x
^3])/(7*b) - (4*a*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/
b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]))/(7*b)))/(13*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 479, normalized size of antiderivative = 0.87

method	result
risch	$\frac{2x^2(7bBx^3+13Ab-10Ba)\sqrt{bx^3+a}}{91b^2} + \frac{8i(13Ab-10Ba)a\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-ab^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-ab^2}{b}}{-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{}}{}}}}{}$
elliptic	$\frac{2Bx^5\sqrt{bx^3+a}}{13b} + \frac{2\left(A-\frac{10Ba}{13b}\right)x^2\sqrt{bx^3+a}}{7b} + \frac{8ia\left(A-\frac{10Ba}{13b}\right)\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-ab^2)^{\frac{1}{3}}}}\sqrt{\frac{x-}{-\frac{3(-ab^2)}{2b}}}}{}$
default	Expression too large to display

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/91*x^2*(7*B*b*x^3+13*A*b-10*B*a)/b^2*(b*x^3+a)^(1/2)+8/273*I*(13*A*b-10*
B*a)*a/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^
(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.14

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = -\frac{2\left(4\,(10\,Ba^2-13\,Aab)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-(7\,Bb^2x^5-(10\right.}{91\,b^3}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/91*(4*(10*B*a^2 - 13*A*a*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstr
assPInverse(0, -4*a/b, x)) - (7*B*b^2*x^5 - (10*B*a*b - 13*A*b^2)*x^2)*sqr
t(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{Ax^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}+\frac{Bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{8}{3}\\\frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**4*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
A*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*s
qrt(a)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((1/2, 8/3), (11/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*gamma(11/3))
```

Maxima [F]

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)x^4}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^4/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)x^4}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^4/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{x^4\,(B\,x^3+A)}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((x^4*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

output
```
int((x^4*(A + B*x^3))/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{\frac{6\sqrt{b\,x^3+a}\,a\,x^2}{91}+\frac{2\sqrt{b\,x^3+a}\,b\,x^5}{13}-\frac{12\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a^2}{91}}{b}$$

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

output
```
(2*(3*sqrt(a + b*x**3)*a*x**2 + 7*sqrt(a + b*x**3)*b*x**5 - 6*int((sqrt(a
+ b*x**3)*x)/(a + b*x**3),x)*a**2))/(91*b)
```

3.203 $\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 517

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2Bx^2\sqrt{a+bx^3}}{7b} + \frac{2(7Ab-4aB)\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{2}\sqrt[3]{a}(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-\right.}{7\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/7*B*x^2*(b*x^3+a)^(1/2)/b+2/7*(7*A*b-4*B*a)*(b*x^3+a)^(1/2)/b^(5/3)/((1+
3^(1/2))*a^(1/3)+b^(1/3)*x)-1/7*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/3)*
(7*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b
^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(5/3)/(a^(1/3)*
(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/
2)+2/21*2^(1/2)*a^(1/3)*(7*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*Elliptic
F(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2
)+2*I)*3^(3/4)/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b
^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.15

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{x^2\left(4B(a+bx^3)+(7Ab-4aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{14b\sqrt{a+bx^3}}$$

input
```
Integrate[(x*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(x^2*(4*B*(a + b*x^3) + (7*A*b - 4*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(14*b*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {959, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(7Ab-4aB)\int \frac{x}{\sqrt{bx^3+a}}dx}{7b}+\frac{2Bx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{832}$$

$$\frac{(7Ab-4aB)\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{7b}+\frac{2Bx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{759}$$

$$\frac{(7Ab-4aB)\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+(1-}{\sqrt[3]{b}x+(1-}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{7b}$$

$$\frac{2Bx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{2416}$$

$$\frac{(7Ab-4aB)\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.}{7b}$$

$$\frac{2Bx^2\sqrt{a+bx^3}}{7b}$$

input
```
Int[(x*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(2*B*x^2*Sqrt[a + b*x^3])/(7*b) + ((7*A*b - 4*a*B)*(((2*Sqrt[a + b*x^3])/(
b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*
a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3
]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[
3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(7*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.90

method	result
risch	$\frac{2B\,x^2\sqrt{b\,x^3+a}}{7b} - \frac{2i(7Ab-4Ba)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-\right.}{}\right.}{}}}{}$
elliptic	$\frac{2B\,x^2\sqrt{b\,x^3+a}}{7b} - \frac{2i\left(A-\frac{4Ba}{7b}\right)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-\right.}{}\right.}{}}}{}$
default	Expression too large to display

input
```
int(x*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*B*x^2*(b*x^3+a)^(1/2)/b-2/21*I*(7*A*b-4*B*a)/b^2*3^(1/2)*(-a*b^2)^(1/3
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.10

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\left(\sqrt{bx^3+a}Bbx^2+(4\,Ba-7\,Ab)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)\right)}{7\,b^2}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/7*(sqrt(b*x^3 + a)*B*b*x^2 + (4*B*a - 7*A*b)*sqrt(b)*weierstrassZeta(0,
-4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b^2
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{Ax^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{Bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
A*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*s
qrt(a)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3))
```

Maxima [F]

$$\int \frac{x(A + Bx^3)}{\sqrt{a + bx^3}} \, dx = \int \frac{(Bx^3 + A)x}{\sqrt{bx^3 + a}} \, dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x(A + Bx^3)}{\sqrt{a + bx^3}} \, dx = \int \frac{(Bx^3 + A)x}{\sqrt{bx^3 + a}} \, dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{x\,(B\,x^3+A)}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((x*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

output
```
int((x*(A + B*x^3))/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\frac{2\sqrt{b\,x^3+a}\,x^2}{7}+\frac{3\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a}{7}$$

input
```
int(x*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*x**2 + 3*int((sqrt(a + b*x**3)*x)/(a + b*x**3),x)*a)/7
```

3.204 $\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 509

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{ax} + \frac{(Ab+2aB)\sqrt{a+bx^3}}{ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{}\right.}{2a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{\sqrt{2}(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right.}{\sqrt[4]{3}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-A*(b*x^3+a)^(1/2)/a/x+(A*b+2*B*a)*(b*x^3+a)^(1/2)/a/b^(2/3)/((1+3^(1/2))*
a^(1/3)+b^(1/3)*x)-1/2*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(A*b+2*B*a)*(a^(1
/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/
3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/
2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/a^(2/3)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^
(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+1/3*2^(1
/2)*(A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)
+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^(2/3)
/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(
1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = \frac{-4A(a+bx^3)+(Ab+2aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{4ax\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^2*Sqrt[a + b*x^3]),x]
```

output
```
(-4*A*(a + b*x^3) + (A*b + 2*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F
1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(4*a*x*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{ 955}$$

$$\frac{(2aB+Ab)\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{A\sqrt{a+bx^3}}{ax}$$

$$\Big\downarrow \text{ 832}$$

$$\frac{(2aB+Ab)\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{A\sqrt{a+bx^3}}{ax}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{(2aB+Ab)\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{\right.}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{\right.}}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{2a}$$

$$\frac{A\sqrt{a+bx^3}}{ax}$$

$$\Big\downarrow \text{ 2416}$$

$$\frac{(2aB+Ab)\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.}{2a}$$

$$\frac{A\sqrt{a+bx^3}}{ax}$$

input
```
Int[(A + B*x^3)/(x^2*Sqrt[a + b*x^3]),x]
```

output
```
-((A*Sqrt[a + b*x^3])/(a*x)) + ((A*b + 2*a*B)*(((2*Sqrt[a + b*x^3])/(b^(1/
3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/
3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a
^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]
])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(2*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.91

method	result
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{a x}-\frac{2i\left(B+\frac{Ab}{2a}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right.}{2b}\right.}{}}}{}$
risch	$-\frac{A\sqrt{b\,x^3+a}}{a x}-\frac{i(Ab+2Ba)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right.}{2b}\right.}{}}}{}$
default	Expression too large to display

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-A*(b*x^3+a)^(1/2)/a/x-2/3*I*(B+1/2*A/a*b)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.11

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = -\frac{(2\,Ba+Ab)\sqrt{b}x\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\sqrt{bx^3+a}Ab}{abx}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-((2*B*a + A*b)*sqrt(b)*x*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0
, -4*a/b, x)) + sqrt(b*x^3 + a)*A*b)/(a*b*x)
```

Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.16

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = \frac{A\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x\Gamma\left(\frac{2}{3}\right)}+\frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**2/(b*x**3+a)**(1/2),x)
```

output
```
A*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*x*gamma(2/3)) + B*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Maxima [F]

$$\int \frac{A + Bx^3}{x^2\sqrt{a + bx^3}}\, dx = \int \frac{Bx^3 + A}{\sqrt{bx^3 + ax^2}}\, dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^2), x)
```

Giac [F]

$$\int \frac{A + Bx^3}{x^2\sqrt{a + bx^3}}\, dx = \int \frac{Bx^3 + A}{\sqrt{bx^3 + ax^2}}\, dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^3}{x^2\sqrt{a + bx^3}}\, dx = \int \frac{B\,x^3 + A}{x^2\,\sqrt{b\,x^3 + a}}\, dx$$

input
```
int((A + B*x^3)/(x^2*(a + b*x^3)^(1/2)),x)
```

output
```
int((A + B*x^3)/(x^2*(a + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{b\,x^3+a}+3\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^5+a\,x^2}dx\right)a x}{x}$$

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3) + 3*int(sqrt(a + b*x**3)/(a*x**2 + b*x**5),x)*a*x)/x
```

3.205 $\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 550

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{4ax^4}+\frac{(5Ab-8aB)\sqrt{a+bx^3}}{8a^2x}-\frac{\sqrt[3]{b}(5Ab-8aB)\sqrt{a+bx^3}}{8a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(5Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\ldots}{16a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\sqrt[3]{b}(5Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{\ldots}\right.}{4\sqrt{2}\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/4*A*(b*x^3+a)^(1/2)/a/x^4+1/8*(5*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^2/x-1/8*b
^(1/3)*(5*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^2/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+1
/16*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*b^(1/3)*(5*A*b-8*B*a)*(a^(1/3)+b^(1/
3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/
3)+b^(1/3)*x),I*3^(1/2)+2*I)/a^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1
/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-1/24*b^(1/3)*(5*A*b-8*B*a
)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2)
)*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((
1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/a^(5/3)/(a^(1
/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx = \frac{-2A(a+bx^3)+(5Ab-8aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a}\right)}{8ax^4\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^5*Sqrt[a + b*x^3]),x]
```

output
```
(-2*A*(a + b*x^3) + (5*A*b - 8*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/(8*a*x^4*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.98 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {955, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(5Ab-8aB)\int \frac{1}{x^2\sqrt{bx^3+a}}dx}{8a}-\frac{A\sqrt{a+bx^3}}{4ax^4}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{(5Ab-8aB)\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{A\sqrt{a+bx^3}}{4ax^4}$$

$$\Big\downarrow \text{ 832}$$

$$-\frac{(5Ab-8aB)\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{A\sqrt{a+bx^3}}{4ax^4}$$

$$\Big\downarrow \text{ 759}$$

$$-\frac{(5Ab-8aB)\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right.}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{2a}}{8a}$$

$$\frac{A\sqrt{a+bx^3}}{4ax^4}$$

$\downarrow$ 2416

$$-\frac{(5Ab-8aB)\left(\frac{b\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}\right.}{8a}$$

$$\frac{A\sqrt{a+bx^3}}{4ax^4}$$

input
```
Int[(A + B*x^3)/(x^5*Sqrt[a + b*x^3]),x]
```

output
```
-1/4*(A*Sqrt[a + b*x^3])/(a*x^4) - ((5*A*b - 8*a*B)*(-(Sqrt[a + b*x^3]/(a*
x)) + (b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) -
 (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
)))/(2*a)))/(8*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 481, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-5Ab\,x^{3}+8Ba\,x^{3}+2Aa\right)}{8a^{2}x^{4}}+$ $\frac{i(5Ab-8Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(}{}}}}{}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{4a\,x^{4}}+\frac{(5Ab-8Ba)\sqrt{b\,x^{3}+a}}{8a^{2}x}+$ $\frac{i(5Ab-8Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}}}}{}$
default	Expression too large to display

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^3+a)^(1/2)*(-5*A*b*x^3+8*B*a*x^3+2*A*a)/a^2/x^4+1/24*I*(5*A*b-8*
B*a)/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1
/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.13

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx = -\frac{(8\,Ba-5\,Ab)\sqrt{b}x^4 \text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+((8\,Ba-5\,Ab)x^3+2\,A}{8\,a^2x^4}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/8*((8*B*a - 5*A*b)*sqrt(b)*x^4*weierstrassZeta(0, -4*a/b, weierstrassPI
nverse(0, -4*a/b, x)) + ((8*B*a - 5*A*b)*x^3 + 2*A*a)*sqrt(b*x^3 + a))/(a^
2*x^4)
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.16

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx = \frac{A\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{B\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**5/(b*x**3+a)**(1/2),x)
```

output
```
A*gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqr
t(a)*x**4*gamma(-1/3)) + B*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{A + Bx^3}{x^5\sqrt{a + bx^3}}\,dx = \int \frac{Bx^3 + A}{\sqrt{bx^3 + a}\,x^5}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^5), x)
```

Giac [F]

$$\int \frac{A + Bx^3}{x^5\sqrt{a + bx^3}}\,dx = \int \frac{Bx^3 + A}{\sqrt{bx^3 + a}\,x^5}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{x^5\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int((A + B*x^3)/(x^5*(a + b*x^3)^(1/2)),x)
```

output
```
int((A + B*x^3)/(x^5*(a + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx=\frac{-2\sqrt{b\,x^3+a}-3\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^8+a\,x^5}dx\right)a\,x^4}{5x^4}$$

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x**3) - 3*int(sqrt(a + b*x**3)/(a*x**5 + b*x**8),x)*a*x**
4)/(5*x**4)
```

3.206 $\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 581

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{7ax^7}+\frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4}$$

$$-\frac{5b(11Ab-14aB)\sqrt{a+bx^3}}{112a^3x}+\frac{5b^{4/3}(11Ab-14aB)\sqrt{a+bx^3}}{112a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{5\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(11Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|\right.}{224a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{5b^{4/3}(11Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-\right.}{56\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/7*A*(b*x^3+a)^(1/2)/a/x^7+1/56*(11*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^2/x^4-
5/112*b*(11*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^3/x+5/112*b^(4/3)*(11*A*b-14*B*a
)*(b*x^3+a)^(1/2)/a^3/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)-5/224*3^(1/4)*(1/2*6
^(1/2)-1/2*2^(1/2))*b^(4/3)*(11*A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*El
lipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*
3^(1/2)+2*I)/a^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(
1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+5/336*b^(4/3)*(11*A*b-14*B*a)*(a^(1/3)+b^
(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(
1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^
(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/a^(8/3)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.13

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx = \frac{-8A(a+bx^3)+(11Ab-14aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},-\frac{1}{3},-\frac{bx^3}{a}\right)}{56ax^7\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^8*Sqrt[a + b*x^3]),x]
```

output
```
(-8*A*(a + b*x^3) + (11*A*b - 14*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometr
ic2F1[-4/3, 1/2, -1/3, -((b*x^3)/a)])/(56*a*x^7*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.05 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 847, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(11Ab-14aB)\int \frac{1}{x^5\sqrt{bx^3+a}}dx}{14a}-\frac{A\sqrt{a+bx^3}}{7ax^7}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(11Ab-14aB)\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+a}}dx}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)}{14a}-\frac{A\sqrt{a+bx^3}}{7ax^7}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(11Ab-14aB)\left(-\frac{5b\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)}{14a}-\frac{A\sqrt{a+bx^3}}{7ax^7}$$

$$\Big\downarrow \text{832}$$

$$-\frac{(11Ab-14aB)\left(-\frac{5b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)}{14a}-\frac{A\sqrt{a+bx^3}}{7ax^7}$$

$$\Big\downarrow \ 759$$

$$-\frac{(11Ab-14aB)\left(-\frac{5b\left(\frac{b\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}\ldots}{\sqrt[3]{b}\ldots}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{2a}\right.}{8a}\right.}{14a}$$

$$\frac{A\sqrt{a+bx^3}}{7ax^7}$$

$\downarrow$ 2416

$$\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{\cdots}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{\cdots}\right.}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$b\,\frac{\left(\cdots\right.}{\sqrt[3]{b}}$$

$$5b\,\frac{\left(\cdots\right.}{\cdots}$$

$$(11Ab-14aB)\left(-\frac{\cdots}{\cdots}\right.$$

$$-\frac{\cdots}{\cdots}$$

$$\frac{A\sqrt{a+bx^3}}{7ax^7}$$

input
```
Int[(A + B*x^3)/(x^8*Sqrt[a + b*x^3]),x]
```

output
```
-1/7*(A*Sqrt[a + b*x^3])/(a*x^7) - ((11*A*b - 14*a*B)*(-1/4*Sqrt[a + b*x^3
]/(a*x^4) - (5*b*(-(Sqrt[a + b*x^3]/(a*x)) + (b*(((2*Sqrt[a + b*x^3])/(b^(
1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(
1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]
*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[
3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(2*a)))/(8*a)))/(14*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(55A\,b^{2}x^{6}-70Bab\,x^{6}-22aAb\,x^{3}+28B\,a^{2}x^{3}+16a^{2}A\right)}{112a^{3}x^{7}}-\frac{5ib(11Ab-14Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right.}{2b}\right.}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{7a\,x^{7}}+\frac{(11Ab-14Ba)\sqrt{b\,x^{3}+a}}{56a^{2}x^{4}}-\frac{5b(11Ab-14Ba)\sqrt{b\,x^{3}+a}}{112a^{3}x}-$
default	Expression too large to display

input
```
int((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/112*(b*x^3+a)^(1/2)*(55*A*b^2*x^6-70*B*a*b*x^6-22*A*a*b*x^3+28*B*a^2*x^
3+16*A*a^2)/a^3/x^7-5/336*I*b*(11*A*b-14*B*a)/a^3*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.17

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx$$
$$= \frac{5\left(14\,Bab-11\,Ab^2\right)\sqrt{b}x^7\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(5\left(14\,Bab-11\,Ab\right.\right.}{112\,a^3x^7}$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
1/112*(5*(14*B*a*b - 11*A*b^2)*sqrt(b)*x^7*weierstrassZeta(0, -4*a/b, weie
rstrassPInverse(0, -4*a/b, x)) + (5*(14*B*a*b - 11*A*b^2)*x^6 - 2*(14*B*a^
2 - 11*A*a*b)*x^3 - 16*A*a^2)*sqrt(b*x^3 + a))/(a^3*x^7)
```

Sympy [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.16

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx = \frac{A\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{B\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**8/(b*x**3+a)**(1/2),x)
```

output
```
A*gamma(-7/3)*hyper((-7/3, 1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqr
t(a)*x**7*gamma(-4/3)) + B*gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), b*x**3*
exp_polar(I*pi)/a)/(3*sqrt(a)*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}x^8}\,dx$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^8), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}x^8}\,dx$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{x^8\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int((A + B*x^3)/(x^8*(a + b*x^3)^(1/2)),x)
```

output
```
int((A + B*x^3)/(x^8*(a + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx=\frac{-2\sqrt{b\,x^3+a}-3\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^{11}+a\,x^8}dx\right)a\,x^7}{11x^7}$$

input
```
int((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x**3) - 3*int(sqrt(a + b*x**3)/(a*x**8 + b*x**11),x)*a*x*
*7)/(11*x**7)
```

3.207 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}} - \frac{2a(2Ab-3aB)\sqrt{a+bx^3}}{3b^4} + \frac{2(Ab-3aB)\left(a+bx^3\right)^{3/2}}{9b^4} + \frac{2B(a+bx^3)^{5/2}}{15b^4}$$

output
```
-2/3*a^2*(A*b-B*a)/b^4/(b*x^3+a)^(1/2)-2/3*a*(2*A*b-3*B*a)*(b*x^3+a)^(1/2)
/b^4+2/9*(A*b-3*B*a)*(b*x^3+a)^(3/2)/b^4+2/15*B*(b*x^3+a)^(5/2)/b^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(48a^3B-8a^2b(5A-3Bx^3)+b^3x^6(5A+3Bx^3)-2ab^2x^3(10A+3Bx^3))}{45b^4\sqrt{a+bx^3}}$$

input
```
Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*(48*a^3*B - 8*a^2*b*(5*A - 3*B*x^3) + b^3*x^6*(5*A + 3*B*x^3) - 2*a*b^2
*x^3*(10*A + 3*B*x^3)))/(45*b^4*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6(Bx^3+A)}{(bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(-\frac{(aB-Ab)a^2}{b^3(bx^3+a)^{3/2}}+\frac{(3aB-2Ab)a}{b^3\sqrt{bx^3+a}}+\frac{B(bx^3+a)^{3/2}}{b^3}+\frac{(Ab-3aB)\sqrt{bx^3+a}}{b^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{2a^2(Ab-aB)}{b^4\sqrt{a+bx^3}}+\frac{2(a+bx^3)^{3/2}(Ab-3aB)}{3b^4}-\frac{2a\sqrt{a+bx^3}(2Ab-3aB)}{b^4}+\frac{2B(a+bx^3)^{5/2}}{5b^4}\right)$$

input
```
Int[(x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
((-2*a^2*(A*b - a*B))/(b^4*Sqrt[a + b*x^3]) - (2*a*(2*A*b - 3*a*B)*Sqrt[a
+ b*x^3])/b^4 + (2*(A*b - 3*a*B)*(a + b*x^3)^(3/2))/(3*b^4) + (2*B*(a + b*
x^3)^(5/2))/(5*b^4))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{16\left(-\frac{\left(\frac{3B x^{3}}{5}+A\right) x^{6} b^{3}}{8}+\frac{a x^{3}\left(\frac{3B x^{3}}{10}+A\right) b^{2}}{2}+a^{2}\left(-\frac{3B x^{3}}{5}+A\right) b-\frac{6a^{3}B}{5}\right)}{9\sqrt{b x^{3}+a}\, b^{4}}$
gosper	$-\frac{2\left(-3b^{3}B x^{9}-5A b^{3}x^{6}+6Ba\, b^{2}x^{6}+20aA\, b^{2}x^{3}-24B\, a^{2}b\, x^{3}+40a^{2}bA-48a^{3}B\right)}{45\sqrt{b x^{3}+a}\, b^{4}}$
trager	$-\frac{2\left(-3b^{3}B x^{9}-5A b^{3}x^{6}+6Ba\, b^{2}x^{6}+20aA\, b^{2}x^{3}-24B\, a^{2}b\, x^{3}+40a^{2}bA-48a^{3}B\right)}{45\sqrt{b x^{3}+a}\, b^{4}}$
orering	$-\frac{2\left(-3b^{3}B x^{9}-5A b^{3}x^{6}+6Ba\, b^{2}x^{6}+20aA\, b^{2}x^{3}-24B\, a^{2}b\, x^{3}+40a^{2}bA-48a^{3}B\right)}{45\sqrt{b x^{3}+a}\, b^{4}}$
risch	$-\frac{2\left(-3b^{2}B x^{6}-5A b^{2}x^{3}+9Bab\, x^{3}+25abA-33a^{2}B\right)\sqrt{b x^{3}+a}}{45b^{4}}-\frac{2a^{2}(Ab-Ba)}{3b^{4}\sqrt{b x^{3}+a}}$
default	$A\left(-\frac{2a^{2}}{3b^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2x^{3}\sqrt{b x^{3}+a}}{9b^{2}}-\frac{10a\sqrt{b x^{3}+a}}{9b^{3}}\right)+B\left(\frac{2a^{3}}{3b^{4}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2x^{6}\sqrt{b x^{3}+a}}{15b^{2}}-\frac{2a\, x^{3}\sqrt{b x^{3}+a}}{5b^{3}}\right.$
elliptic	$-\frac{2a^{2}(Ab-Ba)}{3b^{4}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2B x^{6}\sqrt{b x^{3}+a}}{15b^{2}}+\frac{2\left(\frac{Ab-Ba}{b^{2}}-\frac{4Ba}{5b^{2}}\right)x^{3}\sqrt{b x^{3}+a}}{9b}+\frac{2\left(-\frac{a(Ab-Ba)}{b^{3}}-\frac{2\left(\frac{Ab-Ba}{b^{2}}-\frac{4Ba}{5b^{2}}\right)a}{3b}\right)\sqrt{b x^{3}+}}{3b}$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-16/9*(-1/8*(3/5*B*x^3+A)*x^6*b^3+1/2*a*x^3*(3/10*B*x^3+A)*b^2+a^2*(-3/5*B
*x^3+A)*b-6/5*a^3*B)/(b*x^3+a)^(1/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\,(3\,Bb^3x^9-(6\,Bab^2-5\,Ab^3)x^6+48\,Ba^3-40\,Aa^2b+4\,(6\,Ba^2b-5\,Aab^2)x^3)\sqrt{bx^3}}{45\,(b^5x^3+ab^4)}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/45*(3*B*b^3*x^9 - (6*B*a*b^2 - 5*A*b^3)*x^6 + 48*B*a^3 - 40*A*a^2*b + 4*
(6*B*a^2*b - 5*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/(b^5*x^3 + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.70

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \begin{cases} -\frac{16Aa^2}{9b^3\sqrt{a+bx^3}} - \frac{8Aax^3}{9b^2\sqrt{a+bx^3}} + \frac{2Ax^6}{9b\sqrt{a+bx^3}} + \frac{32Ba^3}{15b^4\sqrt{a+bx^3}} + \frac{16Ba^2x^3}{15b^3\sqrt{a+bx^3}} - \frac{4Bax^6}{15b^2\sqrt{a+bx^3}} + \frac{2Bx^9}{15b\sqrt{a+}} \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{a^{\frac{3}{2}}} \end{cases}$$

input
```
integrate(x**8*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
Piecewise((-16*A*a**2/(9*b**3*sqrt(a + b*x**3)) - 8*A*a*x**3/(9*b**2*sqrt(
a + b*x**3)) + 2*A*x**6/(9*b*sqrt(a + b*x**3)) + 32*B*a**3/(15*b**4*sqrt(a
 + b*x**3)) + 16*B*a**2*x**3/(15*b**3*sqrt(a + b*x**3)) - 4*B*a*x**6/(15*b
**2*sqrt(a + b*x**3)) + 2*B*x**9/(15*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x
**9/9 + B*x**12/12)/a**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2}{15}\,B\left(\frac{(bx^3+a)^{\frac{5}{2}}}{b^4} - \frac{5\,(bx^3+a)^{\frac{3}{2}}a}{b^4} + \frac{15\,\sqrt{bx^3+a}a^2}{b^4} + \frac{5\,a^3}{\sqrt{bx^3+a}b^4}\right) + \frac{2}{9}\,A\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^3} - \frac{6\,\sqrt{bx^3+a}a}{b^3} - \frac{3\,a^2}{\sqrt{bx^3+a}b^3}\right)$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
2/15*B*((b*x^3 + a)^(5/2)/b^4 - 5*(b*x^3 + a)^(3/2)*a/b^4 + 15*sqrt(b*x^3
+ a)*a^2/b^4 + 5*a^3/(sqrt(b*x^3 + a)*b^4)) + 2/9*A*((b*x^3 + a)^(3/2)/b^3
 - 6*sqrt(b*x^3 + a)*a/b^3 - 3*a^2/(sqrt(b*x^3 + a)*b^3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.11

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\,(Ba^3 - Aa^2b)}{3\,\sqrt{bx^3+a}b^4} + \frac{2\,\Big(3\,(bx^3+a)^{\frac{5}{2}}Bb^{16} - 15\,(bx^3+a)^{\frac{3}{2}}Bab^{16} + 45\,\sqrt{bx^3+a}Ba^2b^{16} + 5\,(bx^3+a)^{\frac{3}{2}}Ab^{17} - 30\,\sqrt{bx^3+a}Aab^{17}}{45\,b^{20}}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
2/3*(B*a^3 - A*a^2*b)/(sqrt(b*x^3 + a)*b^4) + 2/45*(3*(b*x^3 + a)^(5/2)*B*
b^16 - 15*(b*x^3 + a)^(3/2)*B*a*b^16 + 45*sqrt(b*x^3 + a)*B*a^2*b^16 + 5*(
b*x^3 + a)^(3/2)*A*b^17 - 30*sqrt(b*x^3 + a)*A*a*b^17)/b^20
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.48

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{\sqrt{b\,x^3+a}\,\left(\frac{2\,(B\,a^2-A\,a\,b)}{b^3}-\frac{2\,a\,\left(\frac{2\,(A\,b^2-B\,a\,b)}{b^3}-\frac{8\,B\,a}{5\,b^2}\right)}{3\,b}\right)}{3\,b} + \frac{x^3\,\sqrt{b\,x^3+a}\,\left(\frac{2\,(A\,b^2-B\,a\,b)}{b^3}-\frac{8\,B\,a}{5\,b^2}\right)}{9\,b} - \frac{a^2\,\left(\frac{2\,A}{3\,b}-\frac{2\,B\,a}{3\,b^2}\right)}{b^2\,\sqrt{b\,x^3+a}} + \frac{2\,B\,x^6\,\sqrt{b\,x^3+a}}{15\,b^2}$$

input
```
int((x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

output
```
((a + b*x^3)^(1/2)*((2*(B*a^2 - A*a*b))/b^3 - (2*a*((2*(A*b^2 - B*a*b))/b^
3 - (8*B*a)/(5*b^2)))/(3*b)))/(3*b) + (x^3*(a + b*x^3)^(1/2)*((2*(A*b^2 -
B*a*b))/b^3 - (8*B*a)/(5*b^2)))/(9*b) - (a^2*((2*A)/(3*b) - (2*B*a)/(3*b^2
)))/(b^2*(a + b*x^3)^(1/2)) + (2*B*x^6*(a + b*x^3)^(1/2))/(15*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.33

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{b\,x^3+a}\,(3b^2x^6-4ab\,x^3+8a^2)}{45b^3}$$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**3)*(8*a**2 - 4*a*b*x**3 + 3*b**2*x**6))/(45*b**3)
```

3.208 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}} + \frac{2(Ab-2aB)\sqrt{a+bx^3}}{3b^3} + \frac{2B(a+bx^3)^{3/2}}{9b^3}$$

output
```
2/3*a*(A*b-B*a)/b^3/(b*x^3+a)^(1/2)+2/3*(A*b-2*B*a)*(b*x^3+a)^(1/2)/b^3+2/
9*B*(b*x^3+a)^(3/2)/b^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(6aAb-8a^2B+3Ab^2x^3-4abBx^3+b^2Bx^6)}{9b^3\sqrt{a+bx^3}}$$

input
```
Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*(6*a*A*b - 8*a^2*B + 3*A*b^2*x^3 - 4*a*b*B*x^3 + b^2*B*x^6))/(9*b^3*Sqr
t[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^3)}{(a+bx^3)^{3/2}} dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^3 (Bx^3+A)}{(bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{3}\int \left(\frac{\sqrt{bx^3+a}B}{b^2}+\frac{Ab-2aB}{b^2\sqrt{bx^3+a}}+\frac{a(aB-Ab)}{b^2 (bx^3+a)^{3/2}}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{2\sqrt{a+bx^3}(Ab-2aB)}{b^3}+\frac{2a(Ab-aB)}{b^3\sqrt{a+bx^3}}+\frac{2B(a+bx^3)^{3/2}}{3b^3}\right)$$

input
```
Int[(x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
((2*a*(A*b - a*B))/(b^3*Sqrt[a + b*x^3]) + (2*(A*b - 2*a*B)*Sqrt[a + b*x^3
])/b^3 + (2*B*(a + b*x^3)^(3/2))/(3*b^3))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{\frac{2\left(\frac{B\,x^{3}}{3}+A\right)x^{3}b^{2}}{3}+\frac{4a\left(-\frac{2B\,x^{3}}{3}+A\right)b}{3}-\frac{16a^{2}B}{9}}{\sqrt{b\,x^{3}+a}\,b^{3}}$	49
gosper	$\frac{\frac{2}{9}b^{2}B\,x^{6}+\frac{2}{3}A\,b^{2}x^{3}-\frac{8}{9}Bab\,x^{3}+\frac{4}{3}abA-\frac{16}{9}a^{2}B}{\sqrt{b\,x^{3}+a}\,b^{3}}$	52
trager	$\frac{\frac{2}{9}b^{2}B\,x^{6}+\frac{2}{3}A\,b^{2}x^{3}-\frac{8}{9}Bab\,x^{3}+\frac{4}{3}abA-\frac{16}{9}a^{2}B}{\sqrt{b\,x^{3}+a}\,b^{3}}$	52
orering	$\frac{\frac{2}{9}b^{2}B\,x^{6}+\frac{2}{3}A\,b^{2}x^{3}-\frac{8}{9}Bab\,x^{3}+\frac{4}{3}abA-\frac{16}{9}a^{2}B}{\sqrt{b\,x^{3}+a}\,b^{3}}$	52
risch	$\frac{2\left(bB\,x^{3}+3Ab-5Ba\right)\sqrt{b\,x^{3}+a}}{9b^{3}}+\frac{2a\left(Ab-Ba\right)}{3b^{3}\sqrt{b\,x^{3}+a}}$	54
elliptic	$\frac{2a\left(Ab-Ba\right)}{3b^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2B\,x^{3}\sqrt{b\,x^{3}+a}}{9b^{2}}+\frac{2\left(\frac{Ab-Ba}{b^{2}}-\frac{2Ba}{3b^{2}}\right)\sqrt{b\,x^{3}+a}}{3b}$	81
default	$A\left(\frac{2a}{3b^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2\sqrt{b\,x^{3}+a}}{3b^{2}}\right)+B\left(-\frac{2a^{2}}{3b^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2x^{3}\sqrt{b\,x^{3}+a}}{9b^{2}}-\frac{10a\sqrt{b\,x^{3}+a}}{9b^{3}}\right)$	94

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4/3*(1/2*(1/3*B*x^3+A)*x^3*b^2+a*(-2/3*B*x^3+A)*b-4/3*a^2*B)/(b*x^3+a)^(1/
2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\,(Bb^2x^6-(4\,Bab-3\,Ab^2)x^3-8\,Ba^2+6\,Aab)\sqrt{bx^3+a}}{9\,(b^4x^3+ab^3)}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/9*(B*b^2*x^6 - (4*B*a*b - 3*A*b^2)*x^3 - 8*B*a^2 + 6*A*a*b)*sqrt(b*x^3 +
 a)/(b^4*x^3 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.70

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \begin{cases} \frac{4Aa}{3b^2\sqrt{a+bx^3}} + \frac{2Ax^3}{3b\sqrt{a+bx^3}} - \frac{16Ba^2}{9b^3\sqrt{a+bx^3}} - \frac{8Bax^3}{9b^2\sqrt{a+bx^3}} + \frac{2Bx^6}{9b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
Piecewise((4*A*a/(3*b**2*sqrt(a + b*x**3)) + 2*A*x**3/(3*b*sqrt(a + b*x**3
)) - 16*B*a**2/(9*b**3*sqrt(a + b*x**3)) - 8*B*a*x**3/(9*b**2*sqrt(a + b*x
**3)) + 2*B*x**6/(9*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**6/6 + B*x**9/9)
/a**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2}{9}\,B\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^3} - \frac{6\,\sqrt{bx^3+a}a}{b^3} - \frac{3\,a^2}{\sqrt{bx^3+a}b^3}\right) + \frac{2}{3}\,A\left(\frac{\sqrt{bx^3+a}}{b^2} + \frac{a}{\sqrt{bx^3+a}b^2}\right)$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
2/9*B*((b*x^3 + a)^(3/2)/b^3 - 6*sqrt(b*x^3 + a)*a/b^3 - 3*a^2/(sqrt(b*x^3
 + a)*b^3)) + 2/3*A*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2\,(Ba^2-Aab)}{3\,\sqrt{bx^3+a}b^3} + \frac{2\left((bx^3+a)^{\frac{3}{2}}Bb^6 - 6\,\sqrt{bx^3+a}Bab^6 + 3\,\sqrt{bx^3+a}Ab^7\right)}{9\,b^9}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
-2/3*(B*a^2 - A*a*b)/(sqrt(b*x^3 + a)*b^3) + 2/9*((b*x^3 + a)^(3/2)*B*b^6
- 6*sqrt(b*x^3 + a)*B*a*b^6 + 3*sqrt(b*x^3 + a)*A*b^7)/b^9
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\,B\,(b\,x^3+a)^2-6\,B\,a^2+6\,A\,b\,(b\,x^3+a)-12\,B\,a\,(b\,x^3+a)+6\,A\,a\,b}{9\,b^3\,\sqrt{b\,x^3+a}}$$

input
```
int((x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

output
```
(2*B*(a + b*x^3)^2 - 6*B*a^2 + 6*A*b*(a + b*x^3) - 12*B*a*(a + b*x^3) + 6*
A*a*b)/(9*b^3*(a + b*x^3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.30

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{b\,x^3+a}\,(b\,x^3-2a)}{9b^2}$$

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**3)*( - 2*a + b*x**3))/(9*b**2)
```

3.209 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}+\frac{2B\sqrt{a+bx^3}}{3b^2}$$

output
```
1/3*(-2*A*b+2*B*a)/b^2/(b*x^3+a)^(1/2)+2/3*B*(b*x^3+a)^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(-Ab+2aB+bBx^3)}{3b^2\sqrt{a+bx^3}}$$

input
```
Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*(-(A*b) + 2*a*B + b*B*x^3))/(3*b^2*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^3)}{(a+bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{3}\int \frac{Bx^3+A}{(bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \; 53$$

$$\frac{1}{3}\int \left(\frac{B}{b\sqrt{bx^3+a}}+\frac{Ab-aB}{b\,(bx^3+a)^{3/2}}\right)dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(\frac{2B\sqrt{a+bx^3}}{b^2}-\frac{2(Ab-aB)}{b^2\sqrt{a+bx^3}}\right)$$

input
```
Int[(x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
((-2*(A*b - a*B))/(b^2*Sqrt[a + b*x^3]) + (2*B*Sqrt[a + b*x^3])/b^2)/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{2(-bB\,x^3+Ab-2Ba)}{3\sqrt{b\,x^3+a}\,b^2}$	30
trager	$-\frac{2(-bB\,x^3+Ab-2Ba)}{3\sqrt{b\,x^3+a}\,b^2}$	30
pseudoelliptic	$-\frac{2((-B\,x^3+A)b-2Ba)}{3\sqrt{b\,x^3+a}\,b^2}$	30
orering	$-\frac{2(-bB\,x^3+Ab-2Ba)}{3\sqrt{b\,x^3+a}\,b^2}$	30
risch	$\frac{2B\sqrt{b\,x^3+a}}{3b^2}-\frac{2(Ab-Ba)}{3b^2\sqrt{b\,x^3+a}}$	39
elliptic	$-\frac{2(Ab-Ba)}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2B\sqrt{b\,x^3+a}}{3b^2}$	43
default	$-\frac{2A}{3b\sqrt{b\,x^3+a}}+B\left(\frac{2a}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2\sqrt{b\,x^3+a}}{3b^2}\right)$	53

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(b*x^3+a)^(1/2)*(-B*b*x^3+A*b-2*B*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\,(Bbx^3+2\,Ba-Ab)\sqrt{bx^3+a}}{3\,(b^3x^3+ab^2)}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(B*b*x^3 + 2*B*a - A*b)*sqrt(b*x^3 + a)/(b^3*x^3 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.63

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \begin{cases} -\frac{2A}{3b\sqrt{a+bx^3}} + \frac{4Ba}{3b^2\sqrt{a+bx^3}} + \frac{2Bx^3}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^3}{3}+\frac{Bx^6}{6}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
Piecewise((-2*A/(3*b*sqrt(a + b*x**3)) + 4*B*a/(3*b**2*sqrt(a + b*x**3)) +
 2*B*x**3/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/a**(3/
2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2}{3}\,B\left(\frac{\sqrt{bx^3+a}}{b^2} + \frac{a}{\sqrt{bx^3+a}\,b^2}\right) - \frac{2\,A}{3\,\sqrt{bx^3+a}\,b}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
2/3*B*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2)) - 2/3*A/(sqrt(b*x^3
+ a)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{bx^3+a}B}{3\,b^2} + \frac{2\,(Ba-Ab)}{3\,\sqrt{bx^3+a}b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
2/3*sqrt(b*x^3 + a)*B/b^2 + 2/3*(B*a - A*b)/(sqrt(b*x^3 + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\,B\,a-2\,A\,b+2\,B\,(b\,x^3+a)}{3\,b^2\,\sqrt{b\,x^3+a}}$$

input
```
int((x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

output
```
(2*B*a - 2*A*b + 2*B*(a + b*x^3))/(3*b^2*(a + b*x^3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.28

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{b\,x^3+a}}{3b}$$

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**3))/(3*b)
```

3.210 $\int \frac{A+Bx^3}{x(a+bx^3)^{3/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \frac{A+Bx^3}{x(a+bx^3)^{3/2}}\,dx = \frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}} - \frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

output
```
2/3*(A*b-B*a)/a/b/(b*x^3+a)^(1/2)-2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a
^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x(a+bx^3)^{3/2}}\,dx = \frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}} - \frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

input
```
Integrate[(A + B*x^3)/(x*(a + b*x^3)^(3/2)),x]
```

output
```
(2*(A*b - a*B))/(3*a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqr
t[a]])/(3*a^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^3\left(bx^3+a\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{A\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{a}+\frac{2(Ab-aB)}{ab\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2A\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{ab}+\frac{2(Ab-aB)}{ab\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{2(Ab-aB)}{ab\sqrt{a+bx^3}}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}\right)$$

input
```
Int[(A + B*x^3)/(x*(a + b*x^3)^(3/2)),x]
```

output
```
((2*(A*b - a*B))/(a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]])/a^(3/2))/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{\frac{2(Ab-Ba)}{3a\sqrt{b\,x^3+a}}-\frac{2Ab\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}}{b}$	49
elliptic	$\frac{\frac{2Ab}{3}-\frac{2Ba}{3}}{ba\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2A\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}$	51
default	$A\left(\frac{2}{3a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}\right)-\frac{2B}{3b\sqrt{b\,x^3+a}}$	57

input
```
int((B*x^3+A)/x/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*((A*b-B*a)/a/(b*x^3+a)^(1/2)-A/a^(3/2)*b*arctanh((b*x^3+a)^(1/2)/a^(1/
2)))/b
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.88

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx = \left[\frac{(Ab^2x^3+Aab)\sqrt{a}\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)-2\sqrt{bx^3+a}(Ba^2-Aab)}{3\left(a^2b^2x^3+a^3b\right)}, \frac{2\left((Ab^2x^3\right.}{}\right.$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/3*((A*b^2*x^3 + A*a*b)*sqrt(a)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) +
 2*a)/x^3) - 2*sqrt(b*x^3 + a)*(B*a^2 - A*a*b))/(a^2*b^2*x^3 + a^3*b), 2/3
*((A*b^2*x^3 + A*a*b)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^3 + a)) - sqrt(b*x
^3 + a)*(B*a^2 - A*a*b))/(a^2*b^2*x^3 + a^3*b)]
```

Sympy [A] (verification not implemented)

Time = 5.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{Ab\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3a\sqrt{-a}}-\frac{-Ab+Ba}{3a\sqrt{a+bx^3}}\right)}{b} & \text{for}\ b\neq 0 \\ \frac{A\log\left(Bx^3\right)+Bx^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x**3+A)/x/(b*x**3+a)**(3/2),x)
```

output
```
Piecewise((2*(A*b*atan(sqrt(a + b*x**3)/sqrt(-a))/(3*a*sqrt(-a)) - (-A*b +
 B*a)/(3*a*sqrt(a + b*x**3)))/b, Ne(b, 0)), ((A*log(B*x**3) + B*x**3)/(3*a
**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx=\frac{1}{3}\,A\left(\frac{\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{bx^3+a}a}\right)-\frac{2\,B}{3\,\sqrt{bx^3+a}b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
1/3*A*(log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2
) + 2/(sqrt(b*x^3 + a)*a)) - 2/3*B/(sqrt(b*x^3 + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx=\frac{2\,A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}a}-\frac{2\,(Ba-Ab)}{3\,\sqrt{bx^3+a}ab}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - 2/3*(B*a - A*b)/(sqr
t(b*x^3 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx=\frac{\frac{2\,A}{3\,a}-\frac{2\,B}{3\,b}}{\sqrt{b\,x^3+a}}+\frac{A\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,a^{3/2}}$$

input
```
int((A + B*x^3)/(x*(a + b*x^3)^(3/2)),x)
```

output
```
((2*A)/(3*a) - (2*B)/(3*b))/(a + b*x^3)^(1/2) + (A*log((((a + b*x^3)^(1/2)
 - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6))/(3*a^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \frac{A+Bx^3}{x\,(a+bx^3)^{3/2}}\,dx = \frac{\sqrt{a}\,\left(\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)-\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)\right)}{3a}$$

input
```
int((B*x^3+A)/x/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(a)*(log(sqrt(a + b*x**3) - sqrt(a)) - log(sqrt(a + b*x**3) + sqrt(a)
)))/(3*a)
```

3.211 $\int \frac{A+Bx^3}{x^4(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{A+Bx^3}{x^4(a+bx^3)^{3/2}}\,dx = -\frac{2(Ab-aB)}{3a^2\sqrt{a+bx^3}} - \frac{A\sqrt{a+bx^3}}{3a^2x^3} + \frac{(3Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

output
```
1/3*(-2*A*b+2*B*a)/a^2/(b*x^3+a)^(1/2)-1/3*A*(b*x^3+a)^(1/2)/a^2/x^3+1/3*(
3*A*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x^4(a+bx^3)^{3/2}}\,dx = \frac{-aA-3Abx^3+2aBx^3}{3a^2x^3\sqrt{a+bx^3}} + \frac{(3Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

input
```
Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x]
```

output
```
(-(a*A) - 3*A*b*x^3 + 2*a*B*x^3)/(3*a^2*x^3*Sqrt[a + b*x^3]) + ((3*A*b - 2
*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {948, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^4 (a+bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3} \int \frac{Bx^3+A}{x^6 (bx^3+a)^{3/2}} dx^3$$

$$\Big\downarrow \; 87$$

$$\frac{1}{3} \left(-\frac{(3Ab-2aB) \int \frac{1}{x^3 (bx^3+a)^{3/2}} dx^3}{2a} - \frac{A}{ax^3\sqrt{a+bx^3}} \right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{3} \left(-\frac{(3Ab-2aB) \left(\frac{\int \frac{1}{x^3\sqrt{bx^3+a}} dx^3}{a} + \frac{2}{a\sqrt{a+bx^3}} \right)}{2a} - \frac{A}{ax^3\sqrt{a+bx^3}} \right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3} \left(-\frac{(3Ab-2aB) \left(\frac{2\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}} d\sqrt{bx^3+a}}{ab} + \frac{2}{a\sqrt{a+bx^3}} \right)}{2a} - \frac{A}{ax^3\sqrt{a+bx^3}} \right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3} \left(-\frac{(3Ab-2aB) \left(\frac{2}{a\sqrt{a+bx^3}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}} \right)}{2a} - \frac{A}{ax^3\sqrt{a+bx^3}} \right)$$

input
```
Int[(A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x]
```

output
```
(-(A/(a*x^3*Sqrt[a + b*x^3])) - ((3*A*b - 2*a*B)*(2/(a*Sqrt[a + b*x^3]) -
(2*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/a^(3/2)))/(2*a))/3
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{-\frac{A\sqrt{b\,x^{3}+a}}{x^{3}}+\frac{(3Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2(Ab-Ba)}{\sqrt{b\,x^{3}+a}}}{3a^{2}}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{3a^{2}x^{3}}-\frac{2(Ab-Ba)}{3a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{(3Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}}$
risch	$-\frac{A\sqrt{b\,x^{3}+a}}{3a^{2}x^{3}}-\frac{-\frac{2bA}{3\sqrt{b\,x^{3}+a}}+a(3Ab-2Ba)\left(\frac{2}{3a\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}\right)}{2a^{2}}$
default	$A\left(-\frac{\sqrt{b\,x^{3}+a}}{3a^{2}x^{3}}-\frac{2b}{3a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)+B\left(\frac{2}{3a\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}\right)$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a^2*(-A*(b*x^3+a)^(1/2)/x^3+(3*A*b-2*B*a)/a^(1/2)*arctanh((b*x^3+a)^(1
/2)/a^(1/2))-2*(A*b-B*a)/(b*x^3+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.71

$$\int \frac{A+Bx^{3}}{x^{4}\left(a+bx^{3}\right)^{3/2}}\,dx=\left[-\frac{((2\,Bab-3\,Ab^{2})x^{6}+(2\,Ba^{2}-3\,Aab)x^{3})\sqrt{a}\log\left(\frac{bx^{3}+2\,\sqrt{bx^{3}+a}\sqrt{a}+2\,a}{x^{3}}\right)-2\,((2\,B\right.}{6\,(a^{3}bx^{6}+a^{4}x^{3})}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/6*(((2*B*a*b - 3*A*b^2)*x^6 + (2*B*a^2 - 3*A*a*b)*x^3)*sqrt(a)*log((b*
x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) - 2*((2*B*a^2 - 3*A*a*b)*x^3 -
 A*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^6 + a^4*x^3), 1/3*(((2*B*a*b - 3*A*b^2)*
x^6 + (2*B*a^2 - 3*A*a*b)*x^3)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^3 + a)) +
 ((2*B*a^2 - 3*A*a*b)*x^3 - A*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^6 + a^4*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(78) = 156$.

Time = 27.58 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.11

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx = A\left(-\frac{1}{3a\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{\sqrt{b}}{a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{a^{\frac{5}{2}}}\right)+B\left(\frac{2a^3\sqrt{1+\frac{bx^3}{a}}}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^2bx^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^2bx^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}\right)$$

input
```
integrate((B*x**3+A)/x**4/(b*x**3+a)**(3/2),x)
```

output
```
A*(-1/(3*a*sqrt(b)*x**(9/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)/(a**2*x**(3/2)
*sqrt(a/(b*x**3) + 1)) + b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/a**(5/2)) + B
*(2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b*
x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a) +
 1)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9/
2) + 3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**
(9/2) + 3*a**(7/2)*b*x**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(69) = 138$.

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.69

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx = -\frac{1}{6}\,A\left(\frac{2\left(3\left(bx^3+a\right)b-2\,ab\right)}{\left(bx^3+a\right)^{\frac{3}{2}}a^2-\sqrt{bx^3+a}a^3}+\frac{3\,b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)+\frac{1}{3}\,B\left(\frac{\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{bx^3+a}a}\right)$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/6*A*(2*(3*(b*x^3 + a)*b - 2*a*b)/((b*x^3 + a)^(3/2)*a^2 - sqrt(b*x^3 +
a)*a^3) + 3*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))
/a^(5/2)) + 1/3*B*(log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt
(a)))/a^(3/2) + 2/(sqrt(b*x^3 + a)*a))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx = \frac{\left(2\,Ba-3\,Ab\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}a^2}+\frac{2\left(bx^3+a\right)Ba-2\,Ba^2-3\left(bx^3+a\right)Ab+2\,Aab}{3\left(\left(bx^3+a\right)^{\frac{3}{2}}-\sqrt{bx^3+a}a\right)a^2}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
1/3*(2*B*a - 3*A*b)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) + 1/3*
(2*(b*x^3 + a)*B*a - 2*B*a^2 - 3*(b*x^3 + a)*A*b + 2*A*a*b)/(((b*x^3 + a)^
(3/2) - sqrt(b*x^3 + a)*a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.54

$$\int \frac{A+Bx^3}{x^4 (a+bx^3)^{3/2}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)(3\,A\,b-2\,B\,a)}{6\,a^{5/2}} - \frac{\frac{2\,B\,a^2-3\,A\,a\,b}{2\,a^3}-\frac{a\left(\frac{A\,b^2}{3\,a^3}+\frac{5\,b\left(2\,B\,a^2-3\,A\,a\,b\right)}{6\,a^4}\right)}{b}}{\sqrt{b\,x^3+a}} - \frac{A\,\sqrt{b\,x^3+a}}{3\,a^2\,x^3}$$

input
```
int((A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x)
```

output
```
(log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*
(3*A*b - 2*B*a))/(6*a^(5/2)) - ((2*B*a^2 - 3*A*a*b)/(2*a^3) - (a*((A*b^2)/
(3*a^3) + (5*b*(2*B*a^2 - 3*A*a*b))/(6*a^4)))/b)/(a + b*x^3)^(1/2) - (A*(a
 + b*x^3)^(1/2))/(3*a^2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx^3}{x^4 (a+bx^3)^{3/2}}\,dx = \frac{-2\sqrt{b\,x^3+a}\,a-\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)b\,x^3+\sqrt{a}\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)b\,x^3}{6a^2x^3}$$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**3)*a - sqrt(a)*log(sqrt(a + b*x**3) - sqrt(a))*b*x**3
+ sqrt(a)*log(sqrt(a + b*x**3) + sqrt(a))*b*x**3)/(6*a**2*x**3)
```

3.212 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx=\frac{2b(Ab-aB)}{3a^3\sqrt{a+bx^3}}-\frac{A\sqrt{a+bx^3}}{6a^2x^6}+\frac{(7Ab-4aB)\sqrt{a+bx^3}}{12a^3x^3}-\frac{b(5Ab-4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}$$

output
```
2/3*b*(A*b-B*a)/a^3/(b*x^3+a)^(1/2)-1/6*A*(b*x^3+a)^(1/2)/a^2/x^6+1/12*(7*
A*b-4*B*a)*(b*x^3+a)^(1/2)/a^3/x^3-1/4*b*(5*A*b-4*B*a)*arctanh((b*x^3+a)^(
1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx=\frac{-2a^2A+5aAbx^3-4a^2Bx^3+15Ab^2x^6-12abBx^6}{12a^3x^6\sqrt{a+bx^3}}+\frac{b(-5Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}$$

input
```
Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x]
```

output
```
(-2*a^2*A + 5*a*A*b*x^3 - 4*a^2*B*x^3 + 15*A*b^2*x^6 - 12*a*b*B*x^6)/(12*a
^3*x^6*Sqrt[a + b*x^3]) + (b*(-5*A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]])/(4*a^(7/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {948, 87, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^7 \left(a+bx^3\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{Bx^3+A}{x^9 \left(bx^3+a\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3} \left(-\frac{(5Ab-4aB) \int \frac{1}{x^6 \left(bx^3+a\right)^{3/2}}dx^3}{4a}-\frac{A}{2ax^6\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3} \left(-\frac{(5Ab-4aB) \left(-\frac{3b \int \frac{1}{x^3 \left(bx^3+a\right)^{3/2}}dx^3}{2a}-\frac{1}{ax^3\sqrt{a+bx^3}}\right)}{4a}-\frac{A}{2ax^6\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(-\frac{(5Ab-4aB)\left(-\frac{3b\left(\frac{\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{a}+\frac{2}{a\sqrt{a+bx^3}}\right)}{2a}-\frac{1}{ax^3\sqrt{a+bx^3}}\right)}{4a}-\frac{A}{2ax^6\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{3}\left(-\frac{(5Ab-4aB)\left(-\frac{3b\left(\frac{2\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{ab}+\frac{2}{a\sqrt{a+bx^3}}\right)}{2a}-\frac{1}{ax^3\sqrt{a+bx^3}}\right)}{4a}-\frac{A}{2ax^6\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \textbf{221}$$

$$\frac{1}{3}\left(-\frac{(5Ab-4aB)\left(-\frac{3b\left(\frac{2}{a\sqrt{a+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{1}{ax^3\sqrt{a+bx^3}}\right)}{4a}-\frac{A}{2ax^6\sqrt{a+bx^3}}\right)$$

input
```
Int[(A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x]
```

output
```
(-1/2*A/(a*x^6*Sqrt[a + b*x^3]) - ((5*A*b - 4*a*B)*(-(1/(a*x^3*Sqrt[a + b*
x^3])) - (3*b*(2/(a*Sqrt[a + b*x^3]) - (2*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]
)/a^(3/2)))/(2*a)))/(4*a))/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{\frac{15 b \sqrt{b x^3+a}\, x^6 \left(A b-\frac{4 B a}{5}\right) \operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)}{2}+\left(\left(2 B x^3+A\right) a^2-\frac{5 \left(-\frac{12 B x^3}{5}+A\right) b x^3 a}{2}-\frac{15 A b^2 x^6}{2}\right) \sqrt{a}}{6 \sqrt{b x^3+a}\, x^6 a^{\frac{7}{2}}}$
elliptic	$-\frac{A\sqrt{b x^3+a}}{6a^2x^6}+\frac{(7Ab-4Ba)\sqrt{b x^3+a}}{12a^3x^3}+\frac{2b(Ab-Ba)}{3a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{b(5Ab-4Ba)\operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)}{4a^{\frac{7}{2}}}$
risch	$-\frac{\sqrt{b x^3+a}\left(-7Ab x^3+4Ba x^3+2Aa\right)}{12a^3x^6}+\frac{b\left(-\frac{2(7Ab-4Ba)}{3\sqrt{b x^3+a}}+3a(5Ab-4Ba)\left(\frac{2}{3a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}\right)\right)}{8a^3}$
default	$A\left(-\frac{\sqrt{b x^3+a}}{6a^2x^6}+\frac{7b\sqrt{b x^3+a}}{12a^3x^3}+\frac{2b^2}{3a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{5b^2\operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)}{4a^{\frac{7}{2}}}\right)+B\left(-\frac{\sqrt{b x^3+a}}{3a^2x^3}-\frac{2b}{3a^2\sqrt{(x^3+}}\right.$

input
```
int((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(15/2*b*(b*x^3+a)^(1/2)*x^6*(A*b-4/5*B*a)*arctanh((b*x^3+a)^(1/2)/a^(
1/2))+((2*B*x^3+A)*a^2-5/2*(-12/5*B*x^3+A)*b*x^3*a-15/2*A*b^2*x^6)*a^(1/2)
)/(b*x^3+a)^(1/2)/x^6/a^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.44

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx=\left[-\frac{3\left((4\,Bab^2-5\,Ab^3)x^9+(4\,Ba^2b-5\,Aab^2)x^6\right)\sqrt{a}\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(}{24\,(a^4bx^9+a^5x^6}\right.$$

$$-\frac{3\left((4\,Bab^2-5\,Ab^3)x^9+(4\,Ba^2b-5\,Aab^2)x^6\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^3+a}}\right)+\left(3\,(4\,Ba^2b-5\,Aab^2)x^6+2\,Aa^3+\right.}{12\,(a^4bx^9+a^5x^6)}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((4*B*a*b^2 - 5*A*b^3)*x^9 + (4*B*a^2*b - 5*A*a*b^2)*x^6)*sqrt(a
)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*(3*(4*B*a^2*b - 5
*A*a*b^2)*x^6 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4
*b*x^9 + a^5*x^6), -1/12*(3*((4*B*a*b^2 - 5*A*b^3)*x^9 + (4*B*a^2*b - 5*A*
a*b^2)*x^6)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^3 + a)) + (3*(4*B*a^2*b - 5*
A*a*b^2)*x^6 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*
b*x^9 + a^5*x^6)]
```

Sympy [A] (verification not implemented)

Time = 68.40 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.64

$$\int \frac{A+Bx^3}{x^7(a+bx^3)^{3/2}}\,dx = A\left(-\frac{1}{6a\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{5\sqrt{b}}{12a^2x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{5b^{\frac{3}{2}}}{4a^3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{5b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4a^{\frac{7}{2}}}\right) + B\left(-\frac{1}{3a\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}}{a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{a^{\frac{5}{2}}}\right)$$

input
```
integrate((B*x**3+A)/x**7/(b*x**3+a)**(3/2),x)
```

output
```
A*(-1/(6*a*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) + 5*sqrt(b)/(12*a**2*x*
*(9/2)*sqrt(a/(b*x**3) + 1)) + 5*b**(3/2)/(4*a**3*x**(3/2)*sqrt(a/(b*x**3)
 + 1)) - 5*b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*a**(7/2))) + B*(-1/(3
*a*sqrt(b)*x**(9/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)/(a**2*x**(3/2)*sqrt(a/
(b*x**3) + 1)) + b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/a**(5/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(97) = 194$.

Time = 0.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.84

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx = \frac{1}{24}\,A\left(\frac{2\left(15\,(bx^3+a)^2b^2 - 25\,(bx^3+a)ab^2 + 8\,a^2b^2\right)}{(bx^3+a)^{\frac{5}{2}}a^3 - 2\,(bx^3+a)^{\frac{3}{2}}a^4 + \sqrt{bx^3+a}a^5} + \frac{15\,b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right) - \frac{1}{6}\,B\left(\frac{2\,(3\,(bx^3+a)b - 2\,ab)}{(bx^3+a)^{\frac{3}{2}}a^2 - \sqrt{bx^3+a}a^3} + \frac{3\,b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
1/24*A*(2*(15*(b*x^3 + a)^2*b^2 - 25*(b*x^3 + a)*a*b^2 + 8*a^2*b^2)/((b*x^
3 + a)^(5/2)*a^3 - 2*(b*x^3 + a)^(3/2)*a^4 + sqrt(b*x^3 + a)*a^5) + 15*b^2
*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(7/2)) - 1
/6*B*(2*(3*(b*x^3 + a)*b - 2*a*b)/((b*x^3 + a)^(3/2)*a^2 - sqrt(b*x^3 + a)
*a^3) + 3*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a
^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx = -\frac{(4\,Bab - 5\,Ab^2)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^3} - \frac{2\,(Bab - Ab^2)}{3\,\sqrt{bx^3+a}a^3} - \frac{4\,(bx^3+a)^{\frac{3}{2}}Bab - 4\,\sqrt{bx^3+a}Ba^2b - 7\,(bx^3+a)^{\frac{3}{2}}Ab^2 + 9\,\sqrt{bx^3+a}Aab^2}{12\,a^3b^2x^6}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
-1/4*(4*B*a*b - 5*A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^3) -
 2/3*(B*a*b - A*b^2)/(sqrt(b*x^3 + a)*a^3) - 1/12*(4*(b*x^3 + a)^(3/2)*B*a
*b - 4*sqrt(b*x^3 + a)*B*a^2*b - 7*(b*x^3 + a)^(3/2)*A*b^2 + 9*sqrt(b*x^3
+ a)*A*a*b^2)/(a^3*b^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.43

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx = \frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\left(5\,A\,b-4\,B\,a\right)}{8\,a^{7/2}} - \frac{\left(4\,B\,a^2-7\,A\,a\,b\right)\sqrt{b\,x^3+a}}{12\,a^4\,x^3} - \frac{A\,\sqrt{b\,x^3+a}}{6\,a^2\,x^6} - \frac{\frac{a\left(\frac{7\,A\,b^3-4\,B\,a\,b^2}{12\,a^4}-\frac{5\,b^2\,\left(5\,A\,b-4\,B\,a\right)}{8\,a^4}\right)}{b}+\frac{3\,b\,\left(5\,A\,b-4\,B\,a\right)}{8\,a^3}}{\sqrt{b\,x^3+a}}$$

input
```
int((A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x)
```

output
```
(b*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6
)*(5*A*b - 4*B*a))/(8*a^(7/2)) - ((4*B*a^2 - 7*A*a*b)*(a + b*x^3)^(1/2))/(
12*a^4*x^3) - (A*(a + b*x^3)^(1/2))/(6*a^2*x^6) - ((a*((7*A*b^3 - 4*B*a*b^
2)/(12*a^4) - (5*b^2*(5*A*b - 4*B*a))/(8*a^4)))/b + (3*b*(5*A*b - 4*B*a))/
(8*a^3))/(a + b*x^3)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx = \frac{-4\sqrt{b\,x^3+a}\,a^2+6\sqrt{b\,x^3+a}\,ab\,x^3+3\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)b^2x^6-3\sqrt{a}\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)b^2x^6}{24a^3x^6}$$

input
```
int((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x)
```

output
```
( - 4*sqrt(a + b*x**3)*a**2 + 6*sqrt(a + b*x**3)*a*b*x**3 + 3*sqrt(a)*log(
sqrt(a + b*x**3) - sqrt(a))*b**2*x**6 - 3*sqrt(a)*log(sqrt(a + b*x**3) + s
qrt(a))*b**2*x**6)/(24*a**3*x**6)
```

3.213 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 298

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2a(Ab-aB)x}{3b^3\sqrt{a+bx^3}}+\frac{2(11Ab-19aB)x\sqrt{a+bx^3}}{55b^3}+\frac{2Bx^4\sqrt{a+bx^3}}{11b^2}$$
$$-\frac{32\sqrt{2+\sqrt{3}}a(11Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{165\sqrt[4]{3}b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*a*(A*b-B*a)*x/b^3/(b*x^3+a)^(1/2)+2/55*(11*A*b-19*B*a)*x*(b*x^3+a)^(1/
2)/b^3+2/11*B*x^4*(b*x^3+a)^(1/2)/b^2-32/495*(1/2*6^(1/2)+1/2*2^(1/2))*a*(
11*A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+
b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(10/3)
/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*
x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.35

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2x\left(-112a^2B+3b^2x^3(11A+5Bx^3)+a(88Ab-42bBx^3)+8a(-11Ab+14aB)\sqrt{1\right.}}{165b^3\sqrt{a+bx^3}}$$

input
```
Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*x*(-112*a^2*B + 3*b^2*x^3*(11*A + 5*B*x^3) + a*(88*A*b - 42*b*B*x^3) +
8*a*(-11*A*b + 14*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3
, -((b*x^3)/a)]))/(165*b^3*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {959, 817, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(11Ab-14aB)\int \frac{x^6}{(bx^3+a)^{3/2}}dx}{11b}+\frac{2Bx^7}{11b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(11Ab-14aB)\left(\frac{8\int \frac{x^3}{\sqrt{bx^3+a}}dx}{3b}-\frac{2x^4}{3b\sqrt{a+bx^3}}\right)}{11b}+\frac{2Bx^7}{11b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{843}$$

$$\frac{(11Ab-14aB)\left(\frac{8\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{2a\int\frac{1}{\sqrt{bx^3+a}}dx}{5b}\right)}{3b}-\frac{2x^4}{3b\sqrt{a+bx^3}}\right)}{11b}+\frac{2Bx^7}{11b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{759}$$

$$\frac{(11Ab-14aB)\left(\frac{8\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{3b}-\ldots\right)}{11b}$$

$$\frac{2Bx^7}{11b\sqrt{a+bx^3}}$$

input
```
Int[(x^6*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*B*x^7)/(11*b*Sqrt[a + b*x^3]) + ((11*A*b - 14*a*B)*((-2*x^4)/(3*b*Sqrt[
a + b*x^3]) + (8*((2*x*Sqrt[a + b*x^3])/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*(a^
(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1
/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(3*b)))/(11*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.36

method	result
elliptic	$\frac{2xa(Ab-Ba)}{3b^3\sqrt{(x^3+\frac{a}{b})b}}+\frac{2Bx^4\sqrt{bx^3+a}}{11b^2}+\frac{2\left(\frac{Ab-Ba}{b^2}-\frac{8Ba}{11b^2}\right)x\sqrt{bx^3+a}}{5b}-\frac{2i\left(-\frac{2a(Ab-Ba)}{3b^3}-\frac{2\left(\frac{Ab-Ba}{b^2}-\frac{8Ba}{11b^2}\right)a}{5b}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i(}{-}}}{}$
default	$A\left(\frac{2xa}{3b^2\sqrt{(x^3+\frac{a}{b})b}}+\frac{2x\sqrt{bx^3+a}}{5b^2}+\frac{32ia\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}}{}\right.$
risch	Expression too large to display

input
```
int(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b^3*x*a*(A*b-B*a)/((x^3+a/b)*b)^(1/2)+2/11*B*x^4*(b*x^3+a)^(1/2)/b^2+2
/5*((A*b-B*a)/b^2-8/11*B/b^2*a)/b*x*(b*x^3+a)^(1/2)-2/3*I*(-2/3*a*(A*b-B*a
)/b^3-2/5*((A*b-B*a)/b^2-8/11*B/b^2*a)/b*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.41

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(16\left(14\,Ba^3-11\,Aa^2b+\left(14\,Ba^2b-11\,Aab^2\right)x^3\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right.}{165\left(b^5x^3+\right.}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/165*(16*(14*B*a^3 - 11*A*a^2*b + (14*B*a^2*b - 11*A*a*b^2)*x^3)*sqrt(b)*
weierstrassPInverse(0, -4*a/b, x) + (15*B*b^3*x^7 - 3*(14*B*a*b^2 - 11*A*b
^3)*x^4 - 8*(14*B*a^2*b - 11*A*a*b^2)*x)*sqrt(b*x^3 + a))/(b^5*x^3 + a*b^4
)
```

Sympy [A] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.27

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{Ax^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)} + \frac{Bx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate(x**6*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
A*x**7*gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*
a**(3/2)*gamma(10/3)) + B*x**10*gamma(10/3)*hyper((3/2, 10/3), (13/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(13/3))
```

Maxima [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x^6}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x^6}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^6\,(B\,x^3+A)}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((x^6*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

output
```
int((x^6*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{-\frac{16\sqrt{b\,x^3+a}\,a x}{55}+\frac{2\sqrt{b\,x^3+a}\,b\,x^4}{11}+\frac{16\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a^2}{55}}{b^2}$$

input
```
int(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

output
```
(2*( - 8*sqrt(a + b*x**3)*a*x + 5*sqrt(a + b*x**3)*b*x**4 + 8*int(sqrt(a +
 b*x**3)/(a + b*x**3),x)*a**2))/(55*b**2)
```

3.214 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 266

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2(Ab-aB)x}{3b^2\sqrt{a+bx^3}}+\frac{2Bx\sqrt{a+bx^3}}{5b^2} + \frac{4\sqrt{2+\sqrt{3}}(5Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*(A*b-B*a)*x/b^2/(b*x^3+a)^(1/2)+2/5*B*x*(b*x^3+a)^(1/2)/b^2+4/45*(1/2
*6^(1/2)+1/2*2^(1/2))*(5*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(
((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+
2*I)*3^(3/4)/b^(7/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(
1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.29

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2x\left(-5Ab+8aB+3bBx^3+(5Ab-8aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\right.\right.}{15b^2\sqrt{a+bx^3}}$$

input
```
Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*x*(-5*A*b + 8*a*B + 3*b*B*x^3 + (5*A*b - 8*a*B)*Sqrt[1 + (b*x^3)/a]*Hyp
ergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(15*b^2*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {959, 817, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{ 959}$$

$$\frac{(5Ab-8aB)\int \frac{x^3}{(bx^3+a)^{3/2}}dx}{5b}+\frac{2Bx^4}{5b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 817}$$

$$\frac{(5Ab-8aB)\left(\frac{2\int \frac{1}{\sqrt{bx^3+a}}dx}{3b}-\frac{2x}{3b\sqrt{a+bx^3}}\right)}{5b}+\frac{2Bx^4}{5b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{(5Ab-8aB)\left(\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x}{3b\sqrt{a+bx^3}}\right)}{5b}$$

$$\frac{2Bx^4}{5b\sqrt{a+bx^3}}$$

input
```
Int[(x^3*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*B*x^4)/(5*b*Sqrt[a + b*x^3]) + ((5*A*b - 8*a*B)*((-2*x)/(3*b*Sqrt[a + b
*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(5*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.31

method	result
elliptic	$-\frac{2x(Ab-Ba)}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2Bx\sqrt{b\,x^3+a}}{5b^2}-\frac{2i\left(\frac{\frac{2Ab}{3}-\frac{2Ba}{3}}{b^2}-\frac{2Ba}{5b^2}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(}{}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}}}}{}$
default	$A\left(-\frac{2x}{3b\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{4i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)}{2b}\right.}{}}}{}\right.$
risch	$\frac{2Bx\sqrt{b\,x^3+a}}{5b^2}+\frac{b(5Ab-7Ba)\left(-\frac{2x}{3b\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{4i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2}{2b}}}}{}\right.}{}$

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/b^2*x*(A*b-B*a)/((x^3+a/b)*b)^(1/2)+2/5*B*x*(b*x^3+a)^(1/2)/b^2-2/3*I
*(2/3*(A*b-B*a)/b^2-2/5*B/b^2*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.36

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx =$$
$$-\frac{2\left(2\left((8\,Bab-5\,Ab^2)x^3+8\,Ba^2-5\,Aab\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\left(3\,Bb^2x^4+(8\,Bab-5\,Ab\right.\right.}{15\,(b^4x^3+ab^3)}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(2*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*sqrt(b)*weierstrass
PInverse(0, -4*a/b, x) - (3*B*b^2*x^4 + (8*B*a*b - 5*A*b^2)*x)*sqrt(b*x^3
+ a))/(b^4*x^3 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.30

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{Ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{Bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**3*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
A*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(3/2)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(10/3))
```

Maxima [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x^3}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x^3}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^3\,(B\,x^3+A)}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((x^3*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

output
```
int((x^3*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{\frac{2\sqrt{b\,x^3+a}\,x}{5}-\frac{2\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a}{5}}{b}$$

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

output
```
(2*(sqrt(a + b*x**3)*x - int(sqrt(a + b*x**3)/(a + b*x**3),x)*a))/(5*b)
```

3.215 $\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 251

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx = \frac{2(Ab-aB)x}{3ab\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-\right.}{3\sqrt[4]{3}ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*(A*b-B*a)*x/a/b/(b*x^3+a)^(1/2)+2/9*(1/2*6^(1/2)+1/2*2^(1/2))*(A*b+2*B
*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/
2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/
((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a/b^(4/3)/(a^(1/3)*
(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.29

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx = \frac{x\left(2Ab-2aB+(Ab+2aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)\right)}{3ab\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(a + b*x^3)^(3/2),x]
```

output
```
(x*(2*A*b - 2*a*B + (A*b + 2*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/
3, 1/2, 4/3, -((b*x^3)/a)]))/(3*a*b*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {910, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(2aB+Ab)\int \frac{1}{\sqrt{bx^3+a}}dx}{3ab}+\frac{2x(Ab-aB)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{759}$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+Ab)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$\frac{2x(Ab-aB)}{3ab\sqrt{a+bx^3}}$$

input
```
Int[(A + B*x^3)/(a + b*x^3)^(3/2),x]
```

output
```
(2*(A*b - a*B)*x)/(3*a*b*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(A*b + 2*
a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(3*3^(1/4)*a*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.34

method	result
elliptic	$\frac{2x(Ab-Ba)}{3ba\sqrt{(x^3+\frac{a}{b})b}}-\frac{2i\left(\frac{B}{b}+\frac{Ab-Ba}{3ab}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\right.}{}}}{}$
default	$A\left(\frac{2x}{3a\sqrt{(x^3+\frac{a}{b})b}}-\frac{2i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}\right.}{(-}}}{9a}\right.$

input

```
int((B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/3/b*x/a*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-2/3*I*(B/b+1/3*(A*b-B*a)/a/b)*3^(1
/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx = -\frac{2\left(\sqrt{bx^3+a}(Bab-Ab^2)x-((2\,Bab+Ab^2)x^3+2\,Ba^2+Aab)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{3\,(ab^3x^3+a^2b^2)}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(b*x^3 + a)*(B*a*b - A*b^2)*x - ((2*B*a*b + A*b^2)*x^3 + 2*B*a^2
 + A*a*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x))/(a*b^3*x^3 + a^2*b^2)
```

Sympy [A] (verification not implemented)

Time = 2.86 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.31

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx = \frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
A*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx = \int \frac{B\,x^3+A}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((A + B*x^3)/(a + b*x^3)^(3/2),x)
```

output
```
int((A + B*x^3)/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx=\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx$$

input
```
int((B*x^3+A)/(b*x^3+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a + b*x**3),x)
```

3.216 $\int \frac{A+Bx^3}{x^3(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 272

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^{3/2}}\,dx = -\frac{A}{2ax^2\sqrt{a+bx^3}}-\frac{(7Ab-4aB)x}{6a^2\sqrt{a+bx^3}} - \frac{\sqrt{2+\sqrt{3}}(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-\right.}{6\sqrt[4]{3}a^2\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-1/2*A/a/x^2/(b*x^3+a)^(1/2)-1/6*(7*A*b-4*B*a)*x/a^2/(b*x^3+a)^(1/2)-1/18*
(1/2*6^(1/2)+1/2*2^(1/2))*(7*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*Ellipt
icF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1
/2)+2*I)*3^(3/4)/a^2/b^(1/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(
1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.32

$$\int \frac{A+Bx^3}{x^3 (a+bx^3)^{3/2}}\,dx = \frac{-6aA-14Abx^3+8aBx^3+(-7Ab+4aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\right.}{12a^2x^2\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^(3/2)),x]
```

output
```
(-6*a*A - 14*A*b*x^3 + 8*a*B*x^3 + (-7*A*b + 4*a*B)*x^3*Sqrt[1 + (b*x^3)/a
]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(12*a^2*x^2*Sqrt[a + b*x
^3])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^3 (a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(7Ab-4aB)\int \frac{1}{(bx^3+a)^{3/2}}dx}{4a}-\frac{A}{2ax^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{(7Ab-4aB)\left(\frac{\int \frac{1}{\sqrt{bx^3+a}}dx}{3a}+\frac{2x}{3a\sqrt{a+bx^3}}\right)}{4a}-\frac{A}{2ax^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{759}$$

$$-\frac{(7Ab-4aB)\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x}{3a\sqrt{a+bx^3}}\right)}{4a}$$
$$\frac{A}{2ax^2\sqrt{a+bx^3}}$$

input
```
Int[(A + B*x^3)/(x^3*(a + b*x^3)^(3/2)),x]
```

output
```
-1/2*A/(a*x^2*Sqrt[a + b*x^3]) - ((7*A*b - 4*a*B)*((2*x)/(3*a*Sqrt[a + b*x
^3]) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(4*a)
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.29

method	result
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{2a^2x^2}-\frac{2x(Ab-Ba)}{3a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i\left(-\frac{Ab}{4a^2}-\frac{Ab-Ba}{3a^2}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{}}{}}}}{}$
default	$B\left(\frac{2x}{3a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}\right.}{(}}}{9a}\right.$
risch	Expression too large to display

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2*A*(b*x^3+a)^(1/2)/x^2-2/3*x/a^2*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-2/3
*I*(-1/4/a^2*A*b-1/3*(A*b-B*a)/a^2)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.38

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx = \frac{\left((4\,Bab-7\,Ab^2)x^5+(4\,Ba^2-7\,Aab)x^2\right)\sqrt{b}\,\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+((4\,\ldots}{6\left(a^2b^2x^5+a^3bx^2\right)}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(((4*B*a*b - 7*A*b^2)*x^5 + (4*B*a^2 - 7*A*a*b)*x^2)*sqrt(b)*weierstra
ssPInverse(0, -4*a/b, x) + ((4*B*a*b - 7*A*b^2)*x^3 - 3*A*a*b)*sqrt(b*x^3
+ a))/(a^2*b^2*x^5 + a^3*b*x^2)
```

Sympy [A] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.30

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^2\Gamma\left(\frac{1}{3}\right)}+\frac{Bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**3/(b*x**3+a)**(3/2),x)
```

output
```
A*gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*x**2*gamma(1/3)) + B*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^{3/2}}\,dx = \int \frac{B\,x^3+A}{x^3\,(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((A + B*x^3)/(x^3*(a + b*x^3)^(3/2)),x)
```

output
```
int((A + B*x^3)/(x^3*(a + b*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^3\,(a+bx^3)^{3/2}}\,dx=\int \frac{\sqrt{b\,x^3+a}}{b\,x^6+a\,x^3}dx$$

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a*x**3 + b*x**6),x)
```

3.217 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 304

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx = -\frac{A}{5ax^5\sqrt{a+bx^3}}-\frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}}+\frac{7(13Ab-10aB)\sqrt{a+bx^3}}{60a^3x^2} + \frac{7\sqrt{2+\sqrt{3}}\,b^{2/3}(13Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{60\sqrt[4]{3}a^3\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/5*A/a/x^5/(b*x^3+a)^(1/2)-1/15*(13*A*b-10*B*a)/a^2/x^2/(b*x^3+a)^(1/2)+
7/60*(13*A*b-10*B*a)*(b*x^3+a)^(1/2)/a^3/x^2+7/180*(1/2*6^(1/2)+1/2*2^(1/2
))*b^(2/3)*(13*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/
2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3
/4)/a^3/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1
/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.24

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx=\frac{-4aA+(13Ab-10aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{20a^2x^5\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^(3/2)),x]
```

output
```
(-4*a*A + (13*A*b - 10*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3
, 3/2, 1/3, -((b*x^3)/a)])/(20*a^2*x^5*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 819, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(13Ab-10aB)\int \frac{1}{x^3\left(bx^3+a\right)^{3/2}}dx}{10a}-\frac{A}{5ax^5\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(13Ab-10aB)\left(\frac{7\int \frac{1}{x^3\sqrt{bx^3+a}}dx}{3a}+\frac{2}{3ax^2\sqrt{a+bx^3}}\right)}{10a}-\frac{A}{5ax^5\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(13Ab-10aB)\left(\frac{7\left(-\frac{b\int\frac{1}{\sqrt{bx^3+a}}dx}{4a}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)}{3a}+\frac{2}{3ax^2\sqrt{a+bx^3}}\right)}{10a}-\frac{A}{5ax^5\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{759}$$

$$-\frac{(13Ab-10aB)\left(\frac{7\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)}{3a}\right)}{10a}$$

$$\frac{A}{5ax^5\sqrt{a+bx^3}}$$

input
```
Int[(A + B*x^3)/(x^6*(a + b*x^3)^(3/2)),x]
```

output
```
-1/5*A/(a*x^5*Sqrt[a + b*x^3]) - ((13*A*b - 10*a*B)*(2/(3*a*x^2*Sqrt[a + b
*x^3]) + (7*(-1/2*Sqrt[a + b*x^3]/(a*x^2) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^
(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1
/4)*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*Sqrt[a + b*x^3])))/(3*a)))/(10*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.27

method	result
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{5a^2x^5}+\frac{(17Ab-10Ba)\sqrt{b\,x^3+a}}{20a^3x^2}+\frac{2bx(Ab-Ba)}{3a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i\left(\frac{b(17Ab-10Ba)}{40a^3}+\frac{(Ab-Ba)b}{3a^3}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i}{}\right.}{\left(-a\right.}}}{}$
default	$A\left(-\frac{\sqrt{b\,x^3+a}}{5a^2x^5}+\frac{17b\sqrt{b\,x^3+a}}{20a^3x^2}+\frac{2b^2x}{3a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{91ib\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(\right.}{}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}}{}\right.$
risch	Expression too large to display

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*A/a^2*(b*x^3+a)^(1/2)/x^5+1/20/a^3*(17*A*b-10*B*a)*(b*x^3+a)^(1/2)/x^
2+2/3*b*x/a^3*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-2/3*I*(1/40*b*(17*A*b-10*B*a)/
a^3+1/3*(A*b-B*a)/a^3*b)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b
*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx^3}{x^6 \left(a+bx^3\right)^{3/2}}\,dx = -\frac{7\left((10\,Bab-13\,Ab^2)x^8+(10\,Ba^2-13\,Aab)x^5\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(7\left(10\,Bab-13\,Ab^2\right.\right.}{60\left(a^3bx^8+a^4x^5\right)}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/60*(7*((10*B*a*b - 13*A*b^2)*x^8 + (10*B*a^2 - 13*A*a*b)*x^5)*sqrt(b)*w
eierstrassPInverse(0, -4*a/b, x) + (7*(10*B*a*b - 13*A*b^2)*x^6 + 3*(10*B*
a^2 - 13*A*a*b)*x^3 + 12*A*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^8 + a^4*x^5)
```

Sympy [A] (verification not implemented)

Time = 26.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.30

$$\int \frac{A+Bx^3}{x^6 \left(a+bx^3\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{3},\frac{3}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{B\Gamma\left(-\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**6/(b*x**3+a)**(3/2),x)
```

output
```
A*gamma(-5/3)*hyper((-5/3, 3/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(3/2)*x**5*gamma(-2/3)) + B*gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(3/2)*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^6), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^{3/2}}\,dx = \int \frac{B\,x^3+A}{x^6\,{(b\,x^3+a)}^{3/2}}\,dx$$

input
```
int((A + B*x^3)/(x^6*(a + b*x^3)^(3/2)),x)
```

output
```
int((A + B*x^3)/(x^6*(a + b*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^6\,(a+bx^3)^{3/2}}\,dx=\int \frac{\sqrt{b\,x^3+a}}{b\,x^9+a\,x^6}dx$$

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a*x**6 + b*x**9),x)
```

3.218 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 546

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2(Ab-aB)x^2}{3b^2\sqrt{a+bx^3}}+\frac{2Bx^2\sqrt{a+bx^3}}{7b^2}+\frac{8(7Ab-10aB)\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4}{7\ 3^{3/4}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{8\sqrt{2}\sqrt[3]{a}(7Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4}{21\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*(A*b-B*a)*x^2/b^2/(b*x^3+a)^(1/2)+2/7*B*x^2*(b*x^3+a)^(1/2)/b^2+8/21*
(7*A*b-10*B*a)*(b*x^3+a)^(1/2)/b^(8/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)-4/2
1*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/3)*(7*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(
1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x),I*3^(1/2)+2*I)*3^(1/4)/b^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/
2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+8/63*2^(1/2)*a^(1/3)*(7*A*
b-10*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1
+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/
3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(8/3)/(a^(1
/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.14

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2x^2\left(7Ab-10aB+bBx^3+(-7Ab+10aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},\right.\right.}{7b^2\sqrt{a+bx^3}}$$

input
```
Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*x^2*(7*A*b - 10*a*B + b*B*x^3 + (-7*A*b + 10*a*B)*Sqrt[1 + (b*x^3)/a]*H
ypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(7*b^2*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {959, 817, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$$

$\big\downarrow$ 959

$$\frac{(7Ab-10aB)\int \frac{x^4}{\left(bx^3+a\right)^{3/2}}dx}{7b}+\frac{2Bx^5}{7b\sqrt{a+bx^3}}$$

$\big\downarrow$ 817

$$\frac{(7Ab-10aB)\left(\frac{4\int\frac{x}{\sqrt{bx^3+a}}dx}{3b}-\frac{2x^2}{3b\sqrt{a+bx^3}}\right)}{7b}+\frac{2Bx^5}{7b\sqrt{a+bx^3}}$$

$\big\downarrow$ 832

$$\frac{(7Ab-10aB)\left(\frac{4\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{3b}-\frac{2x^2}{3b\sqrt{a+bx^3}}\right)}{7b}+\frac{2Bx^5}{7b\sqrt{a+bx^3}}$$

$\big\downarrow$ 759

$$\frac{(7Ab-10aB)\left(\frac{4\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\cdots}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\cdots}\right)\cdots\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{3b}\cdots\right)}{7b}\cdots$$

$$\frac{2Bx^5}{7b\sqrt{a+bx^3}}$$

$\big\downarrow$ 2416

$$\frac{(7Ab-10aB)\left(\frac{4\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|\cdots}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots\right)}{\sqrt[3]{b}}\cdots}{7b\cdots}$$

$$\frac{2Bx^5}{7b\sqrt{a+bx^3}}$$

input
```
Int[(x^4*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*B*x^5)/(7*b*Sqrt[a + b*x^3]) + ((7*A*b - 10*a*B)*((-2*x^2)/(3*b*Sqrt[a
+ b*x^3]) + (4*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(
1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/
3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3])))/(3*b)))/(7*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.92

method	result
elliptic	$-\frac{2x^2(Ab-Ba)}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2B\,x^2\sqrt{b\,x^3+a}}{7b^2}-\frac{2i\left(\frac{\frac{4Ab}{3}-\frac{4Ba}{3}}{b^2}-\frac{4Ba}{7b^2}\right)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\cdots}{-\frac{3\left(-a\,b^2\right)\cdots}{2b}\cdots}}\cdots}{\cdots}$
default	Expression too large to display
risch	Expression too large to display

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/b^2*x^2*(A*b-B*a)/((x^3+a/b)*b)^(1/2)+2/7*B*x^2*(b*x^3+a)^(1/2)/b^2-2
/3*I*(4/3*(A*b-B*a)/b^2-4/7*B/b^2*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I
*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.19

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(4\left((10\,Bab-7\,Ab^2)x^3+10\,Ba^2-7\,Aab\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassP}\right.}{21\,(b^4x^3+ab^3)}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/21*(4*((10*B*a*b - 7*A*b^2)*x^3 + 10*B*a^2 - 7*A*a*b)*sqrt(b)*weierstras
sZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (3*B*b^2*x^5 + (10*B
*a*b - 7*A*b^2)*x^2)*sqrt(b*x^3 + a))/(b^4*x^3 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{Ax^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)} + \frac{Bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**4*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
A*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(3/2)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((3/2, 8/3), (11/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(11/3))
```

Maxima [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx=\int \frac{x^4\,(B\,x^3+A)}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((x^4*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

output
```
int((x^4*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx=\frac{\frac{2\sqrt{b\,x^3+a}\,x^2}{7}-\frac{4\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a}{7}}{b}$$

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

output
```
(2*(sqrt(a + b*x**3)*x**2 - 2*int((sqrt(a + b*x**3)*x)/(a + b*x**3),x)*a))
/(7*b)
```

3.219 $\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 524

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(Ab-aB)x^2}{3ab\sqrt{a+bx^3}} - \frac{2(Ab-4aB)\sqrt{a+bx^3}}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{\sqrt{2-\sqrt{3}}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{2\sqrt{2}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*(A*b-B*a)*x^2/a/b/(b*x^3+a)^(1/2)-2/3*(A*b-4*B*a)*(b*x^3+a)^(1/2)/a/b^
(5/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+1/3*(1/2*6^(1/2)-1/2*2^(1/2))*(A*b-4
*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(
1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x
)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/a^(2/3)/b^(5/3)/(
a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^
3+a)^(1/2)-2/9*2^(1/2)*(A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF((
(1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2
*I)*3^(3/4)/a^(2/3)/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1
/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.14

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{x^2\left(4aB+(Ab-4aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2ab\sqrt{a+bx^3}}$$

input
```
Integrate[(x*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(x^2*(4*a*B + (A*b - 4*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2
, 5/3, -((b*x^3)/a)]))/(2*a*b*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.93 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {957, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \; 957$$

$$\frac{2x^2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{(Ab-4aB)\int \frac{x}{\sqrt{bx^3+a}}dx}{3ab}$$

$$\Big\downarrow \; 832$$

$$\frac{2x^2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{(Ab-4aB)\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{3ab}$$

$$\Big\downarrow \; 759$$

$$\frac{2x^2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{(Ab-4aB)\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\cdots\right.}{\sqrt[3]{b}x+\left(1+\cdots\right.}\right.\right.\cdots}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots\right)}{3ab}$$

$$\Big\downarrow \; 2416$$

$$\frac{2x^2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{(Ab-4aB)\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{\cdots}\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots}{\sqrt[3]{b}}\cdots\right)}{3ab}$$

input
```
Int[(x*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*(A*b - a*B)*x^2)/(3*a*b*Sqrt[a + b*x^3]) - ((A*b - 4*a*B)*(((2*Sqrt[a +
 b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 -
 Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt
[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(3*a*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 490, normalized size of antiderivative = 0.94

method	result
	$2i\left(\frac{B}{b}-\frac{Ab-Ba}{3ab}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\right.}{}}$
elliptic	$\frac{2x^{2}(Ab-Ba)}{3ba\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-$
default	Expression too large to display

input
```
int(x*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b*x^2/a*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-2/3*I*(B/b-1/3*(A*b-B*a)/a/b)*3^
(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipt
icE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.18

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2\left(\sqrt{bx^3+a}(Bab-Ab^2)x^2+((4\,Bab-Ab^2)x^3+4\,Ba^2-Aab)\sqrt{b}\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassP}\right.}{3\,(ab^3x^3+a^2b^2)}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(b*x^3 + a)*(B*a*b - A*b^2)*x^2 + ((4*B*a*b - A*b^2)*x^3 + 4*B*a
^2 - A*a*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a
/b, x)))/(a*b^3*x^3 + a^2*b^2)
```

Sympy [A] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{Ax^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{Bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
A*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(3/2)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3))
```

Maxima [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{x\,(B\,x^3+A)}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((x*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

output
```
int((x*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx$$

input
```
int(x*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

output
```
int((sqrt(a + b*x**3)*x)/(a + b*x**3),x)
```

3.220 $\int \frac{A+Bx^3}{x^2(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 548

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^{3/2}}\,dx = -\frac{A}{ax\sqrt{a+bx^3}} - \frac{(5Ab-2aB)x^2}{3a^2\sqrt{a+bx^3}} + \frac{(5Ab-2aB)\sqrt{a+bx^3}}{3a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{\sqrt{2-\sqrt{3}}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2\ 3^{3/4}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{\sqrt{2}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-A/a/x/(b*x^3+a)^(1/2)-1/3*(5*A*b-2*B*a)*x^2/a^2/(b*x^3+a)^(1/2)+1/3*(5*A*
b-2*B*a)*(b*x^3+a)^(1/2)/a^2/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)-1/6*(
1/2*6^(1/2)-1/2*2^(1/2))*(5*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*Ellipti
cE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/
2)+2*I)*3^(1/4)/a^(5/3)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*
a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+1/9*2^(1/2)*(5*A*b-2*B*a)*(a^(
1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1
/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1
/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^(5/3)/b^(2/3)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.13

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx = \frac{-4aA+(-5Ab+2aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a}\right)}{4a^2x\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^(3/2)),x]
```

output
```
(-4*a*A + (-5*A*b + 2*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3,
3/2, 5/3, -((b*x^3)/a)])/(4*a^2*x*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {955, 819, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^2 \left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(5Ab-2aB)\int \frac{x}{\left(bx^3+a\right)^{3/2}}dx}{2a}-\frac{A}{ax\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(5Ab-2aB)\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\int \frac{x}{\sqrt{bx^3+a}}dx}{3a}\right)}{2a}-\frac{A}{ax\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{832}$$

$$-\frac{(5Ab-2aB)\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}}{3a}\right)}{2a}-\frac{A}{ax\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{759}$$

$$-\frac{(5Ab-2aB)\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\cdots\right)\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{3a}\right)}{2a}$$

$$\frac{A}{ax\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2416}$$

$$-\frac{(5Ab-2aB)\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+(1}{\sqrt[3]{b}x+(1}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}}{\right.}\right)}{2a}$$

$$\frac{A}{ax\sqrt{a+bx^3}}$$

input
```
Int[(A + B*x^3)/(x^2*(a + b*x^3)^(3/2)),x]
```

output
```
-(A/(a*x*Sqrt[a + b*x^3])) - ((5*A*b - 2*a*B)*((2*x^2)/(3*a*Sqrt[a + b*x^3
]) - (((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) -
 (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*
x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*
(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(
3*a)))/(2*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.92

method	result
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{a^{2}x}-\frac{2x^{2}(Ab-Ba)}{3a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2i\left(\frac{Ab}{2a^{2}}+\frac{Ab-Ba}{3a^{2}}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}}}}{}$
default	Expression too large to display
risch	Expression too large to display

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*A*(b*x^3+a)^(1/2)/x-2/3*x^2/a^2*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-2/3*I
*(1/2/a^2*A*b+1/3*(A*b-B*a)/a^2)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.19

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx = \frac{((2\,Bab-5\,Ab^2)x^4+(2\,Ba^2-5\,Aab)x)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPIn}\right.}{3\left(a^2b^2x^4+a^3bx\right)}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*((((2*B*a*b - 5*A*b^2)*x^4 + (2*B*a^2 - 5*A*a*b)*x)*sqrt(b)*weierstrass
Zeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + ((2*B*a*b - 5*A*b^2)*
x^3 - 3*A*a*b)*sqrt(b*x^3 + a))/(a^2*b^2*x^4 + a^3*b*x)
```

Sympy [A] (verification not implemented)

Time = 7.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{3}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x\Gamma\left(\frac{2}{3}\right)}+\frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**2/(b*x**3+a)**(3/2),x)
```

output
```
A*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*x*gamma(2/3)) + B*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^{3/2}}\,dx = \int \frac{B\,x^3+A}{x^2\,(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((A + B*x^3)/(x^2*(a + b*x^3)^(3/2)),x)
```

output
```
int((A + B*x^3)/(x^2*(a + b*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{\sqrt{b\,x^3+a}}{b\,x^5+a\,x^2}dx$$

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a*x**2 + b*x**5),x)
```

3.221 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 580

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx = -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}}$$
$$+\frac{5(11Ab-8aB)\sqrt{a+bx^3}}{24a^3x} - \frac{5\sqrt[3]{b}(11Ab-8aB)\sqrt{a+bx^3}}{24a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}(11Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{\ldots}\right.}{16\ 3^{3/4}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{5\sqrt[3]{b}(11Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{\ldots}\right.}{12\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/4*A/a/x^4/(b*x^3+a)^(1/2)-1/12*(11*A*b-8*B*a)/a^2/x/(b*x^3+a)^(1/2)+5/2
4*(11*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^3/x-5/24*b^(1/3)*(11*A*b-8*B*a)*(b*x^3+
a)^(1/2)/a^3/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+5/48*(1/2*6^(1/2)-1/2*2^(1/2)
)*b^(1/3)*(11*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2)
)*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4
)/a^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^
(1/2)/(b*x^3+a)^(1/2)-5/72*b^(1/3)*(11*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*((a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1
/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)
*x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/a^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((
1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.12

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx = \frac{-2aA+(11Ab-8aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a}\right)}{8a^2x^4\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^(3/2)),x]
```

output
```
(-2*a*A + (11*A*b - 8*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3,
 3/2, 2/3, -((b*x^3)/a)])/(8*a^2*x^4*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.01 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 819, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx$$

$\big\downarrow$ 955

$$-\frac{(11Ab-8aB)\int \frac{1}{x^2\left(bx^3+a\right)^{3/2}}dx}{8a}-\frac{A}{4ax^4\sqrt{a+bx^3}}$$

$\big\downarrow$ 819

$$-\frac{(11Ab-8aB)\left(\frac{5\int \frac{1}{x^2\sqrt{bx^3+a}}dx}{3a}+\frac{2}{3ax\sqrt{a+bx^3}}\right)}{8a}-\frac{A}{4ax^4\sqrt{a+bx^3}}$$

$\big\downarrow$ 847

$$-\frac{(11Ab-8aB)\left(\frac{5\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{3a}+\frac{2}{3ax\sqrt{a+bx^3}}\right)}{8a}-\frac{A}{4ax^4\sqrt{a+bx^3}}$$

$\big\downarrow$ 832

$$-\frac{(11Ab-8aB)\left(\frac{5\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{3a}+\frac{2}{3ax\sqrt{a+bx^3}}\right)}{8a}-\frac{A}{4ax^4\sqrt{a+bx^3}}$$

$\big\downarrow$ 759

$$-\frac{(11Ab-8aB)\left(\frac{5\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+(1}{\sqrt[3]{b}x+(1}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{2a}\right.}{3a}\right.}{8a}$$

$$\frac{A}{4ax^4\sqrt{a+bx^3}}$$

$$\downarrow 2416$$

$$-\frac{(11Ab-8aB)\left(\frac{5\left(\frac{b\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots\right)}{\sqrt[3]{b}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{A}{4ax^4\sqrt{a+bx^3}}$$

input
```
Int[(A + B*x^3)/(x^5*(a + b*x^3)^(3/2)),x]
```

output
```
-1/4*A/(a*x^4*Sqrt[a + b*x^3]) - ((11*A*b - 8*a*B)*(2/(3*a*x*Sqrt[a + b*x^
3]) + (5*(-(Sqrt[a + b*x^3]/(a*x)) + (b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^
(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)
*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(2*a)))/(3*a)))/(8*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 540, normalized size of antiderivative = 0.93

method	result
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{4a^{2}x^{4}}+\frac{(13Ab-8Ba)\sqrt{b\,x^{3}+a}}{8a^{3}x}+\frac{2b\,x^{2}(Ab-Ba)}{3a^{3}\sqrt{(x^{3}+\frac{a}{b})b}}-\frac{2i\left(-\frac{b(13Ab-8Ba)}{16a^{3}}-\frac{(Ab-Ba)b}{3a^{3}}\right)\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i}{\ldots}\right.}{(-a\,\ldots}}}{\ldots}$
default	Expression too large to display
risch	Expression too large to display

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/a^2*(b*x^3+a)^(1/2)/x^4+1/8/a^3*(13*A*b-8*B*a)*(b*x^3+a)^(1/2)/x+2/
3*b*x^2/a^3*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-2/3*I*(-1/16*b*(13*A*b-8*B*a)/a^
3-1/3*(A*b-B*a)/a^3*b)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(
-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-
a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.22

$$\int \frac{A+Bx^3}{x^5 (a+bx^3)^{3/2}}\, dx = -\frac{5\left((8\,Bab-11\,Ab^2)x^7+(8\,Ba^2-11\,Aab)x^4\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right.}{24\left(a^3bx^7+a^4x^4\right)}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/24*(5*((8*B*a*b - 11*A*b^2)*x^7 + (8*B*a^2 - 11*A*a*b)*x^4)*sqrt(b)*wei
erstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (5*(8*B*a*b -
 11*A*b^2)*x^6 + 3*(8*B*a^2 - 11*A*a*b)*x^3 + 6*A*a^2)*sqrt(b*x^3 + a))/(a
^3*b*x^7 + a^4*x^4)
```

Sympy [A] (verification not implemented)

Time = 17.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^3}{x^5 (a+bx^3)^{3/2}}\, dx = \frac{A\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{3}{2} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{B\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{3}{2} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**5/(b*x**3+a)**(3/2),x)
```

output
```
A*gamma(-4/3)*hyper((-4/3, 3/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(3/2)*x**4*gamma(-1/3)) + B*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(3/2)*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^5(a+bx^3)^{3/2}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}x^5}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^5), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^5(a+bx^3)^{3/2}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}x^5}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^5(a+bx^3)^{3/2}}\,dx=\int \frac{B\,x^3+A}{x^5\,(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((A + B*x^3)/(x^5*(a + b*x^3)^(3/2)),x)
```

output
```
int((A + B*x^3)/(x^5*(a + b*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^5 (a+bx^3)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^3+a}}{b\,x^8+a\,x^5}dx$$

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a*x**5 + b*x**8),x)
```

3.222 $\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 611

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx = -\frac{A}{7ax^7\sqrt{a+bx^3}}$$
$$-\frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}}+\frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4}$$
$$-\frac{55b(17Ab-14aB)\sqrt{a+bx^3}}{336a^4x}+\frac{55b^{4/3}(17Ab-14aB)\sqrt{a+bx^3}}{336a^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{55\sqrt{2-\sqrt{3}}b^{4/3}(17Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{224\ 3^{3/4}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{55b^{4/3}(17Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\right.}{168\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/7*A/a/x^7/(b*x^3+a)^(1/2)-1/21*(17*A*b-14*B*a)/a^2/x^4/(b*x^3+a)^(1/2)+
11/168*(17*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^3/x^4-55/336*b*(17*A*b-14*B*a)*(b
*x^3+a)^(1/2)/a^4/x+55/336*b^(4/3)*(17*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^4/((1
+3^(1/2))*a^(1/3)+b^(1/3)*x)-55/672*(1/2*6^(1/2)-1/2*2^(1/2))*b^(4/3)*(17*
A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
(1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(
1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/a^(11/3)/(a
^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3
+a)^(1/2)+55/1008*b^(4/3)*(17*A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*Elli
pticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^
(1/2)+2*I)*2^(1/2)*3^(3/4)/a^(11/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/
2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.12

$$\int \frac{A+Bx^3}{x^8 \left(a+bx^3\right)^{3/2}}\,dx = \frac{-8aA+(17Ab-14aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{3}{2},-\frac{1}{3},-\frac{bx^3}{a}\right)}{56a^2x^7\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^8*(a + b*x^3)^(3/2)),x]
```

output
```
(-8*a*A + (17*A*b - 14*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-4/3
, 3/2, -1/3, -((b*x^3)/a)])/(56*a^2*x^7*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.04 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {955, 819, 847, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx$$

$\big\downarrow$ 955

$$-\frac{(17Ab-14aB)\int \frac{1}{x^5\left(bx^3+a\right)^{3/2}}dx}{14a}-\frac{A}{7ax^7\sqrt{a+bx^3}}$$

$\big\downarrow$ 819

$$-\frac{(17Ab-14aB)\left(\frac{11\int \frac{1}{x^5\sqrt{bx^3+a}}dx}{3a}+\frac{2}{3ax^4\sqrt{a+bx^3}}\right)}{14a}-\frac{A}{7ax^7\sqrt{a+bx^3}}$$

$\big\downarrow$ 847

$$-\frac{(17Ab-14aB)\left(\frac{11\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+a}}dx}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)}{3a}+\frac{2}{3ax^4\sqrt{a+bx^3}}\right)}{14a}-\frac{A}{7ax^7\sqrt{a+bx^3}}$$

$\big\downarrow$ 847

$$-\frac{(17Ab-14aB)\left(\frac{11\left(-\frac{5b\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)}{3a}+\frac{2}{3ax^4\sqrt{a+bx^3}}\right)}{14a}-\frac{A}{7ax^7\sqrt{a+bx^3}}$$

$\big\downarrow$ 832

$$-\frac{(17Ab-14aB)\left(\frac{11\left(-\frac{5b\left(\frac{b\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)}{3a}+\frac{2}{3ax^4\sqrt{a+bx^3}}\right)}{14a}-\frac{A}{7ax^7\sqrt{a+bx^3}}$$

$\Big\downarrow$ 759

$$(17Ab-14aB)\left(\frac{11\left(-\frac{5b\left(\frac{b\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots\right)}{2a}\cdots\right)}{8a}\cdots\right)}{3a}\cdots\right.$$

↓ 2416

$$\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+(\cdots}{\sqrt[3]{b}x+(\cdots}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$b\,\frac{\cdots}{\sqrt[3]{b}}$$

$$5b\,\frac{\cdots}{\cdots}$$

$$11\left(-\frac{\cdots}{\cdots}\right.$$

$$(17Ab-14aB)\,\frac{\cdots}{\cdots}$$

input
```
Int[(A + B*x^3)/(x^8*(a + b*x^3)^(3/2)),x]
```

output
```
-1/7*A/(a*x^7*Sqrt[a + b*x^3]) - ((17*A*b - 14*a*B)*(2/(3*a*x^4*Sqrt[a + b
*x^3]) + (11*(-1/4*Sqrt[a + b*x^3]/(a*x^4) - (5*b*(-(Sqrt[a + b*x^3]/(a*x)
) + (b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (
2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))
)/(2*a)))/(8*a)))/(3*a)))/(14*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 573, normalized size of antiderivative = 0.94

method	result
	$2i\left(\frac{b^2(237Ab-182Ba)}{224a^4}+\frac{b^2(Ab-Ba}{3a^4}\right.$
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{7a^2x^7}+\frac{(25Ab-14Ba)\sqrt{b\,x^3+a}}{56a^3x^4}-\frac{(237Ab-182Ba)b\sqrt{b\,x^3+a}}{112a^4x}-\frac{2b^2x^2(Ab-Ba)}{3a^4\sqrt{(x^3+\frac{a}{b})b}}-$
risch	Expression too large to display
default	Expression too large to display

input
```
int((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7*A/a^2*(b*x^3+a)^(1/2)/x^7+1/56/a^3*(25*A*b-14*B*a)*(b*x^3+a)^(1/2)/x^
4-1/112/a^4*(237*A*b-182*B*a)*b*(b*x^3+a)^(1/2)/x-2/3*b^2*x^2/a^4*(A*b-B*a
)/((x^3+a/b)*b)^(1/2)-2/3*I*(1/224*b^2*(237*A*b-182*B*a)/a^4+1/3*b^2*(A*b-
B*a)/a^4)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.26

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx = \frac{55\left(\left(14\,Bab^2-17\,Ab^3\right)x^{10}+\left(14\,Ba^2b-17\,Aab^2\right)x^7\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{we}\right.}{}$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
1/336*(55*((14*B*a*b^2 - 17*A*b^3)*x^10 + (14*B*a^2*b - 17*A*a*b^2)*x^7)*s
qrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (55
*(14*B*a*b^2 - 17*A*b^3)*x^9 + 33*(14*B*a^2*b - 17*A*a*b^2)*x^6 - 48*A*a^3
 - 6*(14*B*a^3 - 17*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b*x^10 + a^5*x^7)
```

Sympy [A] (verification not implemented)

Time = 46.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},\frac{3}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^7\Gamma\left(-\frac{4}{3}\right)}+\frac{B\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{3}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**8/(b*x**3+a)**(3/2),x)
```

output
```
A*gamma(-7/3)*hyper((-7/3, 3/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(3/2)*x**7*gamma(-4/3)) + B*gamma(-4/3)*hyper((-4/3, 3/2), (-1/3,), b*x**3
*exp_polar(I*pi)/a)/(3*a**(3/2)*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^8(a+bx^3)^{3/2}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}x^8}\,dx$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^8), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^8(a+bx^3)^{3/2}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}x^8}\,dx$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^8(a+bx^3)^{3/2}}\,dx=\int \frac{B\,x^3+A}{x^8\,(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((A + B*x^3)/(x^8*(a + b*x^3)^(3/2)),x)
```

output
```
int((A + B*x^3)/(x^8*(a + b*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A + Bx^3}{x^8 (a + bx^3)^{3/2}} \, dx = \int \frac{\sqrt{b\,x^3 + a}}{b\,x^{11} + a\,x^8} dx$$

input
```
int((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a*x**8 + b*x**11),x)
```

3.223 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=-\frac{2a^2(Ab-aB)}{9b^4(a+bx^3)^{3/2}}+\frac{2a(2Ab-3aB)}{3b^4\sqrt{a+bx^3}}+\frac{2(Ab-3aB)\sqrt{a+bx^3}}{3b^4}+\frac{2B(a+bx^3)^{3/2}}{9b^4}$$

output
```
-2/9*a^2*(A*b-B*a)/b^4/(b*x^3+a)^(3/2)+2/3*a*(2*A*b-3*B*a)/b^4/(b*x^3+a)^(
1/2)+2/3*(A*b-3*B*a)*(b*x^3+a)^(1/2)/b^4+2/9*B*(b*x^3+a)^(3/2)/b^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.71

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\frac{2(-16a^3B+8a^2b(A-3Bx^3)-6ab^2x^3(-2A+Bx^3)+b^3x^6(3A+Bx^3))}{9b^4(a+bx^3)^{3/2}}$$

input
```
Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*(-16*a^3*B + 8*a^2*b*(A - 3*B*x^3) - 6*a*b^2*x^3*(-2*A + B*x^3) + b^3*x
^6*(3*A + B*x^3)))/(9*b^4*(a + b*x^3)^(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (A+Bx^3)}{(a+bx^3)^{5/2}} \, dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{x^6 (Bx^3+A)}{(bx^3+a)^{5/2}}dx^3$$

$$\Big\downarrow \; 86$$

$$\frac{1}{3}\int \left(-\frac{(aB-Ab)a^2}{b^3 (bx^3+a)^{5/2}}+\frac{(3aB-2Ab)a}{b^3 (bx^3+a)^{3/2}}+\frac{B\sqrt{bx^3+a}}{b^3}+\frac{Ab-3aB}{b^3\sqrt{bx^3+a}}\right)dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(-\frac{2a^2(Ab-aB)}{3b^4 (a+bx^3)^{3/2}}+\frac{2a(2Ab-3aB)}{b^4\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}(Ab-3aB)}{b^4}+\frac{2B(a+bx^3)^{3/2}}{3b^4}\right)$$

input
```
Int[(x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
((-2*a^2*(A*b - a*B))/(3*b^4*(a + b*x^3)^(3/2)) + (2*a*(2*A*b - 3*a*B))/(b
^4*Sqrt[a + b*x^3]) + (2*(A*b - 3*a*B)*Sqrt[a + b*x^3])/b^4 + (2*B*(a + b*
x^3)^(3/2))/(3*b^4))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{\left(2B\,x^{9}+6A\,x^{6}\right)b^{3}+24\left(-\frac{B\,x^{3}}{2}+A\right)a\,x^{3}b^{2}+16a^{2}\left(-3B\,x^{3}+A\right)b-32a^{3}B}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}b^{4}}$
risch	$\frac{2\left(bB\,x^{3}+3Ab-8Ba\right)\sqrt{b\,x^{3}+a}}{9b^{4}}+\frac{2a\left(6A\,b^{2}x^{3}-9Bab\,x^{3}+5abA-8a^{2}B\right)}{9b^{4}\left(b\,x^{3}+a\right)^{\frac{3}{2}}}$
gosper	$\frac{\frac{2}{9}b^{3}B\,x^{9}+\frac{2}{3}A\,b^{3}x^{6}-\frac{4}{3}Ba\,b^{2}x^{6}+\frac{8}{3}aA\,b^{2}x^{3}-\frac{16}{3}B\,a^{2}b\,x^{3}+\frac{16}{9}a^{2}bA-\frac{32}{9}a^{3}B}{\left(b\,x^{3}+a\right)^{\frac{3}{2}}b^{4}}$
trager	$\frac{\frac{2}{9}b^{3}B\,x^{9}+\frac{2}{3}A\,b^{3}x^{6}-\frac{4}{3}Ba\,b^{2}x^{6}+\frac{8}{3}aA\,b^{2}x^{3}-\frac{16}{3}B\,a^{2}b\,x^{3}+\frac{16}{9}a^{2}bA-\frac{32}{9}a^{3}B}{\left(b\,x^{3}+a\right)^{\frac{3}{2}}b^{4}}$
orering	$\frac{\frac{2}{9}b^{3}B\,x^{9}+\frac{2}{3}A\,b^{3}x^{6}-\frac{4}{3}Ba\,b^{2}x^{6}+\frac{8}{3}aA\,b^{2}x^{3}-\frac{16}{3}B\,a^{2}b\,x^{3}+\frac{16}{9}a^{2}bA-\frac{32}{9}a^{3}B}{\left(b\,x^{3}+a\right)^{\frac{3}{2}}b^{4}}$
elliptic	$-\frac{2a^{2}\left(Ab-Ba\right)\sqrt{b\,x^{3}+a}}{9b^{6}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{2\left(2Ab-3Ba\right)a}{3b^{4}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2B\,x^{3}\sqrt{b\,x^{3}+a}}{9b^{3}}+\frac{2\left(\frac{Ab-2Ba}{b^{3}}-\frac{2Ba}{3b^{3}}\right)\sqrt{b\,x^{3}+a}}{3b}$
default	$A\left(-\frac{2a^{2}\sqrt{b\,x^{3}+a}}{9b^{5}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{4a}{3b^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2\sqrt{b\,x^{3}+a}}{3b^{3}}\right)+B\left(\frac{2a^{3}\sqrt{b\,x^{3}+a}}{9b^{6}\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{2a^{2}}{b^{4}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2x^{3}\sqrt{b\,x^{3}+a}}{9b^{3}}-1\right.$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*((2*B*x^9+6*A*x^6)*b^3+24*(-1/2*B*x^3+A)*a*x^3*b^2+16*a^2*(-3*B*x^3+A)
*b-32*a^3*B)/(b*x^3+a)^(3/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\left(Bb^3x^9-3\left(2\,Bab^2-Ab^3\right)x^6-16\,Ba^3+8\,Aa^2b-12\left(2\,Ba^2b-Aab^2\right)x^3\right)\sqrt{bx^3+a}}{9\left(b^6x^6+2\,ab^5x^3+a^2b^4\right)}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
2/9*(B*b^3*x^9 - 3*(2*B*a*b^2 - A*b^3)*x^6 - 16*B*a^3 + 8*A*a^2*b - 12*(2*
B*a^2*b - A*a*b^2)*x^3)*sqrt(b*x^3 + a)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(99) = 198.

Time = 0.59 (sec) , antiderivative size = 338, normalized size of antiderivative = 3.28

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \begin{cases} \frac{16Aa^2b}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} + \frac{24Aab^2x^3}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} + \frac{6Ab^3x^6}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} - \frac{\cdots}{9ab^4\sqrt{a+b\cdots}} \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{a^{\frac{5}{2}}} \end{cases}$$

input
```
integrate(x**8*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
Piecewise((16*A*a**2*b/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b
*x**3)) + 24*A*a*b**2*x**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a
 + b*x**3)) + 6*A*b**3*x**6/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(
a + b*x**3)) - 32*B*a**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a +
 b*x**3)) - 48*B*a**2*b*x**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt
(a + b*x**3)) - 12*B*a*b**2*x**6/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*
sqrt(a + b*x**3)) + 2*B*b**3*x**9/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3
*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**9/9 + B*x**12/12)/a**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2}{9}\,B\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^4} - \frac{9\sqrt{bx^3+a}\,a}{b^4} - \frac{9\,a^2}{\sqrt{bx^3+a}\,b^4} + \frac{a^3}{(bx^3+a)^{\frac{3}{2}}b^4}\right) + \frac{2}{9}\,A\left(\frac{3\sqrt{bx^3+a}}{b^3} + \frac{6\,a}{\sqrt{bx^3+a}\,b^3} - \frac{a^2}{(bx^3+a)^{\frac{3}{2}}b^3}\right)$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
2/9*B*((b*x^3 + a)^(3/2)/b^4 - 9*sqrt(b*x^3 + a)*a/b^4 - 9*a^2/(sqrt(b*x^3
 + a)*b^4) + a^3/((b*x^3 + a)^(3/2)*b^4)) + 2/9*A*(3*sqrt(b*x^3 + a)/b^3 +
 6*a/(sqrt(b*x^3 + a)*b^3) - a^2/((b*x^3 + a)^(3/2)*b^3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2\,(9\,(bx^3+a)Ba^2 - Ba^3 - 6\,(bx^3+a)Aab + Aa^2b)}{9\,(bx^3+a)^{\frac{3}{2}}b^4} + \frac{2\left((bx^3+a)^{\frac{3}{2}}Bb^8 - 9\sqrt{bx^3+a}Bab^8 + 3\sqrt{bx^3+a}Ab^9\right)}{9\,b^{12}}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
-2/9*(9*(b*x^3 + a)*B*a^2 - B*a^3 - 6*(b*x^3 + a)*A*a*b + A*a^2*b)/((b*x^3
 + a)^(3/2)*b^4) + 2/9*((b*x^3 + a)^(3/2)*B*b^8 - 9*sqrt(b*x^3 + a)*B*a*b^
8 + 3*sqrt(b*x^3 + a)*A*b^9)/b^12
```

Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.41

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{\sqrt{b\,x^3+a}\,\left(\frac{2\,(A\,b-2\,B\,a)}{b^3}-\frac{4\,B\,a}{3\,b^3}\right)}{3\,b} - \frac{\frac{2\,B\,a^2-2\,A\,a\,b}{3\,b^4}-\frac{a\,\left(\frac{2\,A\,b^2-2\,B\,a\,b}{3\,b^4}-\frac{2\,B\,a}{3\,b^3}\right)}{b}}{\sqrt{b\,x^3+a}} - \frac{a^2\,\left(\frac{2\,A}{9\,b}-\frac{2\,B\,a}{9\,b^2}\right)}{b^2\,{\left(b\,x^3+a\right)}^{3/2}} + \frac{2\,B\,x^3\,\sqrt{b\,x^3+a}}{9\,b^3}$$

input
```
int((x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

output
```
((a + b*x^3)^(1/2)*((2*(A*b - 2*B*a))/b^3 - (4*B*a)/(3*b^3)))/(3*b) - ((2*
B*a^2 - 2*A*a*b)/(3*b^4) - (a*((2*A*b^2 - 2*B*a*b)/(3*b^4) - (2*B*a)/(3*b^
3)))/b)/(a + b*x^3)^(1/2) - (a^2*((2*A)/(9*b) - (2*B*a)/(9*b^2)))/(b^2*(a
+ b*x^3)^(3/2)) + (2*B*x^3*(a + b*x^3)^(1/2))/(9*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.41

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\sqrt{b\,x^3+a}\,(b^2x^6-4ab\,x^3-8a^2)}{9b^3\,(b\,x^3+a)}$$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

output
```
(2*sqrt(a + b*x**3)*( - 8*a**2 - 4*a*b*x**3 + b**2*x**6))/(9*b**3*(a + b*x
**3))
```

3.224 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}} \, dx = \frac{2a(Ab-aB)}{9b^3(a+bx^3)^{3/2}} - \frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}} + \frac{2B\sqrt{a+bx^3}}{3b^3}$$

output
```
2/9*a*(A*b-B*a)/b^3/(b*x^3+a)^(3/2)-2/3*(A*b-2*B*a)/b^3/(b*x^3+a)^(1/2)+2/
3*B*(b*x^3+a)^(1/2)/b^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}} \, dx = \frac{2(-2aAb+8a^2B-3Ab^2x^3+12abBx^3+3b^2Bx^6)}{9b^3(a+bx^3)^{3/2}}$$

input
```
Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*(-2*a*A*b + 8*a^2*B - 3*A*b^2*x^3 + 12*a*b*B*x^3 + 3*b^2*B*x^6))/(9*b^3
*(a + b*x^3)^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^3)}{(a+bx^3)^{5/2}} \, dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3} \int \frac{x^3 (Bx^3+A)}{(bx^3+a)^{5/2}} dx^3$$

$$\Big\downarrow \; 86$$

$$\frac{1}{3} \int \left(\frac{B}{b^2 \sqrt{bx^3+a}} + \frac{Ab-2aB}{b^2 (bx^3+a)^{3/2}} + \frac{a(aB-Ab)}{b^2 (bx^3+a)^{5/2}} \right) dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3} \left(-\frac{2(Ab-2aB)}{b^3 \sqrt{a+bx^3}} + \frac{2a(Ab-aB)}{3b^3 (a+bx^3)^{3/2}} + \frac{2B\sqrt{a+bx^3}}{b^3} \right)$$

input
```
Int[(x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
((2*a*(A*b - a*B))/(3*b^3*(a + b*x^3)^(3/2)) - (2*(A*b - 2*a*B))/(b^3*Sqrt
[a + b*x^3]) + (2*B*Sqrt[a + b*x^3])/b^3)/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$-\frac{4\left(\frac{3x^{3}\left(-B\,x^{3}+A\right)b^{2}}{2}+a\left(-6B\,x^{3}+A\right)b-4a^{2}B\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}b^{3}}$	49
gosper	$-\frac{2\left(-3b^{2}B\,x^{6}+3A\,b^{2}x^{3}-12Bab\,x^{3}+2abA-8a^{2}B\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}b^{3}}$	53
trager	$-\frac{2\left(-3b^{2}B\,x^{6}+3A\,b^{2}x^{3}-12Bab\,x^{3}+2abA-8a^{2}B\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}b^{3}}$	53
orering	$-\frac{2\left(-3b^{2}B\,x^{6}+3A\,b^{2}x^{3}-12Bab\,x^{3}+2abA-8a^{2}B\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}b^{3}}$	53
risch	$\frac{2B\sqrt{b\,x^{3}+a}}{3b^{3}}-\frac{2\left(3A\,b^{2}x^{3}-6Bab\,x^{3}+2abA-5a^{2}B\right)}{9b^{3}\left(b\,x^{3}+a\right)^{\frac{3}{2}}}$	60
elliptic	$\frac{2\left(Ab-Ba\right)a\sqrt{b\,x^{3}+a}}{9b^{5}\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{2\left(Ab-2Ba\right)}{3b^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2B\sqrt{b\,x^{3}+a}}{3b^{3}}$	77
default	$A\left(\frac{2a\sqrt{b\,x^{3}+a}}{9b^{4}\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{2}{3b^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}\right)+B\left(-\frac{2a^{2}\sqrt{b\,x^{3}+a}}{9b^{5}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{4a}{3b^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2\sqrt{b\,x^{3}+a}}{3b^{3}}\right)$	113

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4/9*(3/2*x^3*(-B*x^3+A)*b^2+a*(-6*B*x^3+A)*b-4*a^2*B)/(b*x^3+a)^(3/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\,(3\,Bb^2x^6+3\,(4\,Bab-Ab^2)x^3+8\,Ba^2-2\,Aab)\sqrt{bx^3+a}}{9\,(b^5x^6+2\,ab^4x^3+a^2b^3)}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
2/9*(3*B*b^2*x^6 + 3*(4*B*a*b - A*b^2)*x^3 + 8*B*a^2 - 2*A*a*b)*sqrt(b*x^3
 + a)/(b^5*x^6 + 2*a*b^4*x^3 + a^2*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(70) = 140.

Time = 0.54 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.29

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \begin{cases} -\frac{4Aab}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} - \frac{6Ab^2x^3}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} + \frac{16Ba^2}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} + \frac{}{9ab^3\sqrt{a\cdots}} \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{a^{\frac{5}{2}}} \end{cases}$$

input
```
integrate(x**5*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
Piecewise((-4*A*a*b/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x*
*3)) - 6*A*b**2*x**3/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x
**3)) + 16*B*a**2/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3
)) + 24*B*a*b*x**3/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**
3)) + 6*B*b**2*x**6/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x*
*3)), Ne(b, 0)), ((A*x**6/6 + B*x**9/9)/a**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2}{9}\,B\left(\frac{3\,\sqrt{bx^3+a}}{b^3}+\frac{6\,a}{\sqrt{bx^3+a}b^3}-\frac{a^2}{(bx^3+a)^{\frac{3}{2}}b^3}\right) - \frac{2}{9}\,A\left(\frac{3}{\sqrt{bx^3+a}b^2}-\frac{a}{(bx^3+a)^{\frac{3}{2}}b^2}\right)$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
2/9*B*(3*sqrt(b*x^3 + a)/b^3 + 6*a/(sqrt(b*x^3 + a)*b^3) - a^2/((b*x^3 + a
)^(3/2)*b^3)) - 2/9*A*(3/(sqrt(b*x^3 + a)*b^2) - a/((b*x^3 + a)^(3/2)*b^2)
)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\,\sqrt{bx^3+a}B}{3\,b^3}+\frac{2\,(6\,(bx^3+a)Ba-Ba^2-3\,(bx^3+a)Ab+Aab)}{9\,(bx^3+a)^{\frac{3}{2}}b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
2/3*sqrt(b*x^3 + a)*B/b^3 + 2/9*(6*(b*x^3 + a)*B*a - B*a^2 - 3*(b*x^3 + a)
*A*b + A*a*b)/((b*x^3 + a)^(3/2)*b^3)
```

Mupad [B] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{6\,B\,\left(b\,x^3+a\right)^2-2\,B\,a^2-6\,A\,b\,\left(b\,x^3+a\right)+12\,B\,a\,\left(b\,x^3+a\right)+2\,A\,a\,b}{9\,b^3\,\left(b\,x^3+a\right)^{3/2}}$$

input
```
int((x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

output
```
(6*B*(a + b*x^3)^2 - 2*B*a^2 - 6*A*b*(a + b*x^3) + 12*B*a*(a + b*x^3) + 2*
A*a*b)/(9*b^3*(a + b*x^3)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.42

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\sqrt{b\,x^3+a}\,\left(b\,x^3+2a\right)}{3b^2\left(b\,x^3+a\right)}$$

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

output
```
(2*sqrt(a + b*x**3)*(2*a + b*x**3))/(3*b**2*(a + b*x**3))
```

3.225 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2(Ab-aB)}{9b^2(a+bx^3)^{3/2}} - \frac{2B}{3b^2\sqrt{a+bx^3}}$$

output
```
1/9*(-2*A*b+2*B*a)/b^2/(b*x^3+a)^(3/2)-2/3*B/b^2/(b*x^3+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2(Ab+2aB+3bBx^3)}{9b^2(a+bx^3)^{3/2}}$$

input
```
Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(-2*(A*b + 2*a*B + 3*b*B*x^3))/(9*b^2*(a + b*x^3)^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^3)}{(a+bx^3)^{5/2}} \, dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3} \int \frac{Bx^3+A}{(bx^3+a)^{5/2}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3} \int \left(\frac{B}{b (bx^3+a)^{3/2}}+\frac{Ab-aB}{b (bx^3+a)^{5/2}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{2(Ab-aB)}{3b^2 (a+bx^3)^{3/2}}-\frac{2B}{b^2\sqrt{a+bx^3}}\right)$$

input
```
Int[(x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
((-2*(A*b - a*B))/(3*b^2*(a + b*x^3)^(3/2)) - (2*B)/(b^2*Sqrt[a + b*x^3]))
/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{2\left(3bB\,x^{3}+Ab+2Ba\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}} b^{2}}$	30
trager	$-\frac{2\left(3bB\,x^{3}+Ab+2Ba\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}} b^{2}}$	30
pseudoelliptic	$-\frac{2\left(\left(3B\,x^{3}+A\right)b+2Ba\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}} b^{2}}$	30
orering	$-\frac{2\left(3bB\,x^{3}+Ab+2Ba\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}} b^{2}}$	30
elliptic	$-\frac{2\left(Ab-Ba\right)\sqrt{b\,x^{3}+a}}{9b^{4}\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{2B}{3b^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}$	54
default	$-\frac{2A}{9b\left(b\,x^{3}+a\right)^{\frac{3}{2}}}+B\left(\frac{2a\sqrt{b\,x^{3}+a}}{9b^{4}\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{2}{3b^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}\right)$	64

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9/(b*x^3+a)^(3/2)*(3*B*b*x^3+A*b+2*B*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2\,(3\,Bbx^3+2\,Ba+Ab)\sqrt{bx^3+a}}{9\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/9*(3*B*b*x^3 + 2*B*a + A*b)*sqrt(b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^
2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(44) = 88.

Time = 0.35 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.13

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \begin{cases} -\frac{2Ab}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} - \frac{4Ba}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} - \frac{6Bbx^3}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} & \text{for } b \neq \\ \frac{\frac{Ax^3}{3}+\frac{Bx^6}{6}}{a^{\frac{5}{2}}} & \text{otherw} \end{cases}$$

input
```
integrate(x**2*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
Piecewise((-2*A*b/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3
)) - 4*B*a/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)) - 6*
B*b*x**3/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)), Ne(b,
 0)), ((A*x**3/3 + B*x**6/6)/a**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2}{9}\,B\left(\frac{3}{\sqrt{bx^3+a}b^2} - \frac{a}{(bx^3+a)^{\frac{3}{2}}b^2}\right) - \frac{2\,A}{9\,(bx^3+a)^{\frac{3}{2}}b}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
-2/9*B*(3/(sqrt(b*x^3 + a)*b^2) - a/((b*x^3 + a)^(3/2)*b^2)) - 2/9*A/((b*x
^3 + a)^(3/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2\,(3\,(bx^3+a)B - Ba + Ab)}{9\,(bx^3+a)^{\frac{3}{2}}b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
-2/9*(3*(b*x^3 + a)*B - B*a + A*b)/((b*x^3 + a)^(3/2)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2\,A\,b - 2\,B\,a + 6\,B\,(b\,x^3+a)}{9\,b^2\,(b\,x^3+a)^{3/2}}$$

input
```
int((x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

output
```
-(2*A*b - 2*B*a + 6*B*(a + b*x^3))/(9*b^2*(a + b*x^3)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2\sqrt{b\,x^3+a}}{3b\,(b\,x^3+a)}$$

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

output
```
( - 2*sqrt(a + b*x**3))/(3*b*(a + b*x**3))
```

3.226 $\int \frac{A+Bx^3}{x(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{A+Bx^3}{x(a+bx^3)^{5/2}}\,dx = \frac{2(Ab-aB)}{9ab(a+bx^3)^{3/2}}+\frac{2A}{3a^2\sqrt{a+bx^3}}-\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

output
```
2/9*(A*b-B*a)/a/b/(b*x^3+a)^(3/2)+2/3*A/a^2/(b*x^3+a)^(1/2)-2/3*A*arctanh(
(b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x(a+bx^3)^{5/2}}\,dx = -\frac{2(-4aAb+a^2B-3Ab^2x^3)}{9a^2b(a+bx^3)^{3/2}}-\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

input
```
Integrate[(A + B*x^3)/(x*(a + b*x^3)^(5/2)),x]
```

output
```
(-2*(-4*a*A*b + a^2*B - 3*A*b^2*x^3))/(9*a^2*b*(a + b*x^3)^(3/2)) - (2*A*A
rcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {948, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x\,(a+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^3\,(bx^3+a)^{5/2}}dx^3$$

$$\Big\downarrow \; 87$$

$$\frac{1}{3}\left(\frac{A\int \frac{1}{x^3(bx^3+a)^{3/2}}dx^3}{a}+\frac{2(Ab-aB)}{3ab\,(a+bx^3)^{3/2}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{3}\left(\frac{A\left(\frac{\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{a}+\frac{2}{a\sqrt{a+bx^3}}\right)}{a}+\frac{2(Ab-aB)}{3ab\,(a+bx^3)^{3/2}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{A\left(\frac{2\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{ab}+\frac{2}{a\sqrt{a+bx^3}}\right)}{a}+\frac{2(Ab-aB)}{3ab\,(a+bx^3)^{3/2}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(\frac{A\left(\frac{2}{a\sqrt{a+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{a}+\frac{2(Ab-aB)}{3ab\,(a+bx^3)^{3/2}}\right)$$

input
```
Int[(A + B*x^3)/(x*(a + b*x^3)^(5/2)),x]
```

output
```
((2*(A*b - a*B))/(3*a*b*(a + b*x^3)^(3/2)) + (A*(2/(a*Sqrt[a + b*x^3]) - (
2*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/a^(3/2)))/a)/3
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

method	result	size
pseudoelliptic	$-\frac{2\left(3Ab\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)a^{2}\left(b\,x^{3}+a\right)^{\frac{3}{2}}+a^{\frac{5}{2}}\left(-3A\,b^{2}x^{3}-4abA+a^{2}B\right)\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{\frac{9}{2}}b}$	73
elliptic	$\frac{2(Ab-Ba)\sqrt{b\,x^{3}+a}}{9b^{3}a\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{2A}{3a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2A\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}}$	77
default	$A\left(\frac{2\sqrt{b\,x^{3}+a}}{9a\,b^{2}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{2}{3a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}}\right)-\frac{2B}{9b\left(b\,x^{3}+a\right)^{\frac{3}{2}}}$	85

input
```
int((B*x^3+A)/x/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(3*A*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^2*(b*x^3+a)^(3/2)+a^(5/2)*(
-3*A*b^2*x^3-4*A*a*b+B*a^2))/(b*x^3+a)^(3/2)/a^(9/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.12

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx=\left[\frac{3\left(Ab^3x^6+2\,Aab^2x^3+Aa^2b\right)\sqrt{a}\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\left(3\,Aab^2x^3-Ba^3+4\right.}{9\left(a^3b^3x^6+2\,a^4b^2x^3+a^5b\right)}\right.$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/9*(3*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a^2*b)*sqrt(a)*log((b*x^3 - 2*sqrt(
b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*(3*A*a*b^2*x^3 - B*a^3 + 4*A*a^2*b)*sqr
t(b*x^3 + a))/(a^3*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b), 2/9*(3*(A*b^3*x^6 + 2
*A*a*b^2*x^3 + A*a^2*b)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^3 + a)) + (3*A*a
*b^2*x^3 - B*a^3 + 4*A*a^2*b)*sqrt(b*x^3 + a))/(a^3*b^3*x^6 + 2*a^4*b^2*x^
3 + a^5*b)]
```

Sympy [A] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{Ab}{3a^2\sqrt{a+bx^3}}+\frac{Ab\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3a^2\sqrt{-a}}-\frac{-Ab+Ba}{9a\left(a+bx^3\right)^{\frac{3}{2}}}\right)}{b} & \text{for } b\neq 0 \\ \frac{A\log\left(Bx^3\right)+Bx^3}{3a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x**3+A)/x/(b*x**3+a)**(5/2),x)
```

output
```
Piecewise((2*(A*b/(3*a**2*sqrt(a + b*x**3)) + A*b*atan(sqrt(a + b*x**3)/sq
rt(-a))/(3*a**2*sqrt(-a)) - (-A*b + B*a)/(9*a*(a + b*x**3)**(3/2)))/b, Ne(
b, 0)), ((A*log(B*x**3) + B*x**3)/(3*a**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx = \frac{1}{9}\,A\left(\frac{3\,\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(3\,bx^3+4\,a\right)}{\left(bx^3+a\right)^{\frac{3}{2}}a^2}\right)-\frac{2\,B}{9\left(bx^3+a\right)^{\frac{3}{2}}b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
1/9*A*(3*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5
/2) + 2*(3*b*x^3 + 4*a)/((b*x^3 + a)^(3/2)*a^2)) - 2/9*B/((b*x^3 + a)^(3/2
)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx=\frac{2\,A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}a^2}-\frac{2\,(Ba^2-3\,(bx^3+a)Ab-Aab)}{9\,(bx^3+a)^{\frac{3}{2}}a^2b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 2/9*(B*a^2 - 3*(b*
x^3 + a)*A*b - A*a*b)/((b*x^3 + a)^(3/2)*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx=\frac{\frac{2\,A}{9\,a}-\frac{2\,B}{9\,b}}{\left(b\,x^3+a\right)^{3/2}}+\frac{2\,A}{3\,a^2\,\sqrt{b\,x^3+a}}+\frac{A\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,a^{5/2}}$$

input
```
int((A + B*x^3)/(x*(a + b*x^3)^(5/2)),x)
```

output
```
((2*A)/(9*a) - (2*B)/(9*b))/(a + b*x^3)^(3/2) + (2*A)/(3*a^2*(a + b*x^3)^(
1/2)) + (A*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/
2)))/x^6))/(3*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx=\frac{2\sqrt{b\,x^3+a}\,a+\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)a+\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)b\,x^3-\sqrt{a}\log}{3a^2\left(b\,x^3+a\right)}$$

input
```
int((B*x^3+A)/x/(b*x^3+a)^(5/2),x)
```

output
```
(2*sqrt(a + b*x**3)*a + sqrt(a)*log(sqrt(a + b*x**3) - sqrt(a))*a + sqrt(a
)*log(sqrt(a + b*x**3) - sqrt(a))*b*x**3 - sqrt(a)*log(sqrt(a + b*x**3) +
sqrt(a))*a - sqrt(a)*log(sqrt(a + b*x**3) + sqrt(a))*b*x**3)/(3*a**2*(a +
b*x**3))
```

3.227 $\int \frac{A+Bx^3}{x^4(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx=-\frac{2(Ab-aB)}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{2(2Ab-aB)}{3a^3\sqrt{a+bx^3}}-\frac{A\sqrt{a+bx^3}}{3a^3x^3}+\frac{(5Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}$$

output
```
1/9*(-2*A*b+2*B*a)/a^2/(b*x^3+a)^(3/2)-2/3*(2*A*b-B*a)/a^3/(b*x^3+a)^(1/2)
-1/3*A*(b*x^3+a)^(1/2)/a^3/x^3+1/3*(5*A*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a
^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx=\frac{-3a^2A-20aAbx^3+8a^2Bx^3-15Ab^2x^6+6abBx^6}{9a^3x^3\left(a+bx^3\right)^{3/2}}+\frac{(5Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}$$

input
```
Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x]
```

output
```
(-3*a^2*A - 20*a*A*b*x^3 + 8*a^2*B*x^3 - 15*A*b^2*x^6 + 6*a*b*B*x^6)/(9*a^
3*x^3*(a + b*x^3)^(3/2)) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a
]])/(3*a^(7/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^6\left(bx^3+a\right)^{5/2}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(-\frac{(5Ab-2aB)\int \frac{1}{x^3\left(bx^3+a\right)^{5/2}}dx^3}{2a}-\frac{A}{ax^3\left(a+bx^3\right)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(-\frac{(5Ab-2aB)\left(\frac{\int \frac{1}{x^3\left(bx^3+a\right)^{3/2}}dx^3}{a}+\frac{2}{3a\left(a+bx^3\right)^{3/2}}\right)}{2a}-\frac{A}{ax^3\left(a+bx^3\right)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(-\frac{(5Ab-2aB)\left(\frac{\frac{\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{a}+\frac{2}{a\sqrt{a+bx^3}}}{a}+\frac{2}{3a(a+bx^3)^{3/2}}\right)}{2a}-\frac{A}{ax^3(a+bx^3)^{3/2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{(5Ab-2aB)\left(\frac{\frac{2\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{ab}+\frac{2}{a\sqrt{a+bx^3}}}{a}+\frac{2}{3a(a+bx^3)^{3/2}}\right)}{2a}-\frac{A}{ax^3(a+bx^3)^{3/2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{(5Ab-2aB)\left(\frac{\frac{2}{a\sqrt{a+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a(a+bx^3)^{3/2}}\right)}{2a}-\frac{A}{ax^3(a+bx^3)^{3/2}}\right)$$

input

```
Int[(A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x]
```

output

```
(-(A/(a*x^3*(a + b*x^3)^(3/2))) - ((5*A*b - 2*a*B)*(2/(3*a*(a + b*x^3)^(3/
2)) + (2/(a*Sqrt[a + b*x^3]) - (2*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/a^(3/2
))/a))/(2*a))/3
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{A\sqrt{b\,x^{3}+a}}{3a^{3}x^{3}}-\frac{-\frac{2(5Ab-2Ba)\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{\frac{8Ab}{3}-\frac{4Ba}{3}}{\sqrt{b\,x^{3}+a}}+\frac{4a(Ab-Ba)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}}}{2a^{3}}$
pseudoelliptic	$-\frac{-5\left(Ab-\frac{2Ba}{5}\right)\left(b\,x^{3}+a\right)^{\frac{3}{2}}x^{3}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)+\frac{20b\left(-\frac{3B\,x^{3}}{10}+A\right)x^{3}a^{\frac{3}{2}}}{3}+5A\sqrt{a}\,b^{2}x^{6}+\left(-\frac{8B\,x^{3}}{3}+A\right)a^{\frac{5}{2}}}{3\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{\frac{7}{2}}x^{3}}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{3a^{3}x^{3}}-\frac{2(Ab-Ba)\sqrt{b\,x^{3}+a}}{9a^{2}b^{2}\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{2(2Ab-Ba)}{3a^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{(5Ab-2Ba)\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{7}{2}}}$
default	$A\left(-\frac{\sqrt{b\,x^{3}+a}}{3a^{3}x^{3}}-\frac{2\sqrt{b\,x^{3}+a}}{9a^{2}b\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{4b}{3a^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{5b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{7}{2}}}\right)+B\left(\frac{2\sqrt{b\,x^{3}+a}}{9a\,b^{2}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{}{3a^{2}\sqrt{(}}\right.$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*A*(b*x^3+a)^(1/2)/a^3/x^3-1/2/a^3*(-2/3*(5*A*b-2*B*a)*arctanh((b*x^3+
a)^(1/2)/a^(1/2))/a^(1/2)+4/3*(2*A*b-B*a)/(b*x^3+a)^(1/2)+4/9*a*(A*b-B*a)/
(b*x^3+a)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.11

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx=\left[-\frac{3\left((2\,Bab^2-5\,Ab^3)x^9+2\left(2\,Ba^2b-5\,Aab^2\right)x^6+\left(2\,Ba^3-5\,Aa^2b\right)x^3\right)\sqrt{a}\log\left(\right.}{18\left(a^4b^2x^9\right.}\right.$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/18*(3*((2*B*a*b^2 - 5*A*b^3)*x^9 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^6 + (2*
B*a^3 - 5*A*a^2*b)*x^3)*sqrt(a)*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2
*a)/x^3) - 2*(3*(2*B*a^2*b - 5*A*a*b^2)*x^6 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a
^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3), 1/9*(3*
((2*B*a*b^2 - 5*A*b^3)*x^9 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^6 + (2*B*a^3 - 5*
A*a^2*b)*x^3)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^3 + a)) + (3*(2*B*a^2*b -
5*A*a*b^2)*x^6 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(
a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. $2(109) = 218$.

Time = 131.44 (sec) , antiderivative size = 1608, normalized size of antiderivative = 14.36

$$\int \frac{A+Bx^3}{x^4 \left(a+bx^3\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x**3+A)/x**4/(b*x**3+a)**(5/2),x)
```

output
```
A*(-6*a**17*sqrt(1 + b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 +
54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) - 46*a**16*b*x**3*sqrt(1
 + b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*
x**9 + 18*a**(33/2)*b**3*x**12) - 15*a**16*b*x**3*log(b*x**3/a)/(18*a**(39
/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**
3*x**12) + 30*a**16*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(18*a**(39/2)*x**3
+ 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12)
- 70*a**15*b**2*x**6*sqrt(1 + b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*
b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) - 45*a**15*b**2
*x**6*log(b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2
)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) + 90*a**15*b**2*x**6*log(sqrt(1 + b
*x**3/a) + 1)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2
*x**9 + 18*a**(33/2)*b**3*x**12) - 30*a**14*b**3*x**9*sqrt(1 + b*x**3/a)/(
18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(
33/2)*b**3*x**12) - 45*a**14*b**3*x**9*log(b*x**3/a)/(18*a**(39/2)*x**3 +
54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) +
90*a**14*b**3*x**9*log(sqrt(1 + b*x**3/a) + 1)/(18*a**(39/2)*x**3 + 54*a**
(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) - 15*a**
13*b**4*x**12*log(b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*
a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) + 30*a**13*b**4*x**12*lo...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.52

$$\int \frac{A + Bx^3}{x^4 (a + bx^3)^{5/2}} \, dx = -\frac{1}{18} A \left(\frac{2 \left(15 (bx^3 + a)^2 b - 10 (bx^3 + a)ab - 2 a^2 b\right)}{(bx^3 + a)^{\frac{5}{2}} a^3 - (bx^3 + a)^{\frac{3}{2}} a^4} + \frac{15 b \log\left(\frac{\sqrt{bx^3+a} - \sqrt{a}}{\sqrt{bx^3+a} + \sqrt{a}}\right)}{a^{\frac{7}{2}}} \right) + \frac{1}{9} B \left(\frac{3 \log\left(\frac{\sqrt{bx^3+a} - \sqrt{a}}{\sqrt{bx^3+a} + \sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{2 (3 bx^3 + 4 a)}{(bx^3 + a)^{\frac{3}{2}} a^2} \right)$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/18*A*(2*(15*(b*x^3 + a)^2*b - 10*(b*x^3 + a)*a*b - 2*a^2*b)/((b*x^3 + a
)^(5/2)*a^3 - (b*x^3 + a)^(3/2)*a^4) + 15*b*log((sqrt(b*x^3 + a) - sqrt(a)
)/(sqrt(b*x^3 + a) + sqrt(a)))/a^(7/2)) + 1/9*B*(3*log((sqrt(b*x^3 + a) -
sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/2) + 2*(3*b*x^3 + 4*a)/((b*x^3
+ a)^(3/2)*a^2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^3}{x^4 (a+bx^3)^{5/2}}\,dx = \frac{(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}a^3} + \frac{2\,(3\,(bx^3+a)Ba+Ba^2-6\,(bx^3+a)Ab-Aab)}{9\,(bx^3+a)^{\frac{3}{2}}a^3} - \frac{\sqrt{bx^3+a}A}{3\,a^3x^3}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*(2*B*a - 5*A*b)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2/9*
(3*(b*x^3 + a)*B*a + B*a^2 - 6*(b*x^3 + a)*A*b - A*a*b)/((b*x^3 + a)^(3/2)
*a^3) - 1/3*sqrt(b*x^3 + a)*A/(a^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.77

$$\int \frac{A+Bx^3}{x^4 (a+bx^3)^{5/2}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)\,(5\,A\,b-2\,B\,a)}{6\,a^{7/2}} - \frac{\frac{2\,B\,a^2-5\,A\,a\,b}{2\,a^4}-\frac{a\,\left(\frac{A\,b^2}{3\,a^4}+\frac{5\,b\,\left(2\,B\,a^2-5\,A\,a\,b\right)}{6\,a^5}\right)}{b}}{\sqrt{b\,x^3+a}} - \frac{\frac{2\,B\,a^3-5\,A\,a^2\,b}{4\,a^4}-\frac{a\,\left(\frac{13\,b\,\left(2\,B\,a^3-5\,A\,a^2\,b\right)}{36\,a^5}+\frac{A\,b^2}{3\,a^3}\right)}{b}}{{\left(b\,x^3+a\right)}^{3/2}} - \frac{A\,\sqrt{b\,x^3+a}}{3\,a^3\,x^3}$$

input
```
int((A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x)
```

output
```
(log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*
(5*A*b - 2*B*a))/(6*a^(7/2)) - ((2*B*a^2 - 5*A*a*b)/(2*a^4) - (a*((A*b^2)/
(3*a^4) + (5*b*(2*B*a^2 - 5*A*a*b))/(6*a^5)))/b)/(a + b*x^3)^(1/2) - ((2*B
*a^3 - 5*A*a^2*b)/(4*a^4) - (a*((13*b*(2*B*a^3 - 5*A*a^2*b))/(36*a^5) + (A
*b^2)/(3*a^3)))/b)/(a + b*x^3)^(3/2) - (A*(a + b*x^3)^(1/2))/(3*a^3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^3}{x^4\,(a+bx^3)^{5/2}}\,dx = \frac{-2\sqrt{b\,x^3+a}\,a^2-6\sqrt{b\,x^3+a}\,ab\,x^3-3\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)ab\,x^3-3\sqrt{a}\log(\sqrt{}}{6a^3x}$$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x)
```

output
```
( - 2*sqrt(a + b*x**3)*a**2 - 6*sqrt(a + b*x**3)*a*b*x**3 - 3*sqrt(a)*log(
sqrt(a + b*x**3) - sqrt(a))*a*b*x**3 - 3*sqrt(a)*log(sqrt(a + b*x**3) - sq
rt(a))*b**2*x**6 + 3*sqrt(a)*log(sqrt(a + b*x**3) + sqrt(a))*a*b*x**3 + 3*
sqrt(a)*log(sqrt(a + b*x**3) + sqrt(a))*b**2*x**6)/(6*a**3*x**3*(a + b*x**
3))
```

3.228 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 295

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2a(Ab-aB)x}{9b^3(a+bx^3)^{3/2}} - \frac{2(11Ab-20aB)x}{27b^3\sqrt{a+bx^3}} + \frac{2Bx\sqrt{a+bx^3}}{5b^3} + \frac{32\sqrt{2+\sqrt{3}}(5Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-\right.}{135\sqrt[4]{3}b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*a*(A*b-B*a)*x/b^3/(b*x^3+a)^(3/2)-2/27*(11*A*b-20*B*a)*x/b^3/(b*x^3+a)
^(1/2)+2/5*B*x*(b*x^3+a)^(1/2)/b^3+32/405*(1/2*6^(1/2)+1/2*2^(1/2))*(5*A*b
-14*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+
3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3
)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(10/3)/(a^(1
/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.37

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2x\left(112a^2B+b^2x^3(-55A+27Bx^3)+a(-40Ab+154bBx^3)+8(5Ab-14aB)\,(a+\right.}{135b^3\,(a+bx^3)^{3/2}}$$

input
```
Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*x*(112*a^2*B + b^2*x^3*(-55*A + 27*B*x^3) + a*(-40*A*b + 154*b*B*x^3) +
 8*(5*A*b - 14*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3,
 1/2, 4/3, -((b*x^3)/a)]))/(135*b^3*(a + b*x^3)^(3/2))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {959, 817, 817, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(5Ab-14aB)\int \frac{x^6}{(bx^3+a)^{5/2}}dx}{5b}+\frac{2Bx^7}{5b\,(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(5Ab-14aB)\left(\frac{8\int \frac{x^3}{\left(bx^3+a\right)^{3/2}}dx}{9b}-\frac{2x^4}{9b(a+bx^3)^{3/2}}\right)}{5b}+\frac{2Bx^7}{5b\,(a+bx^3)^{3/2}}$$

$$\Big\downarrow \; 817$$

$$\frac{(5Ab-14aB)\left(\frac{8\left(\frac{2\int \frac{1}{\sqrt{bx^3+a}}dx}{3b}-\frac{2x}{3b\sqrt{a+bx^3}}\right)}{9b}-\frac{2x^4}{9b(a+bx^3)^{3/2}}\right)}{5b}+\frac{2Bx^7}{5b\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 759$$

$$\frac{(5Ab-14aB)\left(\frac{8\left(\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x}{3b\sqrt{a+bx^3}}\right)}{9b}-\frac{}{9b}\right)}{5b}$$

$$\frac{2Bx^7}{5b\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(x^6*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*B*x^7)/(5*b*(a + b*x^3)^(3/2)) + ((5*A*b - 14*a*B)*((-2*x^4)/(9*b*(a +
b*x^3)^(3/2)) + (8*((-2*x)/(3*b*Sqrt[a + b*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(
1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(9*b)))/(5*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.35

method	result
elliptic	$\frac{2xa(Ab-Ba)\sqrt{b\,x^3+a}}{9b^5\left(x^3+\frac{a}{b}\right)^2}-\frac{2x(11Ab-20Ba)}{27b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2Bx\sqrt{b\,x^3+a}}{5b^3}-\frac{2i\left(\frac{Ab-2Ba}{b^3}-\frac{11Ab-20Ba}{27b^3}-\frac{2Ba}{5b^3}\right)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\cdots\right.}{\left(\cdots\right.}}}{\cdots}$
default	$A\left(\frac{2ax\sqrt{b\,x^3+a}}{9b^4\left(x^3+\frac{a}{b}\right)^2}-\frac{22x}{27b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{32i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\cdots}}\cdots}{\cdots}\right.$
risch	Expression too large to display

input
```
int(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x*a/b^5*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2/27/b^3*x*(11*A*b-20*B*
a)/((x^3+a/b)*b)^(1/2)+2/5*B*x*(b*x^3+a)^(1/2)/b^3-2/3*I*((A*b-2*B*a)/b^3-
1/27/b^3*(11*A*b-20*B*a)-2/5*B/b^3*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.52

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2\left(16\left((14\,Bab^2-5\,Ab^3)x^6+14\,Ba^3-5\,Aa^2b+2\,(14\,Ba^2b-5\,Aab^2)x^3\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b}\right.\right.}{135\,(b^6x^6+2\,ab^5x^3+a^2b^4}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/135*(16*((14*B*a*b^2 - 5*A*b^3)*x^6 + 14*B*a^3 - 5*A*a^2*b + 2*(14*B*a^
2*b - 5*A*a*b^2)*x^3)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - (27*B*b^
3*x^7 + 11*(14*B*a*b^2 - 5*A*b^3)*x^4 + 8*(14*B*a^2*b - 5*A*a*b^2)*x)*sqrt
(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [A] (verification not implemented)

Time = 63.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.27

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{Ax^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{7}{3},\frac{5}{2}\\\frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{10}{3}\right)}+\frac{Bx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{2},\frac{10}{3}\\\frac{13}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate(x**6*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
A*x**7*gamma(7/3)*hyper((7/3, 5/2), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*
a**(5/2)*gamma(10/3)) + B*x**10*gamma(10/3)*hyper((5/2, 10/3), (13/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(13/3))
```

Maxima [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^6}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^6}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{x^6\,(B\,x^3+A)}{(b\,x^3+a)^{5/2}}\,dx$$

input
```
int((x^6*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

output
```
int((x^6*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{\frac{16\sqrt{b\,x^3+a}\,ax}{5}+\frac{2\sqrt{b\,x^3+a}\,b\,x^4}{5}-\frac{16\left(\int \frac{\sqrt{b\,x^3+a}}{b^2x^6+2ab\,x^3+a^2}dx\right)a^3}{5}-\frac{16\left(\int \frac{\sqrt{b\,x^3+a}}{b^2x^6+2ab\,x^3+a^2}dx\right)a^2b\,x^3}{5}}{b^2\,(b\,x^3+a)}$$

input
```
int(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

output
```
(2*(8*sqrt(a + b*x**3)*a*x + sqrt(a + b*x**3)*b*x**4 - 8*int(sqrt(a + b*x*
*3)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a**3 - 8*int(sqrt(a + b*x**3)/(a**2
 + 2*a*b*x**3 + b**2*x**6),x)*a**2*b*x**3))/(5*b**2*(a + b*x**3))
```

3.229 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 279

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2(Ab-aB)x}{9b^2(a+bx^3)^{3/2}} + \frac{2(2Ab-11aB)x}{27ab^2\sqrt{a+bx^3}} + \frac{4\sqrt{2+\sqrt{3}}(Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right), -7-}{27\sqrt[4]{3}ab^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-2/9*(A*b-B*a)*x/b^2/(b*x^3+a)^(3/2)+2/27*(2*A*b-11*B*a)*x/a/b^2/(b*x^3+a)
^(1/2)+4/81*(1/2*6^(1/2)+1/2*2^(1/2))*(A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*((a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1
/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)
*x),I*3^(1/2)+2*I)*3^(3/4)/a/b^(7/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1
/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.35

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2x\left(-8a^2B+2Ab^2x^3-ab(A+11Bx^3)+(Ab+8aB)\,(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeo}\right.}{27ab^2\,(a+bx^3)^{3/2}}$$

input
```
Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*x*(-8*a^2*B + 2*A*b^2*x^3 - a*b*(A + 11*B*x^3) + (A*b + 8*a*B)*(a + b*x
^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(
27*a*b^2*(a + b*x^3)^(3/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {957, 817, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(8aB+Ab)\int \frac{x^3}{(bx^3+a)^{3/2}}dx}{9ab}+\frac{2x^4(Ab-aB)}{9ab\,(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(8aB+Ab)\left(\frac{2\int \frac{1}{\sqrt{bx^3+a}}dx}{3b}-\frac{2x}{3b\sqrt{a+bx^3}}\right)}{9ab}+\frac{2x^4(Ab-aB)}{9ab\,(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{759}$$

$$\frac{(8aB+Ab)\left(\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x}{3b\sqrt{a+bx^3}}\right)}{9ab}+\frac{2x^4(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(x^3*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*(A*b - a*B)*x^4)/(9*a*b*(a + b*x^3)^(3/2)) + ((A*b + 8*a*B)*((-2*x)/(3*
b*Sqrt[a + b*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
)))/(9*a*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.33

method	result
elliptic	$-\frac{2x(Ab-Ba)\sqrt{b\,x^3+a}}{9b^4\left(x^3+\frac{a}{b}\right)^2}+\frac{2x(2Ab-11Ba)}{27b^2a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i\left(\frac{B}{b^2}+\frac{2Ab-11Ba}{27b^2a}\right)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{}{-\frac{3(}{}}}}{}$
default	$A\left(-\frac{2x\sqrt{b\,x^3+a}}{9b^3\left(x^3+\frac{a}{b}\right)^2}+\frac{4x}{27ba\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{4i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}}{}\right.$

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*x/b^4*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/27/b^2*x/a*(2*A*b-11*B*
a)/((x^3+a/b)*b)^(1/2)-2/3*I*(B/b^2+1/27/b^2/a*(2*A*b-11*B*a))*3^(1/2)/b*(
-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.51

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\left(2\left((8\,Bab^2+Ab^3)x^6+8\,Ba^3+Aa^2b+2\,(8\,Ba^2b+Aab^2)x^3\right)\sqrt{b}\text{weierstrassPInve}\right.}{27\left(ab^5x^6+2\,a^2b^4x^3\right.}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
2/27*(2*((8*B*a*b^2 + A*b^3)*x^6 + 8*B*a^3 + A*a^2*b + 2*(8*B*a^2*b + A*a*
b^2)*x^3)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - ((11*B*a*b^2 - 2*A*b
^3)*x^4 + (8*B*a^2*b + A*a*b^2)*x)*sqrt(b*x^3 + a))/(a*b^5*x^6 + 2*a^2*b^4
*x^3 + a^3*b^3)
```

Sympy [A] (verification not implemented)

Time = 41.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.29

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{Ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{Bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{7}{3},\frac{5}{2}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**3*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
A*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(5/2)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((7/3, 5/2), (10/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(10/3))
```

Maxima [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^3}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^3}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{x^3\,(B\,x^3+A)}{(b\,x^3+a)^{5/2}}\,dx$$

input
```
int((x^3*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

output
```
int((x^3*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{-2\sqrt{b\,x^3+a}\,x+2\left(\int \frac{\sqrt{b\,x^3+a}}{b^2x^6+2ab\,x^3+a^2}dx\right)a^2+2\left(\int \frac{\sqrt{b\,x^3+a}}{b^2x^6+2ab\,x^3+a^2}dx\right)ab\,x^3}{b\,(b\,x^3+a)}$$

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

output
```
(2*( - sqrt(a + b*x**3)*x + int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2
*x**6),x)*a**2 + int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a
*b*x**3))/(b*(a + b*x**3))
```

3.230 $\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 283

$$\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx = \frac{2(Ab-aB)x}{9ab\,(a+bx^3)^{3/2}} + \frac{2(7Ab+2aB)x}{27a^2b\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}(7Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-\right.}{27\sqrt[4]{3}a^2b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*(A*b-B*a)*x/a/b/(b*x^3+a)^(3/2)+2/27*(7*A*b+2*B*a)*x/a^2/b/(b*x^3+a)^(
1/2)+2/81*(1/2*6^(1/2)+1/2*2^(1/2))*(7*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*((a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1
/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)
*x),I*3^(1/2)+2*I)*3^(3/4)/a^2/b^(4/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^
(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx = \frac{-2a^2Bx+14Ab^2x^4+4abx(5A+Bx^3)+(7Ab+2aB)x(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,\text{Hyperge}}{27a^2b\,(a+bx^3)^{3/2}}$$

input
```
Integrate[(A + B*x^3)/(a + b*x^3)^(5/2),x]
```

output
```
(-2*a^2*B*x + 14*A*b^2*x^4 + 4*a*b*x*(5*A + B*x^3) + (7*A*b + 2*a*B)*x*(a
+ b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)
])/(27*a^2*b*(a + b*x^3)^(3/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {910, 749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(2aB+7Ab)\int \frac{1}{(bx^3+a)^{3/2}}dx}{9ab}+\frac{2x(Ab-aB)}{9ab\,(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(2aB+7Ab)\left(\frac{\int \frac{1}{\sqrt{bx^3+a}}dx}{3a}+\frac{2x}{3a\sqrt{a+bx^3}}\right)}{9ab}+\frac{2x(Ab-aB)}{9ab\,(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{759}$$

$$\frac{(2aB+7Ab)\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x}{3a\sqrt{a+bx^3}}\right)}{9ab}$$
$$\frac{2x(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(A + B*x^3)/(a + b*x^3)^(5/2),x]
```

output
```
(2*(A*b - a*B)*x)/(9*a*b*(a + b*x^3)^(3/2)) + ((7*A*b + 2*a*B)*((2*x)/(3*a
*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(1/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
])))/(9*a*b)
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.29

method	result
elliptic	$\frac{2x(Ab-Ba)\sqrt{b\,x^3+a}}{9a\,b^3\left(x^3+\frac{a}{b}\right)^2}+\frac{2x(7Ab+2Ba)}{27b\,a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i(7Ab+2Ba)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\cdots}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\cdots}}\cdots}{\cdots}$
default	$A\left(\frac{2x\sqrt{b\,x^3+a}}{9a\,b^2\left(x^3+\frac{a}{b}\right)^2}+\frac{14x}{27a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{14i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)\cdots}{2b}\cdots}}\cdots}{\cdots}\cdots\right.$

input
```
int((B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x/a/b^3*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/27/b*x/a^2*(7*A*b+2*B*
a)/((x^3+a/b)*b)^(1/2)-2/81*I*(7*A*b+2*B*a)/a^2/b^2*3^(1/2)*(-a*b^2)^(1/3)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx = \frac{2\left(((2\,Bab^2+7\,Ab^3)x^6+2\,Ba^3+7\,Aa^2b+2\,(2\,Ba^2b+7\,Aab^2)x^3)\sqrt{b}\text{weierstrassPInv}\right.}{27\,(a^2b^4x^6+2\,a^3b^3x^3}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
2/27*(((2*B*a*b^2 + 7*A*b^3)*x^6 + 2*B*a^3 + 7*A*a^2*b + 2*(2*B*a^2*b + 7*
A*a*b^2)*x^3)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + ((2*B*a*b^2 + 7*
A*b^3)*x^4 - (B*a^2*b - 10*A*a*b^2)*x)*sqrt(b*x^3 + a))/(a^2*b^4*x^6 + 2*a
^3*b^3*x^3 + a^4*b^2)
```

Sympy [A] (verification not implemented)

Time = 26.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.28

$$\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx = \frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{5}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{5}{2}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
A*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
5/2)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(5/2)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx = \int \frac{B\,x^3+A}{(b\,x^3+a)^{5/2}}\,dx$$

input
```
int((A + B*x^3)/(a + b*x^3)^(5/2),x)
```

output
```
int((A + B*x^3)/(a + b*x^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{A + Bx^3}{(a + bx^3)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^3 + a}}{b^2x^6 + 2ab\,x^3 + a^2}dx$$

input
```
int((B*x^3+A)/(b*x^3+a)^(5/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
```

3.231 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 300

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx = -\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}} - \frac{(13Ab-4aB)x}{18a^2\left(a+bx^3\right)^{3/2}} - \frac{7(13Ab-4aB)x}{54a^3\sqrt{a+bx^3}}$$
$$-\frac{7\sqrt{2+\sqrt{3}}(13Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7\right.}{54\sqrt[4]{3}a^3\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/2*A/a/x^2/(b*x^3+a)^(3/2)-1/18*(13*A*b-4*B*a)*x/a^2/(b*x^3+a)^(3/2)-7/5
4*(13*A*b-4*B*a)*x/a^3/(b*x^3+a)^(1/2)-7/162*(1/2*6^(1/2)+1/2*2^(1/2))*(13
*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
(1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(
1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^3/b^(1/3)
/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*
x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx^3}{x^3 \left(a+bx^3\right)^{5/2}}\,dx = \frac{-182Ab^2x^6+a^2(-54A+80Bx^3)+a(-260Abx^3+56bBx^6)+7(-13Ab+4aB)x}{108a^3x^2\left(a+bx^3\right)^{3/2}}$$

input
```
Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^(5/2)),x]
```

output
```
(-182*A*b^2*x^6 + a^2*(-54*A + 80*B*x^3) + a*(-260*A*b*x^3 + 56*b*B*x^6) +
 7*(-13*A*b + 4*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1
[1/3, 1/2, 4/3, -((b*x^3)/a)])/(108*a^3*x^2*(a + b*x^3)^(3/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 749, 749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(13Ab-4aB)\int \frac{1}{\left(bx^3+a\right)^{5/2}}dx}{4a}-\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 749}$$

$$-\frac{(13Ab-4aB)\left(\frac{7\int \frac{1}{\left(bx^3+a\right)^{3/2}}dx}{9a}+\frac{2x}{9a\left(a+bx^3\right)^{3/2}}\right)}{4a}-\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{(13Ab-4aB)\left(\frac{7\left(\frac{\int \frac{1}{\sqrt{bx^3+a}}dx}{3a}+\frac{2x}{3a\sqrt{a+bx^3}}\right)}{9a}+\frac{2x}{9a(a+bx^3)^{3/2}}\right)}{4a}-\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{759}$$

$$-\frac{(13Ab-4aB)\left(\frac{7\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x}{3a\sqrt{a+bx^3}}\right)}{9a}+\right.}{4a}$$

$$\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(A + B*x^3)/(x^3*(a + b*x^3)^(5/2)),x]
```

output
```
-1/2*A/(a*x^2*(a + b*x^3)^(3/2)) - ((13*A*b - 4*a*B)*((2*x)/(9*a*(a + b*x^
3)^(3/2)) + (7*((2*x)/(3*a*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*
a*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3])))/(9*a)))/(4*a)
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.30

method	result
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{2a^3x^2}-\frac{2x(Ab-Ba)\sqrt{b\,x^3+a}}{9a^2b^2\left(x^3+\frac{a}{b}\right)^2}-\frac{2x(16Ab-7Ba)}{27a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i\left(-\frac{Ab}{4a^3}-\frac{16Ab-7Ba}{27a^3}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)}{(-a\,b^2)^{\frac{1}{3}}}}}{}$
default	$B\left(\frac{2x\sqrt{b\,x^3+a}}{9a\,b^2\left(x^3+\frac{a}{b}\right)^2}+\frac{14x}{27a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{14i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)}{2b}}}}{}\right.$
risch	Expression too large to display

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^3*A*(b*x^3+a)^(1/2)/x^2-2/9*x/a^2/b^2*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^
3+a/b)^2-2/27*x/a^3*(16*A*b-7*B*a)/((x^3+a/b)*b)^(1/2)-2/3*I*(-1/4/a^3*A*b
-1/27/a^3*(16*A*b-7*B*a))*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.54

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx = \frac{7\left(\left(4\,Bab^2-13\,Ab^3\right)x^8+2\left(4\,Ba^2b-13\,Aab^2\right)x^5+\left(4\,Ba^3-13\,Aa^2b\right)x^2\right)\sqrt{b}\mathrm{weiers}}{54\,(a^3b}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
1/54*(7*((4*B*a*b^2 - 13*A*b^3)*x^8 + 2*(4*B*a^2*b - 13*A*a*b^2)*x^5 + (4*
B*a^3 - 13*A*a^2*b)*x^2)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (7*(4
*B*a*b^2 - 13*A*b^3)*x^6 - 27*A*a^2*b + 10*(4*B*a^2*b - 13*A*a*b^2)*x^3)*s
qrt(b*x^3 + a))/(a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)
```

Sympy [A] (verification not implemented)

Time = 90.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.27

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx = \frac{A\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}x^2\Gamma\left(\frac{1}{3}\right)}+\frac{Bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**3/(b*x**3+a)**(5/2),x)
```

output
```
A*gamma(-2/3)*hyper((-2/3, 5/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
5/2)*x**2*gamma(1/3)) + B*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(5/2)*gamma(4/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^3), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{B\,x^3+A}{x^3\,\left(b\,x^3+a\right)^{5/2}}\,dx$$

input
```
int((A + B*x^3)/(x^3*(a + b*x^3)^(5/2)),x)
```

output
```
int((A + B*x^3)/(x^3*(a + b*x^3)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^3\,(a+bx^3)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^3+a}}{b^2x^9+2ab\,x^6+a^2x^3}dx$$

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a**2*x**3 + 2*a*b*x**6 + b**2*x**9),x)
```

3.232 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 334

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx=-\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}-\frac{19Ab-10aB}{45a^2x^2\left(a+bx^3\right)^{3/2}}-\frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}}+\frac{91(19Ab-10aB)\sqrt{a+bx^3}}{540a^4x^2}+\frac{91\sqrt{2+\sqrt{3}}b^{2/3}(19Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{540\sqrt[4]{3}a^4\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/5*A/a/x^5/(b*x^3+a)^(3/2)-1/45*(19*A*b-10*B*a)/a^2/x^2/(b*x^3+a)^(3/2)-
13/135*(19*A*b-10*B*a)/a^3/x^2/(b*x^3+a)^(1/2)+91/540*(19*A*b-10*B*a)*(b*x
^3+a)^(1/2)/a^4/x^2+91/1620*(1/2*6^(1/2)+1/2*2^(1/2))*b^(2/3)*(19*A*b-10*B
*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/
2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/
((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^4/(a^(1/3)*(a^(1/
3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.25

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx=\frac{-2a^2A+\left(\frac{19Ab}{2}-5aB\right)x^3(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{5}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{10a^3x^5\left(a+bx^3\right)^{3/2}}$$

input
```
Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^(5/2)),x]
```

output
```
(-2*a^2*A + ((19*A*b)/2 - 5*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hyper
geometric2F1[-2/3, 5/2, 1/3, -((b*x^3)/a)])/(10*a^3*x^5*(a + b*x^3)^(3/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {955, 819, 819, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(19Ab-10aB)\int \frac{1}{x^3(bx^3+a)^{5/2}}dx}{10a}-\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(19Ab-10aB)\left(\frac{13\int \frac{1}{x^3\left(bx^3+a\right)^{3/2}}dx}{9a}+\frac{2}{9ax^2(a+bx^3)^{3/2}}\right)}{10a}-\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(19Ab-10aB)\left(\frac{13\left(\frac{7\int \frac{1}{x^3\sqrt{bx^3+a}}dx}{3a}+\frac{2}{3ax^2\sqrt{a+bx^3}}\right)}{9a}+\frac{2}{9ax^2(a+bx^3)^{3/2}}\right)}{10a}-\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(19Ab-10aB)\left(\frac{13\left(\frac{7\left(-\frac{b\int \frac{1}{\sqrt{bx^3+a}}dx}{4a}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)}{3a}+\frac{2}{3ax^2\sqrt{a+bx^3}}\right)}{9a}+\frac{2}{9ax^2(a+bx^3)^{3/2}}\right)}{10a}-\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{759}$$

$$-\frac{(19Ab-10aB)\left(\frac{13\left(\frac{7\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)}{3a}}{9a}\right)}{10a}$$

$$\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(A + B*x^3)/(x^6*(a + b*x^3)^(5/2)),x]
```

output
```
-1/5*A/(a*x^5*(a + b*x^3)^(3/2)) - ((19*A*b - 10*a*B)*(2/(9*a*x^2*(a + b*x
^3)^(3/2)) + (13*(2/(3*a*x^2*Sqrt[a + b*x^3]) + (7*(-1/2*Sqrt[a + b*x^3]/(
a*x^2) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(3*a)))/
(9*a)))/(10*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.27

method	result
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{5a^{3}x^{5}}+\frac{(27Ab-10Ba)\sqrt{b\,x^{3}+a}}{20a^{4}x^{2}}+\frac{2x(Ab-Ba)\sqrt{b\,x^{3}+a}}{9a^{3}b\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{2bx(25Ab-16Ba)}{27a^{4}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2i\left(\frac{b(27Ab-10Ba)}{40a^{4}}+\frac{b(25Ab-16Ba)}{27a^{4}}\right)\sqrt{\cdots}}{\cdots}$
default	$A\left(-\frac{\sqrt{b\,x^{3}+a}}{5a^{3}x^{5}}+\frac{27b\sqrt{b\,x^{3}+a}}{20a^{4}x^{2}}+\frac{2x\sqrt{b\,x^{3}+a}}{9a^{3}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{50b^{2}x}{27a^{4}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{1729ib\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\cdots\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\cdots}{\cdots}\right.$
risch	Expression too large to display

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/a^3*A*(b*x^3+a)^(1/2)/x^5+1/20/a^4*(27*A*b-10*B*a)*(b*x^3+a)^(1/2)/x^
2+2/9*x/a^3/b*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/27*b*x/a^4*(25*A*b-1
6*B*a)/((x^3+a/b)*b)^(1/2)-2/3*I*(1/40*b*(27*A*b-10*B*a)/a^4+1/27*b/a^4*(2
5*A*b-16*B*a))*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.53

$$\int \frac{A + Bx^3}{x^6 (a + bx^3)^{5/2}} \, dx =$$
$$-\frac{91 \left((10 \, Bab^2 - 19 \, Ab^3) x^{11} + 2 \, (10 \, Ba^2 b - 19 \, Aab^2) x^8 + (10 \, Ba^3 - 19 \, Aa^2 b) x^5 \right) \sqrt{b} \text{weierstrassPInverse}(0}{540 \, (a^4 b^2 x}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/540*(91*((10*B*a*b^2 - 19*A*b^3)*x^11 + 2*(10*B*a^2*b - 19*A*a*b^2)*x^8
 + (10*B*a^3 - 19*A*a^2*b)*x^5)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x)
+ (91*(10*B*a*b^2 - 19*A*b^3)*x^9 + 130*(10*B*a^2*b - 19*A*a*b^2)*x^6 + 10
8*A*a^3 + 27*(10*B*a^3 - 19*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b^2*x^11 +
 2*a^5*b*x^8 + a^6*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)/x**6/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^{5/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}x^6}\,dx$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^6), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^{5/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}x^6}\,dx$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{B\,x^3+A}{x^6\,\left(b\,x^3+a\right)^{5/2}}\,dx$$

input
```
int((A + B*x^3)/(x^6*(a + b*x^3)^(5/2)),x)
```

output
```
int((A + B*x^3)/(x^6*(a + b*x^3)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{\sqrt{b\,x^3+a}}{b^2x^{12}+2ab\,x^9+a^2x^6}dx$$

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a**2*x**6 + 2*a*b*x**9 + b**2*x**12),x)
```

3.233 $\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 576

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2(Ab-aB)x^5}{9b^2(a+bx^3)^{3/2}} - \frac{2(10Ab-19aB)x^2}{27b^3\sqrt{a+bx^3}}$$

$$+\frac{2Bx^2\sqrt{a+bx^3}}{7b^3} + \frac{80(7Ab-16aB)\sqrt{a+bx^3}}{189b^{11/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{40\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7Ab-16aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\right.}{63\ 3^{3/4}b^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{80\sqrt{2}\sqrt[3]{a}(7Ab-16aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-\right.}{189\sqrt[4]{3}b^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/9*(A*b-B*a)*x^5/b^2/(b*x^3+a)^(3/2)-2/27*(10*A*b-19*B*a)*x^2/b^3/(b*x^3
+a)^(1/2)+2/7*B*x^2*(b*x^3+a)^(1/2)/b^3+80/189*(7*A*b-16*B*a)*(b*x^3+a)^(1
/2)/b^(11/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)-40/189*(1/2*6^(1/2)-1/2*2^(1/
2))*a^(1/3)*(7*A*b-16*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/
2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(1
/4)/b^(11/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^
2)^(1/2)/(b*x^3+a)^(1/2)+80/567*2^(1/2)*a^(1/3)*(7*A*b-16*B*a)*(a^(1/3)+b^
(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(
1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^
(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(11/3)/(a^(1/3)*(a^(1/3)+b^(1/3)
*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.19

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2x^2\left(-32a^2B+2ab(7A-8Bx^3)+b^2x^3(7A+Bx^3)+2(-7Ab+16aB)\left(a+bx^3\right)\sqrt{\cdots}\right.}{7b^3\left(a+bx^3\right)^{3/2}}$$

input
```
Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*x^2*(-32*a^2*B + 2*a*b*(7*A - 8*B*x^3) + b^2*x^3*(7*A + B*x^3) + 2*(-7*
A*b + 16*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 5/2,
5/3, -((b*x^3)/a)]))/(7*b^3*(a + b*x^3)^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 1.02 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {959, 817, 817, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (A+Bx^3)}{(a+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(7Ab-16aB)\int \frac{x^7}{(bx^3+a)^{5/2}}dx}{7b}+\frac{2Bx^8}{7b (a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(7Ab-16aB)\left(\frac{10\int \frac{x^4}{\left(bx^3+a\right)^{3/2}}dx}{9b}-\frac{2x^5}{9b(a+bx^3)^{3/2}}\right)}{7b}+\frac{2Bx^8}{7b (a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(7Ab-16aB)\left(\frac{10\left(\frac{4\int \frac{x}{\sqrt{bx^3+a}}dx}{3b}-\frac{2x^2}{3b\sqrt{a+bx^3}}\right)}{9b}-\frac{2x^5}{9b(a+bx^3)^{3/2}}\right)}{7b}+\frac{2Bx^8}{7b (a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{832}$$

$$\frac{(7Ab-16aB)\left(\frac{10\left(\frac{4\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{3b}-\frac{2x^2}{3b\sqrt{a+bx^3}}\right)}{9b}-\frac{2x^5}{9b(a+bx^3)^{3/2}}\right)}{7b}+\frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{(7Ab-16aB)\left(\frac{10\left(\frac{4\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\right.}{\sqrt[3]{b}x+\left(1+\right.}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{3b}}{9b}\right.}{7b}$$

$$\frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}}$$

$\downarrow$ 2416

$$(7Ab-16aB)\left(\frac{10\left(\frac{4\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}\cdots\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots\right)}{\sqrt[3]{b}\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(x^7*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*B*x^8)/(7*b*(a + b*x^3)^(3/2)) + ((7*A*b - 16*a*B)*((-2*x^5)/(9*b*(a +
b*x^3)^(3/2)) + (10*((-2*x^2)/(3*b*Sqrt[a + b*x^3]) + (4*(((2*Sqrt[a + b*x
^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqr
t[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 +
 Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(3*b)))/(9*b)))/(7*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 555, normalized size of antiderivative = 0.96

method	result
elliptic	$\frac{2x^2a(Ab-Ba)\sqrt{b\,x^3+a}}{9b^5\left(x^3+\frac{a}{b}\right)^2}-\frac{2x^2(13Ab-22Ba)}{27b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2B\,x^2\sqrt{b\,x^3+a}}{7b^3}-\frac{2i\left(\frac{Ab-2Ba}{b^3}+\frac{13Ab-22Ba}{27b^3}-\frac{4Ba}{7b^3}\right)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b\right.}{2b}\right.}{}}}{}$
default	Expression too large to display
risch	Expression too large to display

input
```
int(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x^2*a/b^5*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2/27/b^3*x^2*(13*A*b-2
2*B*a)/((x^3+a/b)*b)^(1/2)+2/7*B*x^2*(b*x^3+a)^(1/2)/b^3-2/3*I*((A*b-2*B*a
)/b^3+1/27/b^3*(13*A*b-22*B*a)-4/7*B/b^3*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.28

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\left(40\left((16\,Bab^2-7\,Ab^3)x^6+16\,Ba^3-7\,Aa^2b+2\,(16\,Ba^2b-7\,Aab^2)x^3\right)\sqrt{b}\text{weierstr}\right.}{}$$

input
```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
2/189*(40*((16*B*a*b^2 - 7*A*b^3)*x^6 + 16*B*a^3 - 7*A*a^2*b + 2*(16*B*a^2
*b - 7*A*a*b^2)*x^3)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInvers
e(0, -4*a/b, x)) + (27*B*b^3*x^8 + 13*(16*B*a*b^2 - 7*A*b^3)*x^5 + 10*(16*
B*a^2*b - 7*A*a*b^2)*x^2)*sqrt(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^
4)
```

Sympy [A] (verification not implemented)

Time = 82.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{Ax^8\Gamma\left(\frac{8}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{11}{3}\right)} + \frac{Bx^{11}\Gamma\left(\frac{11}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{11}{3}\\ \frac{14}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{14}{3}\right)}$$

input
```
integrate(x**7*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
A*x**8*gamma(8/3)*hyper((5/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*
a**(5/2)*gamma(11/3)) + B*x**11*gamma(11/3)*hyper((5/2, 11/3), (14/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(14/3))
```

Maxima [F]

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^7}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^7/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^7}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^7/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\int \frac{x^7\,(B\,x^3+A)}{(b\,x^3+a)^{5/2}}\,dx$$

input
```
int((x^7*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

output
```
int((x^7*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\frac{-\frac{20\sqrt{b\,x^3+a}\,a\,x^2}{7}+\frac{2\sqrt{b\,x^3+a}\,b\,x^5}{7}+\frac{40\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b^2x^6+2ab\,x^3+a^2}dx\right)a^3}{7}+\frac{40\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b^2x^6+2ab\,x^3+a^2}dx\right)a^2b\,x^3}{7}}{b^2\,(b\,x^3+a)}$$

input
```
int(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

output
```
(2*( - 10*sqrt(a + b*x**3)*a*x**2 + sqrt(a + b*x**3)*b*x**5 + 20*int((sqrt
(a + b*x**3)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a**3 + 20*int((sqrt(a +
 b*x**3)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a**2*b*x**3))/(7*b**2*(a +
b*x**3))
```

3.234 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 557

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}} \, dx = -\frac{2(Ab-aB)x^2}{9b^2(a+bx^3)^{3/2}}$$

$$+\frac{2(4Ab-13aB)x^2}{27ab^2\sqrt{a+bx^3}}-\frac{8(Ab-10aB)\sqrt{a+bx^3}}{27ab^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{4\sqrt{2-\sqrt{3}}(Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{9\,3^{3/4}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{8\sqrt{2}(Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}}{27\sqrt[4]{3}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/9*(A*b-B*a)*x^2/b^2/(b*x^3+a)^(3/2)+2/27*(4*A*b-13*B*a)*x^2/a/b^2/(b*x^
3+a)^(1/2)-8/27*(A*b-10*B*a)*(b*x^3+a)^(1/2)/a/b^(8/3)/((1+3^(1/2))*a^(1/3
)+b^(1/3)*x)+4/27*(1/2*6^(1/2)-1/2*2^(1/2))*(A*b-10*B*a)*(a^(1/3)+b^(1/3)*
x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x
)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+
b^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/a^(2/3)/b^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3
)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-8/81*2^(1/2)
*(A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b
^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^(2/3)/b
^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/
2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.17

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2x^2\left(-aAb+5aB(2a+bx^3)+(Ab-10aB)\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F}\right.}{5ab^2\left(a+bx^3\right)^{3/2}}$$

input
```
Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*x^2*(-(a*A*b) + 5*a*B*(2*a + b*x^3) + (A*b - 10*a*B)*(a + b*x^3)*Sqrt[1
 + (b*x^3)/a]*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)]))/(5*a*b^2*(a
 + b*x^3)^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 1.04 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {957, 817, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{2x^5(Ab-aB)}{9ab(a+bx^3)^{3/2}}-\frac{(Ab-10aB)\int \frac{x^4}{(bx^3+a)^{3/2}}dx}{9ab}$$

$$\Big\downarrow \text{817}$$

$$\frac{2x^5(Ab-aB)}{9ab(a+bx^3)^{3/2}}-\frac{(Ab-10aB)\left(\frac{4\int \frac{x}{\sqrt{bx^3+a}}dx}{3b}-\frac{2x^2}{3b\sqrt{a+bx^3}}\right)}{9ab}$$

$$\Big\downarrow \text{832}$$

$$\frac{2x^5(Ab-aB)}{9ab(a+bx^3)^{3/2}}-\frac{(Ab-10aB)\left(\frac{4\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{3b}-\frac{2x^2}{3b\sqrt{a+bx^3}}\right)}{9ab}$$

$$\Big\downarrow \text{759}$$

$$\frac{2x^5(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}-\frac{(Ab-10aB)\left(\frac{4\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}\right)\cdots\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{3b}\cdots\right)}{9ab}$$

$\downarrow$ 2416

$$\frac{2x^5(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}-\frac{(Ab-10aB)\left(\frac{4\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-\cdots\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\cdots\right)}{3b}\cdots\right)}{9ab}$$

input
```
Int[(x^4*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*(A*b - a*B)*x^5)/(9*a*b*(a + b*x^3)^(3/2)) - ((A*b - 10*a*B)*((-2*x^2)/
(3*b*Sqrt[a + b*x^3]) + (4*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/
3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)
*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])))/(3*b)))/(9*a*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 528, normalized size of antiderivative = 0.95

method	result
	$2i\left(\frac{B}{b^2}-\frac{4Ab-13Ba}{27a\,b^2}\right)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\cdots}$
elliptic	$-\frac{2x^2(Ab-Ba)\sqrt{b\,x^3+a}}{9b^4\left(x^3+\frac{a}{b}\right)^2}+\frac{2x^2(4Ab-13Ba)}{27b^2a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-$
default	Expression too large to display

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*x^2/b^4*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/27/b^2*x^2/a*(4*A*b-1
3*B*a)/((x^3+a/b)*b)^(1/2)-2/3*I*(B/b^2-1/27*(4*A*b-13*B*a)/a/b^2)*3^(1/2)
/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.28

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx =$$
$$-\frac{2\left(4\left((10\,Bab^2-Ab^3)x^6+10\,Ba^3-Aa^2b+2\,(10\,Ba^2b-Aab^2)x^3\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstra}\right.\right.}{27\left(ab^5x^6+2\,a^2b^4x^3+a^3\right.}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/27*(4*((10*B*a*b^2 - A*b^3)*x^6 + 10*B*a^3 - A*a^2*b + 2*(10*B*a^2*b -
A*a*b^2)*x^3)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4
*a/b, x)) + ((13*B*a*b^2 - 4*A*b^3)*x^5 + (10*B*a^2*b - A*a*b^2)*x^2)*sqrt
(b*x^3 + a))/(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)
```

Sympy [A] (verification not implemented)

Time = 41.92 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{Ax^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{5}{2}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)} + \frac{Bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**4*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
A*x**5*gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(5/2)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((5/2, 8/3), (11/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(11/3))
```

Maxima [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{x^4\,(B\,x^3+A)}{(b\,x^3+a)^{5/2}}\,dx$$

input
```
int((x^4*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

output
```
int((x^4*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\sqrt{b\,x^3+a}\,x^2-4\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b^2x^6+2ab\,x^3+a^2}dx\right)a^2-4\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b^2x^6+2ab\,x^3+a^2}dx\right)ab\,x^3}{b\,(b\,x^3+a)}$$

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

output
```
(2*(sqrt(a + b*x**3)*x**2 - 2*int((sqrt(a + b*x**3)*x)/(a**2 + 2*a*b*x**3
+ b**2*x**6),x)*a**2 - 2*int((sqrt(a + b*x**3)*x)/(a**2 + 2*a*b*x**3 + b**
2*x**6),x)*a*b*x**3))/(b*(a + b*x**3))
```

3.235 $\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 563

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2(Ab-aB)x^2}{9ab\,(a+bx^3)^{3/2}}$$

$$+\frac{2(5Ab+4aB)x^2}{27a^2b\sqrt{a+bx^3}}-\frac{2(5Ab+4aB)\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{\sqrt{2-\sqrt{3}}(5Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{2}(5Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*(A*b-B*a)*x^2/a/b/(b*x^3+a)^(3/2)+2/27*(5*A*b+4*B*a)*x^2/a^2/b/(b*x^3+
a)^(1/2)-2/27*(5*A*b+4*B*a)*(b*x^3+a)^(1/2)/a^2/b^(5/3)/((1+3^(1/2))*a^(1/
3)+b^(1/3)*x)+1/27*(1/2*6^(1/2)-1/2*2^(1/2))*(5*A*b+4*B*a)*(a^(1/3)+b^(1/3
)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)
*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3
)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/a^(5/3)/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1
/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-2/81*2^(1/
2)*(5*A*b+4*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3
)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^(5/3
)/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^
(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.14

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{x^2\left(-4a^2B+(5Ab+4aB)\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{2},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{10a^2b\left(a+bx^3\right)^{3/2}}$$

input
```
Integrate[(x*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(x^2*(-4*a^2*B + (5*A*b + 4*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeom
etric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)]))/(10*a^2*b*(a + b*x^3)^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.98 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {957, 819, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(4aB+5Ab)\int \frac{x}{\left(bx^3+a\right)^{3/2}}dx}{9ab}+\frac{2x^2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$\frac{(4aB+5Ab)\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\int \frac{x}{\sqrt{bx^3+a}}dx}{3a}\right)}{9ab}+\frac{2x^2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{832}$$

$$\frac{(4aB+5Ab)\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}}{3a}\right)}{9ab}+\frac{2x^2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{759}$$

$$\frac{(4aB+5Ab)\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}\cdots}{\sqrt[3]{b}\cdots}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{3a}\cdots\right)}{9ab}\cdots$$

$$\frac{2x^2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{(4aB+5Ab)\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{\right.}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{\right.}\right.}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}}{}\right.}{9ab}$$

$$\frac{2x^2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(x*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*(A*b - a*B)*x^2)/(9*a*b*(a + b*x^3)^(3/2)) + ((5*A*b + 4*a*B)*((2*x^2)/
(3*a*Sqrt[a + b*x^3]) - (((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3]))/(3*a)))/(9*a*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 520, normalized size of antiderivative = 0.92

method	result
	$2i(5Ab+4Ba)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\ \sqrt{\frac{x-}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}$
elliptic	$\frac{2x^{2}(Ab-Ba)\sqrt{b\,x^{3}+a}}{9a\,b^{3}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{2x^{2}(5Ab+4Ba)}{27b\,a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+$
default	Expression too large to display

input

```
int(x*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
2/9*x^2/a/b^3*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/27/b*x^2/a^2*(5*A*b+
4*B*a)/((x^3+a/b)*b)^(1/2)+2/81*I/b^2/a^2*(5*A*b+4*B*a)*3^(1/2)*(-a*b^2)^(
1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.27

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\left(((4\,Bab^2+5\,Ab^3)x^6+4\,Ba^3+5\,Aa^2b+2\,(4\,Ba^2b+5\,Aab^2)x^3)\sqrt{b}\text{weierstrassZeta}\right.}{27\,(a^2b^4}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
2/27*(((4*B*a*b^2 + 5*A*b^3)*x^6 + 4*B*a^3 + 5*A*a^2*b + 2*(4*B*a^2*b + 5*
A*a*b^2)*x^3)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4
*a/b, x)) + ((4*B*a*b^2 + 5*A*b^3)*x^5 + (B*a^2*b + 8*A*a*b^2)*x^2)*sqrt(b
*x^3 + a))/(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)
```

Sympy [A] (verification not implemented)

Time = 26.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{Ax^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{Bx^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{5}{2}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
A*x**2*gamma(2/3)*hyper((2/3, 5/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(5/2)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*a**(5/2)*gamma(8/3))
```

Maxima [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{x\,(B\,x^3+A)}{(b\,x^3+a)^{5/2}}\,dx$$

input
```
int((x*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

output
```
int((x*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^3+a}\,x}{b^2x^6+2ab\,x^3+a^2}dx$$

input
```
int(x*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

output
```
int((sqrt(a + b*x**3)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
```

3.236 $\int \frac{A+Bx^3}{x^2(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 578

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^{5/2}}\,dx = -\frac{A}{ax(a+bx^3)^{3/2}} - \frac{(11Ab-2aB)x^2}{9a^2(a+bx^3)^{3/2}}$$
$$-\frac{5(11Ab-2aB)x^2}{27a^3\sqrt{a+bx^3}} + \frac{5(11Ab-2aB)\sqrt{a+bx^3}}{27a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{5\sqrt{2-\sqrt{3}}(11Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right.}{18\ 3^{3/4}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{5\sqrt{2}(11Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right.}{27\sqrt[4]{3}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-A/a/x/(b*x^3+a)^(3/2)-1/9*(11*A*b-2*B*a)*x^2/a^2/(b*x^3+a)^(3/2)-5/27*(11
*A*b-2*B*a)*x^2/a^3/(b*x^3+a)^(1/2)+5/27*(11*A*b-2*B*a)*(b*x^3+a)^(1/2)/a^
3/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)-5/54*(1/2*6^(1/2)-1/2*2^(1/2))*(
11*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b
^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/a^(8/3)/b
^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/
2)/(b*x^3+a)^(1/2)+5/81*2^(1/2)*(11*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)
*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)
,I*3^(1/2)+2*I)*3^(3/4)/a^(8/3)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3
^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx=-\frac{A}{ax\left(a+bx^3\right)^{3/2}}-\frac{\left(\frac{11Ab}{2}-aB\right)x^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{2},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a^3\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^(5/2)),x]
```

output
```
-(A/(a*x*(a + b*x^3)^(3/2))) - (((11*A*b)/2 - a*B)*x^2*Sqrt[1 + (b*x^3)/a]
*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)])/(2*a^3*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.98 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 819, 819, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^2 (a+bx^3)^{5/2}} dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(11Ab-2aB)\int \frac{x}{(bx^3+a)^{5/2}}dx}{2a}-\frac{A}{ax(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{ 819}$$

$$-\frac{(11Ab-2aB)\left(\frac{5\int \frac{x}{(bx^3+a)^{3/2}}dx}{9a}+\frac{2x^2}{9a(a+bx^3)^{3/2}}\right)}{2a}-\frac{A}{ax(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{ 819}$$

$$-\frac{(11Ab-2aB)\left(\frac{5\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\int \frac{x}{\sqrt{bx^3+a}}dx}{3a}\right)}{9a}+\frac{2x^2}{9a(a+bx^3)^{3/2}}\right)}{2a}-\frac{A}{ax(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{ 832}$$

$$-\frac{(11Ab-2aB)\left(\frac{5\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{\int\frac{\sqrt[3]{b}x+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{(1-\sqrt{3})\sqrt[3]{a}\int\frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}}{3a}\right)}{9a}+\frac{2x^2}{9a(a+bx^3)^{3/2}}\right)}{2a}-\frac{A}{ax\,(a+bx^3)^{3/2}}$$

$\downarrow$ 759

$$-\frac{(11Ab-2aB)\left(\frac{5\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{\int\frac{\sqrt[3]{b}x+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{3a}\right)}{9a}}{2a}$$

$$\frac{A}{ax\,(a+bx^3)^{3/2}}$$

$\downarrow$ 2416

$$-\frac{(11Ab-2aB)\left(5\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+}{\sqrt[3]{b}x+}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.}{}$$

$$\frac{A}{ax\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(A + B*x^3)/(x^2*(a + b*x^3)^(5/2)),x]
```

output
```
-(A/(a*x*(a + b*x^3)^(3/2))) - ((11*A*b - 2*a*B)*((2*x^2)/(9*a*(a + b*x^3)
^(3/2)) + (5*((2*x^2)/(3*a*Sqrt[a + b*x^3]) - (((2*Sqrt[a + b*x^3])/(b^(1/
3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/
3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a
^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]
])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(3*a)))/(9*a)))/(2*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 545, normalized size of antiderivative = 0.94

method	result
elliptic	$-\frac{2x^2(Ab-Ba)\sqrt{b\,x^3+a}}{9a^2b^2\left(x^3+\frac{a}{b}\right)^2}-\frac{2x^2(14Ab-5Ba)}{27a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^3+a}}{a^3x}-\frac{2i\left(\frac{14Ab-5Ba}{27a^3}+\frac{Ab}{2a^3}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{}$
default	Expression too large to display
risch	Expression too large to display

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*x^2/a^2/b^2*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2/27*x^2/a^3*(14*A*
b-5*B*a)/((x^3+a/b)*b)^(1/2)-1/a^3*A*(b*x^3+a)^(1/2)/x-2/3*I*(1/27/a^3*(14
*A*b-5*B*a)+1/2/a^3*A*b)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b
*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.29

$$\int \frac{A+Bx^3}{x^2\,(a+bx^3)^{5/2}}\,dx = \frac{5\,((2\,Bab^2-11\,Ab^3)x^7+2\,(2\,Ba^2b-11\,Aab^2)x^4+(2\,Ba^3-11\,Aa^2b)x)\sqrt{b}\text{weierst}}{}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
1/27*(5*((2*B*a*b^2 - 11*A*b^3)*x^7 + 2*(2*B*a^2*b - 11*A*a*b^2)*x^4 + (2*
B*a^3 - 11*A*a^2*b)*x)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInve
rse(0, -4*a/b, x)) + (5*(2*B*a*b^2 - 11*A*b^3)*x^6 - 27*A*a^2*b + 8*(2*B*a
^2*b - 11*A*a*b^2)*x^3)*sqrt(b*x^3 + a))/(a^3*b^3*x^7 + 2*a^4*b^2*x^4 + a^
5*b*x)
```

Sympy [A] (verification not implemented)

Time = 57.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^{5/2}}\,dx = \frac{A\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}x\Gamma\left(\frac{2}{3}\right)} + \frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**2/(b*x**3+a)**(5/2),x)
```

output
```
A*gamma(-1/3)*hyper((-1/3, 5/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
5/2)*x*gamma(2/3)) + B*x**2*gamma(2/3)*hyper((2/3, 5/2), (5/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(5/2)*gamma(5/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^{5/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^{5/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{B\,x^3+A}{x^2\,{\left(b\,x^3+a\right)}^{5/2}} \,d x$$

input
```
int((A + B*x^3)/(x^2*(a + b*x^3)^(5/2)),x)
```

output
```
int((A + B*x^3)/(x^2*(a + b*x^3)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{\sqrt{b\,x^{3}+a}}{b^{2} x^{8}+2 a b\,x^{5}+a^{2} x^{2}}d x$$

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a**2*x**2 + 2*a*b*x**5 + b**2*x**8),x)
```

3.237 $\int \frac{A+Bx^3}{x^5 (a+bx^3)^{5/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 610

$$\int \frac{A+Bx^3}{x^5 (a+bx^3)^{5/2}} \, dx = -\frac{A}{4ax^4 (a+bx^3)^{3/2}} - \frac{17Ab-8aB}{36a^2x (a+bx^3)^{3/2}}$$

$$-\frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}} + \frac{55(17Ab-8aB)\sqrt{a+bx^3}}{216a^4x} - \frac{55\sqrt[3]{b}(17Ab-8aB)\sqrt{a+bx^3}}{216a^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{55\sqrt{2-\sqrt{3}}\sqrt[3]{b}(17Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4}{144\ 3^{3/4}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{55\sqrt[3]{b}(17Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{}}{108\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/4*A/a/x^4/(b*x^3+a)^(3/2)-1/36*(17*A*b-8*B*a)/a^2/x/(b*x^3+a)^(3/2)-11/
108*(17*A*b-8*B*a)/a^3/x/(b*x^3+a)^(1/2)+55/216*(17*A*b-8*B*a)*(b*x^3+a)^(
1/2)/a^4/x-55/216*b^(1/3)*(17*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^4/((1+3^(1/2))*
a^(1/3)+b^(1/3)*x)+55/432*(1/2*6^(1/2)-1/2*2^(1/2))*b^(1/3)*(17*A*b-8*B*a)
*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))
*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1
+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/a^(11/3)/(a^(1/3)*(a^(
1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-5
5/648*b^(1/3)*(17*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(
1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*2^
(1/2)*3^(3/4)/a^(11/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b
^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx^3}{x^5(a+bx^3)^{5/2}}\,dx = \frac{-a^2A+\left(\frac{17Ab}{2}-4aB\right)x^3(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{2},\frac{2}{3},-\frac{bx^3}{a}\right)}{4a^3x^4(a+bx^3)^{3/2}}$$

input
```
Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^(5/2)),x]
```

output
```
(-(a^2*A) + ((17*A*b)/2 - 4*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hyper
geometric2F1[-1/3, 5/2, 2/3, -((b*x^3)/a)])/(4*a^3*x^4*(a + b*x^3)^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 1.05 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {955, 819, 819, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}dx$$

$\big\downarrow$ 955

$$-\frac{(17Ab-8aB)\int \frac{1}{x^2(bx^3+a)^{5/2}}dx}{8a}-\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}$$

$\big\downarrow$ 819

$$-\frac{(17Ab-8aB)\left(\frac{11\int \frac{1}{x^2\left(bx^3+a\right)^{3/2}}dx}{9a}+\frac{2}{9ax(a+bx^3)^{3/2}}\right)}{8a}-\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}$$

$\big\downarrow$ 819

$$-\frac{(17Ab-8aB)\left(\frac{11\left(\frac{5\int \frac{1}{x^2\sqrt{bx^3+a}}dx}{3a}+\frac{2}{3ax\sqrt{a+bx^3}}\right)}{9a}+\frac{2}{9ax(a+bx^3)^{3/2}}\right)}{8a}-\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}$$

$\big\downarrow$ 847

$$-\frac{(17Ab-8aB)\left(\frac{11\left(\frac{5\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{3a}+\frac{2}{3ax\sqrt{a+bx^3}}\right)}{9a}+\frac{2}{9ax(a+bx^3)^{3/2}}\right)}{8a}-\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}$$

$\big\downarrow$ 832

$$-\frac{(17Ab-8aB)\left(\frac{11\left(\frac{5\left(\frac{b\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{3a}+\frac{2}{3ax\sqrt{a+bx^3}}\right)}{9a}+\frac{2}{9ax(a+bx^3)^{3/2}}\right)}{8a}-\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}$$

$\Big\downarrow$ 759

$$(17Ab-8aB)\left(\frac{11\left(\frac{5\left(\frac{b\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{b}x}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{2a}\right)}{3a}\right)}{9a}\right.$$

↓ 2416

$$\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right.}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$b\,\frac{\quad\cdots\quad}{\sqrt[3]{b}}$$

$$5$$

$$11$$

$$(17Ab-8aB)$$

input
```
Int[(A + B*x^3)/(x^5*(a + b*x^3)^(5/2)),x]
```

output
```
-1/4*A/(a*x^4*(a + b*x^3)^(3/2)) - ((17*A*b - 8*a*B)*(2/(9*a*x*(a + b*x^3)
^(3/2)) + (11*(2/(3*a*x*Sqrt[a + b*x^3]) + (5*(-(Sqrt[a + b*x^3]/(a*x)) +
(b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (
3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elli
pticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1
 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(2
*a)))/(3*a)))/(9*a)))/(8*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 3.23 (sec) , antiderivative size = 581, normalized size of antiderivative = 0.95

method	result
	$2i\left(-\frac{b(23Ab-14Ba)}{27a^4}-\frac{b(21Ab-8Ba)}{16a^4}\right)\sqrt{}$
elliptic	$\frac{2x^2(Ab-Ba)\sqrt{b\,x^3+a}}{9a^3b\left(x^3+\frac{a}{b}\right)^2}+\frac{2b\,x^2(23Ab-14Ba)}{27a^4\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^3+a}}{4a^3x^4}+\frac{(21Ab-8Ba)\sqrt{b\,x^3+a}}{8a^4x}-$
default	Expression too large to display
risch	Expression too large to display

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x^2/a^3/b*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/27*b*x^2/a^4*(23*A*b
-14*B*a)/((x^3+a/b)*b)^(1/2)-1/4/a^3*A*(b*x^3+a)^(1/2)/x^4+1/8/a^4*(21*A*b
-8*B*a)*(b*x^3+a)^(1/2)/x-2/3*I*(-1/27*b/a^4*(23*A*b-14*B*a)-1/16*b*(21*A*
b-8*B*a)/a^4)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x
+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*
b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.30

$$\int \frac{A+Bx^3}{x^5 (a+bx^3)^{5/2}} \, dx =$$
$$-\frac{55\left((8\,Bab^2-17\,Ab^3)x^{10}+2\,(8\,Ba^2b-17\,Aab^2)x^7+(8\,Ba^3-17\,Aa^2b)x^4\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{w}\right.}{2}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/216*(55*((8*B*a*b^2 - 17*A*b^3)*x^10 + 2*(8*B*a^2*b - 17*A*a*b^2)*x^7 +
 (8*B*a^3 - 17*A*a^2*b)*x^4)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstras
sPInverse(0, -4*a/b, x)) + (55*(8*B*a*b^2 - 17*A*b^3)*x^9 + 88*(8*B*a^2*b
- 17*A*a*b^2)*x^6 + 54*A*a^3 + 27*(8*B*a^3 - 17*A*a^2*b)*x^3)*sqrt(b*x^3 +
 a))/(a^4*b^2*x^10 + 2*a^5*b*x^7 + a^6*x^4)
```

Sympy [A] (verification not implemented)

Time = 165.61 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx^3}{x^5 (a+bx^3)^{5/2}} \, dx = \frac{A\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{5}{2} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{B\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{5}{2} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**5/(b*x**3+a)**(5/2),x)
```

output
```
A*gamma(-4/3)*hyper((-4/3, 5/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(5/2)*x**4*gamma(-1/3)) + B*gamma(-1/3)*hyper((-1/3, 5/2), (2/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(5/2)*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^5(a+bx^3)^{5/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}x^5}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^5), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^5(a+bx^3)^{5/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}x^5}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^5(a+bx^3)^{5/2}}\,dx = \int \frac{B\,x^3+A}{x^5\,(b\,x^3+a)^{5/2}}\,dx$$

input
```
int((A + B*x^3)/(x^5*(a + b*x^3)^(5/2)),x)
```

output
```
int((A + B*x^3)/(x^5*(a + b*x^3)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A + Bx^3}{x^5 (a + bx^3)^{5/2}} \, dx = \int \frac{\sqrt{b\, x^3 + a}}{b^2 x^{11} + 2ab\, x^8 + a^2 x^5} dx$$

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a**2*x**5 + 2*a*b*x**8 + b**2*x**11),x)
```

3.238 $\int (ex)^{7/2}\sqrt{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 161

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2}+\frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}-\frac{a^2(2Ab-aB)e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{5/2}}$$

output

```
1/24*a*(2*A*b-B*a)*e^2*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b^2+1/12*(2*A*b-B*a)*(e
*x)^(9/2)*(b*x^3+a)^(1/2)/b/e+1/9*B*(e*x)^(9/2)*(b*x^3+a)^(3/2)/b/e-1/24*a
^2*(2*A*b-B*a)*e^(7/2)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a)^(1/2)
)/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.76

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{(ex)^{7/2}\sqrt{a+bx^3}(6aAb-3a^2B+12Ab^2x^3+2abBx^3+8b^2Bx^6)}{72b^2x^2} + \frac{a^2(-2Ab+aB)(ex)^{7/2}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{24b^{5/2}x^{7/2}}$$

input
```
Integrate[(e*x)^(7/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
((e*x)^(7/2)*Sqrt[a + b*x^3]*(6*a*A*b - 3*a^2*B + 12*A*b^2*x^3 + 2*a*b*B*x
^3 + 8*b^2*B*x^6))/(72*b^2*x^2) + (a^2*(-2*A*b + a*B)*(e*x)^(7/2)*Log[Sqrt
[b]*x^(3/2) + Sqrt[a + b*x^3]])/(24*b^(5/2)*x^(7/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {959, 811, 843, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(2Ab-aB)\int (ex)^{7/2}\sqrt{bx^3+a}dx}{2b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\int \frac{(ex)^{7/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)}{2b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

$$\Big\downarrow \text{843}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^3\int\frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx}{2b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)}{2b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int\frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)}{2b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

$$\Big\downarrow \text{807}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int\frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)}{2b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

$$\Big\downarrow \text{224}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int\frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)}{2b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

$$\Big\downarrow \text{219}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3b^{3/2}}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)}{2b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

input
```
Int[(e*x)^(7/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(B*(e*x)^(9/2)*(a + b*x^3)^(3/2))/(9*b*e) + ((2*A*b - a*B)*(((e*x)^(9/2)*S
qrt[a + b*x^3])/(6*e) + (a*((e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*b) - (a*e
^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*b^
(3/2))))/4))/(2*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.86

method	result
risch	$\frac{x^{2}\left(8b^{2}B\,x^{6}+12A\,b^{2}x^{3}+2Bab\,x^{3}+6abA-3a^{2}B\right)\sqrt{b\,x^{3}+a}\,e^{4}}{72b^{2}\sqrt{ex}}-\frac{a^{2}\left(2Ab-Ba\right)\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)e^{4}\sqrt{\left(b\,x^{3}+a\right)ex}}{24b^{2}\sqrt{be}\,\sqrt{ex}\,\sqrt{b\,x^{3}+a}}$
default	$-\frac{e^{3}\sqrt{ex}\,\sqrt{b\,x^{3}+a}\,\left(-8B\sqrt{\left(b\,x^{3}+a\right)ex}\,\sqrt{be}\,b^{2}x^{7}-12A\sqrt{\left(b\,x^{3}+a\right)ex}\,\sqrt{be}\,b^{2}x^{4}-2B\sqrt{\left(b\,x^{3}+a\right)ex}\,\sqrt{be}\,ab\,x^{4}+6A\operatorname{arctanh}\left(\frac{\sqrt{(b\,x^{3}+}}{x^{2}\sqrt{b}}\right.\right.}{72\sqrt{\left(b\,x^{3}+a\right)ex}\,b^{2}\sqrt{be}}$
elliptic	Expression too large to display

input
```
int((e*x)^(7/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/72*x^2*(8*B*b^2*x^6+12*A*b^2*x^3+2*B*a*b*x^3+6*A*a*b-3*B*a^2)*(b*x^3+a)^
(1/2)/b^2*e^4/(e*x)^(1/2)-1/24*a^2*(2*A*b-B*a)/b^2/(b*e)^(1/2)*arctanh(((b
*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*e^4*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/
(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.83

$$\int (ex)^{7/2}\sqrt{a+bx^3}(A+Bx^3)\,dx = \left[-\frac{3\,(Ba^3-2\,Aa^2b)e^3\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e+4\,(2\,b^2x^4+abx)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right.}{288\,b^2}\right.$$

$$-\frac{3\,(Ba^3-2\,Aa^2b)e^3\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\,\sqrt{bx^3+a}\sqrt{ex}bx\sqrt{-\frac{e}{b}}}{2\,bex^3+ae}\right)-2\,(8\,Bb^2e^3x^7+2\,(Bab+6\,Ab^2)e^3x^4-3\,(Ba^2-2}{144\,b^2}$$

input
```
integrate((e*x)^(7/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
[-1/288*(3*(B*a^3 - 2*A*a^2*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^
3 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4
*(8*B*b^2*e^3*x^7 + 2*(B*a*b + 6*A*b^2)*e^3*x^4 - 3*(B*a^2 - 2*A*a*b)*e^3*
x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/144*(3*(B*a^3 - 2*A*a^2*b)*e^3*sqrt(
-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e))
 - 2*(8*B*b^2*e^3*x^7 + 2*(B*a*b + 6*A*b^2)*e^3*x^4 - 3*(B*a^2 - 2*A*a*b)*
e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(141) = 282$.

Time = 18.55 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.85

$$\int (ex)^{7/2}\sqrt{a+bx^3}(A+Bx^3)\,dx = \begin{cases} \frac{2\left(\begin{cases}\text{NaN} \\ \frac{Ae^3\left(\begin{cases} -\frac{a^2e^3\left(\begin{cases}\frac{\log\left(\frac{2b(ex)^{\frac{3}{2}}}{e^3}+2\sqrt{\frac{b}{e^3}}\sqrt{a+bx^3}\right)}{\sqrt{\frac{b}{e^3}}} & \text{for } a\neq 0 \\ \frac{(ex)^{\frac{3}{2}}\log\left((ex)^{\frac{3}{2}}\right)}{\sqrt{bx^3}} & \text{otherwise}\end{cases}\right)}{8b} + \sqrt{a+bx^3}\left(\frac{ae^3(ex)^{\frac{3}{2}}}{8b}+\frac{(ex)^{\frac{9}{2}}}{4}\right) & \text{fo} \\ \frac{\sqrt{a}(ex)^{\frac{9}{2}}}{3} & \text{o}\end{cases}\right)}{} \end{cases}\right)}{} \\ 0 \end{cases}$$

input
```
integrate((e*x)**(7/2)*(b*x**3+a)**(1/2)*(B*x**3+A),x)
```

output
```
Piecewise((2*Piecewise((nan, Eq(e**3, 0)), ((A*e**3*Piecewise((-a**2*e**3*
Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sq
rt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True)
)/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**(3/2)/(8*b) + (e*x)**(9/2)/4), N
e(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, True)) + B*Piecewise((a**3*e**6*Pi
ecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt
(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/
(16*b**2) + sqrt(a + b*x**3)*(-a**2*e**6*(e*x)**(3/2)/(16*b**2) + a*e**3*(
e*x)**(9/2)/(24*b) + (e*x)**(15/2)/6), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(15
/2)/5, True)))/(3*e**3), True))/e, Ne(e, 0)), (0, True))
```

Maxima [F]

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{7}{2}}\,dx$$

input
```
integrate((e*x)^(7/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(126) = 252$.

Time = 0.23 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.05

$$\int (ex)^{7/2}\sqrt{a+bx^3}(A+Bx^3)\,dx=\frac{1}{72}\sqrt{be^4x^3+ae^4}\left(2e^3x^3\left(\frac{4x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3a^2}{b^2e}\right)\sqrt{ex}Bx|e|^2+\frac{\sqrt{be^4x^3+ae^4}\left(2e^3x^3+\frac{ae^3}{b}\right)\sqrt{ex}Ax|e|^2}{12e^4}-\frac{\left(B^2a^6e-4ABa^5be+4A^2a^4b^2e\right)^2e^5\log\left(\left|-\left(\sqrt{ex}Ba^3e^2x-2\sqrt{ex}Aa^2be^2x\right)\sqrt{be}+\sqrt{B^2a^7e^6-4ABa^6b\right.\right.}{24\sqrt{be}b^2|B^2a^6e-4ABa^5be+4A^2a^4b^2e||-Ba^3+2Ac}$$

input
```
integrate((e*x)^(7/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/72*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3*(4*x^3/e^4 + a/(b*e^4)) - 3*a^2/(b
^2*e))*sqrt(e*x)*B*x*abs(e)^2 + 1/12*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3 +
a*e^3/b)*sqrt(e*x)*A*x*abs(e)^2/e^4 - 1/24*(B^2*a^6*e - 4*A*B*a^5*b*e + 4*
A^2*a^4*b^2*e)^2*e^5*log(abs(-(sqrt(e*x)*B*a^3*e^2*x - 2*sqrt(e*x)*A*a^2*b
*e^2*x)*sqrt(b*e) + sqrt(B^2*a^7*e^6 - 4*A*B*a^6*b*e^6 + 4*A^2*a^5*b^2*e^6
 + (sqrt(e*x)*B*a^3*e^2*x - 2*sqrt(e*x)*A*a^2*b*e^2*x)^2*b*e)))/(sqrt(b*e)
*b^2*abs(B^2*a^6*e - 4*A*B*a^5*b*e + 4*A^2*a^4*b^2*e)*abs(-B*a^3 + 2*A*a^2
*b)*abs(e)^2)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,\left(e\,x\right)^{7/2}\,\sqrt{b\,x^3+a}\,dx$$

input
```
int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(1/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{\sqrt{e}\,e^3\left(6\sqrt{x}\,\sqrt{b\,x^3+a}\,a^2bx+28\sqrt{x}\,\sqrt{b\,x^3+a}\,a\,b^2x^4+16\sqrt{x}\,\sqrt{b\,x^3+a}\,b^3x^7+3\sqrt{b}\log\left(\sqrt{b\,x^3\right.\right.}{144b^2}$$

input
```
int((e*x)^(7/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

output
```
(sqrt(e)*e**3*(6*sqrt(x)*sqrt(a + b*x**3)*a**2*b*x + 28*sqrt(x)*sqrt(a + b
*x**3)*a*b**2*x**4 + 16*sqrt(x)*sqrt(a + b*x**3)*b**3*x**7 + 3*sqrt(b)*log
(sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*x)*a**3 - 3*sqrt(b)*log(sqrt(a + b*x**
3) + sqrt(x)*sqrt(b)*x)*a**3))/(144*b**2)
```

3.239 $\int (ex)^{5/2}\sqrt{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 324

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{3a(16Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{320b^2}$$
$$+\frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}$$
$$-\frac{3^{3/4}a^{5/3}(16Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\right.}{640b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
3/320*a*(16*A*b-7*B*a)*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^2+1/80*(16*A*b-7*
B*a)*(e*x)^(7/2)*(b*x^3+a)^(1/2)/b/e+1/8*B*(e*x)^(7/2)*(b*x^3+a)^(3/2)/b/e
-1/640*3^(3/4)*a^(5/3)*(16*A*b-7*B*a)*e^2*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*
((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2
)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1
+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))/b^2/(b^(1/3)*x*(a^(1/3)+b^(
1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

$$\int (ex)^{5/2}\sqrt{a+bx^3}(A+Bx^3)\ dx = \frac{e^2\sqrt{ex}\sqrt{a+bx^3}\left(-\left((a+bx^3)\sqrt{1+\frac{bx^3}{a}}(-16Ab+7aB-10bBx^3)\right)+a(-16Ab+7aB)\,\text{Hyp}$$

$$80b^2\sqrt{1+\frac{bx^3}{a}}$$

input
```
Integrate[(e*x)^(5/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(e^2*Sqrt[e*x]*Sqrt[a + b*x^3]*(-((a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(-16*A*b
 + 7*a*B - 10*b*B*x^3)) + a*(-16*A*b + 7*a*B)*Hypergeometric2F1[-1/2, 1/6,
 7/6, -((b*x^3)/a)]))/(80*b^2*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {959, 811, 843, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\ dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(16Ab-7aB)\int (ex)^{5/2}\sqrt{bx^3+a}dx}{16b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(16Ab-7aB)\left(\frac{3}{10}a\int \frac{(ex)^{5/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{7/2}\sqrt{a+bx^3}}{5e}\right)}{16b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}$$

$$\Big\downarrow \; 843$$

$$\frac{(16Ab-7aB)\left(\frac{3}{10}a\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{ae^3\int\frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{4b}\right)+\frac{(ex)^{7/2}\sqrt{a+bx^3}}{5e}\right)}{16b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}$$

$$\Big\downarrow \; 851$$

$$\frac{(16Ab-7aB)\left(\frac{3}{10}a\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{ae^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b}\right)+\frac{(ex)^{7/2}\sqrt{a+bx^3}}{5e}\right)}{16b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}$$

$$\Big\downarrow \; 766$$

$$\frac{(16Ab-7aB)\left(\frac{3}{10}a\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{a^{2/3}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe-}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe-}\right.\right.}{4\sqrt[4]{3}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right.\right.}{16b}$$

$$\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}$$

input
```
Int[(e*x)^(5/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(B*(e*x)^(7/2)*(a + b*x^3)^(3/2))/(8*b*e) + ((16*A*b - 7*a*B)*(((e*x)^(7/2
)*Sqrt[a + b*x^3])/(5*e) + (3*a*((e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(2*b) - (
a^(2/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*
b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2
]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1
 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*e*
x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sq
rt[a + b*x^3]))/10))/(16*b)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.15 (sec) , antiderivative size = 777, normalized size of antiderivative = 2.40

method	result
risch	$\frac{(40b^2Bx^6+64Ab^2x^3+12Babx^3+48abA-21a^2B)x\sqrt{bx^3+a}\,e^3}{320b^2\sqrt{ex}} - \frac{3a^2(16Ab-7Ba)\left(\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(}{}\right.}{\left(-\frac{(-ab^2)^{\frac{1}{3}}}{2b}\right.}}}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((e*x)^(5/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/320*(40*B*b^2*x^6+64*A*b^2*x^3+12*B*a*b*x^3+48*A*a*b-21*B*a^2)*x*(b*x^3+
a)^(1/2)/b^2*e^3/(e*x)^(1/2)-3/320*a^2*(16*A*b-7*B*a)/b*(1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3
)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*
(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3))
)^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1
/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(
-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3
)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2))*e^3*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x)^(5/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*e^2*x^5 + A*e^2*x^2)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.70 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.30

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{A\sqrt{a}e^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{6}\right)} + \frac{B\sqrt{a}e^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)}$$

input
```
integrate((e*x)**(5/2)*(b*x**3+a)**(1/2)*(B*x**3+A),x)
```

output
```
A*sqrt(a)*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(13/6)) + B*sqrt(a)*e**(5/2)*x**(13/2)*gamma(13
/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(19/6))
```

Maxima [F]

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x)^(5/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(5/2), x)
```

Giac [F]

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x)^(5/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}\,\sqrt{b\,x^3+a}\,dx$$

input
```
int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(1/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int (ex)^{5/2}\sqrt{a+bx^3}(A+Bx^3)\,dx=\frac{\sqrt{e}\,e^2\left(54\sqrt{x}\,\sqrt{b\,x^3+a}\,a^2+152\sqrt{x}\,\sqrt{b\,x^3+a}\,ab\,x^3+80\sqrt{x}\,\sqrt{b\,x^3+a}\,b^2x^6-27\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^4+ax}\right.\right.}{640b}$$

input
```
int((e*x)^(5/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

output
```
(sqrt(e)*e**2*(54*sqrt(x)*sqrt(a + b*x**3)*a**2 + 152*sqrt(x)*sqrt(a + b*x
**3)*a*b*x**3 + 80*sqrt(x)*sqrt(a + b*x**3)*b**2*x**6 - 27*int((sqrt(x)*sq
rt(a + b*x**3))/(a*x + b*x**4),x)*a**3))/(640*b)
```

3.240 $\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 581

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be}$$
$$+\frac{3\left(1+\sqrt{3}\right)a(14Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{112b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}$$
$$-\frac{3\sqrt[4]{3}a^{4/3}(14Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{}\right.\right.}{112b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}(14Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right.}{\sqrt[3]{a}+\left(1+\sqrt{3}\right.}\right.\right.}{224b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/56*(14*A*b-5*B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/b/e+3/112*(1+3^(1/2))*a*(1
4*A*b-5*B*a)*e*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+(1+3^(1/2))*b^
(1/3)*x)+1/7*B*(e*x)^(5/2)*(b*x^3+a)^(3/2)/b/e-3/112*3^(1/4)*a^(4/3)*(14*A
*b-5*B*a)*e*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+
(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1
/2)+1/4*2^(1/2))/b^(5/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2
))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-1/224*3^(3/4)*(1-3^(1/2))*a^(4/3)*(
14*A*b-5*B*a)*e*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(ar
ccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*
6^(1/2)+1/4*2^(1/2))/b^(5/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^
(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.16

$$\int (ex)^{3/2}\sqrt{a+bx^3}(A+Bx^3)\,dx = \frac{x(ex)^{3/2}\sqrt{a+bx^3}\left(5B(a+bx^3)\sqrt{1+\frac{bx^3}{a}}+(14Ab-5aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{6},\frac{11}{6},-\right.\right.}{35b\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(e*x)^(3/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(x*(e*x)^(3/2)*Sqrt[a + b*x^3]*(5*B*(a + b*x^3)*Sqrt[1 + (b*x^3)/a] + (14*
A*b - 5*a*B)*Hypergeometric2F1[-1/2, 5/6, 11/6, -((b*x^3)/a)]))/(35*b*Sqrt
[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {959, 811, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(14Ab-5aB)\int (ex)^{3/2}\sqrt{bx^3+a}dx}{14b}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(14Ab-5aB)\left(\frac{3}{8}a\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)}{14b}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(14Ab-5aB)\left(\frac{3a\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)}{14b}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}$$

$$\Big\downarrow \text{837}$$

$$\frac{(14Ab-5aB)\left(\frac{3a\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)}{14b}+$$
$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}$$

$$\Big\downarrow \text{25}$$

$$\frac{(14Ab-5aB)\left(\frac{3a\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)}{14b}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}$$

$\big\downarrow$ 766

$$\frac{(14Ab-5aB)\left(\frac{3a\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right.}{4e}\right.}{14b}$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}$$

$\big\downarrow$ 2420

$$
(14Ab-5aB)\left(3a\left(\frac{\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}}\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{2b^{2/3}}\right.\right.
$$

$$
\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}
$$

input
```
Int[(e*x)^(3/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(B*(e*x)^(5/2)*(a + b*x^3)^(3/2))/(7*b*e) + ((14*A*b - 5*a*B)*(((e*x)^(5/2
)*Sqrt[a + b*x^3])/(4*e) + (3*a*((((1 + Sqrt[3])*e^3*Sqrt[e*x]*Sqrt[a + b*
x^3])/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x) - (3^(1/4)*a^(1/3)*e*Sqrt[e*
x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b
^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticE[ArcCo
s[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/
3)*e*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/
(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) -
 ((1 - Sqrt[3])*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3
)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3]
)*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)
/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^
(2/3)*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[
3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(4*e)))/(14*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.40 (sec) , antiderivative size = 1140, normalized size of antiderivative = 1.96

method	result	size
risch	Expression too large to display	1140
elliptic	Expression too large to display	1184
default	Expression too large to display	5358

input
```
int((e*x)^(3/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/56*x^3*(8*B*b*x^3+14*A*b+3*B*a)*(b*x^3+a)^(1/2)/b*e^2/(e*x)^(1/2)+3/112*
a*(14*A*b-5*B*a)/b*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^
(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(
1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1
/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a
*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x...
```

Fricas [F]

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^(3/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*e*x^4 + A*e*x)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.17

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{A\sqrt{a}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{6}\right)} + \frac{B\sqrt{a}e^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)}$$

input
```
integrate((e*x)**(3/2)*(b*x**3+a)**(1/2)*(B*x**3+A),x)
```

output
```
A*sqrt(a)*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(11/6)) + B*sqrt(a)*e**(3/2)*x**(11/2)*gamma(11
/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(17/6))
```

Maxima [F]

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^(3/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(3/2), x)
```

Giac [F]

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^(3/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}\,\sqrt{b\,x^3+a}\,dx$$

input
```
int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(1/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{\sqrt{e}\,e\left(34\sqrt{x}\,\sqrt{b\,x^3+a}\,a\,x^2+16\sqrt{x}\,\sqrt{b\,x^3+a}\,b\,x^5+27\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a^2\right)}{112}$$

input
```
int((e*x)^(3/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

output
```
(sqrt(e)*e*(34*sqrt(x)*sqrt(a + b*x**3)*a*x**2 + 16*sqrt(x)*sqrt(a + b*x**
3)*b*x**5 + 27*int((sqrt(x)*sqrt(a + b*x**3)*x)/(a + b*x**3),x)*a**2))/112
```

3.241 $\int \sqrt{ex}\sqrt{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \sqrt{ex}\sqrt{a+bx^3}(A+Bx^3)\,dx = \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}(a+bx^3)^{3/2}}{6be} + \frac{a(4Ab-aB)\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{3/2}}$$

output
```
1/12*(4*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b/e+1/6*B*(e*x)^(3/2)*(b*x^3+
a)^(3/2)/b/e+1/12*a*(4*A*b-B*a)*e^(1/2)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2
)/(b*x^3+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int \sqrt{ex}\sqrt{a+bx^3}(A+Bx^3)\,dx = \frac{x\sqrt{ex}\sqrt{a+bx^3}(4Ab+aB+2bBx^3)}{12b} - \frac{a(-4Ab+aB)\sqrt{ex}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{12b^{3/2}\sqrt{x}}$$

input
```
Integrate[Sqrt[e*x]*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(x*Sqrt[e*x]*Sqrt[a + b*x^3]*(4*A*b + a*B + 2*b*B*x^3))/(12*b) - (a*(-4*A*
b + a*B)*Sqrt[e*x]*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/(12*b^(3/2)*Sqr
t[x])
```

Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {959, 811, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(4Ab-aB)\int \sqrt{ex}\sqrt{bx^3+a}dx}{4b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(4Ab-aB)\left(\frac{1}{2}a\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{4b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(4Ab-aB)\left(\frac{a\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{4b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

$$\Big\downarrow \text{807}$$

$$\frac{(4Ab-aB)\left(\frac{a\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{4b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

$$\Big\downarrow \text{224}$$

$$\frac{(4Ab-aB)\left(\frac{a\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{4b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

$$\Big\downarrow 219$$

$$\frac{(4Ab-aB)\left(\frac{a\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3\sqrt{b}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{4b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

input
```
Int[Sqrt[e*x]*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(B*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(6*b*e) + ((4*A*b - a*B)*(((e*x)^(3/2)*S
qrt[a + b*x^3])/(3*e) + (a*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*
Sqrt[a + (b*x)/e^2])])/(3*Sqrt[b])))/(4*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.90

method	result
risch	$\frac{x^{2}\left(2 b B\,x^{3}+4 A b +B a \right) \sqrt{b\,x^{3}+a}\, e}{12 b \sqrt{e x}}+\frac{a \left(4 A b -B a \right) \operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) e \sqrt{\left(b\,x^{3}+a \right) e x}}{12 b \sqrt{b e}\, \sqrt{e x}\, \sqrt{b\,x^{3}+a}}$
default	$\frac{\sqrt{e x}\, \sqrt{b\,x^{3}+a}\, \left(2 B \sqrt{\left(b\,x^{3}+a \right) e x}\, \sqrt{b e}\, b\,x^{4}+4 A \sqrt{\left(b\,x^{3}+a \right) e x}\, \sqrt{b e}\, b x +4 A \,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) a b e +B \sqrt{\left(b\,x^{3}+a \right) e x}\, \sqrt{b e}\, a x -B\right.}{12 \sqrt{\left(b\,x^{3}+a \right) e x}\, \sqrt{b e}\, b}$
elliptic	Expression too large to display

input
```
int((e*x)^(1/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/12*x^2*(2*B*b*x^3+4*A*b+B*a)*(b*x^3+a)^(1/2)/b*e/(e*x)^(1/2)+1/12*a*(4*A
*b-B*a)/b/(b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*e*((b
*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.83

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\left[-\frac{(Ba^2-4\,Aab)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,b^2x^4+abx)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)-4\,(2\,Bbx^4+\cdots}{48\,b}\right.$$

input
```
integrate((e*x)^(1/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
[-1/48*((B*a^2 - 4*A*a*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e
 - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(2*B*b*x
^4 + (B*a + 4*A*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b, 1/24*((B*a^2 - 4*A*a*b
)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3
+ a*e)) + 2*(2*B*b*x^4 + (B*a + 4*A*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(104) = 208.

Time = 1.62 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.09

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\begin{cases} \frac{2\left(\begin{cases}\text{NaN} \\ \frac{Ae^3\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(\frac{2b(ex)^{\frac{3}{2}}}{e^3}+2\sqrt{\frac{b}{e^3}}\sqrt{a+bx^3}\right)}{\sqrt{\frac{b}{e^3}}} & \text{for } a\neq 0 \\ \frac{(ex)^{\frac{3}{2}}\log\left((ex)^{\frac{3}{2}}\right)}{\sqrt{bx^3}} & \text{otherwise}\end{cases}\right)}{2}+\frac{(ex)^{\frac{3}{2}}\sqrt{a+bx^3}}{2} & \text{for } \frac{b}{e^3}\neq 0 \\ \sqrt{a}(ex)^{\frac{3}{2}} & \text{otherwise}\end{cases}\right)+B\left(\begin{cases} -\frac{a^2e^3\left(\begin{cases}\frac{\log\left(\frac{2b(e\cdots}{e^3\cdots}\right.}{\cdots} \\ \frac{(ex)^{\frac{3}{2}}\log\cdots}{\sqrt{b\cdots}}\end{cases}\right.}{\cdots} \\ \frac{\sqrt{a}(ex)^{\frac{9}{2}}}{3}\end{cases}\right.}{3e^3}\end{cases}\right.}{e} \\ 0 \end{cases}$$

input
```
integrate((e*x)**(1/2)*(b*x**3+a)**(1/2)*(B*x**3+A),x)
```

output
```
Piecewise((2*Piecewise((nan, Eq(e**3, 0)), ((A*e**3*Piecewise((a*Piecewise
((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3
), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/2 + (e*
x)**(3/2)*sqrt(a + b*x**3)/2, Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(3/2), True)
) + B*Piecewise((-a**2*e**3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(
b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)
**(3/2))/sqrt(b*x**3), True))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**(3/2
)/(8*b) + (e*x)**(9/2)/4), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, True))
)/(3*e**3), True))/e, Ne(e, 0)), (0, True))
```

Maxima [F]

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}\sqrt{ex}\,dx$$

input
```
integrate((e*x)^(1/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.46

$$\begin{aligned}&\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx\\&=\frac{Ba^2e\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{12\sqrt{be}b}\\&\quad-\frac{\left(\frac{ae^4\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{\sqrt{be}}-\sqrt{be^4x^3+ae^4}\sqrt{ex}ex\right)A|e|^2}{3\,e^5}\\&\quad+\frac{\sqrt{be^4x^3+ae^4}\left(2\,e^3x^3+\frac{ae^3}{b}\right)\sqrt{ex}Bx|e|^2}{12\,e^7}\end{aligned}$$

input
```
integrate((e*x)^(1/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/12*B*a^2*e*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*e^4)))/
(sqrt(b*e)*b) - 1/3*(a*e^4*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x
^3 + a*e^4)))/sqrt(b*e) - sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*e*x)*A*abs(e)^
2/e^5 + 1/12*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3 + a*e^3/b)*sqrt(e*x)*B*x*a
bs(e)^2/e^7
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,\sqrt{e\,x}\,\sqrt{b\,x^3+a}\,dx$$

input
```
int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(1/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.73

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{\sqrt{e}\left(10\sqrt{x}\,\sqrt{b\,x^3+a}\,abx+4\sqrt{x}\,\sqrt{b\,x^3+a}\,b^2x^4-3\sqrt{b}\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)a^2+3\sqrt{b}\log\left(\sqrt{b\,x^3}\right.\right.}{24b}$$

input
```
int((e*x)^(1/2)*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

output
```
(sqrt(e)*(10*sqrt(x)*sqrt(a + b*x**3)*a*b*x + 4*sqrt(x)*sqrt(a + b*x**3)*b
**2*x**4 - 3*sqrt(b)*log(sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*x)*a**2 + 3*sq
rt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)*x)*a**2))/(24*b)
```

3.242 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 286

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{(10Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{20be} + \frac{B\sqrt{ex}(a+bx^3)^{3/2}}{5be} + \frac{3^{3/4}a^{2/3}(10Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\right.}{40be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/20*(10*A*b-B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b/e+1/5*B*(e*x)^(1/2)*(b*x^3
+a)^(3/2)/b/e+1/40*3^(3/4)*a^(2/3)*(10*A*b-B*a)*(e*x)^(1/2)*(a^(1/3)+b^(1/
3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3
)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1
/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))/b/e/(b^(1/3)*x*(a^(1/
3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{x\sqrt{a+bx^3}\left(B(a+bx^3)\sqrt{1+\frac{bx^3}{a}}+(10Ab-aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{5b\sqrt{ex}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/Sqrt[e*x],x]
```

output
```
(x*Sqrt[a + b*x^3]*(B*(a + b*x^3)*Sqrt[1 + (b*x^3)/a] + (10*A*b - a*B)*Hyp
ergeometric2F1[-1/2, 1/6, 7/6, -((b*x^3)/a)]))/(5*b*Sqrt[e*x]*Sqrt[1 + (b*
x^3)/a])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {959, 811, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx$$

$$\Big\downarrow \ 959$$

$$\frac{(10Ab-aB)\int \frac{\sqrt{bx^3+a}}{\sqrt{ex}}dx}{10b}+\frac{B\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5be}$$

$$\Big\downarrow \ 811$$

$$\frac{(10Ab-aB)\left(\frac{3}{4}a\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)}{10b}+\frac{B\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(10Ab-aB)\left(\frac{3a\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2e}+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)}{10b}+\frac{B\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5be}$$

$$\Big\downarrow \text{766}$$

$$\frac{(10Ab-aB)\left(\frac{3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4e^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}+\right.}{10b}$$

$$\frac{B\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5be}$$

input

```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/Sqrt[e*x],x]
```

output

```
(B*Sqrt[e*x]*(a + b*x^3)^(3/2))/(5*b*e) + ((10*A*b - a*B)*((Sqrt[e*x]*Sqrt
[a + b*x^3])/(2*e) + (3^(3/4)*a^(2/3)*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*
Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e +
(1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*
b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(
4*e^2*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[
3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(10*b)
```

Defintions of rubi rules used

rule 766

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.37 (sec) , antiderivative size = 744, normalized size of antiderivative = 2.60

method	result
risch	$\frac{(4bB\,x^{3}+10Ab+3Ba)x\sqrt{b\,x^{3}+a}}{20b\sqrt{ex}}+\frac{3a(10Ab-Ba)\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}}{}$
elliptic	$\frac{\sqrt{(b\,x^{3}+a)ex}\left(\frac{B\,x^{3}\sqrt{be\,x^{4}+aex}}{5e}+\frac{\left(Ab+\frac{3Ba}{10}\right)\sqrt{be\,x^{4}+aex}}{2be}+\frac{2\left(Aa-\frac{a\left(Ab+\frac{3Ba}{10}\right)}{4b}\right)\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)}{2b}\right.}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}\right.}}}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*(4*B*b*x^3+10*A*b+3*B*a)*x*(b*x^3+a)^(1/2)/b/(e*x)^(1/2)+3/20*a*(10*A
*b-B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(
1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(
b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(e*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.94 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{A\sqrt{a}\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{7}{6}\right)} + \frac{B\sqrt{a}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{13}{6}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)*(B*x**3+A)/(e*x)**(1/2),x)
```

output
```
A*sqrt(a)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I
*pi)/a)/(3*sqrt(e)*gamma(7/6)) + B*sqrt(a)*x**(7/2)*gamma(7/6)*hyper((-1/2
, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/sqrt(e*x), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/sqrt(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{\sqrt{e\,x}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(1/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{\sqrt{e}\left(26\sqrt{x}\,\sqrt{b\,x^3+a}\,a+8\sqrt{x}\,\sqrt{b\,x^3+a}\,b\,x^3+27\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^4+ax}dx\right)a^2\right)}{40e}$$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(1/2),x)
```

output
```
(sqrt(e)*(26*sqrt(x)*sqrt(a + b*x**3)*a + 8*sqrt(x)*sqrt(a + b*x**3)*b*x**
3 + 27*int((sqrt(x)*sqrt(a + b*x**3))/(a*x + b*x**4),x)*a**2))/(40*e)
```

3.243 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 580

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4}$$

$$+\frac{3(1+\sqrt{3})\,(8Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{8b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}-\frac{2A(a+bx^3)^{3/2}}{ae\sqrt{ex}}$$

$$-\frac{3\sqrt[4]{3}\sqrt[3]{a}(8Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{8b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{a}(8Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{16b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/4*(8*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/a/e^4+3/8*(1+3^(1/2))*(8*A*b+B
*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(2/3)/e^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x
)-2*A*(b*x^3+a)^(3/2)/a/e/(e*x)^(1/2)-3/8*3^(1/4)*a^(1/3)*(8*A*b+B*a)*(e*x
)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1
/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1
/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))
/b^(2/3)/e^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x
)^2)^(1/2)/(b*x^3+a)^(1/2)-1/16*3^(3/4)*(1-3^(1/2))*a^(1/3)*(8*A*b+B*a)*(e
*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^
(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3
^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2
))/b^(2/3)/e^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)
*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx = -\frac{2Ax(a+bx^3)^{3/2}}{a(ex)^{3/2}} - \frac{4\left(-4Ab-\frac{aB}{2}\right)x^4\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)}{5a(ex)^{3/2}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(3/2),x]
```

output
```
(-2*A*x*(a + b*x^3)^(3/2))/(a*(e*x)^(3/2)) - (4*(-4*A*b - (a*B)/2)*x^4*Sqr
t[a + b*x^3]*Hypergeometric2F1[-1/2, 5/6, 11/6, -((b*x^3)/a)])/(5*a*(e*x)^
(3/2)*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {955, 811, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{3/2}}dx$$

$$\Big\downarrow \text{ 955}$$

$$\frac{(aB+8Ab)\int (ex)^{3/2}\sqrt{bx^3+a}dx}{ae^3}-\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{(aB+8Ab)\left(\frac{3}{8}a\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{(aB+8Ab)\left(\frac{3a\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 837}$$

$$\frac{(aB+8Ab)\left(\frac{3a\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(aB+8Ab)\left(\frac{3a\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}}$$

$\Big\downarrow$ 766

$$\frac{(aB+8Ab)\left(\frac{3a\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1\cdots}{\left(1\cdots}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right.}{4e}\cdots\right.}{ae^3}$$

$$\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}}$$

$\Big\downarrow$ 2420

$$(aB+8Ab)\left(\frac{3a\left(\frac{\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right)\cdots\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{2b^{2/3}}\cdots\right.}{\cdots}\right.\cdots$$

$$\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(3/2),x]
```

output
```
(-2*A*(a + b*x^3)^(3/2))/(a*e*Sqrt[e*x]) + ((8*A*b + a*B)*(((e*x)^(5/2)*Sq
rt[a + b*x^3])/(4*e) + (3*a*((((1 + Sqrt[3])*e^3*Sqrt[e*x]*Sqrt[a + b*x^3]
)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x) - (3^(1/4)*a^(1/3)*e*Sqrt[e*x]*(
a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/
3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticE[ArcCos[(a
^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e
*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(
1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1
 - Sqrt[3])*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^
2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^
(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^
(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3
)*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*
b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(4*e)))/(a*e^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.69 (sec) , antiderivative size = 1123, normalized size of antiderivative = 1.94

method	result	size
risch	Expression too large to display	1123
elliptic	Expression too large to display	1161
default	Expression too large to display	5736

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x^3+a)^(1/2)*(-B*x^3+8*A)/e/(e*x)^(1/2)+(3*A*b+3/8*B*a)*(x*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^
(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/
3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3))
)^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE
(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),(...
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.78 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{6}\right)} + \frac{B\sqrt{a}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)*(B*x**3+A)/(e*x)**(3/2),x)
```

output
```
A*sqrt(a)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/a
)/(3*e**(3/2)*sqrt(x)*gamma(5/6)) + B*sqrt(a)*x**(5/2)*gamma(5/6)*hyper((-
1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(11/6))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{(e\,x)^{3/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(3/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{\sqrt{e}\left(22\sqrt{b\,x^3+a}\,a+4\sqrt{b\,x^3+a}\,b\,x^3+27\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^5+a\,x^2}dx\right)a^2\right)}{16\sqrt{x}\,e^2}$$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(3/2),x)
```

output
```
(sqrt(e)*(22*sqrt(a + b*x**3)*a + 4*sqrt(a + b*x**3)*b*x**3 + 27*sqrt(x)*i
nt((sqrt(x)*sqrt(a + b*x**3))/(a*x**2 + b*x**5),x)*a**2))/(16*sqrt(x)*e**2
)
```

3.244 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 118

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx = \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}e^{5/2}}$$

output
```
1/3*(2*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/a/e^4-2/3*A*(b*x^3+a)^(3/2)/a/
e/(e*x)^(3/2)+1/3*(2*A*b+B*a)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a
)^(1/2))/b^(1/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx = \frac{x\left(\sqrt{b}\sqrt{a+bx^3}(-2A+Bx^3)+(2Ab+aB)x^{3/2}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{3\sqrt{b}(ex)^{5/2}}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(5/2),x]
```

output
```
(x*(Sqrt[b]*Sqrt[a + b*x^3]*(-2*A + B*x^3) + (2*A*b + a*B)*x^(3/2)*Log[Sqr
t[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(3*Sqrt[b]*(e*x)^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {955, 811, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(aB+2Ab)\int \sqrt{ex}\sqrt{bx^3+a}dx}{ae^3}-\frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{811}$$

$$\frac{(aB+2Ab)\left(\frac{1}{2}a\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{ae^3}-\frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{(aB+2Ab)\left(\frac{a\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{ae^3}-\frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{807}$$

$$\frac{(aB+2Ab)\left(\frac{a\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{ae^3}-\frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{(aB+2Ab)\left(\frac{a\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{ae^3}-\frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(aB+2Ab)\left(\frac{a\sqrt{e}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3\sqrt{b}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{ae^3}-\frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(5/2),x]
```

output
```
(-2*A*(a + b*x^3)^(3/2))/(3*a*e*(e*x)^(3/2)) + ((2*A*b + a*B)*(((e*x)^(3/2
)*Sqrt[a + b*x^3])/(3*e) + (a*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/
2)*Sqrt[a + (b*x)/e^2])])/(3*Sqrt[b])))/(a*e^3)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-B\,x^{3}+2A\right)}{3x\,e^{2}\sqrt{ex}}+\frac{2\left(Ab+\frac{Ba}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)\sqrt{\left(b\,x^{3}+a\right)ex}}{3\sqrt{be}\,e^{2}\sqrt{ex}\,\sqrt{b\,x^{3}+a}}$
default	$\frac{\sqrt{b\,x^{3}+a}\,\left(2A\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)be\,x^{2}+B\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)ae\,x^{2}+B\sqrt{\left(b\,x^{3}+a\right)ex}\,\sqrt{be}\,x^{3}-2A\sqrt{\left(b\,x^{3}+a\right)ex}\,\sqrt{be}\right)}{3x\,e^{2}\sqrt{ex}\,\sqrt{\left(b\,x^{3}+a\right)ex}\,\sqrt{be}}$
elliptic	Expression too large to display

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^3+a)^(1/2)*(-B*x^3+2*A)/x/e^2/(e*x)^(1/2)+2/3*(A*b+1/2*B*a)/(b*e
)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))/e^2*((b*x^3+a)*e*x)
^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx = \left[\frac{(Ba+2\,Ab)\sqrt{be}x^2\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\right.}{12\,be^3x^2}\right.$$
$$\left.-\frac{(Ba+2\,Ab)\sqrt{-be}x^2\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{-be}\sqrt{ex}x}{2\,bex^3+ae}\right)-2\,(Bbx^3-2\,Ab)\sqrt{bx^3+a}\sqrt{ex}}{6\,be^3x^2}\right]$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*((B*a + 2*A*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e
- 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(B*b*x^3 - 2*
A*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2), -1/6*((B*a + 2*A*b)*sqrt(-b*e
)*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) -
 2*(B*b*x^3 - 2*A*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]
```

Sympy [A] (verification not implemented)

Time = 7.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx = -\frac{2A\sqrt{a}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{2A\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3e^{\frac{5}{2}}}$$
$$-\frac{2Abx^{\frac{3}{2}}}{3\sqrt{a}e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{B\sqrt{a}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}}+\frac{Ba\,\mathrm{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3\sqrt{b}e^{\frac{5}{2}}}$$

input
```
integrate((b*x**3+a)**(1/2)*(B*x**3+A)/(e*x)**(5/2),x)
```

output
```
-2*A*sqrt(a)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*A*sqrt(b)*asinh(
sqrt(b)*x**(3/2)/sqrt(a))/(3*e**(5/2)) - 2*A*b*x**(3/2)/(3*sqrt(a)*e**(5/2
)*sqrt(1 + b*x**3/a)) + B*sqrt(a)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2))
 + B*a*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(3*sqrt(b)*e**(5/2))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: Recu
rsive assumption sageVARa>=(-sageVARb*sageVARe/(sageVARe^4*t_nostep^6)) ig
nored1/sageVARe^3/((1/sageVARe)^2)*2*sageVARe*sageVARB/6/sageVARe^6*sqrt(s
ageVAReVARx)*s
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx=\int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{{\left(e\,x\right)}^{5/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(5/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx=\frac{\sqrt{e}\left(-4\sqrt{b\,x^3+a}\,a+2\sqrt{b\,x^3+a}\,b\,x^3-3\sqrt{x}\,\sqrt{b}\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)\cdots}{6\sqrt{x}\,e^3x}$$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(5/2),x)
```

output
```
(sqrt(e)*( - 4*sqrt(a + b*x**3)*a + 2*sqrt(a + b*x**3)*b*x**3 - 3*sqrt(x)*
sqrt(b)*log(sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*x)*a*x + 3*sqrt(x)*sqrt(b)*
log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)*x)*a*x))/(6*sqrt(x)*e**3*x)
```

3.245 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 283

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{(4Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{10ae^4} - \frac{2A(a+bx^3)^{3/2}}{5ae(ex)^{5/2}}$$
$$+ \frac{3^{3/4}(4Ab+5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2+\cdots\right)}{20\sqrt[3]{a}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/10*(4*A*b+5*B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/a/e^4-2/5*A*(b*x^3+a)^(3/2)
/a/e/(e*x)^(5/2)+1/20*3^(3/4)*(4*A*b+5*B*a)*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)
^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+
(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))/a^(1/3)/e^4/(b^(1/3)*x*(a
^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{2x\sqrt{a+bx^3}\left(-A(a+bx^3)\sqrt{1+\frac{bx^3}{a}}+(4Ab+5aB)x^3\,\text{Hypergeometric2F1}\left(\right.\right.}{5a(ex)^{7/2}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(7/2),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(-(A*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]) + (4*A*b + 5*a*
B)*x^3*Hypergeometric2F1[-1/2, 1/6, 7/6, -((b*x^3)/a)]))/(5*a*(e*x)^(7/2)*
Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {955, 811, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(5aB+4Ab)\int \frac{\sqrt{bx^3+a}}{\sqrt{ex}}dx}{5ae^3}-\frac{2A(a+bx^3)^{3/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{811}$$

$$\frac{(5aB+4Ab)\left(\frac{3}{4}a\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)}{5ae^3}-\frac{2A(a+bx^3)^{3/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{(5aB+4Ab)\left(\frac{3a\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2e}+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)}{5ae^3}-\frac{2A(a+bx^3)^{3/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \; 766$$

$$\frac{(5aB+4Ab)\left(\frac{3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4e^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}+\right.}{5ae^3}$$

$$\frac{2A(a+bx^3)^{3/2}}{5ae(ex)^{5/2}}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(7/2),x]
```

output
```
(-2*A*(a + b*x^3)^(3/2))/(5*a*e*(e*x)^(5/2)) + ((4*A*b + 5*a*B)*((Sqrt[e*x
]*Sqrt[a + b*x^3])/(2*e) + (3^(3/4)*a^(2/3)*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)
*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3
)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqr
t[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])
/4])/(4*e^2*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 +
 Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(5*a*e^3)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.26 (sec) , antiderivative size = 743, normalized size of antiderivative = 2.63

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-5B\,x^{3}+4A\right)}{10x^{2}e^{3}\sqrt{ex}}+\frac{2\left(\frac{3Ab}{5}+\frac{3Ba}{4}\right)\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\right.}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*(b*x^3+a)^(1/2)*(-5*B*x^3+4*A)/x^2/e^3/(e*x)^(1/2)+2*(3/5*A*b+3/4*B*
a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))
^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2
)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*b/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))/e^3*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/
(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^4*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.81 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},-\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)} + \frac{B\sqrt{a}\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}$$

input

```
integrate((b*x**3+a)**(1/2)*(B*x**3+A)/(e*x)**(7/2),x)
```

output

```
A*sqrt(a)*gamma(-5/6)*hyper((-5/6, -1/2), (1/6,), b*x**3*exp_polar(I*pi)/a
)/(3*e**(7/2)*x**(5/2)*gamma(1/6)) + B*sqrt(a)*sqrt(x)*gamma(1/6)*hyper((-
1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/6))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{7}{2}}}\,dx$$

input

```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="maxima")
```

output

```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{(e\,x)^{7/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(7/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{\sqrt{e}\left(-14\sqrt{b\,x^3+a}\,a+4\sqrt{b\,x^3+a}\,b\,x^3-27\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^7+a\,x^4}dx\right)a^2x^2\right)}{8\sqrt{x}\,e^4x^2}$$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/(e*x)^(7/2),x)
```

output
```
(sqrt(e)*( - 14*sqrt(a + b*x**3)*a + 4*sqrt(a + b*x**3)*b*x**3 - 27*sqrt(x
)*int((sqrt(x)*sqrt(a + b*x**3))/(a*x**4 + b*x**7),x)*a**2*x**2))/(8*sqrt(
x)*e**4*x**2)
```

3.246 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 564

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx = -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}} + \frac{3(1+\sqrt{3})\sqrt[3]{b}(2Ab+7aB)\sqrt{x}\sqrt{a+bx^3}}{7a\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)} - \frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}}$$
$$- \frac{3\sqrt[4]{3}\sqrt[3]{b}(2Ab+7aB)\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{7a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$- \frac{3^{3/4}(1-\sqrt{3})\sqrt[3]{b}(2Ab+7aB)\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}\right)\right.}{14a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/7*(2*A*b+7*B*a)*(b*x^3+a)^(1/2)/a/x^(1/2)+3/7*(1+3^(1/2))*b^(1/3)*(2*A*
b+7*B*a)*x^(1/2)*(b*x^3+a)^(1/2)/a/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)-2/7*A*(
b*x^3+a)^(3/2)/a/x^(7/2)-3/7*3^(1/4)*b^(1/3)*(2*A*b+7*B*a)*x^(1/2)*(a^(1/3
)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))
*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/
3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/a^(2/3)/(b^(1/
3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a
)^(1/2)-1/14*3^(3/4)*(1-3^(1/2))*b^(1/3)*(2*A*b+7*B*a)*x^(1/2)*(a^(1/3)+b^
(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(
1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a
^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))/a^(2/3)/(b^(1/3)*x
*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx = \frac{2\sqrt{a+bx^3}\left(-A(a+bx^3)-\frac{(2Ab+7aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{7ax^{7/2}}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(9/2),x]
```

output
```
(2*Sqrt[a + b*x^3]*(-(A*(a + b*x^3)) - ((2*A*b + 7*a*B)*x^3*Hypergeometric
2F1[-1/2, -1/6, 5/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(7*a*x^(7/2))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {955, 809, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx$$

$$\Big\downarrow \text{ 955}$$

$$\frac{(7aB+2Ab)\int \frac{\sqrt{bx^3+a}}{x^{3/2}}dx}{7a}-\frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 809}$$

$$\frac{(7aB+2Ab)\left(3b\int \frac{x^{3/2}}{\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}}{\sqrt{x}}\right)}{7a}-\frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{(7aB+2Ab)\left(6b\int \frac{x^2}{\sqrt{bx^3+a}}d\sqrt{x}-\frac{2\sqrt{a+bx^3}}{\sqrt{x}}\right)}{7a}-\frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 837}$$

$$\frac{(7aB+2Ab)\left(6b\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)-\frac{2\sqrt{a+bx^3}}{\sqrt{x}}\right)}{7a}-\frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(7aB+2Ab)\left(6b\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)-\frac{2\sqrt{a+bx^3}}{\sqrt{x}}\right)}{7a}-\frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 766}$$

$$\frac{(7aB+2Ab)\left(6b\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right.}{\left(1+\sqrt{3}\right.}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.}{7a}$$

$$\frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{2420}$$

$$\frac{(7aB+2Ab)\left(6b\left(\frac{\frac{(1+\sqrt{3})\sqrt{x}\sqrt{a+bx^3}}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2b^{2/3}}\right.\right.}{7a}$$

$$\frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}}$$

input

```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(9/2),x]
```

output

```
(-2*A*(a + b*x^3)^(3/2))/(7*a*x^(7/2)) + ((2*A*b + 7*a*B)*((-2*Sqrt[a + b*
x^3])/Sqrt[x] + 6*b*((((1 + Sqrt[3])*Sqrt[x]*Sqrt[a + b*x^3])/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x) - (3^(1/4)*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
 + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*x*(a^(1/3)
+ b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(2*
b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*
(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x
^3]))))/(7*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.52 (sec) , antiderivative size = 1127, normalized size of antiderivative = 2.00

method	result	size
risch	Expression too large to display	1127
elliptic	Expression too large to display	1177
default	Expression too large to display	5911

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*(b*x^3+a)^(1/2)*(3*A*b*x^3+7*B*a*x^3+A*a)/x^(7/2)/a+3/7*b*(2*A*b+7*B*
a)/a*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(
1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*((
(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/
b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*
(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/
(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^...
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^(9/2),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^(9/2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{7}{6},-\frac{1}{2}\\-\frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{\frac{7}{2}}\Gamma\left(-\frac{1}{6}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\\frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{x}\Gamma\left(\frac{5}{6}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)*(B*x**3+A)/x**(9/2),x)
```

output
```
A*sqrt(a)*gamma(-7/6)*hyper((-7/6, -1/2), (-1/6,), b*x**3*exp_polar(I*pi)/
a)/(3*x**(7/2)*gamma(-1/6)) + B*sqrt(a)*gamma(-1/6)*hyper((-1/2, -1/6), (5
/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(x)*gamma(5/6))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^(9/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^(9/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^{9/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(9/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(9/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx = \frac{-10\sqrt{b\,x^3+a}\,a+8\sqrt{b\,x^3+a}\,b\,x^3-27\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^8+a\,x^5}dx\right)a^2x^3}{8\sqrt{x}\,x^3}$$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^(9/2),x)
```

output
```
( - 10*sqrt(a + b*x**3)*a + 8*sqrt(a + b*x**3)*b*x**3 - 27*sqrt(x)*int((sq
rt(x)*sqrt(a + b*x**3))/(a*x**5 + b*x**8),x)*a**2*x**3)/(8*sqrt(x)*x**3)
```

3.247 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + \frac{2}{3}\sqrt{b}B\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)$$

output
```
-2/3*B*(b*x^3+a)^(1/2)/x^(3/2)-2/9*A*(b*x^3+a)^(3/2)/a/x^(9/2)+2/3*b^(1/2)
*B*arctanh(b^(1/2)*x^(3/2)/(b*x^3+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = -\frac{2\sqrt{a+bx^3}(aA+Abx^3+3aBx^3)}{9ax^{9/2}} + \frac{2}{3}\sqrt{b}B\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(11/2),x]
```

output
```
(-2*Sqrt[a + b*x^3]*(a*A + A*b*x^3 + 3*a*B*x^3))/(9*a*x^(9/2)) + (2*Sqrt[b
]*B*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/3
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {953, 809, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{11/2}}dx$$

$$\Big\downarrow \text{953}$$

$$B\int \frac{\sqrt{bx^3+a}}{x^{5/2}}dx-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \text{809}$$

$$B\left(b\int \frac{\sqrt{x}}{\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}}{3x^{3/2}}\right)-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \text{851}$$

$$B\left(2b\int \frac{x}{\sqrt{bx^3+a}}d\sqrt{x}-\frac{2\sqrt{a+bx^3}}{3x^{3/2}}\right)-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \text{807}$$

$$B\left(\frac{2}{3}b\int \frac{1}{\sqrt{a+bx}}dx^{3/2}-\frac{2\sqrt{a+bx^3}}{3x^{3/2}}\right)-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \text{224}$$

$$B\left(\frac{2}{3}b\int \frac{1}{1-bx}d\frac{x^{3/2}}{\sqrt{a+bx}}-\frac{2\sqrt{a+bx^3}}{3x^{3/2}}\right)-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \text{219}$$

$$B\left(\frac{2}{3}\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx}}\right)-\frac{2\sqrt{a+bx^3}}{3x^{3/2}}\right)-\frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(11/2),x]
```

output
```
(-2*A*(a + b*x^3)^(3/2))/(9*a*x^(9/2)) + B*((-2*Sqrt[a + b*x^3])/(3*x^(3/2
)) + (2*Sqrt[b]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x]])/3)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 953
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[d/e^n   Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] &&
(IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && G
tQ[m + n, -1]))
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06

method	result	size
risch	$-\frac{2\sqrt{b\,x^3+a}\,(A b\,x^3+3 B a\,x^3+A a)}{9 x^{\frac{9}{2}} a}+\frac{2 B\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^3+a\right)}}{x^2\sqrt{b}}\right)\sqrt{x(b\,x^3+a)}}{3\sqrt{x}\,\sqrt{b\,x^3+a}}$	84
default	$-\frac{2\sqrt{b\,x^3+a}\left(-3B\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^3+a\right)}}{x^2\sqrt{b}}\right)\sqrt{b}\,a\,x^5+A\sqrt{x(b\,x^3+a)}\,b\,x^3+3B\sqrt{x(b\,x^3+a)}\,a\,x^3+A\sqrt{x(b\,x^3+a)}\,a\right)}{9x^{\frac{9}{2}}\sqrt{x(b\,x^3+a)}\,a}$	108
elliptic	Expression too large to display	1051

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(b*x^3+a)^(1/2)*(A*b*x^3+3*B*a*x^3+A*a)/x^(9/2)/a+2/3*B*b^(1/2)*arcta
nh(1/x^2*(x*(b*x^3+a))^(1/2)/b^(1/2))*(x*(b*x^3+a))^(1/2)/x^(1/2)/(b*x^3+a
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.28

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = \left[\frac{3\,Ba\sqrt{b}x^5\log\left(-8\,b^2x^6-8\,abx^3-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\sqrt{b}\sqrt{x}-a^2\right)-4}{18\,ax^5}\right.$$

$$\left.-\frac{3\,Ba\sqrt{-b}x^5\arctan\left(\frac{2\,\sqrt{bx^3+a}\sqrt{-b}x^{\frac{3}{2}}}{2\,bx^3+a}\right)+2\,((3\,Ba+Ab)x^3+Aa)\sqrt{bx^3+a}\sqrt{x}}{9\,ax^5}\right]$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^(11/2),x, algorithm="fricas")
```

output
```
[1/18*(3*B*a*sqrt(b)*x^5*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*(2*b*x^4 + a*x)*sq
rt(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) - 4*((3*B*a + A*b)*x^3 + A*a)*sqrt(b*
x^3 + a)*sqrt(x))/(a*x^5), -1/9*(3*B*a*sqrt(-b)*x^5*arctan(2*sqrt(b*x^3 +
a)*sqrt(-b)*x^(3/2)/(2*b*x^3 + a)) + 2*((3*B*a + A*b)*x^3 + A*a)*sqrt(b*x^
3 + a)*sqrt(x))/(a*x^5)]
```

Sympy [A] (verification not implemented)

Time = 27.57 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.66

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = -\frac{2A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{9x^3} - \frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{9a} - \frac{2B\sqrt{a}}{3x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{2B\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3} - \frac{2Bbx^{\frac{3}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
integrate((b*x**3+a)**(1/2)*(B*x**3+A)/x**(11/2),x)
```

output
```
-2*A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(9*x**3) - 2*A*b**(3/2)*sqrt(a/(b*x**3)
+ 1)/(9*a) - 2*B*sqrt(a)/(3*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*B*sqrt(b)*asi
nh(sqrt(b)*x**(3/2)/sqrt(a))/3 - 2*B*b*x**(3/2)/(3*sqrt(a)*sqrt(1 + b*x**3
/a))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = -\frac{1}{3}\left(\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right) + \frac{2\sqrt{bx^3+a}}{x^{\frac{3}{2}}}\right)B - \frac{2\,(bx^3+a)^{\frac{3}{2}}A}{9\,ax^{\frac{9}{2}}}$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^(11/2),x, algorithm="maxima")
```

output
```
-1/3*(sqrt(b)*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x
^3 + a)/x^(3/2))) + 2*sqrt(b*x^3 + a)/x^(3/2))*B - 2/9*(b*x^3 + a)^(3/2)*A
/(a*x^(9/2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = -\frac{2\,Bb\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}} + \frac{2\left(3\,Bab\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+3\,Ba\sqrt{-b}\sqrt{b}+A\sqrt{-b}b^{\frac{3}{2}}\right)}{9\,a\sqrt{-b}} - \frac{2\left(3\,Ba^3\sqrt{b+\frac{a}{x^3}}+Aa^2\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}\right)}{9\,a^3}$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^(11/2),x, algorithm="giac")
```

output
```
-2/3*B*b*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b) + 2/9*(3*B*a*b*arctan(s
qrt(b)/sqrt(-b)) + 3*B*a*sqrt(-b)*sqrt(b) + A*sqrt(-b)*b^(3/2))/(a*sqrt(-b
)) - 2/9*(3*B*a^3*sqrt(b + a/x^3) + A*a^2*(b + a/x^3)^(3/2))/a^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = \int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{x^{11/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(11/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(11/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = \frac{-2\sqrt{b\,x^3+a}\,a-8\sqrt{b\,x^3+a}\,b\,x^3-3\sqrt{x}\,\sqrt{b}\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)b\,x^4+}{9\sqrt{x}\,x^4}$$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^(11/2),x)
```

output
```
( - 2*sqrt(a + b*x**3)*a - 8*sqrt(a + b*x**3)*b*x**3 - 3*sqrt(x)*sqrt(b)*l
og(sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*x)*b*x**4 + 3*sqrt(x)*sqrt(b)*log(sq
rt(a + b*x**3) + sqrt(x)*sqrt(b)*x)*b*x**4)/(9*sqrt(x)*x**4)
```

3.248 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 269

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = \frac{2(2Ab-11aB)\sqrt{a+bx^3}}{55ax^{5/2}} - \frac{2A(a+bx^3)^{3/2}}{11ax^{11/2}} - \frac{3^{3/4}b(2Ab-11aB)\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2+}{55a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/55*(2*A*b-11*B*a)*(b*x^3+a)^(1/2)/a/x^(5/2)-2/11*A*(b*x^3+a)^(3/2)/a/x^(
11/2)-1/55*3^(3/4)*b*(2*A*b-11*B*a)*x^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*In
verseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*
b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))/a^(4/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x
)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = \frac{2\sqrt{a+bx^3}\left(-5A(a+bx^3)+\frac{(2Ab-11aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{55ax^{11/2}}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(13/2),x]
```

output
```
(2*Sqrt[a + b*x^3]*(-5*A*(a + b*x^3) + ((2*A*b - 11*a*B)*x^3*Hypergeometri
c2F1[-5/6, -1/2, 1/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(55*a*x^(11/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {955, 809, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(2Ab-11aB)\int \frac{\sqrt{bx^3+a}}{x^{7/2}}dx}{11a}-\frac{2A\left(a+bx^3\right)^{3/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{809}$$

$$-\frac{(2Ab-11aB)\left(\frac{3}{5}b\int \frac{1}{\sqrt{x}\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}}{5x^{5/2}}\right)}{11a}-\frac{2A\left(a+bx^3\right)^{3/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{851}$$

$$-\frac{(2Ab-11aB)\left(\frac{6}{5}b\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}-\frac{2\sqrt{a+bx^3}}{5x^{5/2}}\right)}{11a}-\frac{2A\left(a+bx^3\right)^{3/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{766}$$

$$-\frac{(2Ab-11aB)\left(\frac{3^{3/4}b\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\sqrt{a+bx^3}}{5x^{5/2}}\right)}{11a}$$

$$\frac{2A\left(a+bx^3\right)^{3/2}}{11ax^{11/2}}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(13/2),x]
```

output
```
(-2*A*(a + b*x^3)^(3/2))/(11*a*x^(11/2)) - ((2*A*b - 11*a*B)*((-2*Sqrt[a +
 b*x^3])/(5*x^(5/2)) + (3^(3/4)*b*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3)
 + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(
11*a)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.34 (sec) , antiderivative size = 745, normalized size of antiderivative = 2.77

method	result
risch	$-\frac{2\sqrt{b\,x^3+a}\,(3Ab\,x^3+11Ba\,x^3+5Aa)}{55x^{\frac{11}{2}}\,a}-\frac{6b^2(2Ab-11Ba)\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-}{}\right.}}}{}$
elliptic	$\frac{\sqrt{x(b\,x^3+a)}\left(-\frac{2A\sqrt{b\,x^4+ax}}{11x^6}-\frac{2(3Ab+11Ba)\sqrt{b\,x^4+ax}}{55a\,x^3}+\frac{2\left(Bb-\frac{2b(3Ab+11Ba)}{55a}\right)\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-}{2}\right.}}}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/55*(b*x^3+a)^(1/2)*(3*A*b*x^3+11*B*a*x^3+5*A*a)/x^(11/2)/a-6/55*b^2*(2*
A*b-11*B*a)/a*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*
b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-
1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*(x*(b*x^3+a))^(1/2)/x^(1/2)/(b*
x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx =$$
$$-\frac{2\left(3\left(11\,Bab-2\,Ab^2\right)\sqrt{a}x^6\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)+\left((11\,Ba^2+3\,Aab)x^3+5\,Aa^2\right)\sqrt{bx^3+a}\sqrt{x}\right)}{55\,a^2x^6}$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^(13/2),x, algorithm="fricas")
```

output
```
-2/55*(3*(11*B*a*b - 2*A*b^2)*sqrt(a)*x^6*weierstrassPInverse(0, -4*b/a, 1
/x) + ((11*B*a^2 + 3*A*a*b)*x^3 + 5*A*a^2)*sqrt(b*x^3 + a)*sqrt(x))/(a^2*x
^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 73.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{11}{6},-\frac{1}{2}\\ -\frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{\frac{11}{2}}\Gamma\left(-\frac{5}{6}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},-\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)*(B*x**3+A)/x**(13/2),x)
```

output
```
A*sqrt(a)*gamma(-11/6)*hyper((-11/6, -1/2), (-5/6,), b*x**3*exp_polar(I*pi
)/a)/(3*x**(11/2)*gamma(-5/6)) + B*sqrt(a)*gamma(-5/6)*hyper((-5/6, -1/2),
 (1/6,), b*x**3*exp_polar(I*pi)/a)/(3*x**(5/2)*gamma(1/6))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^(13/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(13/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(B*x^3+A)/x^(13/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^{13/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(13/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(13/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = \frac{2\sqrt{b\,x^3+a}\,a-16\sqrt{b\,x^3+a}\,b\,x^3+27\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^{10}+a\,x^7}dx\right)a^2x^5}{16\sqrt{x}\,x^5}$$

input
```
int((b*x^3+a)^(1/2)*(B*x^3+A)/x^(13/2),x)
```

output
```
(2*sqrt(a + b*x**3)*a - 16*sqrt(a + b*x**3)*b*x**3 + 27*sqrt(x)*int((sqrt(
x)*sqrt(a + b*x**3))/(a*x**7 + b*x**10),x)*a**2*x**5)/(16*sqrt(x)*x**5)
```

3.249 $\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 201

$$\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{72be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be} - \frac{a^3(8Ab-3aB)e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{5/2}}$$

output
```
1/192*a^2*(8*A*b-3*B*a)*e^2*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b^2+1/96*a*(8*A*b-
3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(1/2)/b/e+1/72*(8*A*b-3*B*a)*(e*x)^(9/2)*(b*x
^3+a)^(3/2)/b/e+1/12*B*(e*x)^(9/2)*(b*x^3+a)^(5/2)/b/e-1/192*a^3*(8*A*b-3*
B*a)*e^(7/2)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.72

$$\int (ex)^{7/2} (a+bx^3)^{3/2} (A+Bx^3)\, dx = \frac{e^3\sqrt{ex}\left(\sqrt{b}x^{3/2}\sqrt{a+bx^3}(-9a^3B+6a^2b(4A+Bx^3)+16b^3x^6(4A+3Bx^3)+8ab\right.}{576b^{5/2}\sqrt{x}}$$

input
```
Integrate[(e*x)^(7/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*Sqrt[a + b*x^3]*(-9*a^3*B + 6*a^2*b*(4*A +
 B*x^3) + 16*b^3*x^6*(4*A + 3*B*x^3) + 8*a*b^2*x^3*(14*A + 9*B*x^3)) + 3*a
^3*(-8*A*b + 3*a*B)*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(576*b^(5/2)*
Sqrt[x])
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {959, 811, 811, 843, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{7/2} (a+bx^3)^{3/2} (A+Bx^3)\, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(8Ab-3aB)\int (ex)^{7/2} (bx^3+a)^{3/2}dx}{8b}+\frac{B(ex)^{9/2} (a+bx^3)^{5/2}}{12be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\int (ex)^{7/2}\sqrt{bx^3+a}dx+\frac{(ex)^{9/2}(a+bx^3)^{3/2}}{9e}\right)}{8b}+\frac{B(ex)^{9/2} (a+bx^3)^{5/2}}{12be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\left(\frac{1}{4}a\int\frac{(ex)^{7/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)}{8b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^3\int\frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx}{2b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)}{8b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int\frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)}{8b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int\frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)}{8b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int\frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)}{8b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3b^{3/2}}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}(a+bx^3)^{3/2}}{9e}\right)}{8b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be}$$

input
```
Int[(e*x)^(7/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(B*(e*x)^(9/2)*(a + b*x^3)^(5/2))/(12*b*e) + ((8*A*b - 3*a*B)*(((e*x)^(9/2
)*(a + b*x^3)^(3/2))/(9*e) + (a*(((e*x)^(9/2)*Sqrt[a + b*x^3])/(6*e) + (a*
((e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*b) - (a*e^(7/2)*ArcTanh[(Sqrt[b]*(e*
x)^(3/2))/(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*b^(3/2))))/4))/2))/(8*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.81

method	result
risch	$\frac{x^{2} \left(48 b^{3} B \,x^{9}+64 A \,b^{3} x^{6}+72 B a \,b^{2} x^{6}+112 a A \,b^{2} x^{3}+6 B \,a^{2} b \,x^{3}+24 a^{2} b A -9 a^{3} B \right) \sqrt{b \,x^{3}+a}\, e^{4}}{576 b^{2} \sqrt{e x}}-\frac{a^{3} \left(8 A b -3 B a \right) \operatorname{arctanh}\left(\frac{\sqrt{\left(b \,x^{3}+a \right)}}{x^{2} \sqrt{b e}}\right)}{192 b^{2} \sqrt{b e}\, \sqrt{e x}\, \sqrt{b}}$
default	$-\frac{e^{3} \sqrt{e x}\, \sqrt{b \,x^{3}+a}\, \left(-48 B \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, b^{3} x^{10}-64 A \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, b^{3} x^{7}-72 B \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, a \,b^{2} x^{7}-112 A \sqrt{\left(b \,x^{3}+a \right) e}\right)}{}$
elliptic	Expression too large to display

input
```
int((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/576/b^2*x^2*(48*B*b^3*x^9+64*A*b^3*x^6+72*B*a*b^2*x^6+112*A*a*b^2*x^3+6*
B*a^2*b*x^3+24*A*a^2*b-9*B*a^3)*(b*x^3+a)^(1/2)*e^4/(e*x)^(1/2)-1/192*a^3/
b^2*(8*A*b-3*B*a)/(b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2
))*e^4*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.77

$$\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \left[-\frac{3\,(3\,Ba^4-8\,Aa^3b)e^3\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e+4\,(2\,b^2x^4+abx)\sqrt{bx}\right.}{}\right.$$

input
```
integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
[-1/2304*(3*(3*B*a^4 - 8*A*a^3*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e
*x^3 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b))
- 4*(48*B*b^3*e^3*x^10 + 8*(9*B*a*b^2 + 8*A*b^3)*e^3*x^7 + 2*(3*B*a^2*b +
56*A*a*b^2)*e^3*x^4 - 3*(3*B*a^3 - 8*A*a^2*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(
e*x))/b^2, -1/1152*(3*(3*B*a^4 - 8*A*a^3*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b
*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(48*B*b^3*e^3*x^
10 + 8*(9*B*a*b^2 + 8*A*b^3)*e^3*x^7 + 2*(3*B*a^2*b + 56*A*a*b^2)*e^3*x^4
- 3*(3*B*a^3 - 8*A*a^2*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. 2(180) = 360.

Time = 33.11 (sec) , antiderivative size = 634, normalized size of antiderivative = 3.15

$$\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**(7/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
Piecewise((2*Piecewise((nan, Eq(e**3, 0)), ((A*a*e**3*Piecewise((-a**2*e**
3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/
sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), Tru
e))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**(3/2)/(8*b) + (e*x)**(9/2)/4),
 Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, True)) + A*b*Piecewise((a**3*e**
6*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/
sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), Tru
e))/(16*b**2) + sqrt(a + b*x**3)*(-a**2*e**6*(e*x)**(3/2)/(16*b**2) + a*e*
*3*(e*x)**(9/2)/(24*b) + (e*x)**(15/2)/6), Ne(b/e**3, 0)), (sqrt(a)*(e*x)*
*(15/2)/5, True)) + B*a*Piecewise((a**3*e**6*Piecewise((log(2*b*(e*x)**(3/
2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)
**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/(16*b**2) + sqrt(a + b*x**3
)*(-a**2*e**6*(e*x)**(3/2)/(16*b**2) + a*e**3*(e*x)**(9/2)/(24*b) + (e*x)*
*(15/2)/6), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(15/2)/5, True)) + B*b*Piecewi
se((-5*a**4*e**9*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqr
t(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sq
rt(b*x**3), True))/(128*b**3) + sqrt(a + b*x**3)*(5*a**3*e**9*(e*x)**(3/2)
/(128*b**3) - 5*a**2*e**6*(e*x)**(9/2)/(192*b**2) + a*e**3*(e*x)**(15/2)/(
48*b) + (e*x)**(21/2)/8), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(21/2)/7, True))
/e**3)/(3*e**3), True))/e, Ne(e, 0)), (0, True))
```

Maxima [F]

$$\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right) \left(bx^3+a\right)^{\frac{3}{2}} (ex)^{\frac{7}{2}} \, dx$$

input
```
integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(163) = 326$.

Time = 0.28 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.46

$$\int (ex)^{7/2} (a + bx^3)^{3/2} (A + Bx^3) \, dx = \frac{1}{72} \sqrt{be^4x^3 + ae^4} \left(2 e^3x^3 \left(\frac{4 x^3}{e^4} + \frac{a}{be^4}\right) - \frac{3 a^2}{b^2e}\right) \sqrt{ex} Bax|e|^2 + \frac{1}{72} \sqrt{be^4x^3 + ae^4} \left(2 e^3x^3 \left(\frac{4 x^3}{e^4} + \frac{a}{be^4}\right) - \frac{3 a^2}{b^2e}\right) \sqrt{ex} Abx|e|^2 + \frac{\sqrt{be^4x^3 + ae^4} \left(2 e^3x^3 + \frac{ae^3}{b}\right) \sqrt{ex} Aax|e|^2}{12 e^4} - \frac{(9 B^2a^8e - 48 ABa^7be + 64 A^2a^6b^2e)^2 e^5 \log \left(\left|-(3 \sqrt{ex} Ba^4e^2x - 8 \sqrt{ex} Aa^3be^2x) \sqrt{be} + \sqrt{9 B^2a^9e^6 - 48 \cdots}\right.\right.}{192 \sqrt{be} b^2 |9 B^2a^8e - 48 ABa^7be + 64 A^2a^6b^2e| |-3 B \cdots} + \frac{\left(\frac{15 a^3e^9}{b^3} + 2 \left(4 \left(6 e^3x^3 + \frac{ae^3}{b}\right) e^3x^3 - \frac{5 a^2e^6}{b^2}\right) e^3x^3\right) \sqrt{be^4x^3 + ae^4} \sqrt{ex} Bbx|e|^2}{576 e^{10}}$$

input
```
integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/72*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3*(4*x^3/e^4 + a/(b*e^4)) - 3*a^2/(b
^2*e))*sqrt(e*x)*B*a*x*abs(e)^2 + 1/72*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3*
(4*x^3/e^4 + a/(b*e^4)) - 3*a^2/(b^2*e))*sqrt(e*x)*A*b*x*abs(e)^2 + 1/12*s
qrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3 + a*e^3/b)*sqrt(e*x)*A*a*x*abs(e)^2/e^4
- 1/192*(9*B^2*a^8*e - 48*A*B*a^7*b*e + 64*A^2*a^6*b^2*e)^2*e^5*log(abs(-(
3*sqrt(e*x)*B*a^4*e^2*x - 8*sqrt(e*x)*A*a^3*b*e^2*x)*sqrt(b*e) + sqrt(9*B^
2*a^9*e^6 - 48*A*B*a^8*b*e^6 + 64*A^2*a^7*b^2*e^6 + (3*sqrt(e*x)*B*a^4*e^2
*x - 8*sqrt(e*x)*A*a^3*b*e^2*x)^2*b*e)))/(sqrt(b*e)*b^2*abs(9*B^2*a^8*e -
48*A*B*a^7*b*e + 64*A^2*a^6*b^2*e)*abs(-3*B*a^4 + 8*A*a^3*b)*abs(e)^2) + 1
/576*(15*a^3*e^9/b^3 + 2*(4*(6*e^3*x^3 + a*e^3/b)*e^3*x^3 - 5*a^2*e^6/b^2)
*e^3*x^3)*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*B*b*x*abs(e)^2/e^10
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,\left(e\,x\right)^{7/2}\,{\left(b\,x^3+a\right)}^{3/2} \,dx$$

input
```
int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(3/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.66

$$\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{\sqrt{e}\,e^3 \left(30\sqrt{x}\,\sqrt{b\,x^3+a}\,a^3bx+236\sqrt{x}\,\sqrt{b\,x^3+a}\,a^2b^2x^4+272\sqrt{x}\,\sqrt{b\,x^3+a}\,a\,b^3x^7\right.}{}$$

input
```
int((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

output
```
(sqrt(e)*e**3*(30*sqrt(x)*sqrt(a + b*x**3)*a**3*b*x + 236*sqrt(x)*sqrt(a +
 b*x**3)*a**2*b**2*x**4 + 272*sqrt(x)*sqrt(a + b*x**3)*a*b**3*x**7 + 96*sq
rt(x)*sqrt(a + b*x**3)*b**4*x**10 + 15*sqrt(b)*log(sqrt(a + b*x**3) - sqrt
(x)*sqrt(b)*x)*a**4 - 15*sqrt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)*x)
*a**4))/(1152*b**2)
```

3.250 $\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 364

$$\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{27a^2(22Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{7040b^2} + \frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{1760be} + \frac{(22Ab-7aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{176be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be} - \frac{9\ 3^{3/4}a^{8/3}(22Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)\right.}{14080b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
27/7040*a^2*(22*A*b-7*B*a)*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^2+9/1760*a*(2
2*A*b-7*B*a)*(e*x)^(7/2)*(b*x^3+a)^(1/2)/b/e+1/176*(22*A*b-7*B*a)*(e*x)^(7
/2)*(b*x^3+a)^(3/2)/b/e+1/11*B*(e*x)^(7/2)*(b*x^3+a)^(5/2)/b/e-9/14080*3^(
3/4)*a^(8/3)*(22*A*b-7*B*a)*e^2*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*In
verseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*
b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))/b^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a
^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.32

$$\int (ex)^{5/2} (a+bx^3)^{3/2} (A+Bx^3) \, dx = \frac{e^2\sqrt{ex}\sqrt{a+bx^3}\left(-(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}(-22Ab+7aB-16bBx^3)+a^2(-22Ab\right.}{176b^2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(e*x)^(5/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(e^2*Sqrt[e*x]*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(-22*A
*b + 7*a*B - 16*b*B*x^3)) + a^2*(-22*A*b + 7*a*B)*Hypergeometric2F1[-3/2,
1/6, 7/6, -((b*x^3)/a)]))/(176*b^2*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {959, 811, 811, 843, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(22Ab-7aB)\int (ex)^{5/2}\left(bx^3+a\right)^{3/2}dx}{22b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(22Ab-7aB)\left(\frac{9}{16}a\int (ex)^{5/2}\sqrt{bx^3+a}dx+\frac{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8e}\right)}{22b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be}$$

$$\Big\downarrow \ 811$$

$$\frac{(22Ab-7aB)\left(\frac{9}{16}a\left(\frac{3}{10}a\int\frac{(ex)^{5/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{7/2}\sqrt{a+bx^3}}{5e}\right)+\frac{(ex)^{7/2}(a+bx^3)^{3/2}}{8e}\right)}{22b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be}$$

$$\Big\downarrow \ 843$$

$$\frac{(22Ab-7aB)\left(\frac{9}{16}a\left(\frac{3}{10}a\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{ae^3\int\frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{4b}\right)+\frac{(ex)^{7/2}\sqrt{a+bx^3}}{5e}\right)+\frac{(ex)^{7/2}(a+bx^3)^{3/2}}{8e}\right)}{22b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be}$$

$$\Big\downarrow \ 851$$

$$\frac{(22Ab-7aB)\left(\frac{9}{16}a\left(\frac{3}{10}a\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{ae^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b}\right)+\frac{(ex)^{7/2}\sqrt{a+bx^3}}{5e}\right)+\frac{(ex)^{7/2}(a+bx^3)^{3/2}}{8e}\right)}{22b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be}$$

$$\Big\downarrow \ 766$$

$$\frac{(22Ab-7aB)\left(\frac{9}{16}a\left(\frac{3}{10}a\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{a^{2/3}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{4\sqrt[4]{3}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right.\right.\right.}{22b}$$

$$\frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be}$$

input
```
Int[(e*x)^(5/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(B*(e*x)^(7/2)*(a + b*x^3)^(5/2))/(11*b*e) + ((22*A*b - 7*a*B)*(((e*x)^(7/
2)*(a + b*x^3)^(3/2))/(8*e) + (9*a*(((e*x)^(7/2)*Sqrt[a + b*x^3])/(5*e) +
(3*a*((e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(2*b) - (a^(2/3)*e*Sqrt[e*x]*(a^(1/3
)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2
*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)
*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)],
(2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x)
)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/10))/16))/
(22*b)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.64 (sec) , antiderivative size = 801, normalized size of antiderivative = 2.20

method	result	size
risch	Expression too large to display	801
elliptic	Expression too large to display	976
default	Expression too large to display	4619

input
```
int((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/7040/b^2*(640*B*b^3*x^9+880*A*b^3*x^6+1000*B*a*b^2*x^6+1672*A*a*b^2*x^3+
108*B*a^2*b*x^3+594*A*a^2*b-189*B*a^3)*x*(b*x^3+a)^(1/2)*e^3/(e*x)^(1/2)-2
7/7040*a^3/b*(22*A*b-7*B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/
2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(
1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*e^3*((b*x^3+a)*
e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int (ex)^{5/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*b*e^2*x^8 + (B*a + A*b)*e^2*x^5 + A*a*e^2*x^2)*sqrt(b*x^3 + a)
*sqrt(e*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 70.83 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.55

$$\int (ex)^{5/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{Aa^{\frac{3}{2}}e^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{6}\right)}+\frac{A\sqrt{a}be^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)}+\frac{Ba^{\frac{3}{2}}e^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)}+\frac{B\sqrt{a}be^{\frac{5}{2}}x^{\frac{19}{2}}\Gamma\left(\frac{19}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{19}{6}\\ \frac{25}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{25}{6}\right)}$$

input
```
integrate((e*x)**(5/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
A*a**(3/2)*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3
*exp_polar(I*pi)/a)/(3*gamma(13/6)) + A*sqrt(a)*b*e**(5/2)*x**(13/2)*gamma
(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(19/
6)) + B*a**(3/2)*e**(5/2)*x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(19/6)) + B*sqrt(a)*b*e**(5/2)*x**(19
/2)*gamma(19/6)*hyper((-1/2, 19/6), (25/6,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(25/6))
```

Maxima [F]

$$\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right) \left(bx^3+a\right)^{\frac{3}{2}} (ex)^{\frac{5}{2}} \, dx$$

input
```
integrate((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(5/2), x)
```

Giac [F]

$$\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right) \left(bx^3+a\right)^{\frac{3}{2}} (ex)^{\frac{5}{2}} \, dx$$

input
```
integrate((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}\,{\left(b\,x^3+a\right)}^{3/2} \,dx$$

input
```
int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(3/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{\sqrt{e}\,e^2 \left(162 \sqrt{x}\,\sqrt{b\,x^3+a}\,a^3+712 \sqrt{x}\,\sqrt{b\,x^3+a}\,a^2 b\,x^3+752 \sqrt{x}\,\sqrt{b\,x^3+a}\,a\,b^2 x^6 \right.}{2816 b}$$

input
```
int((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

output
```
(sqrt(e)*e**2*(162*sqrt(x)*sqrt(a + b*x**3)*a**3 + 712*sqrt(x)*sqrt(a + b*
x**3)*a**2*b*x**3 + 752*sqrt(x)*sqrt(a + b*x**3)*a*b**2*x**6 + 256*sqrt(x)
*sqrt(a + b*x**3)*b**3*x**9 - 81*int((sqrt(x)*sqrt(a + b*x**3))/(a*x + b*x
**4),x)*a**4))/(2816*b)
```

3.251 $\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 621

$$\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be}$$

$$+ \frac{27\left(1+\sqrt{3}\right) a^2(4Ab-aB)e\sqrt{ex}\sqrt{a+bx^3}}{448b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$+ \frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be}$$

$$- \frac{27\sqrt[4]{3}a^{7/3}(4Ab-aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{3}}{448b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{9\ 3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}(4Ab-aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}}{\sqrt[3]{a}+\left(1+\sqrt{3}}\right.\right.}{896b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
9/224*a*(4*A*b-B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/b/e+27/448*(1+3^(1/2))*a^2
*(4*A*b-B*a)*e*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+(1+3^(1/2))*b^
(1/3)*x)+1/28*(4*A*b-B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/b/e+1/10*B*(e*x)^(5/
2)*(b*x^3+a)^(5/2)/b/e-27/448*3^(1/4)*a^(7/3)*(4*A*b-B*a)*e*(e*x)^(1/2)*(a
^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(
1/2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(
a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/b^(5/3)/(
b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*
x^3+a)^(1/2)-9/896*3^(3/4)*(1-3^(1/2))*a^(7/3)*(4*A*b-B*a)*e*(e*x)^(1/2)*(
a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^
(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(
1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))/b^(5/3)/
(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b
*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.15

$$\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{x(ex)^{3/2}\sqrt{a+bx^3}\left(B\left(a+bx^3\right)^2\sqrt{1+\frac{bx^3}{a}}+a(4Ab-aB)\,\text{Hypergeometric2F1}\left(-\right.\right.}{10b\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(e*x)^(3/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(x*(e*x)^(3/2)*Sqrt[a + b*x^3]*(B*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a] + a*(4
*A*b - a*B)*Hypergeometric2F1[-3/2, 5/6, 11/6, -((b*x^3)/a)]))/(10*b*Sqrt[
1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {959, 811, 811, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(4Ab-aB)\int (ex)^{3/2} \left(bx^3+a\right)^{3/2}dx}{4b}+\frac{B(ex)^{5/2} \left(a+bx^3\right)^{5/2}}{10be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(4Ab-aB)\left(\frac{9}{14}a\int (ex)^{3/2}\sqrt{bx^3+a}dx+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7e}\right)}{4b}+\frac{B(ex)^{5/2} \left(a+bx^3\right)^{5/2}}{10be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(4Ab-aB)\left(\frac{9}{14}a\left(\frac{3}{8}a\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7e}\right)}{4b}+\frac{B(ex)^{5/2} \left(a+bx^3\right)^{5/2}}{10be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(4Ab-aB)\left(\frac{9}{14}a\left(\frac{3a\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7e}\right)}{4b}+\frac{B(ex)^{5/2} \left(a+bx^3\right)^{5/2}}{10be}$$

$$\Big\downarrow \text{837}$$

$$\frac{(4Ab-aB)\left(\frac{9}{14}a\left(\frac{3a\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int-\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)}{7e}\right.}{4b}$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be}$$

$$\Big\downarrow \; 25$$

$$\frac{(4Ab-aB)\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7e}\right.}{4b}$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be}$$

$$\Big\downarrow \; 766$$

$$\frac{(4Ab-aB)\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\operatorname{EllipticF}\left(\operatorname{arc}\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right.}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right.}}}\right.}{4e}\right.\right.}{4b}$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be}$$

$$\Big\downarrow \; 2420$$

$$(4Ab-aB)\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+}\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right)}{2b^{2/3}}\right.\right.$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be}$$

input
```
Int[(e*x)^(3/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(B*(e*x)^(5/2)*(a + b*x^3)^(5/2))/(10*b*e) + ((4*A*b - a*B)*(((e*x)^(5/2)*
(a + b*x^3)^(3/2))/(7*e) + (9*a*(((e*x)^(5/2)*Sqrt[a + b*x^3])/(4*e) + (3*
a*((((1 + Sqrt[3])*e^3*Sqrt[e*x]*Sqrt[a + b*x^3])/(a^(1/3)*e + (1 + Sqrt[3
])*b^(1/3)*e*x) - (3^(1/4)*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*S
qrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (
1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticE[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b
^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(S
qrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(
1/3)*e*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*e*Sqrt
[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x
+ b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[Ar
cCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^
(1/3)*e*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*e*x*(a^(1/
3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b
*x^3])))/(4*e)))/14))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.41 (sec) , antiderivative size = 1164, normalized size of antiderivative = 1.87

method	result	size
risch	Expression too large to display	1164
elliptic	Expression too large to display	1279
default	Expression too large to display	5790

input
```
int((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/1120/b*x^3*(112*B*b^2*x^6+160*A*b^2*x^3+184*B*a*b*x^3+340*A*a*b+27*B*a^2
)*(b*x^3+a)^(1/2)*e^2/(e*x)^(1/2)+27/448*a^2/b*(4*A*b-B*a)*(x*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)
*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*El
lipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/
2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^...
```

Fricas [F]

$$\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right) \left(bx^3+a\right)^{\frac{3}{2}} (ex)^{\frac{3}{2}} \, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*b*e*x^7 + (B*a + A*b)*e*x^4 + A*a*e*x)*sqrt(b*x^3 + a)*sqrt(e*
x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 23.51 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.32

$$\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\,dx = \frac{Aa^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{6}\right)} + \frac{A\sqrt{a}be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)} + \frac{Ba^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)} + \frac{B\sqrt{a}be^{\frac{3}{2}}x^{\frac{17}{2}}\Gamma\left(\frac{17}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{17}{6}\\ \frac{23}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{23}{6}\right)}$$

input
```
integrate((e*x)**(3/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
A*a**(3/2)*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3
*exp_polar(I*pi)/a)/(3*gamma(11/6)) + A*sqrt(a)*b*e**(3/2)*x**(11/2)*gamma
(11/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(17/
6)) + B*a**(3/2)*e**(3/2)*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(17/6)) + B*sqrt(a)*b*e**(3/2)*x**(17
/2)*gamma(17/6)*hyper((-1/2, 17/6), (23/6,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(23/6))
```

Maxima [F]

$$\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(3/2), x)
```

Giac [F]

$$\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right) \left(bx^3+a\right)^{\frac{3}{2}} (ex)^{\frac{3}{2}} \, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}\,{\left(b\,x^3+a\right)}^{3/2} \,d x$$

input
```
int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(3/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int (ex)^{3/2} (a +bx^3)^{3/2} \left(A+Bx^3\right) \, dx = \frac{\sqrt{e}\,e\Big(734\sqrt{x}\,\sqrt{b\,x^3+a}\,a^2x^2+688\sqrt{x}\,\sqrt{b\,x^3+a}\,ab\,x^5+224\sqrt{x}\,\sqrt{b\,x^3+a}\,b^2x^8+}{2240}$$

input
```
int((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

output
```
(sqrt(e)*e*(734*sqrt(x)*sqrt(a + b*x**3)*a**2*x**2 + 688*sqrt(x)*sqrt(a +
b*x**3)*a*b*x**5 + 224*sqrt(x)*sqrt(a + b*x**3)*b**2*x**8 + 405*int((sqrt(
x)*sqrt(a + b*x**3)*x)/(a + b*x**3),x)*a**3))/2240
```

3.252 $\int \sqrt{ex}(a+bx^3)^{3/2}(A+Bx^3)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 161

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be}+\frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}+\frac{a^2(6Ab-aB)\sqrt{e}\,\mathrm{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{3/2}}$$

output
```
1/24*a*(6*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b/e+1/36*(6*A*b-B*a)*(e*x)^
(3/2)*(b*x^3+a)^(3/2)/b/e+1/9*B*(e*x)^(3/2)*(b*x^3+a)^(5/2)/b/e+1/24*a^2*(
6*A*b-B*a)*e^(1/2)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a)^(1/2))/b^
(3/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.75

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{x\sqrt{ex}\sqrt{a+bx^3}(30aAb+3a^2B+12Ab^2x^3+14abBx^3+8b^2Bx^6)}{72b}-\frac{a^2(-6Ab+aB)\sqrt{ex}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{24b^{3/2}\sqrt{x}}$$

input
```
Integrate[Sqrt[e*x]*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(x*Sqrt[e*x]*Sqrt[a + b*x^3]*(30*a*A*b + 3*a^2*B + 12*A*b^2*x^3 + 14*a*b*B
*x^3 + 8*b^2*B*x^6))/(72*b) - (a^2*(-6*A*b + a*B)*Sqrt[e*x]*Log[Sqrt[b]*x^
(3/2) + Sqrt[a + b*x^3]])/(24*b^(3/2)*Sqrt[x])
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.269$, Rules used = {959, 811, 811, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(6Ab-aB)\int \sqrt{ex}\left(bx^3+a\right)^{3/2}dx}{6b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\int \sqrt{ex}\sqrt{bx^3+a}dx+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)}{6b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)}{6b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(\frac{a\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)}{6b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

$$\Big\downarrow \text{807}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(\frac{a\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)}{6b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

$$\Big\downarrow \text{224}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(\frac{a\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)}{6b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

$$\Big\downarrow \text{219}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(\frac{a\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3\sqrt{b}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)}{6b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

input

```
Int[Sqrt[e*x]*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(B*(e*x)^(3/2)*(a + b*x^3)^(5/2))/(9*b*e) + ((6*A*b - a*B)*(((e*x)^(3/2)*(
a + b*x^3)^(3/2))/(6*e) + (3*a*(((e*x)^(3/2)*Sqrt[a + b*x^3])/(3*e) + (a*S
qrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*Sq
rt[b]))/4))/(6*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.83

method	result
risch	$\frac{x^{2} \left(8 b^{2} B \,x^{6}+12 A \,b^{2} x^{3}+14 B a b \,x^{3}+30 a b A +3 a^{2} B \right) \sqrt{b \,x^{3}+a}\, e}{72 b \sqrt{e x}}+\frac{a^{2} \left(6 A b -B a \right) \operatorname{arctanh}\left(\frac{\sqrt{\left(b \,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) e \sqrt{\left(b \,x^{3}+a \right) e x}}{24 b \sqrt{b e}\, \sqrt{e x}\, \sqrt{b \,x^{3}+a}}$
default	$\frac{\sqrt{e x}\, \sqrt{b \,x^{3}+a}\, \left(8 B \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, b^{2} x^{7}+12 A \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, b^{2} x^{4}+14 B \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, a b \,x^{4}+30 A \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, a b x +\right.}{72 \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, b}$
elliptic	Expression too large to display

input
```
int((e*x)^(1/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/72/b*x^2*(8*B*b^2*x^6+12*A*b^2*x^3+14*B*a*b*x^3+30*A*a*b+3*B*a^2)*(b*x^3
+a)^(1/2)*e/(e*x)^(1/2)+1/24*a^2/b*(6*A*b-B*a)/(b*e)^(1/2)*arctanh(((b*x^3
+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*e*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3
+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.70

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \left[-\frac{3\left(Ba^3-6\,Aa^2b\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)-}{288\,b}\right.$$

input
```
integrate((e*x)^(1/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
[-1/288*(3*(B*a^3 - 6*A*a^2*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 -
a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(8*
B*b^2*x^7 + 2*(7*B*a*b + 6*A*b^2)*x^4 + 3*(B*a^2 + 10*A*a*b)*x)*sqrt(b*x^3
 + a)*sqrt(e*x))/b, 1/144*(3*(B*a^3 - 6*A*a^2*b)*sqrt(-e/b)*arctan(2*sqrt(
b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*(8*B*b^2*x^7 +
2*(7*B*a*b + 6*A*b^2)*x^4 + 3*(B*a^2 + 10*A*a*b)*x)*sqrt(b*x^3 + a)*sqrt(e
*x))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(138) = 276$.

Time = 3.81 (sec) , antiderivative size = 546, normalized size of antiderivative = 3.39

$$\int \sqrt{ex}(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x)**(1/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
Piecewise((2*Piecewise((nan, Eq(e**3, 0)), ((A*a*e**3*Piecewise((a*Piecewi
se((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e*
*3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/2 + (
e*x)**(3/2)*sqrt(a + b*x**3)/2, Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(3/2), Tru
e)) + A*b*Piecewise((-a**2*e**3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*s
qrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((
e*x)**(3/2))/sqrt(b*x**3), True))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**
(3/2)/(8*b) + (e*x)**(9/2)/4), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, Tr
ue)) + B*a*Piecewise((-a**2*e**3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*
sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log(
(e*x)**(3/2))/sqrt(b*x**3), True))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)*
*(3/2)/(8*b) + (e*x)**(9/2)/4), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, T
rue)) + B*b*Piecewise((a**3*e**6*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*
sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log(
(e*x)**(3/2))/sqrt(b*x**3), True))/(16*b**2) + sqrt(a + b*x**3)*(-a**2*e**
6*(e*x)**(3/2)/(16*b**2) + a*e**3*(e*x)**(9/2)/(24*b) + (e*x)**(15/2)/6),
Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(15/2)/5, True))/e**3)/(3*e**3), True))/e,
 Ne(e, 0)), (0, True))
```

Maxima [F]

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}\sqrt{ex}\,dx$$

input
```
integrate((e*x)^(1/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*sqrt(e*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(126) = 252$.

Time = 0.25 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.58

$$\int \sqrt{ex}(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{\sqrt{be^4x^3+ae^4}\left(2e^3x^3\left(\frac{4x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3a^2}{b^2e}\right)\sqrt{ex}Bbx|e|^2}{72e^3} - \frac{(B^2a^6+4ABa^5b+4A^2a^4b^2)e^4\log\left(\left|(\sqrt{ex}Ba^3x+2\sqrt{ex}Aa^2bx)\sqrt{be}+\sqrt{B^2a^7e^2+4ABa^6be^2+4A^2a\cdots}\right|\right)}{24\sqrt{be}b|Ba^3e+2Aa^2be||e|^2} - \frac{\left(\frac{ae^4\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{\sqrt{be}}-\sqrt{be^4x^3+ae^4}\sqrt{ex}ex\right)Aa|e|^2}{3e^5} + \frac{\sqrt{be^4x^3+ae^4}\left(2e^3x^3+\frac{ae^3}{b}\right)\sqrt{ex}Bax|e|^2}{12e^7} + \frac{\sqrt{be^4x^3+ae^4}\left(2e^3x^3+\frac{ae^3}{b}\right)\sqrt{ex}Abx|e|^2}{12e^7}$$

input
```
integrate((e*x)^(1/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/72*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3*(4*x^3/e^4 + a/(b*e^4)) - 3*a^2/(b
^2*e))*sqrt(e*x)*B*b*x*abs(e)^2/e^3 - 1/24*(B^2*a^6 + 4*A*B*a^5*b + 4*A^2*
a^4*b^2)*e^4*log(abs((sqrt(e*x)*B*a^3*x + 2*sqrt(e*x)*A*a^2*b*x)*sqrt(b*e)
 + sqrt(B^2*a^7*e^2 + 4*A*B*a^6*b*e^2 + 4*A^2*a^5*b^2*e^2 + (sqrt(e*x)*B*a
^3*x + 2*sqrt(e*x)*A*a^2*b*x)^2*b*e)))/(sqrt(b*e)*b*abs(B*a^3*e + 2*A*a^2*
b*e)*abs(e)^2) - 1/3*(a*e^4*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*
x^3 + a*e^4)))/sqrt(b*e) - sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*e*x)*A*a*abs(
e)^2/e^5 + 1/12*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3 + a*e^3/b)*sqrt(e*x)*B*
a*x*abs(e)^2/e^7 + 1/12*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3 + a*e^3/b)*sqrt
(e*x)*A*b*x*abs(e)^2/e^7
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,\sqrt{e\,x}\,\left(b\,x^3+a\right)^{3/2}\,dx$$

input
```
int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(3/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.68

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{\sqrt{e}\,\left(66\sqrt{x}\,\sqrt{b\,x^3+a}\,a^2bx+52\sqrt{x}\,\sqrt{b\,x^3+a}\,a\,b^2x^4+16\sqrt{x}\,\sqrt{b\,x^3+a}\,b^3x^7-15\sqrt{b}\log\left(\sqrt{b\,x^3}\right.\right.}{144b}$$

input
```
int((e*x)^(1/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

output
```
(sqrt(e)*(66*sqrt(x)*sqrt(a + b*x**3)*a**2*b*x + 52*sqrt(x)*sqrt(a + b*x**
3)*a*b**2*x**4 + 16*sqrt(x)*sqrt(a + b*x**3)*b**3*x**7 - 15*sqrt(b)*log(sq
rt(a + b*x**3) - sqrt(x)*sqrt(b)*x)*a**3 + 15*sqrt(b)*log(sqrt(a + b*x**3)
 + sqrt(x)*sqrt(b)*x)*a**3))/(144*b)
```

3.253 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 324

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{9a(16Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{320be} + \frac{(16Ab-aB)\sqrt{ex}(a+bx^3)^{3/2}}{80be} + \frac{B\sqrt{ex}(a+bx^3)^{5/2}}{8be} + \frac{9\,3^{3/4}a^{5/3}(16Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\right.}{640be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
9/320*a*(16*A*b-B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b/e+1/80*(16*A*b-B*a)*(e*
x)^(1/2)*(b*x^3+a)^(3/2)/b/e+1/8*B*(e*x)^(1/2)*(b*x^3+a)^(5/2)/b/e+9/640*3
^(3/4)*a^(5/3)*(16*A*b-B*a)*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*Invers
eJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1
/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))/b/e/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/
3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{x\sqrt{a+bx^3}\left(B(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}+a(16Ab-aB)\,\text{Hypergeometric2F1}\left(\right.\right.}{8b\sqrt{ex}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/Sqrt[e*x],x]
```

output
```
(x*Sqrt[a + b*x^3]*(B*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a] + a*(16*A*b - a*B)
*Hypergeometric2F1[-3/2, 1/6, 7/6, -((b*x^3)/a)]))/(8*b*Sqrt[e*x]*Sqrt[1 +
 (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {959, 811, 811, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(16Ab-aB)\int \frac{(bx^3+a)^{3/2}}{\sqrt{ex}}dx}{16b}+\frac{B\sqrt{ex}(a+bx^3)^{5/2}}{8be}$$

$$\Big\downarrow \; 811$$

$$\frac{(16Ab-aB)\left(\frac{9}{10}a\int \frac{\sqrt{bx^3+a}}{\sqrt{ex}}dx+\frac{\sqrt{ex}(a+bx^3)^{3/2}}{5e}\right)}{16b}+\frac{B\sqrt{ex}(a+bx^3)^{5/2}}{8be}$$

$$\Big\downarrow \; 811$$

$$\frac{(16Ab-aB)\left(\frac{9}{10}a\left(\frac{3}{4}a\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)+\frac{\sqrt{ex}(a+bx^3)^{3/2}}{5e}\right)}{16b}+\frac{B\sqrt{ex}(a+bx^3)^{5/2}}{8be}$$

$$\Big\downarrow \ 851$$

$$\frac{(16Ab-aB)\left(\frac{9}{10}a\left(\frac{3a\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2e}+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)+\frac{\sqrt{ex}(a+bx^3)^{3/2}}{5e}\right)}{16b}+\frac{B\sqrt{ex}(a+bx^3)^{5/2}}{8be}$$

$$\Big\downarrow \ 766$$

$$\frac{(16Ab-aB)\left(\frac{9}{10}a\left(\frac{3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+(1+\sqrt{3})\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}xe+\sqrt[3]{a}e}{(1+\sqrt{3})\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{\cdots}\right.\right.}{4e^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+(1+\sqrt{3})\sqrt[3]{b}ex\right)^2}}}\right.\right.}{16b}$$

$$\frac{B\sqrt{ex}(a+bx^3)^{5/2}}{8be}$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/Sqrt[e*x],x]
```

output
```
(B*Sqrt[e*x]*(a + b*x^3)^(5/2))/(8*b*e) + ((16*A*b - a*B)*((Sqrt[e*x]*(a +
 b*x^3)^(3/2))/(5*e) + (9*a*((Sqrt[e*x]*Sqrt[a + b*x^3])/(2*e) + (3^(3/4)*
a^(2/3)*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^
(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*
EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 +
 Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(4*e^2*Sqrt[(b^(1/3)*e*x*(a^(1/
3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b
*x^3])))/10))/(16*b)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.98 (sec) , antiderivative size = 768, normalized size of antiderivative = 2.37

method	result
risch	$\frac{(40b^2Bx^6+64Ab^2x^3+76Babx^3+208abA+27a^2B)x\sqrt{bx^3+a}}{320b\sqrt{ex}}+\frac{27a^2(16Ab-Ba)\left(\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-\cdots}{}\right.}{\left(-\frac{(-ab^2)^{\frac{1}{3}}}{2b}+\cdots\right.}}}{\cdots}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/320/b*(40*B*b^2*x^6+64*A*b^2*x^3+76*B*a*b*x^3+208*A*a*b+27*B*a^2)*x*(b*x
^3+a)^(1/2)/(e*x)^(1/2)+27/320*a^2*(16*A*b-B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*
(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)
^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e
*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*Ell
ipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2
),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(e*x)
, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.34 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{Aa^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{7}{6}\right)} + \frac{A\sqrt{a}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{13}{6}\right)} + \frac{Ba^{\frac{3}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{13}{6}\right)} + \frac{B\sqrt{a}bx^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{19}{6}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(1/2),x)
```

output
```
A*a**(3/2)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(e)*gamma(7/6)) + A*sqrt(a)*b*x**(7/2)*gamma(7/6)*hyper((-
1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6)) + B*
a**(3/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(e)*gamma(13/6)) + B*sqrt(a)*b*x**(13/2)*gamma(13/6)*hyper
((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(19/6))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/sqrt(e*x), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/sqrt(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{\sqrt{e\,x}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(1/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{\sqrt{e}\left(94\sqrt{x}\,\sqrt{b\,x^3+a}\,a^2+56\sqrt{x}\,\sqrt{b\,x^3+a}\,ab\,x^3+16\sqrt{x}\,\sqrt{b\,x^3+a}\,b^2x^6+\right.}{128e}$$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x)
```

output
```
(sqrt(e)*(94*sqrt(x)*sqrt(a + b*x**3)*a**2 + 56*sqrt(x)*sqrt(a + b*x**3)*a
*b*x**3 + 16*sqrt(x)*sqrt(a + b*x**3)*b**2*x**6 + 81*int((sqrt(x)*sqrt(a +
 b*x**3))/(a*x + b*x**4),x)*a**3))/(128*e)
```

3.254 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 614

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4}$$
$$+\frac{27\left(1+\sqrt{3}\right)a(14Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{112b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$
$$+\frac{(14Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7ae^4}-\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}}$$
$$-\frac{27\sqrt[4]{3}a^{4/3}(14Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{3}}{112b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{9\ 3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}(14Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right.}{\sqrt[3]{a}+\left(1+\sqrt{3}\right.}\right.\right.}{224b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
9/56*(14*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/e^4+27/112*(1+3^(1/2))*a*(14
*A*b+B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(2/3)/e^2/(a^(1/3)+(1+3^(1/2))*b^(
1/3)*x)+1/7*(14*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/a/e^4-2*A*(b*x^3+a)^(
5/2)/a/e/(e*x)^(1/2)-27/112*3^(1/4)*a^(4/3)*(14*A*b+B*a)*(e*x)^(1/2)*(a^(1
/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2
))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(
1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/b^(2/3)/e^2/
(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b
*x^3+a)^(1/2)-9/224*3^(3/4)*(1-3^(1/2))*a^(4/3)*(14*A*b+B*a)*(e*x)^(1/2)*(
a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^
(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(
1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))/b^(2/3)/
e^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2
)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.14

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx=\frac{2x\sqrt{a+bx^3}\left(-\frac{5A(a+bx^3)^2}{a}+\frac{(14Ab+aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{5(ex)^{3/2}}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*((-5*A*(a + b*x^3)^2)/a + ((14*A*b + a*B)*x^3*Hyperge
ometric2F1[-3/2, 5/6, 11/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*(e*x)^
(3/2))
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {955, 811, 811, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(aB+14Ab)\int (ex)^{3/2}(bx^3+a)^{3/2}dx}{ae^3}-\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{811}$$

$$\frac{(aB+14Ab)\left(\frac{9}{14}a\int (ex)^{3/2}\sqrt{bx^3+a}dx+\frac{(ex)^{5/2}(a+bx^3)^{3/2}}{7e}\right)}{ae^3}-\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{811}$$

$$\frac{(aB+14Ab)\left(\frac{9}{14}a\left(\frac{3}{8}a\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}(a+bx^3)^{3/2}}{7e}\right)}{ae^3}-\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{851}$$

$$\frac{(aB+14Ab)\left(\frac{9}{14}a\left(\frac{3a\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}(a+bx^3)^{3/2}}{7e}\right)}{ae^3}-\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{837}$$

$$\frac{(aB+14Ab)\left(\frac{9}{14}a\left(\frac{3a\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int-\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}(a+bx^3}{7e}}{ae^3}$$

$$\frac{2A\left(a+bx^3\right)^{5/2}}{ae\sqrt{ex}}$$

$\downarrow$ 25

$$\frac{(aB+14Ab)\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}(a+bx^3)^{3/}}{7e}}{ae^3}$$

$$\frac{2A\left(a+bx^3\right)^{5/2}}{ae\sqrt{ex}}$$

$\downarrow$ 766

$$\frac{(aB+14Ab)\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}\right.}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}\right.}}}\right.}{4e}\right.\right.}{ae^3}$$

$$\frac{2A\left(a+bx^3\right)^{5/2}}{ae\sqrt{ex}}$$

$\downarrow$ 2420

$$\frac{(aB+14Ab)\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe\cdots}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe\cdots}\cdots\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\cdots}\cdots\right)}{2b^{2/3}\cdots}\cdots\right.\right.}{\cdots}$$

$$\frac{2A\left(a+bx^3\right)^{5/2}}{ae\sqrt{ex}}$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

output
```
(-2*A*(a + b*x^3)^(5/2))/(a*e*Sqrt[e*x]) + ((14*A*b + a*B)*(((e*x)^(5/2)*(
a + b*x^3)^(3/2))/(7*e) + (9*a*(((e*x)^(5/2)*Sqrt[a + b*x^3])/(4*e) + (3*a
*((((1 + Sqrt[3])*e^3*Sqrt[e*x]*Sqrt[a + b*x^3])/(a^(1/3)*e + (1 + Sqrt[3]
)*b^(1/3)*e*x) - (3^(1/4)*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sq
rt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1
 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticE[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^
(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(Sq
rt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1
/3)*e*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*e*Sqrt[
e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x +
 b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[Arc
Cos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(
1/3)*e*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*e*x*(a^(1/3
)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*
x^3]))/(4*e)))/14))/(a*e^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.98 (sec) , antiderivative size = 1140, normalized size of antiderivative = 1.86

method	result	size
risch	Expression too large to display	1140
elliptic	Expression too large to display	1233
default	Expression too large to display	6142

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/56*(b*x^3+a)^(1/2)*(-8*B*b*x^6-14*A*b*x^3-17*B*a*x^3+112*A*a)/e/(e*x)^(
1/2)+27/112*a*(14*A*b+B*a)*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(
-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))
^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a
*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(...
```

Fricas [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^2*
x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.72 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.33

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{5}{6}\right)} + \frac{A\sqrt{a}bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)} + \frac{Ba^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)} + \frac{B\sqrt{a}bx^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(3/2),x)
```

output
```
A*a**(3/2)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/
a)/(3*e**(3/2)*sqrt(x)*gamma(5/6)) + A*sqrt(a)*b*x**(5/2)*gamma(5/6)*hyper
((-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(11/6))
+ B*a**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*exp_po
lar(I*pi)/a)/(3*e**(3/2)*gamma(11/6)) + B*sqrt(a)*b*x**(11/2)*gamma(11/6)*
hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(1
7/6))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{(e\,x)^{3/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(3/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{\sqrt{e}\left(362\sqrt{b\,x^3+a}\,a^2+124\sqrt{b\,x^3+a}\,ab\,x^3+32\sqrt{b\,x^3+a}\,b^2x^6+405\sqrt{x}\left(\right.\right.}{224\sqrt{x}\,e^2}$$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x)
```

output
```
(sqrt(e)*(362*sqrt(a + b*x**3)*a**2 + 124*sqrt(a + b*x**3)*a*b*x**3 + 32*s
qrt(a + b*x**3)*b**2*x**6 + 405*sqrt(x)*int((sqrt(x)*sqrt(a + b*x**3))/(a*
x**2 + b*x**5),x)*a**3))/(224*sqrt(x)*e**2)
```

3.255 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 152

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{6ae^4} - \frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}} + \frac{a(4Ab+aB)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{4\sqrt{b}e^{5/2}}$$

output
```
1/4*(4*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/e^4+1/6*(4*A*b+B*a)*(e*x)^(3/2
)*(b*x^3+a)^(3/2)/a/e^4-2/3*A*(b*x^3+a)^(5/2)/a/e/(e*x)^(3/2)+1/4*a*(4*A*b
+B*a)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a)^(1/2))/b^(1/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \frac{x\left(\sqrt{b}\sqrt{a+bx^3}(-8aA+4Abx^3+5aBx^3+2bBx^6)+3a(4Ab+aB)x^{3/2}\ \text{lo}\right.}{12\sqrt{b}(ex)^{5/2}}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

output
```
(x*(Sqrt[b]*Sqrt[a + b*x^3]*(-8*a*A + 4*A*b*x^3 + 5*a*B*x^3 + 2*b*B*x^6) +
 3*a*(4*A*b + a*B)*x^(3/2)*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(12*Sq
rt[b]*(e*x)^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {955, 811, 811, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx$$

$$\Big\downarrow \ 955$$

$$\frac{(aB+4Ab)\int \sqrt{ex}\left(bx^3+a\right)^{3/2}dx}{ae^3}-\frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \ 811$$

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\int \sqrt{ex}\sqrt{bx^3+a}dx+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \ 811$$

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)}{ae^3}-\frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}}$$

$\Big\downarrow$ 851

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\left(\frac{a\int\frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)}{ae^3}-\frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}}$$

$\Big\downarrow$ 807

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\left(\frac{a\int\frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)}{ae^3}-\frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}}$$

$\Big\downarrow$ 224

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\left(\frac{a\int\frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)}{ae^3}-\frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\left(\frac{a\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3\sqrt{b}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)}{ae^3}-\frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}}$$

input

```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

output

```
(-2*A*(a + b*x^3)^(5/2))/(3*a*e*(e*x)^(3/2)) + ((4*A*b + a*B)*((((e*x)^(3/2
)*(a + b*x^3)^(3/2))/(6*e) + (3*a*(((e*x)^(3/2)*Sqrt[a + b*x^3])/(3*e) + (
a*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3
*Sqrt[b])))/4))/(a*e^3)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-2bB\,x^{6}-4Ab\,x^{3}-5Ba\,x^{3}+8Aa\right)}{12x\,e^{2}\sqrt{ex}}+\frac{a(4Ab+Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)\sqrt{(b\,x^{3}+a)ex}}{4\sqrt{be}\,e^{2}\sqrt{ex}\,\sqrt{b\,x^{3}+a}}$
default	$\frac{\sqrt{b\,x^{3}+a}\left(2B\sqrt{(b\,x^{3}+a)ex}\,\sqrt{be}\,b\,x^{6}+12A\,\mathrm{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)abe\,x^{2}+4A\sqrt{(b\,x^{3}+a)ex}\,\sqrt{be}\,b\,x^{3}+3B\,\mathrm{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right.\right.}{12x\,e^{2}\sqrt{ex}\,\sqrt{(b\,x^{3}+a)ex}\,\sqrt{be}}$
elliptic	Expression too large to display

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b*x^3+a)^(1/2)*(-2*B*b*x^6-4*A*b*x^3-5*B*a*x^3+8*A*a)/x/e^2/(e*x)^(
1/2)+1/4*a*(4*A*b+B*a)/(b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)
^(1/2))/e^2*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx^3)^{3/2}\,(A+Bx^3)}{(ex)^{5/2}}\,dx = \left[\frac{3\,(Ba^2+4\,Aab)\sqrt{be}x^2\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,bx^4+ax)\sqrt{b}\right.}{48\,}\right.$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(B*a^2 + 4*A*a*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 -
a^2*e - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(2*B*b^
2*x^6 + (5*B*a*b + 4*A*b^2)*x^3 - 8*A*a*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e
^3*x^2), -1/24*(3*(B*a^2 + 4*A*a*b)*sqrt(-b*e)*x^2*arctan(2*sqrt(b*x^3 + a
)*sqrt(-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) - 2*(2*B*b^2*x^6 + (5*B*a*b +
4*A*b^2)*x^3 - 8*A*a*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(138) = 276$.

Time = 20.53 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.90

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = -\frac{2Aa^{\frac{3}{2}}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{A\sqrt{a}bx^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}} - \frac{2A\sqrt{a}bx^{\frac{3}{2}}}{3e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{Aa\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{e^{\frac{5}{2}}} + \frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}} + \frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}}{12e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{B\sqrt{a}bx^{\frac{9}{2}}}{4e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{4\sqrt{b}e^{\frac{5}{2}}} + \frac{Bb^2x^{\frac{15}{2}}}{6\sqrt{a}e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(5/2),x)
```

output
```
-2*A*a**(3/2)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b*x**(3
/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) - 2*A*sqrt(a)*b*x**(3/2)/(3*e**(5/2)*s
qrt(1 + b*x**3/a)) + A*a*sqrt(b)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/e**(5/2)
+ B*a**(3/2)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) + B*a**(3/2)*x**(3/2
)/(12*e**(5/2)*sqrt(1 + b*x**3/a)) + B*sqrt(a)*b*x**(9/2)/(4*e**(5/2)*sqrt
(1 + b*x**3/a)) + B*a**2*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*sqrt(b)*e**(5/
2)) + B*b**2*x**(15/2)/(6*sqrt(a)*e**(5/2)*sqrt(1 + b*x**3/a))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: Recu
rsive assumption sageVARa>=(-sageVARb*sageVARe/(sageVARe^4*t_nostep^6)) ig
nored1/sageVARe^3/((1/sageVARe)^2)*2*(120*sageVARb^5*sageVARe^6*sageVARB/1
440/sageVARb^4/sage
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{(e\,x)^{5/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(5/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \frac{\sqrt{e}\left(-16\sqrt{b\,x^3+a}\,a^2+18\sqrt{b\,x^3+a}\,ab\,x^3+4\sqrt{b\,x^3+a}\,b^2x^6-15\sqrt{x}\,\sqrt{b}\,1\right.}{24\sqrt{}}$$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x)
```

output
```
(sqrt(e)*( - 16*sqrt(a + b*x**3)*a**2 + 18*sqrt(a + b*x**3)*a*b*x**3 + 4*s
qrt(a + b*x**3)*b**2*x**6 - 15*sqrt(x)*sqrt(b)*log(sqrt(a + b*x**3) - sqrt
(x)*sqrt(b)*x)*a**2*x + 15*sqrt(x)*sqrt(b)*log(sqrt(a + b*x**3) + sqrt(x)*
sqrt(b)*x)*a**2*x))/(24*sqrt(x)*e**3*x)
```

3.256 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 314

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{9(2Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{20e^4} + \frac{(2Ab+aB)\sqrt{ex}(a+bx^3)^{3/2}}{5ae^4} - \frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}} + \frac{9\,3^{3/4}a^{2/3}(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(\right.}{40e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
9/20*(2*A*b+B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/e^4+1/5*(2*A*b+B*a)*(e*x)^(1/
2)*(b*x^3+a)^(3/2)/a/e^4-2/5*A*(b*x^3+a)^(5/2)/a/e/(e*x)^(5/2)+9/40*3^(3/4
)*a^(2/3)*(2*A*b+B*a)*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacob
iAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)
),1/4*6^(1/2)+1/4*2^(1/2))/e^4/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+
3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.27

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx=\frac{2x\sqrt{a+bx^3}\left(-\frac{A(a+bx^3)^2}{a}+\frac{5(2Ab+aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{6},\frac{7}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{5(ex)^{7/2}}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(-((A*(a + b*x^3)^2)/a) + (5*(2*A*b + a*B)*x^3*Hyperg
eometric2F1[-3/2, 1/6, 7/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*(e*x)^
(7/2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {955, 811, 811, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(aB+2Ab)\int \frac{(bx^3+a)^{3/2}}{\sqrt{ex}}dx}{ae^3}-\frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{811}$$

$$\frac{(aB+2Ab)\left(\frac{9}{10}a\int \frac{\sqrt{bx^3+a}}{\sqrt{ex}}dx+\frac{\sqrt{ex}(a+bx^3)^{3/2}}{5e}\right)}{ae^3}-\frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{811}$$

$$\frac{(aB+2Ab)\left(\frac{9}{10}a\left(\frac{3}{4}a\int\frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)+\frac{\sqrt{ex}(a+bx^3)^{3/2}}{5e}\right)}{ae^3}-\frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}}$$

$\downarrow$ 851

$$\frac{(aB+2Ab)\left(\frac{9}{10}a\left(\frac{3a\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2e}+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)+\frac{\sqrt{ex}(a+bx^3)^{3/2}}{5e}\right)}{ae^3}-\frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}}$$

$\downarrow$ 766

$$\frac{(aB+2Ab)\left(\frac{9}{10}a\left(\frac{3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+(1+\sqrt{3})\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}xe+\sqrt[3]{a}e}{(1+\sqrt{3})\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{3}\right.\right.}{4e^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+(1+\sqrt{3})\sqrt[3]{b}ex\right)^2}}}\right.\right.}{ae^3}$$

$$\frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}}$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

output
```
(-2*A*(a + b*x^3)^(5/2))/(5*a*e*(e*x)^(5/2)) + ((2*A*b + a*B)*((Sqrt[e*x]*
(a + b*x^3)^(3/2))/(5*e) + (9*a*((Sqrt[e*x]*Sqrt[a + b*x^3])/(2*e) + (3^(3
/4)*a^(2/3)*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3
)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)
^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e +
(1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(4*e^2*Sqrt[(b^(1/3)*e*x*(a
^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a
 + b*x^3])))/10))/(a*e^3)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.71 (sec) , antiderivative size = 759, normalized size of antiderivative = 2.42

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-4bB\,x^{6}-10Ab\,x^{3}-13Ba\,x^{3}+8Aa\right)}{20x^{2}e^{3}\sqrt{ex}}+\frac{27a(2Ab+Ba)\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-\right.}{2}\right.}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)}}}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(b*x^3+a)^(1/2)*(-4*B*b*x^6-10*A*b*x^3-13*B*a*x^3+8*A*a)/x^2/e^3/(e*
x)^(1/2)+27/20*a*(2*A*b+B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1
/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2
)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))/e^3*((b*x^3+
a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="fricas")
```

output
```
integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^4*
x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.77 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx=\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{6},-\frac{1}{2}\\ \frac{1}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}+\frac{A\sqrt{a}b\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}+\frac{Ba^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}+\frac{B\sqrt{a}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{13}{6}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(7/2),x)
```

output
```
A*a**(3/2)*gamma(-5/6)*hyper((-5/6, -1/2), (1/6,), b*x**3*exp_polar(I*pi)/
a)/(3*e**(7/2)*x**(5/2)*gamma(1/6)) + A*sqrt(a)*b*sqrt(x)*gamma(1/6)*hyper
((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/6)) +
B*a**(3/2)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(
I*pi)/a)/(3*e**(7/2)*gamma(7/6)) + B*sqrt(a)*b*x**(7/2)*gamma(7/6)*hyper((
-1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(13/6))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx=\int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{(e\,x)^{7/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(7/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx=\frac{\sqrt{e}\,\Big(-194\sqrt{b\,x^3+a}\,a^2+92\sqrt{b\,x^3+a}\,ab\,x^3+16\sqrt{b\,x^3+a}\,b^2x^6-405\sqrt{x}\,(}{80\sqrt{x}\,e^4x^2}$$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x)
```

output
```
(sqrt(e)*( - 194*sqrt(a + b*x**3)*a**2 + 92*sqrt(a + b*x**3)*a*b*x**3 + 16
*sqrt(a + b*x**3)*b**2*x**6 - 405*sqrt(x)*int((sqrt(x)*sqrt(a + b*x**3))/(
a*x**4 + b*x**7),x)*a**3*x**2))/(80*sqrt(x)*e**4*x**2)
```

3.257 $\int (ex)^{7/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 241

$$\int (ex)^{7/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2} + \frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be} + \frac{a(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{144be} + \frac{(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{120be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be} - \frac{a^4(10Ab-3aB)e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{384b^{5/2}}$$

output

```
1/384*a^3*(10*A*b-3*B*a)*e^2*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b^2+1/192*a^2*(10
*A*b-3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(1/2)/b/e+1/144*a*(10*A*b-3*B*a)*(e*x)^(
9/2)*(b*x^3+a)^(3/2)/b/e+1/120*(10*A*b-3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(5/2)/
b/e+1/15*B*(e*x)^(9/2)*(b*x^3+a)^(7/2)/b/e-1/384*a^4*(10*A*b-3*B*a)*e^(7/2
)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.68

$$\int (ex)^{7/2} (a + bx^3)^{5/2} (A+Bx^3) \, dx = \frac{e^3\sqrt{ex}\left(\sqrt{b}x^{3/2}\sqrt{a+bx^3}(-45a^4B + 30a^3b(5A+Bx^3) + 96b^4x^9(5A+4Bx^3) + 1\right.}{}$$

input
```
Integrate[(e*x)^(7/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

output
```
(e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*Sqrt[a + b*x^3]*(-45*a^4*B + 30*a^3*b*(5*A
 + B*x^3) + 96*b^4*x^9*(5*A + 4*B*x^3) + 16*a*b^3*x^6*(85*A + 63*B*x^3) +
4*a^2*b^2*x^3*(295*A + 186*B*x^3)) + 15*a^4*(-10*A*b + 3*a*B)*Log[Sqrt[b]*
x^(3/2) + Sqrt[a + b*x^3]]))/(5760*b^(5/2)*Sqrt[x])
```

Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.89, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {959, 811, 811, 811, 843, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{7/2} (a+bx^3)^{5/2} (A+Bx^3) \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(10Ab-3aB)\int (ex)^{7/2} (bx^3+a)^{5/2} dx}{10b} + \frac{B(ex)^{9/2}(a+bx^3)^{7/2}}{15be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\int (ex)^{7/2}(bx^3+a)^{3/2}dx + \frac{(ex)^{9/2}(a+bx^3)^{5/2}}{12e}\right)}{10b} + \frac{B(ex)^{9/2}(a+bx^3)^{7/2}}{15be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\left(\frac{1}{2}a\int (ex)^{7/2}\sqrt{bx^3+a}dx+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12e}\right)}{10b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}$$

$\Big\downarrow$ 811

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\int \frac{(ex)^{7/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12e}\right)}{10b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}$$

$\Big\downarrow$ 843

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^3\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx}{2b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)+\frac{(ex)^{9/2}(a\cdots}{12\cdots}\cdots}{10b}\cdots$$

$$\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}$$

$\Big\downarrow$ 851

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)+\frac{(ex)^{9/2}(\cdots}{\cdots}\cdots}{10b}\cdots$$

$$\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}$$

$\Big\downarrow$ 807

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)+\frac{(e\cdots}{\cdots}\cdots}{10b}\cdots$$

$$\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}$$

$\Big\downarrow$ 224

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int\frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}(a+bx^3)^{3/2}}{9e}\right)+\frac{(ex)}{}\right.}{10b}$$

$$\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}$$

$$\Big\downarrow \ 219$$

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3b^{3/2}}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}(a+bx^3)^{3/2}}{9e}\right.\right.}{10b}$$

$$\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}$$

input
```
Int[(e*x)^(7/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

output
```
(B*(e*x)^(9/2)*(a + b*x^3)^(7/2))/(15*b*e) + ((10*A*b - 3*a*B)*(((e*x)^(9/
2)*(a + b*x^3)^(5/2))/(12*e) + (5*a*(((e*x)^(9/2)*(a + b*x^3)^(3/2))/(9*e)
 + (a*(((e*x)^(9/2)*Sqrt[a + b*x^3])/(6*e) + (a*((e^2*(e*x)^(3/2)*Sqrt[a +
 b*x^3])/(3*b) - (a*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a
+ (b*x)/e^2])])/(3*b^(3/2))))/4))/2))/8))/(10*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 6.42 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.77

method	result
risch	$\frac{x^{2}\left(384 B\,b^{4} x^{12}+480 A\,b^{4} x^{9}+1008 B\,x^{9} a\,b^{3}+1360 A\,x^{6} a\,b^{3}+744 B\,x^{6} a^{2} b^{2}+1180 A\,a^{2} b^{2} x^{3}+30 B\,a^{3} b\,x^{3}+150 A\,a^{3} b-45 B\,a^{4}\right)\sqrt{b\,x^{3}+a}}{5760 b^{2}\sqrt{e x}}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/5760/b^2*x^2*(384*B*b^4*x^12+480*A*b^4*x^9+1008*B*a*b^3*x^9+1360*A*a*b^3
*x^6+744*B*a^2*b^2*x^6+1180*A*a^2*b^2*x^3+30*B*a^3*b*x^3+150*A*a^3*b-45*B*
a^4)*(b*x^3+a)^(1/2)*e^4/(e*x)^(1/2)-1/384*a^4/b^2*(10*A*b-3*B*a)/(b*e)^(1
/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*e^4*((b*x^3+a)*e*x)^(1/
2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.70

$$\int (ex)^{7/2} (a+bx^3)^{5/2} (A+Bx^3)\ dx = \left[-\frac{15\,(3\,Ba^5-10\,Aa^4b)e^3\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e+4\,(2\,b^2x^4+abx)\sqrt{\right.}}{}\right.$$

input
```
integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
[-1/23040*(15*(3*B*a^5 - 10*A*a^4*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*
b*e*x^3 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b
)) - 4*(384*B*b^4*e^3*x^13 + 48*(21*B*a*b^3 + 10*A*b^4)*e^3*x^10 + 8*(93*B
*a^2*b^2 + 170*A*a*b^3)*e^3*x^7 + 10*(3*B*a^3*b + 118*A*a^2*b^2)*e^3*x^4 -
 15*(3*B*a^4 - 10*A*a^3*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/11520
*(15*(3*B*a^5 - 10*A*a^4*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e
*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(384*B*b^4*e^3*x^13 + 48*(21*B*a
*b^3 + 10*A*b^4)*e^3*x^10 + 8*(93*B*a^2*b^2 + 170*A*a*b^3)*e^3*x^7 + 10*(3
*B*a^3*b + 118*A*a^2*b^2)*e^3*x^4 - 15*(3*B*a^4 - 10*A*a^3*b)*e^3*x)*sqrt(
b*x^3 + a)*sqrt(e*x))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(216) = 432$.

Time = 49.73 (sec) , antiderivative size = 1028, normalized size of antiderivative = 4.27

$$\int (ex)^{7/2} \left(a + bx^3\right)^{5/2} \left(A + Bx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**(7/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

output
```
Piecewise((2*Piecewise((nan, Eq(e**3, 0)), ((A*a**2*e**3*Piecewise((-a**2*
e**3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3
))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3),
True))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**(3/2)/(8*b) + (e*x)**(9/2)/
4), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, True)) + 2*A*a*b*Piecewise((a
**3*e**6*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*
x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**
3), True))/(16*b**2) + sqrt(a + b*x**3)*(-a**2*e**6*(e*x)**(3/2)/(16*b**2)
 + a*e**3*(e*x)**(9/2)/(24*b) + (e*x)**(15/2)/6), Ne(b/e**3, 0)), (sqrt(a)
*(e*x)**(15/2)/5, True)) + A*b**2*Piecewise((-5*a**4*e**9*Piecewise((log(2
*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a
, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/(128*b**3) + s
qrt(a + b*x**3)*(5*a**3*e**9*(e*x)**(3/2)/(128*b**3) - 5*a**2*e**6*(e*x)**
(9/2)/(192*b**2) + a*e**3*(e*x)**(15/2)/(48*b) + (e*x)**(21/2)/8), Ne(b/e*
*3, 0)), (sqrt(a)*(e*x)**(21/2)/7, True))/e**3 + B*a**2*Piecewise((a**3*e*
*6*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))
/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), Tr
ue))/(16*b**2) + sqrt(a + b*x**3)*(-a**2*e**6*(e*x)**(3/2)/(16*b**2) + a*e
**3*(e*x)**(9/2)/(24*b) + (e*x)**(15/2)/6), Ne(b/e**3, 0)), (sqrt(a)*(e*x)
**(15/2)/5, True)) + 2*B*a*b*Piecewise((-5*a**4*e**9*Piecewise((log(2*b...
```

Maxima [F]

$$\int (ex)^{7/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{7}{2}}\,dx$$

input
```
integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 701 vs. $2(197) = 394$.

Time = 0.35 (sec) , antiderivative size = 701, normalized size of antiderivative = 2.91

$$\int (ex)^{7/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/72*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3*(4*x^3/e^4 + a/(b*e^4)) - 3*a^2/(b
^2*e))*sqrt(e*x)*B*a^2*x*abs(e)^2 + 1/36*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^
3*(4*x^3/e^4 + a/(b*e^4)) - 3*a^2/(b^2*e))*sqrt(e*x)*A*a*b*x*abs(e)^2 + 1/
5760*sqrt(b*e^4*x^3 + a*e^4)*(2*(4*(6*e^3*x^3*(8*x^3/e^10 + a/(b*e^10)) -
7*a^2/(b^2*e^7))*e^3*x^3 + 35*a^3/(b^3*e^4))*e^3*x^3 - 105*a^4/(b^4*e))*sq
rt(e*x)*B*b^2*x*abs(e)^2 + 1/12*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3 + a*e^3
/b)*sqrt(e*x)*A*a^2*x*abs(e)^2/e^4 - 1/384*(9*B^2*a^10*e - 60*A*B*a^9*b*e
+ 100*A^2*a^8*b^2*e)^2*e^5*log(abs(-(3*sqrt(e*x)*B*a^5*e^2*x - 10*sqrt(e*x
)*A*a^4*b*e^2*x)*sqrt(b*e) + sqrt(9*B^2*a^11*e^6 - 60*A*B*a^10*b*e^6 + 100
*A^2*a^9*b^2*e^6 + (3*sqrt(e*x)*B*a^5*e^2*x - 10*sqrt(e*x)*A*a^4*b*e^2*x)^
2*b*e)))/(sqrt(b*e)*b^2*abs(9*B^2*a^10*e - 60*A*B*a^9*b*e + 100*A^2*a^8*b^
2*e)*abs(-3*B*a^5 + 10*A*a^4*b)*abs(e)^2) + 1/288*(15*a^3*e^9/b^3 + 2*(4*(
6*e^3*x^3 + a*e^3/b)*e^3*x^3 - 5*a^2*e^6/b^2)*e^3*x^3)*sqrt(b*e^4*x^3 + a*
e^4)*sqrt(e*x)*B*a*b*x*abs(e)^2/e^10 + 1/576*(15*a^3*e^9/b^3 + 2*(4*(6*e^3
*x^3 + a*e^3/b)*e^3*x^3 - 5*a^2*e^6/b^2)*e^3*x^3)*sqrt(b*e^4*x^3 + a*e^4)*
sqrt(e*x)*A*b^2*x*abs(e)^2/e^10
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{7/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{7/2}\,{\left(b\,x^3+a\right)}^{5/2} \,d x$$

input
```
int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(5/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.64

$$\int (ex)^{7/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \frac{\sqrt{e}\, e^3 \left(210\sqrt{x}\,\sqrt{b\,x^3+a}\,a^4bx+2420\sqrt{x}\,\sqrt{b\,x^3+a}\,a^3b^2x^4+4208\sqrt{x}\,\sqrt{b\,x^3+a}\,a^2\right.}{}$$

input
```
int((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

output
```
(sqrt(e)*e**3*(210*sqrt(x)*sqrt(a + b*x**3)*a**4*b*x + 2420*sqrt(x)*sqrt(a
 + b*x**3)*a**3*b**2*x**4 + 4208*sqrt(x)*sqrt(a + b*x**3)*a**2*b**3*x**7 +
 2976*sqrt(x)*sqrt(a + b*x**3)*a*b**4*x**10 + 768*sqrt(x)*sqrt(a + b*x**3)
*b**5*x**13 + 105*sqrt(b)*log(sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*x)*a**5 -
 105*sqrt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)*x)*a**5))/(11520*b**2)
```

3.258 $\int (ex)^{5/2} (a+bx^3)^{5/2} (A+Bx^3)\, dx$

Optimal result

Integrand size = 26, antiderivative size = 404

$$\int (ex)^{5/2} (a+bx^3)^{5/2} (A+Bx^3)\, dx = \frac{81a^3(4Ab-aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{5632b^2} + \frac{27a^2(4Ab-aB)(ex)^{7/2}\sqrt{a+bx^3}}{1408be} + \frac{15a(4Ab-aB)(ex)^{7/2}(a+bx^3)^{3/2}}{704be} + \frac{(4Ab-aB)(ex)^{7/2}(a+bx^3)^{5/2}}{44be} + \frac{B(ex)^{7/2}(a+bx^3)^{7/2}}{14be} - \frac{27\, 3^{3/4}a^{11/3}(4Ab-aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)\right.}{11264b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
81/5632*a^3*(4*A*b-B*a)*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^2+27/1408*a^2*(4
*A*b-B*a)*(e*x)^(7/2)*(b*x^3+a)^(1/2)/b/e+15/704*a*(4*A*b-B*a)*(e*x)^(7/2)
*(b*x^3+a)^(3/2)/b/e+1/44*(4*A*b-B*a)*(e*x)^(7/2)*(b*x^3+a)^(5/2)/b/e+1/14
*B*(e*x)^(7/2)*(b*x^3+a)^(7/2)/b/e-27/11264*3^(3/4)*a^(11/3)*(4*A*b-B*a)*e
^2*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)
+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2
^(1/2))/b^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)
^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.29

$$\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \frac{e^2\sqrt{ex}\sqrt{a+bx^3}\left(-(a+bx^3)^3\sqrt{1+\frac{bx^3}{a}}(-28Ab+7aB-22bBx^3)+7a^3(-4Ab \cdots\right.}{308b^2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(e*x)^(5/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

output
```
(e^2*Sqrt[e*x]*Sqrt[a + b*x^3]*(-((a + b*x^3)^3*Sqrt[1 + (b*x^3)/a]*(-28*A
*b + 7*a*B - 22*b*B*x^3)) + 7*a^3*(-4*A*b + a*B)*Hypergeometric2F1[-5/2, 1
/6, 7/6, -((b*x^3)/a)]))/(308*b^2*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {959, 811, 811, 811, 843, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{5/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow\ 959$$

$$\frac{(4Ab-aB)\int (ex)^{5/2}\left(bx^3+a\right)^{5/2}dx}{4b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}$$

$$\Big\downarrow\ 811$$

$$\frac{(4Ab-aB)\left(\frac{15}{22}a\int (ex)^{5/2}\left(bx^3+a\right)^{3/2}dx+\frac{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11e}\right)}{4b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}$$

$$\Big\downarrow\ 811$$

$$\frac{(4Ab-aB)\left(\frac{15}{22}a\left(\frac{9}{16}a\int (ex)^{5/2}\sqrt{bx^3+a}dx+\frac{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8e}\right)+\frac{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11e}\right)}{4b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}$$

$$\Big\downarrow\ 811$$

$$\frac{(4Ab-aB)\left(\frac{15}{22}a\left(\frac{9}{16}a\left(\frac{3}{10}a\int \frac{(ex)^{5/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{7/2}\sqrt{a+bx^3}}{5e}\right)+\frac{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8e}\right)+\frac{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11e}\right)}{4b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}$$

$$\Big\downarrow\ 843$$

$$\frac{(4Ab-aB)\left(\frac{15}{22}a\left(\frac{9}{16}a\left(\frac{3}{10}a\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{ae^3\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{4b}\right)+\frac{(ex)^{7/2}\sqrt{a+bx^3}}{5e}\right)+\frac{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8e}\right)+\frac{(ex)^{7/2}(a}{11}\right.}{4b}$$

$$\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}$$

$$\Big\downarrow\ 851$$

$$\frac{(4Ab-aB)\left(\frac{15}{22}a\left(\frac{9}{16}a\left(\frac{3}{10}a\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{ae^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b}\right)+\frac{(ex)^{7/2}\sqrt{a+bx^3}}{5e}\right)+\frac{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8e}\right)+\frac{(ex)^{7/2}(a-}{11}\right.}{4b}$$

$$\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}$$

$$\Big\downarrow\ 766$$

$$\frac{(4Ab-aB)\left(\frac{15}{22}a\left(\frac{9}{16}a\left(\frac{3}{10}a\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{a^{2/3}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{(1-}{(1+}\right.\right.}{4\sqrt[4]{3}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right.\right.\right.\right.}{4b}$$

$$\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}$$

input
```
Int[(e*x)^(5/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

output
```
(B*(e*x)^(7/2)*(a + b*x^3)^(7/2))/(14*b*e) + ((4*A*b - a*B)*(((e*x)^(7/2)*
(a + b*x^3)^(5/2))/(11*e) + (15*a*(((e*x)^(7/2)*(a + b*x^3)^(3/2))/(8*e) +
 (9*a*(((e*x)^(7/2)*Sqrt[a + b*x^3])/(5*e) + (3*a*((e^2*Sqrt[e*x]*Sqrt[a +
 b*x^3])/(2*b) - (a^(2/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2
/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[
3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*
x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*
b*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*
b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/10))/16))/22))/(4*b)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.14 (sec) , antiderivative size = 825, normalized size of antiderivative = 2.04

method	result	size
risch	Expression too large to display	825
elliptic	Expression too large to display	1134
default	Expression too large to display	5063

input
```
int((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/39424/b^2*(2816*B*b^4*x^12+3584*A*b^4*x^9+7552*B*a*b^3*x^9+10528*A*a*b^3
*x^6+5816*B*a^2*b^2*x^6+9968*A*a^2*b^2*x^3+324*B*a^3*b*x^3+2268*A*a^3*b-56
7*B*a^4)*x*(b*x^3+a)^(1/2)*e^3/(e*x)^(1/2)-81/5632*a^4/b*(4*A*b-B*a)*(1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*
(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3))
)^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*
(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))*e^3*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^
(1/2)
```

Fricas [F]

$$\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*b^2*e^2*x^11 + (2*B*a*b + A*b^2)*e^2*x^8 + (B*a^2 + 2*A*a*b)*e
^2*x^5 + A*a^2*e^2*x^2)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 153.20 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.76

$$\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right)\,dx = \frac{Aa^{\frac{5}{2}}e^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{6}\right)} + \frac{2Aa^{\frac{3}{2}}be^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)} + \frac{A\sqrt{a}b^2e^{\frac{5}{2}}x^{\frac{19}{2}}\Gamma\left(\frac{19}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{19}{6}\\ \frac{25}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{25}{6}\right)} + \frac{Ba^{\frac{5}{2}}e^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)} + \frac{2Ba^{\frac{3}{2}}be^{\frac{5}{2}}x^{\frac{19}{2}}\Gamma\left(\frac{19}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{19}{6}\\ \frac{25}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{25}{6}\right)} + \frac{B\sqrt{a}b^2e^{\frac{5}{2}}x^{\frac{25}{2}}\Gamma\left(\frac{25}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{25}{6}\\ \frac{31}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{31}{6}\right)}$$

input
```
integrate((e*x)**(5/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

output
```
A*a**(5/2)*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3
*exp_polar(I*pi)/a)/(3*gamma(13/6)) + 2*A*a**(3/2)*b*e**(5/2)*x**(13/2)*ga
mma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(
19/6)) + A*sqrt(a)*b**2*e**(5/2)*x**(19/2)*gamma(19/6)*hyper((-1/2, 19/6),
 (25/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(25/6)) + B*a**(5/2)*e**(5/2)*
x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(19/6)) + 2*B*a**(3/2)*b*e**(5/2)*x**(19/2)*gamma(19/6)*hyper((
-1/2, 19/6), (25/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(25/6)) + B*sqrt(a
)*b**2*e**(5/2)*x**(25/2)*gamma(25/6)*hyper((-1/2, 25/6), (31/6,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(31/6))
```

Maxima [F]

$$\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int (Bx^3+A) (bx^3+a)^{\frac{5}{2}} (ex)^{\frac{5}{2}} \, dx$$

input
```
integrate((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(5/2), x)
```

Giac [F]

$$\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int (Bx^3+A) (bx^3+a)^{\frac{5}{2}} (ex)^{\frac{5}{2}} \, dx$$

input
```
integrate((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}\,{\left(b\,x^3+a\right)}^{5/2} \,d x$$

input
```
int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(5/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(5/2), x)
```

Reduce [F]

$$\int (ex)^{5/2} (a+bx^3)^{5/2} (A+Bx^3)\, dx = \frac{\sqrt{e}\, e^2 \Big(3402\sqrt{x}\,\sqrt{b\,x^3+a}\,a^4 + 20584\sqrt{x}\,\sqrt{b\,x^3+a}\,a^3 b\,x^3 + 32688\sqrt{x}\,\sqrt{b\,x^3+a}\,a}{}$$

input
```
int((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

output
```
(sqrt(e)*e**2*(3402*sqrt(x)*sqrt(a + b*x**3)*a**4 + 20584*sqrt(x)*sqrt(a +
 b*x**3)*a**3*b*x**3 + 32688*sqrt(x)*sqrt(a + b*x**3)*a**2*b**2*x**6 + 222
72*sqrt(x)*sqrt(a + b*x**3)*a*b**3*x**9 + 5632*sqrt(x)*sqrt(a + b*x**3)*b*
*4*x**12 - 1701*int((sqrt(x)*sqrt(a + b*x**3))/(a*x + b*x**4),x)*a**5))/(7
8848*b)
```

3.259 $\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 661

$$\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be}$$

$$+ \frac{81\left(1+\sqrt{3}\right)a^3(26Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{11648b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} + \frac{3a(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{728be}$$

$$+ \frac{(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{260be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be}$$

$$- \frac{81\sqrt[4]{3}a^{10/3}(26Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\right.}{11648b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{27\ 3^{3/4}\left(1-\sqrt{3}\right)a^{10/3}(26Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+(}{\sqrt[3]{a}+(}\right.\right.}{23296b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
27/5824*a^2*(26*A*b-5*B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/b/e+81/11648*(1+3^(
1/2))*a^3*(26*A*b-5*B*a)*e*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+(1
+3^(1/2))*b^(1/3)*x)+3/728*a*(26*A*b-5*B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/b/
e+1/260*(26*A*b-5*B*a)*(e*x)^(5/2)*(b*x^3+a)^(5/2)/b/e+1/13*B*(e*x)^(5/2)*
(b*x^3+a)^(7/2)/b/e-81/11648*3^(1/4)*a^(10/3)*(26*A*b-5*B*a)*e*(e*x)^(1/2)
*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+
3^(1/2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^
2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/b^(5/3
)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/
(b*x^3+a)^(1/2)-27/23296*3^(3/4)*(1-3^(1/2))*a^(10/3)*(26*A*b-5*B*a)*e*(e*
x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(
1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^
(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2)
)/b^(5/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2
)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.15

$$\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \frac{x(ex)^{3/2}\sqrt{a+bx^3}\left(5B(a+bx^3)^3\sqrt{1+\frac{bx^3}{a}}+a^2(26Ab-5aB)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)\right)}{65b\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(e*x)^(3/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

output
```
(x*(e*x)^(3/2)*Sqrt[a + b*x^3]*(5*B*(a + b*x^3)^3*Sqrt[1 + (b*x^3)/a] + a^
2*(26*A*b - 5*a*B)*Hypergeometric2F1[-5/2, 5/6, 11/6, -((b*x^3)/a)]))/(65*
b*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {959, 811, 811, 811, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(26Ab-5aB)\int (ex)^{3/2} \left(bx^3+a\right)^{5/2}dx}{26b}+\frac{B(ex)^{5/2} \left(a+bx^3\right)^{7/2}}{13be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(26Ab-5aB)\left(\frac{3}{4}a\int (ex)^{3/2} \left(bx^3+a\right)^{3/2}dx+\frac{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10e}\right)}{26b}+\frac{B(ex)^{5/2} \left(a+bx^3\right)^{7/2}}{13be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(26Ab-5aB)\left(\frac{3}{4}a\left(\frac{9}{14}a\int (ex)^{3/2}\sqrt{bx^3+a}dx+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10e}\right)}{26b}+\frac{B(ex)^{5/2} \left(a+bx^3\right)^{7/2}}{13be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(26Ab-5aB)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3}{8}a\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10e}\right)}{26b}+\frac{B(ex)^{5/2} \left(a+bx^3\right)^{7/2}}{13be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(26Ab-5aB)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3a\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10e}\right)}{26b}+\frac{B(ex)^{5/2} \left(a+bx^3\right)^{7/2}}{13be}$$

$\downarrow$ 837

$$\frac{(26Ab-5aB)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3a\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int-\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}\cdots}{\cdots}\right.\right.\cdots}{26b}\cdots$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be}$$

$\downarrow$ 25

$$\frac{(26Ab-5aB)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}(a\cdots}{7\cdots}\right.\right.\cdots}{26b}\cdots$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be}$$

$\downarrow$ 766

$$\frac{(26Ab-5aB)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\cdots}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}\cdots\right.}{\left(\sqrt[3]{a}e+\left(1+\cdots\right.\right.}}}\cdots\right)}{4e}\cdots\right.\right.\right.}{26b}\cdots$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be}$$

$\downarrow$ 2420

$$\frac{(26Ab-5aB)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(3a\left(\frac{\frac{(1+\sqrt{3})e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+(1+\sqrt{3})\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+(1+\sqrt{3})\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3}}{(1+\sqrt{3}}\right.\right.}{2b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+(1+\sqrt{3})\sqrt[3]{b}ex\right)^2}}}\right.\right.\right.\right.}{}$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be}$$

input
```
Int[(e*x)^(3/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

output
```
(B*(e*x)^(5/2)*(a + b*x^3)^(7/2))/(13*b*e) + ((26*A*b - 5*a*B)*(((e*x)^(5/
2)*(a + b*x^3)^(5/2))/(10*e) + (3*a*(((e*x)^(5/2)*(a + b*x^3)^(3/2))/(7*e)
 + (9*a*(((e*x)^(5/2)*Sqrt[a + b*x^3])/(4*e) + (3*a*((((1 + Sqrt[3])*e^3*S
qrt[e*x]*Sqrt[a + b*x^3])/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x) - (3^(1/
4)*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/
3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x
)^2]*EllipticE[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e +
 (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*e*x*(a^(1/3
)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*
x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/
3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1
/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - S
qrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3
])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^
(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(4*e)))/14))/4)
)/(26*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.38 (sec) , antiderivative size = 1188, normalized size of antiderivative = 1.80

method	result	size
risch	Expression too large to display	1188
elliptic	Expression too large to display	1410
default	Expression too large to display	6202

input
```
int((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/29120/b*x^3*(2240*B*b^3*x^9+2912*A*b^3*x^6+6160*B*a*b^2*x^6+8944*A*a*b^2
*x^3+5000*B*a^2*b*x^3+9542*A*a^2*b+405*B*a^3)*(b*x^3+a)^(1/2)*e^2/(e*x)^(1
/2)+81/11648*a^3/b*(26*A*b-5*B*a)*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2
*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^
(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-
1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/...
```

Fricas [F]

$$\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right) \left(bx^3+a\right)^{\frac{5}{2}} (ex)^{\frac{3}{2}} \, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*b^2*e*x^10 + (2*B*a*b + A*b^2)*e*x^7 + (B*a^2 + 2*A*a*b)*e*x^4
 + A*a^2*e*x)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 54.81 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.47

$$\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx = \frac{Aa^{\frac{5}{2}}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{6}\right)} + \frac{2Aa^{\frac{3}{2}}be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)} + \frac{A\sqrt{a}b^2e^{\frac{3}{2}}x^{\frac{17}{2}}\Gamma\left(\frac{17}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{17}{6}\\ \frac{23}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{23}{6}\right)} + \frac{Ba^{\frac{5}{2}}e^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)} + \frac{2Ba^{\frac{3}{2}}be^{\frac{3}{2}}x^{\frac{17}{2}}\Gamma\left(\frac{17}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{17}{6}\\ \frac{23}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{23}{6}\right)} + \frac{B\sqrt{a}b^2e^{\frac{3}{2}}x^{\frac{23}{2}}\Gamma\left(\frac{23}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{23}{6}\\ \frac{29}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{29}{6}\right)}$$

input
```
integrate((e*x)**(3/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

output
```
A*a**(5/2)*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3
*exp_polar(I*pi)/a)/(3*gamma(11/6)) + 2*A*a**(3/2)*b*e**(3/2)*x**(11/2)*ga
mma(11/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(
17/6)) + A*sqrt(a)*b**2*e**(3/2)*x**(17/2)*gamma(17/6)*hyper((-1/2, 17/6),
 (23/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(23/6)) + B*a**(5/2)*e**(3/2)*
x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(17/6)) + 2*B*a**(3/2)*b*e**(3/2)*x**(17/2)*gamma(17/6)*hyper((
-1/2, 17/6), (23/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(23/6)) + B*sqrt(a
)*b**2*e**(3/2)*x**(23/2)*gamma(23/6)*hyper((-1/2, 23/6), (29/6,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(29/6))
```

Maxima [F]

$$\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}} \, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(3/2), x)
```

Giac [F]

$$\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}} \, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}\,{\left(b\,x^3+a\right)}^{5/2} \,d x$$

input
```
int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(5/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(5/2), x)
```

Reduce [F]

$$\int (ex)^{3/2} (a+bx^3)^{5/2} (A+Bx^3)\, dx = \frac{\sqrt{e}\, e\Big(2842\sqrt{x}\,\sqrt{b\,x^3+a}\,a^3x^2+3984\sqrt{x}\,\sqrt{b\,x^3+a}\,a^2b\,x^5+2592\sqrt{x}\,\sqrt{b\,x^3+a}\,a\,b}{8320}$$

input
```
int((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

output
```
(sqrt(e)*e*(2842*sqrt(x)*sqrt(a + b*x**3)*a**3*x**2 + 3984*sqrt(x)*sqrt(a
+ b*x**3)*a**2*b*x**5 + 2592*sqrt(x)*sqrt(a + b*x**3)*a*b**2*x**8 + 640*sq
rt(x)*sqrt(a + b*x**3)*b**3*x**11 + 1215*int((sqrt(x)*sqrt(a + b*x**3)*x)/
(a + b*x**3),x)*a**4))/8320
```

3.260 $\int \sqrt{ex}(a+bx^3)^{5/2}(A+Bx^3)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 201

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx = \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{72be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be} + \frac{5a^3(8Ab-aB)\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{3/2}}$$

output

```
5/192*a^2*(8*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b/e+5/288*a*(8*A*b-B*a)*
(e*x)^(3/2)*(b*x^3+a)^(3/2)/b/e+1/72*(8*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(5/
2)/b/e+1/12*B*(e*x)^(3/2)*(b*x^3+a)^(7/2)/b/e+5/192*a^3*(8*A*b-B*a)*e^(1/2
)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.70

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\frac{\sqrt{ex}\left(\sqrt{b}x^{3/2}\sqrt{a+bx^3}(15a^3B+16b^3x^6(4A+3Bx^3)+8ab^2x^3(26A+17Bx^3)+2a^2b(132A+\right.}{576b^{3/2}\sqrt{x}}$$

input
```
Integrate[Sqrt[e*x]*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

output
```
(Sqrt[e*x]*(Sqrt[b]*x^(3/2)*Sqrt[a + b*x^3]*(15*a^3*B + 16*b^3*x^6*(4*A +
3*B*x^3) + 8*a*b^2*x^3*(26*A + 17*B*x^3) + 2*a^2*b*(132*A + 59*B*x^3)) - 1
5*a^3*(-8*A*b + a*B)*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(576*b^(3/2)
*Sqrt[x])
```

Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {959, 811, 811, 811, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(8Ab-aB)\int \sqrt{ex}\left(bx^3+a\right)^{5/2}dx}{8b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\int \sqrt{ex}\left(bx^3+a\right)^{3/2}dx+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{8b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\int\sqrt{ex}\sqrt{bx^3+a}dx+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{8b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{8b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\int\frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{8b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\int\frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{8b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\int\frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{8b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3\sqrt{b}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{5/2}}{9e}\right)}{8b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

input
```
Int[Sqrt[e*x]*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

output
```
(B*(e*x)^(3/2)*(a + b*x^3)^(7/2))/(12*b*e) + ((8*A*b - a*B)*(((e*x)^(3/2)*
(a + b*x^3)^(5/2))/(9*e) + (5*a*(((e*x)^(3/2)*(a + b*x^3)^(3/2))/(6*e) + (
3*a*(((e*x)^(3/2)*Sqrt[a + b*x^3])/(3*e) + (a*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*
x)^(3/2))/(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*Sqrt[b])))/4))/6))/(8*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.79

method	result
risch	$\frac{x^2\left(48b^3Bx^9+64Ab^3x^6+136Bab^2x^6+208aAb^2x^3+118Ba^2bx^3+264a^2bA+15a^3B\right)\sqrt{bx^3+a}\,e}{576b\sqrt{ex}}+\frac{5a^3(8Ab-Ba)\operatorname{arctanh}\left(\frac{\sqrt{(b x}}{x^2}\right.}{192b\sqrt{be}\sqrt{ex}}$
default	$\frac{\sqrt{ex}\sqrt{bx^3+a}\left(48B\sqrt{(bx^3+a)ex}\sqrt{be}\,b^3x^{10}+64A\sqrt{(bx^3+a)ex}\sqrt{be}\,b^3x^7+136B\sqrt{(bx^3+a)ex}\sqrt{be}\,ab^2x^7+208A\sqrt{(bx^3+a)ex}\sqrt{be}\right.}{}$
elliptic	Expression too large to display

input
```
int((e*x)^(1/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/576/b*x^2*(48*B*b^3*x^9+64*A*b^3*x^6+136*B*a*b^2*x^6+208*A*a*b^2*x^3+118
*B*a^2*b*x^3+264*A*a^2*b+15*B*a^3)*(b*x^3+a)^(1/2)*e/(e*x)^(1/2)+5/192*a^3
/b*(8*A*b-B*a)/(b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*
e*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.61

$$\int \sqrt{ex}(a+bx^3)^{5/2}(A+Bx^3)\,dx = \left[-\frac{15\,(Ba^4-8\,Aa^3b)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,b^2x^4+abx)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)\right.$$

input
```
integrate((e*x)^(1/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
[-1/2304*(15*(B*a^4 - 8*A*a^3*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3
- a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(
48*B*b^3*x^10 + 8*(17*B*a*b^2 + 8*A*b^3)*x^7 + 2*(59*B*a^2*b + 104*A*a*b^2
)*x^4 + 3*(5*B*a^3 + 88*A*a^2*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b, 1/1152*(
15*(B*a^4 - 8*A*a^3*b)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*s
qrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*(48*B*b^3*x^10 + 8*(17*B*a*b^2 + 8*A*b^3)
*x^7 + 2*(59*B*a^2*b + 104*A*a*b^2)*x^4 + 3*(5*B*a^3 + 88*A*a^2*b)*x)*sqrt
(b*x^3 + a)*sqrt(e*x))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 896 vs. 2(177) = 354.

Time = 8.37 (sec) , antiderivative size = 896, normalized size of antiderivative = 4.46

$$\int \sqrt{ex}(a+bx^3)^{5/2}(A+Bx^3)\,dx = \text{Too large to display}$$

input
```
integrate((e*x)**(1/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

output
```
Piecewise((2*Piecewise((nan, Eq(e**3, 0)), ((A*a**2*e**3*Piecewise((a*Piec
ewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b
/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/2
+ (e*x)**(3/2)*sqrt(a + b*x**3)/2, Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(3/2),
True)) + 2*A*a*b*Piecewise((-a**2*e**3*Piecewise((log(2*b*(e*x)**(3/2)/e**
3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2
)*log((e*x)**(3/2))/sqrt(b*x**3), True))/(8*b) + sqrt(a + b*x**3)*(a*e**3*
(e*x)**(3/2)/(8*b) + (e*x)**(9/2)/4), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2
)/3, True)) + A*b**2*Piecewise((a**3*e**6*Piecewise((log(2*b*(e*x)**(3/2)/
e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(
3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/(16*b**2) + sqrt(a + b*x**3)*(
-a**2*e**6*(e*x)**(3/2)/(16*b**2) + a*e**3*(e*x)**(9/2)/(24*b) + (e*x)**(1
5/2)/6), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(15/2)/5, True))/e**3 + B*a**2*Pi
ecewise((-a**2*e**3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*
sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))
/sqrt(b*x**3), True))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**(3/2)/(8*b)
+ (e*x)**(9/2)/4), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, True)) + 2*B*a
*b*Piecewise((a**3*e**6*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e*
*3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3
/2))/sqrt(b*x**3), True))/(16*b**2) + sqrt(a + b*x**3)*(-a**2*e**6*(e*x...
```

Maxima [F]

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}\sqrt{ex}\,dx$$

input
```
integrate((e*x)^(1/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*sqrt(e*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(159) = 318$.

Time = 0.30 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.94

$$\int \sqrt{ex}(a+bx^3)^{5/2}(A+Bx^3)\,dx = \frac{\sqrt{be^4x^3+ae^4}\left(2e^3x^3\left(\frac{4x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3a^2}{b^2e}\right)\sqrt{ex}Babx|e|^2}{36e^3} + \frac{\sqrt{be^4x^3+ae^4}\left(2e^3x^3\left(\frac{4x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3a^2}{b^2e}\right)\sqrt{ex}Ab^2x|e|^2}{72e^3} - \frac{(25B^2a^8+240ABa^7b+576A^2a^6b^2)e^4\log\left(\left|(5\sqrt{ex}Ba^4x+24\sqrt{ex}Aa^3bx)\sqrt{be}+\sqrt{25B^2a^9e^2+240A}\right.\right.}{192\sqrt{be}b|5Ba^4e+24Aa^3be||e|^2} - \frac{\left(\frac{ae^4\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{\sqrt{be}}-\sqrt{be^4x^3+ae^4}\sqrt{ex}ex\right)Aa^2|e|^2}{3e^5} + \frac{\sqrt{be^4x^3+ae^4}\left(2e^3x^3+\frac{ae^3}{b}\right)\sqrt{ex}Ba^2x|e|^2}{12e^7} + \frac{\sqrt{be^4x^3+ae^4}\left(2e^3x^3+\frac{ae^3}{b}\right)\sqrt{ex}Aabx|e|^2}{6e^7} + \frac{\left(\frac{15a^3e^9}{b^3}+2\left(4\left(6e^3x^3+\frac{ae^3}{b}\right)e^3x^3-\frac{5a^2e^6}{b^2}\right)e^3x^3\right)\sqrt{be^4x^3+ae^4}\sqrt{ex}Bb^2x|e|^2}{576e^{13}}$$

input
```
integrate((e*x)^(1/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/36*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3*(4*x^3/e^4 + a/(b*e^4)) - 3*a^2/(b
^2*e))*sqrt(e*x)*B*a*b*x*abs(e)^2/e^3 + 1/72*sqrt(b*e^4*x^3 + a*e^4)*(2*e^
3*x^3*(4*x^3/e^4 + a/(b*e^4)) - 3*a^2/(b^2*e))*sqrt(e*x)*A*b^2*x*abs(e)^2/
e^3 - 1/192*(25*B^2*a^8 + 240*A*B*a^7*b + 576*A^2*a^6*b^2)*e^4*log(abs((5*
sqrt(e*x)*B*a^4*x + 24*sqrt(e*x)*A*a^3*b*x)*sqrt(b*e) + sqrt(25*B^2*a^9*e^
2 + 240*A*B*a^8*b*e^2 + 576*A^2*a^7*b^2*e^2 + (5*sqrt(e*x)*B*a^4*x + 24*sq
rt(e*x)*A*a^3*b*x)^2*b*e)))/(sqrt(b*e)*b*abs(5*B*a^4*e + 24*A*a^3*b*e)*abs
(e)^2) - 1/3*(a*e^4*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*
e^4)))/sqrt(b*e) - sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*e*x)*A*a^2*abs(e)^2/e
^5 + 1/12*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3 + a*e^3/b)*sqrt(e*x)*B*a^2*x*
abs(e)^2/e^7 + 1/6*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3 + a*e^3/b)*sqrt(e*x)
*A*a*b*x*abs(e)^2/e^7 + 1/576*(15*a^3*e^9/b^3 + 2*(4*(6*e^3*x^3 + a*e^3/b)
*e^3*x^3 - 5*a^2*e^6/b^2)*e^3*x^3)*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*B*b^2
*x*abs(e)^2/e^13
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,\sqrt{e\,x}\,{\left(b\,x^3+a\right)}^{5/2}\,dx$$

input
```
int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(5/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.65

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\frac{\sqrt{e}\left(558\sqrt{x}\,\sqrt{b\,x^3+a}\,a^3bx+652\sqrt{x}\,\sqrt{b\,x^3+a}\,a^2b^2x^4+400\sqrt{x}\,\sqrt{b\,x^3+a}\,a\,b^3x^7+96\sqrt{x}\,\sqrt{b\,x^3+a}\cdots\right.}{115\cdots}$$

input
```
int((e*x)^(1/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

output

```
(sqrt(e)*(558*sqrt(x)*sqrt(a + b*x**3)*a**3*b*x + 652*sqrt(x)*sqrt(a + b*x
**3)*a**2*b**2*x**4 + 400*sqrt(x)*sqrt(a + b*x**3)*a*b**3*x**7 + 96*sqrt(x
)*sqrt(a + b*x**3)*b**4*x**10 - 105*sqrt(b)*log(sqrt(a + b*x**3) - sqrt(x)
*sqrt(b)*x)*a**4 + 105*sqrt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)*x)*a
**4))/(1152*b)
```

3.261 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 364

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{27a^2(22Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{1408be} + \frac{3a(22Ab-aB)\sqrt{ex}(a+bx^3)^{3/2}}{352be} + \frac{(22Ab-aB)\sqrt{ex}(a+bx^3)^{5/2}}{176be} + \frac{B\sqrt{ex}(a+bx^3)^{7/2}}{11be} + \frac{27\,3^{3/4}a^{8/3}(22Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\right.}{2816be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
27/1408*a^2*(22*A*b-B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b/e+3/352*a*(22*A*b-B
*a)*(e*x)^(1/2)*(b*x^3+a)^(3/2)/b/e+1/176*(22*A*b-B*a)*(e*x)^(1/2)*(b*x^3+
a)^(5/2)/b/e+1/11*B*(e*x)^(1/2)*(b*x^3+a)^(7/2)/b/e+27/2816*3^(3/4)*a^(8/3
)*(22*A*b-B*a)*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arc
cos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6
^(1/2)+1/4*2^(1/2))/b/e/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2)
)*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.23

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{x\sqrt{a+bx^3}\left(B(a+bx^3)^3-\frac{a^2(-22Ab+aB)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{6},\frac{7}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{11b\sqrt{ex}}$$

input
```
Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/Sqrt[e*x],x]
```

output
```
(x*Sqrt[a + b*x^3]*(B*(a + b*x^3)^3 - (a^2*(-22*A*b + a*B)*Hypergeometric2
F1[-5/2, 1/6, 7/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(11*b*Sqrt[e*x])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {959, 811, 811, 811, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(22Ab-aB)\int \frac{(bx^3+a)^{5/2}}{\sqrt{ex}}dx}{22b}+\frac{B\sqrt{ex}(a+bx^3)^{7/2}}{11be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(22Ab-aB)\left(\frac{15}{16}a\int \frac{(bx^3+a)^{3/2}}{\sqrt{ex}}dx+\frac{\sqrt{ex}(a+bx^3)^{5/2}}{8e}\right)}{22b}+\frac{B\sqrt{ex}(a+bx^3)^{7/2}}{11be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(22Ab-aB)\left(\frac{15}{16}a\left(\frac{9}{10}a\int\frac{\sqrt{bx^3+a}}{\sqrt{ex}}dx+\frac{\sqrt{ex}(a+bx^3)^{3/2}}{5e}\right)+\frac{\sqrt{ex}(a+bx^3)^{5/2}}{8e}\right)}{22b}+\frac{B\sqrt{ex}\left(a+bx^3\right)^{7/2}}{11be}$$

$\downarrow$ 811

$$\frac{(22Ab-aB)\left(\frac{15}{16}a\left(\frac{9}{10}a\left(\frac{3}{4}a\int\frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)+\frac{\sqrt{ex}(a+bx^3)^{3/2}}{5e}\right)+\frac{\sqrt{ex}(a+bx^3)^{5/2}}{8e}\right)}{22b}+\frac{B\sqrt{ex}\left(a+bx^3\right)^{7/2}}{11be}$$

$\downarrow$ 851

$$\frac{(22Ab-aB)\left(\frac{15}{16}a\left(\frac{9}{10}a\left(\frac{3a\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2e}+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)+\frac{\sqrt{ex}(a+bx^3)^{3/2}}{5e}\right)+\frac{\sqrt{ex}(a+bx^3)^{5/2}}{8e}\right)}{22b}+\frac{B\sqrt{ex}\left(a+bx^3\right)^{7/2}}{11be}$$

$\downarrow$ 766

$$\frac{(22Ab-aB)\left(\frac{15}{16}a\left(\frac{9}{10}a\left(\frac{3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\ldots\right)}{4e^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\ldots\right.\right.\right.}{22b}$$

$$\frac{B\sqrt{ex}\left(a+bx^3\right)^{7/2}}{11be}$$

input
```
Int[((a + b*x^3)^(5/2)*(A + B*x^3))/Sqrt[e*x],x]
```

output
```
(B*Sqrt[e*x]*(a + b*x^3)^(7/2))/(11*b*e) + ((22*A*b - a*B)*((Sqrt[e*x]*(a
+ b*x^3)^(5/2))/(8*e) + (15*a*((Sqrt[e*x]*(a + b*x^3)^(3/2))/(5*e) + (9*a*
((Sqrt[e*x]*Sqrt[a + b*x^3])/(2*e) + (3^(3/4)*a^(2/3)*Sqrt[e*x]*(a^(1/3)*e
 + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^
2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e
+ (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2
+ Sqrt[3])/4])/(4*e^2*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3
)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/10))/16))/(22*b)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.95 (sec) , antiderivative size = 792, normalized size of antiderivative = 2.18

method	result
risch	$\frac{(128b^3 B x^9+176A b^3x^6+376Ba b^2x^6+616aA b^2x^3+356B a^2b x^3+1034a^2bA+81a^3 B)x\sqrt{b x^3+a}}{1408b\sqrt{ex}} + \frac{81a^3(22Ab-Ba)\left(\frac{(-a b^2)^{\frac{1}{3}}}{2b}\right.}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1408/b*(128*B*b^3*x^9+176*A*b^3*x^6+376*B*a*b^2*x^6+616*A*a*b^2*x^3+356*
B*a^2*b*x^3+1034*A*a^2*b+81*B*a^3)*x*(b*x^3+a)^(1/2)/(e*x)^(1/2)+81/1408*a
^3*(22*A*b-B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-
a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*((b*x^3+a)*e*x)^(1/2)/(e*x)
^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^
2)*sqrt(b*x^3 + a)*sqrt(e*x)/(e*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.46 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{Aa^{\frac{5}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{7}{6}\right)} + \frac{2Aa^{\frac{3}{2}}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{13}{6}\right)} + \frac{A\sqrt{a}b^2x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{19}{6}\right)} + \frac{Ba^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{13}{6}\right)} + \frac{2Ba^{\frac{3}{2}}bx^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{19}{6}\right)} + \frac{B\sqrt{a}b^2x^{\frac{19}{2}}\Gamma\left(\frac{19}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{19}{6}\\ \frac{25}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{25}{6}\right)}$$

input
```
integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(1/2),x)
```

output
```
A*a**(5/2)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(e)*gamma(7/6)) + 2*A*a**(3/2)*b*x**(7/2)*gamma(7/6)*hyper
((-1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6)) +
 A*sqrt(a)*b**2*x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*
exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(19/6)) + B*a**(5/2)*x**(7/2)*gamma(7/6
)*hyper((-1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(1
3/6)) + 2*B*a**(3/2)*b*x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,),
b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(19/6)) + B*sqrt(a)*b**2*x**(19/
2)*gamma(19/6)*hyper((-1/2, 19/6), (25/6,), b*x**3*exp_polar(I*pi)/a)/(3*s
qrt(e)*gamma(25/6))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/sqrt(e*x), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/sqrt(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{5/2}}{\sqrt{e\,x}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(1/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{\sqrt{e}\left(2230\sqrt{x}\,\sqrt{b\,x^3+a}\,a^3+1944\sqrt{x}\,\sqrt{b\,x^3+a}\,a^2b\,x^3+1104\sqrt{x}\,\sqrt{b\,x^3+a}\right.}{2816e}$$

input
```
int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x)
```

output
```
(sqrt(e)*(2230*sqrt(x)*sqrt(a + b*x**3)*a**3 + 1944*sqrt(x)*sqrt(a + b*x**
3)*a**2*b*x**3 + 1104*sqrt(x)*sqrt(a + b*x**3)*a*b**2*x**6 + 256*sqrt(x)*s
qrt(a + b*x**3)*b**3*x**9 + 1701*int((sqrt(x)*sqrt(a + b*x**3))/(a*x + b*x
**4),x)*a**4))/(2816*e)
```

3.262 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 650

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4}$$
$$+\frac{81\left(1+\sqrt{3}\right)a^2(20Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{448b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}+\frac{3(20Ab+aB)(ex)^{5/2}(a+bx^3)^{3/2}}{28e^4}$$
$$+\frac{(20Ab+aB)(ex)^{5/2}(a+bx^3)^{5/2}}{10ae^4}-\frac{2A(a+bx^3)^{7/2}}{ae\sqrt{ex}}$$
$$-\frac{81\sqrt[4]{3}a^{7/3}(20Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right.\right.}{448b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{27\ 3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}(20Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{\right.}\right.}{\sqrt[3]{a}+\left(1+\sqrt{\right.}\right.}\right.\right.}{896b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
27/224*a*(20*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/e^4+81/448*(1+3^(1/2))*a
^2*(20*A*b+B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(2/3)/e^2/(a^(1/3)+(1+3^(1/2
))*b^(1/3)*x)+3/28*(20*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/e^4+1/10*(20*A
*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(5/2)/a/e^4-2*A*(b*x^3+a)^(7/2)/a/e/(e*x)^(1
/2)-81/448*3^(1/4)*a^(7/3)*(20*A*b+B*a)*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^
(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/2))*
b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/b^(2/3)/e^2/(b^(1/3)*x*(a^(1/
3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-27/
896*3^(3/4)*(1-3^(1/2))*a^(7/3)*(20*A*b+B*a)*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*
x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x
)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)
+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))/b^(2/3)/e^2/(b^(1/3)*x*(
a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.13

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{2x\sqrt{a+bx^3}\left(-5A(a+bx^3)^3+\frac{a^2(20Ab+aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{5a(ex)^{3/2}}$$

input
```
Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(-5*A*(a + b*x^3)^3 + (a^2*(20*A*b + a*B)*x^3*Hyperge
ometric2F1[-5/2, 5/6, 11/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*a*(e*x
)^(3/2))
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {955, 811, 811, 811, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{3/2}}dx$$

$$\Big\downarrow \text{ 955}$$

$$\frac{(aB+20Ab)\int (ex)^{3/2}\left(bx^3+a\right)^{5/2}dx}{ae^3}-\frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{(aB+20Ab)\left(\frac{3}{4}a\int (ex)^{3/2}\left(bx^3+a\right)^{3/2}dx+\frac{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{(aB+20Ab)\left(\frac{3}{4}a\left(\frac{9}{14}a\int (ex)^{3/2}\sqrt{bx^3+a}dx+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{(aB+20Ab)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3}{8}a\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{(aB+20Ab)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3a\int\frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}}$$

$\downarrow$ 837

$$\frac{(aB+20Ab)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3a\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int-\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}(}{}\right.\right.}{ae^3}$$

$$\frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}}$$

$\downarrow$ 25

$$\frac{(aB+20Ab)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}(a+}{7e}\right.\right.}{ae^3}$$

$$\frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}}$$

$\downarrow$ 766

$$\frac{(aB+20Ab)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{Ellip}}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e\right.}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right.\right.}}}}{4e}}{ae^3}$$

$$\frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \; 2420$$

$$(aB+20Ab)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(3a\left(\frac{\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{2b^{2/3}}\right.\right.\right.\right.$$

$$\frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}}$$

input
```
Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

output
```
(-2*A*(a + b*x^3)^(7/2))/(a*e*Sqrt[e*x]) + ((20*A*b + a*B)*(((e*x)^(5/2)*(
a + b*x^3)^(5/2))/(10*e) + (3*a*(((e*x)^(5/2)*(a + b*x^3)^(3/2))/(7*e) + (
9*a*(((e*x)^(5/2)*Sqrt[a + b*x^3])/(4*e) + (3*a*((((1 + Sqrt[3])*e^3*Sqrt[
e*x]*Sqrt[a + b*x^3])/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x) - (3^(1/4)*a
^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b
^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]
*EllipticE[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1
+ Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*e*x*(a^(1/3)*e
+ b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3]
))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e
*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*
e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[
3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4
])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3
)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(4*e)))/14))/4))/(a
*e^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.57 (sec) , antiderivative size = 1166, normalized size of antiderivative = 1.79

method	result	size
risch	Expression too large to display	1166
elliptic	Expression too large to display	1341
default	Expression too large to display	6530

input
```
int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1120*(b*x^3+a)^(1/2)*(-112*B*b^2*x^9-160*A*b^2*x^6-344*B*a*b*x^6-620*A*
a*b*x^3-367*B*a^2*x^3+2240*A*a^2)/e/(e*x)^(1/2)+81/448*a^2*(20*A*b+B*a)*(x
*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^
(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a
*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*
b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2
)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*E
llipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/...
```

Fricas [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^
2)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.52 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.48

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{Aa^{\frac{5}{2}}\Gamma\left(-\frac{1}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{6} \\ \frac{5}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{6}\right)} + \frac{2Aa^{\frac{3}{2}}bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{6} \\ \frac{11}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)} + \frac{A\sqrt{a}b^2x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{11}{6} \\ \frac{17}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)} + \frac{Ba^{\frac{5}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{6} \\ \frac{11}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)} + \frac{2Ba^{\frac{3}{2}}bx^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{11}{6} \\ \frac{17}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)} + \frac{B\sqrt{a}b^2x^{\frac{17}{2}}\Gamma\left(\frac{17}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{17}{6} \\ \frac{23}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{23}{6}\right)}$$

input
```
integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(3/2),x)
```

output
```
A*a**(5/2)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/
a)/(3*e**(3/2)*sqrt(x)*gamma(5/6)) + 2*A*a**(3/2)*b*x**(5/2)*gamma(5/6)*hy
per((-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(11/6
)) + A*sqrt(a)*b**2*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b*x
**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(17/6)) + B*a**(5/2)*x**(5/2)*gamm
a(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*g
amma(11/6)) + 2*B*a**(3/2)*b*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17
/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(17/6)) + B*sqrt(a)*b**2*
x**(17/2)*gamma(17/6)*hyper((-1/2, 17/6), (23/6,), b*x**3*exp_polar(I*pi)/
a)/(3*e**(3/2)*gamma(23/6))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{5/2}}{(e\,x)^{3/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(3/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{\sqrt{e}\left(1150\sqrt{b\,x^3+a}\,a^3+564\sqrt{b\,x^3+a}\,a^2b\,x^3+288\sqrt{b\,x^3+a}\,a\,b^2x^6+64\sqrt{}\right.}{640\sqrt{x}\,e^2}$$

input
```
int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x)
```

output
```
(sqrt(e)*(1150*sqrt(a + b*x**3)*a**3 + 564*sqrt(a + b*x**3)*a**2*b*x**3 +
288*sqrt(a + b*x**3)*a*b**2*x**6 + 64*sqrt(a + b*x**3)*b**3*x**9 + 1215*sq
rt(x)*int((sqrt(x)*sqrt(a + b*x**3))/(a*x**2 + b*x**5),x)*a**4))/(640*sqrt
(x)*e**2)
```

3.263 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 188

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4} + \frac{5(6Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{36e^4} + \frac{(6Ab+aB)(ex)^{3/2}(a+bx^3)^{5/2}}{9ae^4} - \frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}} + \frac{5a^2(6Ab+aB)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24\sqrt{b}e^{5/2}}$$

output

```
5/24*a*(6*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/e^4+5/36*(6*A*b+B*a)*(e*x)^
(3/2)*(b*x^3+a)^(3/2)/e^4+1/9*(6*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(5/2)/a/e^
4-2/3*A*(b*x^3+a)^(7/2)/a/e/(e*x)^(3/2)+5/24*a^2*(6*A*b+B*a)*arctanh(b^(1/
2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a)^(1/2))/b^(1/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \frac{x\left(\sqrt{b}\sqrt{a+bx^3}(4b^2x^6(3A+2Bx^3)+a^2(-48A+33Bx^3)+a(54Abx^3+2\right.}{72\sqrt{b}(ex)^{5/2}}$$

input
```
Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

output
```
(x*(Sqrt[b]*Sqrt[a + b*x^3]*(4*b^2*x^6*(3*A + 2*B*x^3) + a^2*(-48*A + 33*B
*x^3) + a*(54*A*b*x^3 + 26*b*B*x^6)) + 15*a^2*(6*A*b + a*B)*x^(3/2)*Log[Sq
rt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(72*Sqrt[b]*(e*x)^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {955, 811, 811, 811, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(aB+6Ab)\int \sqrt{ex}(bx^3+a)^{5/2}dx}{ae^3} - \frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{811}$$

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\int \sqrt{ex}(bx^3+a)^{3/2}dx + \frac{(ex)^{3/2}(a+bx^3)^{5/2}}{9e}\right)}{ae^3} - \frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{811}$$

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\int\sqrt{ex}\sqrt{bx^3+a}dx+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{5/2}}{9e}\right)}{ae^3}-\frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}}$$

$\downarrow$ 811

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{5/2}}{9e}\right)}{ae^3}-\frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}}$$

$\downarrow$ 851

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\int\frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{5/2}}{9e}\right)}{ae^3}-\frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}}$$

$\downarrow$ 807

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\int\frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{5/2}}{9e}\right)}{ae^3}-\frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}}$$

$\downarrow$ 224

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\int\frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{5/2}}{9e}\right)}{ae^3}-\frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}}$$

$\downarrow$ 219

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3\sqrt{b}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{5/2}}{9e}\right)}{ae^3}-\frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}}$$

input
```
Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

output
```
(-2*A*(a + b*x^3)^(7/2))/(3*a*e*(e*x)^(3/2)) + ((6*A*b + a*B)*(((e*x)^(3/2
)*(a + b*x^3)^(5/2))/(9*e) + (5*a*(((e*x)^(3/2)*(a + b*x^3)^(3/2))/(6*e) +
 (3*a*(((e*x)^(3/2)*Sqrt[a + b*x^3])/(3*e) + (a*Sqrt[e]*ArcTanh[(Sqrt[b]*(
e*x)^(3/2))/(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*Sqrt[b])))/4))/6))/(a*e^3)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-8 b^{2} B\,x^{9}-12 A\,b^{2} x^{6}-26 B a b\,x^{6}-54 a A b\,x^{3}-33 B\,a^{2} x^{3}+48 a^{2} A\right)}{72 x\,e^{2} \sqrt{e x}}+\frac{5 a^{2} \left(6 A b+B a\right) \operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right) e x}}{x^{2} \sqrt{b e}}\right) \sqrt{\left(b\,x^{3}+ \dots}{24 \sqrt{b e}\,e^{2} \sqrt{e x}\,\sqrt{b\,x^{3}+a}}$
default	$\frac{\sqrt{b\,x^{3}+a}\,\left(8 B \sqrt{b e}\,\sqrt{\left(b\,x^{3}+a\right) e x}\,b^{2} x^{9}+12 A \sqrt{b e}\,\sqrt{\left(b\,x^{3}+a\right) e x}\,b^{2} x^{6}+26 B \sqrt{b e}\,\sqrt{\left(b\,x^{3}+a\right) e x}\,a b\,x^{6}+90 A \operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right) e x}}{x^{2} \sqrt{b e}}\right) a^{2} \dots}{72 x\,e^{2} \sqrt{e x}\,\sqrt{\dots}}$
elliptic	Expression too large to display

input
```
int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/72*(b*x^3+a)^(1/2)*(-8*B*b^2*x^9-12*A*b^2*x^6-26*B*a*b*x^6-54*A*a*b*x^3
-33*B*a^2*x^3+48*A*a^2)/x/e^2/(e*x)^(1/2)+5/24*a^2*(6*A*b+B*a)/(b*e)^(1/2)
*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))/e^2*((b*x^3+a)*e*x)^(1/2)/
(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \left[\frac{15\,(Ba^3+6\,Aa^2b)\sqrt{be}x^2\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,bx^4+ax)\right.}{}\right.$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/288*(15*(B*a^3 + 6*A*a^2*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^
3 - a^2*e - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(8*
B*b^3*x^9 + 2*(13*B*a*b^2 + 6*A*b^3)*x^6 - 48*A*a^2*b + 3*(11*B*a^2*b + 18
*A*a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2), -1/144*(15*(B*a^3 +
 6*A*a^2*b)*sqrt(-b*e)*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b*e)*sqrt(e*x)*x
/(2*b*e*x^3 + a*e)) - 2*(8*B*b^3*x^9 + 2*(13*B*a*b^2 + 6*A*b^3)*x^6 - 48*A
*a^2*b + 3*(11*B*a^2*b + 18*A*a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^
3*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(180) = 360.

Time = 49.34 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.14

$$\begin{aligned}\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx &= -\frac{2Aa^{\frac{5}{2}}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{2Aa^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}} \\ &- \frac{7Aa^{\frac{3}{2}}bx^{\frac{3}{2}}}{12e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{A\sqrt{a}b^2x^{\frac{9}{2}}}{4e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{5Aa^2\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{4e^{\frac{5}{2}}} \\ &+ \frac{Ab^3x^{\frac{15}{2}}}{6\sqrt{a}e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{Ba^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}} + \frac{Ba^{\frac{5}{2}}x^{\frac{3}{2}}}{8e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{35Ba^{\frac{3}{2}}bx^{\frac{9}{2}}}{72e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} \\ &+ \frac{17B\sqrt{a}b^2x^{\frac{15}{2}}}{36e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{5Ba^3\,\mathrm{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{24\sqrt{b}e^{\frac{5}{2}}} + \frac{Bb^3x^{\frac{21}{2}}}{9\sqrt{a}e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}\end{aligned}$$

input
```
integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(5/2),x)
```

output
```
-2*A*a**(5/2)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*A*a**(3/2)*b*x*
*(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) - 7*A*a**(3/2)*b*x**(3/2)/(12*e**(5
/2)*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b**2*x**(9/2)/(4*e**(5/2)*sqrt(1 + b*x
**3/a)) + 5*A*a**2*sqrt(b)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*e**(5/2)) +
A*b**3*x**(15/2)/(6*sqrt(a)*e**(5/2)*sqrt(1 + b*x**3/a)) + B*a**(5/2)*x**(
3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) + B*a**(5/2)*x**(3/2)/(8*e**(5/2)*sqr
t(1 + b*x**3/a)) + 35*B*a**(3/2)*b*x**(9/2)/(72*e**(5/2)*sqrt(1 + b*x**3/a
)) + 17*B*sqrt(a)*b**2*x**(15/2)/(36*e**(5/2)*sqrt(1 + b*x**3/a)) + 5*B*a*
*3*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(24*sqrt(b)*e**(5/2)) + B*b**3*x**(21/2
)/(9*sqrt(a)*e**(5/2)*sqrt(1 + b*x**3/a))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: Recu
rsive assumption sageVARa>=(-sageVARb*sageVARe/(sageVARe^4*t_nostep^6)) ig
nored1/sageVARe^3/((1/sageVARe)^2)*2*((8870400*sageVARb^12*sageVARe^14*sag
eVARB/159667200/sag
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{5/2}}{(e\,x)^{5/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(5/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \frac{\sqrt{e}\left(-96\sqrt{b\,x^3+a}\,a^3+174\sqrt{b\,x^3+a}\,a^2b\,x^3+76\sqrt{b\,x^3+a}\,a\,b^2x^6+16\sqrt{b}\right.}{}$$

input
```
int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x)
```

output
```
(sqrt(e)*( - 96*sqrt(a + b*x**3)*a**3 + 174*sqrt(a + b*x**3)*a**2*b*x**3 +
 76*sqrt(a + b*x**3)*a*b**2*x**6 + 16*sqrt(a + b*x**3)*b**3*x**9 - 105*sqr
t(x)*sqrt(b)*log(sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*x)*a**3*x + 105*sqrt(x
)*sqrt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)*x)*a**3*x))/(144*sqrt(x)*
e**3*x)
```

3.264 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 352

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{27a(16Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{320e^4} + \frac{3(16Ab+5aB)\sqrt{ex}(a+bx^3)^{3/2}}{80e^4} + \frac{(16Ab+5aB)\sqrt{ex}(a+bx^3)^{5/2}}{40ae^4} - \frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}} + \frac{27\ 3^{3/4}a^{5/3}(16Ab+5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)\right.}{640e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
27/320*a*(16*A*b+5*B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/e^4+3/80*(16*A*b+5*B*a
)*(e*x)^(1/2)*(b*x^3+a)^(3/2)/e^4+1/40*(16*A*b+5*B*a)*(e*x)^(1/2)*(b*x^3+a
)^(5/2)/a/e^4-2/5*A*(b*x^3+a)^(7/2)/a/e/(e*x)^(5/2)+27/640*3^(3/4)*a^(5/3)
*(16*A*b+5*B*a)*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(ar
ccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*
6^(1/2)+1/4*2^(1/2))/e^4/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2
))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.25

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{2x\sqrt{a+bx^3}\left(-A(a+bx^3)^3+\frac{a^2(16Ab+5aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{6},\frac{7}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{5a(ex)^{7/2}}$$

input
```
Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(-(A*(a + b*x^3)^3) + (a^2*(16*A*b + 5*a*B)*x^3*Hyper
geometric2F1[-5/2, 1/6, 7/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*a*(e*
x)^(7/2))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {955, 811, 811, 811, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(5aB+16Ab)\int \frac{(bx^3+a)^{5/2}}{\sqrt{ex}}dx}{5ae^3}-\frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{811}$$

$$\frac{(5aB+16Ab)\left(\frac{15}{16}a\int \frac{(bx^3+a)^{3/2}}{\sqrt{ex}}dx+\frac{\sqrt{ex}(a+bx^3)^{5/2}}{8e}\right)}{5ae^3}-\frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{811}$$

$$\frac{(5aB+16Ab)\left(\frac{15}{16}a\left(\frac{9}{10}a\int\frac{\sqrt{bx^3+a}}{\sqrt{ex}}dx+\frac{\sqrt{ex}(a+bx^3)^{3/2}}{5e}\right)+\frac{\sqrt{ex}(a+bx^3)^{5/2}}{8e}\right)}{5ae^3}-\frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}}$$

$\downarrow$ 811

$$\frac{(5aB+16Ab)\left(\frac{15}{16}a\left(\frac{9}{10}a\left(\frac{3}{4}a\int\frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)+\frac{\sqrt{ex}(a+bx^3)^{3/2}}{5e}\right)+\frac{\sqrt{ex}(a+bx^3)^{5/2}}{8e}\right)}{5ae^3}-\frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}}$$

$\downarrow$ 851

$$\frac{(5aB+16Ab)\left(\frac{15}{16}a\left(\frac{9}{10}a\left(\frac{3a\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2e}+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)+\frac{\sqrt{ex}(a+bx^3)^{3/2}}{5e}\right)+\frac{\sqrt{ex}(a+bx^3)^{5/2}}{8e}\right)}{5ae^3}-\frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}}$$

$\downarrow$ 766

$$\frac{(5aB+16Ab)\left(\frac{15}{16}a\left(\frac{9}{10}a\left(\frac{3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right)\right.}{4e^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right.\right.\right.}{5ae^3}$$

$$\frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}}$$

input
```
Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

output
```
(-2*A*(a + b*x^3)^(7/2))/(5*a*e*(e*x)^(5/2)) + ((16*A*b + 5*a*B)*((Sqrt[e*
x]*(a + b*x^3)^(5/2))/(8*e) + (15*a*((Sqrt[e*x]*(a + b*x^3)^(3/2))/(5*e) +
 (9*a*((Sqrt[e*x]*Sqrt[a + b*x^3])/(2*e) + (3^(3/4)*a^(2/3)*Sqrt[e*x]*(a^(
1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*
e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1
/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)
], (2 + Sqrt[3])/4])/(4*e^2*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(
a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/10))/16))/(5*
a*e^3)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.82 (sec) , antiderivative size = 786, normalized size of antiderivative = 2.23

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-40 b^{2} B\,x^{9}-64 A\,b^{2} x^{6}-140 B a b\,x^{6}-368 a A b\,x^{3}-235 B\,a^{2} x^{3}+128 a^{2} A\right)}{320 x^{2} e^{3} \sqrt{e x}}+\frac{81 a^{2} \left(16 A b+5 B a\right) \left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\,\left(-a\,b^{2}\right.}{2 b}\right.}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/320*(b*x^3+a)^(1/2)*(-40*B*b^2*x^9-64*A*b^2*x^6-140*B*a*b*x^6-368*A*a*b
*x^3-235*B*a^2*x^3+128*A*a^2)/x^2/e^3/(e*x)^(1/2)+81/320*a^2*(16*A*b+5*B*a
)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^
2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)
^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x
-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*b/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))/e^3*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(
b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="fricas")
```

output
```
integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^
2)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^4*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 49.85 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{Aa^{\frac{5}{2}}\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},-\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)} + \frac{2Aa^{\frac{3}{2}}b\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)} + \frac{A\sqrt{a}b^2x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{13}{6}\right)} + \frac{Ba^{\frac{5}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)} + \frac{2Ba^{\frac{3}{2}}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{13}{6}\right)} + \frac{B\sqrt{a}b^2x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{19}{6}\right)}$$

input
```
integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(7/2),x)
```

output
```
A*a**(5/2)*gamma(-5/6)*hyper((-5/6, -1/2), (1/6,), b*x**3*exp_polar(I*pi)/
a)/(3*e**(7/2)*x**(5/2)*gamma(1/6)) + 2*A*a**(3/2)*b*sqrt(x)*gamma(1/6)*hy
per((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/6))
 + A*sqrt(a)*b**2*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*e
xp_polar(I*pi)/a)/(3*e**(7/2)*gamma(13/6)) + B*a**(5/2)*sqrt(x)*gamma(1/6)
*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/
6)) + 2*B*a**(3/2)*b*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**
3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(13/6)) + B*sqrt(a)*b**2*x**(13/2)*g
amma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7
/2)*gamma(19/6))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{5/2}}{(e\,x)^{7/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(7/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{\sqrt{e}\left(-3914\sqrt{b\,x^3+a}\,a^3+2412\sqrt{b\,x^3+a}\,a^2b\,x^3+816\sqrt{b\,x^3+a}\,a\,b^2x^6+\right.}{1280\sqrt{x}\,e^4x^2}$$

input
```
int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x)
```

output
```
(sqrt(e)*( - 3914*sqrt(a + b*x**3)*a**3 + 2412*sqrt(a + b*x**3)*a**2*b*x**
3 + 816*sqrt(a + b*x**3)*a*b**2*x**6 + 160*sqrt(a + b*x**3)*b**3*x**9 - 85
05*sqrt(x)*int((sqrt(x)*sqrt(a + b*x**3))/(a*x**4 + b*x**7),x)*a**4*x**2))
/(1280*sqrt(x)*e**4*x**2)
```

3.265 $\int \frac{(ex)^{7/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{a(4Ab-3aB)e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{5/2}}$$

output
```
1/12*(4*A*b-3*B*a)*e^2*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b^2+1/6*B*(e*x)^(9/2)*(
b*x^3+a)^(1/2)/b/e-1/12*a*(4*A*b-3*B*a)*e^(7/2)*arctanh(b^(1/2)*(e*x)^(3/2
)/e^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{(ex)^{7/2}\sqrt{a+bx^3}(4Ab-3aB+2bBx^3)}{12b^2x^2} + \frac{a(-4Ab+3aB)(ex)^{7/2}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{12b^{5/2}x^{7/2}}$$

input
```
Integrate[((e*x)^(7/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
((e*x)^(7/2)*Sqrt[a + b*x^3]*(4*A*b - 3*a*B + 2*b*B*x^3))/(12*b^2*x^2) + (
a*(-4*A*b + 3*a*B)*(e*x)^(7/2)*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/(12
*b^(5/2)*x^(7/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {959, 843, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(4Ab-3aB)\int \frac{(ex)^{7/2}}{\sqrt{bx^3+a}}dx}{4b}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

$$\Big\downarrow \text{843}$$

$$\frac{(4Ab-3aB)\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^3\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx}{2b}\right)}{4b}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(4Ab-3aB)\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{b}\right)}{4b}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

$$\Big\downarrow \text{807}$$

$$\frac{(4Ab-3aB)\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3b}\right)}{4b}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

$$\Big\downarrow \text{224}$$

$$\frac{(4Ab-3aB)\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3b}\right)}{4b}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

$$\Big\downarrow \text{219}$$

$$\frac{(4Ab-3aB)\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3b^{3/2}}\right)}{4b}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

input
```
Int[((e*x)^(7/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(B*(e*x)^(9/2)*Sqrt[a + b*x^3])/(6*b*e) + ((4*A*b - 3*a*B)*((e^2*(e*x)^(3/
2)*Sqrt[a + b*x^3])/(3*b) - (a*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3
/2)*Sqrt[a + (b*x)/e^2])])/(3*b^(3/2))))/(4*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

method	result
risch	$\frac{x^{2}\left(2 b B\,x^{3}+4 A b -3 B a \right) \sqrt{b\,x^{3}+a}\, e^{4}}{12 b^{2} \sqrt{e x}}-\frac{a \left(4 A b -3 B a \right) \operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) e^{4} \sqrt{\left(b\,x^{3}+a \right) e x}}{12 b^{2} \sqrt{b e}\, \sqrt{e x}\, \sqrt{b\,x^{3}+a}}$
default	$-\frac{e^{3} \sqrt{e x}\, \sqrt{b\,x^{3}+a}\, \left(-2 B \sqrt{\left(b\,x^{3}+a \right) e x}\, \sqrt{b e}\, b\,x^{4}+4 A \operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) a b e -4 A \sqrt{\left(b\,x^{3}+a \right) e x}\, \sqrt{b e}\, b x -3 B \operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3} \ldots}{x^{2} \ldots}\right.\right.}{12 \sqrt{\left(b\,x^{3}+a \right) e x}\, b^{2} \sqrt{b e}}$
elliptic	Expression too large to display

input
```
int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*x^2*(2*B*b*x^3+4*A*b-3*B*a)*(b*x^3+a)^(1/2)/b^2*e^4/(e*x)^(1/2)-1/12*
a*(4*A*b-3*B*a)/b^2/(b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1
/2))*e^4*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \left[-\frac{(3\,Ba^2-4\,Aab)e^3\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e+4\,(2\,b^2x^4+abx)\sqrt{bx^3}\right.}{48\,b^2}\right.$$

$$\left.-\frac{(3\,Ba^2-4\,Aab)e^3\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{ex}bx\sqrt{-\frac{e}{b}}}{2\,bex^3+ae}\right)-2\,(2\,Bbe^3x^4-(3\,Ba-4\,Ab)e^3x)\sqrt{bx^3+a}\sqrt{ex}}{24\,b^2}\right]$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*((3*B*a^2 - 4*A*a*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 -
 a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(2
*B*b*e^3*x^4 - (3*B*a - 4*A*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/2
4*((3*B*a^2 - 4*A*a*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b
*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(2*B*b*e^3*x^4 - (3*B*a - 4*A*b)*e^3*
x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2]
```

Sympy [A] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.60

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \begin{cases} \frac{2\left(\begin{cases}\text{NaN} \\ \frac{\begin{cases} -\frac{ae^3\left(Ae^3-\frac{3Bae^3}{4b}\right)\left(\begin{cases}\frac{\log\left(\frac{2b(ex)^{\frac{3}{2}}}{e^3}+2\sqrt{\frac{b}{e^3}}\sqrt{a+bx^3}\right)}{\sqrt{\frac{b}{e^3}}} & \text{for } a\neq 0 \\ \frac{(ex)^{\frac{3}{2}}\log\left((ex)^{\frac{3}{2}}\right)}{\sqrt{bx^3}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{a+bx^3}\Big(\\ \frac{\frac{Ae^3(ex)^{\frac{9}{2}}}{3}+\frac{B(ex)^{\frac{15}{2}}}{5}}{\sqrt{a}}\end{cases}}{3e^3}\end{cases}\right.}{e} \\ 0\end{cases}$$

input
```
integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((2*Piecewise((nan, Eq(e**3, 0)), (Piecewise((-a*e**3*(A*e**3 - 3
*B*a*e**3/(4*b))*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqr
t(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sq
rt(b*x**3), True))/(2*b) + sqrt(a + b*x**3)*(B*e**3*(e*x)**(9/2)/(4*b) + e
**3*(e*x)**(3/2)*(A*e**3 - 3*B*a*e**3/(4*b))/(2*b)), Ne(b/e**3, 0)), ((A*e
**3*(e*x)**(9/2)/3 + B*(e*x)**(15/2)/5)/sqrt(a), True))/(3*e**3), True))/e
, Ne(e, 0)), (0, True))
```

Maxima [F]

$$\int \frac{(ex)^{7/2}\,(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{7}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^(7/2)/sqrt(b*x^3 + a), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.21

$$\int \frac{(ex)^{7/2}\,(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\frac{\sqrt{be^4x^3+ae^4}\sqrt{ex}e^5x\left(\frac{2\,Bx^3}{be^2}-\frac{3\,Bab^3e^5-4\,Ab^4e^5}{b^5e^7}\right)}{12\,|e|^2}-\frac{(3\,Ba^2b^3e^9-4\,Aab^4e^9)\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{12\,\sqrt{be}b^5e|e|^4}$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*e^5*x*(2*B*x^3/(b*e^2) - (3*B*a*b^3
*e^5 - 4*A*b^4*e^5)/(b^5*e^7))/abs(e)^2 - 1/12*(3*B*a^2*b^3*e^9 - 4*A*a*b^
4*e^9)*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*e^4)))/(sqrt(
b*e)*b^5*e*abs(e)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}\,(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(B\,x^3+A)\,(e\,x)^{7/2}}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(1/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.74

$$\int \frac{(ex)^{7/2}\,(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\frac{\sqrt{e}\,e^3\Big(2\sqrt{x}\,\sqrt{b\,x^3+a}\,abx+4\sqrt{x}\,\sqrt{b\,x^3+a}\,b^2x^4+\sqrt{b}\log\Big(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}}{24b^2}$$

input
```
int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(e)*e**3*(2*sqrt(x)*sqrt(a + b*x**3)*a*b*x + 4*sqrt(x)*sqrt(a + b*x**
3)*b**2*x**4 + sqrt(b)*log(sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*x)*a**2 - sq
rt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)*x)*a**2))/(24*b**2)
```

3.266 $\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 286

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{(10Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{20b^2} + \frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be} - \frac{a^{2/3}(10Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2\cdots}{40\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/20*(10*A*b-7*B*a)*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^2+1/5*B*(e*x)^(7/2)*
(b*x^3+a)^(1/2)/b/e-1/120*a^(2/3)*(10*A*b-7*B*a)*e^2*(e*x)^(1/2)*(a^(1/3)+
b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b
^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/
(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b^2/(b^(
1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3
+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.34

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{e^2\sqrt{ex}\left(-((a+bx^3)(-10Ab+7aB-4bBx^3))+a(-10Ab+7aB)\sqrt{1+\frac{bx^3}{a}}\,\mathrm{H}\right.}{20b^2\sqrt{a+bx^3}}$$

input
```
Integrate[((e*x)^(5/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(e^2*Sqrt[e*x]*(-((a + b*x^3)*(-10*A*b + 7*a*B - 4*b*B*x^3)) + a*(-10*A*b
+ 7*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)
]))/(20*b^2*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {959, 843, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(10Ab-7aB)\int \frac{(ex)^{5/2}}{\sqrt{bx^3+a}}dx}{10b}+\frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be}$$

$$\Big\downarrow \text{843}$$

$$\frac{(10Ab-7aB)\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{ae^3\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{4b}\right)}{10b}+\frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(10Ab-7aB)\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{ae^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b}\right)}{10b}+\frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be}$$

$$\Big\downarrow 766$$

$$\frac{(10Ab-7aB)\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{a^{2/3}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right)\right.}{4\sqrt[4]{3}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right.}{10b}$$

$$\frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be}$$

input
```
Int[((e*x)^(5/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(B*(e*x)^(7/2)*Sqrt[a + b*x^3])/(5*b*e) + ((10*A*b - 7*a*B)*((e^2*Sqrt[e*x
]*Sqrt[a + b*x^3])/(2*b) - (a^(2/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*
Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e +
(1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*
b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(
4*3^(1/4)*b*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 +
 Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(10*b)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.74 (sec) , antiderivative size = 753, normalized size of antiderivative = 2.63

method	result
risch	$\frac{(4bB\,x^3+10Ab-7Ba)x\sqrt{b\,x^3+a}\,e^3}{20b^2\sqrt{ex}} - \frac{a(10Ab-7Ba)\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right.}}}{}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^3+a)ex}\left(\frac{B\,e^2x^3\sqrt{be\,x^4+aex}}{5b}+\frac{\left(A\,e^3-\frac{7B\,e^3a}{10b}\right)\sqrt{be\,x^4+aex}}{2be}-\frac{\left(A\,e^3-\frac{7B\,e^3a}{10b}\right)a\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(}{}\right.}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}}}{}\right.}{}$
default	Expression too large to display

input
```
int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*(4*B*b*x^3+10*A*b-7*B*a)*x*(b*x^3+a)^(1/2)/b^2*e^3/(e*x)^(1/2)-1/20*a
*(10*A*b-7*B*a)/b*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*
(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*e^3*((b*x^3+a)*e*x)^(1/2)
/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*e^2*x^5 + A*e^2*x^2)*sqrt(e*x)/sqrt(b*x^3 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.96 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.33

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{Ae^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{13}{6}\right)} + \frac{Be^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{19}{6}\right)}$$

input
```
integrate((e*x)**(5/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
A*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((1/2, 7/6), (13/6,), b*x**3*exp_polar
(I*pi)/a)/(3*sqrt(a)*gamma(13/6)) + B*e**(5/2)*x**(13/2)*gamma(13/6)*hyper
((1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(19/6))
```

Maxima [F]

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^(5/2)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^(5/2)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(B\,x^3+A)\,(e\,x)^{5/2}}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(1/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\frac{\sqrt{e}\,e^2\left(6\sqrt{x}\,\sqrt{b\,x^3+a}\,a+8\sqrt{x}\,\sqrt{b\,x^3+a}\,b\,x^3-3\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^4+ax}dx\right)a^2\right)}{40b}$$

input
```
int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(e)*e**2*(6*sqrt(x)*sqrt(a + b*x**3)*a + 8*sqrt(x)*sqrt(a + b*x**3)*b
*x**3 - 3*int((sqrt(x)*sqrt(a + b*x**3))/(a*x + b*x**4),x)*a**2))/(40*b)
```

3.267 $\int \frac{(ex)^{3/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 543

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be} + \frac{\left(1+\sqrt{3}\right)(8Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}(8Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}(8Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)\right.}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/4*B*(e*x)^(5/2)*(b*x^3+a)^(1/2)/b/e+1/8*(1+3^(1/2))*(8*A*b-5*B*a)*e*(e*x
)^(1/2)*(b*x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)-1/8*3^(1/4
)*a^(1/3)*(8*A*b-5*B*a)*e*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*Elliptic
E((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))/b^(5/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(
1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-1/48*(1-3^(1/2))*a^(1
/3)*(8*A*b-5*B*a)*e*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiA
M(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),
1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b^(5/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a
^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\frac{x(ex)^{3/2}\left(5B(a+bx^3)+(8Ab-5aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\right.\right.}{20b\sqrt{a+bx^3}}$$

input
```
Integrate[((e*x)^(3/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(x*(e*x)^(3/2)*(5*B*(a + b*x^3) + (8*A*b - 5*a*B)*Sqrt[1 + (b*x^3)/a]*Hype
rgeometric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)]))/(20*b*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {959, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(8Ab-5aB)\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx}{8b}+\frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(8Ab-5aB)\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{4be}+\frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}$$

$$\Big\downarrow \text{837}$$

$$\frac{(8Ab-5aB)\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4be}+\frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}$$

$$\Big\downarrow \text{25}$$

$$\frac{(8Ab-5aB)\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4be}+\frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}$$

$$\Big\downarrow \text{766}$$

$$\frac{(8Ab-5aB)\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\text{arcco}\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right.}{4be}$$

$$\frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}$$

$$\Big\downarrow \text{2420}$$

$$\frac{(8Ab-5aB)\left(\frac{\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right)|\frac{1}{4}\left(\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{2b^{2/3}}\right.}{4b}$$

$$\frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}$$

input
```
Int[((e*x)^(3/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(B*(e*x)^(5/2)*Sqrt[a + b*x^3])/(4*b*e) + ((8*A*b - 5*a*B)*((((1 + Sqrt[3]
)*e^3*Sqrt[e*x]*Sqrt[a + b*x^3])/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x) -
 (3^(1/4)*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2
- a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1
/3)*e*x)^2]*EllipticE[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1
/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*e*x*
(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt
[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e
+ b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2
)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e +
 (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 +
 Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*
x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(4*b*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.55 (sec) , antiderivative size = 1124, normalized size of antiderivative = 2.07

method	result	size
risch	Expression too large to display	1124
elliptic	Expression too large to display	1128
default	Expression too large to display	4914

input
```
int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*B*x^3/b*(b*x^3+a)^(1/2)*e^2/(e*x)^(1/2)+1/8*(8*A*b-5*B*a)/b*(x*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^
(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/
3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3))
)^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE
(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),(...
```

Fricas [F]

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*e*x^4 + A*e*x)*sqrt(e*x)/sqrt(b*x^3 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.40 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.17

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{11}{6}\right)}+\frac{Be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{17}{6}\right)}$$

input
```
integrate((e*x)**(3/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
A*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((1/2, 5/6), (11/6,), b*x**3*exp_polar
(I*pi)/a)/(3*sqrt(a)*gamma(11/6)) + B*e**(3/2)*x**(11/2)*gamma(11/6)*hyper
((1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(17/6))
```

Maxima [F]

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^(3/2)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^(3/2)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(B\,x^3+A)\,(e\,x)^{3/2}}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(1/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt{e}\,e\left(2\sqrt{x}\,\sqrt{b\,x^3+a}\,x^2+3\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a\right)}{8}$$

input
```
int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(e)*e*(2*sqrt(x)*sqrt(a + b*x**3)*x**2 + 3*int((sqrt(x)*sqrt(a + b*x*
*3)*x)/(a + b*x**3),x)*a))/8
```

3.268 $\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 83

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}+\frac{(2Ab-aB)\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

output
```
1/3*B*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b/e+1/3*(2*A*b-B*a)*e^(1/2)*arctanh(b^(1
/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx =\frac{\sqrt{ex}\left(\sqrt{b}Bx^{3/2}\sqrt{a+bx^3}+(2Ab-aB)\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{3b^{3/2}\sqrt{x}}$$

input
```
Integrate[(Sqrt[e*x]*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(Sqrt[e*x]*(Sqrt[b]*B*x^(3/2)*Sqrt[a + b*x^3] + (2*A*b - a*B)*Log[Sqrt[b]*
x^(3/2) + Sqrt[a + b*x^3]]))/(3*b^(3/2)*Sqrt[x])
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {959, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{ 959}$$

$$\frac{(2Ab-aB)\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx}{2b}+\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{(2Ab-aB)\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{be}+\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{(2Ab-aB)\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3be}+\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{(2Ab-aB)\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3be}+\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{e}(2Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3b^{3/2}}+\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

input
```
Int[(Sqrt[e*x]*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(B*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*b*e) + ((2*A*b - a*B)*Sqrt[e]*ArcTanh[(
Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*b^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeqQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

method	result	size
risch	$\frac{B\,x^{2}\sqrt{b\,x^{3}+a}\,e}{3b\sqrt{ex}}+\frac{(2Ab-Ba)\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)e\sqrt{(b\,x^{3}+a)ex}}{3b\sqrt{be}\,\sqrt{ex}\,\sqrt{b\,x^{3}+a}}$	94
default	$\frac{\sqrt{ex}\,\sqrt{b\,x^{3}+a}\,\left(2A\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)be+B\sqrt{(b\,x^{3}+a)ex}\,\sqrt{be}\,x-B\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)ae\right)}{3\sqrt{(b\,x^{3}+a)ex}\,b\sqrt{be}}$	112
elliptic	Expression too large to display	1046

input
```
int((e*x)^(1/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*B*x^2/b*(b*x^3+a)^(1/2)*e/(e*x)^(1/2)+1/3*(2*A*b-B*a)/b/(b*e)^(1/2)*ar
ctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*e*((b*x^3+a)*e*x)^(1/2)/(e*x)
^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$$

$$= \left[\frac{4\sqrt{bx^3+a}\sqrt{ex}Bx-(Ba-2\,Ab)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,b^2x^4+abx)\sqrt{bx^3+a}\sqrt{ex}\sqrt{}\right.}{12\,b}\right.$$

input
```
integrate((e*x)^(1/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(4*sqrt(b*x^3 + a)*sqrt(e*x)*B*x - (B*a - 2*A*b)*sqrt(e/b)*log(-8*b^
2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt
(e*x)*sqrt(e/b)))/b, 1/6*(2*sqrt(b*x^3 + a)*sqrt(e*x)*B*x + (B*a - 2*A*b)*
sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 +
a*e)))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(73) = 146$.

Time = 2.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.82

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \begin{cases} \frac{2\left(\begin{cases} \text{NaN} & \text{for } e^3 = \ldots \\ \frac{\begin{cases} \frac{Be^3(ex)^{\frac{3}{2}}\sqrt{a+bx^3}}{2b} + \left(Ae^3 - \frac{Bae^3}{2b}\right)\left(\begin{cases} \frac{\log\left(\frac{2b(ex)^{\frac{3}{2}}}{e^3} + 2\sqrt{\frac{b}{e^3}}\sqrt{a+bx^3}\right)}{\sqrt{\frac{b}{e^3}}} & \text{for } a \neq 0 \\ \frac{(ex)^{\frac{3}{2}}\log\left((ex)^{\frac{3}{2}}\right)}{\sqrt{bx^3}} & \text{otherwise} \end{cases}\right) & \text{for } \frac{b}{e^3} \neq 0 \\ \frac{Ae^3(ex)^{\frac{3}{2}} + \frac{B(ex)^{\frac{9}{2}}}{3}}{\sqrt{a}} & \text{otherwise} \end{cases}}{3e^3} & \text{otherw}\ldots \end{cases}\right)}{e} & \\ 0 & \end{cases}$$

input
```
integrate((e*x)**(1/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((2*Piecewise((nan, Eq(e**3, 0)), (Piecewise((B*e**3*(e*x)**(3/2)
*sqrt(a + b*x**3)/(2*b) + (A*e**3 - B*a*e**3/(2*b))*Piecewise((log(2*b*(e*
x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)),
 ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True)), Ne(b/e**3, 0)), ((A
*e**3*(e*x)**(3/2) + B*(e*x)**(9/2)/3)/sqrt(a), True))/(3*e**3), True))/e,
 Ne(e, 0)), (0, True))
```

Maxima [F]

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)\sqrt{ex}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^(1/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(e*x)/sqrt(b*x^3 + a), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt{be^4x^3+ae^4}\sqrt{ex}Bx}{3\,b|e|^2} + \frac{(Bae^5-2\,Abe^5)\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{3\,\sqrt{be}b|e|^4}$$

input
```
integrate((e*x)^(1/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*B*x/(b*abs(e)^2) + 1/3*(B*a*e^5 - 2*
A*b*e^5)*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*e^4)))/(sqr
t(b*e)*b*abs(e)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{e\,x}}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(1/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{ex}\,(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{\sqrt{e}\left(2\sqrt{x}\,\sqrt{b\,x^3+a}\,bx-\sqrt{b}\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)a+\sqrt{b}\log\left(\sqrt{b\,x^3+a}+\sqrt{x}\,\sqrt{b}\,x\right)a\right)}{6b}$$

input
```
int((e*x)^(1/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(e)*(2*sqrt(x)*sqrt(a + b*x**3)*b*x - sqrt(b)*log(sqrt(a + b*x**3) -
sqrt(x)*sqrt(b)*x)*a + sqrt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)*x)*a
))/(6*b)
```

3.269 $\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 249

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx = \frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be} + \frac{(4Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2+\sqrt{3}}{4\sqrt[4]{3}\sqrt[3]{a}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/2*B*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b/e+1/12*(4*A*b-B*a)*(e*x)^(1/2)*(a^(1/3
)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))
*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x
)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(1/3
)/b/e/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1
/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx = \frac{Bx(a+bx^3)+(4Ab-aB)x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)}{2b\sqrt{ex}\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(Sqrt[e*x]*Sqrt[a + b*x^3]),x]
```

output
```
(B*x*(a + b*x^3) + (4*A*b - a*B)*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1
/6, 1/2, 7/6, -((b*x^3)/a)])/(2*b*Sqrt[e*x]*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {959, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(4Ab-aB)\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{4b}+\frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(4Ab-aB)\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2be}+\frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be}$$

$$\Big\downarrow \text{766}$$

$$\frac{\sqrt{ex}(4Ab-aB)\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{3}\right.\right.}{4\sqrt[4]{3}\sqrt[3]{a}be^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}$$

$$\frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be}$$

input
```
Int[(A + B*x^3)/(Sqrt[e*x]*Sqrt[a + b*x^3]),x]
```

output
```
(B*Sqrt[e*x]*Sqrt[a + b*x^3])/(2*b*e) + ((4*A*b - a*B)*Sqrt[e*x]*(a^(1/3)*
e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x
^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e
 + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2
 + Sqrt[3])/4])/(4*3^(1/4)*a^(1/3)*b*e^2*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^
(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.93 (sec) , antiderivative size = 728, normalized size of antiderivative = 2.92

method	result
risch	$\frac{Bx\sqrt{bx^3+a}}{2b\sqrt{ex}}+\frac{(4Ab-Ba)\left(\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-ab^2)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{(-ab^2)^{\frac{1}{3}}}{b}\right)^2\sqrt{\cdots}}{2\left(-\frac{3(-ab^2\cdots}{2b}\cdots\right)}$
elliptic	$\frac{\sqrt{(bx^3+a)ex}\left(\frac{B\sqrt{bex^4+aex}}{2be}+\frac{2\left(A-\frac{Ba}{4b}\right)\left(\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-ab^2)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{(-ab^2\cdots}{b}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$
default	Expression too large to display

input
```
int((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*B/b*x*(b*x^3+a)^(1/2)/(e*x)^(1/2)+1/2*(4*A*b-B*a)*(1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*
(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-
a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^
(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3
)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3))
)^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b*e*x^4 + a*e*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.38

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx=\frac{A\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{1}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\sqrt{e}\Gamma\left(\frac{7}{6}\right)}+\frac{Bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\sqrt{e}\Gamma\left(\frac{13}{6}\right)}$$

input
```
integrate((B*x**3+A)/(e*x)**(1/2)/(b*x**3+a)**(1/2),x)
```

output
```
A*sqrt(x)*gamma(1/6)*hyper((1/6, 1/2), (7/6,), b*x**3*exp_polar(I*pi)/a)/(
3*sqrt(a)*sqrt(e)*gamma(7/6)) + B*x**(7/2)*gamma(7/6)*hyper((1/2, 7/6), (1
3/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*sqrt(e)*gamma(13/6))
```

Maxima [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*sqrt(e*x)), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{\sqrt{e\,x}\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(1/2)),x)
```

output
```
int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx=\frac{\sqrt{e}\left(2\sqrt{x}\,\sqrt{b\,x^3+a}+3\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^4+ax}dx\right)a\right)}{4e}$$

input
```
int((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(e)*(2*sqrt(x)*sqrt(a + b*x**3) + 3*int((sqrt(x)*sqrt(a + b*x**3))/(a
*x + b*x**4),x)*a))/(4*e)
```

3.270 $\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 542

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx = -\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}+\frac{\left(1+\sqrt{3}\right)(2Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{ab^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{a^{2/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\left(1-\sqrt{3}\right)(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(\right.}{2\sqrt[4]{3}a^{2/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2*A*(b*x^3+a)^(1/2)/a/e/(e*x)^(1/2)+(1+3^(1/2))*(2*A*b+B*a)*(e*x)^(1/2)*(
b*x^3+a)^(1/2)/a/b^(2/3)/e^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)-3^(1/4)*(2*A*
b+B*a)*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^
(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1
/4*2^(1/2))/a^(2/3)/b^(2/3)/e^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1
+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-1/6*(1-3^(1/2))*(2*A*b+B*a)*
(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1
-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1
/2))*3^(3/4)/a^(2/3)/b^(2/3)/e^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(
1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx=\frac{x\left(-10A(a+bx^3)+2(2Ab+aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)\right)}{5a(ex)^{3/2}\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/((e*x)^(3/2)*Sqrt[a + b*x^3]),x]
```

output
```
(x*(-10*A*(a + b*x^3) + 2*(2*A*b + a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeome
tric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)]))/(5*a*(e*x)^(3/2)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {955, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx$$

$\big\downarrow$ 955

$$\frac{(aB+2Ab)\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx}{ae^3}-\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}$$

$\big\downarrow$ 851

$$\frac{2(aB+2Ab)\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{ae^4}-\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}$$

$\big\downarrow$ 837

$$\frac{2(aB+2Ab)\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{ae^4}-\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}$$

$\big\downarrow$ 25

$$\frac{2(aB+2Ab)\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{ae^4}-\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}$$

$\big\downarrow$ 766

$$\frac{2(aB+2Ab)\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\text{arccc}\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right.}{ae^4}$$

$$\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}$$

$\big\downarrow$ 2420

$$\frac{2(aB+2Ab)\left(\frac{\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right)|\frac{1}{4}(\ldots}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{2b^{2/3}}\ldots\right)}{ae\ldots}$$

$$\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}$$

input

```
Int[(A + B*x^3)/((e*x)^(3/2)*Sqrt[a + b*x^3]),x]
```

output

```
(-2*A*Sqrt[a + b*x^3])/(a*e*Sqrt[e*x]) + (2*(2*A*b + a*B)*((((1 + Sqrt[3])
*e^3*Sqrt[e*x]*Sqrt[a + b*x^3])/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x) -
(3^(1/4)*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 -
 a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/
3)*e*x)^2]*EllipticE[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/
3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*e*x*(
a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[
a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e +
 b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)
/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e +
(1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 +
Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x
))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(a*e^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.14 (sec) , antiderivative size = 1119, normalized size of antiderivative = 2.06

method	result	size
risch	Expression too large to display	1119
elliptic	Expression too large to display	1132
default	Expression too large to display	5385

input
```
int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*A*(b*x^3+a)^(1/2)/a/e/(e*x)^(1/2)+(2*A*b+B*a)/a*(x*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*
(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(
((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^...
```

Fricas [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b*e^2*x^5 + a*e^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.72 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.18

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx = \frac{A\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{6},\frac{1}{2} \\ \frac{5}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{6}\right)} + \frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix} \frac{1}{2},\frac{5}{6} \\ \frac{11}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

input
```
integrate((B*x**3+A)/(e*x)**(3/2)/(b*x**3+a)**(1/2),x)
```

output
```
A*gamma(-1/6)*hyper((-1/6, 1/2), (5/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*e**(3/2)*sqrt(x)*gamma(5/6)) + B*x**(5/2)*gamma(5/6)*hyper((1/2, 5/6),
 (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*e**(3/2)*gamma(11/6))
```

Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{{\left(e\,x\right)}^{3/2}\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(1/2)),x)
```

output
```
int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx=\frac{\sqrt{e}\,\left(2\sqrt{b\,x^3+a}+3\sqrt{x}\,\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^5+a\,x^2}dx\right)a\right)}{2\sqrt{x}\,e^2}$$

input
```
int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(e)*(2*sqrt(a + b*x**3) + 3*sqrt(x)*int((sqrt(x)*sqrt(a + b*x**3))/(a
*x**2 + b*x**5),x)*a))/(2*sqrt(x)*e**2)
```

3.271 $\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx=-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{2B\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}e^{5/2}}$$

output
```
-2/3*A*(b*x^3+a)^(1/2)/a/e/(e*x)^(3/2)+2/3*B*arctanh(b^(1/2)*(e*x)^(3/2)/e
^(3/2)/(b*x^3+a)^(1/2))/b^(1/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx=\frac{2x\left(-\frac{A\sqrt{a+bx^3}}{a}+\frac{Bx^{3/2}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{\sqrt{b}}\right)}{3(ex)^{5/2}}$$

input
```
Integrate[(A + B*x^3)/((e*x)^(5/2)*Sqrt[a + b*x^3]),x]
```

output
```
(2*x*(-((A*Sqrt[a + b*x^3])/a) + (B*x^(3/2)*Log[Sqrt[b]*x^(3/2) + Sqrt[a +
 b*x^3]])/Sqrt[b]))/(3*(e*x)^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {953, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{953}$$

$$\frac{B\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx}{e^3}-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{2B\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{e^4}-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{807}$$

$$\frac{2B\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3e^4}-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{2B\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3e^4}-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2B\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3\sqrt{b}e^{5/2}}-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

input
```
Int[(A + B*x^3)/((e*x)^(5/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*A*Sqrt[a + b*x^3])/(3*a*e*(e*x)^(3/2)) + (2*B*ArcTanh[(Sqrt[b]*(e*x)^(
3/2))/(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*Sqrt[b]*e^(5/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 953
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[d/e^n   Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] &&
(IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && G
tQ[m + n, -1]))
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16

method	result	size
risch	$-\frac{2A\sqrt{b\,x^{3}+a}}{3ax\,e^{2}\sqrt{ex}}+\frac{2B\,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)\sqrt{(b\,x^{3}+a)ex}}{3\sqrt{be}\,e^{2}\sqrt{ex}\,\sqrt{b\,x^{3}+a}}$	87
default	$-\frac{2\sqrt{b\,x^{3}+a}\left(-B\,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)ae\,x^{2}+A\sqrt{(b\,x^{3}+a)ex}\,\sqrt{be}\right)}{3x\,e^{2}\sqrt{ex}\,\sqrt{(b\,x^{3}+a)ex}\,a\sqrt{be}}$	93
elliptic	Expression too large to display	1037

input
```
int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/a*A*(b*x^3+a)^(1/2)/x/e^2/(e*x)^(1/2)+2/3*B/(b*e)^(1/2)*arctanh(((b*x
^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))/e^2*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b
*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.44

$$\int\frac{A+Bx^{3}}{(ex)^{5/2}\sqrt{a+bx^{3}}}\,dx=\left[\frac{\sqrt{be}Bax^{2}\log\left(-8\,b^{2}ex^{6}-8\,abex^{3}-a^{2}e-4\,(2\,bx^{4}+ax)\sqrt{bx^{3}+a}\sqrt{be}\sqrt{ex}\right)-}{6\,abe^{3}x^{2}}\right.$$
$$\left.-\frac{\sqrt{-be}Bax^{2}\arctan\left(\frac{2\sqrt{bx^{3}+a}\sqrt{-be}\sqrt{ex}x}{2\,bex^{3}+ae}\right)+2\sqrt{bx^{3}+a}\sqrt{ex}Ab}{3\,abe^{3}x^{2}}\right]$$

input
```
integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(sqrt(b*e)*B*a*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b*x^
4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) - 4*sqrt(b*x^3 + a)*sqrt(e*x
)*A*b)/(a*b*e^3*x^2), -1/3*(sqrt(-b*e)*B*a*x^2*arctan(2*sqrt(b*x^3 + a)*sq
rt(-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) + 2*sqrt(b*x^3 + a)*sqrt(e*x)*A*b)
/(a*b*e^3*x^2)]
```

Sympy [A] (verification not implemented)

Time = 6.84 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx = -\frac{2A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ae^{\frac{5}{2}}} + \frac{2B\,\mathrm{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3\sqrt{b}e^{\frac{5}{2}}}$$

input
```
integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(1/2),x)
```

output
```
-2*A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*e**(5/2)) + 2*B*asinh(sqrt(b)*x**(3
/2)/sqrt(a))/(3*sqrt(b)*e**(5/2))
```

Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
B*integrate(sqrt(x)/sqrt(b*x^3 + a), x)/e^(5/2) - 2/3*(b*sqrt(e)*x^4 + a*s
qrt(e)*x)*A/(sqrt(b*x^3 + a)*a*e^3*x^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx = -\frac{2\,e^3\left(\frac{\frac{B\arctan\left(\frac{\sqrt{be+\frac{ae}{x^3}}}{\sqrt{-be}}\right)}{\sqrt{-be}}+\frac{\sqrt{be+\frac{ae}{x^3}}A}{ae}}{e^3}-\frac{Bae\arctan\left(\frac{\sqrt{be}}{\sqrt{-be}}\right)+\sqrt{be}\sqrt{-be}A}{\sqrt{-be}ae^4}\right)}{3\,|e|^2}$$

input
```
integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
-2/3*e^3*((B*arctan(sqrt(b*e + a*e/x^3)/sqrt(-b*e))/sqrt(-b*e) + sqrt(b*e
+ a*e/x^3)*A/(a*e))/e^3 - (B*a*e*arctan(sqrt(b*e)/sqrt(-b*e)) + sqrt(b*e)*
sqrt(-b*e)*A)/(sqrt(-b*e)*a*e^4))/abs(e)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx = \int \frac{B\,x^3+A}{{\left(e\,x\right)}^{5/2}\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(1/2)),x)
```

output
```
int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx = \frac{\sqrt{e}\left(-2\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)x+\sqrt{x}\,\sqrt{b}\log\left(\sqrt{b\,x^3+a}\right.\right.}{3\sqrt{x}\,e^3x}$$

input
```
int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(e)*( - 2*sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*log(sqrt(a + b*x**3) - s
qrt(x)*sqrt(b)*x)*x + sqrt(x)*sqrt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(
b)*x)*x))/(3*sqrt(x)*e**3*x)
```

3.272 $\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 246

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx=-\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}}$$
$$-\frac{(2Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/5*A*(b*x^3+a)^(1/2)/a/e/(e*x)^(5/2)-1/15*(2*A*b-5*B*a)*(e*x)^(1/2)*(a^(
1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/
2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3
)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(
4/3)/e^4/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)
^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.33

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = -\frac{2x\left(A(a+bx^3)+(2Ab-5aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{5a(ex)^{7/2}\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/((e*x)^(7/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*x*(A*(a + b*x^3) + (2*A*b - 5*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeomet
ric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(5*a*(e*x)^(7/2)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {955, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(2Ab-5aB)\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{5ae^3}-\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{851}$$

$$-\frac{2(2Ab-5aB)\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{5ae^4}-\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{766}$$

$$-\frac{\sqrt{ex}(2Ab-5aB)\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}(2+}{5\sqrt[4]{3}a^{4/3}e^5\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}$$

$$\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}}$$

input
```
Int[(A + B*x^3)/((e*x)^(7/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*A*Sqrt[a + b*x^3])/(5*a*e*(e*x)^(5/2)) - ((2*A*b - 5*a*B)*Sqrt[e*x]*(a
^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3
)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^
(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*
x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*e^5*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e
 + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3
])
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.17 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.01

method	result
risch	$-\frac{2A\sqrt{b\,x^{3}+a}}{5a\,x^{2}e^{3}\sqrt{ex}}-\frac{2(2Ab-5Ba)\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\cdots}{5a\left(-\frac{3(\cdots}{}\cdots\right.}$
elliptic	$\frac{\sqrt{(b\,x^{3}+a)ex}\left(-\frac{2A\sqrt{be\,x^{4}+aex}}{5e^{4}a\,x^{3}}+\frac{2\left(\frac{B}{e^{3}}-\frac{2bA}{5a\,e^{3}}\right)\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/a*A*(b*x^3+a)^(1/2)/x^2/e^3/(e*x)^(1/2)-2/5*(2*A*b-5*B*a)/a*(1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*
b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1
/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^
2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/
(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2))/e^3*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1
/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.24

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = -\frac{2\left((5\,Ba-2\,Ab)\sqrt{ae}x^3\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)+\sqrt{bx^3+a}\sqrt{ex}Aa\right)}{5\,a^2e^4x^3}$$

input
```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*((5*B*a - 2*A*b)*sqrt(a*e)*x^3*weierstrassPInverse(0, -4*b/a, 1/x) +
sqrt(b*x^3 + a)*sqrt(e*x)*A*a)/(a^2*e^4*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = \frac{A\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)} + \frac{B\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{1}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}$$

input
```
integrate((B*x**3+A)/(e*x)**(7/2)/(b*x**3+a)**(1/2),x)
```

output
```
A*gamma(-5/6)*hyper((-5/6, 1/2), (1/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*e**(7/2)*x**(5/2)*gamma(1/6)) + B*sqrt(x)*gamma(1/6)*hyper((1/6, 1/2),
 (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*e**(7/2)*gamma(7/6))
```

Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(7/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = \int \frac{B\,x^3+A}{(e\,x)^{7/2}\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(1/2)),x)
```

output
```
int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = \frac{\sqrt{e}\left(-2\sqrt{b\,x^3+a}-3\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^7+a\,x^4}dx\right)a\,x^2\right)}{2\sqrt{x}\,e^4x^2}$$

input
```
int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(e)*( - 2*sqrt(a + b*x**3) - 3*sqrt(x)*int((sqrt(x)*sqrt(a + b*x**3))
/(a*x**4 + b*x**7),x)*a*x**2))/(2*sqrt(x)*e**4*x**2)
```

3.273 $\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 119

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}} dx = -\frac{2(Ab-aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{Be^2(ex)^{3/2}\sqrt{a+bx^3}}{3b^2} + \frac{(2Ab-3aB)e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

output
```
-2/3*(A*b-B*a)*e^2*(e*x)^(3/2)/b^2/(b*x^3+a)^(1/2)+1/3*B*e^2*(e*x)^(3/2)*(
b*x^3+a)^(1/2)/b^2+1/3*(2*A*b-3*B*a)*e^(7/2)*arctanh(b^(1/2)*(e*x)^(3/2)/e
^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.78

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}} dx = \frac{(ex)^{7/2}\left(\frac{\sqrt{b}x^{3/2}(-2Ab+3aB+bBx^3)}{\sqrt{a+bx^3}} + (2Ab-3aB)\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{3b^{5/2}x^{7/2}}$$

input
```
Integrate[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
((e*x)^(7/2)*((Sqrt[b]*x^(3/2)*(-2*A*b + 3*a*B + b*B*x^3))/Sqrt[a + b*x^3]
 + (2*A*b - 3*a*B)*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(3*b^(5/2)*x^(
7/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {959, 817, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(2Ab-3aB)\int \frac{(ex)^{7/2}}{\left(bx^3+a\right)^{3/2}}dx}{2b}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(2Ab-3aB)\left(\frac{e^3\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx}{b}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{2b}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{851}$$

$$\frac{(2Ab-3aB)\left(\frac{2e^2\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{b}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{2b}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{807}$$

$$\frac{(2Ab-3aB)\left(\frac{2e^2\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3b}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{2b}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{224}$$

$$\frac{(2Ab-3aB)\left(\frac{2e^2\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3b}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{2b}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(2Ab-3aB)\left(\frac{2e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3b^{3/2}}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{2b}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

input
```
Int[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(B*(e*x)^(9/2))/(3*b*e*Sqrt[a + b*x^3]) + ((2*A*b - 3*a*B)*((-2*e^2*(e*x)^
(3/2))/(3*b*Sqrt[a + b*x^3]) + (2*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e
^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*b^(3/2))))/(2*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08

method	result
risch	$\frac{B x^{2} \sqrt{b x^{3}+a}\, e^{4}}{3 b^{2} \sqrt{e x}}+\frac{\left(\frac{2 (2 A b -3 B a) \operatorname{arctanh}\left(\frac{\sqrt{\left(b x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right)}{3 \sqrt{b e}}-\frac{4 (A b -B a) x^{2}}{3 \sqrt{\left(x^{3}+\frac{a}{b}\right) b e x}}\right) e^{4} \sqrt{\left(b x^{3}+a \right) e x}}{2 b^{2} \sqrt{e x}\, \sqrt{b x^{3}+a}}$
default	$\frac{e^{3} \sqrt{e x}\, \left(B \sqrt{b e}\, b \,x^{5}-2 A \sqrt{b e}\, b \,x^{2}+3 B \sqrt{b e}\, a \,x^{2}+2 A \operatorname{arctanh}\left(\frac{\sqrt{\left(b x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) \sqrt{\left(b x^{3}+a \right) e x}\, b -3 B \operatorname{arctanh}\left(\frac{\sqrt{\left(b x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) \sqrt{\left(b x^{3}\right.}}{3 x \sqrt{b x^{3}+a}\, b^{2} \sqrt{b e}}$
elliptic	Expression too large to display

input
```
int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*B*x^2/b^2*(b*x^3+a)^(1/2)*e^4/(e*x)^(1/2)+1/2/b^2*(2/3*(2*A*b-3*B*a)/(
b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))-4/3*(A*b-B*a)*x^
2/((x^3+a/b)*b*e*x)^(1/2))*e^4*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.58

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \left[-\frac{\left((3\,Bab-2\,Ab^2)e^3x^3+(3\,Ba^2-2\,Aab)e^3\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2\right.}{12}\right.$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(((3*B*a*b - 2*A*b^2)*e^3*x^3 + (3*B*a^2 - 2*A*a*b)*e^3)*sqrt(e/b)*
log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3
+ a)*sqrt(e*x)*sqrt(e/b)) - 4*(B*b*e^3*x^4 + (3*B*a - 2*A*b)*e^3*x)*sqrt(b
*x^3 + a)*sqrt(e*x))/(b^3*x^3 + a*b^2), 1/6*(((3*B*a*b - 2*A*b^2)*e^3*x^3
+ (3*B*a^2 - 2*A*a*b)*e^3)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b
*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*(B*b*e^3*x^4 + (3*B*a - 2*A*b)*e^3*x)
*sqrt(b*x^3 + a)*sqrt(e*x))/(b^3*x^3 + a*b^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{7}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^(7/2)/(b*x^3 + a)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.11

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{\left(\frac{Be^4x^3}{b} + \frac{3\,Bab^3e^4-2\,Ab^4e^4}{b^5}\right)\sqrt{ex}ex}{3\,\sqrt{be^4x^3+ae^4}} + \frac{(3\,Bab^3e^4-2\,Ab^4e^4)e^2\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{3\,\sqrt{be}b^5|e|^2}$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
1/3*(B*e^4*x^3/b + (3*B*a*b^3*e^4 - 2*A*b^4*e^4)/b^5)*sqrt(e*x)*e*x/sqrt(b
*e^4*x^3 + a*e^4) + 1/3*(3*B*a*b^3*e^4 - 2*A*b^4*e^4)*e^2*log(abs(-sqrt(b*
e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*e^4)))/(sqrt(b*e)*b^5*abs(e)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(B\,x^3+A)\,(e\,x)^{7/2}}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(3/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.56

$$\int \frac{(ex)^{7/2}\,(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{\sqrt{e}\,e^3\left(2\sqrt{x}\,\sqrt{b\,x^3+a}\,bx+\sqrt{b}\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)a-\sqrt{b}\log\left(\sqrt{b\,x^3+\right.\right.}{6b^2}$$

input
```
int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(e)*e**3*(2*sqrt(x)*sqrt(a + b*x**3)*b*x + sqrt(b)*log(sqrt(a + b*x**
3) - sqrt(x)*sqrt(b)*x)*a - sqrt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)
*x)*a))/(6*b**2)
```

3.274 $\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 285

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2(Ab-aB)e^2\sqrt{ex}}{3b^2\sqrt{a+bx^3}}+\frac{Be^2\sqrt{ex}\sqrt{a+bx^3}}{2b^2}$$
$$+\frac{(4Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\ \text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right.\right.}{12\sqrt[4]{3}\sqrt[3]{a}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*(A*b-B*a)*e^2*(e*x)^(1/2)/b^2/(b*x^3+a)^(1/2)+1/2*B*e^2*(e*x)^(1/2)*(
b*x^3+a)^(1/2)/b^2+1/36*(4*A*b-7*B*a)*e^2*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*
((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2
)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1
+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(1/3)/b^2/(b^(1/3
)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.31

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{e^2\sqrt{ex}\left(-4Ab+7aB+3bBx^3+(4Ab-7aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\right.\right.}{6b^2\sqrt{a+bx^3}}$$

input
```
Integrate[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(e^2*Sqrt[e*x]*(-4*A*b + 7*a*B + 3*b*B*x^3 + (4*A*b - 7*a*B)*Sqrt[1 + (b*x
^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(6*b^2*Sqrt[a + b*
x^3])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {959, 817, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(4Ab-7aB)\int \frac{(ex)^{5/2}}{(bx^3+a)^{3/2}}dx}{4b}+\frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 817$$

$$\frac{(4Ab-7aB)\left(\frac{e^3\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{3b}-\frac{2e^2\sqrt{ex}}{3b\sqrt{a+bx^3}}\right)}{4b}+\frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 851$$

$$\frac{(4Ab-7aB)\left(\frac{2e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{3b}-\frac{2e^2\sqrt{ex}}{3b\sqrt{a+bx^3}}\right)}{4b}+\frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{766}$$

$$\frac{(4Ab-7aB)\left(\frac{e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}\sqrt[3]{a}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}-\frac{2e^2\sqrt{\ldots}}{3b\sqrt{a+\ldots}}\right.}{4b}$$

$$\frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}}$$

input
```
Int[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(B*(e*x)^(7/2))/(2*b*e*Sqrt[a + b*x^3]) + ((4*A*b - 7*a*B)*((-2*e^2*Sqrt[e
*x])/(3*b*Sqrt[a + b*x^3]) + (e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(
a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + S
qrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3
)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(3*3^(1
/4)*a^(1/3)*b*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1
 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(4*b)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.74 (sec) , antiderivative size = 787, normalized size of antiderivative = 2.76

method	result	size
elliptic	Expression too large to display	787
risch	Expression too large to display	2115
default	Expression too large to display	3760

input
```
int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(b*x^3+a)^(1/2)*((b*x^3+a)*e*x)^(1/2)*(-2/3/b^2*e^3*x*(A
*b-B*a)/((x^3+a/b)*b*e*x)^(1/2)+1/2*B*e^2/b^2*(b*e*x^4+a*e*x)^(1/2)+2*(1/3
*(A*b-B*a)*e^3/b^2-1/4*B/b^2*e^3*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/
(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/
3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*
(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [F]

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((B*e^2*x^5 + A*e^2*x^2)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^2*x^6 + 2*a*
b*x^3 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}\,(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(5/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^{5/2}\,(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(ex)^{5/2}\,(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2} (A + Bx^3)}{(a + bx^3)^{3/2}} \, dx = \int \frac{(B\,x^3 + A)\,(e\,x)^{5/2}}{(b\,x^3 + a)^{3/2}} \, dx$$

input
```
int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(3/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{(ex)^{5/2} (A + Bx^3)}{(a + bx^3)^{3/2}} \, dx = \frac{\sqrt{e}\,e^2\left(2\sqrt{x}\,\sqrt{b\,x^3 + a} - \left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^4+ax}dx\right) a\right)}{4b}$$

input
```
int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(e)*e**2*(2*sqrt(x)*sqrt(a + b*x**3) - int((sqrt(x)*sqrt(a + b*x**3))
/(a*x + b*x**4),x)*a))/(4*b)
```

3.275 $\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 553

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} - \frac{\left(1+\sqrt{3}\right)(2Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{3ab^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$
$$+\frac{(2Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{\left(1-\sqrt{3}\right)(2Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\right.}{6\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*(A*b-B*a)*(e*x)^(5/2)/a/b/e/(b*x^3+a)^(1/2)-1/3*(1+3^(1/2))*(2*A*b-5*B
*a)*e*(e*x)^(1/2)*(b*x^3+a)^(1/2)/a/b^(5/3)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x
)+1/3*(2*A*b-5*B*a)*e*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1
-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2
),1/4*6^(1/2)+1/4*2^(1/2))*3^(1/4)/a^(2/3)/b^(5/3)/(b^(1/3)*x*(a^(1/3)+b^(
1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+1/18*(1-3
^(1/2))*(2*A*b-5*B*a)*e*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJac
obiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*
x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(2/3)/b^(5/3)/(b^(1/3)*x*(a^(1/3)+b
^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.14

$$\int \frac{(ex)^{3/2} (A + Bx^3)}{(a + bx^3)^{3/2}} \, dx = \frac{x(ex)^{3/2} \left(5aB + (2Ab - 5aB)\sqrt{1 + \frac{bx^3}{a}} \, \text{Hypergeometric2F1}\left(\frac{5}{6}, \frac{3}{2}, \frac{11}{6}, -\frac{bx^3}{a}\right)\right)}{5ab\sqrt{a + bx^3}}$$

input
```
Integrate[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(x*(e*x)^(3/2)*(5*a*B + (2*A*b - 5*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[5/6, 3/2, 11/6, -((b*x^3)/a)]))/(5*a*b*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 607, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {957, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$$

$\big\downarrow$ 957

$$\frac{2(ex)^{5/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}-\frac{(2Ab-5aB)\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx}{3ab}$$

$\big\downarrow$ 851

$$\frac{2(ex)^{5/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}-\frac{2(2Ab-5aB)\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{3abe}$$

$\big\downarrow$ 837

$$\frac{2(ex)^{5/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}-\frac{2(2Ab-5aB)\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{3abe}$$

$\big\downarrow$ 25

$$\frac{2(ex)^{5/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}-\frac{2(2Ab-5aB)\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{3abe}$$

$\big\downarrow$ 766

$$\frac{2(ex)^{5/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}-\frac{2(2Ab-5aB)\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\text{arc}\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)}}}\right.}{3abe}$$

$\big\downarrow$ 2420

$$\frac{\frac{2(ex)^{5/2}(Ab-aB)}{3abe\sqrt{a+bx^3}} - 2(2Ab-5aB)\left(\frac{\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex} - \frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right)|\ldots\right)}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{2b^{2/3}} \ldots\right)}{3a\ldots}$$

input
```
Int[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*(A*b - a*B)*(e*x)^(5/2))/(3*a*b*e*Sqrt[a + b*x^3]) - (2*(2*A*b - 5*a*B)
*((((1 + Sqrt[3])*e^3*Sqrt[e*x]*Sqrt[a + b*x^3])/(a^(1/3)*e + (1 + Sqrt[3]
)*b^(1/3)*e*x) - (3^(1/4)*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sq
rt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1
 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticE[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^
(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(Sq
rt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1
/3)*e*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*e*Sqrt[
e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x +
 b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[Arc
Cos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(
1/3)*e*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*e*x*(a^(1/3
)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*
x^3]))/(3*a*b*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.98 (sec) , antiderivative size = 1154, normalized size of antiderivative = 2.09

method	result	size
elliptic	Expression too large to display	1154
default	Expression too large to display	5392

input
```
int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(b*x^3+a)^(1/2)*((b*x^3+a)*e*x)^(1/2)*(2/3/b*e^2*x^3/a*(
A*b-B*a)/((x^3+a/b)*b*e*x)^(1/2)+(B*e^2/b-2/3*(A*b-B*a)/a/b*e^2)*(x*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))
^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(
2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1
/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)
))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Elliptic
E(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a...
```

Fricas [F]

$$\int \frac{(ex)^{3/2} (A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((B*e*x^4 + A*e*x)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^2*x^6 + 2*a*b*x^3
+ a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 82.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.17

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}\frac{5}{6},\frac{3}{2}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)} + \frac{Be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)}$$

input
```
integrate((e*x)**(3/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
A*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((5/6, 3/2), (11/6,), b*x**3*exp_polar
(I*pi)/a)/(3*a**(3/2)*gamma(11/6)) + B*e**(3/2)*x**(11/2)*gamma(11/6)*hype
r((3/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(17/6))
```

Maxima [F]

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(B\,x^3+A)\,(e\,x)^{3/2}}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(3/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \sqrt{e}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)e$$

input
```
int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

output
```
sqrt(e)*int((sqrt(x)*sqrt(a + b*x**3)*x)/(a + b*x**3),x)*e
```

3.276 $\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 85

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}} + \frac{2B\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

output
```
2/3*(A*b-B*a)*(e*x)^(3/2)/a/b/e/(b*x^3+a)^(1/2)+2/3*B*e^(1/2)*arctanh(b^(1
/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{ex}\left(\frac{\sqrt{b}(Ab-aB)x^{3/2}}{a\sqrt{a+bx^3}} + B\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{3b^{3/2}\sqrt{x}}$$

input
```
Integrate[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*Sqrt[e*x]*((Sqrt[b]*(A*b - a*B)*x^(3/2))/(a*Sqrt[a + b*x^3]) + B*Log[Sq
rt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(3*b^(3/2)*Sqrt[x])
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {954, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{954}$$

$$\frac{B\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx}{b}+\frac{2(ex)^{3/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{851}$$

$$\frac{2B\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{be}+\frac{2(ex)^{3/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{807}$$

$$\frac{2B\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3be}+\frac{2(ex)^{3/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{224}$$

$$\frac{2B\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3be}+\frac{2(ex)^{3/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2(ex)^{3/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}+\frac{2B\sqrt{e}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3b^{3/2}}$$

input
```
Int[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*(A*b - a*B)*(e*x)^(3/2))/(3*a*b*e*Sqrt[a + b*x^3]) + (2*B*Sqrt[e]*ArcTa
nh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*b^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{2\sqrt{ex}\left(A\sqrt{be}\,b\,x^{2}-B\sqrt{be}\,a\,x^{2}+B\,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)\sqrt{\left(b\,x^{3}+a\right)ex}\,a\right)}{3\sqrt{b\,x^{3}+a}\,x b\sqrt{be}\,a}$	92
elliptic	Expression too large to display	1050

input
```
int((e*x)^(1/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(e*x)^(1/2)/(b*x^3+a)^(1/2)*(A*(b*e)^(1/2)*b*x^2-B*(b*e)^(1/2)*a*x^2+B
*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*((b*x^3+a)*e*x)^(1/2)*a)/x
/b/(b*e)^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.75

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \left[-\frac{4\sqrt{bx^3+a}(Ba-Ab)\sqrt{ex}x-(Babx^3+Ba^2)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2\right.}{6\,(ab^2x^3+a^2b)}\right.$$

$$\left.-\frac{2\sqrt{bx^3+a}(Ba-Ab)\sqrt{ex}x+(Babx^3+Ba^2)\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{ex}bx\sqrt{-\frac{e}{b}}}{2\,bex^3+ae}\right)}{3\,(ab^2x^3+a^2b)}\right]$$

input
```
integrate((e*x)^(1/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/6*(4*sqrt(b*x^3 + a)*(B*a - A*b)*sqrt(e*x)*x - (B*a*b*x^3 + B*a^2)*sqr
t(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt
(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)))/(a*b^2*x^3 + a^2*b), -1/3*(2*sqrt(b*x^3
+ a)*(B*a - A*b)*sqrt(e*x)*x + (B*a*b*x^3 + B*a^2)*sqrt(-e/b)*arctan(2*sqr
t(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)))/(a*b^2*x^3 + a^2
*b)]
```

Sympy [A] (verification not implemented)

Time = 24.92 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2A\sqrt{e}x^{\frac{3}{2}}}{3a^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}} + B\left(\frac{2\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3b^{\frac{3}{2}}} - \frac{2\sqrt{e}x^{\frac{3}{2}}}{3\sqrt{a}b\sqrt{1+\frac{bx^3}{a}}}\right)$$

input
```
integrate((e*x)**(1/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
2*A*sqrt(e)*x**(3/2)/(3*a**(3/2)*sqrt(1 + b*x**3/a)) + B*(2*sqrt(e)*asinh(
sqrt(b)*x**(3/2)/sqrt(a))/(3*b**(3/2)) - 2*sqrt(e)*x**(3/2)/(3*sqrt(a)*b*s
qrt(1 + b*x**3/a)))
```

Maxima [F]

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{ex}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(1/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(e*x)/(b*x^3 + a)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2\,Be^3\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{3\,\sqrt{be}b|e|^2} - \frac{2\,(Bae-Abe)\sqrt{ex}ex}{3\,\sqrt{be^4x^3+ae^4}ab}$$

input
```
integrate((e*x)^(1/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
-2/3*B*e^3*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*e^4)))/(s
qrt(b*e)*b*abs(e)^2) - 2/3*(B*a*e - A*b*e)*sqrt(e*x)*e*x/(sqrt(b*e^4*x^3 +
 a*e^4)*a*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{e\,x}}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(3/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{\sqrt{e}\,\sqrt{b}\left(-\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)+\log\left(\sqrt{b\,x^3+a}+\sqrt{x}\,\sqrt{b}\,x\right)\right)}{3b}$$

input
```
int((e*x)^(1/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(e)*sqrt(b)*( - log(sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*x) + log(sqrt(
a + b*x**3) + sqrt(x)*sqrt(b)*x)))/(3*b)
```

3.277 $\int \frac{A+Bx^3}{\sqrt{ex}(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 258

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx=\frac{2(Ab-aB)\sqrt{ex}}{3abe\sqrt{a+bx^3}}+\frac{(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{4/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*(A*b-B*a)*(e*x)^(1/2)/a/b/e/(b*x^3+a)^(1/2)+1/9*(2*A*b+B*a)*(e*x)^(1/2
)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1
+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*
b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/
4)/a^(4/3)/b/e/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)
*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx = \frac{2x\left(Ab-aB+(2Ab+aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{3ab\sqrt{ex}\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(3/2)),x]
```

output
```
(2*x*(A*b - a*B + (2*A*b + a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6,
 1/2, 7/6, -((b*x^3)/a)]))/(3*a*b*Sqrt[e*x]*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {957, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(aB+2Ab)\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{3ab}+\frac{2\sqrt{ex}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{851}$$

$$\frac{2(aB+2Ab)\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{3abe}+\frac{2\sqrt{ex}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{766}$$

$$\frac{\sqrt{ex}(aB+2Ab)\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}(2+\sqrt{3}}{3\sqrt[4]{3}a^{4/3}be^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}$$
$$\frac{2\sqrt{ex}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

input
```
Int[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(3/2)),x]
```

output
```
(2*(A*b - a*B)*Sqrt[e*x])/(3*a*b*e*Sqrt[a + b*x^3]) + ((2*A*b + a*B)*Sqrt[
e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x +
 b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[Arc
Cos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(
1/3)*e*x)], (2 + Sqrt[3])/4])/(3*3^(1/4)*a^(4/3)*b*e^2*Sqrt[(b^(1/3)*e*x*(
a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[
a + b*x^3])
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.73 (sec) , antiderivative size = 754, normalized size of antiderivative = 2.92

method	result
elliptic	$\frac{\sqrt{(b\,x^3+a)ex}\left(\frac{2x(Ab-Ba)}{3ba\sqrt{\left(x^3+\frac{a}{b}\right)bex}}+\frac{2\left(\frac{B}{b}+\frac{\frac{2Ab}{3}-\frac{2Ba}{3}}{ab}\right)\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}}}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)*(2/3/b*x/a*(A*b-B*a)/((x
^3+a/b)*b*e*x)^(1/2)+2*(B/b+2/3*(A*b-B*a)/a/b)*(1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(
-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^
(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)/(b*
e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*El
lipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/
2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx = -\frac{2\left(((Bab+2\,Ab^2)x^3+Ba^2+2\,Aab)\sqrt{ae}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)+\sqrt{bx^3+a}(Ba^2-Aab)\sqrt{ex}\right)}{3\,(a^2b^2ex^3+a^3be)}$$

input
```
integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*sqrt(a*e)*weierstrassPInve
rse(0, -4*b/a, 1/x) + sqrt(b*x^3 + a)*(B*a^2 - A*a*b)*sqrt(e*x))/(a^2*b^2*
e*x^3 + a^3*b*e)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx = \frac{A\sqrt{x}\,\Gamma\left(\frac{1}{6}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{3}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\sqrt{e}\,\Gamma\left(\frac{7}{6}\right)} + \frac{Bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)\,{}_2F_1\left(\begin{matrix}\frac{7}{6},\frac{3}{2}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\sqrt{e}\,\Gamma\left(\frac{13}{6}\right)}$$

input
```
integrate((B*x**3+A)/(e*x)**(1/2)/(b*x**3+a)**(3/2),x)
```

output
```
A*sqrt(x)*gamma(1/6)*hyper((1/6, 3/2), (7/6,), b*x**3*exp_polar(I*pi)/a)/(
3*a**(3/2)*sqrt(e)*gamma(7/6)) + B*x**(7/2)*gamma(7/6)*hyper((7/6, 3/2), (
13/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*sqrt(e)*gamma(13/6))
```

Maxima [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*sqrt(e*x)), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx = \int \frac{B\,x^3+A}{\sqrt{e\,x}\,(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(3/2)),x)
```

output
```
int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^4+ax}dx\right)}{e}$$

input
```
int((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(e)*int((sqrt(x)*sqrt(a + b*x**3))/(a*x + b*x**4),x))/e
```

3.278 $\int \frac{A+Bx^3}{(ex)^{3/2}(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 585

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx=-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}}$$
$$-\frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}}+\frac{2\left(1+\sqrt{3}\right)(4Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{3a^2b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$
$$-\frac{2(4Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3^{3/4}a^{5/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\left(1-\sqrt{3}\right)(4Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(\right.\right.}{3\sqrt[4]{3}a^{5/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2*A/a/e/(e*x)^(1/2)/(b*x^3+a)^(1/2)-2/3*(4*A*b-B*a)*(e*x)^(5/2)/a^2/e^4/(
b*x^3+a)^(1/2)+2/3*(1+3^(1/2))*(4*A*b-B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/a^2
/b^(2/3)/e^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)-2/3*(4*A*b-B*a)*(e*x)^(1/2)*(
a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^
(1/2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/
(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*3^(1/4)/
a^(5/3)/b^(2/3)/e^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^
(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-1/9*(1-3^(1/2))*(4*A*b-B*a)*(e*x)^(1/2)*
(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3
^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^
(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)
/a^(5/3)/b^(2/3)/e^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b
^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.13

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx = \frac{x\left(-10aA+2(-4Ab+aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{3}{2},\frac{11}{6},-\frac{bx^3}{a}\right)\right)}{5a^2(ex)^{3/2}\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)),x]
```

output
```
(x*(-10*a*A + 2*(-4*A*b + a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[5
/6, 3/2, 11/6, -((b*x^3)/a)]))/(5*a^2*(e*x)^(3/2)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {955, 819, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(4Ab-aB)\int \frac{(ex)^{3/2}}{(bx^3+a)^{3/2}}dx}{ae^3}-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 819}$$

$$-\frac{(4Ab-aB)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{2\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx}{3a}\right)}{ae^3}-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 851}$$

$$-\frac{(4Ab-aB)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{3ae}\right)}{ae^3}-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 837}$$

$$-\frac{(4Ab-aB)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{3ae}\right)}{ae^3}-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{(4Ab-aB)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{3ae}\right)}{ae^3}-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}}$$

$\downarrow$ 766

$$-\frac{(4Ab-aB)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\,\mathrm{E}\cdots}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{\cdots}\right.}{\left(\sqrt[3]{a}e+\left(1+\cdots\right.\right.}}}\cdots\right)}{3ae}\cdots\right)}{ae^3}\cdots$$

$$\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}}$$

$\downarrow$ 2420

$$-\frac{(4Ab-aB)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{\cdots}\right.}{\left(1+\sqrt{\cdots}\right.}\cdots\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\cdots\right)}{2b^{2/3}\cdots}\cdots\right)}{\cdots}$$

$$\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}}$$

input

```
Int[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)),x]
```

output
```
(-2*A)/(a*e*Sqrt[e*x]*Sqrt[a + b*x^3]) - ((4*A*b - a*B)*((2*(e*x)^(5/2))/(
3*a*e*Sqrt[a + b*x^3]) - (4*((((1 + Sqrt[3])*e^3*Sqrt[e*x]*Sqrt[a + b*x^3]
)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x) - (3^(1/4)*a^(1/3)*e*Sqrt[e*x]*(
a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/
3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticE[ArcCos[(a
^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e
*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(
1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1
 - Sqrt[3])*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^
2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^
(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^
(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3
)*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*
b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(3*a*e)))/(a*e^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.63 (sec) , antiderivative size = 1177, normalized size of antiderivative = 2.01

method	result	size
elliptic	Expression too large to display	1177
risch	Expression too large to display	2209
default	Expression too large to display	5563

input
```
int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)*(-2*(b*e*x^3+a*e)/e^2/a^
2*A/(x*(b*e*x^3+a*e))^(1/2)-2/3/e*x^3/a^2*(A*b-B*a)/((x^3+a/b)*b*e*x)^(1/2
)+(2*b/a^2/e*A+2/3/a^2*(A*b-B*a)/e)*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))
^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2
)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.18

$$\int \frac{A+Bx^3}{(ex)^{3/2}\,(a+bx^3)^{3/2}}\,dx = -\frac{2\,\left(\left((Bab-4\,Ab^2)x^4+(Ba^2-4\,Aab)x\right)\sqrt{ae}\,\text{weierstrassZeta}\left(0,-\frac{4\,b}{a},\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)\right)+\cdots\right.}{3\left(a^2b^2e^2x^4+a^3be^2x\right)}$$

input
```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*sqrt(a*e)*weierstrassZ
eta(0, -4*b/a, weierstrassPInverse(0, -4*b/a, 1/x)) + sqrt(b*x^3 + a)*(B*a
^2 - A*a*b)*sqrt(e*x))/(a^2*b^2*e^2*x^4 + a^3*b*e^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 46.48 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.17

$$\int \frac{A+Bx^3}{(ex)^{3/2}(a+bx^3)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{6},\frac{3}{2}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{6}\right)} + \frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}\frac{5}{6},\frac{3}{2}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

input
```
integrate((B*x**3+A)/(e*x)**(3/2)/(b*x**3+a)**(3/2),x)
```

output
```
A*gamma(-1/6)*hyper((-1/6, 3/2), (5/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*e**(3/2)*sqrt(x)*gamma(5/6)) + B*x**(5/2)*gamma(5/6)*hyper((5/6, 3/2)
, (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*e**(3/2)*gamma(11/6))
```

Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}(a+bx^3)^{3/2}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{3/2}(a+bx^3)^{3/2}}\,dx=\int \frac{B\,x^3+A}{(e\,x)^{3/2}\,(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)),x)
```

output
```
int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}(a+bx^3)^{3/2}}\,dx=\frac{\sqrt{e}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^5+a\,x^2}dx\right)}{e^2}$$

input
```
int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(e)*int((sqrt(x)*sqrt(a + b*x**3))/(a*x**2 + b*x**5),x))/e**2
```

3.279 $\int \frac{A+Bx^3}{(ex)^{5/2}(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx=-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}-\frac{2(2Ab-aB)(ex)^{3/2}}{3a^2e^4\sqrt{a+bx^3}}$$

output
```
-2/3*A/a/e/(e*x)^(3/2)/(b*x^3+a)^(1/2)-2/3*(2*A*b-B*a)*(e*x)^(3/2)/a^2/e^4
/(b*x^3+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx=\frac{2x(-aA-2Abx^3+aBx^3)}{3a^2(ex)^{5/2}\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x]
```

output
```
(2*x*(-(a*A) - 2*A*b*x^3 + a*B*x^3))/(3*a^2*(e*x)^(5/2)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {955, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(ex)^{5/2}\,(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \ 955$$

$$-\frac{(2Ab-aB)\int \frac{\sqrt{ex}}{(bx^3+a)^{3/2}}dx}{ae^3}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 796$$

$$-\frac{2(ex)^{3/2}(2Ab-aB)}{3a^2e^4\sqrt{a+bx^3}}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}$$

input
```
Int[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x]
```

output
```
(-2*A)/(3*a*e*(e*x)^(3/2)*Sqrt[a + b*x^3]) - (2*(2*A*b - a*B)*(e*x)^(3/2))
/(3*a^2*e^4*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{2x\left(2Ab\,x^{3}-Ba\,x^{3}+Aa\right)}{3\sqrt{b\,x^{3}+a}\,a^{2}\left(ex\right)^{\frac{5}{2}}}$	39
orering	$-\frac{2x\left(2Ab\,x^{3}-Ba\,x^{3}+Aa\right)}{3\sqrt{b\,x^{3}+a}\,a^{2}\left(ex\right)^{\frac{5}{2}}}$	39
default	$-\frac{2\left(2Ab\,x^{3}-Ba\,x^{3}+Aa\right)}{3x\sqrt{b\,x^{3}+a}\,a^{2}e^{2}\sqrt{ex}}$	44
risch	$-\frac{2A\sqrt{b\,x^{3}+a}}{3a^{2}x\,e^{2}\sqrt{ex}}-\frac{2\left(Ab-Ba\right)x^{2}}{3a^{2}e^{2}\sqrt{ex}\,\sqrt{b\,x^{3}+a}}$	61
elliptic	$\frac{\sqrt{\left(b\,x^{3}+a\right)ex}\,\left(-\frac{2A\sqrt{be\,x^{4}+aex}}{3e^{3}a^{2}x^{2}}-\frac{2x^{2}\left(Ab-Ba\right)}{3e^{2}a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)bex}}\right)}{\sqrt{ex}\,\sqrt{b\,x^{3}+a}}$	88

input
```
int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x*(2*A*b*x^3-B*a*x^3+A*a)/(b*x^3+a)^(1/2)/a^2/(e*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{A + Bx^3}{(ex)^{5/2} (a + bx^3)^{3/2}}\, dx = \frac{2\left((Ba - 2\, Ab)x^3 - Aa\right)\sqrt{bx^3 + a}\sqrt{ex}}{3\left(a^2be^3x^5 + a^3e^3x^2\right)}$$

input
```
integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*((B*a - 2*A*b)*x^3 - A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(a^2*b*e^3*x^5 + a
^3*e^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 86.92 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int \frac{A + Bx^3}{(ex)^{5/2} (a + bx^3)^{3/2}}\, dx = A\left(-\frac{2}{3a\sqrt{b}e^{\frac{5}{2}}x^3\sqrt{\frac{a}{bx^3} + 1}} - \frac{4\sqrt{b}}{3a^2e^{\frac{5}{2}}\sqrt{\frac{a}{bx^3} + 1}}\right) + \frac{2B}{3a\sqrt{b}e^{\frac{5}{2}}\sqrt{\frac{a}{bx^3} + 1}}$$

input
```
integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(3/2),x)
```

output
```
A*(-2/(3*a*sqrt(b)*e**(5/2)*x**3*sqrt(a/(b*x**3) + 1)) - 4*sqrt(b)/(3*a**2
*e**(5/2)*sqrt(a/(b*x**3) + 1))) + 2*B/(3*a*sqrt(b)*e**(5/2)*sqrt(a/(b*x**
3) + 1))
```

Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(5/2)), x)
```

Giac [B] (verification not implemented)

Error detected during grading. Assigning place holder grade for now.

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx = Recursiveaassumption \geq -\frac{2\,Ae^3\left(\frac{\sqrt{be+\frac{ae}{x^3}}}{ae^4}-\frac{\sqrt{be}}{ae^4}\right)}{3\,a|e|^2} + \frac{2\,(Ba-Ab)\sqrt{ex}x}{3\,\sqrt{be^4x^3+ae^4}a^2e} - \frac{bignored}{e^3t_{nostep}^6}$$

input
```
integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
Recursive*a*assumption >= -2/3*A*e^3*(sqrt(b*e + a*e/x^3)/(a*e^4) - sqrt(b
*e)/(a*e^4))/(a*abs(e)^2) + 2/3*(B*a - A*b)*sqrt(e*x)*x/(sqrt(b*e^4*x^3 +
a*e^4)*a^2*e) - b*ignored/(e^3*t_nostep^6)
```

Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx=-\frac{\left(\frac{2\,A}{3\,a\,b\,e^2}+\frac{x^3\,\left(4\,A\,b-2\,B\,a\right)}{3\,a^2\,b\,e^2}\right)\,\sqrt{b\,x^3+a}}{x^4\,\sqrt{e\,x}+\frac{a\,x\,\sqrt{e\,x}}{b}}$$

input
```
int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x)
```

output
```
-(((2*A)/(3*a*b*e^2) + (x^3*(4*A*b - 2*B*a))/(3*a^2*b*e^2))*(a + b*x^3)^(1
/2))/(x^4*(e*x)^(1/2) + (a*x*(e*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.37

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx=-\frac{2\sqrt{e}\,\sqrt{b\,x^3+a}}{3\sqrt{x}\,a\,e^3x}$$

input
```
int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x)
```

output
```
( - 2*sqrt(e)*sqrt(a + b*x**3))/(3*sqrt(x)*a*e**3*x)
```

3.280 $\int \frac{A+Bx^3}{(ex)^{7/2}(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 283

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx=-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}}-\frac{2(8Ab-5aB)\sqrt{ex}}{15a^2e^4\sqrt{a+bx^3}}$$
$$-\frac{2(8Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2+\sqrt{3}\right.}{15\sqrt[4]{3}a^{7/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/5*A/a/e/(e*x)^(5/2)/(b*x^3+a)^(1/2)-2/15*(8*A*b-5*B*a)*(e*x)^(1/2)/a^2/
e^4/(b*x^3+a)^(1/2)-2/45*(8*A*b-5*B*a)*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(
1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^
(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(7/3)/e^4/(b^(1/3)*x
*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.34

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx=\frac{x\left(-2(3aA+8Abx^3-5aBx^3)+4(-8Ab+5aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometri}\right.}{15a^2(ex)^{7/2}\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)),x]
```

output
```
(x*(-2*(3*a*A + 8*A*b*x^3 - 5*a*B*x^3) + 4*(-8*A*b + 5*a*B)*x^3*Sqrt[1 + (
b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(15*a^2*(e*x)^(
7/2)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {955, 819, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(8Ab-5aB)\int \frac{1}{\sqrt{ex}\left(bx^3+a\right)^{3/2}}dx}{5ae^3}-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 819}$$

$$-\frac{(8Ab-5aB)\left(\frac{2\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{3a}+\frac{2\sqrt{ex}}{3ae\sqrt{a+bx^3}}\right)}{5ae^3}-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 851}$$

$$-\frac{(8Ab-5aB)\left(\frac{4\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{3ae}+\frac{2\sqrt{ex}}{3ae\sqrt{a+bx^3}}\right)}{5ae^3}-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 766$$

$$-\frac{(8Ab-5aB)\left(\frac{2\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{4/3}e^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}+\frac{2}{3ae}\right.}{5ae^3}$$

$$\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}}$$

input
```
Int[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)),x]
```

output
```
(-2*A)/(5*a*e*(e*x)^(5/2)*Sqrt[a + b*x^3]) - ((8*A*b - 5*a*B)*((2*Sqrt[e*x
])/(3*a*e*Sqrt[a + b*x^3]) + (2*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(
a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + S
qrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3
)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(3*3^(1
/4)*a^(4/3)*e^2*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e +
(1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(5*a*e^3)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.44 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.77

method	result	size
elliptic	Expression too large to display	784
risch	Expression too large to display	1444
default	Expression too large to display	3783

input
```
int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)*(-2/5/e^4/a^2*A*(b*e*x^4
+a*e*x)^(1/2)/x^3-2/3/e^3*x/a^2*(A*b-B*a)/((x^3+a/b)*b*e*x)^(1/2)+2*(-2/5*
b/a^2/e^3*A-2/3/a^2/e^3*(A*b-B*a))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(
-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3
)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(
-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.41

$$\int \frac{A+Bx^3}{(ex)^{7/2}\,(a+bx^3)^{3/2}}\,dx =$$
$$-\frac{2\left(2\left((5\,Bab-8\,Ab^2)x^6+(5\,Ba^2-8\,Aab)x^3\right)\sqrt{ae}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)-\left((5\,Ba^2-8\,Aab)x^3\right.}{15\left(a^3be^4x^6+a^4e^4x^3\right)}$$

input
```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(2*((5*B*a*b - 8*A*b^2)*x^6 + (5*B*a^2 - 8*A*a*b)*x^3)*sqrt(a*e)*wei
erstrassPInverse(0, -4*b/a, 1/x) - ((5*B*a^2 - 8*A*a*b)*x^3 - 3*A*a^2)*sqr
t(b*x^3 + a)*sqrt(e*x))/(a^3*b*e^4*x^6 + a^4*e^4*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 155.85 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.34

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx=\frac{A\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},\frac{3}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}+\frac{B\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{3}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}$$

input
```
integrate((B*x**3+A)/(e*x)**(7/2)/(b*x**3+a)**(3/2),x)
```

output
```
A*gamma(-5/6)*hyper((-5/6, 3/2), (1/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*e**(7/2)*x**(5/2)*gamma(1/6)) + B*sqrt(x)*gamma(1/6)*hyper((1/6, 3/2)
, (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*e**(7/2)*gamma(7/6))
```

Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(7/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^3}{(ex)^{7/2} (a + bx^3)^{3/2}} \, dx = \int \frac{B\, x^3 + A}{(e\, x)^{7/2} \left(b\, x^3 + a\right)^{3/2}} \, dx$$

input
```
int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)),x)
```

output
```
int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A + Bx^3}{(ex)^{7/2} (a + bx^3)^{3/2}} \, dx = \frac{\sqrt{e} \left(\int \frac{\sqrt{x}\, \sqrt{b\, x^3 + a}}{b\, x^7 + a\, x^4} dx\right)}{e^4}$$

input
```
int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(e)*int((sqrt(x)*sqrt(a + b*x**3))/(a*x**4 + b*x**7),x))/e**4
```

3.281 $\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2(Ab-aB)e^2(ex)^{3/2}}{9b^2(a+bx^3)^{3/2}} + \frac{2(Ab-4aB)e^2(ex)^{3/2}}{9ab^2\sqrt{a+bx^3}} + \frac{2Be^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

output
```
-2/9*(A*b-B*a)*e^2*(e*x)^(3/2)/b^2/(b*x^3+a)^(3/2)+2/9*(A*b-4*B*a)*e^2*(e*
x)^(3/2)/a/b^2/(b*x^3+a)^(1/2)+2/3*B*e^(7/2)*arctanh(b^(1/2)*(e*x)^(3/2)/e
^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2e^3\sqrt{ex}\left(\frac{\sqrt{b}x^{3/2}(-3a^2B+Ab^2x^3-4abBx^3)}{a(a+bx^3)^{3/2}} + 3B\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{9b^{5/2}\sqrt{x}}$$

input
```
Integrate[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*e^3*Sqrt[e*x]*((Sqrt[b]*x^(3/2)*(-3*a^2*B + A*b^2*x^3 - 4*a*b*B*x^3))/(
a*(a + b*x^3)^(3/2)) + 3*B*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(9*b^(
5/2)*Sqrt[x])
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {954, 817, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 954}$$

$$\frac{B\int \frac{(ex)^{7/2}}{\left(bx^3+a\right)^{3/2}}dx}{b}+\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 817}$$

$$\frac{B\left(\frac{e^3\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx}{b}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{b}+\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{B\left(\frac{2e^2\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{b}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{b}+\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{B\left(\frac{2e^2\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3b}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{b}+\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{B\left(\frac{2e^2\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3b}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{b}+\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}+\frac{B\left(\frac{2e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3b^{3/2}}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{b}$$

input
```
Int[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*(A*b - a*B)*(e*x)^(9/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + (B*((-2*e^2*(e*x
)^(3/2))/(3*b*Sqrt[a + b*x^3]) + (2*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/
(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*b^(3/2))))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.30

method	result
default	$\frac{2\left(A\,b^{2} x^{5} \sqrt{b e}-4 B a b\,x^{5} \sqrt{b e}+3 B \,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right) e x}}{x^{2} \sqrt{b e}}\right) a b\,x^{3} \sqrt{\left(b\,x^{3}+a\right) e x}-3 B\,a^{2} x^{2} \sqrt{b e}+3 B \,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right) e x}}{x^{2} \sqrt{b e}}\right) a^{2} \sqrt{(b}}{9 \sqrt{b e}\, b^{2} a \left(b\,x^{3}+a\right)^{\frac{3}{2}} x}$
elliptic	Expression too large to display

input
```
int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(A*b^2*x^5*(b*e)^(1/2)-4*B*a*b*x^5*(b*e)^(1/2)+3*B*arctanh(((b*x^3+a)*
e*x)^(1/2)/x^2/(b*e)^(1/2))*a*b*x^3*((b*x^3+a)*e*x)^(1/2)-3*B*a^2*x^2*(b*e
)^(1/2)+3*B*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*a^2*((b*x^3+a)*
e*x)^(1/2))*(e*x)^(1/2)*e^3/(b*e)^(1/2)/b^2/a/(b*x^3+a)^(3/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.85

$$\int \frac{(ex)^{7/2}\,(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \left[\frac{3\,(Bab^2e^3x^6+2\,Ba^2be^3x^3+Ba^3e^3)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,b\right.}{18\,(ab^4x^6+2}\right.$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/18*(3*(B*a*b^2*e^3*x^6 + 2*B*a^2*b*e^3*x^3 + B*a^3*e^3)*sqrt(e/b)*log(-
8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*
sqrt(e*x)*sqrt(e/b)) - 4*((4*B*a*b - A*b^2)*e^3*x^4 + 3*B*a^2*e^3*x)*sqrt(
b*x^3 + a)*sqrt(e*x))/(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2), -1/9*(3*(B*a*
b^2*e^3*x^6 + 2*B*a^2*b*e^3*x^3 + B*a^3*e^3)*sqrt(-e/b)*arctan(2*sqrt(b*x^
3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*((4*B*a*b - A*b^2)*
e^3*x^4 + 3*B*a^2*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/(a*b^4*x^6 + 2*a^2*b^3
*x^3 + a^3*b^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}\,(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{7}{2}}}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^(7/2)/(b*x^3 + a)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2\,Be^6\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{3\,\sqrt{be}b^2|e|^2} - \frac{2\left(\frac{3\,Bae^8}{b^2}+\frac{(4\,Ba^5b^6e^{24}-Aa^4b^7e^{24})x^3}{a^5b^7e^{16}}\right)\sqrt{ex}ex}{9\,(be^4x^3+ae^4)^{\frac{3}{2}}}$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
-2/3*B*e^6*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*e^4)))/(s
qrt(b*e)*b^2*abs(e)^2) - 2/9*(3*B*a*e^8/b^2 + (4*B*a^5*b^6*e^24 - A*a^4*b^
7*e^24)*x^3/(a^5*b^7*e^16))*sqrt(e*x)*e*x/(b*e^4*x^3 + a*e^4)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(B\,x^3+A)\,(e\,x)^{7/2}}{(b\,x^3+a)^{5/2}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(5/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \frac{(ex)^{7/2}\,(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{\sqrt{e}\,e^3\left(-2\sqrt{x}\,\sqrt{b\,x^3+a}\,bx-\sqrt{b}\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)a-\sqrt{b}\log\left(\sqrt{b\,x^3\right.\right.}{}$$

input
```
int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

output
```
(sqrt(e)*e**3*( - 2*sqrt(x)*sqrt(a + b*x**3)*b*x - sqrt(b)*log(sqrt(a + b*
x**3) - sqrt(x)*sqrt(b)*x)*a - sqrt(b)*log(sqrt(a + b*x**3) - sqrt(x)*sqrt
(b)*x)*b*x**3 + sqrt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)*x)*a + sqrt
(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)*x)*b*x**3))/(3*b**2*(a + b*x**3
))
```

3.282 $\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 295

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2(Ab-aB)e^2\sqrt{ex}}{9b^2\left(a+bx^3\right)^{3/2}}+\frac{2(Ab-10aB)e^2\sqrt{ex}}{27ab^2\sqrt{a+bx^3}}$$
$$+\frac{(2Ab+7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2+\sqrt{3}\cdots\right.}{27\sqrt[4]{3}a^{4/3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/9*(A*b-B*a)*e^2*(e*x)^(1/2)/b^2/(b*x^3+a)^(3/2)+2/27*(A*b-10*B*a)*e^2*(
e*x)^(1/2)/a/b^2/(b*x^3+a)^(1/2)+1/81*(2*A*b+7*B*a)*e^2*(e*x)^(1/2)*(a^(1/
3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2)
)*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*
x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(4/
3)/b^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(
1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.37

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2e^2\sqrt{ex}\left(-7a^2B+Ab^2x^3-2ab(A+5Bx^3)+(2Ab+7aB)(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\right.}{27ab^2(a+bx^3)^{3/2}}$$

input
```
Integrate[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*e^2*Sqrt[e*x]*(-7*a^2*B + A*b^2*x^3 - 2*a*b*(A + 5*B*x^3) + (2*A*b + 7*
a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b
*x^3)/a)]))/(27*a*b^2*(a + b*x^3)^(3/2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {957, 817, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{(7aB+2Ab)\int \frac{(ex)^{5/2}}{(bx^3+a)^{3/2}}dx}{9ab}+\frac{2(ex)^{7/2}(Ab-aB)}{9abe(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{ 817}$$

$$\frac{(7aB+2Ab)\left(\frac{e^3\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{3b}-\frac{2e^2\sqrt{ex}}{3b\sqrt{a+bx^3}}\right)}{9ab}+\frac{2(ex)^{7/2}(Ab-aB)}{9abe(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{(7aB+2Ab)\left(\frac{2e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{3b}-\frac{2e^2\sqrt{ex}}{3b\sqrt{a+bx^3}}\right)}{9ab}+\frac{2(ex)^{7/2}(Ab-aB)}{9abe\,(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{766}$$

$$\frac{(7aB+2Ab)\left(\frac{e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+(1+\sqrt{3})\sqrt[3]{b}ex\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}xe+\sqrt[3]{a}e}{(1+\sqrt{3})\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}\sqrt[3]{a}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+(1+\sqrt{3})\sqrt[3]{b}ex\right)^2}}}-\frac{2e^2\sqrt{ex}}{3b\sqrt{a+bx^3}}\right)}{9ab}+\frac{2(ex)^{7/2}(Ab-aB)}{9abe\,(a+bx^3)^{3/2}}$$

input
```
Int[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*(A*b - a*B)*(e*x)^(7/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + ((2*A*b + 7*a*B)
*((-2*e^2*Sqrt[e*x])/(3*b*Sqrt[a + b*x^3]) + (e*Sqrt[e*x]*(a^(1/3)*e + b^(
1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^
(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 -
 Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt
[3])/4])/(3*3^(1/4)*a^(1/3)*b*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))
/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(9*a*b)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.51 (sec) , antiderivative size = 809, normalized size of antiderivative = 2.74

method	result	size
elliptic	Expression too large to display	809
default	Expression too large to display	7083

input
```
int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(b*x^3+a)^(1/2)*((b*x^3+a)*e*x)^(1/2)*(-2/9*e^2/b^4*(A*b
-B*a)*(b*e*x^4+a*e*x)^(1/2)/(x^3+a/b)^2+2/27/b^2*e^3*x/a*(A*b-10*B*a)/((x^
3+a/b)*b*e*x)^(1/2)+2*(B*e^3/b^2+2/27/b^2/a*e^3*(A*b-10*B*a))*(1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2
)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)
*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^
(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a
*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b
^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.58

$$\int \frac{(ex)^{5/2}\,(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx =$$

$$-\frac{2\,(((7\,Bab^2+2\,Ab^3)e^2x^6+2\,(7\,Ba^2b+2\,Aab^2)e^2x^3+(7\,Ba^3+2\,Aa^2b)e^2)\sqrt{ae}\,\mathrm{weierstrassPInverse}(0,-}{27\,(a^2b^4x^6+2\,a^3b^3x^3+a^4b^2)}$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/27*(((7*B*a*b^2 + 2*A*b^3)*e^2*x^6 + 2*(7*B*a^2*b + 2*A*a*b^2)*e^2*x^3
+ (7*B*a^3 + 2*A*a^2*b)*e^2)*sqrt(a*e)*weierstrassPInverse(0, -4*b/a, 1/x)
 + ((10*B*a^2*b - A*a*b^2)*e^2*x^3 + (7*B*a^3 + 2*A*a^2*b)*e^2)*sqrt(b*x^3
 + a)*sqrt(e*x))/(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(5/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}}{{\left(b\,x^3+a\right)}^{5/2}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(5/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\frac{\sqrt{e}\,e^2\left(-2\sqrt{x}\,\sqrt{b\,x^3+a}+\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b^2x^7+2ab\,x^4+a^2x}dx\right)a^2+\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b^2x^7+2ab\,x^4+a^2x}dx\right)ab\,x

3.283 $\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 596

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\frac{2(Ab-aB)(ex)^{5/2}}{9abe\left(a+bx^3\right)^{3/2}}+\frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}}-\frac{2\left(1+\sqrt{3}\right)(4Ab+5aB)e\sqrt{ex}\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$
$$+\frac{2(4Ab+5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\ 3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{\left(1-\sqrt{3}\right)(4Ab+5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\ldots\right)}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*(A*b-B*a)*(e*x)^(5/2)/a/b/e/(b*x^3+a)^(3/2)+2/27*(4*A*b+5*B*a)*(e*x)^(
5/2)/a^2/b/e/(b*x^3+a)^(1/2)-2/27*(1+3^(1/2))*(4*A*b+5*B*a)*e*(e*x)^(1/2)*
(b*x^3+a)^(1/2)/a^2/b^(5/3)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)+2/27*(4*A*b+5*
B*a)*e*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^
(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1
/4*2^(1/2))*3^(1/4)/a^(5/3)/b^(5/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3
)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+1/81*(1-3^(1/2))*(4*A*b+
5*B*a)*e*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a
^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)
+1/4*2^(1/2))*3^(3/4)/a^(5/3)/b^(5/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1
/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.14

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{x(ex)^{3/2}\left(-5a^2B+(4Ab+5aB)\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\ \text{Hypergeometric2F1}\left(\frac{5}{6},\frac{5}{2}\right.\right.}{10a^2b\left(a+bx^3\right)^{3/2}}$$

input
```
Integrate[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(x*(e*x)^(3/2)*(-5*a^2*B + (4*A*b + 5*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]
*Hypergeometric2F1[5/6, 5/2, 11/6, -((b*x^3)/a)]))/(10*a^2*b*(a + b*x^3)^(
3/2))
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {957, 819, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(5aB+4Ab)\int \frac{(ex)^{3/2}}{(bx^3+a)^{3/2}}dx}{9ab}+\frac{2(ex)^{5/2}(Ab-aB)}{9abe(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$\frac{(5aB+4Ab)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{2\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx}{3a}\right)}{9ab}+\frac{2(ex)^{5/2}(Ab-aB)}{9abe(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{(5aB+4Ab)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{3ae}\right)}{9ab}+\frac{2(ex)^{5/2}(Ab-aB)}{9abe(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{837}$$

$$\frac{(5aB+4Ab)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{3ae}\right)}{9ab}+\frac{2(ex)^{5/2}(Ab-aB)}{9abe(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{(5aB+4Ab)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{3ae}\right)}{9ab}+\frac{2(ex)^{5/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$\downarrow$ 766

$$\frac{(5aB+4Ab)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{Ell}}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}\right.}{\left(\sqrt[3]{a}e+\left(1+\right.\right.}}}\right)}{3ae}\right)}{9ab}$$

$$\frac{2(ex)^{5/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$\downarrow$ 2420

$$\frac{(5aB+4Ab)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(\frac{\frac{(1+\sqrt{3})e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+(1+\sqrt{3})\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+(1+\sqrt{3})\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3}}{(1+\sqrt{3}}\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+(1+\sqrt{3})\sqrt[3]{b}ex\right)^2}}}}{2b^{2/3}}\right.}{}\right)}{}$$

$$\frac{2(ex)^{5/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

input
```
Int[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*(A*b - a*B)*(e*x)^(5/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + ((4*A*b + 5*a*B)
*((2*(e*x)^(5/2))/(3*a*e*Sqrt[a + b*x^3]) - (4*((((1 + Sqrt[3])*e^3*Sqrt[e
*x]*Sqrt[a + b*x^3])/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x) - (3^(1/4)*a^
(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^
(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*
EllipticE[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 +
 Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*e*x*(a^(1/3)*e +
 b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])
)/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*
x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e
 + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3
])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4]
)/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)
*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(3*a*e)))/(9*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.36 (sec) , antiderivative size = 1190, normalized size of antiderivative = 2.00

method	result	size
elliptic	Expression too large to display	1190
default	Expression too large to display	10786

input
```
int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(b*x^3+a)^(1/2)*((b*x^3+a)*e*x)^(1/2)*(2/9*e/a/b^3*x^2*(
A*b-B*a)*(b*e*x^4+a*e*x)^(1/2)/(x^3+a/b)^2+2/27/b*e^2*x^3/a^2*(4*A*b+5*B*a
)/((x^3+a/b)*b*e*x)^(1/2)-2/27/b/a^2*e^2*(4*A*b+5*B*a)*(x*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-
1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*Ellipt
icF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),(
(3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
)+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/...
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.28

$$\int \frac{(ex)^{3/2}\,(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx =$$
$$-\frac{2\left(\left(\left((5\,Bab^2+4\,Ab^3)ex^7+2\,(5\,Ba^2b+4\,Aab^2)ex^4+(5\,Ba^3+4\,Aa^2b)ex\right)\sqrt{ae}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,b}{a},\mathrm{we}\right.\right.\right.}{27\,(a^2b^4x^7+2\,a^3b^3x^4}$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/27*(((5*B*a*b^2 + 4*A*b^3)*e*x^7 + 2*(5*B*a^2*b + 4*A*a*b^2)*e*x^4 + (5
*B*a^3 + 4*A*a^2*b)*e*x)*sqrt(a*e)*weierstrassZeta(0, -4*b/a, weierstrassP
Inverse(0, -4*b/a, 1/x)) + ((8*B*a^2*b + A*a*b^2)*e*x^3 + (5*B*a^3 + 4*A*a
^2*b)*e)*sqrt(b*x^3 + a)*sqrt(e*x))/(a^2*b^4*x^7 + 2*a^3*b^3*x^4 + a^4*b^2
*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(3/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\int \frac{(B\,x^3+A)\,(e\,x)^{3/2}}{(b\,x^3+a)^{5/2}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(5/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\sqrt{e}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}\,x}{b^2x^6+2ab\,x^3+a^2}dx\right)e$$

input
```
int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

output
```
sqrt(e)*int((sqrt(x)*sqrt(a + b*x**3)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x
)*e
```

3.284 $\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2(Ab-aB)(ex)^{3/2}}{9abe\,(a+bx^3)^{3/2}} + \frac{2(2Ab+aB)(ex)^{3/2}}{9a^2be\sqrt{a+bx^3}}$$

output
```
2/9*(A*b-B*a)*(e*x)^(3/2)/a/b/e/(b*x^3+a)^(3/2)+2/9*(2*A*b+B*a)*(e*x)^(3/2
)/a^2/b/e/(b*x^3+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2x\sqrt{ex}(3aA+2Abx^3+aBx^3)}{9a^2\,(a+bx^3)^{3/2}}$$

input
```
Integrate[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*x*Sqrt[e*x]*(3*a*A + 2*A*b*x^3 + a*B*x^3))/(9*a^2*(a + b*x^3)^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {957, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(aB+2Ab)\int \frac{\sqrt{ex}}{(bx^3+a)^{3/2}}dx}{3ab}+\frac{2(ex)^{3/2}(Ab-aB)}{9abe\,(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{796}$$

$$\frac{2(ex)^{3/2}(aB+2Ab)}{9a^2be\sqrt{a+bx^3}}+\frac{2(ex)^{3/2}(Ab-aB)}{9abe\,(a+bx^3)^{3/2}}$$

input
```
Int[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*(A*b - a*B)*(e*x)^(3/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(2*A*b + a*B)
*(e*x)^(3/2))/(9*a^2*b*e*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{2x\left(2Ab\,x^{3}+Ba\,x^{3}+3Aa\right)\sqrt{ex}}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{2}}$	39
default	$\frac{2x\left(2Ab\,x^{3}+Ba\,x^{3}+3Aa\right)\sqrt{ex}}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{2}}$	39
orering	$\frac{2x\left(2Ab\,x^{3}+Ba\,x^{3}+3Aa\right)\sqrt{ex}}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{2}}$	39
elliptic	$\frac{\sqrt{ex}\,\sqrt{\left(b\,x^{3}+a\right)ex}\,\left(\frac{2x(Ab-Ba)\sqrt{be\,x^{4}+aex}}{9a\,b^{3}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{2e\,x^{2}(2Ab+Ba)}{9b\,a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)bex}}\right)}{ex\sqrt{b\,x^{3}+a}}$	111

input
```
int((e*x)^(1/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x*(2*A*b*x^3+B*a*x^3+3*A*a)*(e*x)^(1/2)/(b*x^3+a)^(3/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\left((Ba+2\,Ab)x^4+3\,Aax\right)\sqrt{bx^3+a}\sqrt{ex}}{9\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)}$$

input
```
integrate((e*x)^(1/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
2/9*((B*a + 2*A*b)*x^4 + 3*A*a*x)*sqrt(b*x^3 + a)*sqrt(e*x)/(a^2*b^2*x^6 +
 2*a^3*b*x^3 + a^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(1/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{ex}}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(1/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(e*x)/(b*x^3 + a)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\left(\frac{3\,Ae^5}{a}+\frac{(Ba^5b^5e^{21}+2\,Aa^4b^6e^{21})x^3}{a^6b^5e^{16}}\right)\sqrt{ex}ex}{9\,(be^4x^3+ae^4)^{\frac{3}{2}}}$$

input
```
integrate((e*x)^(1/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
2/9*(3*A*e^5/a + (B*a^5*b^5*e^21 + 2*A*a^4*b^6*e^21)*x^3/(a^6*b^5*e^16))*s
qrt(e*x)*e*x/(b*e^4*x^3 + a*e^4)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{\left(\frac{2\,A\,x\,\sqrt{e\,x}}{3\,a\,b^2}+\frac{x^4\,\sqrt{e\,x}\,(4\,A\,b+2\,B\,a)}{9\,a^2\,b^2}\right)\,\sqrt{b\,x^3+a}}{x^6+\frac{a^2}{b^2}+\frac{2\,a\,x^3}{b}}$$

input
```
int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(5/2),x)
```

output
```
(((2*A*x*(e*x)^(1/2))/(3*a*b^2) + (x^4*(e*x)^(1/2)*(4*A*b + 2*B*a))/(9*a^2
*b^2))*(a + b*x^3)^(1/2))/(x^6 + a^2/b^2 + (2*a*x^3)/b)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{e}\,\sqrt{b\,x^3+a}\,x}{3a\,(b\,x^3+a)}$$

input
```
int((e*x)^(1/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

output
```
(2*sqrt(x)*sqrt(e)*sqrt(a + b*x**3)*x)/(3*a*(a + b*x**3))
```

3.285 $\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 297

$$\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{5/2}}\,dx=\frac{2(Ab-aB)\sqrt{ex}}{9abe\left(a+bx^3\right)^{3/2}}+\frac{2(8Ab+aB)\sqrt{ex}}{27a^2be\sqrt{a+bx^3}}+\frac{2(8Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}a^{7/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*(A*b-B*a)*(e*x)^(1/2)/a/b/e/(b*x^3+a)^(3/2)+2/27*(8*A*b+B*a)*(e*x)^(1/
2)/a^2/b/e/(b*x^3+a)^(1/2)+2/81*(8*A*b+B*a)*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)
^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+
(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(7/3)/b/e/(b^(1
/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+
a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{5/2}}\,dx = \frac{2x\left(-2a^2B+8Ab^2x^3+ab(11A+Bx^3)+2(8Ab+aB)\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\,\text{Hyp}\right.}{27a^2b\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

input
```
Integrate[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(5/2)),x]
```

output
```
(2*x*(-2*a^2*B + 8*A*b^2*x^3 + a*b*(11*A + B*x^3) + 2*(8*A*b + a*B)*(a + b
*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))
/(27*a^2*b*Sqrt[e*x]*(a + b*x^3)^(3/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {957, 819, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(aB+8Ab)\int \frac{1}{\sqrt{ex}\left(bx^3+a\right)^{3/2}}dx}{9ab}+\frac{2\sqrt{ex}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$\frac{(aB+8Ab)\left(\frac{2\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{3a}+\frac{2\sqrt{ex}}{3ae\sqrt{a+bx^3}}\right)}{9ab}+\frac{2\sqrt{ex}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{(aB+8Ab)\left(\frac{4\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{3ae}+\frac{2\sqrt{ex}}{3ae\sqrt{a+bx^3}}\right)}{9ab}+\frac{2\sqrt{ex}(Ab-aB)}{9abe\,(a+bx^3)^{3/2}}$$

$\Big\downarrow$ 766

$$\frac{(aB+8Ab)\left(\frac{2\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{4/3}e^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}+\frac{2\sqrt{e}}{3ae\sqrt{a-}}\right.}{9ab}$$

$$\frac{2\sqrt{ex}(Ab-aB)}{9abe\,(a+bx^3)^{3/2}}$$

input
```
Int[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(5/2)),x]
```

output
```
(2*(A*b - a*B)*Sqrt[e*x])/(9*a*b*e*(a + b*x^3)^(3/2)) + ((8*A*b + a*B)*((2
*Sqrt[e*x])/(3*a*e*Sqrt[a + b*x^3]) + (2*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*
x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e
 + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3
])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4]
)/(3*3^(1/4)*a^(4/3)*e^2*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(
1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(9*a*b)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.72 (sec) , antiderivative size = 785, normalized size of antiderivative = 2.64

method	result	size
elliptic	Expression too large to display	785
default	Expression too large to display	7077

input
```
int((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)*(2/9/e/a/b^3*(A*b-B*a)*(
b*e*x^4+a*e*x)^(1/2)/(x^3+a/b)^2+2/27/b*x/a^2*(8*A*b+B*a)/((x^3+a/b)*b*e*x
)^(1/2)+4/27/a^2*(8*A*b+B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1
/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^
(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx = -\frac{2\,\big(2\,((Bab^2+8\,Ab^3)x^6+Ba^3+8\,Aa^2b+2\,(Ba^2b+8\,Aab^2)x^3)\sqrt{ae}\,\text{weierstrassPInverse}\big(0,-\frac{4\,b}{a},\frac{1}{x}\big)+(2}{27\,(a^3b^3ex^6+2\,a^4b^2ex^3+a^5be)}$$

input
```
integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/27*(2*((B*a*b^2 + 8*A*b^3)*x^6 + B*a^3 + 8*A*a^2*b + 2*(B*a^2*b + 8*A*a
*b^2)*x^3)*sqrt(a*e)*weierstrassPInverse(0, -4*b/a, 1/x) + (2*B*a^3 - 11*A
*a^2*b - (B*a^2*b + 8*A*a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(a^3*b^3*e*
x^6 + 2*a^4*b^2*e*x^3 + a^5*b*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)/(e*x)**(1/2)/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*sqrt(e*x)), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx=\int \frac{B\,x^3+A}{\sqrt{e\,x}\,(b\,x^3+a)^{5/2}}\,dx$$

input
```
int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(5/2)),x)
```

output
```
int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx=\frac{\sqrt{e}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b^2x^7+2ab\,x^4+a^2x}dx\right)}{e}$$

input
```
int((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(5/2),x)
```

output
```
(sqrt(e)*int((sqrt(x)*sqrt(a + b*x**3))/(a**2*x + 2*a*b*x**4 + b**2*x**7),
x))/e
```

3.286 $\int \frac{A+Bx^3}{(ex)^{3/2}(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 624

$$\begin{aligned}
\int \frac{A+Bx^3}{(ex)^{3/2}(a+bx^3)^{5/2}}\,dx &= -\frac{2A}{ae\sqrt{ex}\,(a+bx^3)^{3/2}}-\frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\,(a+bx^3)^{3/2}}\\
&-\frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}}+\frac{8\left(1+\sqrt{3}\right)(10Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{27a^3b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}\\
&-\frac{8(10Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\,3^{3/4}a^{8/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\\
&-\frac{4\left(1-\sqrt{3}\right)(10Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\right.}{27\sqrt[4]{3}a^{8/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}$$

output
```
-2*A/a/e/(e*x)^(1/2)/(b*x^3+a)^(3/2)-2/9*(10*A*b-B*a)*(e*x)^(5/2)/a^2/e^4/
(b*x^3+a)^(3/2)-8/27*(10*A*b-B*a)*(e*x)^(5/2)/a^3/e^4/(b*x^3+a)^(1/2)+8/27
*(1+3^(1/2))*(10*A*b-B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/a^3/b^(2/3)/e^2/(a^(
1/3)+(1+3^(1/2))*b^(1/3)*x)-8/27*(10*A*b-B*a)*(e*x)^(1/2)*(a^(1/3)+b^(1/3)
*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*
x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(
1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*3^(1/4)/a^(8/3)/b^(2/3)
/e^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/
2)/(b*x^3+a)^(1/2)-4/81*(1-3^(1/2))*(10*A*b-B*a)*(e*x)^(1/2)*(a^(1/3)+b^(1
/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/
3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(
1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(8/3)/b^(2
/3)/e^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^
(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx = \frac{2x\left(-5a^2A+(-10Ab+aB)x^3(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{5}{2},\right.\right.}{5a^3(ex)^{3/2}\left(a+bx^3\right)^{3/2}}$$

input
```
Integrate[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)),x]
```

output
```
(2*x*(-5*a^2*A + (-10*A*b + a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hyper
geometric2F1[5/6, 5/2, 11/6, -((b*x^3)/a)]))/(5*a^3*(e*x)^(3/2)*(a + b*x^3
)^(3/2))
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {955, 819, 819, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(10Ab-aB)\int \frac{(ex)^{3/2}}{\left(bx^3+a\right)^{5/2}}dx}{ae^3}-\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 819}$$

$$-\frac{(10Ab-aB)\left(\frac{4\int \frac{(ex)^{3/2}}{\left(bx^3+a\right)^{3/2}}dx}{9a}+\frac{2(ex)^{5/2}}{9ae(a+bx^3)^{3/2}}\right)}{ae^3}-\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 819}$$

$$-\frac{(10Ab-aB)\left(\frac{4\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{2\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx}{3a}\right)}{9a}+\frac{2(ex)^{5/2}}{9ae(a+bx^3)^{3/2}}\right)}{ae^3}-\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 851}$$

$$-\frac{(10Ab-aB)\left(\frac{4\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{3ae}\right)}{9a}+\frac{2(ex)^{5/2}}{9ae(a+bx^3)^{3/2}}\right)}{ae^3}-\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 837}$$

$$-\frac{(10Ab-aB)\left(\frac{4\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int-\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{3ae}\right)}{9a}+\frac{2(ex)^{5/2}}{9ae(a+bx^3)^{3/2}}\right)}{ae^3}-\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

$\downarrow$ 25

$$-\frac{(10Ab-aB)\left(\frac{4\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{3ae}\right)}{9a}+\frac{2(ex)^{5/2}}{9ae(a+bx^3)^{3/2}}\right)}{ae^3}-\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

$\downarrow$ 766

$$-\frac{(10Ab-aB)\left(\frac{4\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{\cdots}\right.}{\left(\sqrt[3]{a}e+\left(1\cdots\right.\right.}}}\right)}{3ae}\right)}{9a}\right)}{ae^3}$$

$$\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

$\downarrow$ 2420

$$-\frac{(10Ab-aB)\left(\frac{4\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\right.}{\left(1+\right.}\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right)}{2b^{2/3}}\right)}{}\right)}{}$$

$$\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)),x]
```

output
```
(-2*A)/(a*e*Sqrt[e*x]*(a + b*x^3)^(3/2)) - ((10*A*b - a*B)*((2*(e*x)^(5/2)
)/(9*a*e*(a + b*x^3)^(3/2)) + (4*((2*(e*x)^(5/2))/(3*a*e*Sqrt[a + b*x^3])
- (4*((((1 + Sqrt[3])*e^3*Sqrt[e*x]*Sqrt[a + b*x^3])/(a^(1/3)*e + (1 + Sqr
t[3])*b^(1/3)*e*x) - (3^(1/4)*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x
)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e
+ (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticE[ArcCos[(a^(1/3)*e + (1 - Sqrt[3]
)*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])
/(Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*
b^(1/3)*e*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*e*S
qrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2
*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF
[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])
*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*e*x*(a^
(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a
+ b*x^3])))/(3*a*e)))/(9*a)))/(a*e^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 1225, normalized size of antiderivative = 1.96

method	result	size
elliptic	Expression too large to display	1225
risch	Expression too large to display	3336
default	Expression too large to display	10961

input
```
int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)*(-2/9/e^2/a^2/b^2*x^2*(A
*b-B*a)*(b*e*x^4+a*e*x)^(1/2)/(x^3+a/b)^2-2/27/e*x^3/a^3*(13*A*b-4*B*a)/((
x^3+a/b)*b*e*x)^(1/2)-2*(b*e*x^3+a*e)/e^2/a^3*A/(x*(b*e*x^3+a*e))^(1/2)+(2
/27/a^3*(13*A*b-4*B*a)/e+2*b/a^3/e*A)*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3
))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b
^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.27

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2\left(4\left((Bab^2-10\,Ab^3)x^7+2\,(Ba^2b-10\,Aab^2)x^4+(Ba^3-10\,Aa^2b)x\right)\sqrt{ae}\text{weierstrassZeta}\left(0,-\frac{4\,b}{a},\text{weier}\right.}{27\left(a^3b^3e^2x^7+2\,a^4b^2e^2x^4+\right.}$$

input
```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/27*(4*((B*a*b^2 - 10*A*b^3)*x^7 + 2*(B*a^2*b - 10*A*a*b^2)*x^4 + (B*a^3
 - 10*A*a^2*b)*x)*sqrt(a*e)*weierstrassZeta(0, -4*b/a, weierstrassPInverse
(0, -4*b/a, 1/x)) + (4*B*a^3 - 13*A*a^2*b + (B*a^2*b - 10*A*a*b^2)*x^3)*sq
rt(b*x^3 + a)*sqrt(e*x))/(a^3*b^3*e^2*x^7 + 2*a^4*b^2*e^2*x^4 + a^5*b*e^2*
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x**3+A)/(e*x)**(3/2)/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{B\,x^3+A}{{\left(e\,x\right)}^{3/2}\,{\left(b\,x^3+a\right)}^{5/2}}\,dx$$

input
```
int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)),x)
```

output
```
int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx=\frac{\sqrt{e}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^{3}+a}}{b^{2}x^{8}+2ab\,x^{5}+a^{2}x^{2}}dx\right)}{e^{2}}$$

input
```
int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x)
```

output
```
(sqrt(e)*int((sqrt(x)*sqrt(a + b*x**3))/(a**2*x**2 + 2*a*b*x**5 + b**2*x**
8),x))/e**2
```

3.287 $\int \frac{A+Bx^3}{(ex)^{5/2}(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 104

$$\int \frac{A+Bx^3}{(ex)^{5/2}(a+bx^3)^{5/2}}\,dx = -\frac{2A}{3ae(ex)^{3/2}(a+bx^3)^{3/2}} - \frac{2(4Ab-aB)(ex)^{3/2}}{9a^2e^4(a+bx^3)^{3/2}} - \frac{4(4Ab-aB)(ex)^{3/2}}{9a^3e^4\sqrt{a+bx^3}}$$

output
```
-2/3*A/a/e/(e*x)^(3/2)/(b*x^3+a)^(3/2)-2/9*(4*A*b-B*a)*(e*x)^(3/2)/a^2/e^4
/(b*x^3+a)^(3/2)-4/9*(4*A*b-B*a)*(e*x)^(3/2)/a^3/e^4/(b*x^3+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx^3}{(ex)^{5/2}(a+bx^3)^{5/2}}\,dx = \frac{2x(-3a^2A-12aAbx^3+3a^2Bx^3-8Ab^2x^6+2abBx^6)}{9a^3(ex)^{5/2}(a+bx^3)^{3/2}}$$

input
```
Integrate[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x]
```

output
```
(2*x*(-3*a^2*A - 12*a*A*b*x^3 + 3*a^2*B*x^3 - 8*A*b^2*x^6 + 2*a*b*B*x^6))/
(9*a^3*(e*x)^(5/2)*(a + b*x^3)^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {955, 805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(ex)^{5/2}(a+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(4Ab-aB)\int \frac{\sqrt{ex}}{(bx^3+a)^{5/2}}dx}{ae^3}-\frac{2A}{3ae(ex)^{3/2}(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{805}$$

$$-\frac{(4Ab-aB)\left(\frac{2\int \frac{\sqrt{ex}}{(bx^3+a)^{3/2}}dx}{3a}+\frac{2(ex)^{3/2}}{9ae(a+bx^3)^{3/2}}\right)}{ae^3}-\frac{2A}{3ae(ex)^{3/2}(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{(4Ab-aB)\left(\frac{4(ex)^{3/2}}{9a^2e\sqrt{a+bx^3}}+\frac{2(ex)^{3/2}}{9ae(a+bx^3)^{3/2}}\right)}{ae^3}-\frac{2A}{3ae(ex)^{3/2}(a+bx^3)^{3/2}}$$

input
```
Int[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x]
```

output
```
(-2*A)/(3*a*e*(e*x)^(3/2)*(a + b*x^3)^(3/2)) - ((4*A*b - a*B)*((2*(e*x)^(3
/2))/(9*a*e*(a + b*x^3)^(3/2)) + (4*(e*x)^(3/2))/(9*a^2*e*Sqrt[a + b*x^3])
))/(a*e^3)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{2x\left(8A\,b^{2}x^{6}-2Bab\,x^{6}+12aAb\,x^{3}-3B\,a^{2}x^{3}+3a^{2}A\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{3}\left(ex\right)^{\frac{5}{2}}}$	62
orering	$-\frac{2x\left(8A\,b^{2}x^{6}-2Bab\,x^{6}+12aAb\,x^{3}-3B\,a^{2}x^{3}+3a^{2}A\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{3}\left(ex\right)^{\frac{5}{2}}}$	62
default	$-\frac{2\left(8A\,b^{2}x^{6}-2Bab\,x^{6}+12aAb\,x^{3}-3B\,a^{2}x^{3}+3a^{2}A\right)}{9a^{3}\sqrt{ex}\,e^{2}\left(b\,x^{3}+a\right)^{\frac{3}{2}}x}$	67
risch	$-\frac{2A\sqrt{b\,x^{3}+a}}{3a^{3}x\,e^{2}\sqrt{ex}}-\frac{2\left(5A\,b^{2}x^{3}-2Bab\,x^{3}+6abA-3a^{2}B\right)x^{2}}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{3}e^{2}\sqrt{ex}}$	82
elliptic	$\frac{\sqrt{\left(b\,x^{3}+a\right)ex}\,\left(-\frac{2x\left(Ab-Ba\right)\sqrt{be\,x^{4}+aex}}{9e^{3}a^{2}b^{2}\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{2x^{2}\left(5Ab-2Ba\right)}{9e^{2}a^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)bex}}-\frac{2A\sqrt{be\,x^{4}+aex}}{3e^{3}a^{3}x^{2}}\right)}{\sqrt{ex}\,\sqrt{b\,x^{3}+a}}$	133

input
```
int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*x*(8*A*b^2*x^6-2*B*a*b*x^6+12*A*a*b*x^3-3*B*a^2*x^3+3*A*a^2)/(b*x^3+a
)^(3/2)/a^3/(e*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx = \frac{2\left(2\left(Bab-4\,Ab^2\right)x^6+3\left(Ba^2-4\,Aab\right)x^3-3\,Aa^2\right)\sqrt{bx^3+a}\sqrt{ex}}{9\left(a^3b^2e^3x^8+2\,a^4be^3x^5+a^5e^3x^2\right)}$$

input
```
integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
2/9*(2*(B*a*b - 4*A*b^2)*x^6 + 3*(B*a^2 - 4*A*a*b)*x^3 - 3*A*a^2)*sqrt(b*x
^3 + a)*sqrt(e*x)/(a^3*b^2*e^3*x^8 + 2*a^4*b*e^3*x^5 + a^5*e^3*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{5/2}(a+bx^3)^{5/2}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx^3}{(ex)^{5/2}(a+bx^3)^{5/2}}\,dx=\text{Exception raised: NotImplementedError}$$

input
```
integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: Recu
rsive assumption sageVARa>=(-sageVARb*sageVARe/(sageVARe^4*t_nostep^6)) ig
nored2*(-(23914845*sageVARb^7*sageVARe^18*sageVARa^6*sageVARA-9565938*sage
VARb^6*sageVARe^18*
```

Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^3}{(ex)^{5/2}(a+bx^3)^{5/2}}\,dx=-\frac{\sqrt{b\,x^3+a}\,\left(\frac{2\,A}{3\,a\,b^2\,e^2}-\frac{x^3\,\left(6\,B\,a^2-24\,A\,a\,b\right)}{9\,a^3\,b^2\,e^2}+\frac{x^6\,\left(16\,A\,b^2-4\,B\,a\,b\right)}{9\,a^3\,b^2\,e^2}\right)}{x^7\,\sqrt{e\,x}+\frac{a^2\,x\,\sqrt{e\,x}}{b^2}+\frac{2\,a\,x^4\,\sqrt{e\,x}}{b}}$$

input
```
int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x)
```

output
```
-((a + b*x^3)^(1/2)*((2*A)/(3*a*b^2*e^2) - (x^3*(6*B*a^2 - 24*A*a*b))/(9*a
^3*b^2*e^2) + (x^6*(16*A*b^2 - 4*B*a*b))/(9*a^3*b^2*e^2)))/(x^7*(e*x)^(1/2
) + (a^2*x*(e*x)^(1/2))/b^2 + (2*a*x^4*(e*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.42

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx=\frac{2\sqrt{e}\,\sqrt{b\,x^3+a}\,(-2b\,x^3-a)}{3\sqrt{x}\,a^2e^3x\,(b\,x^3+a)}$$

input
```
int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x)
```

output
```
(2*sqrt(e)*sqrt(a + b*x**3)*( - a - 2*b*x**3))/(3*sqrt(x)*a**2*e**3*x*(a +
 b*x**3))
```

3.288 $\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 320

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{16(14Ab-5aB)\sqrt{ex}}{135a^3e^4\sqrt{a+bx^3}} - \frac{16(14Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2+\cdots}{135\sqrt[4]{3}a^{10/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/5*A/a/e/(e*x)^(5/2)/(b*x^3+a)^(3/2)-2/45*(14*A*b-5*B*a)*(e*x)^(1/2)/a^2
/e^4/(b*x^3+a)^(3/2)-16/135*(14*A*b-5*B*a)*(e*x)^(1/2)/a^3/e^4/(b*x^3+a)^(
1/2)-16/405*(14*A*b-5*B*a)*(e*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*Inverse
JacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/
3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(10/3)/e^4/(b^(1/3)*x*(a^(1/3)+b
^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.38

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx=\frac{x\left(-224Ab^2x^6+a^2(-54A+110Bx^3)+a(-308Abx^3+80bBx^6)+32(-14A\right.}{135a^3(ex)^{7/2}\,(a+}$$

input
```
Integrate[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)),x]
```

output
```
(x*(-224*A*b^2*x^6 + a^2*(-54*A + 110*B*x^3) + a*(-308*A*b*x^3 + 80*b*B*x^
6) + 32*(-14*A*b + 5*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometr
ic2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(135*a^3*(e*x)^(7/2)*(a + b*x^3)^(3/2
))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {955, 819, 819, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(14Ab-5aB)\int \frac{1}{\sqrt{ex}\left(bx^3+a\right)^{5/2}}dx}{5ae^3}-\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(14Ab-5aB)\left(\frac{8\int \frac{1}{\sqrt{ex}\left(bx^3+a\right)^{3/2}}dx}{9a}+\frac{2\sqrt{ex}}{9ae\left(a+bx^3\right)^{3/2}}\right)}{5ae^3}-\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}}$$

$\downarrow$ 819

$$-\frac{(14Ab-5aB)\left(\frac{8\left(\frac{2\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{3a}+\frac{2\sqrt{ex}}{3ae\sqrt{a+bx^3}}\right)}{9a}+\frac{2\sqrt{ex}}{9ae(a+bx^3)^{3/2}}\right)}{5ae^3}-\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}}$$

$\downarrow$ 851

$$-\frac{(14Ab-5aB)\left(\frac{8\left(\frac{4\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{3ae}+\frac{2\sqrt{ex}}{3ae\sqrt{a+bx^3}}\right)}{9a}+\frac{2\sqrt{ex}}{9ae(a+bx^3)^{3/2}}\right)}{5ae^3}-\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}}$$

$\downarrow$ 766

$$-\frac{(14Ab-5aB)\left(\frac{8\left(\frac{2\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{4/3}e^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}+\frac{2\sqrt{ex}}{3ae\sqrt{a+}}\right.}{9a}\right.}{5ae^3}$$

$$\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)),x]
```

output
```
(-2*A)/(5*a*e*(e*x)^(5/2)*(a + b*x^3)^(3/2)) - ((14*A*b - 5*a*B)*((2*Sqrt[
e*x])/(9*a*e*(a + b*x^3)^(3/2)) + (8*((2*Sqrt[e*x])/(3*a*e*Sqrt[a + b*x^3]
) + (2*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(
1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*E
llipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 +
Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(3*3^(1/4)*a^(4/3)*e^2*Sqrt[(b^(
1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x
)^2]*Sqrt[a + b*x^3])))/(9*a)))/(5*a*e^3)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 12.21 (sec) , antiderivative size = 829, normalized size of antiderivative = 2.59

method	result	size
elliptic	Expression too large to display	829
risch	Expression too large to display	2182
default	Expression too large to display	7299

input
```
int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)*(-2/5/e^4/a^3*A*(b*e*x^4
+a*e*x)^(1/2)/x^3-2/9/e^4/a^2/b^2*(A*b-B*a)*(b*e*x^4+a*e*x)^(1/2)/(x^3+a/b
)^2-2/27/e^3*x/a^3*(17*A*b-8*B*a)/((x^3+a/b)*b*e*x)^(1/2)+2*(-2/5*b/a^3/e^
3*A-2/27/a^3*(17*A*b-8*B*a)/e^3)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3))
)^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a
*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.56

$$\int \frac{A + Bx^3}{(ex)^{7/2} (a + bx^3)^{5/2}} \, dx = -\frac{2 \left(16 \left((5 \, Bab^2 - 14 \, Ab^3)x^9 + 2 \, (5 \, Ba^2b - 14 \, Aab^2)x^6 + (5 \, Ba^3 - 14 \, Aa^2b)x^3\right)\sqrt{ae}\text{weierstrassPInverse}(0,}{135 \left(a^4b^2e^4x^9 + 2 \, a^5be^4x^6\right.}$$

input
```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/135*(16*((5*B*a*b^2 - 14*A*b^3)*x^9 + 2*(5*B*a^2*b - 14*A*a*b^2)*x^6 +
(5*B*a^3 - 14*A*a^2*b)*x^3)*sqrt(a*e)*weierstrassPInverse(0, -4*b/a, 1/x)
- (8*(5*B*a^2*b - 14*A*a*b^2)*x^6 - 27*A*a^3 + 11*(5*B*a^3 - 14*A*a^2*b)*x
^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(a^4*b^2*e^4*x^9 + 2*a^5*b*e^4*x^6 + a^6*e^
4*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^3}{(ex)^{7/2} (a + bx^3)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)/(e*x)**(7/2)/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + Bx^3}{(ex)^{7/2} (a + bx^3)^{5/2}} dx = \int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{5}{2}} (ex)^{\frac{7}{2}}} dx$$

input
```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(7/2)), x)
```

Giac [F]

$$\int \frac{A + Bx^3}{(ex)^{7/2} (a + bx^3)^{5/2}} dx = \int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{5}{2}} (ex)^{\frac{7}{2}}} dx$$

input
```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^3}{(ex)^{7/2} (a + bx^3)^{5/2}} dx = \int \frac{B\,x^3 + A}{(e\,x)^{7/2}\,(b\,x^3 + a)^{5/2}} dx$$

input
```
int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)),x)
```

output
```
int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A + Bx^3}{(ex)^{7/2} (a + bx^3)^{5/2}} \, dx = \frac{\sqrt{e} \left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b^2x^{10}+2ab\,x^7+a^2x^4} dx \right)}{e^4}$$

input
```
int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x)
```

output
```
(sqrt(e)*int((sqrt(x)*sqrt(a + b*x**3))/(a**2*x**4 + 2*a*b*x**7 + b**2*x**
10),x))/e**4
```

3.289 $\int x^8 \sqrt[3]{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int x^8 \sqrt[3]{a+bx^3}(A+Bx^3)\,dx = \frac{a^2(Ab-aB)\,(a+bx^3)^{4/3}}{4b^4} - \frac{a(2Ab-3aB)\,(a+bx^3)^{7/3}}{7b^4} + \frac{(Ab-3aB)\,(a+bx^3)^{10/3}}{10b^4} + \frac{B(a+bx^3)^{13/3}}{13b^4}$$

output
```
1/4*a^2*(A*b-B*a)*(b*x^3+a)^(4/3)/b^4-1/7*a*(2*A*b-3*B*a)*(b*x^3+a)^(7/3)/
b^4+1/10*(A*b-3*B*a)*(b*x^3+a)^(10/3)/b^4+1/13*B*(b*x^3+a)^(13/3)/b^4
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int x^8 \sqrt[3]{a+bx^3}(A+Bx^3)\,dx = \frac{(a+bx^3)^{4/3}\,(117a^2Ab-81a^3B-156aAb^2x^3+108a^2bBx^3+182Ab^3x^6-126ab^2Bx^6+140b^3Bx^9)}{1820b^4}$$

input
```
Integrate[x^8*(a + b*x^3)^(1/3)*(A + B*x^3),x]
```

output
```
((a + b*x^3)^(4/3)*(117*a^2*A*b - 81*a^3*B - 156*a*A*b^2*x^3 + 108*a^2*b*B
*x^3 + 182*A*b^3*x^6 - 126*a*b^2*B*x^6 + 140*b^3*B*x^9))/(1820*b^4)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int x^6 \sqrt[3]{bx^3+a}\left(Bx^3+A\right)dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(\frac{B\left(bx^3+a\right)^{10/3}}{b^3}+\frac{(Ab-3aB)\left(bx^3+a\right)^{7/3}}{b^3}+\frac{a(3aB-2Ab)\left(bx^3+a\right)^{4/3}}{b^3}-\frac{a^2(aB-Ab)\sqrt[3]{bx^3+a}}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3a^2\left(a+bx^3\right)^{4/3}(Ab-aB)}{4b^4}+\frac{3\left(a+bx^3\right)^{10/3}(Ab-3aB)}{10b^4}-\frac{3a\left(a+bx^3\right)^{7/3}(2Ab-3aB)}{7b^4}+\frac{3B\left(a+bx^3\right)^{13/}}{13b^4}\right.$$

input
```
Int[x^8*(a + b*x^3)^(1/3)*(A + B*x^3),x]
```

output
```
((3*a^2*(A*b - a*B)*(a + b*x^3)^(4/3))/(4*b^4) - (3*a*(2*A*b - 3*a*B)*(a +
 b*x^3)^(7/3))/(7*b^4) + (3*(A*b - 3*a*B)*(a + b*x^3)^(10/3))/(10*b^4) + (
3*B*(a + b*x^3)^(13/3))/(13*b^4))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{9\left(\frac{14\left(\frac{10 B\,x^{3}}{13}+A\right) x^{6} b^{3}}{9}-\frac{4 a\left(\frac{21 B\,x^{3}}{26}+A\right) x^{3} b^{2}}{3}+a^{2}\left(\frac{12 B\,x^{3}}{13}+A\right) b-\frac{9 a^{3} B}{13}\right)\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{140 b^{4}}$
gosper	$\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(140 b^{3} B\,x^{9}+182 A\,b^{3} x^{6}-126 B a\,b^{2} x^{6}-156 a A\,b^{2} x^{3}+108 B\,a^{2} b\,x^{3}+117 a^{2} b A-81 a^{3} B\right)}{1820 b^{4}}$
orering	$\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(140 b^{3} B\,x^{9}+182 A\,b^{3} x^{6}-126 B a\,b^{2} x^{6}-156 a A\,b^{2} x^{3}+108 B\,a^{2} b\,x^{3}+117 a^{2} b A-81 a^{3} B\right)}{1820 b^{4}}$
trager	$\frac{\left(140 B\,b^{4} x^{12}+182 A\,b^{4} x^{9}+14 B\,x^{9} a\,b^{3}+26 A\,x^{6} a\,b^{3}-18 B\,x^{6} a^{2} b^{2}-39 A\,a^{2} b^{2} x^{3}+27 B\,a^{3} b\,x^{3}+117 A\,a^{3} b-81 B\,a^{4}\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{1820 b^{4}}$
risch	$\frac{\left(140 B\,b^{4} x^{12}+182 A\,b^{4} x^{9}+14 B\,x^{9} a\,b^{3}+26 A\,x^{6} a\,b^{3}-18 B\,x^{6} a^{2} b^{2}-39 A\,a^{2} b^{2} x^{3}+27 B\,a^{3} b\,x^{3}+117 A\,a^{3} b-81 B\,a^{4}\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{1820 b^{4}}$

input
```
int(x^8*(b*x^3+a)^(1/3)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
9/140*(14/9*(10/13*B*x^3+A)*x^6*b^3-4/3*a*(21/26*B*x^3+A)*x^3*b^2+a^2*(12/
13*B*x^3+A)*b-9/13*a^3*B)*(b*x^3+a)^(4/3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.96

$$\int x^8 \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$= \frac{(140\,Bb^4x^{12} + 14\,(Bab^3 + 13\,Ab^4)x^9 - 2\,(9\,Ba^2b^2 - 13\,Aab^3)x^6 - 81\,Ba^4 + 117\,Aa^3b + 3\,(9\,Ba^3b - 13\,A}{1820\,b^4}$$

input
```
integrate(x^8*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/1820*(140*B*b^4*x^12 + 14*(B*a*b^3 + 13*A*b^4)*x^9 - 2*(9*B*a^2*b^2 - 13
*A*a*b^3)*x^6 - 81*B*a^4 + 117*A*a^3*b + 3*(9*B*a^3*b - 13*A*a^2*b^2)*x^3)
*(b*x^3 + a)^(1/3)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(94) = 188.

Time = 0.51 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.06

$$\int x^8 \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$= \begin{cases} \frac{9Aa^3\sqrt[3]{a+bx^3}}{140b^3} - \frac{3Aa^2x^3\sqrt[3]{a+bx^3}}{140b^2} + \frac{Aax^6\sqrt[3]{a+bx^3}}{70b} + \frac{Ax^9\sqrt[3]{a+bx^3}}{10} - \frac{81Ba^4\sqrt[3]{a+bx^3}}{1820b^4} + \frac{27Ba^3x^3\sqrt[3]{a+b}}{1820b^3} \\ \sqrt[3]{a}\left(\frac{Ax^9}{9} + \frac{Bx^{12}}{12}\right) \end{cases}$$

input
```
integrate(x**8*(b*x**3+a)**(1/3)*(B*x**3+A),x)
```

output
```
Piecewise((9*A*a**3*(a + b*x**3)**(1/3)/(140*b**3) - 3*A*a**2*x**3*(a + b*
x**3)**(1/3)/(140*b**2) + A*a*x**6*(a + b*x**3)**(1/3)/(70*b) + A*x**9*(a
+ b*x**3)**(1/3)/10 - 81*B*a**4*(a + b*x**3)**(1/3)/(1820*b**4) + 27*B*a**
3*x**3*(a + b*x**3)**(1/3)/(1820*b**3) - 9*B*a**2*x**6*(a + b*x**3)**(1/3)
/(910*b**2) + B*a*x**9*(a + b*x**3)**(1/3)/(130*b) + B*x**12*(a + b*x**3)*
*(1/3)/13, Ne(b, 0)), (a**(1/3)*(A*x**9/9 + B*x**12/12), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int x^8 \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{1}{1820}B\left(\frac{140\,(bx^3+a)^{\frac{13}{3}}}{b^4}-\frac{546\,(bx^3+a)^{\frac{10}{3}}a}{b^4}+\frac{780\,(bx^3+a)^{\frac{7}{3}}a^2}{b^4}-\frac{455\,(bx^3+a)^{\frac{4}{3}}a^3}{b^4}\right)+\frac{1}{140}A\left(\frac{14\,(bx^3+a)^{\frac{10}{3}}}{b^3}-\frac{40\,(bx^3+a)^{\frac{7}{3}}a}{b^3}+\frac{35\,(bx^3+a)^{\frac{4}{3}}a^2}{b^3}\right)$$

input
```
integrate(x^8*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/1820*B*(140*(b*x^3 + a)^(13/3)/b^4 - 546*(b*x^3 + a)^(10/3)*a/b^4 + 780*
(b*x^3 + a)^(7/3)*a^2/b^4 - 455*(b*x^3 + a)^(4/3)*a^3/b^4) + 1/140*A*(14*(
b*x^3 + a)^(10/3)/b^3 - 40*(b*x^3 + a)^(7/3)*a/b^3 + 35*(b*x^3 + a)^(4/3)*
a^2/b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int x^8 \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{140\,(bx^3+a)^{\frac{13}{3}}B-546\,(bx^3+a)^{\frac{10}{3}}Ba+780\,(bx^3+a)^{\frac{7}{3}}Ba^2-455\,(bx^3+a)^{\frac{4}{3}}Ba^3+182\,(bx^3+a)^{\frac{10}{3}}Ab}{1820\,b^4}$$

input
```
integrate(x^8*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/1820*(140*(b*x^3 + a)^(13/3)*B - 546*(b*x^3 + a)^(10/3)*B*a + 780*(b*x^3
 + a)^(7/3)*B*a^2 - 455*(b*x^3 + a)^(4/3)*B*a^3 + 182*(b*x^3 + a)^(10/3)*A
*b - 520*(b*x^3 + a)^(7/3)*A*a*b + 455*(b*x^3 + a)^(4/3)*A*a^2*b)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int x^8 \sqrt[3]{a+bx^3} (A+Bx^3) \, dx = (b\,x^3+a)^{1/3} \left(\frac{B\,x^{12}}{13} - \frac{81\,B\,a^4 - 117\,A\,a^3\,b}{1820\,b^4} + \frac{x^9\,(182\,A\,b^4 + 14\,B\,a\,b^3)}{1820\,b^4} - \frac{3\,a^2\,x^3\,(13\,A\,b - 9\,B\,a)}{1820\,b^3} + \frac{a\,x^6\,(13\,A\,b - 9\,B\,a)}{910\,b^2} \right)$$

input
```
int(x^8*(A + B*x^3)*(a + b*x^3)^(1/3),x)
```

output
```
(a + b*x^3)^(1/3)*((B*x^12)/13 - (81*B*a^4 - 117*A*a^3*b)/(1820*b^4) + (x^
9*(182*A*b^4 + 14*B*a*b^3))/(1820*b^4) - (3*a^2*x^3*(13*A*b - 9*B*a))/(182
0*b^3) + (a*x^6*(13*A*b - 9*B*a))/(910*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.55

$$\int x^8 \sqrt[3]{a+bx^3} (A+Bx^3) \, dx = \frac{(b\,x^3+a)^{\frac{1}{3}} (35b^4x^{12} + 49a\,b^3x^9 + 2a^2b^2x^6 - 3a^3b\,x^3 + 9a^4)}{455b^3}$$

input
```
int(x^8*(b*x^3+a)^(1/3)*(B*x^3+A),x)
```

output
```
((a + b*x**3)**(1/3)*(9*a**4 - 3*a**3*b*x**3 + 2*a**2*b**2*x**6 + 49*a*b**
3*x**9 + 35*b**4*x**12))/(455*b**3)
```

3.290 $\int x^5 \sqrt[3]{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int x^5 \sqrt[3]{a+bx^3}(A+Bx^3)\,dx = -\frac{a(Ab-aB)\left(a+bx^3\right)^{4/3}}{4b^3} + \frac{(Ab-2aB)\left(a+bx^3\right)^{7/3}}{7b^3} + \frac{B\left(a+bx^3\right)^{10/3}}{10b^3}$$

output
```
-1/4*a*(A*b-B*a)*(b*x^3+a)^(4/3)/b^3+1/7*(A*b-2*B*a)*(b*x^3+a)^(7/3)/b^3+1
/10*B*(b*x^3+a)^(10/3)/b^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int x^5 \sqrt[3]{a+bx^3}(A+Bx^3)\,dx = \frac{\left(a+bx^3\right)^{4/3}\left(-15aAb+9a^2B+20Ab^2x^3-12abBx^3+14b^2Bx^6\right)}{140b^3}$$

input
```
Integrate[x^5*(a + b*x^3)^(1/3)*(A + B*x^3),x]
```

output
```
((a + b*x^3)^(4/3)*(-15*a*A*b + 9*a^2*B + 20*A*b^2*x^3 - 12*a*b*B*x^3 + 14
*b^2*B*x^6))/(140*b^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \sqrt[3]{a+bx^3}\left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int x^3 \sqrt[3]{bx^3+a}\left(Bx^3+A\right)dx^3$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{3}\int \left(\frac{B\left(bx^3+a\right)^{7/3}}{b^2}+\frac{(Ab-2aB)\left(bx^3+a\right)^{4/3}}{b^2}+\frac{a(aB-Ab)\sqrt[3]{bx^3+a}}{b^2}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{7/3}(Ab-2aB)}{7b^3}-\frac{3a\left(a+bx^3\right)^{4/3}(Ab-aB)}{4b^3}+\frac{3B\left(a+bx^3\right)^{10/3}}{10b^3}\right)$$

input
```
Int[x^5*(a + b*x^3)^(1/3)*(A + B*x^3),x]
```

output
```
((-3*a*(A*b - a*B)*(a + b*x^3)^(4/3))/(4*b^3) + (3*(A*b - 2*a*B)*(a + b*x^
3)^(7/3))/(7*b^3) + (3*B*(a + b*x^3)^(10/3))/(10*b^3))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$-\frac{3\left(-\frac{4x^{3}\left(\frac{7B\,x^{3}}{10}+A\right)b^{2}}{3}+a\left(\frac{4B\,x^{3}}{5}+A\right)b-\frac{3a^{2}B}{5}\right)\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{28b^{3}}$	49
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(-14b^{2}B\,x^{6}-20A\,b^{2}x^{3}+12Bab\,x^{3}+15abA-9a^{2}B\right)}{140b^{3}}$	53
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(-14b^{2}B\,x^{6}-20A\,b^{2}x^{3}+12Bab\,x^{3}+15abA-9a^{2}B\right)}{140b^{3}}$	53
trager	$-\frac{\left(-14b^{3}B\,x^{9}-20A\,b^{3}x^{6}-2Ba\,b^{2}x^{6}-5aA\,b^{2}x^{3}+3B\,a^{2}b\,x^{3}+15a^{2}bA-9a^{3}B\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{140b^{3}}$	77
risch	$-\frac{\left(-14b^{3}B\,x^{9}-20A\,b^{3}x^{6}-2Ba\,b^{2}x^{6}-5aA\,b^{2}x^{3}+3B\,a^{2}b\,x^{3}+15a^{2}bA-9a^{3}B\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{140b^{3}}$	77

input
```
int(x^5*(b*x^3+a)^(1/3)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
-3/28*(-4/3*x^3*(7/10*B*x^3+A)*b^2+a*(4/5*B*x^3+A)*b-3/5*a^2*B)*(b*x^3+a)^
(4/3)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int x^5\sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{(14\,Bb^3x^9+2\,(Bab^2+10\,Ab^3)x^6+9\,Ba^3-15\,Aa^2b-(3\,Ba^2b-5\,Aab^2)x^3)(bx^3+a)^{\frac{1}{3}}}{140\,b^3}$$

input
```
integrate(x^5*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/140*(14*B*b^3*x^9 + 2*(B*a*b^2 + 10*A*b^3)*x^6 + 9*B*a^3 - 15*A*a^2*b -
(3*B*a^2*b - 5*A*a*b^2)*x^3)*(b*x^3 + a)^(1/3)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(65) = 130$.

Time = 0.34 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.22

$$\int x^5\sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\begin{cases}-\frac{3Aa^2\sqrt[3]{a+bx^3}}{28b^2}+\frac{Aax^3\sqrt[3]{a+bx^3}}{28b}+\frac{Ax^6\sqrt[3]{a+bx^3}}{7}+\frac{9Ba^3\sqrt[3]{a+bx^3}}{140b^3}-\frac{3Ba^2x^3\sqrt[3]{a+bx^3}}{140b^2}+\frac{Bax^6\sqrt[3]{a+bx^3}}{70b} \\ \sqrt[3]{a}\left(\frac{Ax^6}{6}+\frac{Bx^9}{9}\right)\end{cases}$$

input
```
integrate(x**5*(b*x**3+a)**(1/3)*(B*x**3+A),x)
```

output
```
Piecewise((-3*A*a**2*(a + b*x**3)**(1/3)/(28*b**2) + A*a*x**3*(a + b*x**3)
**(1/3)/(28*b) + A*x**6*(a + b*x**3)**(1/3)/7 + 9*B*a**3*(a + b*x**3)**(1/
3)/(140*b**3) - 3*B*a**2*x**3*(a + b*x**3)**(1/3)/(140*b**2) + B*a*x**6*(a
 + b*x**3)**(1/3)/(70*b) + B*x**9*(a + b*x**3)**(1/3)/10, Ne(b, 0)), (a**(
1/3)*(A*x**6/6 + B*x**9/9), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int x^5 \sqrt[3]{a+bx^3}\,(A+Bx^3)\ dx = \frac{1}{140}\,B\left(\frac{14\,(bx^3+a)^{\frac{10}{3}}}{b^3} - \frac{40\,(bx^3+a)^{\frac{7}{3}}a}{b^3} + \frac{35\,(bx^3+a)^{\frac{4}{3}}a^2}{b^3}\right) + \frac{1}{28}\,A\left(\frac{4\,(bx^3+a)^{\frac{7}{3}}}{b^2} - \frac{7\,(bx^3+a)^{\frac{4}{3}}a}{b^2}\right)$$

input
```
integrate(x^5*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/140*B*(14*(b*x^3 + a)^(10/3)/b^3 - 40*(b*x^3 + a)^(7/3)*a/b^3 + 35*(b*x^
3 + a)^(4/3)*a^2/b^3) + 1/28*A*(4*(b*x^3 + a)^(7/3)/b^2 - 7*(b*x^3 + a)^(4
/3)*a/b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int x^5 \sqrt[3]{a+bx^3}\,(A+Bx^3)\ dx = \frac{14\,(bx^3+a)^{\frac{10}{3}}B - 40\,(bx^3+a)^{\frac{7}{3}}Ba + 35\,(bx^3+a)^{\frac{4}{3}}Ba^2 + 20\,(bx^3+a)^{\frac{7}{3}}Ab - 35\,(bx^3+a)^{\frac{4}{3}}Aab}{140\,b^3}$$

input
```
integrate(x^5*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/140*(14*(b*x^3 + a)^(10/3)*B - 40*(b*x^3 + a)^(7/3)*B*a + 35*(b*x^3 + a)
^(4/3)*B*a^2 + 20*(b*x^3 + a)^(7/3)*A*b - 35*(b*x^3 + a)^(4/3)*A*a*b)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int x^5 \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx = \left(b\,x^3+a\right)^{1/3}\left(\frac{B\,x^9}{10}+\frac{9\,B\,a^3-15\,A\,a^2\,b}{140\,b^3}+\frac{x^6\,\left(20\,A\,b^3+2\,B\,a\,b^2\right)}{140\,b^3}+\frac{a\,x^3\,\left(5\,A\,b-3\,B\,a\right)}{140\,b^2}\right)$$

input
```
int(x^5*(A + B*x^3)*(a + b*x^3)^(1/3),x)
```

output
```
(a + b*x^3)^(1/3)*((B*x^9)/10 + (9*B*a^3 - 15*A*a^2*b)/(140*b^3) + (x^6*(2
0*A*b^3 + 2*B*a*b^2))/(140*b^3) + (a*x^3*(5*A*b - 3*B*a))/(140*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int x^5 \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(7b^3x^9+11a\,b^2x^6+a^2b\,x^3-3a^3\right)}{70b^2}$$

input
```
int(x^5*(b*x^3+a)^(1/3)*(B*x^3+A),x)
```

output
```
((a + b*x**3)**(1/3)*( - 3*a**3 + a**2*b*x**3 + 11*a*b**2*x**6 + 7*b**3*x*
*9))/(70*b**2)
```

3.291 $\int x^2 \sqrt[3]{a+bx^3}(A+Bx^3)\ dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int x^2 \sqrt[3]{a+bx^3}(A+Bx^3)\ dx = \frac{(Ab-aB)\left(a+bx^3\right)^{4/3}}{4b^2} + \frac{B\left(a+bx^3\right)^{7/3}}{7b^2}$$

output
```
1/4*(A*b-B*a)*(b*x^3+a)^(4/3)/b^2+1/7*B*(b*x^3+a)^(7/3)/b^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int x^2 \sqrt[3]{a+bx^3}(A+Bx^3)\ dx = \frac{\left(a+bx^3\right)^{4/3}(7Ab-3aB+4bBx^3)}{28b^2}$$

input
```
Integrate[x^2*(a + b*x^3)^(1/3)*(A + B*x^3),x]
```

output
```
((a + b*x^3)^(4/3)*(7*A*b - 3*a*B + 4*b*B*x^3))/(28*b^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt[3]{a+bx^3}\left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{3}\int \sqrt[3]{bx^3+a}\left(Bx^3+A\right)dx^3$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{3}\int \left(\frac{B\left(bx^3+a\right)^{4/3}}{b}+\frac{(Ab-aB)\sqrt[3]{bx^3+a}}{b}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}(Ab-aB)}{4b^2}+\frac{3B\left(a+bx^3\right)^{7/3}}{7b^2}\right)$$

input
```
Int[x^2*(a + b*x^3)^(1/3)*(A + B*x^3),x]
```

output
```
((3*(A*b - a*B)*(a + b*x^3)^(4/3))/(4*b^2) + (3*B*(a + b*x^3)^(7/3))/(7*b^
2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(4 b B\,x^{3}+7 A b -3 B a \right)}{28 b^{2}}$	31
orering	$\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(4 b B\,x^{3}+7 A b -3 B a \right)}{28 b^{2}}$	31
pseudoelliptic	$\frac{\left(\left(4 B\,x^{3}+7 A \right) b -3 B a \right) \left(b\,x^{3}+a \right)^{\frac{4}{3}}}{28 b^{2}}$	32
trager	$\frac{\left(4 b^{2} B\,x^{6}+7 A\,b^{2} x^{3}+B a b\,x^{3}+7 a b A -3 a^{2} B \right) \left(b\,x^{3}+a \right)^{\frac{1}{3}}}{28 b^{2}}$	52
risch	$\frac{\left(4 b^{2} B\,x^{6}+7 A\,b^{2} x^{3}+B a b\,x^{3}+7 a b A -3 a^{2} B \right) \left(b\,x^{3}+a \right)^{\frac{1}{3}}}{28 b^{2}}$	52

input
```
int(x^2*(b*x^3+a)^(1/3)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/28*(b*x^3+a)^(4/3)*(4*B*b*x^3+7*A*b-3*B*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int x^2\sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{(4\,Bb^2x^6+(Bab+7\,Ab^2)x^3-3\,Ba^2+7\,Aab)(bx^3+a)^{\frac{1}{3}}}{28\,b^2}$$

input
```
integrate(x^2*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/28*(4*B*b^2*x^6 + (B*a*b + 7*A*b^2)*x^3 - 3*B*a^2 + 7*A*a*b)*(b*x^3 + a)
^(1/3)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(39) = 78$.

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.39

$$\int x^2 \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx = \begin{cases} \frac{Aa\sqrt[3]{a+bx^3}}{4b} + \frac{Ax^3\sqrt[3]{a+bx^3}}{4} - \frac{3Ba^2\sqrt[3]{a+bx^3}}{28b^2} + \frac{Bax^3\sqrt[3]{a+bx^3}}{28b} + \frac{Bx^6\sqrt[3]{a+bx^3}}{7} & \text{for } b \neq 0 \\ \sqrt[3]{a}\left(\frac{Ax^3}{3} + \frac{Bx^6}{6}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(b*x**3+a)**(1/3)*(B*x**3+A),x)
```

output
```
Piecewise((A*a*(a + b*x**3)**(1/3)/(4*b) + A*x**3*(a + b*x**3)**(1/3)/4 -
3*B*a**2*(a + b*x**3)**(1/3)/(28*b**2) + B*a*x**3*(a + b*x**3)**(1/3)/(28*
b) + B*x**6*(a + b*x**3)**(1/3)/7, Ne(b, 0)), (a**(1/3)*(A*x**3/3 + B*x**6
/6), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int x^2 \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{1}{28}\,B\left(\frac{4\,(bx^3+a)^{\frac{7}{3}}}{b^2} - \frac{7\,(bx^3+a)^{\frac{4}{3}}a}{b^2}\right) + \frac{(bx^3+a)^{\frac{4}{3}}A}{4\,b}$$

input
```
integrate(x^2*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/28*B*(4*(b*x^3 + a)^(7/3)/b^2 - 7*(b*x^3 + a)^(4/3)*a/b^2) + 1/4*(b*x^3
+ a)^(4/3)*A/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x^2\sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{4\left(bx^3+a\right)^{\frac{7}{3}}B-7\left(bx^3+a\right)^{\frac{4}{3}}Ba+7\left(bx^3+a\right)^{\frac{4}{3}}Ab}{28\,b^2}$$

input
```
integrate(x^2*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/28*(4*(b*x^3 + a)^(7/3)*B - 7*(b*x^3 + a)^(4/3)*B*a + 7*(b*x^3 + a)^(4/3
)*A*b)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int x^2\sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx=\left(b\,x^3+a\right)^{1/3}\left(\frac{B\,x^6}{7}-\frac{3\,B\,a^2-7\,A\,a\,b}{28\,b^2}+\frac{x^3\,\left(7\,A\,b^2+B\,a\,b\right)}{28\,b^2}\right)$$

input
```
int(x^2*(A + B*x^3)*(a + b*x^3)^(1/3),x)
```

output
```
(a + b*x^3)^(1/3)*((B*x^6)/7 - (3*B*a^2 - 7*A*a*b)/(28*b^2) + (x^3*(7*A*b^
2 + B*a*b))/(28*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int x^2\sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(b^2x^6+2ab\,x^3+a^2\right)}{7b}$$

input
```
int(x^2*(b*x^3+a)^(1/3)*(B*x^3+A),x)
```

output
```
((a + b*x**3)**(1/3)*(a**2 + 2*a*b*x**3 + b**2*x**6))/(7*b)
```

3.292 $\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x}\,dx = A\sqrt[3]{a+bx^3}+\frac{B(a+bx^3)^{4/3}}{4b} - \frac{\sqrt[3]{a}A\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}} - \frac{1}{2}\sqrt[3]{a}A\log(x)+\frac{1}{2}\sqrt[3]{a}A\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)$$

output
```
A*(b*x^3+a)^(1/3)+1/4*B*(b*x^3+a)^(4/3)/b-1/3*a^(1/3)*A*arctan(1/3*(a^(1/3
)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)-1/2*a^(1/3)*A*ln(x)+1/2*a^(1
/3)*A*ln(a^(1/3)-(b*x^3+a)^(1/3))
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{1}{12}\left(\frac{3\sqrt[3]{a+bx^3}(4Ab+B(a+bx^3))}{b} - 4\sqrt{3}\sqrt[3]{a}A\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 4\sqrt[3]{a}A\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right) - 2\sqrt[3]{a}A\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)\right)$$

input
```
Integrate[((a + b*x^3)^(1/3)*(A + B*x^3))/x,x]
```

output
```
((3*(a + b*x^3)^(1/3)*(4*A*b + B*(a + b*x^3)))/b - 4*Sqrt[3]*a^(1/3)*A*Arc
Tan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*A*Log[-a^(1/3
) + (a + b*x^3)^(1/3)] - 2*a^(1/3)*A*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/
3) + (a + b*x^3)^(2/3)])/12
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {948, 90, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x}\,dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{\sqrt[3]{bx^3+a}\left(Bx^3+A\right)}{x^3}dx^3$$

$$\Big\downarrow \ 90$$

$$\frac{1}{3}\left(A\int \frac{\sqrt[3]{bx^3+a}}{x^3}dx^3+\frac{3B\left(a+bx^3\right)^{4/3}}{4b}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{3}\left(A\left(a\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3+3\sqrt[3]{a+bx^3}\right)+\frac{3B\left(a+bx^3\right)^{4/3}}{4b}\right)$$

$$\Big\downarrow \ 69$$

$$\frac{1}{3}\left(A\left(a\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right.\right.$$

$$\Big\downarrow \ 16$$

$$\frac{1}{3}\left(A\left(a\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+\frac{3B}{}\right.$$

$$\Big\downarrow \ 1082$$

$$\frac{1}{3}\left(A\left(a\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+\frac{3B(a+bx}{4b}\right.$$

$$\Big\downarrow \ 217$$

$$\frac{1}{3}\left(A\left(a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+\frac{3B(a+bx}{4b}\right.$$

input
```
Int[((a + b*x^3)^(1/3)*(A + B*x^3))/x,x]
```

output
```
((3*B*(a + b*x^3)^(4/3))/(4*b) + A*(3*(a + b*x^3)^(1/3) + a*(-((Sqrt[3]*Ar
cTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^3]/(2*
a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3)))))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{3\left(\left(B\,x^{3}+4A\right)b+Ba\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2Ab\,a^{\frac{1}{3}}\left(2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-}{12b}$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*((B*x^3+4*A)*b+B*a)*(b*x^3+a)^(1/3)-2*A*b*a^(1/3)*(2*arctan(1/3*(a
^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1
/3)*(b*x^3+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3))))/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x}\,dx = -\frac{4\sqrt{3}Aa^{\frac{1}{3}}b\arctan\left(\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+2Aa^{\frac{1}{3}}b\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4Aa^{\frac{1}{3}}b\log}{12b}$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x,x, algorithm="fricas")
```

output
```
-1/12*(4*sqrt(3)*A*a^(1/3)*b*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*a^(2/
3) + sqrt(3)*a)/a) + 2*A*a^(1/3)*b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/
3)*a^(1/3) + a^(2/3)) - 4*A*a^(1/3)*b*log((b*x^3 + a)^(1/3) - a^(1/3)) - 3
*(B*b*x^3 + B*a + 4*A*b)*(b*x^3 + a)^(1/3))/b
```

Sympy [A] (verification not implemented)

Time = 12.54 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x}\,dx = -\frac{A\sqrt[3]{b}x\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3\Gamma\left(\frac{2}{3}\right)} + B\left(\begin{cases}\frac{\sqrt[3]{a}x^3}{3} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{4}{3}}}{4b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**3+a)**(1/3)*(B*x**3+A)/x,x)
```

output
```
-A*b**(1/3)*x*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), a*exp_polar(I*pi)/(b
*x**3))/(3*gamma(2/3)) + B*Piecewise((a**(1/3)*x**3/3, Eq(b, 0)), ((a + b*
x**3)**(4/3)/(4*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x}\,dx = -\frac{1}{6}\left(2\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) + a^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}} + \left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right) - 2\,a^{\frac{1}{3}}\,\mathrm{lo} \right. + \frac{\left(bx^3+a\right)^{\frac{4}{3}}B}{4\,b}$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x,x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))
/a^(1/3)) + a^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^
(2/3)) - 2*a^(1/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) - 6*(b*x^3 + a)^(1/3))
*A + 1/4*(b*x^3 + a)^(4/3)*B/b
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x}\,dx = -\frac{1}{3}\,\sqrt{3}Aa^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{6}\,Aa^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + \frac{1}{3}\,Aa^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right) + \frac{(bx^3+a)^{\frac{4}{3}}Bb^3+4\,(bx^3+a)^{\frac{1}{3}}Ab^4}{4\,b^4}$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x,x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*A*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/
a^(1/3)) - 1/6*A*a^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3)
 + a^(2/3)) + 1/3*A*a^(1/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3))) + 1/4*((
b*x^3 + a)^(4/3)*B*b^3 + 4*(b*x^3 + a)^(1/3)*A*b^4)/b^4
```

Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x}\,dx$$
$$= A\left(b\,x^3+a\right)^{1/3}+\frac{B\left(b\,x^3+a\right)^{4/3}}{4\,b}+\frac{A\,a^{1/3}\,\ln\left(3\,A\,a^{4/3}-3\,A\,a\left(b\,x^3+a\right)^{1/3}\right)}{3}$$
$$+\frac{A\,a^{1/3}\,\ln\left(3\,A\,a\left(b\,x^3+a\right)^{1/3}-\frac{3\,A\,a^{4/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6}$$
$$-\frac{A\,a^{1/3}\,\ln\left(\frac{3\,A\,a^{4/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3\,A\,a\left(b\,x^3+a\right)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/3))/x,x)
```

output
```
A*(a + b*x^3)^(1/3) + (B*(a + b*x^3)^(4/3))/(4*b) + (A*a^(1/3)*log(3*A*a^(
4/3) - 3*A*a*(a + b*x^3)^(1/3)))/3 + (A*a^(1/3)*log(3*A*a*(a + b*x^3)^(1/3
) - (3*A*a^(4/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/6 - (A*a^(1/3)*log
((3*A*a^(4/3)*(3^(1/2)*1i + 1))/2 + 3*A*a*(a + b*x^3)^(1/3))*(3^(1/2)*1i +
 1))/6
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x}\,dx=\frac{5(b\,x^3+a)^{\frac{1}{3}}\,a}{4}+\frac{(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}{4}+\left(\int\frac{(b\,x^3+a)^{\frac{1}{3}}}{b\,x^4+ax}dx\right)a^2$$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x,x)
```

output
```
(5*(a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3 + 4*int((a + b*x**3)
**(1/3)/(a*x + b*x**4),x)*a**2)/4
```

3.293 $\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 142

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^4}\,dx = B\sqrt[3]{a+bx^3}-\frac{A\sqrt[3]{a+bx^3}}{3x^3} - \frac{(Ab+3aB)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}} - \frac{(Ab+3aB)\log(x)}{6a^{2/3}}+\frac{(Ab+3aB)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}}$$

output

```
B*(b*x^3+a)^(1/3)-1/3*A*(b*x^3+a)^(1/3)/x^3-1/9*(A*b+3*B*a)*arctan(1/3*(a^
(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)-1/6*(A*b+3*B*a)*
ln(x)/a^(2/3)+1/6*(A*b+3*B*a)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(2/3)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^4}\,dx = \frac{1}{18}\left(\frac{6\sqrt[3]{a+bx^3}(-A+3Bx^3)}{x^3} - \frac{2\sqrt{3}(Ab+3aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{2(Ab+3aB)\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{a^{2/3}} - \frac{(Ab+3aB)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{a^{2/3}}\right)$$

input
```
Integrate[((a + b*x^3)^(1/3)*(A + B*x^3))/x^4,x]
```

output
```
((6*(a + b*x^3)^(1/3)*(-A + 3*B*x^3))/x^3 - (2*Sqrt[3]*(A*b + 3*a*B)*ArcTa
n[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*(A*b + 3*a*B)
*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(2/3) - ((A*b + 3*a*B)*Log[a^(2/3) +
 a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/a^(2/3))/18
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {948, 87, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}\left(A+Bx^3\right)}{x^4}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{\sqrt[3]{bx^3+a}\left(Bx^3+A\right)}{x^6}dx^3$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{3}\left(\frac{(3aB+Ab)\int \frac{\sqrt[3]{bx^3+a}}{x^3}dx^3}{3a}-\frac{A\left(a+bx^3\right)^{4/3}}{ax^3}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{3}\left(\frac{(3aB+Ab)\left(a\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3+3\sqrt[3]{a+bx^3}\right)}{3a}-\frac{A\left(a+bx^3\right)^{4/3}}{ax^3}\right)$$

$$\Big\downarrow \text{ 69}$$

$$\frac{1}{3}\left(\frac{(3aB+Ab)\left(a\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a\cdots}\right.}{3a}\right.$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{3}\left(\frac{(3aB+Ab)\left(a\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{3a}\right.$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{3}\left(\frac{(3aB+Ab)\left(a\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{3a}-\frac{A(a+b}{ax}\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}\left(\frac{(3aB+Ab)\left(a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{3a}-\frac{A(a+b}{ax}\right.$$

input
```
Int[((a + b*x^3)^(1/3)*(A + B*x^3))/x^4,x]
```

output
```
(-((A*(a + b*x^3)^(4/3))/(a*x^3)) + ((A*b + 3*a*B)*(3*(a + b*x^3)^(1/3) +
a*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)
) - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3)
))))/(3*a))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{-3\left(b\,x^{3}+a\right)^{\frac{1}{3}} a^{\frac{2}{3}} \left(-3B\,x^{3}+A\right)+\left(Ab+3Ba\right) \left(-\arctan\left(\frac{2\sqrt{3}\,\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{3a^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\cdots\right.}{\cdots}\right.}{9a^{\frac{2}{3}} x^{3}}$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/9*(-3*(b*x^3+a)^(1/3)*a^(2/3)*(-3*B*x^3+A)+(A*b+3*B*a)*(-arctan(2/3*3^(1
/2)/a^(1/3)*(b*x^3+a)^(1/3)+1/3*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(1/3)-a^(1/3
))-1/2*ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3)))*x^3)/a^(2/3)/x
^3
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^4}\,dx =$$

$$-\frac{6\sqrt{\frac{1}{3}}(3\,Ba^2+Aab)(a^2)^{\frac{1}{6}}x^3\arctan\left(\frac{\sqrt{\frac{1}{3}}(a^2)^{\frac{1}{6}}\left((a^2)^{\frac{1}{3}}a+2\,(bx^3+a)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}\right)}{a^2}\right)+(3\,Ba+Ab)(a^2)^{\frac{2}{3}}x^3\log\left((bx^{\cdots}\right.}{\cdots}$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^4,x, algorithm="fricas")
```

output
```
-1/18*(6*sqrt(1/3)*(3*B*a^2 + A*a*b)*(a^2)^(1/6)*x^3*arctan(sqrt(1/3)*(a^2
)^(1/6)*((a^2)^(1/3)*a + 2*(b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) + (3*B*a +
A*b)*(a^2)^(2/3)*x^3*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)
^(1/3)*(a^2)^(2/3)) - 2*(3*B*a + A*b)*(a^2)^(2/3)*x^3*log((b*x^3 + a)^(1/3
)*a - (a^2)^(2/3)) - 6*(3*B*a^2*x^3 - A*a^2)*(b*x^3 + a)^(1/3))/(a^2*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.86 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^4}\,dx = -\frac{A\sqrt[3]{b}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3x^2\Gamma\left(\frac{5}{3}\right)} - \frac{B\sqrt[3]{b}x\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)*(B*x**3+A)/x**4,x)
```

output
```
-A*b**(1/3)*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**
3))/(3*x**2*gamma(5/3)) - B*b**(1/3)*x*gamma(-1/3)*hyper((-1/3, -1/3), (2/
3,), a*exp_polar(I*pi)/(b*x**3))/(3*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^4}\,dx = -\frac{1}{18}\left(\frac{2\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{b\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{2\,b\log\left((bx^3+\right.}{a^{\frac{2}{3}}}\right.$$
$$-\frac{1}{6}\left(2\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)+a^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\,a^{\frac{1}{3}}\text{lo}\right.$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^4,x, algorithm="maxima")
```

output
```
-1/18*(2*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(2/3) + b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/
3))/a^(2/3) - 2*b*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(2/3) + 6*(b*x^3 + a)
^(1/3)/x^3)*A - 1/6*(2*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(
1/3) + a^(1/3))/a^(1/3)) + a^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/
3)*a^(1/3) + a^(2/3)) - 2*a^(1/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) - 6*(b*
x^3 + a)^(1/3))*B
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^4}\,dx = -\frac{1}{18}b\left(\frac{2\sqrt{3}(3\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}b}-\frac{18\,(bx^3+a)^{\frac{1}{3}}B}{b}+\frac{(3\,Ba+Ab)\log\left((bx^3+a)\right.}{a^{\frac{2}{3}}b}\right.$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^4,x, algorithm="giac")
```

output
```
-1/18*b*(2*sqrt(3)*(3*B*a + A*b)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) +
 a^(1/3))/a^(1/3))/(a^(2/3)*b) - 18*(b*x^3 + a)^(1/3)*B/b + (3*B*a + A*b)*
log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(2/3)*b) -
 2*(3*B*a + A*b)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/3)*b) + 6*(b*
x^3 + a)^(1/3)*A/(b*x^3))
```

Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^4}\,dx$$
$$= B\left(b\,x^3+a\right)^{1/3} - \frac{A\left(b\,x^3+a\right)^{1/3}}{3\,x^3} + \frac{B\,a^{1/3}\,\ln\left(3\,B\,a^{4/3}-3\,B\,a\left(b\,x^3+a\right)^{1/3}\right)}{3}$$
$$- \frac{\ln\left(\frac{a^{1/3}\left(A\,b-\sqrt{3}\,A\,b\,1\mathrm{i}\right)}{2}+A\,b\left(b\,x^3+a\right)^{1/3}\right)\left(A\,b-\sqrt{3}\,A\,b\,1\mathrm{i}\right)}{18\,a^{2/3}}$$
$$- \frac{\ln\left(\frac{a^{1/3}\left(A\,b+\sqrt{3}\,A\,b\,1\mathrm{i}\right)}{2}+A\,b\left(b\,x^3+a\right)^{1/3}\right)\left(A\,b+\sqrt{3}\,A\,b\,1\mathrm{i}\right)}{18\,a^{2/3}}$$
$$+ \frac{A\,b\,\ln\left(A\,b\left(b\,x^3+a\right)^{1/3}-A\,a^{1/3}\,b\right)}{9\,a^{2/3}}$$
$$+ \frac{B\,a^{1/3}\,\ln\left(3\,B\,a\left(b\,x^3+a\right)^{1/3}-\frac{3\,B\,a^{4/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6}$$
$$- \frac{B\,a^{1/3}\,\ln\left(\frac{3\,B\,a^{4/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3\,B\,a\left(b\,x^3+a\right)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/3))/x^4,x)
```

output
```
B*(a + b*x^3)^(1/3) - (A*(a + b*x^3)^(1/3))/(3*x^3) + (B*a^(1/3)*log(3*B*a
^(4/3) - 3*B*a*(a + b*x^3)^(1/3)))/3 - (log((a^(1/3)*(A*b - 3^(1/2)*A*b*1i
))/2 + A*b*(a + b*x^3)^(1/3))*(A*b - 3^(1/2)*A*b*1i))/(18*a^(2/3)) - (log(
(a^(1/3)*(A*b + 3^(1/2)*A*b*1i))/2 + A*b*(a + b*x^3)^(1/3))*(A*b + 3^(1/2)
*A*b*1i))/(18*a^(2/3)) + (A*b*log(A*b*(a + b*x^3)^(1/3) - A*a^(1/3)*b))/(9
*a^(2/3)) + (B*a^(1/3)*log(3*B*a*(a + b*x^3)^(1/3) - (3*B*a^(4/3)*(3^(1/2)
*1i - 1))/2)*(3^(1/2)*1i - 1))/6 - (B*a^(1/3)*log((3*B*a^(4/3)*(3^(1/2)*1i
 + 1))/2 + 3*B*a*(a + b*x^3)^(1/3))*(3^(1/2)*1i + 1))/6
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^4}\,dx = \frac{-(b\,x^3+a)^{\frac{1}{3}}\,a+3(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3+4\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{b\,x^4+ax}dx\right)ab\,x^3}{3x^3}$$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^4,x)
```

output
```
( - (a + b*x**3)**(1/3)*a + 3*(a + b*x**3)**(1/3)*b*x**3 + 4*int((a + b*x*
*3)**(1/3)/(a*x + b*x**4),x)*a*b*x**3)/(3*x**3)
```

3.294 $\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^7}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 161

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^7}\,dx = -\frac{A\sqrt[3]{a+bx^3}}{6x^6} - \frac{(Ab+6aB)\sqrt[3]{a+bx^3}}{18ax^3} + \frac{b(Ab-3aB)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}} + \frac{b(Ab-3aB)\log(x)}{18a^{5/3}} - \frac{b(Ab-3aB)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{5/3}}$$

output
```
-1/6*A*(b*x^3+a)^(1/3)/x^6-1/18*(A*b+6*B*a)*(b*x^3+a)^(1/3)/a/x^3+1/27*b*(
A*b-3*B*a)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)
/a^(5/3)+1/18*b*(A*b-3*B*a)*ln(x)/a^(5/3)-1/18*b*(A*b-3*B*a)*ln(a^(1/3)-(b
*x^3+a)^(1/3))/a^(5/3)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^7}\,dx$$
$$= \frac{-\frac{3a^{2/3}\sqrt[3]{a+bx^3}(Abx^3+3a(A+2Bx^3))}{x^6} + 2\sqrt{3}b(Ab-3aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2b(Ab-3aB)\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right) + b(Ab-3aB)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{54a^{5/3}}$$

input
```
Integrate[((a + b*x^3)^(1/3)*(A + B*x^3))/x^7,x]
```

output
```
((-3*a^(2/3)*(a + b*x^3)^(1/3)*(A*b*x^3 + 3*a*(A + 2*B*x^3)))/x^6 + 2*Sqrt
[3]*b*(A*b - 3*a*B)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] -
2*b*(A*b - 3*a*B)*Log[-a^(1/3) + (a + b*x^3)^(1/3)] + b*(A*b - 3*a*B)*Log[
a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*a^(5/3))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {948, 87, 51, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^7}\,dx$$
$$\Big\downarrow \text{948}$$
$$\frac{1}{3}\int \frac{\sqrt[3]{bx^3+a}(Bx^3+A)}{x^9}dx^3$$
$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(-\frac{(Ab-3aB)\int \frac{\sqrt[3]{bx^3+a}}{x^6}dx^3}{3a}-\frac{A\left(a+bx^3\right)^{4/3}}{2ax^6}\right)$$

$\Big\downarrow$ 51

$$\frac{1}{3}\left(-\frac{(Ab-3aB)\left(\frac{1}{3}b\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3-\frac{\sqrt[3]{a+bx^3}}{x^3}\right)}{3a}-\frac{A\left(a+bx^3\right)^{4/3}}{2ax^6}\right)$$

$\Big\downarrow$ 69

$$\frac{1}{3}\left(-\frac{(Ab-3aB)\left(\frac{1}{3}b\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)-\sqrt[3]{\ }\right.}{3a}\right.$$

$\Big\downarrow$ 16

$$\frac{1}{3}\left(-\frac{(Ab-3aB)\left(\frac{1}{3}b\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx^3}}{x^3}\right.}{3a}\right.$$

$\Big\downarrow$ 1082

$$\frac{1}{3}\left(-\frac{(Ab-3aB)\left(\frac{1}{3}b\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx^3}}{x^3}\right)}{3a}-\frac{A(a+}{2}\right.$$

$\Big\downarrow$ 217

$$\frac{1}{3}\left(-\frac{(Ab-3aB)\left(\frac{1}{3}b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx^3}}{x^3}\right)}{3a}-\frac{A(a+\right.$$

input
```
Int[((a + b*x^3)^(1/3)*(A + B*x^3))/x^7,x]
```

output
```
(-1/2*(A*(a + b*x^3)^(4/3))/(a*x^6) - ((A*b - 3*a*B)*(-((a + b*x^3)^(1/3)/
x^3) + (b*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])
/a^(2/3)) - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2
*a^(2/3))))/3))/(3*a))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{\frac{3\left(-3\left(2B\,x^{3}+A\right)a^{\frac{5}{3}}-A\,a^{\frac{2}{3}}b\,x^{3}\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{2}+\left(Ab-3Ba\right)b\left(\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}-\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{\ln(\cdots}{\cdots}\right.}{27a^{\frac{5}{3}}x^{6}}$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/27/a^(5/3)*(3/2*(-3*(2*B*x^3+A)*a^(5/3)-A*a^(2/3)*b*x^3)*(b*x^3+a)^(1/3)
+(A*b-3*B*a)*b*(arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^
(1/2)-ln((b*x^3+a)^(1/3)-a^(1/3))+1/2*ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)
^(1/3)+a^(2/3)))*x^6)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^7}\,dx =$$

$$-\frac{6\sqrt{\frac{1}{3}}(3\,Ba^2b-Aab^2)x^6\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(-a^2\right)^{\frac{1}{3}}a-2\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{a^2}\right)+(3\,Bab-A}{}$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^7,x, algorithm="fricas")
```

output
```
-1/54*(6*sqrt(1/3)*(3*B*a^2*b - A*a*b^2)*x^6*sqrt(-(-a^2)^(1/3))*arctan(-s
qrt(1/3)*((-a^2)^(1/3)*a - 2*(b*x^3 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^
(1/3))/a^2) + (3*B*a*b - A*b^2)*(-a^2)^(2/3)*x^6*log((b*x^3 + a)^(2/3)*a -
 (-a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(-a^2)^(2/3)) - 2*(3*B*a*b - A*b^2)*(-
a^2)^(2/3)*x^6*log((b*x^3 + a)^(1/3)*a - (-a^2)^(2/3)) + 3*(3*A*a^3 + (6*B
*a^3 + A*a^2*b)*x^3)*(b*x^3 + a)^(1/3))/(a^3*x^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 40.57 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^7}\,dx = -\frac{A\sqrt[3]{b}\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3x^5\Gamma\left(\frac{8}{3}\right)} - \frac{B\sqrt[3]{b}\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3x^2\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)*(B*x**3+A)/x**7,x)
```

output
```
-A*b**(1/3)*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), a*exp_polar(I*pi)/(b*x**
3))/(3*x**5*gamma(8/3)) - B*b**(1/3)*gamma(2/3)*hyper((-1/3, 2/3), (5/3,),
 a*exp_polar(I*pi)/(b*x**3))/(3*x**2*gamma(5/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(131) = 262.

Time = 0.12 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.63

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^7}\,dx = \frac{1}{54}\left(\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}} + \frac{b^2\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}} - \frac{2\,b^2\log\left((bx^3+\right.}{a^{\frac{5}{3}}}\right.$$
$$-\frac{1}{18}\left(\frac{2\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}} + \frac{b\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}} - \frac{2\,b\log\left((bx^3+\right.}{a^{\frac{2}{3}}}\right.$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^7,x, algorithm="maxima")
```

output
```
1/54*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(5/3) + b^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^
(2/3))/a^(5/3) - 2*b^2*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(5/3) - 3*((b*x^
3 + a)^(4/3)*b^2 + 2*(b*x^3 + a)^(1/3)*a*b^2)/((b*x^3 + a)^2*a - 2*(b*x^3
+ a)*a^2 + a^3))*A - 1/18*(2*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(
1/3) + a^(1/3))/a^(1/3))/a^(2/3) + b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(
1/3)*a^(1/3) + a^(2/3))/a^(2/3) - 2*b*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(
2/3) + 6*(b*x^3 + a)^(1/3)/x^3)*B
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^7}\,dx =$$

$$-\frac{\frac{2\sqrt{3}\left(3\,Ba^{\frac{4}{3}}b^2-Aa^{\frac{1}{3}}b^3\right)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^2}+\frac{\left(3\,Bab^2-Ab^3\right)\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2\left(3\,Bab^2-Ab^3\right)\log\cdots}{\cdots}}{54\,b}$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^7,x, algorithm="giac")
```

output
```
-1/54*(2*sqrt(3)*(3*B*a^(4/3)*b^2 - A*a^(1/3)*b^3)*arctan(1/3*sqrt(3)*(2*(
b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^2 + (3*B*a*b^2 - A*b^3)*log((b*x^3
+ a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(5/3) - 2*(3*B*a*b^2 -
 A*b^3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(5/3) + 3*(6*(b*x^3 + a)^(
4/3)*B*a*b^2 - 6*(b*x^3 + a)^(1/3)*B*a^2*b^2 + (b*x^3 + a)^(4/3)*A*b^3 + 2
*(b*x^3 + a)^(1/3)*A*a*b^3)/(a*b^2*x^6))/b
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.12

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^7}\,dx$$
$$= \frac{B\,b\,\ln\left(B\,b\,(b\,x^3+a)^{1/3}-B\,a^{1/3}\,b\right)}{9\,a^{2/3}} - \frac{B\,(b\,x^3+a)^{1/3}}{3\,x^3}$$
$$- \frac{\ln\left(\frac{a^{1/3}\,\left(B\,b-\sqrt{3}\,B\,b\,1\mathrm{i}\right)}{2}+B\,b\,(b\,x^3+a)^{1/3}\right)\,\left(B\,b-\sqrt{3}\,B\,b\,1\mathrm{i}\right)}{18\,a^{2/3}}$$
$$- \frac{\ln\left(\frac{a^{1/3}\,\left(B\,b+\sqrt{3}\,B\,b\,1\mathrm{i}\right)}{2}+B\,b\,(b\,x^3+a)^{1/3}\right)\,\left(B\,b+\sqrt{3}\,B\,b\,1\mathrm{i}\right)}{18\,a^{2/3}}$$
$$+ \frac{\ln\left(\frac{A\,b^2-\sqrt{3}\,A\,b^2\,1\mathrm{i}}{6\,a^{2/3}}+\frac{A\,b^2\,(b\,x^3+a)^{1/3}}{3\,a}\right)\,\left(A\,b^2-\sqrt{3}\,A\,b^2\,1\mathrm{i}\right)}{54\,a^{5/3}}$$
$$- \frac{\frac{A\,b^2\,(b\,x^3+a)^{1/3}}{9}+\frac{A\,b^2\,(b\,x^3+a)^{4/3}}{18\,a}}{(b\,x^3+a)^2-2\,a\,(b\,x^3+a)+a^2} - \frac{A\,b^2\,\ln\left((b\,x^3+a)^{1/3}-a^{1/3}\right)}{27\,a^{5/3}}$$
$$+ \frac{A\,b^2\,\ln\left(\frac{A\,b^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{2/3}}+\frac{A\,b^2\,(b\,x^3+a)^{1/3}}{3\,a}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27\,a^{5/3}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/3))/x^7,x)
```

output
```
(log((A*b^2 - 3^(1/2)*A*b^2*1i)/(6*a^(2/3)) + (A*b^2*(a + b*x^3)^(1/3))/(3
*a))*(A*b^2 - 3^(1/2)*A*b^2*1i))/(54*a^(5/3)) - (B*(a + b*x^3)^(1/3))/(3*x
^3) - (log((a^(1/3)*(B*b - 3^(1/2)*B*b*1i))/2 + B*b*(a + b*x^3)^(1/3))*(B*
b - 3^(1/2)*B*b*1i))/(18*a^(2/3)) - (log((a^(1/3)*(B*b + 3^(1/2)*B*b*1i))/
2 + B*b*(a + b*x^3)^(1/3))*(B*b + 3^(1/2)*B*b*1i))/(18*a^(2/3)) - ((A*b^2*
(a + b*x^3)^(1/3))/9 + (A*b^2*(a + b*x^3)^(4/3))/(18*a))/((a + b*x^3)^2 -
2*a*(a + b*x^3) + a^2) + (B*b*log(B*b*(a + b*x^3)^(1/3) - B*a^(1/3)*b))/(9
*a^(2/3)) - (A*b^2*log((a + b*x^3)^(1/3) - a^(1/3)))/(27*a^(5/3)) + (A*b^2
*log((A*b^2*((3^(1/2)*1i)/2 + 1/2))/(3*a^(2/3)) + (A*b^2*(a + b*x^3)^(1/3)
)/(3*a))*((3^(1/2)*1i)/2 + 1/2))/(27*a^(5/3))
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^7}\,dx$$
$$= \frac{-3(b\,x^3+a)^{\frac{1}{3}}\,a-7(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3+4\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{b\,x^4+ax}dx\right)b^2x^6}{18x^6}$$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^7,x)
```

output
```
( - 3*(a + b*x**3)**(1/3)*a - 7*(a + b*x**3)**(1/3)*b*x**3 + 4*int((a + b*
x**3)**(1/3)/(a*x + b*x**4),x)*b**2*x**6)/(18*x**6)
```

3.295 $\int x^4 \sqrt[3]{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 181

$$\int x^4 \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{a(9Ab-5aB)x^2\sqrt[3]{a+bx^3}}{162b^2} + \frac{(9Ab-5aB)x^5\sqrt[3]{a+bx^3}}{54b} + \frac{Bx^5\left(a+bx^3\right)^{4/3}}{9b} + \frac{a^2(9Ab-5aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{81\sqrt{3}b^{8/3}} + \frac{a^2(9Ab-5aB)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{162b^{8/3}}$$

output
```
1/162*a*(9*A*b-5*B*a)*x^2*(b*x^3+a)^(1/3)/b^2+1/54*(9*A*b-5*B*a)*x^5*(b*x^
3+a)^(1/3)/b+1/9*B*x^5*(b*x^3+a)^(4/3)/b+1/243*a^2*(9*A*b-5*B*a)*arctan(1/
3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(8/3)+1/162*a^2*(9*A*
b-5*B*a)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(8/3)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.17

$$\int x^4\sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{3b^{2/3}x^2\sqrt[3]{a+bx^3}(-5a^2B+3ab(3A+Bx^3)+9b^2x^3(3A+2Bx^3))-2\sqrt{3}a^2(-9Ab+5aB)\arctan\left(\frac{}{\sqrt[3]{bx}}\right.}{}$$

input
```
Integrate[x^4*(a + b*x^3)^(1/3)*(A + B*x^3),x]
```

output
```
(3*b^(2/3)*x^2*(a + b*x^3)^(1/3)*(-5*a^2*B + 3*a*b*(3*A + B*x^3) + 9*b^2*x
^3*(3*A + 2*B*x^3)) - 2*Sqrt[3]*a^2*(-9*A*b + 5*a*B)*ArcTan[(Sqrt[3]*b^(1/
3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*a^2*(-9*A*b + 5*a*B)*Log[-(b^
(1/3)*x) + (a + b*x^3)^(1/3)] + a^2*(-9*A*b + 5*a*B)*Log[b^(2/3)*x^2 + b^(
1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(486*b^(8/3))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {959, 811, 843, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$\Big\downarrow \text{959}$$
$$\frac{(9Ab-5aB)\int x^4\sqrt[3]{bx^3+a}dx}{9b}+\frac{Bx^5\left(a+bx^3\right)^{4/3}}{9b}$$
$$\Big\downarrow \text{811}$$
$$\frac{(9Ab-5aB)\left(\frac{1}{6}a\int\frac{x^4}{\left(bx^3+a\right)^{2/3}}dx+\frac{1}{6}x^5\sqrt[3]{a+bx^3}\right)}{9b}+\frac{Bx^5\left(a+bx^3\right)^{4/3}}{9b}$$

$$\Big\downarrow \; 843$$

$$\frac{(9Ab-5aB)\left(\frac{1}{6}a\left(\frac{x^2\sqrt[3]{a+bx^3}}{3b}-\frac{2a\int \frac{x}{(bx^3+a)^{2/3}}dx}{3b}\right)+\frac{1}{6}x^5\sqrt[3]{a+bx^3}\right)}{9b}+\frac{Bx^5(a+bx^3)^{4/3}}{9b}$$

$$\Big\downarrow \; 853$$

$$\frac{(9Ab-5aB)\left(\frac{1}{6}a\left(\frac{x^2\sqrt[3]{a+bx^3}}{3b}-\frac{2a\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\right)}{3b}\right)+\frac{1}{6}x^5\sqrt[3]{a+bx^3}\right)}{9b}+\frac{Bx^5(a+bx^3)^{4/3}}{9b}$$

input

```
Int[x^4*(a + b*x^3)^(1/3)*(A + B*x^3),x]
```

output

```
(B*x^5*(a + b*x^3)^(4/3))/(9*b) + ((9*A*b - 5*a*B)*((x^5*(a + b*x^3)^(1/3)
)/6 + (a*((x^2*(a + b*x^3)^(1/3))/(3*b) - (2*a*(-(ArcTan[(1 + (2*b^(1/3)*x
)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3))) - Log[b^(1/3)*x - (a + b*
x^3)^(1/3)]/(2*b^(2/3))))/(3*b)))/6))/(9*b)
```

Defintions of rubi rules used

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$-\frac{-3\left(-\frac{5Ba^2b^{\frac{2}{3}}}{9}+a\left(\frac{Bx^3}{3}+A\right)b^{\frac{5}{3}}+3\left(\frac{2Bx^3}{3}+A\right)b^{\frac{8}{3}}x^3\right)x^2\left(bx^3+a\right)^{\frac{1}{3}}+a^2\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\right.}{54b^{\frac{8}{3}}}$

input
```
int(x^4*(b*x^3+a)^(1/3)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
-1/54*(-3*(-5/9*B*a^2*b^(2/3)+a*(1/3*B*x^3+A)*b^(5/3)+3*(2/3*B*x^3+A)*b^(8
/3)*x^3)*x^2*(b*x^3+a)^(1/3)+a^2*(2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+
2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b
*x^3+a)^(2/3))/x^2)-2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x))*(A*b-5/9*B*a))/b
^(8/3)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.35

$$\int x^4\sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$= \frac{6\sqrt{\frac{1}{3}}(5\,Ba^3b-9\,Aa^2b^2)(b^2)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left((b^2)^{\frac{1}{3}}bx+2\,(bx^3+a)^{\frac{1}{3}}(b^2)^{\frac{2}{3}}\right)(b^2)^{\frac{1}{6}}}{b^2x}\right)-2\,(5\,Ba^3-9\,Aa^2b)(b^2)^{\frac{2}{3}}\log\left(\right.}{}$$

input
```
integrate(x^4*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/486*(6*sqrt(1/3)*(5*B*a^3*b - 9*A*a^2*b^2)*(b^2)^(1/6)*arctan(sqrt(1/3)*
((b^2)^(1/3)*b*x + 2*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) -
 2*(5*B*a^3 - 9*A*a^2*b)*(b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/
3)*b)/x) + (5*B*a^3 - 9*A*a^2*b)*(b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 + (b*x
^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2) + 3*(18*B*b^4*x^8
+ 3*(B*a*b^3 + 9*A*b^4)*x^5 - (5*B*a^2*b^2 - 9*A*a*b^3)*x^2)*(b*x^3 + a)^(
1/3))/b^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.46

$$\int x^4 \sqrt[3]{a+bx^3}(A+Bx^3)\ dx = \frac{A\sqrt[3]{a}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{B\sqrt[3]{a}x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

input

```
integrate(x**4*(b*x**3+a)**(1/3)*(B*x**3+A),x)
```

output

```
A*a**(1/3)*x**5*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(8/3)) + B*a**(1/3)*x**8*gamma(8/3)*hyper((-1/3, 8/3), (11/3
,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(150) = 300.

Time = 0.12 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.26

$$\int x^4\sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx = -\frac{1}{54}\left(\frac{2\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{5}{3}}}+\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{5}{3}}}-\frac{2a^2\log\left(-b^{\frac{1}{3}}+\right.}{b^{\frac{5}{3}}}\right. + \frac{1}{486}\left(\frac{10\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{8}{3}}}+\frac{5a^3\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{8}{3}}}-\frac{10a^3\log\left(-\right.}{}\right.$$

input
```
integrate(x^4*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="maxima")
```

output
```
-1/54*(2*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/
b^(1/3))/b^(5/3) + a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3
+ a)^(2/3)/x^2)/b^(5/3) - 2*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(5/3
) - 3*(2*(b*x^3 + a)^(1/3)*a^2*b/x + (b*x^3 + a)^(4/3)*a^2/x^4)/(b^3 - 2*(
b*x^3 + a)*b^2/x^3 + (b*x^3 + a)^2*b/x^6))*A + 1/486*(10*sqrt(3)*a^3*arcta
n(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(8/3) + 5*a^3*l
og(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(8/3)
- 10*a^3*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(8/3) - 3*(10*(b*x^3 + a)^(
1/3)*a^3*b^2/x + 13*(b*x^3 + a)^(4/3)*a^3*b/x^4 - 5*(b*x^3 + a)^(7/3)*a^3/
x^7)/(b^5 - 3*(b*x^3 + a)*b^4/x^3 + 3*(b*x^3 + a)^2*b^3/x^6 - (b*x^3 + a)^
3*b^2/x^9))*B
```

Giac [F]

$$\int x^4 \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{1}{3}}x^4\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx=\int x^4\left(B\,x^3+A\right)\left(b\,x^3+a\right)^{1/3}\,dx$$

input
```
int(x^4*(A + B*x^3)*(a + b*x^3)^(1/3),x)
```

output
```
int(x^4*(A + B*x^3)*(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int x^4 \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{2(b\,x^3+a)^{\frac{1}{3}}\,a^2x^2+15(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^5+9(b\,x^3+a)^{\frac{1}{3}}\,b^2x^8-4\left(\int\frac{x}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a^3}{81b}$$

input
```
int(x^4*(b*x^3+a)^(1/3)*(B*x^3+A),x)
```

output
```
(2*(a + b*x**3)**(1/3)*a**2*x**2 + 15*(a + b*x**3)**(1/3)*a*b*x**5 + 9*(a
+ b*x**3)**(1/3)*b**2*x**8 - 4*int(((a + b*x**3)**(1/3)*x)/(a + b*x**3),x)
*a**3)/(81*b)
```

3.296 $\int x\sqrt[3]{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 146

$$\int x\sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{(3Ab-aB)x^2\sqrt[3]{a+bx^3}}{9b}+\frac{Bx^2(a+bx^3)^{4/3}}{6b}-\frac{a(3Ab-aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{5/3}}-\frac{a(3Ab-aB)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}}$$

output
```
1/9*(3*A*b-B*a)*x^2*(b*x^3+a)^(1/3)/b+1/6*B*x^2*(b*x^3+a)^(4/3)/b-1/27*a*(
3*A*b-B*a)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(
5/3)-1/18*a*(3*A*b-B*a)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.24

$$\int x\sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{3b^{2/3}x^2\sqrt[3]{a+bx^3}(6Ab+B(a+3bx^3))+2\sqrt{3}a(-3Ab+aB)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+2a(-3Ab+a}{54b^{5/3}}$$

input
```
Integrate[x*(a + b*x^3)^(1/3)*(A + B*x^3),x]
```

output
```
(3*b^(2/3)*x^2*(a + b*x^3)^(1/3)*(6*A*b + B*(a + 3*b*x^3)) + 2*Sqrt[3]*a*(
-3*A*b + a*B)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))
] + 2*a*(-3*A*b + a*B)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - a*(-3*A*b +
 a*B)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/
(54*b^(5/3))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {959, 811, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(3Ab-aB)\int x\sqrt[3]{bx^3+a}dx}{3b}+\frac{Bx^2\left(a+bx^3\right)^{4/3}}{6b}$$

$$\Big\downarrow \text{811}$$

$$\frac{(3Ab-aB)\left(\frac{1}{3}a\int\frac{x}{(bx^3+a)^{2/3}}dx+\frac{1}{3}x^2\sqrt[3]{a+bx^3}\right)}{3b}+\frac{Bx^2\left(a+bx^3\right)^{4/3}}{6b}$$

$$\Big\downarrow \text{853}$$

$$\frac{(3Ab-aB)\left(\frac{1}{3}a\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\right)+\frac{1}{3}x^2\sqrt[3]{a+bx^3}\right)}{3b}+\frac{Bx^2\left(a+bx^3\right)^{4/3}}{6b}$$

input
```
Int[x*(a + b*x^3)^(1/3)*(A + B*x^3),x]
```

output
```
(B*x^2*(a + b*x^3)^(4/3))/(6*b) + ((3*A*b - a*B)*((x^2*(a + b*x^3)^(1/3))/
3 + (a*(-(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b
^(2/3))) - Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b^(2/3))))/3))/(3*b)
```

Defintions of rubi rules used

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(119) = 238.

Time = 1.18 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.87

method	result
pseudoelliptic	$\frac{9B\,b^{\frac{5}{3}} x^{5} \left(b\,x^{3}+a\right)^{\frac{1}{3}}+18A\,b^{\frac{5}{3}} x^{2} \left(b\,x^{3}+a\right)^{\frac{1}{3}}+3Ba\,x^{2} b^{\frac{2}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}}+6A\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}} x}\right)\sqrt{3}\,ab-2B\arctan}{}$

input
```
int(x*(b*x^3+a)^(1/3)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/54*(9*B*b^(5/3)*x^5*(b*x^3+a)^(1/3)+18*A*b^(5/3)*x^2*(b*x^3+a)^(1/3)+3*B
*a*x^2*b^(2/3)*(b*x^3+a)^(1/3)+6*A*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+
a)^(1/3))/b^(1/3)/x)*3^(1/2)*a*b-2*B*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^
3+a)^(1/3))/b^(1/3)/x)*3^(1/2)*a^2-6*A*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*
a*b+3*A*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a*
b+2*B*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^2-B*ln((b^(2/3)*x^2+b^(1/3)*(b*
x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a^2)/b^(5/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(116) = 232.

Time = 0.14 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.66

$$\int x\sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx =$$

$$-\frac{6\sqrt{\frac{1}{3}}(Ba^2b-3\,Aab^2)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(-b^2\right)^{\frac{1}{3}}bx-2\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}}{b^2x}\right)-2\left(Ba^2-3\,Ac\right.}{}$$

input
```
integrate(x*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="fricas")
```

output
```
-1/54*(6*sqrt(1/3)*(B*a^2*b - 3*A*a*b^2)*sqrt(-(-b^2)^(1/3))*arctan(-sqrt(
1/3)*((-b^2)^(1/3)*b*x - 2*(b*x^3 + a)^(1/3)*(-b^2)^(2/3))*sqrt(-(-b^2)^(1
/3))/(b^2*x)) - 2*(B*a^2 - 3*A*a*b)*(-b^2)^(2/3)*log(-((-b^2)^(2/3)*x - (b
*x^3 + a)^(1/3)*b)/x) + (B*a^2 - 3*A*a*b)*(-b^2)^(2/3)*log(-((-b^2)^(1/3)*
b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) - 3*(
3*B*b^3*x^5 + (B*a*b^2 + 6*A*b^3)*x^2)*(b*x^3 + a)^(1/3))/b^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.57

$$\int x\sqrt[3]{a+bx^3}\,(A+Bx^3)\;dx = \frac{A\sqrt[3]{a}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{B\sqrt[3]{a}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x*(b*x**3+a)**(1/3)*(B*x**3+A),x)
```

output
```
A*a**(1/3)*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(5/3)) + B*a**(1/3)*x**5*gamma(5/3)*hyper((-1/3, 5/3), (8/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(116) = 232.

Time = 0.11 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.19

$$\int x\sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$=\frac{1}{18}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{2}{3}}}+\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{2}{3}}}-\frac{2a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{2}{3}}}\right.$$

$$-\frac{1}{54}\left(\frac{2\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{5}{3}}}+\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{5}{3}}}-\frac{2a^2\log\left(-b^{\frac{1}{3}}+\right.}{b^{\frac{5}{3}}}\right.$$

input
```
integrate(x*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/18*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(
1/3))/b^(2/3) + a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^
(2/3)/x^2)/b^(2/3) - 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(2/3) - 6*(
b*x^3 + a)^(1/3)*a/((b - (b*x^3 + a)/x^3)*x))*A - 1/54*(2*sqrt(3)*a^2*arct
an(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(5/3) + a^2*lo
g(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(5/3) -
 2*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(5/3) - 3*(2*(b*x^3 + a)^(1/3
)*a^2*b/x + (b*x^3 + a)^(4/3)*a^2/x^4)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x
^3 + a)^2*b/x^6))*B
```

Giac [F]

$$\int x\sqrt[3]{a+bx^3}\,(A+Bx^3)\ dx=\int (Bx^3+A)\,(bx^3+a)^{\frac{1}{3}}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt[3]{a+bx^3}\,(A+Bx^3)\ dx=\int x\,\left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{1/3}\,dx$$

input
```
int(x*(A + B*x^3)*(a + b*x^3)^(1/3),x)
```

output
```
int(x*(A + B*x^3)*(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int x\sqrt[3]{a+bx^3}\,(A+Bx^3)\ dx=\frac{7(b\,x^3+a)^{\frac{1}{3}}\,a\,x^2}{18}+\frac{(b\,x^3+a)^{\frac{1}{3}}\,b\,x^5}{6}+\frac{2\left(\int\frac{x}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a^2}{9}$$

input
```
int(x*(b*x^3+a)^(1/3)*(B*x^3+A),x)
```

output
```
(7*(a + b*x**3)**(1/3)*a*x**2 + 3*(a + b*x**3)**(1/3)*b*x**5 + 4*int(((a +
 b*x**3)**(1/3)*x)/(a + b*x**3),x)*a**2)/18
```

3.297 $\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 139

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^2}\,dx = \frac{(3Ab+aB)x^2\sqrt[3]{a+bx^3}}{3a} - \frac{A(a+bx^3)^{4/3}}{ax} - \frac{(3Ab+aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}} - \frac{(3Ab+aB)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}}$$

output
```
1/3*(3*A*b+B*a)*x^2*(b*x^3+a)^(1/3)/a-A*(b*x^3+a)^(4/3)/a/x-1/9*(3*A*b+B*a
)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)-1/6*
(3*A*b+B*a)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^2}\,dx$$
$$= \frac{6b^{2/3}\sqrt[3]{a+bx^3}(-3A+Bx^3)-2\sqrt{3}(3Ab+aB)x\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-2(3Ab+aB)x\log\left(-\sqrt[3]{b}\right.}{18b^{2/3}x}$$

input
```
Integrate[((a + b*x^3)^(1/3)*(A + B*x^3))/x^2,x]
```

output
```
(6*b^(2/3)*(a + b*x^3)^(1/3)*(-3*A + B*x^3) - 2*Sqrt[3]*(3*A*b + a*B)*x*Ar
cTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*(3*A*b + a
*B)*x*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + (3*A*b + a*B)*x*Log[b^(2/3)*
x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*b^(2/3)*x)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 811, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^2}\,dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(aB+3Ab)\int x\sqrt[3]{bx^3+a}dx}{a}-\frac{A\left(a+bx^3\right)^{4/3}}{ax}$$

$$\Big\downarrow \text{811}$$

$$\frac{(aB+3Ab)\left(\frac{1}{3}a\int \frac{x}{(bx^3+a)^{2/3}}dx+\frac{1}{3}x^2\sqrt[3]{a+bx^3}\right)}{a}-\frac{A\left(a+bx^3\right)^{4/3}}{ax}$$

$$\Big\downarrow \text{853}$$

$$\frac{(aB+3Ab)\left(\frac{1}{3}a\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\right)+\frac{1}{3}x^2\sqrt[3]{a+bx^3}\right)}{a}-\frac{A\left(a+bx^3\right)^{4/3}}{ax}$$

input
```
Int[((a + b*x^3)^(1/3)*(A + B*x^3))/x^2,x]
```

output
```
-((A*(a + b*x^3)^(4/3))/(a*x)) + ((3*A*b + a*B)*((x^2*(a + b*x^3)^(1/3))/3
 + (a*(-(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^
(2/3))) - Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b^(2/3))))/3))/a
```

Defintions of rubi rules used

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$\frac{-6\left(b\,x^{3}+a\right)^{\frac{1}{3}} b^{\frac{2}{3}} \left(-\frac{B\,x^{3}}{3}+A\right)+\left(A b+\frac{B a}{3}\right) \left(2 \arctan\left(\frac{\sqrt{3}\,\left(\frac{2 \left(b\,x^{3}+a\right)^{\frac{1}{3}}}{b^{\frac{1}{3}}}+x\right)}{3 x}\right) \sqrt{3}+\ln\left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right.\right.}{6 b^{\frac{2}{3}} x}$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(-6*(b*x^3+a)^(1/3)*b^(2/3)*(-1/3*B*x^3+A)+(A*b+1/3*B*a)*(2*arctan(1/3
*3^(1/2)*(2*(b*x^3+a)^(1/3)/b^(1/3)+x)/x)*3^(1/2)+ln((b^(2/3)*x^2+b^(1/3)*
(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/
x))*x)/b^(2/3)/x
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^2}\,dx = \frac{A\sqrt[3]{a}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{B\sqrt[3]{a}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)*(B*x**3+A)/x**2,x)
```

output
```
A*a**(1/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x*gamma(2/3)) + B*a**(1/3)*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(114) = 228.

Time = 0.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.82

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^2}\,dx$$
$$=\frac{1}{6}\left(2\sqrt{3}b^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)+b^{\frac{1}{3}}\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)-2\,b^{\frac{1}{3}}\log\left(-\right.\right.$$
$$+\frac{1}{18}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{2}{3}}}+\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{2}{3}}}-\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^{3}}{}\right.}{b^{\frac{2}{3}}}\right.$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^2,x, algorithm="maxima")
```

output
```
1/6*(2*sqrt(3)*b^(1/3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x
)/b^(1/3)) + b^(1/3)*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 +
a)^(2/3)/x^2) - 2*b^(1/3)*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x) - 6*(b*x^3 +
 a)^(1/3)/x)*A + 1/18*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3
+ a)^(1/3)/x)/b^(1/3))/b^(2/3) + a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)
/x + (b*x^3 + a)^(2/3)/x^2)/b^(2/3) - 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)
/x)/b^(2/3) - 6*(b*x^3 + a)^(1/3)*a/((b - (b*x^3 + a)/x^3)*x))*B
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^2}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{1}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^2,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^2}\,dx=\int \frac{\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{1/3}}{x^2} \,d x$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/3))/x^2,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/3))/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^2}\,dx=\frac{-3(b\,x^{3}+a)^{\frac{1}{3}} a+(b\,x^{3}+a)^{\frac{1}{3}} b\,x^{3}+4\left(\int \frac{x}{(b\,x^{3}+a)^{\frac{2}{3}}}d x\right) a b x}{3x}$$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^2,x)
```

output
```
( - 3*(a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3 + 4*int(((a + b*x
**3)**(1/3)*x)/(a + b*x**3),x)*a*b*x)/(3*x)
```

3.298 $\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^5}\,dx = -\frac{B\sqrt[3]{a+bx^3}}{x}-\frac{A(a+bx^3)^{4/3}}{4ax^4} - \frac{\sqrt[3]{b}B\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{2}\sqrt[3]{b}B\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)$$

output
```
-B*(b*x^3+a)^(1/3)/x-1/4*A*(b*x^3+a)^(4/3)/a/x^4-1/3*b^(1/3)*B*arctan(1/3*
(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)-1/2*b^(1/3)*B*ln(b^(1/3)*
x-(b*x^3+a)^(1/3))
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^5}\,dx = \frac{1}{12}\left(-\frac{3\sqrt[3]{a+bx^3}(Abx^3+a(A+4Bx^3))}{ax^4} - 4\sqrt{3}\sqrt[3]{b}B\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - 4\sqrt[3]{b}B\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right) + 2\sqrt[3]{b}B\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)\right)$$

input

```
Integrate[((a + b*x^3)^(1/3)*(A + B*x^3))/x^5,x]
```

output

```
((-3*(a + b*x^3)^(1/3)*(A*b*x^3 + a*(A + 4*B*x^3)))/(a*x^4) - 4*Sqrt[3]*b^
(1/3)*B*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 4*
b^(1/3)*B*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 2*b^(1/3)*B*Log[b^(2/3)*
x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/12
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {953, 809, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^5}\,dx$$

$$\Big\downarrow \text{953}$$

$$B\int \frac{\sqrt[3]{bx^3+a}}{x^2}dx - \frac{A\left(a+bx^3\right)^{4/3}}{4ax^4}$$

$$\Big\downarrow \text{809}$$

$$B\left(b\int \frac{x}{(bx^3+a)^{2/3}}dx-\frac{\sqrt[3]{a+bx^3}}{x}\right)-\frac{A(a+bx^3)^{4/3}}{4ax^4}$$

$$\Big\downarrow \text{853}$$

$$B\left(b\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\right)-\frac{\sqrt[3]{a+bx^3}}{x}\right)-\frac{A(a+bx^3)^{4/3}}{4ax^4}$$

input
```
Int[((a + b*x^3)^(1/3)*(A + B*x^3))/x^5,x]
```

output
```
-1/4*(A*(a + b*x^3)^(4/3))/(a*x^4) + B*(-((a + b*x^3)^(1/3)/x) + b*(-(ArcT
an[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3))) - Log
[b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b^(2/3))))
```

Defintions of rubi rules used

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 953
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[d/e^n   Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] &&
(IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && G
tQ[m + n, -1]))
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.27

method	result
pseudoelliptic	$\frac{((-3Ab-12Ba)x^3-3Aa)\left(b\,x^3+a\right)^{\frac{1}{3}}+4a\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)-\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)+\frac{\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}\cdots}{\cdots}\right)}{\cdots}\cdots\right)}{12a\,x^4}$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/12*(((-3*A*b-12*B*a)*x^3-3*A*a)*(b*x^3+a)^(1/3)+4*a*(3^(1/2)*arctan(1/3*
3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)-ln((-b^(1/3)*x+(b*x^3+a)^
(1/3))/x)+1/2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x
^2))*b^(1/3)*B*x^4)/a/x^4
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^5}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^5,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^5}\,dx = \frac{A\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{4}{3}\right)}{3x^3\Gamma\left(-\frac{1}{3}\right)} + \frac{Ab^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{4}{3}\right)}{3a\Gamma\left(-\frac{1}{3}\right)} + \frac{B\sqrt[3]{a}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)*(B*x**3+A)/x**5,x)
```

output
```
A*b**(1/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-4/3)/(3*x**3*gamma(-1/3)) + A*b*
*(4/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-4/3)/(3*a*gamma(-1/3)) + B*a**(1/3)*
gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gam
ma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^5}\,dx = \frac{1}{6}\left(2\sqrt{3}b^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right) + b^{\frac{1}{3}}\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right) - 2\,b^{\frac{1}{3}}\log\left(-\right.\right. - \frac{(bx^3+a)^{\frac{4}{3}}A}{4\,ax^4}$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^5,x, algorithm="maxima")
```

output
```
1/6*(2*sqrt(3)*b^(1/3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x
)/b^(1/3)) + b^(1/3)*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 +
a)^(2/3)/x^2) - 2*b^(1/3)*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x) - 6*(b*x^3 +
 a)^(1/3)/x)*B - 1/4*(b*x^3 + a)^(4/3)*A/(a*x^4)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^5}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{1}{3}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^5,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3)/x^5, x)
```

Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^5}\,dx = -\frac{A\,(b\,x^3+a)^{4/3}}{4\,a\,x^4} - \frac{B\,(b\,x^3+a)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},-\frac{1}{3};\ \frac{2}{3};\ -\frac{b\,x^3}{a}\right)}{x\,\left(\frac{b\,x^3}{a}+1\right)^{1/3}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/3))/x^5,x)
```

output
```
- (A*(a + b*x^3)^(4/3))/(4*a*x^4) - (B*(a + b*x^3)^(1/3)*hypergeom([-1/3,
-1/3], 2/3, -(b*x^3)/a))/(x*((b*x^3)/a + 1)^(1/3))
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^5}\,dx = \frac{-(b\,x^3+a)^{\frac{1}{3}}\,a-(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3+4\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{x^2}dx\right)b\,x^4}{4x^4}$$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^5,x)
```

output
```
( - (a + b*x**3)**(1/3)*a - (a + b*x**3)**(1/3)*b*x**3 + 4*int((a + b*x**3
)**(1/3)/x**2,x)*b*x**4)/(4*x**4)
```

3.299 $\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^8}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^8}\,dx = -\frac{A(a+bx^3)^{4/3}}{7ax^7}+\frac{(3Ab-7aB)\,(a+bx^3)^{4/3}}{28a^2x^4}$$

output
```
-1/7*A*(b*x^3+a)^(4/3)/a/x^7+1/28*(3*A*b-7*B*a)*(b*x^3+a)^(4/3)/a^2/x^4
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^8}\,dx = \frac{(a+bx^3)^{4/3}\,(-4aA+3Abx^3-7aBx^3)}{28a^2x^7}$$

input
```
Integrate[((a + b*x^3)^(1/3)*(A + B*x^3))/x^8,x]
```

output
```
((a + b*x^3)^(4/3)*(-4*a*A + 3*A*b*x^3 - 7*a*B*x^3))/(28*a^2*x^7)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {955, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}\,(A+Bx^3)}{x^8}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(3Ab-7aB)\int \frac{\sqrt[3]{bx^3+a}}{x^5}dx}{7a}-\frac{A(a+bx^3)^{4/3}}{7ax^7}$$

$$\Big\downarrow \text{796}$$

$$\frac{(a+bx^3)^{4/3}(3Ab-7aB)}{28a^2x^4}-\frac{A(a+bx^3)^{4/3}}{7ax^7}$$

input
```
Int[((a + b*x^3)^(1/3)*(A + B*x^3))/x^8,x]
```

output
```
-1/7*(A*(a + b*x^3)^(4/3))/(a*x^7) + ((3*A*b - 7*a*B)*(a + b*x^3)^(4/3))/(
28*a^2*x^4)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result	size
pseudoelliptic	$-\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(\left(\frac{7 B\,x^{3}}{4}+A\right) a-\frac{3 A b\,x^{3}}{4}\right)}{7 a^{2} x^{7}}$	36
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(-3 A b\,x^{3}+7 B a\,x^{3}+4 A a\right)}{28 a^{2} x^{7}}$	37
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(-3 A b\,x^{3}+7 B a\,x^{3}+4 A a\right)}{28 a^{2} x^{7}}$	37
trager	$-\frac{\left(-3 A\,b^{2} x^{6}+7 B a b\,x^{6}+a A b\,x^{3}+7 B\,a^{2} x^{3}+4 a^{2} A\right) \left(b\,x^{3}+a\right)^{\frac{1}{3}}}{28 a^{2} x^{7}}$	58
risch	$-\frac{\left(-3 A\,b^{2} x^{6}+7 B a b\,x^{6}+a A b\,x^{3}+7 B\,a^{2} x^{3}+4 a^{2} A\right) \left(b\,x^{3}+a\right)^{\frac{1}{3}}}{28 a^{2} x^{7}}$	58

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*(b*x^3+a)^(4/3)*((7/4*B*x^3+A)*a-3/4*A*b*x^3)/a^2/x^7
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^8}\,dx = -\frac{((7\,Bab-3\,Ab^2)x^6+(7\,Ba^2+Aab)x^3+4\,Aa^2)(bx^3+a)^{\frac{1}{3}}}{28\,a^2x^7}$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^8,x, algorithm="fricas")
```

output
```
-1/28*((7*B*a*b - 3*A*b^2)*x^6 + (7*B*a^2 + A*a*b)*x^3 + 4*A*a^2)*(b*x^3 +
 a)^(1/3)/(a^2*x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(46) = 92.

Time = 1.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.53

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^8}\,dx = -\frac{4A\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)} - \frac{Ab^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{7}{3}\right)}{9ax^3\Gamma\left(-\frac{1}{3}\right)} + \frac{Ab^{\frac{7}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{7}{3}\right)}{3a^2\Gamma\left(-\frac{1}{3}\right)} + \frac{B\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{4}{3}\right)}{3x^3\Gamma\left(-\frac{1}{3}\right)} + \frac{Bb^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{4}{3}\right)}{3a\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)*(B*x**3+A)/x**8,x)
```

output
```
-4*A*b**(1/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(9*x**6*gamma(-1/3)) - A
*b**(4/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(9*a*x**3*gamma(-1/3)) + A*b
**(7/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(3*a**2*gamma(-1/3)) + B*b**(1
/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-4/3)/(3*x**3*gamma(-1/3)) + B*b**(4/3)*
(a/(b*x**3) + 1)**(1/3)*gamma(-4/3)/(3*a*gamma(-1/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^8}\,dx = \frac{A\left(\frac{7\,(bx^3+a)^{\frac{4}{3}}b}{x^4} - \frac{4\,(bx^3+a)^{\frac{7}{3}}}{x^7}\right)}{28\,a^2} - \frac{(bx^3+a)^{\frac{4}{3}}B}{4\,ax^4}$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^8,x, algorithm="maxima")
```

output
```
1/28*A*(7*(b*x^3 + a)^(4/3)*b/x^4 - 4*(b*x^3 + a)^(7/3)/x^7)/a^2 - 1/4*(b*
x^3 + a)^(4/3)*B/(a*x^4)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^8}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{1}{3}}}{x^8}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^8,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3)/x^8, x)
```

Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^8}\,dx = \frac{(A\,b^2+B\,a\,b)\,\left(b\,x^3+a\right)^{1/3}}{7\,a^2\,x} - \frac{(7\,B\,a^2+A\,b\,a)\,\left(b\,x^3+a\right)^{1/3}}{28\,a^2\,x^4} - \frac{A\,\left(b\,x^3+a\right)^{1/3}}{7\,x^7} - \frac{b\,\left(b\,x^3+a\right)^{1/3}\,(A\,b+11\,B\,a)}{28\,a^2\,x}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/3))/x^8,x)
```

output
```
((A*b^2 + B*a*b)*(a + b*x^3)^(1/3))/(7*a^2*x) - ((7*B*a^2 + A*a*b)*(a + b*
x^3)^(1/3))/(28*a^2*x^4) - (A*(a + b*x^3)^(1/3))/(7*x^7) - (b*(a + b*x^3)^
(1/3)*(A*b + 11*B*a))/(28*a^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^8}\,dx = \frac{(b\,x^3+a)^{\frac{1}{3}}\,(-b^2x^6-2ab\,x^3-a^2)}{7a\,x^7}$$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^8,x)
```

output
```
((a + b*x**3)**(1/3)*( - a**2 - 2*a*b*x**3 - b**2*x**6))/(7*a*x**7)
```

3.300 $\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{11}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = -\frac{A(a+bx^3)^{4/3}}{10ax^{10}}+\frac{(3Ab-5aB)\,(a+bx^3)^{4/3}}{35a^2x^7} - \frac{3b(3Ab-5aB)\,(a+bx^3)^{4/3}}{140a^3x^4}$$

output
```
-1/10*A*(b*x^3+a)^(4/3)/a/x^10+1/35*(3*A*b-5*B*a)*(b*x^3+a)^(4/3)/a^2/x^7-
3/140*b*(3*A*b-5*B*a)*(b*x^3+a)^(4/3)/a^3/x^4
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \frac{(a+bx^3)^{4/3}\,(-14a^2A+12aAbx^3-20a^2Bx^3-9Ab^2x^6+15abBx^6)}{140a^3x^{10}}$$

input
```
Integrate[((a + b*x^3)^(1/3)*(A + B*x^3))/x^11,x]
```

output
```
((a + b*x^3)^(4/3)*(-14*a^2*A + 12*a*A*b*x^3 - 20*a^2*B*x^3 - 9*A*b^2*x^6
+ 15*a*b*B*x^6))/(140*a^3*x^10)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{11}}dx$$

$$\Big\downarrow \ 955$$

$$-\frac{(3Ab-5aB)\int \frac{\sqrt[3]{bx^3+a}}{x^8}dx}{5a}-\frac{A(a+bx^3)^{4/3}}{10ax^{10}}$$

$$\Big\downarrow \ 803$$

$$-\frac{(3Ab-5aB)\left(-\frac{3b\int \frac{\sqrt[3]{bx^3+a}}{x^5}dx}{7a}-\frac{(a+bx^3)^{4/3}}{7ax^7}\right)}{5a}-\frac{A(a+bx^3)^{4/3}}{10ax^{10}}$$

$$\Big\downarrow \ 796$$

$$-\frac{\left(\frac{3b(a+bx^3)^{4/3}}{28a^2x^4}-\frac{(a+bx^3)^{4/3}}{7ax^7}\right)(3Ab-5aB)}{5a}-\frac{A(a+bx^3)^{4/3}}{10ax^{10}}$$

input
```
Int[((a + b*x^3)^(1/3)*(A + B*x^3))/x^11,x]
```

output
```
-1/10*(A*(a + b*x^3)^(4/3))/(a*x^10) - ((3*A*b - 5*a*B)*(-1/7*(a + b*x^3)^
(4/3)/(a*x^7) + (3*b*(a + b*x^3)^(4/3))/(28*a^2*x^4)))/(5*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$-\frac{\left(\left(\frac{10 B\,x^{3}}{7}+A\right) a^{2}-\frac{6 \left(\frac{5 B\,x^{3}}{4}+A\right) b\,x^{3} a}{7}+\frac{9 A\,b^{2} x^{6}}{14}\right) \left(b\,x^{3}+a\right)^{\frac{4}{3}}}{10 a^{3} x^{10}}$	55
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(9 A\,b^{2} x^{6}-15 B a b\,x^{6}-12 a A b\,x^{3}+20 B\,a^{2} x^{3}+14 a^{2} A\right)}{140 a^{3} x^{10}}$	59
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(9 A\,b^{2} x^{6}-15 B a b\,x^{6}-12 a A b\,x^{3}+20 B\,a^{2} x^{3}+14 a^{2} A\right)}{140 a^{3} x^{10}}$	59
trager	$-\frac{\left(9 A\,x^{9} b^{3}-15 B\,x^{9} a\,b^{2}-3 A\,x^{6} a\,b^{2}+5 B\,x^{6} a^{2} b+2 a^{2} A b\,x^{3}+20 B\,x^{3} a^{3}+14 a^{3} A\right) \left(b\,x^{3}+a\right)^{\frac{1}{3}}}{140 a^{3} x^{10}}$	83
risch	$-\frac{\left(9 A\,x^{9} b^{3}-15 B\,x^{9} a\,b^{2}-3 A\,x^{6} a\,b^{2}+5 B\,x^{6} a^{2} b+2 a^{2} A b\,x^{3}+20 B\,x^{3} a^{3}+14 a^{3} A\right) \left(b\,x^{3}+a\right)^{\frac{1}{3}}}{140 a^{3} x^{10}}$	83

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*((10/7*B*x^3+A)*a^2-6/7*(5/4*B*x^3+A)*b*x^3*a+9/14*A*b^2*x^6)*(b*x^3
+a)^(4/3)/a^3/x^10
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{11}}\,dx$$
$$=\frac{(3\,(5\,Bab^2-3\,Ab^3)x^9-(5\,Ba^2b-3\,Aab^2)x^6-14\,Aa^3-2\,(10\,Ba^3+Aa^2b)x^3)(bx^3+a)^{\frac{1}{3}}}{140\,a^3x^{10}}$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^11,x, algorithm="fricas")
```

output
```
1/140*(3*(5*B*a*b^2 - 3*A*b^3)*x^9 - (5*B*a^2*b - 3*A*a*b^2)*x^6 - 14*A*a^
3 - 2*(10*B*a^3 + A*a^2*b)*x^3)*(b*x^3 + a)^(1/3)/(a^3*x^10)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. 2(78) = 156.

Time = 1.96 (sec) , antiderivative size = 646, normalized size of antiderivative = 7.69

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**3+a)**(1/3)*(B*x**3+A)/x**11,x)
```

output
```
28*A*a**5*b**(13/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(27*a**5*b**4*x**
9*gamma(-1/3) + 54*a**4*b**5*x**12*gamma(-1/3) + 27*a**3*b**6*x**15*gamma(
-1/3)) + 60*A*a**4*b**(16/3)*x**3*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(27
*a**5*b**4*x**9*gamma(-1/3) + 54*a**4*b**5*x**12*gamma(-1/3) + 27*a**3*b**
6*x**15*gamma(-1/3)) + 30*A*a**3*b**(19/3)*x**6*(a/(b*x**3) + 1)**(1/3)*ga
mma(-10/3)/(27*a**5*b**4*x**9*gamma(-1/3) + 54*a**4*b**5*x**12*gamma(-1/3)
 + 27*a**3*b**6*x**15*gamma(-1/3)) + 10*A*a**2*b**(22/3)*x**9*(a/(b*x**3)
+ 1)**(1/3)*gamma(-10/3)/(27*a**5*b**4*x**9*gamma(-1/3) + 54*a**4*b**5*x**
12*gamma(-1/3) + 27*a**3*b**6*x**15*gamma(-1/3)) + 30*A*a*b**(25/3)*x**12*
(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(27*a**5*b**4*x**9*gamma(-1/3) + 54*a
**4*b**5*x**12*gamma(-1/3) + 27*a**3*b**6*x**15*gamma(-1/3)) + 18*A*b**(28
/3)*x**15*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(27*a**5*b**4*x**9*gamma(-1
/3) + 54*a**4*b**5*x**12*gamma(-1/3) + 27*a**3*b**6*x**15*gamma(-1/3)) - 4
*B*b**(1/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(9*x**6*gamma(-1/3)) - B*b
**(4/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(9*a*x**3*gamma(-1/3)) + B*b**
(7/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(3*a**2*gamma(-1/3))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \frac{B\left(\frac{7\,(bx^3+a)^{\frac{4}{3}}b}{x^4} - \frac{4\,(bx^3+a)^{\frac{7}{3}}}{x^7}\right)}{28\,a^2} - \frac{A\left(\frac{35\,(bx^3+a)^{\frac{4}{3}}b^2}{x^4} - \frac{40\,(bx^3+a)^{\frac{7}{3}}b}{x^7} + \frac{14\,(bx^3+a)^{\frac{10}{3}}}{x^{10}}\right)}{140\,a^3}$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^11,x, algorithm="maxima")
```

output
```
1/28*B*(7*(b*x^3 + a)^(4/3)*b/x^4 - 4*(b*x^3 + a)^(7/3)/x^7)/a^2 - 1/140*A
*(35*(b*x^3 + a)^(4/3)*b^2/x^4 - 40*(b*x^3 + a)^(7/3)*b/x^7 + 14*(b*x^3 +
a)^(10/3)/x^10)/a^3
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{1}{3}}}{x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^11,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3)/x^11, x)
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.57

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \frac{3\,A\,b^2\,(b\,x^3+a)^{1/3}}{140\,a^2\,x^4} - \frac{B\,(b\,x^3+a)^{1/3}}{7\,x^7} - \frac{A\,b\,(b\,x^3+a)^{1/3}}{70\,a\,x^7} - \frac{B\,b\,(b\,x^3+a)^{1/3}}{28\,a\,x^4} - \frac{9\,A\,b^3\,(b\,x^3+a)^{1/3}}{140\,a^3\,x} - \frac{A\,(b\,x^3+a)^{1/3}}{10\,x^{10}} + \frac{3\,B\,b^2\,(b\,x^3+a)^{1/3}}{28\,a^2\,x}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/3))/x^11,x)
```

output
```
(3*A*b^2*(a + b*x^3)^(1/3))/(140*a^2*x^4) - (B*(a + b*x^3)^(1/3))/(7*x^7)
- (A*b*(a + b*x^3)^(1/3))/(70*a*x^7) - (B*b*(a + b*x^3)^(1/3))/(28*a*x^4)
- (9*A*b^3*(a + b*x^3)^(1/3))/(140*a^3*x) - (A*(a + b*x^3)^(1/3))/(10*x^10
) + (3*B*b^2*(a + b*x^3)^(1/3))/(28*a^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \frac{(b\,x^3+a)^{\frac{1}{3}}\,(3b^3x^9-a\,b^2x^6-11a^2b\,x^3-7a^3)}{70a^2x^{10}}$$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^11,x)
```

output
```
((a + b*x**3)**(1/3)*( - 7*a**3 - 11*a**2*b*x**3 - a*b**2*x**6 + 3*b**3*x*
*9))/(70*a**2*x**10)
```

3.301 $\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{14}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{14}}\,dx = -\frac{A(a+bx^3)^{4/3}}{13ax^{13}} + \frac{(9Ab-13aB)(a+bx^3)^{4/3}}{130a^2x^{10}} - \frac{3b(9Ab-13aB)(a+bx^3)^{4/3}}{455a^3x^7} + \frac{9b^2(9Ab-13aB)(a+bx^3)^{4/3}}{1820a^4x^4}$$

output
```
-1/13*A*(b*x^3+a)^(4/3)/a/x^13+1/130*(9*A*b-13*B*a)*(b*x^3+a)^(4/3)/a^2/x^
10-3/455*b*(9*A*b-13*B*a)*(b*x^3+a)^(4/3)/a^3/x^7+9/1820*b^2*(9*A*b-13*B*a
)*(b*x^3+a)^(4/3)/a^4/x^4
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{14}}\,dx = \frac{(a+bx^3)^{4/3}\left(-140a^3A+126a^2Abx^3-182a^3Bx^3-108aAb^2x^6+156a^2bBx^6+81Ab^3x^9-117ab^2Bx^9\right)}{1820a^4x^{13}}$$

input
```
Integrate[((a + b*x^3)^(1/3)*(A + B*x^3))/x^14,x]
```

output
```
((a + b*x^3)^(4/3)*(-140*a^3*A + 126*a^2*A*b*x^3 - 182*a^3*B*x^3 - 108*a*A
*b^2*x^6 + 156*a^2*b*B*x^6 + 81*A*b^3*x^9 - 117*a*b^2*B*x^9))/(1820*a^4*x^
13)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}\left(A+Bx^3\right)}{x^{14}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(9Ab-13aB)\int \frac{\sqrt[3]{bx^3+a}}{x^{11}}dx}{13a}-\frac{A\left(a+bx^3\right)^{4/3}}{13ax^{13}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(9Ab-13aB)\left(-\frac{3b\int \frac{\sqrt[3]{bx^3+a}}{x^8}dx}{5a}-\frac{\left(a+bx^3\right)^{4/3}}{10ax^{10}}\right)}{13a}-\frac{A\left(a+bx^3\right)^{4/3}}{13ax^{13}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(9Ab-13aB)\left(-\frac{3b\left(-\frac{3b\int \frac{\sqrt[3]{bx^3+a}}{x^5}dx}{7a}-\frac{\left(a+bx^3\right)^{4/3}}{7ax^7}\right)}{5a}-\frac{\left(a+bx^3\right)^{4/3}}{10ax^{10}}\right)}{13a}-\frac{A\left(a+bx^3\right)^{4/3}}{13ax^{13}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{\left(-\frac{3b\left(\frac{3b\left(a+bx^3\right)^{4/3}}{28a^2x^4}-\frac{\left(a+bx^3\right)^{4/3}}{7ax^7}\right)}{5a}-\frac{\left(a+bx^3\right)^{4/3}}{10ax^{10}}\right)(9Ab-13aB)}{13a}-\frac{A\left(a+bx^3\right)^{4/3}}{13ax^{13}}$$

input
```
Int[((a + b*x^3)^(1/3)*(A + B*x^3))/x^14,x]
```

output
```
-1/13*(A*(a + b*x^3)^(4/3))/(a*x^13) - ((9*A*b - 13*a*B)*(-1/10*(a + b*x^3
)^(4/3)/(a*x^10) - (3*b*(-1/7*(a + b*x^3)^(4/3)/(a*x^7) + (3*b*(a + b*x^3)
^(4/3))/(28*a^2*x^4)))/(5*a)))/(13*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$-\frac{\left(\left(\frac{13 B x^3}{10}+A\right) a^3-\frac{9 b \left(\frac{26 B x^3}{21}+A\right) x^3 a^2}{10}+\frac{27 \left(\frac{13 B x^3}{12}+A\right) b^2 x^6 a}{35}-\frac{81 A x^9 b^3}{140}\right) \left(b x^3+a\right)^{\frac{4}{3}}}{13 x^{13} a^4}$
gosper	$-\frac{\left(b x^3+a\right)^{\frac{4}{3}} \left(-81 A x^9 b^3+117 B x^9 a b^2+108 A x^6 a b^2-156 B x^6 a^2 b-126 a^2 A b x^3+182 B x^3 a^3+140 a^3 A\right)}{1820 x^{13} a^4}$
orering	$-\frac{\left(b x^3+a\right)^{\frac{4}{3}} \left(-81 A x^9 b^3+117 B x^9 a b^2+108 A x^6 a b^2-156 B x^6 a^2 b-126 a^2 A b x^3+182 B x^3 a^3+140 a^3 A\right)}{1820 x^{13} a^4}$
trager	$-\frac{\left(-81 A b^4 x^{12}+117 B a b^3 x^{12}+27 A a b^3 x^9-39 B a^2 b^2 x^9-18 A a^2 b^2 x^6+26 B a^3 b x^6+14 A a^3 b x^3+182 B a^4 x^3+140 A a^4\right) (b}{1820 x^{13} a^4}$
risch	$-\frac{\left(-81 A b^4 x^{12}+117 B a b^3 x^{12}+27 A a b^3 x^9-39 B a^2 b^2 x^9-18 A a^2 b^2 x^6+26 B a^3 b x^6+14 A a^3 b x^3+182 B a^4 x^3+140 A a^4\right) (b}{1820 x^{13} a^4}$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/13*((13/10*B*x^3+A)*a^3-9/10*b*(26/21*B*x^3+A)*x^3*a^2+27/35*(13/12*B*x
^3+A)*b^2*x^6*a-81/140*A*x^9*b^3)*(b*x^3+a)^(4/3)/x^13/a^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{14}}\,dx =$$
$$-\frac{(9\,(13\,Bab^3-9\,Ab^4)x^{12}-3\,(13\,Ba^2b^2-9\,Aab^3)x^9+2\,(13\,Ba^3b-9\,Aa^2b^2)x^6+140\,Aa^4+14\,(13\,Ba}{1820\,a^4x^{13}}$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^14,x, algorithm="fricas")
```

output
```
-1/1820*(9*(13*B*a*b^3 - 9*A*b^4)*x^12 - 3*(13*B*a^2*b^2 - 9*A*a*b^3)*x^9
+ 2*(13*B*a^3*b - 9*A*a^2*b^2)*x^6 + 140*A*a^4 + 14*(13*B*a^4 + A*a^3*b)*x
^3)*(b*x^3 + a)^(1/3)/(a^4*x^13)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1392 vs. $2(112) = 224$.

Time = 2.38 (sec) , antiderivative size = 1392, normalized size of antiderivative = 11.90

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{14}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**3+a)**(1/3)*(B*x**3+A)/x**14,x)
```

output
```
-280*A*a**7*b**(28/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-13/3)/(81*a**7*b**9*x
**12*gamma(-1/3) + 243*a**6*b**10*x**15*gamma(-1/3) + 243*a**5*b**11*x**18
*gamma(-1/3) + 81*a**4*b**12*x**21*gamma(-1/3)) - 868*A*a**6*b**(31/3)*x**
3*(a/(b*x**3) + 1)**(1/3)*gamma(-13/3)/(81*a**7*b**9*x**12*gamma(-1/3) + 2
43*a**6*b**10*x**15*gamma(-1/3) + 243*a**5*b**11*x**18*gamma(-1/3) + 81*a*
*4*b**12*x**21*gamma(-1/3)) - 888*A*a**5*b**(34/3)*x**6*(a/(b*x**3) + 1)**
(1/3)*gamma(-13/3)/(81*a**7*b**9*x**12*gamma(-1/3) + 243*a**6*b**10*x**15*
gamma(-1/3) + 243*a**5*b**11*x**18*gamma(-1/3) + 81*a**4*b**12*x**21*gamma
(-1/3)) - 310*A*a**4*b**(37/3)*x**9*(a/(b*x**3) + 1)**(1/3)*gamma(-13/3)/(
81*a**7*b**9*x**12*gamma(-1/3) + 243*a**6*b**10*x**15*gamma(-1/3) + 243*a*
*5*b**11*x**18*gamma(-1/3) + 81*a**4*b**12*x**21*gamma(-1/3)) + 80*A*a**3*
b**(40/3)*x**12*(a/(b*x**3) + 1)**(1/3)*gamma(-13/3)/(81*a**7*b**9*x**12*g
amma(-1/3) + 243*a**6*b**10*x**15*gamma(-1/3) + 243*a**5*b**11*x**18*gamma
(-1/3) + 81*a**4*b**12*x**21*gamma(-1/3)) + 360*A*a**2*b**(43/3)*x**15*(a/
(b*x**3) + 1)**(1/3)*gamma(-13/3)/(81*a**7*b**9*x**12*gamma(-1/3) + 243*a*
*6*b**10*x**15*gamma(-1/3) + 243*a**5*b**11*x**18*gamma(-1/3) + 81*a**4*b*
*12*x**21*gamma(-1/3)) + 432*A*a*b**(46/3)*x**18*(a/(b*x**3) + 1)**(1/3)*g
amma(-13/3)/(81*a**7*b**9*x**12*gamma(-1/3) + 243*a**6*b**10*x**15*gamma(-
1/3) + 243*a**5*b**11*x**18*gamma(-1/3) + 81*a**4*b**12*x**21*gamma(-1/3))
 + 162*A*b**(49/3)*x**21*(a/(b*x**3) + 1)**(1/3)*gamma(-13/3)/(81*a**7*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{14}}\,dx = -\frac{B\left(\frac{35\,(bx^3+a)^{\frac{4}{3}}b^2}{x^4} - \frac{40\,(bx^3+a)^{\frac{7}{3}}b}{x^7} + \frac{14\,(bx^3+a)^{\frac{10}{3}}}{x^{10}}\right)}{140\,a^3} + \frac{A\left(\frac{455\,(bx^3+a)^{\frac{4}{3}}b^3}{x^4} - \frac{780\,(bx^3+a)^{\frac{7}{3}}b^2}{x^7} + \frac{546\,(bx^3+a)^{\frac{10}{3}}b}{x^{10}} - \frac{140\,(bx^3+a)^{\frac{13}{3}}}{x^{13}}\right)}{1820\,a^4}$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^14,x, algorithm="maxima")
```

output
```
-1/140*B*(35*(b*x^3 + a)^(4/3)*b^2/x^4 - 40*(b*x^3 + a)^(7/3)*b/x^7 + 14*(
b*x^3 + a)^(10/3)/x^10)/a^3 + 1/1820*A*(455*(b*x^3 + a)^(4/3)*b^3/x^4 - 78
0*(b*x^3 + a)^(7/3)*b^2/x^7 + 546*(b*x^3 + a)^(10/3)*b/x^10 - 140*(b*x^3 +
 a)^(13/3)/x^13)/a^4
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{14}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{1}{3}}}{x^{14}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^14,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3)/x^14, x)
```

Mupad [B] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{14}}\,dx = \frac{81\,A\,b^4\,(b\,x^3+a)^{1/3}}{1820\,a^4\,x} - \frac{B\,(b\,x^3+a)^{1/3}}{10\,x^{10}} - \frac{A\,b\,(b\,x^3+a)^{1/3}}{130\,a\,x^{10}} - \frac{B\,b\,(b\,x^3+a)^{1/3}}{70\,a\,x^7} - \frac{A\,(b\,x^3+a)^{1/3}}{13\,x^{13}} - \frac{27\,A\,b^3\,(b\,x^3+a)^{1/3}}{1820\,a^3\,x^4} + \frac{9\,A\,b^2\,(b\,x^3+a)^{1/3}}{910\,a^2\,x^7} - \frac{9\,B\,b^3\,(b\,x^3+a)^{1/3}}{140\,a^3\,x} + \frac{3\,B\,b^2\,(b\,x^3+a)^{1/3}}{140\,a^2\,x^4}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/3))/x^14,x)
```

output
```
(81*A*b^4*(a + b*x^3)^(1/3))/(1820*a^4*x) - (B*(a + b*x^3)^(1/3))/(10*x^10
) - (A*b*(a + b*x^3)^(1/3))/(130*a*x^10) - (B*b*(a + b*x^3)^(1/3))/(70*a*x
^7) - (A*(a + b*x^3)^(1/3))/(13*x^13) - (27*A*b^3*(a + b*x^3)^(1/3))/(1820
*a^3*x^4) + (9*A*b^2*(a + b*x^3)^(1/3))/(910*a^2*x^7) - (9*B*b^3*(a + b*x^
3)^(1/3))/(140*a^3*x) + (3*B*b^2*(a + b*x^3)^(1/3))/(140*a^2*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^{14}}\,dx = \frac{(b\,x^3+a)^{\frac{1}{3}}\,(-9b^4x^{12}+3a\,b^3x^9-2a^2b^2x^6-49a^3b\,x^3-35a^4)}{455a^3x^{13}}$$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^14,x)
```

output
```
((a + b*x**3)**(1/3)*( - 35*a**4 - 49*a**3*b*x**3 - 2*a**2*b**2*x**6 + 3*a
*b**3*x**9 - 9*b**4*x**12))/(455*a**3*x**13)
```

3.302 $\int x^3 \sqrt[3]{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int x^3 \sqrt[3]{a+bx^3}(A+Bx^3)\,dx = \frac{Bx^4(a+bx^3)^{4/3}}{8b} + \frac{(2Ab-aB)x^4\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{8b\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/8*B*x^4*(b*x^3+a)^(4/3)/b+1/8*(2*A*b-B*a)*x^4*(b*x^3+a)^(1/3)*hypergeom(
[-1/3, 4/3],[7/3],-b*x^3/a)/b/(1+b*x^3/a)^(1/3)
```

Mathematica [A] (verified)

Time = 6.76 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int x^3 \sqrt[3]{a+bx^3}(A+Bx^3)\,dx = \frac{\sqrt[3]{a+bx^3}\left(7Ax^4\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)+4Bx^7\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{7}{3},\frac{10}{3},-\frac{bx^3}{a}\right)\right)}{28\sqrt[3]{1+\frac{bx^3}{a}}}$$

input
```
Integrate[x^3*(a + b*x^3)^(1/3)*(A + B*x^3),x]
```

output
```
((a + b*x^3)^(1/3)*(7*A*x^4*Hypergeometric2F1[-1/3, 4/3, 7/3, -((b*x^3)/a)
] + 4*B*x^7*Hypergeometric2F1[-1/3, 7/3, 10/3, -((b*x^3)/a)]))/(28*(1 + (b
*x^3)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt[3]{a+bx^3} \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(2Ab-aB)\int x^3 \sqrt[3]{bx^3+a}dx}{2b}+\frac{Bx^4\left(a+bx^3\right)^{4/3}}{8b}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt[3]{a+bx^3}(2Ab-aB)\int x^3 \sqrt[3]{\frac{bx^3}{a}+1}dx}{2b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{Bx^4\left(a+bx^3\right)^{4/3}}{8b}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^4\sqrt[3]{a+bx^3}(2Ab-aB)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{8b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{Bx^4\left(a+bx^3\right)^{4/3}}{8b}$$

input
```
Int[x^3*(a + b*x^3)^(1/3)*(A + B*x^3),x]
```

output
```
(B*x^4*(a + b*x^3)^(4/3))/(8*b) + ((2*A*b - a*B)*x^4*(a + b*x^3)^(1/3)*Hyp
ergeometric2F1[-1/3, 4/3, 7/3, -((b*x^3)/a)])/(8*b*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^{3} \left(b \,x^{3}+a \right)^{\frac{1}{3}} \left(B \,x^{3}+A \right)d x$$

input
```
int(x^3*(b*x^3+a)^(1/3)*(B*x^3+A),x)
```

output
```
int(x^3*(b*x^3+a)^(1/3)*(B*x^3+A),x)
```

Fricas [F]

$$\int x^3 \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{1}{3}}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*x^6 + A*x^3)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int x^3 \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{A\sqrt[3]{a}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{B\sqrt[3]{a}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**3*(b*x**3+a)**(1/3)*(B*x**3+A),x)
```

output
```
A*a**(1/3)*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(7/3)) + B*a**(1/3)*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/3
,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Maxima [F]

$$\int x^3\sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{1}{3}}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3)*x^3, x)
```

Giac [F]

$$\int x^3\sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{1}{3}}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx=\int x^3\,\left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{1/3}\,dx$$

input
```
int(x^3*(A + B*x^3)*(a + b*x^3)^(1/3),x)
```

output
```
int(x^3*(A + B*x^3)*(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int x^3\sqrt[3]{a+bx^3}\,(A+Bx^3)\ dx$$
$$=\frac{2(b\,x^3+a)^{\frac{1}{3}}\,a^2x+9(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^4+5(b\,x^3+a)^{\frac{1}{3}}\,b^2x^7-2\left(\int\frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a^3}{40b}$$

input
```
int(x^3*(b*x^3+a)^(1/3)*(B*x^3+A),x)
```

output
```
(2*(a + b*x**3)**(1/3)*a**2*x + 9*(a + b*x**3)**(1/3)*a*b*x**4 + 5*(a + b*
x**3)**(1/3)*b**2*x**7 - 2*int((a + b*x**3)**(1/3)/(a + b*x**3),x)*a**3)/(
40*b)
```

3.303 $\int \sqrt[3]{a+bx^3}\,(A+Bx^3)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \sqrt[3]{a+bx^3}\,(A+Bx^3)\,dx = \frac{Bx(a+bx^3)^{4/3}}{5b} + \frac{(5Ab-aB)x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/5*B*x*(b*x^3+a)^(4/3)/b+1/5*(5*A*b-B*a)*x*(b*x^3+a)^(1/3)*hypergeom([-1/
3, 1/3],[4/3],-b*x^3/a)/b/(1+b*x^3/a)^(1/3)
```

Mathematica [A] (verified)

Time = 7.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \sqrt[3]{a+bx^3}\,(A+Bx^3)\,dx = \frac{x\sqrt[3]{a+bx^3}\left(B(a+bx^3)+\frac{(5Ab-aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}\right)}{5b}$$

input
```
Integrate[(a + b*x^3)^(1/3)*(A + B*x^3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*(B*(a + b*x^3) + ((5*A*b - a*B)*Hypergeometric2F1[-1/
3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(5*b)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{ 913}$$

$$\frac{(5Ab-aB)\int \sqrt[3]{bx^3+a}dx}{5b}+\frac{Bx\left(a+bx^3\right)^{4/3}}{5b}$$

$$\Big\downarrow \text{ 779}$$

$$\frac{\sqrt[3]{a+bx^3}(5Ab-aB)\int \sqrt[3]{\frac{bx^3}{a}+1}dx}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{Bx\left(a+bx^3\right)^{4/3}}{5b}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{x\sqrt[3]{a+bx^3}(5Ab-aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{Bx\left(a+bx^3\right)^{4/3}}{5b}$$

input
```
Int[(a + b*x^3)^(1/3)*(A + B*x^3),x]
```

output
```
(B*x*(a + b*x^3)^(4/3))/(5*b) + ((5*A*b - a*B)*x*(a + b*x^3)^(1/3)*Hyperge
ometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(5*b*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(B\,x^{3}+A\right)dx$$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A),x)
```

output
```
int((b*x^3+a)^(1/3)*(B*x^3+A),x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx^3}(A+Bx^3)\ dx=\int (Bx^3+A)(bx^3+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{A\sqrt[3]{a}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{B\sqrt[3]{a}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)*(B*x**3+A),x)
```

output
```
A*a**(1/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + B*a**(1/3)*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Maxima [F]

$$\int \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{1/3}\,dx$$

input
```
int((A + B*x^3)*(a + b*x^3)^(1/3),x)
```

output
```
int((A + B*x^3)*(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{3(b\,x^3+a)^{\frac{1}{3}}\,ax}{5}+\frac{(b\,x^3+a)^{\frac{1}{3}}\,b\,x^4}{5}+\frac{2\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a^2}{5}$$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A),x)
```

output
```
(3*(a + b*x**3)**(1/3)*a*x + (a + b*x**3)**(1/3)*b*x**4 + 2*int((a + b*x**
3)**(1/3)/(a + b*x**3),x)*a**2)/5
```

3.304 $\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^3} \, dx = -\frac{A(a+bx^3)^{4/3}}{2ax^2}+\frac{(Ab+aB)x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{a\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-1/2*A*(b*x^3+a)^(4/3)/a/x^2+(A*b+B*a)*x*(b*x^3+a)^(1/3)*hypergeom([-1/3,
1/3],[4/3],-b*x^3/a)/a/(1+b*x^3/a)^(1/3)
```

Mathematica [A] (verified)

Time = 7.36 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^3} \, dx = \frac{\sqrt[3]{a+bx^3}\left(-A(a+bx^3)+\frac{2(Ab+aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}\right)}{2ax^2}$$

input
```
Integrate[((a + b*x^3)^(1/3)*(A + B*x^3))/x^3,x]
```

output
```
((a + b*x^3)^(1/3)*(-(A*(a + b*x^3)) + (2*(A*b + a*B)*x^3*Hypergeometric2F
1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(2*a*x^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^3}\,dx$$

$$\Big\downarrow \ 955$$

$$\frac{(aB+Ab)\int \sqrt[3]{bx^3+a}dx}{a}-\frac{A(a+bx^3)^{4/3}}{2ax^2}$$

$$\Big\downarrow \ 779$$

$$\frac{\sqrt[3]{a+bx^3}(aB+Ab)\int \sqrt[3]{\frac{bx^3}{a}+1}dx}{a\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{A(a+bx^3)^{4/3}}{2ax^2}$$

$$\Big\downarrow \ 778$$

$$\frac{x\sqrt[3]{a+bx^3}(aB+Ab)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{a\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{A(a+bx^3)^{4/3}}{2ax^2}$$

input
```
Int[((a + b*x^3)^(1/3)*(A + B*x^3))/x^3,x]
```

output
```
-1/2*(A*(a + b*x^3)^(4/3))/(a*x^2) + ((A*b + a*B)*x*(a + b*x^3)^(1/3)*Hype
rgeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(a*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}\,(B\,x^{3}+A)}{x^{3}}dx$$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^3,x)
```

output
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^3,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^3}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{1}{3}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^3,x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(1/3)/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^3}\,dx = \frac{A\sqrt[3]{a}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{B\sqrt[3]{a}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)*(B*x**3+A)/x**3,x)
```

output
```
A*a**(1/3)*gamma(-2/3)*hyper((-2/3, -1/3), (1/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**2*gamma(1/3)) + B*a**(1/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^3}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{1}{3}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^3,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^3}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{1}{3}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^3,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^3}\,dx=\int \frac{(B\,x^3+A)\,\left(b\,x^3+a\right)^{1/3}}{x^3}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/3))/x^3,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/3))/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^3}\,dx = \frac{-3(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3-4\left(\int\frac{(b\,x^3+a)^{\frac{1}{3}}}{b\,x^6+a\,x^3}dx\right)a^2x^2}{2x^2}$$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^3,x)
```

output
```
( - 3*(a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3 - 4*int((a + b*x*
*3)**(1/3)/(a*x**3 + b*x**6),x)*a**2*x**2)/(2*x**2)
```

3.305 $\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^6}\,dx = -\frac{A(a+bx^3)^{4/3}}{5ax^5}+\frac{(Ab-5aB)\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{10ax^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-1/5*A*(b*x^3+a)^(4/3)/a/x^5+1/10*(A*b-5*B*a)*(b*x^3+a)^(1/3)*hypergeom([-
2/3, -1/3],[1/3],-b*x^3/a)/a/x^2/(1+b*x^3/a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^6}\,dx = \frac{\sqrt[3]{a+bx^3}\left(-2A(a+bx^3)+\frac{(Ab-5aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}\right)}{10ax^5}$$

input
```
Integrate[((a + b*x^3)^(1/3)*(A + B*x^3))/x^6,x]
```

output
```
((a + b*x^3)^(1/3)*(-2*A*(a + b*x^3) + ((A*b - 5*a*B)*x^3*Hypergeometric2F
1[-2/3, -1/3, 1/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(10*a*x^5)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}\left(A+Bx^3\right)}{x^6}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(Ab-5aB)\int \frac{\sqrt[3]{bx^3+a}}{x^3}dx}{5a}-\frac{A\left(a+bx^3\right)^{4/3}}{5ax^5}$$

$$\Big\downarrow \text{889}$$

$$-\frac{\sqrt[3]{a+bx^3}(Ab-5aB)\int \frac{\sqrt[3]{\frac{bx^3}{a}+1}}{x^3}dx}{5a\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{A\left(a+bx^3\right)^{4/3}}{5ax^5}$$

$$\Big\downarrow \text{888}$$

$$\frac{\sqrt[3]{a+bx^3}(Ab-5aB)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{10ax^2\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{A\left(a+bx^3\right)^{4/3}}{5ax^5}$$

input
```
Int[((a + b*x^3)^(1/3)*(A + B*x^3))/x^6,x]
```

output
```
-1/5*(A*(a + b*x^3)^(4/3))/(a*x^5) + ((A*b - 5*a*B)*(a + b*x^3)^(1/3)*Hype
rgeometric2F1[-2/3, -1/3, 1/3, -((b*x^3)/a)])/(10*a*x^2*(1 + (b*x^3)/a)^(1
/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}\,(B\,x^3+A)}{x^6}dx$$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^6,x)
```

output
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^6,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^6}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{1}{3}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^6,x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(1/3)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^6}\,dx = \frac{A\sqrt[3]{b}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{B\sqrt[3]{a}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)*(B*x**3+A)/x**6,x)
```

output
```
A*b**(1/3)*gamma(-4/3)*hyper((-1/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**
3))/(3*x**4*gamma(-1/3)) + B*a**(1/3)*gamma(-2/3)*hyper((-2/3, -1/3), (1/3
,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^6}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{1}{3}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^6,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^6}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{1}{3}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^6,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^6}\,dx = \int \frac{(B\,x^3+A)\,\left(b\,x^3+a\right)^{1/3}}{x^6}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/3))/x^6,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/3))/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^6}\,dx = \frac{-(b\,x^3+a)^{\frac{1}{3}}\,a-5(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3-4\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{b\,x^6+a\,x^3}dx\right)ab\,x^5}{5x^5}$$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^6,x)
```

output
```
( - (a + b*x**3)**(1/3)*a - 5*(a + b*x**3)**(1/3)*b*x**3 - 4*int((a + b*x*
*3)**(1/3)/(a*x**3 + b*x**6),x)*a*b*x**5)/(5*x**5)
```

3.306 $\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^9}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^9}\,dx = -\frac{A(a+bx^3)^{4/3}}{8ax^8}+\frac{(Ab-2aB)\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{1}{3},-\frac{2}{3},-\frac{bx^3}{a}\right)}{10ax^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-1/8*A*(b*x^3+a)^(4/3)/a/x^8+1/10*(A*b-2*B*a)*(b*x^3+a)^(1/3)*hypergeom([-
5/3, -1/3],[-2/3],-b*x^3/a)/a/x^5/(1+b*x^3/a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^9}\,dx = \frac{\sqrt[3]{a+bx^3}\left(-5A(a+bx^3)+\frac{4(Ab-2aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{1}{3},-\frac{2}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}\right)}{40ax^8}$$

input
```
Integrate[((a + b*x^3)^(1/3)*(A + B*x^3))/x^9,x]
```

output
```
((a + b*x^3)^(1/3)*(-5*A*(a + b*x^3) + (4*(A*b - 2*a*B)*x^3*Hypergeometric
2F1[-5/3, -1/3, -2/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(40*a*x^8)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^9}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(Ab-2aB)\int \frac{\sqrt[3]{bx^3+a}}{x^6}dx}{2a}-\frac{A(a+bx^3)^{4/3}}{8ax^8}$$

$$\Big\downarrow \text{889}$$

$$-\frac{\sqrt[3]{a+bx^3}(Ab-2aB)\int \frac{\sqrt[3]{\frac{bx^3}{a}+1}}{x^6}dx}{2a\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{A(a+bx^3)^{4/3}}{8ax^8}$$

$$\Big\downarrow \text{888}$$

$$\frac{\sqrt[3]{a+bx^3}(Ab-2aB)\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{1}{3},-\frac{2}{3},-\frac{bx^3}{a}\right)}{10ax^5\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{A(a+bx^3)^{4/3}}{8ax^8}$$

input
```
Int[((a + b*x^3)^(1/3)*(A + B*x^3))/x^9,x]
```

output
```
-1/8*(A*(a + b*x^3)^(4/3))/(a*x^8) + ((A*b - 2*a*B)*(a + b*x^3)^(1/3)*Hype
rgeometric2F1[-5/3, -1/3, -2/3, -((b*x^3)/a)])/(10*a*x^5*(1 + (b*x^3)/a)^(
1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}\,(B\,x^3+A)}{x^9}dx$$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^9,x)
```

output
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^9,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^9}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{1}{3}}}{x^9}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^9,x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(1/3)/x^9, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^9}\,dx=\frac{A\sqrt[3]{a}\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{3}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)}+\frac{B\sqrt[3]{b}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)*(B*x**3+A)/x**9,x)
```

output
```
A*a**(1/3)*gamma(-8/3)*hyper((-8/3, -1/3), (-5/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**8*gamma(-5/3)) + B*b**(1/3)*gamma(-4/3)*hyper((-1/3, 4/3), (7/3,
), a*exp_polar(I*pi)/(b*x**3))/(3*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^9}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{1}{3}}}{x^9}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^9,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3)/x^9, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^9}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{1}{3}}}{x^9}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(B*x^3+A)/x^9,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3)/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^9}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{1/3}}{x^9}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/3))/x^9,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/3))/x^9, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}(A+Bx^3)}{x^9}\,dx$$
$$= \frac{-3(b\,x^3+a)^{\frac{1}{3}}\,a-7(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3+4\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{b\,x^{12}+a\,x^9}dx\right)a^2x^8}{28x^8}$$

input
```
int((b*x^3+a)^(1/3)*(B*x^3+A)/x^9,x)
```

output
```
( - 3*(a + b*x**3)**(1/3)*a - 7*(a + b*x**3)**(1/3)*b*x**3 + 4*int((a + b*
x**3)**(1/3)/(a*x**9 + b*x**12),x)*a**2*x**8)/(28*x**8)
```

3.307 $\int x^8(a+bx^3)^{2/3}\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int x^8\left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)\,dx=\frac{a^2(Ab-aB)\left(a+bx^3\right)^{5/3}}{5b^4}-\frac{a(2Ab-3aB)\left(a+bx^3\right)^{8/3}}{8b^4}+\frac{(Ab-3aB)\left(a+bx^3\right)^{11/3}}{11b^4}+\frac{B\left(a+bx^3\right)^{14/3}}{14b^4}$$

output
```
1/5*a^2*(A*b-B*a)*(b*x^3+a)^(5/3)/b^4-1/8*a*(2*A*b-3*B*a)*(b*x^3+a)^(8/3)/
b^4+1/11*(A*b-3*B*a)*(b*x^3+a)^(11/3)/b^4+1/14*B*(b*x^3+a)^(14/3)/b^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int x^8\left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)\,dx=\frac{\left(a+bx^3\right)^{5/3}\left(126a^2Ab-81a^3B-210aAb^2x^3+135a^2bBx^3+280Ab^3x^6-180ab^2Bx^6+220b^3\right.}{3080b^4}$$

input
```
Integrate[x^8*(a + b*x^3)^(2/3)*(A + B*x^3),x]
```

output
```
((a + b*x^3)^(5/3)*(126*a^2*A*b - 81*a^3*B - 210*a*A*b^2*x^3 + 135*a^2*b*B
*x^3 + 280*A*b^3*x^6 - 180*a*b^2*B*x^6 + 220*b^3*B*x^9))/(3080*b^4)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int x^6 \left(bx^3+a\right)^{2/3} \left(Bx^3+A\right) dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3} \int \left(\frac{B\left(bx^3+a\right)^{11/3}}{b^3}+\frac{(Ab-3aB)\left(bx^3+a\right)^{8/3}}{b^3}+\frac{a(3aB-2Ab)\left(bx^3+a\right)^{5/3}}{b^3}-\frac{a^2(aB-Ab)\left(bx^3+a\right)^{2/3}}{b^3}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3a^2\left(a+bx^3\right)^{5/3}(Ab-aB)}{5b^4}+\frac{3\left(a+bx^3\right)^{11/3}(Ab-3aB)}{11b^4}-\frac{3a\left(a+bx^3\right)^{8/3}(2Ab-3aB)}{8b^4}+\frac{3B\left(a+bx^3\right)^{14/3}}{14b^4}\right.$$

input
```
Int[x^8*(a + b*x^3)^(2/3)*(A + B*x^3),x]
```

output
```
((3*a^2*(A*b - a*B)*(a + b*x^3)^(5/3))/(5*b^4) - (3*a*(2*A*b - 3*a*B)*(a +
 b*x^3)^(8/3))/(8*b^4) + (3*(A*b - 3*a*B)*(a + b*x^3)^(11/3))/(11*b^4) + (
3*B*(a + b*x^3)^(14/3))/(14*b^4))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{9\left(\frac{20\left(\frac{11 B x^3}{14}+A\right) x^6 b^3}{9}-\frac{5 a\left(\frac{6 B x^3}{7}+A\right) x^3 b^2}{3}+a^2\left(\frac{15 B x^3}{14}+A\right) b-\frac{9 a^3 B}{14}\right)\left(b x^3+a\right)^{\frac{5}{3}}}{220 b^4}$
gosper	$\frac{\left(b x^3+a\right)^{\frac{5}{3}}\left(220 b^3 B x^9+280 A b^3 x^6-180 B a b^2 x^6-210 a A b^2 x^3+135 B a^2 b x^3+126 a^2 b A-81 a^3 B\right)}{3080 b^4}$
orering	$\frac{\left(b x^3+a\right)^{\frac{5}{3}}\left(220 b^3 B x^9+280 A b^3 x^6-180 B a b^2 x^6-210 a A b^2 x^3+135 B a^2 b x^3+126 a^2 b A-81 a^3 B\right)}{3080 b^4}$
trager	$\frac{\left(220 B b^4 x^{12}+280 A b^4 x^9+40 B x^9 a b^3+70 A x^6 a b^3-45 B x^6 a^2 b^2-84 A a^2 b^2 x^3+54 B a^3 b x^3+126 A a^3 b-81 B a^4\right)\left(b x^3+a\right)^{\frac{2}{3}}}{3080 b^4}$
risch	$\frac{\left(220 B b^4 x^{12}+280 A b^4 x^9+40 B x^9 a b^3+70 A x^6 a b^3-45 B x^6 a^2 b^2-84 A a^2 b^2 x^3+54 B a^3 b x^3+126 A a^3 b-81 B a^4\right)\left(b x^3+a\right)^{\frac{2}{3}}}{3080 b^4}$

input
```
int(x^8*(b*x^3+a)^(2/3)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
9/220*(20/9*(11/14*B*x^3+A)*x^6*b^3-5/3*a*(6/7*B*x^3+A)*x^3*b^2+a^2*(15/14
*B*x^3+A)*b-9/14*a^3*B)*(b*x^3+a)^(5/3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.96

$$\int x^8 (a+bx^3)^{2/3} (A+Bx^3)\,dx = \frac{(220\,Bb^4x^{12} + 40\,(Bab^3 + 7\,Ab^4)x^9 - 5\,(9\,Ba^2b^2 - 14\,Aab^3)x^6 - 81\,Ba^4 + 126\,Aa^3b + 6\,(9\,Ba$$
$$3080\,b^4$$

input
```
integrate(x^8*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/3080*(220*B*b^4*x^12 + 40*(B*a*b^3 + 7*A*b^4)*x^9 - 5*(9*B*a^2*b^2 - 14*
A*a*b^3)*x^6 - 81*B*a^4 + 126*A*a^3*b + 6*(9*B*a^3*b - 14*A*a^2*b^2)*x^3)*
(b*x^3 + a)^(2/3)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(94) = 188$.

Time = 0.52 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.06

$$\int x^8 (a+bx^3)^{2/3} (A+Bx^3)\,dx = \begin{cases} \frac{9Aa^3(a+bx^3)^{\frac{2}{3}}}{220b^3} - \frac{3Aa^2x^3(a+bx^3)^{\frac{2}{3}}}{110b^2} + \frac{Aax^6(a+bx^3)^{\frac{2}{3}}}{44b} + \frac{Ax^9(a+bx^3)^{\frac{2}{3}}}{11} - \frac{81Ba^4(a+bx^3)^{\frac{2}{3}}}{3080b^4} + \frac{27Ba^3x^3(a+bx^3)^{\frac{2}{3}}}{1540b^3} \\ a^{\frac{2}{3}}\left(\frac{Ax^9}{9} + \frac{Bx^{12}}{12}\right) \end{cases}$$

input
```
integrate(x**8*(b*x**3+a)**(2/3)*(B*x**3+A),x)
```

output
```
Piecewise((9*A*a**3*(a + b*x**3)**(2/3)/(220*b**3) - 3*A*a**2*x**3*(a + b*
x**3)**(2/3)/(110*b**2) + A*a*x**6*(a + b*x**3)**(2/3)/(44*b) + A*x**9*(a
+ b*x**3)**(2/3)/11 - 81*B*a**4*(a + b*x**3)**(2/3)/(3080*b**4) + 27*B*a**
3*x**3*(a + b*x**3)**(2/3)/(1540*b**3) - 9*B*a**2*x**6*(a + b*x**3)**(2/3)
/(616*b**2) + B*a*x**9*(a + b*x**3)**(2/3)/(77*b) + B*x**12*(a + b*x**3)**
(2/3)/14, Ne(b, 0)), (a**(2/3)*(A*x**9/9 + B*x**12/12), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int x^8 \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right)\,dx = \frac{1}{3080}\,B\left(\frac{220\,(bx^3+a)^{\frac{14}{3}}}{b^4} - \frac{840\,(bx^3+a)^{\frac{11}{3}}a}{b^4} + \frac{1155\,(bx^3+a)^{\frac{8}{3}}a^2}{b^4} - \frac{616\,(bx^3+a)^{\frac{5}{3}}a^3}{b^4}\right) + \frac{1}{220}\,A\left(\frac{20\,(bx^3+a)^{\frac{11}{3}}}{b^3} - \frac{55\,(bx^3+a)^{\frac{8}{3}}a}{b^3} + \frac{44\,(bx^3+a)^{\frac{5}{3}}a^2}{b^3}\right)$$

input
```
integrate(x^8*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/3080*B*(220*(b*x^3 + a)^(14/3)/b^4 - 840*(b*x^3 + a)^(11/3)*a/b^4 + 1155
*(b*x^3 + a)^(8/3)*a^2/b^4 - 616*(b*x^3 + a)^(5/3)*a^3/b^4) + 1/220*A*(20*
(b*x^3 + a)^(11/3)/b^3 - 55*(b*x^3 + a)^(8/3)*a/b^3 + 44*(b*x^3 + a)^(5/3)
*a^2/b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int x^8 \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right)\,dx = \frac{220\,(bx^3+a)^{\frac{14}{3}}B - 840\,(bx^3+a)^{\frac{11}{3}}Ba + 1155\,(bx^3+a)^{\frac{8}{3}}Ba^2 - 616\,(bx^3+a)^{\frac{5}{3}}Ba^3 + 280\,(bx}{3080\,b^4}$$

input
```
integrate(x^8*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/3080*(220*(b*x^3 + a)^(14/3)*B - 840*(b*x^3 + a)^(11/3)*B*a + 1155*(b*x^
3 + a)^(8/3)*B*a^2 - 616*(b*x^3 + a)^(5/3)*B*a^3 + 280*(b*x^3 + a)^(11/3)*
A*b - 770*(b*x^3 + a)^(8/3)*A*a*b + 616*(b*x^3 + a)^(5/3)*A*a^2*b)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int x^8 \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx = \left(b\,x^3+a\right)^{2/3} \left(\frac{B\,x^{12}}{14} - \frac{81\,B\,a^4-126\,A\,a^3\,b}{3080\,b^4} + \frac{x^9\,\left(280\,A\,b^4+40\,B\,a\,b^3\right)}{3080\,b^4} - \frac{3\,a^2\,x^3\,\left(14\,A\,b-9\,B\right.}{1540\,b^3}\right.$$

input
```
int(x^8*(A + B*x^3)*(a + b*x^3)^(2/3),x)
```

output
```
(a + b*x^3)^(2/3)*((B*x^12)/14 - (81*B*a^4 - 126*A*a^3*b)/(3080*b^4) + (x^
9*(280*A*b^4 + 40*B*a*b^3))/(3080*b^4) - (3*a^2*x^3*(14*A*b - 9*B*a))/(154
0*b^3) + (a*x^6*(14*A*b - 9*B*a))/(616*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.55

$$\int x^8 \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx = \frac{\left(b\,x^3+a\right)^{\frac{2}{3}} \left(44b^4x^{12}+64a\,b^3x^9+5a^2b^2x^6-6a^3b\,x^3+9a^4\right)}{616b^3}$$

input
```
int(x^8*(b*x^3+a)^(2/3)*(B*x^3+A),x)
```

output
```
((a + b*x**3)**(2/3)*(9*a**4 - 6*a**3*b*x**3 + 5*a**2*b**2*x**6 + 64*a*b**
3*x**9 + 44*b**4*x**12))/(616*b**3)
```

3.308 $\int x^5(a+bx^3)^{2/3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int x^5(a+bx^3)^{2/3}(A+Bx^3)\,dx = -\frac{a(Ab-aB)(a+bx^3)^{5/3}}{5b^3} + \frac{(Ab-2aB)(a+bx^3)^{8/3}}{8b^3} + \frac{B(a+bx^3)^{11/3}}{11b^3}$$

output
```
-1/5*a*(A*b-B*a)*(b*x^3+a)^(5/3)/b^3+1/8*(A*b-2*B*a)*(b*x^3+a)^(8/3)/b^3+1
/11*B*(b*x^3+a)^(11/3)/b^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int x^5(a+bx^3)^{2/3}(A+Bx^3)\,dx = \frac{(a+bx^3)^{5/3}(-33aAb+18a^2B+55Ab^2x^3-30abBx^3+40b^2Bx^6)}{440b^3}$$

input
```
Integrate[x^5*(a + b*x^3)^(2/3)*(A + B*x^3),x]
```

output
```
((a + b*x^3)^(5/3)*(-33*a*A*b + 18*a^2*B + 55*A*b^2*x^3 - 30*a*b*B*x^3 + 4
0*b^2*B*x^6))/(440*b^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int x^3 \left(bx^3+a\right)^{2/3} \left(Bx^3+A\right) dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3} \int \left(\frac{B\left(bx^3+a\right)^{8/3}}{b^2}+\frac{(Ab-2aB)\left(bx^3+a\right)^{5/3}}{b^2}+\frac{a(aB-Ab)\left(bx^3+a\right)^{2/3}}{b^2}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{3\left(a+bx^3\right)^{8/3} (Ab-2aB)}{8b^3}-\frac{3a\left(a+bx^3\right)^{5/3} (Ab-aB)}{5b^3}+\frac{3B\left(a+bx^3\right)^{11/3}}{11b^3}\right)$$

input
```
Int[x^5*(a + b*x^3)^(2/3)*(A + B*x^3),x]
```

output
```
((-3*a*(A*b - a*B)*(a + b*x^3)^(5/3))/(5*b^3) + (3*(A*b - 2*a*B)*(a + b*x^
3)^(8/3))/(8*b^3) + (3*B*(a + b*x^3)^(11/3))/(11*b^3))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$-\frac{3\left(b\,x^{3}+a\right)^{\frac{5}{3}}\left(-\frac{5\left(\frac{8B\,x^{3}}{11}+A\right)x^{3}b^{2}}{3}+a\left(\frac{10B\,x^{3}}{11}+A\right)b-\frac{6a^{2}B}{11}\right)}{40b^{3}}$	49
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}}\left(-40b^{2}B\,x^{6}-55A\,b^{2}x^{3}+30Bab\,x^{3}+33abA-18a^{2}B\right)}{440b^{3}}$	53
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}}\left(-40b^{2}B\,x^{6}-55A\,b^{2}x^{3}+30Bab\,x^{3}+33abA-18a^{2}B\right)}{440b^{3}}$	53
trager	$-\frac{\left(-40b^{3}B\,x^{9}-55A\,b^{3}x^{6}-10Ba\,b^{2}x^{6}-22aA\,b^{2}x^{3}+12B\,a^{2}b\,x^{3}+33a^{2}bA-18a^{3}B\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{440b^{3}}$	77
risch	$-\frac{\left(-40b^{3}B\,x^{9}-55A\,b^{3}x^{6}-10Ba\,b^{2}x^{6}-22aA\,b^{2}x^{3}+12B\,a^{2}b\,x^{3}+33a^{2}bA-18a^{3}B\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{440b^{3}}$	77

input
```
int(x^5*(b*x^3+a)^(2/3)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
-3/40*(b*x^3+a)^(5/3)*(-5/3*(8/11*B*x^3+A)*x^3*b^2+a*(10/11*B*x^3+A)*b-6/1
1*a^2*B)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int x^5 \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx = \frac{(40\, Bb^3x^9 + 5\,(2\, Bab^2 + 11\, Ab^3)x^6 + 18\, Ba^3 - 33\, Aa^2b - 2\,(6\, Ba^2b - 11\, Aab^2)x^3)(bx^3 + a)^{\frac{2}{3}}}{440\, b^3}$$

input
```
integrate(x^5*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/440*(40*B*b^3*x^9 + 5*(2*B*a*b^2 + 11*A*b^3)*x^6 + 18*B*a^3 - 33*A*a^2*b
 - 2*(6*B*a^2*b - 11*A*a*b^2)*x^3)*(b*x^3 + a)^(2/3)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(63) = 126$.

Time = 0.56 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.22

$$\int x^5 \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx = \begin{cases} -\frac{3Aa^2(a+bx^3)^{\frac{2}{3}}}{40b^2} + \frac{Aax^3(a+bx^3)^{\frac{2}{3}}}{20b} + \frac{Ax^6(a+bx^3)^{\frac{2}{3}}}{8} + \frac{9Ba^3(a+bx^3)^{\frac{2}{3}}}{220b^3} - \frac{3Ba^2x^3(a+bx^3)^{\frac{2}{3}}}{110b^2} + \frac{Bax^6(a+bx^3)^{\frac{2}{3}}}{44b} \\ a^{\frac{2}{3}}\left(\frac{Ax^6}{6} + \frac{Bx^9}{9}\right) \end{cases}$$

input
```
integrate(x**5*(b*x**3+a)**(2/3)*(B*x**3+A),x)
```

output
```
Piecewise((-3*A*a**2*(a + b*x**3)**(2/3)/(40*b**2) + A*a*x**3*(a + b*x**3)
**(2/3)/(20*b) + A*x**6*(a + b*x**3)**(2/3)/8 + 9*B*a**3*(a + b*x**3)**(2/
3)/(220*b**3) - 3*B*a**2*x**3*(a + b*x**3)**(2/3)/(110*b**2) + B*a*x**6*(a
 + b*x**3)**(2/3)/(44*b) + B*x**9*(a + b*x**3)**(2/3)/11, Ne(b, 0)), (a**(
2/3)*(A*x**6/6 + B*x**9/9), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int x^5 \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx = \frac{1}{220} B \left(\frac{20 \left(bx^3+a\right)^{\frac{11}{3}}}{b^3} - \frac{55 \left(bx^3+a\right)^{\frac{8}{3}} a}{b^3} + \frac{44 \left(bx^3+a\right)^{\frac{5}{3}} a^2}{b^3}\right) + \frac{1}{40} A \left(\frac{5 \left(bx^3+a\right)^{\frac{8}{3}}}{b^2} - \frac{8 \left(bx^3+a\right)^{\frac{5}{3}} a}{b^2}\right)$$

input
```
integrate(x^5*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/220*B*(20*(b*x^3 + a)^(11/3)/b^3 - 55*(b*x^3 + a)^(8/3)*a/b^3 + 44*(b*x^
3 + a)^(5/3)*a^2/b^3) + 1/40*A*(5*(b*x^3 + a)^(8/3)/b^2 - 8*(b*x^3 + a)^(5
/3)*a/b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int x^5 \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx = \frac{40 \left(bx^3+a\right)^{\frac{11}{3}} B - 110 \left(bx^3+a\right)^{\frac{8}{3}} Ba + 88 \left(bx^3+a\right)^{\frac{5}{3}} Ba^2 + 55 \left(bx^3+a\right)^{\frac{8}{3}} Ab - 88 \left(bx^3+a\right)^{\frac{5}{3}}}{440 b^3}$$

input
```
integrate(x^5*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/440*(40*(b*x^3 + a)^(11/3)*B - 110*(b*x^3 + a)^(8/3)*B*a + 88*(b*x^3 + a
)^(5/3)*B*a^2 + 55*(b*x^3 + a)^(8/3)*A*b - 88*(b*x^3 + a)^(5/3)*A*a*b)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int x^5\left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)\,dx=\left(b\,x^3+a\right)^{2/3}\left(\frac{B\,x^9}{11}+\frac{18\,B\,a^3-33\,A\,a^2\,b}{440\,b^3}+\frac{x^6\,\left(55\,A\,b^3+10\,B\,a\,b^2\right)}{440\,b^3}+\frac{a\,x^3\,\left(11\,A\,b-6\,B\,a\right)}{220\,b^2}\right)$$

input
```
int(x^5*(A + B*x^3)*(a + b*x^3)^(2/3),x)
```

output
```
(a + b*x^3)^(2/3)*((B*x^9)/11 + (18*B*a^3 - 33*A*a^2*b)/(440*b^3) + (x^6*(
55*A*b^3 + 10*B*a*b^2))/(440*b^3) + (a*x^3*(11*A*b - 6*B*a))/(220*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

$$\int x^5\left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)\,dx=\frac{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(8b^3x^9+13a\,b^2x^6+2a^2b\,x^3-3a^3\right)}{88b^2}$$

input
```
int(x^5*(b*x^3+a)^(2/3)*(B*x^3+A),x)
```

output
```
((a + b*x**3)**(2/3)*( - 3*a**3 + 2*a**2*b*x**3 + 13*a*b**2*x**6 + 8*b**3*
x**9))/(88*b**2)
```

3.309 $\int x^2(a+bx^3)^{2/3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int x^2(a+bx^3)^{2/3}(A+Bx^3)\,dx=\frac{(Ab-aB)(a+bx^3)^{5/3}}{5b^2}+\frac{B(a+bx^3)^{8/3}}{8b^2}$$

output
```
1/5*(A*b-B*a)*(b*x^3+a)^(5/3)/b^2+1/8*B*(b*x^3+a)^(8/3)/b^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int x^2(a+bx^3)^{2/3}(A+Bx^3)\,dx=\frac{(a+bx^3)^{5/3}(8Ab-3aB+5bBx^3)}{40b^2}$$

input
```
Integrate[x^2*(a + b*x^3)^(2/3)*(A + B*x^3),x]
```

output
```
((a + b*x^3)^(5/3)*(8*A*b - 3*a*B + 5*b*B*x^3))/(40*b^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3} \int \left(bx^3+a\right)^{2/3} \left(Bx^3+A\right) dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3} \int \left(\frac{B\left(bx^3+a\right)^{5/3}}{b}+\frac{(Ab-aB)\left(bx^3+a\right)^{2/3}}{b}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{3\left(a+bx^3\right)^{5/3} (Ab-aB)}{5b^2}+\frac{3B\left(a+bx^3\right)^{8/3}}{8b^2}\right)$$

input
```
Int[x^2*(a + b*x^3)^(2/3)*(A + B*x^3),x]
```

output
```
((3*(A*b - a*B)*(a + b*x^3)^(5/3))/(5*b^2) + (3*B*(a + b*x^3)^(8/3))/(8*b^
2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(5bB\,x^{3}+8Ab-3Ba\right)}{40b^{2}}$	31
orering	$\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(5bB\,x^{3}+8Ab-3Ba\right)}{40b^{2}}$	31
pseudoelliptic	$\frac{\left(\left(5B\,x^{3}+8A\right)b-3Ba\right) \left(b\,x^{3}+a\right)^{\frac{5}{3}}}{40b^{2}}$	32
trager	$\frac{\left(5b^{2}B\,x^{6}+8A\,b^{2}x^{3}+2Bab\,x^{3}+8abA-3a^{2}B\right) \left(b\,x^{3}+a\right)^{\frac{2}{3}}}{40b^{2}}$	53
risch	$\frac{\left(5b^{2}B\,x^{6}+8A\,b^{2}x^{3}+2Bab\,x^{3}+8abA-3a^{2}B\right) \left(b\,x^{3}+a\right)^{\frac{2}{3}}}{40b^{2}}$	53

input
```
int(x^2*(b*x^3+a)^(2/3)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/40*(b*x^3+a)^(5/3)*(5*B*b*x^3+8*A*b-3*B*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int x^2\left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)\,dx=\frac{(5\,Bb^2x^6+2\,(Bab+4\,Ab^2)x^3-3\,Ba^2+8\,Aab)(bx^3+a)^{\frac{2}{3}}}{40\,b^2}$$

input
```
integrate(x^2*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/40*(5*B*b^2*x^6 + 2*(B*a*b + 4*A*b^2)*x^3 - 3*B*a^2 + 8*A*a*b)*(b*x^3 +
a)^(2/3)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(39) = 78$.

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.39

$$\int x^2(a+bx^3)^{2/3}(A+Bx^3)\,dx = \begin{cases} \frac{Aa(a+bx^3)^{\frac{2}{3}}}{5b} + \frac{Ax^3(a+bx^3)^{\frac{2}{3}}}{5} - \frac{3Ba^2(a+bx^3)^{\frac{2}{3}}}{40b^2} + \frac{Bax^3(a+bx^3)^{\frac{2}{3}}}{20b} + \frac{Bx^6(a+bx^3)^{\frac{2}{3}}}{8} & \text{for } b \neq 0 \\ a^{\frac{2}{3}}\left(\frac{Ax^3}{3} + \frac{Bx^6}{6}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(b*x**3+a)**(2/3)*(B*x**3+A),x)
```

output
```
Piecewise((A*a*(a + b*x**3)**(2/3)/(5*b) + A*x**3*(a + b*x**3)**(2/3)/5 -
3*B*a**2*(a + b*x**3)**(2/3)/(40*b**2) + B*a*x**3*(a + b*x**3)**(2/3)/(20*
b) + B*x**6*(a + b*x**3)**(2/3)/8, Ne(b, 0)), (a**(2/3)*(A*x**3/3 + B*x**6
/6), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int x^2(a+bx^3)^{2/3}(A+Bx^3)\,dx = \frac{1}{40}B\left(\frac{5(bx^3+a)^{\frac{8}{3}}}{b^2} - \frac{8(bx^3+a)^{\frac{5}{3}}a}{b^2}\right) + \frac{(bx^3+a)^{\frac{5}{3}}A}{5b}$$

input
```
integrate(x^2*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/40*B*(5*(b*x^3 + a)^(8/3)/b^2 - 8*(b*x^3 + a)^(5/3)*a/b^2) + 1/5*(b*x^3
+ a)^(5/3)*A/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x^2\left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)\,dx=\frac{5\left(bx^3+a\right)^{\frac{8}{3}}B-8\left(bx^3+a\right)^{\frac{5}{3}}Ba+8\left(bx^3+a\right)^{\frac{5}{3}}Ab}{40\,b^2}$$

input
```
integrate(x^2*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/40*(5*(b*x^3 + a)^(8/3)*B - 8*(b*x^3 + a)^(5/3)*B*a + 8*(b*x^3 + a)^(5/3
)*A*b)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int x^2\left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)\,dx=\left(b\,x^3+a\right)^{2/3}\left(\frac{B\,x^6}{8}-\frac{3\,B\,a^2-8\,A\,a\,b}{40\,b^2}+\frac{x^3\,\left(8\,A\,b^2+2\,B\,a\,b\right)}{40\,b^2}\right)$$

input
```
int(x^2*(A + B*x^3)*(a + b*x^3)^(2/3),x)
```

output
```
(a + b*x^3)^(2/3)*((B*x^6)/8 - (3*B*a^2 - 8*A*a*b)/(40*b^2) + (x^3*(8*A*b^
2 + 2*B*a*b))/(40*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int x^2 \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right)\,dx=\frac{(b\,x^3+a)^{\frac{2}{3}}\,(b^2x^6+2ab\,x^3+a^2)}{8b}$$

input
```
int(x^2*(b*x^3+a)^(2/3)*(B*x^3+A),x)
```

output
```
((a + b*x**3)**(2/3)*(a**2 + 2*a*b*x**3 + b**2*x**6))/(8*b)
```

3.310 $\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x}\,dx = \frac{1}{2}A\left(a+bx^3\right)^{2/3}+\frac{B(a+bx^3)^{5/3}}{5b} + \frac{a^{2/3}A\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}} - \frac{1}{2}a^{2/3}A\log(x)+\frac{1}{2}a^{2/3}A\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)$$

output
```
1/2*A*(b*x^3+a)^(2/3)+1/5*B*(b*x^3+a)^(5/3)/b+1/3*a^(2/3)*A*arctan(1/3*(a^
(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)-1/2*a^(2/3)*A*ln(x)+1/2*
a^(2/3)*A*ln(a^(1/3)-(b*x^3+a)^(1/3))
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x}\,dx = \frac{(a+bx^3)^{2/3}(5Ab+2B(a+bx^3))}{10b} + \frac{a^{2/3}A\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}a^{2/3}A\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right) - \frac{1}{6}a^{2/3}A\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)$$

input
```
Integrate[((a + b*x^3)^(2/3)*(A + B*x^3))/x,x]
```

output
```
((a + b*x^3)^(2/3)*(5*A*b + 2*B*(a + b*x^3)))/(10*b) + (a^(2/3)*A*ArcTan[(
1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/Sqrt[3] + (a^(2/3)*A*Log[-a^(
1/3) + (a + b*x^3)^(1/3)])/3 - (a^(2/3)*A*Log[a^(2/3) + a^(1/3)*(a + b*x^3
)^(1/3) + (a + b*x^3)^(2/3)])/6
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {948, 90, 60, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{2/3}(Bx^3+A)}{x^3}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(A\int \frac{\left(bx^3+a\right)^{2/3}}{x^3}dx^3+\frac{3B\left(a+bx^3\right)^{5/3}}{5b}\right)$$

$\big\downarrow$ 60

$$\frac{1}{3}\left(A\left(a\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)+\frac{3B\left(a+bx^3\right)^{5/3}}{5b}\right)$$

$\big\downarrow$ 67

$$\frac{1}{3}\left(A\left(a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx\right.\right.\right.$$

$\big\downarrow$ 16

$$\frac{1}{3}\left(A\left(a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)\right.$$

$\big\downarrow$ 1082

$$\frac{1}{3}\left(A\left(a\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)+\frac{3B(a}{}\right.$$

$\big\downarrow$ 217

$$\frac{1}{3}\left(A\left(a\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)+\frac{3B(a+}{5b}\right.$$

input
```
Int[((a + b*x^3)^(2/3)*(A + B*x^3))/x,x]
```

output
```
((3*B*(a + b*x^3)^(5/3))/(5*b) + A*((3*(a + b*x^3)^(2/3))/2 + a*((Sqrt[3]*
ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^3]/(2
*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3)))))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$-\frac{\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)\right)A b\,a^{\frac{2}{3}}+3\left(\left(\right.\right.}{6b}$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
```

output
```
1/6*(-(-2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)+
ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a
^(1/3)))*A*b*a^(2/3)+3*((2/5*B*x^3+A)*b+2/5*B*a)*(b*x^3+a)^(2/3))/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^3)^{2/3}\,(A+Bx^3)}{x}\,dx = \frac{10\sqrt{3}A(a^2)^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(a^2)^{\frac{1}{3}}}{3a}\right)-5A(a^2)^{\frac{1}{3}}b\log\left((bx^3+a)^{\frac{2}{3}}\right.}{}$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x,x, algorithm="fricas")
```

output
```
1/30*(10*sqrt(3)*A*(a^2)^(1/3)*b*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b*x^3
+ a)^(1/3)*(a^2)^(1/3))/a) - 5*A*(a^2)^(1/3)*b*log((b*x^3 + a)^(2/3)*a + (
a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) + 10*A*(a^2)^(1/3)*b*log((b*
x^3 + a)^(1/3)*a - (a^2)^(2/3)) + 3*(2*B*b*x^3 + 2*B*a + 5*A*b)*(b*x^3 + a
)^(2/3))/b
```

Sympy [A] (verification not implemented)

Time = 11.45 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x}\,dx = -\frac{Ab^{\frac{2}{3}}x^2\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3\Gamma\left(\frac{1}{3}\right)} + B\left(\begin{cases}\frac{a^{\frac{2}{3}}x^3}{3} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{5}{3}}}{5b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**3+a)**(2/3)*(B*x**3+A)/x,x)
```

output
```
-A*b**(2/3)*x**2*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), a*exp_polar(I*pi)
/(b*x**3))/(3*gamma(1/3)) + B*Piecewise((a**(2/3)*x**3/3, Eq(b, 0)), ((a +
 b*x**3)**(5/3)/(5*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x}\,dx = \frac{1}{6}\left(2\sqrt{3}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}} + (b\right.\right.$$
$$+ \frac{(bx^3+a)^{\frac{5}{3}}B}{5\,b}$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x,x, algorithm="maxima")
```

output
```
1/6*(2*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/
a^(1/3)) - a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(
2/3)) + 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + 3*(b*x^3 + a)^(2/3))*
A + 1/5*(b*x^3 + a)^(5/3)*B/b
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x}\,dx = \frac{1}{3}\sqrt{3}Aa^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{6}\,Aa^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + \frac{1}{3}\,Aa^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right) + \frac{2\,(bx^3+a)^{\frac{5}{3}}Bb^4+5\,(bx^3+a)^{\frac{2}{3}}Ab^5}{10\,b^5}$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x,x, algorithm="giac")
```

output
```
1/3*sqrt(3)*A*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a
^(1/3)) - 1/6*A*a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3)
+ a^(2/3)) + 1/3*A*a^(2/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3))) + 1/10*(2
*(b*x^3 + a)^(5/3)*B*b^4 + 5*(b*x^3 + a)^(2/3)*A*b^5)/b^5
```

Mupad [B] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x}\,dx = \frac{A\,(b\,x^3+a)^{2/3}}{2} + \frac{B\,(b\,x^3+a)^{5/3}}{5\,b} + \frac{A\,a^{2/3}\,\ln\left(A^2\,a^2\,(b\,x^3+a)^{1/3}-A^2\,a^{7/3}\right)}{3} + \frac{A\,a^{2/3}\,\ln\left(A^2\,a^2\,(b\,x^3+a)^{1/3}-\frac{A^2\,a^{7/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6} - \frac{A\,a^{2/3}\,\ln\left(A^2\,a^2\,(b\,x^3+a)^{1/3}-\frac{A^2\,a^{7/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(2/3))/x,x)
```

output
```
(A*(a + b*x^3)^(2/3))/2 + (B*(a + b*x^3)^(5/3))/(5*b) + (A*a^(2/3)*log(A^2
*a^2*(a + b*x^3)^(1/3) - A^2*a^(7/3)))/3 + (A*a^(2/3)*log(A^2*a^2*(a + b*x
^3)^(1/3) - (A^2*a^(7/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/6 - (A*a
^(2/3)*log(A^2*a^2*(a + b*x^3)^(1/3) - (A^2*a^(7/3)*(3^(1/2)*1i + 1)^2)/4)
*(3^(1/2)*1i + 1))/6
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x}\,dx = \frac{7(b\,x^3+a)^{\frac{2}{3}}\,a}{10} + \frac{(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3}{5} + \left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{b\,x^4+ax}dx\right)a^2$$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x,x)
```

output
```
(7*(a + b*x**3)**(2/3)*a + 2*(a + b*x**3)**(2/3)*b*x**3 + 10*int((a + b*x*
*3)**(2/3)/(a*x + b*x**4),x)*a**2)/10
```

3.311 $\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 148

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^4} \, dx = \frac{1}{2}B(a+bx^3)^{2/3} - \frac{A(a+bx^3)^{2/3}}{3x^3} + \frac{(2Ab+3aB)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}} - \frac{(2Ab+3aB)\log(x)}{6\sqrt[3]{a}} + \frac{(2Ab+3aB)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{a}}$$

output
```
1/2*B*(b*x^3+a)^(2/3)-1/3*A*(b*x^3+a)^(2/3)/x^3+1/9*(2*A*b+3*B*a)*arctan(1
/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(1/3)-1/6*(2*A*b
+3*B*a)*ln(x)/a^(1/3)+1/6*(2*A*b+3*B*a)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(1/3
)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^4}\,dx = \frac{1}{18}\left(\frac{3(a+bx^3)^{2/3}(-2A+3Bx^3)}{x^3} + \frac{2\sqrt{3}(2Ab+3aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{2(2Ab+3aB)\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{a}} - \frac{(2Ab+3aB)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{\sqrt[3]{a}}\right)$$

input
```
Integrate[((a + b*x^3)^(2/3)*(A + B*x^3))/x^4,x]
```

output
```
((3*(a + b*x^3)^(2/3)*(-2*A + 3*B*x^3))/x^3 + (2*Sqrt[3]*(2*A*b + 3*a*B)*A
rcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) + (2*(2*A*b +
3*a*B)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(1/3) - ((2*A*b + 3*a*B)*Log[a
^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/a^(1/3))/18
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {948, 87, 60, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^4}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{2/3}(Bx^3+A)}{x^6}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{(3aB+2Ab)\int \frac{(bx^3+a)^{2/3}}{x^3}dx^3}{3a}-\frac{A(a+bx^3)^{5/3}}{ax^3}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{(3aB+2Ab)\left(a\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3+\frac{3}{2}(a+bx^3)^{2/3}\right)}{3a}-\frac{A(a+bx^3)^{5/3}}{ax^3}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\left(\frac{(3aB+2Ab)\left(a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)+\frac{3}{2}(a -$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{(3aB+2Ab)\left(a\left(\frac{3}{2}\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/\cdots}\right.}{3a}\right.$$

$\downarrow$ 1082

$$\frac{1}{3}\left(\frac{(3aB+2Ab)\left(a\left(-\frac{3\int\frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)}{3a}-\frac{A(\cdots}{\cdots}\right.$$

$\downarrow$ 217

$$\frac{1}{3}\left(\frac{(3aB+2Ab)\left(a\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)}{3a}-\frac{A(a+\cdots}{\cdots}\right.$$

input
```
Int[((a + b*x^3)^(2/3)*(A + B*x^3))/x^4,x]
```

output
```
(-((A*(a + b*x^3)^(5/3))/(a*x^3)) + ((2*A*b + 3*a*B)*((3*(a + b*x^3)^(2/3)
)/2 + a*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(
1/3) - Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1
/3)))))/(3*a))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(-\frac{3B\,x^{3}}{2}+A\right)+\frac{\left(-2\arctan\left(\frac{2\sqrt{3}\,\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{3a^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}\cdots\right.\right.}{3}}{3a^{\frac{1}{3}}x^{3}}$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a^(1/3)*(b*x^3+a)^(2/3)*(-3/2*B*x^3+A)+1/3*(-2*arctan(2/3*3^(1/2)/a^
(1/3)*(b*x^3+a)^(1/3)+1/3*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x
^3+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*(A*b+3/2*B*a)*x^3)/a^(
1/3)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.46

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^4}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}(3\,Ba^2+2\,Aab)x^3\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3+3\sqrt{\frac{1}{3}}\left(2\,(bx^3+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right.}{x^3}\right.}{}\right.$$

$$-\frac{(3\,Ba+2\,Ab)a^{\frac{2}{3}}x^3\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\,(3\,Ba+2\,Ab)a^{\frac{2}{3}}x^3\log\left((bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{18\,ax^3}$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^4,x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(1/3)*(3*B*a^2 + 2*A*a*b)*x^3*sqrt(-1/a^(2/3))*log((2*b*x^3 +
 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))
*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) - (3*B*a + 2*A
*b)*a^(2/3)*x^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3
)) + 2*(3*B*a + 2*A*b)*a^(2/3)*x^3*log((b*x^3 + a)^(1/3) - a^(1/3)) + 3*(3
*B*a*x^3 - 2*A*a)*(b*x^3 + a)^(2/3))/(a*x^3), -1/18*((3*B*a + 2*A*b)*a^(2/
3)*x^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*(3
*B*a + 2*A*b)*a^(2/3)*x^3*log((b*x^3 + a)^(1/3) - a^(1/3)) - 6*sqrt(1/3)*(
3*B*a^2 + 2*A*a*b)*x^3*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^
(1/3))/a^(1/3) - 3*(3*B*a*x^3 - 2*A*a)*(b*x^3 + a)^(2/3))/(a*x^3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.46 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^4}\,dx = -\frac{Ab^{\frac{2}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^3}\right)}{3x\Gamma\left(\frac{4}{3}\right)} - \frac{Bb^{\frac{2}{3}}x^2\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^3}\right)}{3\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)*(B*x**3+A)/x**4,x)
```

output
```
-A*b**(2/3)*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), a*exp_polar(I*pi)/(b*x**
3))/(3*x*gamma(4/3)) - B*b**(2/3)*x**2*gamma(-2/3)*hyper((-2/3, -2/3), (1/
3,), a*exp_polar(I*pi)/(b*x**3))/(3*gamma(1/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^4}\,dx = \frac{1}{9}\left(\frac{2\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}} - \frac{b\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a\right.}{a^{\frac{1}{3}}}\right.$$

$$+\frac{1}{6}\left(2\sqrt{3}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}\log\left(\right.\right.$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^4,x, algorithm="maxima")
```

output
```
1/9*(2*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(1/3) - b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)
)/a^(1/3) + 2*b*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(1/3) - 3*(b*x^3 + a)^(
2/3)/x^3)*A + 1/6*(2*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/
3) + a^(1/3))/a^(1/3)) - a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)
*a^(1/3) + a^(2/3)) + 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + 3*(b*x^
3 + a)^(2/3))*B
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^4}\,dx = \frac{1}{18}\left(\frac{2\sqrt{3}(3\,Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}b}+\frac{9\,(bx^3+a)^{\frac{2}{3}}B}{b}-\right.$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^4,x, algorithm="giac")
```

output
```
1/18*(2*sqrt(3)*(3*B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) +
a^(1/3))/a^(1/3))/(a^(1/3)*b) + 9*(b*x^3 + a)^(2/3)*B/b - (3*B*a + 2*A*b)*
log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(1/3)*b) +
 2*(3*B*a^(4/3) + 2*A*a^(1/3)*b)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^
(2/3)*b) - 6*(b*x^3 + a)^(2/3)*A/(b*x^3))*b
```

Mupad [B] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.07

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^4}\,dx = \frac{B\,(b\,x^3+a)^{2/3}}{2} - \frac{A\,(b\,x^3+a)^{2/3}}{3\,x^3} + \frac{B\,a^{2/3}\,\ln\left(B^2\,a^2\,(b\,x^3+a)^{1/3}-B^2\,a^{7/3}\right)}{3} - \frac{\ln\left(\frac{a^{1/3}\,\left(A\,b-\sqrt{3}\,A\,b\,1\mathrm{i}\right)^2}{9}-\frac{4\,A^2\,b^2\,(b\,x^3+a)^{1/3}}{9}\right)\,\left(A\,b-\sqrt{3}\,A\,b\,1\mathrm{i}\right)}{9\,a^{1/3}} - \frac{\ln\left(\frac{a^{1/3}\,\left(A\,b+\sqrt{3}\,A\,b\,1\mathrm{i}\right)^2}{9}-\frac{4\,A^2\,b^2\,(b\,x^3+a)^{1/3}}{9}\right)\,\left(A\,b+\sqrt{3}\,A\,b\,1\mathrm{i}\right)}{9\,a^{1/3}} + \frac{B\,a^{2/3}\,\ln\left(B^2\,a^2\,(b\,x^3+a)^{1/3}-\frac{B^2\,a^{7/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6} - \frac{B\,a^{2/3}\,\ln\left(B^2\,a^2\,(b\,x^3+a)^{1/3}-\frac{B^2\,a^{7/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6} + \frac{2\,A\,b\,\ln\left(\frac{4\,A^2\,a^{1/3}\,b^2}{9}-\frac{4\,A^2\,b^2\,(b\,x^3+a)^{1/3}}{9}\right)}{9\,a^{1/3}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(2/3))/x^4,x)
```

output
```
(B*(a + b*x^3)^(2/3))/2 - (A*(a + b*x^3)^(2/3))/(3*x^3) + (B*a^(2/3)*log(B
^2*a^2*(a + b*x^3)^(1/3) - B^2*a^(7/3)))/3 - (log((a^(1/3)*(A*b - 3^(1/2)*
A*b*1i)^2)/9 - (4*A^2*b^2*(a + b*x^3)^(1/3))/9)*(A*b - 3^(1/2)*A*b*1i))/(9
*a^(1/3)) - (log((a^(1/3)*(A*b + 3^(1/2)*A*b*1i)^2)/9 - (4*A^2*b^2*(a + b*
x^3)^(1/3))/9)*(A*b + 3^(1/2)*A*b*1i))/(9*a^(1/3)) + (B*a^(2/3)*log(B^2*a^
2*(a + b*x^3)^(1/3) - (B^2*a^(7/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1)
)/6 - (B*a^(2/3)*log(B^2*a^2*(a + b*x^3)^(1/3) - (B^2*a^(7/3)*(3^(1/2)*1i
+ 1)^2)/4)*(3^(1/2)*1i + 1))/6 + (2*A*b*log((4*A^2*a^(1/3)*b^2)/9 - (4*A^2
*b^2*(a + b*x^3)^(1/3))/9))/(9*a^(1/3))
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^4}\,dx = \frac{-2(b\,x^3+a)^{\frac{2}{3}}\,a+3(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3+10\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{b\,x^4+ax}dx\right)ab\,x^3}{6x^3}$$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^4,x)
```

output
```
( - 2*(a + b*x**3)**(2/3)*a + 3*(a + b*x**3)**(2/3)*b*x**3 + 10*int((a + b
*x**3)**(2/3)/(a*x + b*x**4),x)*a*b*x**3)/(6*x**3)
```

3.312 $\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^7}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 161

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^7}\,dx = -\frac{A(a+bx^3)^{2/3}}{6x^6} - \frac{(Ab+3aB)(a+bx^3)^{2/3}}{9ax^3} - \frac{b(Ab-6aB)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}} + \frac{b(Ab-6aB)\log(x)}{18a^{4/3}} - \frac{b(Ab-6aB)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{4/3}}$$

output
```
-1/6*A*(b*x^3+a)^(2/3)/x^6-1/9*(A*b+3*B*a)*(b*x^3+a)^(2/3)/a/x^3-1/27*b*(A
*b-6*B*a)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/
a^(4/3)+1/18*b*(A*b-6*B*a)*ln(x)/a^(4/3)-1/18*b*(A*b-6*B*a)*ln(a^(1/3)-(b*
x^3+a)^(1/3))/a^(4/3)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^7}\,dx = \frac{-\frac{3\sqrt[3]{a}(a+bx^3)^{2/3}(2Abx^3+3a(A+2Bx^3))}{x^6} - 2\sqrt{3}b(Ab-6aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)$$

input
```
Integrate[((a + b*x^3)^(2/3)*(A + B*x^3))/x^7,x]
```

output
```
((-3*a^(1/3)*(a + b*x^3)^(2/3)*(2*A*b*x^3 + 3*a*(A + 2*B*x^3)))/x^6 - 2*Sq
rt[3]*b*(A*b - 6*a*B)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]]
- 2*b*(A*b - 6*a*B)*Log[-a^(1/3) + (a + b*x^3)^(1/3)] + b*(A*b - 6*a*B)*Lo
g[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*a^(4/3))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {948, 87, 51, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^7}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{2/3}(Bx^3+A)}{x^9}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(-\frac{(Ab-6aB)\int \frac{(bx^3+a)^{2/3}}{x^6}dx^3}{6a} - \frac{A(a+bx^3)^{5/3}}{2ax^6}\right)$$

$$\Big\downarrow \; 51$$

$$\frac{1}{3}\left(-\frac{(Ab-6aB)\left(\frac{2}{3}b\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3-\frac{\left(a+bx^3\right)^{2/3}}{x^3}\right)}{6a}-\frac{A\left(a+bx^3\right)^{5/3}}{2ax^6}\right)$$

$$\Big\downarrow \; 67$$

$$\frac{1}{3}\left(-\frac{(Ab-6aB)\left(\frac{2}{3}b\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)-\frac{(a+}{}\right.}{6a}\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(-\frac{(Ab-6aB)\left(\frac{2}{3}b\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)-\frac{\left(a+bx^3\right)^{2/3}}{x^3}\right)}{6a}\right.$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{3}\left(-\frac{(Ab-6aB)\left(\frac{2}{3}b\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)-\frac{\left(a+bx^3\right)^{2/3}}{x^3}\right)}{6a}-\frac{A(a}{}\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}\left(-\frac{(Ab-6aB)\left(\frac{2}{3}b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)-\frac{(a+bx^3)^{2/3}}{x^3}\right)}{6a}-\frac{A(a+b}{2ax}\right.$$

input
```
Int[((a + b*x^3)^(2/3)*(A + B*x^3))/x^7,x]
```

output
```
(-1/2*(A*(a + b*x^3)^(5/3))/(a*x^6) - ((A*b - 6*a*B)*(-((a + b*x^3)^(2/3)/
x^3) + (2*b*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])
/a^(1/3) - Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*
a^(1/3))))/3))/(6*a))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{(b\,x^3+a)^{\frac{2}{3}}(2B\,x^3+A)a^{\frac{4}{3}}+\frac{2\left(A\left(b\,x^3+a\right)^{\frac{2}{3}}a^{\frac{1}{3}}-\frac{\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^3+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}+a\right.}{6}}{3}}{6a^{\frac{4}{3}}x^6}$

input

```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
```

output

```
-1/6*((b*x^3+a)^(2/3)*(2*B*x^3+A)*a^(4/3)+2/3*(A*(b*x^3+a)^(2/3)*a^(1/3)-1
/6*(-2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)+ln(
(b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1
/3)))*(A*b-6*B*a)*x^3)*b*x^3)/a^(4/3)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^7}\,dx = \left[-\frac{3\sqrt{\frac{1}{3}}(6\,Ba^2b-Aab^2)x^6\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx^3-3\sqrt{\frac{1}{3}}\left(2\,(bx^3+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}\right.}{}\right.}{}\right.$$

input

```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^7,x, algorithm="fricas")
```

output
```
[-1/54*(3*sqrt(1/3)*(6*B*a^2*b - A*a*b^2)*x^6*sqrt((-a)^(1/3)/a)*log((2*b*
x^3 - 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a +
(-a)^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x
^3) + (6*B*a*b - A*b^2)*(-a)^(2/3)*x^6*log((b*x^3 + a)^(2/3) - (b*x^3 + a)
^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(6*B*a*b - A*b^2)*(-a)^(2/3)*x^6*log((
b*x^3 + a)^(1/3) + (-a)^(1/3)) + 3*(2*(3*B*a^2 + A*a*b)*x^3 + 3*A*a^2)*(b*
x^3 + a)^(2/3))/(a^2*x^6), 1/54*(6*sqrt(1/3)*(6*B*a^2*b - A*a*b^2)*x^6*sqr
t(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) - (-a)^(1/3))*sqrt(
-(-a)^(1/3)/a)) - (6*B*a*b - A*b^2)*(-a)^(2/3)*x^6*log((b*x^3 + a)^(2/3) -
 (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(6*B*a*b - A*b^2)*(-a)^(2/
3)*x^6*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 3*(2*(3*B*a^2 + A*a*b)*x^3 +
3*A*a^2)*(b*x^3 + a)^(2/3))/(a^2*x^6)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 37.94 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^7}\,dx = -\frac{Ab^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3x^4\Gamma\left(\frac{7}{3}\right)} - \frac{Bb^{\frac{2}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3x\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)*(B*x**3+A)/x**7,x)
```

output
```
-A*b**(2/3)*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**
3))/(3*x**4*gamma(7/3)) - B*b**(2/3)*gamma(1/3)*hyper((-2/3, 1/3), (4/3,),
 a*exp_polar(I*pi)/(b*x**3))/(3*x*gamma(4/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(131) = 262.

Time = 0.12 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.65

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^7}\,dx =$$

$$-\frac{1}{54}\left(\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b^2\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2\,b^2\log\left((bx^3+\right.}{a^{\frac{4}{3}}}\right.$$

$$+\frac{1}{9}\left(\frac{2\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{b\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}}+\frac{2\,b\log\left((bx^3+a)^{\frac{1}{3}}\right.}{a^{\frac{1}{3}}}\right.$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^7,x, algorithm="maxima")
```

output
```
-1/54*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^
(1/3))/a^(4/3) - b^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a
^(2/3))/a^(4/3) + 2*b^2*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(4/3) + 3*(2*(b
*x^3 + a)^(5/3)*b^2 + (b*x^3 + a)^(2/3)*a*b^2)/((b*x^3 + a)^2*a - 2*(b*x^3
 + a)*a^2 + a^3))*A + 1/9*(2*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(
1/3) + a^(1/3))/a^(1/3))/a^(1/3) - b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(
1/3)*a^(1/3) + a^(2/3))/a^(1/3) + 2*b*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(
1/3) - 3*(b*x^3 + a)^(2/3)/x^3)*B
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^7}\,dx =$$

$$-\frac{\frac{(6\,Bab^2-Ab^3)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}-\frac{2\sqrt{3}\left(6\,Ba^{\frac{5}{3}}b^2-Aa^{\frac{2}{3}}b^3\right)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^2}-\frac{2\left(6\,Ba^{\frac{4}{3}}b^2-Aa^{\frac{1}{3}}b^3\right)\ldots}{a\ldots}}{54\,b}$$

input

```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^7,x, algorithm="giac")
```

output

```
-1/54*((6*B*a*b^2 - A*b^3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/
3) + a^(2/3))/a^(4/3) - 2*sqrt(3)*(6*B*a^(5/3)*b^2 - A*a^(2/3)*b^3)*arctan
(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^2 - 2*(6*B*a^(4/3)
*b^2 - A*a^(1/3)*b^3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(5/3) + 3*(6
*(b*x^3 + a)^(5/3)*B*a*b^2 - 6*(b*x^3 + a)^(2/3)*B*a^2*b^2 + 2*(b*x^3 + a)
^(5/3)*A*b^3 + (b*x^3 + a)^(2/3)*A*a*b^3)/(a*b^2*x^6))/b
```

Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.33

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^7}\,dx = \frac{2\,B\,b\,\ln\left(\frac{4\,B^2\,a^{1/3}\,b^2}{9}-\frac{4\,B^2\,b^2\,(b\,x^3+a)^{1/3}}{9}\right)}{9\,a^{1/3}} - \frac{B\,(b\,x^3+a)^{2/3}}{3\,x^3} - \frac{\ln\left(\frac{a^{1/3}\,\left(B\,b-\sqrt{3}\,B\,b\,1\mathrm{i}\right)^2}{9}-\frac{4\,B^2\,b^2\,(b\,x^3+a)^{1/3}}{9}\right)\,\left(B\,b-\sqrt{3}\,B\,b\,1\mathrm{i}\right)}{9\,a^{1/3}} - \frac{\ln\left(\frac{a^{1/3}\,\left(B\,b+\sqrt{3}\,B\,b\,1\mathrm{i}\right)^2}{9}-\frac{4\,B^2\,b^2\,(b\,x^3+a)^{1/3}}{9}\right)\,\left(B\,b+\sqrt{3}\,B\,b\,1\mathrm{i}\right)}{9\,a^{1/3}} + \frac{\ln\left(\frac{A^2\,b^4\,(b\,x^3+a)^{1/3}}{81\,a^2}-\frac{\left(A\,b^2-\sqrt{3}\,A\,b^2\,1\mathrm{i}\right)^2}{324\,a^{5/3}}\right)\,\left(A\,b^2-\sqrt{3}\,A\,b^2\,1\mathrm{i}\right)}{54\,a^{4/3}} - \frac{\frac{A\,b^2\,(b\,x^3+a)^{2/3}}{18}+\frac{A\,b^2\,(b\,x^3+a)^{5/3}}{9\,a}}{(b\,x^3+a)^2-2\,a\,(b\,x^3+a)+a^2} - \frac{A\,b^2\,\ln\left((b\,x^3+a)^{1/3}-a^{1/3}\right)}{27\,a^{4/3}} + \frac{A\,b^2\,\ln\left(\frac{A^2\,b^4\,(b\,x^3+a)^{1/3}}{81\,a^2}-\frac{A^2\,b^4\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{81\,a^{5/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27\,a^{4/3}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(2/3))/x^7,x)
```

output
```
(log((A^2*b^4*(a + b*x^3)^(1/3))/(81*a^2) - (A*b^2 - 3^(1/2)*A*b^2*1i)^2/(
324*a^(5/3)))*(A*b^2 - 3^(1/2)*A*b^2*1i))/(54*a^(4/3)) - (B*(a + b*x^3)^(2
/3))/(3*x^3) - (log((a^(1/3)*(B*b - 3^(1/2)*B*b*1i)^2)/9 - (4*B^2*b^2*(a +
 b*x^3)^(1/3))/9)*(B*b - 3^(1/2)*B*b*1i))/(9*a^(1/3)) - (log((a^(1/3)*(B*b
 + 3^(1/2)*B*b*1i)^2)/9 - (4*B^2*b^2*(a + b*x^3)^(1/3))/9)*(B*b + 3^(1/2)*
B*b*1i))/(9*a^(1/3)) - ((A*b^2*(a + b*x^3)^(2/3))/18 + (A*b^2*(a + b*x^3)^
(5/3))/(9*a))/((a + b*x^3)^2 - 2*a*(a + b*x^3) + a^2) + (2*B*b*log((4*B^2*
a^(1/3)*b^2)/9 - (4*B^2*b^2*(a + b*x^3)^(1/3))/9))/(9*a^(1/3)) - (A*b^2*lo
g((a + b*x^3)^(1/3) - a^(1/3)))/(27*a^(4/3)) + (A*b^2*log((A^2*b^4*(a + b*
x^3)^(1/3))/(81*a^2) - (A^2*b^4*((3^(1/2)*1i)/2 + 1/2)^2)/(81*a^(5/3)))*((
3^(1/2)*1i)/2 + 1/2))/(27*a^(4/3))
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^7}\,dx = \frac{-3(b\,x^3+a)^{\frac{2}{3}}\,a-8(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3+10\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{b\,x^4+ax}dx\right)b^2x^6}{18x^6}$$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^7,x)
```

output
```
( - 3*(a + b*x**3)**(2/3)*a - 8*(a + b*x**3)**(2/3)*b*x**3 + 10*int((a + b
*x**3)**(2/3)/(a*x + b*x**4),x)*b**2*x**6)/(18*x**6)
```

3.313 $\int x^3(a+bx^3)^{2/3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 178

$$\int x^3(a+bx^3)^{2/3}(A+Bx^3)\,dx=\frac{a(9Ab-4aB)x(a+bx^3)^{2/3}}{81b^2}+\frac{(9Ab-4aB)x^4(a+bx^3)^{2/3}}{54b}+\frac{Bx^4(a+bx^3)^{5/3}}{9b}-\frac{a^2(9Ab-4aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{81\sqrt{3}b^{7/3}}+\frac{a^2(9Ab-4aB)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{162b^{7/3}}$$

output

```
1/81*a*(9*A*b-4*B*a)*x*(b*x^3+a)^(2/3)/b^2+1/54*(9*A*b-4*B*a)*x^4*(b*x^3+a
)^(2/3)/b+1/9*B*x^4*(b*x^3+a)^(5/3)/b-1/243*a^2*(9*A*b-4*B*a)*arctan(1/3*(
1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(7/3)+1/162*a^2*(9*A*b-4
*B*a)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.19

$$\int x^3\left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)\,dx=\frac{3\sqrt[3]{b}\left(a+bx^3\right)^{2/3}\left(-8a^2Bx+6abx(3A+Bx^3)+9b^2x^4(3A+2Bx^3)\right)+2\sqrt{3}a^2(-9Ab+4aB)\,a}{}$$

input
```
Integrate[x^3*(a + b*x^3)^(2/3)*(A + B*x^3),x]
```

output
```
(3*b^(1/3)*(a + b*x^3)^(2/3)*(-8*a^2*B*x + 6*a*b*x*(3*A + B*x^3) + 9*b^2*x
^4*(3*A + 2*B*x^3)) + 2*Sqrt[3]*a^2*(-9*A*b + 4*a*B)*ArcTan[(Sqrt[3]*b^(1/
3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*a^2*(-9*A*b + 4*a*B)*Log[-(b^
(1/3)*x) + (a + b*x^3)^(1/3)] + a^2*(-9*A*b + 4*a*B)*Log[b^(2/3)*x^2 + b^(
1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(486*b^(7/3))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {959, 811, 843, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(9Ab-4aB)\int x^3\left(bx^3+a\right)^{2/3}dx}{9b}+\frac{Bx^4\left(a+bx^3\right)^{5/3}}{9b}$$

$$\Big\downarrow \text{811}$$

$$\frac{(9Ab-4aB)\left(\frac{1}{3}a\int \frac{x^3}{\sqrt[3]{bx^3+a}}dx+\frac{1}{6}x^4\left(a+bx^3\right)^{2/3}\right)}{9b}+\frac{Bx^4\left(a+bx^3\right)^{5/3}}{9b}$$

$$\Big\downarrow \; 843$$

$$\frac{(9Ab-4aB)\left(\frac{1}{3}a\left(\frac{x(a+bx^3)^{2/3}}{3b}-\frac{a\int\frac{1}{\sqrt[3]{bx^3+a}}dx}{3b}\right)+\frac{1}{6}x^4(a+bx^3)^{2/3}\right)}{9b}+\frac{Bx^4(a+bx^3)^{5/3}}{9b}$$

$$\Big\downarrow \; 769$$

$$\frac{(9Ab-4aB)\left(\frac{1}{3}a\left(\frac{x(a+bx^3)^{2/3}}{3b}-\frac{a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{3b}\right)+\frac{1}{6}x^4(a+bx^3)^{2/3}\right)}{9b}+\frac{Bx^4(a+bx^3)^{5/3}}{9b}$$

input
```
Int[x^3*(a + b*x^3)^(2/3)*(A + B*x^3),x]
```

output
```
(B*x^4*(a + b*x^3)^(5/3))/(9*b) + ((9*A*b - 4*a*B)*((x^4*(a + b*x^3)^(2/3)
)/6 + (a*((x*(a + b*x^3)^(2/3))/(3*b) - (a*(ArcTan[(1 + (2*b^(1/3)*x)/(a +
 b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)
^(1/3)]/(2*b^(1/3))))/(3*b)))/3))/(9*b)
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{-6a\left(\frac{Bx^3}{3}+A\right)\left(bx^3+a\right)^{\frac{2}{3}} x b^{\frac{4}{3}}-9\left(\frac{2Bx^3}{3}+A\right)\left(bx^3+a\right)^{\frac{2}{3}} x^4 b^{\frac{7}{3}}+a^2\left(\frac{8\left(bx^3+a\right)^{\frac{2}{3}} xBb^{\frac{1}{3}}}{3}+\left(Ab-\frac{4Ba}{9}\right)\left(-2\sqrt{3}\arctan\left(\right.\right.\right.}{54b^{\frac{7}{3}}}$

input
```
int(x^3*(b*x^3+a)^(2/3)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
-1/54*(-6*a*(1/3*B*x^3+A)*(b*x^3+a)^(2/3)*x*b^(4/3)-9*(2/3*B*x^3+A)*(b*x^3
+a)^(2/3)*x^4*b^(7/3)+a^2*(8/3*(b*x^3+a)^(2/3)*x*B*b^(1/3)+(A*b-4/9*B*a)*(
-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln(
(b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-b^(1/3
)*x+(b*x^3+a)^(1/3))/x))))/b^(7/3)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.71

$$\int x^3 (a+bx^3)^{2/3} (A+Bx^3)\ dx = \left[-\frac{3\sqrt{\frac{1}{3}}(4\,Ba^3b - 9\,Aa^2b^2)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3 - 3\,(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2 - 3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3 + (bx^3+a)^{\frac{1}{3}}\right.\right.}{}\right.$$

$$-\frac{2\,(4\,Ba^3 - 9\,Aa^2b)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right) - (4\,Ba^3 - 9\,Aa^2b)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right) + \frac{6\sqrt{}}{}}{}$$

input
```
integrate(x^3*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="fricas")
```

output
```
[-1/486*(3*sqrt(1/3)*(4*B*a^3*b - 9*A*a^2*b^2)*sqrt(-1/b^(2/3))*log(3*b*x^
3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 +
a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) +
2*(4*B*a^3 - 9*A*a^2*b)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) -
(4*B*a^3 - 9*A*a^2*b)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)
*x + (b*x^3 + a)^(2/3))/x^2) - 3*(18*B*b^3*x^7 + 3*(2*B*a*b^2 + 9*A*b^3)*x
^4 - 2*(4*B*a^2*b - 9*A*a*b^2)*x)*(b*x^3 + a)^(2/3))/b^3, -1/486*(2*(4*B*a
^3 - 9*A*a^2*b)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (4*B*a^3
 - 9*A*a^2*b)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*
x^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(4*B*a^3*b - 9*A*a^2*b^2)*arctan(sqrt(1
/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(18*B*b^3*x
^7 + 3*(2*B*a*b^2 + 9*A*b^3)*x^4 - 2*(4*B*a^2*b - 9*A*a*b^2)*x)*(b*x^3 + a
)^(2/3))/b^3]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.62 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.47

$$\int x^3(a+bx^3)^{2/3}\left(A+Bx^3\right)\,dx = \frac{Aa^{\frac{2}{3}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{Ba^{\frac{2}{3}}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**3*(b*x**3+a)**(2/3)*(B*x**3+A),x)
```

output
```
A*a**(2/3)*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(7/3)) + B*a**(2/3)*x**7*gamma(7/3)*hyper((-2/3, 7/3), (10/3
,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(147) = 294$.

Time = 0.14 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.30

$$\int x^3(a+bx^3)^{2/3}(A+Bx^3)\,dx = \frac{1}{54}\left(\frac{2\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} - \frac{a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2a^2\log\left(\right.}{}\right.$$

$$-\frac{1}{243}\left(\frac{4\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}} - \frac{2a^3\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}} + \frac{4a^3\log\left(-b^{\frac{1}{3}}+\right.}{b^{\frac{7}{3}}}\right.$$

input
```
integrate(x^3*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/54*(2*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b
^(1/3))/b^(4/3) - a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 +
 a)^(2/3)/x^2)/b^(4/3) + 2*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3)
 + 3*((b*x^3 + a)^(2/3)*a^2*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*
(b*x^3 + a)*b^2/x^3 + (b*x^3 + a)^2*b/x^6))*A - 1/243*(4*sqrt(3)*a^3*arcta
n(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^3*l
og(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3)
+ 4*a^3*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) + 3*(2*(b*x^3 + a)^(2/
3)*a^3*b^2/x^2 + 11*(b*x^3 + a)^(5/3)*a^3*b/x^5 - 4*(b*x^3 + a)^(8/3)*a^3/
x^8)/(b^5 - 3*(b*x^3 + a)*b^4/x^3 + 3*(b*x^3 + a)^2*b^3/x^6 - (b*x^3 + a)^
3*b^2/x^9))*B
```

Giac [F]

$$\int x^3 (a+bx^3)^{2/3} (A+Bx^3) \, dx = \int (Bx^3+A)(bx^3+a)^{\frac{2}{3}}x^3 \, dx$$

input
```
integrate(x^3*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (a+bx^3)^{2/3} (A+Bx^3) \, dx = \int x^3 \left(B\,x^3+A\right) \left(b\,x^3+a\right)^{2/3} \, dx$$

input
```
int(x^3*(A + B*x^3)*(a + b*x^3)^(2/3),x)
```

output
```
int(x^3*(A + B*x^3)*(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int x^3 (a+bx^3)^{2/3} (A+Bx^3) \, dx = \frac{10(b\,x^3+a)^{\frac{2}{3}} a^2 x + 33(b\,x^3+a)^{\frac{2}{3}} ab\,x^4 + 18(b\,x^3+a)^{\frac{2}{3}} b^2 x^7 - 10\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}} dx\right) a^3}{162b}$$

input
```
int(x^3*(b*x^3+a)^(2/3)*(B*x^3+A),x)
```

output
```
(10*(a + b*x**3)**(2/3)*a**2*x + 33*(a + b*x**3)**(2/3)*a*b*x**4 + 18*(a +
 b*x**3)**(2/3)*b**2*x**7 - 10*int((a + b*x**3)**(2/3)/(a + b*x**3),x)*a**
3)/(162*b)
```

3.314 $\int (a+bx^3)^{2/3} (A+Bx^3)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 141

$$\int (a+bx^3)^{2/3} (A+Bx^3)\, dx = \frac{(6Ab-aB)x(a+bx^3)^{2/3}}{18b} + \frac{Bx(a+bx^3)^{5/3}}{6b} + \frac{a(6Ab-aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{4/3}} - \frac{a(6Ab-aB)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{4/3}}$$

output
```
1/18*(6*A*b-B*a)*x*(b*x^3+a)^(2/3)/b+1/6*B*x*(b*x^3+a)^(5/3)/b+1/27*a*(6*A
*b-B*a)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(4/3
)-1/18*a*(6*A*b-B*a)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.28

$$\int (a+bx^3)^{2/3} (A+Bx^3)\, dx = \frac{3\sqrt[3]{b}x(a+bx^3)^{2/3}(6Ab+2aB+3bBx^3) - 2\sqrt{3}a(-6Ab+aB)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) + \cdots}{\cdots}$$

input
```
Integrate[(a + b*x^3)^(2/3)*(A + B*x^3),x]
```

output
```
(3*b^(1/3)*x*(a + b*x^3)^(2/3)*(6*A*b + 2*a*B + 3*b*B*x^3) - 2*Sqrt[3]*a*(
-6*A*b + a*B)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))
] + 2*a*(-6*A*b + a*B)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - a*(-6*A*b +
 a*B)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/
(54*b^(4/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {913, 748, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(6Ab-aB)\int \left(bx^3+a\right)^{2/3}dx}{6b}+\frac{Bx\left(a+bx^3\right)^{5/3}}{6b}$$

$$\Big\downarrow \text{748}$$

$$\frac{(6Ab-aB)\left(\frac{2}{3}a\int \frac{1}{\sqrt[3]{bx^3+a}}dx+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)}{6b}+\frac{Bx\left(a+bx^3\right)^{5/3}}{6b}$$

$$\Big\downarrow \text{769}$$

$$\frac{(6Ab-aB)\left(\frac{2}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)}{6b}+\frac{Bx\left(a+bx^3\right)^{5/3}}{6b}$$

input
```
Int[(a + b*x^3)^(2/3)*(A + B*x^3),x]
```

output
```
(B*x*(a + b*x^3)^(5/3))/(6*b) + ((6*A*b - a*B)*((x*(a + b*x^3)^(2/3))/3 +
(2*a*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/
3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/3))/(6*b)
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(114) = 228$.

Time = 1.20 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.91

method	result
pseudoelliptic	$\frac{9B\,b^{\frac{4}{3}} x^{4} \left(b\,x^{3}+a\right)^{\frac{2}{3}}+18A\,b^{\frac{4}{3}} x\left(b\,x^{3}+a\right)^{\frac{2}{3}}+6Bax\,b^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{2}{3}}-12A\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}} x}\right)\sqrt{3}\,ab+2B\arctan}{}$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/54*(9*B*b^(4/3)*x^4*(b*x^3+a)^(2/3)+18*A*b^(4/3)*x*(b*x^3+a)^(2/3)+6*B*a
*x*b^(1/3)*(b*x^3+a)^(2/3)-12*A*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^
(1/3))/b^(1/3)/x)*3^(1/2)*a*b+2*B*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a
)^(1/3))/b^(1/3)/x)*3^(1/2)*a^2-12*A*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a*
b+6*A*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a*b+
2*B*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^2-B*ln((b^(2/3)*x^2+b^(1/3)*(b*x^
3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a^2)/b^(4/3)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 470, normalized size of antiderivative = 3.33

$$\int \left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)\,dx = \left[-\frac{3\sqrt{\frac{1}{3}}(Ba^2b-6\,Aab^2)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3-(bx\right.\right.}{}\right.$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="fricas")
```

output
```
[-1/54*(3*sqrt(1/3)*(B*a^2*b - 6*A*a*b^2)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 -
 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x
^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b)
 + 2*a) - 2*(B*a^2 - 6*A*a*b)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(
1/3))/x) + (B*a^2 - 6*A*a*b)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^
(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(3*B*b^2*x^4 + 2*(B*a*b +
 3*A*b^2)*x)*(b*x^3 + a)^(2/3))/b^2, 1/54*(6*sqrt(1/3)*(B*a^2*b - 6*A*a*b^
2)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/
3))*sqrt(-(-b)^(1/3)/b)/x) + 2*(B*a^2 - 6*A*a*b)*(-b)^(2/3)*log(((-b)^(1/3
)*x + (b*x^3 + a)^(1/3))/x) - (B*a^2 - 6*A*a*b)*(-b)^(2/3)*log(((-b)^(2/3)
*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(3*B*b
^2*x^4 + 2*(B*a*b + 3*A*b^2)*x)*(b*x^3 + a)^(2/3))/b^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.58

$$\int \left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)\,dx = \frac{Aa^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{Ba^{\frac{2}{3}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)*(B*x**3+A),x)
```

output
```
A*a**(2/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + B*a**(2/3)*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(111) = 222$.

Time = 0.12 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.28

$$\int (a+bx^3)^{2/3} (A+Bx^3)\, dx =$$

$$-\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)\right.}{x}}{b^{\frac{1}{3}}}\right.$$

$$+\frac{1}{54}\left(\frac{2\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2a^2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3\right.}{}}{b^{\frac{4}{3}}}\right.$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="maxima")
```

output
```
-1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(
1/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^
(2/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(
b*x^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*A + 1/54*(2*sqrt(3)*a^2*ar
ctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*
log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3)
 + 2*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3
)*a^2*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 +
(b*x^3 + a)^2*b/x^6))*B
```

Giac [F]

$$\int \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right)\,dx = \int \left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{2/3}\,dx$$

input
```
int((A + B*x^3)*(a + b*x^3)^(2/3),x)
```

output
```
int((A + B*x^3)*(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right)\,dx = \frac{4(b\,x^3+a)^{\frac{2}{3}}\,ax}{9}+\frac{(b\,x^3+a)^{\frac{2}{3}}\,b\,x^4}{6}+\frac{5\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)a^2}{9}$$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A),x)
```

output
```
(8*(a + b*x**3)**(2/3)*a*x + 3*(a + b*x**3)**(2/3)*b*x**4 + 10*int((a + b*
x**3)**(2/3)/(a + b*x**3),x)*a**2)/18
```

3.315 $\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 141

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^3}\,dx = \frac{(3Ab+2aB)x(a+bx^3)^{2/3}}{6a} - \frac{A(a+bx^3)^{5/3}}{2ax^2} + \frac{(3Ab+2aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}} - \frac{(3Ab+2aB)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{b}}$$

output
```
1/6*(3*A*b+2*B*a)*x*(b*x^3+a)^(2/3)/a-1/2*A*(b*x^3+a)^(5/3)/a/x^2+1/9*(3*A
*b+2*B*a)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(1
/3)-1/6*(3*A*b+2*B*a)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^3}\,dx = \frac{1}{18}\left(\frac{3(a+bx^3)^{2/3}(-3A+2Bx^3)}{x^2} + \frac{2\sqrt{3}(3Ab+2aB)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}} - \frac{2(3Ab+2aB)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{b}} + \frac{(3Ab+2aB)\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{\sqrt[3]{b}}\right)$$

input
```
Integrate[((a + b*x^3)^(2/3)*(A + B*x^3))/x^3,x]
```

output
```
((3*(a + b*x^3)^(2/3)*(-3*A + 2*B*x^3))/x^2 + (2*Sqrt[3]*(3*A*b + 2*a*B)*A
rcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/b^(1/3) - (2
*(3*A*b + 2*a*B)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(1/3) + ((3*A*b
+ 2*a*B)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)
])/b^(1/3))/18
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 748, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)}{x^3}\,dx$$

$$\Big\downarrow \text{ 955}$$

$$\frac{(2aB+3Ab)\int \left(bx^3+a\right)^{2/3}dx}{2a}-\frac{A\left(a+bx^3\right)^{5/3}}{2ax^2}$$

$$\Big\downarrow \text{ 748}$$

$$\frac{(2aB+3Ab)\left(\frac{2}{3}a\int \frac{1}{\sqrt[3]{bx^3+a}}dx+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)}{2a}-\frac{A\left(a+bx^3\right)^{5/3}}{2ax^2}$$

$$\Big\downarrow \text{ 769}$$

$$\frac{(2aB+3Ab)\left(\frac{2}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)}{2a}-\frac{A\left(a+bx^3\right)^{5/3}}{2ax^2}$$

input
```
Int[((a + b*x^3)^(2/3)*(A + B*x^3))/x^3,x]
```

output
```
-1/2*(A*(a + b*x^3)^(5/3))/(a*x^2) + ((3*A*b + 2*a*B)*((x*(a + b*x^3)^(2/3
))/3 + (2*a*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3
]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/3))/(2*a)
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$-\frac{\frac{3b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(-\frac{2B\,x^{3}}{3}+A\right)}{2}+\left(\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{b^{\frac{1}{3}}}+x\right)}{3x}\right)\sqrt{3}+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\cdots}{x^{2}}\right)}{2}\cdots}{3b^{\frac{1}{3}}x^{2}}$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3/2*b^(1/3)*(b*x^3+a)^(2/3)*(-2/3*B*x^3+A)+(arctan(1/3*3^(1/2)*(2*(b
*x^3+a)^(1/3)/b^(1/3)+x)/x)*3^(1/2)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2
*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(A*b+2/3
*B*a)*x^2)/b^(1/3)/x^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.77 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.60

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^3}\,dx = \frac{Aa^{\frac{2}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{2}{3} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{Ba^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)*(B*x**3+A)/x**3,x)
```

output
```
A*a**(2/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**2*gamma(1/3)) + B*a**(2/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(114) = 228$.

Time = 0.12 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^3}\,dx =$$

$$-\frac{1}{6}\left(2\sqrt{3}b^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)-b^{\frac{2}{3}}\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)+2\,b^{\frac{2}{3}}\log\left(-\right.\right.$$

$$-\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)}{x}\right.}{b^{\frac{1}{3}}}\right.$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^3,x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*b^(2/3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/
x)/b^(1/3)) - b^(2/3)*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 +
 a)^(2/3)/x^2) + 2*b^(2/3)*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x) + 3*(b*x^3
+ a)^(2/3)/x^2)*A - 1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^
3 + a)^(1/3)/x)/b^(1/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/
3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/
3)/x)/b^(1/3) + 3*(b*x^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*B
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^3}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{2}{3}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^3,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^3}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{2/3}}{x^3}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(2/3))/x^3,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(2/3))/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^3}\,dx = \frac{-3(b\,x^3+a)^{\frac{2}{3}}\,a+2(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3+10\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)ab\,x^2}{6x^2}$$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^3,x)
```

output
```
( - 3*(a + b*x**3)**(2/3)*a + 2*(a + b*x**3)**(2/3)*b*x**3 + 10*int((a + b
*x**3)**(2/3)/(a + b*x**3),x)*a*b*x**2)/(6*x**2)
```

3.316 $\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^6}\,dx = -\frac{B(a+bx^3)^{2/3}}{2x^2} - \frac{A(a+bx^3)^{5/3}}{5ax^5} + \frac{b^{2/3}B\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{2}b^{2/3}B\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)$$

output
```
-1/2*B*(b*x^3+a)^(2/3)/x^2-1/5*A*(b*x^3+a)^(5/3)/a/x^5+1/3*b^(2/3)*B*arcta
n(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)-1/2*b^(2/3)*B*ln(-b
^(1/3)*x+(b*x^3+a)^(1/3))
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.50

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^6}\,dx = \frac{(a+bx^3)^{2/3}(-2aA-2Abx^3-5aBx^3)}{10ax^5} + \frac{b^{2/3}B\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}} - \frac{1}{3}b^{2/3}B\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right) + \frac{1}{6}b^{2/3}B\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)$$

input
```
Integrate[((a + b*x^3)^(2/3)*(A + B*x^3))/x^6,x]
```

output
```
((a + b*x^3)^(2/3)*(-2*a*A - 2*A*b*x^3 - 5*a*B*x^3))/(10*a*x^5) + (b^(2/3)
*B*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/Sqrt[3]
- (b^(2/3)*B*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/3 + (b^(2/3)*B*Log[b^(
2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/6
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {953, 809, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^6}\,dx$$

$$\Big\downarrow \text{953}$$

$$B\int \frac{(bx^3+a)^{2/3}}{x^3}dx - \frac{A(a+bx^3)^{5/3}}{5ax^5}$$

$$\Big\downarrow \text{809}$$

$$B\left(b\int \frac{1}{\sqrt[3]{bx^3+a}}dx-\frac{\left(a+bx^3\right)^{2/3}}{2x^2}\right)-\frac{A\left(a+bx^3\right)^{5/3}}{5ax^5}$$

$$\Big\downarrow \text{769}$$

$$B\left(b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)-\frac{\left(a+bx^3\right)^{2/3}}{2x^2}\right)-\frac{A\left(a+bx^3\right)^{5/3}}{5ax^5}$$

input
```
Int[((a + b*x^3)^(2/3)*(A + B*x^3))/x^6,x]
```

output
```
-1/5*(A*(a + b*x^3)^(5/3))/(a*x^5) + B*(-1/2*(a + b*x^3)^(2/3)/x^2 + b*(Ar
cTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Lo
g[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 953
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[d/e^n   Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] &&
(IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && G
tQ[m + n, -1]))
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.25

method	result
pseudoelliptic	$\frac{5a\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)-2\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)\right)B\,x^{5}}{30a\,x^{5}}$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/30*(5*a*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(
1/3)/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*
ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x))*B*x^5*b^(2/3)-6*((A*b+5/2*B*a)*x^3+A*a
)*(b*x^3+a)^(2/3))/a/x^5
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^6,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^6}\,dx = \frac{Ab^{\frac{2}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)} + \frac{Ab^{\frac{5}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3a\Gamma\left(-\frac{2}{3}\right)} + \frac{Ba^{\frac{2}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)*(B*x**3+A)/x**6,x)
```

output
```
A*b**(2/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-5/3)/(3*x**3*gamma(-2/3)) + A*b*
*(5/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-5/3)/(3*a*gamma(-2/3)) + B*a**(2/3)*
gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*
gamma(1/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^6}\,dx = -\frac{1}{6}\left(2\sqrt{3}b^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right) - b^{\frac{2}{3}}\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right) + 2\,b^{\frac{2}{3}}\log\left(-\right.\right.$$

$$-\frac{(bx^3+a)^{\frac{5}{3}}A}{5\,ax^5}$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^6,x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*b^(2/3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/
x)/b^(1/3)) - b^(2/3)*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 +
 a)^(2/3)/x^2) + 2*b^(2/3)*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x) + 3*(b*x^3
+ a)^(2/3)/x^2)*B - 1/5*(b*x^3 + a)^(5/3)*A/(a*x^5)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^6}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{2}{3}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^6,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^6}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{2/3}}{x^6}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(2/3))/x^6,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(2/3))/x^6, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^6}\,dx = \frac{-(b\,x^3+a)^{\frac{2}{3}}\,a-(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3+5\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{x^3}dx\right)b\,x^5}{5x^5}$$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^6,x)
```

output
```
( - (a + b*x**3)**(2/3)*a - (a + b*x**3)**(2/3)*b*x**3 + 5*int((a + b*x**3
)**(2/3)/x**3,x)*b*x**5)/(5*x**5)
```

3.317 $\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^9}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^9}\,dx = -\frac{A(a+bx^3)^{5/3}}{8ax^8} + \frac{(3Ab-8aB)(a+bx^3)^{5/3}}{40a^2x^5}$$

output
```
-1/8*A*(b*x^3+a)^(5/3)/a/x^8+1/40*(3*A*b-8*B*a)*(b*x^3+a)^(5/3)/a^2/x^5
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^9}\,dx = \frac{(a+bx^3)^{5/3}(-5aA+3Abx^3-8aBx^3)}{40a^2x^8}$$

input
```
Integrate[((a + b*x^3)^(2/3)*(A + B*x^3))/x^9,x]
```

output
```
((a + b*x^3)^(5/3)*(-5*a*A + 3*A*b*x^3 - 8*a*B*x^3))/(40*a^2*x^8)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {955, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^9}dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(3Ab-8aB)\int \frac{(bx^3+a)^{2/3}}{x^6}dx}{8a}-\frac{A(a+bx^3)^{5/3}}{8ax^8}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{(a+bx^3)^{5/3}(3Ab-8aB)}{40a^2x^5}-\frac{A(a+bx^3)^{5/3}}{8ax^8}$$

input
```
Int[((a + b*x^3)^(2/3)*(A + B*x^3))/x^9,x]
```

output
```
-1/8*(A*(a + b*x^3)^(5/3))/(a*x^8) + ((3*A*b - 8*a*B)*(a + b*x^3)^(5/3))/(
40*a^2*x^5)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result	size
pseudoelliptic	$-\frac{(b\,x^{3}+a)^{\frac{5}{3}}\left(\left(\frac{8B\,x^{3}}{5}+A\right)a-\frac{3Ab\,x^{3}}{5}\right)}{8a^{2}x^{8}}$	36
gosper	$-\frac{(b\,x^{3}+a)^{\frac{5}{3}}\left(-3Ab\,x^{3}+8Ba\,x^{3}+5Aa\right)}{40a^{2}x^{8}}$	37
orering	$-\frac{(b\,x^{3}+a)^{\frac{5}{3}}\left(-3Ab\,x^{3}+8Ba\,x^{3}+5Aa\right)}{40a^{2}x^{8}}$	37
trager	$-\frac{\left(-3A\,b^{2}x^{6}+8Bab\,x^{6}+2aAb\,x^{3}+8B\,a^{2}x^{3}+5a^{2}A\right)(b\,x^{3}+a)^{\frac{2}{3}}}{40a^{2}x^{8}}$	59
risch	$-\frac{\left(-3A\,b^{2}x^{6}+8Bab\,x^{6}+2aAb\,x^{3}+8B\,a^{2}x^{3}+5a^{2}A\right)(b\,x^{3}+a)^{\frac{2}{3}}}{40a^{2}x^{8}}$	59

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^3+a)^(5/3)*((8/5*B*x^3+A)*a-3/5*A*b*x^3)/a^2/x^8
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^9}\,dx = -\frac{((8\,Bab-3\,Ab^2)x^6+2\,(4\,Ba^2+Aab)x^3+5\,Aa^2)(bx^3+a)^{\frac{2}{3}}}{40\,a^2x^8}$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^9,x, algorithm="fricas")
```

output
```
-1/40*((8*B*a*b - 3*A*b^2)*x^6 + 2*(4*B*a^2 + A*a*b)*x^3 + 5*A*a^2)*(b*x^3
 + a)^(2/3)/(a^2*x^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(46) = 92.

Time = 1.69 (sec) , antiderivative size = 189, normalized size of antiderivative = 3.57

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^9}\,dx = -\frac{5Ab^{\frac{2}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^6\Gamma\left(-\frac{2}{3}\right)} - \frac{2Ab^{\frac{5}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9ax^3\Gamma\left(-\frac{2}{3}\right)} + \frac{Ab^{\frac{8}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{3a^2\Gamma\left(-\frac{2}{3}\right)} + \frac{Bb^{\frac{2}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)} + \frac{Bb^{\frac{5}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3a\Gamma\left(-\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)*(B*x**3+A)/x**9,x)
```

output
```
-5*A*b**(2/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(9*x**6*gamma(-2/3)) - 2
*A*b**(5/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(9*a*x**3*gamma(-2/3)) + A
*b**(8/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(3*a**2*gamma(-2/3)) + B*b**
(2/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-5/3)/(3*x**3*gamma(-2/3)) + B*b**(5/3
)*(a/(b*x**3) + 1)**(2/3)*gamma(-5/3)/(3*a*gamma(-2/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^9}\,dx = \frac{A\left(\frac{8\,(bx^3+a)^{\frac{5}{3}}b}{x^5} - \frac{5\,(bx^3+a)^{\frac{8}{3}}}{x^8}\right)}{40\,a^2} - \frac{(bx^3+a)^{\frac{5}{3}}B}{5\,ax^5}$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^9,x, algorithm="maxima")
```

output
```
1/40*A*(8*(b*x^3 + a)^(5/3)*b/x^5 - 5*(b*x^3 + a)^(8/3)/x^8)/a^2 - 1/5*(b*
x^3 + a)^(5/3)*B/(a*x^5)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^9}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{2}{3}}}{x^9}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^9,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)/x^9, x)
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^9}\,dx = \frac{(A\,b^2+B\,a\,b)\,(b\,x^3+a)^{2/3}}{8\,a^2\,x^2} - \frac{(4\,B\,a^2+A\,b\,a)\,(b\,x^3+a)^{2/3}}{20\,a^2\,x^5} - \frac{A\,(b\,x^3+a)^{2/3}}{8\,x^8} - \frac{b\,(b\,x^3+a)^{2/3}\,(2\,A\,b+13\,B\,a)}{40\,a^2\,x^2}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(2/3))/x^9,x)
```

output
```
((A*b^2 + B*a*b)*(a + b*x^3)^(2/3))/(8*a^2*x^2) - ((4*B*a^2 + A*a*b)*(a +
b*x^3)^(2/3))/(20*a^2*x^5) - (A*(a + b*x^3)^(2/3))/(8*x^8) - (b*(a + b*x^3
)^(2/3)*(2*A*b + 13*B*a))/(40*a^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^9}\,dx = \frac{(b\,x^3+a)^{\frac{2}{3}}\left(-b^2x^6-2ab\,x^3-a^2\right)}{8a\,x^8}$$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^9,x)
```

output
```
((a + b*x**3)**(2/3)*( - a**2 - 2*a*b*x**3 - b**2*x**6))/(8*a*x**8)
```

3.318 $\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{12}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{12}}\,dx = -\frac{A(a+bx^3)^{5/3}}{11ax^{11}} + \frac{(6Ab-11aB)(a+bx^3)^{5/3}}{88a^2x^8} - \frac{3b(6Ab-11aB)(a+bx^3)^{5/3}}{440a^3x^5}$$

output
```
-1/11*A*(b*x^3+a)^(5/3)/a/x^11+1/88*(6*A*b-11*B*a)*(b*x^3+a)^(5/3)/a^2/x^8
-3/440*b*(6*A*b-11*B*a)*(b*x^3+a)^(5/3)/a^3/x^5
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{12}}\,dx = \frac{(a+bx^3)^{5/3}(-40a^2A+30aAbx^3-55a^2Bx^3-18Ab^2x^6+33abBx^6)}{440a^3x^{11}}$$

input
```
Integrate[((a + b*x^3)^(2/3)*(A + B*x^3))/x^12,x]
```

output
```
((a + b*x^3)^(5/3)*(-40*a^2*A + 30*a*A*b*x^3 - 55*a^2*B*x^3 - 18*A*b^2*x^6
 + 33*a*b*B*x^6))/(440*a^3*x^11)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {955, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{12}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(6Ab-11aB)\int \frac{(bx^3+a)^{2/3}}{x^9}dx}{11a}-\frac{A(a+bx^3)^{5/3}}{11ax^{11}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(6Ab-11aB)\left(-\frac{3b\int \frac{(bx^3+a)^{2/3}}{x^6}dx}{8a}-\frac{(a+bx^3)^{5/3}}{8ax^8}\right)}{11a}-\frac{A(a+bx^3)^{5/3}}{11ax^{11}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{\left(\frac{3b(a+bx^3)^{5/3}}{40a^2x^5}-\frac{(a+bx^3)^{5/3}}{8ax^8}\right)(6Ab-11aB)}{11a}-\frac{A(a+bx^3)^{5/3}}{11ax^{11}}$$

input
```
Int[((a + b*x^3)^(2/3)*(A + B*x^3))/x^12,x]
```

output
```
-1/11*(A*(a + b*x^3)^(5/3))/(a*x^11) - ((6*A*b - 11*a*B)*(-1/8*(a + b*x^3)
^(5/3)/(a*x^8) + (3*b*(a + b*x^3)^(5/3))/(40*a^2*x^5)))/(11*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$-\frac{\left(\left(\frac{11 B x^3}{8}+A\right) a^2-\frac{3\left(\frac{11 B x^3}{10}+A\right) b x^3 a}{4}+\frac{9 A b^2 x^6}{20}\right)\left(b x^3+a\right)^{\frac{5}{3}}}{11 a^3 x^{11}}$	55
gosper	$-\frac{\left(b x^3+a\right)^{\frac{5}{3}}\left(18 A b^2 x^6-33 B a b x^6-30 a A b x^3+55 B a^2 x^3+40 a^2 A\right)}{440 a^3 x^{11}}$	59
orering	$-\frac{\left(b x^3+a\right)^{\frac{5}{3}}\left(18 A b^2 x^6-33 B a b x^6-30 a A b x^3+55 B a^2 x^3+40 a^2 A\right)}{440 a^3 x^{11}}$	59
trager	$-\frac{\left(18 A x^9 b^3-33 B x^9 a b^2-12 A x^6 a b^2+22 B x^6 a^2 b+10 a^2 A b x^3+55 B x^3 a^3+40 a^3 A\right)\left(b x^3+a\right)^{\frac{2}{3}}}{440 a^3 x^{11}}$	83
risch	$-\frac{\left(18 A x^9 b^3-33 B x^9 a b^2-12 A x^6 a b^2+22 B x^6 a^2 b+10 a^2 A b x^3+55 B x^3 a^3+40 a^3 A\right)\left(b x^3+a\right)^{\frac{2}{3}}}{440 a^3 x^{11}}$	83

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/11*((11/8*B*x^3+A)*a^2-3/4*(11/10*B*x^3+A)*b*x^3*a+9/20*A*b^2*x^6)*(b*x
^3+a)^(5/3)/a^3/x^11
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{12}}\,dx = \frac{(3\,(11\,Bab^2-6\,Ab^3)x^9-2\,(11\,Ba^2b-6\,Aab^2)x^6-40\,Aa^3-5\,(11\,Ba^3+}{440\,a^3x^{11}}$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^12,x, algorithm="fricas")
```

output
```
1/440*(3*(11*B*a*b^2 - 6*A*b^3)*x^9 - 2*(11*B*a^2*b - 6*A*a*b^2)*x^6 - 40*
A*a^3 - 5*(11*B*a^3 + 2*A*a^2*b)*x^3)*(b*x^3 + a)^(2/3)/(a^3*x^11)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. 2(78) = 156.

Time = 2.35 (sec) , antiderivative size = 648, normalized size of antiderivative = 7.71

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{12}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**3+a)**(2/3)*(B*x**3+A)/x**12,x)
```

output
```
40*A*a**5*b**(14/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(27*a**5*b**4*x**
9*gamma(-2/3) + 54*a**4*b**5*x**12*gamma(-2/3) + 27*a**3*b**6*x**15*gamma(
-2/3)) + 90*A*a**4*b**(17/3)*x**3*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(27
*a**5*b**4*x**9*gamma(-2/3) + 54*a**4*b**5*x**12*gamma(-2/3) + 27*a**3*b**
6*x**15*gamma(-2/3)) + 48*A*a**3*b**(20/3)*x**6*(a/(b*x**3) + 1)**(2/3)*ga
mma(-11/3)/(27*a**5*b**4*x**9*gamma(-2/3) + 54*a**4*b**5*x**12*gamma(-2/3)
 + 27*a**3*b**6*x**15*gamma(-2/3)) + 4*A*a**2*b**(23/3)*x**9*(a/(b*x**3) +
 1)**(2/3)*gamma(-11/3)/(27*a**5*b**4*x**9*gamma(-2/3) + 54*a**4*b**5*x**1
2*gamma(-2/3) + 27*a**3*b**6*x**15*gamma(-2/3)) + 24*A*a*b**(26/3)*x**12*(
a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(27*a**5*b**4*x**9*gamma(-2/3) + 54*a*
*4*b**5*x**12*gamma(-2/3) + 27*a**3*b**6*x**15*gamma(-2/3)) + 18*A*b**(29/
3)*x**15*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(27*a**5*b**4*x**9*gamma(-2/
3) + 54*a**4*b**5*x**12*gamma(-2/3) + 27*a**3*b**6*x**15*gamma(-2/3)) - 5*
B*b**(2/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(9*x**6*gamma(-2/3)) - 2*B*
b**(5/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(9*a*x**3*gamma(-2/3)) + B*b*
*(8/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(3*a**2*gamma(-2/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{12}}\,dx = \frac{B\left(\frac{8\,(bx^3+a)^{\frac{5}{3}}b}{x^5} - \frac{5\,(bx^3+a)^{\frac{8}{3}}}{x^8}\right)}{40\,a^2} - \frac{A\left(\frac{44\,(bx^3+a)^{\frac{5}{3}}b^2}{x^5} - \frac{55\,(bx^3+a)^{\frac{8}{3}}b}{x^8} + \frac{20\,(bx^3+a)^{\frac{11}{3}}}{x^{11}}\right)}{220\,a^3}$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^12,x, algorithm="maxima")
```

output
```
1/40*B*(8*(b*x^3 + a)^(5/3)*b/x^5 - 5*(b*x^3 + a)^(8/3)/x^8)/a^2 - 1/220*A
*(44*(b*x^3 + a)^(5/3)*b^2/x^5 - 55*(b*x^3 + a)^(8/3)*b/x^8 + 20*(b*x^3 +
a)^(11/3)/x^11)/a^3
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{12}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{2}{3}}}{x^{12}}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^12,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)/x^12, x)
```

Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{12}}\,dx = \frac{3\,A\,b^2\,(b\,x^3+a)^{2/3}}{110\,a^2\,x^5} - \frac{B\,(b\,x^3+a)^{2/3}}{8\,x^8} - \frac{A\,b\,(b\,x^3+a)^{2/3}}{44\,a\,x^8} - \frac{B\,b\,(b\,x^3+a)^{2/3}}{20\,a\,x^5} - \frac{9\,A\,b^3\,(b\,x^3+a)^{2/3}}{220\,a^3\,x^2} - \frac{A\,(b\,x^3+a)^{2/3}}{11\,x^{11}} + \frac{3\,B\,b^2\,(b\,x^3+a)^{2/3}}{40\,a^2\,x^2}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(2/3))/x^12,x)
```

output
```
(3*A*b^2*(a + b*x^3)^(2/3))/(110*a^2*x^5) - (B*(a + b*x^3)^(2/3))/(8*x^8)
- (A*b*(a + b*x^3)^(2/3))/(44*a*x^8) - (B*b*(a + b*x^3)^(2/3))/(20*a*x^5)
- (9*A*b^3*(a + b*x^3)^(2/3))/(220*a^3*x^2) - (A*(a + b*x^3)^(2/3))/(11*x^
11) + (3*B*b^2*(a + b*x^3)^(2/3))/(40*a^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{12}}\,dx = \frac{(b\,x^3+a)^{\frac{2}{3}}\,(3b^3x^9-2a\,b^2x^6-13a^2b\,x^3-8a^3)}{88a^2x^{11}}$$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^12,x)
```

output
```
((a + b*x**3)**(2/3)*( - 8*a**3 - 13*a**2*b*x**3 - 2*a*b**2*x**6 + 3*b**3*
x**9))/(88*a**2*x**11)
```

3.319 $\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{15}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{15}}\,dx = -\frac{A(a+bx^3)^{5/3}}{14ax^{14}} + \frac{(9Ab-14aB)(a+bx^3)^{5/3}}{154a^2x^{11}} - \frac{3b(9Ab-14aB)(a+bx^3)^{5/3}}{616a^3x^8} + \frac{9b^2(9Ab-14aB)(a+bx^3)^{5/3}}{3080a^4x^5}$$

output
```
-1/14*A*(b*x^3+a)^(5/3)/a/x^14+1/154*(9*A*b-14*B*a)*(b*x^3+a)^(5/3)/a^2/x^
11-3/616*b*(9*A*b-14*B*a)*(b*x^3+a)^(5/3)/a^3/x^8+9/3080*b^2*(9*A*b-14*B*a
)*(b*x^3+a)^(5/3)/a^4/x^5
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{15}}\,dx = \frac{(a+bx^3)^{5/3}(-220a^3A+180a^2Abx^3-280a^3Bx^3-135aAb^2x^6+210a^2bB}{3080a^4x^{14}}$$

input
```
Integrate[((a + b*x^3)^(2/3)*(A + B*x^3))/x^15,x]
```

output
```
((a + b*x^3)^(5/3)*(-220*a^3*A + 180*a^2*A*b*x^3 - 280*a^3*B*x^3 - 135*a*A
*b^2*x^6 + 210*a^2*b*B*x^6 + 81*A*b^3*x^9 - 126*a*b^2*B*x^9))/(3080*a^4*x^
14)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)}{x^{15}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(9Ab-14aB)\int \frac{\left(bx^3+a\right)^{2/3}}{x^{12}}dx}{14a}-\frac{A\left(a+bx^3\right)^{5/3}}{14ax^{14}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(9Ab-14aB)\left(-\frac{6b\int \frac{\left(bx^3+a\right)^{2/3}}{x^9}dx}{11a}-\frac{\left(a+bx^3\right)^{5/3}}{11ax^{11}}\right)}{14a}-\frac{A\left(a+bx^3\right)^{5/3}}{14ax^{14}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(9Ab-14aB)\left(-\frac{6b\left(-\frac{3b\int \frac{\left(bx^3+a\right)^{2/3}}{x^6}dx}{8a}-\frac{\left(a+bx^3\right)^{5/3}}{8ax^8}\right)}{11a}-\frac{\left(a+bx^3\right)^{5/3}}{11ax^{11}}\right)}{14a}-\frac{A\left(a+bx^3\right)^{5/3}}{14ax^{14}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{\left(-\frac{6b\left(\frac{3b\left(a+bx^3\right)^{5/3}}{40a^2x^5}-\frac{\left(a+bx^3\right)^{5/3}}{8ax^8}\right)}{11a}-\frac{\left(a+bx^3\right)^{5/3}}{11ax^{11}}\right)(9Ab-14aB)}{14a}-\frac{A\left(a+bx^3\right)^{5/3}}{14ax^{14}}$$

input
```
Int[((a + b*x^3)^(2/3)*(A + B*x^3))/x^15,x]
```

output
```
-1/14*(A*(a + b*x^3)^(5/3))/(a*x^14) - ((9*A*b - 14*a*B)*(-1/11*(a + b*x^3
)^(5/3)/(a*x^11) - (6*b*(-1/8*(a + b*x^3)^(5/3)/(a*x^8) + (3*b*(a + b*x^3)
^(5/3))/(40*a^2*x^5)))/(11*a)))/(14*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$-\frac{(b x^3+a)^{\frac{5}{3}}\left(\left(\frac{14B x^3}{11}+A\right)a^3-\frac{9b x^3\left(\frac{7B x^3}{6}+A\right)a^2}{11}+\frac{27b^2\left(\frac{14B x^3}{15}+A\right)x^6 a}{44}-\frac{81A x^9 b^3}{220}\right)}{14x^{14}a^4}$
gosper	$-\frac{(b x^3+a)^{\frac{5}{3}}(-81A x^9b^3+126B x^9a b^2+135A x^6a b^2-210B x^6a^2b-180a^2Ab x^3+280B x^3a^3+220a^3A)}{3080x^{14}a^4}$
orering	$-\frac{(b x^3+a)^{\frac{5}{3}}(-81A x^9b^3+126B x^9a b^2+135A x^6a b^2-210B x^6a^2b-180a^2Ab x^3+280B x^3a^3+220a^3A)}{3080x^{14}a^4}$
trager	$-\frac{(-81A b^4x^{12}+126Ba b^3x^{12}+54Aa b^3x^9-84B a^2b^2x^9-45A a^2b^2x^6+70B a^3b x^6+40A a^3b x^3+280B a^4x^3+220A a^4)(b}{3080x^{14}a^4}$
risch	$-\frac{(-81A b^4x^{12}+126Ba b^3x^{12}+54Aa b^3x^9-84B a^2b^2x^9-45A a^2b^2x^6+70B a^3b x^6+40A a^3b x^3+280B a^4x^3+220A a^4)(b}{3080x^{14}a^4}$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/14*(b*x^3+a)^(5/3)*((14/11*B*x^3+A)*a^3-9/11*b*x^3*(7/6*B*x^3+A)*a^2+27
/44*b^2*(14/15*B*x^3+A)*x^6*a-81/220*A*x^9*b^3)/x^14/a^4
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{15}}\,dx =$$

$$-\frac{(9(14Bab^3-9Ab^4)x^{12}-6(14Ba^2b^2-9Aab^3)x^9+5(14Ba^3b-9Aa^2b^2)x^6+220Aa^4+40(7Ba^4+}{3080a^4x^{14}}$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^15,x, algorithm="fricas")
```

output
```
-1/3080*(9*(14*B*a*b^3 - 9*A*b^4)*x^12 - 6*(14*B*a^2*b^2 - 9*A*a*b^3)*x^9
+ 5*(14*B*a^3*b - 9*A*a^2*b^2)*x^6 + 220*A*a^4 + 40*(7*B*a^4 + A*a^3*b)*x^
3)*(b*x^3 + a)^(2/3)/(a^4*x^14)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1392 vs. $2(112) = 224$.

Time = 3.13 (sec) , antiderivative size = 1392, normalized size of antiderivative = 11.90

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{15}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**3+a)**(2/3)*(B*x**3+A)/x**15,x)
```

output
```
-440*A*a**7*b**(29/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-14/3)/(81*a**7*b**9*x
**12*gamma(-2/3) + 243*a**6*b**10*x**15*gamma(-2/3) + 243*a**5*b**11*x**18
*gamma(-2/3) + 81*a**4*b**12*x**21*gamma(-2/3)) - 1400*A*a**6*b**(32/3)*x*
*3*(a/(b*x**3) + 1)**(2/3)*gamma(-14/3)/(81*a**7*b**9*x**12*gamma(-2/3) +
243*a**6*b**10*x**15*gamma(-2/3) + 243*a**5*b**11*x**18*gamma(-2/3) + 81*a
**4*b**12*x**21*gamma(-2/3)) - 1470*A*a**5*b**(35/3)*x**6*(a/(b*x**3) + 1)
**(2/3)*gamma(-14/3)/(81*a**7*b**9*x**12*gamma(-2/3) + 243*a**6*b**10*x**1
5*gamma(-2/3) + 243*a**5*b**11*x**18*gamma(-2/3) + 81*a**4*b**12*x**21*gam
ma(-2/3)) - 518*A*a**4*b**(38/3)*x**9*(a/(b*x**3) + 1)**(2/3)*gamma(-14/3)
/(81*a**7*b**9*x**12*gamma(-2/3) + 243*a**6*b**10*x**15*gamma(-2/3) + 243*
a**5*b**11*x**18*gamma(-2/3) + 81*a**4*b**12*x**21*gamma(-2/3)) + 28*A*a**
3*b**(41/3)*x**12*(a/(b*x**3) + 1)**(2/3)*gamma(-14/3)/(81*a**7*b**9*x**12
*gamma(-2/3) + 243*a**6*b**10*x**15*gamma(-2/3) + 243*a**5*b**11*x**18*gam
ma(-2/3) + 81*a**4*b**12*x**21*gamma(-2/3)) + 252*A*a**2*b**(44/3)*x**15*(
a/(b*x**3) + 1)**(2/3)*gamma(-14/3)/(81*a**7*b**9*x**12*gamma(-2/3) + 243*
a**6*b**10*x**15*gamma(-2/3) + 243*a**5*b**11*x**18*gamma(-2/3) + 81*a**4*
b**12*x**21*gamma(-2/3)) + 378*A*a*b**(47/3)*x**18*(a/(b*x**3) + 1)**(2/3)
*gamma(-14/3)/(81*a**7*b**9*x**12*gamma(-2/3) + 243*a**6*b**10*x**15*gamma
(-2/3) + 243*a**5*b**11*x**18*gamma(-2/3) + 81*a**4*b**12*x**21*gamma(-2/3
)) + 162*A*b**(50/3)*x**21*(a/(b*x**3) + 1)**(2/3)*gamma(-14/3)/(81*a**...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{15}}\,dx = -\frac{B\left(\frac{44\,(bx^3+a)^{\frac{5}{3}}b^2}{x^5} - \frac{55\,(bx^3+a)^{\frac{8}{3}}b}{x^8} + \frac{20\,(bx^3+a)^{\frac{11}{3}}}{x^{11}}\right)}{220\,a^3} + \frac{A\left(\frac{616\,(bx^3+a)^{\frac{5}{3}}b^3}{x^5} - \frac{1155\,(bx^3+a)^{\frac{8}{3}}b^2}{x^8} + \frac{840\,(bx^3+a)^{\frac{11}{3}}b}{x^{11}} - \frac{220\,(bx^3+a)^{\frac{14}{3}}}{x^{14}}\right)}{3080\,a^4}$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^15,x, algorithm="maxima")
```

output
```
-1/220*B*(44*(b*x^3 + a)^(5/3)*b^2/x^5 - 55*(b*x^3 + a)^(8/3)*b/x^8 + 20*(
b*x^3 + a)^(11/3)/x^11)/a^3 + 1/3080*A*(616*(b*x^3 + a)^(5/3)*b^3/x^5 - 11
55*(b*x^3 + a)^(8/3)*b^2/x^8 + 840*(b*x^3 + a)^(11/3)*b/x^11 - 220*(b*x^3
+ a)^(14/3)/x^14)/a^4
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{15}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{2}{3}}}{x^{15}}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^15,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)/x^15, x)
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{15}}\,dx = \frac{81\,A\,b^4\,(b\,x^3+a)^{2/3}}{3080\,a^4\,x^2} - \frac{B\,(b\,x^3+a)^{2/3}}{11\,x^{11}} - \frac{A\,b\,(b\,x^3+a)^{2/3}}{77\,a\,x^{11}} - \frac{B\,b\,(b\,x^3+a)^{2/3}}{44\,a\,x^8} - \frac{A\,(b\,x^3+a)^{2/3}}{14\,x^{14}} - \frac{27\,A\,b^3\,(b\,x^3+a)^{2/3}}{1540\,a^3\,x^5} + \frac{9\,A\,b^2\,(b\,x^3+a)^{2/3}}{616\,a^2\,x^8} - \frac{9\,B\,b^3\,(b\,x^3+a)^{2/3}}{220\,a^3\,x^2} + \frac{3\,B\,b^2\,(b\,x^3+a)^{2/3}}{110\,a^2\,x^5}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(2/3))/x^15,x)
```

output
```
(81*A*b^4*(a + b*x^3)^(2/3))/(3080*a^4*x^2) - (B*(a + b*x^3)^(2/3))/(11*x^
11) - (A*b*(a + b*x^3)^(2/3))/(77*a*x^11) - (B*b*(a + b*x^3)^(2/3))/(44*a*
x^8) - (A*(a + b*x^3)^(2/3))/(14*x^14) - (27*A*b^3*(a + b*x^3)^(2/3))/(154
0*a^3*x^5) + (9*A*b^2*(a + b*x^3)^(2/3))/(616*a^2*x^8) - (9*B*b^3*(a + b*x
^3)^(2/3))/(220*a^3*x^2) + (3*B*b^2*(a + b*x^3)^(2/3))/(110*a^2*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.51

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{15}}\,dx = \frac{(b\,x^3+a)^{\frac{2}{3}}\,(-9b^4x^{12}+6a\,b^3x^9-5a^2b^2x^6-64a^3b\,x^3-44a^4)}{616a^3x^{14}}$$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^15,x)
```

output
```
((a + b*x**3)**(2/3)*( - 44*a**4 - 64*a**3*b*x**3 - 5*a**2*b**2*x**6 + 6*a
*b**3*x**9 - 9*b**4*x**12))/(616*a**3*x**14)
```

3.320 $\int x^4(a+bx^3)^{2/3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int x^4(a+bx^3)^{2/3}(A+Bx^3)\,dx = \frac{Bx^5(a+bx^3)^{5/3}}{10b} + \frac{(2Ab-aB)x^5(a+bx^3)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{10b\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
1/10*B*x^5*(b*x^3+a)^(5/3)/b+1/10*(2*A*b-B*a)*x^5*(b*x^3+a)^(2/3)*hypergeo
m([-2/3, 5/3],[8/3],-b*x^3/a)/b/(1+b*x^3/a)^(2/3)
```

Mathematica [A] (verified)

Time = 7.96 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int x^4(a+bx^3)^{2/3}(A+Bx^3)\,dx = \frac{(a+bx^3)^{2/3}\left(8Ax^5\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)+5Bx^8\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{8}{3}\right.\right.}{40\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

input
```
Integrate[x^4*(a + b*x^3)^(2/3)*(A + B*x^3),x]
```

output
```
((a + b*x^3)^(2/3)*(8*A*x^5*Hypergeometric2F1[-2/3, 5/3, 8/3, -((b*x^3)/a)
] + 5*B*x^8*Hypergeometric2F1[-2/3, 8/3, 11/3, -((b*x^3)/a)]))/(40*(1 + (b
*x^3)/a)^(2/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(2Ab-aB)\int x^4 \left(bx^3+a\right)^{2/3}dx}{2b}+\frac{Bx^5 \left(a+bx^3\right)^{5/3}}{10b}$$

$$\Big\downarrow \text{889}$$

$$\frac{\left(a+bx^3\right)^{2/3}(2Ab-aB)\int x^4 \left(\frac{bx^3}{a}+1\right)^{2/3}dx}{2b\left(\frac{bx^3}{a}+1\right)^{2/3}}+\frac{Bx^5 \left(a+bx^3\right)^{5/3}}{10b}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^5 \left(a+bx^3\right)^{2/3}(2Ab-aB) \text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{10b\left(\frac{bx^3}{a}+1\right)^{2/3}}+\frac{Bx^5 \left(a+bx^3\right)^{5/3}}{10b}$$

input
```
Int[x^4*(a + b*x^3)^(2/3)*(A + B*x^3),x]
```

output
```
(B*x^5*(a + b*x^3)^(5/3))/(10*b) + ((2*A*b - a*B)*x^5*(a + b*x^3)^(2/3)*Hy
pergeometric2F1[-2/3, 5/3, 8/3, -((b*x^3)/a)])/(10*b*(1 + (b*x^3)/a)^(2/3)
)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^{4} \left(b \,x^{3}+a \right)^{\frac{2}{3}} \left(B \,x^{3}+A \right)d x$$

input
```
int(x^4*(b*x^3+a)^(2/3)*(B*x^3+A),x)
```

output
```
int(x^4*(b*x^3+a)^(2/3)*(B*x^3+A),x)
```

Fricas [F]

$$\int x^4 \left(a+b x^3\right)^{2/3} \left(A+B x^3\right) \, dx=\int { {\left(B x^{3} + A\right)} {\left(b x^{3} + a\right)}^{\frac{2}{3}} x^{4} \,d x }$$

input
```
integrate(x^4*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*x^7 + A*x^4)*(b*x^3 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int x^4 (a+bx^3)^{2/3} (A+Bx^3)\,dx = \frac{Aa^{\frac{2}{3}}x^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{Ba^{\frac{2}{3}}x^8\Gamma\left(\frac{8}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{8}{3} \\ \frac{11}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**4*(b*x**3+a)**(2/3)*(B*x**3+A),x)
```

output
```
A*a**(2/3)*x**5*gamma(5/3)*hyper((-2/3, 5/3), (8/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(8/3)) + B*a**(2/3)*x**8*gamma(8/3)*hyper((-2/3, 8/3), (11/3
,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3))
```

Maxima [F]

$$\int x^4 (a+bx^3)^{2/3} (A+Bx^3)\,dx = \int (Bx^3+A)(bx^3+a)^{\frac{2}{3}}x^4\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)*x^4, x)
```

Giac [F]

$$\int x^4 \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx=\int (Bx^3+A)\left(bx^3+a\right)^{\frac{2}{3}}x^4 \, dx$$

input
```
integrate(x^4*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx=\int x^4\,\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{2/3} \,d x$$

input
```
int(x^4*(A + B*x^3)*(a + b*x^3)^(2/3),x)
```

output
```
int(x^4*(A + B*x^3)*(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int x^4 \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx=\frac{5(b\,x^{3}+a)^{\frac{2}{3}} a^{2}x^{2}+24(b\,x^{3}+a)^{\frac{2}{3}} a b\,x^{5}+14(b\,x^{3}+a)^{\frac{2}{3}} b^{2}x^{8}-10\left(\int \frac{x}{(b\,x^{3}+a)^{\frac{1}{3}}}d x \right) a^{3}}{140 b}$$

input
```
int(x^4*(b*x^3+a)^(2/3)*(B*x^3+A),x)
```

output
```
(5*(a + b*x**3)**(2/3)*a**2*x**2 + 24*(a + b*x**3)**(2/3)*a*b*x**5 + 14*(a
 + b*x**3)**(2/3)*b**2*x**8 - 10*int(((a + b*x**3)**(2/3)*x)/(a + b*x**3),
x)*a**3)/(140*b)
```

3.321 $\int x(a+bx^3)^{2/3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 86

$$\int x\left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)\,dx=\frac{Bx^2(a+bx^3)^{5/3}}{7b}+\frac{(7Ab-2aB)x^2(a+bx^3)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{14b\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
1/7*B*x^2*(b*x^3+a)^(5/3)/b+1/14*(7*A*b-2*B*a)*x^2*(b*x^3+a)^(2/3)*hyperge
om([-2/3, 2/3],[5/3],-b*x^3/a)/b/(1+b*x^3/a)^(2/3)
```

Mathematica [A] (verified)

Time = 7.74 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int x\left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)\,dx=\frac{\left(a+bx^3\right)^{2/3}\left(5Ax^2\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)+2Bx^5\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{5}{3}\right.\right.}{10\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

input
```
Integrate[x*(a + b*x^3)^(2/3)*(A + B*x^3),x]
```

output
```
((a + b*x^3)^(2/3)*(5*A*x^2*Hypergeometric2F1[-2/3, 2/3, 5/3, -((b*x^3)/a)
] + 2*B*x^5*Hypergeometric2F1[-2/3, 5/3, 8/3, -((b*x^3)/a)]))/(10*(1 + (b*
x^3)/a)^(2/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(7Ab-2aB)\int x\left(bx^3+a\right)^{2/3}dx}{7b}+\frac{Bx^2\left(a+bx^3\right)^{5/3}}{7b}$$

$$\Big\downarrow \text{889}$$

$$\frac{\left(a+bx^3\right)^{2/3}(7Ab-2aB)\int x\left(\frac{bx^3}{a}+1\right)^{2/3}dx}{7b\left(\frac{bx^3}{a}+1\right)^{2/3}}+\frac{Bx^2\left(a+bx^3\right)^{5/3}}{7b}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2\left(a+bx^3\right)^{2/3}(7Ab-2aB)\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{14b\left(\frac{bx^3}{a}+1\right)^{2/3}}+\frac{Bx^2\left(a+bx^3\right)^{5/3}}{7b}$$

input
```
Int[x*(a + b*x^3)^(2/3)*(A + B*x^3),x]
```

output
```
(B*x^2*(a + b*x^3)^(5/3))/(7*b) + ((7*A*b - 2*a*B)*x^2*(a + b*x^3)^(2/3)*H
ypergeometric2F1[-2/3, 2/3, 5/3, -((b*x^3)/a)])/(14*b*(1 + (b*x^3)/a)^(2/3
))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(B\,x^{3}+A\right)dx$$

input
```
int(x*(b*x^3+a)^(2/3)*(B*x^3+A),x)
```

output
```
int(x*(b*x^3+a)^(2/3)*(B*x^3+A),x)
```

Fricas [F]

$$\int x\left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)\,dx=\int\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{2}{3}}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*x^4 + A*x)*(b*x^3 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int x\left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)\,dx = \frac{Aa^{\frac{2}{3}}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{Ba^{\frac{2}{3}}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x*(b*x**3+a)**(2/3)*(B*x**3+A),x)
```

output
```
A*a**(2/3)*x**2*gamma(2/3)*hyper((-2/3, 2/3), (5/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(5/3)) + B*a**(2/3)*x**5*gamma(5/3)*hyper((-2/3, 5/3), (8/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3))
```

Maxima [F]

$$\int x\left(a+bx^3\right)^{2/3}\left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{2}{3}}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)*x, x)
```

Giac [F]

$$\int x(a+bx^3)^{2/3}(A+Bx^3)\,dx=\int (Bx^3+A)(bx^3+a)^{\frac{2}{3}}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx^3)^{2/3}(A+Bx^3)\,dx=\int x\left(B\,x^3+A\right)\left(b\,x^3+a\right)^{2/3}\,dx$$

input
```
int(x*(A + B*x^3)*(a + b*x^3)^(2/3),x)
```

output
```
int(x*(A + B*x^3)*(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int x(a+bx^3)^{2/3}(A+Bx^3)\,dx=\frac{9(b\,x^3+a)^{\frac{2}{3}}\,a\,x^2}{28}+\frac{(b\,x^3+a)^{\frac{2}{3}}\,b\,x^5}{7}+\frac{5\left(\int\frac{x}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)a^2}{14}$$

input
```
int(x*(b*x^3+a)^(2/3)*(B*x^3+A),x)
```

output
```
(9*(a + b*x**3)**(2/3)*a*x**2 + 4*(a + b*x**3)**(2/3)*b*x**5 + 10*int(((a
+ b*x**3)**(2/3)*x)/(a + b*x**3),x)*a**2)/28
```

3.322 $\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^2}\,dx = -\frac{A(a+bx^3)^{5/3}}{ax} + \frac{(4Ab+aB)x^2(a+bx^3)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
-A*(b*x^3+a)^(5/3)/a/x+1/2*(4*A*b+B*a)*x^2*(b*x^3+a)^(2/3)*hypergeom([-2/3
, 2/3],[5/3],-b*x^3/a)/a/(1+b*x^3/a)^(2/3)
```

Mathematica [A] (verified)

Time = 7.96 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^2}\,dx = \frac{(a+bx^3)^{2/3}\left(-2A(a+bx^3)+\frac{(4Ab+aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\right)}{2ax}$$

input
```
Integrate[((a + b*x^3)^(2/3)*(A + B*x^3))/x^2,x]
```

output
```
((a + b*x^3)^(2/3)*(-2*A*(a + b*x^3) + ((4*A*b + a*B)*x^3*Hypergeometric2F
1[-2/3, 2/3, 5/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(2/3)))/(2*a*x)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^2}\,dx$$

$$\Big\downarrow \text{ 955}$$

$$\frac{(aB+4Ab)\int x(bx^3+a)^{2/3}dx}{a}-\frac{A(a+bx^3)^{5/3}}{ax}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{(a+bx^3)^{2/3}(aB+4Ab)\int x\left(\frac{bx^3}{a}+1\right)^{2/3}dx}{a\left(\frac{bx^3}{a}+1\right)^{2/3}}-\frac{A(a+bx^3)^{5/3}}{ax}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^2(a+bx^3)^{2/3}(aB+4Ab)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a\left(\frac{bx^3}{a}+1\right)^{2/3}}-\frac{A(a+bx^3)^{5/3}}{ax}$$

input
```
Int[((a + b*x^3)^(2/3)*(A + B*x^3))/x^2,x]
```

output
```
-((A*(a + b*x^3)^(5/3))/(a*x)) + ((4*A*b + a*B)*x^2*(a + b*x^3)^(2/3)*Hype
rgeometric2F1[-2/3, 2/3, 5/3, -((b*x^3)/a)])/(2*a*(1 + (b*x^3)/a)^(2/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{2}{3}}\,(B\,x^{3}+A)}{x^{2}}dx$$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^2,x)
```

output
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^2}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^2,x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(2/3)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^2}\,dx = \frac{Aa^{\frac{2}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{Ba^{\frac{2}{3}}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)*(B*x**3+A)/x**2,x)
```

output
```
A*a**(2/3)*gamma(-1/3)*hyper((-2/3, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x*gamma(2/3)) + B*a**(2/3)*x**2*gamma(2/3)*hyper((-2/3, 2/3), (5/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^2}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^2,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^2}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^2,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^2}\,dx=\int \frac{(B\,x^3+A)\,(b\,x^3+a)^{2/3}}{x^2}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(2/3))/x^2,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(2/3))/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^2}\,dx = \frac{6(b\,x^3+a)^{\frac{2}{3}}\,a+(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3+10\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{b\,x^5+a\,x^2}dx\right)a^2x}{4x}$$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^2,x)
```

output
```
(6*(a + b*x**3)**(2/3)*a + (a + b*x**3)**(2/3)*b*x**3 + 10*int((a + b*x**3
)**(2/3)/(a*x**2 + b*x**5),x)*a**2*x)/(4*x)
```

3.323 $\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^5} dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^5} dx = -\frac{A(a+bx^3)^{5/3}}{4ax^4} - \frac{(Ab+4aB)(a+bx^3)^{2/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{4ax\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
-1/4*A*(b*x^3+a)^(5/3)/a/x^4-1/4*(A*b+4*B*a)*(b*x^3+a)^(2/3)*hypergeom([-2
/3, -1/3],[2/3],-b*x^3/a)/a/x/(1+b*x^3/a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^5} dx = \frac{(a+bx^3)^{2/3}\left(-A(a+bx^3) - \frac{(Ab+4aB)x^3\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\right)}{4ax^4}$$

input
```
Integrate[((a + b*x^3)^(2/3)*(A + B*x^3))/x^5,x]
```

output
```
((a + b*x^3)^(2/3)*(-(A*(a + b*x^3)) - ((A*b + 4*a*B)*x^3*Hypergeometric2F
1[-2/3, -1/3, 2/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(2/3)))/(4*a*x^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^5}\,dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(4aB+Ab)\int \frac{(bx^3+a)^{2/3}}{x^2}dx}{4a}-\frac{A(a+bx^3)^{5/3}}{4ax^4}$$

$$\Big\downarrow \text{889}$$

$$\frac{(a+bx^3)^{2/3}(4aB+Ab)\int \frac{\left(\frac{bx^3}{a}+1\right)^{2/3}}{x^2}dx}{4a\left(\frac{bx^3}{a}+1\right)^{2/3}}-\frac{A(a+bx^3)^{5/3}}{4ax^4}$$

$$\Big\downarrow \text{888}$$

$$-\frac{(a+bx^3)^{2/3}(4aB+Ab)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{4ax\left(\frac{bx^3}{a}+1\right)^{2/3}}-\frac{A(a+bx^3)^{5/3}}{4ax^4}$$

input
```
Int[((a + b*x^3)^(2/3)*(A + B*x^3))/x^5,x]
```

output
```
-1/4*(A*(a + b*x^3)^(5/3))/(a*x^4) - ((A*b + 4*a*B)*(a + b*x^3)^(2/3)*Hype
rgeometric2F1[-2/3, -1/3, 2/3, -((b*x^3)/a)])/(4*a*x*(1 + (b*x^3)/a)^(2/3)
)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(B\,x^{3}+A\right)}{x^{5}}dx$$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^5,x)
```

output
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^5,x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^5}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{2}{3}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^5,x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(2/3)/x^5, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^5}\,dx = \frac{Aa^{\frac{2}{3}}\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, -\frac{2}{3} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{Ba^{\frac{2}{3}}\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)*(B*x**3+A)/x**5,x)
```

output
```
A*a**(2/3)*gamma(-4/3)*hyper((-4/3, -2/3), (-1/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**4*gamma(-1/3)) + B*a**(2/3)*gamma(-1/3)*hyper((-2/3, -1/3), (2/3
,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^5}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{2}{3}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^5,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)/x^5, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^5}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{2}{3}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^5,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^5}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{2/3}}{x^5}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(2/3))/x^5,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(2/3))/x^5, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^5}\,dx = \frac{-3(b\,x^3+a)^{\frac{2}{3}}\,a+2(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3-10\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{b\,x^8+a\,x^5}dx\right)a^2x^4}{2x^4}$$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^5,x)
```

output
```
( - 3*(a + b*x**3)**(2/3)*a + 2*(a + b*x**3)**(2/3)*b*x**3 - 10*int((a + b
*x**3)**(2/3)/(a*x**5 + b*x**8),x)*a**2*x**4)/(2*x**4)
```

3.324 $\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^8}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^8}\,dx = -\frac{A(a+bx^3)^{5/3}}{7ax^7} + \frac{(2Ab-7aB)(a+bx^3)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{2}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{28ax^4\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
-1/7*A*(b*x^3+a)^(5/3)/a/x^7+1/28*(2*A*b-7*B*a)*(b*x^3+a)^(2/3)*hypergeom(
[-4/3, -2/3],[-1/3],-b*x^3/a)/a/x^4/(1+b*x^3/a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^8}\,dx = \frac{(a+bx^3)^{2/3}\left(-4A(a+bx^3)+\frac{(2Ab-7aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{2}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\right)}{28ax^7}$$

input
```
Integrate[((a + b*x^3)^(2/3)*(A + B*x^3))/x^8,x]
```

output
```
((a + b*x^3)^(2/3)*(-4*A*(a + b*x^3) + ((2*A*b - 7*a*B)*x^3*Hypergeometric
2F1[-4/3, -2/3, -1/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(2/3)))/(28*a*x^7)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^8}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(2Ab-7aB)\int \frac{(bx^3+a)^{2/3}}{x^5}dx}{7a}-\frac{A(a+bx^3)^{5/3}}{7ax^7}$$

$$\Big\downarrow \text{889}$$

$$-\frac{(a+bx^3)^{2/3}(2Ab-7aB)\int \frac{\left(\frac{bx^3}{a}+1\right)^{2/3}}{x^5}dx}{7a\left(\frac{bx^3}{a}+1\right)^{2/3}}-\frac{A(a+bx^3)^{5/3}}{7ax^7}$$

$$\Big\downarrow \text{888}$$

$$\frac{(a+bx^3)^{2/3}(2Ab-7aB)\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{2}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{28ax^4\left(\frac{bx^3}{a}+1\right)^{2/3}}-\frac{A(a+bx^3)^{5/3}}{7ax^7}$$

input
```
Int[((a + b*x^3)^(2/3)*(A + B*x^3))/x^8,x]
```

output
```
-1/7*(A*(a + b*x^3)^(5/3))/(a*x^7) + ((2*A*b - 7*a*B)*(a + b*x^3)^(2/3)*Hy
pergeometric2F1[-4/3, -2/3, -1/3, -((b*x^3)/a)])/(28*a*x^4*(1 + (b*x^3)/a)
^(2/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{2}{3}} \left(B\,x^{3}+A\right)}{x^{8}}dx$$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^8,x)
```

output
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^8,x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^8}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{2}{3}}}{x^8}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^8,x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(2/3)/x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.83 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^8}\,dx=\frac{Ab^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{Ba^{\frac{2}{3}}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{2}{3}\\ -\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)*(B*x**3+A)/x**8,x)
```

output
```
A*b**(2/3)*gamma(-5/3)*hyper((-2/3, 5/3), (8/3,), a*exp_polar(I*pi)/(b*x**
3))/(3*x**5*gamma(-2/3)) + B*a**(2/3)*gamma(-4/3)*hyper((-4/3, -2/3), (-1/
3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^8}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{2}{3}}}{x^8}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^8,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)/x^8, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^8}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{2}{3}}}{x^8}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^8,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^8}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{2/3}}{x^8}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(2/3))/x^8,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(2/3))/x^8, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^8}\,dx = \frac{-2(b\,x^3+a)^{\frac{2}{3}}\,a-7(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3-10\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{b\,x^8+a\,x^5}dx\right)ab\,x^7}{14x^7}$$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^8,x)
```

output
```
( - 2*(a + b*x**3)**(2/3)*a - 7*(a + b*x**3)**(2/3)*b*x**3 - 10*int((a + b
*x**3)**(2/3)/(a*x**5 + b*x**8),x)*a*b*x**7)/(14*x**7)
```

3.325 $\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{11}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{11}}\,dx = -\frac{A(a+bx^3)^{5/3}}{10ax^{10}} + \frac{(Ab-2aB)(a+bx^3)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{7}{3},-\frac{2}{3},-\frac{4}{3},-\frac{bx^3}{a}\right)}{14ax^7\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
-1/10*A*(b*x^3+a)^(5/3)/a/x^10+1/14*(A*b-2*B*a)*(b*x^3+a)^(2/3)*hypergeom(
[-7/3, -2/3],[-4/3],-b*x^3/a)/a/x^7/(1+b*x^3/a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{11}}\,dx = \frac{(a+bx^3)^{2/3}\left(-7A(a+bx^3)+\frac{5(Ab-2aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{7}{3},-\frac{2}{3},-\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\right)}{70ax^{10}}$$

input
```
Integrate[((a + b*x^3)^(2/3)*(A + B*x^3))/x^11,x]
```

output
```
((a + b*x^3)^(2/3)*(-7*A*(a + b*x^3) + (5*(A*b - 2*a*B)*x^3*Hypergeometric
2F1[-7/3, -2/3, -4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(2/3)))/(70*a*x^10)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{11}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(Ab-2aB)\int \frac{(bx^3+a)^{2/3}}{x^8}dx}{2a}-\frac{A(a+bx^3)^{5/3}}{10ax^{10}}$$

$$\Big\downarrow \text{889}$$

$$-\frac{(a+bx^3)^{2/3}(Ab-2aB)\int \frac{\left(\frac{bx^3}{a}+1\right)^{2/3}}{x^8}dx}{2a\left(\frac{bx^3}{a}+1\right)^{2/3}}-\frac{A(a+bx^3)^{5/3}}{10ax^{10}}$$

$$\Big\downarrow \text{888}$$

$$\frac{(a+bx^3)^{2/3}(Ab-2aB)\,\text{Hypergeometric2F1}\left(-\frac{7}{3},-\frac{2}{3},-\frac{4}{3},-\frac{bx^3}{a}\right)}{14ax^7\left(\frac{bx^3}{a}+1\right)^{2/3}}-\frac{A(a+bx^3)^{5/3}}{10ax^{10}}$$

input
```
Int[((a + b*x^3)^(2/3)*(A + B*x^3))/x^11,x]
```

output
```
-1/10*(A*(a + b*x^3)^(5/3))/(a*x^10) + ((A*b - 2*a*B)*(a + b*x^3)^(2/3)*Hy
pergeometric2F1[-7/3, -2/3, -4/3, -((b*x^3)/a)])/(14*a*x^7*(1 + (b*x^3)/a)
^(2/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{2}{3}}\,(B\,x^{3}+A)}{x^{11}}dx$$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^11,x)
```

output
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^11,x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{11}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{2}{3}}}{x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^11,x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(2/3)/x^11, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.39 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{11}}\,dx = \frac{Aa^{\frac{2}{3}}\Gamma\left(-\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{10}{3},-\frac{2}{3}\\-\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{10}\Gamma\left(-\frac{7}{3}\right)} + \frac{Bb^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)*(B*x**3+A)/x**11,x)
```

output
```
A*a**(2/3)*gamma(-10/3)*hyper((-10/3, -2/3), (-7/3,), b*x**3*exp_polar(I*p
i)/a)/(3*x**10*gamma(-7/3)) + B*b**(2/3)*gamma(-5/3)*hyper((-2/3, 5/3), (8
/3,), a*exp_polar(I*pi)/(b*x**3))/(3*x**5*gamma(-2/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{11}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{2}{3}}}{x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^11,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)/x^11, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{11}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{2}{3}}}{x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(B*x^3+A)/x^11,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)/x^11, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{11}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{2/3}}{x^{11}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(2/3))/x^11,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(2/3))/x^11, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}(A+Bx^3)}{x^{11}}\,dx = \frac{-3(b\,x^3+a)^{\frac{2}{3}}\,a-8(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3+10\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{b\,x^{14}+a\,x^{11}}dx\right)a^2x^{10}}{40x^{10}}$$

input
```
int((b*x^3+a)^(2/3)*(B*x^3+A)/x^11,x)
```

output
```
( - 3*(a + b*x**3)**(2/3)*a - 8*(a + b*x**3)**(2/3)*b*x**3 + 10*int((a + b
*x**3)**(2/3)/(a*x**11 + b*x**14),x)*a**2*x**10)/(40*x**10)
```

3.326 $\int \frac{x^8(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{x^8(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{a^2(Ab-aB)\left(a+bx^3\right)^{2/3}}{2b^4} - \frac{a(2Ab-3aB)\left(a+bx^3\right)^{5/3}}{5b^4} + \frac{(Ab-3aB)\left(a+bx^3\right)^{8/3}}{8b^4} + \frac{B\left(a+bx^3\right)^{11/3}}{11b^4}$$

output
```
1/2*a^2*(A*b-B*a)*(b*x^3+a)^(2/3)/b^4-1/5*a*(2*A*b-3*B*a)*(b*x^3+a)^(5/3)/
b^4+1/8*(A*b-3*B*a)*(b*x^3+a)^(8/3)/b^4+1/11*B*(b*x^3+a)^(11/3)/b^4
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int \frac{x^8(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{\left(a+bx^3\right)^{2/3}\left(99a^2Ab-81a^3B-66aAb^2x^3+54a^2bBx^3+55Ab^3x^6-45ab^2Bx^6+40b^3Bx^9\right)}{440b^4}$$

input
```
Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(2/3)*(99*a^2*A*b - 81*a^3*B - 66*a*A*b^2*x^3 + 54*a^2*b*B*x^
3 + 55*A*b^3*x^6 - 45*a*b^2*B*x^6 + 40*b^3*B*x^9))/(440*b^4)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(A+Bx^3)}{\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6(Bx^3+A)}{\sqrt[3]{bx^3+a}}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(\frac{B(bx^3+a)^{8/3}}{b^3}+\frac{(Ab-3aB)(bx^3+a)^{5/3}}{b^3}+\frac{a(3aB-2Ab)(bx^3+a)^{2/3}}{b^3}-\frac{a^2(aB-Ab)}{b^3\sqrt[3]{bx^3+a}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3a^2(a+bx^3)^{2/3}(Ab-aB)}{2b^4}+\frac{3(a+bx^3)^{8/3}(Ab-3aB)}{8b^4}-\frac{3a(a+bx^3)^{5/3}(2Ab-3aB)}{5b^4}+\frac{3B(a+bx^3)^{11/3}}{11b^4}\right)$$

input
```
Int[(x^8*(A + B*x^3))/(a + b*x^3)^(1/3),x]
```

output
```
((3*a^2*(A*b - a*B)*(a + b*x^3)^(2/3))/(2*b^4) - (3*a*(2*A*b - 3*a*B)*(a +
 b*x^3)^(5/3))/(5*b^4) + (3*(A*b - 3*a*B)*(a + b*x^3)^(8/3))/(8*b^4) + (3*
B*(a + b*x^3)^(11/3))/(11*b^4))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$\frac{9\left(\frac{5\left(\frac{8B\,x^{3}}{11}+A\right)x^{6}b^{3}}{9}-\frac{2a\left(\frac{15B\,x^{3}}{22}+A\right)x^{3}b^{2}}{3}+a^{2}\left(\frac{6B\,x^{3}}{11}+A\right)b-\frac{9a^{3}B}{11}\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{40b^{4}}$	68
gosper	$\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(40b^{3}B\,x^{9}+55A\,b^{3}x^{6}-45Ba\,b^{2}x^{6}-66aA\,b^{2}x^{3}+54B\,a^{2}b\,x^{3}+99a^{2}bA-81a^{3}B\right)}{440b^{4}}$	77
trager	$\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(40b^{3}B\,x^{9}+55A\,b^{3}x^{6}-45Ba\,b^{2}x^{6}-66aA\,b^{2}x^{3}+54B\,a^{2}b\,x^{3}+99a^{2}bA-81a^{3}B\right)}{440b^{4}}$	77
risch	$\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(40b^{3}B\,x^{9}+55A\,b^{3}x^{6}-45Ba\,b^{2}x^{6}-66aA\,b^{2}x^{3}+54B\,a^{2}b\,x^{3}+99a^{2}bA-81a^{3}B\right)}{440b^{4}}$	77
orering	$\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(40b^{3}B\,x^{9}+55A\,b^{3}x^{6}-45Ba\,b^{2}x^{6}-66aA\,b^{2}x^{3}+54B\,a^{2}b\,x^{3}+99a^{2}bA-81a^{3}B\right)}{440b^{4}}$	77

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
9/40*(5/9*(8/11*B*x^3+A)*x^6*b^3-2/3*a*(15/22*B*x^3+A)*x^3*b^2+a^2*(6/11*B
*x^3+A)*b-9/11*a^3*B)*(b*x^3+a)^(2/3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int \frac{x^8(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{(40\,Bb^3x^9 - 5\,(9\,Bab^2 - 11\,Ab^3)x^6 - 81\,Ba^3 + 99\,Aa^2b + 6\,(9\,Ba^2b - 11\,Aab^2)x^3)(bx^3+a)^{\frac{2}{3}}}{440\,b^4}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
1/440*(40*B*b^3*x^9 - 5*(9*B*a*b^2 - 11*A*b^3)*x^6 - 81*B*a^3 + 99*A*a^2*b
 + 6*(9*B*a^2*b - 11*A*a*b^2)*x^3)*(b*x^3 + a)^(2/3)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.67

$$\int \frac{x^8(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \begin{cases} \frac{9Aa^2(a+bx^3)^{\frac{2}{3}}}{40b^3} - \frac{3Aax^3(a+bx^3)^{\frac{2}{3}}}{20b^2} + \frac{Ax^6(a+bx^3)^{\frac{2}{3}}}{8b} - \frac{81Ba^3(a+bx^3)^{\frac{2}{3}}}{440b^4} + \frac{27Ba^2x^3(a+bx^3)^{\frac{2}{3}}}{220b^3} - \frac{9Bax^6(a+bx^3)^{\frac{2}{3}}}{88b^2} + \frac{Bx^9(a+b}{11b} \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{\sqrt[3]{a}} \end{cases}$$

input
```
integrate(x**8*(B*x**3+A)/(b*x**3+a)**(1/3),x)
```

output
```
Piecewise((9*A*a**2*(a + b*x**3)**(2/3)/(40*b**3) - 3*A*a*x**3*(a + b*x**3
)**(2/3)/(20*b**2) + A*x**6*(a + b*x**3)**(2/3)/(8*b) - 81*B*a**3*(a + b*x
**3)**(2/3)/(440*b**4) + 27*B*a**2*x**3*(a + b*x**3)**(2/3)/(220*b**3) - 9
*B*a*x**6*(a + b*x**3)**(2/3)/(88*b**2) + B*x**9*(a + b*x**3)**(2/3)/(11*b
), Ne(b, 0)), ((A*x**9/9 + B*x**12/12)/a**(1/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int \frac{x^8(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{1}{440}\,B\left(\frac{40\,(bx^3+a)^{\frac{11}{3}}}{b^4} - \frac{165\,(bx^3+a)^{\frac{8}{3}}a}{b^4} + \frac{264\,(bx^3+a)^{\frac{5}{3}}a^2}{b^4} - \frac{220\,(bx^3+a)^{\frac{2}{3}}a^3}{b^4}\right) + \frac{1}{40}\,A\left(\frac{5\,(bx^3+a)^{\frac{8}{3}}}{b^3} - \frac{16\,(bx^3+a)^{\frac{5}{3}}a}{b^3} + \frac{20\,(bx^3+a)^{\frac{2}{3}}a^2}{b^3}\right)$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/440*B*(40*(b*x^3 + a)^(11/3)/b^4 - 165*(b*x^3 + a)^(8/3)*a/b^4 + 264*(b*
x^3 + a)^(5/3)*a^2/b^4 - 220*(b*x^3 + a)^(2/3)*a^3/b^4) + 1/40*A*(5*(b*x^3
 + a)^(8/3)/b^3 - 16*(b*x^3 + a)^(5/3)*a/b^3 + 20*(b*x^3 + a)^(2/3)*a^2/b^
3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.07

$$\int \frac{x^8(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{40\,(bx^3+a)^{\frac{11}{3}}B - 165\,(bx^3+a)^{\frac{8}{3}}Ba + 264\,(bx^3+a)^{\frac{5}{3}}Ba^2 + 55\,(bx^3+a)^{\frac{8}{3}}Ab - 176\,(bx^3+a)^{\frac{5}{3}}Aab}{440\,b^4} - \frac{(bx^3+a)^{\frac{2}{3}}Ba^3 - (bx^3+a)^{\frac{2}{3}}Aa^2b}{2\,b^4}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
1/440*(40*(b*x^3 + a)^(11/3)*B - 165*(b*x^3 + a)^(8/3)*B*a + 264*(b*x^3 +
a)^(5/3)*B*a^2 + 55*(b*x^3 + a)^(8/3)*A*b - 176*(b*x^3 + a)^(5/3)*A*a*b)/b
^4 - 1/2*((b*x^3 + a)^(2/3)*B*a^3 - (b*x^3 + a)^(2/3)*A*a^2*b)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int \frac{x^8(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = -\left(b\,x^3+a\right)^{2/3}\left(\frac{81\,B\,a^3-99\,A\,a^2\,b}{440\,b^4}-\frac{B\,x^9}{11\,b}-\frac{x^6\,\left(55\,A\,b^3-45\,B\,a\,b^2\right)}{440\,b^4}+\frac{3\,a\,x^3\,\left(11\,A\,b-9\,B\,a\right)}{220\,b^3}\right)$$

input
```
int((x^8*(A + B*x^3))/(a + b*x^3)^(1/3),x)
```

output
```
-(a + b*x^3)^(2/3)*((81*B*a^3 - 99*A*a^2*b)/(440*b^4) - (B*x^9)/(11*b) - (
x^6*(55*A*b^3 - 45*B*a*b^2))/(440*b^4) + (3*a*x^3*(11*A*b - 9*B*a))/(220*b
^3))
```

Reduce [F]

$$\int \frac{x^8(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{x^{11}}{\left(b\,x^3+a\right)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{x^8}{\left(b\,x^3+a\right)^{\frac{1}{3}}}dx\right)a$$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^(1/3),x)
```

output
```
int(x**11/(a + b*x**3)**(1/3),x)*b + int(x**8/(a + b*x**3)**(1/3),x)*a
```

3.327 $\int \frac{x^5(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x^5(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = -\frac{a(Ab-aB)\left(a+bx^3\right)^{2/3}}{2b^3} + \frac{(Ab-2aB)\left(a+bx^3\right)^{5/3}}{5b^3} + \frac{B\left(a+bx^3\right)^{8/3}}{8b^3}$$

output
```
-1/2*a*(A*b-B*a)*(b*x^3+a)^(2/3)/b^3+1/5*(A*b-2*B*a)*(b*x^3+a)^(5/3)/b^3+1
/8*B*(b*x^3+a)^(8/3)/b^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{x^5(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{\left(a+bx^3\right)^{2/3}\left(-12aAb+9a^2B+8Ab^2x^3-6abBx^3+5b^2Bx^6\right)}{40b^3}$$

input
```
Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(2/3)*(-12*a*A*b + 9*a^2*B + 8*A*b^2*x^3 - 6*a*b*B*x^3 + 5*b^
2*B*x^6))/(40*b^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(A+Bx^3)}{\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{x^3(Bx^3+A)}{\sqrt[3]{bx^3+a}}dx^3$$

$$\Big\downarrow \ 86$$

$$\frac{1}{3}\int \left(\frac{B(bx^3+a)^{5/3}}{b^2}+\frac{(Ab-2aB)(bx^3+a)^{2/3}}{b^2}+\frac{a(aB-Ab)}{b^2\sqrt[3]{bx^3+a}}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(\frac{3(a+bx^3)^{5/3}(Ab-2aB)}{5b^3}-\frac{3a(a+bx^3)^{2/3}(Ab-aB)}{2b^3}+\frac{3B(a+bx^3)^{8/3}}{8b^3}\right)$$

input
```
Int[(x^5*(A + B*x^3))/(a + b*x^3)^(1/3),x]
```

output
```
((-3*a*(A*b - a*B)*(a + b*x^3)^(2/3))/(2*b^3) + (3*(A*b - 2*a*B)*(a + b*x^
3)^(5/3))/(5*b^3) + (3*B*(a + b*x^3)^(8/3))/(8*b^3))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$-\frac{3\left(-\frac{2\left(\frac{5Bx^{3}}{8}+A\right)x^{3}b^{2}}{3}+a\left(\frac{Bx^{3}}{2}+A\right)b-\frac{3a^{2}B}{4}\right)\left(bx^{3}+a\right)^{\frac{2}{3}}}{10b^{3}}$	49
gosper	$-\frac{\left(bx^{3}+a\right)^{\frac{2}{3}}\left(-5b^{2}Bx^{6}-8Ab^{2}x^{3}+6Babx^{3}+12abA-9a^{2}B\right)}{40b^{3}}$	53
trager	$-\frac{\left(bx^{3}+a\right)^{\frac{2}{3}}\left(-5b^{2}Bx^{6}-8Ab^{2}x^{3}+6Babx^{3}+12abA-9a^{2}B\right)}{40b^{3}}$	53
risch	$-\frac{\left(bx^{3}+a\right)^{\frac{2}{3}}\left(-5b^{2}Bx^{6}-8Ab^{2}x^{3}+6Babx^{3}+12abA-9a^{2}B\right)}{40b^{3}}$	53
orering	$-\frac{\left(bx^{3}+a\right)^{\frac{2}{3}}\left(-5b^{2}Bx^{6}-8Ab^{2}x^{3}+6Babx^{3}+12abA-9a^{2}B\right)}{40b^{3}}$	53

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/10*(-2/3*(5/8*B*x^3+A)*x^3*b^2+a*(1/2*B*x^3+A)*b-3/4*a^2*B)*(b*x^3+a)^(
2/3)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \frac{x^5(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{(5\,Bb^2x^6 - 2\,(3\,Bab - 4\,Ab^2)x^3 + 9\,Ba^2 - 12\,Aab)(bx^3+a)^{\frac{2}{3}}}{40\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
1/40*(5*B*b^2*x^6 - 2*(3*B*a*b - 4*A*b^2)*x^3 + 9*B*a^2 - 12*A*a*b)*(b*x^3
 + a)^(2/3)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.66

$$\int \frac{x^5(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \begin{cases} -\frac{3Aa(a+bx^3)^{\frac{2}{3}}}{10b^2} + \frac{Ax^3(a+bx^3)^{\frac{2}{3}}}{5b} + \frac{9Ba^2(a+bx^3)^{\frac{2}{3}}}{40b^3} - \frac{3Bax^3(a+bx^3)^{\frac{2}{3}}}{20b^2} + \frac{Bx^6(a+bx^3)^{\frac{2}{3}}}{8b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^6}{6} + \frac{Bx^9}{9}}{\sqrt[3]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(B*x**3+A)/(b*x**3+a)**(1/3),x)
```

output
```
Piecewise((-3*A*a*(a + b*x**3)**(2/3)/(10*b**2) + A*x**3*(a + b*x**3)**(2/
3)/(5*b) + 9*B*a**2*(a + b*x**3)**(2/3)/(40*b**3) - 3*B*a*x**3*(a + b*x**3
)**(2/3)/(20*b**2) + B*x**6*(a + b*x**3)**(2/3)/(8*b), Ne(b, 0)), ((A*x**6
/6 + B*x**9/9)/a**(1/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \frac{x^5(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{1}{40}\,B\left(\frac{5\,(bx^3+a)^{\frac{8}{3}}}{b^3}-\frac{16\,(bx^3+a)^{\frac{5}{3}}a}{b^3}+\frac{20\,(bx^3+a)^{\frac{2}{3}}a^2}{b^3}\right)+\frac{1}{10}\,A\left(\frac{2\,(bx^3+a)^{\frac{5}{3}}}{b^2}-\frac{5\,(bx^3+a)^{\frac{2}{3}}a}{b^2}\right)$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/40*B*(5*(b*x^3 + a)^(8/3)/b^3 - 16*(b*x^3 + a)^(5/3)*a/b^3 + 20*(b*x^3 +
 a)^(2/3)*a^2/b^3) + 1/10*A*(2*(b*x^3 + a)^(5/3)/b^2 - 5*(b*x^3 + a)^(2/3)
*a/b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{x^5(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{5\,(bx^3+a)^{\frac{8}{3}}B-16\,(bx^3+a)^{\frac{5}{3}}Ba+8\,(bx^3+a)^{\frac{5}{3}}Ab}{40\,b^3}+\frac{(bx^3+a)^{\frac{2}{3}}Ba^2-(bx^3+a)^{\frac{2}{3}}Aab}{2\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
1/40*(5*(b*x^3 + a)^(8/3)*B - 16*(b*x^3 + a)^(5/3)*B*a + 8*(b*x^3 + a)^(5/
3)*A*b)/b^3 + 1/2*((b*x^3 + a)^(2/3)*B*a^2 - (b*x^3 + a)^(2/3)*A*a*b)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \frac{x^5(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \left(b\,x^3+a\right)^{2/3}\left(\frac{9\,B\,a^2-12\,A\,a\,b}{40\,b^3}+\frac{x^3\,\left(8\,A\,b^2-6\,B\,a\,b\right)}{40\,b^3}+\frac{B\,x^6}{8\,b}\right)$$

input
```
int((x^5*(A + B*x^3))/(a + b*x^3)^(1/3),x)
```

output
```
(a + b*x^3)^(2/3)*((9*B*a^2 - 12*A*a*b)/(40*b^3) + (x^3*(8*A*b^2 - 6*B*a*b
))/(40*b^3) + (B*x^6)/(8*b))
```

Reduce [F]

$$\int \frac{x^5(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{x^8}{\left(b\,x^3+a\right)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{x^5}{\left(b\,x^3+a\right)^{\frac{1}{3}}}dx\right)a$$

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^(1/3),x)
```

output
```
int(x**8/(a + b*x**3)**(1/3),x)*b + int(x**5/(a + b*x**3)**(1/3),x)*a
```

3.328 $\int \frac{x^2(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{x^2(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{(Ab-aB)\left(a+bx^3\right)^{2/3}}{2b^2}+\frac{B\left(a+bx^3\right)^{5/3}}{5b^2}$$

output
```
1/2*(A*b-B*a)*(b*x^3+a)^(2/3)/b^2+1/5*B*(b*x^3+a)^(5/3)/b^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{x^2(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{\left(a+bx^3\right)^{2/3}\left(5Ab-3aB+2bBx^3\right)}{10b^2}$$

input
```
Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(2/3)*(5*A*b - 3*a*B + 2*b*B*x^3))/(10*b^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^3)}{\sqrt[3]{a+bx^3}} \, dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{\sqrt[3]{bx^3+a}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\frac{(bx^3+a)^{2/3} B}{b}+\frac{Ab-aB}{b\sqrt[3]{bx^3+a}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3(a+bx^3)^{2/3} (Ab-aB)}{2b^2}+\frac{3B(a+bx^3)^{5/3}}{5b^2}\right)$$

input
```
Int[(x^2*(A + B*x^3))/(a + b*x^3)^(1/3),x]
```

output
```
((3*(A*b - a*B)*(a + b*x^3)^(2/3))/(2*b^2) + (3*B*(a + b*x^3)^(5/3))/(5*b^
2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{(b\,x^3+a)^{\frac{2}{3}}\,(2bB\,x^3+5Ab-3Ba)}{10b^2}$	31
trager	$\frac{(b\,x^3+a)^{\frac{2}{3}}\,(2bB\,x^3+5Ab-3Ba)}{10b^2}$	31
risch	$\frac{(b\,x^3+a)^{\frac{2}{3}}\,(2bB\,x^3+5Ab-3Ba)}{10b^2}$	31
orering	$\frac{(b\,x^3+a)^{\frac{2}{3}}\,(2bB\,x^3+5Ab-3Ba)}{10b^2}$	31
pseudoelliptic	$\frac{((2B\,x^3+5A)b-3Ba)\,(b\,x^3+a)^{\frac{2}{3}}}{10b^2}$	32

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/10*(b*x^3+a)^(2/3)*(2*B*b*x^3+5*A*b-3*B*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

$$\int \frac{x^2(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx=\frac{(2\,Bbx^3-3\,Ba+5\,Ab)(bx^3+a)^{\frac{2}{3}}}{10\,b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
1/10*(2*B*b*x^3 - 3*B*a + 5*A*b)*(b*x^3 + a)^(2/3)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.54

$$\int \frac{x^2(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx=\begin{cases}\frac{A(a+bx^3)^{\frac{2}{3}}}{2b}-\frac{3Ba(a+bx^3)^{\frac{2}{3}}}{10b^2}+\frac{Bx^3(a+bx^3)^{\frac{2}{3}}}{5b} & \text{for } b\neq 0\\ \frac{\frac{Ax^3}{3}+\frac{Bx^6}{6}}{\sqrt[3]{a}} & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(B*x**3+A)/(b*x**3+a)**(1/3),x)
```

output
```
Piecewise((A*(a + b*x**3)**(2/3)/(2*b) - 3*B*a*(a + b*x**3)**(2/3)/(10*b**
2) + B*x**3*(a + b*x**3)**(2/3)/(5*b), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/a
**(1/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{x^2(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx=\frac{1}{10}\,B\left(\frac{2\,(bx^3+a)^{\frac{5}{3}}}{b^2}-\frac{5\,(bx^3+a)^{\frac{2}{3}}a}{b^2}\right)+\frac{(bx^3+a)^{\frac{2}{3}}A}{2\,b}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/10*B*(2*(b*x^3 + a)^(5/3)/b^2 - 5*(b*x^3 + a)^(2/3)*a/b^2) + 1/2*(b*x^3
+ a)^(2/3)*A/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{x^2(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{(bx^3+a)^{\frac{5}{3}}B}{5\,b^2} - \frac{(bx^3+a)^{\frac{2}{3}}Ba - (bx^3+a)^{\frac{2}{3}}Ab}{2\,b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
1/5*(b*x^3 + a)^(5/3)*B/b^2 - 1/2*((b*x^3 + a)^(2/3)*B*a - (b*x^3 + a)^(2/
3)*A*b)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{x^2(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \left(\frac{5\,A\,b-3\,B\,a}{10\,b^2}+\frac{B\,x^3}{5\,b}\right)\,{\left(b\,x^3+a\right)}^{2/3}$$

input
```
int((x^2*(A + B*x^3))/(a + b*x^3)^(1/3),x)
```

output
```
((5*A*b - 3*B*a)/(10*b^2) + (B*x^3)/(5*b))*(a + b*x^3)^(2/3)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{x^5}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{x^2}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)a$$

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^(1/3),x)
```

output
```
int(x**5/(a + b*x**3)**(1/3),x)*b + int(x**2/(a + b*x**3)**(1/3),x)*a
```

3.329 $\int \frac{A+Bx^3}{x\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{A+Bx^3}{x\sqrt[3]{a+bx^3}}\,dx = \frac{B(a+bx^3)^{2/3}}{2b}+\frac{A\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}} - \frac{A\log(x)}{2\sqrt[3]{a}}+\frac{A\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}$$

output
```
1/2*B*(b*x^3+a)^(2/3)/b+1/3*A*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/
2)/a^(1/3))*3^(1/2)/a^(1/3)-1/2*A*ln(x)/a^(1/3)+1/2*A*ln(a^(1/3)-(b*x^3+a)
^(1/3))/a^(1/3)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx^3}{x\sqrt[3]{a+bx^3}}\,dx$$
$$= \frac{3\sqrt[3]{a}B(a+bx^3)^{2/3}+2\sqrt{3}Ab\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2Ab\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)-Ab\log\left(a^{2/3}+\sqrt[3]{\ldots}\right.}{6\sqrt[3]{a}b}$$

input
```
Integrate[(A + B*x^3)/(x*(a + b*x^3)^(1/3)),x]
```

output
```
(3*a^(1/3)*B*(a + b*x^3)^(2/3) + 2*Sqrt[3]*A*b*ArcTan[(1 + (2*(a + b*x^3)^
(1/3))/a^(1/3))/Sqrt[3]] + 2*A*b*Log[-a^(1/3) + (a + b*x^3)^(1/3)] - A*b*L
og[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*a^(1/3)*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 90, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^3\sqrt[3]{bx^3+a}}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(A\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3+\frac{3B\left(a+bx^3\right)^{2/3}}{2b}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\left(A\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3B\left(a+bx^3\right)^{2/3}}{2b}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(A\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3B\left(a+bx^3\right)^{2/3}}{2b}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(A\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3B\left(a+bx^3\right)^{2/3}}{2b}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(A\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3B\left(a+bx^3\right)^{2/3}}{2b}\right)$$

input
```
Int[(A + B*x^3)/(x*(a + b*x^3)^(1/3)),x]
```

output
```
((3*B*(a + b*x^3)^(2/3))/(2*b) + A*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1
/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) -
(a + b*x^3)^(1/3)])/(2*a^(1/3))))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{2A\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\,(b\,x^3+a)^{\frac{1}{3}}}{3a^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)b+3B(b\,x^3+a)^{\frac{2}{3}}a^{\frac{1}{3}}+2A\ln\left((b\,x^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)b-A\ln\left((b\,x^3+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(b\,x^3+a)^{\frac{1}{3}}+\right.}{6b\,a^{\frac{1}{3}}}$

input
```
int((B*x^3+A)/x/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*A*3^(1/2)*arctan(2/3*3^(1/2)/a^(1/3)*(b*x^3+a)^(1/3)+1/3*3^(1/2))*b
+3*B*(b*x^3+a)^(2/3)*a^(1/3)+2*A*ln((b*x^3+a)^(1/3)-a^(1/3))*b-A*ln((b*x^3
+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))*b)/b/a^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.67

$$\int \frac{A+Bx^3}{x\sqrt[3]{a+bx^3}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}Aab\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3+3\sqrt{\frac{1}{3}}\left(2\,(bx^3+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx^3+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^3}\right)-Aa^{\frac{2}{3}}b\log\left((bx^3+\right.}{6\,ab}\right.$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*A*a*b*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*
x^3 + a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) -
 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) - A*a^(2/3)*b*log((b*x^3 + a)^(2/
3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*A*a^(2/3)*b*log((b*x^3 + a)^
(1/3) - a^(1/3)) + 3*(b*x^3 + a)^(2/3)*B*a)/(a*b), 1/6*(6*sqrt(1/3)*A*a^(2
/3)*b*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)) - A*a^(2/3
)*b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*A*a^(
2/3)*b*log((b*x^3 + a)^(1/3) - a^(1/3)) + 3*(b*x^3 + a)^(2/3)*B*a)/(a*b)]
```

Sympy [A] (verification not implemented)

Time = 6.48 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx^3}{x\sqrt[3]{a+bx^3}}\,dx = -\frac{A\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3\sqrt[3]{b}x\Gamma\left(\frac{4}{3}\right)}+B\left(\begin{cases}\frac{x^3}{3\sqrt[3]{a}} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{2}{3}}}{2b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x**3+A)/x/(b*x**3+a)**(1/3),x)
```

output
```
-A*gamma(1/3)*hyper((1/3, 1/3), (4/3,), a*exp_polar(I*pi)/(b*x**3))/(3*b**
(1/3)*x*gamma(4/3)) + B*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x
**3)**(2/3)/(2*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x\sqrt[3]{a+bx^3}}\,dx = \frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}} - \frac{\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}} + \frac{2\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-\right.}{a^{\frac{1}{3}}}\right. + \frac{\left(bx^3+a\right)^{\frac{2}{3}}B}{2b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(1/3) - log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(1/3) + 2*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(1/3))*A + 1/2*(b*x^3 + a)^(2
/3)*B/b
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x\sqrt[3]{a+bx^3}}\,dx = \frac{\sqrt{3}A\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{1}{3}}} - \frac{A\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}} + \frac{A\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3a^{\frac{1}{3}}} + \frac{\left(bx^3+a\right)^{\frac{2}{3}}B}{2b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*A*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(1/3) - 1/6*A*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3
))/a^(1/3) + 1/3*A*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(1/3) + 1/2*(b*
x^3 + a)^(2/3)*B/b
```

Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx^3}{x\sqrt[3]{a+bx^3}}\,dx = \frac{B\,{\left(b\,x^3+a\right)}^{2/3}}{2\,b} - \frac{\ln\left(A^2\,{\left(b\,x^3+a\right)}^{1/3}-\frac{a^{1/3}\,{\left(A-\sqrt{3}\,A\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(A-\sqrt{3}\,A\,1{}\mathrm{i}\right)}{6\,a^{1/3}} - \frac{\ln\left(A^2\,{\left(b\,x^3+a\right)}^{1/3}-\frac{a^{1/3}\,{\left(A+\sqrt{3}\,A\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(A+\sqrt{3}\,A\,1{}\mathrm{i}\right)}{6\,a^{1/3}} + \frac{A\,\ln\left(A^2\,{\left(b\,x^3+a\right)}^{1/3}-A^2\,a^{1/3}\right)}{3\,a^{1/3}}$$

input
```
int((A + B*x^3)/(x*(a + b*x^3)^(1/3)),x)
```

output
```
(B*(a + b*x^3)^(2/3))/(2*b) - (log(A^2*(a + b*x^3)^(1/3) - (a^(1/3)*(A - 3
^(1/2)*A*1i)^2)/4)*(A - 3^(1/2)*A*1i))/(6*a^(1/3)) - (log(A^2*(a + b*x^3)^
(1/3) - (a^(1/3)*(A + 3^(1/2)*A*1i)^2)/4)*(A + 3^(1/2)*A*1i))/(6*a^(1/3))
+ (A*log(A^2*(a + b*x^3)^(1/3) - A^2*a^(1/3)))/(3*a^(1/3))
```

Reduce [F]

$$\int \frac{A+Bx^3}{x\sqrt[3]{a+bx^3}}\,dx=\left(\int \frac{x^2}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,x}dx\right)a$$

input
```
int((B*x^3+A)/x/(b*x^3+a)^(1/3),x)
```

output
```
int(x**2/(a + b*x**3)**(1/3),x)*b + int(1/((a + b*x**3)**(1/3)*x),x)*a
```

3.330 $\int \frac{A+Bx^3}{x^4 \sqrt[3]{a+bx^3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 132

$$\int \frac{A+Bx^3}{x^4\sqrt[3]{a+bx^3}}\,dx = -\frac{A(a+bx^3)^{2/3}}{3ax^3} - \frac{(Ab-3aB)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}} + \frac{(Ab-3aB)\log(x)}{6a^{4/3}} - \frac{(Ab-3aB)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}}$$

output
```
-1/3*A*(b*x^3+a)^(2/3)/a/x^3-1/9*(A*b-3*B*a)*arctan(1/3*(a^(1/3)+2*(b*x^3+
a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(4/3)+1/6*(A*b-3*B*a)*ln(x)/a^(4/3)-1
/6*(A*b-3*B*a)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(4/3)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^3}{x^4\sqrt[3]{a+bx^3}}\,dx$$

$$= \frac{-\frac{6\sqrt[3]{a}A(a+bx^3)^{2/3}}{x^3} - 2\sqrt{3}(Ab-3aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2(Ab-3aB)\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right) + }{18a^{4/3}}$$

input
```
Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(1/3)),x]
```

output
```
((-6*a^(1/3)*A*(a + b*x^3)^(2/3))/x^3 - 2*Sqrt[3]*(A*b - 3*a*B)*ArcTan[(1
+ (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*(A*b - 3*a*B)*Log[-a^(1/3) +
 (a + b*x^3)^(1/3)] + (A*b - 3*a*B)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3
) + (a + b*x^3)^(2/3)])/(18*a^(4/3))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^4 \sqrt[3]{a+bx^3}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{Bx^3+A}{x^6 \sqrt[3]{bx^3+a}} dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3} \left(-\frac{(Ab-3aB) \int \frac{1}{x^3 \sqrt[3]{bx^3+a}} dx^3}{3a} - \frac{A(a+bx^3)^{2/3}}{ax^3} \right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3} \left(-\frac{(Ab-3aB) \left(\frac{3}{2} \int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}} d\sqrt[3]{bx^3+a} - \frac{3 \int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}} d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}} - \frac{\log(x^3)}{2\sqrt[3]{a}} \right)}{3a} - \frac{A(a+}{ax} \right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(-\frac{(Ab-3aB)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{3a}-\frac{A\left(a+bx^3\right)^{2/3}}{ax^3}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(-\frac{(Ab-3aB)\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{3a}-\frac{A\left(a+bx^3\right)^{2/3}}{ax^3}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(-\frac{(Ab-3aB)\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{3a}-\frac{A\left(a+bx^3\right)^{2/3}}{ax^3}\right)$$

input
```
Int[(A + B*x^3)/(x^4*(a + b*x^3)^(1/3)),x]
```

output
```
(-((A*(a + b*x^3)^(2/3))/(a*x^3)) - ((A*b - 3*a*B)*((Sqrt[3]*ArcTan[(1 + (
2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^3]/(2*a^(1/3)) + (
3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3))))/(3*a))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$\frac{-6A\left(b\,x^{3}+a\right)^{\frac{2}{3}}a^{\frac{1}{3}}+\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\right.\right.}{18a^{\frac{4}{3}}x^{3}}$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/18*(-6*A*(b*x^3+a)^(2/3)*a^(1/3)+(-2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/
3))*3^(1/2)/a^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^
(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*(A*b-3*B*a)*x^3)/a^(4/3)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 398, normalized size of antiderivative = 3.02

$$\int \frac{A+Bx^3}{x^4\sqrt[3]{a+bx^3}}\,dx$$

$$= \left[-\frac{3\sqrt{\frac{1}{3}}(3\,Ba^2-Aab)x^3\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx^3-3\sqrt{\frac{1}{3}}\left(2\,(bx^3+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}}{x^3}\right.}{}\right.$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[-1/18*(3*sqrt(1/3)*(3*B*a^2 - A*a*b)*x^3*sqrt((-a)^(1/3)/a)*log((2*b*x^3
- 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)
^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3)
+ (3*B*a - A*b)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(
-a)^(1/3) + (-a)^(2/3)) - 2*(3*B*a - A*b)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(
1/3) + (-a)^(1/3)) + 6*(b*x^3 + a)^(2/3)*A*a)/(a^2*x^3), 1/18*(6*sqrt(1/3)
*(3*B*a^2 - A*a*b)*x^3*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^3 + a)
^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - (3*B*a - A*b)*(-a)^(2/3)*x^3*l
og((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(3*B
*a - A*b)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 6*(b*x^3 +
a)^(2/3)*A*a)/(a^2*x^3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.63 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx^3}{x^4\sqrt[3]{a+bx^3}}\,dx = -\frac{A\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3\sqrt[3]{b}x^4\Gamma\left(\frac{7}{3}\right)} - \frac{B\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3\sqrt[3]{b}x\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**4/(b*x**3+a)**(1/3),x)
```

output
```
-A*gamma(4/3)*hyper((1/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**3))/(3*b**
(1/3)*x**4*gamma(7/3)) - B*gamma(1/3)*hyper((1/3, 1/3), (4/3,), a*exp_pola
r(I*pi)/(b*x**3))/(3*b**(1/3)*x*gamma(4/3))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.60

$$\int \frac{A+Bx^3}{x^4\sqrt[3]{a+bx^3}}\,dx = -\frac{1}{18}\left(\frac{2\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}+\frac{6\,(bx^3+a)^{\frac{2}{3}}b}{(bx^3+a)a-a^2}-\frac{b\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right.}{a^{\frac{4}{3}}}\right. + \frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}}+\frac{2\,\log\left((bx^3+a)^{\frac{1}{3}}\right.}{a^{\frac{1}{3}}}\right.$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/18*(2*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(4/3) + 6*(b*x^3 + a)^(2/3)*b/((b*x^3 + a)*a - a^2) - b*log((b*x^3
+ a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) + 2*b*log((b*x^3
 + a)^(1/3) - a^(1/3))/a^(4/3))*A + 1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(
b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - log((b*x^3 + a)^(2/3) + (b*
x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) + 2*log((b*x^3 + a)^(1/3) - a^(1
/3))/a^(1/3))*B
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx^3}{x^4\sqrt[3]{a+bx^3}}\,dx = \frac{1}{18}b\left(\frac{2\sqrt{3}(3\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}b}-\frac{(3\,Ba-Ab)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a\right.}{a^{\frac{4}{3}}b}\right.$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
1/18*b*(2*sqrt(3)*(3*B*a - A*b)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) +
a^(1/3))/a^(1/3))/(a^(4/3)*b) - (3*B*a - A*b)*log((b*x^3 + a)^(2/3) + (b*x
^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(4/3)*b) + 2*(3*B*a^(4/3) - A*a^(1/3)*
b)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5/3)*b) - 6*(b*x^3 + a)^(2/3)
*A/(a*b*x^3))
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.10

$$\int \frac{A+Bx^3}{x^4\sqrt[3]{a+bx^3}}\,dx = \frac{\ln\left(\frac{\left(A\,b-\sqrt{3}\,A\,b\,1\mathrm{i}\right)^2}{36\,a^{5/3}}-\frac{A^2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}}{9\,a^2}\right)\,\left(A\,b-\sqrt{3}\,A\,b\,1\mathrm{i}\right)}{18\,a^{4/3}} + \frac{\ln\left(\frac{\left(A\,b+\sqrt{3}\,A\,b\,1\mathrm{i}\right)^2}{36\,a^{5/3}}-\frac{A^2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}}{9\,a^2}\right)\,\left(A\,b+\sqrt{3}\,A\,b\,1\mathrm{i}\right)}{18\,a^{4/3}} - \frac{\ln\left(B^2\,{\left(b\,x^3+a\right)}^{1/3}-\frac{a^{1/3}\,{\left(B-\sqrt{3}\,B\,1\mathrm{i}\right)}^2}{4}\right)\,\left(B-\sqrt{3}\,B\,1\mathrm{i}\right)}{6\,a^{1/3}} - \frac{\ln\left(B^2\,{\left(b\,x^3+a\right)}^{1/3}-\frac{a^{1/3}\,{\left(B+\sqrt{3}\,B\,1\mathrm{i}\right)}^2}{4}\right)\,\left(B+\sqrt{3}\,B\,1\mathrm{i}\right)}{6\,a^{1/3}} + \frac{B\,\ln\left(B^2\,{\left(b\,x^3+a\right)}^{1/3}-B^2\,a^{1/3}\right)}{3\,a^{1/3}} - \frac{A\,b\,\ln\left({\left(b\,x^3+a\right)}^{1/3}-a^{1/3}\right)}{9\,a^{4/3}} - \frac{A\,{\left(b\,x^3+a\right)}^{2/3}}{3\,a\,x^3}$$

input
```
int((A + B*x^3)/(x^4*(a + b*x^3)^(1/3)),x)
```

output
```
(log((A*b - 3^(1/2)*A*b*1i)^2/(36*a^(5/3)) - (A^2*b^2*(a + b*x^3)^(1/3))/(
9*a^2))*(A*b - 3^(1/2)*A*b*1i))/(18*a^(4/3)) + (log((A*b + 3^(1/2)*A*b*1i)
^2/(36*a^(5/3)) - (A^2*b^2*(a + b*x^3)^(1/3))/(9*a^2))*(A*b + 3^(1/2)*A*b*
1i))/(18*a^(4/3)) - (log(B^2*(a + b*x^3)^(1/3) - (a^(1/3)*(B - 3^(1/2)*B*1
i)^2)/4)*(B - 3^(1/2)*B*1i))/(6*a^(1/3)) - (log(B^2*(a + b*x^3)^(1/3) - (a
^(1/3)*(B + 3^(1/2)*B*1i)^2)/4)*(B + 3^(1/2)*B*1i))/(6*a^(1/3)) + (B*log(B
^2*(a + b*x^3)^(1/3) - B^2*a^(1/3)))/(3*a^(1/3)) - (A*b*log((a + b*x^3)^(1
/3) - a^(1/3)))/(9*a^(4/3)) - (A*(a + b*x^3)^(2/3))/(3*a*x^3)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^4\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,x^4}dx\right)a+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,x}dx\right)b$$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^(1/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**4),x)*a + int(1/((a + b*x**3)**(1/3)*x),x)*b
```

3.331 $\int \frac{A+Bx^3}{x^7 \sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 168

$$\int \frac{A+Bx^3}{x^7 \sqrt[3]{a+bx^3}}\,dx = -\frac{A(a+bx^3)^{2/3}}{6ax^6} + \frac{(2Ab-3aB)\,(a+bx^3)^{2/3}}{9a^2x^3} + \frac{b(2Ab-3aB)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}} - \frac{b(2Ab-3aB)\log(x)}{18a^{7/3}} + \frac{b(2Ab-3aB)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{7/3}}$$

output
```
-1/6*A*(b*x^3+a)^(2/3)/a/x^6+1/9*(2*A*b-3*B*a)*(b*x^3+a)^(2/3)/a^2/x^3+1/2
7*b*(2*A*b-3*B*a)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*
3^(1/2)/a^(7/3)-1/18*b*(2*A*b-3*B*a)*ln(x)/a^(7/3)+1/18*b*(2*A*b-3*B*a)*ln
(a^(1/3)-(b*x^3+a)^(1/3))/a^(7/3)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx^3}{x^7\sqrt[3]{a+bx^3}}\,dx = \frac{-\frac{3\sqrt[3]{a}(a+bx^3)^{2/3}(-4Abx^3+3a(A+2Bx^3))}{x^6}+2\sqrt{3}b(2Ab-3aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2b(2Ab-3aB)\log (}{54a^{7/3}}$$

input
```
Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^(1/3)),x]
```

output
```
((-3*a^(1/3)*(a + b*x^3)^(2/3)*(-4*A*b*x^3 + 3*a*(A + 2*B*x^3)))/x^6 + 2*S
qrt[3]*b*(2*A*b - 3*a*B)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3
]] + 2*b*(2*A*b - 3*a*B)*Log[-a^(1/3) + (a + b*x^3)^(1/3)] + b*(-2*A*b + 3
*a*B)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*a^
(7/3))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {948, 87, 52, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^7\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^9\sqrt[3]{bx^3+a}}dx^3$$

$$\Big\downarrow \; 87$$

$$\frac{1}{3}\left(-\frac{(2Ab-3aB)\int \frac{1}{x^6\sqrt[3]{bx^3+a}}dx^3}{3a}-\frac{A\left(a+bx^3\right)^{2/3}}{2ax^6}\right)$$

$\Big\downarrow$ 52

$$\frac{1}{3}\left(-\frac{(2Ab-3aB)\left(-\frac{b\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3}{3a}-\frac{\left(a+bx^3\right)^{2/3}}{ax^3}\right)}{3a}-\frac{A\left(a+bx^3\right)^{2/3}}{2ax^6}\right)$$

$\Big\downarrow$ 67

$$\frac{1}{3}\left(-\frac{(2Ab-3aB)\left(-\frac{b\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{3a}-\frac{\left(a+bx^3\right)^{2}}{ax^3}\right.}{3a}\right.$$

$\Big\downarrow$ 16

$$\frac{1}{3}\left(-\frac{(2Ab-3aB)\left(-\frac{b\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{3a}-\frac{\left(a+bx^3\right)^{2/3}}{ax^3}\right)}{3a}-\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(-\frac{(2Ab-3aB)\left(-\frac{b\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{3a}-\frac{\left(a+bx^3\right)^{2/3}}{ax^3}\right)}{3a}-\frac{A\left(a+bx^3\right.}{2ax^6}\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(-\frac{(2Ab-3aB)\left(-\frac{b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{3a}-\frac{\left(a+bx^3\right)^{2/3}}{ax^3}\right)}{3a}-\frac{A\left(a+bx^3\right)^2}{2ax^6}$$

input
```
Int[(A + B*x^3)/(x^7*(a + b*x^3)^(1/3)),x]
```

output
```
(-1/2*(A*(a + b*x^3)^(2/3))/(a*x^6) - ((2*A*b - 3*a*B)*(-((a + b*x^3)^(2/3
)/(a*x^3)) - (b*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[
3]])/a^(1/3) - Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])
/(2*a^(1/3))))/(3*a)))/(3*a))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{\frac{9\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(2B\,x^{3}+A\right)a^{\frac{4}{3}}}{2}+\left(-6A\left(b\,x^{3}+a\right)^{\frac{2}{3}}a^{\frac{1}{3}}+\left(Ab-\frac{3Ba}{2}\right)x^{3}\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+\right.\right.\right.\right.}{27a^{\frac{7}{3}}x^{6}}$

input
```
int((B*x^3+A)/x^7/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/27*(9/2*(b*x^3+a)^(2/3)*(2*B*x^3+A)*a^(4/3)+(-6*A*(b*x^3+a)^(2/3)*a^(1/
3)+(A*b-3/2*B*a)*x^3*(-2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^
(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))-2*ln((b
*x^3+a)^(1/3)-a^(1/3))))*b*x^3)/a^(7/3)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.42

$$\int \frac{A+Bx^3}{x^7\sqrt[3]{a+bx^3}}\,dx$$
$$=\left[-\frac{3\sqrt{\frac{1}{3}}(3\,Ba^2b-2\,Aab^2)x^6\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3+3\,\sqrt{\frac{1}{3}}\left(2\,(bx^3+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx^3+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^3}\right)-\right.$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[-1/54*(3*sqrt(1/3)*(3*B*a^2*b - 2*A*a*b^2)*x^6*sqrt(-1/a^(2/3))*log((2*b*
x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(
4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) - (3*B*a*
b - 2*A*b^2)*a^(2/3)*x^6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3)
 + a^(2/3)) + 2*(3*B*a*b - 2*A*b^2)*a^(2/3)*x^6*log((b*x^3 + a)^(1/3) - a^
(1/3)) + 3*(2*(3*B*a^2 - 2*A*a*b)*x^3 + 3*A*a^2)*(b*x^3 + a)^(2/3))/(a^3*x
^6), 1/54*((3*B*a*b - 2*A*b^2)*a^(2/3)*x^6*log((b*x^3 + a)^(2/3) + (b*x^3
+ a)^(1/3)*a^(1/3) + a^(2/3)) - 2*(3*B*a*b - 2*A*b^2)*a^(2/3)*x^6*log((b*x
^3 + a)^(1/3) - a^(1/3)) - 6*sqrt(1/3)*(3*B*a^2*b - 2*A*a*b^2)*x^6*arctan(
sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - 3*(2*(3*B*a^2
 - 2*A*a*b)*x^3 + 3*A*a^2)*(b*x^3 + a)^(2/3))/(a^3*x^6)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.54 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.49

$$\int \frac{A+Bx^3}{x^7\sqrt[3]{a+bx^3}}\,dx = -\frac{A\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3\sqrt[3]{b}x^7\Gamma\left(\frac{10}{3}\right)} - \frac{B\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3\sqrt[3]{b}x^4\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**7/(b*x**3+a)**(1/3),x)
```

output
```
-A*gamma(7/3)*hyper((1/3, 7/3), (10/3,), a*exp_polar(I*pi)/(b*x**3))/(3*b*
*(1/3)*x**7*gamma(10/3)) - B*gamma(4/3)*hyper((1/3, 4/3), (7/3,), a*exp_po
lar(I*pi)/(b*x**3))/(3*b**(1/3)*x**4*gamma(7/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(135) = 270$.

Time = 0.11 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.68

$$\int \frac{A + Bx^3}{x^7 \sqrt[3]{a + bx^3}} \, dx$$
$$= \frac{1}{54} \left(\frac{4\sqrt{3}b^2 \arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}} - \frac{2\,b^2 \log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{a^{\frac{7}{3}}} + \frac{4\,b^2 \log\left((bx^3\right.}{a} \right.$$
$$- \frac{1}{18} \left(\frac{2\sqrt{3}b \arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}} + \frac{6\,(bx^3+a)^{\frac{2}{3}}b}{(bx^3+a)a - a^2} - \frac{b \log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}} \right.$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/54*(4*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(7/3) - 2*b^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) +
a^(2/3))/a^(7/3) + 4*b^2*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(7/3) + 3*(4*(
b*x^3 + a)^(5/3)*b^2 - 7*(b*x^3 + a)^(2/3)*a*b^2)/((b*x^3 + a)^2*a^2 - 2*(
b*x^3 + a)*a^3 + a^4))*A - 1/18*(2*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3
+ a)^(1/3) + a^(1/3))/a^(1/3))/a^(4/3) + 6*(b*x^3 + a)^(2/3)*b/((b*x^3 + a
)*a - a^2) - b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)
)/a^(4/3) + 2*b*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(4/3))*B
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.31

$$\int \frac{A+Bx^3}{x^7\sqrt[3]{a+bx^3}}\,dx$$

$$= \frac{\frac{(3\,Bab^2-2\,Ab^3)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{7}{3}}}-\frac{2\sqrt{3}\left(3\,Ba^{\frac{5}{3}}b^2-2\,Aa^{\frac{2}{3}}b^3\right)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^3}-\frac{2\left(3\,Ba^{\frac{4}{3}}b^2-2\,Aa^{\frac{1}{3}}\right.}{}}{54\,b}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
1/54*((3*B*a*b^2 - 2*A*b^3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1
/3) + a^(2/3))/a^(7/3) - 2*sqrt(3)*(3*B*a^(5/3)*b^2 - 2*A*a^(2/3)*b^3)*arc
tan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^3 - 2*(3*B*a^(4
/3)*b^2 - 2*A*a^(1/3)*b^3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(8/3) -
 3*(6*(b*x^3 + a)^(5/3)*B*a*b^2 - 6*(b*x^3 + a)^(2/3)*B*a^2*b^2 - 4*(b*x^3
 + a)^(5/3)*A*b^3 + 7*(b*x^3 + a)^(2/3)*A*a*b^3)/(a^2*b^2*x^6))/b
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.18

$$\int \frac{A+Bx^3}{x^7\sqrt[3]{a+bx^3}}\,dx = \frac{2\,A\,b^2\,\ln\left(\left(b\,x^3+a\right)^{1/3}-a^{1/3}\right)}{27\,a^{7/3}} + \frac{\ln\left(\frac{\left(B\,b-\sqrt{3}\,B\,b\,1\mathrm{i}\right)^2}{36\,a^{5/3}}-\frac{B^2\,b^2\,\left(b\,x^3+a\right)^{1/3}}{9\,a^2}\right)\,\left(B\,b-\sqrt{3}\,B\,b\,1\mathrm{i}\right)}{18\,a^{4/3}} + \frac{\ln\left(\frac{\left(B\,b+\sqrt{3}\,B\,b\,1\mathrm{i}\right)^2}{36\,a^{5/3}}-\frac{B^2\,b^2\,\left(b\,x^3+a\right)^{1/3}}{9\,a^2}\right)\,\left(B\,b+\sqrt{3}\,B\,b\,1\mathrm{i}\right)}{18\,a^{4/3}} - \frac{B\,b\,\ln\left(\left(b\,x^3+a\right)^{1/3}-a^{1/3}\right)}{9\,a^{4/3}} - \frac{\frac{7\,A\,b^2\,\left(b\,x^3+a\right)^{2/3}}{18\,a}-\frac{2\,A\,b^2\,\left(b\,x^3+a\right)^{5/3}}{9\,a^2}}{\left(b\,x^3+a\right)^2-2\,a\,\left(b\,x^3+a\right)+a^2} - \frac{B\,\left(b\,x^3+a\right)^{2/3}}{3\,a\,x^3} + \frac{2\,A\,b^2\,\ln\left(\frac{4\,A^2\,b^4\,\left(b\,x^3+a\right)^{1/3}}{81\,a^4}-\frac{4\,A^2\,b^4\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{81\,a^{11/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27\,a^{7/3}} - \frac{2\,A\,b^2\,\ln\left(\frac{4\,A^2\,b^4\,\left(b\,x^3+a\right)^{1/3}}{81\,a^4}-\frac{4\,A^2\,b^4\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{81\,a^{11/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27\,a^{7/3}}$$

input
```
int((A + B*x^3)/(x^7*(a + b*x^3)^(1/3)),x)
```

output
```
(log((B*b - 3^(1/2)*B*b*1i)^2/(36*a^(5/3)) - (B^2*b^2*(a + b*x^3)^(1/3))/(
9*a^2))*(B*b - 3^(1/2)*B*b*1i))/(18*a^(4/3)) - ((7*A*b^2*(a + b*x^3)^(2/3)
)/(18*a) - (2*A*b^2*(a + b*x^3)^(5/3))/(9*a^2))/((a + b*x^3)^2 - 2*a*(a +
b*x^3) + a^2) + (log((B*b + 3^(1/2)*B*b*1i)^2/(36*a^(5/3)) - (B^2*b^2*(a +
 b*x^3)^(1/3))/(9*a^2))*(B*b + 3^(1/2)*B*b*1i))/(18*a^(4/3)) - (B*b*log((a
 + b*x^3)^(1/3) - a^(1/3)))/(9*a^(4/3)) + (2*A*b^2*log((a + b*x^3)^(1/3) -
 a^(1/3)))/(27*a^(7/3)) - (B*(a + b*x^3)^(2/3))/(3*a*x^3) + (2*A*b^2*log((
4*A^2*b^4*(a + b*x^3)^(1/3))/(81*a^4) - (4*A^2*b^4*((3^(1/2)*1i)/2 - 1/2)^
2)/(81*a^(11/3)))*((3^(1/2)*1i)/2 - 1/2))/(27*a^(7/3)) - (2*A*b^2*log((4*A
^2*b^4*(a + b*x^3)^(1/3))/(81*a^4) - (4*A^2*b^4*((3^(1/2)*1i)/2 + 1/2)^2)/
(81*a^(11/3)))*((3^(1/2)*1i)/2 + 1/2))/(27*a^(7/3))
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^7\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,x^7}dx\right)a+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,x^4}dx\right)b$$

input
```
int((B*x^3+A)/x^7/(b*x^3+a)^(1/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**7),x)*a + int(1/((a + b*x**3)**(1/3)*x**4),x
)*b
```

3.332 $\int \frac{x^3(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 143

$$\int \frac{x^3(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{(3Ab-2aB)x(a+bx^3)^{2/3}}{9b^2} + \frac{Bx^4(a+bx^3)^{2/3}}{6b} - \frac{a(3Ab-2aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{7/3}} + \frac{a(3Ab-2aB)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{7/3}}$$

output
```
1/9*(3*A*b-2*B*a)*x*(b*x^3+a)^(2/3)/b^2+1/6*B*x^4*(b*x^3+a)^(2/3)/b-1/27*a
*(3*A*b-2*B*a)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)
/b^(7/3)+1/18*a*(3*A*b-2*B*a)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.27

$$\int \frac{x^3(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{3\sqrt[3]{b}x(a+bx^3)^{2/3}(6Ab-4aB+3bBx^3)+2\sqrt{3}a(-3Ab+2aB)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-2a(-3Ab}{54b^{7/3}}$$

input
```
Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^(1/3),x]
```

output
```
(3*b^(1/3)*x*(a + b*x^3)^(2/3)*(6*A*b - 4*a*B + 3*b*B*x^3) + 2*Sqrt[3]*a*(
-3*A*b + 2*a*B)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3
))] - 2*a*(-3*A*b + 2*a*B)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + a*(-3*A
*b + 2*a*B)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2
/3)])/(54*b^(7/3))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 843, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(3Ab-2aB)\int \frac{x^3}{\sqrt[3]{bx^3+a}}dx}{3b}+\frac{Bx^4(a+bx^3)^{2/3}}{6b}$$

$$\Big\downarrow \text{843}$$

$$\frac{(3Ab-2aB)\left(\frac{x(a+bx^3)^{2/3}}{3b}-\frac{a\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{3b}\right)}{3b}+\frac{Bx^4(a+bx^3)^{2/3}}{6b}$$

$$\Big\downarrow \text{769}$$

$$\frac{(3Ab-2aB)\left(\frac{x(a+bx^3)^{2/3}}{3b}-\frac{a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{3b}\right)}{3b}+\frac{Bx^4(a+bx^3)^{2/3}}{6b}$$

input

```
Int[(x^3*(A + B*x^3))/(a + b*x^3)^(1/3),x]
```

output

```
(B*x^4*(a + b*x^3)^(2/3))/(6*b) + ((3*A*b - 2*a*B)*((x*(a + b*x^3)^(2/3))/
(3*b) - (a*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]
*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/(3*b)))/(3
*b)
```

Defintions of rubi rules used

rule 769

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(116) = 232$.

Time = 1.27 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.88

method	result
pseudoelliptic	$\frac{9B b^{\frac{4}{3}} x^4 (b x^3+a)^{\frac{2}{3}}+18A b^{\frac{4}{3}} x (b x^3+a)^{\frac{2}{3}}-12B a x b^{\frac{1}{3}} (b x^3+a)^{\frac{2}{3}}+6A \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(b x^3+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right)\sqrt{3}\, a b-4B \arctan}{}$

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/54*(9*B*b^(4/3)*x^4*(b*x^3+a)^(2/3)+18*A*b^(4/3)*x*(b*x^3+a)^(2/3)-12*B*
a*x*b^(1/3)*(b*x^3+a)^(2/3)+6*A*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^
(1/3))/b^(1/3)/x)*3^(1/2)*a*b-4*B*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a
)^(1/3))/b^(1/3)/x)*3^(1/2)*a^2+6*A*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a*b
-3*A*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a*b-4
*B*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^2+2*B*ln((b^(2/3)*x^2+b^(1/3)*(b*x
^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a^2)/b^(7/3)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.98

$$\int \frac{x^3(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \left[-\frac{3\sqrt{\frac{1}{3}}(2\,Ba^2b-3\,Aab^2)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\,\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+(bx^3+a)^{\frac{1}{3}}bx^2-2\,(bx^3\right.\right.}{} \right.$$

$$-\frac{2\,(2\,Ba^2-3\,Aab)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right)-(2\,Ba^2-3\,Aab)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)+\frac{6\,\sqrt{}}{}}{54\,b^3}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[-1/54*(3*sqrt(1/3)*(2*B*a^2*b - 3*A*a*b^2)*sqrt(-1/b^(2/3))*log(3*b*x^3 -
 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^
(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(
2*B*a^2 - 3*A*a*b)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (2*B*
a^2 - 3*A*a*b)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b
*x^3 + a)^(2/3))/x^2) - 3*(3*B*b^2*x^4 - 2*(2*B*a*b - 3*A*b^2)*x)*(b*x^3 +
 a)^(2/3))/b^3, -1/54*(2*(2*B*a^2 - 3*A*a*b)*b^(2/3)*log(-(b^(1/3)*x - (b*
x^3 + a)^(1/3))/x) - (2*B*a^2 - 3*A*a*b)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3
 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(2*B*a^2*b -
 3*A*a*b^2)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x)
)/b^(1/3) - 3*(3*B*b^2*x^4 - 2*(2*B*a*b - 3*A*b^2)*x)*(b*x^3 + a)^(2/3))/b
^3]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.65 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.56

$$\int \frac{x^3(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{Ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)} + \frac{Bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**3*(B*x**3+A)/(b*x**3+a)**(1/3),x)
```

output
```
A*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(1/3)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((1/3, 7/3), (10/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(10/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(116) = 232.

Time = 0.14 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.29

$$\int \frac{x^3(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx$$
$$= \frac{1}{18}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} - \frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a}{x}\right.}{b^{\frac{4}{3}}}\right.$$
$$- \frac{1}{54}\left(\frac{4\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}} - \frac{2\,a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}} + \frac{4\,a^2\log\left(-b^{\frac{1}{3}}\right.}{b^{\frac{7}{3}}}\right.$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/18*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(
1/3))/b^(4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^
(2/3)/x^2)/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(
b*x^3 + a)^(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*A - 1/54*(4*sqrt(3)*a^
2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) -
2*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b
^(7/3) + 4*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) - 3*(7*(b*x^3 +
 a)^(2/3)*a^2*b/x^2 - 4*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^4 - 2*(b*x^3 + a)*b^
3/x^3 + (b*x^3 + a)^2*b^2/x^6))*B
```

Giac [F]

$$\int \frac{x^3(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(Bx^3+A)x^3}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^3\,(B\,x^3+A)}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((x^3*(A + B*x^3))/(a + b*x^3)^(1/3),x)
```

output
```
int((x^3*(A + B*x^3))/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)b + \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)a$$

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^(1/3),x)
```

output
```
int(x**6/(a + b*x**3)**(1/3),x)*b + int(x**3/(a + b*x**3)**(1/3),x)*a
```

3.333 $\int \frac{A+Bx^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \frac{A+Bx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{Bx(a+bx^3)^{2/3}}{3b} + \frac{(3Ab-aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}} - \frac{(3Ab-aB)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}$$

output
```
1/3*B*x*(b*x^3+a)^(2/3)/b+1/9*(3*A*b-B*a)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3
+a)^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)-1/6*(3*A*b-B*a)*ln(-b^(1/3)*x+(b*x^3+a
)^(1/3))/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.47

$$\int \frac{A+Bx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{6\sqrt[3]{b}Bx(a+bx^3)^{2/3}+2\sqrt{3}(3Ab-aB)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+2(-3Ab+aB)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+ \right.}{18b^{4/3}}$$

input
```
Integrate[(A + B*x^3)/(a + b*x^3)^(1/3),x]
```

output
```
(6*b^(1/3)*B*x*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(3*A*b - a*B)*ArcTan[(Sqrt[3]
*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 2*(-3*A*b + a*B)*Log[-(b^
(1/3)*x) + (a + b*x^3)^(1/3)] + (3*A*b - a*B)*Log[b^(2/3)*x^2 + b^(1/3)*x*
(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*b^(4/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(3Ab-aB)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{3b}+\frac{Bx\left(a+bx^3\right)^{2/3}}{3b}$$

$$\Big\downarrow \text{769}$$

$$\frac{(3Ab-aB)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{3b}+\frac{Bx\left(a+bx^3\right)^{2/3}}{3b}$$

input
```
Int[(A + B*x^3)/(a + b*x^3)^(1/3),x]
```

output
```
(B*x*(a + b*x^3)^(2/3))/(3*b) + ((3*A*b - a*B)*(ArcTan[(1 + (2*b^(1/3)*x)/
(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*
x^3)^(1/3)]/(2*b^(1/3))))/(3*b)
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(88) = 176$.

Time = 1.15 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.03

method	result
pseudoelliptic	$\frac{6\left(b\,x^{3}+a\right)^{\frac{2}{3}} x B\, b^{\frac{1}{3}}-6 A \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right)\sqrt{3}\, b+2 B \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right)\sqrt{3}\, a-6 A \ln\left(\frac{-b^{\frac{1}{3}}\ldots}{}\right)}{}$

input
```
int((B*x^3+A)/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/18*(6*(b*x^3+a)^(2/3)*x*B*b^(1/3)-6*A*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b
*x^3+a)^(1/3))/b^(1/3)/x)*3^(1/2)*b+2*B*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b
*x^3+a)^(1/3))/b^(1/3)/x)*3^(1/2)*a-6*A*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)
*b+3*A*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*b+2
*B*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a-B*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a
)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a)/b^(4/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(86) = 172$.

Time = 0.09 (sec) , antiderivative size = 411, normalized size of antiderivative = 3.70

$$\int \frac{A + Bx^3}{\sqrt[3]{a + bx^3}}\,dx = \left[\frac{6\,(bx^3+a)^{\frac{2}{3}}Bbx - 3\,\sqrt{\frac{1}{3}}(Bab - 3\,Ab^2)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3 - 3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2 - 3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3 - \right.\right.}{}\right.$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/18*(6*(b*x^3 + a)^(2/3)*B*b*x - 3*sqrt(1/3)*(B*a*b - 3*A*b^2)*sqrt((-b)
^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*(
(-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3
)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*(B*a - 3*A*b)*(-b)^(2/3)*log(((-b)^(1/3
)*x + (b*x^3 + a)^(1/3))/x) - (B*a - 3*A*b)*(-b)^(2/3)*log(((-b)^(2/3)*x^2
 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/b^2, 1/18*(6*
(b*x^3 + a)^(2/3)*B*b*x + 6*sqrt(1/3)*(B*a*b - 3*A*b^2)*sqrt(-(-b)^(1/3)/b
)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/
b)/x) + 2*(B*a - 3*A*b)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/
x) - (B*a - 3*A*b)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)
^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/b^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{Ax\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{Bx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((B*x**3+A)/(b*x**3+a)**(1/3),x)
```

output
```
A*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
1/3)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(1/3)*gamma(7/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(86) = 172$.

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.20

$$\int \frac{A+Bx^3}{\sqrt[3]{a+bx^3}}\,dx = -\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right.}{b^{\frac{1}{3}}}\right.$$
$$+\frac{1}{18}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3\ldots\right.}{b^{\frac{4}{3}}}\right.$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*A + 1/18*(
2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/
b^(4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3
+ a)^(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*B
```

Giac [F]

$$\int \frac{A+Bx^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{B\,x^3+A}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((A + B*x^3)/(a + b*x^3)^(1/3),x)
```

output
```
int((A + B*x^3)/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)b + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)a$$

input
```
int((B*x^3+A)/(b*x^3+a)^(1/3),x)
```

output
```
int(x**3/(a + b*x**3)**(1/3),x)*b + int(1/(a + b*x**3)**(1/3),x)*a
```

3.334 $\int \frac{A+Bx^3}{x^3 \sqrt[3]{a+bx^3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{A+Bx^3}{x^3 \sqrt[3]{a+bx^3}} \, dx = -\frac{A(a+bx^3)^{2/3}}{2ax^2} + \frac{B \arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{B \log\left(-\sqrt[3]{b}x + \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

output
```
-1/2*A*(b*x^3+a)^(2/3)/a/x^2+1/3*B*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/
3))*3^(1/2))*3^(1/2)/b^(1/3)-1/2*B*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.62

$$\int \frac{A+Bx^3}{x^3\sqrt[3]{a+bx^3}}\,dx = -\frac{A(a+bx^3)^{2/3}}{2ax^2} + \frac{B\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{B\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}} + \frac{B\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{6\sqrt[3]{b}}$$

input
```
Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^(1/3)),x]
```

output
```
-1/2*(A*(a + b*x^3)^(2/3))/(a*x^2) + (B*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3
)*x + 2*(a + b*x^3)^(1/3))])/(Sqrt[3]*b^(1/3)) - (B*Log[-(b^(1/3)*x) + (a
+ b*x^3)^(1/3)])/(3*b^(1/3)) + (B*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^
(1/3) + (a + b*x^3)^(2/3)])/(6*b^(1/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {953, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^3\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \text{953}$$

$$B\int \frac{1}{\sqrt[3]{bx^3+a}}dx - \frac{A(a+bx^3)^{2/3}}{2ax^2}$$

$$\Big\downarrow \text{769}$$

$$B\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)-\frac{A(a+bx^3)^{2/3}}{2ax^2}$$

input
```
Int[(A + B*x^3)/(x^3*(a + b*x^3)^(1/3)),x]
```

output
```
-1/2*(A*(a + b*x^3)^(2/3))/(a*x^2) + B*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x
^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/
3)]/(2*b^(1/3)))
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 953
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[d/e^n   Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] &&
(IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && G
tQ[m + n, -1]))
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.41

method	result
pseudoelliptic	$-\frac{\frac{2B\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx^3+a)^{\frac{1}{3}}}{b^{\frac{1}{3}}}+x\right)}{3x}\right)ax^2}{3}+\frac{2B\ln\left(\frac{-b^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)ax^2}{3}-\frac{B\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)a\,x \cdots}{3}\cdots}{2b^{\frac{1}{3}}a\,x^2}$

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2/3*B*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)/b^(1/3)+x)/x)*a*
x^2+2/3*B*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a*x^2-1/3*B*ln((b^(2/3)*x^2+b
^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a*x^2+A*(b*x^3+a)^(2/3)*b^(
1/3))/b^(1/3)/a/x^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^3\sqrt[3]{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx^3}{x^3\sqrt[3]{a+bx^3}}\,dx = \frac{Ab^{\frac{2}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{2}{3}\right)}{3a\Gamma\left(\frac{1}{3}\right)} + \frac{Bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**3/(b*x**3+a)**(1/3),x)
```

output
```
A*b**(2/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-2/3)/(3*a*gamma(1/3)) + B*x*gamm
a(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gam
ma(4/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx^3}{x^3\sqrt[3]{a+bx^3}}\,dx = -\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right.}{b^{\frac{1}{3}}}\right. - \frac{(bx^3+a)^{\frac{2}{3}}A}{2\,ax^2}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*B - 1/2*(b
*x^3 + a)^(2/3)*A/(a*x^2)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^3\sqrt[3]{a+bx^3}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{1}{3}}x^3}\,dx$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(1/3)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx^3}{x^3\sqrt[3]{a+bx^3}}\,dx=\frac{B\,x\,\left(\frac{b\,x^3}{a}+1\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{(b\,x^3+a)^{1/3}}-\frac{A\,(b\,x^3+a)^{2/3}}{2\,a\,x^2}$$

input
```
int((A + B*x^3)/(x^3*(a + b*x^3)^(1/3)),x)
```

output
```
(B*x*((b*x^3)/a + 1)^(1/3)*hypergeom([1/3, 1/3], 4/3, -(b*x^3)/a))/(a + b*
x^3)^(1/3) - (A*(a + b*x^3)^(2/3))/(2*a*x^2)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^3\sqrt[3]{a+bx^3}}\,dx=\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,x^3}dx\right)a$$

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^(1/3),x)
```

output
```
int(1/(a + b*x**3)**(1/3),x)*b + int(1/((a + b*x**3)**(1/3)*x**3),x)*a
```

3.335 $\int \frac{A+Bx^3}{x^6 \sqrt[3]{a+bx^3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{A+Bx^3}{x^6 \sqrt[3]{a+bx^3}} \, dx = -\frac{A(a+bx^3)^{2/3}}{5ax^5} + \frac{(3Ab-5aB)(a+bx^3)^{2/3}}{10a^2x^2}$$

output
```
-1/5*A*(b*x^3+a)^(2/3)/a/x^5+1/10*(3*A*b-5*B*a)*(b*x^3+a)^(2/3)/a^2/x^2
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx^3}{x^6 \sqrt[3]{a+bx^3}} \, dx = \frac{(a+bx^3)^{2/3}(-2aA+3Abx^3-5aBx^3)}{10a^2x^5}$$

input
```
Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^(1/3)),x]
```

output
```
((a + b*x^3)^(2/3)*(-2*a*A + 3*A*b*x^3 - 5*a*B*x^3))/(10*a^2*x^5)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {955, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^6 \sqrt[3]{a+bx^3}} \, dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(3Ab-5aB)\int \frac{1}{x^3 \sqrt[3]{bx^3+a}}dx}{5a}-\frac{A\left(a+bx^3\right)^{2/3}}{5ax^5}$$

$$\Big\downarrow \; 796$$

$$\frac{\left(a+bx^3\right)^{2/3}(3Ab-5aB)}{10a^2x^2}-\frac{A\left(a+bx^3\right)^{2/3}}{5ax^5}$$

input
```
Int[(A + B*x^3)/(x^6*(a + b*x^3)^(1/3)),x]
```

output
```
-1/5*(A*(a + b*x^3)^(2/3))/(a*x^5) + ((3*A*b - 5*a*B)*(a + b*x^3)^(2/3))/(
10*a^2*x^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result	size
pseudoelliptic	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(\left(\frac{5 B\,x^{3}}{2}+A\right) a-\frac{3 A b\,x^{3}}{2}\right)}{5 a^{2} x^{5}}$	36
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(-3 A b\,x^{3}+5 B a\,x^{3}+2 A a\right)}{10 a^{2} x^{5}}$	37
trager	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(-3 A b\,x^{3}+5 B a\,x^{3}+2 A a\right)}{10 a^{2} x^{5}}$	37
risch	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(-3 A b\,x^{3}+5 B a\,x^{3}+2 A a\right)}{10 a^{2} x^{5}}$	37
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(-3 A b\,x^{3}+5 B a\,x^{3}+2 A a\right)}{10 a^{2} x^{5}}$	37

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(b*x^3+a)^(2/3)*((5/2*B*x^3+A)*a-3/2*A*b*x^3)/a^2/x^5
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx^3}{x^6\sqrt[3]{a+bx^3}}\,dx = -\frac{((5\,Ba-3\,Ab)x^3+2\,Aa)(bx^3+a)^{\frac{2}{3}}}{10\,a^2x^5}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
-1/10*((5*B*a - 3*A*b)*x^3 + 2*A*a)*(b*x^3 + a)^(2/3)/(a^2*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(46) = 92.

Time = 1.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{A+Bx^3}{x^6\sqrt[3]{a+bx^3}}\,dx = -\frac{2Ab^{\frac{2}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{9ax^3\Gamma\left(\frac{1}{3}\right)} + \frac{Ab^{\frac{5}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3a^2\Gamma\left(\frac{1}{3}\right)} + \frac{Bb^{\frac{2}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{2}{3}\right)}{3a\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**6/(b*x**3+a)**(1/3),x)
```

output
```
-2*A*b**(2/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-5/3)/(9*a*x**3*gamma(1/3)) +
A*b**(5/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-5/3)/(3*a**2*gamma(1/3)) + B*b**
(2/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-2/3)/(3*a*gamma(1/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{x^6\sqrt[3]{a+bx^3}}\,dx = \frac{A\left(\frac{5\,(bx^3+a)^{\frac{2}{3}}b}{x^2}-\frac{2\,(bx^3+a)^{\frac{5}{3}}}{x^5}\right)}{10\,a^2}-\frac{(bx^3+a)^{\frac{2}{3}}B}{2\,ax^2}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/10*A*(5*(b*x^3 + a)^(2/3)*b/x^2 - 2*(b*x^3 + a)^(5/3)/x^5)/a^2 - 1/2*(b*
x^3 + a)^(2/3)*B/(a*x^2)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^6\sqrt[3]{a+bx^3}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{1}{3}}x^6}\,dx$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(1/3)*x^6), x)
```

Mupad [B] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^3}{x^6\sqrt[3]{a+bx^3}}\,dx = -\frac{(b\,x^3+a)^{2/3}\,(2\,A\,a-3\,A\,b\,x^3+5\,B\,a\,x^3)}{10\,a^2\,x^5}$$

input
```
int((A + B*x^3)/(x^6*(a + b*x^3)^(1/3)),x)
```

output
```
-((a + b*x^3)^(2/3)*(2*A*a - 3*A*b*x^3 + 5*B*a*x^3))/(10*a^2*x^5)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^6\sqrt[3]{a+bx^3}}\,dx=\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,x^6}dx\right)a+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,x^3}dx\right)b$$

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^(1/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**6),x)*a + int(1/((a + b*x**3)**(1/3)*x**3),x
)*b
```

3.336 $\int \frac{A+Bx^3}{x^9\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{A+Bx^3}{x^9\sqrt[3]{a+bx^3}}\,dx = -\frac{A(a+bx^3)^{2/3}}{8ax^8}+\frac{(3Ab-4aB)\left(a+bx^3\right)^{2/3}}{20a^2x^5} - \frac{3b(3Ab-4aB)\left(a+bx^3\right)^{2/3}}{40a^3x^2}$$

output
```
-1/8*A*(b*x^3+a)^(2/3)/a/x^8+1/20*(3*A*b-4*B*a)*(b*x^3+a)^(2/3)/a^2/x^5-3/
40*b*(3*A*b-4*B*a)*(b*x^3+a)^(2/3)/a^3/x^2
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^3}{x^9\sqrt[3]{a+bx^3}}\,dx = \frac{\left(a+bx^3\right)^{2/3}\left(-5a^2A+6aAbx^3-8a^2Bx^3-9Ab^2x^6+12abBx^6\right)}{40a^3x^8}$$

input
```
Integrate[(A + B*x^3)/(x^9*(a + b*x^3)^(1/3)),x]
```

output
```
((a + b*x^3)^(2/3)*(-5*a^2*A + 6*a*A*b*x^3 - 8*a^2*B*x^3 - 9*A*b^2*x^6 + 1
2*a*b*B*x^6))/(40*a^3*x^8)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^9\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \ 955$$

$$-\frac{(3Ab-4aB)\int \frac{1}{x^6\sqrt[3]{bx^3+a}}dx}{4a}-\frac{A\left(a+bx^3\right)^{2/3}}{8ax^8}$$

$$\Big\downarrow \ 803$$

$$-\frac{(3Ab-4aB)\left(-\frac{3b\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx}{5a}-\frac{\left(a+bx^3\right)^{2/3}}{5ax^5}\right)}{4a}-\frac{A\left(a+bx^3\right)^{2/3}}{8ax^8}$$

$$\Big\downarrow \ 796$$

$$-\frac{\left(\frac{3b\left(a+bx^3\right)^{2/3}}{10a^2x^2}-\frac{\left(a+bx^3\right)^{2/3}}{5ax^5}\right)(3Ab-4aB)}{4a}-\frac{A\left(a+bx^3\right)^{2/3}}{8ax^8}$$

input
```
Int[(A + B*x^3)/(x^9*(a + b*x^3)^(1/3)),x]
```

output
```
-1/8*(A*(a + b*x^3)^(2/3))/(a*x^8) - ((3*A*b - 4*a*B)*(-1/5*(a + b*x^3)^(2
/3)/(a*x^5) + (3*b*(a + b*x^3)^(2/3))/(10*a^2*x^2)))/(4*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(\left(\frac{8 B\,x^{3}}{5}+A\right) a^{2}-\frac{6 b\,x^{3} \left(2 B\,x^{3}+A\right) a}{5}+\frac{9 A\,b^{2} x^{6}}{5}\right)}{8 a^{3} x^{8}}$	55
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(9 A\,b^{2} x^{6}-12 B a b\,x^{6}-6 a A b\,x^{3}+8 B\,a^{2} x^{3}+5 a^{2} A\right)}{40 a^{3} x^{8}}$	59
trager	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(9 A\,b^{2} x^{6}-12 B a b\,x^{6}-6 a A b\,x^{3}+8 B\,a^{2} x^{3}+5 a^{2} A\right)}{40 a^{3} x^{8}}$	59
risch	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(9 A\,b^{2} x^{6}-12 B a b\,x^{6}-6 a A b\,x^{3}+8 B\,a^{2} x^{3}+5 a^{2} A\right)}{40 a^{3} x^{8}}$	59
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(9 A\,b^{2} x^{6}-12 B a b\,x^{6}-6 a A b\,x^{3}+8 B\,a^{2} x^{3}+5 a^{2} A\right)}{40 a^{3} x^{8}}$	59

input
```
int((B*x^3+A)/x^9/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^3+a)^(2/3)*((8/5*B*x^3+A)*a^2-6/5*b*x^3*(2*B*x^3+A)*a+9/5*A*b^2*
x^6)/a^3/x^8
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^3}{x^9\sqrt[3]{a+bx^3}}\,dx = \frac{(3\,(4\,Bab-3\,Ab^2)x^6-2\,(4\,Ba^2-3\,Aab)x^3-5\,Aa^2)(bx^3+a)^{\frac{2}{3}}}{40\,a^3x^8}$$

input
```
integrate((B*x^3+A)/x^9/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
1/40*(3*(4*B*a*b - 3*A*b^2)*x^6 - 2*(4*B*a^2 - 3*A*a*b)*x^3 - 5*A*a^2)*(b*
x^3 + a)^(2/3)/(a^3*x^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(78) = 156.

Time = 1.49 (sec) , antiderivative size = 490, normalized size of antiderivative = 5.83

$$\begin{aligned}\int \frac{A+Bx^3}{x^9\sqrt[3]{a+bx^3}}\,dx &= \frac{10Aa^4b^{\frac{14}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{1}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{1}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{1}{3}\right)} \\ &+ \frac{8Aa^3b^{\frac{17}{3}}x^3\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{1}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{1}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{1}{3}\right)} \\ &+ \frac{4Aa^2b^{\frac{20}{3}}x^6\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{1}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{1}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{1}{3}\right)} \\ &+ \frac{24Aab^{\frac{23}{3}}x^9\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{1}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{1}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{1}{3}\right)} \\ &+ \frac{18Ab^{\frac{26}{3}}x^{12}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{1}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{1}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{1}{3}\right)} \\ &- \frac{2Bb^{\frac{2}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{9ax^3\Gamma\left(\frac{1}{3}\right)} + \frac{Bb^{\frac{5}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3a^2\Gamma\left(\frac{1}{3}\right)}\end{aligned}$$

input
```
integrate((B*x**3+A)/x**9/(b*x**3+a)**(1/3),x)
```

output
```
10*A*a**4*b**(14/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(27*a**5*b**4*x**6
*gamma(1/3) + 54*a**4*b**5*x**9*gamma(1/3) + 27*a**3*b**6*x**12*gamma(1/3)
) + 8*A*a**3*b**(17/3)*x**3*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(27*a**5*b
**4*x**6*gamma(1/3) + 54*a**4*b**5*x**9*gamma(1/3) + 27*a**3*b**6*x**12*ga
mma(1/3)) + 4*A*a**2*b**(20/3)*x**6*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(2
7*a**5*b**4*x**6*gamma(1/3) + 54*a**4*b**5*x**9*gamma(1/3) + 27*a**3*b**6*
x**12*gamma(1/3)) + 24*A*a*b**(23/3)*x**9*(a/(b*x**3) + 1)**(2/3)*gamma(-8
/3)/(27*a**5*b**4*x**6*gamma(1/3) + 54*a**4*b**5*x**9*gamma(1/3) + 27*a**3
*b**6*x**12*gamma(1/3)) + 18*A*b**(26/3)*x**12*(a/(b*x**3) + 1)**(2/3)*gam
ma(-8/3)/(27*a**5*b**4*x**6*gamma(1/3) + 54*a**4*b**5*x**9*gamma(1/3) + 27
*a**3*b**6*x**12*gamma(1/3)) - 2*B*b**(2/3)*(a/(b*x**3) + 1)**(2/3)*gamma(
-5/3)/(9*a*x**3*gamma(1/3)) + B*b**(5/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-5/
3)/(3*a**2*gamma(1/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^3}{x^9\sqrt[3]{a+bx^3}}\,dx = \frac{B\left(\frac{5\,(bx^3+a)^{\frac{2}{3}}b}{x^2}-\frac{2\,(bx^3+a)^{\frac{5}{3}}}{x^5}\right)}{10\,a^2} - \frac{A\left(\frac{20\,(bx^3+a)^{\frac{2}{3}}b^2}{x^2}-\frac{16\,(bx^3+a)^{\frac{5}{3}}b}{x^5}+\frac{5\,(bx^3+a)^{\frac{8}{3}}}{x^8}\right)}{40\,a^3}$$

input
```
integrate((B*x^3+A)/x^9/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/10*B*(5*(b*x^3 + a)^(2/3)*b/x^2 - 2*(b*x^3 + a)^(5/3)/x^5)/a^2 - 1/40*A*
(20*(b*x^3 + a)^(2/3)*b^2/x^2 - 16*(b*x^3 + a)^(5/3)*b/x^5 + 5*(b*x^3 + a)
^(8/3)/x^8)/a^3
```

Giac [F]

$$\int \frac{A+Bx^3}{x^9\sqrt[3]{a+bx^3}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{1}{3}}x^9}\,dx$$

input
```
integrate((B*x^3+A)/x^9/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(1/3)*x^9), x)
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^3}{x^9\sqrt[3]{a+bx^3}}\,dx = -\frac{(b\,x^3+a)^{2/3}\,(8\,B\,a^2\,x^3+5\,A\,a^2-12\,B\,a\,b\,x^6-6\,A\,a\,b\,x^3+9\,A\,b^2\,x^6)}{40\,a^3\,x^8}$$

input
```
int((A + B*x^3)/(x^9*(a + b*x^3)^(1/3)),x)
```

output
```
-((a + b*x^3)^(2/3)*(5*A*a^2 + 8*B*a^2*x^3 + 9*A*b^2*x^6 - 6*A*a*b*x^3 - 1
2*B*a*b*x^6))/(40*a^3*x^8)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^9\sqrt[3]{a+bx^3}}\,dx=\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,x^9}dx\right)a+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,x^6}dx\right)b$$

input
```
int((B*x^3+A)/x^9/(b*x^3+a)^(1/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**9),x)*a + int(1/((a + b*x**3)**(1/3)*x**6),x
)*b
```

3.337 $\int \frac{A+Bx^3}{x^{12}\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \frac{A+Bx^3}{x^{12}\sqrt[3]{a+bx^3}}\,dx = -\frac{A(a+bx^3)^{2/3}}{11ax^{11}} + \frac{(9Ab-11aB)\,(a+bx^3)^{2/3}}{88a^2x^8} - \frac{3b(9Ab-11aB)\,(a+bx^3)^{2/3}}{220a^3x^5} + \frac{9b^2(9Ab-11aB)\,(a+bx^3)^{2/3}}{440a^4x^2}$$

output
```
-1/11*A*(b*x^3+a)^(2/3)/a/x^11+1/88*(9*A*b-11*B*a)*(b*x^3+a)^(2/3)/a^2/x^8
-3/220*b*(9*A*b-11*B*a)*(b*x^3+a)^(2/3)/a^3/x^5+9/440*b^2*(9*A*b-11*B*a)*(
b*x^3+a)^(2/3)/a^4/x^2
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^3}{x^{12}\sqrt[3]{a+bx^3}}\,dx = \frac{(a+bx^3)^{2/3}\left(-40a^3A+45a^2Abx^3-55a^3Bx^3-54aAb^2x^6+66a^2bBx^6+81Ab^3x^9-99ab^2Bx^9\right)}{440a^4x^{11}}$$

input
```
Integrate[(A + B*x^3)/(x^12*(a + b*x^3)^(1/3)),x]
```

output
```
((a + b*x^3)^(2/3)*(-40*a^3*A + 45*a^2*A*b*x^3 - 55*a^3*B*x^3 - 54*a*A*b^2
*x^6 + 66*a^2*b*B*x^6 + 81*A*b^3*x^9 - 99*a*b^2*B*x^9))/(440*a^4*x^11)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{12}\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(9Ab-11aB)\int \frac{1}{x^9\sqrt[3]{bx^3+a}}dx}{11a}-\frac{A\left(a+bx^3\right)^{2/3}}{11ax^{11}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(9Ab-11aB)\left(-\frac{3b\int \frac{1}{x^6\sqrt[3]{bx^3+a}}dx}{4a}-\frac{\left(a+bx^3\right)^{2/3}}{8ax^8}\right)}{11a}-\frac{A\left(a+bx^3\right)^{2/3}}{11ax^{11}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(9Ab-11aB)\left(-\frac{3b\left(-\frac{3b\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx}{5a}-\frac{\left(a+bx^3\right)^{2/3}}{5ax^5}\right)}{4a}-\frac{\left(a+bx^3\right)^{2/3}}{8ax^8}\right)}{11a}-\frac{A\left(a+bx^3\right)^{2/3}}{11ax^{11}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{\left(-\frac{3b\left(\frac{3b\left(a+bx^3\right)^{2/3}}{10a^2x^2}-\frac{\left(a+bx^3\right)^{2/3}}{5ax^5}\right)}{4a}-\frac{\left(a+bx^3\right)^{2/3}}{8ax^8}\right)(9Ab-11aB)}{11a}-\frac{A\left(a+bx^3\right)^{2/3}}{11ax^{11}}$$

input
```
Int[(A + B*x^3)/(x^12*(a + b*x^3)^(1/3)),x]
```

output
```
-1/11*(A*(a + b*x^3)^(2/3))/(a*x^11) - ((9*A*b - 11*a*B)*(-1/8*(a + b*x^3)
^(2/3)/(a*x^8) - (3*b*(-1/5*(a + b*x^3)^(2/3)/(a*x^5) + (3*b*(a + b*x^3)^(
2/3))/(10*a^2*x^2)))/(4*a)))/(11*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.63

method	result	size
pseudoelliptic	$-\frac{\left(\left(\frac{11 B\,x^{3}}{8}+A\right) a^{3}-\frac{9 b \left(\frac{22 B\,x^{3}}{15}+A\right) x^{3} a^{2}}{8}+\frac{27 b^{2} x^{6} \left(\frac{11 B\,x^{3}}{6}+A\right) a}{20}-\frac{81 A\,x^{9} b^{3}}{40}\right) \left(b\,x^{3}+a\right)^{\frac{2}{3}}}{11 x^{11} a^{4}}$	74
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(-81 A\,x^{9} b^{3}+99 B\,x^{9} a\,b^{2}+54 A\,x^{6} a\,b^{2}-66 B\,x^{6} a^{2} b-45 a^{2} A b\,x^{3}+55 B\,x^{3} a^{3}+40 a^{3} A\right)}{440 x^{11} a^{4}}$	83
trager	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(-81 A\,x^{9} b^{3}+99 B\,x^{9} a\,b^{2}+54 A\,x^{6} a\,b^{2}-66 B\,x^{6} a^{2} b-45 a^{2} A b\,x^{3}+55 B\,x^{3} a^{3}+40 a^{3} A\right)}{440 x^{11} a^{4}}$	83
risch	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(-81 A\,x^{9} b^{3}+99 B\,x^{9} a\,b^{2}+54 A\,x^{6} a\,b^{2}-66 B\,x^{6} a^{2} b-45 a^{2} A b\,x^{3}+55 B\,x^{3} a^{3}+40 a^{3} A\right)}{440 x^{11} a^{4}}$	83
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(-81 A\,x^{9} b^{3}+99 B\,x^{9} a\,b^{2}+54 A\,x^{6} a\,b^{2}-66 B\,x^{6} a^{2} b-45 a^{2} A b\,x^{3}+55 B\,x^{3} a^{3}+40 a^{3} A\right)}{440 x^{11} a^{4}}$	83

input
```
int((B*x^3+A)/x^12/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/11*((11/8*B*x^3+A)*a^3-9/8*b*(22/15*B*x^3+A)*x^3*a^2+27/20*b^2*x^6*(11/
6*B*x^3+A)*a-81/40*A*x^9*b^3)*(b*x^3+a)^(2/3)/x^11/a^4
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^3}{x^{12}\sqrt[3]{a+bx^3}}\,dx = -\frac{(9\,(11\,Bab^2-9\,Ab^3)x^9-6\,(11\,Ba^2b-9\,Aab^2)x^6+40\,Aa^3+5\,(11\,Ba^3-9\,Aa^2b)x^3)(bx^3+a)^{\frac{2}{3}}}{440\,a^4x^{11}}$$

input
```
integrate((B*x^3+A)/x^12/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
-1/440*(9*(11*B*a*b^2 - 9*A*b^3)*x^9 - 6*(11*B*a^2*b - 9*A*a*b^2)*x^6 + 40
*A*a^3 + 5*(11*B*a^3 - 9*A*a^2*b)*x^3)*(b*x^3 + a)^(2/3)/(a^4*x^11)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1120 vs. 2(112) = 224.

Time = 2.72 (sec) , antiderivative size = 1120, normalized size of antiderivative = 9.57

$$\int \frac{A+Bx^3}{x^{12}\sqrt[3]{a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x**3+A)/x**12/(b*x**3+a)**(1/3),x)
```

output
```
-80*A*a**6*b**(29/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(81*a**7*b**9*x*
*9*gamma(1/3) + 243*a**6*b**10*x**12*gamma(1/3) + 243*a**5*b**11*x**15*gam
ma(1/3) + 81*a**4*b**12*x**18*gamma(1/3)) - 150*A*a**5*b**(32/3)*x**3*(a/(
b*x**3) + 1)**(2/3)*gamma(-11/3)/(81*a**7*b**9*x**9*gamma(1/3) + 243*a**6*
b**10*x**12*gamma(1/3) + 243*a**5*b**11*x**15*gamma(1/3) + 81*a**4*b**12*x
**18*gamma(1/3)) - 78*A*a**4*b**(35/3)*x**6*(a/(b*x**3) + 1)**(2/3)*gamma(
-11/3)/(81*a**7*b**9*x**9*gamma(1/3) + 243*a**6*b**10*x**12*gamma(1/3) + 2
43*a**5*b**11*x**15*gamma(1/3) + 81*a**4*b**12*x**18*gamma(1/3)) + 28*A*a*
*3*b**(38/3)*x**9*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(81*a**7*b**9*x**9*
gamma(1/3) + 243*a**6*b**10*x**12*gamma(1/3) + 243*a**5*b**11*x**15*gamma(
1/3) + 81*a**4*b**12*x**18*gamma(1/3)) + 252*A*a**2*b**(41/3)*x**12*(a/(b*
x**3) + 1)**(2/3)*gamma(-11/3)/(81*a**7*b**9*x**9*gamma(1/3) + 243*a**6*b*
*10*x**12*gamma(1/3) + 243*a**5*b**11*x**15*gamma(1/3) + 81*a**4*b**12*x**
18*gamma(1/3)) + 378*A*a*b**(44/3)*x**15*(a/(b*x**3) + 1)**(2/3)*gamma(-11
/3)/(81*a**7*b**9*x**9*gamma(1/3) + 243*a**6*b**10*x**12*gamma(1/3) + 243*
a**5*b**11*x**15*gamma(1/3) + 81*a**4*b**12*x**18*gamma(1/3)) + 162*A*b**(
47/3)*x**18*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(81*a**7*b**9*x**9*gamma(
1/3) + 243*a**6*b**10*x**12*gamma(1/3) + 243*a**5*b**11*x**15*gamma(1/3) +
 81*a**4*b**12*x**18*gamma(1/3)) + 10*B*a**4*b**(14/3)*(a/(b*x**3) + 1)**(
2/3)*gamma(-8/3)/(27*a**5*b**4*x**6*gamma(1/3) + 54*a**4*b**5*x**9*gamm...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x^{12}\sqrt[3]{a+bx^3}}\,dx = -\frac{B\left(\frac{20\,(bx^3+a)^{\frac{2}{3}}b^2}{x^2}-\frac{16\,(bx^3+a)^{\frac{5}{3}}b}{x^5}+\frac{5\,(bx^3+a)^{\frac{8}{3}}}{x^8}\right)}{40\,a^3} + \frac{\left(\frac{220\,(bx^3+a)^{\frac{2}{3}}b^3}{x^2}-\frac{264\,(bx^3+a)^{\frac{5}{3}}b^2}{x^5}+\frac{165\,(bx^3+a)^{\frac{8}{3}}b}{x^8}-\frac{40\,(bx^3+a)^{\frac{11}{3}}}{x^{11}}\right)A}{440\,a^4}$$

input
```
integrate((B*x^3+A)/x^12/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/40*B*(20*(b*x^3 + a)^(2/3)*b^2/x^2 - 16*(b*x^3 + a)^(5/3)*b/x^5 + 5*(b*
x^3 + a)^(8/3)/x^8)/a^3 + 1/440*(220*(b*x^3 + a)^(2/3)*b^3/x^2 - 264*(b*x^
3 + a)^(5/3)*b^2/x^5 + 165*(b*x^3 + a)^(8/3)*b/x^8 - 40*(b*x^3 + a)^(11/3)
/x^11)*A/a^4
```

Giac [F]

$$\int \frac{A+Bx^3}{x^{12}\sqrt[3]{a+bx^3}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{1}{3}}x^{12}}\,dx$$

input
```
integrate((B*x^3+A)/x^12/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(1/3)*x^12), x)
```

Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^3}{x^{12}\sqrt[3]{a+bx^3}}\,dx = \frac{\left(b\,x^3+a\right)^{2/3}\,\left(9\,A\,b-11\,B\,a\right)}{88\,a^2\,x^8}+\frac{\left(b\,x^3+a\right)^{2/3}\,\left(81\,A\,b^3-99\,B\,a\,b^2\right)}{440\,a^4\,x^2}-\frac{\left(27\,A\,b^2-33\,B\,a\,b\right)\,\left(b\,x^3+a\right)^{2/3}}{220\,a^3\,x^5}-\frac{A\,\left(b\,x^3+a\right)^{2/3}}{11\,a\,x^{11}}$$

input
```
int((A + B*x^3)/(x^12*(a + b*x^3)^(1/3)),x)
```

output
```
((a + b*x^3)^(2/3)*(9*A*b - 11*B*a))/(88*a^2*x^8) + ((a + b*x^3)^(2/3)*(81
*A*b^3 - 99*B*a*b^2))/(440*a^4*x^2) - ((27*A*b^2 - 33*B*a*b)*(a + b*x^3)^(
2/3))/(220*a^3*x^5) - (A*(a + b*x^3)^(2/3))/(11*a*x^11)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^{12}\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\,x^{12}}dx\right)a+\left(\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\,x^9}dx\right)b$$

input
```
int((B*x^3+A)/x^12/(b*x^3+a)^(1/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**12),x)*a + int(1/((a + b*x**3)**(1/3)*x**9),
x)*b
```

3.338 $\int \frac{x^4(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{x^4(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{Bx^5(a+bx^3)^{2/3}}{7b} + \frac{(7Ab-5aB)x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{35b\sqrt[3]{a+bx^3}}$$

output
```
1/7*B*x^5*(b*x^3+a)^(2/3)/b+1/35*(7*A*b-5*B*a)*x^5*(1+b*x^3/a)^(1/3)*hyper
geom([1/3, 5/3],[8/3],-b*x^3/a)/b/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{x^4(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\left(8Ax^5\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)+5Bx^8\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{11}{3},-\frac{bx^3}{a}\right)\right)}{40\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^(1/3),x]
```

output
```
((1 + (b*x^3)/a)^(1/3)*(8*A*x^5*Hypergeometric2F1[1/3, 5/3, 8/3, -((b*x^3)
/a)] + 5*B*x^8*Hypergeometric2F1[1/3, 8/3, 11/3, -((b*x^3)/a)]))/(40*(a +
b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^3)}{\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(7Ab-5aB)\int \frac{x^4}{\sqrt[3]{bx^3+a}}dx}{7b}+\frac{Bx^5(a+bx^3)^{2/3}}{7b}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}(7Ab-5aB)\int \frac{x^4}{\sqrt[3]{\frac{bx^3}{a}+1}}dx}{7b\sqrt[3]{a+bx^3}}+\frac{Bx^5(a+bx^3)^{2/3}}{7b}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}(7Ab-5aB)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{35b\sqrt[3]{a+bx^3}}+\frac{Bx^5(a+bx^3)^{2/3}}{7b}$$

input
```
Int[(x^4*(A + B*x^3))/(a + b*x^3)^(1/3),x]
```

output
```
(B*x^5*(a + b*x^3)^(2/3))/(7*b) + ((7*A*b - 5*a*B)*x^5*(1 + (b*x^3)/a)^(1/
3)*Hypergeometric2F1[1/3, 5/3, 8/3, -((b*x^3)/a)])/(35*b*(a + b*x^3)^(1/3)
)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^4(B\,x^3+A)}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^(1/3),x)
```

output
```
int(x^4*(B*x^3+A)/(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{x^4(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((B*x^7 + A*x^4)/(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{x^4(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{Ax^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{8}{3}\right)} + \frac{Bx^8\Gamma\left(\frac{8}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**4*(B*x**3+A)/(b*x**3+a)**(1/3),x)
```

output
```
A*x**5*gamma(5/3)*hyper((1/3, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(1/3)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((1/3, 8/3), (11/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(11/3))
```

Maxima [F]

$$\int \frac{x^4(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^4\,(B\,x^3+A)}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((x^4*(A + B*x^3))/(a + b*x^3)^(1/3),x)
```

output
```
int((x^4*(A + B*x^3))/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{x^7}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)b + \left(\int \frac{x^4}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)a$$

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^(1/3),x)
```

output
```
int(x**7/(a + b*x**3)**(1/3),x)*b + int(x**4/(a + b*x**3)**(1/3),x)*a
```

3.339 $\int \frac{x(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 86

$$\int \frac{x(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{Bx^2(a+bx^3)^{2/3}}{4b} + \frac{(2Ab-aB)x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{4b\sqrt[3]{a+bx^3}}$$

output
```
1/4*B*x^2*(b*x^3+a)^(2/3)/b+1/4*(2*A*b-B*a)*x^2*(1+b*x^3/a)^(1/3)*hypergeo
m([1/3, 2/3],[5/3],-b*x^3/a)/b/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{x(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\left(5Ax^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)+2Bx^5\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)\right)}{10\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(x*(A + B*x^3))/(a + b*x^3)^(1/3),x]
```

output
```
((1 + (b*x^3)/a)^(1/3)*(5*A*x^2*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)
/a)] + 2*B*x^5*Hypergeometric2F1[1/3, 5/3, 8/3, -((b*x^3)/a)]))/(10*(a + b
*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(2Ab-aB)\int \frac{x}{\sqrt[3]{bx^3+a}}dx}{2b}+\frac{Bx^2(a+bx^3)^{2/3}}{4b}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}(2Ab-aB)\int \frac{x}{\sqrt[3]{\frac{bx^3}{a}+1}}dx}{2b\sqrt[3]{a+bx^3}}+\frac{Bx^2(a+bx^3)^{2/3}}{4b}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}(2Ab-aB)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{4b\sqrt[3]{a+bx^3}}+\frac{Bx^2(a+bx^3)^{2/3}}{4b}$$

input
```
Int[(x*(A + B*x^3))/(a + b*x^3)^(1/3),x]
```

output
```
(B*x^2*(a + b*x^3)^(2/3))/(4*b) + ((2*A*b - a*B)*x^2*(1 + (b*x^3)/a)^(1/3)
*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(4*b*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x(B\,x^{3}+A)}{(b\,x^{3}+a)^{\frac{1}{3}}}dx$$

input
```
int(x*(B*x^3+A)/(b*x^3+a)^(1/3),x)
```

output
```
int(x*(B*x^3+A)/(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{x(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((B*x^4 + A*x)/(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{x(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{Ax^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)} + \frac{Bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x*(B*x**3+A)/(b*x**3+a)**(1/3),x)
```

output
```
A*x**2*gamma(2/3)*hyper((1/3, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(1/3)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((1/3, 5/3), (8/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*a**(1/3)*gamma(8/3))
```

Maxima [F]

$$\int \frac{x(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x/(b*x^3 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{x(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x\,(B\,x^3+A)}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((x*(A + B*x^3))/(a + b*x^3)^(1/3),x)
```

output
```
int((x*(A + B*x^3))/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{x(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{x^4}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)b + \left(\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)a$$

input
```
int(x*(B*x^3+A)/(b*x^3+a)^(1/3),x)
```

output
```
int(x**4/(a + b*x**3)**(1/3),x)*b + int(x/(a + b*x**3)**(1/3),x)*a
```

3.340 $\int \frac{A+Bx^3}{x^2\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{A+Bx^3}{x^2\sqrt[3]{a+bx^3}}\,dx = -\frac{A(a+bx^3)^{2/3}}{ax} + \frac{(Ab+aB)x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a\sqrt[3]{a+bx^3}}$$

output
```
-A*(b*x^3+a)^(2/3)/a/x+1/2*(A*b+B*a)*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3,
 2/3],[5/3],-b*x^3/a)/a/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^3}{x^2\sqrt[3]{a+bx^3}}\,dx = \frac{-2A(a+bx^3)+(Ab+aB)x^3\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2ax\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^(1/3)),x]
```

output
```
(-2*A*(a + b*x^3) + (A*b + a*B)*x^3*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F
1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(2*a*x*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^2\sqrt[3]{a+bx^3}}dx$$
$$\Big\downarrow \text{ 955}$$
$$\frac{(aB+Ab)\int \frac{x}{\sqrt[3]{bx^3+a}}dx}{a}-\frac{A\left(a+bx^3\right)^{2/3}}{ax}$$
$$\Big\downarrow \text{ 889}$$
$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}(aB+Ab)\int \frac{x}{\sqrt[3]{\frac{bx^3}{a}+1}}dx}{a\sqrt[3]{a+bx^3}}-\frac{A\left(a+bx^3\right)^{2/3}}{ax}$$
$$\Big\downarrow \text{ 888}$$
$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}(aB+Ab)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a\sqrt[3]{a+bx^3}}-\frac{A\left(a+bx^3\right)^{2/3}}{ax}$$

input
```
Int[(A + B*x^3)/(x^2*(a + b*x^3)^(1/3)),x]
```

output
```
-((A*(a + b*x^3)^(2/3))/(a*x)) + ((A*b + a*B)*x^2*(1 + (b*x^3)/a)^(1/3)*Hy
pergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(2*a*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{B\,x^{3}+A}{x^{2} \left(b\,x^{3}+a \right)^{\frac{1}{3}}}d x$$

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^(1/3),x)
```

output
```
int((B*x^3+A)/x^2/(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{A+Bx^3}{x^2\sqrt[3]{a+bx^3}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(2/3)/(b*x^5 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x^2\sqrt[3]{a+bx^3}}\,dx=\frac{A\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}x\Gamma\left(\frac{2}{3}\right)}+\frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**2/(b*x**3+a)**(1/3),x)
```

output
```
A*gamma(-1/3)*hyper((-1/3, 1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
1/3)*x*gamma(2/3)) + B*x**2*gamma(2/3)*hyper((1/3, 2/3), (5/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(1/3)*gamma(5/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^2\sqrt[3]{a+bx^3}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(1/3)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^2\sqrt[3]{a+bx^3}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(1/3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^2\sqrt[3]{a+bx^3}}\,dx = \int \frac{B\,x^3+A}{x^2\,{(b\,x^3+a)}^{1/3}}\,dx$$

input
```
int((A + B*x^3)/(x^2*(a + b*x^3)^(1/3)),x)
```

output
```
int((A + B*x^3)/(x^2*(a + b*x^3)^(1/3)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^2\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,x^2}dx\right)a$$

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^(1/3),x)
```

output
```
int(x/(a + b*x**3)**(1/3),x)*b + int(1/((a + b*x**3)**(1/3)*x**2),x)*a
```

3.341 $\int \frac{A+Bx^3}{x^5 \sqrt[3]{a+bx^3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{A+Bx^3}{x^5 \sqrt[3]{a+bx^3}} \, dx = -\frac{A(a+bx^3)^{2/3}}{4ax^4} + \frac{(Ab-2aB)\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{2ax\sqrt[3]{a+bx^3}}$$

output
```
-1/4*A*(b*x^3+a)^(2/3)/a/x^4+1/2*(A*b-2*B*a)*(1+b*x^3/a)^(1/3)*hypergeom([
-1/3, 1/3],[2/3],-b*x^3/a)/a/x/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x^5 \sqrt[3]{a+bx^3}} \, dx = \frac{-A(a+bx^3)+2(Ab-2aB)x^3\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{4ax^4\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^(1/3)),x]
```

output
```
(-(A*(a + b*x^3)) + 2*(A*b - 2*a*B)*x^3*(1 + (b*x^3)/a)^(1/3)*Hypergeometr
ic2F1[-1/3, 1/3, 2/3, -((b*x^3)/a)])/(4*a*x^4*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^5\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(Ab-2aB)\int \frac{1}{x^2\sqrt[3]{bx^3+a}}dx}{2a}-\frac{A\left(a+bx^3\right)^{2/3}}{4ax^4}$$

$$\Big\downarrow \text{889}$$

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}(Ab-2aB)\int \frac{1}{x^2\sqrt[3]{\frac{bx^3}{a}+1}}dx}{2a\sqrt[3]{a+bx^3}}-\frac{A\left(a+bx^3\right)^{2/3}}{4ax^4}$$

$$\Big\downarrow \text{888}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}(Ab-2aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{2ax\sqrt[3]{a+bx^3}}-\frac{A\left(a+bx^3\right)^{2/3}}{4ax^4}$$

input
```
Int[(A + B*x^3)/(x^5*(a + b*x^3)^(1/3)),x]
```

output
```
-1/4*(A*(a + b*x^3)^(2/3))/(a*x^4) + ((A*b - 2*a*B)*(1 + (b*x^3)/a)^(1/3)*
Hypergeometric2F1[-1/3, 1/3, 2/3, -((b*x^3)/a)])/(2*a*x*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{B\,x^{3}+A}{x^{5} \left(b\,x^{3}+a \right)^{\frac{1}{3}}}d x$$

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^(1/3),x)
```

output
```
int((B*x^3+A)/x^5/(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{A+Bx^3}{x^5\sqrt[3]{a+bx^3}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{1}{3}}x^5}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(2/3)/(b*x^8 + a*x^5), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{x^5\sqrt[3]{a+bx^3}}\,dx=\frac{A\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{1}{3}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{B\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**5/(b*x**3+a)**(1/3),x)
```

output
```
A*gamma(-4/3)*hyper((-4/3, 1/3), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(1/3)*x**4*gamma(-1/3)) + B*gamma(-1/3)*hyper((-1/3, 1/3), (2/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(1/3)*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^5\sqrt[3]{a+bx^3}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{1}{3}}x^5}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(1/3)*x^5), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^5\sqrt[3]{a+bx^3}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{1}{3}}x^5}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(1/3)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^5\sqrt[3]{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{x^5\,{\left(b\,x^3+a\right)}^{1/3}}\,dx$$

input
```
int((A + B*x^3)/(x^5*(a + b*x^3)^(1/3)),x)
```

output
```
int((A + B*x^3)/(x^5*(a + b*x^3)^(1/3)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^5\sqrt[3]{a+bx^3}}\,dx=\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,x^5}dx\right)a+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,x^2}dx\right)b$$

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^(1/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**5),x)*a + int(1/((a + b*x**3)**(1/3)*x**2),x
)*b
```

3.342 $\int \frac{A+Bx^3}{x^8 \sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{A+Bx^3}{x^8 \sqrt[3]{a+bx^3}}\,dx = -\frac{A(a+bx^3)^{2/3}}{7ax^7} + \frac{(5Ab-7aB)\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{28ax^4\sqrt[3]{a+bx^3}}$$

output
```
-1/7*A*(b*x^3+a)^(2/3)/a/x^7+1/28*(5*A*b-7*B*a)*(1+b*x^3/a)^(1/3)*hypergeo
m([-4/3, 1/3],[-1/3],-b*x^3/a)/a/x^4/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x^8 \sqrt[3]{a+bx^3}}\,dx = \frac{-4A(a+bx^3)+(5Ab-7aB)x^3\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{28ax^7\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^8*(a + b*x^3)^(1/3)),x]
```

output
```
(-4*A*(a + b*x^3) + (5*A*b - 7*a*B)*x^3*(1 + (b*x^3)/a)^(1/3)*Hypergeometr
ic2F1[-4/3, 1/3, -1/3, -((b*x^3)/a)])/(28*a*x^7*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^8 \sqrt[3]{a+bx^3}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(5Ab-7aB)\int \frac{1}{x^5 \sqrt[3]{bx^3+a}}dx}{7a}-\frac{A\left(a+bx^3\right)^{2/3}}{7ax^7}$$

$$\Big\downarrow \text{889}$$

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}(5Ab-7aB)\int \frac{1}{x^5 \sqrt[3]{\frac{bx^3}{a}+1}}dx}{7a\sqrt[3]{a+bx^3}}-\frac{A\left(a+bx^3\right)^{2/3}}{7ax^7}$$

$$\Big\downarrow \text{888}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}(5Ab-7aB)\, \text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{28ax^4\sqrt[3]{a+bx^3}}-\frac{A\left(a+bx^3\right)^{2/3}}{7ax^7}$$

input
```
Int[(A + B*x^3)/(x^8*(a + b*x^3)^(1/3)),x]
```

output
```
-1/7*(A*(a + b*x^3)^(2/3))/(a*x^7) + ((5*A*b - 7*a*B)*(1 + (b*x^3)/a)^(1/3
)*Hypergeometric2F1[-4/3, 1/3, -1/3, -((b*x^3)/a)])/(28*a*x^4*(a + b*x^3)^
(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{B\,x^{3}+A}{x^{8}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}d x$$

input
```
int((B*x^3+A)/x^8/(b*x^3+a)^(1/3),x)
```

output
```
int((B*x^3+A)/x^8/(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{A+Bx^3}{x^8\sqrt[3]{a+bx^3}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{1}{3}}x^8}\,dx$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(2/3)/(b*x^11 + a*x^8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^3}{x^8\sqrt[3]{a+bx^3}}\,dx=\frac{A\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},\frac{1}{3}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}x^7\Gamma\left(-\frac{4}{3}\right)}+\frac{B\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{1}{3}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**8/(b*x**3+a)**(1/3),x)
```

output
```
A*gamma(-7/3)*hyper((-7/3, 1/3), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(1/3)*x**7*gamma(-4/3)) + B*gamma(-4/3)*hyper((-4/3, 1/3), (-1/3,), b*x**3
*exp_polar(I*pi)/a)/(3*a**(1/3)*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^8\sqrt[3]{a+bx^3}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{1}{3}}x^8}\,dx$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(1/3)*x^8), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^8\sqrt[3]{a+bx^3}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{1}{3}}x^8}\,dx$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(1/3)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^8\sqrt[3]{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{x^8\,{\left(b\,x^3+a\right)}^{1/3}}\,dx$$

input
```
int((A + B*x^3)/(x^8*(a + b*x^3)^(1/3)),x)
```

output
```
int((A + B*x^3)/(x^8*(a + b*x^3)^(1/3)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^8\sqrt[3]{a+bx^3}}\,dx=\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,x^8}dx\right)a+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,x^5}dx\right)b$$

input
```
int((B*x^3+A)/x^8/(b*x^3+a)^(1/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**8),x)*a + int(1/((a + b*x**3)**(1/3)*x**5),x
)*b
```

3.343 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{a^2(Ab-aB)\sqrt[3]{a+bx^3}}{b^4} - \frac{a(2Ab-3aB)\,(a+bx^3)^{4/3}}{4b^4} + \frac{(Ab-3aB)\,(a+bx^3)^{7/3}}{7b^4} + \frac{B(a+bx^3)^{10/3}}{10b^4}$$

output
```
a^2*(A*b-B*a)*(b*x^3+a)^(1/3)/b^4-1/4*a*(2*A*b-3*B*a)*(b*x^3+a)^(4/3)/b^4+
1/7*(A*b-3*B*a)*(b*x^3+a)^(7/3)/b^4+1/10*B*(b*x^3+a)^(10/3)/b^4
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{\sqrt[3]{a+bx^3}(90a^2Ab - 81a^3B - 30aAb^2x^3 + 27a^2bBx^3 + 20Ab^3x^6 - 18ab^2Bx^6 + 14b^3}{140b^4}$$

input
```
Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(2/3),x]
```

output
```
((a + b*x^3)^(1/3)*(90*a^2*A*b - 81*a^3*B - 30*a*A*b^2*x^3 + 27*a^2*b*B*x^
3 + 20*A*b^3*x^6 - 18*a*b^2*B*x^6 + 14*b^3*B*x^9))/(140*b^4)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (A+Bx^3)}{(a+bx^3)^{2/3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6 (Bx^3+A)}{(bx^3+a)^{2/3}}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(\frac{B(bx^3+a)^{7/3}}{b^3}+\frac{(Ab-3aB)(bx^3+a)^{4/3}}{b^3}+\frac{a(3aB-2Ab)\sqrt[3]{bx^3+a}}{b^3}-\frac{a^2(aB-Ab)}{b^3(bx^3+a)^{2/3}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3a^2\sqrt[3]{a+bx^3}(Ab-aB)}{b^4}+\frac{3(a+bx^3)^{7/3}(Ab-3aB)}{7b^4}-\frac{3a(a+bx^3)^{4/3}(2Ab-3aB)}{4b^4}+\frac{3B(a+bx^3)^{10/3}}{10b^4}\right)$$

input
```
Int[(x^8*(A + B*x^3))/(a + b*x^3)^(2/3),x]
```

output
```
((3*a^2*(A*b - a*B)*(a + b*x^3)^(1/3))/b^4 - (3*a*(2*A*b - 3*a*B)*(a + b*x
^3)^(4/3))/(4*b^4) + (3*(A*b - 3*a*B)*(a + b*x^3)^(7/3))/(7*b^4) + (3*B*(a
 + b*x^3)^(10/3))/(10*b^4))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.68

method	result	size
pseudoelliptic	$\frac{9\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(\frac{2x^{6}\left(\frac{7B\,x^{3}}{10}+A\right)b^{3}}{9}-\frac{\left(\frac{3B\,x^{3}}{5}+A\right)a\,x^{3}b^{2}}{3}+a^{2}\left(\frac{3B\,x^{3}}{10}+A\right)b-\frac{9a^{3}B}{10}\right)}{14b^{4}}$	68
gosper	$\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(14b^{3}B\,x^{9}+20A\,b^{3}x^{6}-18Ba\,b^{2}x^{6}-30aA\,b^{2}x^{3}+27B\,a^{2}b\,x^{3}+90a^{2}bA-81a^{3}B\right)}{140b^{4}}$	77
trager	$\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(14b^{3}B\,x^{9}+20A\,b^{3}x^{6}-18Ba\,b^{2}x^{6}-30aA\,b^{2}x^{3}+27B\,a^{2}b\,x^{3}+90a^{2}bA-81a^{3}B\right)}{140b^{4}}$	77
risch	$\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(14b^{3}B\,x^{9}+20A\,b^{3}x^{6}-18Ba\,b^{2}x^{6}-30aA\,b^{2}x^{3}+27B\,a^{2}b\,x^{3}+90a^{2}bA-81a^{3}B\right)}{140b^{4}}$	77
orering	$\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(14b^{3}B\,x^{9}+20A\,b^{3}x^{6}-18Ba\,b^{2}x^{6}-30aA\,b^{2}x^{3}+27B\,a^{2}b\,x^{3}+90a^{2}bA-81a^{3}B\right)}{140b^{4}}$	77

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
9/14*(b*x^3+a)^(1/3)*(2/9*x^6*(7/10*B*x^3+A)*b^3-1/3*(3/5*B*x^3+A)*a*x^3*b
^2+a^2*(3/10*B*x^3+A)*b-9/10*a^3*B)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{(14\,Bb^3x^9 - 2\,(9\,Bab^2 - 10\,Ab^3)x^6 - 81\,Ba^3 + 90\,Aa^2b + 3\,(9\,Ba^2b - 10\,Aab^2)x^3)(b}{140\,b^4}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/140*(14*B*b^3*x^9 - 2*(9*B*a*b^2 - 10*A*b^3)*x^6 - 81*B*a^3 + 90*A*a^2*b
 + 3*(9*B*a^2*b - 10*A*a*b^2)*x^3)*(b*x^3 + a)^(1/3)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.72

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \begin{cases} \frac{9Aa^2\sqrt[3]{a+bx^3}}{14b^3} - \frac{3Aax^3\sqrt[3]{a+bx^3}}{14b^2} + \frac{Ax^6\sqrt[3]{a+bx^3}}{7b} - \frac{81Ba^3\sqrt[3]{a+bx^3}}{140b^4} + \frac{27Ba^2x^3\sqrt[3]{a+}}{140b^3} \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{a^{\frac{2}{3}}} \end{cases}$$

input
```
integrate(x**8*(B*x**3+A)/(b*x**3+a)**(2/3),x)
```

output
```
Piecewise((9*A*a**2*(a + b*x**3)**(1/3)/(14*b**3) - 3*A*a*x**3*(a + b*x**3
)**(1/3)/(14*b**2) + A*x**6*(a + b*x**3)**(1/3)/(7*b) - 81*B*a**3*(a + b*x
**3)**(1/3)/(140*b**4) + 27*B*a**2*x**3*(a + b*x**3)**(1/3)/(140*b**3) - 9
*B*a*x**6*(a + b*x**3)**(1/3)/(70*b**2) + B*x**9*(a + b*x**3)**(1/3)/(10*b
), Ne(b, 0)), ((A*x**9/9 + B*x**12/12)/a**(2/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.18

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{1}{140}\,B\left(\frac{14\,(bx^3+a)^{\frac{10}{3}}}{b^4} - \frac{60\,(bx^3+a)^{\frac{7}{3}}a}{b^4} + \frac{105\,(bx^3+a)^{\frac{4}{3}}a^2}{b^4} - \frac{140\,(bx^3+a)^{\frac{1}{3}}a^3}{b^4}\right) + \frac{1}{14}\,A\left(\frac{2\,(bx^3+a)^{\frac{7}{3}}}{b^3} - \frac{7\,(bx^3+a)^{\frac{4}{3}}a}{b^3} + \frac{14\,(bx^3+a)^{\frac{1}{3}}a^2}{b^3}\right)$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/140*B*(14*(b*x^3 + a)^(10/3)/b^4 - 60*(b*x^3 + a)^(7/3)*a/b^4 + 105*(b*x
^3 + a)^(4/3)*a^2/b^4 - 140*(b*x^3 + a)^(1/3)*a^3/b^4) + 1/14*A*(2*(b*x^3
+ a)^(7/3)/b^3 - 7*(b*x^3 + a)^(4/3)*a/b^3 + 14*(b*x^3 + a)^(1/3)*a^2/b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = -\frac{(Ba^3 - Aa^2b)(bx^3+a)^{\frac{1}{3}}}{b^4} + \frac{14\,(bx^3+a)^{\frac{10}{3}}B - 60\,(bx^3+a)^{\frac{7}{3}}Ba + 105\,(bx^3+a)^{\frac{4}{3}}Ba^2 + 20\,(bx^3+a)^{\frac{7}{3}}Ab - 70\,(bx^3+a)^{\frac{4}{3}}Aab}{140\,b^4}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
-(B*a^3 - A*a^2*b)*(b*x^3 + a)^(1/3)/b^4 + 1/140*(14*(b*x^3 + a)^(10/3)*B
- 60*(b*x^3 + a)^(7/3)*B*a + 105*(b*x^3 + a)^(4/3)*B*a^2 + 20*(b*x^3 + a)^
(7/3)*A*b - 70*(b*x^3 + a)^(4/3)*A*a*b)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx =$$
$$-\left(b\,x^3+a\right)^{1/3}\left(\frac{81\,B\,a^3-90\,A\,a^2\,b}{140\,b^4}-\frac{B\,x^9}{10\,b}-\frac{x^6\,\left(20\,A\,b^3-18\,B\,a\,b^2\right)}{140\,b^4}+\frac{3\,a\,x^3\,\left(10\,A\,b-9\,B\,a\right)}{140\,b^3}\right)$$

input
```
int((x^8*(A + B*x^3))/(a + b*x^3)^(2/3),x)
```

output
```
-(a + b*x^3)^(1/3)*((81*B*a^3 - 90*A*a^2*b)/(140*b^4) - (B*x^9)/(10*b) - (
x^6*(20*A*b^3 - 18*B*a*b^2))/(140*b^4) + (3*a*x^3*(10*A*b - 9*B*a))/(140*b
^3))
```

Reduce [F]

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \left(\int \frac{x^{11}}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)b+\left(\int \frac{x^8}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a$$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^(2/3),x)
```

output
```
int(x**11/(a + b*x**3)**(2/3),x)*b + int(x**8/(a + b*x**3)**(2/3),x)*a
```

3.344 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{2/3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{2/3}} \, dx = -\frac{a(Ab-aB)\sqrt[3]{a+bx^3}}{b^3}+\frac{(Ab-2aB)\left(a+bx^3\right)^{4/3}}{4b^3}+\frac{B\left(a+bx^3\right)^{7/3}}{7b^3}$$

output
```
-a*(A*b-B*a)*(b*x^3+a)^(1/3)/b^3+1/4*(A*b-2*B*a)*(b*x^3+a)^(4/3)/b^3+1/7*B
*(b*x^3+a)^(7/3)/b^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{2/3}} \, dx = \frac{\sqrt[3]{a+bx^3}(-21aAb+18a^2B+7Ab^2x^3-6abBx^3+4b^2Bx^6)}{28b^3}$$

input
```
Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(2/3),x]
```

output
```
((a + b*x^3)^(1/3)*(-21*a*A*b + 18*a^2*B + 7*A*b^2*x^3 - 6*a*b*B*x^3 + 4*b
^2*B*x^6))/(28*b^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^3)}{(a+bx^3)^{2/3}} \, dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{x^3 (Bx^3+A)}{(bx^3+a)^{2/3}}dx^3$$

$$\Big\downarrow \ 86$$

$$\frac{1}{3}\int \left(\frac{B(bx^3+a)^{4/3}}{b^2}+\frac{(Ab-2aB)\sqrt[3]{bx^3+a}}{b^2}+\frac{a(aB-Ab)}{b^2(bx^3+a)^{2/3}}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(\frac{3(a+bx^3)^{4/3}(Ab-2aB)}{4b^3}-\frac{3a\sqrt[3]{a+bx^3}(Ab-aB)}{b^3}+\frac{3B(a+bx^3)^{7/3}}{7b^3}\right)$$

input
```
Int[(x^5*(A + B*x^3))/(a + b*x^3)^(2/3),x]
```

output
```
((-3*a*(A*b - a*B)*(a + b*x^3)^(1/3))/b^3 + (3*(A*b - 2*a*B)*(a + b*x^3)^(
4/3))/(4*b^3) + (3*B*(a + b*x^3)^(7/3))/(7*b^3))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$-\frac{3\left(-\frac{x^{3}\left(\frac{4B\,x^{3}}{7}+A\right)b^{2}}{3}+a\left(\frac{2B\,x^{3}}{7}+A\right)b-\frac{6a^{2}B}{7}\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{4b^{3}}$	49
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(-4b^{2}B\,x^{6}-7A\,b^{2}x^{3}+6Bab\,x^{3}+21abA-18a^{2}B\right)}{28b^{3}}$	53
trager	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(-4b^{2}B\,x^{6}-7A\,b^{2}x^{3}+6Bab\,x^{3}+21abA-18a^{2}B\right)}{28b^{3}}$	53
risch	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(-4b^{2}B\,x^{6}-7A\,b^{2}x^{3}+6Bab\,x^{3}+21abA-18a^{2}B\right)}{28b^{3}}$	53
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(-4b^{2}B\,x^{6}-7A\,b^{2}x^{3}+6Bab\,x^{3}+21abA-18a^{2}B\right)}{28b^{3}}$	53

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/4*(-1/3*x^3*(4/7*B*x^3+A)*b^2+a*(2/7*B*x^3+A)*b-6/7*a^2*B)*(b*x^3+a)^(1
/3)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{(4\,Bb^2x^6-(6\,Bab-7\,Ab^2)x^3+18\,Ba^2-21\,Aab)(bx^3+a)^{\frac{1}{3}}}{28\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/28*(4*B*b^2*x^6 - (6*B*a*b - 7*A*b^2)*x^3 + 18*B*a^2 - 21*A*a*b)*(b*x^3
+ a)^(1/3)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.70

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \begin{cases} -\frac{3Aa\sqrt[3]{a+bx^3}}{4b^2}+\frac{Ax^3\sqrt[3]{a+bx^3}}{4b}+\frac{9Ba^2\sqrt[3]{a+bx^3}}{14b^3}-\frac{3Bax^3\sqrt[3]{a+bx^3}}{14b^2}+\frac{Bx^6\sqrt[3]{a+bx^3}}{7b} \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{a^{\frac{2}{3}}} \end{cases}$$

input
```
integrate(x**5*(B*x**3+A)/(b*x**3+a)**(2/3),x)
```

output
```
Piecewise((-3*A*a*(a + b*x**3)**(1/3)/(4*b**2) + A*x**3*(a + b*x**3)**(1/3
)/(4*b) + 9*B*a**2*(a + b*x**3)**(1/3)/(14*b**3) - 3*B*a*x**3*(a + b*x**3)
**(1/3)/(14*b**2) + B*x**6*(a + b*x**3)**(1/3)/(7*b), Ne(b, 0)), ((A*x**6/
6 + B*x**9/9)/a**(2/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.17

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{1}{14}\,B\left(\frac{2\,(bx^3+a)^{\frac{7}{3}}}{b^3} - \frac{7\,(bx^3+a)^{\frac{4}{3}}a}{b^3} + \frac{14\,(bx^3+a)^{\frac{1}{3}}a^2}{b^3}\right) + \frac{1}{4}\,A\left(\frac{(bx^3+a)^{\frac{4}{3}}}{b^2} - \frac{4\,(bx^3+a)^{\frac{1}{3}}a}{b^2}\right)$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/14*B*(2*(b*x^3 + a)^(7/3)/b^3 - 7*(b*x^3 + a)^(4/3)*a/b^3 + 14*(b*x^3 +
a)^(1/3)*a^2/b^3) + 1/4*A*((b*x^3 + a)^(4/3)/b^2 - 4*(b*x^3 + a)^(1/3)*a/b
^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{(bx^3+a)^{\frac{1}{3}}(Ba^2-Aab)}{b^3} + \frac{4\,(bx^3+a)^{\frac{7}{3}}B - 14\,(bx^3+a)^{\frac{4}{3}}Ba + 7\,(bx^3+a)^{\frac{4}{3}}Ab}{28\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
(b*x^3 + a)^(1/3)*(B*a^2 - A*a*b)/b^3 + 1/28*(4*(b*x^3 + a)^(7/3)*B - 14*(
b*x^3 + a)^(4/3)*B*a + 7*(b*x^3 + a)^(4/3)*A*b)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = (b\,x^3+a)^{1/3}\left(\frac{18\,B\,a^2-21\,A\,a\,b}{28\,b^3}+\frac{x^3\,(7\,A\,b^2-6\,B\,a\,b)}{28\,b^3}+\frac{B\,x^6}{7\,b}\right)$$

input
```
int((x^5*(A + B*x^3))/(a + b*x^3)^(2/3),x)
```

output
```
(a + b*x^3)^(1/3)*((18*B*a^2 - 21*A*a*b)/(28*b^3) + (x^3*(7*A*b^2 - 6*B*a*
b))/(28*b^3) + (B*x^6)/(7*b))
```

Reduce [F]

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \left(\int \frac{x^8}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)b+\left(\int \frac{x^5}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a$$

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^(2/3),x)
```

output
```
int(x**8/(a + b*x**3)**(2/3),x)*b + int(x**5/(a + b*x**3)**(2/3),x)*a
```

3.345 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{(Ab-aB)\sqrt[3]{a+bx^3}}{b^2}+\frac{B(a+bx^3)^{4/3}}{4b^2}$$

output
```
(A*b-B*a)*(b*x^3+a)^(1/3)/b^2+1/4*B*(b*x^3+a)^(4/3)/b^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{\sqrt[3]{a+bx^3}(4Ab-3aB+bBx^3)}{4b^2}$$

input
```
Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(2/3),x]
```

output
```
((a + b*x^3)^(1/3)*(4*A*b - 3*a*B + b*B*x^3))/(4*b^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^3)}{(a+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{(bx^3+a)^{2/3}}dx^3$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{3}\int \left(\frac{\sqrt[3]{bx^3+a}B}{b}+\frac{Ab-aB}{b(bx^3+a)^{2/3}}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}(Ab-aB)}{b^2}+\frac{3B(a+bx^3)^{4/3}}{4b^2}\right)$$

input
```
Int[(x^2*(A + B*x^3))/(a + b*x^3)^(2/3),x]
```

output
```
((3*(A*b - a*B)*(a + b*x^3)^(1/3))/b^2 + (3*B*(a + b*x^3)^(4/3))/(4*b^2))/
3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{(b\,x^3+a)^{\frac{1}{3}}\,(bB\,x^3+4Ab-3Ba)}{4b^2}$	30
trager	$\frac{(b\,x^3+a)^{\frac{1}{3}}\,(bB\,x^3+4Ab-3Ba)}{4b^2}$	30
risch	$\frac{(b\,x^3+a)^{\frac{1}{3}}\,(bB\,x^3+4Ab-3Ba)}{4b^2}$	30
orering	$\frac{(b\,x^3+a)^{\frac{1}{3}}\,(bB\,x^3+4Ab-3Ba)}{4b^2}$	30
pseudoelliptic	$\frac{((B\,x^3+4A)b-3Ba)\,(b\,x^3+a)^{\frac{1}{3}}}{4b^2}$	31

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/4*(b*x^3+a)^(1/3)*(B*b*x^3+4*A*b-3*B*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{(Bbx^3-3\,Ba+4\,Ab)(bx^3+a)^{\frac{1}{3}}}{4\,b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/4*(B*b*x^3 - 3*B*a + 4*A*b)*(b*x^3 + a)^(1/3)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \begin{cases} \frac{A\sqrt[3]{a+bx^3}}{b} - \frac{3Ba\sqrt[3]{a+bx^3}}{4b^2} + \frac{Bx^3\sqrt[3]{a+bx^3}}{4b} & \text{for } b\neq 0 \\ \frac{\frac{Ax^3}{3}+\frac{Bx^6}{6}}{a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(B*x**3+A)/(b*x**3+a)**(2/3),x)
```

output
```
Piecewise((A*(a + b*x**3)**(1/3)/b - 3*B*a*(a + b*x**3)**(1/3)/(4*b**2) +
B*x**3*(a + b*x**3)**(1/3)/(4*b), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/a**(2/
3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{1}{4}\,B\left(\frac{(bx^3+a)^{\frac{4}{3}}}{b^2} - \frac{4\,(bx^3+a)^{\frac{1}{3}}a}{b^2}\right) + \frac{(bx^3+a)^{\frac{1}{3}}A}{b}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/4*B*((b*x^3 + a)^(4/3)/b^2 - 4*(b*x^3 + a)^(1/3)*a/b^2) + (b*x^3 + a)^(1
/3)*A/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{(bx^3+a)^{\frac{4}{3}}B}{4\,b^2} - \frac{(bx^3+a)^{\frac{1}{3}}(Ba-Ab)}{b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
1/4*(b*x^3 + a)^(4/3)*B/b^2 - (b*x^3 + a)^(1/3)*(B*a - A*b)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \left(\frac{4\,A\,b-3\,B\,a}{4\,b^2}+\frac{B\,x^3}{4\,b}\right)\,\left(b\,x^3+a\right)^{1/3}$$

input
```
int((x^2*(A + B*x^3))/(a + b*x^3)^(2/3),x)
```

output
```
((4*A*b - 3*B*a)/(4*b^2) + (B*x^3)/(4*b))*(a + b*x^3)^(1/3)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{(b\,x^3+a)^{\frac{1}{3}}\,a+\left(\int \frac{x^5}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)b^2}{b}$$

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^(2/3),x)
```

output
```
((a + b*x**3)**(1/3)*a + int(x**5/(a + b*x**3)**(2/3),x)*b**2)/b
```

3.346 $\int \frac{A+Bx^3}{x(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{A+Bx^3}{x(a+bx^3)^{2/3}}\,dx = \frac{B\sqrt[3]{a+bx^3}}{b} - \frac{A\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}} - \frac{A\log(x)}{2a^{2/3}} + \frac{A\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}$$

output
```
B*(b*x^3+a)^(1/3)/b-1/3*A*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a
^(1/3))*3^(1/2)/a^(2/3)-1/2*A*ln(x)/a^(2/3)+1/2*A*ln(a^(1/3)-(b*x^3+a)^(1/
3))/a^(2/3)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx^3}{x(a+bx^3)^{2/3}}\,dx = \frac{6a^{2/3}B\sqrt[3]{a+bx^3} - 2\sqrt{3}Ab\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2Ab\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{6a^{2/3}b}$$

input
```
Integrate[(A + B*x^3)/(x*(a + b*x^3)^(2/3)),x]
```

output
```
(6*a^(2/3)*B*(a + b*x^3)^(1/3) - 2*Sqrt[3]*A*b*ArcTan[(1 + (2*(a + b*x^3)^
(1/3))/a^(1/3))/Sqrt[3]] + 2*A*b*Log[-a^(1/3) + (a + b*x^3)^(1/3)] - A*b*L
og[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*a^(2/3)*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 90, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{2/3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^3\left(bx^3+a\right)^{2/3}}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(A\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3+\frac{3B\sqrt[3]{a+bx^3}}{b}\right)$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{3}\left(A\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+\frac{3B\sqrt[3]{a+bx^3}}{b}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(A\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+\frac{3B\sqrt[3]{a+bx^3}}{b}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(A\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+\frac{3B\sqrt[3]{a+bx^3}}{b}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(A\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+\frac{3B\sqrt[3]{a+bx^3}}{b}\right)$$

input
```
Int[(A + B*x^3)/(x*(a + b*x^3)^(2/3)),x]
```

output
```
((3*B*(a + b*x^3)^(1/3))/b + A*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3
))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (
a + b*x^3)^(1/3)])/(2*a^(2/3))))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{-A\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{3a^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)b+A\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)b-\frac{A\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)b}{2}+3B\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{3a^{\frac{2}{3}}b}$

input
```
int((B*x^3+A)/x/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-A*3^(1/2)*arctan(2/3*3^(1/2)/a^(1/3)*(b*x^3+a)^(1/3)+1/3*3^(1/2))*b+
A*ln((b*x^3+a)^(1/3)-a^(1/3))*b-1/2*A*ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)
^(1/3)+a^(2/3))*b+3*B*(b*x^3+a)^(1/3)*a^(2/3))/a^(2/3)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{2/3}}\,dx = -\frac{6\sqrt{\frac{1}{3}}A(a^2)^{\frac{1}{6}}ab\arctan\left(\frac{\sqrt{\frac{1}{3}}(a^2)^{\frac{1}{6}}\left((a^2)^{\frac{1}{3}}a+2\,(bx^3+a)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}\right)}{a^2}\right)+A(a^2)^{\frac{2}{3}}b\log\left((bx^3+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(bx^3+\right.}{6\,a^2b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
-1/6*(6*sqrt(1/3)*A*(a^2)^(1/6)*a*b*arctan(sqrt(1/3)*(a^2)^(1/6)*((a^2)^(1
/3)*a + 2*(b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) + A*(a^2)^(2/3)*b*log((b*x^3
 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 2*A*(a^2)
^(2/3)*b*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)) - 6*(b*x^3 + a)^(1/3)*B*a^
2)/(a^2*b)
```

Sympy [A] (verification not implemented)

Time = 6.79 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{2/3}}\,dx = -\frac{A\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^3}\right)}{3b^{\frac{2}{3}}x^2\Gamma\left(\frac{5}{3}\right)}+B\left(\begin{cases}\frac{x^3}{3a^{\frac{2}{3}}} & \text{for } b=0\\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x**3+A)/x/(b*x**3+a)**(2/3),x)
```

output
```
-A*gamma(2/3)*hyper((2/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**3))/(3*b**
(2/3)*x**2*gamma(5/3)) + B*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a +
b*x**3)**(1/3)/b, True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{2/3}}\,dx = -\frac{1}{6}A\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{2\,\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-\right.}{a^{\frac{2}{3}}}\right.$$
$$+\frac{\left(bx^3+a\right)^{\frac{1}{3}}B}{b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
-1/6*A*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(2/3) + log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(2/3) - 2*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(2/3)) + (b*x^3 + a)^(1/3)
*B/b
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{2/3}}\,dx = -\frac{\sqrt{3}A\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}} - \frac{A\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{2}{3}}} + \frac{A\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{2}{3}}} + \frac{\left(bx^3+a\right)^{\frac{1}{3}}B}{b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*A*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(2/3) - 1/6*A*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/
3))/a^(2/3) + 1/3*A*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(2/3) + (b*x^3
 + a)^(1/3)*B/b
```

Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^3}{x\,(a+bx^3)^{2/3}}\,dx = \frac{B\,(b\,x^3+a)^{1/3}}{b} - \frac{\ln\left(3\,A\,(b\,x^3+a)^{1/3}+\frac{3\,a^{1/3}\,\left(A-\sqrt{3}\,A\,1\mathrm{i}\right)}{2}\right)\,\left(A-\sqrt{3}\,A\,1\mathrm{i}\right)}{6\,a^{2/3}} - \frac{\ln\left(3\,A\,(b\,x^3+a)^{1/3}+\frac{3\,a^{1/3}\,\left(A+\sqrt{3}\,A\,1\mathrm{i}\right)}{2}\right)\,\left(A+\sqrt{3}\,A\,1\mathrm{i}\right)}{6\,a^{2/3}} + \frac{A\,\ln\left(3\,A\,a^{1/3}-3\,A\,(b\,x^3+a)^{1/3}\right)}{3\,a^{2/3}}$$

input
```
int((A + B*x^3)/(x*(a + b*x^3)^(2/3)),x)
```

output
```
(B*(a + b*x^3)^(1/3))/b - (log(3*A*(a + b*x^3)^(1/3) + (3*a^(1/3)*(A - 3^(
1/2)*A*1i))/2)*(A - 3^(1/2)*A*1i))/(6*a^(2/3)) - (log(3*A*(a + b*x^3)^(1/3
) + (3*a^(1/3)*(A + 3^(1/2)*A*1i))/2)*(A + 3^(1/2)*A*1i))/(6*a^(2/3)) + (A
*log(3*A*a^(1/3) - 3*A*(a + b*x^3)^(1/3)))/(3*a^(2/3))
```

Reduce [F]

$$\int \frac{A+Bx^3}{x\,(a+bx^3)^{2/3}}\,dx = (b\,x^3+a)^{\frac{1}{3}} + \left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,x}dx\right) a$$

input
```
int((B*x^3+A)/x/(b*x^3+a)^(2/3),x)
```

output
```
(a + b*x**3)**(1/3) + int(1/((a + b*x**3)**(2/3)*x),x)*a
```

3.347 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{2/3}}\,dx = -\frac{A\sqrt[3]{a+bx^3}}{3ax^3}+\frac{(2Ab-3aB)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}} + \frac{(2Ab-3aB)\log(x)}{6a^{5/3}}-\frac{(2Ab-3aB)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{5/3}}$$

output
```
-1/3*A*(b*x^3+a)^(1/3)/a/x^3+1/9*(2*A*b-3*B*a)*arctan(1/3*(a^(1/3)+2*(b*x^
3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)+1/6*(2*A*b-3*B*a)*ln(x)/a^(5/
3)-1/6*(2*A*b-3*B*a)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(5/3)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{2/3}}\,dx = \frac{-\frac{6a^{2/3}A\sqrt[3]{a+bx^3}}{x^3}+2\sqrt{3}(2Ab-3aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2(-2Ab+3aB)}{18a}$$

input
```
Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(2/3)),x]
```

output
```
((-6*a^(2/3)*A*(a + b*x^3)^(1/3))/x^3 + 2*Sqrt[3]*(2*A*b - 3*a*B)*ArcTan[(
1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 2*(-2*A*b + 3*a*B)*Log[-a^(1
/3) + (a + b*x^3)^(1/3)] + (2*A*b - 3*a*B)*Log[a^(2/3) + a^(1/3)*(a + b*x^
3)^(1/3) + (a + b*x^3)^(2/3)])/(18*a^(5/3))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^4 \left(a+bx^3\right)^{2/3}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{Bx^3+A}{x^6 \left(bx^3+a\right)^{2/3}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3} \left(-\frac{(2Ab-3aB) \int \frac{1}{x^3 (bx^3+a)^{2/3}}dx^3}{3a}-\frac{A \sqrt[3]{a+bx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{3} \left(-\frac{(2Ab-3aB) \left(-\frac{3 \int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3 \int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log \left(x^3\right)}{2a^{2/3}}\right)}{3a}-\frac{A\sqrt[3]{a}}{a}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(-\frac{(2Ab-3aB)\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)}{3a}-\frac{A\sqrt[3]{a+bx^3}}{ax^3}\right)$$

$\Big\downarrow$ 1082

$$\frac{1}{3}\left(-\frac{(2Ab-3aB)\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)}{3a}-\frac{A\sqrt[3]{a+bx^3}}{ax^3}\right)$$

$\Big\downarrow$ 217

$$\frac{1}{3}\left(-\frac{(2Ab-3aB)\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)}{3a}-\frac{A\sqrt[3]{a+bx^3}}{ax^3}\right)$$

input
```
Int[(A + B*x^3)/(x^4*(a + b*x^3)^(2/3)),x]
```

output
```
(-((A*(a + b*x^3)^(1/3))/(a*x^3)) - ((2*A*b - 3*a*B)*(-((Sqrt[3]*ArcTan[(1
 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^3]/(2*a^(2/3)
) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3))))/(3*a))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{-3A\left(b\,x^{3}+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+\left(Ab-\frac{3Ba}{2}\right)\left(2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x\right.\right.\right.}{9a^{\frac{5}{3}}x^{3}}$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/9*(-3*A*(b*x^3+a)^(1/3)*a^(2/3)+(A*b-3/2*B*a)*(2*arctan(1/3*(a^(1/3)+2*(
b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3
+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*x^3)/a^(5/3)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.53

$$\int \frac{A+Bx^3}{x^4\,(a+bx^3)^{2/3}}\,dx =$$

$$-\frac{6\sqrt{\frac{1}{3}}(3\,Ba^2-2\,Aab)x^3\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(-a^2\right)^{\frac{1}{3}}a-2\,(bx^3+a)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{a^2}\right)+(3\,Ba-2\,Ab)(\cdots}{\cdots}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
-1/18*(6*sqrt(1/3)*(3*B*a^2 - 2*A*a*b)*x^3*sqrt(-(-a^2)^(1/3))*arctan(-sqr
t(1/3)*((-a^2)^(1/3)*a - 2*(b*x^3 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1
/3))/a^2) + (3*B*a - 2*A*b)*(-a^2)^(2/3)*x^3*log((b*x^3 + a)^(2/3)*a - (-a
^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(-a^2)^(2/3)) - 2*(3*B*a - 2*A*b)*(-a^2)^(
2/3)*x^3*log((b*x^3 + a)^(1/3)*a - (-a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)*A*a
^2)/(a^3*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.61 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{2/3}}\,dx = -\frac{A\Gamma\left(\frac{5}{3}\right)\,{}_{2}F_{1}\left(\left.\begin{matrix}\frac{2}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^3}\right)}{3b^{\frac{2}{3}}x^5\Gamma\left(\frac{8}{3}\right)} - \frac{B\Gamma\left(\frac{2}{3}\right)\,{}_{2}F_{1}\left(\left.\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^3}\right)}{3b^{\frac{2}{3}}x^2\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**4/(b*x**3+a)**(2/3),x)
```

output
```
-A*gamma(5/3)*hyper((2/3, 5/3), (8/3,), a*exp_polar(I*pi)/(b*x**3))/(3*b**
(2/3)*x**5*gamma(8/3)) - B*gamma(2/3)*hyper((2/3, 2/3), (5/3,), a*exp_pola
r(I*pi)/(b*x**3))/(3*b**(2/3)*x**2*gamma(5/3))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.55

$$\int \frac{A+Bx^3}{x^4(a+bx^3)^{2/3}}\,dx = -\frac{1}{6}B\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{2\log\left((bx^3+a)^{\frac{1}{3}}-\right.}{a^{\frac{2}{3}}}\right.$$

$$+\frac{1}{9}A\left(\frac{2\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}-\frac{3(bx^3+a)^{\frac{1}{3}}b}{(bx^3+a)a-a^2}+\frac{b\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}\right.$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
-1/6*B*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(2/3) + log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(2/3) - 2*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(2/3)) + 1/9*A*(2*sqrt(3)*
b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(5/3) - 3*
(b*x^3 + a)^(1/3)*b/((b*x^3 + a)*a - a^2) + b*log((b*x^3 + a)^(2/3) + (b*x
^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(5/3) - 2*b*log((b*x^3 + a)^(1/3) - a^(
1/3))/a^(5/3))
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^3}{x^4(a+bx^3)^{2/3}}\,dx = -\frac{1}{18}b\left(\frac{2\sqrt{3}(3Ba-2Ab)\arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}b}+\frac{(3Ba-2Ab)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}\right.}{a^{\frac{5}{3}}b}\right.$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
-1/18*b*(2*sqrt(3)*(3*B*a - 2*A*b)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3)
 + a^(1/3))/a^(1/3))/(a^(5/3)*b) + (3*B*a - 2*A*b)*log((b*x^3 + a)^(2/3) +
 (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(5/3)*b) - 2*(3*B*a - 2*A*b)*log(
abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5/3)*b) + 6*(b*x^3 + a)^(1/3)*A/(a*b
*x^3))
```

Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.88

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{2/3}}\,dx = \frac{\ln\left(\frac{A\,b-\sqrt{3}\,A\,b\,1\mathrm{i}}{a^{2/3}}+\frac{2\,A\,b\,{\left(b\,x^3+a\right)}^{1/3}}{a}\right)\,\left(A\,b-\sqrt{3}\,A\,b\,1\mathrm{i}\right)}{9\,a^{5/3}} + \frac{\ln\left(\frac{A\,b+\sqrt{3}\,A\,b\,1\mathrm{i}}{a^{2/3}}+\frac{2\,A\,b\,{\left(b\,x^3+a\right)}^{1/3}}{a}\right)\,\left(A\,b+\sqrt{3}\,A\,b\,1\mathrm{i}\right)}{9\,a^{5/3}} - \frac{\ln\left(3\,B\,{\left(b\,x^3+a\right)}^{1/3}+\frac{3\,a^{1/3}\,\left(B-\sqrt{3}\,B\,1\mathrm{i}\right)}{2}\right)\,\left(B-\sqrt{3}\,B\,1\mathrm{i}\right)}{6\,a^{2/3}} - \frac{\ln\left(3\,B\,{\left(b\,x^3+a\right)}^{1/3}+\frac{3\,a^{1/3}\,\left(B+\sqrt{3}\,B\,1\mathrm{i}\right)}{2}\right)\,\left(B+\sqrt{3}\,B\,1\mathrm{i}\right)}{6\,a^{2/3}} + \frac{B\,\ln\left(3\,B\,a^{1/3}-3\,B\,{\left(b\,x^3+a\right)}^{1/3}\right)}{3\,a^{2/3}} - \frac{2\,A\,b\,\ln\left({\left(b\,x^3+a\right)}^{1/3}-a^{1/3}\right)}{9\,a^{5/3}} - \frac{A\,{\left(b\,x^3+a\right)}^{1/3}}{3\,a\,x^3}$$

input
```
int((A + B*x^3)/(x^4*(a + b*x^3)^(2/3)),x)
```

output
```
(log((A*b - 3^(1/2)*A*b*1i)/a^(2/3) + (2*A*b*(a + b*x^3)^(1/3))/a)*(A*b -
3^(1/2)*A*b*1i))/(9*a^(5/3)) + (log((A*b + 3^(1/2)*A*b*1i)/a^(2/3) + (2*A*
b*(a + b*x^3)^(1/3))/a)*(A*b + 3^(1/2)*A*b*1i))/(9*a^(5/3)) - (log(3*B*(a
+ b*x^3)^(1/3) + (3*a^(1/3)*(B - 3^(1/2)*B*1i))/2)*(B - 3^(1/2)*B*1i))/(6*
a^(2/3)) - (log(3*B*(a + b*x^3)^(1/3) + (3*a^(1/3)*(B + 3^(1/2)*B*1i))/2)*
(B + 3^(1/2)*B*1i))/(6*a^(2/3)) + (B*log(3*B*a^(1/3) - 3*B*(a + b*x^3)^(1/
3)))/(3*a^(2/3)) - (2*A*b*log((a + b*x^3)^(1/3) - a^(1/3)))/(9*a^(5/3)) -
(A*(a + b*x^3)^(1/3))/(3*a*x^3)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^4 (a+bx^3)^{2/3}}\,dx = \left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,x^4}dx\right)a + \left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,x}dx\right)b$$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^(2/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*x**4),x)*a + int(1/((a + b*x**3)**(2/3)*x),x)*b
```

3.348 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 168

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{2/3}}\,dx=-\frac{A\sqrt[3]{a+bx^3}}{6ax^6}+\frac{(5Ab-6aB)\sqrt[3]{a+bx^3}}{18a^2x^3}-\frac{b(5Ab-6aB)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}}-\frac{b(5Ab-6aB)\log(x)}{18a^{8/3}}+\frac{b(5Ab-6aB)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{8/3}}$$

output

```
-1/6*A*(b*x^3+a)^(1/3)/a/x^6+1/18*(5*A*b-6*B*a)*(b*x^3+a)^(1/3)/a^2/x^3-1/
27*b*(5*A*b-6*B*a)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))
*3^(1/2)/a^(8/3)-1/18*b*(5*A*b-6*B*a)*ln(x)/a^(8/3)+1/18*b*(5*A*b-6*B*a)*l
n(a^(1/3)-(b*x^3+a)^(1/3))/a^(8/3)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{2/3}}\,dx=\frac{-\frac{3a^{2/3}\sqrt[3]{a+bx^3}(-5Abx^3+3a(A+2Bx^3))}{x^6}+2\sqrt{3}b(-5Ab+6aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{}$$

input
```
Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^(2/3)),x]
```

output
```
((-3*a^(2/3)*(a + b*x^3)^(1/3)*(-5*A*b*x^3 + 3*a*(A + 2*B*x^3)))/x^6 + 2*S
qrt[3]*b*(-5*A*b + 6*a*B)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[
3]] + 2*b*(5*A*b - 6*a*B)*Log[-a^(1/3) + (a + b*x^3)^(1/3)] + b*(-5*A*b +
6*a*B)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*a
^(8/3))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {948, 87, 52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{2/3}}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^9\left(bx^3+a\right)^{2/3}}dx^3$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{3}\left(-\frac{(5Ab-6aB)\int \frac{1}{x^6\left(bx^3+a\right)^{2/3}}dx^3}{6a}-\frac{A\sqrt[3]{a+bx^3}}{2ax^6}\right)$$

$$\Big\downarrow \; 52$$

$$\frac{1}{3}\left(-\frac{(5Ab-6aB)\left(-\frac{2b\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3}{3a}-\frac{\sqrt[3]{a+bx^3}}{ax^3}\right)}{6a}-\frac{A\sqrt[3]{a+bx^3}}{2ax^6}\right)$$

$$\Big\downarrow \; 69$$

$$\frac{1}{3}\left(-\frac{(5Ab-6aB)\left(-\frac{2b\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a-}}{ax}\right.}{6a}\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(-\frac{(5Ab-6aB)\left(-\frac{2b\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^3}}{ax^3}\right)}{6a}\right.$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{3}\left(-\frac{(5Ab-6aB)\left(-\frac{2b\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^3}}{ax^3}\right)}{6a}-\frac{A\sqrt[3]{a+b}}{2ax^6}\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(-\frac{(5Ab-6aB)\left(-\frac{2b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^3}}{ax^3}\right)}{6a}-\frac{A\sqrt[3]{a+b}}{2ax^6}\right.$$

input
```
Int[(A + B*x^3)/(x^7*(a + b*x^3)^(2/3)),x]
```

output
```
(-1/2*(A*(a + b*x^3)^(1/3))/(a*x^6) - ((5*A*b - 6*a*B)*(-((a + b*x^3)^(1/3
)/(a*x^3)) - (2*b*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/S
qrt[3]])/a^(2/3)) - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1
/3)])/(2*a^(2/3))))/(3*a)))/(6*a))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$-\frac{5\left(\left(\left(-\frac{3 A a^{\frac{2}{3}} b x^3}{2}+\frac{9\left(2 B x^3+A\right) a^{\frac{5}{3}}}{10}\right)\left(b x^3+a\right)^{\frac{1}{3}}+\left(A b-\frac{6 B a}{5}\right)\left(\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b x^3+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3 a^{\frac{1}{3}}}\right)\sqrt{3}-\ln\left(\left(b x^3+a\right)^{\frac{1}{3}}-a\right.\right.\right.\right.}{27 a^{\frac{8}{3}} x^6}$

input
```
int((B*x^3+A)/x^7/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-5/27*((-3/2*A*a^(2/3)*b*x^3+9/10*(2*B*x^3+A)*a^(5/3))*(b*x^3+a)^(1/3)+(A*
b-6/5*B*a)*(arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2
)-ln((b*x^3+a)^(1/3)-a^(1/3))+1/2*ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/
3)+a^(2/3)))*b*x^6)/a^(8/3)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{2/3}}\,dx = \frac{6\sqrt{\frac{1}{3}}(6\,Ba^2b-5\,Aab^2)(a^2)^{\frac{1}{6}}x^6\arctan\left(\frac{\sqrt{\frac{1}{3}}(a^2)^{\frac{1}{6}}\left((a^2)^{\frac{1}{3}}a+2\,(bx^3+a)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}\right)}{a^2}\right)+(6\,Bab\cdots}{\cdots}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/54*(6*sqrt(1/3)*(6*B*a^2*b - 5*A*a*b^2)*(a^2)^(1/6)*x^6*arctan(sqrt(1/3)
*(a^2)^(1/6)*((a^2)^(1/3)*a + 2*(b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) + (6*B
*a*b - 5*A*b^2)*(a^2)^(2/3)*x^6*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a +
(b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 2*(6*B*a*b - 5*A*b^2)*(a^2)^(2/3)*x^6*log
((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)) - 3*(3*A*a^3 + (6*B*a^3 - 5*A*a^2*b)*x
^3)*(b*x^3 + a)^(1/3))/(a^4*x^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.89 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.49

$$\int \frac{A+Bx^3}{x^7 (a+bx^3)^{2/3}}\,dx = -\frac{A\Gamma\left(\frac{8}{3}\right) {}_2F_1\left(\begin{matrix}\frac{2}{3}, \frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle| \frac{ae^{i\pi}}{bx^3}\right)}{3b^{\frac{2}{3}}x^8\Gamma\left(\frac{11}{3}\right)} - \frac{B\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix}\frac{2}{3}, \frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle| \frac{ae^{i\pi}}{bx^3}\right)}{3b^{\frac{2}{3}}x^5\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**7/(b*x**3+a)**(2/3),x)
```

output
```
-A*gamma(8/3)*hyper((2/3, 8/3), (11/3,), a*exp_polar(I*pi)/(b*x**3))/(3*b*
*(2/3)*x**8*gamma(11/3)) - B*gamma(5/3)*hyper((2/3, 5/3), (8/3,), a*exp_po
lar(I*pi)/(b*x**3))/(3*b**(2/3)*x**5*gamma(8/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(135) = 270$.

Time = 0.11 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.68

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{2/3}}\,dx = \frac{1}{9}\,B\left(\frac{2\,\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}} - \frac{3\,\left(bx^3+a\right)^{\frac{1}{3}}b}{(bx^3+a)a-a^2} + \frac{b\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\right.}{\cdots}\right.$$

$$-\frac{1}{54}\,A\left(\frac{10\,\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{8}{3}}} + \frac{5\,b^2\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{8}{3}}} - \frac{10\,b^2\log\left(\left(\cdots\right.\right.}{\cdots}\right.$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/9*B*(2*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(5/3) - 3*(b*x^3 + a)^(1/3)*b/((b*x^3 + a)*a - a^2) + b*log((b*x^3
+ a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(5/3) - 2*b*log((b*x^3
 + a)^(1/3) - a^(1/3))/a^(5/3)) - 1/54*A*(10*sqrt(3)*b^2*arctan(1/3*sqrt(3
)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(8/3) + 5*b^2*log((b*x^3 + a)
^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(8/3) - 10*b^2*log((b*x^3
+ a)^(1/3) - a^(1/3))/a^(8/3) - 3*(5*(b*x^3 + a)^(4/3)*b^2 - 8*(b*x^3 + a)
^(1/3)*a*b^2)/((b*x^3 + a)^2*a^2 - 2*(b*x^3 + a)*a^3 + a^4))
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{2/3}}\,dx = \frac{\frac{2\,\sqrt{3}\left(6\,Ba^{\frac{4}{3}}b^2-5\,Aa^{\frac{1}{3}}b^3\right)\arctan\left(\frac{\sqrt{3}\left(2\,\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^3} + \frac{\left(6\,Bab^2-5\,Ab^3\right)\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\cdots\right.}{a^{\frac{8}{3}}}}{\cdots}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
1/54*(2*sqrt(3)*(6*B*a^(4/3)*b^2 - 5*A*a^(1/3)*b^3)*arctan(1/3*sqrt(3)*(2*
(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^3 + (6*B*a*b^2 - 5*A*b^3)*log((b*x
^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(8/3) - 2*(6*B*a*b^
2 - 5*A*b^3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(8/3) - 3*(6*(b*x^3 +
 a)^(4/3)*B*a*b^2 - 6*(b*x^3 + a)^(1/3)*B*a^2*b^2 - 5*(b*x^3 + a)^(4/3)*A*
b^3 + 8*(b*x^3 + a)^(1/3)*A*a*b^3)/(a^2*b^2*x^6))/b
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.03

$$\int \frac{A+Bx^3}{x^7\,(a+bx^3)^{2/3}}\,dx = \frac{5\,A\,b^2\,\ln\left(\left(b\,x^3+a\right)^{1/3}-a^{1/3}\right)}{27\,a^{8/3}} + \frac{\ln\left(\frac{B\,b-\sqrt{3}\,B\,b\,1\mathrm{i}}{a^{2/3}}+\frac{2\,B\,b\,\left(b\,x^3+a\right)^{1/3}}{a}\right)\,\left(B\,b-\sqrt{3}\,B\,b\,1\mathrm{i}\right)}{9\,a^{5/3}} + \frac{\ln\left(\frac{B\,b+\sqrt{3}\,B\,b\,1\mathrm{i}}{a^{2/3}}+\frac{2\,B\,b\,\left(b\,x^3+a\right)^{1/3}}{a}\right)\,\left(B\,b+\sqrt{3}\,B\,b\,1\mathrm{i}\right)}{9\,a^{5/3}} - \frac{2\,B\,b\,\ln\left(\left(b\,x^3+a\right)^{1/3}-a^{1/3}\right)}{9\,a^{5/3}} - \frac{\frac{4\,A\,b^2\,\left(b\,x^3+a\right)^{1/3}}{9\,a}-\frac{5\,A\,b^2\,\left(b\,x^3+a\right)^{4/3}}{18\,a^2}}{\left(b\,x^3+a\right)^2-2\,a\,\left(b\,x^3+a\right)+a^2} - \frac{B\,\left(b\,x^3+a\right)^{1/3}}{3\,a\,x^3} + \frac{5\,A\,b^2\,\ln\left(\frac{5\,A\,b^2\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{5/3}}-\frac{5\,A\,b^2\,\left(b\,x^3+a\right)^{1/3}}{3\,a^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27\,a^{8/3}} - \frac{5\,A\,b^2\,\ln\left(\frac{5\,A\,b^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{5/3}}+\frac{5\,A\,b^2\,\left(b\,x^3+a\right)^{1/3}}{3\,a^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27\,a^{8/3}}$$

input
```
int((A + B*x^3)/(x^7*(a + b*x^3)^(2/3)),x)
```

output
```
(log((B*b - 3^(1/2)*B*b*1i)/a^(2/3) + (2*B*b*(a + b*x^3)^(1/3))/a)*(B*b -
3^(1/2)*B*b*1i))/(9*a^(5/3)) - ((4*A*b^2*(a + b*x^3)^(1/3))/(9*a) - (5*A*b
^2*(a + b*x^3)^(4/3))/(18*a^2))/((a + b*x^3)^2 - 2*a*(a + b*x^3) + a^2) +
(log((B*b + 3^(1/2)*B*b*1i)/a^(2/3) + (2*B*b*(a + b*x^3)^(1/3))/a)*(B*b +
3^(1/2)*B*b*1i))/(9*a^(5/3)) - (2*B*b*log((a + b*x^3)^(1/3) - a^(1/3)))/(9
*a^(5/3)) + (5*A*b^2*log((a + b*x^3)^(1/3) - a^(1/3)))/(27*a^(8/3)) - (B*(
a + b*x^3)^(1/3))/(3*a*x^3) + (5*A*b^2*log((5*A*b^2*((3^(1/2)*1i)/2 - 1/2)
)/(3*a^(5/3)) - (5*A*b^2*(a + b*x^3)^(1/3))/(3*a^2))*((3^(1/2)*1i)/2 - 1/2
))/(27*a^(8/3)) - (5*A*b^2*log((5*A*b^2*((3^(1/2)*1i)/2 + 1/2))/(3*a^(5/3)
) + (5*A*b^2*(a + b*x^3)^(1/3))/(3*a^2))*((3^(1/2)*1i)/2 + 1/2))/(27*a^(8/
3))
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^7(a+bx^3)^{2/3}}\,dx = \left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,x^7}dx\right)a + \left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,x^4}dx\right)b$$

input
```
int((B*x^3+A)/x^7/(b*x^3+a)^(2/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*x**7),x)*a + int(1/((a + b*x**3)**(2/3)*x**4),x
)*b
```

3.349 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 146

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{(6Ab-5aB)x^2\sqrt[3]{a+bx^3}}{18b^2}+\frac{Bx^5\sqrt[3]{a+bx^3}}{6b} + \frac{a(6Ab-5aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{8/3}}+\frac{a(6Ab-5aB)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{8/3}}$$

output
```
1/18*(6*A*b-5*B*a)*x^2*(b*x^3+a)^(1/3)/b^2+1/6*B*x^5*(b*x^3+a)^(1/3)/b+1/2
7*a*(6*A*b-5*B*a)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1
/2)/b^(8/3)+1/18*a*(6*A*b-5*B*a)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(8/3)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.26

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{3b^{2/3}x^2\sqrt[3]{a+bx^3}(6Ab-5aB+3bBx^3)-2\sqrt{3}a(-6Ab+5aB)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+}}\right.}{}$$

input
```
Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^(2/3),x]
```

output
```
(3*b^(2/3)*x^2*(a + b*x^3)^(1/3)*(6*A*b - 5*a*B + 3*b*B*x^3) - 2*Sqrt[3]*a
*(-6*A*b + 5*a*B)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1
/3))] - 2*a*(-6*A*b + 5*a*B)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + a*(-6
*A*b + 5*a*B)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^
(2/3)])/(54*b^(8/3))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 843, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(6Ab-5aB)\int \frac{x^4}{(bx^3+a)^{2/3}}dx}{6b}+\frac{Bx^5\sqrt[3]{a+bx^3}}{6b}$$

$$\Big\downarrow \text{843}$$

$$\frac{(6Ab-5aB)\left(\frac{x^2\sqrt[3]{a+bx^3}}{3b}-\frac{2a\int \frac{x}{(bx^3+a)^{2/3}}dx}{3b}\right)}{6b}+\frac{Bx^5\sqrt[3]{a+bx^3}}{6b}$$

$$\downarrow 853$$

$$\frac{(6Ab-5aB)\left(\frac{x^2\sqrt[3]{a+bx^3}}{3b}-\frac{2a\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\right)}{3b}\right)}{6b}+\frac{Bx^5\sqrt[3]{a+bx^3}}{6b}$$

input
```
Int[(x^4*(A + B*x^3))/(a + b*x^3)^(2/3),x]
```

output
```
(B*x^5*(a + b*x^3)^(1/3))/(6*b) + ((6*A*b - 5*a*B)*((x^2*(a + b*x^3)^(1/3)
)/(3*b) - (2*a*(-(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(S
qrt[3]*b^(2/3))) - Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b^(2/3))))/(3*b))
)/(6*b)
```

Defintions of rubi rules used

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(119) = 238$.

Time = 1.26 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.87

method	result
pseudoelliptic	$\frac{9B\,b^{\frac{5}{3}}x^{5}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+18A\,b^{\frac{5}{3}}x^{2}\left(b\,x^{3}+a\right)^{\frac{1}{3}}-15Ba\,x^{2}b^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}-12A\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)\sqrt{3}\,ab+10B\,\mathrm{ar}}{}$

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/54*(9*B*b^(5/3)*x^5*(b*x^3+a)^(1/3)+18*A*b^(5/3)*x^2*(b*x^3+a)^(1/3)-15*
B*a*x^2*b^(2/3)*(b*x^3+a)^(1/3)-12*A*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^
3+a)^(1/3))/b^(1/3)/x)*3^(1/2)*a*b+10*B*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b
*x^3+a)^(1/3))/b^(1/3)/x)*3^(1/2)*a^2+12*A*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))
/x)*a*b-6*A*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2
)*a*b-10*B*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^2+5*B*ln((b^(2/3)*x^2+b^(1
/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a^2)/b^(8/3)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.49

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{6\sqrt{\frac{1}{3}}(5\,Ba^2b-6\,Aab^2)(b^2)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left((b^2)^{\frac{1}{3}}bx+2\,(bx^3+a)^{\frac{1}{3}}(b^2)^{\frac{2}{3}}\right)(b^2)^{\frac{1}{6}}}{b^2x}\right)-2\,(5\,Ba^2-}{}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/54*(6*sqrt(1/3)*(5*B*a^2*b - 6*A*a*b^2)*(b^2)^(1/6)*arctan(sqrt(1/3)*((b
^2)^(1/3)*b*x + 2*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 2*
(5*B*a^2 - 6*A*a*b)*(b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)
/x) + (5*B*a^2 - 6*A*a*b)*(b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a)
^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2) + 3*(3*B*b^3*x^5 - (5*B*a
*b^2 - 6*A*b^3)*x^2)*(b*x^3 + a)^(1/3))/b^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.55

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{Ax^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)}+\frac{Bx^8\Gamma\left(\frac{8}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**4*(B*x**3+A)/(b*x**3+a)**(2/3),x)
```

output
```
A*x**5*gamma(5/3)*hyper((2/3, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(2/3)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((2/3, 8/3), (11/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(11/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(119) = 238.

Time = 0.12 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.24

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{1}{54}\,B\left(\frac{10\,\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{8}{3}}}+\frac{5\,a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right.}{b^{\frac{8}{3}}}\right.$$

$$-\frac{1}{9}\,A\left(\frac{2\,\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{5}{3}}}+\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{5}{3}}}-\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+}{x}\right.}{b^{\frac{5}{3}}}\right.$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/54*B*(10*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x
)/b^(1/3))/b^(8/3) + 5*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*
x^3 + a)^(2/3)/x^2)/b^(8/3) - 10*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b
^(8/3) + 3*(8*(b*x^3 + a)^(1/3)*a^2*b/x - 5*(b*x^3 + a)^(4/3)*a^2/x^4)/(b^
4 - 2*(b*x^3 + a)*b^3/x^3 + (b*x^3 + a)^2*b^2/x^6)) - 1/9*A*(2*sqrt(3)*a*a
rctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(5/3) + a*l
og(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(5/3)
- 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(5/3) + 3*(b*x^3 + a)^(1/3)*a/
((b^2 - (b*x^3 + a)*b/x^3)*x))
```

Giac [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^4\,(B\,x^3+A)}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int((x^4*(A + B*x^3))/(a + b*x^3)^(2/3),x)
```

output
```
int((x^4*(A + B*x^3))/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \left(\int \frac{x^7}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)b + \left(\int \frac{x^4}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a$$

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^(2/3),x)
```

output
```
int(x**7/(a + b*x**3)**(2/3),x)*b + int(x**4/(a + b*x**3)**(2/3),x)*a
```

3.350 $\int \frac{x(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 114

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx=\frac{Bx^2\sqrt[3]{a+bx^3}}{3b}-\frac{(3Ab-2aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}}-\frac{(3Ab-2aB)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{5/3}}$$

output
```
1/3*B*x^2*(b*x^3+a)^(1/3)/b-1/9*(3*A*b-2*B*a)*arctan(1/3*(1+2*b^(1/3)*x/(b
*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(5/3)-1/6*(3*A*b-2*B*a)*ln(b^(1/3)*x-(b*
x^3+a)^(1/3))/b^(5/3)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.46

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{6b^{2/3}Bx^2\sqrt[3]{a+bx^3} - 2\sqrt{3}(3Ab-2aB)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) + 2(-3Ab+2aB)}{18}$$

input
```
Integrate[(x*(A + B*x^3))/(a + b*x^3)^(2/3),x]
```

output
```
(6*b^(2/3)*B*x^2*(a + b*x^3)^(1/3) - 2*Sqrt[3]*(3*A*b - 2*a*B)*ArcTan[(Sqr
t[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 2*(-3*A*b + 2*a*B)*Lo
g[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + (3*A*b - 2*a*B)*Log[b^(2/3)*x^2 + b^
(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*b^(5/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {959, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(3Ab-2aB)\int \frac{x}{(bx^3+a)^{2/3}}dx}{3b} + \frac{Bx^2\sqrt[3]{a+bx^3}}{3b}$$

$$\Big\downarrow \text{853}$$

$$\frac{(3Ab-2aB)\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\right)}{3b}+\frac{Bx^2\sqrt[3]{a+bx^3}}{3b}$$

input
```
Int[(x*(A + B*x^3))/(a + b*x^3)^(2/3),x]
```

output
```
(B*x^2*(a + b*x^3)^(1/3))/(3*b) + ((3*A*b - 2*a*B)*(-(ArcTan[(1 + (2*b^(1/
3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3))) - Log[b^(1/3)*x - (a
+ b*x^3)^(1/3)]/(2*b^(2/3))))/(3*b)
```

Defintions of rubi rules used

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(91) = 182$.

Time = 1.24 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.99

method	result
pseudoelliptic	$\frac{6\left(b\,x^{3}+a\right)^{\frac{1}{3}} b^{\frac{2}{3}} x^{2} B+6 A \arctan\left(\frac{\sqrt{3}\,\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right)\sqrt{3}\, b-4 B \arctan\left(\frac{\sqrt{3}\,\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right)\sqrt{3}\, a-6 A \ln\left(\frac{-b^{\frac{1}{3}}}{}\right.}{}$

input
```
int(x*(B*x^3+A)/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/18*(6*(b*x^3+a)^(1/3)*b^(2/3)*x^2*B+6*A*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*
(b*x^3+a)^(1/3))/b^(1/3)/x)*3^(1/2)*b-4*B*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*
(b*x^3+a)^(1/3))/b^(1/3)/x)*3^(1/2)*a-6*A*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/
x)*b+3*A*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*b
+4*B*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a-2*B*ln((b^(2/3)*x^2+b^(1/3)*(b*x
^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a)/b^(5/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(91) = 182.

Time = 0.10 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.91

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{6\,(bx^3+a)^{\frac{1}{3}}Bb^2x^2-6\,\sqrt{\frac{1}{3}}(2\,Bab-3\,Ab^2)\sqrt{-(-b^2)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b^2)^{\frac{1}{3}}bx-2\,(bx^3+\cdots\right.}{b^2\cdots}\right.}{\cdots}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/18*(6*(b*x^3 + a)^(1/3)*B*b^2*x^2 - 6*sqrt(1/3)*(2*B*a*b - 3*A*b^2)*sqrt
(-(-b^2)^(1/3))*arctan(-sqrt(1/3)*((-b^2)^(1/3)*b*x - 2*(b*x^3 + a)^(1/3)*
(-b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) + 2*(2*B*a - 3*A*b)*(-b^2)^(2/3
)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) - (2*B*a - 3*A*b)*(-b^2)^
(2/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3
 + a)^(2/3)*b)/x^2))/b^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{Ax^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)} + \frac{Bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)}$$

input

```
integrate(x*(B*x**3+A)/(b*x**3+a)**(2/3),x)
```

output

```
A*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(2/3)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((2/3, 5/3), (8/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*a**(2/3)*gamma(8/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(91) = 182$.

Time = 0.12 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.12

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = -\frac{1}{9}\,B\left(\frac{2\,\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{5}{3}}}+\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{5}{3}}}-\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+\cdots}{x}\right.}{b^{\frac{5}{3}}}\right.$$
$$+\frac{1}{6}\,A\left(\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{2}{3}}}+\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{2}{3}}}-\frac{2\,\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{2}{3}}}\right.$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
-1/9*B*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b
^(1/3))/b^(5/3) + a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a
)^(2/3)/x^2)/b^(5/3) - 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(5/3) + 3
*(b*x^3 + a)^(1/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x)) + 1/6*A*(2*sqrt(3)*arc
tan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(2/3) + log(b
^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(2/3) - 2*
log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(2/3))
```

Giac [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \int \frac{x\,(B\,x^3+A)}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int((x*(A + B*x^3))/(a + b*x^3)^(2/3),x)
```

output
```
int((x*(A + B*x^3))/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \left(\int \frac{x^4}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)b + \left(\int \frac{x}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a$$

input
```
int(x*(B*x^3+A)/(b*x^3+a)^(2/3),x)
```

output
```
int(x**4/(a + b*x**3)**(2/3),x)*b + int(x/(a + b*x**3)**(2/3),x)*a
```

3.351 $\int \frac{A+Bx^3}{x^2(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^{2/3}}\,dx = -\frac{A\sqrt[3]{a+bx^3}}{ax} - \frac{B\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} - \frac{B\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}$$

output
```
-A*(b*x^3+a)^(1/3)/a/x-1/3*B*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^
(1/2))*3^(1/2)/b^(2/3)-1/2*B*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.61

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^{2/3}}\,dx = -\frac{A\sqrt[3]{a+bx^3}}{ax} - \frac{B\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}b^{2/3}} - \frac{B\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3b^{2/3}} + \frac{B\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{6b^{2/3}}$$

input
```
Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^(2/3)),x]
```

output
```
-((A*(a + b*x^3)^(1/3))/(a*x)) - (B*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x
+ 2*(a + b*x^3)^(1/3))])/(Sqrt[3]*b^(2/3)) - (B*Log[-(b^(1/3)*x) + (a + b*
x^3)^(1/3)])/(3*b^(2/3)) + (B*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3
) + (a + b*x^3)^(2/3)])/(6*b^(2/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {953, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{2/3}}\,dx$$

$$\Big\downarrow \text{ 953}$$

$$B\int \frac{x}{\left(bx^3+a\right)^{2/3}}dx-\frac{A\sqrt[3]{a+bx^3}}{ax}$$

$$\Big\downarrow \text{ 853}$$

$$B\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\right)-\frac{A\sqrt[3]{a+bx^3}}{ax}$$

input
```
Int[(A + B*x^3)/(x^2*(a + b*x^3)^(2/3)),x]
```

output
```
-((A*(a + b*x^3)^(1/3))/(a*x)) + B*(-(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3
)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3))) - Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/
(2*b^(2/3)))
```

Defintions of rubi rules used

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 953
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[d/e^n   Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] &&
(IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && G
tQ[m + n, -1]))
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.35

method	result
pseudoelliptic	$-\frac{-\frac{B\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{b^{\frac{1}{3}}}+x\right)}{3x}\right)a x}{3}+\frac{B\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a x}{3}-\frac{B\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)a x}{6}+A}{b^{\frac{2}{3}}x a}$

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-(-1/3*B*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)/b^(1/3)+x)/x)*a*x+1
/3*B*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a*x-1/6*B*ln((b^(2/3)*x^2+b^(1/3)*
(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a*x+A*(b*x^3+a)^(1/3)*b^(2/3))/b^(
2/3)/x/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{2/3}}\,dx = \frac{A\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)} + \frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**2/(b*x**3+a)**(2/3),x)
```

output
```
A*b**(1/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-1/3)/(3*a*gamma(2/3)) + B*x**2*g
amma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*
gamma(5/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{2/3}}\,dx=\frac{1}{6}\,B\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{2}{3}}}+\frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{2}{3}}}-\,2\right.$$

$$-\frac{\left(bx^3+a\right)^{\frac{1}{3}}A}{ax}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/6*B*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(2/3) + log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/
3)/x^2)/b^(2/3) - 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(2/3)) - (b*x^3
+ a)^(1/3)*A/(a*x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{2}{3}}x^2}\,dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(2/3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^2 (a+bx^3)^{2/3}}\,dx = \int \frac{B\,x^3+A}{x^2\,{\left(b\,x^3+a\right)}^{2/3}}\,dx$$

input
```
int((A + B*x^3)/(x^2*(a + b*x^3)^(2/3)),x)
```

output
```
int((A + B*x^3)/(x^2*(a + b*x^3)^(2/3)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^2 (a+bx^3)^{2/3}}\,dx = \left(\int \frac{x}{(b\,x^3+a)^{\frac{2}{3}}}dx\right) b+\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,x^2}dx\right) a$$

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^(2/3),x)
```

output
```
int(x/(a + b*x**3)**(2/3),x)*b + int(1/((a + b*x**3)**(2/3)*x**2),x)*a
```

3.352 $\int \frac{A+Bx^3}{x^5(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{2/3}}\,dx=-\frac{A\sqrt[3]{a+bx^3}}{4ax^4}+\frac{(3Ab-4aB)\sqrt[3]{a+bx^3}}{4a^2x}$$

output
```
-1/4*A*(b*x^3+a)^(1/3)/a/x^4+1/4*(3*A*b-4*B*a)*(b*x^3+a)^(1/3)/a^2/x
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{2/3}}\,dx=\frac{\sqrt[3]{a+bx^3}(-aA+3Abx^3-4aBx^3)}{4a^2x^4}$$

input
```
Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^(2/3)),x]
```

output
```
((a + b*x^3)^(1/3)*(-(a*A) + 3*A*b*x^3 - 4*a*B*x^3))/(4*a^2*x^4)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {955, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^5 (a+bx^3)^{2/3}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(3Ab-4aB)\int \frac{1}{x^2(bx^3+a)^{2/3}}dx}{4a}-\frac{A\sqrt[3]{a+bx^3}}{4ax^4}$$

$$\Big\downarrow \text{796}$$

$$\frac{\sqrt[3]{a+bx^3}(3Ab-4aB)}{4a^2x}-\frac{A\sqrt[3]{a+bx^3}}{4ax^4}$$

input
```
Int[(A + B*x^3)/(x^5*(a + b*x^3)^(2/3)),x]
```

output
```
-1/4*(A*(a + b*x^3)^(1/3))/(a*x^4) + ((3*A*b - 4*a*B)*(a + b*x^3)^(1/3))/(
4*a^2*x)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{(b\,x^3+a)^{\frac{1}{3}}\left(-3Ab\,x^3+4Ba\,x^3+Aa\right)}{4a^2x^4}$	36
trager	$-\frac{(b\,x^3+a)^{\frac{1}{3}}\left(-3Ab\,x^3+4Ba\,x^3+Aa\right)}{4a^2x^4}$	36
risch	$-\frac{(b\,x^3+a)^{\frac{1}{3}}\left(-3Ab\,x^3+4Ba\,x^3+Aa\right)}{4a^2x^4}$	36
pseudoelliptic	$-\frac{\left((4B\,x^3+A)a-3Ab\,x^3\right)(b\,x^3+a)^{\frac{1}{3}}}{4a^2x^4}$	36
orering	$-\frac{(b\,x^3+a)^{\frac{1}{3}}\left(-3Ab\,x^3+4Ba\,x^3+Aa\right)}{4a^2x^4}$	36

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x^3+a)^(1/3)*(-3*A*b*x^3+4*B*a*x^3+A*a)/a^2/x^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{2/3}}\,dx = -\frac{\left((4\,Ba-3\,Ab)x^3+Aa\right)\left(bx^3+a\right)^{\frac{1}{3}}}{4\,a^2x^4}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
-1/4*((4*B*a - 3*A*b)*x^3 + A*a)*(b*x^3 + a)^(1/3)/(a^2*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(42) = 84.

Time = 1.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.98

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{2/3}}\,dx = -\frac{A\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{4}{3}\right)}{9ax^3\Gamma\left(\frac{2}{3}\right)} + \frac{Ab^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{4}{3}\right)}{3a^2\Gamma\left(\frac{2}{3}\right)} + \frac{B\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**5/(b*x**3+a)**(2/3),x)
```

output
```
-A*b**(1/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-4/3)/(9*a*x**3*gamma(2/3)) + A*
b**(4/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-4/3)/(3*a**2*gamma(2/3)) + B*b**(1
/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-1/3)/(3*a*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{2/3}}\,dx=\frac{A\left(\frac{4\,(bx^3+a)^{\frac{1}{3}}b}{x}-\frac{(bx^3+a)^{\frac{4}{3}}}{x^4}\right)}{4\,a^2}-\frac{(bx^3+a)^{\frac{1}{3}}B}{ax}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/4*A*(4*(b*x^3 + a)^(1/3)*b/x - (b*x^3 + a)^(4/3)/x^4)/a^2 - (b*x^3 + a)^
(1/3)*B/(a*x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{2}{3}}x^5}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(2/3)*x^5), x)
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{2/3}}\,dx=-\frac{\left(b\,x^3+a\right)^{1/3}\,\left(A\,a-3\,A\,b\,x^3+4\,B\,a\,x^3\right)}{4\,a^2\,x^4}$$

input
```
int((A + B*x^3)/(x^5*(a + b*x^3)^(2/3)),x)
```

output
```
-((a + b*x^3)^(1/3)*(A*a - 3*A*b*x^3 + 4*B*a*x^3))/(4*a^2*x^4)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^5 (a+bx^3)^{2/3}}\,dx = \left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,x^5}dx\right)a+\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,x^2}dx\right)b$$

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^(2/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*x**5),x)*a + int(1/((a + b*x**3)**(2/3)*x**2),x
)*b
```

3.353 $\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{2/3}}\,dx=-\frac{A\sqrt[3]{a+bx^3}}{7ax^7}+\frac{(6Ab-7aB)\sqrt[3]{a+bx^3}}{28a^2x^4}-\frac{3b(6Ab-7aB)\sqrt[3]{a+bx^3}}{28a^3x}$$

output
```
-1/7*A*(b*x^3+a)^(1/3)/a/x^7+1/28*(6*A*b-7*B*a)*(b*x^3+a)^(1/3)/a^2/x^4-3/
28*b*(6*A*b-7*B*a)*(b*x^3+a)^(1/3)/a^3/x
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{2/3}}\,dx=\frac{\sqrt[3]{a+bx^3}(-4a^2A+6aAbx^3-7a^2Bx^3-18Ab^2x^6+21abBx^6)}{28a^3x^7}$$

input
```
Integrate[(A + B*x^3)/(x^8*(a + b*x^3)^(2/3)),x]
```

output
```
((a + b*x^3)^(1/3)*(-4*a^2*A + 6*a*A*b*x^3 - 7*a^2*B*x^3 - 18*A*b^2*x^6 +
21*a*b*B*x^6))/(28*a^3*x^7)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^8 \left(a+bx^3\right)^{2/3}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(6Ab-7aB)\int \frac{1}{x^5 (bx^3+a)^{2/3}}dx}{7a}-\frac{A\sqrt[3]{a+bx^3}}{7ax^7}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(6Ab-7aB)\left(-\frac{3b\int \frac{1}{x^2 \left(bx^3+a\right)^{2/3}}dx}{4a}-\frac{\sqrt[3]{a+bx^3}}{4ax^4}\right)}{7a}-\frac{A\sqrt[3]{a+bx^3}}{7ax^7}$$

$$\Big\downarrow \text{796}$$

$$-\frac{\left(\frac{3b\sqrt[3]{a+bx^3}}{4a^2x}-\frac{\sqrt[3]{a+bx^3}}{4ax^4}\right)(6Ab-7aB)}{7a}-\frac{A\sqrt[3]{a+bx^3}}{7ax^7}$$

input
```
Int[(A + B*x^3)/(x^8*(a + b*x^3)^(2/3)),x]
```

output
```
-1/7*(A*(a + b*x^3)^(1/3))/(a*x^7) - ((6*A*b - 7*a*B)*(-1/4*(a + b*x^3)^(1
/3)/(a*x^4) + (3*b*(a + b*x^3)^(1/3))/(4*a^2*x)))/(7*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(\left(\frac{7B\,x^{3}}{4}+A\right) a^{2}-\frac{3\left(\frac{7B\,x^{3}}{2}+A\right) b\,x^{3} a}{2}+\frac{9A\,b^{2} x^{6}}{2}\right)}{7a^{3} x^{7}}$	55
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(18A\,b^{2} x^{6}-21Bab\,x^{6}-6aAb\,x^{3}+7B\,a^{2} x^{3}+4a^{2} A\right)}{28a^{3} x^{7}}$	59
trager	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(18A\,b^{2} x^{6}-21Bab\,x^{6}-6aAb\,x^{3}+7B\,a^{2} x^{3}+4a^{2} A\right)}{28a^{3} x^{7}}$	59
risch	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(18A\,b^{2} x^{6}-21Bab\,x^{6}-6aAb\,x^{3}+7B\,a^{2} x^{3}+4a^{2} A\right)}{28a^{3} x^{7}}$	59
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(18A\,b^{2} x^{6}-21Bab\,x^{6}-6aAb\,x^{3}+7B\,a^{2} x^{3}+4a^{2} A\right)}{28a^{3} x^{7}}$	59

input
```
int((B*x^3+A)/x^8/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-1/7*(b*x^3+a)^(1/3)*((7/4*B*x^3+A)*a^2-3/2*(7/2*B*x^3+A)*b*x^3*a+9/2*A*b^
2*x^6)/a^3/x^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^3}{x^8\,(a+bx^3)^{2/3}}\,dx = \frac{(3\,(7\,Bab-6\,Ab^2)x^6-(7\,Ba^2-6\,Aab)x^3-4\,Aa^2)(bx^3+a)^{\frac{1}{3}}}{28\,a^3x^7}$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/28*(3*(7*B*a*b - 6*A*b^2)*x^6 - (7*B*a^2 - 6*A*a*b)*x^3 - 4*A*a^2)*(b*x^
3 + a)^(1/3)/(a^3*x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(76) = 152.

Time = 2.03 (sec) , antiderivative size = 488, normalized size of antiderivative = 5.81

$$\int \frac{A+Bx^3}{x^8\,(a+bx^3)^{2/3}}\,dx = \frac{4Aa^4b^{\frac{13}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{7}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{2}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{2}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{2}{3}\right)} + \frac{2Aa^3b^{\frac{16}{3}}x^3\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{7}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{2}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{2}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{2}{3}\right)} + \frac{10Aa^2b^{\frac{19}{3}}x^6\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{7}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{2}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{2}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{2}{3}\right)} + \frac{30Aab^{\frac{22}{3}}x^9\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{7}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{2}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{2}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{2}{3}\right)} + \frac{18Ab^{\frac{25}{3}}x^{12}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{7}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{2}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{2}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{2}{3}\right)} - \frac{B\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{4}{3}\right)}{9ax^3\Gamma\left(\frac{2}{3}\right)} + \frac{Bb^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{4}{3}\right)}{3a^2\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**8/(b*x**3+a)**(2/3),x)
```

output
```
4*A*a**4*b**(13/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(27*a**5*b**4*x**6*
gamma(2/3) + 54*a**4*b**5*x**9*gamma(2/3) + 27*a**3*b**6*x**12*gamma(2/3))
 + 2*A*a**3*b**(16/3)*x**3*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(27*a**5*b*
*4*x**6*gamma(2/3) + 54*a**4*b**5*x**9*gamma(2/3) + 27*a**3*b**6*x**12*gam
ma(2/3)) + 10*A*a**2*b**(19/3)*x**6*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(2
7*a**5*b**4*x**6*gamma(2/3) + 54*a**4*b**5*x**9*gamma(2/3) + 27*a**3*b**6*
x**12*gamma(2/3)) + 30*A*a*b**(22/3)*x**9*(a/(b*x**3) + 1)**(1/3)*gamma(-7
/3)/(27*a**5*b**4*x**6*gamma(2/3) + 54*a**4*b**5*x**9*gamma(2/3) + 27*a**3
*b**6*x**12*gamma(2/3)) + 18*A*b**(25/3)*x**12*(a/(b*x**3) + 1)**(1/3)*gam
ma(-7/3)/(27*a**5*b**4*x**6*gamma(2/3) + 54*a**4*b**5*x**9*gamma(2/3) + 27
*a**3*b**6*x**12*gamma(2/3)) - B*b**(1/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-4
/3)/(9*a*x**3*gamma(2/3)) + B*b**(4/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-4/3)
/(3*a**2*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^3}{x^8 \left(a+bx^3\right)^{2/3}}\,dx = \frac{B\left(\frac{4\,\left(bx^3+a\right)^{\frac{1}{3}}b}{x} - \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{x^4}\right)}{4\,a^2} - \frac{\left(\frac{14\,\left(bx^3+a\right)^{\frac{1}{3}}b^2}{x} - \frac{7\,\left(bx^3+a\right)^{\frac{4}{3}}b}{x^4} + \frac{2\,\left(bx^3+a\right)^{\frac{7}{3}}}{x^7}\right)A}{14\,a^3}$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/4*B*(4*(b*x^3 + a)^(1/3)*b/x - (b*x^3 + a)^(4/3)/x^4)/a^2 - 1/14*(14*(b*
x^3 + a)^(1/3)*b^2/x - 7*(b*x^3 + a)^(4/3)*b/x^4 + 2*(b*x^3 + a)^(7/3)/x^7
)*A/a^3
```

Giac [F]

$$\int \frac{A+Bx^3}{x^8 \left(a+bx^3\right)^{2/3}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{2}{3}}x^8}\,dx$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(2/3)*x^8), x)
```

Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^3}{x^8 (a+bx^3)^{2/3}}\,dx = -\frac{(b\,x^3+a)^{1/3}\,(7\,B\,a^2\,x^3+4\,A\,a^2-21\,B\,a\,b\,x^6-6\,A\,a\,b\,x^3+18\,A\,b^2\,x^6)}{28\,a^3\,x^7}$$

input
```
int((A + B*x^3)/(x^8*(a + b*x^3)^(2/3)),x)
```

output
```
-((a + b*x^3)^(1/3)*(4*A*a^2 + 7*B*a^2*x^3 + 18*A*b^2*x^6 - 6*A*a*b*x^3 -
21*B*a*b*x^6))/(28*a^3*x^7)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^8 (a+bx^3)^{2/3}}\,dx = \left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,x^8}dx\right)a+\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,x^5}dx\right)b$$

input
```
int((B*x^3+A)/x^8/(b*x^3+a)^(2/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*x**8),x)*a + int(1/((a + b*x**3)**(2/3)*x**5),x
)*b
```

3.354 $\int \frac{A+Bx^3}{x^{11}\left(a+bx^3\right)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \frac{A+Bx^3}{x^{11}\left(a+bx^3\right)^{2/3}}\,dx = -\frac{A\sqrt[3]{a+bx^3}}{10ax^{10}}+\frac{(9Ab-10aB)\sqrt[3]{a+bx^3}}{70a^2x^7} -\frac{3b(9Ab-10aB)\sqrt[3]{a+bx^3}}{140a^3x^4}+\frac{9b^2(9Ab-10aB)\sqrt[3]{a+bx^3}}{140a^4x}$$

output
```
-1/10*A*(b*x^3+a)^(1/3)/a/x^10+1/70*(9*A*b-10*B*a)*(b*x^3+a)^(1/3)/a^2/x^7
-3/140*b*(9*A*b-10*B*a)*(b*x^3+a)^(1/3)/a^3/x^4+9/140*b^2*(9*A*b-10*B*a)*(
b*x^3+a)^(1/3)/a^4/x
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^3}{x^{11}\left(a+bx^3\right)^{2/3}}\,dx = \frac{\sqrt[3]{a+bx^3}\left(-14a^3A+18a^2Abx^3-20a^3Bx^3-27aAb^2x^6+30a^2bBx^6+81Ab^3x^9-\right.}{140a^4x^{10}}$$

input
```
Integrate[(A + B*x^3)/(x^11*(a + b*x^3)^(2/3)),x]
```

output
```
((a + b*x^3)^(1/3)*(-14*a^3*A + 18*a^2*A*b*x^3 - 20*a^3*B*x^3 - 27*a*A*b^2
*x^6 + 30*a^2*b*B*x^6 + 81*A*b^3*x^9 - 90*a*b^2*B*x^9))/(140*a^4*x^10)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{11} (a+bx^3)^{2/3}} \, dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(9Ab-10aB)\int \frac{1}{x^8(bx^3+a)^{2/3}}dx}{10a}-\frac{A\sqrt[3]{a+bx^3}}{10ax^{10}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{(9Ab-10aB)\left(-\frac{6b\int \frac{1}{x^5\left(bx^3+a\right)^{2/3}}dx}{7a}-\frac{\sqrt[3]{a+bx^3}}{7ax^7}\right)}{10a}-\frac{A\sqrt[3]{a+bx^3}}{10ax^{10}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{(9Ab-10aB)\left(-\frac{6b\left(-\frac{3b\int \frac{1}{x^2\left(bx^3+a\right)^{2/3}}dx}{4a}-\frac{\sqrt[3]{a+bx^3}}{4ax^4}\right)}{7a}-\frac{\sqrt[3]{a+bx^3}}{7ax^7}\right)}{10a}-\frac{A\sqrt[3]{a+bx^3}}{10ax^{10}}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{\left(-\frac{6b\left(\frac{3b\sqrt[3]{a+bx^3}}{4a^2x}-\frac{\sqrt[3]{a+bx^3}}{4ax^4}\right)}{7a}-\frac{\sqrt[3]{a+bx^3}}{7ax^7}\right)(9Ab-10aB)}{10a}-\frac{A\sqrt[3]{a+bx^3}}{10ax^{10}}$$

input
```
Int[(A + B*x^3)/(x^11*(a + b*x^3)^(2/3)),x]
```

output
```
-1/10*(A*(a + b*x^3)^(1/3))/(a*x^10) - ((9*A*b - 10*a*B)*(-1/7*(a + b*x^3)
^(1/3)/(a*x^7) - (6*b*(-1/4*(a + b*x^3)^(1/3)/(a*x^4) + (3*b*(a + b*x^3)^(
1/3))/(4*a^2*x)))/(7*a)))/(10*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.63

method	result	size
pseudoelliptic	$-\frac{\left(\left(\frac{10B\,x^{3}}{7}+A\right)a^{3}-\frac{9b\left(\frac{5B\,x^{3}}{3}+A\right)x^{3}a^{2}}{7}+\frac{27b^{2}\left(\frac{10B\,x^{3}}{3}+A\right)x^{6}a}{14}-\frac{81A\,x^{9}b^{3}}{14}\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{10x^{10}a^{4}}$	74
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(-81A\,x^{9}b^{3}+90B\,x^{9}a\,b^{2}+27A\,x^{6}a\,b^{2}-30B\,x^{6}a^{2}b-18a^{2}Ab\,x^{3}+20B\,x^{3}a^{3}+14a^{3}A\right)}{140x^{10}a^{4}}$	83
trager	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(-81A\,x^{9}b^{3}+90B\,x^{9}a\,b^{2}+27A\,x^{6}a\,b^{2}-30B\,x^{6}a^{2}b-18a^{2}Ab\,x^{3}+20B\,x^{3}a^{3}+14a^{3}A\right)}{140x^{10}a^{4}}$	83
risch	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(-81A\,x^{9}b^{3}+90B\,x^{9}a\,b^{2}+27A\,x^{6}a\,b^{2}-30B\,x^{6}a^{2}b-18a^{2}Ab\,x^{3}+20B\,x^{3}a^{3}+14a^{3}A\right)}{140x^{10}a^{4}}$	83
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(-81A\,x^{9}b^{3}+90B\,x^{9}a\,b^{2}+27A\,x^{6}a\,b^{2}-30B\,x^{6}a^{2}b-18a^{2}Ab\,x^{3}+20B\,x^{3}a^{3}+14a^{3}A\right)}{140x^{10}a^{4}}$	83

input
```
int((B*x^3+A)/x^11/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-1/10*((10/7*B*x^3+A)*a^3-9/7*b*(5/3*B*x^3+A)*x^3*a^2+27/14*b^2*(10/3*B*x^
3+A)*x^6*a-81/14*A*x^9*b^3)*(b*x^3+a)^(1/3)/x^10/a^4
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^3}{x^{11}\left(a+bx^3\right)^{2/3}}\,dx =$$
$$-\frac{(9\,(10\,Bab^2-9\,Ab^3)x^9-3\,(10\,Ba^2b-9\,Aab^2)x^6+14\,Aa^3+2\,(10\,Ba^3-9\,Aa^2b)x^3)(bx^3+a)^{\frac{1}{3}}}{140\,a^4x^{10}}$$

input
```
integrate((B*x^3+A)/x^11/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
-1/140*(9*(10*B*a*b^2 - 9*A*b^3)*x^9 - 3*(10*B*a^2*b - 9*A*a*b^2)*x^6 + 14
*A*a^3 + 2*(10*B*a^3 - 9*A*a^2*b)*x^3)*(b*x^3 + a)^(1/3)/(a^4*x^10)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1120 vs. $2(110) = 220$.

Time = 2.51 (sec) , antiderivative size = 1120, normalized size of antiderivative = 9.57

$$\int \frac{A + Bx^3}{x^{11} (a + bx^3)^{2/3}} \, dx = \text{Too large to display}$$

input
```
integrate((B*x**3+A)/x**11/(b*x**3+a)**(2/3),x)
```

output
```
-28*A*a**6*b**(28/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x*
*9*gamma(2/3) + 243*a**6*b**10*x**12*gamma(2/3) + 243*a**5*b**11*x**15*gam
ma(2/3) + 81*a**4*b**12*x**18*gamma(2/3)) - 48*A*a**5*b**(31/3)*x**3*(a/(b
*x**3) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**9*gamma(2/3) + 243*a**6*b
**10*x**12*gamma(2/3) + 243*a**5*b**11*x**15*gamma(2/3) + 81*a**4*b**12*x*
*18*gamma(2/3)) - 30*A*a**4*b**(34/3)*x**6*(a/(b*x**3) + 1)**(1/3)*gamma(-
10/3)/(81*a**7*b**9*x**9*gamma(2/3) + 243*a**6*b**10*x**12*gamma(2/3) + 24
3*a**5*b**11*x**15*gamma(2/3) + 81*a**4*b**12*x**18*gamma(2/3)) + 80*A*a**
3*b**(37/3)*x**9*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**9*g
amma(2/3) + 243*a**6*b**10*x**12*gamma(2/3) + 243*a**5*b**11*x**15*gamma(2
/3) + 81*a**4*b**12*x**18*gamma(2/3)) + 360*A*a**2*b**(40/3)*x**12*(a/(b*x
**3) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**9*gamma(2/3) + 243*a**6*b**
10*x**12*gamma(2/3) + 243*a**5*b**11*x**15*gamma(2/3) + 81*a**4*b**12*x**1
8*gamma(2/3)) + 432*A*a*b**(43/3)*x**15*(a/(b*x**3) + 1)**(1/3)*gamma(-10/
3)/(81*a**7*b**9*x**9*gamma(2/3) + 243*a**6*b**10*x**12*gamma(2/3) + 243*a
**5*b**11*x**15*gamma(2/3) + 81*a**4*b**12*x**18*gamma(2/3)) + 162*A*b**(4
6/3)*x**18*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**9*gamma(2
/3) + 243*a**6*b**10*x**12*gamma(2/3) + 243*a**5*b**11*x**15*gamma(2/3) +
81*a**4*b**12*x**18*gamma(2/3)) + 4*B*a**4*b**(13/3)*(a/(b*x**3) + 1)**(1/
3)*gamma(-7/3)/(27*a**5*b**4*x**6*gamma(2/3) + 54*a**4*b**5*x**9*gamma(...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x^{11}\left(a+bx^3\right)^{2/3}}\,dx = -\frac{\left(\frac{14\,\left(bx^3+a\right)^{\frac{1}{3}}b^2}{x} - \frac{7\,\left(bx^3+a\right)^{\frac{4}{3}}b}{x^4} + \frac{2\,\left(bx^3+a\right)^{\frac{7}{3}}}{x^7}\right)B}{14\,a^3} + \frac{\left(\frac{140\,\left(bx^3+a\right)^{\frac{1}{3}}b^3}{x} - \frac{105\,\left(bx^3+a\right)^{\frac{4}{3}}b^2}{x^4} + \frac{60\,\left(bx^3+a\right)^{\frac{7}{3}}b}{x^7} - \frac{14\,\left(bx^3+a\right)^{\frac{10}{3}}}{x^{10}}\right)A}{140\,a^4}$$

input
```
integrate((B*x^3+A)/x^11/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
-1/14*(14*(b*x^3 + a)^(1/3)*b^2/x - 7*(b*x^3 + a)^(4/3)*b/x^4 + 2*(b*x^3 +
 a)^(7/3)/x^7)*B/a^3 + 1/140*(140*(b*x^3 + a)^(1/3)*b^3/x - 105*(b*x^3 + a
)^(4/3)*b^2/x^4 + 60*(b*x^3 + a)^(7/3)*b/x^7 - 14*(b*x^3 + a)^(10/3)/x^10)
*A/a^4
```

Giac [F]

$$\int \frac{A+Bx^3}{x^{11}\left(a+bx^3\right)^{2/3}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{2}{3}}x^{11}}\,dx$$

input
```
integrate((B*x^3+A)/x^11/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(2/3)*x^11), x)
```

Mupad [B] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^3}{x^{11}\left(a+bx^3\right)^{2/3}}\,dx = \frac{\left(b\,x^3+a\right)^{1/3}\,(9\,A\,b-10\,B\,a)}{70\,a^2\,x^7} + \frac{\left(b\,x^3+a\right)^{1/3}\,(81\,A\,b^3-90\,B\,a\,b^2)}{140\,a^4\,x} - \frac{(27\,A\,b^2-30\,B\,a\,b)\,\left(b\,x^3+a\right)^{1/3}}{140\,a^3\,x^4} - \frac{A\,\left(b\,x^3+a\right)^{1/3}}{10\,a\,x^{10}}$$

input
```
int((A + B*x^3)/(x^11*(a + b*x^3)^(2/3)),x)
```

output
```
((a + b*x^3)^(1/3)*(9*A*b - 10*B*a))/(70*a^2*x^7) + ((a + b*x^3)^(1/3)*(81
*A*b^3 - 90*B*a*b^2))/(140*a^4*x) - ((27*A*b^2 - 30*B*a*b)*(a + b*x^3)^(1/
3))/(140*a^3*x^4) - (A*(a + b*x^3)^(1/3))/(10*a*x^10)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^{11}\left(a+bx^3\right)^{2/3}}\,dx = \left(\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}\,x^{11}}dx\right)a + \left(\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}\,x^{8}}dx\right)b$$

input
```
int((B*x^3+A)/x^11/(b*x^3+a)^(2/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*x**11),x)*a + int(1/((a + b*x**3)**(2/3)*x**8),
x)*b
```

3.355 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{Bx^7\sqrt[3]{a+bx^3}}{8b} + \frac{(8Ab-7aB)x^7\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{10}{3},-\frac{bx^3}{a}\right)}{56b\,(a+bx^3)^{2/3}}$$

output
```
1/8*B*x^7*(b*x^3+a)^(1/3)/b+1/56*(8*A*b-7*B*a)*x^7*(1+b*x^3/a)^(2/3)*hyper
geom([2/3, 7/3],[10/3],-b*x^3/a)/b/(b*x^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{x^7\left(1+\frac{bx^3}{a}\right)^{2/3}\left(10A\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{10}{3},-\frac{bx^3}{a}\right)+7Bx^3\,\text{Hypergeometric2}\right.}{70\,(a+bx^3)^{2/3}}$$

input
```
Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^(2/3),x]
```

output
```
(x^7*(1 + (b*x^3)/a)^(2/3)*(10*A*Hypergeometric2F1[2/3, 7/3, 10/3, -((b*x^
3)/a)] + 7*B*x^3*Hypergeometric2F1[2/3, 10/3, 13/3, -((b*x^3)/a)]))/(70*(a
 + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (A+Bx^3)}{(a+bx^3)^{2/3}} \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(8Ab-7aB)\int \frac{x^6}{(bx^3+a)^{2/3}}dx}{8b}+\frac{Bx^7\sqrt[3]{a+bx^3}}{8b}$$

$$\Big\downarrow \text{889}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(8Ab-7aB)\int \frac{x^6}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{8b\,(a+bx^3)^{2/3}}+\frac{Bx^7\sqrt[3]{a+bx^3}}{8b}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^7\left(\frac{bx^3}{a}+1\right)^{2/3}(8Ab-7aB)\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{10}{3},-\frac{bx^3}{a}\right)}{56b\,(a+bx^3)^{2/3}}+\frac{Bx^7\sqrt[3]{a+bx^3}}{8b}$$

input
```
Int[(x^6*(A + B*x^3))/(a + b*x^3)^(2/3),x]
```

output
```
(B*x^7*(a + b*x^3)^(1/3))/(8*b) + ((8*A*b - 7*a*B)*x^7*(1 + (b*x^3)/a)^(2/
3)*Hypergeometric2F1[2/3, 7/3, 10/3, -((b*x^3)/a)])/(56*b*(a + b*x^3)^(2/3
))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^6(B\,x^3+A)}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(x^6*(B*x^3+A)/(b*x^3+a)^(2/3),x)
```

output
```
int(x^6*(B*x^3+A)/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \int \frac{(Bx^3+A)x^6}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((B*x^9 + A*x^6)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.85 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{Ax^7\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{10}{3}\right)} + \frac{Bx^{10}\Gamma\left(\frac{10}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate(x**6*(B*x**3+A)/(b*x**3+a)**(2/3),x)
```

output
```
A*x**7*gamma(7/3)*hyper((2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*
a**(2/3)*gamma(10/3)) + B*x**10*gamma(10/3)*hyper((2/3, 10/3), (13/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(13/3))
```

Maxima [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \int \frac{(Bx^3+A)x^6}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \int \frac{(Bx^3+A)x^6}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^6\,(B\,x^3+A)}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int((x^6*(A + B*x^3))/(a + b*x^3)^(2/3),x)
```

output
```
int((x^6*(A + B*x^3))/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \left(\int \frac{x^9}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)b + \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a$$

input
```
int(x^6*(B*x^3+A)/(b*x^3+a)^(2/3),x)
```

output
```
int(x**9/(a + b*x**3)**(2/3),x)*b + int(x**6/(a + b*x**3)**(2/3),x)*a
```

3.356 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{Bx^4\sqrt[3]{a+bx^3}}{5b} + \frac{(5Ab-4aB)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{20b\,(a+bx^3)^{2/3}}$$

output
```
1/5*B*x^4*(b*x^3+a)^(1/3)/b+1/20*(5*A*b-4*B*a)*x^4*(1+b*x^3/a)^(2/3)*hyper
geom([2/3, 4/3],[7/3],-b*x^3/a)/b/(b*x^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\left(7Ax^4\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)+4Bx^7\,\text{Hypergeometric2F1}\right.}{28\,(a+bx^3)^{2/3}}$$

input
```
Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^(2/3),x]
```

output
```
((1 + (b*x^3)/a)^(2/3)*(7*A*x^4*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^3)
/a)] + 4*B*x^7*Hypergeometric2F1[2/3, 7/3, 10/3, -((b*x^3)/a)]))/(28*(a +
b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (A+Bx^3)}{(a+bx^3)^{2/3}} \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(5Ab-4aB)\int \frac{x^3}{(bx^3+a)^{2/3}}dx}{5b}+\frac{Bx^4\sqrt[3]{a+bx^3}}{5b}$$

$$\Big\downarrow \text{889}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(5Ab-4aB)\int \frac{x^3}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{5b\,(a+bx^3)^{2/3}}+\frac{Bx^4\sqrt[3]{a+bx^3}}{5b}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^4\left(\frac{bx^3}{a}+1\right)^{2/3}(5Ab-4aB)\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{20b\,(a+bx^3)^{2/3}}+\frac{Bx^4\sqrt[3]{a+bx^3}}{5b}$$

input
```
Int[(x^3*(A + B*x^3))/(a + b*x^3)^(2/3),x]
```

output
```
(B*x^4*(a + b*x^3)^(1/3))/(5*b) + ((5*A*b - 4*a*B)*x^4*(1 + (b*x^3)/a)^(2/
3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])/(20*b*(a + b*x^3)^(2/3)
)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^3(B\,x^3+A)}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^(2/3),x)
```

output
```
int(x^3*(B*x^3+A)/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx=\int \frac{(Bx^3+A)x^3}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((B*x^6 + A*x^3)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx=\frac{Ax^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}+\frac{Bx^7\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**3*(B*x**3+A)/(b*x**3+a)**(2/3),x)
```

output
```
A*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(2/3)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((2/3, 7/3), (10/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(10/3))
```

Maxima [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx=\int \frac{(Bx^3+A)x^3}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \int \frac{(Bx^3+A)x^3}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^3\,(B\,x^3+A)}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int((x^3*(A + B*x^3))/(a + b*x^3)^(2/3),x)
```

output
```
int((x^3*(A + B*x^3))/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)b + \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a$$

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^(2/3),x)
```

output
```
int(x**6/(a + b*x**3)**(2/3),x)*b + int(x**3/(a + b*x**3)**(2/3),x)*a
```

3.357 $\int \frac{A+Bx^3}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{2/3}}\,dx=\frac{Bx\sqrt[3]{a+bx^3}}{2b}+\frac{(2Ab-aB)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}}$$

output
```
1/2*B*x*(b*x^3+a)^(1/3)/b+1/2*(2*A*b-B*a)*x*(1+b*x^3/a)^(2/3)*hypergeom([1
/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{2/3}}\,dx=\frac{Bx\left(a+bx^3\right)+(2Ab-aB)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(A + B*x^3)/(a + b*x^3)^(2/3),x]
```

output
```
(B*x*(a + b*x^3) + (2*A*b - a*B)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1
[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(a+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(2Ab-aB)\int \frac{1}{(bx^3+a)^{2/3}}dx}{2b}+\frac{Bx\sqrt[3]{a+bx^3}}{2b}$$

$$\Big\downarrow \text{779}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(2Ab-aB)\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2b\,(a+bx^3)^{2/3}}+\frac{Bx\sqrt[3]{a+bx^3}}{2b}$$

$$\Big\downarrow \text{778}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2Ab-aB)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}}+\frac{Bx\sqrt[3]{a+bx^3}}{2b}$$

input
```
Int[(A + B*x^3)/(a + b*x^3)^(2/3),x]
```

output
```
(B*x*(a + b*x^3)^(1/3))/(2*b) + ((2*A*b - a*B)*x*(1 + (b*x^3)/a)^(2/3)*Hyp
ergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{B\,x^{3}+A}{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}dx$$

input
```
int((B*x^3+A)/(b*x^3+a)^(2/3),x)
```

output
```
int((B*x^3+A)/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{A+Bx^3}{(a+bx^3)^{2/3}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^3}{(a+bx^3)^{2/3}}\,dx=\frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((B*x**3+A)/(b*x**3+a)**(2/3),x)
```

output
```
A*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
2/3)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(2/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{(a+bx^3)^{2/3}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{B\,x^3+A}{\left(b\,x^3+a\right)^{2/3}}\,dx$$

input
```
int((A + B*x^3)/(a + b*x^3)^(2/3),x)
```

output
```
int((A + B*x^3)/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{2/3}}\,dx=\left(\int \frac{x^3}{\left(b\,x^3+a\right)^{\frac{2}{3}}}dx\right)b+\left(\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}}dx\right)a$$

input
```
int((B*x^3+A)/(b*x^3+a)^(2/3),x)
```

output
```
int(x**3/(a + b*x**3)**(2/3),x)*b + int(1/(a + b*x**3)**(2/3),x)*a
```

3.358 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{2/3}}\,dx=-\frac{A\sqrt[3]{a+bx^3}}{2ax^2}-\frac{(Ab-2aB)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

output
```
-1/2*A*(b*x^3+a)^(1/3)/a/x^2-1/2*(A*b-2*B*a)*x*(1+b*x^3/a)^(2/3)*hypergeom
([1/3, 2/3],[4/3],-b*x^3/a)/a/(b*x^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{2/3}}\,dx=\frac{-A(a+bx^3)+(-Ab+2aB)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2ax^2\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^(2/3)),x]
```

output
```
(-(A*(a + b*x^3)) + (-(A*b) + 2*a*B)*x^3*(1 + (b*x^3)/a)^(2/3)*Hypergeomet
ric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*x^2*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^3 (a+bx^3)^{2/3}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(Ab-2aB)\int \frac{1}{(bx^3+a)^{2/3}}dx}{2a}-\frac{A\sqrt[3]{a+bx^3}}{2ax^2}$$

$$\Big\downarrow \text{779}$$

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(Ab-2aB)\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a(a+bx^3)^{2/3}}-\frac{A\sqrt[3]{a+bx^3}}{2ax^2}$$

$$\Big\downarrow \text{778}$$

$$-\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(Ab-2aB)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(a+bx^3)^{2/3}}-\frac{A\sqrt[3]{a+bx^3}}{2ax^2}$$

input
```
Int[(A + B*x^3)/(x^3*(a + b*x^3)^(2/3)),x]
```

output
```
-1/2*(A*(a + b*x^3)^(1/3))/(a*x^2) - ((A*b - 2*a*B)*x*(1 + (b*x^3)/a)^(2/3
)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{B\,x^{3}+A}{x^{3}\left(b\,x^{3}+a\right)^{\frac{2}{3}}}dx$$

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^(2/3),x)
```

output
```
int((B*x^3+A)/x^3/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{A + Bx^3}{x^3 (a + bx^3)^{2/3}} \, dx = \int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{2}{3}} x^3} \, dx$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(1/3)/(b*x^6 + a*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \frac{A + Bx^3}{x^3 (a + bx^3)^{2/3}} \, dx = \frac{A\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{2}{3} \\ \frac{1}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}} x^2 \Gamma\left(\frac{1}{3}\right)} + \frac{Bx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{2}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}} \Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**3/(b*x**3+a)**(2/3),x)
```

output
```
A*gamma(-2/3)*hyper((-2/3, 2/3), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
2/3)*x**2*gamma(1/3)) + B*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(2/3)*gamma(4/3))
```

Maxima [F]

$$\int \frac{A + Bx^3}{x^3 (a + bx^3)^{2/3}} \, dx = \int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{2}{3}} x^3} \, dx$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(2/3)*x^3), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{2}{3}}x^3}\,dx$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(2/3)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{B\,x^3+A}{x^3\,\left(b\,x^3+a\right)^{2/3}}\,dx$$

input
```
int((A + B*x^3)/(x^3*(a + b*x^3)^(2/3)),x)
```

output
```
int((A + B*x^3)/(x^3*(a + b*x^3)^(2/3)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{2/3}}\,dx=\left(\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}}dx\right)b+\left(\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}\,x^3}dx\right)a$$

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^(2/3),x)
```

output
```
int(1/(a + b*x**3)**(2/3),x)*b + int(1/((a + b*x**3)**(2/3)*x**3),x)*a
```

3.359 $\int \frac{A+Bx^3}{x^6 \left(a+bx^3\right)^{2/3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{A+Bx^3}{x^6 \left(a+bx^3\right)^{2/3}} \, dx = -\frac{A\sqrt[3]{a+bx^3}}{5ax^5} + \frac{(4Ab-5aB)\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{10ax^2\left(a+bx^3\right)^{2/3}}$$

output
```
-1/5*A*(b*x^3+a)^(1/3)/a/x^5+1/10*(4*A*b-5*B*a)*(1+b*x^3/a)^(2/3)*hypergeo
m([-2/3, 2/3],[1/3],-b*x^3/a)/a/x^2/(b*x^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x^6 \left(a+bx^3\right)^{2/3}} \, dx = \frac{-2A(a+bx^3)+(4Ab-5aB)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{10ax^5\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^(2/3)),x]
```

output
```
(-2*A*(a + b*x^3) + (4*A*b - 5*a*B)*x^3*(1 + (b*x^3)/a)^(2/3)*Hypergeometr
ic2F1[-2/3, 2/3, 1/3, -((b*x^3)/a)])/(10*a*x^5*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^6 \left(a+bx^3\right)^{2/3}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(4Ab-5aB)\int \frac{1}{x^3(bx^3+a)^{2/3}}dx}{5a}-\frac{A\sqrt[3]{a+bx^3}}{5ax^5}$$

$$\Big\downarrow \text{889}$$

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(4Ab-5aB)\int \frac{1}{x^3\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{5a\left(a+bx^3\right)^{2/3}}-\frac{A\sqrt[3]{a+bx^3}}{5ax^5}$$

$$\Big\downarrow \text{888}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(4Ab-5aB)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{10ax^2\left(a+bx^3\right)^{2/3}}-\frac{A\sqrt[3]{a+bx^3}}{5ax^5}$$

input
```
Int[(A + B*x^3)/(x^6*(a + b*x^3)^(2/3)),x]
```

output
```
-1/5*(A*(a + b*x^3)^(1/3))/(a*x^5) + ((4*A*b - 5*a*B)*(1 + (b*x^3)/a)^(2/3
)*Hypergeometric2F1[-2/3, 2/3, 1/3, -((b*x^3)/a)])/(10*a*x^2*(a + b*x^3)^(
2/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{B\,x^{3}+A}{x^{6} \left(b\,x^{3}+a \right)^{\frac{2}{3}}}d x$$

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^(2/3),x)
```

output
```
int((B*x^3+A)/x^6/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^{2/3}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{2}{3}}x^6}\,dx$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(1/3)/(b*x^9 + a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.49 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^{2/3}}\,dx = \frac{A\Gamma\left(-\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{3}, \frac{2}{3} \\ -\frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{B\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{2}{3} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**6/(b*x**3+a)**(2/3),x)
```

output
```
A*gamma(-5/3)*hyper((-5/3, 2/3), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(2/3)*x**5*gamma(-2/3)) + B*gamma(-2/3)*hyper((-2/3, 2/3), (1/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(2/3)*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^{2/3}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{2}{3}}x^6}\,dx$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(2/3)*x^6), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^{2/3}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{2}{3}}x^6}\,dx$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(2/3)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^{2/3}}\,dx = \int \frac{B\,x^3+A}{x^6\,(b\,x^3+a)^{2/3}}\,dx$$

input
```
int((A + B*x^3)/(x^6*(a + b*x^3)^(2/3)),x)
```

output
```
int((A + B*x^3)/(x^6*(a + b*x^3)^(2/3)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^{2/3}}\,dx = \left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,x^6}dx\right) a + \left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,x^3}dx\right) b$$

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^(2/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*x**6),x)*a + int(1/((a + b*x**3)**(2/3)*x**3),x
)*b
```

3.360 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 101

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = -\frac{a^2(Ab-aB)}{b^4\sqrt[3]{a+bx^3}} - \frac{a(2Ab-3aB)\left(a+bx^3\right)^{2/3}}{2b^4} + \frac{(Ab-3aB)\left(a+bx^3\right)^{5/3}}{5b^4} + \frac{B\left(a+bx^3\right)^{8/3}}{8b^4}$$

output
```
-a^2*(A*b-B*a)/b^4/(b*x^3+a)^(1/3)-1/2*a*(2*A*b-3*B*a)*(b*x^3+a)^(2/3)/b^4
+1/5*(A*b-3*B*a)*(b*x^3+a)^(5/3)/b^4+1/8*B*(b*x^3+a)^(8/3)/b^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{81a^3B-9a^2b(8A-3Bx^3)-3ab^2x^3(8A+3Bx^3)+b^3x^6(8A+5Bx^3)}{40b^4\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(4/3),x]
```

output
```
(81*a^3*B - 9*a^2*b*(8*A - 3*B*x^3) - 3*a*b^2*x^3*(8*A + 3*B*x^3) + b^3*x^
6*(8*A + 5*B*x^3))/(40*b^4*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (A+Bx^3)}{(a+bx^3)^{4/3}} dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{x^6 (Bx^3+A)}{(bx^3+a)^{4/3}} dx^3$$

$$\Big\downarrow \; 86$$

$$\frac{1}{3}\int \left(-\frac{(aB-Ab)a^2}{b^3 (bx^3+a)^{4/3}}+\frac{(3aB-2Ab)a}{b^3 \sqrt[3]{bx^3+a}}+\frac{B(bx^3+a)^{5/3}}{b^3}+\frac{(Ab-3aB)(bx^3+a)^{2/3}}{b^3}\right) dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(-\frac{3a^2(Ab-aB)}{b^4 \sqrt[3]{a+bx^3}}+\frac{3(a+bx^3)^{5/3}(Ab-3aB)}{5b^4}-\frac{3a(a+bx^3)^{2/3}(2Ab-3aB)}{2b^4}+\frac{3B(a+bx^3)^{8/3}}{8b^4}\right)$$

input
```
Int[(x^8*(A + B*x^3))/(a + b*x^3)^(4/3),x]
```

output
```
((-3*a^2*(A*b - a*B))/(b^4*(a + b*x^3)^(1/3)) - (3*a*(2*A*b - 3*a*B)*(a +
b*x^3)^(2/3))/(2*b^4) + (3*(A*b - 3*a*B)*(a + b*x^3)^(5/3))/(5*b^4) + (3*B
*(a + b*x^3)^(8/3))/(8*b^4))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$-\frac{9\left(-\frac{\left(\frac{5B\,x^{3}}{8}+A\right)x^{6}b^{3}}{9}+\frac{\left(\frac{3B\,x^{3}}{8}+A\right)a\,x^{3}b^{2}}{3}+a^{2}\left(-\frac{3B\,x^{3}}{8}+A\right)b-\frac{9a^{3}B}{8}\right)}{5\left(b\,x^{3}+a\right)^{\frac{1}{3}}b^{4}}$	68
gosper	$-\frac{-5b^{3}B\,x^{9}-8A\,b^{3}x^{6}+9Ba\,b^{2}x^{6}+24aA\,b^{2}x^{3}-27B\,a^{2}b\,x^{3}+72a^{2}bA-81a^{3}B}{40\left(b\,x^{3}+a\right)^{\frac{1}{3}}b^{4}}$	77
trager	$-\frac{-5b^{3}B\,x^{9}-8A\,b^{3}x^{6}+9Ba\,b^{2}x^{6}+24aA\,b^{2}x^{3}-27B\,a^{2}b\,x^{3}+72a^{2}bA-81a^{3}B}{40\left(b\,x^{3}+a\right)^{\frac{1}{3}}b^{4}}$	77
orering	$-\frac{-5b^{3}B\,x^{9}-8A\,b^{3}x^{6}+9Ba\,b^{2}x^{6}+24aA\,b^{2}x^{3}-27B\,a^{2}b\,x^{3}+72a^{2}bA-81a^{3}B}{40\left(b\,x^{3}+a\right)^{\frac{1}{3}}b^{4}}$	77
risch	$-\frac{\left(-5b^{2}B\,x^{6}-8A\,b^{2}x^{3}+14Bab\,x^{3}+32abA-41a^{2}B\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{40b^{4}}-\frac{a^{2}\left(Ab-Ba\right)}{b^{4}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}$	79

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-9/5*(-1/9*(5/8*B*x^3+A)*x^6*b^3+1/3*(3/8*B*x^3+A)*a*x^3*b^2+a^2*(-3/8*B*x
^3+A)*b-9/8*a^3*B)/(b*x^3+a)^(1/3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{(5\,Bb^3x^9-(9\,Bab^2-8\,Ab^3)x^6+81\,Ba^3-72\,Aa^2b+3\,(9\,Ba^2b-8\,Aab^2)x^3)(bx^3+}{40\,(b^5x^3+ab^4)}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
1/40*(5*B*b^3*x^9 - (9*B*a*b^2 - 8*A*b^3)*x^6 + 81*B*a^3 - 72*A*a^2*b + 3*
(9*B*a^2*b - 8*A*a*b^2)*x^3)*(b*x^3 + a)^(2/3)/(b^5*x^3 + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.70

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \begin{cases} -\frac{9Aa^2}{5b^3\sqrt[3]{a+bx^3}} - \frac{3Aax^3}{5b^2\sqrt[3]{a+bx^3}} + \frac{Ax^6}{5b\sqrt[3]{a+bx^3}} + \frac{81Ba^3}{40b^4\sqrt[3]{a+bx^3}} + \frac{27Ba^2x^3}{40b^3\sqrt[3]{a+bx^3}} - \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{a^{\frac{4}{3}}} \end{cases}$$

input
```
integrate(x**8*(B*x**3+A)/(b*x**3+a)**(4/3),x)
```

output
```
Piecewise((-9*A*a**2/(5*b**3*(a + b*x**3)**(1/3)) - 3*A*a*x**3/(5*b**2*(a
+ b*x**3)**(1/3)) + A*x**6/(5*b*(a + b*x**3)**(1/3)) + 81*B*a**3/(40*b**4*
(a + b*x**3)**(1/3)) + 27*B*a**2*x**3/(40*b**3*(a + b*x**3)**(1/3)) - 9*B*
a*x**6/(40*b**2*(a + b*x**3)**(1/3)) + B*x**9/(8*b*(a + b*x**3)**(1/3)), N
e(b, 0)), ((A*x**9/9 + B*x**12/12)/a**(4/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.16

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{1}{40}\,B\left(\frac{5\,(bx^3+a)^{\frac{8}{3}}}{b^4} - \frac{24\,(bx^3+a)^{\frac{5}{3}}a}{b^4} + \frac{60\,(bx^3+a)^{\frac{2}{3}}a^2}{b^4} + \frac{40\,a^3}{(bx^3+a)^{\frac{1}{3}}b^4}\right) + \frac{1}{5}\,A\left(\frac{(bx^3+a)^{\frac{5}{3}}}{b^3} - \frac{5\,(bx^3+a)^{\frac{2}{3}}a}{b^3} - \frac{5\,a^2}{(bx^3+a)^{\frac{1}{3}}b^3}\right)$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
1/40*B*(5*(b*x^3 + a)^(8/3)/b^4 - 24*(b*x^3 + a)^(5/3)*a/b^4 + 60*(b*x^3 +
 a)^(2/3)*a^2/b^4 + 40*a^3/((b*x^3 + a)^(1/3)*b^4)) + 1/5*A*((b*x^3 + a)^(
5/3)/b^3 - 5*(b*x^3 + a)^(2/3)*a/b^3 - 5*a^2/((b*x^3 + a)^(1/3)*b^3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{Ba^3-Aa^2b}{(bx^3+a)^{\frac{1}{3}}b^4} + \frac{5\,(bx^3+a)^{\frac{8}{3}}Bb^{28} - 24\,(bx^3+a)^{\frac{5}{3}}Bab^{28} + 60\,(bx^3+a)^{\frac{2}{3}}Ba^2b^{28} + 8\,(bx^3+a)^{\frac{5}{3}}Ab^{29} - 40\,(bx^3+a)^{\frac{2}{3}}Aab^{29}}{40\,b^{32}}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
(B*a^3 - A*a^2*b)/((b*x^3 + a)^(1/3)*b^4) + 1/40*(5*(b*x^3 + a)^(8/3)*B*b^
28 - 24*(b*x^3 + a)^(5/3)*B*a*b^28 + 60*(b*x^3 + a)^(2/3)*B*a^2*b^28 + 8*(
b*x^3 + a)^(5/3)*A*b^29 - 40*(b*x^3 + a)^(2/3)*A*a*b^29)/b^32
```

Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{5\,B\,(b\,x^3+a)^3+40\,B\,a^3+8\,A\,b\,(b\,x^3+a)^2-24\,B\,a\,(b\,x^3+a)^2+60\,B\,a^2\,(b\,x^3+a)}{40\,b^4\,(b\,x^3+a)^{1/3}}$$

input
```
int((x^8*(A + B*x^3))/(a + b*x^3)^(4/3),x)
```

output
```
(5*B*(a + b*x^3)^3 + 40*B*a^3 + 8*A*b*(a + b*x^3)^2 - 24*B*a*(a + b*x^3)^2
 + 60*B*a^2*(a + b*x^3) - 40*A*a^2*b - 40*A*a*b*(a + b*x^3))/(40*b^4*(a +
b*x^3)^(1/3))
```

Reduce [F]

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{x^8}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^(4/3),x)
```

output
```
int(x**8/(a + b*x**3)**(1/3),x)
```

3.361 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{4/3}} dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx=\frac{a(Ab-aB)}{b^3\sqrt[3]{a+bx^3}}+\frac{(Ab-2aB)\,(a+bx^3)^{2/3}}{2b^3}+\frac{B(a+bx^3)^{5/3}}{5b^3}$$

output
```
a*(A*b-B*a)/b^3/(b*x^3+a)^(1/3)+1/2*(A*b-2*B*a)*(b*x^3+a)^(2/3)/b^3+1/5*B*
(b*x^3+a)^(5/3)/b^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx=\frac{15aAb-18a^2B+5Ab^2x^3-6abBx^3+2b^2Bx^6}{10b^3\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(4/3),x]
```

output
```
(15*a*A*b - 18*a^2*B + 5*A*b^2*x^3 - 6*a*b*B*x^3 + 2*b^2*B*x^6)/(10*b^3*(a
 + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^3)}{(a+bx^3)^{4/3}} \, dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3} \int \frac{x^3 (Bx^3+A)}{(bx^3+a)^{4/3}} dx^3$$

$$\Big\downarrow \ 86$$

$$\frac{1}{3} \int \left(\frac{(bx^3+a)^{2/3} B}{b^2} + \frac{Ab-2aB}{b^2 \sqrt[3]{bx^3+a}} + \frac{a(aB-Ab)}{b^2 (bx^3+a)^{4/3}} \right) dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3} \left(\frac{3(a+bx^3)^{2/3} (Ab-2aB)}{2b^3} + \frac{3a(Ab-aB)}{b^3 \sqrt[3]{a+bx^3}} + \frac{3B(a+bx^3)^{5/3}}{5b^3} \right)$$

input
```
Int[(x^5*(A + B*x^3))/(a + b*x^3)^(4/3),x]
```

output
```
((3*a*(A*b - a*B))/(b^3*(a + b*x^3)^(1/3)) + (3*(A*b - 2*a*B)*(a + b*x^3)^
(2/3))/(2*b^3) + (3*B*(a + b*x^3)^(5/3))/(5*b^3))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

method	result	size
pseudoelliptic	$\frac{\frac{\left(\frac{2B\,x^{3}}{5}+A\right)x^{3}b^{2}}{2}+\frac{3a\left(-\frac{2B\,x^{3}}{5}+A\right)b}{2}-\frac{9a^{2}B}{5}}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}b^{3}}$	49
gosper	$\frac{2b^{2}B\,x^{6}+5A\,b^{2}x^{3}-6Bab\,x^{3}+15abA-18a^{2}B}{10\left(b\,x^{3}+a\right)^{\frac{1}{3}}b^{3}}$	53
trager	$\frac{2b^{2}B\,x^{6}+5A\,b^{2}x^{3}-6Bab\,x^{3}+15abA-18a^{2}B}{10\left(b\,x^{3}+a\right)^{\frac{1}{3}}b^{3}}$	53
orering	$\frac{2b^{2}B\,x^{6}+5A\,b^{2}x^{3}-6Bab\,x^{3}+15abA-18a^{2}B}{10\left(b\,x^{3}+a\right)^{\frac{1}{3}}b^{3}}$	53
risch	$\frac{\left(2bB\,x^{3}+5Ab-8Ba\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{10b^{3}}+\frac{a\left(Ab-Ba\right)}{b^{3}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}$	54

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3/2*(1/3*(2/5*B*x^3+A)*x^3*b^2+a*(-2/5*B*x^3+A)*b-6/5*a^2*B)/(b*x^3+a)^(1/
3)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{(2\,Bb^2x^6-(6\,Bab-5\,Ab^2)x^3-18\,Ba^2+15\,Aab)(bx^3+a)^{\frac{2}{3}}}{10\,(b^4x^3+ab^3)}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
1/10*(2*B*b^2*x^6 - (6*B*a*b - 5*A*b^2)*x^3 - 18*B*a^2 + 15*A*a*b)*(b*x^3
+ a)^(2/3)/(b^4*x^3 + a*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(60) = 120.

Time = 0.36 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.73

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \begin{cases} \frac{3Aa}{2b^2\sqrt[3]{a+bx^3}}+\frac{Ax^3}{2b\sqrt[3]{a+bx^3}}-\frac{9Ba^2}{5b^3\sqrt[3]{a+bx^3}}-\frac{3Bax^3}{5b^2\sqrt[3]{a+bx^3}}+\frac{Bx^6}{5b\sqrt[3]{a+bx^3}} & \text{for } b \neq \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{a^{\frac{4}{3}}} & \text{otherw} \end{cases}$$

input
```
integrate(x**5*(B*x**3+A)/(b*x**3+a)**(4/3),x)
```

output
```
Piecewise((3*A*a/(2*b**2*(a + b*x**3)**(1/3)) + A*x**3/(2*b*(a + b*x**3)**
(1/3)) - 9*B*a**2/(5*b**3*(a + b*x**3)**(1/3)) - 3*B*a*x**3/(5*b**2*(a + b
*x**3)**(1/3)) + B*x**6/(5*b*(a + b*x**3)**(1/3)), Ne(b, 0)), ((A*x**6/6 +
 B*x**9/9)/a**(4/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{1}{5}\,B\left(\frac{(bx^3+a)^{\frac{5}{3}}}{b^3} - \frac{5\,(bx^3+a)^{\frac{2}{3}}a}{b^3} - \frac{5\,a^2}{(bx^3+a)^{\frac{1}{3}}b^3}\right) + \frac{1}{2}\,A\left(\frac{(bx^3+a)^{\frac{2}{3}}}{b^2} + \frac{2\,a}{(bx^3+a)^{\frac{1}{3}}b^2}\right)$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
1/5*B*((b*x^3 + a)^(5/3)/b^3 - 5*(b*x^3 + a)^(2/3)*a/b^3 - 5*a^2/((b*x^3 +
 a)^(1/3)*b^3)) + 1/2*A*((b*x^3 + a)^(2/3)/b^2 + 2*a/((b*x^3 + a)^(1/3)*b^
2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = -\frac{Ba^2-Aab}{(bx^3+a)^{\frac{1}{3}}b^3} + \frac{2\,(bx^3+a)^{\frac{5}{3}}Bb^{12} - 10\,(bx^3+a)^{\frac{2}{3}}Bab^{12} + 5\,(bx^3+a)^{\frac{2}{3}}Ab^{13}}{10\,b^{15}}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
-(B*a^2 - A*a*b)/((b*x^3 + a)^(1/3)*b^3) + 1/10*(2*(b*x^3 + a)^(5/3)*B*b^1
2 - 10*(b*x^3 + a)^(2/3)*B*a*b^12 + 5*(b*x^3 + a)^(2/3)*A*b^13)/b^15
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{2\,B\,{\left(b\,x^3+a\right)}^2-10\,B\,a^2+5\,A\,b\,\left(b\,x^3+a\right)-10\,B\,a\,\left(b\,x^3+a\right)+10\,A\,a\,b}{10\,b^3\,{\left(b\,x^3+a\right)}^{1/3}}$$

input
```
int((x^5*(A + B*x^3))/(a + b*x^3)^(4/3),x)
```

output
```
(2*B*(a + b*x^3)^2 - 10*B*a^2 + 5*A*b*(a + b*x^3) - 10*B*a*(a + b*x^3) + 1
0*A*a*b)/(10*b^3*(a + b*x^3)^(1/3))
```

Reduce [F]

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{x^5}{\left(b\,x^3+a\right)^{\frac{1}{3}}}dx$$

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^(4/3),x)
```

output
```
int(x**5/(a + b*x**3)**(1/3),x)
```

3.362 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{4/3}} dx$

Optimal result

Integrand size = 22, antiderivative size = 44

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = -\frac{Ab-aB}{b^2\sqrt[3]{a+bx^3}}+\frac{B(a+bx^3)^{2/3}}{2b^2}$$

output
```
-(A*b-B*a)/b^2/(b*x^3+a)^(1/3)+1/2*B*(b*x^3+a)^(2/3)/b^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{-2Ab+3aB+bBx^3}{2b^2\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(4/3),x]
```

output
```
(-2*A*b + 3*a*B + b*B*x^3)/(2*b^2*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^3)}{(a+bx^3)^{4/3}} \, dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{(bx^3+a)^{4/3}}dx^3$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{3}\int \left(\frac{B}{b\sqrt[3]{bx^3+a}}+\frac{Ab-aB}{b(bx^3+a)^{4/3}}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{3B(a+bx^3)^{2/3}}{2b^2}-\frac{3(Ab-aB)}{b^2\sqrt[3]{a+bx^3}}\right)$$

input
```
Int[(x^2*(A + B*x^3))/(a + b*x^3)^(4/3),x]
```

output
```
((-3*(A*b - a*B))/(b^2*(a + b*x^3)^(1/3)) + (3*B*(a + b*x^3)^(2/3))/(2*b^2
))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

method	result	size
pseudoelliptic	$-\frac{\left(-\frac{B\,x^{3}}{2}+A\right)b-\frac{3Ba}{2}}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}b^{2}}$	30
gosper	$-\frac{-bB\,x^{3}+2Ab-3Ba}{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}b^{2}}$	31
trager	$-\frac{-bB\,x^{3}+2Ab-3Ba}{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}b^{2}}$	31
orering	$-\frac{-bB\,x^{3}+2Ab-3Ba}{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}b^{2}}$	31
risch	$-\frac{Ab-Ba}{b^{2}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}+\frac{B\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{2b^{2}}$	39

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-((-1/2*B*x^3+A)*b-3/2*B*a)/(b*x^3+a)^(1/3)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{(Bbx^3+3\,Ba-2\,Ab)(bx^3+a)^{\frac{2}{3}}}{2\,(b^3x^3+ab^2)}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
1/2*(B*b*x^3 + 3*B*a - 2*A*b)*(b*x^3 + a)^(2/3)/(b^3*x^3 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.59

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \begin{cases} -\frac{A}{b\sqrt[3]{a+bx^3}} + \frac{3Ba}{2b^2\sqrt[3]{a+bx^3}} + \frac{Bx^3}{2b\sqrt[3]{a+bx^3}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^3}{3}+\frac{Bx^6}{6}}{a^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(B*x**3+A)/(b*x**3+a)**(4/3),x)
```

output
```
Piecewise((-A/(b*(a + b*x**3)**(1/3)) + 3*B*a/(2*b**2*(a + b*x**3)**(1/3))
 + B*x**3/(2*b*(a + b*x**3)**(1/3)), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/a**
(4/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{1}{2}\,B\left(\frac{(bx^3+a)^{\frac{2}{3}}}{b^2} + \frac{2\,a}{(bx^3+a)^{\frac{1}{3}}b^2}\right) - \frac{A}{(bx^3+a)^{\frac{1}{3}}b}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
1/2*B*((b*x^3 + a)^(2/3)/b^2 + 2*a/((b*x^3 + a)^(1/3)*b^2)) - A/((b*x^3 +
a)^(1/3)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{(bx^3+a)^{\frac{2}{3}}B}{2\,b^2} + \frac{Ba-Ab}{(bx^3+a)^{\frac{1}{3}}b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
1/2*(b*x^3 + a)^(2/3)*B/b^2 + (B*a - A*b)/((b*x^3 + a)^(1/3)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{2\,B\,a-2\,A\,b+B\,(b\,x^3+a)}{2\,b^2\,(b\,x^3+a)^{1/3}}$$

input
```
int((x^2*(A + B*x^3))/(a + b*x^3)^(4/3),x)
```

output
```
(2*B*a - 2*A*b + B*(a + b*x^3))/(2*b^2*(a + b*x^3)^(1/3))
```

Reduce [F]

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{x^2}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^(4/3),x)
```

output
```
int(x**2/(a + b*x**3)**(1/3),x)
```

3.363 $\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{4/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{4/3}}\,dx=\frac{Ab-aB}{ab\sqrt[3]{a+bx^3}}+\frac{A\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}}-\frac{A\log(x)}{2a^{4/3}}+\frac{A\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{4/3}}$$

output
```
(A*b-B*a)/a/b/(b*x^3+a)^(1/3)+1/3*A*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))
*3^(1/2)/a^(1/3))*3^(1/2)/a^(4/3)-1/2*A*ln(x)/a^(4/3)+1/2*A*ln(a^(1/3)-(b*
x^3+a)^(1/3))/a^(4/3)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{4/3}}\,dx=\frac{\frac{6\sqrt[3]{a}(Ab-aB)}{b\sqrt[3]{a+bx^3}}+2\sqrt{3}A\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2A\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)-A\,\mathrm{lo}}{6a^{4/3}}$$

input
```
Integrate[(A + B*x^3)/(x*(a + b*x^3)^(4/3)),x]
```

output
```
((6*a^(1/3)*(A*b - a*B))/(b*(a + b*x^3)^(1/3)) + 2*Sqrt[3]*A*ArcTan[(1 + (
2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 2*A*Log[-a^(1/3) + (a + b*x^3)^(1
/3)] - A*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*
a^(4/3))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{4/3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^3\left(bx^3+a\right)^{4/3}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{A\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3}{a}+\frac{3(Ab-aB)}{ab\sqrt[3]{a+bx^3}}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\left(\frac{A\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{a}+\frac{3(Ab-aB)}{ab\sqrt[3]{a+bx^3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{A\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{a}+\frac{3(Ab-aB)}{ab\sqrt[3]{a+bx^3}}\right)$$

$$\Big\downarrow \textbf{1082}$$

$$\frac{1}{3}\left(\frac{A\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{a}+\frac{3(Ab-aB)}{ab\sqrt[3]{a+bx^3}}\right)$$

$$\Big\downarrow \textbf{217}$$

$$\frac{1}{3}\left(\frac{A\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{a}+\frac{3(Ab-aB)}{ab\sqrt[3]{a+bx^3}}\right)$$

input
```
Int[(A + B*x^3)/(x*(a + b*x^3)^(4/3)),x]
```

output
```
((3*(A*b - a*B))/(a*b*(a + b*x^3)^(1/3)) + (A*((Sqrt[3]*ArcTan[(1 + (2*(a
+ b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^3]/(2*a^(1/3)) + (3*Log
[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3))))/a)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$\frac{A}{a(b\,x^3+a)^{\frac{1}{3}}}-\frac{B}{b(b\,x^3+a)^{\frac{1}{3}}}+\frac{A\ln\left((b\,x^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{4}{3}}}-\frac{A\ln\left((b\,x^3+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(b\,x^3+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{4}{3}}}+\frac{A\sqrt{3}\,\arctan\left(\frac{2\cdots}{\cdots}\right.}{\cdots}$

input
```
int((B*x^3+A)/x/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
1/a/(b*x^3+a)^(1/3)*A-1/b/(b*x^3+a)^(1/3)*B+1/3*A/a^(4/3)*ln((b*x^3+a)^(1/
3)-a^(1/3))-1/6*A/a^(4/3)*ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/
3))+1/3*A/a^(4/3)*3^(1/2)*arctan(2/3*3^(1/2)/a^(1/3)*(b*x^3+a)^(1/3)+1/3*3
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 392, normalized size of antiderivative = 3.50

$$\int \frac{A+Bx^3}{x\,(a+bx^3)^{4/3}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}(Aab^2x^3+Aa^2b)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3+3\sqrt{\frac{1}{3}}\left(2\,(bx^3+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx\cdots}{x^3}\right.}{\cdots}\right.$$

$$-\frac{(Ab^2x^3+Aab)a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\,(Ab^2x^3+Aab)a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\cdots}{6\,(a^2b^2x^3+a^3b)}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*(A*a*b^2*x^3 + A*a^2*b)*sqrt(-1/a^(2/3))*log((2*b*x^3 +
3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*
sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) - (A*b^2*x^3 +
A*a*b)*a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)
) + 2*(A*b^2*x^3 + A*a*b)*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) - 6*(b*
x^3 + a)^(2/3)*(B*a^2 - A*a*b))/(a^2*b^2*x^3 + a^3*b), -1/6*((A*b^2*x^3 +
A*a*b)*a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)
) - 2*(A*b^2*x^3 + A*a*b)*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) - 6*sqr
t(1/3)*(A*a*b^2*x^3 + A*a^2*b)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(
1/3))/a^(1/3))/a^(1/3) + 6*(b*x^3 + a)^(2/3)*(B*a^2 - A*a*b))/(a^2*b^2*x^3
 + a^3*b)]
```

Sympy [A] (verification not implemented)

Time = 10.50 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{4/3}}\,dx = -\frac{A\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3b^{\frac{4}{3}}x^4\Gamma\left(\frac{7}{3}\right)}+B\left(\begin{cases}-\frac{1}{b\sqrt[3]{a+bx^3}} & \text{for } b\neq 0\\ \frac{x^3}{3a^{\frac{4}{3}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x**3+A)/x/(b*x**3+a)**(4/3),x)
```

output
```
-A*gamma(4/3)*hyper((4/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**3))/(3*b**
(4/3)*x**4*gamma(7/3)) + B*Piecewise((-1/(b*(a + b*x**3)**(1/3)), Ne(b, 0)
), (x**3/(3*a**(4/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{4/3}}\,dx = \frac{1}{6}\,A\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}} - \frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}} + \right.$$

$$- \frac{B}{(bx^3+a)^{\frac{1}{3}}b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
1/6*A*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(4/3) - log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(4/3) + 2*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(4/3) + 6/((b*x^3 + a)^(1/3
)*a)) - B/((b*x^3 + a)^(1/3)*b)
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{4/3}}\,dx = \frac{\sqrt{3}A\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{4}{3}}} - \frac{A\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{4}{3}}} + \frac{A\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{4}{3}}} - \frac{Ba-Ab}{(bx^3+a)^{\frac{1}{3}}ab}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*A*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(4/3) - 1/6*A*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3
))/a^(4/3) + 1/3*A*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(4/3) - (B*a -
A*b)/((b*x^3 + a)^(1/3)*a*b)
```

Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{4/3}}\,dx = \frac{A\ \ln\left(A^2\,a\,{\left(b\,x^3+a\right)}^{1/3}-A^2\,a^{4/3}\right)}{3\,a^{4/3}}+\frac{A}{a\,{\left(b\,x^3+a\right)}^{1/3}} - \frac{B}{b\,{\left(b\,x^3+a\right)}^{1/3}}-\frac{\ln\left(A^2\,a\,{\left(b\,x^3+a\right)}^{1/3}-\frac{a^{4/3}\,{\left(A-\sqrt{3}\,A\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(A-\sqrt{3}\,A\,1{}\mathrm{i}\right)}{6\,a^{4/3}} - \frac{\ln\left(A^2\,a\,{\left(b\,x^3+a\right)}^{1/3}-\frac{a^{4/3}\,{\left(A+\sqrt{3}\,A\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(A+\sqrt{3}\,A\,1{}\mathrm{i}\right)}{6\,a^{4/3}}$$

input
```
int((A + B*x^3)/(x*(a + b*x^3)^(4/3)),x)
```

output
```
(A*log(A^2*a*(a + b*x^3)^(1/3) - A^2*a^(4/3)))/(3*a^(4/3)) + A/(a*(a + b*x
^3)^(1/3)) - B/(b*(a + b*x^3)^(1/3)) - (log(A^2*a*(a + b*x^3)^(1/3) - (a^(
4/3)*(A - 3^(1/2)*A*1i)^2)/4)*(A - 3^(1/2)*A*1i))/(6*a^(4/3)) - (log(A^2*a
*(a + b*x^3)^(1/3) - (a^(4/3)*(A + 3^(1/2)*A*1i)^2)/4)*(A + 3^(1/2)*A*1i))
/(6*a^(4/3))
```

Reduce [F]

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{4/3}}\,dx = \int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\,x}dx$$

input
```
int((B*x^3+A)/x/(b*x^3+a)^(4/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x),x)
```

3.364 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{4/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 159

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{4/3}}\,dx = -\frac{Ab-aB}{a^2\sqrt[3]{a+bx^3}}-\frac{A\left(a+bx^3\right)^{2/3}}{3a^2x^3} -\frac{(4Ab-3aB)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}} +\frac{(4Ab-3aB)\log(x)}{6a^{7/3}}-\frac{(4Ab-3aB)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{7/3}}$$

output
```
-(A*b-B*a)/a^2/(b*x^3+a)^(1/3)-1/3*A*(b*x^3+a)^(2/3)/a^2/x^3-1/9*(4*A*b-3*
B*a)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(7/
3)+1/6*(4*A*b-3*B*a)*ln(x)/a^(7/3)-1/6*(4*A*b-3*B*a)*ln(a^(1/3)-(b*x^3+a)^
(1/3))/a^(7/3)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x^4 (a+bx^3)^{4/3}}\,dx = \frac{-\frac{6\sqrt[3]{a}(4Abx^3+a(A-3Bx^3))}{x^3\sqrt[3]{a+bx^3}} + 2\sqrt{3}(-4Ab+3aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2(-4Ab$$

input
```
Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(4/3)),x]
```

output
```
((-6*a^(1/3)*(4*A*b*x^3 + a*(A - 3*B*x^3)))/(x^3*(a + b*x^3)^(1/3)) + 2*Sq
rt[3]*(-4*A*b + 3*a*B)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]]
 + 2*(-4*A*b + 3*a*B)*Log[-a^(1/3) + (a + b*x^3)^(1/3)] + (4*A*b - 3*a*B)*
Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*a^(7/3))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {948, 87, 61, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^4 (a+bx^3)^{4/3}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^6 (bx^3+a)^{4/3}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(-\frac{(4Ab-3aB)\int \frac{1}{x^3(bx^3+a)^{4/3}}dx^3}{3a} - \frac{A}{ax^3\sqrt[3]{a+bx^3}}\right)$$

$$\Big\downarrow 61$$

$$\frac{1}{3}\left(-\frac{(4Ab-3aB)\left(\frac{\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{3a}-\frac{A}{ax^3\sqrt[3]{a+bx^3}}\right)$$

$$\Big\downarrow 67$$

$$\frac{1}{3}\left(-\frac{(4Ab-3aB)\left(\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{3a}\right.$$

$$\Big\downarrow 16$$

$$\frac{1}{3}\left(-\frac{(4Ab-3aB)\left(\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{3a}-\frac{}{ax^3}\right.$$

$$\Big\downarrow 1082$$

$$\frac{1}{3}\left(-\frac{(4Ab-3aB)\left(\frac{-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{3a}-\frac{A}{ax^3\sqrt[3]{a+bx^3}}\right.$$

$\downarrow$ 217

$$\frac{1}{3}\left(-\frac{(4Ab-3aB)\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{3a}-\frac{A}{ax^3\sqrt[3]{a+bx^3}}\right)$$

input
```
Int[(A + B*x^3)/(x^4*(a + b*x^3)^(4/3)),x]
```

output
```
(-(A/(a*x^3*(a + b*x^3)^(1/3))) - ((4*A*b - 3*a*B)*(3/(a*(a + b*x^3)^(1/3)
) + ((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3)
 - Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3))
)/a))/(3*a))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{\frac{2\left(Ab-\frac{3Ba}{4}\right)\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)\right)x^{3}\left(b\,x^{3}+a\cdots\right.}{9}}{a^{\frac{7}{3}}x^{3}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
2/9/(b*x^3+a)^(1/3)*((A*b-3/4*B*a)*(-2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/
3))*3^(1/2)/a^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^
(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*x^3*(b*x^3+a)^(1/3)+3/2*(3*B*x^3-A)*
a^(4/3)-6*A*b*x^3*a^(1/3))/a^(7/3)/x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(127) = 254.

Time = 0.15 (sec) , antiderivative size = 594, normalized size of antiderivative = 3.74

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
[-1/18*(3*sqrt(1/3)*((3*B*a^2*b - 4*A*a*b^2)*x^6 + (3*B*a^3 - 4*A*a^2*b)*x
^3)*sqrt((-a)^(1/3)/a)*log((2*b*x^3 - 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a
)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^
3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) + ((3*B*a*b - 4*A*b^2)*x^6 + (3*B*a^2
- 4*A*a*b)*x^3)*(-a)^(2/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^
(1/3) + (-a)^(2/3)) - 2*((3*B*a*b - 4*A*b^2)*x^6 + (3*B*a^2 - 4*A*a*b)*x^3
)*(-a)^(2/3)*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 6*((3*B*a^2 - 4*A*a*b)*
x^3 - A*a^2)*(b*x^3 + a)^(2/3))/(a^3*b*x^6 + a^4*x^3), 1/18*(6*sqrt(1/3)*(
(3*B*a^2*b - 4*A*a*b^2)*x^6 + (3*B*a^3 - 4*A*a^2*b)*x^3)*sqrt(-(-a)^(1/3)/
a)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)
) - ((3*B*a*b - 4*A*b^2)*x^6 + (3*B*a^2 - 4*A*a*b)*x^3)*(-a)^(2/3)*log((b*
x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*((3*B*a*b
- 4*A*b^2)*x^6 + (3*B*a^2 - 4*A*a*b)*x^3)*(-a)^(2/3)*log((b*x^3 + a)^(1/3)
 + (-a)^(1/3)) + 6*((3*B*a^2 - 4*A*a*b)*x^3 - A*a^2)*(b*x^3 + a)^(2/3))/(a
^3*b*x^6 + a^4*x^3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.58 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.52

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{4/3}}\,dx = -\frac{A\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^3}\right)}{3b^{\frac{4}{3}}x^7\Gamma\left(\frac{10}{3}\right)} - \frac{B\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^3}\right)}{3b^{\frac{4}{3}}x^4\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**4/(b*x**3+a)**(4/3),x)
```

output
```
-A*gamma(7/3)*hyper((4/3, 7/3), (10/3,), a*exp_polar(I*pi)/(b*x**3))/(3*b*
*(4/3)*x**7*gamma(10/3)) - B*gamma(4/3)*hyper((4/3, 4/3), (7/3,), a*exp_po
lar(I*pi)/(b*x**3))/(3*b**(4/3)*x**4*gamma(7/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.53

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{4/3}}\,dx = \frac{1}{6}\,B\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}} - \frac{\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}\right.$$

$$-\frac{1}{9}\,A\left(\frac{4\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}} + \frac{3\left(4\left(bx^3+a\right)b-3\,ab\right)}{\left(bx^3+a\right)^{\frac{4}{3}}a^2-\left(bx^3+a\right)^{\frac{1}{3}}a^3} - \frac{2\,b\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+\right.\right.}{a^{\frac{7}{3}}}\right.$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
1/6*B*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(4/3) - log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(4/3) + 2*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(4/3) + 6/((b*x^3 + a)^(1/3
)*a)) - 1/9*A*(4*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/
3))/a^(1/3))/a^(7/3) + 3*(4*(b*x^3 + a)*b - 3*a*b)/((b*x^3 + a)^(4/3)*a^2
- (b*x^3 + a)^(1/3)*a^3) - 2*b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a
^(1/3) + a^(2/3))/a^(7/3) + 4*b*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(7/3))
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx^3}{x^4 (a+bx^3)^{4/3}}\,dx = -\frac{(3\,Ba-4\,Ab)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{7}{3}}} + \frac{\sqrt{3}\left(3\,Ba^{\frac{5}{3}}-4\,Aa^{\frac{2}{3}}b\right)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^3} + \frac{\left(3\,Ba^{\frac{4}{3}}-4\,Aa^{\frac{1}{3}}b\right)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{8}{3}}} + \frac{3\,(bx^3+a)Ba-3\,Ba^2-4\,(bx^3+a)Ab+3\,Aab}{3\left((bx^3+a)^{\frac{4}{3}}-(bx^3+a)^{\frac{1}{3}}a\right)a^2}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
-1/18*(3*B*a - 4*A*b)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) +
a^(2/3))/a^(7/3) + 1/9*sqrt(3)*(3*B*a^(5/3) - 4*A*a^(2/3)*b)*arctan(1/3*sq
rt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^3 + 1/9*(3*B*a^(4/3) - 4*
A*a^(1/3)*b)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(8/3) + 1/3*(3*(b*x^3
 + a)*B*a - 3*B*a^2 - 4*(b*x^3 + a)*A*b + 3*A*a*b)/(((b*x^3 + a)^(4/3) - (
b*x^3 + a)^(1/3)*a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.17

$$\int \frac{A+Bx^3}{x^4 (a+bx^3)^{4/3}}\,dx = \frac{B\ln\left(B^2\,a\,(b\,x^3+a)^{1/3}-B^2\,a^{4/3}\right)}{3\,a^{4/3}} - \frac{\frac{A\,b}{a}-\frac{4\,A\,b\,(b\,x^3+a)}{3\,a^2}}{a\,(b\,x^3+a)^{1/3}-(b\,x^3+a)^{4/3}} + \frac{B}{a\,(b\,x^3+a)^{1/3}} + \frac{\ln\left(3\,a^{7/3}\,\left(2\,A\,b-\sqrt{3}\,A\,b\,2\mathrm{i}\right)^2-48\,A^2\,a^2\,b^2\,(b\,x^3+a)^{1/3}\right)\,\left(2\,A\,b-\sqrt{3}\,A\,b\,2\mathrm{i}\right)}{9\,a^{7/3}} + \frac{\ln\left(3\,a^{7/3}\,\left(2\,A\,b+\sqrt{3}\,A\,b\,2\mathrm{i}\right)^2-48\,A^2\,a^2\,b^2\,(b\,x^3+a)^{1/3}\right)\,\left(2\,A\,b+\sqrt{3}\,A\,b\,2\mathrm{i}\right)}{9\,a^{7/3}} - \frac{\ln\left(B^2\,a\,(b\,x^3+a)^{1/3}-\frac{a^{4/3}\,\left(B-\sqrt{3}\,B\,1\mathrm{i}\right)^2}{4}\right)\,\left(B-\sqrt{3}\,B\,1\mathrm{i}\right)}{6\,a^{4/3}} - \frac{\ln\left(B^2\,a\,(b\,x^3+a)^{1/3}-\frac{a^{4/3}\,\left(B+\sqrt{3}\,B\,1\mathrm{i}\right)^2}{4}\right)\,\left(B+\sqrt{3}\,B\,1\mathrm{i}\right)}{6\,a^{4/3}} - \frac{4\,A\,b\,\ln\left(48\,A^2\,a^{7/3}\,b^2-48\,A^2\,a^2\,b^2\,(b\,x^3+a)^{1/3}\right)}{9\,a^{7/3}}$$

input
```
int((A + B*x^3)/(x^4*(a + b*x^3)^(4/3)),x)
```

output
```
(B*log(B^2*a*(a + b*x^3)^(1/3) - B^2*a^(4/3)))/(3*a^(4/3)) - ((A*b)/a - (4
*A*b*(a + b*x^3))/(3*a^2))/(a*(a + b*x^3)^(1/3) - (a + b*x^3)^(4/3)) + B/(
a*(a + b*x^3)^(1/3)) + (log(3*a^(7/3)*(2*A*b - 3^(1/2)*A*b*2i)^2 - 48*A^2*
a^2*b^2*(a + b*x^3)^(1/3))*(2*A*b - 3^(1/2)*A*b*2i))/(9*a^(7/3)) + (log(3*
a^(7/3)*(2*A*b + 3^(1/2)*A*b*2i)^2 - 48*A^2*a^2*b^2*(a + b*x^3)^(1/3))*(2*
A*b + 3^(1/2)*A*b*2i))/(9*a^(7/3)) - (log(B^2*a*(a + b*x^3)^(1/3) - (a^(4/
3)*(B - 3^(1/2)*B*1i)^2)/4)*(B - 3^(1/2)*B*1i))/(6*a^(4/3)) - (log(B^2*a*(
a + b*x^3)^(1/3) - (a^(4/3)*(B + 3^(1/2)*B*1i)^2)/4)*(B + 3^(1/2)*B*1i))/(
6*a^(4/3)) - (4*A*b*log(48*A^2*a^(7/3)*b^2 - 48*A^2*a^2*b^2*(a + b*x^3)^(1
/3)))/(9*a^(7/3))
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{4/3}}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}x^4}dx$$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^(4/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**4),x)
```

3.365 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 168

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{a(Ab-aB)x}{b^3\sqrt[3]{a+bx^3}}+\frac{(3Ab-5aB)x(a+bx^3)^{2/3}}{9b^3} + \frac{Bx^4(a+bx^3)^{2/3}}{6b^2} - \frac{2a(6Ab-7aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{10/3}} + \frac{a(6Ab-7aB)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{9b^{10/3}}$$

output
```
a*(A*b-B*a)*x/b^3/(b*x^3+a)^(1/3)+1/9*(3*A*b-5*B*a)*x*(b*x^3+a)^(2/3)/b^3+
1/6*B*x^4*(b*x^3+a)^(2/3)/b^2-2/27*a*(6*A*b-7*B*a)*arctan(1/3*(1+2*b^(1/3)
*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(10/3)+1/9*a*(6*A*b-7*B*a)*ln(-b^(1
/3)*x+(b*x^3+a)^(1/3))/b^(10/3)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.22

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{\frac{3\sqrt[3]{b}(-28a^2Bx+abx(24A-7Bx^3)+3b^2x^4(2A+Bx^3))}{\sqrt[3]{a+bx^3}} + 4\sqrt{3}a(-6Ab+7aB)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}}{\sqrt[3]{b}x+2\sqrt[3]{a}}\right.}{}$$

input
```
Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^(4/3),x]
```

output
```
((3*b^(1/3)*(-28*a^2*B*x + a*b*x*(24*A - 7*B*x^3) + 3*b^2*x^4*(2*A + B*x^3
)))/(a + b*x^3)^(1/3) + 4*Sqrt[3]*a*(-6*A*b + 7*a*B)*ArcTan[(Sqrt[3]*b^(1/
3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 4*a*(-6*A*b + 7*a*B)*Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)] + 2*a*(-6*A*b + 7*a*B)*Log[b^(2/3)*x^2 + b^(1/
3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(10/3))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {957, 843, 843, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{4/3}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^7(Ab-aB)}{ab\sqrt[3]{a+bx^3}} - \frac{(6Ab-7aB)\int \frac{x^6}{\sqrt[3]{bx^3+a}}dx}{ab}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^7(Ab-aB)}{ab\sqrt[3]{a+bx^3}}-\frac{(6Ab-7aB)\left(\frac{x^4\left(a+bx^3\right)^{2/3}}{6b}-\frac{2a\int\frac{x^3}{\sqrt[3]{bx^3+a}}dx}{3b}\right)}{ab}$$

$\downarrow$ 843

$$\frac{x^7(Ab-aB)}{ab\sqrt[3]{a+bx^3}}-\frac{(6Ab-7aB)\left(\frac{x^4\left(a+bx^3\right)^{2/3}}{6b}-\frac{2a\left(\frac{x\left(a+bx^3\right)^{2/3}}{3b}-\frac{a\int\frac{1}{\sqrt[3]{bx^3+a}}dx}{3b}\right)}{3b}\right)}{ab}$$

$\downarrow$ 769

$$\frac{\frac{x^7(Ab-aB)}{ab\sqrt[3]{a+bx^3}}-(6Ab-7aB)\left(\frac{x^4(a+bx^3)^{2/3}}{6b}-\frac{2a\left(\frac{x(a+bx^3)^{2/3}}{3b}-\frac{a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{3b}\right)}{3b}\right)}{ab}$$

input

```
Int[(x^6*(A + B*x^3))/(a + b*x^3)^(4/3),x]
```

output

```
((A*b - a*B)*x^7)/(a*b*(a + b*x^3)^(1/3)) - ((6*A*b - 7*a*B)*((x^4*(a + b*
x^3)^(2/3))/(6*b) - (2*a*((x*(a + b*x^3)^(2/3))/(3*b) - (a*(ArcTan[(1 + (2
*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*
x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/(3*b)))/(3*b)))/(a*b)
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$\frac{\frac{4a\left(\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{2}\right)\left(Ab-\frac{7Ba}{6}\right)\left(bx\right.}{9}}{b^{\frac{10}{3}}\left(bx^3+a\right)^{\frac{1}{3}}}$

input
```
int(x^6*(B*x^3+A)/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
4/9/b^(10/3)*(a*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/
b^(1/3)/x)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((b^(2/3)*x^2+b^(1/3)*
(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(A*b-7/6*B*a)*(b*x^3+a)^(1/3)+3*(
a*(-7/24*B*x^3+A)*b^(4/3)+1/4*(1/2*B*x^3+A)*x^3*b^(7/3)-7/6*B*a^2*b^(1/3))
*x)/(b*x^3+a)^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(140) = 280$.

Time = 0.11 (sec) , antiderivative size = 624, normalized size of antiderivative = 3.71

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
[-1/54*(6*sqrt(1/3)*(7*B*a^3*b - 6*A*a^2*b^2 + (7*B*a^2*b^2 - 6*A*a*b^3)*x
^3)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqr
t(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3
)*x)*sqrt(-1/b^(2/3)) + 2*a) + 4*(7*B*a^3 - 6*A*a^2*b + (7*B*a^2*b - 6*A*a
*b^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 2*(7*B*a^3 -
6*A*a^2*b + (7*B*a^2*b - 6*A*a*b^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3
 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(3*B*b^3*x^7 - (7*B*a*
b^2 - 6*A*b^3)*x^4 - 4*(7*B*a^2*b - 6*A*a*b^2)*x)*(b*x^3 + a)^(2/3))/(b^5*
x^3 + a*b^4), -1/54*(4*(7*B*a^3 - 6*A*a^2*b + (7*B*a^2*b - 6*A*a*b^2)*x^3)
*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 2*(7*B*a^3 - 6*A*a^2*b
+ (7*B*a^2*b - 6*A*a*b^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3
)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 12*sqrt(1/3)*(7*B*a^3*b - 6*A*a^2*
b^2 + (7*B*a^2*b^2 - 6*A*a*b^3)*x^3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^
3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(3*B*b^3*x^7 - (7*B*a*b^2 - 6*A*b^3
)*x^4 - 4*(7*B*a^2*b - 6*A*a*b^2)*x)*(b*x^3 + a)^(2/3))/(b^5*x^3 + a*b^4)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.48

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{Ax^7\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{10}{3}\right)}+\frac{Bx^{10}\Gamma\left(\frac{10}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate(x**6*(B*x**3+A)/(b*x**3+a)**(4/3),x)
```

output
```
A*x**7*gamma(7/3)*hyper((4/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*
a**(4/3)*gamma(10/3)) + B*x**10*gamma(10/3)*hyper((4/3, 10/3), (13/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gamma(13/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(140) = 280.

Time = 0.12 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.21

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx =$$

$$-\frac{1}{54}B\left(\frac{28\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{10}{3}}}+\frac{3\left(18a^2b^2-\frac{49(bx^3+a)a^2b}{x^3}+\frac{28(bx^3+a)^2a^2}{x^6}\right)}{\frac{(bx^3+a)^{\frac{1}{3}}b^5}{x}-\frac{2(bx^3+a)^{\frac{4}{3}}b^4}{x^4}+\frac{(bx^3+a)^{\frac{7}{3}}b^3}{x^7}}-\frac{14a^2\log\left(b^{\frac{2}{3}}\right.}{}\right.$$

$$+\frac{1}{9}A\left(\frac{4\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}+\frac{3\left(3ab-\frac{4(bx^3+a)a}{x^3}\right)}{\frac{(bx^3+a)^{\frac{1}{3}}b^3}{x}-\frac{(bx^3+a)^{\frac{4}{3}}b^2}{x^4}}-\frac{2a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right.}{b^{\frac{7}{3}}}\right.$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
-1/54*B*(28*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/
x)/b^(1/3))/b^(10/3) + 3*(18*a^2*b^2 - 49*(b*x^3 + a)*a^2*b/x^3 + 28*(b*x^
3 + a)^2*a^2/x^6)/((b*x^3 + a)^(1/3)*b^5/x - 2*(b*x^3 + a)^(4/3)*b^4/x^4 +
 (b*x^3 + a)^(7/3)*b^3/x^7) - 14*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/
3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(10/3) + 28*a^2*log(-b^(1/3) + (b*x^3 + a)
^(1/3)/x)/b^(10/3)) + 1/9*A*(4*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(
b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) + 3*(3*a*b - 4*(b*x^3 + a)*a/x^3)/((b
*x^3 + a)^(1/3)*b^3/x - (b*x^3 + a)^(4/3)*b^2/x^4) - 2*a*log(b^(2/3) + (b*
x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a*log(-b^(1/
3) + (b*x^3 + a)^(1/3)/x)/b^(7/3))
```

Giac [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{(Bx^3+A)x^6}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{x^6\,(B\,x^3+A)}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((x^6*(A + B*x^3))/(a + b*x^3)^(4/3),x)
```

output
```
int((x^6*(A + B*x^3))/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int(x^6*(B*x^3+A)/(b*x^3+a)^(4/3),x)
```

output
```
int(x**6/(a + b*x**3)**(1/3),x)
```

3.366 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 136

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = -\frac{(Ab-aB)x}{b^2\sqrt[3]{a+bx^3}}+\frac{Bx(a+bx^3)^{2/3}}{3b^2} + \frac{(3Ab-4aB)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{7/3}} - \frac{(3Ab-4aB)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{7/3}}$$

output

```
-(A*b-B*a)*x/b^2/(b*x^3+a)^(1/3)+1/3*B*x*(b*x^3+a)^(2/3)/b^2+1/9*(3*A*b-4*
B*a)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(7/3)-1
/6*(3*A*b-4*B*a)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.30

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{\frac{6\sqrt[3]{b}x(-3Ab+4aB+bBx^3)}{\sqrt[3]{a+bx^3}}+2\sqrt{3}(3Ab-4aB)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+(-6Ab+8a}{1}$$

input
```
Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^(4/3),x]
```

output
```
((6*b^(1/3)*x*(-3*A*b + 4*a*B + b*B*x^3))/(a + b*x^3)^(1/3) + 2*Sqrt[3]*(3
*A*b - 4*a*B)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))
] + (-6*A*b + 8*a*B)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + (3*A*b - 4*a*
B)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18
*b^(7/3))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {957, 843, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{4/3}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^4(Ab-aB)}{ab\sqrt[3]{a+bx^3}}-\frac{(3Ab-4aB)\int \frac{x^3}{\sqrt[3]{bx^3+a}}dx}{ab}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^4(Ab-aB)}{ab\sqrt[3]{a+bx^3}}-\frac{(3Ab-4aB)\left(\frac{x(a+bx^3)^{2/3}}{3b}-\frac{a\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{3b}\right)}{ab}$$

$$\downarrow 769$$

$$\frac{x^4(Ab-aB)}{ab\sqrt[3]{a+bx^3}}-\frac{(3Ab-4aB)\left(\frac{x(a+bx^3)^{2/3}}{3b}-\frac{a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{3b}\right)}{ab}$$

input
```
Int[(x^3*(A + B*x^3))/(a + b*x^3)^(4/3),x]
```

output
```
((A*b - a*B)*x^4)/(a*b*(a + b*x^3)^(1/3)) - ((3*A*b - 4*a*B)*((x*(a + b*x^
3)^(2/3))/(3*b) - (a*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]
]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/
(3*b)))/(a*b)
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.09

method	result
pseudoelliptic	$-\frac{-\frac{b^{2}\left(Ab-\frac{4Ba}{3}\right)\left(-2\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{b^{\frac{1}{3}}}+x\right)}{3x}\right)\sqrt{3}+\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)-2\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)\right)}{6}}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}b^{\frac{13}{3}}}$

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-1/(b*x^3+a)^(1/3)*(-1/6*b^2*(A*b-4/3*B*a)*(-2*arctan(1/3*3^(1/2)*(2*(b*x^
3+a)^(1/3)/b^(1/3)+x)/x)*3^(1/2)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x
+(b*x^3+a)^(2/3))/x^2)-2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x))*(b*x^3+a)^(1/
3)+b^(7/3)*((-1/3*B*x^3+A)*b-4/3*B*a)*x)/b^(13/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(110) = 220.

Time = 0.13 (sec) , antiderivative size = 598, normalized size of antiderivative = 4.40

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
[-1/18*(3*sqrt(1/3)*(4*B*a^2*b - 3*A*a*b^2 + (4*B*a*b^2 - 3*A*b^3)*x^3)*sq
rt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt
(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-
b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 2*((4*B*a*b - 3*A*b^2)*x^3 + 4*B*a
^2 - 3*A*a*b)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + ((4*B
*a*b - 3*A*b^2)*x^3 + 4*B*a^2 - 3*A*a*b)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 -
(b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 6*(B*b^2*x^4 +
(4*B*a*b - 3*A*b^2)*x)*(b*x^3 + a)^(2/3))/(b^4*x^3 + a*b^3), 1/18*(6*sqrt(
1/3)*(4*B*a^2*b - 3*A*a*b^2 + (4*B*a*b^2 - 3*A*b^3)*x^3)*sqrt(-(-b)^(1/3)/
b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)
/b)/x) + 2*((4*B*a*b - 3*A*b^2)*x^3 + 4*B*a^2 - 3*A*a*b)*(-b)^(2/3)*log(((
-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - ((4*B*a*b - 3*A*b^2)*x^3 + 4*B*a^2 -
 3*A*a*b)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x
+ (b*x^3 + a)^(2/3))/x^2) + 6*(B*b^2*x^4 + (4*B*a*b - 3*A*b^2)*x)*(b*x^3 +
 a)^(2/3))/(b^4*x^3 + a*b^3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.92 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.59

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{Ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}+\frac{Bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**3*(B*x**3+A)/(b*x**3+a)**(4/3),x)
```

output
```
A*x**4*gamma(4/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(4/3)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((4/3, 7/3), (10/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(4/3)*gamma(10/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(110) = 220$.

Time = 0.11 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.07

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{1}{9}\,B\left(\frac{4\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}+\frac{3\left(3\,ab-\frac{4\,(bx^3+a)a}{x^3}\right)}{\frac{(bx^3+a)^{\frac{1}{3}}b^3}{x}-\frac{(bx^3+a)^{\frac{4}{3}}b^2}{x^4}}-\frac{2\,a\log\left(b^{\frac{2}{3}}+\right.}{}\right.$$

$$-\frac{1}{6}\,A\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}+\frac{6\,x}{(bx^3+a)^{\frac{1}{3}}b}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,\log\left(-\right.}{}\right.$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
1/9*B*(4*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^
(1/3))/b^(7/3) + 3*(3*a*b - 4*(b*x^3 + a)*a/x^3)/((b*x^3 + a)^(1/3)*b^3/x
- (b*x^3 + a)^(4/3)*b^2/x^4) - 2*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)
/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)
/x)/b^(7/3)) - 1/6*A*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a
)^(1/3)/x)/b^(1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b
*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3
) + (b*x^3 + a)^(1/3)/x)/b^(4/3))
```

Giac [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{(Bx^3+A)x^3}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{x^3\,(B\,x^3+A)}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((x^3*(A + B*x^3))/(a + b*x^3)^(4/3),x)
```

output
```
int((x^3*(A + B*x^3))/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^(4/3),x)
```

output
```
int(x**3/(a + b*x**3)**(1/3),x)
```

3.367 $\int \frac{A+Bx^3}{(a+bx^3)^{4/3}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 99

$$\int \frac{A+Bx^3}{(a+bx^3)^{4/3}} \, dx = \frac{(Ab-aB)x}{ab\sqrt[3]{a+bx^3}} + \frac{B\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}} - \frac{B\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2b^{4/3}}$$

output

```
(A*b-B*a)*x/a/b/(b*x^3+a)^(1/3)+1/3*B*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^
(1/3))*3^(1/2))*3^(1/2)/b^(4/3)-1/2*B*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/
3)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.52

$$\int \frac{A+Bx^3}{(a+bx^3)^{4/3}}\,dx = \frac{\frac{6\sqrt[3]{b}(Ab-aB)x}{a\sqrt[3]{a+bx^3}} + 2\sqrt{3}B\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - 2B\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right) + B}{6b^{4/3}}$$

input
```
Integrate[(A + B*x^3)/(a + b*x^3)^(4/3),x]
```

output
```
((6*b^(1/3)*(A*b - a*B)*x)/(a*(a + b*x^3)^(1/3)) + 2*Sqrt[3]*B*ArcTan[(Sqr
t[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*B*Log[-(b^(1/3)*x)
+ (a + b*x^3)^(1/3)] + B*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (
a + b*x^3)^(2/3)])/(6*b^(4/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {910, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(a+bx^3)^{4/3}}dx$$

$$\Big\downarrow \ 910$$

$$\frac{B\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{b} + \frac{x(Ab-aB)}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \ 769$$

$$\frac{x(Ab-aB)}{ab\sqrt[3]{a+bx^3}}+\frac{B\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{b}$$

input
```
Int[(A + B*x^3)/(a + b*x^3)^(4/3),x]
```

output
```
((A*b - a*B)*x)/(a*b*(a + b*x^3)^(1/3)) + (B*(ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^
3)^(1/3)]/(2*b^(1/3))))/b
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.37

method	result
pseudoelliptic	$\frac{xA}{a(bx^3+a)^{\frac{1}{3}}} - \frac{xB}{b(bx^3+a)^{\frac{1}{3}}} - \frac{B\ln\left(\frac{-b^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{4}{3}}} + \frac{B\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{6b^{\frac{4}{3}}} - \frac{B\sqrt{3}\,\text{arcta}}{}$

input
```
int((B*x^3+A)/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
1/a*x/(b*x^3+a)^(1/3)*A-1/b*x/(b*x^3+a)^(1/3)*B-1/3*B/b^(4/3)*ln((-b^(1/3)
*x+(b*x^3+a)^(1/3))/x)+1/6*B/b^(4/3)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/
3)*x+(b*x^3+a)^(2/3))/x^2)-1/3*B/b^(4/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*
x^3+a)^(1/3)/b^(1/3)+x)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(82) = 164$.

Time = 0.11 (sec) , antiderivative size = 488, normalized size of antiderivative = 4.93

$$\int \frac{A+Bx^3}{(a+bx^3)^{4/3}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}(Bab^2x^3+Ba^2b)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^{3}\right.\right.}{}\right.$$

$$-\frac{6\sqrt{\frac{1}{3}}(Bab^2x^3+Ba^2b)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+6\,(bx^3+a)^{\frac{2}{3}}(Bab-Ab^2)x}{6\,(ab}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*(B*a*b^2*x^3 + B*a^2*b)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 -
 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x
^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b)
 + 2*a) - 6*(b*x^3 + a)^(2/3)*(B*a*b - A*b^2)*x - 2*(B*a*b*x^3 + B*a^2)*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (B*a*b*x^3 + B*a^2)*(
-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 +
a)^(2/3))/x^2))/(a*b^3*x^3 + a^2*b^2), -1/6*(6*sqrt(1/3)*(B*a*b^2*x^3 + B*
a^2*b)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)
^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 6*(b*x^3 + a)^(2/3)*(B*a*b - A*b^2)*x + 2
*(B*a*b*x^3 + B*a^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x)
- (B*a*b*x^3 + B*a^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(
-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(a*b^3*x^3 + a^2*b^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx^3}{(a+bx^3)^{4/3}}\,dx = \frac{Ax\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{4}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{4}{3}\right)} + \frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((B*x**3+A)/(b*x**3+a)**(4/3),x)
```

output
```
A*x*gamma(1/3)/(3*a**(4/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) + B*x**4*gamm
a(4/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gam
ma(7/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx^3}{(a+bx^3)^{4/3}}\,dx = -\frac{1}{6}\,B\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}+\frac{6\,x}{(bx^3+a)^{\frac{1}{3}}b}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,\log\left(-\right.}{}\right.$$

$$+\frac{Ax}{(bx^3+a)^{\frac{1}{3}}a}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
-1/6*B*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(
1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x^3 + a)^(1/3
)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) + (b*x^3 + a
)^(1/3)/x)/b^(4/3)) + A*x/((b*x^3 + a)^(1/3)*a)
```

Giac [F]

$$\int \frac{A+Bx^3}{(a+bx^3)^{4/3}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(a+bx^3)^{4/3}}\,dx=\int \frac{B\,x^3+A}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((A + B*x^3)/(a + b*x^3)^(4/3),x)
```

output
```
int((A + B*x^3)/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{(a+bx^3)^{4/3}}\,dx=\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int((B*x^3+A)/(b*x^3+a)^(4/3),x)
```

output
```
int(1/(a + b*x**3)**(1/3),x)
```

3.368 $\int \frac{A+Bx^3}{x^3(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^{4/3}}\,dx = -\frac{A}{2ax^2\sqrt[3]{a+bx^3}} - \frac{(3Ab-2aB)x}{2a^2\sqrt[3]{a+bx^3}}$$

output
```
-1/2*A/a/x^2/(b*x^3+a)^(1/3)-1/2*(3*A*b-2*B*a)*x/a^2/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^{4/3}}\,dx = \frac{-aA-3Abx^3+2aBx^3}{2a^2x^2\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^(4/3)),x]
```

output
```
(-(a*A) - 3*A*b*x^3 + 2*a*B*x^3)/(2*a^2*x^2*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {955, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^3 (a+bx^3)^{4/3}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(3Ab-2aB)\int \frac{1}{(bx^3+a)^{4/3}}dx}{2a}-\frac{A}{2ax^2\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{746}$$

$$-\frac{x(3Ab-2aB)}{2a^2\sqrt[3]{a+bx^3}}-\frac{A}{2ax^2\sqrt[3]{a+bx^3}}$$

input
```
Int[(A + B*x^3)/(x^3*(a + b*x^3)^(4/3)),x]
```

output
```
-1/2*A/(a*x^2*(a + b*x^3)^(1/3)) - ((3*A*b - 2*a*B)*x)/(2*a^2*(a + b*x^3)^
(1/3))
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{3Ab\,x^{3}-2Ba\,x^{3}+Aa}{2x^{2}\left(b\,x^{3}+a\right)^{\frac{1}{3}}a^{2}}$	36
trager	$-\frac{3Ab\,x^{3}-2Ba\,x^{3}+Aa}{2x^{2}\left(b\,x^{3}+a\right)^{\frac{1}{3}}a^{2}}$	36
pseudoelliptic	$-\frac{\left(-2B\,x^{3}+A\right)a+3Ab\,x^{3}}{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}x^{2}a^{2}}$	36
orering	$-\frac{3Ab\,x^{3}-2Ba\,x^{3}+Aa}{2x^{2}\left(b\,x^{3}+a\right)^{\frac{1}{3}}a^{2}}$	36
risch	$-\frac{A\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{2a^{2}x^{2}}-\frac{x\left(Ab-Ba\right)}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}a^{2}}$	43

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(3*A*b*x^3-2*B*a*x^3+A*a)/x^2/(b*x^3+a)^(1/3)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{4/3}}\,dx=\frac{\left((2\,Ba-3\,Ab)x^3-Aa\right)\left(bx^3+a\right)^{\frac{2}{3}}}{2\left(a^2bx^5+a^3x^2\right)}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
1/2*((2*B*a - 3*A*b)*x^3 - A*a)*(b*x^3 + a)^(2/3)/(a^2*b*x^5 + a^3*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(44) = 88$.

Time = 7.61 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.00

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{4/3}}\,dx = A\left(\frac{\Gamma\left(-\frac{2}{3}\right)}{9a\sqrt[3]{b}x^3\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(\frac{4}{3}\right)}+\frac{b^{\frac{2}{3}}\Gamma\left(-\frac{2}{3}\right)}{3a^2\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(\frac{4}{3}\right)}\right) + \frac{Bx\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{4}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**3/(b*x**3+a)**(4/3),x)
```

output
```
A*(gamma(-2/3)/(9*a*b**(1/3)*x**3*(a/(b*x**3) + 1)**(1/3)*gamma(4/3)) + b*
*(2/3)*gamma(-2/3)/(3*a**2*(a/(b*x**3) + 1)**(1/3)*gamma(4/3))) + B*x*gamm
a(1/3)/(3*a**(4/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{4/3}}\,dx = -\frac{1}{2}A\left(\frac{2\,bx}{\left(bx^3+a\right)^{\frac{1}{3}}a^2}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{a^2x^2}\right)+\frac{Bx}{\left(bx^3+a\right)^{\frac{1}{3}}a}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
-1/2*A*(2*b*x/((b*x^3 + a)^(1/3)*a^2) + (b*x^3 + a)^(2/3)/(a^2*x^2)) + B*x
/((b*x^3 + a)^(1/3)*a)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{4/3}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{4}{3}}x^3}\,dx$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(4/3)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{4/3}}\,dx=\frac{2\,A\,a-3\,A\,\left(b\,x^3+a\right)+2\,B\,a\,x^3}{2\,a^2\,x^2\,{\left(b\,x^3+a\right)}^{1/3}}$$

input
```
int((A + B*x^3)/(x^3*(a + b*x^3)^(4/3)),x)
```

output
```
(2*A*a - 3*A*(a + b*x^3) + 2*B*a*x^3)/(2*a^2*x^2*(a + b*x^3)^(1/3))
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{4/3}}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}x^3}dx$$

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^(4/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**3),x)
```

3.369 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{4/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{4/3}}\,dx=-\frac{A}{5ax^5\sqrt[3]{a+bx^3}}-\frac{6Ab-5aB}{5a^2x^2\sqrt[3]{a+bx^3}}+\frac{3(6Ab-5aB)\left(a+bx^3\right)^{2/3}}{10a^3x^2}$$

output
```
-1/5*A/a/x^5/(b*x^3+a)^(1/3)-1/5*(6*A*b-5*B*a)/a^2/x^2/(b*x^3+a)^(1/3)+3/1
0*(6*A*b-5*B*a)*(b*x^3+a)^(2/3)/a^3/x^2
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{4/3}}\,dx=\frac{-2a^2A+6aAbx^3-5a^2Bx^3+18Ab^2x^6-15abBx^6}{10a^3x^5\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^(4/3)),x]
```

output
```
(-2*a^2*A + 6*a*A*b*x^3 - 5*a^2*B*x^3 + 18*A*b^2*x^6 - 15*a*b*B*x^6)/(10*a
^3*x^5*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {955, 803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{4/3}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(6Ab-5aB)\int \frac{1}{x^3(bx^3+a)^{4/3}}dx}{5a}-\frac{A}{5ax^5\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(6Ab-5aB)\left(-\frac{3b\int \frac{1}{\left(bx^3+a\right)^{4/3}}dx}{2a}-\frac{1}{2ax^2\sqrt[3]{a+bx^3}}\right)}{5a}-\frac{A}{5ax^5\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{746}$$

$$-\frac{\left(-\frac{3bx}{2a^2\sqrt[3]{a+bx^3}}-\frac{1}{2ax^2\sqrt[3]{a+bx^3}}\right)(6Ab-5aB)}{5a}-\frac{A}{5ax^5\sqrt[3]{a+bx^3}}$$

input
```
Int[(A + B*x^3)/(x^6*(a + b*x^3)^(4/3)),x]
```

output
```
-1/5*A/(a*x^5*(a + b*x^3)^(1/3)) - ((6*A*b - 5*a*B)*(-1/2*1/(a*x^2*(a + b*
x^3)^(1/3)) - (3*b*x)/(2*a^2*(a + b*x^3)^(1/3))))/(5*a)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$\frac{\left(-5B\,x^{3}-2A\right)a^{2}+6b\left(-\frac{5B\,x^{3}}{2}+A\right)x^{3}a+18A\,b^{2}x^{6}}{10\left(b\,x^{3}+a\right)^{\frac{1}{3}}x^{5}a^{3}}$	57
gosper	$-\frac{-18A\,b^{2}x^{6}+15Bab\,x^{6}-6aAb\,x^{3}+5B\,a^{2}x^{3}+2a^{2}A}{10x^{5}\left(b\,x^{3}+a\right)^{\frac{1}{3}}a^{3}}$	59
trager	$-\frac{-18A\,b^{2}x^{6}+15Bab\,x^{6}-6aAb\,x^{3}+5B\,a^{2}x^{3}+2a^{2}A}{10x^{5}\left(b\,x^{3}+a\right)^{\frac{1}{3}}a^{3}}$	59
orering	$-\frac{-18A\,b^{2}x^{6}+15Bab\,x^{6}-6aAb\,x^{3}+5B\,a^{2}x^{3}+2a^{2}A}{10x^{5}\left(b\,x^{3}+a\right)^{\frac{1}{3}}a^{3}}$	59
risch	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(-8Ab\,x^{3}+5Ba\,x^{3}+2Aa\right)}{10a^{3}x^{5}}+\frac{x\left(Ab-Ba\right)b}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}a^{3}}$	61

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
1/10*((-5*B*x^3-2*A)*a^2+6*b*(-5/2*B*x^3+A)*x^3*a+18*A*b^2*x^6)/(b*x^3+a)^
(1/3)/x^5/a^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{4/3}}\,dx = -\frac{(3\,(5\,Bab-6\,Ab^2)x^6+(5\,Ba^2-6\,Aab)x^3+2\,Aa^2)(bx^3+a)^{\frac{2}{3}}}{10\,(a^3bx^8+a^4x^5)}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
-1/10*(3*(5*B*a*b - 6*A*b^2)*x^6 + (5*B*a^2 - 6*A*a*b)*x^3 + 2*A*a^2)*(b*x
^3 + a)^(2/3)/(a^3*b*x^8 + a^4*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(75) = 150.

Time = 21.63 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.77

$$\begin{aligned}\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{4/3}}\,dx = {} & A\left(-\frac{2a^3b^{\frac{14}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{27a^5b^4x^3\Gamma\left(\frac{4}{3}\right)+54a^4b^5x^6\Gamma\left(\frac{4}{3}\right)+27a^3b^6x^9\Gamma\left(\frac{4}{3}\right)}\right.\\
&+\frac{4a^2b^{\frac{17}{3}}x^3\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{27a^5b^4x^3\Gamma\left(\frac{4}{3}\right)+54a^4b^5x^6\Gamma\left(\frac{4}{3}\right)+27a^3b^6x^9\Gamma\left(\frac{4}{3}\right)}\\
&+\frac{24ab^{\frac{20}{3}}x^6\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{27a^5b^4x^3\Gamma\left(\frac{4}{3}\right)+54a^4b^5x^6\Gamma\left(\frac{4}{3}\right)+27a^3b^6x^9\Gamma\left(\frac{4}{3}\right)}\\
&\left.+\frac{18b^{\frac{23}{3}}x^9\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{27a^5b^4x^3\Gamma\left(\frac{4}{3}\right)+54a^4b^5x^6\Gamma\left(\frac{4}{3}\right)+27a^3b^6x^9\Gamma\left(\frac{4}{3}\right)}\right)\\
&+B\left(\frac{\Gamma\left(-\frac{2}{3}\right)}{9a\sqrt[3]{b}x^3\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(\frac{4}{3}\right)}+\frac{b^{\frac{2}{3}}\Gamma\left(-\frac{2}{3}\right)}{3a^2\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(\frac{4}{3}\right)}\right)\end{aligned}$$

input
```
integrate((B*x**3+A)/x**6/(b*x**3+a)**(4/3),x)
```

output
```
A*(-2*a**3*b**(14/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-5/3)/(27*a**5*b**4*x**
3*gamma(4/3) + 54*a**4*b**5*x**6*gamma(4/3) + 27*a**3*b**6*x**9*gamma(4/3)
) + 4*a**2*b**(17/3)*x**3*(a/(b*x**3) + 1)**(2/3)*gamma(-5/3)/(27*a**5*b**
4*x**3*gamma(4/3) + 54*a**4*b**5*x**6*gamma(4/3) + 27*a**3*b**6*x**9*gamma
(4/3)) + 24*a*b**(20/3)*x**6*(a/(b*x**3) + 1)**(2/3)*gamma(-5/3)/(27*a**5*
b**4*x**3*gamma(4/3) + 54*a**4*b**5*x**6*gamma(4/3) + 27*a**3*b**6*x**9*ga
mma(4/3)) + 18*b**(23/3)*x**9*(a/(b*x**3) + 1)**(2/3)*gamma(-5/3)/(27*a**5
*b**4*x**3*gamma(4/3) + 54*a**4*b**5*x**6*gamma(4/3) + 27*a**3*b**6*x**9*g
amma(4/3))) + B*(gamma(-2/3)/(9*a*b**(1/3)*x**3*(a/(b*x**3) + 1)**(1/3)*ga
mma(4/3)) + b**(2/3)*gamma(-2/3)/(3*a**2*(a/(b*x**3) + 1)**(1/3)*gamma(4/3
)))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^{4/3}}\,dx = -\frac{1}{2}\,B\left(\frac{2\,bx}{(bx^3+a)^{\frac{1}{3}}a^2} + \frac{(bx^3+a)^{\frac{2}{3}}}{a^2x^2}\right) + \frac{1}{5}\,A\left(\frac{5\,b^2x}{(bx^3+a)^{\frac{1}{3}}a^3} + \frac{\frac{5\,(bx^3+a)^{\frac{2}{3}}b}{x^2} - \frac{(bx^3+a)^{\frac{5}{3}}}{x^5}}{a^3}\right)$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
-1/2*B*(2*b*x/((b*x^3 + a)^(1/3)*a^2) + (b*x^3 + a)^(2/3)/(a^2*x^2)) + 1/5
*A*(5*b^2*x/((b*x^3 + a)^(1/3)*a^3) + (5*(b*x^3 + a)^(2/3)*b/x^2 - (b*x^3
+ a)^(5/3)/x^5)/a^3)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{4/3}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{4}{3}}x^6}\,dx$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(4/3)*x^6), x)
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{4/3}}\,dx=\frac{18\,A\,\left(b\,x^3+a\right)^2+10\,A\,a^2+10\,B\,a^2\,x^3-30\,A\,a\,\left(b\,x^3+a\right)-15\,B\,a\,x^3\,\left(b\,x^3+a\right)}{10\,a^3\,x^5\,\left(b\,x^3+a\right)^{1/3}}$$

input
```
int((A + B*x^3)/(x^6*(a + b*x^3)^(4/3)),x)
```

output
```
(18*A*(a + b*x^3)^2 + 10*A*a^2 + 10*B*a^2*x^3 - 30*A*a*(a + b*x^3) - 15*B*
a*x^3*(a + b*x^3))/(10*a^3*x^5*(a + b*x^3)^(1/3))
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{4/3}}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}x^6}dx$$

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^(4/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**6),x)
```

3.370 $\int \frac{A+Bx^3}{x^9\left(a+bx^3\right)^{4/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 114

$$\int \frac{A+Bx^3}{x^9\left(a+bx^3\right)^{4/3}}\,dx=-\frac{A}{8ax^8\sqrt[3]{a+bx^3}}-\frac{9Ab-8aB}{8a^2x^5\sqrt[3]{a+bx^3}}+\frac{3(9Ab-8aB)\left(a+bx^3\right)^{2/3}}{20a^3x^5}-\frac{9b(9Ab-8aB)\left(a+bx^3\right)^{2/3}}{40a^4x^2}$$

output
```
-1/8*A/a/x^8/(b*x^3+a)^(1/3)-1/8*(9*A*b-8*B*a)/a^2/x^5/(b*x^3+a)^(1/3)+3/2
0*(9*A*b-8*B*a)*(b*x^3+a)^(2/3)/a^3/x^5-9/40*b*(9*A*b-8*B*a)*(b*x^3+a)^(2/
3)/a^4/x^2
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx^3}{x^9\left(a+bx^3\right)^{4/3}}\,dx=\frac{-5a^3A+9a^2Abx^3-8a^3Bx^3-27aAb^2x^6+24a^2bBx^6-81Ab^3x^9+72ab^2Bx^9}{40a^4x^8\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^9*(a + b*x^3)^(4/3)),x]
```

output
```
(-5*a^3*A + 9*a^2*A*b*x^3 - 8*a^3*B*x^3 - 27*a*A*b^2*x^6 + 24*a^2*b*B*x^6
- 81*A*b^3*x^9 + 72*a*b^2*B*x^9)/(40*a^4*x^8*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 803, 803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^9 (a+bx^3)^{4/3}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(9Ab-8aB)\int \frac{1}{x^6(bx^3+a)^{4/3}}dx}{8a}-\frac{A}{8ax^8\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(9Ab-8aB)\left(-\frac{6b\int \frac{1}{x^3\left(bx^3+a\right)^{4/3}}dx}{5a}-\frac{1}{5ax^5\sqrt[3]{a+bx^3}}\right)}{8a}-\frac{A}{8ax^8\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(9Ab-8aB)\left(-\frac{6b\left(-\frac{3b\int \frac{1}{\left(bx^3+a\right)^{4/3}}dx}{2a}-\frac{1}{2ax^2\sqrt[3]{a+bx^3}}\right)}{5a}-\frac{1}{5ax^5\sqrt[3]{a+bx^3}}\right)}{8a}-\frac{A}{8ax^8\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{746}$$

$$-\frac{\left(-\frac{6b\left(-\frac{3bx}{2a^2\sqrt[3]{a+bx^3}}-\frac{1}{2ax^2\sqrt[3]{a+bx^3}}\right)}{5a}-\frac{1}{5ax^5\sqrt[3]{a+bx^3}}\right)(9Ab-8aB)}{8a}-\frac{A}{8ax^8\sqrt[3]{a+bx^3}}$$

input
```
Int[(A + B*x^3)/(x^9*(a + b*x^3)^(4/3)),x]
```

output
```
-1/8*A/(a*x^8*(a + b*x^3)^(1/3)) - ((9*A*b - 8*a*B)*(-1/5*1/(a*x^5*(a + b*
x^3)^(1/3)) - (6*b*(-1/2*1/(a*x^2*(a + b*x^3)^(1/3)) - (3*b*x)/(2*a^2*(a +
 b*x^3)^(1/3))))/(5*a)))/(8*a)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$-\frac{\left(\frac{8B\,x^{3}}{5}+A\right)a^{3}-\frac{9b\left(\frac{8B\,x^{3}}{3}+A\right)x^{3}a^{2}}{5}+\frac{27b^{2}\left(-\frac{8B\,x^{3}}{3}+A\right)x^{6}a}{5}+\frac{81A\,x^{9}b^{3}}{5}}{8(b\,x^{3}+a)^{\frac{1}{3}}x^{8}a^{4}}$	74
gosper	$-\frac{81A\,x^{9}b^{3}-72B\,x^{9}a\,b^{2}+27A\,x^{6}a\,b^{2}-24B\,x^{6}a^{2}b-9a^{2}Ab\,x^{3}+8B\,x^{3}a^{3}+5a^{3}A}{40x^{8}(b\,x^{3}+a)^{\frac{1}{3}}a^{4}}$	83
trager	$-\frac{81A\,x^{9}b^{3}-72B\,x^{9}a\,b^{2}+27A\,x^{6}a\,b^{2}-24B\,x^{6}a^{2}b-9a^{2}Ab\,x^{3}+8B\,x^{3}a^{3}+5a^{3}A}{40x^{8}(b\,x^{3}+a)^{\frac{1}{3}}a^{4}}$	83
orering	$-\frac{81A\,x^{9}b^{3}-72B\,x^{9}a\,b^{2}+27A\,x^{6}a\,b^{2}-24B\,x^{6}a^{2}b-9a^{2}Ab\,x^{3}+8B\,x^{3}a^{3}+5a^{3}A}{40x^{8}(b\,x^{3}+a)^{\frac{1}{3}}a^{4}}$	83
risch	$-\frac{(b\,x^{3}+a)^{\frac{2}{3}}(41A\,b^{2}x^{6}-32Bab\,x^{6}-14aAb\,x^{3}+8B\,a^{2}x^{3}+5a^{2}A)}{40a^{4}x^{8}}-\frac{x(Ab-Ba)b^{2}}{(b\,x^{3}+a)^{\frac{1}{3}}a^{4}}$	86

input
```
int((B*x^3+A)/x^9/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-1/8*((8/5*B*x^3+A)*a^3-9/5*b*(8/3*B*x^3+A)*x^3*a^2+27/5*b^2*(-8/3*B*x^3+A
)*x^6*a+81/5*A*x^9*b^3)/(b*x^3+a)^(1/3)/x^8/a^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx^3}{x^9\,(a+bx^3)^{4/3}}\,dx = \frac{(9\,(8\,Bab^2-9\,Ab^3)x^9+3\,(8\,Ba^2b-9\,Aab^2)x^6-5\,Aa^3-(8\,Ba^3-9\,Aa^2b)x^3)(bx^3}{40\,(a^4bx^{11}+a^5x^8)}$$

input
```
integrate((B*x^3+A)/x^9/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
1/40*(9*(8*B*a*b^2 - 9*A*b^3)*x^9 + 3*(8*B*a^2*b - 9*A*a*b^2)*x^6 - 5*A*a^
3 - (8*B*a^3 - 9*A*a^2*b)*x^3)*(b*x^3 + a)^(2/3)/(a^4*b*x^11 + a^5*x^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(107) = 214$.

Time = 41.41 (sec) , antiderivative size = 920, normalized size of antiderivative = 8.07

$$\int \frac{A+Bx^3}{x^9 (a+bx^3)^{4/3}} \, dx = \text{Too large to display}$$

input
```
integrate((B*x**3+A)/x**9/(b*x**3+a)**(4/3),x)
```

output
```
A*(10*a**5*b**(29/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(81*a**7*b**9*x**
6*gamma(4/3) + 243*a**6*b**10*x**9*gamma(4/3) + 243*a**5*b**11*x**12*gamma
(4/3) + 81*a**4*b**12*x**15*gamma(4/3)) + 2*a**4*b**(32/3)*x**3*(a/(b*x**3
) + 1)**(2/3)*gamma(-8/3)/(81*a**7*b**9*x**6*gamma(4/3) + 243*a**6*b**10*x
**9*gamma(4/3) + 243*a**5*b**11*x**12*gamma(4/3) + 81*a**4*b**12*x**15*gam
ma(4/3)) + 28*a**3*b**(35/3)*x**6*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(81*
a**7*b**9*x**6*gamma(4/3) + 243*a**6*b**10*x**9*gamma(4/3) + 243*a**5*b**1
1*x**12*gamma(4/3) + 81*a**4*b**12*x**15*gamma(4/3)) + 252*a**2*b**(38/3)*
x**9*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(81*a**7*b**9*x**6*gamma(4/3) + 2
43*a**6*b**10*x**9*gamma(4/3) + 243*a**5*b**11*x**12*gamma(4/3) + 81*a**4*
b**12*x**15*gamma(4/3)) + 378*a*b**(41/3)*x**12*(a/(b*x**3) + 1)**(2/3)*ga
mma(-8/3)/(81*a**7*b**9*x**6*gamma(4/3) + 243*a**6*b**10*x**9*gamma(4/3) +
 243*a**5*b**11*x**12*gamma(4/3) + 81*a**4*b**12*x**15*gamma(4/3)) + 162*b
**(44/3)*x**15*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(81*a**7*b**9*x**6*gamm
a(4/3) + 243*a**6*b**10*x**9*gamma(4/3) + 243*a**5*b**11*x**12*gamma(4/3)
+ 81*a**4*b**12*x**15*gamma(4/3))) + B*(-2*a**3*b**(14/3)*(a/(b*x**3) + 1)
**(2/3)*gamma(-5/3)/(27*a**5*b**4*x**3*gamma(4/3) + 54*a**4*b**5*x**6*gamm
a(4/3) + 27*a**3*b**6*x**9*gamma(4/3)) + 4*a**2*b**(17/3)*x**3*(a/(b*x**3)
 + 1)**(2/3)*gamma(-5/3)/(27*a**5*b**4*x**3*gamma(4/3) + 54*a**4*b**5*x**6
*gamma(4/3) + 27*a**3*b**6*x**9*gamma(4/3)) + 24*a*b**(20/3)*x**6*(a/(b...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^3}{x^9(a+bx^3)^{4/3}}\,dx = \frac{1}{5}B\left(\frac{5\,b^2x}{(bx^3+a)^{\frac{1}{3}}a^3} + \frac{\frac{5\,(bx^3+a)^{\frac{2}{3}}b}{x^2} - \frac{(bx^3+a)^{\frac{5}{3}}}{x^5}}{a^3}\right) - \frac{1}{40}A\left(\frac{40\,b^3x}{(bx^3+a)^{\frac{1}{3}}a^4} + \frac{\frac{60\,(bx^3+a)^{\frac{2}{3}}b^2}{x^2} - \frac{24\,(bx^3+a)^{\frac{5}{3}}b}{x^5} + \frac{5\,(bx^3+a)^{\frac{8}{3}}}{x^8}}{a^4}\right)$$

input
```
integrate((B*x^3+A)/x^9/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
1/5*B*(5*b^2*x/((b*x^3 + a)^(1/3)*a^3) + (5*(b*x^3 + a)^(2/3)*b/x^2 - (b*x
^3 + a)^(5/3)/x^5)/a^3) - 1/40*A*(40*b^3*x/((b*x^3 + a)^(1/3)*a^4) + (60*(
b*x^3 + a)^(2/3)*b^2/x^2 - 24*(b*x^3 + a)^(5/3)*b/x^5 + 5*(b*x^3 + a)^(8/3
)/x^8)/a^4)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^9(a+bx^3)^{4/3}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{4}{3}}x^9}\,dx$$

input
```
integrate((B*x^3+A)/x^9/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(4/3)*x^9), x)
```

Mupad [B] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^3}{x^9 \left(a+bx^3\right)^{4/3}}\,dx = -\frac{\left(4\,B\,a^2-7\,A\,a\,b\right)\,\left(b\,x^3+a\right)^{2/3}}{20\,a^4\,x^5} - \frac{x\,\left(A\,b^3-B\,a\,b^2\right)}{a^4\,\left(b\,x^3+a\right)^{1/3}} - \frac{A\,\left(b\,x^3+a\right)^{2/3}}{8\,a^2\,x^8} - \frac{b\,\left(b\,x^3+a\right)^{2/3}\,\left(41\,A\,b-32\,B\,a\right)}{40\,a^4\,x^2}$$

input
```
int((A + B*x^3)/(x^9*(a + b*x^3)^(4/3)),x)
```

output
```
- ((4*B*a^2 - 7*A*a*b)*(a + b*x^3)^(2/3))/(20*a^4*x^5) - (x*(A*b^3 - B*a*b
^2))/(a^4*(a + b*x^3)^(1/3)) - (A*(a + b*x^3)^(2/3))/(8*a^2*x^8) - (b*(a +
 b*x^3)^(2/3)*(41*A*b - 32*B*a))/(40*a^4*x^2)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^9 \left(a+bx^3\right)^{4/3}}\,dx = \int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\,x^9}dx$$

input
```
int((B*x^3+A)/x^9/(b*x^3+a)^(4/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**9),x)
```

3.371 $\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{Bx^8}{7b\sqrt[3]{a+bx^3}} + \frac{(7Ab-8aB)x^8\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{8}{3},\frac{11}{3},-\frac{bx^3}{a}\right)}{56ab\sqrt[3]{a+bx^3}}$$

output
```
1/7*B*x^8/b/(b*x^3+a)^(1/3)+1/56*(7*A*b-8*B*a)*x^8*(1+b*x^3/a)^(1/3)*hyper
geom([4/3, 8/3],[11/3],-b*x^3/a)/a/b/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}\left(11A\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{8}{3},\frac{11}{3},-\frac{bx^3}{a}\right)+8Bx^3\,\text{Hypergeometric2F1}\right.}{88a\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^(4/3),x]
```

output
```
(x^8*(1 + (b*x^3)/a)^(1/3)*(11*A*Hypergeometric2F1[4/3, 8/3, 11/3, -((b*x^
3)/a)] + 8*B*x^3*Hypergeometric2F1[4/3, 11/3, 14/3, -((b*x^3)/a)]))/(88*a*
(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {957, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (A+Bx^3)}{(a+bx^3)^{4/3}} dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^8(Ab-aB)}{ab\sqrt[3]{a+bx^3}}-\frac{(7Ab-8aB)\int \frac{x^7}{\sqrt[3]{bx^3+a}}dx}{ab}$$

$$\Big\downarrow \text{889}$$

$$\frac{x^8(Ab-aB)}{ab\sqrt[3]{a+bx^3}}-\frac{\sqrt[3]{\frac{bx^3}{a}+1}(7Ab-8aB)\int \frac{x^7}{\sqrt[3]{\frac{bx^3}{a}+1}}dx}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^8(Ab-aB)}{ab\sqrt[3]{a+bx^3}}-\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}(7Ab-8aB)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{11}{3},-\frac{bx^3}{a}\right)}{8ab\sqrt[3]{a+bx^3}}$$

input
```
Int[(x^7*(A + B*x^3))/(a + b*x^3)^(4/3),x]
```

output
```
((A*b - a*B)*x^8)/(a*b*(a + b*x^3)^(1/3)) - ((7*A*b - 8*a*B)*x^8*(1 + (b*x
^3)/a)^(1/3)*Hypergeometric2F1[1/3, 8/3, 11/3, -((b*x^3)/a)])/(8*a*b*(a +
b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^{7}(B\,x^{3}+A)}{(b\,x^{3}+a)^{\frac{4}{3}}}dx$$

input
```
int(x^7*(B*x^3+A)/(b*x^3+a)^(4/3),x)
```

output
```
int(x^7*(B*x^3+A)/(b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{(Bx^3+A)x^7}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((B*x^10 + A*x^7)*(b*x^3 + a)^(2/3)/(b^2*x^6 + 2*a*b*x^3 + a^2), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{Ax^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{11}{3}\right)} + \frac{Bx^{11}\Gamma\left(\frac{11}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{11}{3}\\ \frac{14}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{14}{3}\right)}$$

input
```
integrate(x**7*(B*x**3+A)/(b*x**3+a)**(4/3),x)
```

output
```
A*x**8*gamma(8/3)*hyper((4/3, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*
a**(4/3)*gamma(11/3)) + B*x**11*gamma(11/3)*hyper((4/3, 11/3), (14/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gamma(14/3))
```

Maxima [F]

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{(Bx^3+A)x^7}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^7/(b*x^3 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{(Bx^3+A)x^7}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^7/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{x^7\,(B\,x^3+A)}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((x^7*(A + B*x^3))/(a + b*x^3)^(4/3),x)
```

output
```
int((x^7*(A + B*x^3))/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx=\int \frac{x^7}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int(x^7*(B*x^3+A)/(b*x^3+a)^(4/3),x)
```

output
```
int(x**7/(a + b*x**3)**(1/3),x)
```

3.372 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{Bx^5}{4b\sqrt[3]{a+bx^3}} + \frac{(4Ab-5aB)x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{20ab\sqrt[3]{a+bx^3}}$$

output
```
1/4*B*x^5/b/(b*x^3+a)^(1/3)+1/20*(4*A*b-5*B*a)*x^5*(1+b*x^3/a)^(1/3)*hyper
geom([4/3, 5/3],[8/3],-b*x^3/a)/a/b/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\left(8Ax^5\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)+5Bx^8\,\text{Hypergeometric2F1}\left(\right.\right.}{40a\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^(4/3),x]
```

output
```
((1 + (b*x^3)/a)^(1/3)*(8*A*x^5*Hypergeometric2F1[4/3, 5/3, 8/3, -((b*x^3)
/a)] + 5*B*x^8*Hypergeometric2F1[4/3, 8/3, 11/3, -((b*x^3)/a)]))/(40*a*(a
+ b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {957, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^5(Ab-aB)}{ab\sqrt[3]{a+bx^3}}-\frac{(4Ab-5aB)\int \frac{x^4}{\sqrt[3]{bx^3+a}}dx}{ab}$$

$$\Big\downarrow \text{889}$$

$$\frac{x^5(Ab-aB)}{ab\sqrt[3]{a+bx^3}}-\frac{\sqrt[3]{\frac{bx^3}{a}+1}(4Ab-5aB)\int \frac{x^4}{\sqrt[3]{\frac{bx^3}{a}+1}}dx}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^5(Ab-aB)}{ab\sqrt[3]{a+bx^3}}-\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}(4Ab-5aB)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{5ab\sqrt[3]{a+bx^3}}$$

input
```
Int[(x^4*(A + B*x^3))/(a + b*x^3)^(4/3),x]
```

output
```
((A*b - a*B)*x^5)/(a*b*(a + b*x^3)^(1/3)) - ((4*A*b - 5*a*B)*x^5*(1 + (b*x
^3)/a)^(1/3)*Hypergeometric2F1[1/3, 5/3, 8/3, -((b*x^3)/a)])/(5*a*b*(a + b
*x^3)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^4(B\,x^3+A)}{(b\,x^3+a)^{\frac{4}{3}}}dx$$

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^(4/3),x)
```

output
```
int(x^4*(B*x^3+A)/(b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((B*x^7 + A*x^4)*(b*x^3 + a)^(2/3)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{Ax^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{8}{3}\right)} + \frac{Bx^8\Gamma\left(\frac{8}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**4*(B*x**3+A)/(b*x**3+a)**(4/3),x)
```

output
```
A*x**5*gamma(5/3)*hyper((4/3, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(4/3)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((4/3, 8/3), (11/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(4/3)*gamma(11/3))
```

Maxima [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx=\int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx=\int \frac{x^4\,(B\,x^3+A)}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((x^4*(A + B*x^3))/(a + b*x^3)^(4/3),x)
```

output
```
int((x^4*(A + B*x^3))/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx=\int \frac{x^4}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^(4/3),x)
```

output
```
int(x**4/(a + b*x**3)**(1/3),x)
```

3.373 $\int \frac{x(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{Bx^2}{b\sqrt[3]{a+bx^3}} + \frac{(Ab-2aB)x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2ab\sqrt[3]{a+bx^3}}$$

output
```
B*x^2/b/(b*x^3+a)^(1/3)+1/2*(A*b-2*B*a)*x^2*(1+b*x^3/a)^(1/3)*hypergeom([2
/3, 4/3],[5/3],-b*x^3/a)/a/b/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\left(5Ax^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},-\frac{bx^3}{a}\right)+2Bx^5\,\text{Hypergeometric2F1}\left(\frac{4}{3}\right.\right.}{10a\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(x*(A + B*x^3))/(a + b*x^3)^(4/3),x]
```

output
```
((1 + (b*x^3)/a)^(1/3)*(5*A*x^2*Hypergeometric2F1[2/3, 4/3, 5/3, -((b*x^3)
/a)] + 2*B*x^5*Hypergeometric2F1[4/3, 5/3, 8/3, -((b*x^3)/a)]))/(10*a*(a +
 b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {957, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{4/3}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^2(Ab-aB)}{ab\sqrt[3]{a+bx^3}}-\frac{(Ab-2aB)\int \frac{x}{\sqrt[3]{bx^3+a}}dx}{ab}$$

$$\Big\downarrow \text{889}$$

$$\frac{x^2(Ab-aB)}{ab\sqrt[3]{a+bx^3}}-\frac{\sqrt[3]{\frac{bx^3}{a}+1}(Ab-2aB)\int \frac{x}{\sqrt[3]{\frac{bx^3}{a}+1}}dx}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2(Ab-aB)}{ab\sqrt[3]{a+bx^3}}-\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}(Ab-2aB)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2ab\sqrt[3]{a+bx^3}}$$

input
```
Int[(x*(A + B*x^3))/(a + b*x^3)^(4/3),x]
```

output
```
((A*b - a*B)*x^2)/(a*b*(a + b*x^3)^(1/3)) - ((A*b - 2*a*B)*x^2*(1 + (b*x^3
)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(2*a*b*(a + b*x
^3)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x(B\,x^{3}+A)}{(b\,x^{3}+a)^{\frac{4}{3}}}dx$$

input
```
int(x*(B*x^3+A)/(b*x^3+a)^(4/3),x)
```

output
```
int(x*(B*x^3+A)/(b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((B*x^4 + A*x)*(b*x^3 + a)^(2/3)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.43 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{Ax^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)} + \frac{Bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x*(B*x**3+A)/(b*x**3+a)**(4/3),x)
```

output
```
A*x**2*gamma(2/3)*hyper((2/3, 4/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(4/3)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((4/3, 5/3), (8/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*a**(4/3)*gamma(8/3))
```

Maxima [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x/(b*x^3 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{x\,(B\,x^3+A)}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((x*(A + B*x^3))/(a + b*x^3)^(4/3),x)
```

output
```
int((x*(A + B*x^3))/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int(x*(B*x^3+A)/(b*x^3+a)^(4/3),x)
```

output
```
int(x/(a + b*x**3)**(1/3),x)
```

3.374 $\int \frac{A+Bx^3}{x^2(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^{4/3}}\,dx = -\frac{A}{ax\sqrt[3]{a+bx^3}} - \frac{(2Ab-aB)x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a^2\sqrt[3]{a+bx^3}}$$

output
```
-A/a/x/(b*x^3+a)^(1/3)-1/2*(2*A*b-B*a)*x^2*(1+b*x^3/a)^(1/3)*hypergeom([2/
3, 4/3],[5/3],-b*x^3/a)/a^2/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^{4/3}}\,dx = \frac{-2aA+(-2Ab+aB)x^3\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a^2x\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^(4/3)),x]
```

output
```
(-2*a*A + (-2*A*b + a*B)*x^3*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[2/3,
4/3, 5/3, -((b*x^3)/a)])/(2*a^2*x*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^{4/3}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(2Ab-aB)\int \frac{x}{(bx^3+a)^{4/3}}dx}{a}-\frac{A}{ax\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{889}$$

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}(2Ab-aB)\int \frac{x}{\left(\frac{bx^3}{a}+1\right)^{4/3}}dx}{a^2\sqrt[3]{a+bx^3}}-\frac{A}{ax\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}(2Ab-aB)\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a^2\sqrt[3]{a+bx^3}}-\frac{A}{ax\sqrt[3]{a+bx^3}}$$

input
```
Int[(A + B*x^3)/(x^2*(a + b*x^3)^(4/3)),x]
```

output
```
-(A/(a*x*(a + b*x^3)^(1/3))) - ((2*A*b - a*B)*x^2*(1 + (b*x^3)/a)^(1/3)*Hy
pergeometric2F1[2/3, 4/3, 5/3, -((b*x^3)/a)])/(2*a^2*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{B\,x^{3}+A}{x^{2}\left(b\,x^{3}+a\right)^{\frac{4}{3}}}dx$$

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^(4/3),x)
```

output
```
int((B*x^3+A)/x^2/(b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{4/3}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{4}{3}}x^2}\,dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(2/3)/(b^2*x^8 + 2*a*b*x^5 + a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{4/3}}\,dx=\frac{A\Gamma\left(-\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}x\Gamma\left(\frac{2}{3}\right)}+\frac{Bx^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**2/(b*x**3+a)**(4/3),x)
```

output
```
A*gamma(-1/3)*hyper((-1/3, 4/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
4/3)*x*gamma(2/3)) + B*x**2*gamma(2/3)*hyper((2/3, 4/3), (5/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(4/3)*gamma(5/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{4/3}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{4}{3}}x^2}\,dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(4/3)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{4/3}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{4}{3}}x^2}\,dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(4/3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{4/3}}\,dx = \int \frac{B\,x^3+A}{x^2\,\left(b\,x^3+a\right)^{4/3}}\,dx$$

input
```
int((A + B*x^3)/(x^2*(a + b*x^3)^(4/3)),x)
```

output
```
int((A + B*x^3)/(x^2*(a + b*x^3)^(4/3)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{4/3}}\,dx = \int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\,x^2}dx$$

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^(4/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**2),x)
```

3.375 $\int \frac{A+Bx^3}{x^5(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{A+Bx^3}{x^5(a+bx^3)^{4/3}}\,dx = -\frac{A}{4ax^4\sqrt[3]{a+bx^3}} + \frac{(5Ab-4aB)\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{4a^2x\sqrt[3]{a+bx^3}}$$

output
```
-1/4*A/a/x^4/(b*x^3+a)^(1/3)+1/4*(5*A*b-4*B*a)*(1+b*x^3/a)^(1/3)*hypergeom
([-1/3, 4/3],[2/3],-b*x^3/a)/a^2/x/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^3}{x^5(a+bx^3)^{4/3}}\,dx = \frac{-aA+(5Ab-4aB)x^3\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{4a^2x^4\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^(4/3)),x]
```

output
```
(-(a*A) + (5*A*b - 4*a*B)*x^3*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[-1/3
, 4/3, 2/3, -((b*x^3)/a)])/(4*a^2*x^4*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^5 (a+bx^3)^{4/3}} \, dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(5Ab-4aB)\int \frac{1}{x^2(bx^3+a)^{4/3}}dx}{4a}-\frac{A}{4ax^4\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{ 889}$$

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}(5Ab-4aB)\int \frac{1}{x^2\left(\frac{bx^3}{a}+1\right)^{4/3}}dx}{4a^2\sqrt[3]{a+bx^3}}-\frac{A}{4ax^4\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}(5Ab-4aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{4a^2x\sqrt[3]{a+bx^3}}-\frac{A}{4ax^4\sqrt[3]{a+bx^3}}$$

input
```
Int[(A + B*x^3)/(x^5*(a + b*x^3)^(4/3)),x]
```

output
```
-1/4*A/(a*x^4*(a + b*x^3)^(1/3)) + ((5*A*b - 4*a*B)*(1 + (b*x^3)/a)^(1/3)*
Hypergeometric2F1[-1/3, 4/3, 2/3, -((b*x^3)/a)])/(4*a^2*x*(a + b*x^3)^(1/3
))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{B\,x^{3}+A}{x^{5}\left(b\,x^{3}+a\right)^{\frac{4}{3}}}d x$$

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^(4/3),x)
```

output
```
int((B*x^3+A)/x^5/(b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{4/3}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{4}{3}}x^5}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(2/3)/(b^2*x^11 + 2*a*b*x^8 + a^2*x^5), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{4/3}}\,dx=\frac{A\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{4}{3}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{B\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**5/(b*x**3+a)**(4/3),x)
```

output
```
A*gamma(-4/3)*hyper((-4/3, 4/3), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(4/3)*x**4*gamma(-1/3)) + B*gamma(-1/3)*hyper((-1/3, 4/3), (2/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(4/3)*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{4/3}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{4}{3}}x^5}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(4/3)*x^5), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{4/3}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{4}{3}}x^5}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(4/3)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{4/3}}\,dx=\int \frac{B\,x^3+A}{x^5\,\left(b\,x^3+a\right)^{4/3}}\,dx$$

input
```
int((A + B*x^3)/(x^5*(a + b*x^3)^(4/3)),x)
```

output
```
int((A + B*x^3)/(x^5*(a + b*x^3)^(4/3)), x)
```

Reduce [F]

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{4/3}}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}x^5}dx$$

input

```
int((B*x^3+A)/x^5/(b*x^3+a)^(4/3),x)
```

output

```
int(1/((a + b*x**3)**(1/3)*x**5),x)
```

3.376 $\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{4/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{4/3}}\,dx = -\frac{A}{7ax^7\sqrt[3]{a+bx^3}} + \frac{(8Ab-7aB)\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{4}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{28a^2x^4\sqrt[3]{a+bx^3}}$$

output
```
-1/7*A/a/x^7/(b*x^3+a)^(1/3)+1/28*(8*A*b-7*B*a)*(1+b*x^3/a)^(1/3)*hypergeo
m([-4/3, 4/3],[-1/3],-b*x^3/a)/a^2/x^4/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{4/3}}\,dx = \frac{-4aA+(8Ab-7aB)x^3\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{4}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{28a^2x^7\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^8*(a + b*x^3)^(4/3)),x]
```

output
```
(-4*a*A + (8*A*b - 7*a*B)*x^3*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[-4/3
, 4/3, -1/3, -((b*x^3)/a)])/(28*a^2*x^7*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^8 (a+bx^3)^{4/3}} \, dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(8Ab-7aB)\int \frac{1}{x^5 (bx^3+a)^{4/3}}dx}{7a}-\frac{A}{7ax^7\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{ 889}$$

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}(8Ab-7aB)\int \frac{1}{x^5 \left(\frac{bx^3}{a}+1\right)^{4/3}}dx}{7a^2\sqrt[3]{a+bx^3}}-\frac{A}{7ax^7\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}(8Ab-7aB)\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{4}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{28a^2x^4\sqrt[3]{a+bx^3}}-\frac{A}{7ax^7\sqrt[3]{a+bx^3}}$$

input
```
Int[(A + B*x^3)/(x^8*(a + b*x^3)^(4/3)),x]
```

output
```
-1/7*A/(a*x^7*(a + b*x^3)^(1/3)) + ((8*A*b - 7*a*B)*(1 + (b*x^3)/a)^(1/3)*
Hypergeometric2F1[-4/3, 4/3, -1/3, -((b*x^3)/a)])/(28*a^2*x^4*(a + b*x^3)^
(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{B\,x^{3}+A}{x^{8} \left(b\,x^{3}+a \right)^{\frac{4}{3}}}d x$$

input
```
int((B*x^3+A)/x^8/(b*x^3+a)^(4/3),x)
```

output
```
int((B*x^3+A)/x^8/(b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{A+Bx^3}{x^8 (a+bx^3)^{4/3}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{4}{3}}x^8}\,dx$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(2/3)/(b^2*x^14 + 2*a*b*x^11 + a^2*x^8),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 39.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^3}{x^8 (a+bx^3)^{4/3}}\,dx = \frac{A\Gamma\left(-\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{3}, \frac{4}{3} \\ -\frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{B\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{4}{3} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**8/(b*x**3+a)**(4/3),x)
```

output
```
A*gamma(-7/3)*hyper((-7/3, 4/3), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(4/3)*x**7*gamma(-4/3)) + B*gamma(-4/3)*hyper((-4/3, 4/3), (-1/3,), b*x**3
*exp_polar(I*pi)/a)/(3*a**(4/3)*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{A + Bx^3}{x^8 (a + bx^3)^{4/3}} \, dx = \int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{4}{3}} x^8} \, dx$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(4/3)*x^8), x)
```

Giac [F]

$$\int \frac{A + Bx^3}{x^8 (a + bx^3)^{4/3}} \, dx = \int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{4}{3}} x^8} \, dx$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(4/3)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^3}{x^8 (a + bx^3)^{4/3}} \, dx = \int \frac{B\,x^3 + A}{x^8\,(b\,x^3 + a)^{4/3}} \, dx$$

input
```
int((A + B*x^3)/(x^8*(a + b*x^3)^(4/3)),x)
```

output
```
int((A + B*x^3)/(x^8*(a + b*x^3)^(4/3)), x)
```

Reduce [F]

$$\int \frac{A + Bx^3}{x^8 (a + bx^3)^{4/3}} \, dx = \int \frac{1}{(b\,x^3 + a)^{\frac{1}{3}}\, x^8} dx$$

input
```
int((B*x^3+A)/x^8/(b*x^3+a)^(4/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**8),x)
```

3.377 $\int x^m(a+bx^3)^5(A+Bx^3)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 148

$$\int x^m(a+bx^3)^5(A+Bx^3)\,dx=\frac{a^5Ax^{1+m}}{1+m}+\frac{a^4(5Ab+aB)x^{4+m}}{4+m}+\frac{5a^3b(2Ab+aB)x^{7+m}}{7+m}+\frac{10a^2b^2(Ab+aB)x^{10+m}}{10+m}+\frac{5ab^3(Ab+2aB)x^{13+m}}{13+m}+\frac{b^4(Ab+5aB)x^{16+m}}{16+m}+\frac{b^5Bx^{19+m}}{19+m}$$

output
```
a^5*A*x^(1+m)/(1+m)+a^4*(5*A*b+B*a)*x^(4+m)/(4+m)+5*a^3*b*(2*A*b+B*a)*x^(7
+m)/(7+m)+10*a^2*b^2*(A*b+B*a)*x^(10+m)/(10+m)+5*a*b^3*(A*b+2*B*a)*x^(13+m
)/(13+m)+b^4*(A*b+5*B*a)*x^(16+m)/(16+m)+b^5*B*x^(19+m)/(19+m)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

$$\int x^m(a+bx^3)^5(A+Bx^3)\,dx=x^{1+m}\left(\frac{a^5A}{1+m}+\frac{a^4(5Ab+aB)x^3}{4+m}+\frac{5a^3b(2Ab+aB)x^6}{7+m}+\frac{10a^2b^2(Ab+aB)x^9}{10+m}+\frac{5ab^3(Ab+2aB)x^{12}}{13+m}+\frac{b^4(Ab+5aB)x^{15}}{16+m}+\frac{b^5Bx^{18}}{19+m}\right)$$

input
```
Integrate[x^m*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
x^(1 + m)*((a^5*A)/(1 + m) + (a^4*(5*A*b + a*B)*x^3)/(4 + m) + (5*a^3*b*(2
*A*b + a*B)*x^6)/(7 + m) + (10*a^2*b^2*(A*b + a*B)*x^9)/(10 + m) + (5*a*b^
3*(A*b + 2*a*B)*x^12)/(13 + m) + (b^4*(A*b + 5*a*B)*x^15)/(16 + m) + (b^5*
B*x^18)/(19 + m))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \; 950$$

$$\int \left(a^5Ax^m+a^4x^{m+3}(aB+5Ab)+5a^3bx^{m+6}(aB+2Ab)+10a^2b^2x^{m+9}(aB+Ab)+b^4x^{m+15}(5aB+Ab)+5al\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{a^5Ax^{m+1}}{m+1}+\frac{a^4x^{m+4}(aB+5Ab)}{m+4}+\frac{5a^3bx^{m+7}(aB+2Ab)}{m+7}+\frac{10a^2b^2x^{m+10}(aB+Ab)}{m+10}+\frac{b^4x^{m+16}(5aB+Ab)}{m+16}+\frac{5ab^3x^{m+13}(2aB+Ab)}{m+13}+\frac{b^5Bx^{m+19}}{m+19}$$

input
```
Int[x^m*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^(1 + m))/(1 + m) + (a^4*(5*A*b + a*B)*x^(4 + m))/(4 + m) + (5*a^3
*b*(2*A*b + a*B)*x^(7 + m))/(7 + m) + (10*a^2*b^2*(A*b + a*B)*x^(10 + m))/
(10 + m) + (5*a*b^3*(A*b + 2*a*B)*x^(13 + m))/(13 + m) + (b^4*(A*b + 5*a*B
)*x^(16 + m))/(16 + m) + (b^5*B*x^(19 + m))/(19 + m)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(148) = 296$.

Time = 1.49 (sec) , antiderivative size = 1077, normalized size of antiderivative = 7.28

method	result	size
risch	Expression too large to display	1077
orering	Expression too large to display	1077
gosper	Expression too large to display	1078
parallelrisch	Expression too large to display	1332

input
```
int(x^m*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output

```
x*(B*b^5*m^6*x^18+51*B*b^5*m^5*x^18+1005*B*b^5*m^4*x^18+A*b^5*m^6*x^15+5*B
*a*b^4*m^6*x^15+9605*B*b^5*m^3*x^18+54*A*b^5*m^5*x^15+270*B*a*b^4*m^5*x^15
+45474*B*b^5*m^2*x^18+1110*A*b^5*m^4*x^15+5550*B*a*b^4*m^4*x^15+95064*B*b^
5*m*x^18+5*A*a*b^4*m^6*x^12+10940*A*b^5*m^3*x^15+10*B*a^2*b^3*m^6*x^12+547
00*B*a*b^4*m^3*x^15+58240*B*b^5*x^18+285*A*a*b^4*m^5*x^12+52929*A*b^5*m^2*
x^15+570*B*a^2*b^3*m^5*x^12+264645*B*a*b^4*m^2*x^15+6165*A*a*b^4*m^4*x^12+
112206*A*b^5*m*x^15+12330*B*a^2*b^3*m^4*x^12+561030*B*a*b^4*m*x^15+10*A*a^
2*b^3*m^6*x^9+63355*A*a*b^4*m^3*x^12+69160*A*b^5*x^15+10*B*a^3*b^2*m^6*x^9
+126710*B*a^2*b^3*m^3*x^12+345800*B*a*b^4*x^15+600*A*a^2*b^3*m^5*x^9+31623
0*A*a*b^4*m^2*x^12+600*B*a^3*b^2*m^5*x^9+632460*B*a^2*b^3*m^2*x^12+13740*A
*a^2*b^3*m^4*x^9+684360*A*a*b^4*m*x^12+13740*B*a^3*b^2*m^4*x^9+1368720*B*a
^2*b^3*m*x^12+10*A*a^3*b^2*m^6*x^6+149600*A*a^2*b^3*m^3*x^9+425600*A*a*b^4
*x^12+5*B*a^4*b*m^6*x^6+149600*B*a^3*b^2*m^3*x^9+851200*B*a^2*b^3*x^12+630
*A*a^3*b^2*m^5*x^6+783690*A*a^2*b^3*m^2*x^9+315*B*a^4*b*m^5*x^6+783690*B*a
^3*b^2*m^2*x^9+15330*A*a^3*b^2*m^4*x^6+1753800*A*a^2*b^3*m*x^9+7665*B*a^4*
b*m^4*x^6+1753800*B*a^3*b^2*m*x^9+5*A*a^4*b*m^6*x^3+179690*A*a^3*b^2*m^3*x
^6+1106560*A*a^2*b^3*x^9+B*a^5*m^6*x^3+89845*B*a^4*b*m^3*x^6+1106560*B*a^3
*b^2*x^9+330*A*a^4*b*m^5*x^3+1021860*A*a^3*b^2*m^2*x^6+66*B*a^5*m^5*x^3+51
0930*B*a^4*b*m^2*x^6+8550*A*a^4*b*m^4*x^3+2437680*A*a^3*b^2*m*x^6+1710*B*a
^5*m^4*x^3+1218840*B*a^4*b*m*x^6+A*a^5*m^6+109300*A*a^4*b*m^3*x^3+15808...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(148) = 296$.

Time = 0.12 (sec) , antiderivative size = 851, normalized size of antiderivative = 5.75

$$\int x^m (a + bx^3)^5 (A + Bx^3)\, dx = \text{Too large to display}$$

input

```
integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
((B*b^5*m^6 + 51*B*b^5*m^5 + 1005*B*b^5*m^4 + 9605*B*b^5*m^3 + 45474*B*b^5
*m^2 + 95064*B*b^5*m + 58240*B*b^5)*x^19 + ((5*B*a*b^4 + A*b^5)*m^6 + 3458
00*B*a*b^4 + 69160*A*b^5 + 54*(5*B*a*b^4 + A*b^5)*m^5 + 1110*(5*B*a*b^4 +
A*b^5)*m^4 + 10940*(5*B*a*b^4 + A*b^5)*m^3 + 52929*(5*B*a*b^4 + A*b^5)*m^2
 + 112206*(5*B*a*b^4 + A*b^5)*m)*x^16 + 5*((2*B*a^2*b^3 + A*a*b^4)*m^6 + 1
70240*B*a^2*b^3 + 85120*A*a*b^4 + 57*(2*B*a^2*b^3 + A*a*b^4)*m^5 + 1233*(2
*B*a^2*b^3 + A*a*b^4)*m^4 + 12671*(2*B*a^2*b^3 + A*a*b^4)*m^3 + 63246*(2*B
*a^2*b^3 + A*a*b^4)*m^2 + 136872*(2*B*a^2*b^3 + A*a*b^4)*m)*x^13 + 10*((B*
a^3*b^2 + A*a^2*b^3)*m^6 + 110656*B*a^3*b^2 + 110656*A*a^2*b^3 + 60*(B*a^3
*b^2 + A*a^2*b^3)*m^5 + 1374*(B*a^3*b^2 + A*a^2*b^3)*m^4 + 14960*(B*a^3*b^
2 + A*a^2*b^3)*m^3 + 78369*(B*a^3*b^2 + A*a^2*b^3)*m^2 + 175380*(B*a^3*b^2
 + A*a^2*b^3)*m)*x^10 + 5*((B*a^4*b + 2*A*a^3*b^2)*m^6 + 158080*B*a^4*b +
316160*A*a^3*b^2 + 63*(B*a^4*b + 2*A*a^3*b^2)*m^5 + 1533*(B*a^4*b + 2*A*a^
3*b^2)*m^4 + 17969*(B*a^4*b + 2*A*a^3*b^2)*m^3 + 102186*(B*a^4*b + 2*A*a^3
*b^2)*m^2 + 243768*(B*a^4*b + 2*A*a^3*b^2)*m)*x^7 + ((B*a^5 + 5*A*a^4*b)*m
^6 + 276640*B*a^5 + 1383200*A*a^4*b + 66*(B*a^5 + 5*A*a^4*b)*m^5 + 1710*(B
*a^5 + 5*A*a^4*b)*m^4 + 21860*(B*a^5 + 5*A*a^4*b)*m^3 + 140529*(B*a^5 + 5*
A*a^4*b)*m^2 + 396954*(B*a^5 + 5*A*a^4*b)*m)*x^4 + (A*a^5*m^6 + 69*A*a^5*m
^5 + 1905*A*a^5*m^4 + 26795*A*a^5*m^3 + 201174*A*a^5*m^2 + 757896*A*a^5*m
+ 1106560*A*a^5)*x)*x^m/(m^7 + 70*m^6 + 1974*m^5 + 28700*m^4 + 227969*m...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5418 vs. $2(138) = 276$.

Time = 1.73 (sec) , antiderivative size = 5418, normalized size of antiderivative = 36.61

$$\int x^m (a + bx^3)^5 (A + Bx^3) \, dx = \text{Too large to display}$$

input
```
integrate(x**m*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
Piecewise((-A*a**5/(18*x**18) - A*a**4*b/(3*x**15) - 5*A*a**3*b**2/(6*x**1
2) - 10*A*a**2*b**3/(9*x**9) - 5*A*a*b**4/(6*x**6) - A*b**5/(3*x**3) - B*a
**5/(15*x**15) - 5*B*a**4*b/(12*x**12) - 10*B*a**3*b**2/(9*x**9) - 5*B*a**
2*b**3/(3*x**6) - 5*B*a*b**4/(3*x**3) + B*b**5*log(x), Eq(m, -19)), (-A*a*
*5/(15*x**15) - 5*A*a**4*b/(12*x**12) - 10*A*a**3*b**2/(9*x**9) - 5*A*a**2
*b**3/(3*x**6) - 5*A*a*b**4/(3*x**3) + A*b**5*log(x) - B*a**5/(12*x**12) -
 5*B*a**4*b/(9*x**9) - 5*B*a**3*b**2/(3*x**6) - 10*B*a**2*b**3/(3*x**3) +
5*B*a*b**4*log(x) + B*b**5*x**3/3, Eq(m, -16)), (-A*a**5/(12*x**12) - 5*A*
a**4*b/(9*x**9) - 5*A*a**3*b**2/(3*x**6) - 10*A*a**2*b**3/(3*x**3) + 5*A*a
*b**4*log(x) + A*b**5*x**3/3 - B*a**5/(9*x**9) - 5*B*a**4*b/(6*x**6) - 10*
B*a**3*b**2/(3*x**3) + 10*B*a**2*b**3*log(x) + 5*B*a*b**4*x**3/3 + B*b**5*
x**6/6, Eq(m, -13)), (-A*a**5/(9*x**9) - 5*A*a**4*b/(6*x**6) - 10*A*a**3*b
**2/(3*x**3) + 10*A*a**2*b**3*log(x) + 5*A*a*b**4*x**3/3 + A*b**5*x**6/6 -
 B*a**5/(6*x**6) - 5*B*a**4*b/(3*x**3) + 10*B*a**3*b**2*log(x) + 10*B*a**2
*b**3*x**3/3 + 5*B*a*b**4*x**6/6 + B*b**5*x**9/9, Eq(m, -10)), (-A*a**5/(6
*x**6) - 5*A*a**4*b/(3*x**3) + 10*A*a**3*b**2*log(x) + 10*A*a**2*b**3*x**3
/3 + 5*A*a*b**4*x**6/6 + A*b**5*x**9/9 - B*a**5/(3*x**3) + 5*B*a**4*b*log(
x) + 10*B*a**3*b**2*x**3/3 + 5*B*a**2*b**3*x**6/3 + 5*B*a*b**4*x**9/9 + B*
b**5*x**12/12, Eq(m, -7)), (-A*a**5/(3*x**3) + 5*A*a**4*b*log(x) + 10*A*a*
*3*b**2*x**3/3 + 5*A*a**2*b**3*x**6/3 + 5*A*a*b**4*x**9/9 + A*b**5*x**1...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.39

$$\int x^m (a+bx^3)^5 (A+Bx^3)\ dx = \frac{Bb^5x^{m+19}}{m+19} + \frac{5\,Bab^4x^{m+16}}{m+16} + \frac{Ab^5x^{m+16}}{m+16} + \frac{10\,Ba^2b^3x^{m+13}}{m+13} + \frac{5\,Aab^4x^{m+13}}{m+13} + \frac{10\,Ba^3b^2x^{m+10}}{m+10} + \frac{10\,Aa^2b^3x^{m+10}}{m+10} + \frac{5\,Ba^4bx^{m+7}}{m+7} + \frac{10\,Aa^3b^2x^{m+7}}{m+7} + \frac{Ba^5x^{m+4}}{m+4} + \frac{5\,Aa^4bx^{m+4}}{m+4} + \frac{Aa^5x^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
B*b^5*x^(m + 19)/(m + 19) + 5*B*a*b^4*x^(m + 16)/(m + 16) + A*b^5*x^(m + 1
6)/(m + 16) + 10*B*a^2*b^3*x^(m + 13)/(m + 13) + 5*A*a*b^4*x^(m + 13)/(m +
 13) + 10*B*a^3*b^2*x^(m + 10)/(m + 10) + 10*A*a^2*b^3*x^(m + 10)/(m + 10)
 + 5*B*a^4*b*x^(m + 7)/(m + 7) + 10*A*a^3*b^2*x^(m + 7)/(m + 7) + B*a^5*x^
(m + 4)/(m + 4) + 5*A*a^4*b*x^(m + 4)/(m + 4) + A*a^5*x^(m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1331 vs. 2(148) = 296.

Time = 0.17 (sec) , antiderivative size = 1331, normalized size of antiderivative = 8.99

$$\int x^m (a + bx^3)^5 (A + Bx^3) \, dx = \text{Too large to display}$$

input
```
integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
(B*b^5*m^6*x^19*x^m + 51*B*b^5*m^5*x^19*x^m + 1005*B*b^5*m^4*x^19*x^m + 5*
B*a*b^4*m^6*x^16*x^m + A*b^5*m^6*x^16*x^m + 9605*B*b^5*m^3*x^19*x^m + 270*
B*a*b^4*m^5*x^16*x^m + 54*A*b^5*m^5*x^16*x^m + 45474*B*b^5*m^2*x^19*x^m +
5550*B*a*b^4*m^4*x^16*x^m + 1110*A*b^5*m^4*x^16*x^m + 95064*B*b^5*m*x^19*x
^m + 10*B*a^2*b^3*m^6*x^13*x^m + 5*A*a*b^4*m^6*x^13*x^m + 54700*B*a*b^4*m^
3*x^16*x^m + 10940*A*b^5*m^3*x^16*x^m + 58240*B*b^5*x^19*x^m + 570*B*a^2*b
^3*m^5*x^13*x^m + 285*A*a*b^4*m^5*x^13*x^m + 264645*B*a*b^4*m^2*x^16*x^m +
 52929*A*b^5*m^2*x^16*x^m + 12330*B*a^2*b^3*m^4*x^13*x^m + 6165*A*a*b^4*m^
4*x^13*x^m + 561030*B*a*b^4*m*x^16*x^m + 112206*A*b^5*m*x^16*x^m + 10*B*a^
3*b^2*m^6*x^10*x^m + 10*A*a^2*b^3*m^6*x^10*x^m + 126710*B*a^2*b^3*m^3*x^13
*x^m + 63355*A*a*b^4*m^3*x^13*x^m + 345800*B*a*b^4*x^16*x^m + 69160*A*b^5*
x^16*x^m + 600*B*a^3*b^2*m^5*x^10*x^m + 600*A*a^2*b^3*m^5*x^10*x^m + 63246
0*B*a^2*b^3*m^2*x^13*x^m + 316230*A*a*b^4*m^2*x^13*x^m + 13740*B*a^3*b^2*m
^4*x^10*x^m + 13740*A*a^2*b^3*m^4*x^10*x^m + 1368720*B*a^2*b^3*m*x^13*x^m
+ 684360*A*a*b^4*m*x^13*x^m + 5*B*a^4*b*m^6*x^7*x^m + 10*A*a^3*b^2*m^6*x^7
*x^m + 149600*B*a^3*b^2*m^3*x^10*x^m + 149600*A*a^2*b^3*m^3*x^10*x^m + 851
200*B*a^2*b^3*x^13*x^m + 425600*A*a*b^4*x^13*x^m + 315*B*a^4*b*m^5*x^7*x^m
 + 630*A*a^3*b^2*m^5*x^7*x^m + 783690*B*a^3*b^2*m^2*x^10*x^m + 783690*A*a^
2*b^3*m^2*x^10*x^m + 7665*B*a^4*b*m^4*x^7*x^m + 15330*A*a^3*b^2*m^4*x^7*x^
m + 1753800*B*a^3*b^2*m*x^10*x^m + 1753800*A*a^2*b^3*m*x^10*x^m + B*a^5...
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.78

$$\int x^m (a+bx^3)^5 (A+Bx^3)\,dx$$
$$= \frac{B\,b^5\,x^m\,x^{19}\,(m^6+51\,m^5+1005\,m^4+9605\,m^3+45474\,m^2+95064\,m+58240)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}$$
$$+\frac{a^4\,x^m\,x^4\,(5\,A\,b+B\,a)\,(m^6+66\,m^5+1710\,m^4+21860\,m^3+140529\,m^2+396954\,m+276640)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}$$
$$+\frac{b^4\,x^m\,x^{16}\,(A\,b+5\,B\,a)\,(m^6+54\,m^5+1110\,m^4+10940\,m^3+52929\,m^2+112206\,m+69160)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}$$
$$+\frac{A\,a^5\,x\,x^m\,(m^6+69\,m^5+1905\,m^4+26795\,m^3+201174\,m^2+757896\,m+1106560)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}$$
$$+\frac{10\,a^2\,b^2\,x^m\,x^{10}\,(A\,b+B\,a)\,(m^6+60\,m^5+1374\,m^4+14960\,m^3+78369\,m^2+175380\,m+110656)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}$$
$$+\frac{5\,a\,b^3\,x^m\,x^{13}\,(A\,b+2\,B\,a)\,(m^6+57\,m^5+1233\,m^4+12671\,m^3+63246\,m^2+136872\,m+85120)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}$$
$$+\frac{5\,a^3\,b\,x^m\,x^7\,(2\,A\,b+B\,a)\,(m^6+63\,m^5+1533\,m^4+17969\,m^3+102186\,m^2+243768\,m+158080)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}$$

input
```
int(x^m*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
(B*b^5*x^m*x^19*(95064*m + 45474*m^2 + 9605*m^3 + 1005*m^4 + 51*m^5 + m^6
+ 58240))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70
*m^6 + m^7 + 1106560) + (a^4*x^m*x^4*(5*A*b + B*a)*(396954*m + 140529*m^2
+ 21860*m^3 + 1710*m^4 + 66*m^5 + m^6 + 276640))/(1864456*m + 959070*m^2 +
 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (b^4*x^m*x^
16*(A*b + 5*B*a)*(112206*m + 52929*m^2 + 10940*m^3 + 1110*m^4 + 54*m^5 + m
^6 + 69160))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 +
 70*m^6 + m^7 + 1106560) + (A*a^5*x*x^m*(757896*m + 201174*m^2 + 26795*m^3
 + 1905*m^4 + 69*m^5 + m^6 + 1106560))/(1864456*m + 959070*m^2 + 227969*m^
3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (10*a^2*b^2*x^m*x^10*
(A*b + B*a)*(175380*m + 78369*m^2 + 14960*m^3 + 1374*m^4 + 60*m^5 + m^6 +
110656))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*
m^6 + m^7 + 1106560) + (5*a*b^3*x^m*x^13*(A*b + 2*B*a)*(136872*m + 63246*m
^2 + 12671*m^3 + 1233*m^4 + 57*m^5 + m^6 + 85120))/(1864456*m + 959070*m^2
 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (5*a^3*b*
x^m*x^7*(2*A*b + B*a)*(243768*m + 102186*m^2 + 17969*m^3 + 1533*m^4 + 63*m
^5 + m^6 + 158080))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 197
4*m^5 + 70*m^6 + m^7 + 1106560)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 599, normalized size of antiderivative = 4.05

$$\int x^m\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx$$
$$= \frac{x^m x(b^6m^6x^{18}+51b^6m^5x^{18}+1005b^6m^4x^{18}+6a\,b^5m^6x^{15}+9605b^6m^3x^{18}+324a\,b^5m^5x^{15}+45474b^6m^2x}{}$$

input
```
int(x^m*(b*x^3+a)^5*(B*x^3+A),x)
```

output
```
(x**m*x*(a**6*m**6 + 69*a**6*m**5 + 1905*a**6*m**4 + 26795*a**6*m**3 + 201
174*a**6*m**2 + 757896*a**6*m + 1106560*a**6 + 6*a**5*b*m**6*x**3 + 396*a*
*5*b*m**5*x**3 + 10260*a**5*b*m**4*x**3 + 131160*a**5*b*m**3*x**3 + 843174
*a**5*b*m**2*x**3 + 2381724*a**5*b*m*x**3 + 1659840*a**5*b*x**3 + 15*a**4*
b**2*m**6*x**6 + 945*a**4*b**2*m**5*x**6 + 22995*a**4*b**2*m**4*x**6 + 269
535*a**4*b**2*m**3*x**6 + 1532790*a**4*b**2*m**2*x**6 + 3656520*a**4*b**2*
m*x**6 + 2371200*a**4*b**2*x**6 + 20*a**3*b**3*m**6*x**9 + 1200*a**3*b**3*
m**5*x**9 + 27480*a**3*b**3*m**4*x**9 + 299200*a**3*b**3*m**3*x**9 + 15673
80*a**3*b**3*m**2*x**9 + 3507600*a**3*b**3*m*x**9 + 2213120*a**3*b**3*x**9
 + 15*a**2*b**4*m**6*x**12 + 855*a**2*b**4*m**5*x**12 + 18495*a**2*b**4*m*
*4*x**12 + 190065*a**2*b**4*m**3*x**12 + 948690*a**2*b**4*m**2*x**12 + 205
3080*a**2*b**4*m*x**12 + 1276800*a**2*b**4*x**12 + 6*a*b**5*m**6*x**15 + 3
24*a*b**5*m**5*x**15 + 6660*a*b**5*m**4*x**15 + 65640*a*b**5*m**3*x**15 +
317574*a*b**5*m**2*x**15 + 673236*a*b**5*m*x**15 + 414960*a*b**5*x**15 + b
**6*m**6*x**18 + 51*b**6*m**5*x**18 + 1005*b**6*m**4*x**18 + 9605*b**6*m**
3*x**18 + 45474*b**6*m**2*x**18 + 95064*b**6*m*x**18 + 58240*b**6*x**18))/
(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 186
4456*m + 1106560)
```

3.378 $\int x^m(a+bx^3)^2(A+Bx^3)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 71

$$\int x^m(a+bx^3)^2(A+Bx^3)\,dx=\frac{a^2Ax^{1+m}}{1+m}+\frac{a(2Ab+aB)x^{4+m}}{4+m}+\frac{b(Ab+2aB)x^{7+m}}{7+m}+\frac{b^2Bx^{10+m}}{10+m}$$

output
```
a^2*A*x^(1+m)/(1+m)+a*(2*A*b+B*a)*x^(4+m)/(4+m)+b*(A*b+2*B*a)*x^(7+m)/(7+m
)+b^2*B*x^(10+m)/(10+m)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int x^m(a+bx^3)^2(A+Bx^3)\,dx=x^{1+m}\left(\frac{a^2A}{1+m}+\frac{a(2Ab+aB)x^3}{4+m}+\frac{b(Ab+2aB)x^6}{7+m}+\frac{b^2Bx^9}{10+m}\right)$$

input
```
Integrate[x^m*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
x^(1 + m)*((a^2*A)/(1 + m) + (a*(2*A*b + a*B)*x^3)/(4 + m) + (b*(A*b + 2*a
*B)*x^6)/(7 + m) + (b^2*B*x^9)/(10 + m))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (a+bx^3)^2 (A+Bx^3)\, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^2Ax^m+ax^{m+3}(aB+2Ab)+bx^{m+6}(2aB+Ab)+b^2Bx^{m+9}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2Ax^{m+1}}{m+1}+\frac{ax^{m+4}(aB+2Ab)}{m+4}+\frac{bx^{m+7}(2aB+Ab)}{m+7}+\frac{b^2Bx^{m+10}}{m+10}$$

input
```
Int[x^m*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(a^2*A*x^(1 + m))/(1 + m) + (a*(2*A*b + a*B)*x^(4 + m))/(4 + m) + (b*(A*b
+ 2*a*B)*x^(7 + m))/(7 + m) + (b^2*B*x^(10 + m))/(10 + m)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(71) = 142$.

Time = 0.69 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.68

method	result
risch	$x(B\,b^2m^3x^9+12B\,b^2m^2x^9+39m\,x^9B\,b^2+A\,b^2m^3x^6+2Bab\,m^3x^6+28b^2B\,x^9+15A\,b^2m^2x^6+30Bab\,m^2x^6+54A\,b^2x^6m+108$
orering	$x(B\,b^2m^3x^9+12B\,b^2m^2x^9+39m\,x^9B\,b^2+A\,b^2m^3x^6+2Bab\,m^3x^6+28b^2B\,x^9+15A\,b^2m^2x^6+30Bab\,m^2x^6+54A\,b^2x^6m+108$
gosper	$x^{1+m}(B\,b^2m^3x^9+12B\,b^2m^2x^9+39m\,x^9B\,b^2+A\,b^2m^3x^6+2Bab\,m^3x^6+28b^2B\,x^9+15A\,b^2m^2x^6+30Bab\,m^2x^6+54A\,b^2x^6m$
parallelrisch	$80B\,x^7x^mab+B\,x^4x^ma^2m^3+28B\,x^{10}x^mb^2+40A\,x^7x^mb^2+70B\,x^4x^ma^2+280Ax\,x^ma^2+39B\,x^{10}x^mb^2m+A\,x^7x^mb^2m^3+15$

input
```
int(x^m*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
x*(B*b^2*m^3*x^9+12*B*b^2*m^2*x^9+39*B*b^2*m*x^9+A*b^2*m^3*x^6+2*B*a*b*m^3
*x^6+28*B*b^2*x^9+15*A*b^2*m^2*x^6+30*B*a*b*m^2*x^6+54*A*b^2*m*x^6+108*B*a
*b*m*x^6+2*A*a*b*m^3*x^3+40*A*b^2*x^6+B*a^2*m^3*x^3+80*B*a*b*x^6+36*A*a*b*
m^2*x^3+18*B*a^2*m^2*x^3+174*A*a*b*m*x^3+87*B*a^2*m*x^3+A*a^2*m^3+140*A*a*
b*x^3+70*B*a^2*x^3+21*A*a^2*m^2+138*A*a^2*m+280*A*a^2)*x^m/(10+m)/(7+m)/(4
+m)/(1+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(71) = 142$.

Time = 0.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.03

$$\int x^m \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx = \frac{((Bb^2m^3+12\,Bb^2m^2+39\,Bb^2m+28\,Bb^2)x^{10}+((2\,Bab+Ab^2)m^3+80\,Bab+40\,Ab^2+15\,(2\,Bab+A}{}$$

input
```
integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

output
```
((B*b^2*m^3 + 12*B*b^2*m^2 + 39*B*b^2*m + 28*B*b^2)*x^10 + ((2*B*a*b + A*b
^2)*m^3 + 80*B*a*b + 40*A*b^2 + 15*(2*B*a*b + A*b^2)*m^2 + 54*(2*B*a*b + A
*b^2)*m)*x^7 + ((B*a^2 + 2*A*a*b)*m^3 + 70*B*a^2 + 140*A*a*b + 18*(B*a^2 +
 2*A*a*b)*m^2 + 87*(B*a^2 + 2*A*a*b)*m)*x^4 + (A*a^2*m^3 + 21*A*a^2*m^2 +
138*A*a^2*m + 280*A*a^2)*x)*x^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1057 vs. $2(63) = 126$.

Time = 0.62 (sec) , antiderivative size = 1057, normalized size of antiderivative = 14.89

$$\int x^m (a + bx^3)^2 (A + Bx^3)\, dx = \text{Too large to display}$$

input
```
integrate(x**m*(b*x**3+a)**2*(B*x**3+A),x)
```

output
```
Piecewise((-A*a**2/(9*x**9) - A*a*b/(3*x**6) - A*b**2/(3*x**3) - B*a**2/(6
*x**6) - 2*B*a*b/(3*x**3) + B*b**2*log(x), Eq(m, -10)), (-A*a**2/(6*x**6)
- 2*A*a*b/(3*x**3) + A*b**2*log(x) - B*a**2/(3*x**3) + 2*B*a*b*log(x) + B*
b**2*x**3/3, Eq(m, -7)), (-A*a**2/(3*x**3) + 2*A*a*b*log(x) + A*b**2*x**3/
3 + B*a**2*log(x) + 2*B*a*b*x**3/3 + B*b**2*x**6/6, Eq(m, -4)), (A*a**2*lo
g(x) + 2*A*a*b*x**3/3 + A*b**2*x**6/6 + B*a**2*x**3/3 + B*a*b*x**6/3 + B*b
**2*x**9/9, Eq(m, -1)), (A*a**2*m**3*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 4
18*m + 280) + 21*A*a**2*m**2*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 2
80) + 138*A*a**2*m*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 280*
A*a**2*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 2*A*a*b*m**3*x**
4*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 36*A*a*b*m**2*x**4*x**m
/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 174*A*a*b*m*x**4*x**m/(m**4 +
 22*m**3 + 159*m**2 + 418*m + 280) + 140*A*a*b*x**4*x**m/(m**4 + 22*m**3 +
 159*m**2 + 418*m + 280) + A*b**2*m**3*x**7*x**m/(m**4 + 22*m**3 + 159*m**
2 + 418*m + 280) + 15*A*b**2*m**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 4
18*m + 280) + 54*A*b**2*m*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 2
80) + 40*A*b**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + B*a*
*2*m**3*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 18*B*a**2*m*
*2*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 87*B*a**2*m*x**4*
x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 70*B*a**2*x**4*x**m/(m...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.28

$$\int x^m (a+bx^3)^2 (A+Bx^3)\,dx = \frac{Bb^2x^{m+10}}{m+10} + \frac{2\,Babx^{m+7}}{m+7} + \frac{Ab^2x^{m+7}}{m+7} + \frac{Ba^2x^{m+4}}{m+4} + \frac{2\,Aabx^{m+4}}{m+4} + \frac{Aa^2x^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

output
```
B*b^2*x^(m + 10)/(m + 10) + 2*B*a*b*x^(m + 7)/(m + 7) + A*b^2*x^(m + 7)/(m
 + 7) + B*a^2*x^(m + 4)/(m + 4) + 2*A*a*b*x^(m + 4)/(m + 4) + A*a^2*x^(m +
 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(71) = 142.

Time = 0.13 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.68

$$\int x^m (a+bx^3)^2 (A+Bx^3)\,dx = \frac{Bb^2m^3x^{10}x^m + 12\,Bb^2m^2x^{10}x^m + 39\,Bb^2mx^{10}x^m + 2\,Babm^3x^7x^m + Ab^2m^3x^7x^m + 28\,Bb^2x^{10}x^m + 30\,B}{}$$

input
```
integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

output
```
(B*b^2*m^3*x^10*x^m + 12*B*b^2*m^2*x^10*x^m + 39*B*b^2*m*x^10*x^m + 2*B*a*
b*m^3*x^7*x^m + A*b^2*m^3*x^7*x^m + 28*B*b^2*x^10*x^m + 30*B*a*b*m^2*x^7*x
^m + 15*A*b^2*m^2*x^7*x^m + 108*B*a*b*m*x^7*x^m + 54*A*b^2*m*x^7*x^m + B*a
^2*m^3*x^4*x^m + 2*A*a*b*m^3*x^4*x^m + 80*B*a*b*x^7*x^m + 40*A*b^2*x^7*x^m
 + 18*B*a^2*m^2*x^4*x^m + 36*A*a*b*m^2*x^4*x^m + 87*B*a^2*m*x^4*x^m + 174*
A*a*b*m*x^4*x^m + A*a^2*m^3*x*x^m + 70*B*a^2*x^4*x^m + 140*A*a*b*x^4*x^m +
 21*A*a^2*m^2*x*x^m + 138*A*a^2*m*x*x^m + 280*A*a^2*x*x^m)/(m^4 + 22*m^3 +
 159*m^2 + 418*m + 280)
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.49

$$\int x^m (a+bx^3)^2 (A+Bx^3)\,dx = x^m \left(\frac{B\,b^2\,x^{10}\,(m^3+12\,m^2+39\,m+28)}{m^4+22\,m^3+159\,m^2+418\,m+280} + \frac{A\,a^2\,x\,(m^3+21\,m^2+138\,m+280)}{m^4+22\,m^3+159\,m^2+418\,m+280} + \frac{a\,x^4\,(2\,A\,b+B\,a)\,(m^3+18\,m^2+87\,m+70)}{m^4+22\,m^3+159\,m^2+418\,m+280} + \frac{b\,x^7\,(A\,b+2\,B\,a)\,(m^3+15\,m^2+54\,m+40)}{m^4+22\,m^3+159\,m^2+418\,m+280}\right)$$

input
```
int(x^m*(A + B*x^3)*(a + b*x^3)^2,x)
```

output
```
x^m*((B*b^2*x^10*(39*m + 12*m^2 + m^3 + 28))/(418*m + 159*m^2 + 22*m^3 + m
^4 + 280) + (A*a^2*x*(138*m + 21*m^2 + m^3 + 280))/(418*m + 159*m^2 + 22*m
^3 + m^4 + 280) + (a*x^4*(2*A*b + B*a)*(87*m + 18*m^2 + m^3 + 70))/(418*m
+ 159*m^2 + 22*m^3 + m^4 + 280) + (b*x^7*(A*b + 2*B*a)*(54*m + 15*m^2 + m^
3 + 40))/(418*m + 159*m^2 + 22*m^3 + m^4 + 280))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.48

$$\int x^m (a+bx^3)^2 (A+Bx^3)\,dx = \frac{x^m x(b^3m^3x^9+12b^3m^2x^9+39b^3m\,x^9+3a\,b^2m^3x^6+28b^3x^9+45a\,b^2m^2x^6+162a\,b^2m\,x^6+3a^2b\,m^3x^3+}{m^4+22m^3+159m^2+4}$$

input
```
int(x^m*(b*x^3+a)^2*(B*x^3+A),x)
```

output
```
(x**m*x*(a**3*m**3 + 21*a**3*m**2 + 138*a**3*m + 280*a**3 + 3*a**2*b*m**3*
x**3 + 54*a**2*b*m**2*x**3 + 261*a**2*b*m*x**3 + 210*a**2*b*x**3 + 3*a*b**
2*m**3*x**6 + 45*a*b**2*m**2*x**6 + 162*a*b**2*m*x**6 + 120*a*b**2*x**6 +
b**3*m**3*x**9 + 12*b**3*m**2*x**9 + 39*b**3*m*x**9 + 28*b**3*x**9))/(m**4
 + 22*m**3 + 159*m**2 + 418*m + 280)
```

3.379 $\int x^m(a+bx^3)(A+Bx^3)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 45

$$\int x^m(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{aAx^{1+m}}{1+m}+\frac{(Ab+aB)x^{4+m}}{4+m}+\frac{bBx^{7+m}}{7+m}$$

output
```
a*A*x^(1+m)/(1+m)+(A*b+B*a)*x^(4+m)/(4+m)+b*B*x^(7+m)/(7+m)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x^m(a+bx^3)\left(A+Bx^3\right)\,dx=x^{1+m}\left(\frac{aA}{1+m}+\frac{(Ab+aB)x^3}{4+m}+\frac{bBx^6}{7+m}\right)$$

input
```
Integrate[x^m*(a + b*x^3)*(A + B*x^3),x]
```

output
```
x^(1 + m)*((a*A)/(1 + m) + ((A*b + a*B)*x^3)/(4 + m) + (b*B*x^6)/(7 + m))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m\left(a+bx^3\right)\left(A+Bx^3\right)dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^{m+3}(aB+Ab)+aAx^m+bBx^{m+6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{m+4}(aB+Ab)}{m+4}+\frac{aAx^{m+1}}{m+1}+\frac{bBx^{m+7}}{m+7}$$

input
```
Int[x^m*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(a*A*x^(1 + m))/(1 + m) + ((A*b + a*B)*x^(4 + m))/(4 + m) + (b*B*x^(7 + m)
)/(7 + m)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

method	result
norman	$\frac{(Ab+Ba)x^4 e^{m\ln(x)}}{4+m}+\frac{Aax\,e^{m\ln(x)}}{1+m}+\frac{Bb\,x^7 e^{m\ln(x)}}{7+m}$
risch	$\frac{x(Bb\,m^2x^6+5Bbm\,x^6+4bB\,x^6+Ab\,m^2x^3+Ba\,m^2x^3+8Abm\,x^3+8Bam\,x^3+7Ab\,x^3+7Ba\,x^3+Aa\,m^2+11Aam+28Aa)x^m}{(7+m)(4+m)(1+m)}$
orering	$\frac{x(Bb\,m^2x^6+5Bbm\,x^6+4bB\,x^6+Ab\,m^2x^3+Ba\,m^2x^3+8Abm\,x^3+8Bam\,x^3+7Ab\,x^3+7Ba\,x^3+Aa\,m^2+11Aam+28Aa)x^m}{(7+m)(4+m)(1+m)}$
gosper	$\frac{x^{1+m}(Bb\,m^2x^6+5Bbm\,x^6+4bB\,x^6+Ab\,m^2x^3+Ba\,m^2x^3+8Abm\,x^3+8Bam\,x^3+7Ab\,x^3+7Ba\,x^3+Aa\,m^2+11Aam+28Aa)}{(1+m)(4+m)(7+m)}$
parallelrisch	$\frac{B\,x^7x^m b\,m^2+5B\,x^7x^m bm+4B\,x^7x^m b+A\,x^4x^m b\,m^2+B\,x^4x^m a\,m^2+8A\,x^4x^m bm+8B\,x^4x^m am+7A\,x^4x^m b+7B\,x^4x^m a+A}{(7+m)(4+m)(1+m)}$

input
```
int(x^m*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
(A*b+B*a)/(4+m)*x^4*exp(m*ln(x))+A*a/(1+m)*x*exp(m*ln(x))+B*b/(7+m)*x^7*ex
p(m*ln(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(45) = 90.

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.04

$$\int x^m(a+bx^3)\left(A+Bx^3\right)\,dx = \frac{((Bbm^2+5\,Bbm+4\,Bb)x^7+((Ba+Ab)m^2+7\,Ba+7\,Ab+8\,(Ba+Ab)m)x^4+(Aam^2+11\,Aam+}{m^3+12\,m^2+39\,m+28}$$

input
```
integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

output
```
((B*b*m^2 + 5*B*b*m + 4*B*b)*x^7 + ((B*a + A*b)*m^2 + 7*B*a + 7*A*b + 8*(B
*a + A*b)*m)*x^4 + (A*a*m^2 + 11*A*a*m + 28*A*a)*x)*x^m/(m^3 + 12*m^2 + 39
*m + 28)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(37) = 74$.

Time = 0.47 (sec) , antiderivative size = 410, normalized size of antiderivative = 9.11

$$\int x^m(a+bx^3)\,(A+Bx^3)\,dx = \begin{cases} -\frac{Aa}{6x^6} - \frac{Ab}{3x^3} - \frac{Ba}{3x^3} + Bb\log(x) \\ -\frac{Aa}{3x^3} + Ab\log(x) + Ba\log(x) + \frac{Bbx^3}{3} \\ Aa\log(x) + \frac{Abx^3}{3} + \frac{Bax^3}{3} + \frac{Bbx^6}{6} \\ \frac{Aam^2xx^m}{m^3+12m^2+39m+28} + \frac{11Aamxx^m}{m^3+12m^2+39m+28} + \frac{28Aaxx^m}{m^3+12m^2+39m+28} + \frac{Abm^2x^4x^m}{m^3+12m^2+39m+28} + \frac{8Abmx^4x^m}{m^3+12m^2+39m+28} + \frac{7Abx}{m^3+12m^2} \end{cases}$$

input
```
integrate(x**m*(b*x**3+a)*(B*x**3+A),x)
```

output
```
Piecewise((-A*a/(6*x**6) - A*b/(3*x**3) - B*a/(3*x**3) + B*b*log(x), Eq(m,
 -7)), (-A*a/(3*x**3) + A*b*log(x) + B*a*log(x) + B*b*x**3/3, Eq(m, -4)),
(A*a*log(x) + A*b*x**3/3 + B*a*x**3/3 + B*b*x**6/6, Eq(m, -1)), (A*a*m**2*
x*x**m/(m**3 + 12*m**2 + 39*m + 28) + 11*A*a*m*x*x**m/(m**3 + 12*m**2 + 39
*m + 28) + 28*A*a*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + A*b*m**2*x**4*x**m
/(m**3 + 12*m**2 + 39*m + 28) + 8*A*b*m*x**4*x**m/(m**3 + 12*m**2 + 39*m +
 28) + 7*A*b*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + B*a*m**2*x**4*x**m/(
m**3 + 12*m**2 + 39*m + 28) + 8*B*a*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 2
8) + 7*B*a*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + B*b*m**2*x**7*x**m/(m*
*3 + 12*m**2 + 39*m + 28) + 5*B*b*m*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28)
 + 4*B*b*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int x^m(a+bx^3)\,(A+Bx^3)\,dx = \frac{Bbx^{m+7}}{m+7} + \frac{Bax^{m+4}}{m+4} + \frac{Abx^{m+4}}{m+4} + \frac{Aax^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

output
```
B*b*x^(m + 7)/(m + 7) + B*a*x^(m + 4)/(m + 4) + A*b*x^(m + 4)/(m + 4) + A*
a*x^(m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(45) = 90$.

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.18

$$\int x^m(a+bx^3)(A+Bx^3)\,dx = \frac{Bbm^2x^7x^m + 5\,Bbmx^7x^m + 4\,Bbx^7x^m + Bam^2x^4x^m + Abm^2x^4x^m + 8\,Bamx^4x^m + 8\,Abmx^4x^m + 7\,Ba}{m^3 + 12\,m^2 + 39\,m + 28}$$

input
```
integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

output
```
(B*b*m^2*x^7*x^m + 5*B*b*m*x^7*x^m + 4*B*b*x^7*x^m + B*a*m^2*x^4*x^m + A*b
*m^2*x^4*x^m + 8*B*a*m*x^4*x^m + 8*A*b*m*x^4*x^m + 7*B*a*x^4*x^m + 7*A*b*x
^4*x^m + A*a*m^2*x*x^m + 11*A*a*m*x*x^m + 28*A*a*x*x^m)/(m^3 + 12*m^2 + 39
*m + 28)
```

Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.11

$$\int x^m(a+bx^3)(A+Bx^3)\,dx = x^m\left(\frac{x^4\,(A\,b+B\,a)\,(m^2+8\,m+7)}{m^3+12\,m^2+39\,m+28} + \frac{B\,b\,x^7\,(m^2+5\,m+4)}{m^3+12\,m^2+39\,m+28} + \frac{A\,a\,x\,(m^2+11\,m+28)}{m^3+12\,m^2+39\,m+28}\right)$$

input
```
int(x^m*(A + B*x^3)*(a + b*x^3),x)
```

output
```
x^m*((x^4*(A*b + B*a)*(8*m + m^2 + 7))/(39*m + 12*m^2 + m^3 + 28) + (B*b*x
^7*(5*m + m^2 + 4))/(39*m + 12*m^2 + m^3 + 28) + (A*a*x*(11*m + m^2 + 28))
/(39*m + 12*m^2 + m^3 + 28))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.02

$$\int x^m (a + bx^3) (A + Bx^3) \, dx = \frac{x^m x (b^2 m^2 x^6 + 5b^2 m\, x^6 + 4b^2 x^6 + 2ab\, m^2 x^3 + 16abm\, x^3 + 14ab\, x^3 + a^2 m^2 + 11a^2 m + 28a^2)}{m^3 + 12m^2 + 39m + 28}$$

input
```
int(x^m*(b*x^3+a)*(B*x^3+A),x)
```

output
```
(x**m*x*(a**2*m**2 + 11*a**2*m + 28*a**2 + 2*a*b*m**2*x**3 + 16*a*b*m*x**3
 + 14*a*b*x**3 + b**2*m**2*x**6 + 5*b**2*m*x**6 + 4*b**2*x**6))/(m**3 + 12
*m**2 + 39*m + 28)
```

3.380 $\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^{1+m}}{b(1+m)} + \frac{(Ab-aB)x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{ab(1+m)}$$

output
```
B*x^(1+m)/b/(1+m)+(A*b-B*a)*x^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-
b*x^3/a)/a/b/(1+m)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \frac{x^{1+m}\left(aB+(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)\right)}{ab(1+m)}$$

input
```
Integrate[(x^m*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(x^(1 + m)*(a*B + (A*b - a*B)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -
((b*x^3)/a)]))/(a*b*(1 + m))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (A+Bx^3)}{a+bx^3} \, dx$$

$$\Big\downarrow \text{ 959}$$

$$\frac{(Ab-aB)\int \frac{x^m}{bx^3+a}dx}{b}+\frac{Bx^{m+1}}{b(m+1)}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^{m+1}(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{ab(m+1)}+\frac{Bx^{m+1}}{b(m+1)}$$

input
```
Int[(x^m*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(B*x^(1 + m))/(b*(1 + m)) + ((A*b - a*B)*x^(1 + m)*Hypergeometric2F1[1, (1
 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(a*b*(1 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^{m}\left(B\,x^{3}+A\right)}{b\,x^{3}+a}dx$$

input
```
int(x^m*(B*x^3+A)/(b*x^3+a),x)
```

output
```
int(x^m*(B*x^3+A)/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx=\int \frac{(Bx^3+A)x^m}{bx^3+a}\,dx$$

input
```
integrate(x^m*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*x^m/(b*x^3 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.76 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.83

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \frac{Amx^{m+1}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{Ax^{m+1}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{Bmx^{m+4}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{4}{3}\right)\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)} + \frac{4Bx^{m+4}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{4}{3}\right)\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

input
```
integrate(x**m*(B*x**3+A)/(b*x**3+a),x)
```

output
```
A*m*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3
+ 1/3)/(9*a*gamma(m/3 + 4/3)) + A*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*p
i)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(9*a*gamma(m/3 + 4/3)) + B*m*x**(m +
4)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma(m/3 + 4/3)/(9*a*
gamma(m/3 + 7/3)) + 4*B*x**(m + 4)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m
/3 + 4/3)*gamma(m/3 + 4/3)/(9*a*gamma(m/3 + 7/3))
```

Maxima [F]

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \int \frac{(Bx^3+A)x^m}{bx^3+a}\,dx$$

input
```
integrate(x^m*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^m/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \int \frac{(Bx^3+A)x^m}{bx^3+a}\,dx$$

input
```
integrate(x^m*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^m/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \int \frac{x^m\,(B\,x^3+A)}{b\,x^3+a}\,dx$$

input
```
int((x^m*(A + B*x^3))/(a + b*x^3),x)
```

output
```
int((x^m*(A + B*x^3))/(a + b*x^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.15

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \frac{x^m x}{m+1}$$

input
```
int(x^m*(B*x^3+A)/(b*x^3+a),x)
```

output
```
(x**m*x)/(m + 1)
```

3.381 $\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ab-aB)x^{1+m}}{3ab(a+bx^3)} + \frac{(Ab(2-m)+aB(1+m))x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{3a^2b(1+m)}$$

output
```
1/3*(A*b-B*a)*x^(1+m)/a/b/(b*x^3+a)+1/3*(A*b*(2-m)+a*B*(1+m))*x^(1+m)*hype
rgeom([1, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^2/b/(1+m)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{x^{1+m}\left(aB\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+(Ab-aB)\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right.\right.}{a^2b(1+m)}$$

input
```
Integrate[(x^m*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(x^(1 + m)*(a*B*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)] +
 (A*b - a*B)*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)]))/(a
^2*b*(1 + m))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(A+Bx^3\right)}{\left(a+bx^3\right)^2} \, dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(aB(m+1)+Ab(2-m)) \int \frac{x^m}{bx^3+a}dx}{3ab}+\frac{x^{m+1}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m+1}(aB(m+1)+Ab(2-m)) \text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{3a^2b(m+1)}+\frac{x^{m+1}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

input
```
Int[(x^m*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^(1 + m))/(3*a*b*(a + b*x^3)) + ((A*b*(2 - m) + a*B*(1 + m))
*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(3*a^
2*b*(1 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^m(B\,x^3+A)}{(b\,x^3+a)^2}dx$$

input
```
int(x^m*(B*x^3+A)/(b*x^3+a)^2,x)
```

output
```
int(x^m*(B*x^3+A)/(b*x^3+a)^2,x)
```

Fricas [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx=\int \frac{(Bx^3+A)x^m}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^m*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*x^m/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 152.79 (sec) , antiderivative size = 1049, normalized size of antiderivative = 11.28

$$\int \frac{x^m(A + Bx^3)}{(a + bx^3)^2} dx = \text{Too large to display}$$

input
```
integrate(x**m*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
A*(-a*m**2*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gam
ma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3))
 + a*m*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m
/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) + 3
*a*m*x**(m + 1)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**
3*gamma(m/3 + 4/3)) + 2*a*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1,
 m/3 + 1/3)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*ga
mma(m/3 + 4/3)) + 3*a*x**(m + 1)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3
) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) - b*m**2*x**3*x**(m + 1)*lerchphi(b*x
**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 +
 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) + b*m*x**3*x**(m + 1)*lerchphi(b*
x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3
+ 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) + 2*b*x**3*x**(m + 1)*lerchphi(b
*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3
 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3))) + B*(-a*m**2*x**(m + 4)*lerchp
hi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma(m/3 + 4/3)/(27*a**3*gamma
(m/3 + 7/3) + 27*a**2*b*x**3*gamma(m/3 + 7/3)) - 5*a*m*x**(m + 4)*lerchphi
(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma(m/3 + 4/3)/(27*a**3*gamma(m
/3 + 7/3) + 27*a**2*b*x**3*gamma(m/3 + 7/3)) + 3*a*m*x**(m + 4)*gamma(m/3
+ 4/3)/(27*a**3*gamma(m/3 + 7/3) + 27*a**2*b*x**3*gamma(m/3 + 7/3)) - 4...
```

Maxima [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx = \int \frac{(Bx^3+A)x^m}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^m*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^m/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx = \int \frac{(Bx^3+A)x^m}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^m*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^m/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx = \int \frac{x^m\,(B\,x^3+A)}{(b\,x^3+a)^2}\,dx$$

input
```
int((x^m*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
int((x^m*(A + B*x^3))/(a + b*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx = \int \frac{x^m}{b\,x^3+a}dx$$

input
```
int(x^m*(B*x^3+A)/(b*x^3+a)^2,x)
```

output
```
int(x**m/(a + b*x**3),x)
```

3.382 $\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ab-aB)x^{1+m}}{6ab\,(a+bx^3)^2} + \frac{(Ab(5-m)+aB(1+m))x^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{6a^3b(1+m)}$$

output
```
1/6*(A*b-B*a)*x^(1+m)/a/b/(b*x^3+a)^2+1/6*(A*b*(5-m)+a*B*(1+m))*x^(1+m)*hy
pergeom([2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^3/b/(1+m)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^{1+m}\left(aB\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+(Ab-aB)\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx}{a}\right.\right.}{a^3b(1+m)}$$

input
```
Integrate[(x^m*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
(x^(1 + m)*(a*B*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)] +
 (A*b - a*B)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)]))/(a
^3*b*(1 + m))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(aB(m+1)+Ab(5-m))\int \frac{x^m}{\left(bx^3+a\right)^2}dx}{6ab}+\frac{x^{m+1}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m+1}(aB(m+1)+Ab(5-m))\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{6a^3b(m+1)}+\frac{x^{m+1}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input
```
Int[(x^m*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^(1 + m))/(6*a*b*(a + b*x^3)^2) + ((A*b*(5 - m) + a*B*(1 + m
))*x^(1 + m)*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(6*
a^3*b*(1 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^m(B\,x^3+A)}{(b\,x^3+a)^3}dx$$

input
```
int(x^m*(B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
int(x^m*(B*x^3+A)/(b*x^3+a)^3,x)
```

Fricas [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx=\int \frac{(Bx^3+A)x^m}{(bx^3+a)^3}\,dx$$

input
```
integrate(x^m*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*x^m/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \int \frac{(Bx^3+A)x^m}{(bx^3+a)^3}\,dx$$

input
```
integrate(x^m*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^m/(b*x^3 + a)^3, x)
```

Giac [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \int \frac{(Bx^3+A)x^m}{(bx^3+a)^3}\,dx$$

input
```
integrate(x^m*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^m/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \int \frac{x^m\,(B\,x^3+A)}{(b\,x^3+a)^3}\,dx$$

input
```
int((x^m*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
int((x^m*(A + B*x^3))/(a + b*x^3)^3, x)
```

Reduce [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \int \frac{x^m}{b^2x^6+2ab\,x^3+a^2}dx$$

input
```
int(x^m*(B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
int(x**m/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
```

3.383 $\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \frac{2B(ex)^{1+m} \left(a+bx^3\right)^{7/2}}{be(23+2m)} + \frac{a^2 \left(\frac{A}{1+m}-\frac{2aB}{23b+2bm}\right) (ex)^{1+m} \sqrt{a+bx^3} \,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{e\sqrt{1+\frac{bx^3}{a}}}$$

output
```
2*B*(e*x)^(1+m)*(b*x^3+a)^(7/2)/b/e/(23+2*m)+a^2*(A/(1+m)-2*a*B/(2*b*m+23*
b))*(e*x)^(1+m)*(b*x^3+a)^(1/2)*hypergeom([-5/2, 1/3+1/3*m],[4/3+1/3*m],-b
*x^3/a)/e/(1+b*x^3/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.92

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \frac{a^2 x (ex)^m \sqrt{a+bx^3} \left(A(4+m) \,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m)x^3 \,\text{Hyp}\right.}{(1+m)(4+m)\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(e*x)^m*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

output
```
(a^2*x*(e*x)^m*Sqrt[a + b*x^3]*(A*(4 + m)*Hypergeometric2F1[-5/2, (1 + m)/
3, (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-5/2, (4 + m
)/3, (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) (ex)^m \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{2B\left(a+bx^3\right)^{7/2} (ex)^{m+1}}{be(2m+23)}-\frac{(2aB(m+1)-Ab(2m+23)) \int (ex)^m \left(bx^3+a\right)^{5/2}dx}{b(2m+23)}$$

$$\Big\downarrow \text{889}$$

$$\frac{2B\left(a+bx^3\right)^{7/2} (ex)^{m+1}}{be(2m+23)}-\frac{a^2\sqrt{a+bx^3}(2aB(m+1)-Ab(2m+23)) \int (ex)^m \left(\frac{bx^3}{a}+1\right)^{5/2}dx}{b(2m+23)\sqrt{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2B\left(a+bx^3\right)^{7/2} (ex)^{m+1}}{be(2m+23)}-\frac{a^2\sqrt{a+bx^3}(ex)^{m+1}(2aB(m+1)-Ab(2m+23)) \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{be(m+1)(2m+23)\sqrt{\frac{bx^3}{a}+1}}$$

input
```
Int[(e*x)^m*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

output
```
(2*B*(e*x)^(1 + m)*(a + b*x^3)^(7/2))/(b*e*(23 + 2*m)) - (a^2*(2*a*B*(1 +
m) - A*b*(23 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-5/2,
 (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(23 + 2*m)*Sqrt[1 + (b*
x^3)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (ex)^m \left(b\,x^3+a\right)^{\frac{5}{2}} \left(B\,x^3+A\right)dx$$

input
```
int((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

output
```
int((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

Fricas [F]

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^
2)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.41 (sec) , antiderivative size = 379, normalized size of antiderivative = 3.08

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) dx = \frac{Aa^{\frac{5}{2}}e^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{2Aa^{\frac{3}{2}}be^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{4}{3} \\ \frac{m}{3}+\frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)} + \frac{A\sqrt{a}b^2 e^m x^{m+7}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{7}{3} \\ \frac{m}{3}+\frac{10}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{10}{3}\right)} + \frac{Ba^{\frac{5}{2}}e^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{4}{3} \\ \frac{m}{3}+\frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)} + \frac{2Ba^{\frac{3}{2}}be^m x^{m+7}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{7}{3} \\ \frac{m}{3}+\frac{10}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{10}{3}\right)} + \frac{B\sqrt{a}b^2 e^m x^{m+10}\Gamma\left(\frac{m}{3}+\frac{10}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{10}{3} \\ \frac{m}{3}+\frac{13}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{13}{3}\right)}$$

input
```
integrate((e*x)**m*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

output
```
A*a**(5/2)*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3
+ 4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + 2*A*a**(3/2)*b*e
**m*x**(m + 4)*gamma(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 7/3)) + A*sqrt(a)*b**2*e**m*x**(m +
7)*gamma(m/3 + 7/3)*hyper((-1/2, m/3 + 7/3), (m/3 + 10/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(m/3 + 10/3)) + B*a**(5/2)*e**m*x**(m + 4)*gamma(m/3 +
 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(m/3 + 7/3)) + 2*B*a**(3/2)*b*e**m*x**(m + 7)*gamma(m/3 + 7/3)*hyper(
(-1/2, m/3 + 7/3), (m/3 + 10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 +
 10/3)) + B*sqrt(a)*b**2*e**m*x**(m + 10)*gamma(m/3 + 10/3)*hyper((-1/2, m
/3 + 10/3), (m/3 + 13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 13/3))
```

Maxima [F]

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^m\,{\left(b\,x^3+a\right)}^{5/2} \,d x$$

input
```
int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(5/2),x)
```

output
```
int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(5/2), x)
```

Reduce [F]

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A +Bx^3\right) \, dx = \frac{e^m \left(16x^m \sqrt{b\,x^3+a}\,a^3m^3x+432x^m \sqrt{b\,x^3+a}\,a^3m^2x+3912x^m \sqrt{b\,x^3+a}\,a^3mx+13000x^m \sqrt{b}\right.}{}$$

input
```
int((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

output
```
(e**m*(16*x**m*sqrt(a + b*x**3)*a**3*m**3*x + 432*x**m*sqrt(a + b*x**3)*a*
*3*m**2*x + 3912*x**m*sqrt(a + b*x**3)*a**3*m*x + 13000*x**m*sqrt(a + b*x*
*3)*a**3*x + 48*x**m*sqrt(a + b*x**3)*a**2*b*m**3*x**4 + 1128*x**m*sqrt(a
+ b*x**3)*a**2*b*m**2*x**4 + 7872*x**m*sqrt(a + b*x**3)*a**2*b*m*x**4 + 13
380*x**m*sqrt(a + b*x**3)*a**2*b*x**4 + 48*x**m*sqrt(a + b*x**3)*a*b**2*m*
*3*x**7 + 960*x**m*sqrt(a + b*x**3)*a*b**2*m**2*x**7 + 5268*x**m*sqrt(a +
b*x**3)*a*b**2*m*x**7 + 7920*x**m*sqrt(a + b*x**3)*a*b**2*x**7 + 16*x**m*s
qrt(a + b*x**3)*b**3*m**3*x**10 + 264*x**m*sqrt(a + b*x**3)*b**3*m**2*x**1
0 + 1308*x**m*sqrt(a + b*x**3)*b**3*m*x**10 + 1870*x**m*sqrt(a + b*x**3)*b
**3*x**10 + 136080*int((x**m*sqrt(a + b*x**3))/(16*a*m**4 + 448*a*m**3 + 4
344*a*m**2 + 16912*a*m + 21505*a + 16*b*m**4*x**3 + 448*b*m**3*x**3 + 4344
*b*m**2*x**3 + 16912*b*m*x**3 + 21505*b*x**3),x)*a**4*m**4 + 3810240*int((
x**m*sqrt(a + b*x**3))/(16*a*m**4 + 448*a*m**3 + 4344*a*m**2 + 16912*a*m +
 21505*a + 16*b*m**4*x**3 + 448*b*m**3*x**3 + 4344*b*m**2*x**3 + 16912*b*m
*x**3 + 21505*b*x**3),x)*a**4*m**3 + 36945720*int((x**m*sqrt(a + b*x**3))/
(16*a*m**4 + 448*a*m**3 + 4344*a*m**2 + 16912*a*m + 21505*a + 16*b*m**4*x*
*3 + 448*b*m**3*x**3 + 4344*b*m**2*x**3 + 16912*b*m*x**3 + 21505*b*x**3),x
)*a**4*m**2 + 143836560*int((x**m*sqrt(a + b*x**3))/(16*a*m**4 + 448*a*m**
3 + 4344*a*m**2 + 16912*a*m + 21505*a + 16*b*m**4*x**3 + 448*b*m**3*x**3 +
 4344*b*m**2*x**3 + 16912*b*m*x**3 + 21505*b*x**3),x)*a**4*m + 18290002...
```

3.384 $\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 121

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{5/2}}{be(17+2m)} + \frac{a\left(\frac{A}{1+m}-\frac{2aB}{17b+2bm}\right)(ex)^{1+m}\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{e\sqrt{1+\frac{bx^3}{a}}}$$

output
```
2*B*(e*x)^(1+m)*(b*x^3+a)^(5/2)/b/e/(17+2*m)+a*(A/(1+m)-2*a*B/(2*b*m+17*b)
)*(e*x)^(1+m)*(b*x^3+a)^(1/2)*hypergeom([-3/2, 1/3+1/3*m],[4/3+1/3*m],-b*x
^3/a)/e/(1+b*x^3/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{ax(ex)^m\sqrt{a+bx^3}\left(A(4+m)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m)x^3\,\text{Hype}\right.}{(1+m)(4+m)\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(e*x)^m*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(a*x*(e*x)^m*Sqrt[a + b*x^3]*(A*(4 + m)*Hypergeometric2F1[-3/2, (1 + m)/3,
 (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-3/2, (4 + m)/
3, (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) (ex)^m \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{2B\left(a+bx^3\right)^{5/2} (ex)^{m+1}}{be(2m+17)}-\frac{(2aB(m+1)-Ab(2m+17)) \int (ex)^m \left(bx^3+a\right)^{3/2}dx}{b(2m+17)}$$

$$\Big\downarrow \text{889}$$

$$\frac{2B\left(a+bx^3\right)^{5/2} (ex)^{m+1}}{be(2m+17)}-\frac{a\sqrt{a+bx^3}(2aB(m+1)-Ab(2m+17)) \int (ex)^m \left(\frac{bx^3}{a}+1\right)^{3/2}dx}{b(2m+17)\sqrt{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2B\left(a+bx^3\right)^{5/2} (ex)^{m+1}}{be(2m+17)}-\frac{a\sqrt{a+bx^3}(ex)^{m+1}(2aB(m+1)-Ab(2m+17)) \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{be(m+1)(2m+17)\sqrt{\frac{bx^3}{a}+1}}$$

input
```
Int[(e*x)^m*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*B*(e*x)^(1 + m)*(a + b*x^3)^(5/2))/(b*e*(17 + 2*m)) - (a*(2*a*B*(1 + m)
 - A*b*(17 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, (
1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(17 + 2*m)*Sqrt[1 + (b*x^
3)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (ex)^m \left(b\,x^3+a\right)^{\frac{3}{2}} \left(B\,x^3+A\right)dx$$

input
```
int((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

output
```
int((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

Fricas [F]

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.24 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.03

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{Aa^{\frac{3}{2}}e^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{A\sqrt{a}be^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{4}{3} \\ \frac{m}{3}+\frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)} + \frac{Ba^{\frac{3}{2}}e^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{4}{3} \\ \frac{m}{3}+\frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)} + \frac{B\sqrt{a}be^m x^{m+7}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{7}{3} \\ \frac{m}{3}+\frac{10}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{10}{3}\right)}$$

input
```
integrate((e*x)**m*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
A*a**(3/2)*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3
+ 4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + A*sqrt(a)*b*e**m
*x**(m + 4)*gamma(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3
*exp_polar(I*pi)/a)/(3*gamma(m/3 + 7/3)) + B*a**(3/2)*e**m*x**(m + 4)*gamm
a(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(m/3 + 7/3)) + B*sqrt(a)*b*e**m*x**(m + 7)*gamma(m/3 + 7/3)*hy
per((-1/2, m/3 + 7/3), (m/3 + 10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m
/3 + 10/3))
```

Maxima [F]

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx = \int \left(Bx^3+A\right) \left(bx^3+a\right)^{\frac{3}{2}} (ex)^m dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx = \int \left(Bx^3+A\right) \left(bx^3+a\right)^{\frac{3}{2}} (ex)^m dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^m\,{\left(b\,x^3+a\right)}^{3/2} \,d x$$

input
```
int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(3/2),x)
```

output
```
int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{e^m\Big(8x^m\sqrt{b\,x^3+a}\,a^2m^2x+124x^m\sqrt{b\,x^3+a}\,a^2mx+530x^m\sqrt{b\,x^3+a}\,a^2x+16x^m\sqrt{b\,x^3+a}\,a$$

input
```
int((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

output
```
(e**m*(8*x**m*sqrt(a + b*x**3)*a**2*m**2*x + 124*x**m*sqrt(a + b*x**3)*a**
2*m*x + 530*x**m*sqrt(a + b*x**3)*a**2*x + 16*x**m*sqrt(a + b*x**3)*a*b*m*
*2*x**4 + 188*x**m*sqrt(a + b*x**3)*a*b*m*x**4 + 370*x**m*sqrt(a + b*x**3)
*a*b*x**4 + 8*x**m*sqrt(a + b*x**3)*b**2*m**2*x**7 + 64*x**m*sqrt(a + b*x*
*3)*b**2*m*x**7 + 110*x**m*sqrt(a + b*x**3)*b**2*x**7 + 3240*int((x**m*sqr
t(a + b*x**3))/(8*a*m**3 + 132*a*m**2 + 654*a*m + 935*a + 8*b*m**3*x**3 +
132*b*m**2*x**3 + 654*b*m*x**3 + 935*b*x**3),x)*a**3*m**3 + 53460*int((x**
m*sqrt(a + b*x**3))/(8*a*m**3 + 132*a*m**2 + 654*a*m + 935*a + 8*b*m**3*x*
*3 + 132*b*m**2*x**3 + 654*b*m*x**3 + 935*b*x**3),x)*a**3*m**2 + 264870*in
t((x**m*sqrt(a + b*x**3))/(8*a*m**3 + 132*a*m**2 + 654*a*m + 935*a + 8*b*m
**3*x**3 + 132*b*m**2*x**3 + 654*b*m*x**3 + 935*b*x**3),x)*a**3*m + 378675
*int((x**m*sqrt(a + b*x**3))/(8*a*m**3 + 132*a*m**2 + 654*a*m + 935*a + 8*
b*m**3*x**3 + 132*b*m**2*x**3 + 654*b*m*x**3 + 935*b*x**3),x)*a**3))/(8*m*
*3 + 132*m**2 + 654*m + 935)
```

3.385 $\int (ex)^m \sqrt{a+bx^3}(A+Bx^3)\, dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int (ex)^m \sqrt{a+bx^3}(A+Bx^3)\, dx = \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{3/2}}{be(11+2m)} + \frac{\left(\frac{A}{1+m}-\frac{2aB}{11b+2bm}\right)(ex)^{1+m}\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{e\sqrt{1+\frac{bx^3}{a}}}$$

output

```
2*B*(e*x)^(1+m)*(b*x^3+a)^(3/2)/b/e/(11+2*m)+(A/(1+m)-2*a*B/(2*b*m+11*b))*
(e*x)^(1+m)*(b*x^3+a)^(1/2)*hypergeom([-1/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3
/a)/e/(1+b*x^3/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int (ex)^m\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{x(ex)^m\sqrt{a+bx^3}\left(A(4+m)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m)x^3\,\text{Hypergeometric2}\right.}{(1+m)(4+m)\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(e*x)^m*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(x*(e*x)^m*Sqrt[a + b*x^3]*(A*(4 + m)*Hypergeometric2F1[-1/2, (1 + m)/3, (
4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-1/2, (4 + m)/3,
 (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)(ex)^m\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{2B\left(a+bx^3\right)^{3/2}(ex)^{m+1}}{be(2m+11)}-\frac{(2aB(m+1)-Ab(2m+11))\int (ex)^m\sqrt{bx^3+a}dx}{b(2m+11)}$$

$$\Big\downarrow \; 889$$

$$\frac{2B\left(a+bx^3\right)^{3/2}(ex)^{m+1}}{be(2m+11)}-\frac{\sqrt{a+bx^3}(2aB(m+1)-Ab(2m+11))\int (ex)^m\sqrt{\frac{bx^3}{a}+1}dx}{b(2m+11)\sqrt{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \; 888$$

$$\frac{2B\left(a+bx^3\right)^{3/2}(ex)^{m+1}}{be(2m+11)}-\frac{\sqrt{a+bx^3}(ex)^{m+1}(2aB(m+1)-Ab(2m+11))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{be(m+1)(2m+11)\sqrt{\frac{bx^3}{a}+1}}$$

input
```
Int[(e*x)^m*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*B*(e*x)^(1 + m)*(a + b*x^3)^(3/2))/(b*e*(11 + 2*m)) - ((2*a*B*(1 + m) -
 A*b*(11 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, (1
+ m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(11 + 2*m)*Sqrt[1 + (b*x^3)
/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (ex)^m \sqrt{b\,x^3+a}\,\left(B\,x^3+A\right)dx$$

input
```
int((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

output
```
int((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

Fricas [F]

$$\int (ex)^m\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^m\,dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01

$$\int (ex)^m\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{A\sqrt{a}e^mx^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{B\sqrt{a}e^mx^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{4}{3}\\ \frac{m}{3}+\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

input
```
integrate((e*x)**m*(b*x**3+a)**(1/2)*(B*x**3+A),x)
```

output
```
A*sqrt(a)*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3 +
 4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + B*sqrt(a)*e**m*x*
*(m + 4)*gamma(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*gamma(m/3 + 7/3))
```

Maxima [F]

$$\int (ex)^m\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^m\,dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^m\,dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^m\,\sqrt{b\,x^3+a}\,dx$$

input
```
int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(1/2),x)
```

output
```
int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int (ex)^m \sqrt{a+bx^3}\left(A+Bx^3\right) \, dx = \frac{e^m \left(4x^m \sqrt{b\,x^3+a}\,amx + 28x^m \sqrt{b\,x^3+a}\,ax + 4x^m \sqrt{b\,x^3+a}\,bm\,x^4 + 10x^m \sqrt{b\,x^3+a}\,b\,x^4 + 108\left(\int \frac{}{4b\,m^2}\right.\right.}{}$$

input
```
int((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

output
```
(e**m*(4*x**m*sqrt(a + b*x**3)*a*m*x + 28*x**m*sqrt(a + b*x**3)*a*x + 4*x*
*m*sqrt(a + b*x**3)*b*m*x**4 + 10*x**m*sqrt(a + b*x**3)*b*x**4 + 108*int((
x**m*sqrt(a + b*x**3))/(4*a*m**2 + 32*a*m + 55*a + 4*b*m**2*x**3 + 32*b*m*
x**3 + 55*b*x**3),x)*a**2*m**2 + 864*int((x**m*sqrt(a + b*x**3))/(4*a*m**2
 + 32*a*m + 55*a + 4*b*m**2*x**3 + 32*b*m*x**3 + 55*b*x**3),x)*a**2*m + 14
85*int((x**m*sqrt(a + b*x**3))/(4*a*m**2 + 32*a*m + 55*a + 4*b*m**2*x**3 +
 32*b*m*x**3 + 55*b*x**3),x)*a**2))/(4*m**2 + 32*m + 55)
```

3.386 $\int \frac{(ex)^m(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{(ex)^m(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2B(ex)^{1+m}\sqrt{a+bx^3}}{be(5+2m)} + \frac{\left(\frac{A}{1+m}-\frac{2aB}{5b+2bm}\right)(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{e\sqrt{a+bx^3}}$$

output

```
2*B*(e*x)^(1+m)*(b*x^3+a)^(1/2)/b/e/(5+2*m)+(A/(1+m)-2*a*B/(2*b*m+5*b))*(e
*x)^(1+m)*(1+b*x^3/a)^(1/2)*hypergeom([1/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/
a)/e/(b*x^3+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{(ex)^m (A + Bx^3)}{\sqrt{a + bx^3}} dx$$
$$= \frac{x(ex)^m \sqrt{1 + \frac{bx^3}{a}} \left(A(4+m) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{3}, \frac{4+m}{3}, -\frac{bx^3}{a}\right) + B(1+m)x^3 \text{Hypergeometric2F}\right.}{(1+m)(4+m)\sqrt{a+bx^3}}$$

input
```
Integrate[((e*x)^m*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (b*x^3)/a]*(A*(4 + m)*Hypergeometric2F1[1/2, (1 + m)/3
, (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[1/2, (4 + m)/
3, (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A + Bx^3)(ex)^m}{\sqrt{a + bx^3}} dx$$
$$\Big\downarrow \text{959}$$
$$\frac{2B\sqrt{a+bx^3}(ex)^{m+1}}{be(2m+5)} - \frac{(2aB(m+1) - Ab(2m+5)) \int \frac{(ex)^m}{\sqrt{bx^3+a}}dx}{b(2m+5)}$$
$$\Big\downarrow \text{889}$$
$$\frac{2B\sqrt{a+bx^3}(ex)^{m+1}}{be(2m+5)} - \frac{\sqrt{\frac{bx^3}{a}+1}(2aB(m+1) - Ab(2m+5)) \int \frac{(ex)^m}{\sqrt{\frac{bx^3}{a}+1}}dx}{b(2m+5)\sqrt{a+bx^3}}$$
$$\Big\downarrow \text{888}$$

$$\frac{2B\sqrt{a+bx^3}(ex)^{m+1}}{be(2m+5)}-\frac{\sqrt{\frac{bx^3}{a}+1}(ex)^{m+1}(2aB(m+1)-Ab(2m+5))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{be(m+1)(2m+5)\sqrt{a+bx^3}}$$

input
```
Int[((e*x)^m*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(2*B*(e*x)^(1 + m)*Sqrt[a + b*x^3])/(b*e*(5 + 2*m)) - ((2*a*B*(1 + m) - A*
b*(5 + 2*m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, (1 +
 m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(5 + 2*m)*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(ex)^m (B\,x^3 + A)}{\sqrt{b\,x^3 + a}}dx$$

input
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

output
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^3)}{\sqrt{a + bx^3}}\,dx = \int \frac{(Bx^3 + A)(ex)^m}{\sqrt{bx^3 + a}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(e*x)^m/sqrt(b*x^3 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \frac{(ex)^m (A + Bx^3)}{\sqrt{a + bx^3}}\,dx = \frac{Ae^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{Be^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{m}{3}+\frac{4}{3} \\ \frac{m}{3}+\frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

input
```
integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
A*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((1/2, m/3 + 1/3), (m/3 + 4/3,), b
*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(m/3 + 4/3)) + B*e**m*x**(m + 4)*
gamma(m/3 + 4/3)*hyper((1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*
pi)/a)/(3*sqrt(a)*gamma(m/3 + 7/3))
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)(ex)^m}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^m/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)(ex)^m}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^m/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(B\,x^3+A)\,(e\,x)^m}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(1/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A + Bx^3)}{\sqrt{a + bx^3}}\, dx = \frac{e^m \left(2x^m \sqrt{b\,x^3 + a}\,x + 6\left(\int \frac{x^m \sqrt{b\,x^3+a}}{2bm\,x^3+5b\,x^3+2am+5a}dx\right) am + 15\left(\int \frac{x^m \sqrt{b\,x^3+a}}{2bm\,x^3+5b\,x^3+2am+5a}dx\right) a\right)}{2m+5}$$

input
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

output
```
(e**m*(2*x**m*sqrt(a + b*x**3)*x + 6*int((x**m*sqrt(a + b*x**3))/(2*a*m +
5*a + 2*b*m*x**3 + 5*b*x**3),x)*a*m + 15*int((x**m*sqrt(a + b*x**3))/(2*a*
m + 5*a + 2*b*m*x**3 + 5*b*x**3),x)*a))/(2*m + 5)
```

3.387 $\int \frac{(ex)^m(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{(ex)^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=-\frac{2B(ex)^{1+m}}{be(1-2m)\sqrt{a+bx^3}}+\frac{(2aB(1+m)+A(b-2bm))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{abe(1-2m)(1+m)\sqrt{a+bx^3}}$$

output
```
-2*B*(e*x)^(1+m)/b/e/(1-2*m)/(b*x^3+a)^(1/2)+(2*a*B*(1+m)+A*(-2*b*m+b))*(e
*x)^(1+m)*(1+b*x^3/a)^(1/2)*hypergeom([3/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/
a)/a/b/e/(1-2*m)/(1+m)/(b*x^3+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

$$\int \frac{(ex)^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\frac{x(ex)^m\sqrt{1+\frac{bx^3}{a}}\left(A(4+m)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+\right.}{a(1+m)(4+m)\sqrt{a+bx^3}}$$

input
```
Integrate[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (b*x^3)/a]*(A*(4 + m)*Hypergeometric2F1[3/2, (1 + m)/3
, (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[3/2, (4 + m)/
3, (7 + m)/3, -((b*x^3)/a)]))/(a*(1 + m)*(4 + m)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {957, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^3)(ex)^m}{(a+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(2aB(m+1)+A(b-2bm))\int \frac{(ex)^m}{\sqrt{bx^3+a}}dx}{3ab}+\frac{2(ex)^{m+1}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}(2aB(m+1)+A(b-2bm))\int \frac{(ex)^m}{\sqrt{\frac{bx^3}{a}+1}}dx}{3ab\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}(ex)^{m+1}(2aB(m+1)+A(b-2bm))\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{3abe(m+1)\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

input
```
Int[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*(A*b - a*B)*(e*x)^(1 + m))/(3*a*b*e*Sqrt[a + b*x^3]) + ((2*a*B*(1 + m)
+ A*(b - 2*b*m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2,
(1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(3*a*b*e*(1 + m)*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{(ex)^{m} (B\,x^{3}+A)}{(b\,x^{3}+a)^{\frac{3}{2}}}dx$$

input
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

output
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)(ex)^m}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.78 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{Ae^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\begin{matrix}\frac{3}{2}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{Be^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right) {}_2F_1\left(\begin{matrix}\frac{3}{2}, \frac{m}{3}+\frac{4}{3} \\ \frac{m}{3}+\frac{7}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

input
```
integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
A*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((3/2, m/3 + 1/3), (m/3 + 4/3,), b
*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(m/3 + 4/3)) + B*e**m*x**(m + 4)
*gamma(m/3 + 4/3)*hyper((3/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I
*pi)/a)/(3*a**(3/2)*gamma(m/3 + 7/3))
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)(ex)^m}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)(ex)^m}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(B\,x^3+A)\,(e\,x)^m}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(3/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = e^m \left(\int \frac{x^m\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)$$

input
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

output
```
e**m*int((x**m*sqrt(a + b*x**3))/(a + b*x**3),x)
```

3.388 $\int \frac{(ex)^m(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2B(ex)^{1+m}}{be(7-2m)(a+bx^3)^{3/2}} + \frac{\left(\frac{A}{1+m}+\frac{2aB}{7b-2bm}\right)(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a^2e\sqrt{a+bx^3}}$$

output
```
-2*B*(e*x)^(1+m)/b/e/(7-2*m)/(b*x^3+a)^(3/2)+(A/(1+m)+2*a*B/(-2*b*m+7*b))*
(e*x)^(1+m)*(1+b*x^3/a)^(1/2)*hypergeom([5/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^
3/a)/a^2/e/(b*x^3+a)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.92

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{x(ex)^m\sqrt{1+\frac{bx^3}{a}}\left(A(4+m)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m\right.}{a^2(1+m)(4+m)\sqrt{a+bx^3}}$$

input
```
Integrate[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (b*x^3)/a]*(A*(4 + m)*Hypergeometric2F1[5/2, (1 + m)/3
, (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[5/2, (4 + m)/
3, (7 + m)/3, -((b*x^3)/a)]))/(a^2*(1 + m)*(4 + m)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {957, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^3)(ex)^m}{(a+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(2aB(m+1)+Ab(7-2m))\int \frac{(ex)^m}{(bx^3+a)^{3/2}}dx}{9ab}+\frac{2(ex)^{m+1}(Ab-aB)}{9abe\,(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}(2aB(m+1)+Ab(7-2m))\int \frac{(ex)^m}{\left(\frac{bx^3}{a}+1\right)^{3/2}}dx}{9a^2b\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{9abe\,(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}(ex)^{m+1}(2aB(m+1)+Ab(7-2m))\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{9a^2be(m+1)\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{9abe\,(a+bx^3)^{3/2}}$$

input
```
Int[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*(A*b - a*B)*(e*x)^(1 + m))/(9*a*b*e*(a + b*x^3)^(3/2)) + ((A*b*(7 - 2*m
) + 2*a*B*(1 + m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2
, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(9*a^2*b*e*(1 + m)*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{(ex)^m\,(B\,x^3+A)}{(b\,x^3+a)^{\frac{5}{2}}}dx$$

input
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

output
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^3)}{(a + bx^3)^{5/2}} \, dx = \int \frac{(Bx^3 + A)(ex)^m}{(bx^3 + a)^{\frac{5}{2}}} \, dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^
2*b*x^3 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A + Bx^3)}{(a + bx^3)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^3)}{(a + bx^3)^{5/2}} \, dx = \int \frac{(Bx^3 + A)(ex)^m}{(bx^3 + a)^{\frac{5}{2}}} \, dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\int \frac{(Bx^3+A)(ex)^m}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\int \frac{(B\,x^3+A)\,(e\,x)^m}{(b\,x^3+a)^{5/2}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(5/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=e^m\left(\int \frac{x^m\sqrt{b\,x^3+a}}{b^2x^6+2ab\,x^3+a^2}dx\right)$$

input
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

output
```
e**m*int((x**m*sqrt(a + b*x**3))/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
```

3.389 $\int (ex)^m \left(a+bx^3\right)^{4/3} \left(A+Bx^3\right) dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int (ex)^m \left(a+bx^3\right)^{4/3} \left(A+Bx^3\right) dx = \frac{B(ex)^{1+m} \left(a+bx^3\right)^{7/3}}{be(8+m)} + \frac{a\left(\frac{A}{1+m}-\frac{aB}{b(8+m)}\right)(ex)^{1+m}\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{e\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
B*(e*x)^(1+m)*(b*x^3+a)^(7/3)/b/e/(8+m)+a*(A/(1+m)-a*B/b/(8+m))*(e*x)^(1+m
)*(b*x^3+a)^(1/3)*hypergeom([-4/3, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/e/(1+b
*x^3/a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

$$\int (ex)^m \left(a+bx^3\right)^{4/3} \left(A+Bx^3\right) dx = \frac{ax(ex)^m\sqrt[3]{a+bx^3}\left(A(4+m)\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m)x^3\,\text{Hype}\right.}{(1+m)(4+m)\sqrt[3]{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(e*x)^m*(a + b*x^3)^(4/3)*(A + B*x^3),x]
```

output
```
(a*x*(e*x)^m*(a + b*x^3)^(1/3)*(A*(4 + m)*Hypergeometric2F1[-4/3, (1 + m)/
3, (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-4/3, (4 + m
)/3, (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{4/3} \left(A+Bx^3\right) (ex)^m \, dx$$

$$\Big\downarrow \text{ 959}$$

$$\left(A-\frac{aB(m+1)}{b(m+8)}\right) \int (ex)^m \left(bx^3+a\right)^{4/3}dx+\frac{B\left(a+bx^3\right)^{7/3} (ex)^{m+1}}{be(m+8)}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{a\sqrt[3]{a+bx^3}\left(A-\frac{aB(m+1)}{b(m+8)}\right)\int (ex)^m \left(\frac{bx^3}{a}+1\right)^{4/3}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{B\left(a+bx^3\right)^{7/3} (ex)^{m+1}}{be(m+8)}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{a\sqrt[3]{a+bx^3}(ex)^{m+1}\left(A-\frac{aB(m+1)}{b(m+8)}\right)\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{e(m+1)\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{B\left(a+bx^3\right)^{7/3} (ex)^{m+1}}{be(m+8)}$$

input
```
Int[(e*x)^m*(a + b*x^3)^(4/3)*(A + B*x^3),x]
```

output
```
(B*(e*x)^(1 + m)*(a + b*x^3)^(7/3))/(b*e*(8 + m)) + (a*(A - (a*B*(1 + m))/
(b*(8 + m)))*(e*x)^(1 + m)*(a + b*x^3)^(1/3)*Hypergeometric2F1[-4/3, (1 +
m)/3, (4 + m)/3, -((b*x^3)/a)])/(e*(1 + m)*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (ex)^m \left(b\,x^3+a\right)^{\frac{4}{3}} \left(B\,x^3+A\right) dx$$

input
```
int((e*x)^m*(b*x^3+a)^(4/3)*(B*x^3+A),x)
```

output
```
int((e*x)^m*(b*x^3+a)^(4/3)*(B*x^3+A),x)
```

Fricas [F]

$$\int (ex)^m \left(a+bx^3\right)^{4/3} \left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{4}{3}}(ex)^m\,dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(4/3)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*(b*x^3 + a)^(1/3)*(e*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.65 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.12

$$\int (ex)^m \left(a+bx^3\right)^{4/3} \left(A+Bx^3\right)\,dx = \frac{Aa^{\frac{4}{3}}e^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{A\sqrt[3]{a}be^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{m}{3}+\frac{4}{3} \\ \frac{m}{3}+\frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)} + \frac{Ba^{\frac{4}{3}}e^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{m}{3}+\frac{4}{3} \\ \frac{m}{3}+\frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)} + \frac{B\sqrt[3]{a}be^m x^{m+7}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{m}{3}+\frac{7}{3} \\ \frac{m}{3}+\frac{10}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{10}{3}\right)}$$

input
```
integrate((e*x)**m*(b*x**3+a)**(4/3)*(B*x**3+A),x)
```

output
```
A*a**(4/3)*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((-1/3, m/3 + 1/3), (m/3
+ 4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + A*a**(1/3)*b*e**
m*x**(m + 4)*gamma(m/3 + 4/3)*hyper((-1/3, m/3 + 4/3), (m/3 + 7/3,), b*x**
3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 7/3)) + B*a**(4/3)*e**m*x**(m + 4)*gam
ma(m/3 + 4/3)*hyper((-1/3, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(m/3 + 7/3)) + B*a**(1/3)*b*e**m*x**(m + 7)*gamma(m/3 + 7/3)*
hyper((-1/3, m/3 + 7/3), (m/3 + 10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma
(m/3 + 10/3))
```

Maxima [F]

$$\int (ex)^m \left(a+bx^3\right)^{4/3} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{4}{3}}(ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(4/3)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(4/3)*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m \left(a+bx^3\right)^{4/3} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{4}{3}}(ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(4/3)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(4/3)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^3\right)^{4/3} \left(A+Bx^3\right) dx = \int \left(B\,x^3+A\right) \left(e\,x\right)^m \left(b\,x^3+a\right)^{4/3} dx$$

input
```
int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(4/3),x)
```

output
```
int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int (ex)^m \left(a+bx^3\right)^{4/3} \left(A +Bx^3\right) dx = \frac{e^m \left(x^m (b\,x^3+a)^{\frac{1}{3}}\, a^2 m^2 x + 14 x^m (b\,x^3+a)^{\frac{1}{3}}\, a^2 m x + 52 x^m (b\,x^3+a)^{\frac{1}{3}}\, a^2 x + 2 x^m (b\,x^3+a)^{\frac{1}{3}}\, a\right.}{}$$

input
```
int((e*x)^m*(b*x^3+a)^(4/3)*(B*x^3+A),x)
```

output
```
(e**m*(x**m*(a + b*x**3)**(1/3)*a**2*m**2*x + 14*x**m*(a + b*x**3)**(1/3)*
a**2*m*x + 52*x**m*(a + b*x**3)**(1/3)*a**2*x + 2*x**m*(a + b*x**3)**(1/3)
*a*b*m**2*x**4 + 21*x**m*(a + b*x**3)**(1/3)*a*b*m*x**4 + 34*x**m*(a + b*x
**3)**(1/3)*a*b*x**4 + x**m*(a + b*x**3)**(1/3)*b**2*m**2*x**7 + 7*x**m*(a
 + b*x**3)**(1/3)*b**2*m*x**7 + 10*x**m*(a + b*x**3)**(1/3)*b**2*x**7 + 28
*int((x**m*(a + b*x**3)**(1/3))/(a*m**3 + 15*a*m**2 + 66*a*m + 80*a + b*m*
*3*x**3 + 15*b*m**2*x**3 + 66*b*m*x**3 + 80*b*x**3),x)*a**3*m**3 + 420*int
((x**m*(a + b*x**3)**(1/3))/(a*m**3 + 15*a*m**2 + 66*a*m + 80*a + b*m**3*x
**3 + 15*b*m**2*x**3 + 66*b*m*x**3 + 80*b*x**3),x)*a**3*m**2 + 1848*int((x
**m*(a + b*x**3)**(1/3))/(a*m**3 + 15*a*m**2 + 66*a*m + 80*a + b*m**3*x**3
 + 15*b*m**2*x**3 + 66*b*m*x**3 + 80*b*x**3),x)*a**3*m + 2240*int((x**m*(a
 + b*x**3)**(1/3))/(a*m**3 + 15*a*m**2 + 66*a*m + 80*a + b*m**3*x**3 + 15*
b*m**2*x**3 + 66*b*m*x**3 + 80*b*x**3),x)*a**3))/(m**3 + 15*m**2 + 66*m +
80)
```

3.390 $\int (ex)^m \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int (ex)^m \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx = \frac{B(ex)^{1+m} \left(a+bx^3\right)^{5/3}}{be(6+m)} + \frac{\left(\frac{A}{1+m}-\frac{aB}{b(6+m)}\right) (ex)^{1+m} \left(a+bx^3\right)^{2/3} \text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{e \left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
B*(e*x)^(1+m)*(b*x^3+a)^(5/3)/b/e/(6+m)+(A/(1+m)-a*B/b/(6+m))*(e*x)^(1+m)*
(b*x^3+a)^(2/3)*hypergeom([-2/3, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/e/(1+b*x
^3/a)^(2/3)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int (ex)^m \left(a+bx^3\right)^{2/3} \left(A +Bx^3\right) \, dx = \frac{x(ex)^m \left(a+bx^3\right)^{2/3} \left(A(4+m) \text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m)x^3 \text{Hy}\right.}{(1+m)(4+m) \left(1+\frac{bx^3}{a}\right)^{2/3}}$$

input
```
Integrate[(e*x)^m*(a + b*x^3)^(2/3)*(A + B*x^3),x]
```

output
```
(x*(e*x)^m*(a + b*x^3)^(2/3)*(A*(4 + m)*Hypergeometric2F1[-2/3, (1 + m)/3,
 (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-2/3, (4 + m)/
3, (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*(1 + (b*x^3)/a)^(2/3))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) (ex)^m \, dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{aB(m+1)}{b(m+6)}\right) \int (ex)^m \left(bx^3+a\right)^{2/3}dx+\frac{B\left(a+bx^3\right)^{5/3} (ex)^{m+1}}{be(m+6)}$$

$$\Big\downarrow \text{889}$$

$$\frac{\left(a+bx^3\right)^{2/3} \left(A-\frac{aB(m+1)}{b(m+6)}\right) \int (ex)^m \left(\frac{bx^3}{a}+1\right)^{2/3}dx}{\left(\frac{bx^3}{a}+1\right)^{2/3}}+\frac{B\left(a+bx^3\right)^{5/3} (ex)^{m+1}}{be(m+6)}$$

$$\Big\downarrow \text{888}$$

$$\frac{\left(a+bx^3\right)^{2/3} (ex)^{m+1} \left(A-\frac{aB(m+1)}{b(m+6)}\right) \text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{e(m+1) \left(\frac{bx^3}{a}+1\right)^{2/3}}+\frac{B\left(a+bx^3\right)^{5/3} (ex)^{m+1}}{be(m+6)}$$

input
```
Int[(e*x)^m*(a + b*x^3)^(2/3)*(A + B*x^3),x]
```

output
```
(B*(e*x)^(1 + m)*(a + b*x^3)^(5/3))/(b*e*(6 + m)) + ((A - (a*B*(1 + m))/(b
*(6 + m)))*(e*x)^(1 + m)*(a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, (1 + m)
/3, (4 + m)/3, -((b*x^3)/a)])/(e*(1 + m)*(1 + (b*x^3)/a)^(2/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (ex)^m \left(b\,x^3+a\right)^{\frac{2}{3}} \left(B\,x^3+A\right)dx$$

input
```
int((e*x)^m*(b*x^3+a)^(2/3)*(B*x^3+A),x)
```

output
```
int((e*x)^m*(b*x^3+a)^(2/3)*(B*x^3+A),x)
```

Fricas [F]

$$\int (ex)^m \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{2}{3}}(ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(2/3)*(e*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.01 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int (ex)^m \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx = \frac{Aa^{\frac{2}{3}}e^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{Ba^{\frac{2}{3}}e^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{m}{3}+\frac{4}{3} \\ \frac{m}{3}+\frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

input
```
integrate((e*x)**m*(b*x**3+a)**(2/3)*(B*x**3+A),x)
```

output
```
A*a**(2/3)*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((-2/3, m/3 + 1/3), (m/3
+ 4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + B*a**(2/3)*e**m*
x**(m + 4)*gamma(m/3 + 4/3)*hyper((-2/3, m/3 + 4/3), (m/3 + 7/3,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(m/3 + 7/3))
```

Maxima [F]

$$\int (ex)^m \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right) \left(bx^3+a\right)^{\frac{2}{3}} (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right) \left(bx^3+a\right)^{\frac{2}{3}} (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(2/3)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(2/3)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^m\,{\left(b\,x^3+a\right)}^{2/3} \,d x$$

input
```
int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(2/3),x)
```

output
```
int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int (ex)^m \left(a+bx^3\right)^{2/3} \left(A+Bx^3\right) \, dx = \frac{e^m \left(x^m (b\,x^3+a)^{\frac{2}{3}} amx + 8x^m (b\,x^3+a)^{\frac{2}{3}} ax + x^m (b\,x^3+a)^{\frac{2}{3}} bm\,x^4 + 3x^m (b\,x^3+a)^{\frac{2}{3}} b\,x^4 + \right.}{}$$

input
```
int((e*x)^m*(b*x^3+a)^(2/3)*(B*x^3+A),x)
```

output
```
(e**m*(x**m*(a + b*x**3)**(2/3)*a*m*x + 8*x**m*(a + b*x**3)**(2/3)*a*x + x
**m*(a + b*x**3)**(2/3)*b*m*x**4 + 3*x**m*(a + b*x**3)**(2/3)*b*x**4 + 10*
int((x**m*(a + b*x**3)**(2/3))/(a*m**2 + 9*a*m + 18*a + b*m**2*x**3 + 9*b*
m*x**3 + 18*b*x**3),x)*a**2*m**2 + 90*int((x**m*(a + b*x**3)**(2/3))/(a*m*
*2 + 9*a*m + 18*a + b*m**2*x**3 + 9*b*m*x**3 + 18*b*x**3),x)*a**2*m + 180*
int((x**m*(a + b*x**3)**(2/3))/(a*m**2 + 9*a*m + 18*a + b*m**2*x**3 + 9*b*
m*x**3 + 18*b*x**3),x)*a**2))/(m**2 + 9*m + 18)
```

3.391 $\int (ex)^m \sqrt[3]{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int (ex)^m \sqrt[3]{a+bx^3}(A+Bx^3)\,dx = \frac{B(ex)^{1+m}\left(a+bx^3\right)^{4/3}}{be(5+m)} + \frac{\left(\frac{A}{1+m}-\frac{aB}{b(5+m)}\right)(ex)^{1+m}\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{e\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
B*(e*x)^(1+m)*(b*x^3+a)^(4/3)/b/e/(5+m)+(A/(1+m)-a*B/b/(5+m))*(e*x)^(1+m)*
(b*x^3+a)^(1/3)*hypergeom([-1/3, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/e/(1+b*x
^3/a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int (ex)^m \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\, dx$$
$$= \frac{x(ex)^m \sqrt[3]{a+bx^3}\left(A(4+m)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m)x^3\,\text{Hypergeometric2}\right.}{(1+m)(4+m)\sqrt[3]{1+\frac{bx^3}{a}}}$$

input

```
Integrate[(e*x)^m*(a + b*x^3)^(1/3)*(A + B*x^3),x]
```

output

```
(x*(e*x)^m*(a + b*x^3)^(1/3)*(A*(4 + m)*Hypergeometric2F1[-1/3, (1 + m)/3,
 (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-1/3, (4 + m)/
3, (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}\left(A+Bx^3\right)(ex)^m\, dx$$
$$\Big\downarrow \text{ 959}$$
$$\left(A-\frac{aB(m+1)}{b(m+5)}\right)\int (ex)^m \sqrt[3]{bx^3+a}dx+\frac{B\left(a+bx^3\right)^{4/3}(ex)^{m+1}}{be(m+5)}$$
$$\Big\downarrow \text{ 889}$$
$$\frac{\sqrt[3]{a+bx^3}\left(A-\frac{aB(m+1)}{b(m+5)}\right)\int (ex)^m \sqrt[3]{\frac{bx^3}{a}+1}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{B\left(a+bx^3\right)^{4/3}(ex)^{m+1}}{be(m+5)}$$

$$\Big\downarrow \text{888}$$

$$\frac{\sqrt[3]{a+bx^3}(ex)^{m+1}\left(A-\frac{aB(m+1)}{b(m+5)}\right)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{e(m+1)\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{B\left(a+bx^3\right)^{4/3}(ex)^{m+1}}{be(m+5)}$$

input
```
Int[(e*x)^m*(a + b*x^3)^(1/3)*(A + B*x^3),x]
```

output
```
(B*(e*x)^(1 + m)*(a + b*x^3)^(4/3))/(b*e*(5 + m)) + ((A - (a*B*(1 + m))/(b
*(5 + m)))*(e*x)^(1 + m)*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, (1 + m)
/3, (4 + m)/3, -((b*x^3)/a)])/(e*(1 + m)*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (ex)^m \left(b\,x^3+a\right)^{\frac{1}{3}} \left(B\,x^3+A\right) dx$$

input
```
int((e*x)^m*(b*x^3+a)^(1/3)*(B*x^3+A),x)
```

output
```
int((e*x)^m*(b*x^3+a)^(1/3)*(B*x^3+A),x)
```

Fricas [F]

$$\int (ex)^m \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{1}{3}}(ex)^m\,dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(1/3)*(e*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int (ex)^m \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{A\sqrt[3]{a}e^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{m}{3}+\frac{1}{3}\\\frac{m}{3}+\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{B\sqrt[3]{a}e^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{m}{3}+\frac{4}{3}\\\frac{m}{3}+\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

input
```
integrate((e*x)**m*(b*x**3+a)**(1/3)*(B*x**3+A),x)
```

output
```
A*a**(1/3)*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((-1/3, m/3 + 1/3), (m/3
+ 4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + B*a**(1/3)*e**m*
x**(m + 4)*gamma(m/3 + 4/3)*hyper((-1/3, m/3 + 4/3), (m/3 + 7/3,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(m/3 + 7/3))
```

Maxima [F]

$$\int (ex)^m \sqrt[3]{a+bx^3}(A+Bx^3)\,dx = \int (Bx^3+A)(bx^3+a)^{\frac{1}{3}}(ex)^m\,dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3)*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m \sqrt[3]{a+bx^3}(A+Bx^3)\,dx = \int (Bx^3+A)(bx^3+a)^{\frac{1}{3}}(ex)^m\,dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(1/3)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(1/3)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \sqrt[3]{a+bx^3}(A+Bx^3)\,dx = \int (B\,x^3+A)\,(e\,x)^m\,(b\,x^3+a)^{1/3}\,dx$$

input
```
int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(1/3),x)
```

output
```
int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int (ex)^m \sqrt[3]{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$= \frac{e^m \left(x^m (b\,x^3+a)^{\frac{1}{3}}\,amx + 6x^m (b\,x^3+a)^{\frac{1}{3}}\,ax + x^m (b\,x^3+a)^{\frac{1}{3}}\,bm\,x^4 + 2x^m (b\,x^3+a)^{\frac{1}{3}}\,b\,x^4 + 4\left(\int \frac{}{b\,m^2x^3\cdots}\right.\right.}{}$$

input
```
int((e*x)^m*(b*x^3+a)^(1/3)*(B*x^3+A),x)
```

output
```
(e**m*(x**m*(a + b*x**3)**(1/3)*a*m*x + 6*x**m*(a + b*x**3)**(1/3)*a*x + x
**m*(a + b*x**3)**(1/3)*b*m*x**4 + 2*x**m*(a + b*x**3)**(1/3)*b*x**4 + 4*i
nt((x**m*(a + b*x**3)**(1/3))/(a*m**2 + 7*a*m + 10*a + b*m**2*x**3 + 7*b*m
*x**3 + 10*b*x**3),x)*a**2*m**2 + 28*int((x**m*(a + b*x**3)**(1/3))/(a*m**
2 + 7*a*m + 10*a + b*m**2*x**3 + 7*b*m*x**3 + 10*b*x**3),x)*a**2*m + 40*in
t((x**m*(a + b*x**3)**(1/3))/(a*m**2 + 7*a*m + 10*a + b*m**2*x**3 + 7*b*m*
x**3 + 10*b*x**3),x)*a**2))/(m**2 + 7*m + 10)
```

3.392 $\int \frac{(ex)^m(A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{B(ex)^{1+m}(a+bx^3)^{2/3}}{be(3+m)} + \frac{\left(\frac{A}{1+m}-\frac{aB}{b(3+m)}\right)(ex)^{1+m}\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{e\sqrt[3]{a+bx^3}}$$

output

```
B*(e*x)^(1+m)*(b*x^3+a)^(2/3)/b/e/(3+m)+(A/(1+m)-a*B/b/(3+m))*(e*x)^(1+m)*
(1+b*x^3/a)^(1/3)*hypergeom([1/3, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/e/(b*x^
3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx$$
$$= \frac{x(ex)^m \sqrt[3]{1+\frac{bx^3}{a}}\left(A(4+m)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m)x^3\,\text{Hypergeometric2}\right.}{(1+m)(4+m)\sqrt[3]{a+bx^3}}$$

input
```
Integrate[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(1/3),x]
```

output
```
(x*(e*x)^m*(1 + (b*x^3)/a)^(1/3)*(A*(4 + m)*Hypergeometric2F1[1/3, (1 + m)
/3, (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[1/3, (4 + m
)/3, (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^3)(ex)^m}{\sqrt[3]{a+bx^3}}\,dx$$
$$\Big\downarrow \text{959}$$
$$\left(A-\frac{aB(m+1)}{b(m+3)}\right)\int \frac{(ex)^m}{\sqrt[3]{bx^3+a}}dx+\frac{B(a+bx^3)^{2/3}(ex)^{m+1}}{be(m+3)}$$
$$\Big\downarrow \text{889}$$
$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\left(A-\frac{aB(m+1)}{b(m+3)}\right)\int \frac{(ex)^m}{\sqrt[3]{\frac{bx^3}{a}+1}}dx}{\sqrt[3]{a+bx^3}}+\frac{B(a+bx^3)^{2/3}(ex)^{m+1}}{be(m+3)}$$

$$\Big\downarrow \text{888}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}(ex)^{m+1}\left(A-\frac{aB(m+1)}{b(m+3)}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{e(m+1)\sqrt[3]{a+bx^3}}+\frac{B\left(a+bx^3\right)^{2/3}(ex)^{m+1}}{be(m+3)}$$

input
```
Int[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(1/3),x]
```

output
```
(B*(e*x)^(1 + m)*(a + b*x^3)^(2/3))/(b*e*(3 + m)) + ((A - (a*B*(1 + m))/(b
*(3 + m)))*(e*x)^(1 + m)*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, (1 +
 m)/3, (4 + m)/3, -((b*x^3)/a)])/(e*(1 + m)*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(ex)^m (B\,x^3+A)}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/3),x)
```

output
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(Bx^3+A)(ex)^m}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.92 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \frac{Ae^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{Be^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{m}{3}+\frac{4}{3}\\ \frac{m}{3}+\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

input
```
integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(1/3),x)
```

output
```
A*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((1/3, m/3 + 1/3), (m/3 + 4/3,), b
*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(m/3 + 4/3)) + B*e**m*x**(m + 4)
*gamma(m/3 + 4/3)*hyper((1/3, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I
*pi)/a)/(3*a**(1/3)*gamma(m/3 + 7/3))
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^3)}{\sqrt[3]{a + bx^3}} dx = \int \frac{(Bx^3 + A)(ex)^m}{(bx^3 + a)^{\frac{1}{3}}} dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^3)}{\sqrt[3]{a + bx^3}} dx = \int \frac{(Bx^3 + A)(ex)^m}{(bx^3 + a)^{\frac{1}{3}}} dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(B\,x^3+A)\;(e\,x)^m}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(1/3),x)
```

output
```
int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt[3]{a+bx^3}}\,dx = e^m\left(\left(\int \frac{x^m}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)a+\left(\int \frac{x^m x^3}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)b\right)$$

input
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/3),x)
```

output
```
e**m*(int(x**m/(a + b*x**3)**(1/3),x)*a + int((x**m*x**3)/(a + b*x**3)**(1
/3),x)*b)
```

3.393 $\int \frac{(ex)^m(A+Bx^3)}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{B(ex)^{1+m}\sqrt[3]{a+bx^3}}{be(2+m)} + \frac{\left(\frac{A}{1+m}-\frac{aB}{b(2+m)}\right)(ex)^{1+m}\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{e\left(a+bx^3\right)^{2/3}}$$

output
```
B*(e*x)^(1+m)*(b*x^3+a)^(1/3)/b/e/(2+m)+(A/(1+m)-a*B/b/(2+m))*(e*x)^(1+m)*
(1+b*x^3/a)^(2/3)*hypergeom([2/3, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/e/(b*x^
3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \frac{x(ex)^m\left(1+\frac{bx^3}{a}\right)^{2/3}\left(A(4+m)\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1\right.}{(1+m)(4+m)\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(2/3),x]
```

output
```
(x*(e*x)^m*(1 + (b*x^3)/a)^(2/3)*(A*(4 + m)*Hypergeometric2F1[2/3, (1 + m)
/3, (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[2/3, (4 + m
)/3, (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^3)(ex)^m}{(a+bx^3)^{2/3}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{aB(m+1)}{b(m+2)}\right)\int \frac{(ex)^m}{(bx^3+a)^{2/3}}dx+\frac{B\sqrt[3]{a+bx^3}(ex)^{m+1}}{be(m+2)}$$

$$\Big\downarrow \text{889}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\left(A-\frac{aB(m+1)}{b(m+2)}\right)\int \frac{(ex)^m}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}+\frac{B\sqrt[3]{a+bx^3}(ex)^{m+1}}{be(m+2)}$$

$$\Big\downarrow \text{888}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(ex)^{m+1}\left(A-\frac{aB(m+1)}{b(m+2)}\right)\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{e(m+1)(a+bx^3)^{2/3}}+\frac{B\sqrt[3]{a+bx^3}(ex)^{m+1}}{be(m+2)}$$

input
```
Int[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(2/3),x]
```

output
```
(B*(e*x)^(1 + m)*(a + b*x^3)^(1/3))/(b*e*(2 + m)) + ((A - (a*B*(1 + m))/(b
*(2 + m)))*(e*x)^(1 + m)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, (1 +
 m)/3, (4 + m)/3, -((b*x^3)/a)])/(e*(1 + m)*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(ex)^m (B\,x^3 + A)}{(b\,x^3 + a)^{\frac{2}{3}}}dx$$

input
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(2/3),x)
```

output
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{2/3}}\,dx=\int \frac{(Bx^3+A)(ex)^m}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{2/3}}\,dx=\frac{Ae^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{Be^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{m}{3}+\frac{4}{3}\\ \frac{m}{3}+\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

input
```
integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(2/3),x)
```

output
```
A*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((2/3, m/3 + 1/3), (m/3 + 4/3,), b
*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(m/3 + 4/3)) + B*e**m*x**(m + 4)
*gamma(m/3 + 4/3)*hyper((2/3, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I
*pi)/a)/(3*a**(2/3)*gamma(m/3 + 7/3))
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \int \frac{(Bx^3+A)(ex)^m}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \int \frac{(Bx^3+A)(ex)^m}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{2/3}}\,dx = \int \frac{(B\,x^3+A)\,(e\,x)^m}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(2/3),x)
```

output
```
int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A + Bx^3)}{(a + bx^3)^{2/3}} \, dx = e^m \left(\left(\int \frac{x^m}{(b\,x^3 + a)^{\frac{2}{3}}} dx \right) a + \left(\int \frac{x^m x^3}{(b\,x^3 + a)^{\frac{2}{3}}} dx \right) b \right)$$

input
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(2/3),x)
```

output
```
e**m*(int(x**m/(a + b*x**3)**(2/3),x)*a + int((x**m*x**3)/(a + b*x**3)**(2
/3),x)*b)
```

3.394 $\int \frac{(ex)^m(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 113

$$\int \frac{(ex)^m(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{B(ex)^{1+m}}{bem\sqrt[3]{a+bx^3}} - \frac{\left(\frac{B}{bm}-\frac{A}{a+am}\right)(ex)^{1+m}\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{e\sqrt[3]{a+bx^3}}$$

output
```
B*(e*x)^(1+m)/b/e/m/(b*x^3+a)^(1/3)-(B/b/m-A/(a*m+a))*(e*x)^(1+m)*(1+b*x^3
/a)^(1/3)*hypergeom([4/3, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/e/(b*x^3+a)^(1/
3)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int \frac{(ex)^m(A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{x(ex)^m\sqrt[3]{1+\frac{bx^3}{a}}\left(A(4+m)\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+\right.}{a(1+m)(4+m)\sqrt[3]{a+bx^3}}$$

input
```
Integrate[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(4/3),x]
```

output
```
(x*(e*x)^m*(1 + (b*x^3)/a)^(1/3)*(A*(4 + m)*Hypergeometric2F1[4/3, (1 + m)
/3, (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[4/3, (4 + m
)/3, (7 + m)/3, -((b*x^3)/a)]))/(a*(1 + m)*(4 + m)*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {957, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^3)(ex)^m}{(a+bx^3)^{4/3}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{abe\sqrt[3]{a+bx^3}}-\frac{(Abm-aB(m+1))\int \frac{(ex)^m}{\sqrt[3]{bx^3+a}}dx}{ab}$$

$$\Big\downarrow \text{889}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{abe\sqrt[3]{a+bx^3}}-\frac{\sqrt[3]{\frac{bx^3}{a}+1}(Abm-aB(m+1))\int \frac{(ex)^m}{\sqrt[3]{\frac{bx^3}{a}+1}}dx}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{abe\sqrt[3]{a+bx^3}}-\frac{\sqrt[3]{\frac{bx^3}{a}+1}(ex)^{m+1}(Abm-aB(m+1))\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{abe(m+1)\sqrt[3]{a+bx^3}}$$

input
```
Int[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(4/3),x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m))/(a*b*e*(a + b*x^3)^(1/3)) - ((A*b*m - a*B*(1 +
 m))*(e*x)^(1 + m)*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, (1 + m)/3,
 (4 + m)/3, -((b*x^3)/a)])/(a*b*e*(1 + m)*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{(ex)^m (B\,x^3 + A)}{(b\,x^3 + a)^{\frac{4}{3}}}dx$$

input
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(4/3),x)
```

output
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{(Bx^3+A)(ex)^m}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(b*x^3 + a)^(2/3)*(e*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)
, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.70 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \frac{Ae^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{Be^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{m}{3}+\frac{4}{3}\\ \frac{m}{3}+\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

input
```
integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(4/3),x)
```

output
```
A*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((4/3, m/3 + 1/3), (m/3 + 4/3,), b
*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gamma(m/3 + 4/3)) + B*e**m*x**(m + 4)
*gamma(m/3 + 4/3)*hyper((4/3, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I
*pi)/a)/(3*a**(4/3)*gamma(m/3 + 7/3))
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{(Bx^3+A)(ex)^m}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{(Bx^3+A)(ex)^m}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{4/3}}\,dx = \int \frac{(B\,x^3+A)\,(e\,x)^m}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(4/3),x)
```

output
```
int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A + Bx^3)}{(a + bx^3)^{4/3}}\, dx = e^m \left(\int \frac{x^m}{(b\,x^3 + a)^{\frac{1}{3}}} dx \right)$$

input

```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(4/3),x)
```

output

```
e**m*int(x**m/(a + b*x**3)**(1/3),x)
```

3.395 $\int x^8(a+bx^3)^p\,(c+dx^3)\;dx$

Optimal result

Integrand size = 20, antiderivative size = 123

$$\int x^8(a+bx^3)^p\,(c+dx^3)\;dx = \frac{a^2(bc-ad)\,(a+bx^3)^{1+p}}{3b^4(1+p)} - \frac{a(2bc-3ad)\,(a+bx^3)^{2+p}}{3b^4(2+p)} + \frac{(bc-3ad)\,(a+bx^3)^{3+p}}{3b^4(3+p)} + \frac{d(a+bx^3)^{4+p}}{3b^4(4+p)}$$

output
```
1/3*a^2*(-a*d+b*c)*(b*x^3+a)^(p+1)/b^4/(p+1)-1/3*a*(-3*a*d+2*b*c)*(b*x^3+a
)^(2+p)/b^4/(2+p)+1/3*(-3*a*d+b*c)*(b*x^3+a)^(3+p)/b^4/(3+p)+1/3*d*(b*x^3+
a)^(4+p)/b^4/(4+p)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int x^8(a+bx^3)^p\,(c+dx^3)\;dx = \frac{(a+bx^3)^{1+p}\left(\frac{a^2(bc-ad)}{1+p}+\frac{a(-2bc+3ad)(a+bx^3)}{2+p}+\frac{(bc-3ad)(a+bx^3)^2}{3+p}+\frac{d(a+bx^3)^3}{4+p}\right)}{3b^4}$$

input
```
Integrate[x^8*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
((a + b*x^3)^(1 + p)*((a^2*(b*c - a*d))/(1 + p) + (a*(-2*b*c + 3*a*d)*(a +
 b*x^3))/(2 + p) + ((b*c - 3*a*d)*(a + b*x^3)^2)/(3 + p) + (d*(a + b*x^3)^
3)/(4 + p)))/(3*b^4)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(c+dx^3\right) \left(a+bx^3\right)^p \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3} \int x^6 \left(bx^3+a\right)^p \left(dx^3+c\right) dx^3$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{3} \int \left(-\frac{a^2(ad-bc) \left(bx^3+a\right)^p}{b^3}+\frac{a(3ad-2bc) \left(bx^3+a\right)^{p+1}}{b^3}+\frac{(bc-3ad) \left(bx^3+a\right)^{p+2}}{b^3}+\frac{d\left(bx^3+a\right)^{p+3}}{b^3}\right) dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} \left(\frac{a^2(bc-ad) \left(a+bx^3\right)^{p+1}}{b^4(p+1)}-\frac{a(2bc-3ad) \left(a+bx^3\right)^{p+2}}{b^4(p+2)}+\frac{(bc-3ad) \left(a+bx^3\right)^{p+3}}{b^4(p+3)}+\frac{d\left(a+bx^3\right)^{p+4}}{b^4(p+4)}\right)$$

input
```
Int[x^8*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
((a^2*(b*c - a*d)*(a + b*x^3)^(1 + p))/(b^4*(1 + p)) - (a*(2*b*c - 3*a*d)*
(a + b*x^3)^(2 + p))/(b^4*(2 + p)) + ((b*c - 3*a*d)*(a + b*x^3)^(3 + p))/(
b^4*(3 + p)) + (d*(a + b*x^3)^(4 + p))/(b^4*(4 + p)))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(115) = 230.

Time = 1.02 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.90

method	result
gosper	$-\frac{(b x^3+a)^{p+1}(-b^3 d p^3 x^9-6b^3 d p^2 x^9-11b^3 dp x^9-b^3 c p^3 x^6-6b^3 d x^9+3a b^2 d p^2 x^6-7b^3 c p^2 x^6+9a b^2 dp x^6-14b^3 cp x^6+6a b}{3b^4(p^4+10p^3+35p^2+50p+}$
orering	$-\frac{(b x^3+a)^{p}(-b^3 d p^3 x^9-6b^3 d p^2 x^9-11b^3 dp x^9-b^3 c p^3 x^6-6b^3 d x^9+3a b^2 d p^2 x^6-7b^3 c p^2 x^6+9a b^2 dp x^6-14b^3 cp x^6+6a b^2 d}{3b^4(p^4+10p^3+35p^2+50}$
risch	$-\frac{(-b^4 d p^3 x^{12}-6b^4 d p^2 x^{12}-11b^4 dp x^{12}-a b^3 d p^3 x^9-b^4 c p^3 x^9-6d x^{12} b^4-3a b^3 d p^2 x^9-7b^4 c p^2 x^9-2a b^3 dp x^9-14b^4 cp x^9-a}{3(3}$
parallelrisch	$\frac{x^9 (b x^3+a)^p a^2 b^3 d p^3+x^9 (b x^3+a)^p a b^4 c p^3+3x^9 (b x^3+a)^p a^2 b^3 d p^2+7x^9 (b x^3+a)^p a b^4 c p^2+2x^9 (b x^3+a)^p a^2 b^3 dp+14x^9 (b}{}$

input
```
int(x^8*(b*x^3+a)^p*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^4*(b*x^3+a)^(p+1)/(p^4+10*p^3+35*p^2+50*p+24)*(-b^3*d*p^3*x^9-6*b^3
*d*p^2*x^9-11*b^3*d*p*x^9-b^3*c*p^3*x^6-6*b^3*d*x^9+3*a*b^2*d*p^2*x^6-7*b^
3*c*p^2*x^6+9*a*b^2*d*p*x^6-14*b^3*c*p*x^6+6*a*b^2*d*x^6-8*b^3*c*x^6+2*a*b
^2*c*p^2*x^3-6*a^2*b*d*p*x^3+10*a*b^2*c*p*x^3-6*a^2*b*d*x^3+8*a*b^2*c*x^3-
2*a^2*b*c*p+6*a^3*d-8*a^2*b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(115) = 230$.

Time = 0.12 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.08

$$\int x^8\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx$$
$$= \frac{((b^4dp^3+6\,b^4dp^2+11\,b^4dp+6\,b^4d)x^{12}+(8\,b^4c+(b^4c+ab^3d)p^3+(7\,b^4c+3\,ab^3d)p^2+2\,(7\,b^4c+ab^3d)p}{3\,(b^4p}$$

input
```
integrate(x^8*(b*x^3+a)^p*(d*x^3+c),x, algorithm="fricas")
```

output
```
1/3*((b^4*d*p^3 + 6*b^4*d*p^2 + 11*b^4*d*p + 6*b^4*d)*x^12 + (8*b^4*c + (b
^4*c + a*b^3*d)*p^3 + (7*b^4*c + 3*a*b^3*d)*p^2 + 2*(7*b^4*c + a*b^3*d)*p)
*x^9 + (a*b^3*c*p^3 + (5*a*b^3*c - 3*a^2*b^2*d)*p^2 + (4*a*b^3*c - 3*a^2*b
^2*d)*p)*x^6 + 2*a^3*b*c*p + 8*a^3*b*c - 6*a^4*d - 2*(a^2*b^2*c*p^2 + (4*a
^2*b^2*c - 3*a^3*b*d)*p)*x^3)*(b*x^3 + a)^p/(b^4*p^4 + 10*b^4*p^3 + 35*b^4
*p^2 + 50*b^4*p + 24*b^4)
```

Sympy [F(-1)]

Timed out.

$$\int x^8\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx = \text{Timed out}$$

input
```
integrate(x**8*(b*x**3+a)**p*(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.48

$$\int x^8\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx = \frac{((p^2+3\,p+2)b^3x^9+(p^2+p)ab^2x^6-2\,a^2bpx^3+2\,a^3)(bx^3+a)^pc}{3\,(p^3+6\,p^2+11\,p+6)b^3} + \frac{((p^3+6\,p^2+11\,p+6)b^4x^{12}+(p^3+3\,p^2+2\,p)ab^3x^9-3\,(p^2+p)a^2b^2x^6+6\,a^3bpx^3-6\,a^4)(bx^3+a)^pd}{3\,(p^4+10\,p^3+35\,p^2+50\,p+24)b^4}$$

input
```
integrate(x^8*(b*x^3+a)^p*(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*((p^2 + 3*p + 2)*b^3*x^9 + (p^2 + p)*a*b^2*x^6 - 2*a^2*b*p*x^3 + 2*a^3
)*(b*x^3 + a)^p*c/((p^3 + 6*p^2 + 11*p + 6)*b^3) + 1/3*((p^3 + 6*p^2 + 11*
p + 6)*b^4*x^12 + (p^3 + 3*p^2 + 2*p)*a*b^3*x^9 - 3*(p^2 + p)*a^2*b^2*x^6
+ 6*a^3*b*p*x^3 - 6*a^4)*(b*x^3 + a)^p*d/((p^4 + 10*p^3 + 35*p^2 + 50*p +
24)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. 2(115) = 230.

Time = 0.14 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.55

$$\int x^8\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx = \frac{(bx^3+a)^3(bx^3+a)^pbcp^2-2\,(bx^3+a)^2(bx^3+a)^pabcp^2+(bx^3+a)^4(bx^3+a)^pdp^2-3\,(bx^3+a)^3(bx^3+}{} + \frac{\frac{(bx^3+a)^{p+1}a^2bc}{p+1}-\frac{(bx^3+a)^{p+1}a^3d}{p+1}}{3\,b^4}$$

input
```
integrate(x^8*(b*x^3+a)^p*(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*((b*x^3 + a)^3*(b*x^3 + a)^p*b*c*p^2 - 2*(b*x^3 + a)^2*(b*x^3 + a)^p*a
*b*c*p^2 + (b*x^3 + a)^4*(b*x^3 + a)^p*d*p^2 - 3*(b*x^3 + a)^3*(b*x^3 + a)
^p*a*d*p^2 + 3*(b*x^3 + a)^2*(b*x^3 + a)^p*a^2*d*p^2 + 6*(b*x^3 + a)^3*(b*
x^3 + a)^p*b*c*p - 14*(b*x^3 + a)^2*(b*x^3 + a)^p*a*b*c*p + 5*(b*x^3 + a)^
4*(b*x^3 + a)^p*d*p - 18*(b*x^3 + a)^3*(b*x^3 + a)^p*a*d*p + 21*(b*x^3 + a
)^2*(b*x^3 + a)^p*a^2*d*p + 8*(b*x^3 + a)^3*(b*x^3 + a)^p*b*c - 24*(b*x^3
+ a)^2*(b*x^3 + a)^p*a*b*c + 6*(b*x^3 + a)^4*(b*x^3 + a)^p*d - 24*(b*x^3 +
 a)^3*(b*x^3 + a)^p*a*d + 36*(b*x^3 + a)^2*(b*x^3 + a)^p*a^2*d)/(b^4*p^3 +
 9*b^4*p^2 + 26*b^4*p + 24*b^4) + 1/3*((b*x^3 + a)^(p + 1)*a^2*b*c/(p + 1)
 - (b*x^3 + a)^(p + 1)*a^3*d/(p + 1))/b^4
```

Mupad [B] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.89

$$\int x^8 (a+bx^3)^p (c+dx^3)\,dx = (b\,x^3+a)^p\left(\frac{d\,x^{12}\,(p^3+6\,p^2+11\,p+6)}{3\,(p^4+10\,p^3+35\,p^2+50\,p+24)} + \frac{2\,a^3\,(4\,b\,c-3\,a\,d+b\,c\,p)}{3\,b^4\,(p^4+10\,p^3+35\,p^2+50\,p+24)} + \frac{x^9\,(4\,b\,c+a\,d\,p+b\,c\,p)\,(p^2+3\,p+2)}{3\,b\,(p^4+10\,p^3+35\,p^2+50\,p+24)} - \frac{2\,a^2\,p\,x^3\,(4\,b\,c-3\,a\,d+b\,c\,p)}{3\,b^3\,(p^4+10\,p^3+35\,p^2+50\,p+24)} + \frac{a\,p\,x^6\,(p+1)\,(4\,b\,c-3\,a\,d+b\,c\,p)}{3\,b^2\,(p^4+10\,p^3+35\,p^2+50\,p+24)}\right)$$

input
```
int(x^8*(a + b*x^3)^p*(c + d*x^3),x)
```

output
```
(a + b*x^3)^p*((d*x^12*(11*p + 6*p^2 + p^3 + 6))/(3*(50*p + 35*p^2 + 10*p^
3 + p^4 + 24)) + (2*a^3*(4*b*c - 3*a*d + b*c*p))/(3*b^4*(50*p + 35*p^2 + 1
0*p^3 + p^4 + 24)) + (x^9*(4*b*c + a*d*p + b*c*p)*(3*p + p^2 + 2))/(3*b*(5
0*p + 35*p^2 + 10*p^3 + p^4 + 24)) - (2*a^2*p*x^3*(4*b*c - 3*a*d + b*c*p))
/(3*b^3*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (a*p*x^6*(p + 1)*(4*b*c - 3
*a*d + b*c*p))/(3*b^2*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.27

$$\int x^8(a+bx^3)^p(c+dx^3)\,dx$$
$$= \frac{(b\,x^3+a)^p\,(b^4d\,p^3x^{12}+6b^4d\,p^2x^{12}+11b^4dp\,x^{12}+a\,b^3d\,p^3x^9+b^4c\,p^3x^9+6b^4d\,x^{12}+3a\,b^3d\,p^2x^9+7b^4c\,p}{}$$

input
```
int(x^8*(b*x^3+a)^p*(d*x^3+c),x)
```

output
```
((a + b*x**3)**p*( - 6*a**4*d + 2*a**3*b*c*p + 8*a**3*b*c + 6*a**3*b*d*p*x
**3 - 2*a**2*b**2*c*p**2*x**3 - 8*a**2*b**2*c*p*x**3 - 3*a**2*b**2*d*p**2*
x**6 - 3*a**2*b**2*d*p*x**6 + a*b**3*c*p**3*x**6 + 5*a*b**3*c*p**2*x**6 +
4*a*b**3*c*p*x**6 + a*b**3*d*p**3*x**9 + 3*a*b**3*d*p**2*x**9 + 2*a*b**3*d
*p*x**9 + b**4*c*p**3*x**9 + 7*b**4*c*p**2*x**9 + 14*b**4*c*p*x**9 + 8*b**
4*c*x**9 + b**4*d*p**3*x**12 + 6*b**4*d*p**2*x**12 + 11*b**4*d*p*x**12 + 6
*b**4*d*x**12))/(3*b**4*(p**4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.396 $\int x^5(a+bx^3)^p(c+dx^3)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int x^5(a+bx^3)^p(c+dx^3)\,dx = -\frac{a(bc-ad)(a+bx^3)^{1+p}}{3b^3(1+p)} + \frac{(bc-2ad)(a+bx^3)^{2+p}}{3b^3(2+p)} + \frac{d(a+bx^3)^{3+p}}{3b^3(3+p)}$$

output
```
-1/3*a*(-a*d+b*c)*(b*x^3+a)^(p+1)/b^3/(p+1)+1/3*(-2*a*d+b*c)*(b*x^3+a)^(2+
p)/b^3/(2+p)+1/3*d*(b*x^3+a)^(3+p)/b^3/(3+p)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95

$$\int x^5(a+bx^3)^p(c+dx^3)\,dx = \frac{1}{3}\left(-\frac{a(bc-ad)(a+bx^3)^{1+p}}{b^3(1+p)} + \frac{(bc-2ad)(a+bx^3)^{2+p}}{b^3(2+p)} + \frac{d(a+bx^3)^{3+p}}{b^3(3+p)}\right)$$

input
```
Integrate[x^5*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
(-((a*(b*c - a*d)*(a + b*x^3)^(1 + p))/(b^3*(1 + p))) + ((b*c - 2*a*d)*(a
+ b*x^3)^(2 + p))/(b^3*(2 + p)) + (d*(a + b*x^3)^(3 + p))/(b^3*(3 + p)))/3
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(c+dx^3\right) \left(a+bx^3\right)^p dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int x^3 \left(bx^3+a\right)^p \left(dx^3+c\right) dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3} \int \left(\frac{a(ad-bc) \left(bx^3+a\right)^p}{b^2}+\frac{(bc-2ad) \left(bx^3+a\right)^{p+1}}{b^2}+\frac{d \left(bx^3+a\right)^{p+2}}{b^2}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{a(bc-ad) \left(a+bx^3\right)^{p+1}}{b^3(p+1)}+\frac{(bc-2ad) \left(a+bx^3\right)^{p+2}}{b^3(p+2)}+\frac{d \left(a+bx^3\right)^{p+3}}{b^3(p+3)}\right)$$

input
```
Int[x^5*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
(-((a*(b*c - a*d)*(a + b*x^3)^(1 + p))/(b^3*(1 + p))) + ((b*c - 2*a*d)*(a
+ b*x^3)^(2 + p))/(b^3*(2 + p)) + (d*(a + b*x^3)^(3 + p))/(b^3*(3 + p)))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.44

method	result
gosper	$\frac{\left(b\,x^{3}+a\right)^{p+1}\left(b^{2}d\,p^{2}x^{6}+3b^{2}dp\,x^{6}+2x^{6}b^{2}d+b^{2}c\,p^{2}x^{3}-2abdp\,x^{3}+4b^{2}cp\,x^{3}-2abd\,x^{3}+3b^{2}c\,x^{3}-abcp+2d\,a^{2}-3abc\right)}{3b^{3}\left(p^{3}+6p^{2}+11p+6\right)}$
orering	$\frac{\left(b\,x^{3}+a\right)\left(b^{2}d\,p^{2}x^{6}+3b^{2}dp\,x^{6}+2x^{6}b^{2}d+b^{2}c\,p^{2}x^{3}-2abdp\,x^{3}+4b^{2}cp\,x^{3}-2abd\,x^{3}+3b^{2}c\,x^{3}-abcp+2d\,a^{2}-3abc\right)\left(b\,x^{3}+a\right)^{p}}{3b^{3}\left(p^{3}+6p^{2}+11p+6\right)}$
risch	$\frac{\left(b^{3}d\,p^{2}x^{9}+3b^{3}dp\,x^{9}+2b^{3}d\,x^{9}+a\,b^{2}d\,p^{2}x^{6}+b^{3}c\,p^{2}x^{6}+a\,b^{2}dp\,x^{6}+4b^{3}cp\,x^{6}+3b^{3}c\,x^{6}+a\,b^{2}c\,p^{2}x^{3}-2a^{2}bdp\,x^{3}+3a\,b^{2}cp\,x^{3}-a^{2}bc\right.}{3(2+p)(3+p)(p+1)b^{3}}$
parallelrisch	$\frac{x^{9}\left(b\,x^{3}+a\right)^{p}b^{3}d\,p^{2}+3x^{9}\left(b\,x^{3}+a\right)^{p}b^{3}dp+2x^{9}\left(b\,x^{3}+a\right)^{p}b^{3}d+x^{6}\left(b\,x^{3}+a\right)^{p}a\,b^{2}d\,p^{2}+x^{6}\left(b\,x^{3}+a\right)^{p}b^{3}c\,p^{2}+x^{6}\left(b\,x^{3}+a\right)^{p}a\,b^{2}dp+}{}$

input
```
int(x^5*(b*x^3+a)^p*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/b^3*(b*x^3+a)^(p+1)/(p^3+6*p^2+11*p+6)*(b^2*d*p^2*x^6+3*b^2*d*p*x^6+2*
b^2*d*x^6+b^2*c*p^2*x^3-2*a*b*d*p*x^3+4*b^2*c*p*x^3-2*a*b*d*x^3+3*b^2*c*x^
3-a*b*c*p+2*a^2*d-3*a*b*c)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.83

$$\int x^5 (a + bx^3)^p (c + dx^3)\ dx$$
$$= \frac{((b^3dp^2 + 3\,b^3dp + 2\,b^3d)x^9 + (3\,b^3c + (b^3c + ab^2d)p^2 + (4\,b^3c + ab^2d)p)x^6 - a^2bcp - 3\,a^2bc + 2\,a^3d + (a}{3\,(b^3p^3 + 6\,b^3p^2 + 11\,b^3p + 6\,b^3)}$$

input
```
integrate(x^5*(b*x^3+a)^p*(d*x^3+c),x, algorithm="fricas")
```

output
```
1/3*((b^3*d*p^2 + 3*b^3*d*p + 2*b^3*d)*x^9 + (3*b^3*c + (b^3*c + a*b^2*d)*
p^2 + (4*b^3*c + a*b^2*d)*p)*x^6 - a^2*b*c*p - 3*a^2*b*c + 2*a^3*d + (a*b^
2*c*p^2 + (3*a*b^2*c - 2*a^2*b*d)*p)*x^3)*(b*x^3 + a)^p/(b^3*p^3 + 6*b^3*p
^2 + 11*b^3*p + 6*b^3)
```

Sympy [F(-1)]

Timed out.

$$\int x^5 (a + bx^3)^p (c + dx^3)\ dx = \text{Timed out}$$

input
```
integrate(x**5*(b*x**3+a)**p*(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.40

$$\int x^5 (a + bx^3)^p (c + dx^3)\ dx$$
$$= \frac{(b^2(p+1)x^6 + abpx^3 - a^2)(bx^3 + a)^p c}{3\,(p^2 + 3\,p + 2)b^2}$$
$$+ \frac{((p^2 + 3\,p + 2)b^3x^9 + (p^2 + p)ab^2x^6 - 2\,a^2bpx^3 + 2\,a^3)(bx^3 + a)^p d}{3\,(p^3 + 6\,p^2 + 11\,p + 6)b^3}$$

input
```
integrate(x^5*(b*x^3+a)^p*(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*(b^2*(p + 1)*x^6 + a*b*p*x^3 - a^2)*(b*x^3 + a)^p*c/((p^2 + 3*p + 2)*b
^2) + 1/3*((p^2 + 3*p + 2)*b^3*x^9 + (p^2 + p)*a*b^2*x^6 - 2*a^2*b*p*x^3 +
 2*a^3)*(b*x^3 + a)^p*d/((p^3 + 6*p^2 + 11*p + 6)*b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(82) = 164$.

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.32

$$\int x^5 (a + bx^3)^p (c + dx^3) \, dx$$
$$= \frac{(bx^3+a)^2 (bx^3+a)^p bcp + (bx^3+a)^3 (bx^3+a)^p dp - 2\,(bx^3+a)^2 (bx^3+a)^p adp + 3\,(bx^3+a)^2 (bx^3+a)^p b}{3\,(b^3p^2 + 5\,b^3p + 6\,b^3)}$$
$$- \frac{\frac{(bx^3+a)^{p+1}abc}{p+1} - \frac{(bx^3+a)^{p+1}a^2d}{p+1}}{3\,b^3}$$

input
```
integrate(x^5*(b*x^3+a)^p*(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*((b*x^3 + a)^2*(b*x^3 + a)^p*b*c*p + (b*x^3 + a)^3*(b*x^3 + a)^p*d*p -
 2*(b*x^3 + a)^2*(b*x^3 + a)^p*a*d*p + 3*(b*x^3 + a)^2*(b*x^3 + a)^p*b*c +
 2*(b*x^3 + a)^3*(b*x^3 + a)^p*d - 6*(b*x^3 + a)^2*(b*x^3 + a)^p*a*d)/(b^3
*p^2 + 5*b^3*p + 6*b^3) - 1/3*((b*x^3 + a)^(p + 1)*a*b*c/(p + 1) - (b*x^3
+ a)^(p + 1)*a^2*d/(p + 1))/b^3
```

Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.75

$$\int x^5 \left(a+bx^3\right)^p \left(c+dx^3\right)\,dx = \left(b\,x^3+a\right)^p\,\left(\frac{d\,x^9\,\left(p^2+3\,p+2\right)}{3\,\left(p^3+6\,p^2+11\,p+6\right)}-\frac{a^2\,\left(3\,b\,c-2\,a\,d+b\,c\,p\right)}{3\,b^3\,\left(p^3+6\,p^2+11\,p+6\right)}+\frac{x^6\,\left(p+1\right)\,\left(3\,b\,c+a\,d\,p+b\,c\,p\right)}{3\,b\,\left(p^3+6\,p^2+11\,p+6\right)}+\frac{a\,p\,x^3\,\left(3\,b\,c-2\,a\,d+b\,c\,p\right)}{3\,b^2\,\left(p^3+6\,p^2+11\,p+6\right)}\right)$$

input
```
int(x^5*(a + b*x^3)^p*(c + d*x^3),x)
```

output
```
(a + b*x^3)^p*((d*x^9*(3*p + p^2 + 2))/(3*(11*p + 6*p^2 + p^3 + 6)) - (a^2
*(3*b*c - 2*a*d + b*c*p))/(3*b^3*(11*p + 6*p^2 + p^3 + 6)) + (x^6*(p + 1)*
(3*b*c + a*d*p + b*c*p))/(3*b*(11*p + 6*p^2 + p^3 + 6)) + (a*p*x^3*(3*b*c
- 2*a*d + b*c*p))/(3*b^2*(11*p + 6*p^2 + p^3 + 6)))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.90

$$\int x^5 \left(a+bx^3\right)^p \left(c+dx^3\right)\,dx = \frac{\left(b\,x^3+a\right)^p \left(b^3 d\,p^2 x^9+3b^3 dp\,x^9+2b^3 d\,x^9+a\,b^2 d\,p^2 x^6+b^3 c\,p^2 x^6+a\,b^2 dp\,x^6+4b^3 cp\,x^6+3b^3 c\,x^6+a\,b^2 \cdots\right.}{3b^3\left(p^3+6p^2+11p+6\right)}$$

input
```
int(x^5*(b*x^3+a)^p*(d*x^3+c),x)
```

output
```
((a + b*x**3)**p*(2*a**3*d - a**2*b*c*p - 3*a**2*b*c - 2*a**2*b*d*p*x**3 +
 a*b**2*c*p**2*x**3 + 3*a*b**2*c*p*x**3 + a*b**2*d*p**2*x**6 + a*b**2*d*p*
x**6 + b**3*c*p**2*x**6 + 4*b**3*c*p*x**6 + 3*b**3*c*x**6 + b**3*d*p**2*x*
*9 + 3*b**3*d*p*x**9 + 2*b**3*d*x**9))/(3*b**3*(p**3 + 6*p**2 + 11*p + 6))
```

3.397 $\int x^2(a+bx^3)^p\,(c+dx^3)\;dx$

Optimal result

Integrand size = 20, antiderivative size = 56

$$\int x^2\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx=\frac{(bc-ad)\left(a+bx^3\right)^{1+p}}{3b^2(1+p)}+\frac{d\left(a+bx^3\right)^{2+p}}{3b^2(2+p)}$$

output
```
1/3*(-a*d+b*c)*(b*x^3+a)^(p+1)/b^2/(p+1)+1/3*d*(b*x^3+a)^(2+p)/b^2/(2+p)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int x^2\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx=\frac{\left(a+bx^3\right)^{1+p}\left(-ad+bc(2+p)+bd(1+p)x^3\right)}{3b^2(1+p)(2+p)}$$

input
```
Integrate[x^2*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
((a + b*x^3)^(1 + p)*(-(a*d) + b*c*(2 + p) + b*d*(1 + p)*x^3))/(3*b^2*(1 +
 p)*(2 + p))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(c+dx^3\right) \left(a+bx^3\right)^p \, dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3} \int \left(bx^3+a\right)^p \left(dx^3+c\right)dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3} \int \left(\frac{(bc-ad) \left(bx^3+a\right)^p}{b}+\frac{d \left(bx^3+a\right)^{p+1}}{b}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{(bc-ad) \left(a+bx^3\right)^{p+1}}{b^2 (p+1)}+\frac{d \left(a+bx^3\right)^{p+2}}{b^2 (p+2)}\right)$$

input
```
Int[x^2*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
(((b*c - a*d)*(a + b*x^3)^(1 + p))/(b^2*(1 + p)) + (d*(a + b*x^3)^(2 + p))
/(b^2*(2 + p)))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

method	result
gosper	$-\frac{\left(b\,x^{3}+a\right)^{p+1}\left(-bdp\,x^{3}-bd\,x^{3}-bcp+ad-2bc\right)}{3b^{2}\left(p^{2}+3p+2\right)}$
orering	$-\frac{\left(b\,x^{3}+a\right)^{p}\left(-bdp\,x^{3}-bd\,x^{3}-bcp+ad-2bc\right)\left(b\,x^{3}+a\right)}{3b^{2}\left(p^{2}+3p+2\right)}$
risch	$-\frac{\left(-b^{2}dp\,x^{6}-x^{6}b^{2}d-abdp\,x^{3}-b^{2}cp\,x^{3}-2b^{2}c\,x^{3}-abcp+d\,a^{2}-2abc\right)\left(b\,x^{3}+a\right)^{p}}{3b^{2}\left(2+p\right)\left(p+1\right)}$
norman	$\frac{d\,x^{6}{\mathrm e}^{p\ln\left(b\,x^{3}+a\right)}}{3p+6}-\frac{a\left(-bcp+ad-2bc\right){\mathrm e}^{p\ln\left(b\,x^{3}+a\right)}}{3b^{2}\left(p^{2}+3p+2\right)}+\frac{\left(adp+bcp+2bc\right)x^{3}{\mathrm e}^{p\ln\left(b\,x^{3}+a\right)}}{3b\left(p^{2}+3p+2\right)}$
parallelrisch	$\frac{x^{6}\left(b\,x^{3}+a\right)^{p}a\,b^{2}dp+x^{6}\left(b\,x^{3}+a\right)^{p}a\,b^{2}d+x^{3}\left(b\,x^{3}+a\right)^{p}a^{2}bdp+x^{3}\left(b\,x^{3}+a\right)^{p}a\,b^{2}cp+2x^{3}\left(b\,x^{3}+a\right)^{p}a\,b^{2}c+\left(b\,x^{3}+a\right)^{p}a^{2}bcp-(b\,x}{3b^{2}\left(p^{2}+3p+2\right)a}$

input
```
int(x^2*(b*x^3+a)^p*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^2*(b*x^3+a)^(p+1)/(p^2+3*p+2)*(-b*d*p*x^3-b*d*x^3-b*c*p+a*d-2*b*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.57

$$\int x^2\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx = \frac{((b^2dp+b^2d)x^6+abcp+(2\,b^2c+(b^2c+abd)p)x^3+2\,abc-a^2d)(bx^3+a)^p}{3\,(b^2p^2+3\,b^2p+2\,b^2)}$$

input
```
integrate(x^2*(b*x^3+a)^p*(d*x^3+c),x, algorithm="fricas")
```

output
```
1/3*((b^2*d*p + b^2*d)*x^6 + a*b*c*p + (2*b^2*c + (b^2*c + a*b*d)*p)*x^3 +
 2*a*b*c - a^2*d)*(b*x^3 + a)^p/(b^2*p^2 + 3*b^2*p + 2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(46) = 92$.

Time = 121.66 (sec) , antiderivative size = 683, normalized size of antiderivative = 12.20

$$\int x^2(a+bx^3)^p\,(c+dx^3)\;dx$$

$$= \begin{cases} a^p\left(\frac{cx^3}{3}+\frac{dx^6}{6}\right) \\ \frac{ad\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^2+3b^3x^3}+\frac{ad\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4(-\frac{a}{b})^{\frac{2}{3}}\right)}{3ab^2+3b^3x^3}-\frac{2ad\log(2)}{3ab^2+3b^3x^3}+\frac{ad}{3ab^2+3b^3x^3}-\frac{bc}{3ab^2+3b^3x^3}+\frac{bdx^3\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^2+3b^3x^3} \\ -\frac{ad\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{3b^2}-\frac{ad\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4(-\frac{a}{b})^{\frac{2}{3}}\right)}{3b^2}+\frac{c\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{3b}+\frac{c\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4(-\frac{a}{b})^{\frac{2}{3}}\right)}{3b}+\frac{dx^3}{3b} \\ -\frac{a^2d(a+bx^3)^p}{3b^2p^2+9b^2p+6b^2}+\frac{abcp(a+bx^3)^p}{3b^2p^2+9b^2p+6b^2}+\frac{2abc(a+bx^3)^p}{3b^2p^2+9b^2p+6b^2}+\frac{abdpx^3(a+bx^3)^p}{3b^2p^2+9b^2p+6b^2}+\frac{b^2cpx^3(a+bx^3)^p}{3b^2p^2+9b^2p+6b^2}+\frac{2b^2cx^3(a+bx^3)^p}{3b^2p^2+9b^2p+6b^2}+\frac{b^2dpx}{3b^2p^2} \end{cases}$$

input
```
integrate(x**2*(b*x**3+a)**p*(d*x**3+c),x)
```

output
```
Piecewise((a**p*(c*x**3/3 + d*x**6/6), Eq(b, 0)), (a*d*log(x - (-a/b)**(1/
3))/(3*a*b**2 + 3*b**3*x**3) + a*d*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/
b)**(2/3))/(3*a*b**2 + 3*b**3*x**3) - 2*a*d*log(2)/(3*a*b**2 + 3*b**3*x**3
) + a*d/(3*a*b**2 + 3*b**3*x**3) - b*c/(3*a*b**2 + 3*b**3*x**3) + b*d*x**3
*log(x - (-a/b)**(1/3))/(3*a*b**2 + 3*b**3*x**3) + b*d*x**3*log(4*x**2 + 4
*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b**2 + 3*b**3*x**3) - 2*b*d*x**3*
log(2)/(3*a*b**2 + 3*b**3*x**3), Eq(p, -2)), (-a*d*log(x - (-a/b)**(1/3))/
(3*b**2) - a*d*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*b**2)
+ c*log(x - (-a/b)**(1/3))/(3*b) + c*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-
a/b)**(2/3))/(3*b) + d*x**3/(3*b), Eq(p, -1)), (-a**2*d*(a + b*x**3)**p/(3
*b**2*p**2 + 9*b**2*p + 6*b**2) + a*b*c*p*(a + b*x**3)**p/(3*b**2*p**2 + 9
*b**2*p + 6*b**2) + 2*a*b*c*(a + b*x**3)**p/(3*b**2*p**2 + 9*b**2*p + 6*b*
*2) + a*b*d*p*x**3*(a + b*x**3)**p/(3*b**2*p**2 + 9*b**2*p + 6*b**2) + b**
2*c*p*x**3*(a + b*x**3)**p/(3*b**2*p**2 + 9*b**2*p + 6*b**2) + 2*b**2*c*x*
*3*(a + b*x**3)**p/(3*b**2*p**2 + 9*b**2*p + 6*b**2) + b**2*d*p*x**6*(a +
b*x**3)**p/(3*b**2*p**2 + 9*b**2*p + 6*b**2) + b**2*d*x**6*(a + b*x**3)**p
/(3*b**2*p**2 + 9*b**2*p + 6*b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.27

$$\int x^2 (a + bx^3)^p (c + dx^3) \, dx = \frac{(b^2(p+1)x^6 + abpx^3 - a^2)(bx^3 + a)^p d}{3(p^2 + 3p + 2)b^2} + \frac{(bx^3 + a)^{p+1} c}{3b(p+1)}$$

input
```
integrate(x^2*(b*x^3+a)^p*(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*(b^2*(p + 1)*x^6 + a*b*p*x^3 - a^2)*(b*x^3 + a)^p*d/((p^2 + 3*p + 2)*b
^2) + 1/3*(b*x^3 + a)^(p + 1)*c/(b*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int x^2(a+bx^3)^p(c+dx^3)\,dx = \frac{(bx^3+a)^2(bx^3+a)^p d}{3\,b^2(p+2)} + \frac{\frac{(bx^3+a)^{p+1}bc}{p+1} - \frac{(bx^3+a)^{p+1}ad}{p+1}}{3\,b^2}$$

input
```
integrate(x^2*(b*x^3+a)^p*(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*(b*x^3 + a)^2*(b*x^3 + a)^p*d/(b^2*(p + 2)) + 1/3*((b*x^3 + a)^(p + 1)
*b*c/(p + 1) - (b*x^3 + a)^(p + 1)*a*d/(p + 1))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.71

$$\int x^2(a+bx^3)^p(c+dx^3)\,dx = (b\,x^3+a)^p\left(\frac{a\,(2\,b\,c-a\,d+b\,c\,p)}{3\,b^2\,(p^2+3\,p+2)} + \frac{x^3\,(2\,b^2\,c+b^2\,c\,p+a\,b\,d\,p)}{3\,b^2\,(p^2+3\,p+2)} + \frac{d\,x^6\,(p+1)}{3\,(p^2+3\,p+2)}\right)$$

input
```
int(x^2*(a + b*x^3)^p*(c + d*x^3),x)
```

output
```
(a + b*x^3)^p*((a*(2*b*c - a*d + b*c*p))/(3*b^2*(3*p + p^2 + 2)) + (x^3*(2
*b^2*c + b^2*c*p + a*b*d*p))/(3*b^2*(3*p + p^2 + 2)) + (d*x^6*(p + 1))/(3*
(3*p + p^2 + 2)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.50

$$\int x^2(a+bx^3)^p(c+dx^3)\,dx = \frac{(b\,x^3+a)^p\,(b^2dp\,x^6+b^2d\,x^6+abdp\,x^3+b^2cp\,x^3+2b^2c\,x^3+abcp-a^2d+2abc)}{3b^2\,(p^2+3p+2)}$$

input
```
int(x^2*(b*x^3+a)^p*(d*x^3+c),x)
```

output
```
((a + b*x**3)**p*( - a**2*d + a*b*c*p + 2*a*b*c + a*b*d*p*x**3 + b**2*c*p*
x**3 + 2*b**2*c*x**3 + b**2*d*p*x**6 + b**2*d*x**6))/(3*b**2*(p**2 + 3*p +
 2))
```

3.398 $\int \frac{(a+bx^3)^p(c+dx^3)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{(a+bx^3)^p(c+dx^3)}{x}\,dx = \frac{d(a+bx^3)^{1+p}}{3b(1+p)} - \frac{c(a+bx^3)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^3}{a}\right)}{3a(1+p)}$$

output
```
1/3*d*(b*x^3+a)^(p+1)/b/(p+1)-1/3*c*(b*x^3+a)^(p+1)*hypergeom([1, p+1],[2+
p],1+b*x^3/a)/a/(p+1)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)^p(c+dx^3)}{x}\,dx = \frac{(a+bx^3)^{1+p}\left(ad-bc\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^3}{a}\right)\right)}{3ab(1+p)}$$

input
```
Integrate[((a + b*x^3)^p*(c + d*x^3))/x,x]
```

output
```
((a + b*x^3)^(1 + p)*(a*d - b*c*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*
x^3)/a]))/(3*a*b*(1 + p))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 90, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)(a+bx^3)^p}{x}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^p(dx^3+c)}{x^3}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(c\int \frac{(bx^3+a)^p}{x^3}dx^3+\frac{d(a+bx^3)^{p+1}}{b(p+1)}\right)$$

$$\Big\downarrow \text{75}$$

$$\frac{1}{3}\left(\frac{d(a+bx^3)^{p+1}}{b(p+1)}-\frac{c(a+bx^3)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^3}{a}+1\right)}{a(p+1)}\right)$$

input
```
Int[((a + b*x^3)^p*(c + d*x^3))/x,x]
```

output
```
((d*(a + b*x^3)^(1 + p))/(b*(1 + p)) - (c*(a + b*x^3)^(1 + p)*Hypergeometr
ic2F1[1, 1 + p, 2 + p, 1 + (b*x^3)/a])/(a*(1 + p)))/3
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{p}\,(d\,x^{3}+c)}{x}dx$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/x,x)
```

output
```
int((b*x^3+a)^p*(d*x^3+c)/x,x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{x}\,dx=\int \frac{(dx^3+c)(bx^3+a)^p}{x}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/x,x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*(b*x^3 + a)^p/x, x)
```

Sympy [A] (verification not implemented)

Time = 13.47 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{x}\,dx = -\frac{b^p c x^{3p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3\Gamma(1-p)} + d\left(\begin{cases}\frac{a^p x^3}{3} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^3)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log(a+bx^3) & \text{otherwise}\end{cases}}{3b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**3+a)**p*(d*x**3+c)/x,x)
```

output
```
-b**p*c*x**(3*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*
x**3))/(3*gamma(1 - p)) + d*Piecewise((a**p*x**3/3, Eq(b, 0)), (Piecewise(
((a + b*x**3)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**3), True))/(3*b)
, True))
```

Maxima [F]

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{x}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{x}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/x,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/x, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{x}\,dx=\int \frac{(dx^3+c)(bx^3+a)^p}{x}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/x,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{x}\,dx=\int \frac{(b\,x^3+a)^p\,(d\,x^3+c)}{x}\,dx$$

input
```
int(((a + b*x^3)^p*(c + d*x^3))/x,x)
```

output
```
int(((a + b*x^3)^p*(c + d*x^3))/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{x}\,dx$$
$$=\frac{(b\,x^3+a)^p\,adp+(b\,x^3+a)^p\,bcp+(b\,x^3+a)^p\,bc+(b\,x^3+a)^p\,bdp\,x^3+3\left(\int\frac{(b\,x^3+a)^p}{b\,x^4+ax}dx\right)abc\,p^2+3\left(\int \cdots\right.}{3bp\,(p+1)}$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/x,x)
```

output
```
((a + b*x**3)**p*a*d*p + (a + b*x**3)**p*b*c*p + (a + b*x**3)**p*b*c + (a
+ b*x**3)**p*b*d*p*x**3 + 3*int((a + b*x**3)**p/(a*x + b*x**4),x)*a*b*c*p*
*2 + 3*int((a + b*x**3)**p/(a*x + b*x**4),x)*a*b*c*p)/(3*b*p*(p + 1))
```

3.399 $\int \frac{(a+bx^3)^p(c+dx^3)}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 72

$$\int \frac{(a+bx^3)^p(c+dx^3)}{x^4}\,dx = -\frac{c(a+bx^3)^{1+p}}{3ax^3} - \frac{(ad+bcp)(a+bx^3)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^3}{a}\right)}{3a^2(1+p)}$$

output
```
-1/3*c*(b*x^3+a)^(p+1)/a/x^3-1/3*(b*c*p+a*d)*(b*x^3+a)^(p+1)*hypergeom([1,
 p+1],[2+p],1+b*x^3/a)/a^2/(p+1)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^p(c+dx^3)}{x^4}\,dx = -\frac{(a+bx^3)^{1+p}\left(ac(1+p)+(ad+bcp)x^3\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^3}{a}\right)\right)}{3a^2(1+p)x^3}$$

input
```
Integrate[((a + b*x^3)^p*(c + d*x^3))/x^4,x]
```

output
```
-1/3*((a + b*x^3)^(1 + p)*(a*c*(1 + p) + (a*d + b*c*p)*x^3*Hypergeometric2
F1[1, 1 + p, 2 + p, 1 + (b*x^3)/a]))/(a^2*(1 + p)*x^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)(a+bx^3)^p}{x^4}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^p(dx^3+c)}{x^6}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{(ad+bcp)\int \frac{(bx^3+a)^p}{x^3}dx^3}{a}-\frac{c(a+bx^3)^{p+1}}{ax^3}\right)$$

$$\Big\downarrow \text{75}$$

$$\frac{1}{3}\left(-\frac{(a+bx^3)^{p+1}(ad+bcp)\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^3}{a}+1\right)}{a^2(p+1)}-\frac{c(a+bx^3)^{p+1}}{ax^3}\right)$$

input
```
Int[((a + b*x^3)^p*(c + d*x^3))/x^4,x]
```

output
```
(-((c*(a + b*x^3)^(1 + p))/(a*x^3)) - ((a*d + b*c*p)*(a + b*x^3)^(1 + p)*H
ypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^3)/a])/(a^2*(1 + p)))/3
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(b\,x^3+a)^p\,(d\,x^3+c)}{x^4}dx$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/x^4,x)
```

output
```
int((b*x^3+a)^p*(d*x^3+c)/x^4,x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{x^4}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{x^4}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/x^4,x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*(b*x^3 + a)^p/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 58.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{x^4}\,dx = -\frac{b^p c x^{3p-3}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3\Gamma(2-p)} - \frac{b^p d x^{3p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3\Gamma(1-p)}$$

input
```
integrate((b*x**3+a)**p*(d*x**3+c)/x**4,x)
```

output
```
-b**p*c*x**(3*p - 3)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar
(I*pi)/(b*x**3))/(3*gamma(2 - p)) - b**p*d*x**(3*p)*gamma(-p)*hyper((-p, -
p), (1 - p,), a*exp_polar(I*pi)/(b*x**3))/(3*gamma(1 - p))
```

Maxima [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{x^4}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{x^4}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/x^4,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{x^4}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{x^4}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/x^4,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^p (c+dx^3)}{x^4}\,dx = \int \frac{(b\,x^3+a)^p\,(d\,x^3+c)}{x^4}\,dx$$

input
```
int(((a + b*x^3)^p*(c + d*x^3))/x^4,x)
```

output
```
int(((a + b*x^3)^p*(c + d*x^3))/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{x^4} \, dx$$
$$= \frac{-(b\,x^3+a)^p\,cp+(b\,x^3+a)^p\,d\,x^3+3\left(\int \frac{(b\,x^3+a)^p}{b\,x^4+ax}dx\right)adp\,x^3+3\left(\int \frac{(b\,x^3+a)^p}{b\,x^4+ax}dx\right)bc\,p^2x^3}{3p\,x^3}$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/x^4,x)
```

output
```
( - (a + b*x**3)**p*c*p + (a + b*x**3)**p*d*x**3 + 3*int((a + b*x**3)**p/(
a*x + b*x**4),x)*a*d*p*x**3 + 3*int((a + b*x**3)**p/(a*x + b*x**4),x)*b*c*
p**2*x**3)/(3*p*x**3)
```

3.400 $\int x^3(a+bx^3)^p\,(c+dx^3)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int x^3\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx=\frac{dx^4\left(a+bx^3\right)^{1+p}}{b(7+3p)}+\frac{1}{4}\left(c-\frac{4ad}{7b+3bp}\right)x^4\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{4}{3},-p,\frac{7}{3},-\frac{bx^3}{a}\right)$$

output
```
d*x^4*(b*x^3+a)^(p+1)/b/(7+3*p)+1/4*(c-4*a*d/(3*b*p+7*b))*x^4*(b*x^3+a)^p*
hypergeom([4/3, -p],[7/3],-b*x^3/a)/((1+b*x^3/a)^p)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

$$\int x^3\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx=\frac{1}{28}x^4\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(7c\,\text{Hypergeometric2F1}\left(\frac{4}{3},-p,\frac{7}{3},-\frac{bx^3}{a}\right)+4dx^3\,\text{Hypergeometric2F1}\left(\frac{7}{3},-p,\frac{10}{3},-\frac{bx^3}{a}\right)\right)$$

input
```
Integrate[x^3*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
(x^4*(a + b*x^3)^p*(7*c*Hypergeometric2F1[4/3, -p, 7/3, -((b*x^3)/a)] + 4*
d*x^3*Hypergeometric2F1[7/3, -p, 10/3, -((b*x^3)/a)]))/(28*(1 + (b*x^3)/a)
^p)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(c+dx^3\right)\left(a+bx^3\right)^p\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{4ad}{3bp+7b}\right)\int x^3\left(bx^3+a\right)^p\,dx+\frac{dx^4\left(a+bx^3\right)^{p+1}}{b(3p+7)}$$

$$\Big\downarrow \text{889}$$

$$\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(c-\frac{4ad}{3bp+7b}\right)\int x^3\left(\frac{bx^3}{a}+1\right)^p\,dx+\frac{dx^4\left(a+bx^3\right)^{p+1}}{b(3p+7)}$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{4}x^4\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(c-\frac{4ad}{3bp+7b}\right)\text{Hypergeometric2F1}\left(\frac{4}{3},-p,\frac{7}{3},-\frac{bx^3}{a}\right)+\frac{dx^4\left(a+bx^3\right)^{p+1}}{b(3p+7)}$$

input
```
Int[x^3*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
(d*x^4*(a + b*x^3)^(1 + p))/(b*(7 + 3*p)) + ((c - (4*a*d)/(7*b + 3*b*p))*x
^4*(a + b*x^3)^p*Hypergeometric2F1[4/3, -p, 7/3, -((b*x^3)/a)])/(4*(1 + (b
*x^3)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^{3}\left(b\,x^{3}+a\right)^{p}\left(d\,x^{3}+c\right)dx$$

input
```
int(x^3*(b*x^3+a)^p*(d*x^3+c),x)
```

output
```
int(x^3*(b*x^3+a)^p*(d*x^3+c),x)
```

Fricas [F]

$$\int x^3\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx=\int\left(dx^3+c\right)\left(bx^3+a\right)^px^3\,dx$$

input
```
integrate(x^3*(b*x^3+a)^p*(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((d*x^6 + c*x^3)*(b*x^3 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 96.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int x^3(a+bx^3)^p\,(c+dx^3)\;dx = \frac{a^p c x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix}\frac{4}{3}, -p \\ \frac{7}{3}\end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^p d x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix}\frac{7}{3}, -p \\ \frac{10}{3}\end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**3*(b*x**3+a)**p*(d*x**3+c),x)
```

output
```
a**p*c*x**4*gamma(4/3)*hyper((4/3, -p), (7/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(7/3)) + a**p*d*x**7*gamma(7/3)*hyper((7/3, -p), (10/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*gamma(10/3))
```

Maxima [F]

$$\int x^3(a+bx^3)^p\,(c+dx^3)\;dx = \int (dx^3+c)\,(bx^3+a)^p x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a)^p*(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p*x^3, x)
```

Giac [F]

$$\int x^3 (a + bx^3)^p (c + dx^3) dx = \int (dx^3 + c) (bx^3 + a)^p x^3 dx$$

input
```
integrate(x^3*(b*x^3+a)^p*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (a + bx^3)^p (c + dx^3) dx = \int x^3 (b x^3 + a)^p (d x^3 + c) dx$$

input
```
int(x^3*(a + b*x^3)^p*(c + d*x^3),x)
```

output
```
int(x^3*(a + b*x^3)^p*(c + d*x^3), x)
```

Reduce [F]

$$\int x^3 (a + bx^3)^p (c + dx^3) dx = \text{Too large to display}$$

input
```
int(x^3*(b*x^3+a)^p*(d*x^3+c),x)
```

output
```
( - 12*(a + b*x**3)**p*a**2*d*p*x + 9*(a + b*x**3)**p*a*b*c*p**2*x + 21*(a
 + b*x**3)**p*a*b*c*p*x + 9*(a + b*x**3)**p*a*b*d*p**2*x**4 + 3*(a + b*x**
3)**p*a*b*d*p*x**4 + 9*(a + b*x**3)**p*b**2*c*p**2*x**4 + 24*(a + b*x**3)*
*p*b**2*c*p*x**4 + 7*(a + b*x**3)**p*b**2*c*x**4 + 9*(a + b*x**3)**p*b**2*
d*p**2*x**7 + 15*(a + b*x**3)**p*b**2*d*p*x**7 + 4*(a + b*x**3)**p*b**2*d*
x**7 + 324*int((a + b*x**3)**p/(27*a*p**3 + 108*a*p**2 + 117*a*p + 28*a +
27*b*p**3*x**3 + 108*b*p**2*x**3 + 117*b*p*x**3 + 28*b*x**3),x)*a**3*d*p**
4 + 1296*int((a + b*x**3)**p/(27*a*p**3 + 108*a*p**2 + 117*a*p + 28*a + 27
*b*p**3*x**3 + 108*b*p**2*x**3 + 117*b*p*x**3 + 28*b*x**3),x)*a**3*d*p**3
 + 1404*int((a + b*x**3)**p/(27*a*p**3 + 108*a*p**2 + 117*a*p + 28*a + 27*b
*p**3*x**3 + 108*b*p**2*x**3 + 117*b*p*x**3 + 28*b*x**3),x)*a**3*d*p**2 +
336*int((a + b*x**3)**p/(27*a*p**3 + 108*a*p**2 + 117*a*p + 28*a + 27*b*p*
*3*x**3 + 108*b*p**2*x**3 + 117*b*p*x**3 + 28*b*x**3),x)*a**3*d*p - 243*in
t((a + b*x**3)**p/(27*a*p**3 + 108*a*p**2 + 117*a*p + 28*a + 27*b*p**3*x**
3 + 108*b*p**2*x**3 + 117*b*p*x**3 + 28*b*x**3),x)*a**2*b*c*p**5 - 1539*in
t((a + b*x**3)**p/(27*a*p**3 + 108*a*p**2 + 117*a*p + 28*a + 27*b*p**3*x**
3 + 108*b*p**2*x**3 + 117*b*p*x**3 + 28*b*x**3),x)*a**2*b*c*p**4 - 3321*in
t((a + b*x**3)**p/(27*a*p**3 + 108*a*p**2 + 117*a*p + 28*a + 27*b*p**3*x**
3 + 108*b*p**2*x**3 + 117*b*p*x**3 + 28*b*x**3),x)*a**2*b*c*p**3 - 2709*in
t((a + b*x**3)**p/(27*a*p**3 + 108*a*p**2 + 117*a*p + 28*a + 27*b*p**3*...
```

3.401 $\int x(a+bx^3)^p (c+dx^3) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 92

$$\int x(a+bx^3)^p (c+dx^3) \, dx = \frac{dx^2(a+bx^3)^{1+p}}{b(5+3p)} + \frac{1}{2}\left(c-\frac{2ad}{5b+3bp}\right)x^2(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},-\frac{bx^3}{a}\right)$$

output
```
d*x^2*(b*x^3+a)^(p+1)/b/(5+3*p)+1/2*(c-2*a*d/(3*b*p+5*b))*x^2*(b*x^3+a)^p*
hypergeom([2/3, -p],[5/3],-b*x^3/a)/((1+b*x^3/a)^p)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

$$\int x(a+bx^3)^p (c+dx^3) \, dx = \frac{1}{10}x^2(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(5c\,\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},-\frac{bx^3}{a}\right)+2dx^3\,\text{Hypergeometric2F1}\left(\frac{5}{3},-p,\frac{8}{3},-\frac{bx^3}{a}\right)\right)$$

input
```
Integrate[x*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
(x^2*(a + b*x^3)^p*(5*c*Hypergeometric2F1[2/3, -p, 5/3, -((b*x^3)/a)] + 2*
d*x^3*Hypergeometric2F1[5/3, -p, 8/3, -((b*x^3)/a)]))/(10*(1 + (b*x^3)/a)^
p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(c+dx^3)\left(a+bx^3\right)^p dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{2ad}{3bp+5b}\right)\int x\left(bx^3+a\right)^p dx+\frac{dx^2\left(a+bx^3\right)^{p+1}}{b(3p+5)}$$

$$\Big\downarrow \text{889}$$

$$\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(c-\frac{2ad}{3bp+5b}\right)\int x\left(\frac{bx^3}{a}+1\right)^p dx+\frac{dx^2\left(a+bx^3\right)^{p+1}}{b(3p+5)}$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{2}x^2\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(c-\frac{2ad}{3bp+5b}\right)\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},-\frac{bx^3}{a}\right)+\frac{dx^2\left(a+bx^3\right)^{p+1}}{b(3p+5)}$$

input
```
Int[x*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
(d*x^2*(a + b*x^3)^(1 + p))/(b*(5 + 3*p)) + ((c - (2*a*d)/(5*b + 3*b*p))*x
^2*(a + b*x^3)^p*Hypergeometric2F1[2/3, -p, 5/3, -((b*x^3)/a)])/(2*(1 + (b
*x^3)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x\left(b\,x^{3}+a\right)^{p}\left(d\,x^{3}+c\right)dx$$

input
```
int(x*(b*x^3+a)^p*(d*x^3+c),x)
```

output
```
int(x*(b*x^3+a)^p*(d*x^3+c),x)
```

Fricas [F]

$$\int x\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx=\int\left(dx^3+c\right)\left(bx^3+a\right)^px\,dx$$

input
```
integrate(x*(b*x^3+a)^p*(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((d*x^4 + c*x)*(b*x^3 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 64.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int x(a+bx^3)^p(c+dx^3)\,dx = \frac{a^p c x^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{2}{3}, -p\\ \frac{5}{3}\end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{a^p d x^5 \Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{5}{3}, -p\\ \frac{8}{3}\end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x*(b*x**3+a)**p*(d*x**3+c),x)
```

output
```
a**p*c*x**2*gamma(2/3)*hyper((2/3, -p), (5/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(5/3)) + a**p*d*x**5*gamma(5/3)*hyper((5/3, -p), (8/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*gamma(8/3))
```

Maxima [F]

$$\int x(a+bx^3)^p(c+dx^3)\,dx = \int (dx^3+c)(bx^3+a)^p x\,dx$$

input
```
integrate(x*(b*x^3+a)^p*(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p*x, x)
```

Giac [F]

$$\int x\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx=\int \left(dx^3+c\right)\left(bx^3+a\right)^p x\,dx$$

input
```
integrate(x*(b*x^3+a)^p*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx=\int x\,\left(b\,x^3+a\right)^p\,\left(d\,x^3+c\right)\,dx$$

input
```
int(x*(a + b*x^3)^p*(c + d*x^3),x)
```

output
```
int(x*(a + b*x^3)^p*(c + d*x^3), x)
```

Reduce [F]

$$\int x\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx$$
$$=\frac{3(b\,x^3+a)^p\,adp\,x^2+3(b\,x^3+a)^p\,bcp\,x^2+5(b\,x^3+a)^p\,bc\,x^2+3(b\,x^3+a)^p\,bdp\,x^5+2(b\,x^3+a)^p\,bd\,x^5-}{}$$

input
```
int(x*(b*x^3+a)^p*(d*x^3+c),x)
```

output

```
(3*(a + b*x**3)**p*a*d*p*x**2 + 3*(a + b*x**3)**p*b*c*p*x**2 + 5*(a + b*x*
*3)**p*b*c*x**2 + 3*(a + b*x**3)**p*b*d*p*x**5 + 2*(a + b*x**3)**p*b*d*x**
5 - 54*int(((a + b*x**3)**p*x)/(9*a*p**2 + 21*a*p + 10*a + 9*b*p**2*x**3 +
 21*b*p*x**3 + 10*b*x**3),x)*a**2*d*p**3 - 126*int(((a + b*x**3)**p*x)/(9*
a*p**2 + 21*a*p + 10*a + 9*b*p**2*x**3 + 21*b*p*x**3 + 10*b*x**3),x)*a**2*
d*p**2 - 60*int(((a + b*x**3)**p*x)/(9*a*p**2 + 21*a*p + 10*a + 9*b*p**2*x
**3 + 21*b*p*x**3 + 10*b*x**3),x)*a**2*d*p + 81*int(((a + b*x**3)**p*x)/(9
*a*p**2 + 21*a*p + 10*a + 9*b*p**2*x**3 + 21*b*p*x**3 + 10*b*x**3),x)*a*b*
c*p**4 + 324*int(((a + b*x**3)**p*x)/(9*a*p**2 + 21*a*p + 10*a + 9*b*p**2*
x**3 + 21*b*p*x**3 + 10*b*x**3),x)*a*b*c*p**3 + 405*int(((a + b*x**3)**p*x
)/(9*a*p**2 + 21*a*p + 10*a + 9*b*p**2*x**3 + 21*b*p*x**3 + 10*b*x**3),x)*
a*b*c*p**2 + 150*int(((a + b*x**3)**p*x)/(9*a*p**2 + 21*a*p + 10*a + 9*b*p
**2*x**3 + 21*b*p*x**3 + 10*b*x**3),x)*a*b*c*p)/(b*(9*p**2 + 21*p + 10))
```

3.402 $\int (a+bx^3)^p (c+dx^3)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int (a+bx^3)^p (c+dx^3)\,dx = \frac{dx(a+bx^3)^{1+p}}{b(4+3p)} + \left(c-\frac{ad}{4b+3bp}\right)x(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},-\frac{bx^3}{a}\right)$$

output
```
d*x*(b*x^3+a)^(p+1)/b/(4+3*p)+(c-a*d/(3*b*p+4*b))*x*(b*x^3+a)^p*hypergeom(
[1/3, -p],[4/3],-b*x^3/a)/((1+b*x^3/a)^p)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

$$\int (a+bx^3)^p (c+dx^3)\,dx = \frac{x(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(d(a+bx^3)\left(1+\frac{bx^3}{a}\right)^p+(-ad+bc(4+3p))\,\text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},-\right.\right.}{b(4+3p)}$$

input
```
Integrate[(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^p*(d*(a + b*x^3)*(1 + (b*x^3)/a)^p + (-(a*d) + b*c*(4 + 3*p
))*Hypergeometric2F1[1/3, -p, 4/3, -((b*x^3)/a)]))/(b*(4 + 3*p)*(1 + (b*x^
3)/a)^p)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^3) (a+bx^3)^p \, dx$$

$$\Big\downarrow \text{913}$$

$$\left(c-\frac{ad}{3bp+4b}\right)\int (bx^3+a)^p dx+\frac{dx(a+bx^3)^{p+1}}{b(3p+4)}$$

$$\Big\downarrow \text{779}$$

$$(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(c-\frac{ad}{3bp+4b}\right)\int \left(\frac{bx^3}{a}+1\right)^p dx+\frac{dx(a+bx^3)^{p+1}}{b(3p+4)}$$

$$\Big\downarrow \text{778}$$

$$x(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(c-\frac{ad}{3bp+4b}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},-\frac{bx^3}{a}\right)+\frac{dx(a+bx^3)^{p+1}}{b(3p+4)}$$

input
```
Int[(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
(d*x*(a + b*x^3)^(1 + p))/(b*(4 + 3*p)) + ((c - (a*d)/(4*b + 3*b*p))*x*(a
+ b*x^3)^p*Hypergeometric2F1[1/3, -p, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^
p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \left(b\,x^{3}+a \right)^{p} \left(d\,x^{3}+c \right)d x$$

input
```
int((b*x^3+a)^p*(d*x^3+c),x)
```

output
```
int((b*x^3+a)^p*(d*x^3+c),x)
```

Fricas [F]

$$\int (a+bx^3)^p \left(c+dx^3\right)\,dx=\int \left(dx^{3}+c\right)\left(bx^{3}+a\right)^{p}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*(b*x^3 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 43.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \left(a+bx^3\right)^p \left(c+dx^3\right)\,dx=\frac{a^p c x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},-p\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^p d x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},-p\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**3+a)**p*(d*x**3+c),x)
```

output
```
a**p*c*x*gamma(1/3)*hyper((1/3, -p), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(4/3)) + a**p*d*x**4*gamma(4/3)*hyper((4/3, -p), (7/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*gamma(7/3))
```

Maxima [F]

$$\int \left(a+bx^3\right)^p \left(c+dx^3\right)\,dx=\int \left(dx^3+c\right)\left(bx^3+a\right)^p\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p, x)
```

Giac [F]

$$\int (a+bx^3)^p (c+dx^3)\ dx = \int (dx^3+c)(bx^3+a)^p\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^3)^p (c+dx^3)\ dx = \int (b\,x^3+a)^p\,(d\,x^3+c)\ dx$$

input
```
int((a + b*x^3)^p*(c + d*x^3),x)
```

output
```
int((a + b*x^3)^p*(c + d*x^3), x)
```

Reduce [F]

$$\int (a+bx^3)^p (c+dx^3)\ dx$$
$$= \frac{3(b\,x^3+a)^p\,adpx+3(b\,x^3+a)^p\,bcpx+4(b\,x^3+a)^p\,bcx+3(b\,x^3+a)^p\,bdp\,x^4+(b\,x^3+a)^p\,bd\,x^4-27\Big(\int}{}$$

input
```
int((b*x^3+a)^p*(d*x^3+c),x)
```

output
```
(3*(a + b*x**3)**p*a*d*p*x + 3*(a + b*x**3)**p*b*c*p*x + 4*(a + b*x**3)**p
*b*c*x + 3*(a + b*x**3)**p*b*d*p*x**4 + (a + b*x**3)**p*b*d*x**4 - 27*int(
(a + b*x**3)**p/(9*a*p**2 + 15*a*p + 4*a + 9*b*p**2*x**3 + 15*b*p*x**3 + 4
*b*x**3),x)*a**2*d*p**3 - 45*int((a + b*x**3)**p/(9*a*p**2 + 15*a*p + 4*a
+ 9*b*p**2*x**3 + 15*b*p*x**3 + 4*b*x**3),x)*a**2*d*p**2 - 12*int((a + b*x
**3)**p/(9*a*p**2 + 15*a*p + 4*a + 9*b*p**2*x**3 + 15*b*p*x**3 + 4*b*x**3)
,x)*a**2*d*p + 81*int((a + b*x**3)**p/(9*a*p**2 + 15*a*p + 4*a + 9*b*p**2*
x**3 + 15*b*p*x**3 + 4*b*x**3),x)*a*b*c*p**4 + 243*int((a + b*x**3)**p/(9*
a*p**2 + 15*a*p + 4*a + 9*b*p**2*x**3 + 15*b*p*x**3 + 4*b*x**3),x)*a*b*c*p
**3 + 216*int((a + b*x**3)**p/(9*a*p**2 + 15*a*p + 4*a + 9*b*p**2*x**3 + 1
5*b*p*x**3 + 4*b*x**3),x)*a*b*c*p**2 + 48*int((a + b*x**3)**p/(9*a*p**2 +
15*a*p + 4*a + 9*b*p**2*x**3 + 15*b*p*x**3 + 4*b*x**3),x)*a*b*c*p)/(b*(9*p
**2 + 15*p + 4))
```

3.403 $\int \frac{(a+bx^3)^p(c+dx^3)}{x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{(a+bx^3)^p(c+dx^3)}{x^2} dx = -\frac{c(a+bx^3)^{1+p}}{ax} + \frac{(ad+bc(2+3p))x^2(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},-\frac{bx^3}{a}\right)}{2a}$$

output
```
-c*(b*x^3+a)^(p+1)/a/x+1/2*(a*d+b*c*(2+3*p))*x^2*(b*x^3+a)^p*hypergeom([2/
3, -p],[5/3],-b*x^3/a)/a/((1+b*x^3/a)^p)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^p(c+dx^3)}{x^2} dx = \frac{(a+bx^3)^p\left(-2c(a+bx^3)+(ad+bc(2+3p))x^3\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2ax}$$

input
```
Integrate[((a + b*x^3)^p*(c + d*x^3))/x^2,x]
```

output
```
((a + b*x^3)^p*(-2*c*(a + b*x^3) + ((a*d + b*c*(2 + 3*p))*x^3*Hypergeometr
ic2F1[2/3, -p, 5/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^p))/(2*a*x)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)(a+bx^3)^p}{x^2}dx$$

$$\Big\downarrow \text{ 955}$$

$$\frac{(ad+bc(3p+2))\int x(bx^3+a)^p dx}{a} - \frac{c(a+bx^3)^{p+1}}{ax}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}(ad+bc(3p+2))\int x\left(\frac{bx^3}{a}+1\right)^p dx}{a} - \frac{c(a+bx^3)^{p+1}}{ax}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^2(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}(ad+bc(3p+2))\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},-\frac{bx^3}{a}\right)}{2a} - \frac{c(a+bx^3)^{p+1}}{ax}$$

input
```
Int[((a + b*x^3)^p*(c + d*x^3))/x^2,x]
```

output
```
-((c*(a + b*x^3)^(1 + p))/(a*x)) + ((a*d + b*c*(2 + 3*p))*x^2*(a + b*x^3)^
p*Hypergeometric2F1[2/3, -p, 5/3, -((b*x^3)/a)])/(2*a*(1 + (b*x^3)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{(b\,x^3+a)^p\,(d\,x^3+c)}{x^2}dx$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/x^2,x)
```

output
```
int((b*x^3+a)^p*(d*x^3+c)/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{x^2}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/x^2,x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*(b*x^3 + a)^p/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 42.81 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^p (c+dx^3)}{x^2}\,dx = \frac{a^p c\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{3}, -p \\ \frac{2}{3} \end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{a^p dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\left.\begin{matrix} \frac{2}{3}, -p \\ \frac{5}{3} \end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((b*x**3+a)**p*(d*x**3+c)/x**2,x)
```

output
```
a**p*c*gamma(-1/3)*hyper((-1/3, -p), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*
x*gamma(2/3)) + a**p*d*x**2*gamma(2/3)*hyper((2/3, -p), (5/3,), b*x**3*exp
_polar(I*pi)/a)/(3*gamma(5/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{x^2}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/x^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{x^2}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/x^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{x^2}\,dx = \int \frac{(b\,x^3+a)^p\,(d\,x^3+c)}{x^2}\,dx$$

input
```
int(((a + b*x^3)^p*(c + d*x^3))/x^2,x)
```

output
```
int(((a + b*x^3)^p*(c + d*x^3))/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{x^2}\,dx$$
$$= \frac{3(b\,x^3+a)^p\,adp + 3(b\,x^3+a)^p\,bcp + 2(b\,x^3+a)^p\,bc + 3(b\,x^3+a)^p\,bdp\,x^3 - (b\,x^3+a)^p\,bd\,x^3 + 27\Big(\int \frac{}{9b\,p^{}}}{}$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/x^2,x)
```

output
```
(3*(a + b*x**3)**p*a*d*p + 3*(a + b*x**3)**p*b*c*p + 2*(a + b*x**3)**p*b*c
 + 3*(a + b*x**3)**p*b*d*p*x**3 - (a + b*x**3)**p*b*d*x**3 + 27*int((a + b
*x**3)**p/(9*a*p**2*x**2 + 3*a*p*x**2 - 2*a*x**2 + 9*b*p**2*x**5 + 3*b*p*x
**5 - 2*b*x**5),x)*a**2*d*p**3*x + 9*int((a + b*x**3)**p/(9*a*p**2*x**2 +
3*a*p*x**2 - 2*a*x**2 + 9*b*p**2*x**5 + 3*b*p*x**5 - 2*b*x**5),x)*a**2*d*p
**2*x - 6*int((a + b*x**3)**p/(9*a*p**2*x**2 + 3*a*p*x**2 - 2*a*x**2 + 9*b
*p**2*x**5 + 3*b*p*x**5 - 2*b*x**5),x)*a**2*d*p*x + 81*int((a + b*x**3)**p
/(9*a*p**2*x**2 + 3*a*p*x**2 - 2*a*x**2 + 9*b*p**2*x**5 + 3*b*p*x**5 - 2*b
*x**5),x)*a*b*c*p**4*x + 81*int((a + b*x**3)**p/(9*a*p**2*x**2 + 3*a*p*x**
2 - 2*a*x**2 + 9*b*p**2*x**5 + 3*b*p*x**5 - 2*b*x**5),x)*a*b*c*p**3*x - 12
*int((a + b*x**3)**p/(9*a*p**2*x**2 + 3*a*p*x**2 - 2*a*x**2 + 9*b*p**2*x**
5 + 3*b*p*x**5 - 2*b*x**5),x)*a*b*c*p*x)/(b*x*(9*p**2 + 3*p - 2))
```

3.404 $\int \frac{(a+bx^3)^p(c+dx^3)}{x^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 86

$$\int \frac{(a+bx^3)^p(c+dx^3)}{x^3} dx = -\frac{c(a+bx^3)^{1+p}}{2ax^2} + \frac{(2ad+b(c+3cp))x(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},-\frac{bx^3}{a}\right)}{2a}$$

output
```
-1/2*c*(b*x^3+a)^(p+1)/a/x^2+1/2*(2*a*d+b*(3*c*p+c))*x*(b*x^3+a)^p*hyperge
om([1/3, -p],[4/3],-b*x^3/a)/a/((1+b*x^3/a)^p)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^p(c+dx^3)}{x^3} dx = \frac{(a+bx^3)^p\left(-c(a+bx^3)+(2ad+b(c+3cp))x^3\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},-\frac{bx^3}{a}\right)\right)}{2ax^2}$$

input
```
Integrate[((a + b*x^3)^p*(c + d*x^3))/x^3,x]
```

output
```
((a + b*x^3)^p*(-(c*(a + b*x^3)) + ((2*a*d + b*(c + 3*c*p))*x^3*Hypergeome
tric2F1[1/3, -p, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^p))/(2*a*x^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {955, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)(a+bx^3)^p}{x^3}\,dx$$

$$\Big\downarrow \text{ 955}$$

$$\frac{(2ad+b(3cp+c))\int (bx^3+a)^p\,dx}{2a}-\frac{c(a+bx^3)^{p+1}}{2ax^2}$$

$$\Big\downarrow \text{ 779}$$

$$\frac{(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}(2ad+b(3cp+c))\int \left(\frac{bx^3}{a}+1\right)^p dx}{2a}-\frac{c(a+bx^3)^{p+1}}{2ax^2}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{x(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}(2ad+b(3cp+c))\,\text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},-\frac{bx^3}{a}\right)}{2a}-\frac{c(a+bx^3)^{p+1}}{2ax^2}$$

input
```
Int[((a + b*x^3)^p*(c + d*x^3))/x^3,x]
```

output
```
-1/2*(c*(a + b*x^3)^(1 + p))/(a*x^2) + ((2*a*d + b*(c + 3*c*p))*x*(a + b*x
^3)^p*Hypergeometric2F1[1/3, -p, 4/3, -((b*x^3)/a)])/(2*a*(1 + (b*x^3)/a)^
p)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{(b\,x^3+a)^p\,(d\,x^3+c)}{x^3}dx$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/x^3,x)
```

output
```
int((b*x^3+a)^p*(d*x^3+c)/x^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^p(c+dx^3)}{x^3}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/x^3,x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*(b*x^3 + a)^p/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 46.88 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^p(c+dx^3)}{x^3}\,dx = \frac{a^p c\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-p\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{a^p dx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},-p\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b*x**3+a)**p*(d*x**3+c)/x**3,x)
```

output
```
a**p*c*gamma(-2/3)*hyper((-2/3, -p), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*
x**2*gamma(1/3)) + a**p*d*x*gamma(1/3)*hyper((1/3, -p), (4/3,), b*x**3*exp
_polar(I*pi)/a)/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{x^3}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/x^3,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{x^3}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/x^3,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^p (c+dx^3)}{x^3}\,dx = \int \frac{(b\,x^3+a)^p\,(d\,x^3+c)}{x^3}\,dx$$

input
```
int(((a + b*x^3)^p*(c + d*x^3))/x^3,x)
```

output
```
int(((a + b*x^3)^p*(c + d*x^3))/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{x^3}\,dx$$
$$= \frac{3(b\,x^3+a)^p\,adp+3(b\,x^3+a)^p\,bcp+(b\,x^3+a)^p\,bc+3(b\,x^3+a)^p\,bdp\,x^3-2(b\,x^3+a)^p\,bd\,x^3+54\Big(\int \frac{}{9b\,p^{}}}{}$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/x^3,x)
```

output
```
(3*(a + b*x**3)**p*a*d*p + 3*(a + b*x**3)**p*b*c*p + (a + b*x**3)**p*b*c +
 3*(a + b*x**3)**p*b*d*p*x**3 - 2*(a + b*x**3)**p*b*d*x**3 + 54*int((a + b
*x**3)**p/(9*a*p**2*x**3 - 3*a*p*x**3 - 2*a*x**3 + 9*b*p**2*x**6 - 3*b*p*x
**6 - 2*b*x**6),x)*a**2*d*p**3*x**2 - 18*int((a + b*x**3)**p/(9*a*p**2*x**
3 - 3*a*p*x**3 - 2*a*x**3 + 9*b*p**2*x**6 - 3*b*p*x**6 - 2*b*x**6),x)*a**2
*d*p**2*x**2 - 12*int((a + b*x**3)**p/(9*a*p**2*x**3 - 3*a*p*x**3 - 2*a*x*
*3 + 9*b*p**2*x**6 - 3*b*p*x**6 - 2*b*x**6),x)*a**2*d*p*x**2 + 81*int((a +
 b*x**3)**p/(9*a*p**2*x**3 - 3*a*p*x**3 - 2*a*x**3 + 9*b*p**2*x**6 - 3*b*p
*x**6 - 2*b*x**6),x)*a*b*c*p**4*x**2 - 27*int((a + b*x**3)**p/(9*a*p**2*x*
*3 - 3*a*p*x**3 - 2*a*x**3 + 9*b*p**2*x**6 - 3*b*p*x**6 - 2*b*x**6),x)*a*b
*c*p**2*x**2 - 6*int((a + b*x**3)**p/(9*a*p**2*x**3 - 3*a*p*x**3 - 2*a*x**
3 + 9*b*p**2*x**6 - 3*b*p*x**6 - 2*b*x**6),x)*a*b*c*p*x**2)/(b*x**2*(9*p**
2 - 3*p - 2))
```

3.405 $\int \frac{(a+bx^3)^p (c+dx^3)}{x^5} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{(a+bx^3)^p (c+dx^3)}{x^5} \, dx = -\frac{c(a+bx^3)^{1+p}}{4ax^4} - \frac{(4ad-bc(1-3p))(a+bx^3)^p \left(1+\frac{bx^3}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{3},-p,\frac{2}{3},-\frac{bx^3}{a}\right)}{4ax}$$

output
```
-1/4*c*(b*x^3+a)^(p+1)/a/x^4-1/4*(4*a*d-b*c*(1-3*p))*(b*x^3+a)^p*hypergeom
([-1/3, -p],[2/3],-b*x^3/a)/a/x/((1+b*x^3/a)^p)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^p (c+dx^3)}{x^5} \, dx = \frac{(a+bx^3)^p \left(-c(a+bx^3)-(4ad+bc(-1+3p))x^3\left(1+\frac{bx^3}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{3},-p,\frac{2}{3},-\frac{bx^3}{a}\right)\right)}{4ax^4}$$

input
```
Integrate[((a + b*x^3)^p*(c + d*x^3))/x^5,x]
```

output
```
((a + b*x^3)^p*(-(c*(a + b*x^3)) - ((4*a*d + b*c*(-1 + 3*p))*x^3*Hypergeom
etric2F1[-1/3, -p, 2/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^p))/(4*a*x^4)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)(a+bx^3)^p}{x^5}\,dx$$

$$\Big\downarrow \text{ 955}$$

$$\frac{(4ad-bc(1-3p))\int \frac{(bx^3+a)^p}{x^2}dx}{4a}-\frac{c(a+bx^3)^{p+1}}{4ax^4}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}(4ad-bc(1-3p))\int \frac{\left(\frac{bx^3}{a}+1\right)^p}{x^2}dx}{4a}-\frac{c(a+bx^3)^{p+1}}{4ax^4}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}(4ad-bc(1-3p))\,\text{Hypergeometric2F1}\left(-\frac{1}{3},-p,\frac{2}{3},-\frac{bx^3}{a}\right)}{4ax}-\frac{c(a+bx^3)^{p+1}}{4ax^4}$$

input
```
Int[((a + b*x^3)^p*(c + d*x^3))/x^5,x]
```

output
```
-1/4*(c*(a + b*x^3)^(1 + p))/(a*x^4) - ((4*a*d - b*c*(1 - 3*p))*(a + b*x^3
)^p*Hypergeometric2F1[-1/3, -p, 2/3, -((b*x^3)/a)])/(4*a*x*(1 + (b*x^3)/a)
^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{(b\,x^3+a)^p\,(d\,x^3+c)}{x^5}dx$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/x^5,x)
```

output
```
int((b*x^3+a)^p*(d*x^3+c)/x^5,x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{x^5}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/x^5,x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*(b*x^3 + a)^p/x^5, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 103.96 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^p (c+dx^3)}{x^5}\,dx = \frac{a^p c\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, -p \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{a^p d\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, -p \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**p*(d*x**3+c)/x**5,x)
```

output
```
a**p*c*gamma(-4/3)*hyper((-4/3, -p), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3
*x**4*gamma(-1/3)) + a**p*d*gamma(-1/3)*hyper((-1/3, -p), (2/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{x^5}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/x^5,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/x^5, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{x^5}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/x^5,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{x^5}\,dx = \int \frac{(b\,x^3+a)^p\,(d\,x^3+c)}{x^5}\,dx$$

input
```
int(((a + b*x^3)^p*(c + d*x^3))/x^5,x)
```

output
```
int(((a + b*x^3)^p*(c + d*x^3))/x^5, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{x^5} \, dx$$
$$= \frac{3(b\,x^3+a)^p\,adp+3(b\,x^3+a)^p\,bcp-(b\,x^3+a)^p\,bc+3(b\,x^3+a)^p\,bdp\,x^3-4(b\,x^3+a)^p\,bd\,x^3+108\left(\int \frac{}{9b}\right.}{}$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/x^5,x)
```

output
```
(3*(a + b*x**3)**p*a*d*p + 3*(a + b*x**3)**p*b*c*p - (a + b*x**3)**p*b*c +
 3*(a + b*x**3)**p*b*d*p*x**3 - 4*(a + b*x**3)**p*b*d*x**3 + 108*int((a +
b*x**3)**p/(9*a*p**2*x**5 - 15*a*p*x**5 + 4*a*x**5 + 9*b*p**2*x**8 - 15*b*
p*x**8 + 4*b*x**8),x)*a**2*d*p**3*x**4 - 180*int((a + b*x**3)**p/(9*a*p**2
*x**5 - 15*a*p*x**5 + 4*a*x**5 + 9*b*p**2*x**8 - 15*b*p*x**8 + 4*b*x**8),x
)*a**2*d*p**2*x**4 + 48*int((a + b*x**3)**p/(9*a*p**2*x**5 - 15*a*p*x**5 +
 4*a*x**5 + 9*b*p**2*x**8 - 15*b*p*x**8 + 4*b*x**8),x)*a**2*d*p*x**4 + 81*
int((a + b*x**3)**p/(9*a*p**2*x**5 - 15*a*p*x**5 + 4*a*x**5 + 9*b*p**2*x**
8 - 15*b*p*x**8 + 4*b*x**8),x)*a*b*c*p**4*x**4 - 162*int((a + b*x**3)**p/(
9*a*p**2*x**5 - 15*a*p*x**5 + 4*a*x**5 + 9*b*p**2*x**8 - 15*b*p*x**8 + 4*b
*x**8),x)*a*b*c*p**3*x**4 + 81*int((a + b*x**3)**p/(9*a*p**2*x**5 - 15*a*p
*x**5 + 4*a*x**5 + 9*b*p**2*x**8 - 15*b*p*x**8 + 4*b*x**8),x)*a*b*c*p**2*x
**4 - 12*int((a + b*x**3)**p/(9*a*p**2*x**5 - 15*a*p*x**5 + 4*a*x**5 + 9*b
*p**2*x**8 - 15*b*p*x**8 + 4*b*x**8),x)*a*b*c*p*x**4)/(b*x**4*(9*p**2 - 15
*p + 4))
```

3.406 $\int (ex)^{3/2} \left(a+bx^3\right)^p \left(c+dx^3\right) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int (ex)^{3/2} \left(a+bx^3\right)^p \left(c+dx^3\right) \, dx = \frac{2d(ex)^{5/2} \left(a+bx^3\right)^{1+p}}{be(11+6p)} + \frac{2\left(c-\frac{5ad}{11b+6bp}\right) (ex)^{5/2} \left(a+bx^3\right)^p \left(1+\frac{bx^3}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{6},-p,\frac{11}{6},-\frac{bx^3}{a}\right)}{5e}$$

output
```
2*d*(e*x)^(5/2)*(b*x^3+a)^(p+1)/b/e/(11+6*p)+2/5*(c-5*a*d/(6*b*p+11*b))*(e
*x)^(5/2)*(b*x^3+a)^p*hypergeom([5/6, -p],[11/6],-b*x^3/a)/e/((1+b*x^3/a)^
p)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

$$\int (ex)^{3/2} \left(a+bx^3\right)^p \left(c+dx^3\right) \, dx = \frac{2}{55} x (ex)^{3/2} \left(a+bx^3\right)^p \left(1+\frac{bx^3}{a}\right)^{-p} \left(11c\,\text{Hypergeometric2F1}\left(\frac{5}{6},-p,\frac{11}{6},-\frac{bx^3}{a}\right)+5dx^3\,\text{Hype}\right.$$

input
```
Integrate[(e*x)^(3/2)*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
(2*x*(e*x)^(3/2)*(a + b*x^3)^p*(11*c*Hypergeometric2F1[5/6, -p, 11/6, -((b
*x^3)/a)] + 5*d*x^3*Hypergeometric2F1[11/6, -p, 17/6, -((b*x^3)/a)]))/(55*
(1 + (b*x^3)/a)^p)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3/2} \left(c+dx^3\right) \left(a+bx^3\right)^p \, dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{5ad}{6bp+11b}\right) \int (ex)^{3/2} \left(bx^3+a\right)^p dx+\frac{2d(ex)^{5/2} \left(a+bx^3\right)^{p+1}}{be(6p+11)}$$

$$\Big\downarrow \text{889}$$

$$\left(a+bx^3\right)^p \left(\frac{bx^3}{a}+1\right)^{-p} \left(c-\frac{5ad}{6bp+11b}\right) \int (ex)^{3/2} \left(\frac{bx^3}{a}+1\right)^p dx+\frac{2d(ex)^{5/2} \left(a+bx^3\right)^{p+1}}{be(6p+11)}$$

$$\Big\downarrow \text{888}$$

$$\frac{2(ex)^{5/2} \left(a+bx^3\right)^p \left(\frac{bx^3}{a}+1\right)^{-p} \left(c-\frac{5ad}{6bp+11b}\right) \text{Hypergeometric2F1}\left(\frac{5}{6},-p,\frac{11}{6},-\frac{bx^3}{a}\right)}{5e}+\frac{2d(ex)^{5/2} \left(a+bx^3\right)^{p+1}}{be(6p+11)}$$

input
```
Int[(e*x)^(3/2)*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
(2*d*(e*x)^(5/2)*(a + b*x^3)^(1 + p))/(b*e*(11 + 6*p)) + (2*(c - (5*a*d)/(
11*b + 6*b*p))*(e*x)^(5/2)*(a + b*x^3)^p*Hypergeometric2F1[5/6, -p, 11/6,
-((b*x^3)/a)])/(5*e*(1 + (b*x^3)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (ex)^{\frac{3}{2}} \left(b\,x^{3}+a\right)^{p} \left(d\,x^{3}+c\right) dx$$

input
```
int((e*x)^(3/2)*(b*x^3+a)^p*(d*x^3+c),x)
```

output
```
int((e*x)^(3/2)*(b*x^3+a)^p*(d*x^3+c),x)
```

Fricas [F]

$$\int (ex)^{3/2} \left(a+bx^3\right)^p \left(c+dx^3\right) \, dx = \int \left(dx^3+c\right)(ex)^{\frac{3}{2}} \left(bx^3+a\right)^p \, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^3+a)^p*(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((d*e*x^4 + c*e*x)*sqrt(e*x)*(b*x^3 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{3/2} \left(a+bx^3\right)^p \left(c+dx^3\right) \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(3/2)*(b*x**3+a)**p*(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{3/2} \left(a+bx^3\right)^p \left(c+dx^3\right) \, dx = \int \left(dx^3+c\right)(ex)^{\frac{3}{2}} \left(bx^3+a\right)^p \, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^3+a)^p*(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(e*x)^(3/2)*(b*x^3 + a)^p, x)
```

Giac [F]

$$\int (ex)^{3/2} \left(a+bx^3\right)^p \left(c+dx^3\right) \, dx = \int \left(dx^3+c\right)(ex)^{\frac{3}{2}} \left(bx^3+a\right)^p \, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^3+a)^p*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(e*x)^(3/2)*(b*x^3 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2} (a+bx^3)^p (c+dx^3) \, dx = \int (e\,x)^{3/2} (b\,x^3+a)^p (d\,x^3+c) \, dx$$

input
```
int((e*x)^(3/2)*(a + b*x^3)^p*(c + d*x^3),x)
```

output
```
int((e*x)^(3/2)*(a + b*x^3)^p*(c + d*x^3), x)
```

Reduce [F]

$$\int (ex)^{3/2} (a+bx^3)^p (c+dx^3) \, dx = \frac{2\sqrt{e}\,e\Big(6\sqrt{x}\,(b\,x^3+a)^p\,adp\,x^2+6\sqrt{x}\,(b\,x^3+a)^p\,bcp\,x^2+11\sqrt{x}\,(b\,x^3+a)^p\,bc\,x^2+6\sqrt{x}\,(b\,x^3+$$

input
```
int((e*x)^(3/2)*(b*x^3+a)^p*(d*x^3+c),x)
```

output
```
(2*sqrt(e)*e*(6*sqrt(x)*(a + b*x**3)**p*a*d*p*x**2 + 6*sqrt(x)*(a + b*x**3
)**p*b*c*p*x**2 + 11*sqrt(x)*(a + b*x**3)**p*b*c*x**2 + 6*sqrt(x)*(a + b*x
**3)**p*b*d*p*x**5 + 5*sqrt(x)*(a + b*x**3)**p*b*d*x**5 - 540*int((sqrt(x)
*(a + b*x**3)**p*x)/(36*a*p**2 + 96*a*p + 55*a + 36*b*p**2*x**3 + 96*b*p*x
**3 + 55*b*x**3),x)*a**2*d*p**3 - 1440*int((sqrt(x)*(a + b*x**3)**p*x)/(36
*a*p**2 + 96*a*p + 55*a + 36*b*p**2*x**3 + 96*b*p*x**3 + 55*b*x**3),x)*a**
2*d*p**2 - 825*int((sqrt(x)*(a + b*x**3)**p*x)/(36*a*p**2 + 96*a*p + 55*a
+ 36*b*p**2*x**3 + 96*b*p*x**3 + 55*b*x**3),x)*a**2*d*p + 648*int((sqrt(x)
*(a + b*x**3)**p*x)/(36*a*p**2 + 96*a*p + 55*a + 36*b*p**2*x**3 + 96*b*p*x
**3 + 55*b*x**3),x)*a*b*c*p**4 + 2916*int((sqrt(x)*(a + b*x**3)**p*x)/(36*
a*p**2 + 96*a*p + 55*a + 36*b*p**2*x**3 + 96*b*p*x**3 + 55*b*x**3),x)*a*b*
c*p**3 + 4158*int((sqrt(x)*(a + b*x**3)**p*x)/(36*a*p**2 + 96*a*p + 55*a +
 36*b*p**2*x**3 + 96*b*p*x**3 + 55*b*x**3),x)*a*b*c*p**2 + 1815*int((sqrt(
x)*(a + b*x**3)**p*x)/(36*a*p**2 + 96*a*p + 55*a + 36*b*p**2*x**3 + 96*b*p
*x**3 + 55*b*x**3),x)*a*b*c*p))/(b*(36*p**2 + 96*p + 55))
```

3.407 $\int \sqrt{ex}(a+bx^3)^p\left(c+dx^3\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \sqrt{ex}\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx = \frac{2d(ex)^{3/2}\left(a+bx^3\right)^{1+p}}{3be(3+2p)} + \frac{2\left(c-\frac{ad}{3b+2bp}\right)(ex)^{3/2}\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^3}{a}\right)}{3e}$$

output
```
2/3*d*(e*x)^(3/2)*(b*x^3+a)^(p+1)/b/e/(3+2*p)+2/3*(c-a*d/(2*b*p+3*b))*(e*x
)^(3/2)*(b*x^3+a)^p*hypergeom([1/2, -p],[3/2],-b*x^3/a)/e/((1+b*x^3/a)^p)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

$$\int \sqrt{ex}\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx = \frac{2x\sqrt{ex}(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(d(a+bx^3)\left(1+\frac{bx^3}{a}\right)^p+(-ad+bc(3+2p))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p\right.\right.}{3b(3+2p)}$$

input
```
Integrate[Sqrt[e*x]*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
(2*x*Sqrt[e*x]*(a + b*x^3)^p*(d*(a + b*x^3)*(1 + (b*x^3)/a)^p + (-(a*d) +
b*c*(3 + 2*p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^3)/a)]))/(3*b*(3 + 2
*p)*(1 + (b*x^3)/a)^p)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}\left(c+dx^3\right)\left(a+bx^3\right)^p\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{ad}{2bp+3b}\right)\int \sqrt{ex}\left(bx^3+a\right)^p dx+\frac{2d(ex)^{3/2}\left(a+bx^3\right)^{p+1}}{3be(2p+3)}$$

$$\Big\downarrow \text{889}$$

$$\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(c-\frac{ad}{2bp+3b}\right)\int \sqrt{ex}\left(\frac{bx^3}{a}+1\right)^p dx+\frac{2d(ex)^{3/2}\left(a+bx^3\right)^{p+1}}{3be(2p+3)}$$

$$\Big\downarrow \text{888}$$

$$\frac{2(ex)^{3/2}\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(c-\frac{ad}{2bp+3b}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^3}{a}\right)}{3e}+\frac{2d(ex)^{3/2}\left(a+bx^3\right)^{p+1}}{3be(2p+3)}$$

input
```
Int[Sqrt[e*x]*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
(2*d*(e*x)^(3/2)*(a + b*x^3)^(1 + p))/(3*b*e*(3 + 2*p)) + (2*(c - (a*d)/(3
*b + 2*b*p))*(e*x)^(3/2)*(a + b*x^3)^p*Hypergeometric2F1[1/2, -p, 3/2, -((
b*x^3)/a)])/(3*e*(1 + (b*x^3)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \sqrt{ex}\,(b\,x^{3}+a)^{p}\,(d\,x^{3}+c)\,dx$$

input
```
int((e*x)^(1/2)*(b*x^3+a)^p*(d*x^3+c),x)
```

output
```
int((e*x)^(1/2)*(b*x^3+a)^p*(d*x^3+c),x)
```

Fricas [F]

$$\int \sqrt{ex}\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx=\int \left(dx^3+c\right)\sqrt{ex}\left(bx^3+a\right)^p\,dx$$

input
```
integrate((e*x)^(1/2)*(b*x^3+a)^p*(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*sqrt(e*x)*(b*x^3 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{ex}\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(1/2)*(b*x**3+a)**p*(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{ex}\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx=\int \left(dx^3+c\right)\sqrt{ex}\left(bx^3+a\right)^p\,dx$$

input
```
integrate((e*x)^(1/2)*(b*x^3+a)^p*(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*sqrt(e*x)*(b*x^3 + a)^p, x)
```

Giac [F]

$$\int \sqrt{ex}\left(a+bx^3\right)^p \left(c+dx^3\right)\,dx=\int \left(dx^3+c\right)\sqrt{ex}\left(bx^3+a\right)^p\,dx$$

input
```
integrate((e*x)^(1/2)*(b*x^3+a)^p*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*sqrt(e*x)*(b*x^3 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}\left(a+bx^3\right)^p \left(c+dx^3\right)\,dx=\int \sqrt{e\,x}\,\left(b\,x^3+a\right)^p\,\left(d\,x^3+c\right)\,dx$$

input
```
int((e*x)^(1/2)*(a + b*x^3)^p*(c + d*x^3),x)
```

output
```
int((e*x)^(1/2)*(a + b*x^3)^p*(c + d*x^3), x)
```

Reduce [F]

$$\int \sqrt{ex}\left(a+bx^3\right)^p \left(c+dx^3\right)\,dx$$
$$=\frac{2\sqrt{e}\left(2\sqrt{x}\,\left(b\,x^3+a\right)^p adpx+2\sqrt{x}\,\left(b\,x^3+a\right)^p bcpx+3\sqrt{x}\,\left(b\,x^3+a\right)^p bcx+2\sqrt{x}\,\left(b\,x^3+a\right)^p bdp\,x^4+\sqrt{x}\right.}{}$$

input
```
int((e*x)^(1/2)*(b*x^3+a)^p*(d*x^3+c),x)
```

output

```
(2*sqrt(e)*(2*sqrt(x)*(a + b*x**3)**p*a*d*p*x + 2*sqrt(x)*(a + b*x**3)**p*
b*c*p*x + 3*sqrt(x)*(a + b*x**3)**p*b*c*x + 2*sqrt(x)*(a + b*x**3)**p*b*d*
p*x**4 + sqrt(x)*(a + b*x**3)**p*b*d*x**4 - 12*int((sqrt(x)*(a + b*x**3)**
p)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**3 + 8*b*p*x**3 + 3*b*x**3),x)*a**
2*d*p**3 - 24*int((sqrt(x)*(a + b*x**3)**p)/(4*a*p**2 + 8*a*p + 3*a + 4*b*
p**2*x**3 + 8*b*p*x**3 + 3*b*x**3),x)*a**2*d*p**2 - 9*int((sqrt(x)*(a + b*
x**3)**p)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**3 + 8*b*p*x**3 + 3*b*x**3)
,x)*a**2*d*p + 24*int((sqrt(x)*(a + b*x**3)**p)/(4*a*p**2 + 8*a*p + 3*a +
4*b*p**2*x**3 + 8*b*p*x**3 + 3*b*x**3),x)*a*b*c*p**4 + 84*int((sqrt(x)*(a
+ b*x**3)**p)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**3 + 8*b*p*x**3 + 3*b*x
**3),x)*a*b*c*p**3 + 90*int((sqrt(x)*(a + b*x**3)**p)/(4*a*p**2 + 8*a*p +
3*a + 4*b*p**2*x**3 + 8*b*p*x**3 + 3*b*x**3),x)*a*b*c*p**2 + 27*int((sqrt(
x)*(a + b*x**3)**p)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**3 + 8*b*p*x**3 +
 3*b*x**3),x)*a*b*c*p))/(3*b*(4*p**2 + 8*p + 3))
```

3.408 $\int \frac{(a+bx^3)^p(c+dx^3)}{\sqrt{ex}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \frac{(a+bx^3)^p(c+dx^3)}{\sqrt{ex}}\,dx = \frac{2d\sqrt{ex}(a+bx^3)^{1+p}}{be(7+6p)} + \frac{2\left(c-\frac{ad}{7b+6bp}\right)\sqrt{ex}(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{6},-p,\frac{7}{6},-\frac{bx^3}{a}\right)}{e}$$

output
```
2*d*(e*x)^(1/2)*(b*x^3+a)^(p+1)/b/e/(7+6*p)+2*(c-a*d/(6*b*p+7*b))*(e*x)^(1
/2)*(b*x^3+a)^p*hypergeom([1/6, -p],[7/6],-b*x^3/a)/e/((1+b*x^3/a)^p)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^p(c+dx^3)}{\sqrt{ex}}\,dx = \frac{2x(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(d(a+bx^3)\left(1+\frac{bx^3}{a}\right)^p+(-ad+bc(7+6p))\,\text{Hypergeometric2F1}\left(\frac{1}{6},-p,\frac{7}{6},-\right.\right.}{b(7+6p)\sqrt{ex}}$$

input
```
Integrate[((a + b*x^3)^p*(c + d*x^3))/Sqrt[e*x],x]
```

output
```
(2*x*(a + b*x^3)^p*(d*(a + b*x^3)*(1 + (b*x^3)/a)^p + (-(a*d) + b*c*(7 + 6
*p))*Hypergeometric2F1[1/6, -p, 7/6, -((b*x^3)/a)]))/(b*(7 + 6*p)*Sqrt[e*x
]*(1 + (b*x^3)/a)^p)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)(a+bx^3)^p}{\sqrt{ex}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{ad}{6bp+7b}\right)\int \frac{(bx^3+a)^p}{\sqrt{ex}}dx+\frac{2d\sqrt{ex}(a+bx^3)^{p+1}}{be(6p+7)}$$

$$\Big\downarrow \text{889}$$

$$(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(c-\frac{ad}{6bp+7b}\right)\int \frac{\left(\frac{bx^3}{a}+1\right)^p}{\sqrt{ex}}dx+\frac{2d\sqrt{ex}(a+bx^3)^{p+1}}{be(6p+7)}$$

$$\Big\downarrow \text{888}$$

$$\frac{2\sqrt{ex}(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(c-\frac{ad}{6bp+7b}\right)\text{Hypergeometric2F1}\left(\frac{1}{6},-p,\frac{7}{6},-\frac{bx^3}{a}\right)}{e}+\frac{2d\sqrt{ex}(a+bx^3)^{p+1}}{be(6p+7)}$$

input
```
Int[((a + b*x^3)^p*(c + d*x^3))/Sqrt[e*x],x]
```

output
```
(2*d*Sqrt[e*x]*(a + b*x^3)^(1 + p))/(b*e*(7 + 6*p)) + (2*(c - (a*d)/(7*b +
 6*b*p))*Sqrt[e*x]*(a + b*x^3)^p*Hypergeometric2F1[1/6, -p, 7/6, -((b*x^3)
/a)])/(e*(1 + (b*x^3)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(b\,x^3+a)^p\,(d\,x^3+c)}{\sqrt{ex}}dx$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/(e*x)^(1/2),x)
```

output
```
int((b*x^3+a)^p*(d*x^3+c)/(e*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{\sqrt{ex}}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*sqrt(e*x)*(b*x^3 + a)^p/(e*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^p (c+dx^3)}{\sqrt{ex}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**p*(d*x**3+c)/(e*x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{\sqrt{ex}}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/sqrt(e*x), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{\sqrt{ex}}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/sqrt(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^p (c+dx^3)}{\sqrt{ex}}\,dx = \int \frac{(b\,x^3+a)^p\,(d\,x^3+c)}{\sqrt{e\,x}}\,dx$$

input
```
int(((a + b*x^3)^p*(c + d*x^3))/(e*x)^(1/2),x)
```

output
```
int(((a + b*x^3)^p*(c + d*x^3))/(e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{\sqrt{ex}}\,dx$$
$$= \frac{2\sqrt{e}\left(6\sqrt{x}\,(b\,x^3+a)^p\,adp + 6\sqrt{x}\,(b\,x^3+a)^p\,bcp + 7\sqrt{x}\,(b\,x^3+a)^p\,bc + 6\sqrt{x}\,(b\,x^3+a)^p\,bdp\,x^3 + \sqrt{x}\,(b\,x$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/(e*x)^(1/2),x)
```

output

```
(2*sqrt(e)*(6*sqrt(x)*(a + b*x**3)**p*a*d*p + 6*sqrt(x)*(a + b*x**3)**p*b*
c*p + 7*sqrt(x)*(a + b*x**3)**p*b*c + 6*sqrt(x)*(a + b*x**3)**p*b*d*p*x**3
 + sqrt(x)*(a + b*x**3)**p*b*d*x**3 - 108*int((sqrt(x)*(a + b*x**3)**p)/(3
6*a*p**2*x + 48*a*p*x + 7*a*x + 36*b*p**2*x**4 + 48*b*p*x**4 + 7*b*x**4),x
)*a**2*d*p**3 - 144*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x + 48*a*p*x
+ 7*a*x + 36*b*p**2*x**4 + 48*b*p*x**4 + 7*b*x**4),x)*a**2*d*p**2 - 21*int
((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x + 48*a*p*x + 7*a*x + 36*b*p**2*x**
4 + 48*b*p*x**4 + 7*b*x**4),x)*a**2*d*p + 648*int((sqrt(x)*(a + b*x**3)**p
)/(36*a*p**2*x + 48*a*p*x + 7*a*x + 36*b*p**2*x**4 + 48*b*p*x**4 + 7*b*x**
4),x)*a*b*c*p**4 + 1620*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x + 48*a*
p*x + 7*a*x + 36*b*p**2*x**4 + 48*b*p*x**4 + 7*b*x**4),x)*a*b*c*p**3 + 113
4*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x + 48*a*p*x + 7*a*x + 36*b*p**
2*x**4 + 48*b*p*x**4 + 7*b*x**4),x)*a*b*c*p**2 + 147*int((sqrt(x)*(a + b*x
**3)**p)/(36*a*p**2*x + 48*a*p*x + 7*a*x + 36*b*p**2*x**4 + 48*b*p*x**4 +
7*b*x**4),x)*a*b*c*p))/(b*e*(36*p**2 + 48*p + 7))
```

3.409 $\int \frac{(a+bx^3)^p(c+dx^3)}{(ex)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{(a+bx^3)^p(c+dx^3)}{(ex)^{3/2}}\,dx = -\frac{2c(a+bx^3)^{1+p}}{ae\sqrt{ex}} + \frac{2(ad+bc(5+6p))(ex)^{5/2}(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{6},-p,\frac{11}{6},-\frac{bx^3}{a}\right)}{5ae^4}$$

output
```
-2*c*(b*x^3+a)^(p+1)/a/e/(e*x)^(1/2)+2/5*(a*d+b*c*(5+6*p))*(e*x)^(5/2)*(b*
x^3+a)^p*hypergeom([5/6, -p],[11/6],-b*x^3/a)/a/e^4/((1+b*x^3/a)^p)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^p(c+dx^3)}{(ex)^{3/2}}\,dx = -\frac{2x(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(5c(a+bx^3)\left(1+\frac{bx^3}{a}\right)^{p}-(ad+bc(5+6p))x^3\,\text{Hypergeometric2F1}\left(\frac{5}{6},-p,\frac{11}{6}\right.\right.}{5a(ex)^{3/2}}$$

input
```
Integrate[((a + b*x^3)^p*(c + d*x^3))/(e*x)^(3/2),x]
```

output
```
(-2*x*(a + b*x^3)^p*(5*c*(a + b*x^3)*(1 + (b*x^3)/a)^p - (a*d + b*c*(5 + 6
*p))*x^3*Hypergeometric2F1[5/6, -p, 11/6, -((b*x^3)/a)]))/(5*a*(e*x)^(3/2)
*(1 + (b*x^3)/a)^p)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)(a+bx^3)^p}{(ex)^{3/2}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(ad+bc(6p+5))\int (ex)^{3/2}(bx^3+a)^p dx}{ae^3}-\frac{2c(a+bx^3)^{p+1}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{889}$$

$$\frac{(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}(ad+bc(6p+5))\int (ex)^{3/2}\left(\frac{bx^3}{a}+1\right)^p dx}{ae^3}-\frac{2c(a+bx^3)^{p+1}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2(ex)^{5/2}(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}(ad+bc(6p+5))\text{Hypergeometric2F1}\left(\frac{5}{6},-p,\frac{11}{6},-\frac{bx^3}{a}\right)}{5ae^4}-\frac{2c(a+bx^3)^{p+1}}{ae\sqrt{ex}}$$

input
```
Int[((a + b*x^3)^p*(c + d*x^3))/(e*x)^(3/2),x]
```

output
```
(-2*c*(a + b*x^3)^(1 + p))/(a*e*Sqrt[e*x]) + (2*(a*d + b*c*(5 + 6*p))*(e*x
)^(5/2)*(a + b*x^3)^p*Hypergeometric2F1[5/6, -p, 11/6, -((b*x^3)/a)])/(5*a
*e^4*(1 + (b*x^3)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{p}\,(d\,x^{3}+c)}{(ex)^{\frac{3}{2}}}dx$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/(e*x)^(3/2),x)
```

output
```
int((b*x^3+a)^p*(d*x^3+c)/(e*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{(ex)^{3/2}}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/(e*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*sqrt(e*x)*(b*x^3 + a)^p/(e^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{(ex)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**p*(d*x**3+c)/(e*x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{(ex)^{3/2}}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/(e*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{(ex)^{3/2}} \, dx = \int \frac{(dx^3+c)(bx^3+a)^p}{(ex)^{\frac{3}{2}}} \, dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/(e*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/(e*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^p (c+dx^3)}{(ex)^{3/2}} \, dx = \int \frac{(b\,x^3+a)^p\,(d\,x^3+c)}{(e\,x)^{3/2}} \, dx$$

input
```
int(((a + b*x^3)^p*(c + d*x^3))/(e*x)^(3/2),x)
```

output
```
int(((a + b*x^3)^p*(c + d*x^3))/(e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{(ex)^{3/2}} \, dx = \frac{2\sqrt{e}\left(6(b\,x^3+a)^p\,adp + 6(b\,x^3+a)^p\,bcp + 5(b\,x^3+a)^p\,bc + 6(b\,x^3+a)^p\,bdp\,x\right.}{}$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/(e*x)^(3/2),x)
```

output

```
(2*sqrt(e)*(6*(a + b*x**3)**p*a*d*p + 6*(a + b*x**3)**p*b*c*p + 5*(a + b*x
**3)**p*b*c + 6*(a + b*x**3)**p*b*d*p*x**3 - (a + b*x**3)**p*b*d*x**3 + 10
8*sqrt(x)*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x**2 + 24*a*p*x**2 - 5*
a*x**2 + 36*b*p**2*x**5 + 24*b*p*x**5 - 5*b*x**5),x)*a**2*d*p**3 + 72*sqrt
(x)*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x**2 + 24*a*p*x**2 - 5*a*x**2
 + 36*b*p**2*x**5 + 24*b*p*x**5 - 5*b*x**5),x)*a**2*d*p**2 - 15*sqrt(x)*in
t((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x**2 + 24*a*p*x**2 - 5*a*x**2 + 36*
b*p**2*x**5 + 24*b*p*x**5 - 5*b*x**5),x)*a**2*d*p + 648*sqrt(x)*int((sqrt(
x)*(a + b*x**3)**p)/(36*a*p**2*x**2 + 24*a*p*x**2 - 5*a*x**2 + 36*b*p**2*x
**5 + 24*b*p*x**5 - 5*b*x**5),x)*a*b*c*p**4 + 972*sqrt(x)*int((sqrt(x)*(a
+ b*x**3)**p)/(36*a*p**2*x**2 + 24*a*p*x**2 - 5*a*x**2 + 36*b*p**2*x**5 +
24*b*p*x**5 - 5*b*x**5),x)*a*b*c*p**3 + 270*sqrt(x)*int((sqrt(x)*(a + b*x*
*3)**p)/(36*a*p**2*x**2 + 24*a*p*x**2 - 5*a*x**2 + 36*b*p**2*x**5 + 24*b*p
*x**5 - 5*b*x**5),x)*a*b*c*p**2 - 75*sqrt(x)*int((sqrt(x)*(a + b*x**3)**p)
/(36*a*p**2*x**2 + 24*a*p*x**2 - 5*a*x**2 + 36*b*p**2*x**5 + 24*b*p*x**5 -
 5*b*x**5),x)*a*b*c*p))/(sqrt(x)*b*e**2*(36*p**2 + 24*p - 5))
```

3.410 $\int \frac{(a+bx^3)^p(c+dx^3)}{(ex)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 101

$$\int \frac{(a+bx^3)^p(c+dx^3)}{(ex)^{5/2}}\,dx = -\frac{2c(a+bx^3)^{1+p}}{3ae(ex)^{3/2}} + \frac{2(ad+b(c+2cp))(ex)^{3/2}(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^3}{a}\right)}{3ae^4}$$

output
```
-2/3*c*(b*x^3+a)^(p+1)/a/e/(e*x)^(3/2)+2/3*(a*d+b*(2*c*p+c))*(e*x)^(3/2)*(
b*x^3+a)^p*hypergeom([1/2, -p],[3/2],-b*x^3/a)/a/e^4/((1+b*x^3/a)^p)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^p(c+dx^3)}{(ex)^{5/2}}\,dx = -\frac{2x(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(c(a+bx^3)\left(1+\frac{bx^3}{a}\right)^p-(ad+b(c+2cp))x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\right.\right.}{3a(ex)^{5/2}}$$

input
```
Integrate[((a + b*x^3)^p*(c + d*x^3))/(e*x)^(5/2),x]
```

output
```
(-2*x*(a + b*x^3)^p*(c*(a + b*x^3)*(1 + (b*x^3)/a)^p - (a*d + b*(c + 2*c*p
))*x^3*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^3)/a)]))/(3*a*(e*x)^(5/2)*(1
 + (b*x^3)/a)^p)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)(a+bx^3)^p}{(ex)^{5/2}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(ad+b(2cp+c))\int \sqrt{ex}(bx^3+a)^pdx}{ae^3}-\frac{2c(a+bx^3)^{p+1}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{889}$$

$$\frac{(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}(ad+b(2cp+c))\int \sqrt{ex}\left(\frac{bx^3}{a}+1\right)^pdx}{ae^3}-\frac{2c(a+bx^3)^{p+1}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2(ex)^{3/2}(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}(ad+b(2cp+c))\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^3}{a}\right)}{3ae^4}-\frac{2c(a+bx^3)^{p+1}}{3ae(ex)^{3/2}}$$

input
```
Int[((a + b*x^3)^p*(c + d*x^3))/(e*x)^(5/2),x]
```

output
```
(-2*c*(a + b*x^3)^(1 + p))/(3*a*e*(e*x)^(3/2)) + (2*(a*d + b*(c + 2*c*p))*
(e*x)^(3/2)*(a + b*x^3)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^3)/a)])/(
3*a*e^4*(1 + (b*x^3)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{(b\,x^3+a)^p\,(d\,x^3+c)}{(ex)^{\frac{5}{2}}}dx$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/(e*x)^(5/2),x)
```

output
```
int((b*x^3+a)^p*(d*x^3+c)/(e*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{(ex)^{5/2}} dx = \int \frac{(dx^3+c)(bx^3+a)^p}{(ex)^{\frac{5}{2}}} dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*sqrt(e*x)*(b*x^3 + a)^p/(e^3*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^p (c+dx^3)}{(ex)^{5/2}} dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**p*(d*x**3+c)/(e*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{(ex)^{5/2}} dx = \int \frac{(dx^3+c)(bx^3+a)^p}{(ex)^{\frac{5}{2}}} dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/(e*x)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{(ex)^{5/2}} \, dx = \int \frac{(dx^3+c)(bx^3+a)^p}{(ex)^{\frac{5}{2}}} \, dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/(e*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/(e*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^p (c+dx^3)}{(ex)^{5/2}} \, dx = \int \frac{(b\,x^3+a)^p\,(d\,x^3+c)}{(e\,x)^{5/2}} \, dx$$

input
```
int(((a + b*x^3)^p*(c + d*x^3))/(e*x)^(5/2),x)
```

output
```
int(((a + b*x^3)^p*(c + d*x^3))/(e*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{(ex)^{5/2}} \, dx = \frac{2\sqrt{e}\left(2(b\,x^3+a)^p\,adp+2(b\,x^3+a)^p\,bcp+(b\,x^3+a)^p\,bc+2(b\,x^3+a)^p\,bdp\,x^3\right.}{}$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/(e*x)^(5/2),x)
```

output
```
(2*sqrt(e)*(2*(a + b*x**3)**p*a*d*p + 2*(a + b*x**3)**p*b*c*p + (a + b*x**
3)**p*b*c + 2*(a + b*x**3)**p*b*d*p*x**3 - (a + b*x**3)**p*b*d*x**3 + 12*s
qrt(x)*int((sqrt(x)*(a + b*x**3)**p)/(4*a*p**2*x**3 - a*x**3 + 4*b*p**2*x*
*6 - b*x**6),x)*a**2*d*p**3*x - 3*sqrt(x)*int((sqrt(x)*(a + b*x**3)**p)/(4
*a*p**2*x**3 - a*x**3 + 4*b*p**2*x**6 - b*x**6),x)*a**2*d*p*x + 24*sqrt(x)
*int((sqrt(x)*(a + b*x**3)**p)/(4*a*p**2*x**3 - a*x**3 + 4*b*p**2*x**6 - b
*x**6),x)*a*b*c*p**4*x + 12*sqrt(x)*int((sqrt(x)*(a + b*x**3)**p)/(4*a*p**
2*x**3 - a*x**3 + 4*b*p**2*x**6 - b*x**6),x)*a*b*c*p**3*x - 6*sqrt(x)*int(
(sqrt(x)*(a + b*x**3)**p)/(4*a*p**2*x**3 - a*x**3 + 4*b*p**2*x**6 - b*x**6
),x)*a*b*c*p**2*x - 3*sqrt(x)*int((sqrt(x)*(a + b*x**3)**p)/(4*a*p**2*x**3
 - a*x**3 + 4*b*p**2*x**6 - b*x**6),x)*a*b*c*p*x))/(3*sqrt(x)*b*e**3*x*(4*
p**2 - 1))
```

3.411 $\int \frac{(a+bx^3)^p(c+dx^3)}{(ex)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 102

$$\int \frac{(a+bx^3)^p(c+dx^3)}{(ex)^{7/2}}\,dx = -\frac{2c(a+bx^3)^{1+p}}{5ae(ex)^{5/2}} + \frac{2(5ad+b(c+6cp))\sqrt{ex}(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{6},-p,\frac{7}{6},-\frac{bx^3}{a}\right)}{5ae^4}$$

output
```
-2/5*c*(b*x^3+a)^(p+1)/a/e/(e*x)^(5/2)+2/5*(5*a*d+b*(6*c*p+c))*(e*x)^(1/2)
*(b*x^3+a)^p*hypergeom([1/6, -p],[7/6],-b*x^3/a)/a/e^4/((1+b*x^3/a)^p)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^p(c+dx^3)}{(ex)^{7/2}}\,dx = -\frac{2x(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(c(a+bx^3)\left(1+\frac{bx^3}{a}\right)^p-(5ad+b(c+6cp))x^3\,\text{Hypergeometric2F1}\left(\frac{1}{6},-p,\frac{7}{6},\right.\right.}{5a(ex)^{7/2}}$$

input
```
Integrate[((a + b*x^3)^p*(c + d*x^3))/(e*x)^(7/2),x]
```

output
```
(-2*x*(a + b*x^3)^p*(c*(a + b*x^3)*(1 + (b*x^3)/a)^p - (5*a*d + b*(c + 6*c
*p))*x^3*Hypergeometric2F1[1/6, -p, 7/6, -((b*x^3)/a)]))/(5*a*(e*x)^(7/2)*
(1 + (b*x^3)/a)^p)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)(a+bx^3)^p}{(ex)^{7/2}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(5ad+b(6cp+c))\int \frac{(bx^3+a)^p}{\sqrt{ex}}dx}{5ae^3}-\frac{2c(a+bx^3)^{p+1}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{889}$$

$$\frac{(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}(5ad+b(6cp+c))\int \frac{\left(\frac{bx^3}{a}+1\right)^p}{\sqrt{ex}}dx}{5ae^3}-\frac{2c(a+bx^3)^{p+1}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2\sqrt{ex}(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}(5ad+b(6cp+c))\,\text{Hypergeometric2F1}\left(\frac{1}{6},-p,\frac{7}{6},-\frac{bx^3}{a}\right)}{5ae^4}-\frac{2c(a+bx^3)^{p+1}}{5ae(ex)^{5/2}}$$

input
```
Int[((a + b*x^3)^p*(c + d*x^3))/(e*x)^(7/2),x]
```

output
```
(-2*c*(a + b*x^3)^(1 + p))/(5*a*e*(e*x)^(5/2)) + (2*(5*a*d + b*(c + 6*c*p)
)*Sqrt[e*x]*(a + b*x^3)^p*Hypergeometric2F1[1/6, -p, 7/6, -((b*x^3)/a)])/(
5*a*e^4*(1 + (b*x^3)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{p}\,(d\,x^{3}+c)}{(ex)^{\frac{7}{2}}}dx$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/(e*x)^(7/2),x)
```

output
```
int((b*x^3+a)^p*(d*x^3+c)/(e*x)^(7/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^p(c+dx^3)}{(ex)^{7/2}}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/(e*x)^(7/2),x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*sqrt(e*x)*(b*x^3 + a)^p/(e^4*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^p(c+dx^3)}{(ex)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**p*(d*x**3+c)/(e*x)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^p(c+dx^3)}{(ex)^{7/2}}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/(e*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/(e*x)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{(ex)^{7/2}} \, dx = \int \frac{(dx^3+c)(bx^3+a)^p}{(ex)^{\frac{7}{2}}} \, dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/(e*x)^(7/2),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/(e*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^p (c+dx^3)}{(ex)^{7/2}} \, dx = \int \frac{(b\,x^3+a)^p\,(d\,x^3+c)}{(e\,x)^{7/2}} \, dx$$

input
```
int(((a + b*x^3)^p*(c + d*x^3))/(e*x)^(7/2),x)
```

output
```
int(((a + b*x^3)^p*(c + d*x^3))/(e*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{(ex)^{7/2}} \, dx = \frac{2\sqrt{e}\left(6(b\,x^3+a)^p\,adp+6(b\,x^3+a)^p\,bcp+(b\,x^3+a)^p\,bc+6(b\,x^3+a)^p\,bdp\,x^3\right.}{}$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/(e*x)^(7/2),x)
```

output
```
(2*sqrt(e)*(6*(a + b*x**3)**p*a*d*p + 6*(a + b*x**3)**p*b*c*p + (a + b*x**
3)**p*b*c + 6*(a + b*x**3)**p*b*d*p*x**3 - 5*(a + b*x**3)**p*b*d*x**3 + 54
0*sqrt(x)*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x**4 - 24*a*p*x**4 - 5*
a*x**4 + 36*b*p**2*x**7 - 24*b*p*x**7 - 5*b*x**7),x)*a**2*d*p**3*x**2 - 36
0*sqrt(x)*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x**4 - 24*a*p*x**4 - 5*
a*x**4 + 36*b*p**2*x**7 - 24*b*p*x**7 - 5*b*x**7),x)*a**2*d*p**2*x**2 - 75
*sqrt(x)*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x**4 - 24*a*p*x**4 - 5*a
*x**4 + 36*b*p**2*x**7 - 24*b*p*x**7 - 5*b*x**7),x)*a**2*d*p*x**2 + 648*sq
rt(x)*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x**4 - 24*a*p*x**4 - 5*a*x*
*4 + 36*b*p**2*x**7 - 24*b*p*x**7 - 5*b*x**7),x)*a*b*c*p**4*x**2 - 324*sqr
t(x)*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x**4 - 24*a*p*x**4 - 5*a*x**
4 + 36*b*p**2*x**7 - 24*b*p*x**7 - 5*b*x**7),x)*a*b*c*p**3*x**2 - 162*sqrt
(x)*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x**4 - 24*a*p*x**4 - 5*a*x**4
 + 36*b*p**2*x**7 - 24*b*p*x**7 - 5*b*x**7),x)*a*b*c*p**2*x**2 - 15*sqrt(x
)*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x**4 - 24*a*p*x**4 - 5*a*x**4 +
 36*b*p**2*x**7 - 24*b*p*x**7 - 5*b*x**7),x)*a*b*c*p*x**2))/(sqrt(x)*b*e**
4*x**2*(36*p**2 - 24*p - 5))
```

3.412 $\int \frac{(a+bx^3)^p(c+dx^3)}{(ex)^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{(a+bx^3)^p(c+dx^3)}{(ex)^{9/2}}\,dx = -\frac{2c(a+bx^3)^{1+p}}{7ae(ex)^{7/2}} - \frac{2(7ad-bc(1-6p))(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{6},-p,\frac{5}{6},-\frac{bx^3}{a}\right)}{7ae^4\sqrt{ex}}$$

output
```
-2/7*c*(b*x^3+a)^(p+1)/a/e/(e*x)^(7/2)-2/7*(7*a*d-b*c*(1-6*p))*(b*x^3+a)^p
*hypergeom([-1/6, -p],[5/6],-b*x^3/a)/a/e^4/(e*x)^(1/2)/((1+b*x^3/a)^p)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^p(c+dx^3)}{(ex)^{9/2}}\,dx = -\frac{2\sqrt{ex}(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(c(a+bx^3)\left(1+\frac{bx^3}{a}\right)^p+(7ad+bc(-1+6p))x^3\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\right.\right.}{7ae^5x^4}$$

input
```
Integrate[((a + b*x^3)^p*(c + d*x^3))/(e*x)^(9/2),x]
```

output
```
(-2*Sqrt[e*x]*(a + b*x^3)^p*(c*(a + b*x^3)*(1 + (b*x^3)/a)^p + (7*a*d + b*
c*(-1 + 6*p))*x^3*Hypergeometric2F1[-1/6, -p, 5/6, -((b*x^3)/a)]))/(7*a*e^
5*x^4*(1 + (b*x^3)/a)^p)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {955, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)(a+bx^3)^p}{(ex)^{9/2}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(7ad-bc(1-6p))\int \frac{(bx^3+a)^p}{(ex)^{3/2}}dx}{7ae^3}-\frac{2c(a+bx^3)^{p+1}}{7ae(ex)^{7/2}}$$

$$\Big\downarrow \text{889}$$

$$\frac{(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}(7ad-bc(1-6p))\int \frac{\left(\frac{bx^3}{a}+1\right)^p}{(ex)^{3/2}}dx}{7ae^3}-\frac{2c(a+bx^3)^{p+1}}{7ae(ex)^{7/2}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}(7ad-bc(1-6p))\text{Hypergeometric2F1}\left(-\frac{1}{6},-p,\frac{5}{6},-\frac{bx^3}{a}\right)}{7ae^4\sqrt{ex}}-\frac{2c(a+bx^3)^{p+1}}{7ae(ex)^{7/2}}$$

input
```
Int[((a + b*x^3)^p*(c + d*x^3))/(e*x)^(9/2),x]
```

output
```
(-2*c*(a + b*x^3)^(1 + p))/(7*a*e*(e*x)^(7/2)) - (2*(7*a*d - b*c*(1 - 6*p)
)*(a + b*x^3)^p*Hypergeometric2F1[-1/6, -p, 5/6, -((b*x^3)/a)])/(7*a*e^4*S
qrt[e*x]*(1 + (b*x^3)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{(b\,x^3+a)^p\,(d\,x^3+c)}{(ex)^{\frac{9}{2}}}dx$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/(e*x)^(9/2),x)
```

output
```
int((b*x^3+a)^p*(d*x^3+c)/(e*x)^(9/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{(ex)^{9/2}}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/(e*x)^(9/2),x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*sqrt(e*x)*(b*x^3 + a)^p/(e^5*x^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{(ex)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**p*(d*x**3+c)/(e*x)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^p\,(c+dx^3)}{(ex)^{9/2}}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/(e*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/(e*x)^(9/2), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{(ex)^{9/2}}\,dx = \int \frac{(dx^3+c)(bx^3+a)^p}{(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^p*(d*x^3+c)/(e*x)^(9/2),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p/(e*x)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^p (c+dx^3)}{(ex)^{9/2}}\,dx = \int \frac{(b\,x^3+a)^p\,(d\,x^3+c)}{(e\,x)^{9/2}}\,dx$$

input
```
int(((a + b*x^3)^p*(c + d*x^3))/(e*x)^(9/2),x)
```

output
```
int(((a + b*x^3)^p*(c + d*x^3))/(e*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^p (c+dx^3)}{(ex)^{9/2}}\,dx = \frac{2\sqrt{e}\left(6(b\,x^3+a)^p\,adp + 6(b\,x^3+a)^p\,bcp - (b\,x^3+a)^p\,bc + 6(b\,x^3+a)^p\,bdp\,x^3\right.}{}$$

input
```
int((b*x^3+a)^p*(d*x^3+c)/(e*x)^(9/2),x)
```

output

```
(2*sqrt(e)*(6*(a + b*x**3)**p*a*d*p + 6*(a + b*x**3)**p*b*c*p - (a + b*x**
3)**p*b*c + 6*(a + b*x**3)**p*b*d*p*x**3 - 7*(a + b*x**3)**p*b*d*x**3 + 75
6*sqrt(x)*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x**5 - 48*a*p*x**5 + 7*
a*x**5 + 36*b*p**2*x**8 - 48*b*p*x**8 + 7*b*x**8),x)*a**2*d*p**3*x**3 - 10
08*sqrt(x)*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x**5 - 48*a*p*x**5 + 7
*a*x**5 + 36*b*p**2*x**8 - 48*b*p*x**8 + 7*b*x**8),x)*a**2*d*p**2*x**3 + 1
47*sqrt(x)*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x**5 - 48*a*p*x**5 + 7
*a*x**5 + 36*b*p**2*x**8 - 48*b*p*x**8 + 7*b*x**8),x)*a**2*d*p*x**3 + 648*
sqrt(x)*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x**5 - 48*a*p*x**5 + 7*a*
x**5 + 36*b*p**2*x**8 - 48*b*p*x**8 + 7*b*x**8),x)*a*b*c*p**4*x**3 - 972*s
qrt(x)*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x**5 - 48*a*p*x**5 + 7*a*x
**5 + 36*b*p**2*x**8 - 48*b*p*x**8 + 7*b*x**8),x)*a*b*c*p**3*x**3 + 270*sq
rt(x)*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x**5 - 48*a*p*x**5 + 7*a*x*
*5 + 36*b*p**2*x**8 - 48*b*p*x**8 + 7*b*x**8),x)*a*b*c*p**2*x**3 - 21*sqrt
(x)*int((sqrt(x)*(a + b*x**3)**p)/(36*a*p**2*x**5 - 48*a*p*x**5 + 7*a*x**5
 + 36*b*p**2*x**8 - 48*b*p*x**8 + 7*b*x**8),x)*a*b*c*p*x**3))/(sqrt(x)*b*e
**5*x**3*(36*p**2 - 48*p + 7))
```

3.413 $\int (ex)^m (a+bx^3)^p (c+dx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int (ex)^m (a+bx^3)^p (c+dx^3)\,dx = \frac{d(ex)^{1+m}(a+bx^3)^{1+p}}{be(4+m+3p)} + \frac{\left(\frac{c}{1+m}-\frac{ad}{b(4+m+3p)}\right)(ex)^{1+m}(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{3},-p,\frac{4+m}{3},-\frac{bx^3}{a}\right)}{e}$$

output
```
d*(e*x)^(1+m)*(b*x^3+a)^(p+1)/b/e/(4+m+3*p)+(c/(1+m)-a*d/b/(4+m+3*p))*(e*x
)^(1+m)*(b*x^3+a)^p*hypergeom([-p, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/e/((1+
b*x^3/a)^p)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.91

$$\int (ex)^m (a+bx^3)^p (c+dx^3)\,dx = \frac{x(ex)^m(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(c(4+m)\text{Hypergeometric2F1}\left(\frac{1+m}{3},-p,\frac{4+m}{3},-\frac{bx^3}{a}\right)+d(1+m)x^3\,\text{Hyp}\right.}{(1+m)(4+m)}$$

input
```
Integrate[(e*x)^m*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
(x*(e*x)^m*(a + b*x^3)^p*(c*(4 + m)*Hypergeometric2F1[(1 + m)/3, -p, (4 +
m)/3, -((b*x^3)/a)] + d*(1 + m)*x^3*Hypergeometric2F1[(4 + m)/3, -p, (7 +
m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*(1 + (b*x^3)/a)^p)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^3\right)(ex)^m \left(a+bx^3\right)^p \, dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{ad(m+1)}{b(m+3p+4)}\right)\int (ex)^m \left(bx^3+a\right)^p dx+\frac{d(ex)^{m+1}\left(a+bx^3\right)^{p+1}}{be(m+3p+4)}$$

$$\Big\downarrow \text{889}$$

$$\left(a+bx^3\right)^p \left(\frac{bx^3}{a}+1\right)^{-p}\left(c-\frac{ad(m+1)}{b(m+3p+4)}\right)\int (ex)^m \left(\frac{bx^3}{a}+1\right)^p dx+\frac{d(ex)^{m+1}\left(a+bx^3\right)^{p+1}}{be(m+3p+4)}$$

$$\Big\downarrow \text{888}$$

$$\frac{(ex)^{m+1}\left(a+bx^3\right)^p \left(\frac{bx^3}{a}+1\right)^{-p}\left(c-\frac{ad(m+1)}{b(m+3p+4)}\right)\text{Hypergeometric2F1}\left(\frac{m+1}{3},-p,\frac{m+4}{3},-\frac{bx^3}{a}\right)}{e(m+1)}+\frac{d(ex)^{m+1}\left(a+bx^3\right)^{p+1}}{be(m+3p+4)}$$

input
```
Int[(e*x)^m*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
(d*(e*x)^(1 + m)*(a + b*x^3)^(1 + p))/(b*e*(4 + m + 3*p)) + ((c - (a*d*(1
+ m))/(b*(4 + m + 3*p)))*(e*x)^(1 + m)*(a + b*x^3)^p*Hypergeometric2F1[(1
+ m)/3, -p, (4 + m)/3, -((b*x^3)/a)])/(e*(1 + m)*(1 + (b*x^3)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (ex)^m \left(b\,x^3+a\right)^p \left(d\,x^3+c\right)dx$$

input
```
int((e*x)^m*(b*x^3+a)^p*(d*x^3+c),x)
```

output
```
int((e*x)^m*(b*x^3+a)^p*(d*x^3+c),x)
```

Fricas [F]

$$\int (ex)^m (a+bx^3)^p (c+dx^3) \, dx = \int (dx^3+c)(bx^3+a)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^p*(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*(b*x^3 + a)^p*(e*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m (a+bx^3)^p (c+dx^3) \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(b*x**3+a)**p*(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m (a+bx^3)^p (c+dx^3) \, dx = \int (dx^3+c)(bx^3+a)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^p*(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m \left(a+bx^3\right)^p \left(c+dx^3\right) \, dx = \int \left(dx^3+c\right)\left(bx^3+a\right)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^p*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^3\right)^p \left(c+dx^3\right) \, dx = \int {(e\,x)}^m\,{\left(b\,x^3+a\right)}^p\,\left(d\,x^3+c\right) \,d x$$

input
```
int((e*x)^m*(a + b*x^3)^p*(c + d*x^3),x)
```

output
```
int((e*x)^m*(a + b*x^3)^p*(c + d*x^3), x)
```

Reduce [F]

$$\int (ex)^m \left(a+bx^3\right)^p \left(c+dx^3\right) \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^3+a)^p*(d*x^3+c),x)
```

output

```
(e**m*(3*x**m*(a + b*x**3)**p*a*d*p*x + x**m*(a + b*x**3)**p*b*c*m*x + 3*x
**m*(a + b*x**3)**p*b*c*p*x + 4*x**m*(a + b*x**3)**p*b*c*x + x**m*(a + b*x
**3)**p*b*d*m*x**4 + 3*x**m*(a + b*x**3)**p*b*d*p*x**4 + x**m*(a + b*x**3)
**p*b*d*x**4 - 3*int((x**m*(a + b*x**3)**p)/(a*m**2 + 6*a*m*p + 5*a*m + 9*
a*p**2 + 15*a*p + 4*a + b*m**2*x**3 + 6*b*m*p*x**3 + 5*b*m*x**3 + 9*b*p**2
*x**3 + 15*b*p*x**3 + 4*b*x**3),x)*a**2*d*m**3*p - 18*int((x**m*(a + b*x**
3)**p)/(a*m**2 + 6*a*m*p + 5*a*m + 9*a*p**2 + 15*a*p + 4*a + b*m**2*x**3 +
 6*b*m*p*x**3 + 5*b*m*x**3 + 9*b*p**2*x**3 + 15*b*p*x**3 + 4*b*x**3),x)*a*
*2*d*m**2*p**2 - 18*int((x**m*(a + b*x**3)**p)/(a*m**2 + 6*a*m*p + 5*a*m +
 9*a*p**2 + 15*a*p + 4*a + b*m**2*x**3 + 6*b*m*p*x**3 + 5*b*m*x**3 + 9*b*p
**2*x**3 + 15*b*p*x**3 + 4*b*x**3),x)*a**2*d*m**2*p - 27*int((x**m*(a + b*
x**3)**p)/(a*m**2 + 6*a*m*p + 5*a*m + 9*a*p**2 + 15*a*p + 4*a + b*m**2*x**
3 + 6*b*m*p*x**3 + 5*b*m*x**3 + 9*b*p**2*x**3 + 15*b*p*x**3 + 4*b*x**3),x)
*a**2*d*m*p**3 - 63*int((x**m*(a + b*x**3)**p)/(a*m**2 + 6*a*m*p + 5*a*m +
 9*a*p**2 + 15*a*p + 4*a + b*m**2*x**3 + 6*b*m*p*x**3 + 5*b*m*x**3 + 9*b*p
**2*x**3 + 15*b*p*x**3 + 4*b*x**3),x)*a**2*d*m*p**2 - 27*int((x**m*(a + b*
x**3)**p)/(a*m**2 + 6*a*m*p + 5*a*m + 9*a*p**2 + 15*a*p + 4*a + b*m**2*x**
3 + 6*b*m*p*x**3 + 5*b*m*x**3 + 9*b*p**2*x**3 + 15*b*p*x**3 + 4*b*x**3),x)
*a**2*d*m*p - 27*int((x**m*(a + b*x**3)**p)/(a*m**2 + 6*a*m*p + 5*a*m + 9*
a*p**2 + 15*a*p + 4*a + b*m**2*x**3 + 6*b*m*p*x**3 + 5*b*m*x**3 + 9*b*p...
```

3.414 $\int x^{-4-3p}(a+bx^3)^p\,(c+dx^3)\ dx$

Optimal result

Integrand size = 24, antiderivative size = 89

$$\int x^{-4-3p}(a+bx^3)^p\,(c+dx^3)\ dx = -\frac{cx^{-3(1+p)}(a+bx^3)^{1+p}}{3a(1+p)} - \frac{dx^{-3p}(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^3}{a}\right)}{3p}$$

output
```
-1/3*c*(b*x^3+a)^(p+1)/a/(p+1)/(x^(3*p+3))-1/3*d*(b*x^3+a)^p*hypergeom([-p
, -p],[1-p],-b*x^3/a)/p/(x^(3*p))/((1+b*x^3/a)^p)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int x^{-4-3p}(a+bx^3)^p (c+dx^3) \, dx = \frac{1}{3}x^{-3p}(a+bx^3)^p \left(-\frac{c(a+bx^3)}{a(1+p)x^3} - \frac{d\left(1+\frac{bx^3}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^3}{a}\right)}{p} \right)$$

input
```
Integrate[x^(-4 - 3*p)*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
((a + b*x^3)^p*(-((c*(a + b*x^3))/(a*(1 + p)*x^3)) - (d*Hypergeometric2F1[
-p, -p, 1 - p, -((b*x^3)/a)])/(p*(1 + (b*x^3)/a)^p)))/(3*x^(3*p))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {954, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3p-4}(c+dx^3)(a+bx^3)^p \, dx$$

$$\Big\downarrow \text{954}$$

$$\frac{d\int x^{-3p-4}(bx^3+a)^{p+1}dx}{b} - \frac{x^{-3(p+1)}(bc-ad)(a+bx^3)^{p+1}}{3ab(p+1)}$$

$$\Big\downarrow \text{882}$$

$$\frac{dx^{-3(p+1)}\left(\frac{x^3}{a+bx^3}\right)^{p+1}\left(a+bx^3\right)^{p+1}\int\frac{\left(\frac{x^3}{bx^3+a}\right)^{-p-2}}{1-\frac{bx^3}{bx^3+a}}d\frac{x^3}{bx^3+a}}{3b}-\frac{x^{-3(p+1)}(bc-ad)\left(a+bx^3\right)^{p+1}}{3ab(p+1)}$$

$$\Big\downarrow \; 74$$

$$-\frac{x^{-3(p+1)}(bc-ad)\left(a+bx^3\right)^{p+1}}{3ab(p+1)}-\frac{dx^{-3(p+1)}\left(a+bx^3\right)^{p+1}\text{Hypergeometric2F1}\left(1,-p-1,-p,\frac{bx^3}{bx^3+a}\right)}{3b(p+1)}$$

input
```
Int[x^(-4 - 3*p)*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
-1/3*((b*c - a*d)*(a + b*x^3)^(1 + p))/(a*b*(1 + p)*x^(3*(1 + p))) - (d*(a
 + b*x^3)^(1 + p)*Hypergeometric2F1[1, -1 - p, -p, (b*x^3)/(a + b*x^3)])/(
3*b*(1 + p)*x^(3*(1 + p)))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

Maple [F]

$$\int x^{-4-3p} (b\,x^3+a)^p \left(d\,x^3+c\right) dx$$

input
```
int(x^(-4-3*p)*(b*x^3+a)^p*(d*x^3+c),x)
```

output
```
int(x^(-4-3*p)*(b*x^3+a)^p*(d*x^3+c),x)
```

Fricas [F]

$$\int x^{-4-3p} \left(a+bx^3\right)^p \left(c+dx^3\right)\,dx = \int \left(dx^3+c\right) \left(bx^3+a\right)^p x^{-3\,p-4}\,dx$$

input
```
integrate(x^(-4-3*p)*(b*x^3+a)^p*(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*(b*x^3 + a)^p*x^(-3*p - 4), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-4-3p} \left(a+bx^3\right)^p \left(c+dx^3\right)\,dx = \text{Timed out}$$

input
```
integrate(x**(-4-3*p)*(b*x**3+a)**p*(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-4-3p}\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx=\int \left(dx^3+c\right)\left(bx^3+a\right)^p x^{-3\,p-4}\,dx$$

input
```
integrate(x^(-4-3*p)*(b*x^3+a)^p*(d*x^3+c),x, algorithm="maxima")
```

output
```
d*integrate(e^(p*log(b*x^3 + a) - 3*p*log(x))/x, x) - 1/3*(b*x^3 + a)*c*e^
(p*log(b*x^3 + a) - 3*p*log(x))/(a*(p + 1)*x^3)
```

Giac [F]

$$\int x^{-4-3p}\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx=\int \left(dx^3+c\right)\left(bx^3+a\right)^p x^{-3\,p-4}\,dx$$

input
```
integrate(x^(-4-3*p)*(b*x^3+a)^p*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p*x^(-3*p - 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-4-3p}\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx=\int \frac{\left(b\,x^3+a\right)^p\,\left(d\,x^3+c\right)}{x^{3\,p+4}}\,dx$$

input
```
int(((a + b*x^3)^p*(c + d*x^3))/x^(3*p + 4),x)
```

output
```
int(((a + b*x^3)^p*(c + d*x^3))/x^(3*p + 4), x)
```

Reduce [F]

$$\int x^{-4-3p}(a+bx^3)^p\left(c+dx^3\right)\,dx$$
$$=\frac{-(b\,x^3+a)^p\,ac-(b\,x^3+a)^p\,bc\,x^3+3x^{3p}\left(\int\frac{(b\,x^3+a)^p}{x^{3p}x}dx\right)adp\,x^3+3x^{3p}\left(\int\frac{(b\,x^3+a)^p}{x^{3p}x}dx\right)ad\,x^3}{3x^{3p}a\,x^3\,(p+1)}$$

input
```
int(x^(-4-3*p)*(b*x^3+a)^p*(d*x^3+c),x)
```

output
```
( - (a + b*x**3)**p*a*c - (a + b*x**3)**p*b*c*x**3 + 3*x**(3*p)*int((a + b
*x**3)**p/(x**(3*p)*x),x)*a*d*p*x**3 + 3*x**(3*p)*int((a + b*x**3)**p/(x**
(3*p)*x),x)*a*d*x**3)/(3*x**(3*p)*a*x**3*(p + 1))
```

3.415 $\int (ex)^m \left(a+bx^3\right)^p \left(a(1+m)+b(1+m+3(1+p))\right.$

Optimal result

Integrand size = 34, antiderivative size = 22

$$\int (ex)^m \left(a+bx^3\right)^p \left(a(1+m)+b(1+m+3(1+p))x^3\right) \, dx = \frac{(ex)^{1+m} \left(a+bx^3\right)^{1+p}}{e}$$

output
```
(e*x)^(1+m)*(b*x^3+a)^(p+1)/e
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 4.82

$$\int (ex)^m \left(a+bx^3\right)^p \left(a(1+m)+b(1+m+3(1+p))x^3\right) \, dx$$
$$= \frac{x(ex)^m \left(a+bx^3\right)^p \left(1+\frac{bx^3}{a}\right)^{-p} \left(a(4+m)\,\text{Hypergeometric2F1}\left(\frac{1+m}{3},-p,\frac{4+m}{3},-\frac{bx^3}{a}\right)+b(4+m+3p)x\right.}{4+m}$$

input
```
Integrate[(e*x)^m*(a + b*x^3)^p*(a*(1 + m) + b*(1 + m + 3*(1 + p))*x^3),x]
```

output
```
(x*(e*x)^m*(a + b*x^3)^p*(a*(4 + m)*Hypergeometric2F1[(1 + m)/3, -p, (4 +
m)/3, -((b*x^3)/a)] + b*(4 + m + 3*p)*x^3*Hypergeometric2F1[(4 + m)/3, -p,
 (7 + m)/3, -((b*x^3)/a)]))/((4 + m)*(1 + (b*x^3)/a)^p)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+bx^3\right)^p \left(a(m+1)+bx^3(m+3(p+1)+1)\right) \, dx$$

$$\Big\downarrow \text{951}$$

$$\frac{(ex)^{m+1} \left(a+bx^3\right)^{p+1}}{e}$$

input
```
Int[(e*x)^m*(a + b*x^3)^p*(a*(1 + m) + b*(1 + m + 3*(1 + p))*x^3),x]
```

output
```
((e*x)^(1 + m)*(a + b*x^3)^(1 + p))/e
```

Defintions of rubi rules used

rule 951
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
gosper	$x(ex)^{m}\left(b\,x^{3}+a\right)^{p+1}$	19
parallelrisch	$\frac{x^{4}(ex)^{m}\left(b\,x^{3}+a\right)^{p}ab+x(ex)^{m}\left(b\,x^{3}+a\right)^{p}a^{2}}{a}$	45
risch	$\left(b\,x^{3}+a\right)^{p}x\left(b\,x^{3}+a\right)x^{m}e^{m}\mathrm{e}^{\frac{i\,\mathrm{csgn}(iex)\pi m(\mathrm{csgn}(iex)-\mathrm{csgn}(ix))(-\mathrm{csgn}(iex)+\mathrm{csgn}(ie))}{2}}$	65
orering	$\frac{x\left(b\,x^{3}+a\right)(ex)^{m}\left(b\,x^{3}+a\right)^{p}\left(a(1+m)+b(4+m+3p)x^{3}\right)}{b\,x^{3}m+3b\,x^{3}p+4b\,x^{3}+am+a}$	67

input
```
int((e*x)^m*(b*x^3+a)^p*(a*(1+m)+b*(4+m+3*p)*x^3),x,method=_RETURNVERBOSE)
```

output
```
x*(e*x)^m*(b*x^3+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^m\left(a+bx^3\right)^p\left(a(1+m)+b(1+m+3(1+p))x^3\right)\,dx=\left(bx^4+ax\right)\left(bx^3+a\right)^p(ex)^m$$

input
```
integrate((e*x)^m*(b*x^3+a)^p*(a*(1+m)+b*(4+m+3*p)*x^3),x, algorithm="fric
as")
```

output
```
(b*x^4 + a*x)*(b*x^3 + a)^p*(e*x)^m
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^3\right)^p \left(a(1+m)+b(1+m+3(1+p))x^3\right)\,dx=\text{Timed out}$$

input
```
integrate((e*x)**m*(b*x**3+a)**p*(a*(1+m)+b*(4+m+3*p)*x**3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int (ex)^m \left(a+bx^3\right)^p \left(a(1+m)+b(1+m+3(1+p))x^3\right)\,dx = \left(be^m x^4+ae^m x\right)e^{\left(p\log\left(bx^3+a\right)+m\log(x)\right)}$$

input
```
integrate((e*x)^m*(b*x^3+a)^p*(a*(1+m)+b*(4+m+3*p)*x^3),x, algorithm="maxi
ma")
```

output
```
(b*e^m*x^4 + a*e^m*x)*e^(p*log(b*x^3 + a) + m*log(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int (ex)^m \left(a+bx^3\right)^p \left(a(1+m)+b(1+m+3(1+p))x^3\right)\,dx = \left(bx^3+a\right)^p(ex)^m bx^4+\left(bx^3+a\right)^p(ex)^m ax$$

input
```
integrate((e*x)^m*(b*x^3+a)^p*(a*(1+m)+b*(4+m+3*p)*x^3),x, algorithm="giac
")
```

output
```
(b*x^3 + a)^p*(e*x)^m*b*x^4 + (b*x^3 + a)^p*(e*x)^m*a*x
```

Mupad [B] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (ex)^m \left(a+bx^3\right)^p \left(a(1+m)+b(1+m+3(1+p))x^3\right) \, dx = x\,(e\,x)^m\,\left(b\,x^3+a\right)^{p+1}$$

input
```
int((a*(m + 1) + b*x^3*(m + 3*p + 4))*(e*x)^m*(a + b*x^3)^p,x)
```

output
```
x*(e*x)^m*(a + b*x^3)^(p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^m \left(a+bx^3\right)^p \left(a(1+m)+b(1+m+3(1+p))x^3\right) \, dx = x^m e^m \left(b\,x^3+a\right)^p x \left(b\,x^3+a\right)$$

input
```
int((e*x)^m*(b*x^3+a)^p*(a*(1+m)+b*(4+m+3*p)*x^3),x)
```

output
```
x**m*e**m*(a + b*x**3)**p*x*(a + b*x**3)
```

3.416 $\int \frac{x^{11}}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{x^{11}}{(a+bx^3)(c+dx^3)}\,dx = -\frac{(bc+ad)x^3}{3b^2d^2}+\frac{x^6}{6bd}-\frac{a^3\log(a+bx^3)}{3b^3(bc-ad)}+\frac{c^3\log(c+dx^3)}{3d^3(bc-ad)}$$

output
```
-1/3*(a*d+b*c)*x^3/b^2/d^2+1/6*x^6/b/d-1/3*a^3*ln(b*x^3+a)/b^3/(-a*d+b*c)+
1/3*c^3*ln(d*x^3+c)/d^3/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{(a+bx^3)(c+dx^3)}\,dx = \frac{bd(bc-ad)x^3(-2bc-2ad+bdx^3)-2a^3d^3\log(a+bx^3)+2b^3c^3\log(c+dx^3)}{6b^3d^3(bc-ad)}$$

input
```
Integrate[x^11/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(b*d*(b*c - a*d)*x^3*(-2*b*c - 2*a*d + b*d*x^3) - 2*a^3*d^3*Log[a + b*x^3]
 + 2*b^3*c^3*Log[c + d*x^3])/(6*b^3*d^3*(b*c - a*d))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^3)(c+dx^3)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^9}{(bx^3+a)(dx^3+c)}dx^3$$

$$\Big\downarrow \text{ 93}$$

$$\frac{1}{3}\int \left(-\frac{a^3}{b^2(bc-ad)(bx^3+a)}+\frac{x^3}{bd}+\frac{-bc-ad}{b^2d^2}-\frac{c^3}{d^2(ad-bc)(dx^3+c)}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{a^3\log(a+bx^3)}{b^3(bc-ad)}-\frac{x^3(ad+bc)}{b^2d^2}+\frac{c^3\log(c+dx^3)}{d^3(bc-ad)}+\frac{x^6}{2bd}\right)$$

input
```
Int[x^11/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(-(((b*c + a*d)*x^3)/(b^2*d^2)) + x^6/(2*b*d) - (a^3*Log[a + b*x^3])/(b^3*
(b*c - a*d)) + (c^3*Log[c + d*x^3])/(d^3*(b*c - a*d)))/3
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\left(-b d\,x^{3}+a d+b c\right)^{2}}{6 b^{3} d^{3}}-\frac{c^{3} \ln \left(d\,x^{3}+c\right)}{3 d^{3} \left(a d-b c\right)}+\frac{a^{3} \ln \left(b\,x^{3}+a\right)}{3 b^{3} \left(a d-b c\right)}$	78
norman	$\frac{x^{6}}{6 b d}-\frac{\left(a d+b c\right) x^{3}}{3 b^{2} d^{2}}+\frac{a^{3} \ln \left(b\,x^{3}+a\right)}{3 b^{3} \left(a d-b c\right)}-\frac{c^{3} \ln \left(d\,x^{3}+c\right)}{3 d^{3} \left(a d-b c\right)}$	83
parallelrisch	$\frac{a\,b^{2} d^{3} x^{6}-b^{3} c\,d^{2} x^{6}-2 a^{2} b\,d^{3} x^{3}+2 b^{3} c^{2} d\,x^{3}+2 a^{3} \ln \left(b\,x^{3}+a\right) d^{3}-2 c^{3} \ln \left(d\,x^{3}+c\right) b^{3}}{6 b^{3} d^{3} \left(a d-b c\right)}$	99
risch	$\frac{x^{6}}{6 b d}-\frac{x^{3} a}{3 b^{2} d}-\frac{x^{3} c}{3 b\,d^{2}}+\frac{a^{2}}{6 b^{3} d}+\frac{a c}{3 b^{2} d^{2}}+\frac{c^{2}}{6 b\,d^{3}}-\frac{c^{3} \ln \left(d\,x^{3}+c\right)}{3 d^{3} \left(a d-b c\right)}+\frac{a^{3} \ln \left(-b\,x^{3}-a\right)}{3 b^{3} \left(a d-b c\right)}$	124

input
```
int(x^11/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-b*d*x^3+a*d+b*c)^2/b^3/d^3-1/3*c^3/d^3/(a*d-b*c)*ln(d*x^3+c)+1/3*a^3
/b^3/(a*d-b*c)*ln(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int \frac{x^{11}}{(a+bx^3)(c+dx^3)}\,dx = \frac{(b^3cd^2-ab^2d^3)x^6-2\,a^3d^3\log\left(bx^3+a\right)+2\,b^3c^3\log\left(dx^3+c\right)-2\,(b^3c^2d-a^2bd^3)x^3}{6\,(b^4cd^3-ab^3d^4)}$$

input
```
integrate(x^11/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/6*((b^3*c*d^2 - a*b^2*d^3)*x^6 - 2*a^3*d^3*log(b*x^3 + a) + 2*b^3*c^3*lo
g(d*x^3 + c) - 2*(b^3*c^2*d - a^2*b*d^3)*x^3)/(b^4*c*d^3 - a*b^3*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(x**11/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

$$\int \frac{x^{11}}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a^3\log(bx^3+a)}{3(b^4c-ab^3d)} + \frac{c^3\log(dx^3+c)}{3(bcd^3-ad^4)} + \frac{bdx^6-2(bc+ad)x^3}{6\,b^2d^2}$$

input
```
integrate(x^11/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/3*a^3*log(b*x^3 + a)/(b^4*c - a*b^3*d) + 1/3*c^3*log(d*x^3 + c)/(b*c*d^
3 - a*d^4) + 1/6*(b*d*x^6 - 2*(b*c + a*d)*x^3)/(b^2*d^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a^3\log(|bx^3+a|)}{3(b^4c-ab^3d)} + \frac{c^3\log(|dx^3+c|)}{3(bcd^3-ad^4)} + \frac{bdx^6-2\,bcx^3-2\,adx^3}{6\,b^2d^2}$$

input
```
integrate(x^11/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*a^3*log(abs(b*x^3 + a))/(b^4*c - a*b^3*d) + 1/3*c^3*log(abs(d*x^3 + c
))/(b*c*d^3 - a*d^4) + 1/6*(b*d*x^6 - 2*b*c*x^3 - 2*a*d*x^3)/(b^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}}{(a+bx^3)(c+dx^3)}\,dx = \frac{x^6}{6\,b\,d} - \frac{c^3\,\ln\left(d\,x^3+c\right)}{3\,\left(a\,d^4-b\,c\,d^3\right)} - \frac{a^3\,\ln\left(b\,x^3+a\right)}{3\,\left(b^4\,c-a\,b^3\,d\right)} - \frac{x^3\,\left(a\,d+b\,c\right)}{3\,b^2\,d^2}$$

input
```
int(x^11/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
x^6/(6*b*d) - (c^3*log(c + d*x^3))/(3*(a*d^4 - b*c*d^3)) - (a^3*log(a + b*
x^3))/(3*(b^4*c - a*b^3*d)) - (x^3*(a*d + b*c))/(3*b^2*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.78

$$\int \frac{x^{11}}{(a+bx^3)(c+dx^3)}\,dx = \frac{2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^3d^3+2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a^3d^3-2\log\left(c^{\frac{2}{3}}-d^{\frac{1}{3}}c^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2\right)b^3c^3-2\log\left(c^{\frac{1}{3}}+\right.}{6b^3d^3\left(ad-bc\right)}$$

input
```
int(x^11/(b*x^3+a)/(d*x^3+c),x)
```

output
```
(2*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3*d**3 + 2*log(a
**(1/3) + b**(1/3)*x)*a**3*d**3 - 2*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d
**(2/3)*x**2)*b**3*c**3 - 2*log(c**(1/3) + d**(1/3)*x)*b**3*c**3 - 2*a**2*
b*d**3*x**3 + a*b**2*d**3*x**6 + 2*b**3*c**2*d*x**3 - b**3*c*d**2*x**6)/(6
*b**3*d**3*(a*d - b*c))
```

3.417 $\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx = \frac{x^3}{3bd} + \frac{a^2\log(a+bx^3)}{3b^2(bc-ad)} - \frac{c^2\log(c+dx^3)}{3d^2(bc-ad)}$$

output
```
1/3*x^3/b/d+1/3*a^2*ln(b*x^3+a)/b^2/(-a*d+b*c)-1/3*c^2*ln(d*x^3+c)/d^2/(-a
*d+b*c)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx = \frac{a^2d^2\log(a+bx^3) - b(d(-bc+ad)x^3 + bc^2\log(c+dx^3))}{3b^2d^2(bc-ad)}$$

input
```
Integrate[x^8/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(a^2*d^2*Log[a + b*x^3] - b*(d*(-(b*c) + a*d)*x^3 + b*c^2*Log[c + d*x^3]))
/(3*b^2*d^2*(b*c - a*d))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^6}{(bx^3+a)(dx^3+c)}dx^3$$

$$\Big\downarrow \text{ 93}$$

$$\frac{1}{3}\int \left(\frac{a^2}{b(bc-ad)(bx^3+a)}+\frac{1}{bd}+\frac{c^2}{d(ad-bc)(dx^3+c)}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{a^2\log(a+bx^3)}{b^2(bc-ad)}-\frac{c^2\log(c+dx^3)}{d^2(bc-ad)}+\frac{x^3}{bd}\right)$$

input
```
Int[x^8/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(x^3/(b*d) + (a^2*Log[a + b*x^3])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x^3])
/(d^2*(b*c - a*d)))/3
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{x^{3}}{3bd}+\frac{c^{2}\ln\left(d\,x^{3}+c\right)}{3\left(ad-bc\right)d^{2}}-\frac{a^{2}\ln\left(b\,x^{3}+a\right)}{3\left(ad-bc\right)b^{2}}$	65
norman	$\frac{x^{3}}{3bd}+\frac{c^{2}\ln\left(d\,x^{3}+c\right)}{3\left(ad-bc\right)d^{2}}-\frac{a^{2}\ln\left(b\,x^{3}+a\right)}{3\left(ad-bc\right)b^{2}}$	65
risch	$\frac{x^{3}}{3bd}+\frac{c^{2}\ln\left(d\,x^{3}+c\right)}{3\left(ad-bc\right)d^{2}}-\frac{a^{2}\ln\left(-b\,x^{3}-a\right)}{3b^{2}\left(ad-bc\right)}$	68
parallelrisch	$-\frac{-x^{3}ab\,d^{2}+x^{3}b^{2}cd+a^{2}\ln\left(b\,x^{3}+a\right)d^{2}-c^{2}\ln\left(d\,x^{3}+c\right)b^{2}}{3b^{2}d^{2}\left(ad-bc\right)}$	70

input
```
int(x^8/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/b/d+1/3*c^2/(a*d-b*c)/d^2*ln(d*x^3+c)-1/3*a^2/(a*d-b*c)/b^2*ln(b*x
^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx = \frac{a^2d^2\log\left(bx^3+a\right)-b^2c^2\log\left(dx^3+c\right)+(b^2cd-abd^2)x^3}{3\left(b^3cd^2-ab^2d^3\right)}$$

input
```
integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/3*(a^2*d^2*log(b*x^3 + a) - b^2*c^2*log(d*x^3 + c) + (b^2*c*d - a*b*d^2)
*x^3)/(b^3*c*d^2 - a*b^2*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(54) = 108$.

Time = 150.85 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.87

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a^2\log\left(x^3+\frac{\frac{a^4d^3}{b(ad-bc)}-\frac{2a^3cd^2}{ad-bc}+\frac{a^2bc^2d}{ad-bc}+a^2cd+abc^2}{a^2d^2+b^2c^2}\right)}{3b^2(ad-bc)} + \frac{c^2\log\left(x^3+\frac{-\frac{a^2bc^2d}{ad-bc}+a^2cd+\frac{2ab^2c^3}{ad-bc}+abc^2-\frac{b^3c^4}{d(ad-bc)}}{a^2d^2+b^2c^2}\right)}{3d^2(ad-bc)} + \frac{x^3}{3bd}$$

input
```
integrate(x**8/(b*x**3+a)/(d*x**3+c),x)
```

output
```
-a**2*log(x**3 + (a**4*d**3/(b*(a*d - b*c)) - 2*a**3*c*d**2/(a*d - b*c) +
a**2*b*c**2*d/(a*d - b*c) + a**2*c*d + a*b*c**2)/(a**2*d**2 + b**2*c**2))/
(3*b**2*(a*d - b*c)) + c**2*log(x**3 + (-a**2*b*c**2*d/(a*d - b*c) + a**2*
c*d + 2*a*b**2*c**3/(a*d - b*c) + a*b*c**2 - b**3*c**4/(d*(a*d - b*c)))/(a
**2*d**2 + b**2*c**2))/(3*d**2*(a*d - b*c)) + x**3/(3*b*d)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx = \frac{a^2\log(bx^3+a)}{3(b^3c-ab^2d)} - \frac{c^2\log(dx^3+c)}{3(bcd^2-ad^3)} + \frac{x^3}{3\,bd}$$

input
```
integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*a^2*log(b*x^3 + a)/(b^3*c - a*b^2*d) - 1/3*c^2*log(d*x^3 + c)/(b*c*d^2
 - a*d^3) + 1/3*x^3/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx = \frac{a^2\log(|bx^3+a|)}{3(b^3c-ab^2d)} - \frac{c^2\log(|dx^3+c|)}{3(bcd^2-ad^3)} + \frac{x^3}{3\,bd}$$

input
```
integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*a^2*log(abs(b*x^3 + a))/(b^3*c - a*b^2*d) - 1/3*c^2*log(abs(d*x^3 + c)
)/(b*c*d^2 - a*d^3) + 1/3*x^3/(b*d)
```

Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx = \frac{a^2\,\ln(b\,x^3+a)}{3\,b^3\,c-3\,a\,b^2\,d} + \frac{c^2\,\ln(d\,x^3+c)}{3\,a\,d^3-3\,b\,c\,d^2} + \frac{x^3}{3\,b\,d}$$

input
```
int(x^8/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
(a^2*log(a + b*x^3))/(3*b^3*c - 3*a*b^2*d) + (c^2*log(c + d*x^3))/(3*a*d^3
 - 3*b*c*d^2) + x^3/(3*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.86

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx = \frac{-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^2d^2-\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a^2d^2+\log\left(c^{\frac{2}{3}}-d^{\frac{1}{3}}c^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2\right)b^2c^2+\log\left(c^{\frac{1}{3}}+d^{\frac{1}{3}}x\right.}{3b^2d^2(ad-bc)}$$

input
```
int(x^8/(b*x^3+a)/(d*x^3+c),x)
```

output
```
( - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*d**2 - log(a*
*(1/3) + b**(1/3)*x)*a**2*d**2 + log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(
2/3)*x**2)*b**2*c**2 + log(c**(1/3) + d**(1/3)*x)*b**2*c**2 + a*b*d**2*x**
3 - b**2*c*d*x**3)/(3*b**2*d**2*(a*d - b*c))
```

3.418 $\int \frac{x^5}{(a+bx^3)(c+dx^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)} dx = -\frac{a\log(a+bx^3)}{3b(bc-ad)} + \frac{c\log(c+dx^3)}{3d(bc-ad)}$$

output
```
-1/3*a*ln(b*x^3+a)/b/(-a*d+b*c)+1/3*c*ln(d*x^3+c)/d/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)} dx = -\frac{ad\log(a+bx^3) - bc\log(c+dx^3)}{3b^2cd - 3abd^2}$$

input
```
Integrate[x^5/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
-((a*d*Log[a + b*x^3] - b*c*Log[c + d*x^3])/(3*b^2*c*d - 3*a*b*d^2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{(bx^3+a)(dx^3+c)}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(\frac{c}{(bc-ad)(dx^3+c)}-\frac{a}{(bc-ad)(bx^3+a)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{c\log(c+dx^3)}{d(bc-ad)}-\frac{a\log(a+bx^3)}{b(bc-ad)}\right)$$

input
```
Int[x^5/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(-((a*Log[a + b*x^3])/(b*(b*c - a*d))) + (c*Log[c + d*x^3])/(d*(b*c - a*d)
))/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$\frac{a\ln(b\,x^3+a)d-c\ln(d\,x^3+c)b}{3(ad-bc)bd}$	43
default	$-\frac{c\ln(d\,x^3+c)}{3(ad-bc)d}+\frac{a\ln(b\,x^3+a)}{3(ad-bc)b}$	50
norman	$-\frac{c\ln(d\,x^3+c)}{3(ad-bc)d}+\frac{a\ln(b\,x^3+a)}{3(ad-bc)b}$	50
risch	$-\frac{c\ln(-d\,x^3-c)}{3(ad-bc)d}+\frac{a\ln(b\,x^3+a)}{3(ad-bc)b}$	53

input
```
int(x^5/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*ln(b*x^3+a)*d-c*ln(d*x^3+c)*b)/(a*d-b*c)/b/d
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx=-\frac{ad\log(bx^3+a)-bc\log(dx^3+c)}{3(b^2cd-abd^2)}$$

input
```
integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/3*(a*d*log(b*x^3 + a) - b*c*log(d*x^3 + c))/(b^2*c*d - a*b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(39) = 78.

Time = 2.54 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.72

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx = \frac{a\log\left(x^3 + \frac{\frac{a^3d^2}{b(ad-bc)} - \frac{2a^2cd}{ad-bc} + \frac{abc^2}{ad-bc} + 2ac}{ad+bc}\right)}{3b(ad-bc)} - \frac{c\log\left(x^3 + \frac{-\frac{a^2cd}{ad-bc} + \frac{2abc^2}{ad-bc} + 2ac - \frac{b^2c^3}{d(ad-bc)}}{ad+bc}\right)}{3d(ad-bc)}$$

input
```
integrate(x**5/(b*x**3+a)/(d*x**3+c),x)
```

output
```
a*log(x**3 + (a**3*d**2/(b*(a*d - b*c)) - 2*a**2*c*d/(a*d - b*c) + a*b*c**
2/(a*d - b*c) + 2*a*c)/(a*d + b*c))/(3*b*(a*d - b*c)) - c*log(x**3 + (-a**
2*c*d/(a*d - b*c) + 2*a*b*c**2/(a*d - b*c) + 2*a*c - b**2*c**3/(d*(a*d - b
*c)))/(a*d + b*c))/(3*d*(a*d - b*c))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a\log(bx^3+a)}{3(b^2c-abd)} + \frac{c\log(dx^3+c)}{3(bcd-ad^2)}$$

input
```
integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/3*a*log(b*x^3 + a)/(b^2*c - a*b*d) + 1/3*c*log(d*x^3 + c)/(b*c*d - a*d^
2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a\log(|bx^3+a|)}{3(b^2c-abd)} + \frac{c\log(|dx^3+c|)}{3(bcd-ad^2)}$$

input
```
integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*a*log(abs(b*x^3 + a))/(b^2*c - a*b*d) + 1/3*c*log(abs(d*x^3 + c))/(b*
c*d - a*d^2)
```

Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a\,\ln(b\,x^3+a)}{3\,b^2\,c-3\,a\,b\,d} - \frac{c\,\ln(d\,x^3+c)}{3\,a\,d^2-3\,b\,c\,d}$$

input
```
int(x^5/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
- (a*log(a + b*x^3))/(3*b^2*c - 3*a*b*d) - (c*log(c + d*x^3))/(3*a*d^2 - 3
*b*c*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.79

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx = \frac{\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)ad+\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)ad-\log\left(c^{\frac{2}{3}}-d^{\frac{1}{3}}c^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2\right)bc-\log\left(c^{\frac{1}{3}}+d^{\frac{1}{3}}x\right)bc}{3bd(ad-bc)}$$

input
```
int(x^5/(b*x^3+a)/(d*x^3+c),x)
```

output
```
(log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*d + log(a**(1/3) +
b**(1/3)*x)*a*d - log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*b*c
- log(c**(1/3) + d**(1/3)*x)*b*c)/(3*b*d*(a*d - b*c))
```

3.419 $\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{\log(a+bx^3)}{3(bc-ad)} - \frac{\log(c+dx^3)}{3(bc-ad)}$$

output
```
ln(b*x^3+a)/(-3*a*d+3*b*c)-ln(d*x^3+c)/(-3*a*d+3*b*c)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{\log(a+bx^3)-\log(c+dx^3)}{3bc-3ad}$$

input
```
Integrate[x^2/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(Log[a + b*x^3] - Log[c + d*x^3])/(3*b*c - 3*a*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{3}\int \frac{1}{(bx^3+a)(dx^3+c)}dx^3$$

$$\Big\downarrow \text{ 47}$$

$$\frac{1}{3}\left(\frac{b\int \frac{1}{bx^3+a}dx^3}{bc-ad}-\frac{d\int \frac{1}{dx^3+c}dx^3}{bc-ad}\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{3}\left(\frac{\log(a+bx^3)}{bc-ad}-\frac{\log(c+dx^3)}{bc-ad}\right)$$

input
```
Int[x^2/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(Log[a + b*x^3]/(b*c - a*d) - Log[c + d*x^3]/(b*c - a*d))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

method	result	size
parallelrisch	$-\frac{\ln(b\,x^3+a)-\ln(d\,x^3+c)}{3(ad-bc)}$	32
default	$\frac{\ln(d\,x^3+c)}{3ad-3bc}-\frac{\ln(b\,x^3+a)}{3(ad-bc)}$	42
norman	$\frac{\ln(d\,x^3+c)}{3ad-3bc}-\frac{\ln(b\,x^3+a)}{3(ad-bc)}$	42
risch	$-\frac{\ln(-b\,x^3-a)}{3(ad-bc)}+\frac{\ln(d\,x^3+c)}{3ad-3bc}$	45

input
```
int(x^2/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(ln(b*x^3+a)-ln(d*x^3+c))/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{\log(bx^3+a)-\log(dx^3+c)}{3(bc-ad)}$$

input
```
integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/3*(log(b*x^3 + a) - log(d*x^3 + c))/(b*c - a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(36) = 72$.

Time = 0.77 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.07

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{\log\left(x^3 + \frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{3(ad-bc)} - \frac{\log\left(x^3 + \frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{3(ad-bc)}$$

input
```
integrate(x**2/(b*x**3+a)/(d*x**3+c),x)
```

output
```
log(x**3 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c*
*2/(a*d - b*c) + b*c)/(2*b*d))/(3*(a*d - b*c)) - log(x**3 + (a**2*d**2/(a*
d - b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b
*d))/(3*(a*d - b*c))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{\log(bx^3+a)}{3(bc-ad)} - \frac{\log(dx^3+c)}{3(bc-ad)}$$

input
```
integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*log(b*x^3 + a)/(b*c - a*d) - 1/3*log(d*x^3 + c)/(b*c - a*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{b\log\left(|bx^3+a|\right)}{3\left(b^2c-abd\right)} - \frac{d\log\left(|dx^3+c|\right)}{3\left(bcd-ad^2\right)}$$

input
```
integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*b*log(abs(b*x^3 + a))/(b^2*c - a*b*d) - 1/3*d*log(abs(d*x^3 + c))/(b*c
*d - a*d^2)
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 602, normalized size of antiderivative = 13.38

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = -\frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{x^3\left(36\,c\,b^4\,d^3+36\,a\,b^3\,d^4\right)+\frac{x^3\left(54\,a^2\,b^3\,d^5+108\,a\,b^4\,c\,d^4+54\,b^5\,c^2\,d^3\right)+108\,a\,b^4\,c^2\,d^3+108\,a^2\,b^3\,c\,d^4}{3\,a\,d-3\,b\,c}+36\,a\,b^3\,c\,d^3}{3\,a\,d-3\,b\,c}+6\,b^3\,d^3\,x^3\right)1\mathrm{i}}{3\,a\,d-3\,b\,c}-\frac{\left(\frac{x^3\left(36\,c\cdots\right.}{\cdots}\right.}{\cdots}}{\frac{\frac{x^3\left(36\,c\,b^4\,d^3+36\,a\,b^3\,d^4\right)+\frac{x^3\left(54\,a^2\,b^3\,d^5+108\,a\,b^4\,c\,d^4+54\,b^5\,c^2\,d^3\right)+108\,a\,b^4\,c^2\,d^3+108\,a^2\,b^3\,c\,d^4}{3\,a\,d-3\,b\,c}+36\,a\,b^3\,c\,d^3}{3\,a\,d-3\,b\,c}+6\,b^3\,d^3\,x^3}{3\,a\,d-3\,b\,c}+\frac{x^3\left(36\,c\,b\cdots\right.}{\cdots}}\right)\cdots}{3\,a\,d-3\,b\,c\cdots}$$

input
```
int(x^2/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
-(atan(((((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) + (x^3*(54*a^2*b^3*d^5 + 54*b
^5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*
a*d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) + 6*b^3*d^3*x^3)*1i)/(3*a*d
 - 3*b*c) - (((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) - (x^3*(54*a^2*b^3*d^5 +
54*b^5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)
/(3*a*d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) - 6*b^3*d^3*x^3)*1i)/(3
*a*d - 3*b*c))/(((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) + (x^3*(54*a^2*b^3*d^5
 + 54*b^5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d
^4)/(3*a*d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) + 6*b^3*d^3*x^3)/(3*
a*d - 3*b*c) + ((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) - (x^3*(54*a^2*b^3*d^5
+ 54*b^5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^
4)/(3*a*d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) - 6*b^3*d^3*x^3)/(3*a
*d - 3*b*c)))*2i)/(3*a*d - 3*b*c)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.76

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)+\log\left(c^{\frac{2}{3}}-d^{\frac{1}{3}}c^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2\right)+\log\left(c^{\frac{1}{3}}+d^{\frac{1}{3}}x\right)}{3ad-3bc}$$

input
```
int(x^2/(b*x^3+a)/(d*x^3+c),x)
```

output
```
( - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) - log(a**(1/3) + b
**(1/3)*x) + log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2) + log(c**
(1/3) + d**(1/3)*x))/(3*(a*d - b*c))
```

3.420 $\int \frac{1}{x(a+bx^3)(c+dx^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \frac{1}{x(a+bx^3)(c+dx^3)} dx = \frac{\log(x)}{ac} - \frac{b\log(a+bx^3)}{3a(bc-ad)} + \frac{d\log(c+dx^3)}{3c(bc-ad)}$$

output
```
ln(x)/a/c-1/3*b*ln(b*x^3+a)/a/(-a*d+b*c)+1/3*d*ln(d*x^3+c)/c/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(a+bx^3)(c+dx^3)} dx = \frac{3bc\log(x) - 3ad\log(x) - bc\log(a+bx^3) + ad\log(c+dx^3)}{3abc^2 - 3a^2cd}$$

input
```
Integrate[1/(x*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
(3*b*c*Log[x] - 3*a*d*Log[x] - b*c*Log[a + b*x^3] + a*d*Log[c + d*x^3])/(3
*a*b*c^2 - 3*a^2*c*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{1}{x^3\left(bx^3+a\right)\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \text{ 93}$$

$$\frac{1}{3}\int \left(\frac{b^2}{a(ad-bc)\left(bx^3+a\right)}+\frac{d^2}{c(bc-ad)\left(dx^3+c\right)}+\frac{1}{acx^3}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{b\log\left(a+bx^3\right)}{a(bc-ad)}+\frac{d\log\left(c+dx^3\right)}{c(bc-ad)}+\frac{\log\left(x^3\right)}{ac}\right)$$

input
```
Int[1/(x*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
(Log[x^3]/(a*c) - (b*Log[a + b*x^3])/(a*(b*c - a*d)) + (d*Log[c + d*x^3])/
(c*(b*c - a*d)))/3
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$\frac{3\ln(x)ad-3\ln(x)bc+b\ln(b\,x^{3}+a)c-d\ln(d\,x^{3}+c)a}{3ac(ad-bc)}$	55
default	$-\frac{d\ln(d\,x^{3}+c)}{3(ad-bc)c}+\frac{\ln(x)}{ac}+\frac{b\ln(b\,x^{3}+a)}{3(ad-bc)a}$	59
norman	$-\frac{d\ln(d\,x^{3}+c)}{3(ad-bc)c}+\frac{\ln(x)}{ac}+\frac{b\ln(b\,x^{3}+a)}{3(ad-bc)a}$	59
risch	$\frac{\ln(x)}{ac}-\frac{d\ln(-d\,x^{3}-c)}{3c(ad-bc)}+\frac{b\ln(-b\,x^{3}-a)}{3(ad-bc)a}$	65

input
```
int(1/x/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*ln(x)*a*d-3*ln(x)*b*c+b*ln(b*x^3+a)*c-d*ln(d*x^3+c)*a)/a/c/(a*d-b*c
)
```

Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)}\,dx=-\frac{bc\log\left(bx^3+a\right)-ad\log\left(dx^3+c\right)-3\left(bc-ad\right)\log\left(x\right)}{3\left(abc^2-a^2cd\right)}$$

input
```
integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/3*(b*c*log(b*x^3 + a) - a*d*log(d*x^3 + c) - 3*(b*c - a*d)*log(x))/(a*b
*c^2 - a^2*c*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\,(a+bx^3)\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{1}{x\,(a+bx^3)\,(c+dx^3)}\,dx = -\frac{b\log\left(bx^3+a\right)}{3\,(abc-a^2d)} + \frac{d\log\left(dx^3+c\right)}{3\,(bc^2-acd)} + \frac{\log\left(x^3\right)}{3\,ac}$$

input
```
integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/3*b*log(b*x^3 + a)/(a*b*c - a^2*d) + 1/3*d*log(d*x^3 + c)/(b*c^2 - a*c*
d) + 1/3*log(x^3)/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\,(a+bx^3)\,(c+dx^3)}\,dx = -\frac{b^2\log\left(|bx^3+a|\right)}{3\,(ab^2c-a^2bd)} + \frac{d^2\log\left(|dx^3+c|\right)}{3\,(bc^2d-acd^2)} + \frac{\log\left(|x|\right)}{ac}$$

input
```
integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*b^2*log(abs(b*x^3 + a))/(a*b^2*c - a^2*b*d) + 1/3*d^2*log(abs(d*x^3 +
 c))/(b*c^2*d - a*c*d^2) + log(abs(x))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)}\,dx=\frac{b\,\ln\left(b\,x^3+a\right)}{3\,a^2\,d-3\,a\,b\,c}+\frac{d\,\ln\left(d\,x^3+c\right)}{3\,b\,c^2-3\,a\,c\,d}+\frac{\ln\left(x\right)}{a\,c}$$

input
```
int(1/(x*(a + b*x^3)*(c + d*x^3)),x)
```

output
```
(b*log(a + b*x^3))/(3*a^2*d - 3*a*b*c) + (d*log(c + d*x^3))/(3*b*c^2 - 3*a
*c*d) + log(x)/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.73

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$
$$=\frac{\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)bc+\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)bc-\log\left(c^{\frac{2}{3}}-d^{\frac{1}{3}}c^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2\right)ad-\log\left(c^{\frac{1}{3}}+d^{\frac{1}{3}}x\right)ad+3}{3ac\left(ad-bc\right)}$$

input
```
int(1/x/(b*x^3+a)/(d*x^3+c),x)
```

output
```
(log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*c + log(a**(1/3) +
b**(1/3)*x)*b*c - log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a*d
- log(c**(1/3) + d**(1/3)*x)*a*d + 3*log(x)*a*d - 3*log(x)*b*c)/(3*a*c*(a*
d - b*c))
```

3.421 $\int \frac{1}{x^4(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{1}{x^4(a+bx^3)(c+dx^3)}\,dx = -\frac{1}{3acx^3} - \frac{(bc+ad)\log(x)}{a^2c^2} + \frac{b^2\log(a+bx^3)}{3a^2(bc-ad)} - \frac{d^2\log(c+dx^3)}{3c^2(bc-ad)}$$

output
```
-1/3/a/c/x^3-(a*d+b*c)*ln(x)/a^2/c^2+1/3*b^2*ln(b*x^3+a)/a^2/(-a*d+b*c)-1/
3*d^2*ln(d*x^3+c)/c^2/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^4(a+bx^3)(c+dx^3)}\,dx = -\frac{1}{3acx^3} + \frac{(-bc-ad)\log(x)}{a^2c^2} - \frac{b^2\log(a+bx^3)}{3a^2(-bc+ad)} - \frac{d^2\log(c+dx^3)}{3c^2(bc-ad)}$$

input
```
Integrate[1/(x^4*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
-1/3*1/(a*c*x^3) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^3]
)/(3*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^3])/(3*c^2*(b*c - a*d))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^3)(c+dx^3)} dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{1}{x^6 (bx^3+a)(dx^3+c)} dx^3$$

$$\Big\downarrow \text{93}$$

$$\frac{1}{3} \int \left(-\frac{b^3}{a^2(ad-bc)(bx^3+a)} - \frac{d^3}{c^2(bc-ad)(dx^3+c)} + \frac{-bc-ad}{a^2c^2x^3} + \frac{1}{acx^6}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{b^2 \log(a+bx^3)}{a^2(bc-ad)} - \frac{\log(x^3)(ad+bc)}{a^2c^2} - \frac{d^2\log(c+dx^3)}{c^2(bc-ad)} - \frac{1}{acx^3}\right)$$

input
```
Int[1/(x^4*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
(-(1/(a*c*x^3)) - ((b*c + a*d)*Log[x^3])/(a^2*c^2) + (b^2*Log[a + b*x^3])/
(a^2*(b*c - a*d)) - (d^2*Log[c + d*x^3])/(c^2*(b*c - a*d)))/3
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

method	result	size
norman	$-\frac{1}{3 a c \,x^{3}}-\frac{b^{2} \ln \left(b \,x^{3}+a \right)}{3 a^{2} \left(a d -b c \right)}+\frac{d^{2} \ln \left(d \,x^{3}+c \right)}{3 c^{2} \left(a d -b c \right)}-\frac{\left(a d +b c \right) \ln \left(x \right)}{a^{2} c^{2}}$	82
default	$\frac{d^{2} \ln \left(d \,x^{3}+c \right)}{3 c^{2} \left(a d -b c \right)}-\frac{1}{3 a c \,x^{3}}+\frac{\left(-a d -b c \right) \ln \left(x \right)}{a^{2} c^{2}}-\frac{b^{2} \ln \left(b \,x^{3}+a \right)}{3 a^{2} \left(a d -b c \right)}$	83
risch	$-\frac{1}{3 a c \,x^{3}}-\frac{\ln \left(x \right) d}{a \,c^{2}}-\frac{\ln \left(x \right) b}{a^{2} c}-\frac{b^{2} \ln \left(-b \,x^{3}-a \right)}{3 \left(a d -b c \right) a^{2}}+\frac{d^{2} \ln \left(d \,x^{3}+c \right)}{3 c^{2} \left(a d -b c \right)}$	90
parallelrisch	$-\frac{3 \ln \left(x \right) x^{3} a^{2} d^{2}-3 \ln \left(x \right) x^{3} b^{2} c^{2}+b^{2} \ln \left(b \,x^{3}+a \right) c^{2} x^{3}-d^{2} \ln \left(d \,x^{3}+c \right) a^{2} x^{3}+a^{2} c d -b \,c^{2} a}{3 a^{2} c^{2} x^{3} \left(a d -b c \right)}$	99

input
```
int(1/x^4/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/c/x^3-1/3*b^2/a^2/(a*d-b*c)*ln(b*x^3+a)+1/3*d^2/c^2/(a*d-b*c)*ln(d*
x^3+c)-(a*d+b*c)*ln(x)/a^2/c^2
```

Fricas [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^4 (a + bx^3) (c + dx^3)} dx = \frac{b^2c^2x^3 \log(bx^3 + a) - a^2d^2x^3 \log(dx^3 + c) - 3(b^2c^2 - a^2d^2)x^3 \log(x) - abc^2 + a^2cd}{3(a^2bc^3 - a^3c^2d)x^3}$$

input
```
integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/3*(b^2*c^2*x^3*log(b*x^3 + a) - a^2*d^2*x^3*log(d*x^3 + c) - 3*(b^2*c^2
- a^2*d^2)*x^3*log(x) - a*b*c^2 + a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^3) (c + dx^3)} dx = \text{Timed out}$$

input
```
integrate(1/x**4/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (a + bx^3) (c + dx^3)} dx = \frac{b^2 \log(bx^3 + a)}{3(a^2bc - a^3d)} - \frac{d^2 \log(dx^3 + c)}{3(bc^3 - ac^2d)} - \frac{(bc + ad) \log(x^3)}{3a^2c^2} - \frac{1}{3acx^3}$$

input
```
integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*b^2*log(b*x^3 + a)/(a^2*b*c - a^3*d) - 1/3*d^2*log(d*x^3 + c)/(b*c^3 -
 a*c^2*d) - 1/3*(b*c + a*d)*log(x^3)/(a^2*c^2) - 1/3/(a*c*x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^4 (a + bx^3) (c + dx^3)} \, dx = \frac{b^3 \log (|bx^3 + a|)}{3 (a^2b^2c - a^3bd)} - \frac{d^3 \log (|dx^3 + c|)}{3 (bc^3d - ac^2d^2)} - \frac{(bc + ad) \log (|x|)}{a^2c^2} + \frac{bcx^3 + adx^3 - ac}{3 a^2c^2x^3}$$

input
```
integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*b^3*log(abs(b*x^3 + a))/(a^2*b^2*c - a^3*b*d) - 1/3*d^3*log(abs(d*x^3
+ c))/(b*c^3*d - a*c^2*d^2) - (b*c + a*d)*log(abs(x))/(a^2*c^2) + 1/3*(b*c
*x^3 + a*d*x^3 - a*c)/(a^2*c^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (a + bx^3) (c + dx^3)} \, dx = -\frac{b^2 \ln (b x^3 + a)}{3 (a^3 d - a^2 b c)} - \frac{d^2 \ln (d x^3 + c)}{3 (b c^3 - a c^2 d)} - \frac{1}{3 a c x^3} - \frac{\ln (x) (a d + b c)}{a^2 c^2}$$

input
```
int(1/(x^4*(a + b*x^3)*(c + d*x^3)),x)
```

output
```
- (b^2*log(a + b*x^3))/(3*(a^3*d - a^2*b*c)) - (d^2*log(c + d*x^3))/(3*(b*
c^3 - a*c^2*d)) - 1/(3*a*c*x^3) - (log(x)*(a*d + b*c))/(a^2*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.90

$$\int \frac{1}{x^4 (a+bx^3) (c+dx^3)} dx$$
$$= \frac{-\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}} a^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2\right) b^2 c^2 x^3 - \log\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right) b^2 c^2 x^3 + \log\left(c^{\frac{2}{3}} - d^{\frac{1}{3}} c^{\frac{1}{3}} x + d^{\frac{2}{3}} x^2\right) a^2 d^2 x^3 + \log\left(c^{\frac{1}{3}}\right.}{3a^2 c^2 x^3 (ad - bc)}$$

input
```
int(1/x^4/(b*x^3+a)/(d*x^3+c),x)
```

output
```
( - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**2*c**2*x**3 - l
og(a**(1/3) + b**(1/3)*x)*b**2*c**2*x**3 + log(c**(2/3) - d**(1/3)*c**(1/3
)*x + d**(2/3)*x**2)*a**2*d**2*x**3 + log(c**(1/3) + d**(1/3)*x)*a**2*d**2
*x**3 - 3*log(x)*a**2*d**2*x**3 + 3*log(x)*b**2*c**2*x**3 - a**2*c*d + a*b
*c**2)/(3*a**2*c**2*x**3*(a*d - b*c))
```

3.422 $\int \frac{x^9}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 317

$$\begin{aligned}\int \frac{x^9}{(a+bx^3)(c+dx^3)}\,dx = &-\frac{(bc+ad)x}{b^2d^2}+\frac{x^4}{4bd}+\frac{a^{7/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}(bc-ad)}\\ &-\frac{c^{7/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{7/3}(bc-ad)}\\ &-\frac{a^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}(bc-ad)}+\frac{c^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{7/3}(bc-ad)}\\ &+\frac{a^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{7/3}(bc-ad)}\\ &-\frac{c^{7/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{7/3}(bc-ad)}\end{aligned}$$

output
```
-(a*d+b*c)*x/b^2/d^2+1/4*x^4/b/d+1/3*a^(7/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)
*x)*3^(1/2)/a^(1/3))*3^(1/2)/b^(7/3)/(-a*d+b*c)-1/3*c^(7/3)*arctan(1/3*(c^
(1/3)-2*d^(1/3)*x)*3^(1/2)/c^(1/3))*3^(1/2)/d^(7/3)/(-a*d+b*c)-1/3*a^(7/3)
*ln(a^(1/3)+b^(1/3)*x)/b^(7/3)/(-a*d+b*c)+1/3*c^(7/3)*ln(c^(1/3)+d^(1/3)*x
)/d^(7/3)/(-a*d+b*c)+1/6*a^(7/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/b^(7/3)/(-a*d+b*c)-1/6*c^(7/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/
d^(7/3)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.82

$$\int \frac{x^9}{(a+bx^3)(c+dx^3)}\,dx$$

$$= \frac{\frac{12a^2x}{b^2} - \frac{12c^2x}{d^2} - \frac{3ax^4}{b} + \frac{3cx^4}{d} + \frac{4\sqrt{3}a^{7/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{7/3}} - \frac{4\sqrt{3}c^{7/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{d^{7/3}} - \frac{4a^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{7/3}} + \frac{4c}{\ldots}}{12bc-12ad}$$

input
```
Integrate[x^9/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
((12*a^2*x)/b^2 - (12*c^2*x)/d^2 - (3*a*x^4)/b + (3*c*x^4)/d + (4*Sqrt[3]*
a^(7/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(7/3) - (4*Sqrt[3]*
c^(7/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(7/3) - (4*a^(7/3)*
Log[a^(1/3) + b^(1/3)*x])/b^(7/3) + (4*c^(7/3)*Log[c^(1/3) + d^(1/3)*x])/d
^(7/3) + (2*a^(7/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(7/3
) - (2*c^(7/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(7/3))/(1
2*b*c - 12*a*d)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {979, 27, 1052, 1020, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^3)(c+dx^3)}dx$$

$$\Big\downarrow \text{979}$$

$$\frac{x^4}{4bd}-\frac{\int \frac{4x^3((bc+ad)x^3+ac)}{(bx^3+a)(dx^3+c)}dx}{4bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4}{4bd}-\frac{\int \frac{x^3((bc+ad)x^3+ac)}{(bx^3+a)(dx^3+c)}dx}{bd}$$

$$\Big\downarrow \text{1052}$$

$$\frac{x^4}{4bd}-\frac{\frac{x(ad+bc)}{bd}-\frac{\int \frac{\left(b^2c^2+abdc+a^2d^2\right)x^3+ac(bc+ad)}{\left(bx^3+a\right)\left(dx^3+c\right)}dx}{bd}}{bd}$$

$$\Big\downarrow \text{1020}$$

$$\frac{x^4}{4bd}-\frac{\frac{x(ad+bc)}{bd}-\frac{\frac{b^2c^3\int \frac{1}{dx^3+c}dx}{bc-ad}-\frac{a^3d^2\int \frac{1}{bx^3+a}dx}{bc-ad}}{bd}}{bd}$$

$$\Big\downarrow \text{750}$$

$$\frac{x^4}{4bd}-\frac{\frac{x(ad+bc)}{bd}-\frac{\frac{b^2c^3\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}-\frac{a^3d^2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad}}{bd}}{bd}$$

$$\Big\downarrow \text{16}$$

$$\frac{x^4}{4bd}-\frac{\frac{x(ad+bc)}{bd}-\frac{\frac{b^2c^3\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a^3d^2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}}{bd}}{bd}$$

$\downarrow$ 1142

$$\frac{x^4}{4bd}-\frac{\frac{x(ad+bc)}{bd}-\frac{\frac{b^2c^3\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-a^3d^2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x \cdots}}{\cdots}\right.}{bd}}{bd}$$

$\downarrow$ 25

$$\frac{x^4}{4bd}-\frac{\frac{x(ad+bc)}{bd}-\frac{\frac{b^2c^3\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int \frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-a^3d^2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+\cdots}}{\cdots}\right.}{bd}}{bd}$$

$\downarrow$ 27

$$\frac{x^4}{4bd}-\frac{\frac{x(ad+bc)}{bd}-\frac{\frac{b^2c^3\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-a^3d^2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x \cdots}}{\cdots}\right.}{bd}}{bd}$$

$\downarrow$ 1082

$$\frac{x^4}{4bd}-\frac{\frac{x(ad+bc)}{bd}-\frac{\frac{b^2c^3\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a^3d^2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+\cdots}\cdots\right)\cdots}{\cdots}}{bd}}{bd}$$

$\Big\downarrow$ 217

$$\frac{x^4}{4bd}-\frac{\frac{x(ad+bc)}{bd}-\frac{\frac{b^2c^3\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a^3d^2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\,\mathrm{arc}\cdots}{\cdots}}{3a^{2/3}}\cdots\right)}{bc-ad\cdots}}{bd}}{bd}$$

$\Big\downarrow$ 1103

$$\frac{x^4}{4bd}-\frac{\frac{x(ad+bc)}{bd}-\frac{\frac{b^2c^3\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a^3d^2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\cdots}{\cdots}}{3a^{2/3}}\cdots\right)}{bc-\cdots}}{bd}}{bd}$$

input

```
Int[x^9/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
x^4/(4*b*d) - (((b*c + a*d)*x)/(b*d) - (-((a^3*d^2*(Log[a^(1/3) + b^(1/3)*
x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/
3)))/(3*a^(2/3))))/(b*c - a*d)) + (b^2*c^3*(Log[c^(1/3) + d^(1/3)*x]/(3*c^
(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/
d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(2*d^(1/3)))/(3*
c^(2/3))))/(b*c - a*d))/(b*d))/(b*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.76

method	result
default	$-\frac{-\frac{1}{4}bd\,x^4+adx+bcx}{b^2d^2}-\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)c^3}{d^2(ad-bc)}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln(}{}\right.}{}$
risch	$\frac{x^4}{4bd}-\frac{ax}{b^2d}-\frac{cx}{b\,d^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\left(d^4a^3-3a^2c\,d^3b+3a\,c^2d^2b^2-d\,c^3b^3\right)\_Z^3+b^6c^7\right)}\_R\ln\left(\left(\left(-a^5b^2c\,d^6+4a^4b^3c^2d^5-6a^3b^4c^3d^4+\right.\right.\right.}{}$

input
```
int(x^9/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2/d^2*(-1/4*b*d*x^4+a*d*x+b*c*x)-(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))
-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))/d^2*c^3/(a*d-b*c)+(1/3/b/(a/
b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/b^
2*a^3/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.89

$$\int \frac{x^9}{(a+bx^3)(c+dx^3)}\,dx =$$
$$-\frac{4\sqrt{3}a^2d^2\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)+4\sqrt{3}b^2c^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(-\frac{c}{d}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right)-2\,a^2d^2\left(\frac{a}{b}\right)^{\frac{1}{3}}\log}{}$$

input
```
integrate(x^9/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/12*(4*sqrt(3)*a^2*d^2*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3)
 - sqrt(3)*a)/a) + 4*sqrt(3)*b^2*c^2*(-c/d)^(1/3)*arctan(1/3*(2*sqrt(3)*d*
x*(-c/d)^(2/3) - sqrt(3)*c)/c) - 2*a^2*d^2*(a/b)^(1/3)*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3)) - 2*b^2*c^2*(-c/d)^(1/3)*log(x^2 + x*(-c/d)^(1/3) + (-
c/d)^(2/3)) + 4*a^2*d^2*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 4*b^2*c^2*(-c/d
)^(1/3)*log(x - (-c/d)^(1/3)) - 3*(b^2*c*d - a*b*d^2)*x^4 + 12*(b^2*c^2 -
a^2*d^2)*x)/(b^3*c*d^2 - a*b^2*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(x**9/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.15

$$\int \frac{x^9}{(a+bx^3)(c+dx^3)}\,dx = -\frac{\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^4c\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^3d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3}c^3\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd^3\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^4\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{a^3\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^4c\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab^3d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{c^3\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^4\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} - \frac{a^3\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^4c\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab^3d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{c^3\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^4\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{bdx^4-4\,(bc+ad)x}{4\,b^2d^2}$$

input
```
integrate(x^9/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*a^3*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^4
*c*(a/b)^(1/3) - a*b^3*d*(a/b)^(1/3))*(a/b)^(1/3)) + 1/3*sqrt(3)*c^3*arcta
n(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*d^3*(c/d)^(1/3) - a*d
^4*(c/d)^(1/3))*(c/d)^(1/3)) + 1/6*a^3*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/
3))/(b^4*c*(a/b)^(2/3) - a*b^3*d*(a/b)^(2/3)) - 1/6*c^3*log(x^2 - x*(c/d)^
(1/3) + (c/d)^(2/3))/(b*c*d^3*(c/d)^(2/3) - a*d^4*(c/d)^(2/3)) - 1/3*a^3*l
og(x + (a/b)^(1/3))/(b^4*c*(a/b)^(2/3) - a*b^3*d*(a/b)^(2/3)) + 1/3*c^3*lo
g(x + (c/d)^(1/3))/(b*c*d^3*(c/d)^(2/3) - a*d^4*(c/d)^(2/3)) + 1/4*(b*d*x^
4 - 4*(b*c + a*d)*x)/(b^2*d^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.11

$$\int \frac{x^9}{(a+bx^3)(c+dx^3)}\,dx = \frac{a^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(ab^3c-a^2b^2d)} - \frac{c^3\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2d^2-acd^3)} - \frac{(-ab^2)^{\frac{1}{3}}\,a^2\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^4c-\sqrt{3}ab^3d} + \frac{(-cd^2)^{\frac{1}{3}}\,c^2\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^3-\sqrt{3}ad^4} - \frac{(-ab^2)^{\frac{1}{3}}\,a^2\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(b^4c-ab^3d)} + \frac{(-cd^2)^{\frac{1}{3}}\,c^2\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^3-ad^4)} + \frac{b^3d^3x^4-4\,b^3cd^2x-4\,ab^2d^3x}{4\,b^4d^4}$$

input
```
integrate(x^9/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*a^3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3*c - a^2*b^2*d) - 1/
3*c^3*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2*d^2 - a*c*d^3) - (-a*
b^2)^(1/3)*a^2*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt
(3)*b^4*c - sqrt(3)*a*b^3*d) + (-c*d^2)^(1/3)*c^2*arctan(1/3*sqrt(3)*(2*x
+ (-c/d)^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) - 1/6*(-a*
b^2)^(1/3)*a^2*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^4*c - a*b^3*d)
+ 1/6*(-c*d^2)^(1/3)*c^2*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^3
 - a*d^4) + 1/4*(b^3*d^3*x^4 - 4*b^3*c*d^2*x - 4*a*b^2*d^3*x)/(b^4*d^4)
```

Mupad [B] (verification not implemented)

Time = 10.83 (sec) , antiderivative size = 1665, normalized size of antiderivative = 5.25

$$\int \frac{x^9}{(a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int(x^9/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
log((((81*a^2*b^2*c^2*d^2*x*(a*d - b*c)^4 + 81*a*b^3*c*d^3*(a*d + b*c)*(a
*d - b*c)^4*(a^7/(b^7*(a*d - b*c)^3))^(1/3))*(a^7/(b^7*(a*d - b*c)^3))^(2/
3))/9 - (9*(a*b^9*c^10 + a^10*c*d^9 - a^2*b^8*c^9*d - a^9*b*c^2*d^8))/(b^4
*d^4))*(a^7/(b^7*(a*d - b*c)^3))^(1/3))/3 + (3*a^3*c^3*x*(a^6*d^6 + b^6*c^
6))/(b^4*d^4))*(-a^7/(27*b^10*c^3 - 27*a^3*b^7*d^3 + 81*a^2*b^8*c*d^2 - 81
*a*b^9*c^2*d))^(1/3) + log((((81*a^2*b^2*c^2*d^2*x*(a*d - b*c)^4 + 81*a*b
^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*(-c^7/(d^7*(a*d - b*c)^3))^(1/3))*(-c^7
/(d^7*(a*d - b*c)^3))^(2/3))/9 - (9*(a*b^9*c^10 + a^10*c*d^9 - a^2*b^8*c^9
*d - a^9*b*c^2*d^8))/(b^4*d^4))*(-c^7/(d^7*(a*d - b*c)^3))^(1/3))/3 + (3*a
^3*c^3*x*(a^6*d^6 + b^6*c^6))/(b^4*d^4))*(-c^7/(27*a^3*d^10 - 27*b^3*c^3*d
^7 + 81*a*b^2*c^2*d^8 - 81*a^2*b*c*d^9))^(1/3) + (log(((3^(1/2)*1i - 1)*((
(81*a^2*b^2*c^2*d^2*x*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*
d + b*c)*(a*d - b*c)^4*(-c^7/(d^7*(a*d - b*c)^3))^(1/3))/2)*(3^(1/2)*1i -
1)^2*(-c^7/(d^7*(a*d - b*c)^3))^(2/3))/36 - (9*(a*b^9*c^10 + a^10*c*d^9 -
a^2*b^8*c^9*d - a^9*b*c^2*d^8))/(b^4*d^4))*(-c^7/(d^7*(a*d - b*c)^3))^(1/3
))/6 + (3*a^3*c^3*x*(a^6*d^6 + b^6*c^6))/(b^4*d^4))*(-c^7/(27*a^3*d^10 - 2
7*b^3*c^3*d^7 + 81*a*b^2*c^2*d^8 - 81*a^2*b*c*d^9))^(1/3)*(3^(1/2)*1i - 1)
)/2 - (log(((3^(1/2)*1i + 1)*(((81*a^2*b^2*c^2*d^2*x*(a*d - b*c)^4 - (81*a
*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(-c^7/(d^7*(a*d - b*
c)^3))^(1/3))/2)*(3^(1/2)*1i + 1)^2*(-c^7/(d^7*(a*d - b*c)^3))^(2/3))/3...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.69

$$\int \frac{x^9}{(a+bx^3)(c+dx^3)}\,dx$$

$$= \frac{-4d^{\frac{7}{3}}a^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)+4c^{\frac{7}{3}}b^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)-2d^{\frac{7}{3}}a^{\frac{7}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)+4d^{\frac{7}{3}}a^{\frac{7}{3}}\log\left(\right.}{}$$

input
```
int(x^9/(b*x^3+a)/(d*x^3+c),x)
```

output
```
( - 4*d**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*s
qrt(3)))*a**2*d**2 + 4*c**(1/3)*b**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/
3)*x)/(c**(1/3)*sqrt(3)))*b**2*c**2 - 2*d**(1/3)*a**(1/3)*log(a**(2/3) - b
**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*d**2 + 4*d**(1/3)*a**(1/3)*log(a*
*(1/3) + b**(1/3)*x)*a**2*d**2 + 2*c**(1/3)*b**(1/3)*log(c**(2/3) - d**(1/
3)*c**(1/3)*x + d**(2/3)*x**2)*b**2*c**2 - 4*c**(1/3)*b**(1/3)*log(c**(1/3
) + d**(1/3)*x)*b**2*c**2 - 12*d**(1/3)*b**(1/3)*a**2*d**2*x + 3*d**(1/3)*
b**(1/3)*a*b*d**2*x**4 + 12*d**(1/3)*b**(1/3)*b**2*c**2*x - 3*d**(1/3)*b**
(1/3)*b**2*c*d*x**4)/(12*d**(1/3)*b**(1/3)*b**2*d**2*(a*d - b*c))
```

3.423 $\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 301

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = \frac{x^2}{2bd} - \frac{a^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}(bc-ad)} + \frac{c^{5/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{5/3}(bc-ad)} - \frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)} + \frac{c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{5/3}(bc-ad)} + \frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}(bc-ad)} - \frac{c^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{5/3}(bc-ad)}$$

output
```
1/2*x^2/b/d-1/3*a^(5/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*
3^(1/2)/b^(5/3)/(-a*d+b*c)+1/3*c^(5/3)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^
(1/2)/c^(1/3))*3^(1/2)/d^(5/3)/(-a*d+b*c)-1/3*a^(5/3)*ln(a^(1/3)+b^(1/3)*x
)/b^(5/3)/(-a*d+b*c)+1/3*c^(5/3)*ln(c^(1/3)+d^(1/3)*x)/d^(5/3)/(-a*d+b*c)+
1/6*a^(5/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(5/3)/(-a*d+b*c)-1
/6*c^(5/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/d^(5/3)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.80

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = \frac{-\frac{3ax^2}{b}+\frac{3cx^2}{d}-\frac{2\sqrt{3}a^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}}+\frac{2\sqrt{3}c^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{d^{5/3}}-\frac{2a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{5/3}}+\frac{2c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^{5/3}}}{6bc-6ad}$$

input
```
Integrate[x^7/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
((-3*a*x^2)/b + (3*c*x^2)/d - (2*Sqrt[3]*a^(5/3)*ArcTan[(1 - (2*b^(1/3)*x)
/a^(1/3))/Sqrt[3]])/b^(5/3) + (2*Sqrt[3]*c^(5/3)*ArcTan[(1 - (2*d^(1/3)*x)
/c^(1/3))/Sqrt[3]])/d^(5/3) - (2*a^(5/3)*Log[a^(1/3) + b^(1/3)*x])/b^(5/3)
 + (2*c^(5/3)*Log[c^(1/3) + d^(1/3)*x])/d^(5/3) + (a^(5/3)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - (c^(5/3)*Log[c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2])/d^(5/3))/(6*b*c - 6*a*d)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {979, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx$$

$$\Big\downarrow \; 979$$

$$\frac{x^2}{2bd}-\frac{\int \frac{2x((bc+ad)x^3+ac)}{(bx^3+a)(dx^3+c)}dx}{2bd}$$

$$\Big\downarrow \; 27$$

$$\frac{x^2}{2bd}-\frac{\int \frac{x((bc+ad)x^3+ac)}{(bx^3+a)(dx^3+c)}dx}{bd}$$

$\Big\downarrow$ 1054

$$\frac{x^2}{2bd}-\frac{\int \left(\frac{dxa^2}{(ad-bc)(bx^3+a)}+\frac{bc^2x}{(bc-ad)(dx^3+c)}\right)dx}{bd}$$

$\Big\downarrow$ 2009

$$\frac{x^2}{2bd}-$$

$$\frac{\frac{a^{5/3}d\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}(bc-ad)}-\frac{a^{5/3}d\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}(bc-ad)}+\frac{a^{5/3}d\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)}-\frac{bc^{5/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{2/3}(bc-ad)}+\frac{bc^{5/3}}{}}{bd}$$

input
```
Int[x^7/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
x^2/(2*b*d) - ((a^(5/3)*d*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*b^(2/3)*(b*c - a*d)) - (b*c^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*
x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*d^(2/3)*(b*c - a*d)) + (a^(5/3)*d*Log[a^(1
/3) + b^(1/3)*x])/(3*b^(2/3)*(b*c - a*d)) - (b*c^(5/3)*Log[c^(1/3) + d^(1/
3)*x])/(3*d^(2/3)*(b*c - a*d)) - (a^(5/3)*d*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(6*b^(2/3)*(b*c - a*d)) + (b*c^(5/3)*Log[c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2])/(6*d^(2/3)*(b*c - a*d)))/(b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.76

method	result
default	$\frac{x^2}{2bd}+\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)c^2}{(ad-bc)d}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right.\right.}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{(ad}$
risch	$\frac{x^2}{2bd}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\left(a^3b^2d^3-3a^2b^3c\,d^2+3a\,b^4c^2d-b^5c^3\right)\_Z^3-a^5d^3\right)}\_R\ln\left(\left(\left(-a^5b^2c\,d^6+2a^4b^3c^2d^5-2a^3b^4c^3d^4+2a^2b^5c^4d^3-a\,b^6\right.\right.\right.}{}$

input
```
int(x^7/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/b/d+(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(x^2
-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(
2/(c/d)^(1/3)*x-1)))*c^2/(a*d-b*c)/d-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))
+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(
1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a^2/(a*d-b*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.91

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = \frac{2\sqrt{3}ad\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3a}\right)-2\sqrt{3}bc\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}c}{3c}\right)+ad\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(\right.}{}$$

input
```
integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*a*d*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/
3) - sqrt(3)*a)/a) - 2*sqrt(3)*b*c*(-c^2/d^2)^(1/3)*arctan(1/3*(2*sqrt(3)*
d*x*(-c^2/d^2)^(1/3) + sqrt(3)*c)/c) + a*d*(a^2/b^2)^(1/3)*log(a*x^2 - b*x
*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) + b*c*(-c^2/d^2)^(1/3)*log(c*x^2 - d
*x*(-c^2/d^2)^(2/3) - c*(-c^2/d^2)^(1/3)) - 2*a*d*(a^2/b^2)^(1/3)*log(a*x
+ b*(a^2/b^2)^(2/3)) - 2*b*c*(-c^2/d^2)^(1/3)*log(c*x + d*(-c^2/d^2)^(2/3)
) + 3*(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(x**7/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.08

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = \frac{\sqrt{3}a^2 \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^3c-ab^2d\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}c^2 \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd^2-ad^3\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{a^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^3c\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} - \frac{c^2\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} - \frac{a^2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^3c\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} + \frac{c^2\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} + \frac{x^2}{2\,bd}$$

input
```
integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^3*
c - a*b^2*d)*(a/b)^(1/3)) - 1/3*sqrt(3)*c^2*arctan(1/3*sqrt(3)*(2*x - (c/d
)^(1/3))/(c/d)^(1/3))/((b*c*d^2 - a*d^3)*(c/d)^(1/3)) + 1/6*a^2*log(x^2 -
x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*c*(a/b)^(1/3) - a*b^2*d*(a/b)^(1/3)) - 1
/6*c^2*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*d^2*(c/d)^(1/3) - a*d^3
*(c/d)^(1/3)) - 1/3*a^2*log(x + (a/b)^(1/3))/(b^3*c*(a/b)^(1/3) - a*b^2*d*
(a/b)^(1/3)) + 1/3*c^2*log(x + (c/d)^(1/3))/(b*c*d^2*(c/d)^(1/3) - a*d^3*(
c/d)^(1/3)) + 1/2*x^2/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.03

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(ab^2c-a^2bd)} + \frac{c^2\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2d-acd^2)} - \frac{(-ab^2)^{\frac{2}{3}}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^4c-\sqrt{3}ab^3d} + \frac{(-cd^2)^{\frac{2}{3}}\,c\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^3-\sqrt{3}ad^4} + \frac{(-ab^2)^{\frac{2}{3}}\,a\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(b^4c-ab^3d)} - \frac{(-cd^2)^{\frac{2}{3}}\,c\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^3-ad^4)} + \frac{x^2}{2\,bd}$$

input
```
integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*a^2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c - a^2*b*d) + 1/3
*c^2*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2*d - a*c*d^2) - (-a*b^2
)^(2/3)*a*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b
^4*c - sqrt(3)*a*b^3*d) + (-c*d^2)^(2/3)*c*arctan(1/3*sqrt(3)*(2*x + (-c/d
)^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) + 1/6*(-a*b^2)^(2
/3)*a*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^4*c - a*b^3*d) - 1/6*(-c
*d^2)^(2/3)*c*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^3 - a*d^4) +
 1/2*x^2/(b*d)
```

Mupad [B] (verification not implemented)

Time = 10.70 (sec) , antiderivative size = 1751, normalized size of antiderivative = 5.82

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int(x^7/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(((((27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + 27*a*b^3*c*d^
3*(a*d + b*c)*(a*d - b*c)^4*(a^5/(b^5*(a*d - b*c)^3))^(2/3))*(a^5/(b^5*(a*
d - b*c)^3))^(1/3))/3 - (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*
c^2*d^6))/(b^2*d^2))*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/9 - (a^4*c^4*x*(a^2*
d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-a^5/(27*b^8*c^3 - 27*a^3*b^5*d^3 +
81*a^2*b^6*c*d^2 - 81*a*b^7*c^2*d))^(1/3) + log(((((27*a^2*b*c^2*d*x*(a^2*
d^2 + b^2*c^2)*(a*d - b*c)^2 + 27*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*(-
c^5/(d^5*(a*d - b*c)^3))^(2/3))*(-c^5/(d^5*(a*d - b*c)^3))^(1/3))/3 - (9*(
a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))*(-c^5/(
d^5*(a*d - b*c)^3))^(2/3))/9 - (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(
b^2*d^2))*(-c^5/(27*a^3*d^8 - 27*b^3*c^3*d^5 + 81*a*b^2*c^2*d^6 - 81*a^2*b
*c*d^7))^(1/3) - (log(((3^(1/2)*1i + 1)^2*(((3^(1/2)*1i + 1)*(27*a^2*b*c^2
*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i + 1)^
2*(a*d + b*c)*(a*d - b*c)^4*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/4)*(a^5/(b^5*
(a*d - b*c)^3))^(1/3))/6 + (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7
*b*c^2*d^6))/(b^2*d^2))*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/36 + (a^4*c^4*x*(
a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-a^5/(27*b^8*c^3 - 27*a^3*b^5*d^
3 + 81*a^2*b^6*c*d^2 - 81*a*b^7*c^2*d))^(1/3)*(3^(1/2)*1i + 1))/2 + (log((
(3^(1/2)*1i - 1)^2*(((3^(1/2)*1i - 1)*(27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2
)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d -...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.76

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx$$
$$= \frac{2d^{\frac{5}{3}}c^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a^2 - 2b^{\frac{5}{3}}a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)c^2 + 3d^{\frac{5}{3}}c^{\frac{1}{3}}b^{\frac{2}{3}}a^{\frac{4}{3}}x^2 - 3d^{\frac{2}{3}}c^{\frac{4}{3}}b^{\frac{5}{3}}a^{\frac{1}{3}}x^2 + b^{\frac{5}{3}}a^{\frac{1}{3}}\log\Big(c \ldots}{6d^{\frac{5}{3}}c^{\frac{1}{3}}b \ldots}$$

input
```
int(x^7/(b*x^3+a)/(d*x^3+c),x)
```

output
```
(2*d**(2/3)*c**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt
(3)))*a**2*d - 2*b**(2/3)*a**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/
(c**(1/3)*sqrt(3)))*b*c**2 + 3*d**(2/3)*c**(1/3)*b**(2/3)*a**(1/3)*a*d*x**
2 - 3*d**(2/3)*c**(1/3)*b**(2/3)*a**(1/3)*b*c*x**2 + b**(2/3)*a**(1/3)*log
(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*b*c**2 - 2*b**(2/3)*a**(1
/3)*log(c**(1/3) + d**(1/3)*x)*b*c**2 - d**(2/3)*c**(1/3)*log(a**(2/3) - b
**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*d + 2*d**(2/3)*c**(1/3)*log(a**(1
/3) + b**(1/3)*x)*a**2*d)/(6*d**(2/3)*c**(1/3)*b**(2/3)*a**(1/3)*b*d*(a*d
- b*c))
```

3.424 $\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 296

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = \frac{x}{bd} - \frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}(bc-ad)} + \frac{c^{4/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{4/3}(bc-ad)} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{4/3}(bc-ad)} - \frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}(bc-ad)} + \frac{c^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{4/3}(bc-ad)}$$

output
```
x/b/d-1/3*a^(4/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2
)/b^(4/3)/(-a*d+b*c)+1/3*c^(4/3)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)/
c^(1/3))*3^(1/2)/d^(4/3)/(-a*d+b*c)+1/3*a^(4/3)*ln(a^(1/3)+b^(1/3)*x)/b^(4
/3)/(-a*d+b*c)-1/3*c^(4/3)*ln(c^(1/3)+d^(1/3)*x)/d^(4/3)/(-a*d+b*c)-1/6*a^
(4/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(4/3)/(-a*d+b*c)+1/6*c^(
4/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/d^(4/3)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.80

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = \frac{-\frac{6ax}{b}+\frac{6cx}{d}-\frac{2\sqrt{3}a^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}}+\frac{2\sqrt{3}c^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{d^{4/3}}+\frac{2a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{4/3}}-\frac{2c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right.}{d^{4/3}}\cdots}{6bc-6ad}$$

input
```
Integrate[x^6/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
((-6*a*x)/b + (6*c*x)/d - (2*Sqrt[3]*a^(4/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]])/b^(4/3) + (2*Sqrt[3]*c^(4/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(
1/3))/Sqrt[3]])/d^(4/3) + (2*a^(4/3)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) - (
2*c^(4/3)*Log[c^(1/3) + d^(1/3)*x])/d^(4/3) - (a^(4/3)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/b^(4/3) + (c^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2])/d^(4/3))/(6*b*c - 6*a*d)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {979, 1020, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx$$

$$\Big\downarrow \; 979$$

$$\frac{x}{bd}-\frac{\int \frac{(bc+ad)x^3+ac}{(bx^3+a)(dx^3+c)}dx}{bd}$$

$$\Big\downarrow \; 1020$$

$$\frac{x}{bd}-\frac{\frac{bc^2\int\frac{1}{dx^3+c}dx}{bc-ad}-\frac{a^2d\int\frac{1}{bx^3+a}dx}{bc-ad}}{bd}$$

$\downarrow$ 750

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad}}{bd}$$

$\downarrow$ 16

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}}{bd}$$

$\downarrow$ 1142

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\cdots}{3a^{2/3}}\cdots\right)}{\cdots}}{bd}$$

$\downarrow$ 25

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\cdots}{3a^{2/3}}\cdots\right)}{bc-\cdots}}{bd}$$

$\downarrow$ 27

$$\frac{\frac{x}{bd}-\frac{bc^2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\cdots}{3a^{2/3}}\cdots\right)}{b\cdots}}{bd}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{x}{bd}-\frac{bc^2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\cdots}{\cdots}}{3a^{2/3}}\cdots\right)}{\cdots}}{bd}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{x}{bd}-\frac{bc^2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}\cdots\right)}{bc-ad\cdots}}{bd}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{x}{bd}-\frac{bc^2\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a^2d\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\cdots}{\cdots}}{\sqrt{\cdots}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}\cdots\right)}{bc-ad}}{bd}$$

input
```
Int[x^6/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
x/(b*d) - (-((a^2*d*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sq
rt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/(b*c - a*d))
 + (b*c^2*(Log[c^(1/3) + d^(1/3)*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcT
an[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2]/(2*d^(1/3)))/(3*c^(2/3))))/(b*c - a*d))/(b*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.76

method	result
default	$\frac{x}{bd}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)c^2}{d(ad-bc)}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{b(ad-b}$
risch	$\frac{x}{bd}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\left(b\,d^3a^3-3a^2b^2c\,d^2+3a\,b^3c^2d-b^4c^3\right)\_Z^3+a^4d^3\right)}\_R\ln\left(\left(-a^5bc\,d^5-a\,b^5c^5d\right)x+\left(-a^5b\,d^6+3a^4b^2c\,d^5-2a^3b^3c^2d^4\right.\right.}{3db}$

input
```
int(x^6/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
x/b/d+(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^
(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)
^(1/3)*x-1)))/d*c^2/(a*d-b*c)-(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(
a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/b*a^2/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.77

$$\int\frac{x^6}{(a+bx^3)\,(c+dx^3)}\,dx=$$
$$-\frac{2\sqrt{3}ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)+2\sqrt{3}bc\left(\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(\frac{c}{d}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right)-ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2\right.}{}$$

input
```
integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*a*d*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) -
sqrt(3)*a)/a) + 2*sqrt(3)*b*c*(c/d)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(c/d)^
(2/3) - sqrt(3)*c)/c) - a*d*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)
^(2/3)) - b*c*(c/d)^(1/3)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3)) + 2*a*d*(
-a/b)^(1/3)*log(x - (-a/b)^(1/3)) + 2*b*c*(c/d)^(1/3)*log(x + (c/d)^(1/3))
 - 6*(b*c - a*d)*x)/(b^2*c*d - a*b*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(x**6/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.18

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = \frac{\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^3c\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}c^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{a^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^3c\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{c^2\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{a^2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^3c\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{c^2\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{x}{bd}$$

input
```
integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^3*
c*(a/b)^(1/3) - a*b^2*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*c^2*arctan
(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*d^2*(c/d)^(1/3) - a*d^
3*(c/d)^(1/3))*(c/d)^(1/3)) - 1/6*a^2*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3
))/(b^3*c*(a/b)^(2/3) - a*b^2*d*(a/b)^(2/3)) + 1/6*c^2*log(x^2 - x*(c/d)^(
1/3) + (c/d)^(2/3))/(b*c*d^2*(c/d)^(2/3) - a*d^3*(c/d)^(2/3)) + 1/3*a^2*lo
g(x + (a/b)^(1/3))/(b^3*c*(a/b)^(2/3) - a*b^2*d*(a/b)^(2/3)) - 1/3*c^2*log
(x + (c/d)^(1/3))/(b*c*d^2*(c/d)^(2/3) - a*d^3*(c/d)^(2/3)) + x/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.04

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(ab^2c-a^2bd)} + \frac{c^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2d-acd^2)} + \frac{(-ab^2)^{\frac{1}{3}}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^3c-\sqrt{3}ab^2d} - \frac{(-cd^2)^{\frac{1}{3}}\,c\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3} + \frac{(-ab^2)^{\frac{1}{3}}\,a\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(b^3c-ab^2d)} - \frac{(-cd^2)^{\frac{1}{3}}\,c\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^2-ad^3)} + \frac{x}{bd}$$

input
```
integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*a^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c - a^2*b*d) + 1/3
*c^2*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2*d - a*c*d^2) + (-a*b^2
)^(1/3)*a*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b
^3*c - sqrt(3)*a*b^2*d) - (-c*d^2)^(1/3)*c*arctan(1/3*sqrt(3)*(2*x + (-c/d
)^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3) + 1/6*(-a*b^2)^(1
/3)*a*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^3*c - a*b^2*d) - 1/6*(-c
*d^2)^(1/3)*c*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^2 - a*d^3) +
 x/(b*d)
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 873, normalized size of antiderivative = 2.95

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int(x^6/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(a*x + b^2*c*(-a^4/(b^4*(a*d - b*c)^3))^(1/3) - a*b*d*(-a^4/(b^4*(a*d -
 b*c)^3))^(1/3))*(a^4/(27*b^7*c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^2 - 81
*a*b^6*c^2*d))^(1/3) + log(c*x + a*d^2*(c^4/(d^4*(a*d - b*c)^3))^(1/3) - b
*c*d*(c^4/(d^4*(a*d - b*c)^3))^(1/3))*(c^4/(27*a^3*d^7 - 27*b^3*c^3*d^4 +
81*a*b^2*c^2*d^5 - 81*a^2*b*c*d^6))^(1/3) + x/(b*d) + (log((3*x*(a^2*b^4*c
^6 + a^6*c^2*d^4))/(b*d) - (3*a*c^2*(3^(1/2)*1i - 1)*(-a^4/(b^4*(a*d - b*c
)^3))^(1/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d - a^4*b*c*d^4))/(2*d))*(a^4/(
27*b^7*c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^2 - 81*a*b^6*c^2*d))^(1/3)*(3
^(1/2)*1i - 1))/2 - (log((3*x*(a^2*b^4*c^6 + a^6*c^2*d^4))/(b*d) + (3*a*c^
2*(3^(1/2)*1i + 1)*(-a^4/(b^4*(a*d - b*c)^3))^(1/3)*(a^5*d^5 - b^5*c^5 + a
*b^4*c^4*d - a^4*b*c*d^4))/(2*d))*(a^4/(27*b^7*c^3 - 27*a^3*b^4*d^3 + 81*a
^2*b^5*c*d^2 - 81*a*b^6*c^2*d))^(1/3)*(3^(1/2)*1i + 1))/2 + (log((3*x*(a^2
*b^4*c^6 + a^6*c^2*d^4))/(b*d) + (3*a^2*c*(3^(1/2)*1i - 1)*(c^4/(d^4*(a*d
- b*c)^3))^(1/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d - a^4*b*c*d^4))/(2*b))*(
c^4/(27*a^3*d^7 - 27*b^3*c^3*d^4 + 81*a*b^2*c^2*d^5 - 81*a^2*b*c*d^6))^(1/
3)*(3^(1/2)*1i - 1))/2 - (log((3*x*(a^2*b^4*c^6 + a^6*c^2*d^4))/(b*d) - (3
*a^2*c*(3^(1/2)*1i + 1)*(c^4/(d^4*(a*d - b*c)^3))^(1/3)*(a^5*d^5 - b^5*c^5
 + a*b^4*c^4*d - a^4*b*c*d^4))/(2*b))*(c^4/(27*a^3*d^7 - 27*b^3*c^3*d^4 +
81*a*b^2*c^2*d^5 - 81*a^2*b*c*d^6))^(1/3)*(3^(1/2)*1i + 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.64

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = \frac{2d^{\frac{4}{3}}a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)-2c^{\frac{4}{3}}b^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)+d^{\frac{4}{3}}a^{\frac{4}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-2d^{\frac{4}{3}}a^{\frac{4}{3}}\log\left(a^{\frac{1}{3}}\right.}{6d^{\frac{4}{3}}b^{\frac{4}{3}}(ad-bc)}$$

input
```
int(x^6/(b*x^3+a)/(d*x^3+c),x)
```

output
```
(2*d**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt
(3)))*a*d - 2*c**(1/3)*b**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c*
*(1/3)*sqrt(3)))*b*c + d**(1/3)*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*
x + b**(2/3)*x**2)*a*d - 2*d**(1/3)*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a*
d - c**(1/3)*b**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*
b*c + 2*c**(1/3)*b**(1/3)*log(c**(1/3) + d**(1/3)*x)*b*c + 6*d**(1/3)*b**(
1/3)*a*d*x - 6*d**(1/3)*b**(1/3)*b*c*x)/(6*d**(1/3)*b**(1/3)*b*d*(a*d - b*
c))
```

3.425 $\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 288

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx = \frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}(bc-ad)} - \frac{c^{2/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{2/3}(bc-ad)} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)} - \frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{2/3}(bc-ad)} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}(bc-ad)} + \frac{c^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{2/3}(bc-ad)}$$

output
```
1/3*a^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/b^(2
/3)/(-a*d+b*c)-1/3*c^(2/3)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)/c^(1/3
))*3^(1/2)/d^(2/3)/(-a*d+b*c)+1/3*a^(2/3)*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)/(-
a*d+b*c)-1/3*c^(2/3)*ln(c^(1/3)+d^(1/3)*x)/d^(2/3)/(-a*d+b*c)-1/6*a^(2/3)*
ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)/(-a*d+b*c)+1/6*c^(2/3)*l
n(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/d^(2/3)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx$$
$$= \frac{\frac{2\sqrt{3}a^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} - \frac{2\sqrt{3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{d^{2/3}} + \frac{2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} - \frac{2c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^{2/3}} - \cdots}{6bc-6ad}$$

input
```
Integrate[x^4/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
((2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) -
 (2*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(2/3) +
 (2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (2*c^(2/3)*Log[c^(1/3) + d
^(1/3)*x])/d^(2/3) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/b^(2/3) + (c^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(
2/3))/(6*b*c - 6*a*d)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {981, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx$$
$$\Big\downarrow \text{ 981}$$
$$\frac{c\int \frac{x}{dx^3+c}dx}{bc-ad} - \frac{a\int \frac{x}{bx^3+a}dx}{bc-ad}$$
$$\Big\downarrow \text{ 821}$$

$$\frac{c\left(\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{c\left(\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{c\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{c\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int \frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{c\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}-\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{c\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}-\frac{a\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{c\left(\frac{\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}$$

input
```
Int[x^4/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
-((a*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan
[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/(b*c - a*d)) + (
c*(-1/3*Log[c^(1/3) + d^(1/3)*x]/(c^(1/3)*d^(2/3)) + (-((Sqrt[3]*ArcTan[(1
 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3)) + Log[c^(2/3) - c^(1/3)*d^(1/
3)*x + d^(2/3)*x^2]/(2*d^(1/3)))/(3*c^(1/3)*d^(1/3))))/(b*c - a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 981
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[(-a)*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(a + b*x^n),
x], x] + Simp[c*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(c + d*x^n), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
 m, 2*n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72

method	result
default	$-\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)c}{ad-bc}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\right.}{ad-bc}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^3d^5-3a^2bc\,d^4+3a\,b^2c^2d^3-b^3c^3d^2\right)\_Z^3-c^2\right)}\_R\ln\left(\left(\left(-2a^3b^2c\,d^4+4a^2b^3c^2d^3-2a\,b^4c^3d^2\right)\_R^3-a^2cd-b\,c^2a\right)x+\right.\right.}{3}$

input
```
int(x^4/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3
)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/
3)*x-1)))*c/(a*d-b*c)+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx =$$

$$-\frac{2\sqrt{3}\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}a}{3a}\right)-2\sqrt{3}\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}c}{3c}\right)-\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax\right.}{}$$

input
```
integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3
) + sqrt(3)*a)/a) - 2*sqrt(3)*(c^2/d^2)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(c
^2/d^2)^(1/3) - sqrt(3)*c)/c) - (-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2
)^(2/3) - a*(-a^2/b^2)^(1/3)) - (c^2/d^2)^(1/3)*log(c*x^2 - d*x*(c^2/d^2)^
(2/3) + c*(c^2/d^2)^(1/3)) + 2*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2/b^2)^(2/
3)) + 2*(c^2/d^2)^(1/3)*log(c*x + d*(c^2/d^2)^(2/3)))/(b*c - a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(260) = 520$.

Time = 67.21 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.99

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx$$
$$= \mathrm{RootSum}\left(t^3\cdot\left(27a^3d^5-81a^2bcd^4+81ab^2c^2d^3-27b^3c^3d^2\right)-c^2,\left(t\mapsto t\log\left(x+\frac{243t^5a^6b^2d^8-1458t\right.\right.\right.$$
$$+\mathrm{RootSum}\left(t^3\cdot\left(27a^3b^2d^3-81a^2b^3cd^2+81ab^4c^2d-27b^5c^3\right)+a^2,\left(t\mapsto t\log\left(x+\frac{243t^5a^6b^2d^8-145\right.\right.\right.$$

input
```
integrate(x**4/(b*x**3+a)/(d*x**3+c),x)
```

output
```
RootSum(_t**3*(27*a**3*d**5 - 81*a**2*b*c*d**4 + 81*a*b**2*c**2*d**3 - 27*
b**3*c**3*d**2) - c**2, Lambda(_t, _t*log(x + (243*_t**5*a**6*b**2*d**8 -
1458*_t**5*a**5*b**3*c*d**7 + 3645*_t**5*a**4*b**4*c**2*d**6 - 4860*_t**5*
a**3*b**5*c**3*d**5 + 3645*_t**5*a**2*b**6*c**4*d**4 - 1458*_t**5*a*b**7*c
**5*d**3 + 243*_t**5*b**8*c**6*d**2 + 9*_t**2*a**5*d**5 - 18*_t**2*a**4*b*
c*d**4 + 9*_t**2*a**3*b**2*c**2*d**3 + 9*_t**2*a**2*b**3*c**3*d**2 - 18*_t
**2*a*b**4*c**4*d + 9*_t**2*b**5*c**5)/(a**3*c*d**2 + a*b**2*c**3)))) + Ro
otSum(_t**3*(27*a**3*b**2*d**3 - 81*a**2*b**3*c*d**2 + 81*a*b**4*c**2*d -
27*b**5*c**3) + a**2, Lambda(_t, _t*log(x + (243*_t**5*a**6*b**2*d**8 - 14
58*_t**5*a**5*b**3*c*d**7 + 3645*_t**5*a**4*b**4*c**2*d**6 - 4860*_t**5*a*
*3*b**5*c**3*d**5 + 3645*_t**5*a**2*b**6*c**4*d**4 - 1458*_t**5*a*b**7*c**
5*d**3 + 243*_t**5*b**8*c**6*d**2 + 9*_t**2*a**5*d**5 - 18*_t**2*a**4*b*c*
d**4 + 9*_t**2*a**3*b**2*c**2*d**3 + 9*_t**2*a**2*b**3*c**3*d**2 - 18*_t**
2*a*b**4*c**4*d + 9*_t**2*b**5*c**5)/(a**3*c*d**2 + a*b**2*c**3))))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx = -\frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c-abd\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}c\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd-ad^2\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{c\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} + \frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{c\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}$$

input
```
integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c
 - a*b*d)*(a/b)^(1/3)) + 1/3*sqrt(3)*c*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/
3))/(c/d)^(1/3))/((b*c*d - a*d^2)*(c/d)^(1/3)) - 1/6*a*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3))/(b^2*c*(a/b)^(1/3) - a*b*d*(a/b)^(1/3)) + 1/6*c*log(x^
2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*d*(c/d)^(1/3) - a*d^2*(c/d)^(1/3)) +
 1/3*a*log(x + (a/b)^(1/3))/(b^2*c*(a/b)^(1/3) - a*b*d*(a/b)^(1/3)) - 1/3*
c*log(x + (c/d)^(1/3))/(b*c*d*(c/d)^(1/3) - a*d^2*(c/d)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.99

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx = \frac{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(abc-a^2d)} - \frac{c\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} + \frac{(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^3c-\sqrt{3}ab^2d} - \frac{(-cd^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3} - \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(b^3c-ab^2d)} + \frac{(-cd^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^2-ad^3)}$$

input

```
integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output

```
1/3*a*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) - 1/3*c*(-c/
d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(2/3)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^3*c - sqrt(3)*
a*b^2*d) - (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(
1/3))/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3) - 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(
-a/b)^(1/3) + (-a/b)^(2/3))/(b^3*c - a*b^2*d) + 1/6*(-c*d^2)^(2/3)*log(x^2
 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^2 - a*d^3)
```

Mupad [B] (verification not implemented)

Time = 7.89 (sec) , antiderivative size = 1364, normalized size of antiderivative = 4.74

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int(x^4/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(a*x + b^3*c^2*(-a^2/(b^2*(a*d - b*c)^3))^(2/3) + a^2*b*d^2*(-a^2/(b^2*
(a*d - b*c)^3))^(2/3) - 2*a*b^2*c*d*(-a^2/(b^2*(a*d - b*c)^3))^(2/3))*(a^2
/(27*b^5*c^3 - 27*a^3*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^(1/3)
+ log(c*x + a^2*d^3*(c^2/(d^2*(a*d - b*c)^3))^(2/3) + b^2*c^2*d*(c^2/(d^2*
(a*d - b*c)^3))^(2/3) - 2*a*b*c*d^2*(c^2/(d^2*(a*d - b*c)^3))^(2/3))*(c^2/
(27*a^3*d^5 - 27*b^3*c^3*d^2 + 81*a*b^2*c^2*d^3 - 81*a^2*b*c*d^4))^(1/3) +
 (log(((3^(1/2)*1i - 1)^2*(-a^2/(b^2*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i -
 1)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1)
^2*(a*d + b*c)*(a*d - b*c)^4*(-a^2/(b^2*(a*d - b*c)^3))^(2/3))/4)*(-a^2/(b
^2*(a*d - b*c)^3))^(1/3))/6 - 9*a^2*b^4*c^4*d^2 - 9*a^4*b^2*c^2*d^4 + 9*a*
b^5*c^5*d + 9*a^5*b*c*d^5))/36 + a^2*b*c^2*d*x*(a*d + b*c))*(a^2/(27*b^5*c
^3 - 27*a^3*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^(1/3)*(3^(1/2)*1
i - 1))/2 - (log(((3^(1/2)*1i + 1)^2*(-a^2/(b^2*(a*d - b*c)^3))^(2/3)*(((3
^(1/2)*1i + 1)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1
/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-a^2/(b^2*(a*d - b*c)^3))^(2/3))/
4)*(-a^2/(b^2*(a*d - b*c)^3))^(1/3))/6 + 9*a^2*b^4*c^4*d^2 + 9*a^4*b^2*c^2
*d^4 - 9*a*b^5*c^5*d - 9*a^5*b*c*d^5))/36 - a^2*b*c^2*d*x*(a*d + b*c))*(a^
2/(27*b^5*c^3 - 27*a^3*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^(1/3)
*(3^(1/2)*1i + 1))/2 + (log(((3^(1/2)*1i - 1)^2*(c^2/(d^2*(a*d - b*c)^3))^
(2/3)*(((3^(1/2)*1i - 1)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.63

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx = \frac{-2d^{\frac{2}{3}}c^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a+2b^{\frac{2}{3}}a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)c-b^{\frac{2}{3}}a^{\frac{1}{3}}\log\left(c^{\frac{2}{3}}-d^{\frac{1}{3}}c^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2\right)c+2b^{\frac{2}{3}}a^{\frac{1}{3}}\log}{6d^{\frac{2}{3}}c^{\frac{1}{3}}b^{\frac{2}{3}}a^{\frac{1}{3}}(ad-bc)}$$

input
```
int(x^4/(b*x^3+a)/(d*x^3+c),x)
```

output
```
( - 2*d**(2/3)*c**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*s
qrt(3)))*a + 2*b**(2/3)*a**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c
**(1/3)*sqrt(3)))*c - b**(2/3)*a**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x
 + d**(2/3)*x**2)*c + 2*b**(2/3)*a**(1/3)*log(c**(1/3) + d**(1/3)*x)*c + d
**(2/3)*c**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a - 2
*d**(2/3)*c**(1/3)*log(a**(1/3) + b**(1/3)*x)*a)/(6*d**(2/3)*c**(1/3)*b**(
2/3)*a**(1/3)*(a*d - b*c))
```

3.426 $\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 288

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx = \frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}(bc-ad)} - \frac{\sqrt[3]{c}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)} + \frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{b}(bc-ad)} - \frac{\sqrt[3]{c}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{d}(bc-ad)}$$

output
```
1/3*a^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/b^(1
/3)/(-a*d+b*c)-1/3*c^(1/3)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)/c^(1/3
))*3^(1/2)/d^(1/3)/(-a*d+b*c)-1/3*a^(1/3)*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)/(-
a*d+b*c)+1/3*c^(1/3)*ln(c^(1/3)+d^(1/3)*x)/d^(1/3)/(-a*d+b*c)+1/6*a^(1/3)*
ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(1/3)/(-a*d+b*c)-1/6*c^(1/3)*l
n(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/d^(1/3)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$$
$$= \frac{\frac{2\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} - \frac{2\sqrt{3}\sqrt[3]{c}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}} - \frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} + \frac{2\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt[3]{d}} + \frac{\sqrt[3]{a}\log\left(a\right.}{}}{6bc-6ad}$$

input
```
Integrate[x^3/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
((2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) -
 (2*Sqrt[3]*c^(1/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3) -
 (2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (2*c^(1/3)*Log[c^(1/3) + d
^(1/3)*x])/d^(1/3) + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/b^(1/3) - (c^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(
1/3))/(6*b*c - 6*a*d)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {981, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$$
$$\Big\downarrow \text{981}$$
$$\frac{c\int \frac{1}{dx^3+c}dx}{bc-ad} - \frac{a\int \frac{1}{bx^3+a}dx}{bc-ad}$$
$$\Big\downarrow \text{750}$$

$$\frac{c\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad}$$

$\big\downarrow$ 16

$$\frac{c\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}$$

$\big\downarrow$ 1142

$$\frac{c\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}$$

$\big\downarrow$ 25

$$\frac{c\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}$$

$$\Big\downarrow \ 27$$

$$\frac{c\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}$$

$$\Big\downarrow \ 1082$$

$$\frac{c\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}$$

$$\Big\downarrow \; 217$$

$$\frac{c\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1103$$

$$\frac{c\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}$$

input
```
Int[x^3/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
-((a*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1
 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/(b*c - a*d)) + (c*(Log[c^(1
/3) + d^(1/3)*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*
x)/c^(1/3))/Sqrt[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2]/(2*d^(1/3)))/(3*c^(2/3))))/(b*c - a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 981
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[(-a)*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(a + b*x^n),
x], x] + Simp[c*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(c + d*x^n), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
 m, 2*n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72

method	result
default	$-\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)c}{ad-bc}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\sqrt{3}\ldots}{ad-bc}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^{4}a^{3}-3a^{2}c\,d^{3}b+3a\,c^{2}d^{2}b^{2}-d\,c^{3}b^{3}\right)\_Z^{3}+c\right)}\_R\ln\left(\left(\left(a^{4}b\,d^{5}-4a^{3}b^{2}c\,d^{4}+6a^{2}b^{3}c^{2}d^{3}-4a\,b^{4}c^{3}d^{2}+b^{5}c^{4}d\right)\_R^{3}-a^{2}d^{2}\ldots\right.\right.\right.}{3}$

input
```
int(x^3/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)
*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3
)*x-1)))*c/(a*d-b*c)+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1)))*a/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.69

$$\int \frac{x^{3}}{(a+bx^{3})\,(c+dx^{3})}\,dx =$$

$$-\frac{2\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)+2\sqrt{3}\left(-\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(-\frac{c}{d}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}\ldots\right.}{6\,(bc\ldots}$$

input
```
integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3
)*a)/a) + 2*sqrt(3)*(-c/d)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(-c/d)^(2/3) -
sqrt(3)*c)/c) - (a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - (-c/d
)^(1/3)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3)) + 2*(a/b)^(1/3)*log(x + (
a/b)^(1/3)) + 2*(-c/d)^(1/3)*log(x - (-c/d)^(1/3)))/(b*c - a*d)
```

Sympy [A] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$$
$$= \text{RootSum}\left(t^3\cdot\left(27a^3d^4-81a^2bcd^3+81ab^2c^2d^2-27b^3c^3d\right)+c,\left(t\mapsto t\log\left(x+\frac{162t^4a^4bd^5-648t^4a^3b\right.\right.\right.$$
$$+\text{RootSum}\left(t^3\cdot\left(27a^3bd^3-81a^2b^2cd^2+81ab^3c^2d-27b^4c^3\right)-a,\left(t\mapsto t\log\left(x+\frac{162t^4a^4bd^5-648t^4a\right.\right.\right.$$

input
```
integrate(x**3/(b*x**3+a)/(d*x**3+c),x)
```

output
```
RootSum(_t**3*(27*a**3*d**4 - 81*a**2*b*c*d**3 + 81*a*b**2*c**2*d**2 - 27*
b**3*c**3*d) + c, Lambda(_t, _t*log(x + (162*_t**4*a**4*b*d**5 - 648*_t**4
*a**3*b**2*c*d**4 + 972*_t**4*a**2*b**3*c**2*d**3 - 648*_t**4*a*b**4*c**3*
d**2 + 162*_t**4*b**5*c**4*d - 3*_t*a**2*d**2 + 6*_t*a*b*c*d - 3*_t*b**2*c
**2)/(a*d + b*c))) + RootSum(_t**3*(27*a**3*b*d**3 - 81*a**2*b**2*c*d**2
+ 81*a*b**3*c**2*d - 27*b**4*c**3) - a, Lambda(_t, _t*log(x + (162*_t**4*a
**4*b*d**5 - 648*_t**4*a**3*b**2*c*d**4 + 972*_t**4*a**2*b**3*c**2*d**3 -
648*_t**4*a*b**4*c**3*d**2 + 162*_t**4*b**5*c**4*d - 3*_t*a**2*d**2 + 6*_t
*a*b*c*d - 3*_t*b**2*c**2)/(a*d + b*c)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.10

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx = -\frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3}c\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{c\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} - \frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{c\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

input
```
integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c
*(a/b)^(1/3) - a*b*d*(a/b)^(1/3))*(a/b)^(1/3)) + 1/3*sqrt(3)*c*arctan(1/3*
sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*d*(c/d)^(1/3) - a*d^2*(c/d)
^(1/3))*(c/d)^(1/3)) + 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*c
*(a/b)^(2/3) - a*b*d*(a/b)^(2/3)) - 1/6*c*log(x^2 - x*(c/d)^(1/3) + (c/d)^
(2/3))/(b*c*d*(c/d)^(2/3) - a*d^2*(c/d)^(2/3)) - 1/3*a*log(x + (a/b)^(1/3)
)/(b^2*c*(a/b)^(2/3) - a*b*d*(a/b)^(2/3)) + 1/3*c*log(x + (c/d)^(1/3))/(b*
c*d*(c/d)^(2/3) - a*d^2*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx = \frac{a\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(abc-a^2d)} - \frac{c\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} - \frac{(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c-\sqrt{3}abd} + \frac{(-cd^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd-\sqrt{3}ad^2} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(b^2c-abd)} + \frac{(-cd^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd-ad^2)}$$

input
```
integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*a*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) - 1/3*c*(-c/
d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) - (-a*b^2)^(1/3)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^2*c - sqrt(3)*
a*b*d) + (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/
3))/(sqrt(3)*b*c*d - sqrt(3)*a*d^2) - 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b
)^(1/3) + (-a/b)^(2/3))/(b^2*c - a*b*d) + 1/6*(-c*d^2)^(1/3)*log(x^2 + x*(
-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d - a*d^2)
```

Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 1265, normalized size of antiderivative = 4.39

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int(x^3/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(x + a*d*(a/(b*(a*d - b*c)^3))^(1/3) - b*c*(a/(b*(a*d - b*c)^3))^(1/3))
*(-a/(27*b^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^(1/3
) + log(x - a*d*(-c/(d*(a*d - b*c)^3))^(1/3) + b*c*(-c/(d*(a*d - b*c)^3))^
(1/3))*(-c/(27*a^3*d^4 - 27*b^3*c^3*d + 81*a*b^2*c^2*d^2 - 81*a^2*b*c*d^3)
)^(1/3) + (log(((3^(1/2)*1i - 1)*(a/(b*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i
 - 1)^2*(81*a*b^3*c*d^3*x*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i - 1)
*(a*d + b*c)*(a*d - b*c)^4*(a/(b*(a*d - b*c)^3))^(1/3))/2)*(a/(b*(a*d - b*
c)^3))^(2/3))/36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 -
 9*a^3*b^3*c^2*d^4))/6 - 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i
- 1)*(-a/(27*b^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^
(1/3))/2 - (log(((3^(1/2)*1i + 1)*(a/(b*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1
i + 1)^2*(81*a*b^3*c*d^3*x*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(1/2)*1i + 1
)*(a*d + b*c)*(a*d - b*c)^4*(a/(b*(a*d - b*c)^3))^(1/3))/2)*(a/(b*(a*d - b
*c)^3))^(2/3))/36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3
- 9*a^3*b^3*c^2*d^4))/6 + 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i
 + 1)*(-a/(27*b^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))
^(1/3))/2 + (log(((3^(1/2)*1i - 1)*(-c/(d*(a*d - b*c)^3))^(1/3)*(((3^(1/2)
*1i - 1)^2*(81*a*b^3*c*d^3*x*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i -
 1)*(a*d + b*c)*(a*d - b*c)^4*(-c/(d*(a*d - b*c)^3))^(1/3))/2)*(-c/(d*(a*d
 - b*c)^3))^(2/3))/36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.59

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$$
$$= \frac{-2d^{\frac{1}{3}}a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)+2c^{\frac{1}{3}}b^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)-d^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)+2d^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(a\right.}{6d^{\frac{1}{3}}b^{\frac{1}{3}}(ad-bc)}$$

input
```
int(x^3/(b*x^3+a)/(d*x^3+c),x)
```

output
```
( - 2*d**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*s
qrt(3))) + 2*c**(1/3)*b**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**
(1/3)*sqrt(3))) - d**(1/3)*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b
**(2/3)*x**2) + 2*d**(1/3)*a**(1/3)*log(a**(1/3) + b**(1/3)*x) + c**(1/3)*
b**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2) - 2*c**(1/3)*
b**(1/3)*log(c**(1/3) + d**(1/3)*x))/(6*d**(1/3)*b**(1/3)*(a*d - b*c))
```

3.427 $\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 288

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx = -\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}(bc-ad)}+\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)} + \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)} - \frac{\sqrt[3]{d}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{c}(bc-ad)}$$

output
```
-1/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(
1/3)/(-a*d+b*c)+1/3*d^(1/3)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)/c^(1/
3))*3^(1/2)/c^(1/3)/(-a*d+b*c)-1/3*b^(1/3)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/(
-a*d+b*c)+1/3*d^(1/3)*ln(c^(1/3)+d^(1/3)*x)/c^(1/3)/(-a*d+b*c)+1/6*b^(1/3)
*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/(-a*d+b*c)-1/6*d^(1/3)*
ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(1/3)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx = \frac{\frac{2\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} - \frac{2\sqrt{3}\sqrt[3]{d}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{c}} + \frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} - \frac{2\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt[3]{c}} - \frac{\sqrt[3]{b}\log\left(a^{\cdots}\right.}{\cdots}}{-6bc+6ad}$$

input
```
Integrate[x/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
((2*Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) -
 (2*Sqrt[3]*d^(1/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(1/3) +
 (2*b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) - (2*d^(1/3)*Log[c^(1/3) + d
^(1/3)*x])/c^(1/3) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/a^(1/3) + (d^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(
1/3))/(-6*b*c + 6*a*d)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx$$

$$\Big\downarrow \text{ 982}$$

$$\frac{b\int \frac{x}{bx^3+a}dx}{bc-ad} - \frac{d\int \frac{x}{dx^3+c}dx}{bc-ad}$$

$$\Big\downarrow \text{ 821}$$

$$\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}\right)}{bc-ad}$$

$\downarrow$ 16

$$\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}$$

$\downarrow$ 1142

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}$$

$\downarrow$ 25

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1082$$

$$\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}-\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \textbf{217}$$

$$\frac{b\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}-\frac{d\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}$$

input
```
Int[x/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(b*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(
1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/(b*c - a*d) - (d*(
-1/3*Log[c^(1/3) + d^(1/3)*x]/(c^(1/3)*d^(2/3)) + (-((Sqrt[3]*ArcTan[(1 -
(2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3)) + Log[c^(2/3) - c^(1/3)*d^(1/3)*
x + d^(2/3)*x^2]/(2*d^(1/3)))/(3*c^(1/3)*d^(1/3))))/(b*c - a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72

method	result
default	$\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)d}{ad-bc}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\cdots\right.}{ad-bc}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{3}c\,d^{3}-3a^{2}b\,c^{2}d^{2}+3a\,b^{2}c^{3}d-c^{4}b^{3}\right)\_Z^{3}+d\right)}\_R\ln\left(\left(\left(-d^{4}a^{4}+2a^{3}bc\,d^{3}-2a^{2}b^{2}c^{2}d^{2}+2a\,b^{3}c^{3}d-c^{4}b^{4}\right)\_R^{3}+bd\right)x+\left(\cdots\right.\right.\right.}{3}$

input
```
int(x/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)
*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3
)*x-1)))*d/(a*d-b*c)-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.70

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx = \frac{2\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-2\sqrt{3}\left(-\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-\cdots\right.}{\cdots}$$

input
```
integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3))
 - 2*sqrt(3)*(-d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(-d/c)^(1/3) + 1/3*sqrt(3))
 + (b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) + (-d/c)^(1/3)
*log(d*x^2 - c*x*(-d/c)^(2/3) - c*(-d/c)^(1/3)) - 2*(b/a)^(1/3)*log(b*x +
a*(b/a)^(2/3)) - 2*(-d/c)^(1/3)*log(d*x + c*(-d/c)^(2/3)))/(b*c - a*d)
```

Sympy [A] (verification not implemented)

Time = 5.56 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.79

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx$$
$$= \text{RootSum}\left(t^3\cdot\left(27a^4d^3-81a^3bcd^2+81a^2b^2c^2d-27ab^3c^3\right)-b,\left(t\mapsto t\log\left(x+\frac{243t^5a^7cd^6-1458t^5a^6\right.\right.\right.$$
$$+\text{RootSum}\left(t^3\cdot\left(27a^3cd^3-81a^2bc^2d^2+81ab^2c^3d-27b^3c^4\right)+d,\left(t\mapsto t\log\left(x+\frac{243t^5a^7cd^6-1458t^5\right.\right.\right.$$

input
```
integrate(x/(b*x**3+a)/(d*x**3+c),x)
```

output
```
RootSum(_t**3*(27*a**4*d**3 - 81*a**3*b*c*d**2 + 81*a**2*b**2*c**2*d - 27*
a*b**3*c**3) - b, Lambda(_t, _t*log(x + (243*_t**5*a**7*c*d**6 - 1458*_t**
5*a**6*b*c**2*d**5 + 3645*_t**5*a**5*b**2*c**3*d**4 - 4860*_t**5*a**4*b**3
*c**4*d**3 + 3645*_t**5*a**3*b**4*c**5*d**2 - 1458*_t**5*a**2*b**5*c**6*d
 + 243*_t**5*a*b**6*c**7 + 9*_t**2*a**4*d**4 - 18*_t**2*a**3*b*c*d**3 + 18*
_t**2*a**2*b**2*c**2*d**2 - 18*_t**2*a*b**3*c**3*d + 9*_t**2*b**4*c**4)/(a
*b*d**2 + b**2*c*d)))) + RootSum(_t**3*(27*a**3*c*d**3 - 81*a**2*b*c**2*d*
*2 + 81*a*b**2*c**3*d - 27*b**3*c**4) + d, Lambda(_t, _t*log(x + (243*_t**
5*a**7*c*d**6 - 1458*_t**5*a**6*b*c**2*d**5 + 3645*_t**5*a**5*b**2*c**3*d*
*4 - 4860*_t**5*a**4*b**3*c**4*d**3 + 3645*_t**5*a**3*b**4*c**5*d**2 - 145
8*_t**5*a**2*b**5*c**6*d + 243*_t**5*a*b**6*c**7 + 9*_t**2*a**4*d**4 - 18*
_t**2*a**3*b*c*d**3 + 18*_t**2*a**2*b**2*c**2*d**2 - 18*_t**2*a*b**3*c**3*
d + 9*_t**2*b**4*c**4)/(a*b*d**2 + b**2*c*d))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.92

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3(bc-ad)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3(bc-ad)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} - \frac{\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} - \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} + \frac{\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}$$

input
```
integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b*c - a*
d)*(a/b)^(1/3)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)
^(1/3))/((b*c - a*d)*(c/d)^(1/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2
/3))/(b*c*(a/b)^(1/3) - a*d*(a/b)^(1/3)) - 1/6*log(x^2 - x*(c/d)^(1/3) + (
c/d)^(2/3))/(b*c*(c/d)^(1/3) - a*d*(c/d)^(1/3)) - 1/3*log(x + (a/b)^(1/3))
/(b*c*(a/b)^(1/3) - a*d*(a/b)^(1/3)) + 1/3*log(x + (c/d)^(1/3))/(b*c*(c/d)
^(1/3) - a*d*(c/d)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.01

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx = -\frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(abc-a^2d)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} - \frac{(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab^2c-\sqrt{3}a^2bd} + \frac{(-cd^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2d-\sqrt{3}acd^2} + \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(ab^2c-a^2bd)} - \frac{(-cd^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^2d-acd^2)}$$

input
```
integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c
/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) - (-a*b^2)^(2/3)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b^2*c - sqrt(
3)*a^2*b*d) + (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d
)^(1/3))/(sqrt(3)*b*c^2*d - sqrt(3)*a*c*d^2) + 1/6*(-a*b^2)^(2/3)*log(x^2
+ x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2*c - a^2*b*d) - 1/6*(-c*d^2)^(2/3)*
log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^2*d - a*c*d^2)
```

Mupad [B] (verification not implemented)

Time = 3.22 (sec) , antiderivative size = 982, normalized size of antiderivative = 3.41

$$\int \frac{x}{(a+bx^3)(c+dx^3)} dx = \text{Too large to display}$$

input
```
int(x/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(b*x + a^3*d^2*(b/(a*(a*d - b*c)^3))^(2/3) + a*b^2*c^2*(b/(a*(a*d - b*c
)^3))^(2/3) - 2*a^2*b*c*d*(b/(a*(a*d - b*c)^3))^(2/3))*(b/(27*a^4*d^3 - 27
*a*b^3*c^3 + 81*a^2*b^2*c^2*d - 81*a^3*b*c*d^2))^(1/3) + log(d*x + b^2*c^3
*(-d/(c*(a*d - b*c)^3))^(2/3) + a^2*c*d^2*(-d/(c*(a*d - b*c)^3))^(2/3) - 2
*a*b*c^2*d*(-d/(c*(a*d - b*c)^3))^(2/3))*(d/(27*b^3*c^4 - 27*a^3*c*d^3 + 8
1*a^2*b*c^2*d^2 - 81*a*b^2*c^3*d))^(1/3) + (log(b^4*d^4*x - (b*(3^(1/2)*1i
 - 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*
(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b/(a*(a*d - b*c)^3))^(2/3))/
4))/(216*a*(a*d - b*c)^3))*(3^(1/2)*1i - 1)*(b/(27*a^4*d^3 - 27*a*b^3*c^3
+ 81*a^2*b^2*c^2*d - 81*a^3*b*c*d^2))^(1/3))/2 - (log(b^4*d^4*x + (b*(3^(1
/2)*1i + 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*
c*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b/(a*(a*d - b*c)^3))^(
2/3))/4))/(216*a*(a*d - b*c)^3))*(3^(1/2)*1i + 1)*(b/(27*a^4*d^3 - 27*a*b^
3*c^3 + 81*a^2*b^2*c^2*d - 81*a^3*b*c*d^2))^(1/3))/2 + (log(b^4*d^4*x + (d
*(3^(1/2)*1i - 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*
a*b^3*c*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d/(c*(a*d - b*c
)^3))^(2/3))/4))/(216*c*(a*d - b*c)^3))*(3^(1/2)*1i - 1)*(d/(27*b^3*c^4 -
27*a^3*c*d^3 + 81*a^2*b*c^2*d^2 - 81*a*b^2*c^3*d))^(1/3))/2 - (log(b^4*d^4
*x - (d*(3^(1/2)*1i + 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2
 + (27*a*b^3*c*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d/(c*...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.63

$$\int \frac{x}{(a+bx^3)(c+dx^3)} dx = \frac{2d^{\frac{2}{3}}c^{\frac{1}{3}}\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b - 2b^{\frac{2}{3}}a^{\frac{1}{3}}\sqrt{3}\, atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)d + b^{\frac{2}{3}}a^{\frac{1}{3}}\log\left(c^{\frac{2}{3}} - d^{\frac{1}{3}}c^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2\right)d - 2b^{\frac{2}{3}}a^{\frac{1}{3}}\log(}{6d^{\frac{2}{3}}c^{\frac{1}{3}}b^{\frac{2}{3}}a^{\frac{1}{3}}(ad-bc)}$$

input
```
int(x/(b*x^3+a)/(d*x^3+c),x)
```

output
```
(2*d**(2/3)*c**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt
(3)))*b - 2*b**(2/3)*a**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(
1/3)*sqrt(3)))*d + b**(2/3)*a**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x +
d**(2/3)*x**2)*d - 2*b**(2/3)*a**(1/3)*log(c**(1/3) + d**(1/3)*x)*d - d**(
2/3)*c**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b + 2*d*
*(2/3)*c**(1/3)*log(a**(1/3) + b**(1/3)*x)*b)/(6*d**(2/3)*c**(1/3)*b**(2/3
)*a**(1/3)*(a*d - b*c))
```

3.428 $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 288

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = -\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)}+\frac{d^{2/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)}-\frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}+\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)}$$

output

```
-1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(
2/3)/(-a*d+b*c)+1/3*d^(2/3)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)/c^(1/
3))*3^(1/2)/c^(2/3)/(-a*d+b*c)+1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/(
-a*d+b*c)-1/3*d^(2/3)*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/(-a*d+b*c)-1/6*b^(2/3)
*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/(-a*d+b*c)+1/6*d^(2/3)*
ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(2/3)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.11 (sec), antiderivative size = 224, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$$
$$= \frac{\frac{2\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{2\sqrt{3}d^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{2/3}} - \frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{2d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{2/3}} + \frac{b^{2/3}\log\left(a^{2/}\right.}{}}{-6bc+6ad}$$

input
```
Integrate[1/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
((2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) -
 (2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(2/3) -
 (2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (2*d^(2/3)*Log[c^(1/3) + d
^(1/3)*x])/c^(2/3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/a^(2/3) - (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(
2/3))/(-6*b*c + 6*a*d)
```

Rubi [A] (verified)

Time = 0.67 (sec), antiderivative size = 248, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {917, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$$
$$\Big\downarrow \; 917$$
$$\frac{b\int \frac{1}{bx^3+a}dx}{bc-ad} - \frac{d\int \frac{1}{dx^3+c}dx}{bc-ad}$$
$$\Big\downarrow \; 750$$

$$\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \text{16}$$

$$\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1142}$$

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int \frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1082$$

$$\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\downarrow 217$$

$$\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\downarrow 1103$$

$$\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

input
```
Int[1/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(b*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/(b*c - a*d) - (d*(Log[c^(1/3)
 + d^(1/3)*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/
c^(1/3))/Sqrt[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^
2]/(2*d^(1/3)))/(3*c^(2/3))))/(b*c - a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 917
```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^n), x], x] - Simp[d/(b*c - a*d)   Int[1/(c
 + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72

method	result
default	$\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)d}{ad-bc}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\sqrt{3}\ \text{ar}\cdots\right.}{ad-bc}$
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(a^{3}c^{2}d^{3}-3a^{2}bc^{3}d^{2}+3ab^{2}c^{4}d-c^{5}b^{3}\right)\_Z^{3}-d^{2}\right)}\_R\ln\left(\left(\left(-a^{5}d^{5}+3a^{4}bcd^{4}-2a^{3}b^{2}c^{2}d^{3}-2a^{2}b^{3}c^{3}d^{2}+3ab^{4}c^{4}d-c^{5}b^{5}\right)\_\cdots\right.\right.\right.}{3}$

input
```
int(1/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*
x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)
*x-1)))*d/(a*d-b*c)-(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)
*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1)))*b/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^{3})(c+dx^{3})}\,dx =$$
$$-\frac{2\sqrt{3}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)+2\sqrt{3}\left(\frac{d^{2}}{c^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^{2}}{c^{2}}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}\cdots\right.}{\cdots}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3
) - sqrt(3)*b)/b) + 2*sqrt(3)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d
^2/c^2)^(2/3) - sqrt(3)*d)/d) - (-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2
/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - (d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*
(d^2/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*(-b^2/a^2)^(1/3)*log(b*x - a*(-
b^2/a^2)^(1/3)) + 2*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)))/(b*c - a
*d)
```

Sympy [A] (verification not implemented)

Time = 57.84 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.55

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$$
$$= \text{RootSum}\left(t^3\cdot\left(27a^5d^3-81a^4bcd^2+81a^3b^2c^2d-27a^2b^3c^3\right)+b^2,\left(t\mapsto t\log\left(x+\frac{81t^4a^7c^2d^5-243t^4a^6\right.\right.\right.$$
$$+\text{RootSum}\left(t^3\cdot\left(27a^3c^2d^3-81a^2bc^3d^2+81ab^2c^4d-27b^3c^5\right)-d^2,\left(t\mapsto t\log\left(x+\frac{81t^4a^7c^2d^5-243t\right.\right.\right.$$

input
```
integrate(1/(b*x**3+a)/(d*x**3+c),x)
```

output
```
RootSum(_t**3*(27*a**5*d**3 - 81*a**4*b*c*d**2 + 81*a**3*b**2*c**2*d - 27*
a**2*b**3*c**3) + b**2, Lambda(_t, _t*log(x + (81*_t**4*a**7*c**2*d**5 - 2
43*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**5*b**2*c**4*d**3 + 162*_t**4*a**4
*b**3*c**5*d**2 - 243*_t**4*a**3*b**4*c**6*d + 81*_t**4*a**2*b**5*c**7 - 3
*_t*a**4*d**4 + 3*_t*a**3*b*c*d**3 + 3*_t*a*b**3*c**3*d - 3*_t*b**4*c**4)/
(a**2*b*d**3 + b**3*c**2*d)))) + RootSum(_t**3*(27*a**3*c**2*d**3 - 81*a**
2*b*c**3*d**2 + 81*a*b**2*c**4*d - 27*b**3*c**5) - d**2, Lambda(_t, _t*log
(x + (81*_t**4*a**7*c**2*d**5 - 243*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**
5*b**2*c**4*d**3 + 162*_t**4*a**4*b**3*c**5*d**2 - 243*_t**4*a**3*b**4*c**
6*d + 81*_t**4*a**2*b**5*c**7 - 3*_t*a**4*d**4 + 3*_t*a**3*b*c*d**3 + 3*_t
*a*b**3*c**3*d - 3*_t*b**4*c**4)/(a**2*b*d**3 + b**3*c**2*d))))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b*c*(a/b
)^(1/3) - a*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(
2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*(c/d)^(1/3) - a*d*(c/d)^(1/3))*(c/d)
^(1/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*c*(a/b)^(2/3) - a*
d*(a/b)^(2/3)) + 1/6*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*(c/d)^(2/
3) - a*d*(c/d)^(2/3)) + 1/3*log(x + (a/b)^(1/3))/(b*c*(a/b)^(2/3) - a*d*(a
/b)^(2/3)) - 1/3*log(x + (c/d)^(1/3))/(b*c*(c/d)^(2/3) - a*d*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = -\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(abc-a^2d)} + \frac{d\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} + \frac{(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}abc-\sqrt{3}a^2d} - \frac{(-cd^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2-\sqrt{3}acd} + \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(abc-a^2d)} - \frac{(-cd^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^2-acd)}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c
/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(1/3)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b*c - sqrt(3)
*a^2*d) - (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1
/3))/(sqrt(3)*b*c^2 - sqrt(3)*a*c*d) + 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a*b*c - a^2*d) - 1/6*(-c*d^2)^(1/3)*log(x^2 + x*
(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^2 - a*c*d)
```

Mupad [B] (verification not implemented)

Time = 7.84 (sec) , antiderivative size = 1364, normalized size of antiderivative = 4.74

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(((-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*
a*b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))*(a*d +
b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-b
^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3
) + log(((d^2/(c^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 -
18*a*b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(d^2/(c^2*(a*d - b*c)^3))^(1/3))*(a*d
+ b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-
d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/
3) + (log(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)
*(((3^(1/2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*a*b^3*
c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3)
)^(1/3))/2)*(-b^2/(a^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c
^2*d^4 + 18*a*b^5*c*d^5))/6)*(-b^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b
^2*c^2*d - 81*a^4*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x -
 ((3^(1/2)*1i + 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(
81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*
(a*d + b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)*(-b^2/(a^2*
(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5
))/6)*(-b^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d
^2))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \frac{2d^{\frac{1}{3}}a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)bc-2c^{\frac{1}{3}}b^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)ad+d^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)bc-2d^{\frac{1}{3}}a^{\frac{1}{3}}\cdots}{6d^{\frac{1}{3}}b^{\frac{1}{3}}ac\,(ad-bc)}$$

input
```
int(1/(b*x^3+a)/(d*x^3+c),x)
```

output
```
(2*d**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt
(3)))*b*c - 2*c**(1/3)*b**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c*
*(1/3)*sqrt(3)))*a*d + d**(1/3)*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*
x + b**(2/3)*x**2)*b*c - 2*d**(1/3)*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b*
c - c**(1/3)*b**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*
a*d + 2*c**(1/3)*b**(1/3)*log(c**(1/3) + d**(1/3)*x)*a*d)/(6*d**(1/3)*b**(
1/3)*a*c*(a*d - b*c))
```

3.429 $\int \frac{1}{x^2(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 299

$$\int \frac{1}{x^2(a+bx^3)(c+dx^3)}\,dx = -\frac{1}{acx} + \frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}(bc-ad)} - \frac{d^{4/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{4/3}(bc-ad)} + \frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)} - \frac{d^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{4/3}(bc-ad)} - \frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}(bc-ad)} + \frac{d^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{4/3}(bc-ad)}$$

output
```
-1/a/c/x+1/3*b^(4/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(
1/2)/a^(4/3)/(-a*d+b*c)-1/3*d^(4/3)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/
2)/c^(1/3))*3^(1/2)/c^(4/3)/(-a*d+b*c)+1/3*b^(4/3)*ln(a^(1/3)+b^(1/3)*x)/a
^(4/3)/(-a*d+b*c)-1/3*d^(4/3)*ln(c^(1/3)+d^(1/3)*x)/c^(4/3)/(-a*d+b*c)-1/6
*b^(4/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/(-a*d+b*c)+1/6*
d^(4/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(4/3)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2 (a+bx^3) (c+dx^3)} \, dx$$
$$= \frac{\frac{6b}{a} - \frac{6d}{c} - \frac{2\sqrt{3}b^{4/3}x \arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} + \frac{2\sqrt{3}d^{4/3}x \arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{4/3}} - \frac{2b^{4/3}x \log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{4/3}} + \frac{2d^{4/3}x \log\left(\sqrt[3]{c}+\sqrt[3]{d}x \ldots}{c^{4/3}}}{-6bcx+6adx}$$

input
```
Integrate[1/(x^2*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
((6*b)/a - (6*d)/c - (2*Sqrt[3]*b^(4/3)*x*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3
))/Sqrt[3]])/a^(4/3) + (2*Sqrt[3]*d^(4/3)*x*ArcTan[(1 - (2*d^(1/3)*x)/c^(1
/3))/Sqrt[3]])/c^(4/3) - (2*b^(4/3)*x*Log[a^(1/3) + b^(1/3)*x])/a^(4/3) +
(2*d^(4/3)*x*Log[c^(1/3) + d^(1/3)*x])/c^(4/3) + (b^(4/3)*x*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3) - (d^(4/3)*x*Log[c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2])/c^(4/3))/(-6*b*c*x + 6*a*d*x)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {980, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^3) (c+dx^3)} \, dx$$
$$\Big\downarrow \text{ 980}$$
$$\frac{\int -\frac{x(bdx^3+bc+ad)}{(bx^3+a)(dx^3+c)}dx}{ac} - \frac{1}{acx}$$
$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{x(bdx^3+bc+ad)}{(bx^3+a)(dx^3+c)}dx}{ac}-\frac{1}{acx}$$

$\Big\downarrow$ 1054

$$-\frac{\int \left(\frac{cxb^2}{(bc-ad)(bx^3+a)}+\frac{ad^2x}{(ad-bc)(dx^3+c)}\right)dx}{ac}-\frac{1}{acx}$$

$\Big\downarrow$ 2009

$$-\frac{\frac{b^{4/3}c\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)}-\frac{b^{4/3}c\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}(bc-ad)}+\frac{ad^{4/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)}-\frac{b^{4/3}c\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)}-\frac{ad^4}{}}{ac}$$

$$\frac{1}{acx}$$

input
```
Int[1/(x^2*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
-(1/(a*c*x)) - (-((b^(4/3)*c*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(Sqrt[3]*a^(1/3)*(b*c - a*d))) + (a*d^(4/3)*ArcTan[(c^(1/3) - 2*d^(1
/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(1/3)*(b*c - a*d)) - (b^(4/3)*c*Log[
a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*(b*c - a*d)) + (a*d^(4/3)*Log[c^(1/3) + d
^(1/3)*x])/(3*c^(1/3)*(b*c - a*d)) + (b^(4/3)*c*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*(b*c - a*d)) - (a*d^(4/3)*Log[c^(2/3) - c
^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(1/3)*(b*c - a*d)))/(a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)d^{2}}{(ad-bc)c}-\frac{1}{acx}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\right.}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{(}$
risch	$-\frac{1}{acx}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^{3}c^{4}a^{3}-3a^{2}bc^{5}d^{2}+3ab^{2}c^{6}d-b^{3}c^{7}\right)\_Z^{3}-d^{4}\right)}\_R\ln\left(\left(\left(-4a^{10}c^{4}d^{6}+22a^{9}bc^{5}d^{5}-52a^{8}b^{2}c^{6}d^{4}+68a^{7}b^{3}c^{7}d\right.\right.\right.\right.}{}$

input
```
int(1/x^2/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3
)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/
3)*x-1)))*d^2/(a*d-b*c)/c-1/a/c/x+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/
6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3
)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b^2/(a*d-b*c)/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx =$$

$$-\frac{2\sqrt{3}bcx\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-2\sqrt{3}adx\left(\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{d}{c}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-bcx(-}{}$$

input
```
integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*b*c*x*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3
*sqrt(3)) - 2*sqrt(3)*a*d*x*(d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(d/c)^(1/3) -
 1/3*sqrt(3)) - b*c*x*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)
^(1/3)) - a*d*x*(d/c)^(1/3)*log(d*x^2 - c*x*(d/c)^(2/3) + c*(d/c)^(1/3)) +
 2*b*c*x*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)) + 2*a*d*x*(d/c)^(1/3)*log(
d*x + c*(d/c)^(2/3)) + 6*b*c - 6*a*d)/((a*b*c^2 - a^2*c*d)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+bx^3)(c+dx^3)}\,dx = -\frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3(abc-a^2d)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3}d\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3(bc^2-acd)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{b\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(abc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} + \frac{d\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-acd\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} + \frac{b\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(abc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} - \frac{d\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-acd\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} - \frac{1}{acx}$$

input
```
integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a*b*c
 - a^2*d)*(a/b)^(1/3)) + 1/3*sqrt(3)*d*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/
3))/(c/d)^(1/3))/((b*c^2 - a*c*d)*(c/d)^(1/3)) - 1/6*b*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3))/(a*b*c*(a/b)^(1/3) - a^2*d*(a/b)^(1/3)) + 1/6*d*log(x^
2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c^2*(c/d)^(1/3) - a*c*d*(c/d)^(1/3)) +
 1/3*b*log(x + (a/b)^(1/3))/(a*b*c*(a/b)^(1/3) - a^2*d*(a/b)^(1/3)) - 1/3*
d*log(x + (c/d)^(1/3))/(b*c^2*(c/d)^(1/3) - a*c*d*(c/d)^(1/3)) - 1/(a*c*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^2(a+bx^3)(c+dx^3)}\,dx = \frac{b^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3(a^2bc-a^3d)} - \frac{d^2\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3(bc^3-ac^2d)} + \frac{(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^2bc-\sqrt{3}a^3d} - \frac{(-cd^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^3-\sqrt{3}ac^2d} - \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(a^2bc-a^3d)} + \frac{(-cd^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(bc^3-ac^2d)} - \frac{1}{acx}$$

input
```
integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*b^2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b*c - a^3*d) - 1/3*d^
2*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^3 - a*c^2*d) + (-a*b^2)^(2/
3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b*c
- sqrt(3)*a^3*d) - (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/
(-c/d)^(1/3))/(sqrt(3)*b*c^3 - sqrt(3)*a*c^2*d) - 1/6*(-a*b^2)^(2/3)*log(x
^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b*c - a^3*d) + 1/6*(-c*d^2)^(2/3)
*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^3 - a*c^2*d) - 1/(a*c*x)
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 716, normalized size of antiderivative = 2.39

$$\int \frac{1}{x^2 (a+bx^3) (c+dx^3)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(b - a^2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^(1/3) + a*b*c*x*(-b^4/(a^4*(a*d
 - b*c)^3))^(1/3))*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d -
 81*a^6*b*c*d^2))^(1/3) + log(d - b*c^2*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3)
+ a*c*d*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3))*(-d^4/(27*b^3*c^7 - 27*a^3*c^4*
d^3 + 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^(1/3) - 1/(a*c*x) - (log(b - 3^(
1/2)*b*1i + 2*a^2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^(1/3) - 2*a*b*c*x*(-b^4/(
a^4*(a*d - b*c)^3))^(1/3))*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2
*c^2*d - 81*a^6*b*c*d^2))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(b + 3^(1/2)*b*1
i + 2*a^2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^(1/3) - 2*a*b*c*x*(-b^4/(a^4*(a*d
 - b*c)^3))^(1/3))*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d -
 81*a^6*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(d - 3^(1/2)*d*1i + 2*b*
c^2*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3) - 2*a*c*d*x*(d^4/(c^4*(a*d - b*c)^3)
)^(1/3))*(-d^4/(27*b^3*c^7 - 27*a^3*c^4*d^3 + 81*a^2*b*c^5*d^2 - 81*a*b^2*
c^6*d))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(d + 3^(1/2)*d*1i + 2*b*c^2*x*(d^4
/(c^4*(a*d - b*c)^3))^(1/3) - 2*a*c*d*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3))*(
-d^4/(27*b^3*c^7 - 27*a^3*c^4*d^3 + 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^(1
/3)*(3^(1/2)*1i - 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^2 (a+bx^3) (c+dx^3)} \, dx = \frac{-2d^{\frac{2}{3}}c^{\frac{4}{3}}\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) b^2x+2b^{\frac{2}{3}}a^{\frac{4}{3}}\sqrt{3}\, atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right) d^2x-6d^{\frac{5}{3}}c^{\frac{1}{3}}b^{\frac{2}{3}}a^{\frac{4}{3}}+6d^{\frac{2}{3}}c^{\frac{4}{3}}b^{\frac{5}{3}}a^{\frac{1}{3}}-b^{\frac{2}{3}}a^{\frac{4}{3}}\log\left(c\right.}{6d^{\frac{2}{3}}c}$$

input
```
int(1/x^2/(b*x^3+a)/(d*x^3+c),x)
```

output
```
( - 2*d**(2/3)*c**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*s
qrt(3)))*b**2*c*x + 2*b**(2/3)*a**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3
)*x)/(c**(1/3)*sqrt(3)))*a*d**2*x - 6*d**(2/3)*c**(1/3)*b**(2/3)*a**(1/3)*
a*d + 6*d**(2/3)*c**(1/3)*b**(2/3)*a**(1/3)*b*c - b**(2/3)*a**(1/3)*log(c*
*(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a*d**2*x + 2*b**(2/3)*a**(1/
3)*log(c**(1/3) + d**(1/3)*x)*a*d**2*x + d**(2/3)*c**(1/3)*log(a**(2/3) -
b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**2*c*x - 2*d**(2/3)*c**(1/3)*log(a*
*(1/3) + b**(1/3)*x)*b**2*c*x)/(6*d**(2/3)*c**(1/3)*b**(2/3)*a**(1/3)*a*c*
x*(a*d - b*c))
```

3.430 $\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 301

$$\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx = -\frac{1}{2acx^2}+\frac{b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}(bc-ad)} - \frac{d^{5/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{5/3}(bc-ad)} - \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}(bc-ad)} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{5/3}(bc-ad)} + \frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}(bc-ad)} - \frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{5/3}(bc-ad)}$$

output
```
-1/2/a/c/x^2+1/3*b^(5/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))
*3^(1/2)/a^(5/3)/(-a*d+b*c)-1/3*d^(5/3)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3
^(1/2)/c^(1/3))*3^(1/2)/c^(5/3)/(-a*d+b*c)-1/3*b^(5/3)*ln(a^(1/3)+b^(1/3)*
x)/a^(5/3)/(-a*d+b*c)+1/3*d^(5/3)*ln(c^(1/3)+d^(1/3)*x)/c^(5/3)/(-a*d+b*c)
+1/6*b^(5/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/(-a*d+b*c)-
1/6*d^(5/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(5/3)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3 (a+bx^3)(c+dx^3)}\,dx$$
$$= \frac{\frac{3b}{a} - \frac{3d}{c} - \frac{2\sqrt{3}b^{5/3}x^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} + \frac{2\sqrt{3}d^{5/3}x^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}} + \frac{2b^{5/3}x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}} - \frac{2d^{5/3}x^2\log\left(\sqrt[3]{c}+ \sqrt[3]{\ldots}\right.}{c^{5/3}}}{6(-bc+ad)x^2}$$

input
```
Integrate[1/(x^3*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
((3*b)/a - (3*d)/c - (2*Sqrt[3]*b^(5/3)*x^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/a^(5/3) + (2*Sqrt[3]*d^(5/3)*x^2*ArcTan[(1 - (2*d^(1/3)*x)/
c^(1/3))/Sqrt[3]])/c^(5/3) + (2*b^(5/3)*x^2*Log[a^(1/3) + b^(1/3)*x])/a^(5
/3) - (2*d^(5/3)*x^2*Log[c^(1/3) + d^(1/3)*x])/c^(5/3) - (b^(5/3)*x^2*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + (d^(5/3)*x^2*Log[c^(
2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(5/3))/(6*(-(b*c) + a*d)*x^2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {980, 27, 1020, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^3)(c+dx^3)}\,dx$$

$$\Big\downarrow \; 980$$

$$\frac{\int -\frac{2(bdx^3+bc+ad)}{(bx^3+a)(dx^3+c)}dx}{2ac} - \frac{1}{2acx^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{bdx^3+bc+ad}{(bx^3+a)(dx^3+c)}dx}{ac}-\frac{1}{2acx^2}$$

$\Big\downarrow$ 1020

$$-\frac{\frac{b^2c\int \frac{1}{bx^3+a}dx}{bc-ad}-\frac{ad^2\int \frac{1}{dx^3+c}dx}{bc-ad}}{ac}-\frac{1}{2acx^2}$$

$\Big\downarrow$ 750

$$-\frac{\frac{b^2c\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}}{ac}-\frac{1}{2acx^2}$$

$\Big\downarrow$ 16

$$-\frac{\frac{b^2c\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}}{ac}-\frac{1}{2acx^2}$$

$\Big\downarrow$ 1142

$$-\frac{\frac{b^2c\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\cdots}{3c^{2/3}}\cdots\right)}{\cdots}}{ac}-\frac{1}{2acx^2}$$

$\Big\downarrow$ 25

$$-\frac{\frac{b^2c\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\cdots}{3c^{2/3}}\cdots\right)}{\cdots}}{ac}$$

$$\frac{1}{2acx^2}$$

$\downarrow$ 27

$$-\frac{\frac{b^2c\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\cdots}{3c^{2/3}}\cdots\right)}{\cdots}}{ac}$$

$$\frac{1}{2acx^2}$$

$\downarrow$ 1082

$$-\frac{\frac{b^2c\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int\cdots}{\cdots}}{3c^{2/3}}\cdots\right)}{\cdots}}{ac}$$

$$\frac{1}{2acx^2}$$

$\downarrow$ 217

$$-\frac{\frac{b^2c\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}\right.}{bc-ad}}{ac}$$

$$\frac{1}{2acx^2}$$

$$\Big\downarrow \; 1103$$

$$-\frac{\frac{b^2c\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{ad^2\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x\right.}{2\sqrt[3]{d}}}{3c^{2/3}}\right.}{bc-ad}}{ac}$$

$$\frac{1}{2acx^2}$$

input
```
Int[1/(x^3*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
-1/2*1/(a*c*x^2) - ((b^2*c*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) +
 (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/(b*c
- a*d) - (a*d^2*(Log[c^(1/3) + d^(1/3)*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3
]*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3)) - Log[c^(2/3) - c^
(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(2*d^(1/3))))/(3*c^(2/3))))/(b*c - a*d))/(a*
c)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.76

method	result
default	$-\frac{1}{2ac\,x^2}-\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)d^2}{c(ad-bc)}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\right.}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{c}$
risch	$-\frac{1}{2ac\,x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(c^5d^3a^3-3a^2b\,c^6d^2+3a\,b^2c^7d-b^3c^8\right)\_Z^3+d^5\right)}\_R\ln\left(\left(\left(-4a^{11}c^5d^6+22a^{10}b\,c^6d^5-52a^9b^2c^7d^4+68a^8b^3\right.\right.\right.\right.}{}$

input
```
int(1/x^3/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/c/x^2-(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2
-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(c/d)^(1/3)*x-1)))/c*d^2/(a*d-b*c)+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-
1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/a*b^2/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)} \, dx = -\frac{2\sqrt{3}bcx^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}} \arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right) + 2\sqrt{3}adx^2\left(-\frac{d^2}{c^2}\right)^{\frac{1}{3}} \arctan\left(\frac{2\sqrt{3}cx\left(-\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right) - bcx^2}{}$$

input
```
integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*b*c*x^2*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2
)^(2/3) - sqrt(3)*b)/b) + 2*sqrt(3)*a*d*x^2*(-d^2/c^2)^(1/3)*arctan(1/3*(2
*sqrt(3)*c*x*(-d^2/c^2)^(2/3) - sqrt(3)*d)/d) - b*c*x^2*(b^2/a^2)^(1/3)*lo
g(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) - a*d*x^2*(-d^2/c
^2)^(1/3)*log(d^2*x^2 + c*d*x*(-d^2/c^2)^(1/3) + c^2*(-d^2/c^2)^(2/3)) + 2
*b*c*x^2*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 2*a*d*x^2*(-d^2/c^
2)^(1/3)*log(d*x - c*(-d^2/c^2)^(1/3)) + 3*b*c - 3*a*d)/((a*b*c^2 - a^2*c*
d)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (a + bx^3) (c + dx^3)} \, dx = -\frac{\sqrt{3} b \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 \left(abc\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^2 d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3} d \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3 \left(bc^2\left(\frac{c}{d}\right)^{\frac{1}{3}} - acd\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{b \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(abc\left(\frac{a}{b}\right)^{\frac{2}{3}} - a^2 d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{d \log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6 \left(bc^2\left(\frac{c}{d}\right)^{\frac{2}{3}} - acd\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} - \frac{b \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(abc\left(\frac{a}{b}\right)^{\frac{2}{3}} - a^2 d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{d \log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3 \left(bc^2\left(\frac{c}{d}\right)^{\frac{2}{3}} - acd\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} - \frac{1}{2 acx^2}$$

input
```
integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a*b*c
*(a/b)^(1/3) - a^2*d*(a/b)^(1/3))*(a/b)^(1/3)) + 1/3*sqrt(3)*d*arctan(1/3*
sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c^2*(c/d)^(1/3) - a*c*d*(c/d)
^(1/3))*(c/d)^(1/3)) + 1/6*b*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*c
*(a/b)^(2/3) - a^2*d*(a/b)^(2/3)) - 1/6*d*log(x^2 - x*(c/d)^(1/3) + (c/d)^
(2/3))/(b*c^2*(c/d)^(2/3) - a*c*d*(c/d)^(2/3)) - 1/3*b*log(x + (a/b)^(1/3)
)/(a*b*c*(a/b)^(2/3) - a^2*d*(a/b)^(2/3)) + 1/3*d*log(x + (c/d)^(1/3))/(b*
c^2*(c/d)^(2/3) - a*c*d*(c/d)^(2/3)) - 1/2/(a*c*x^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)} dx = \frac{b^2 \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log \left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3 (a^2bc-a^3d)} - \frac{d^2 \left(-\frac{c}{d}\right)^{\frac{1}{3}} \log \left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3 (bc^3-ac^2d)} - \frac{(-ab^2)^{\frac{1}{3}} b \arctan \left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^2bc-\sqrt{3}a^3d} + \frac{(-cd^2)^{\frac{1}{3}} d \arctan \left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^3-\sqrt{3}ac^2d} - \frac{(-ab^2)^{\frac{1}{3}} b \log \left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 (a^2bc-a^3d)} + \frac{(-cd^2)^{\frac{1}{3}} d \log \left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6 (bc^3-ac^2d)} - \frac{1}{2\,acx^2}$$

input
```
integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*b^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b*c - a^3*d) - 1/3*d^
2*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^3 - a*c^2*d) - (-a*b^2)^(1/
3)*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b*
c - sqrt(3)*a^3*d) + (-c*d^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/
3))/(-c/d)^(1/3))/(sqrt(3)*b*c^3 - sqrt(3)*a*c^2*d) - 1/6*(-a*b^2)^(1/3)*b
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b*c - a^3*d) + 1/6*(-c*d^2)
^(1/3)*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^3 - a*c^2*d) - 1/2/
(a*c*x^2)
```

Mupad [B] (verification not implemented)

Time = 10.63 (sec) , antiderivative size = 1829, normalized size of antiderivative = 6.08

$$\int \frac{1}{x^3 (a + bx^3) (c + dx^3)} dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(((b^5/(a^5*(a*d - b*c)^3))^(1/3)*(((81*a^10*b^3*c^10*d^3*(a*d + b*c)*(
a*d - b*c)^4*(b^5/(a^5*(a*d - b*c)^3))^(1/3) - 81*a^8*b^3*c^8*d^3*x*(a*d -
 b*c)^4*(a^2*d^2 + b^2*c^2 + a*b*c*d))*(b^5/(a^5*(a*d - b*c)^3))^(2/3))/9
+ 9*a^6*b^9*c^11*d^4 - 9*a^7*b^8*c^10*d^5 - 9*a^10*b^5*c^7*d^8 + 9*a^11*b^
4*c^6*d^9))/3 + 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(b^5/(27*a^8*d^3
- 27*a^5*b^3*c^3 + 81*a^6*b^2*c^2*d - 81*a^7*b*c*d^2))^(1/3) + log(((-d^5/
(c^5*(a*d - b*c)^3))^(1/3)*(((81*a^10*b^3*c^10*d^3*(a*d + b*c)*(a*d - b*c)
^4*(-d^5/(c^5*(a*d - b*c)^3))^(1/3) - 81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(
a^2*d^2 + b^2*c^2 + a*b*c*d))*(-d^5/(c^5*(a*d - b*c)^3))^(2/3))/9 + 9*a^6*
b^9*c^11*d^4 - 9*a^7*b^8*c^10*d^5 - 9*a^10*b^5*c^7*d^8 + 9*a^11*b^4*c^6*d^
9))/3 + 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(d^5/(27*b^3*c^8 - 27*a^3
*c^5*d^3 + 81*a^2*b*c^6*d^2 - 81*a*b^2*c^7*d))^(1/3) + (log(((3^(1/2)*1i -
 1)*(b^5/(a^5*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)^2*(81*a^8*b^3*c^8*d
^3*x*(a*d - b*c)^4*(a^2*d^2 + b^2*c^2 + a*b*c*d) - (81*a^10*b^3*c^10*d^3*(
3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(b^5/(a^5*(a*d - b*c)^3))^(1/3))
/2)*(b^5/(a^5*(a*d - b*c)^3))^(2/3))/36 - 9*a^6*b^9*c^11*d^4 + 9*a^7*b^8*c
^10*d^5 + 9*a^10*b^5*c^7*d^8 - 9*a^11*b^4*c^6*d^9))/6 - 3*a^6*b^6*c^6*d^6*
x*(a^2*d^2 + b^2*c^2))*(b^5/(27*a^8*d^3 - 27*a^5*b^3*c^3 + 81*a^6*b^2*c^2*
d - 81*a^7*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)*(b
^5/(a^5*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*a^8*b^3*c^8*d^3*...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3 (a + bx^3) (c + dx^3)} dx$$
$$= \frac{-2d^{\frac{1}{3}}a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2c^2x^2 + 2c^{\frac{1}{3}}b^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)a^2d^2x^2 - d^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)b^2}{}$$

input
```
int(1/x^3/(b*x^3+a)/(d*x^3+c),x)
```

output
```
( - 2*d**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*s
qrt(3)))*b**2*c**2*x**2 + 2*c**(1/3)*b**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d
**(1/3)*x)/(c**(1/3)*sqrt(3)))*a**2*d**2*x**2 - d**(1/3)*a**(1/3)*log(a**(
2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**2*c**2*x**2 + 2*d**(1/3)*a*
*(1/3)*log(a**(1/3) + b**(1/3)*x)*b**2*c**2*x**2 + c**(1/3)*b**(1/3)*log(c
**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**2*d**2*x**2 - 2*c**(1/3)
*b**(1/3)*log(c**(1/3) + d**(1/3)*x)*a**2*d**2*x**2 - 3*d**(1/3)*b**(1/3)*
a**2*c*d + 3*d**(1/3)*b**(1/3)*a*b*c**2)/(6*d**(1/3)*b**(1/3)*a**2*c**2*x*
*2*(a*d - b*c))
```

3.431 $\int \frac{1}{x^5(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 318

$$\begin{aligned}\int \frac{1}{x^5(a+bx^3)(c+dx^3)}\,dx &= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{7/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}(bc-ad)}\\&+\frac{d^{7/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{7/3}(bc-ad)}\\&-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}(bc-ad)}+\frac{d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{7/3}(bc-ad)}\\&+\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}(bc-ad)}\\&-\frac{d^{7/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{7/3}(bc-ad)}\end{aligned}$$

output
```
-1/4/a/c/x^4+(a*d+b*c)/a^2/c^2/x-1/3*b^(7/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)
*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(7/3)/(-a*d+b*c)+1/3*d^(7/3)*arctan(1/3*(c^
(1/3)-2*d^(1/3)*x)*3^(1/2)/c^(1/3))*3^(1/2)/c^(7/3)/(-a*d+b*c)-1/3*b^(7/3)
*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/(-a*d+b*c)+1/3*d^(7/3)*ln(c^(1/3)+d^(1/3)*x
)/c^(7/3)/(-a*d+b*c)+1/6*b^(7/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/a^(7/3)/(-a*d+b*c)-1/6*d^(7/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/
c^(7/3)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^5 (a+bx^3) (c+dx^3)} \, dx$$

$$= \frac{\frac{3b}{a} - \frac{3d}{c} - \frac{12b^2x^3}{a^2} + \frac{12d^2x^3}{c^2} + \frac{4\sqrt{3}b^{7/3}x^4 \arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}} - \frac{4\sqrt{3}d^{7/3}x^4 \arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{7/3}} + \frac{4b^{7/3}x^4 \log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{7/3}}}{12(-bc+ad)x^4}$$

input
```
Integrate[1/(x^5*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
((3*b)/a - (3*d)/c - (12*b^2*x^3)/a^2 + (12*d^2*x^3)/c^2 + (4*Sqrt[3]*b^(7
/3)*x^4*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(7/3) - (4*Sqrt[3]*
d^(7/3)*x^4*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(7/3) + (4*b^(7
/3)*x^4*Log[a^(1/3) + b^(1/3)*x])/a^(7/3) - (4*d^(7/3)*x^4*Log[c^(1/3) + d
^(1/3)*x])/c^(7/3) - (2*b^(7/3)*x^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/a^(7/3) + (2*d^(7/3)*x^4*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2])/c^(7/3))/(12*(-(b*c) + a*d)*x^4)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {980, 27, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^3) (c+dx^3)} \, dx$$

$$\Big\downarrow \text{980}$$

$$\frac{\int -\frac{4(bdx^3+bc+ad)}{x^2(bx^3+a)(dx^3+c)}dx}{4ac}-\frac{1}{4acx^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{bdx^3+bc+ad}{x^2(bx^3+a)(dx^3+c)}dx}{ac}-\frac{1}{4acx^4}$$

$$\Big\downarrow \text{1053}$$

$$-\frac{-\frac{\int \frac{x\left(bd(bc+ad)x^3+b^2c^2+a^2d^2+abcd\right)}{\left(bx^3+a\right)\left(dx^3+c\right)}dx}{ac}-\frac{ad+bc}{acx}}{ac}-\frac{1}{4acx^4}$$

$$\Big\downarrow \text{1054}$$

$$-\frac{-\frac{\int \left(\frac{c^2xb^3}{(bc-ad)\left(bx^3+a\right)}+\frac{a^2d^3x}{(ad-bc)\left(dx^3+c\right)}\right)dx}{ac}-\frac{ad+bc}{acx}}{ac}-\frac{1}{4acx^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{\frac{b^{7/3}c^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)}+\frac{a^2d^{7/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)}-\frac{a^2d^{7/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{c}(bc-ad)}+\frac{a^2d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)}-\frac{b^{7/}}{}}{ac}}{ac}$$

$$\frac{1}{4acx^4}$$

input
```
Int[1/(x^5*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
-1/4*1/(a*c*x^4) - (-((b*c + a*d)/(a*c*x)) - (-((b^(7/3)*c^2*ArcTan[(a^(1/
3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*(b*c - a*d))) + (a^
2*d^(7/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(1
/3)*(b*c - a*d)) - (b^(7/3)*c^2*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*(b*c
- a*d)) + (a^2*d^(7/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(1/3)*(b*c - a*d)) +
 (b^(7/3)*c^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*(
b*c - a*d)) - (a^2*d^(7/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])
/(6*c^(1/3)*(b*c - a*d)))/(a*c))/(a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.78

method	result
default	$\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)d^{3}}{c^{2}(ad-bc)}-\frac{1}{4ac\,x^{4}}-\frac{-ad-bc}{a^{2}c^{2}x}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln(\ldots}{\ldots}\right.}{\ldots}$
risch	$\frac{\frac{(ad+bc)x^{3}}{a^{2}c^{2}}-\frac{1}{4ac}}{x^{4}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^{3}a^{10}-3c\,d^{2}a^{9}b+3c^{2}d\,a^{8}b^{2}-a^{7}b^{3}c^{3}\right)\_Z^{3}-b^{7}\right)}\_R\ln\left(\left(\left(-4a^{13}c^{7}d^{6}+22a^{12}b\,c^{8}d^{5}-52a^{11}b^{2}c^{9}d^{4}\ldots\right.\right.\right.\right.}{\ldots}$

input
```
int(1/x^5/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)
*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3
)*x-1)))*d^3/c^2/(a*d-b*c)-1/4/a/c/x^4-1/a^2/c^2*(-a*d-b*c)/x-(-1/3/b/(a/b
)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b^3
/a^2/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^5 (a+bx^3) (c+dx^3)} dx = \frac{4\sqrt{3}b^2c^2x^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-4\sqrt{3}a^2d^2x^4\left(-\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+2b^2$$

input
```
integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/12*(4*sqrt(3)*b^2*c^2*x^4*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) -
 1/3*sqrt(3)) - 4*sqrt(3)*a^2*d^2*x^4*(-d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(-
d/c)^(1/3) + 1/3*sqrt(3)) + 2*b^2*c^2*x^4*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a
)^(2/3) + a*(b/a)^(1/3)) + 2*a^2*d^2*x^4*(-d/c)^(1/3)*log(d*x^2 - c*x*(-d/
c)^(2/3) - c*(-d/c)^(1/3)) - 4*b^2*c^2*x^4*(b/a)^(1/3)*log(b*x + a*(b/a)^(
2/3)) - 4*a^2*d^2*x^4*(-d/c)^(1/3)*log(d*x + c*(-d/c)^(2/3)) - 3*a*b*c^2 +
 3*a^2*c*d + 12*(b^2*c^2 - a^2*d^2)*x^3)/((a^2*b*c^3 - a^3*c^2*d)*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^3) (c+dx^3)} dx = \text{Timed out}$$

input
```
integrate(1/x**5/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^5 (a+bx^3)(c+dx^3)}\,dx = \frac{\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(a^2bc-a^3d)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}d^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,(bc^3-ac^2d)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{b^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^2bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^3d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} - \frac{d^2\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^3\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^2d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} - \frac{b^2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a^2bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^3d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} + \frac{d^2\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^3\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^2d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} + \frac{4\,(bc+ad)x^3-ac}{4\,a^2c^2x^4}$$

input
```
integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a^2*
b*c - a^3*d)*(a/b)^(1/3)) - 1/3*sqrt(3)*d^2*arctan(1/3*sqrt(3)*(2*x - (c/d
)^(1/3))/(c/d)^(1/3))/((b*c^3 - a*c^2*d)*(c/d)^(1/3)) + 1/6*b^2*log(x^2 -
x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*c*(a/b)^(1/3) - a^3*d*(a/b)^(1/3)) - 1
/6*d^2*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c^3*(c/d)^(1/3) - a*c^2*d
*(c/d)^(1/3)) - 1/3*b^2*log(x + (a/b)^(1/3))/(a^2*b*c*(a/b)^(1/3) - a^3*d*
(a/b)^(1/3)) + 1/3*d^2*log(x + (c/d)^(1/3))/(b*c^3*(c/d)^(1/3) - a*c^2*d*(
c/d)^(1/3)) + 1/4*(4*(b*c + a*d)*x^3 - a*c)/(a^2*c^2*x^4)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^5(a+bx^3)(c+dx^3)}\,dx = -\frac{b^3\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3(a^3bc-a^4d)} + \frac{d^3\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3(bc^4-ac^3d)} - \frac{(-ab^2)^{\frac{2}{3}}\,b\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^3bc-\sqrt{3}a^4d} + \frac{(-cd^2)^{\frac{2}{3}}\,d\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^4-\sqrt{3}ac^3d} + \frac{(-ab^2)^{\frac{2}{3}}\,b\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(a^3bc-a^4d)} - \frac{(-cd^2)^{\frac{2}{3}}\,d\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(bc^4-ac^3d)} + \frac{4\,bcx^3+4\,adx^3-ac}{4\,a^2c^2x^4}$$

input
```
integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*b^3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b*c - a^4*d) + 1/3*d
^3*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^4 - a*c^3*d) - (-a*b^2)^(2
/3)*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^3*b
*c - sqrt(3)*a^4*d) + (-c*d^2)^(2/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1
/3))/(-c/d)^(1/3))/(sqrt(3)*b*c^4 - sqrt(3)*a*c^3*d) + 1/6*(-a*b^2)^(2/3)*
b*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b*c - a^4*d) - 1/6*(-c*d^2
)^(2/3)*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^4 - a*c^3*d) + 1/4
*(4*b*c*x^3 + 4*a*d*x^3 - a*c)/(a^2*c^2*x^4)
```

Mupad [B] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 1734, normalized size of antiderivative = 5.45

$$\int \frac{1}{x^5 (a+bx^3) (c+dx^3)} \, dx = \text{Too large to display}$$

input

```
int(1/(x^5*(a + b*x^3)*(c + d*x^3)),x)
```

output

```
log(((b^7/(a^7*(a*d - b*c)^3))^(2/3)*(((27*a^14*b^3*c^14*d^3*x*(a^6*d^6 +
b^6*c^6)*(a*d - b*c)^2 + 27*a^19*b^3*c^19*d^3*(a*d + b*c)*(a*d - b*c)^4*(b
^7/(a^7*(a*d - b*c)^3))^(2/3))*(b^7/(a^7*(a*d - b*c)^3))^(1/3))/3 + 9*a^13
*b^11*c^20*d^4 - 9*a^14*b^10*c^19*d^5 - 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*
c^13*d^11))/9 + a^13*b^9*c^13*d^9*x)*(b^7/(27*a^10*d^3 - 27*a^7*b^3*c^3 +
81*a^8*b^2*c^2*d - 81*a^9*b*c*d^2))^(1/3) + log(((-d^7/(c^7*(a*d - b*c)^3)
)^(2/3)*(((27*a^14*b^3*c^14*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + 27*a
^19*b^3*c^19*d^3*(a*d + b*c)*(a*d - b*c)^4*(-d^7/(c^7*(a*d - b*c)^3))^(2/3
))*(-d^7/(c^7*(a*d - b*c)^3))^(1/3))/3 + 9*a^13*b^11*c^20*d^4 - 9*a^14*b^1
0*c^19*d^5 - 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13*d^11))/9 + a^13*b^9*c^
13*d^9*x)*(d^7/(27*b^3*c^10 - 27*a^3*c^7*d^3 + 81*a^2*b*c^8*d^2 - 81*a*b^2
*c^9*d))^(1/3) - (1/(4*a*c) - (x^3*(a*d + b*c))/(a^2*c^2))/x^4 + (log(((3^
(1/2)*1i - 1)^2*(b^7/(a^7*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i - 1)*(27*a^1
4*b^3*c^14*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + (27*a^19*b^3*c^19*d^3
*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b^7/(a^7*(a*d - b*c)^3))^(2
/3))/4)*(b^7/(a^7*(a*d - b*c)^3))^(1/3))/6 + 9*a^13*b^11*c^20*d^4 - 9*a^14
*b^10*c^19*d^5 - 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13*d^11))/36 + a^13*b
^9*c^13*d^9*x)*(b^7/(27*a^10*d^3 - 27*a^7*b^3*c^3 + 81*a^8*b^2*c^2*d - 81*
a^9*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)^2*(b^7/(a
^7*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i + 1)*(27*a^14*b^3*c^14*d^3*x*(a^...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^5 (a+bx^3) (c+dx^3)} \, dx$$

$$= \frac{4d^{\frac{2}{3}}c^{\frac{7}{3}}\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) b^3x^4 - 4b^{\frac{2}{3}}a^{\frac{7}{3}}\sqrt{3}\, atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right) d^3x^4 - 3d^{\frac{5}{3}}c^{\frac{4}{3}}b^{\frac{2}{3}}a^{\frac{7}{3}} + 12d^{\frac{8}{3}}c^{\frac{1}{3}}b^{\frac{2}{3}}a^{\frac{7}{3}}x^3 + 3d^{\frac{2}{3}}c^{\frac{7}{3}}b}{}$$

input
```
int(1/x^5/(b*x^3+a)/(d*x^3+c),x)
```

output
```
(4*d**(2/3)*c**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt
(3)))*b**3*c**2*x**4 - 4*b**(2/3)*a**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(
1/3)*x)/(c**(1/3)*sqrt(3)))*a**2*d**3*x**4 - 3*d**(2/3)*c**(1/3)*b**(2/3)*
a**(1/3)*a**2*c*d + 12*d**(2/3)*c**(1/3)*b**(2/3)*a**(1/3)*a**2*d**2*x**3
+ 3*d**(2/3)*c**(1/3)*b**(2/3)*a**(1/3)*a*b*c**2 - 12*d**(2/3)*c**(1/3)*b*
*(2/3)*a**(1/3)*b**2*c**2*x**3 + 2*b**(2/3)*a**(1/3)*log(c**(2/3) - d**(1/
3)*c**(1/3)*x + d**(2/3)*x**2)*a**2*d**3*x**4 - 4*b**(2/3)*a**(1/3)*log(c*
*(1/3) + d**(1/3)*x)*a**2*d**3*x**4 - 2*d**(2/3)*c**(1/3)*log(a**(2/3) - b
**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**3*c**2*x**4 + 4*d**(2/3)*c**(1/3)*l
og(a**(1/3) + b**(1/3)*x)*b**3*c**2*x**4)/(12*d**(2/3)*c**(1/3)*b**(2/3)*a
**(1/3)*a**2*c**2*x**4*(a*d - b*c))
```

3.432 $\int \frac{1}{x^6(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 321

$$\begin{aligned}\int \frac{1}{x^6(a+bx^3)(c+dx^3)}\,dx = &-\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}-\frac{b^{8/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}(bc-ad)}\\&+\frac{d^{8/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{8/3}(bc-ad)}\\&+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}(bc-ad)}-\frac{d^{8/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{8/3}(bc-ad)}\\&-\frac{b^{8/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}(bc-ad)}\\&+\frac{d^{8/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{8/3}(bc-ad)}\end{aligned}$$

output
```
-1/5/a/c/x^5+1/2*(a*d+b*c)/a^2/c^2/x^2-1/3*b^(8/3)*arctan(1/3*(a^(1/3)-2*b
^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3)/(-a*d+b*c)+1/3*d^(8/3)*arctan(1
/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)/c^(1/3))*3^(1/2)/c^(8/3)/(-a*d+b*c)+1/3*b
^(8/3)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/(-a*d+b*c)-1/3*d^(8/3)*ln(c^(1/3)+d^(
1/3)*x)/c^(8/3)/(-a*d+b*c)-1/6*b^(8/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/a^(8/3)/(-a*d+b*c)+1/6*d^(8/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)
*x^2)/c^(8/3)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^6 (a+bx^3)(c+dx^3)} dx = \frac{\frac{6b}{a} - \frac{6d}{c} - \frac{15b^2x^3}{a^2} + \frac{15d^2x^3}{c^2} + \frac{10\sqrt{3}b^{8/3}x^5 \arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}} - \frac{10\sqrt{3}d^{8/3}x^5 \arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{8/3}} - \frac{10b^{8/3}x^5 \log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{8/3}}}{30(-bc+ad)x}$$

input
```
Integrate[1/(x^6*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
((6*b)/a - (6*d)/c - (15*b^2*x^3)/a^2 + (15*d^2*x^3)/c^2 + (10*Sqrt[3]*b^(
8/3)*x^5*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(8/3) - (10*Sqrt[3
]*d^(8/3)*x^5*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(8/3) - (10*b
^(8/3)*x^5*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) + (10*d^(8/3)*x^5*Log[c^(1/3)
 + d^(1/3)*x])/c^(8/3) + (5*b^(8/3)*x^5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/a^(8/3) - (5*d^(8/3)*x^5*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2])/c^(8/3))/(30*(-(b*c) + a*d)*x^5)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {980, 27, 1053, 27, 1020, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^3) (c+dx^3)} \, dx$$

$$\Big\downarrow \text{ 980}$$

$$\frac{\int -\frac{5(bdx^3+bc+ad)}{x^3(bx^3+a)(dx^3+c)}dx}{5ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{bdx^3+bc+ad}{x^3(bx^3+a)(dx^3+c)}dx}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{ 1053}$$

$$-\frac{-\frac{\int \frac{2\left(bd(bc+ad)x^3+b^2c^2+a^2d^2+abcd\right)}{\left(bx^3+a\right)\left(dx^3+c\right)}dx}{2ac}-\frac{ad+bc}{2acx^2}}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{-\frac{\int \frac{bd(bc+ad)x^3+b^2c^2+a^2d^2+abcd}{\left(bx^3+a\right)\left(dx^3+c\right)}dx}{ac}-\frac{ad+bc}{2acx^2}}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{ 1020}$$

$$-\frac{-\frac{\frac{b^3c^2\int \frac{1}{bx^3+a}dx}{bc-ad}-\frac{a^2d^3\int \frac{1}{dx^3+c}dx}{bc-ad}}{ac}-\frac{ad+bc}{2acx^2}}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{ 750}$$

$$-\frac{-\frac{\frac{b^3c^2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad}-\frac{a^2d^3\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}}{ac}-\frac{ad+bc}{2acx^2}}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \; 16$$

$$-\frac{-\frac{\frac{b^3c^2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{a^2d^3\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}}{ac}-\frac{ad+bc}{2acx^2}}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \; 1142$$

$$-\frac{-\frac{\frac{b^3c^2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{a^2d^3\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx \cdots}{3c^{2/\cdots}}\cdots\right.}{\cdots}}{ac}\cdots}{ac}$$

$$\frac{1}{5acx^5}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{b^3c^2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{a^2d^3\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\cdots}{3c^{2/3}}\cdots\right)}{\cdots}}{ac}\cdots}{ac}\cdots$$

$$\frac{1}{5acx^5}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{b^3c^2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{a^2d^3\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\cdots}{3c^{2/3}}\cdots\right)}{\cdots}}{ac}\cdots}{ac}\cdots$$

$$\frac{1}{5acx^5}$$

$$\Big\downarrow \; 1082$$

$$-\frac{-\frac{b^3c^2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{a^2d^3\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\cdots}{\cdots}}{3\cdots}\cdots\right)}{\cdots}}{ac}\cdots}{ac}\cdots$$

$$\frac{1}{5acx^5}$$

$$\Big\downarrow \; 217$$

$$-\frac{-\frac{b^3c^2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{a^2d^3\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}\cdots\right)}{bc-ad}}{ac}}{ac}\cdots \frac{1}{5acx^5}$$

$\downarrow$ 1103

$$-\frac{-\frac{b^3c^2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{a^2d^3\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x\cdots\right)}{2\sqrt[3]{d}}}{3c^{2/3}}\cdots\right)}{bc-ad}}{ac}}{ac}\cdots \frac{1}{5acx^5}$$

input
```
Int[1/(x^6*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
-1/5*1/(a*c*x^5) - (-1/2*(b*c + a*d)/(a*c*x^2) - ((b^3*c^2*(Log[a^(1/3) +
b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/
(2*b^(1/3)))/(3*a^(2/3))))/(b*c - a*d) - (a^2*d^3*(Log[c^(1/3) + d^(1/3)*x
]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqr
t[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(2*d^(1/3
)))/(3*c^(2/3))))/(b*c - a*d))/(a*c))/(a*c)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.77

method	result
default	$\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)d^3}{c^2(ad-bc)}-\frac{1}{5ac\,x^5}-\frac{-ad-bc}{2x^2a^2c^2}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2 \right.}{}\right.}{}$
risch	$\frac{\frac{(ad+bc)x^3}{2a^2c^2}-\frac{1}{5ac}}{x^5}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(d^3c^8a^3-3d^2c^9a^2b+3d\,c^{10}a\,b^2-b^3c^{11}\right)\_Z^3-d^8\right)}\_R\ln\left(\left(\left(-4a^{14}c^8d^6+22a^{13}b\,c^9d^5-52a^{12}b^2c^{10}\right.\right.\right.\right.}{}$

input
```
int(1/x^6/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*
x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)
*x-1)))/c^2*d^3/(a*d-b*c)-1/5/a/c/x^5-1/2*(-a*d-b*c)/x^2/a^2/c^2-(1/3/b/(a
/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/a
^2*b^3/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^6(a+bx^3)(c+dx^3)}\,dx = -\frac{10\sqrt{3}b^2c^2x^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)+10\sqrt{3}a^2d^2x^5\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3\,d}\right)-5}{}$$

input
```
integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/30*(10*sqrt(3)*b^2*c^2*x^5*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(
-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 10*sqrt(3)*a^2*d^2*x^5*(d^2/c^2)^(1/3)*a
rctan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2/3) - sqrt(3)*d)/d) - 5*b^2*c^2*x^5*(
-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3
)) - 5*a^2*d^2*x^5*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c
^2*(d^2/c^2)^(2/3)) + 10*b^2*c^2*x^5*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^
2)^(1/3)) + 10*a^2*d^2*x^5*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) +
6*a*b*c^2 - 6*a^2*c*d - 15*(b^2*c^2 - a^2*d^2)*x^3)/((a^2*b*c^3 - a^3*c^2*
d)*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^3) (c+dx^3)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**6/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^6(a+bx^3)(c+dx^3)}\,dx = \frac{\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(a^2bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^3d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}d^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^3\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^2d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{b^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^2bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^3d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{d^2\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^3\left(\frac{c}{d}\right)^{\frac{2}{3}}-ac^2d\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{b^2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a^2bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^3d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{d^2\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^3\left(\frac{c}{d}\right)^{\frac{2}{3}}-ac^2d\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{5(bc+ad)x^3-2ac}{10a^2c^2x^5}$$

input
```
integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a^2*
b*c*(a/b)^(1/3) - a^3*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*d^2*arctan
(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c^3*(c/d)^(1/3) - a*c^2*
d*(c/d)^(1/3))*(c/d)^(1/3)) - 1/6*b^2*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3
))/(a^2*b*c*(a/b)^(2/3) - a^3*d*(a/b)^(2/3)) + 1/6*d^2*log(x^2 - x*(c/d)^(
1/3) + (c/d)^(2/3))/(b*c^3*(c/d)^(2/3) - a*c^2*d*(c/d)^(2/3)) + 1/3*b^2*lo
g(x + (a/b)^(1/3))/(a^2*b*c*(a/b)^(2/3) - a^3*d*(a/b)^(2/3)) - 1/3*d^2*log
(x + (c/d)^(1/3))/(b*c^3*(c/d)^(2/3) - a*c^2*d*(c/d)^(2/3)) + 1/10*(5*(b*c
 + a*d)*x^3 - 2*a*c)/(a^2*c^2*x^5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^6 (a+bx^3)(c+dx^3)}\,dx = -\frac{b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(a^3bc-a^4d)} + \frac{d^3\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^4-ac^3d)} + \frac{(-ab^2)^{\frac{1}{3}}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^3bc-\sqrt{3}a^4d} - \frac{(-cd^2)^{\frac{1}{3}}\,d^2\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^4-\sqrt{3}ac^3d} + \frac{(-ab^2)^{\frac{1}{3}}\,b^2\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(a^3bc-a^4d)} - \frac{(-cd^2)^{\frac{1}{3}}\,d^2\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^4-ac^3d)} + \frac{5\,bcx^3+5\,adx^3-2\,ac}{10\,a^2c^2x^5}$$

input
```
integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*b^3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b*c - a^4*d) + 1/3*d
^3*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^4 - a*c^3*d) + (-a*b^2)^(1
/3)*b^2*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^3
*b*c - sqrt(3)*a^4*d) - (-c*d^2)^(1/3)*d^2*arctan(1/3*sqrt(3)*(2*x + (-c/d
)^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b*c^4 - sqrt(3)*a*c^3*d) + 1/6*(-a*b^2)^(1
/3)*b^2*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b*c - a^4*d) - 1/6*(
-c*d^2)^(1/3)*d^2*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^4 - a*c^3*
d) + 1/10*(5*b*c*x^3 + 5*a*d*x^3 - 2*a*c)/(a^2*c^2*x^5)
```

Mupad [B] (verification not implemented)

Time = 10.42 (sec) , antiderivative size = 1860, normalized size of antiderivative = 5.79

$$\int \frac{1}{x^6 (a+bx^3)(c+dx^3)} dx = \text{Too large to display}$$

input
```
int(1/(x^6*(a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(((-b^8/(a^8*(a*d - b*c)^3))^(1/3)*(9*a^13*b^11*c^19*d^5 - 9*a^12*b^12*
c^20*d^4 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12 + 9*a^16*b^3*c^16*d
^3*(a*d + b*c)*(a*d - b*c)^4*(a^3*c^3*(-b^8/(a^8*(a*d - b*c)^3))^(1/3) + a
^2*d^2*x + b^2*c^2*x)*(-b^8/(a^8*(a*d - b*c)^3))^(2/3)))/3 + 3*a^12*b^7*c^
12*d^7*x*(a^4*d^4 + b^4*c^4))*(-b^8/(27*a^11*d^3 - 27*a^8*b^3*c^3 + 81*a^9
*b^2*c^2*d - 81*a^10*b*c*d^2))^(1/3) - (1/(5*a*c) - (x^3*(a*d + b*c))/(2*a
^2*c^2))/x^5 + log(((d^8/(c^8*(a*d - b*c)^3))^(1/3)*(9*a^13*b^11*c^19*d^5
- 9*a^12*b^12*c^20*d^4 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12 + 9*a
^16*b^3*c^16*d^3*(a*d + b*c)*(a*d - b*c)^4*(a^3*c^3*(d^8/(c^8*(a*d - b*c)^
3))^(1/3) + a^2*d^2*x + b^2*c^2*x)*(d^8/(c^8*(a*d - b*c)^3))^(2/3)))/3 + 3
*a^12*b^7*c^12*d^7*x*(a^4*d^4 + b^4*c^4))*(-d^8/(27*b^3*c^11 - 27*a^3*c^8*
d^3 + 81*a^2*b*c^9*d^2 - 81*a*b^2*c^10*d))^(1/3) + (log(((3^(1/2)*1i - 1)*
(-b^8/(a^8*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)^2*(81*a^16*b^3*c^16*d^
3*x*(a*d - b*c)^4*(a^3*d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2) + (81*a^
19*b^3*c^19*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^8/(a^8*(a*d
 - b*c)^3))^(1/3))/2)*(-b^8/(a^8*(a*d - b*c)^3))^(2/3))/36 - 9*a^12*b^12*c
^20*d^4 + 9*a^13*b^11*c^19*d^5 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^
12))/6 + 3*a^12*b^7*c^12*d^7*x*(a^4*d^4 + b^4*c^4))*(-b^8/(27*a^11*d^3 - 2
7*a^8*b^3*c^3 + 81*a^9*b^2*c^2*d - 81*a^10*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1
))/2 - (log(((3^(1/2)*1i + 1)*(-b^8/(a^8*(a*d - b*c)^3))^(1/3)*(((3^(1/...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^6 (a+bx^3)(c+dx^3)} dx$$

$$= \frac{10d^{\frac{1}{3}}a^{\frac{1}{3}}\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) b^3c^3x^5 - 10c^{\frac{1}{3}}b^{\frac{1}{3}}\sqrt{3}\, atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right) a^3d^3x^5 + 5d^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{}$$

input
```
int(1/x^6/(b*x^3+a)/(d*x^3+c),x)
```

output
```
(10*d**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqr
t(3)))*b**3*c**3*x**5 - 10*c**(1/3)*b**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d*
*(1/3)*x)/(c**(1/3)*sqrt(3)))*a**3*d**3*x**5 + 5*d**(1/3)*a**(1/3)*log(a**
(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**3*c**3*x**5 - 10*d**(1/3)*
a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b**3*c**3*x**5 - 5*c**(1/3)*b**(1/3)*l
og(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**3*d**3*x**5 + 10*c**
(1/3)*b**(1/3)*log(c**(1/3) + d**(1/3)*x)*a**3*d**3*x**5 - 6*d**(1/3)*b**(
1/3)*a**3*c**2*d + 15*d**(1/3)*b**(1/3)*a**3*c*d**2*x**3 + 6*d**(1/3)*b**(
1/3)*a**2*b*c**3 - 15*d**(1/3)*b**(1/3)*a*b**2*c**3*x**3)/(30*d**(1/3)*b**
(1/3)*a**3*c**3*x**5*(a*d - b*c))
```

3.433 $\int \frac{1}{x^8(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 352

$$\int \frac{1}{x^8(a+bx^3)(c+dx^3)}\,dx = -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^{10/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}(bc-ad)}-\frac{d^{10/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{10/3}(bc-ad)}+\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}(bc-ad)}-\frac{d^{10/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{10/3}(bc-ad)}-\frac{b^{10/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}(bc-ad)}+\frac{d^{10/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{10/3}(bc-ad)}$$

output
```
-1/7/a/c/x^7+1/4*(a*d+b*c)/a^2/c^2/x^4-(a^2*d^2+a*b*c*d+b^2*c^2)/a^3/c^3/x
+1/3*b^(10/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^
(10/3)/(-a*d+b*c)-1/3*d^(10/3)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)/c^
(1/3))*3^(1/2)/c^(10/3)/(-a*d+b*c)+1/3*b^(10/3)*ln(a^(1/3)+b^(1/3)*x)/a^(1
0/3)/(-a*d+b*c)-1/3*d^(10/3)*ln(c^(1/3)+d^(1/3)*x)/c^(10/3)/(-a*d+b*c)-1/6
*b^(10/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/(-a*d+b*c)+1/
6*d^(10/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(10/3)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^8 (a+bx^3) (c+dx^3)} \, dx$$

$$= \frac{\frac{12b}{a} - \frac{12d}{c} - \frac{21b^2x^3}{a^2} + \frac{21d^2x^3}{c^2} + \frac{84b^3x^6}{a^3} - \frac{84d^3x^6}{c^3} - \frac{28\sqrt{3}b^{10/3}x^7 \arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{10/3}} + \frac{28\sqrt{3}d^{10/3}x^7 \arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{10/3}} -}{84}$$

input
```
Integrate[1/(x^8*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
((12*b)/a - (12*d)/c - (21*b^2*x^3)/a^2 + (21*d^2*x^3)/c^2 + (84*b^3*x^6)/
a^3 - (84*d^3*x^6)/c^3 - (28*Sqrt[3]*b^(10/3)*x^7*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]])/a^(10/3) + (28*Sqrt[3]*d^(10/3)*x^7*ArcTan[(1 - (2*d^
(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(10/3) - (28*b^(10/3)*x^7*Log[a^(1/3) + b^(1
/3)*x])/a^(10/3) + (28*d^(10/3)*x^7*Log[c^(1/3) + d^(1/3)*x])/c^(10/3) + (
14*b^(10/3)*x^7*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(10/3) -
 (14*d^(10/3)*x^7*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(10/3)
)/(84*(-(b*c) + a*d)*x^7)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {980, 27, 1053, 27, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 (a+bx^3) (c+dx^3)} \, dx$$

$$\Big\downarrow \; 980$$

$$\frac{\int -\frac{7(bdx^3+bc+ad)}{x^5(bx^3+a)(dx^3+c)}dx}{7ac} - \frac{1}{7acx^7}$$

$\downarrow$ 27

$$-\frac{\int \frac{bdx^3+bc+ad}{x^5(bx^3+a)(dx^3+c)}dx}{ac}-\frac{1}{7acx^7}$$

$\downarrow$ 1053

$$-\frac{-\frac{\int \frac{4\left(bd(bc+ad)x^3+b^2c^2+a^2d^2+abcd\right)}{x^2\left(bx^3+a\right)\left(dx^3+c\right)}dx}{4ac}-\frac{ad+bc}{4acx^4}}{ac}-\frac{1}{7acx^7}$$

$\downarrow$ 27

$$-\frac{-\frac{\int \frac{bd(bc+ad)x^3+b^2c^2+a^2d^2+abcd}{x^2\left(bx^3+a\right)\left(dx^3+c\right)}dx}{ac}-\frac{ad+bc}{4acx^4}}{ac}-\frac{1}{7acx^7}$$

$\downarrow$ 1053

$$-\frac{-\frac{-\frac{\int \frac{x\left(bd\left(b^2c^2+abdc+a^2d^2\right)x^3+(bc+ad)\left(b^2c^2+a^2d^2\right)\right)}{\left(bx^3+a\right)\left(dx^3+c\right)}dx}{ac}-\frac{\frac{b^2c}{a}+\frac{ad^2}{c}+bd}{x}}{ac}-\frac{ad+bc}{4acx^4}}{ac}-\frac{1}{7acx^7}$$

$\downarrow$ 1054

$$-\frac{-\frac{-\frac{\int \left(\frac{c^3xb^4}{(bc-ad)\left(bx^3+a\right)}+\frac{a^3d^4x}{(ad-bc)\left(dx^3+c\right)}\right)dx}{ac}-\frac{\frac{b^2c}{a}+\frac{ad^2}{c}+bd}{x}}{ac}-\frac{ad+bc}{4acx^4}}{ac}-\frac{1}{7acx^7}$$

$\downarrow$ 2009

$$-\frac{-\frac{-\frac{\frac{b^{10/3}c^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)}+\frac{a^3d^{10/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)}-\frac{a^3d^{10/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{c}(bc-ad)}+\frac{a^3d^{10/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)}}{ac}}{ac}}{ac}$$

$$\frac{1}{7acx^7}$$

input
```
Int[1/(x^8*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
-1/7*1/(a*c*x^7) - (-1/4*(b*c + a*d)/(a*c*x^4) - (-(((b^2*c)/a + b*d + (a*
d^2)/c)/x) - (-((b^(10/3)*c^3*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(1/3)*(b*c - a*d))) + (a^3*d^(10/3)*ArcTan[(c^(1/3) - 2*
d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(1/3)*(b*c - a*d)) - (b^(10/3)*c
^3*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*(b*c - a*d)) + (a^3*d^(10/3)*Log[c
^(1/3) + d^(1/3)*x])/(3*c^(1/3)*(b*c - a*d)) + (b^(10/3)*c^3*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*(b*c - a*d)) - (a^3*d^(10/3)
*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(1/3)*(b*c - a*d)))/
(a*c))/(a*c))/(a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.79

method	result
default	$-\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)d^4}{c^3(ad-bc)}-\frac{1}{7acx^7}-\frac{-ad-bc}{4a^2c^2x^4}-\frac{a^2d^2+abcd+b^2c^2}{a^3c^3x}+\ldots$
risch	$\frac{-\frac{\left(a^2d^2+abcd+b^2c^2\right)x^6}{c^3a^3}+\frac{(ad+bc)x^3}{4a^2c^2}-\frac{1}{7ac}}{x^7}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^3a^{13}-3c\,d^2a^{12}b+3c^2d\,a^{11}b^2-a^{10}b^3c^3\right)\_Z^3+b^{10}\right)}\_R\ln\left(\left(\left(-4a^{16}c^{10}\ldots\right.\right.\right.\right.$

input
```
int(1/x^8/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3
)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/
3)*x-1)))*d^4/c^3/(a*d-b*c)-1/7/a/c/x^7-1/4*(-a*d-b*c)/a^2/c^2/x^4-(a^2*d^
2+a*b*c*d+b^2*c^2)/a^3/c^3/x+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(
a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b^4/a^3/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^8 (a+bx^3)(c+dx^3)}\,dx = -\frac{28\sqrt{3}b^3c^3x^7\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-28\sqrt{3}a^3d^3x^7\left(\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{d}{c}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)}{}$$

input
```
integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/84*(28*sqrt(3)*b^3*c^3*x^7*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/
3) + 1/3*sqrt(3)) - 28*sqrt(3)*a^3*d^3*x^7*(d/c)^(1/3)*arctan(2/3*sqrt(3)*
x*(d/c)^(1/3) - 1/3*sqrt(3)) - 14*b^3*c^3*x^7*(-b/a)^(1/3)*log(b*x^2 - a*x
*(-b/a)^(2/3) - a*(-b/a)^(1/3)) - 14*a^3*d^3*x^7*(d/c)^(1/3)*log(d*x^2 - c
*x*(d/c)^(2/3) + c*(d/c)^(1/3)) + 28*b^3*c^3*x^7*(-b/a)^(1/3)*log(b*x + a*
(-b/a)^(2/3)) + 28*a^3*d^3*x^7*(d/c)^(1/3)*log(d*x + c*(d/c)^(2/3)) + 84*(
b^3*c^3 - a^3*d^3)*x^6 + 12*a^2*b*c^3 - 12*a^3*c^2*d - 21*(a*b^2*c^3 - a^3
*c*d^2)*x^3)/((a^3*b*c^4 - a^4*c^3*d)*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^8 (a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**8/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^8(a+bx^3)(c+dx^3)}\,dx = -\frac{\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(a^3bc-a^4d\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3}d^3\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^4-ac^3d\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{b^3\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^3bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^4d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} + \frac{d^3\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^4\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^3d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} + \frac{b^3\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a^3bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^4d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} - \frac{d^3\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^4\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^3d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} - \frac{28\left(b^2c^2+abcd+a^2d^2\right)x^6+4\,a^2c^2-7\left(abc^2+a^2cd\right)x^3}{28\,a^3c^3x^7}$$

input
```
integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a^3
*b*c - a^4*d)*(a/b)^(1/3)) + 1/3*sqrt(3)*d^3*arctan(1/3*sqrt(3)*(2*x - (c/
d)^(1/3))/(c/d)^(1/3))/((b*c^4 - a*c^3*d)*(c/d)^(1/3)) - 1/6*b^3*log(x^2 -
 x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*c*(a/b)^(1/3) - a^4*d*(a/b)^(1/3)) +
1/6*d^3*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c^4*(c/d)^(1/3) - a*c^3*
d*(c/d)^(1/3)) + 1/3*b^3*log(x + (a/b)^(1/3))/(a^3*b*c*(a/b)^(1/3) - a^4*d
*(a/b)^(1/3)) - 1/3*d^3*log(x + (c/d)^(1/3))/(b*c^4*(c/d)^(1/3) - a*c^3*d*
(c/d)^(1/3)) - 1/28*(28*(b^2*c^2 + a*b*c*d + a^2*d^2)*x^6 + 4*a^2*c^2 - 7*
(a*b*c^2 + a^2*c*d)*x^3)/(a^3*c^3*x^7)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^8(a+bx^3)(c+dx^3)}\,dx$$
$$= \frac{b^4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3(a^4bc-a^5d)} - \frac{d^4\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3(bc^5-ac^4d)}$$
$$+ \frac{(-ab^2)^{\frac{2}{3}}\,b^2\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^4bc-\sqrt{3}a^5d} - \frac{(-cd^2)^{\frac{2}{3}}\,d^2\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^5-\sqrt{3}ac^4d}$$
$$- \frac{(-ab^2)^{\frac{2}{3}}\,b^2\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(a^4bc-a^5d)}$$
$$+ \frac{(-cd^2)^{\frac{2}{3}}\,d^2\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(bc^5-ac^4d)}$$
$$- \frac{28\,b^2c^2x^6+28\,abcdx^6+28\,a^2d^2x^6-7\,abc^2x^3-7\,a^2cdx^3+4\,a^2c^2}{28\,a^3c^3x^7}$$

input
```
integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*b^4*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b*c - a^5*d) - 1/3*d^
4*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^5 - a*c^4*d) + (-a*b^2)^(2/
3)*b^2*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^4*
b*c - sqrt(3)*a^5*d) - (-c*d^2)^(2/3)*d^2*arctan(1/3*sqrt(3)*(2*x + (-c/d)
^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b*c^5 - sqrt(3)*a*c^4*d) - 1/6*(-a*b^2)^(2/
3)*b^2*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b*c - a^5*d) + 1/6*(-
c*d^2)^(2/3)*d^2*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^5 - a*c^4*d
) - 1/28*(28*b^2*c^2*x^6 + 28*a*b*c*d*x^6 + 28*a^2*d^2*x^6 - 7*a*b*c^2*x^3
 - 7*a^2*c*d*x^3 + 4*a^2*c^2)/(a^3*c^3*x^7)
```

Mupad [B] (verification not implemented)

Time = 10.78 (sec) , antiderivative size = 1814, normalized size of antiderivative = 5.15

$$\int \frac{1}{x^8 (a+bx^3)(c+dx^3)} dx = \text{Too large to display}$$

input
```
int(1/(x^8*(a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(((-b^10/(a^10*(a*d - b*c)^3))^(2/3)*(((27*a^21*b^3*c^21*d^3*x*(a^8*d^8
 + b^8*c^8)*(a*d - b*c)^2 + 27*a^28*b^3*c^28*d^3*(a*d + b*c)*(a*d - b*c)^4
*(-b^10/(a^10*(a*d - b*c)^3))^(2/3))*(-b^10/(a^10*(a*d - b*c)^3))^(1/3))/3
 - 9*a^19*b^14*c^29*d^4 + 9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*
a^29*b^4*c^19*d^14))/9 - a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-b^10/(27*a^1
3*d^3 - 27*a^10*b^3*c^3 + 81*a^11*b^2*c^2*d - 81*a^12*b*c*d^2))^(1/3) + lo
g(((d^10/(c^10*(a*d - b*c)^3))^(2/3)*(((27*a^21*b^3*c^21*d^3*x*(a^8*d^8 +
b^8*c^8)*(a*d - b*c)^2 + 27*a^28*b^3*c^28*d^3*(a*d + b*c)*(a*d - b*c)^4*(d
^10/(c^10*(a*d - b*c)^3))^(2/3))*(d^10/(c^10*(a*d - b*c)^3))^(1/3))/3 - 9*
a^19*b^14*c^29*d^4 + 9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*a^29*
b^4*c^19*d^14))/9 - a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-d^10/(27*b^3*c^13
 - 27*a^3*c^10*d^3 + 81*a^2*b*c^11*d^2 - 81*a*b^2*c^12*d))^(1/3) - (1/(7*a
*c) - (x^3*(a*d + b*c))/(4*a^2*c^2) + (x^6*(a^2*d^2 + b^2*c^2 + a*b*c*d))/
(a^3*c^3))/x^7 - (log(((3^(1/2)*1i + 1)^2*(-b^10/(a^10*(a*d - b*c)^3))^(2/
3)*(((3^(1/2)*1i + 1)*(27*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b
*c)^2 + (27*a^28*b^3*c^28*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4
*(-b^10/(a^10*(a*d - b*c)^3))^(2/3))/4)*(-b^10/(a^10*(a*d - b*c)^3))^(1/3)
)/6 + 9*a^19*b^14*c^29*d^4 - 9*a^20*b^13*c^28*d^5 - 9*a^28*b^5*c^20*d^13 +
 9*a^29*b^4*c^19*d^14))/36 + a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-b^10/(27
*a^13*d^3 - 27*a^10*b^3*c^3 + 81*a^11*b^2*c^2*d - 81*a^12*b*c*d^2))^(1/...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^8 (a+bx^3)(c+dx^3)} dx = \frac{-28d^{\frac{2}{3}}c^{\frac{10}{3}}\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) b^4x^7 + 28b^{\frac{2}{3}}a^{\frac{10}{3}}\sqrt{3}\, atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right) d^4x^7 - 12d^{\frac{5}{3}}c^{\frac{7}{3}}b^{\frac{2}{3}}a^{\frac{10}{3}} + 21d^{\frac{8}{3}}c^{\frac{4}{3}}b^{\frac{2}{3}}a^{\frac{10}{3}}x^3 - \cdots}{}$$

input
```
int(1/x^8/(b*x^3+a)/(d*x^3+c),x)
```

output
```
( - 28*d**(2/3)*c**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*
sqrt(3)))*b**4*c**3*x**7 + 28*b**(2/3)*a**(1/3)*sqrt(3)*atan((c**(1/3) - 2
*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a**3*d**4*x**7 - 12*d**(2/3)*c**(1/3)*b**
(2/3)*a**(1/3)*a**3*c**2*d + 21*d**(2/3)*c**(1/3)*b**(2/3)*a**(1/3)*a**3*c
*d**2*x**3 - 84*d**(2/3)*c**(1/3)*b**(2/3)*a**(1/3)*a**3*d**3*x**6 + 12*d*
*(2/3)*c**(1/3)*b**(2/3)*a**(1/3)*a**2*b*c**3 - 21*d**(2/3)*c**(1/3)*b**(2
/3)*a**(1/3)*a*b**2*c**3*x**3 + 84*d**(2/3)*c**(1/3)*b**(2/3)*a**(1/3)*b**
3*c**3*x**6 - 14*b**(2/3)*a**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d*
*(2/3)*x**2)*a**3*d**4*x**7 + 28*b**(2/3)*a**(1/3)*log(c**(1/3) + d**(1/3)
*x)*a**3*d**4*x**7 + 14*d**(2/3)*c**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)
*x + b**(2/3)*x**2)*b**4*c**3*x**7 - 28*d**(2/3)*c**(1/3)*log(a**(1/3) + b
**(1/3)*x)*b**4*c**3*x**7)/(84*d**(2/3)*c**(1/3)*b**(2/3)*a**(1/3)*a**3*c*
*3*x**7*(a*d - b*c))
```

3.434 $\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 97

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{32c^2\sqrt{c+dx^3}}{3d^3} - \frac{10c(c+dx^3)^{3/2}}{9d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3} - \frac{32c^{5/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^3}$$

output
```
32/3*c^2*(d*x^3+c)^(1/2)/d^3-10/9*c*(d*x^3+c)^(3/2)/d^3+2/15*(d*x^3+c)^(5/
2)/d^3-32/3*c^(5/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*3^(1/2)/d^
3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2\sqrt{c+dx^3}(218c^2-19cdx^3+3d^2x^6)}{45d^3} - \frac{32c^{5/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^3}$$

input
```
Integrate[(x^8*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
(2*Sqrt[c + d*x^3]*(218*c^2 - 19*c*d*x^3 + 3*d^2*x^6))/(45*d^3) - (32*c^(5
/2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6\sqrt{dx^3+c}}{dx^3+4c}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{16\sqrt{dx^3+c}c^2}{d^2\left(dx^3+4c\right)}-\frac{5\sqrt{dx^3+c}c}{d^2}+\frac{\left(dx^3+c\right)^{3/2}}{d^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{32\sqrt{3}c^{5/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{d^3}+\frac{32c^2\sqrt{c+dx^3}}{d^3}-\frac{10c\left(c+dx^3\right)^{3/2}}{3d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{5d^3}\right)$$

input
```
Int[(x^8*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
((32*c^2*Sqrt[c + d*x^3])/d^3 - (10*c*(c + d*x^3)^(3/2))/(3*d^3) + (2*(c +
 d*x^3)^(5/2))/(5*d^3) - (32*Sqrt[3]*c^(5/2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[
3]*Sqrt[c])])/d^3)/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 10.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{-480c^{\frac{5}{2}}\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)+\left(6d^{2}x^{6}-38cd\,x^{3}+436c^{2}\right)\sqrt{d\,x^{3}+c}}{45d^{3}}$
risch	$\frac{2\left(3d^{2}x^{6}-19cd\,x^{3}+218c^{2}\right)\sqrt{d\,x^{3}+c}}{45d^{3}}-\frac{32c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{3d^{3}}$
default	$\frac{\frac{2x^{6}\sqrt{d\,x^{3}+c}}{15}+\frac{2c\,x^{3}\sqrt{d\,x^{3}+c}}{45d}-\frac{4c^{2}\sqrt{d\,x^{3}+c}}{45d^{2}}}{d}-\frac{8c\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9d^{3}}+\frac{16c^{2}\left(2\sqrt{d\,x^{3}+c}-2\sqrt{c}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\right)}{3d^{3}}$
elliptic	$\frac{2x^{6}\sqrt{d\,x^{3}+c}}{15d}-\frac{38c\,x^{3}\sqrt{d\,x^{3}+c}}{45d^{2}}+\frac{436c^{2}\sqrt{d\,x^{3}+c}}{45d^{3}}+\frac{16ic^{2}\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(- \cdots\right.}{\cdots}\right)}{\left(- \cdots\right.}}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
1/45*(-480*c^(5/2)*3^(1/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))+(6*
d^2*x^6-38*c*d*x^3+436*c^2)*(d*x^3+c)^(1/2))/d^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.60

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

$$=\left[\frac{2\left(360\sqrt{\frac{1}{3}}\sqrt{-c}c^{2}\log\left(\frac{dx^{3}-6\sqrt{\frac{1}{3}}\sqrt{dx^{3}+c}\sqrt{-c}-2c}{dx^{3}+4c}\right)+\left(3d^{2}x^{6}-19cdx^{3}+218c^{2}\right)\sqrt{dx^{3}+c}\right)}{45d^{3}},\ \frac{2\left(720\sqrt{\frac{1}{3}}c^{\cdots}\right.}{\cdots}\right.$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
[2/45*(360*sqrt(1/3)*sqrt(-c)*c^2*log((d*x^3 - 6*sqrt(1/3)*sqrt(d*x^3 + c)
*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) + (3*d^2*x^6 - 19*c*d*x^3 + 218*c^2)*sqrt(
d*x^3 + c))/d^3, 2/45*(720*sqrt(1/3)*c^(5/2)*arctan(3*sqrt(1/3)*sqrt(c)/sq
rt(d*x^3 + c)) + (3*d^2*x^6 - 19*c*d*x^3 + 218*c^2)*sqrt(d*x^3 + c))/d^3]
```

Sympy [A] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{16\sqrt{3}c^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}+\frac{16c^2\sqrt{c+dx^3}}{3}-\frac{5c\left(c+dx^3\right)^{\frac{3}{2}}}{9}+\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15}\right)}{d^3} & \text{for } d\neq 0 \\ \frac{x^9}{36\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Piecewise((2*(-16*sqrt(3)*c**(5/2)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c
)))/3 + 16*c**2*sqrt(c + d*x**3)/3 - 5*c*(c + d*x**3)**(3/2)/9 + (c + d*x*
*3)**(5/2)/15)/d**3, Ne(d, 0)), (x**9/(36*sqrt(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.71

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = -\frac{2\left(240\sqrt{3}c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-3\left(dx^3+c\right)^{\frac{5}{2}}+25\left(dx^3+c\right)^{\frac{3}{2}}c-240\sqrt{dx^3+c}c^2\right)}{45\,d^3}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
-2/45*(240*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3
*(d*x^3 + c)^(5/2) + 25*(d*x^3 + c)^(3/2)*c - 240*sqrt(d*x^3 + c)*c^2)/d^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = -\frac{32\sqrt{3}c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3\,d^3} + \frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}d^{12} - 25\left(dx^3+c\right)^{\frac{3}{2}}cd^{12} + 240\sqrt{dx^3+c}c^2d^{12}\right)}{45\,d^{15}}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
-32/3*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^3 + 2/
45*(3*(d*x^3 + c)^(5/2)*d^12 - 25*(d*x^3 + c)^(3/2)*c*d^12 + 240*sqrt(d*x^
3 + c)*c^2*d^12)/d^15
```

Mupad [B] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.12

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{436\,c^2\sqrt{d\,x^3+c}}{45\,d^3} + \frac{2\,x^6\sqrt{d\,x^3+c}}{15\,d} - \frac{38\,c\,x^3\sqrt{d\,x^3+c}}{45\,d^2} + \frac{\sqrt{3}\,c^{5/2}\,\ln\left(\frac{2\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,16\mathrm{i}}{3\,d^3}$$

input
```
int((x^8*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

output
```
(436*c^2*(c + d*x^3)^(1/2))/(45*d^3) + (2*x^6*(c + d*x^3)^(1/2))/(15*d) -
(38*c*x^3*(c + d*x^3)^(1/2))/(45*d^2) + (3^(1/2)*c^(5/2)*log((2*3^(1/2)*c
+ c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*16i)/(3*d^3
)
```

Reduce [F]

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{\frac{76\sqrt{d\,x^3+c}\,c^2}{45} - \frac{38\sqrt{d\,x^3+c}\,cd\,x^3}{45} + \frac{2\sqrt{d\,x^3+c}\,d^2x^6}{15} + 12\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{d^2x^6+5cd\,x^3+4c^2}dx\right)c^2d^2}{d^3}$$

input
```
int(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

output
```
(2*(38*sqrt(c + d*x**3)*c**2 - 19*sqrt(c + d*x**3)*c*d*x**3 + 3*sqrt(c + d
*x**3)*d**2*x**6 + 270*int((sqrt(c + d*x**3)*x**5)/(4*c**2 + 5*c*d*x**3 +
d**2*x**6),x)*c**2*d**2))/(45*d**3)
```

3.435 $\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 76

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx = -\frac{8c\sqrt{c+dx^3}}{3d^2}+\frac{2(c+dx^3)^{3/2}}{9d^2}+\frac{8c^{3/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^2}$$

output
```
-8/3*c*(d*x^3+c)^(1/2)/d^2+2/9*(d*x^3+c)^(3/2)/d^2+8/3*c^(3/2)*arctan(1/3*
(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*3^(1/2)/d^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2(-11c+dx^3)\sqrt{c+dx^3}}{9d^2}+\frac{8c^{3/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^2}$$

input
```
Integrate[(x^5*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
(2*(-11*c + d*x^3)*Sqrt[c + d*x^3])/(9*d^2) + (8*c^(3/2)*ArcTan[Sqrt[c + d
*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {948, 90, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3\sqrt{dx^3+c}}{dx^3+4c}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{3/2}}{3d^2}-\frac{4c\int \frac{\sqrt{dx^3+c}}{dx^3+4c}dx^3}{d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{3/2}}{3d^2}-\frac{4c\left(\frac{2\sqrt{c+dx^3}}{d}-3c\int \frac{1}{\sqrt{dx^3+c}(dx^3+4c)}dx^3\right)}{d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{3/2}}{3d^2}-\frac{4c\left(\frac{2\sqrt{c+dx^3}}{d}-\frac{6c\int \frac{1}{x^6+3c}d\sqrt{dx^3+c}}{d}\right)}{d}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{3/2}}{3d^2}-\frac{4c\left(\frac{2\sqrt{c+dx^3}}{d}-\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{d}\right)}{d}\right)$$

input
```
Int[(x^5*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
((2*(c + d*x^3)^(3/2))/(3*d^2) - (4*c*((2*Sqrt[c + d*x^3])/d - (2*Sqrt[3]*
Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/d))/d)/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{24c^{\frac{3}{2}}\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)-2\sqrt{d\,x^{3}+c}\,\left(-d\,x^{3}+11c\right)}{9d^{2}}$
risch	$-\frac{2\left(-d\,x^{3}+11c\right)\sqrt{d\,x^{3}+c}}{9d^{2}}+\frac{8c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{3d^{2}}$
default	$\frac{2\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9d^{2}}-\frac{4c\left(2\sqrt{d\,x^{3}+c}-2\sqrt{c}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\right)}{3d^{2}}$
elliptic	$\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d}-\frac{22c\sqrt{d\,x^{3}+c}}{9d^{2}}-\frac{4ic\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\,\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{i d\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\cdots}\cdots}{\cdots}\right)}{\cdots}$

input
```
int(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
1/9*(24*c^(3/2)*3^(1/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))-2*(d*x
^3+c)^(1/2)*(-d*x^3+11*c))/d^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.71

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx = \left[\frac{2\left(18\sqrt{\frac{1}{3}}\sqrt{-c}c\log\left(\frac{dx^3+6\sqrt{\frac{1}{3}}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+\sqrt{dx^3+c}(dx^3-11c)\right)}{9d^2}, -\frac{2\left(36\sqrt{\frac{1}{3}}c^{\frac{3}{2}}\arctan\left(\frac{3\sqrt{\frac{1}{3}}\sqrt{c}}{\sqrt{dx^3+c}}\right)-\sqrt{dx^3+c}(dx^3-11c)\right)}{9d^2}\right]$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
[2/9*(18*sqrt(1/3)*sqrt(-c)*c*log((d*x^3 + 6*sqrt(1/3)*sqrt(d*x^3 + c)*sqr
t(-c) - 2*c)/(d*x^3 + 4*c)) + sqrt(d*x^3 + c)*(d*x^3 - 11*c))/d^2, -2/9*(3
6*sqrt(1/3)*c^(3/2)*arctan(3*sqrt(1/3)*sqrt(c)/sqrt(d*x^3 + c)) - sqrt(d*x
^3 + c)*(d*x^3 - 11*c))/d^2]
```

Sympy [A] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx = \begin{cases} \frac{2\cdot\left(\frac{4\sqrt{3}c^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}-\frac{4c\sqrt{c+dx^3}}{3}+\frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d^2} & \text{for } d\neq 0 \\ \frac{x^6}{24\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Piecewise((2*(4*sqrt(3)*c**(3/2)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c))
)/3 - 4*c*sqrt(c + d*x**3)/3 + (c + d*x**3)**(3/2)/9)/d**2, Ne(d, 0)), (x*
*6/(24*sqrt(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2\left(12\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+(dx^3+c)^{\frac{3}{2}}-12\sqrt{dx^3+c}c\right)}{9\,d^2}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
2/9*(12*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + (d*x
^3 + c)^(3/2) - 12*sqrt(d*x^3 + c)*c)/d^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{8\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3\,d^2}+\frac{2\left((dx^3+c)^{\frac{3}{2}}d^4-12\sqrt{dx^3+c}cd^4\right)}{9\,d^6}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
8/3*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^2 + 2/9*
((d*x^3 + c)^(3/2)*d^4 - 12*sqrt(d*x^3 + c)*c*d^4)/d^6
```

Mupad [B] (verification not implemented)

Time = 2.85 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d}-\frac{22\,c\,\sqrt{d\,x^3+c}}{9\,d^2}+\frac{\sqrt{3}\,c^{3/2}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,4\mathrm{i}}{3\,d^2}$$

input
```
int((x^5*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

output
```
(2*x^3*(c + d*x^3)^(1/2))/(9*d) - (22*c*(c + d*x^3)^(1/2))/(9*d^2) + (3^(1
/2)*c^(3/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^
3)/(4*c + d*x^3))*4i)/(3*d^2)
```

Reduce [F]

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{-4\sqrt{d\,x^3+c}\,c+2\sqrt{d\,x^3+c}\,d\,x^3-27\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{d^2x^6+5cd\,x^3+4c^2}dx\right)c\,d^2}{9d^2}$$

input
```
int(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

output
```
( - 4*sqrt(c + d*x**3)*c + 2*sqrt(c + d*x**3)*d*x**3 - 27*int((sqrt(c + d*
x**3)*x**5)/(4*c**2 + 5*c*d*x**3 + d**2*x**6),x)*c*d**2)/(9*d**2)
```

3.436 $\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx=\frac{2\sqrt{c+dx^3}}{3d}-\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d}$$

output
```
2/3*(d*x^3+c)^(1/2)/d-2/3*c^(1/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/
2))*3^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx=\frac{2\left(\sqrt{c+dx^3}-\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)\right)}{3d}$$

input
```
Integrate[(x^2*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
(2*(Sqrt[c + d*x^3] - Sqrt[3]*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt
[c])]))/(3*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {946, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{dx^3+4c}dx^3$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{d}-3c\int \frac{1}{\sqrt{dx^3+c}\,(dx^3+4c)}dx^3\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{d}-\frac{6c\int \frac{1}{x^6+3c}d\sqrt{dx^3+c}}{d}\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{d}-\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{d}\right)$$

input
```
Int[(x^2*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
((2*Sqrt[c + d*x^3])/d - (2*Sqrt[3]*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Sqrt[c])])/d)/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result
default	$\frac{2\sqrt{d\,x^{3}+c}-2\sqrt{c}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)}{3d}$
pseudoelliptic	$\frac{2\sqrt{d\,x^{3}+c}-2\sqrt{c}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)}{3d}$
risch	$\frac{2\sqrt{d\,x^{3}+c}}{3d}-\frac{2\sqrt{c}\,\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{3d}$
elliptic	$\frac{2\sqrt{d\,x^{3}+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{i d\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right.}}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(2*(d*x^3+c)^(1/2)-2*c^(1/2)*3^(1/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1
/2)/c^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.93

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx = \left[\frac{3\,\sqrt{\frac{1}{3}}\sqrt{-c}\log\left(\frac{dx^3-6\,\sqrt{\frac{1}{3}}\sqrt{dx^3+c}\sqrt{-c}-2\,c}{dx^3+4\,c}\right)+2\,\sqrt{dx^3+c}}{3\,d},\ \frac{2\left(3\,\sqrt{\frac{1}{3}}\sqrt{c}\arctan\left(\frac{3\,\sqrt{\frac{1}{3}}\sqrt{c}}{\sqrt{dx^3+c}}\right)+\sqrt{dx^3+c}\right)}{3\,d}\right]$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
[1/3*(3*sqrt(1/3)*sqrt(-c)*log((d*x^3 - 6*sqrt(1/3)*sqrt(d*x^3 + c)*sqrt(-
c) - 2*c)/(d*x^3 + 4*c)) + 2*sqrt(d*x^3 + c))/d, 2/3*(3*sqrt(1/3)*sqrt(c)*
arctan(3*sqrt(1/3)*sqrt(c)/sqrt(d*x^3 + c)) + sqrt(d*x^3 + c))/d]
```

Sympy [A] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{\sqrt{3}\sqrt{c}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}+\frac{\sqrt{c+dx^3}}{3}\right)}{d} & \text{for } d\neq 0 \\ \frac{x^3}{12\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Piecewise((2*(-sqrt(3)*sqrt(c)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/
3 + sqrt(c + d*x**3)/3)/d, Ne(d, 0)), (x**3/(12*sqrt(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx = -\frac{2\left(\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-\sqrt{dx^3+c}\right)}{3\,d}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
-2/3*(sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - sqrt(d
*x^3 + c))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx = -\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3\,d}+\frac{2\sqrt{dx^3+c}}{3\,d}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
-2/3*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d + 2/3*s
qrt(d*x^3 + c)/d
```

Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,d}+\frac{\sqrt{3}\,\sqrt{c}\,\ln\left(\frac{2\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,1\mathrm{i}}{3\,d}$$

input
```
int((x^2*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*d) + (3^(1/2)*c^(1/2)*log((2*3^(1/2)*c + c^(1/2)*
(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(3*d)
```

Reduce [F]

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2\sqrt{d\,x^3+c}+9\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{d^2x^6+5cd\,x^3+4c^2}dx\right)d^2}{12d}$$

input
```
int(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

output
```
(2*sqrt(c + d*x**3) + 9*int((sqrt(c + d*x**3)*x**5)/(4*c**2 + 5*c*d*x**3 +
 d**2*x**6),x)*d**2)/(12*d)
```

3.437 $\int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 65

$$\int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)}\,dx = \frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

output
```
1/6*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*3^(1/2)/c^(1/2)-1/6*arctan
h((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right) - \operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x*(4*c + d*x^3)),x]
```

output
```
(Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])] - ArcTanh[Sqrt[c + d*x^
3]/Sqrt[c]])/(6*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {948, 94, 73, 216, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x\left(4c+dx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^3\left(dx^3+4c\right)}dx^3$$

$$\Big\downarrow \text{94}$$

$$\frac{1}{3}\left(\frac{1}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{3}{4}d\int \frac{1}{\sqrt{dx^3+c}\left(dx^3+4c\right)}dx^3\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+3c}d\sqrt{dx^3+c}+\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{c}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x*(4*c + d*x^3)),x]
```

output
```
((Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(2*Sqrt[c]) - ArcTanh
[Sqrt[c + d*x^3]/Sqrt[c]]/(2*Sqrt[c]))/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}-\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6\sqrt{c}}$	45
default	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}}{4c}-\frac{2\sqrt{d\,x^{3}+c}-2\sqrt{c}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)}{12c}$	81
elliptic	Expression too large to display	1502

input
```
int((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*3^(1/2)-arctanh((d*x^3+c)
^(1/2)/c^(1/2)))/c^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)}\,dx = \left[-\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{c}}{\sqrt{dx^3+c}}\right)-\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{12\,c}, -\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)-2\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)}{12\,c}\right]$$

input
```
integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
[-1/12*(2*sqrt(3)*sqrt(c)*arctan(sqrt(3)*sqrt(c)/sqrt(d*x^3 + c)) - sqrt(c
)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c, -1/12*(sqrt(3)*sq
rt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c
)) - 2*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)))/c]
```

Sympy [A] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)}\,dx = \begin{cases} \frac{2\left(\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12\sqrt{-c}}+\frac{\sqrt{3}d\,\mathrm{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{12\sqrt{c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{\log(x^3)}{12\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**3+c)**(1/2)/x/(d*x**3+4*c),x)
```

output
```
Piecewise((2*(d*atan(sqrt(c + d*x**3)/sqrt(-c))/(12*sqrt(-c)) + sqrt(3)*d*
atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/(12*sqrt(c)))/d, Ne(d, 0)), (lo
g(x**3)/(12*sqrt(c)), True))
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x\,(4c+dx^3)}\,dx = \int \frac{\sqrt{dx^3+c}}{(dx^3+4\,c)x}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{c+dx^3}}{x\,(4c+dx^3)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{6\,\sqrt{c}} + \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6\,\sqrt{-c}}$$

input
```
integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/sqrt(c) + 1/6*arct
an(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c)
```

Mupad [B] (verification not implemented)

Time = 3.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{12\,\sqrt{c}} + \frac{\sqrt{3}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,1\mathrm{i}}{12\,\sqrt{c}}$$

input
```
int((c + d*x^3)^(1/2)/(x*(4*c + d*x^3)),x)
```

output
```
log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(
12*c^(1/2)) + (3^(1/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3
^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(12*c^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)}\,dx = \frac{\sqrt{c}\log\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)-\sqrt{c}\log\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)+9\left(\int \frac{\sqrt{d\,x^3+c}\,x^2}{d^2x^6+5cd\,x^3+4c^2}dx\right)cd}{12c}$$

input
```
int((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x)
```

output
```
(sqrt(c)*log(sqrt(c + d*x**3) - sqrt(c)) - sqrt(c)*log(sqrt(c + d*x**3) +
sqrt(c)) + 9*int((sqrt(c + d*x**3)*x**2)/(4*c**2 + 5*c*d*x**3 + d**2*x**6)
,x)*c*d)/(12*c)
```

3.438 $\int \frac{\sqrt{c+dx^3}}{x^4(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{\sqrt{c+dx^3}}{x^4(4c+dx^3)}\,dx=-\frac{\sqrt{c+dx^3}}{12cx^3}-\frac{d\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}c^{3/2}}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}$$

output
```
-1/12*(d*x^3+c)^(1/2)/c/x^3-1/24*d*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1
/2))*3^(1/2)/c^(3/2)-1/24*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+dx^3}}{x^4(4c+dx^3)}\,dx=-\frac{\sqrt{c+dx^3}}{12cx^3}-\frac{d\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}c^{3/2}}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^4*(4*c + d*x^3)),x]
```

output
```
-1/12*Sqrt[c + d*x^3]/(c*x^3) - (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c]
)])/(8*Sqrt[3]*c^(3/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(24*c^(3/2)
)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {948, 110, 27, 174, 73, 216, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(4c+dx^3\right)}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^6\left(dx^3+4c\right)}dx^3$$

$$\Big\downarrow \; 110$$

$$\frac{1}{3}\left(\frac{\int \frac{d(2c-dx^3)}{2x^3\sqrt{dx^3+c}(dx^3+4c)}dx^3}{4c}-\frac{\sqrt{c+dx^3}}{4cx^3}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{d\int \frac{2c-dx^3}{x^3\sqrt{dx^3+c}(dx^3+4c)}dx^3}{8c}-\frac{\sqrt{c+dx^3}}{4cx^3}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{3}\left(\frac{d\left(\frac{1}{2}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3-\frac{3}{2}d\int \frac{1}{\sqrt{dx^3+c}(dx^3+4c)}dx^3\right)}{8c}-\frac{\sqrt{c+dx^3}}{4cx^3}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{d\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{d}-3\int \frac{1}{x^6+3c}d\sqrt{dx^3+c}\right)}{8c}-\frac{\sqrt{c+dx^3}}{4cx^3}\right)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{3}\left(\frac{d\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{d}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{c}}\right)}{8c}-\frac{\sqrt{c+dx^3}}{4cx^3}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(\frac{d\left(-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{8c}-\frac{\sqrt{c+dx^3}}{4cx^3}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x^4*(4*c + d*x^3)),x]
```

output
```
(-1/4*Sqrt[c + d*x^3]/(c*x^3) + (d*(-((Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqr
t[3]*Sqrt[c])])/Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/Sqrt[c]))/(8*c
))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}}{12c\,x^{3}}-\frac{d\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{24c^{\frac{3}{2}}}-\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{24c^{\frac{3}{2}}}$
pseudoelliptic	$-\frac{\sqrt{d\,x^{3}+c}}{12c\,x^{3}}-\frac{d\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{24c^{\frac{3}{2}}}-\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{24c^{\frac{3}{2}}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3x^{3}}-\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}}{4c}-\frac{d\left(\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}\right)}{16c^{2}}+\frac{d\left(2\sqrt{d\,x^{3}+c}-2\sqrt{c}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^{3}+}}{3\sqrt{}}\right.\right.}{48c^{2}}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(d*x^3+c)^(1/2)/c/x^3-1/24*d*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1
/2))*3^(1/2)/c^(3/2)-1/24*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.15

$$\int \frac{\sqrt{c+dx^3}}{x^4\,(4c+dx^3)}\,dx = \left[\frac{2\,\sqrt{3}\sqrt{c}dx^3\arctan\left(\frac{\sqrt{3}\sqrt{c}}{\sqrt{dx^3+c}}\right)+\sqrt{c}dx^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-4\,\sqrt{dx^3+c}c}{48\,c^2x^3},\right.$$
$$\left.-\frac{\sqrt{3}\sqrt{-c}dx^3\log\left(\frac{dx^3+2\,\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2\,c}{dx^3+4\,c}\right)-2\,\sqrt{-c}dx^3\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)+4\,\sqrt{dx^3+c}c}{48\,c^2x^3}\right]$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
[1/48*(2*sqrt(3)*sqrt(c)*d*x^3*arctan(sqrt(3)*sqrt(c)/sqrt(d*x^3 + c)) + s
qrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 4*sqrt(d
*x^3 + c)*c)/(c^2*x^3), -1/48*(sqrt(3)*sqrt(-c)*d*x^3*log((d*x^3 + 2*sqrt(
3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) - 2*sqrt(-c)*d*x^3*arcta
n(sqrt(-c)/sqrt(d*x^3 + c)) + 4*sqrt(d*x^3 + c)*c)/(c^2*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(4c+dx^3\right)}\,dx=\int \frac{\sqrt{c+dx^3}}{x^4\cdot\left(4c+dx^3\right)}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**4/(d*x**3+4*c),x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**4*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(4c+dx^3\right)}\,dx=\int \frac{\sqrt{dx^3+c}}{(dx^3+4\,c)x^4}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{c+dx^3}}{x^4(4c+dx^3)}\,dx = -\frac{\sqrt{3}d\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{24\,c^{\frac{3}{2}}} + \frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{24\sqrt{-c}c} - \frac{\sqrt{dx^3+c}}{12\,cx^3}$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="giac")
```

output
```
-1/24*sqrt(3)*d*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(3/2) + 1/24
*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/12*sqrt(d*x^3 + c)/(c
*x^3)
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{c+dx^3}}{x^4(4c+dx^3)}\,dx = \frac{d\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{48\,c^{3/2}} - \frac{\sqrt{d\,x^3+c}}{12\,c\,x^3} + \frac{\sqrt{3}\,d\,\ln\left(\frac{2\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,1\mathrm{i}}{48\,c^{3/2}}$$

input
```
int((c + d*x^3)^(1/2)/(x^4*(4*c + d*x^3)),x)
```

output
```
(d*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6
))/(48*c^(3/2)) - (c + d*x^3)^(1/2)/(12*c*x^3) + (3^(1/2)*d*log((2*3^(1/2)
*c + c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(48*
c^(3/2))
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(4c+dx^3\right)}\,dx$$
$$= \frac{-32\sqrt{d\,x^3+c}\,c+6\sqrt{d\,x^3+c}\,d\,x^3+8\sqrt{c}\log\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)d\,x^3-8\sqrt{c}\log\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)d\,x^3-9}{384c^2x^3}$$

input
```
int((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x)
```

output
```
( - 32*sqrt(c + d*x**3)*c + 6*sqrt(c + d*x**3)*d*x**3 + 8*sqrt(c)*log(sqrt
(c + d*x**3) - sqrt(c))*d*x**3 - 8*sqrt(c)*log(sqrt(c + d*x**3) + sqrt(c))
*d*x**3 - 9*int((sqrt(c + d*x**3)*x**5)/(4*c**2 + 5*c*d*x**3 + d**2*x**6),
x)*d**3*x**3 - 108*int((sqrt(c + d*x**3)*x**2)/(4*c**2 + 5*c*d*x**3 + d**2
*x**6),x)*c*d**2*x**3)/(384*c**2*x**3)
```

3.439 $\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 689

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{50c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{2\sqrt[3]{2}c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{\sqrt{3}d^{5/3}} + \frac{2\sqrt[3]{2}c^{7/6}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^{5/3}} - \frac{2\sqrt[3]{2}c^{7/6}\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} + \frac{2\sqrt[3]{2}c^{7/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3d^{5/3}} + \frac{25\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{50\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
2/7*x^2*(d*x^3+c)^(1/2)/d-50/7*c*(d*x^3+c)^(1/2)/d^(5/3)/((1+3^(1/2))*c^(1
/3)+d^(1/3)*x)-2/3*2^(1/3)*c^(7/6)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+2^(1/3)
*d^(1/3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/d^(5/3)+2/3*2^(1/3)*c^(7/6)*arctan(1/
3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*3^(1/2)/d^(5/3)-2*2^(1/3)*c^(7/6)*arcta
nh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))/d^(5/3)+2/3*2^(1/3
)*c^(7/6)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)+25/7*3^(1/4)*(1/2*6^(1/
2)-1/2*2^(1/2))*c^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^
(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))
*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/d^(5/3)
/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*
x^3+c)^(1/2)-50/21*2^(1/2)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d
^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF((
(1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2
*I)*3^(3/4)/d^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1
/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.51 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.19

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{8x^2(c+dx^3)-8cx^2\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-5dx^5\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\right.}{28d\sqrt{c+dx^3}}$$

input
```
Integrate[(x^4*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
(8*x^2*(c + d*x^3) - 8*c*x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3
, -((d*x^3)/c), -1/4*(d*x^3)/c] - 5*d*x^5*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3
, 1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(28*d*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 691, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {978, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}dx$$

$$\Big\downarrow \text{978}$$

$$\frac{2x^2\sqrt{c+dx^3}}{7d}-\frac{2\int \frac{cx(25dx^3+16c)}{2\sqrt{dx^3+c}(dx^3+4c)}dx}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x^2\sqrt{c+dx^3}}{7d}-\frac{c\int \frac{x(25dx^3+16c)}{\sqrt{dx^3+c}(dx^3+4c)}dx}{7d}$$

$$\Big\downarrow \text{1054}$$

$$\frac{2x^2\sqrt{c+dx^3}}{7d}-\frac{c\int \left(\frac{25x}{\sqrt{dx^3+c}}-\frac{84cx}{\sqrt{dx^3+c}(dx^3+4c)}\right)dx}{7d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x^2\sqrt{c+dx^3}}{7d}-$$

$$c\left(\frac{50\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{25\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{\right.}}{}\right.$$

input
```
Int[(x^4*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
(2*x^2*Sqrt[c + d*x^3])/(7*d) - (c*((50*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)) + (14*2^(1/3)*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)
*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) - (14*
2^(1/3)*c^(1/6)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^(2/3
)) + (14*2^(1/3)*c^(1/6)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/S
qrt[c + d*x^3]])/d^(2/3) - (14*2^(1/3)*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/Sqr
t[c]])/(3*d^(2/3)) - (25*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1
/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1
/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
 d*x^3]) + (50*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/
3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(7*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.88 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.26

method	result	size
elliptic	Expression too large to display	867
risch	Expression too large to display	872
default	Expression too large to display	1309

input
```
int(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
2/7*x^2*(d*x^3+c)^(1/2)/d+50/21*I*c/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),
(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)))-4/3*I*c/d^4*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d
*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^
(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^
2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/
2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*
3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2442 vs. $2(488) = 976$.

Time = 3.01 (sec) , antiderivative size = 2442, normalized size of antiderivative = 3.54

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = \text{Too large to display}$$

input

```
integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output

```
1/42*(12*sqrt(d*x^3 + c)*d*x^2 - 14*(4/27)^(1/6)*d^2*(-c^7/d^10)^(1/6)*log
(32*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c^3*d^
8)*(-c^7/d^10)^(5/6) - 96*sqrt(1/3)*(c^3*d^7*x^7 - c^4*d^6*x^4 - 2*c^5*d^5
*x)*sqrt(-c^7/d^10) + 4*(9*4^(2/3)*c^2*d^8*x^5*(-c^7/d^10)^(2/3) + 2*c^6*d
^2*x^7 - 32*c^7*d*x^4 - 16*c^8*x + 4^(1/3)*(5*c^4*d^5*x^6 - 20*c^5*d^4*x^3
 - 16*c^6*d^3)*(-c^7/d^10)^(1/3))*sqrt(d*x^3 + c) - 24*(4/27)^(1/6)*(c^5*d
^4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2)*(-c^7/d^10)^(1/6))/(d^3*x^9 + 12*c
*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 14*(4/27)^(1/6)*d^2*(-c^7/d^10)^(1/6)
*log(-32*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c
^3*d^8)*(-c^7/d^10)^(5/6) - 96*sqrt(1/3)*(c^3*d^7*x^7 - c^4*d^6*x^4 - 2*c^
5*d^5*x)*sqrt(-c^7/d^10) - 4*(9*4^(2/3)*c^2*d^8*x^5*(-c^7/d^10)^(2/3) + 2*
c^6*d^2*x^7 - 32*c^7*d*x^4 - 16*c^8*x + 4^(1/3)*(5*c^4*d^5*x^6 - 20*c^5*d^
4*x^3 - 16*c^6*d^3)*(-c^7/d^10)^(1/3))*sqrt(d*x^3 + c) - 24*(4/27)^(1/6)*(
c^5*d^4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2)*(-c^7/d^10)^(1/6))/(d^3*x^9 +
 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 300*c*sqrt(d)*weierstrassZeta(0,
 -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 7*(4/27)^(1/6)*(sqrt(-3)*d^2
 - d^2)*(-c^7/d^10)^(1/6)*log(32*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6
 - 72*c^2*d^9*x^3 - 32*c^3*d^8 + sqrt(-3)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c
^2*d^9*x^3 - 32*c^3*d^8))*(-c^7/d^10)^(5/6) + 192*sqrt(1/3)*(c^3*d^7*x^7 -
 c^4*d^6*x^4 - 2*c^5*d^5*x)*sqrt(-c^7/d^10) + 4*(4*c^6*d^2*x^7 - 64*c^7...
```

Sympy [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx=\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

input
```
integrate(x**4*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Integral(x**4*sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx=\int \frac{\sqrt{dx^3+c}x^4}{dx^3+4\,c}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 + 4*c), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx=\int \frac{\sqrt{dx^3+c}x^4}{dx^3+4\,c}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 + 4*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx=\int \frac{x^4\,\sqrt{d\,x^3+c}}{d\,x^3+4\,c} \,dx$$

input
```
int((x^4*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

output
```
int((x^4*(c + d*x^3)^(1/2))/(4*c + d*x^3), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx=\frac{2\sqrt{d\,x^3+c}\,x^2-25\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{d^2x^6+5cd\,x^3+4c^2}dx\right)cd-16\left(\int \frac{\sqrt{d\,x^3+c}\,x}{d^2x^6+5cd\,x^3+4c^2}dx\right)c^2}{7d}$$

input
```
int(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

output
```
(2*sqrt(c + d*x**3)*x**2 - 25*int((sqrt(c + d*x**3)*x**4)/(4*c**2 + 5*c*d*
x**3 + d**2*x**6),x)*c*d - 16*int((sqrt(c + d*x**3)*x)/(4*c**2 + 5*c*d*x**
3 + d**2*x**6),x)*c**2)/(7*d)
```

3.440 $\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 659

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}\sqrt{3}d^{2/3}}$$
$$- \frac{\sqrt[6]{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2^{2/3}\sqrt{3}d^{2/3}} + \frac{\sqrt[6]{c}\,\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}d^{2/3}} - \frac{\sqrt[6]{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3\ 2^{2/3}d^{2/3}}$$
$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+ \frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
2*(d*x^3+c)^(1/2)/d^(2/3)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)+1/6*c^(1/6)*arct
an(3^(1/2)*c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*2^(1/3)*3^
(1/2)/d^(2/3)-1/6*c^(1/6)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*2^(1
/3)*3^(1/2)/d^(2/3)+1/2*c^(1/6)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x
)/(d*x^3+c)^(1/2))*2^(1/3)/d^(2/3)-1/6*c^(1/6)*arctanh((d*x^3+c)^(1/2)/c^(
1/2))*2^(1/3)/d^(2/3)-3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*c^(1/3)*(c^(1/3)+d
^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^
(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c
^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/d^(2/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+
3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+2/3*2^(1/2)*c^(1/3)*(
c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c
^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3
^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/d^(2/3)/(c^(1/3)*(c^(1/3
)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.78 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.10

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx=\frac{x^2\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{8\sqrt{c+dx^3}}$$

input
```
Integrate[(x*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
(x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, -1/2, 1, 5/3, -((d*x^3)/c), -1/4*(d
*x^3)/c])/(8*Sqrt[c + d*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.15 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {984, 832, 759, 986, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

$$\Big\downarrow \text{984}$$

$$\int \frac{x}{\sqrt{dx^3+c}}dx-3c\int \frac{x}{\sqrt{dx^3+c}\,(dx^3+4c)}dx$$

$$\Big\downarrow \text{832}$$

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}\int \frac{1}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}+\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-3c\int \frac{x}{\sqrt{dx^3+c}\,(dx^3+4c)}dx$$

$$\Big\downarrow \text{759}$$

$$\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-3c\int \frac{x}{\sqrt{dx^3+c}\,(dx^3+4c)}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{986}$$

$$\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-$$

$$\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$3c\left(-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}\right.$$

$\downarrow$ 2416

$$-\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}-$$

$$3c\left(-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}\right.$$

input
```
Int[(x*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
-3*c*(-1/3*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c +
 d*x^3]]/(2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTan[Sqrt[c + d*x^3]/(Sqrt[
3]*Sqrt[c])]/(3*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) - ArcTanh[(c^(1/6)*(c^(1/
3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]]/(3*2^(2/3)*c^(5/6)*d^(2/3)) + Ar
cTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(9*2^(2/3)*c^(5/6)*d^(2/3))) + ((2*Sqrt[c +
 d*x^3])/(d^(1/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (3^(1/4)*Sqrt[2 -
 Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x
+ d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1
 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7
- 4*Sqrt[3]])/(d^(1/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/d^(1/3) - (2*(1 - Sqrt[3])*Sqrt
[2 + Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 986
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.02 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.29

method	result	size
default	Expression too large to display	848
elliptic	Expression too large to display	848

input
```
int(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1
/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/d^3*2^(
1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(
-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-
(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha
*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2202 vs. $2(470) = 940$.

Time = 1.07 (sec) , antiderivative size = 2202, normalized size of antiderivative = 3.34

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
-1/12*((1/432)^(1/6)*(sqrt(-3)*d - d)*(-c/d^4)^(1/6)*log(1/2*(36*(1/432)^(
5/6)*(d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 + sqrt(-3)*(d^6
*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3))*(-c/d^4)^(5/6) + 24*sq
rt(1/3)*(c*d^4*x^7 - c^2*d^3*x^4 - 2*c^3*d^2*x)*sqrt(-c/d^4) + (2*c*d^2*x^
7 - 32*c^2*d*x^4 - 16*c^3*x + 18*(1/2)^(2/3)*(sqrt(-3)*c*d^4*x^5 - c*d^4*x
^5)*(-c/d^4)^(2/3) - (1/2)^(1/3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d
+ sqrt(-3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d))*(-c/d^4)^(1/3))*sqrt
(d*x^3 + c) - 6*(1/432)^(1/6)*(c*d^3*x^8 - 7*c^2*d^2*x^5 - 8*c^3*d*x^2 - s
qrt(-3)*(c*d^3*x^8 - 7*c^2*d^2*x^5 - 8*c^3*d*x^2))*(-c/d^4)^(1/6))/(d^3*x^
9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - (1/432)^(1/6)*(sqrt(-3)*d - d
)*(-c/d^4)^(1/6)*log(-1/2*(36*(1/432)^(5/6)*(d^6*x^9 - 66*c*d^5*x^6 - 72*c
^2*d^4*x^3 - 32*c^3*d^3 + sqrt(-3)*(d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^
3 - 32*c^3*d^3))*(-c/d^4)^(5/6) + 24*sqrt(1/3)*(c*d^4*x^7 - c^2*d^3*x^4 -
2*c^3*d^2*x)*sqrt(-c/d^4) - (2*c*d^2*x^7 - 32*c^2*d*x^4 - 16*c^3*x + 18*(1
/2)^(2/3)*(sqrt(-3)*c*d^4*x^5 - c*d^4*x^5)*(-c/d^4)^(2/3) - (1/2)^(1/3)*(5
*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d + sqrt(-3)*(5*c*d^3*x^6 - 20*c^2*d^
2*x^3 - 16*c^3*d))*(-c/d^4)^(1/3))*sqrt(d*x^3 + c) - 6*(1/432)^(1/6)*(c*d^
3*x^8 - 7*c^2*d^2*x^5 - 8*c^3*d*x^2 - sqrt(-3)*(c*d^3*x^8 - 7*c^2*d^2*x^5
- 8*c^3*d*x^2))*(-c/d^4)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 6
4*c^3)) - (1/432)^(1/6)*(sqrt(-3)*d + d)*(-c/d^4)^(1/6)*log(1/2*(36*(1/...
```

Sympy [F]

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx=\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

input
```
integrate(x*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Integral(x*sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{dx^3+cx}}{dx^3+4\,c}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x/(d*x^3 + 4*c), x)
```

Giac [F]

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{dx^3+cx}}{dx^3+4\,c}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x/(d*x^3 + 4*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{x\,\sqrt{d\,x^3+c}}{d\,x^3+4\,c}\,dx$$

input
```
int((x*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

output
```
int((x*(c + d*x^3)^(1/2))/(4*c + d*x^3), x)
```

Reduce [F]

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x}{d\,x^3+4c}dx$$

input
```
int(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

output
```
int((sqrt(c + d*x**3)*x)/(4*c + d*x**3),x)
```

3.441 $\int \frac{\sqrt{c+dx^3}}{x^2(4c+dx^3)} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 697

$$\int \frac{\sqrt{c+dx^3}}{x^2(4c+dx^3)}\,dx$$

$$= -\frac{\sqrt{c+dx^3}}{4cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{4c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4\,2^{2/3}\sqrt{3}c^{5/6}}$$

$$+ \frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{4\,2^{2/3}\sqrt{3}c^{5/6}} - \frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4\,2^{2/3}c^{5/6}} + \frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\,2^{2/3}c^{5/6}}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{8c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{2\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-1/4*(d*x^3+c)^(1/2)/c/x+1/4*d^(1/3)*(d*x^3+c)^(1/2)/c/((1+3^(1/2))*c^(1/3
)+d^(1/3)*x)-1/24*d^(1/3)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*
x)/(d*x^3+c)^(1/2))*2^(1/3)*3^(1/2)/c^(5/6)+1/24*d^(1/3)*arctan(1/3*(d*x^3
+c)^(1/2)*3^(1/2)/c^(1/2))*2^(1/3)*3^(1/2)/c^(5/6)-1/8*d^(1/3)*arctanh(c^(
1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*2^(1/3)/c^(5/6)+1/24*d^(
1/3)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*2^(1/3)/c^(5/6)-1/8*3^(1/4)*(1/2*6^(
1/2)-1/2*2^(1/2))*d^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+
d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2
))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/c^(2/
3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(
d*x^3+c)^(1/2)+1/12*d^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*
x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1
/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*2^(
1/2)*3^(3/4)/c^(2/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(
1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{c+dx^3}}{x^2\,(4c+dx^3)}\,dx$$
$$= \frac{-40c(c+dx^3)+25cdx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+d^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\right.}{160c^2x\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^2*(4*c + d*x^3)),x]
```

output
```
(-40*c*(c + d*x^3) + 25*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1,
5/3, -((d*x^3)/c), -1/4*(d*x^3)/c] + d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[
5/3, 1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(160*c^2*x*Sqrt[c + d*x^3
])
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 690, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {975, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^2(4c+dx^3)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{dx(dx^3+10c)}{2\sqrt{dx^3+c}(dx^3+4c)}dx}{4c}-\frac{\sqrt{c+dx^3}}{4cx}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{x(dx^3+10c)}{\sqrt{dx^3+c}(dx^3+4c)}dx}{8c}-\frac{\sqrt{c+dx^3}}{4cx}$$

$$\Big\downarrow \text{1054}$$

$$\frac{d\int \left(\frac{x}{\sqrt{dx^3+c}}+\frac{6cx}{\sqrt{dx^3+c}(dx^3+4c)}\right)dx}{8c}-\frac{\sqrt{c+dx^3}}{4cx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\cdots}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{\sqrt{c+dx^3}}{4cx}$$

input
```
Int[Sqrt[c + d*x^3]/(x^2*(4*c + d*x^3)),x]
```

output
```
-1/4*Sqrt[c + d*x^3]/(c*x) + (d*((2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)) - (2^(1/3)*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/
3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) + (2^(1/3)*c^
(1/6)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^(2/3)) - (2^(1
/3)*c^(1/6)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3
]])/d^(2/3) + (2^(1/3)*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*d^(2/3
)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3
) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2
]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1
/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[
2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqr
t[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(8*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.80 (sec) , antiderivative size = 868, normalized size of antiderivative = 1.25

method	result	size
elliptic	Expression too large to display	868
risch	Expression too large to display	872
default	Expression too large to display	1306

input
```
int((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(d*x^3+c)^(1/2)/c/x-1/12*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*
((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1
/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2)))-1/12*I/d^2/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*
(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)
*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-
c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/
2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_al...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2253 vs. $2(490) = 980$.

Time = 0.71 (sec) , antiderivative size = 2253, normalized size of antiderivative = 3.23

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
1/48*(2*(1/432)^(1/6)*c*x*(-d^2/c^5)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6 - 7
2*c^2*d^2*x^3 - 32*c^3*d + 48*(1/2)^(2/3)*(c^4*d^2*x^7 - c^5*d*x^4 - 2*c^6
*x)*(-d^2/c^5)^(2/3) + 12*(1/2)^(1/3)*(c^2*d^3*x^8 - 7*c^3*d^2*x^5 - 8*c^4
*d*x^2)*(-d^2/c^5)^(1/3) + 6*(1296*(1/432)^(5/6)*c^5*d*x^5*(-d^2/c^5)^(5/6
) + sqrt(1/3)*(5*c^3*d^2*x^6 - 20*c^4*d*x^3 - 16*c^5)*sqrt(-d^2/c^5) + 2*(
1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-d^2/c^5)^(1/6))*sq
rt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 2*(1/43
2)^(1/6)*c*x*(-d^2/c^5)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6 - 72*c^2*d^2*x^3
 - 32*c^3*d + 48*(1/2)^(2/3)*(c^4*d^2*x^7 - c^5*d*x^4 - 2*c^6*x)*(-d^2/c^5
)^(2/3) + 12*(1/2)^(1/3)*(c^2*d^3*x^8 - 7*c^3*d^2*x^5 - 8*c^4*d*x^2)*(-d^2
/c^5)^(1/3) - 6*(1296*(1/432)^(5/6)*c^5*d*x^5*(-d^2/c^5)^(5/6) + sqrt(1/3)
*(5*c^3*d^2*x^6 - 20*c^4*d*x^3 - 16*c^5)*sqrt(-d^2/c^5) + 2*(1/432)^(1/6)*
(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-d^2/c^5)^(1/6))*sqrt(d*x^3 + c)
)/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 12*sqrt(d)*x*weierst
rassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - (1/432)^(1/6)*(sq
rt(-3)*c*x + c*x)*(-d^2/c^5)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6 - 72*c^2*d^
2*x^3 - 32*c^3*d - 24*(1/2)^(2/3)*(c^4*d^2*x^7 - c^5*d*x^4 - 2*c^6*x + sqr
t(-3)*(c^4*d^2*x^7 - c^5*d*x^4 - 2*c^6*x))*(-d^2/c^5)^(2/3) - 6*(1/2)^(1/3
)*(c^2*d^3*x^8 - 7*c^3*d^2*x^5 - 8*c^4*d*x^2 - sqrt(-3)*(c^2*d^3*x^8 - 7*c
^3*d^2*x^5 - 8*c^4*d*x^2))*(-d^2/c^5)^(1/3) + 6*sqrt(d*x^3 + c)*(648*(1...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}\,dx=\int \frac{\sqrt{c+dx^3}}{x^2\cdot\left(4c+dx^3\right)}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**2/(d*x**3+4*c),x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**2*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}\,dx=\int \frac{\sqrt{dx^3+c}}{(dx^3+4\,c)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}\,dx=\int \frac{\sqrt{dx^3+c}}{(dx^3+4\,c)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^2\,(4c+dx^3)}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^2\,\left(d\,x^3+4\,c\right)}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^2*(4*c + d*x^3)),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^2*(4*c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\,(4c+dx^3)}\,dx=\int \frac{\sqrt{d\,x^3+c}}{d\,x^5+4c\,x^2}dx$$

input
```
int((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x)
```

output
```
int(sqrt(c + d*x**3)/(4*c*x**2 + d*x**5),x)
```

3.442 $\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx=\frac{x^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},1,-\frac{1}{2},\frac{7}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/16*x^4*(d*x^3+c)^(1/2)*AppellF1(4/3,-1/2,1,7/3,-d*x^3/c,-1/4*d*x^3/c)/c/
(1+d*x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 236 vs. $2(66) = 132$.

Time = 6.37 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.58

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx$$
$$=\frac{x\left(-17x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+32\left(\frac{c}{d}+x^3+\frac{\cdots}{d(4c+dx^3)\left(-16c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\cdots\right.\right.}\right.\right.}{80\sqrt{c+dx^3}}$$

input
```
Integrate[(x^3*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
(x*(-17*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -
1/4*(d*x^3)/c] + 32*(c/d + x^3 + (64*c^3*AppellF1[1/3, 1/2, 1, 4/3, -((d*x
^3)/c), -1/4*(d*x^3)/c])/(d*(4*c + d*x^3)*(-16*c*AppellF1[1/3, 1/2, 1, 4/3
, -((d*x^3)/c), -1/4*(d*x^3)/c] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d
*x^3)/c), -1/4*(d*x^3)/c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -1/
4*(d*x^3)/c]))))))/(80*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

$$\Big\downarrow \; 1013$$

$$\frac{\sqrt{c+dx^3}\int \frac{x^3\sqrt{\frac{dx^3}{c}+1}}{dx^3+4c}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \; 1012$$

$$\frac{x^4\sqrt{c+dx^3}\operatorname{AppellF1}\left(\frac{4}{3},1,-\frac{1}{2},\frac{7}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(x^3*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
(x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -1/2, 7/3, -1/4*(d*x^3)/c, -((d*x^3)
/c)])/(16*c*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.69 (sec) , antiderivative size = 713, normalized size of antiderivative = 10.80

method	result	size
elliptic	Expression too large to display	713
risch	Expression too large to display	718
default	Expression too large to display	1003

input
```
int(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
2/5/d*x*(d*x^3+c)^(1/2)+34/15*I*c/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-4/3*I*c/d^4*2^(1/2)*sum(1/_alpha^2*(-c*d^2
)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(
-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3
^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(
-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)
^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*
(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2387 vs. $2(52) = 104$.

Time = 1.77 (sec) , antiderivative size = 2387, normalized size of antiderivative = 36.17

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
-1/60*(10*(16/27)^(1/6)*d^2*(-c^5/d^8)^(1/6)*log((27*(16/27)^(5/6)*(d^9*x^
8 - 7*c*d^8*x^5 - 8*c^2*d^7*x^2)*(-c^5/d^8)^(5/6) + 96*sqrt(1/3)*(c^2*d^6*
x^7 - c^3*d^5*x^4 - 2*c^4*d^4*x)*sqrt(-c^5/d^8) + 4*(2*c^4*d^2*x^7 - 18*2^
(1/3)*c^3*d^4*x^5*(-c^5/d^8)^(1/3) - 32*c^5*d*x^4 - 16*c^6*x - 2^(2/3)*(5*
c*d^7*x^6 - 20*c^2*d^6*x^3 - 16*c^3*d^5)*(-c^5/d^8)^(2/3))*sqrt(d*x^3 + c)
 - 2*(16/27)^(1/6)*(c^3*d^4*x^9 - 66*c^4*d^3*x^6 - 72*c^5*d^2*x^3 - 32*c^6
*d)*(-c^5/d^8)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) -
10*(16/27)^(1/6)*d^2*(-c^5/d^8)^(1/6)*log(-(27*(16/27)^(5/6)*(d^9*x^8 - 7*
c*d^8*x^5 - 8*c^2*d^7*x^2)*(-c^5/d^8)^(5/6) + 96*sqrt(1/3)*(c^2*d^6*x^7 -
c^3*d^5*x^4 - 2*c^4*d^4*x)*sqrt(-c^5/d^8) - 4*(2*c^4*d^2*x^7 - 18*2^(1/3)*
c^3*d^4*x^5*(-c^5/d^8)^(1/3) - 32*c^5*d*x^4 - 16*c^6*x - 2^(2/3)*(5*c*d^7*
x^6 - 20*c^2*d^6*x^3 - 16*c^3*d^5)*(-c^5/d^8)^(2/3))*sqrt(d*x^3 + c) - 2*(
16/27)^(1/6)*(c^3*d^4*x^9 - 66*c^4*d^3*x^6 - 72*c^5*d^2*x^3 - 32*c^6*d)*(-
c^5/d^8)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 24*sqr
t(d*x^3 + c)*d*x + 168*c*sqrt(d)*weierstrassPInverse(0, -4*c/d, x) - 5*(16
/27)^(1/6)*(sqrt(-3)*d^2 - d^2)*(-c^5/d^8)^(1/6)*log((27*(16/27)^(5/6)*(d^
9*x^8 - 7*c*d^8*x^5 - 8*c^2*d^7*x^2 + sqrt(-3)*(d^9*x^8 - 7*c*d^8*x^5 - 8*
c^2*d^7*x^2))*(-c^5/d^8)^(5/6) - 192*sqrt(1/3)*(c^2*d^6*x^7 - c^3*d^5*x^4
- 2*c^4*d^4*x)*sqrt(-c^5/d^8) + 4*(4*c^4*d^2*x^7 - 64*c^5*d*x^4 - 32*c^6*x
 + 2^(2/3)*(5*c*d^7*x^6 - 20*c^2*d^6*x^3 - 16*c^3*d^5 - sqrt(-3)*(5*c*d...
```

Sympy [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx=\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

input
```
integrate(x**3*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Integral(x**3*sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{dx^3+c}x^3}{dx^3+4\,c}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x^3/(d*x^3 + 4*c), x)
```

Giac [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{dx^3+c}x^3}{dx^3+4\,c}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x^3/(d*x^3 + 4*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{x^3\,\sqrt{d\,x^3+c}}{d\,x^3+4\,c}\,dx$$

input
```
int((x^3*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

output
```
int((x^3*(c + d*x^3)^(1/2))/(4*c + d*x^3), x)
```

Reduce [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx$$
$$= \frac{2\sqrt{d\,x^3+c}\,x - 8\left(\int \frac{\sqrt{d\,x^3+c}}{d^2x^6+5cd\,x^3+4c^2}dx\right)c^2 - 17\left(\int \frac{\sqrt{d\,x^3+c}\,x^3}{d^2x^6+5cd\,x^3+4c^2}dx\right)cd}{5d}$$

input
```
int(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

output
```
(2*sqrt(c + d*x**3)*x - 8*int(sqrt(c + d*x**3)/(4*c**2 + 5*c*d*x**3 + d**2
*x**6),x)*c**2 - 17*int((sqrt(c + d*x**3)*x**3)/(4*c**2 + 5*c*d*x**3 + d**
2*x**6),x)*c*d)/(5*d)
```

3.443 $\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{x\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},1,-\frac{1}{2},\frac{4}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/4*x*(d*x^3+c)^(1/2)*AppellF1(1/3,-1/2,1,4/3,-d*x^3/c,-1/4*d*x^3/c)/c/(1+
d*x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 165 vs. $2(64) = 128$.

Time = 10.18 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.58

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx$$
$$= \frac{16cx\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{(4c+dx^3)\left(16c\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-3dx^3\left(\text{AppellF1}\left(\frac{4}{3},-\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-2\,\text{Appe}\right.\right.}$$

input
```
Integrate[Sqrt[c + d*x^3]/(4*c + d*x^3),x]
```

output
```
(16*c*x*Sqrt[c + d*x^3]*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -1/4*(d*
x^3)/c])/((4*c + d*x^3)*(16*c*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -1
/4*(d*x^3)/c] - 3*d*x^3*(AppellF1[4/3, -1/2, 2, 7/3, -((d*x^3)/c), -1/4*(d
*x^3)/c] - 2*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -1/4*(d*x^3)/c])))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt{c+dx^3}\int \frac{\sqrt{\frac{dx^3}{c}+1}}{dx^3+4c}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},1,-\frac{1}{2},\frac{4}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^3]/(4*c + d*x^3),x]
```

output
```
(x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -1/2, 4/3, -1/4*(d*x^3)/c, -((d*x^3)/c
)])/(4*c*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.02 (sec) , antiderivative size = 696, normalized size of antiderivative = 10.88

method	result
default	$-\frac{2i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{3d\sqrt{d\,x^3+c}}$
elliptic	$-\frac{2i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{3d\sqrt{d\,x^3+c}}$

input
```
int((d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(
-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
))+1/3*I/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c
*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d
*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/
3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c
*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,
(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2240 vs. $2(50) = 100$.

Time = 0.57 (sec) , antiderivative size = 2240, normalized size of antiderivative = 35.00

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output

```
1/24*((1/108)^(1/6)*(sqrt(-3)*d + d)*(-1/(c*d^2))^(1/6)*log((d^3*x^9 - 66*
c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 + 12*(1/4)^(2/3)*(c*d^4*x^8 - 7*c^2*d^3*
x^5 - 8*c^3*d^2*x^2 + sqrt(-3)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2)
)*(-1/(c*d^2))^(2/3) + 24*(1/4)^(1/3)*(c*d^3*x^7 - c^2*d^2*x^4 - 2*c^3*d*x
 - sqrt(-3)*(c*d^3*x^7 - c^2*d^2*x^4 - 2*c^3*d*x))*(-1/(c*d^2))^(1/3) + 6*
sqrt(d*x^3 + c)*(18*(1/108)^(5/6)*(c*d^4*x^7 - 16*c^2*d^3*x^4 - 8*c^3*d^2*
x - sqrt(-3)*(c*d^4*x^7 - 16*c^2*d^3*x^4 - 8*c^3*d^2*x))*(-1/(c*d^2))^(5/6
) - sqrt(1/3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d)*sqrt(-1/(c*d^2)) +
 9*(1/108)^(1/6)*(sqrt(-3)*c*d^2*x^5 + c*d^2*x^5)*(-1/(c*d^2))^(1/6)))/(d^
3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - (1/108)^(1/6)*(sqrt(-3)*d
 + d)*(-1/(c*d^2))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c
^3 + 12*(1/4)^(2/3)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2 + sqrt(-3)*
(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2))*(-1/(c*d^2))^(2/3) + 24*(1/4)
^(1/3)*(c*d^3*x^7 - c^2*d^2*x^4 - 2*c^3*d*x - sqrt(-3)*(c*d^3*x^7 - c^2*d^
2*x^4 - 2*c^3*d*x))*(-1/(c*d^2))^(1/3) - 6*sqrt(d*x^3 + c)*(18*(1/108)^(5/
6)*(c*d^4*x^7 - 16*c^2*d^3*x^4 - 8*c^3*d^2*x - sqrt(-3)*(c*d^4*x^7 - 16*c^
2*d^3*x^4 - 8*c^3*d^2*x))*(-1/(c*d^2))^(5/6) - sqrt(1/3)*(5*c*d^3*x^6 - 20
*c^2*d^2*x^3 - 16*c^3*d)*sqrt(-1/(c*d^2)) + 9*(1/108)^(1/6)*(sqrt(-3)*c*d^
2*x^5 + c*d^2*x^5)*(-1/(c*d^2))^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d
*x^3 + 64*c^3)) - (1/108)^(1/6)*(sqrt(-3)*d - d)*(-1/(c*d^2))^(1/6)*log...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx=\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

input

```
integrate((d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output

```
Integral(sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{dx^3+c}}{dx^3+4\,c}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/(d*x^3 + 4*c), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{dx^3+c}}{dx^3+4\,c}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/(d*x^3 + 4*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{d\,x^3+c}}{d\,x^3+4\,c}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(4*c + d*x^3),x)
```

output
```
int((c + d*x^3)^(1/2)/(4*c + d*x^3), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{d\,x^3+c}}{d\,x^3+4c}dx$$

input
```
int((d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

output
```
int(sqrt(c + d*x**3)/(4*c + d*x**3),x)
```

3.444 $\int \frac{\sqrt{c+dx^3}}{x^3(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \frac{\sqrt{c+dx^3}}{x^3(4c+dx^3)}\,dx = -\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},1,-\frac{1}{2},\frac{1}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
-1/8*(d*x^3+c)^(1/2)*AppellF1(-2/3,-1/2,1,1/3,-d*x^3/c,-1/4*d*x^3/c)/c/x^2
/(1+d*x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 244 vs. 2(66) = 132.

Time = 11.16 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.70

$$\int \frac{\sqrt{c+dx^3}}{x^3(4c+dx^3)}\,dx$$
$$= \frac{-32c(c+dx^3)-d^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+\frac{2048c}{(4c+dx^3)\left(16c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right.}}{256c^2x^2\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^3*(4*c + d*x^3)),x]
```

output
```
(-32*c*(c + d*x^3) - d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3
, -((d*x^3)/c), -1/4*(d*x^3)/c] + (2048*c^3*d*x^3*AppellF1[1/3, 1/2, 1, 4/
3, -((d*x^3)/c), -1/4*(d*x^3)/c])/((4*c + d*x^3)*(16*c*AppellF1[1/3, 1/2,
1, 4/3, -((d*x^3)/c), -1/4*(d*x^3)/c] - 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3
, -((d*x^3)/c), -1/4*(d*x^3)/c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c
), -1/4*(d*x^3)/c]))))/(256*c^2*x^2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{c+dx^3}\int \frac{\sqrt{\frac{dx^3}{c}+1}}{x^3(dx^3+4c)}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},1,-\frac{1}{2},\frac{1}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^3]/(x^3*(4*c + d*x^3)),x]
```

output
```
-1/8*(Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -1/2, 1/3, -1/4*(d*x^3)/c, -((d*x^
3)/c)])/(c*x^2*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.84 (sec) , antiderivative size = 716, normalized size of antiderivative = 10.85

method	result	size
elliptic	Expression too large to display	716
risch	Expression too large to display	720
default	Expression too large to display	1002

input
```
int((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
-1/8/c/x^2*(d*x^3+c)^(1/2)+1/24*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/12*I/d^2/c*2^(1/2)*sum(1/_alpha^2*(-c*d^2)
^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d
^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^
(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-
c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^
(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(
-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2361 vs. $2(52) = 104$.

Time = 1.15 (sec) , antiderivative size = 2361, normalized size of antiderivative = 35.77

$$\int \frac{\sqrt{c + dx^3}}{x^3 (4c + dx^3)} \, dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
-1/96*(2*(1/108)^(1/6)*c*x^2*(-d^4/c^7)^(1/6)*log((d^6*x^9 - 66*c*d^5*x^6
- 72*c^2*d^4*x^3 - 32*c^3*d^3 - 24*(1/4)^(2/3)*(c^5*d^3*x^8 - 7*c^6*d^2*x^
5 - 8*c^7*d*x^2)*(-d^4/c^7)^(2/3) - 48*(1/4)^(1/3)*(c^3*d^4*x^7 - c^4*d^3*
x^4 - 2*c^5*d^2*x)*(-d^4/c^7)^(1/3) + 6*(18*(1/108)^(1/6)*c^2*d^4*x^5*(-d^
4/c^7)^(1/6) + 36*(1/108)^(5/6)*(c^6*d^2*x^7 - 16*c^7*d*x^4 - 8*c^8*x)*(-d
^4/c^7)^(5/6) + sqrt(1/3)*(5*c^4*d^3*x^6 - 20*c^5*d^2*x^3 - 16*c^6*d)*sqrt
(-d^4/c^7))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c
^3)) - 2*(1/108)^(1/6)*c*x^2*(-d^4/c^7)^(1/6)*log((d^6*x^9 - 66*c*d^5*x^6
- 72*c^2*d^4*x^3 - 32*c^3*d^3 - 24*(1/4)^(2/3)*(c^5*d^3*x^8 - 7*c^6*d^2*x^
5 - 8*c^7*d*x^2)*(-d^4/c^7)^(2/3) - 48*(1/4)^(1/3)*(c^3*d^4*x^7 - c^4*d^3*
x^4 - 2*c^5*d^2*x)*(-d^4/c^7)^(1/3) - 6*(18*(1/108)^(1/6)*c^2*d^4*x^5*(-d^
4/c^7)^(1/6) + 36*(1/108)^(5/6)*(c^6*d^2*x^7 - 16*c^7*d*x^4 - 8*c^8*x)*(-d
^4/c^7)^(5/6) + sqrt(1/3)*(5*c^4*d^3*x^6 - 20*c^5*d^2*x^3 - 16*c^6*d)*sqrt
(-d^4/c^7))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c
^3)) - 12*sqrt(d)*x^2*weierstrassPInverse(0, -4*c/d, x) + (1/108)^(1/6)*(s
qrt(-3)*c*x^2 + c*x^2)*(-d^4/c^7)^(1/6)*log((d^6*x^9 - 66*c*d^5*x^6 - 72*c
^2*d^4*x^3 - 32*c^3*d^3 + 12*(1/4)^(2/3)*(c^5*d^3*x^8 - 7*c^6*d^2*x^5 - 8*
c^7*d*x^2 + sqrt(-3)*(c^5*d^3*x^8 - 7*c^6*d^2*x^5 - 8*c^7*d*x^2))*(-d^4/c^
7)^(2/3) + 24*(1/4)^(1/3)*(c^3*d^4*x^7 - c^4*d^3*x^4 - 2*c^5*d^2*x - sqrt(
-3)*(c^3*d^4*x^7 - c^4*d^3*x^4 - 2*c^5*d^2*x))*(-d^4/c^7)^(1/3) + 6*sqr...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3(4c+dx^3)}\,dx = \int \frac{\sqrt{c+dx^3}}{x^3\cdot(4c+dx^3)}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**3/(d*x**3+4*c),x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**3*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)}\,dx = \int \frac{\sqrt{dx^3+c}}{(dx^3+4\,c)x^3}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^3), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)}\,dx = \int \frac{\sqrt{dx^3+c}}{(dx^3+4\,c)x^3}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)}\,dx = \int \frac{\sqrt{d\,x^3+c}}{x^3\,\left(d\,x^3+4\,c\right)}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^3*(4*c + d*x^3)),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^3*(4*c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3(4c+dx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}}{d\,x^6+4c\,x^3}dx$$

input
```
int((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x)
```

output
```
int(sqrt(c + d*x**3)/(4*c*x**3 + d*x**6),x)
```

3.445 $\int \frac{x^8}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 78

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=-\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{32c^{3/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^3}$$

output
```
-10/3*c*(d*x^3+c)^(1/2)/d^3+2/9*(d*x^3+c)^(3/2)/d^3+32/9*c^(3/2)*arctan(1/
3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*3^(1/2)/d^3
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{2(-14c+dx^3)\sqrt{c+dx^3}+32\sqrt{3}c^{3/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{9d^3}$$

input
```
Integrate[x^8/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(2*(-14*c + d*x^3)*Sqrt[c + d*x^3] + 32*Sqrt[3]*c^(3/2)*ArcTan[Sqrt[c + d*
x^3]/(Sqrt[3]*Sqrt[c])])/(9*d^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{x^6}{\sqrt{dx^3+c}\,(dx^3+4c)}dx^3$$

$$\Big\downarrow \ 99$$

$$\frac{1}{3}\int \left(\frac{16c^2}{d^2\sqrt{dx^3+c}\,(dx^3+4c)}-\frac{5c}{d^2\sqrt{dx^3+c}}+\frac{\sqrt{dx^3+c}}{d^2}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(\frac{32c^{3/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^3}-\frac{10c\sqrt{c+dx^3}}{d^3}+\frac{2\left(c+dx^3\right)^{3/2}}{3d^3}\right)$$

input
```
Int[x^8/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
((-10*c*Sqrt[c + d*x^3])/d^3 + (2*(c + d*x^3)^(3/2))/(3*d^3) + (32*c^(3/2)
*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^3))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{32c^{\frac{3}{2}}\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)-2\sqrt{d\,x^{3}+c}\,\left(-d\,x^{3}+14c\right)}{9d^{3}}$
risch	$-\frac{2\left(-d\,x^{3}+14c\right)\sqrt{d\,x^{3}+c}}{9d^{3}}+\frac{32c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{9d^{3}}$
default	$\frac{\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d}-\frac{4c\sqrt{d\,x^{3}+c}}{9d^{2}}}{d}-\frac{8c\sqrt{d\,x^{3}+c}}{3d^{3}}+\frac{32c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{9d^{3}}$
elliptic	$\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d^{2}}-\frac{28c\sqrt{d\,x^{3}+c}}{9d^{3}}-\frac{16ic\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\cdots}}{\cdots}\right)}{\cdots}$

input
```
int(x^8/(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
1/9*(32*c^(3/2)*3^(1/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))-2*(d*x
^3+c)^(1/2)*(-d*x^3+14*c))/d^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.67

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \left[\frac{2\left(24\sqrt{\frac{1}{3}}\sqrt{-c}c\log\left(\frac{dx^3+6\sqrt{\frac{1}{3}}\sqrt{dx^3+c}\sqrt{-c}-2\,c}{dx^3+4\,c}\right)+\sqrt{dx^3+c}(dx^3-14\,c)\right)}{9\,d^3},\ -\frac{2\left(48\sqrt{\frac{1}{3}}c^{\frac{3}{2}}\arctan\left(\frac{3\sqrt{\frac{1}{3}}\sqrt{c}}{\sqrt{dx^3+c}}\right)-\sqrt{dx^3+c}(dx^3-14\,c)\right)}{9\,d^3}\right]$$

input
```
integrate(x^8/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
[2/9*(24*sqrt(1/3)*sqrt(-c)*c*log((d*x^3 + 6*sqrt(1/3)*sqrt(d*x^3 + c)*sqr
t(-c) - 2*c)/(d*x^3 + 4*c)) + sqrt(d*x^3 + c)*(d*x^3 - 14*c))/d^3, -2/9*(4
8*sqrt(1/3)*c^(3/2)*arctan(3*sqrt(1/3)*sqrt(c)/sqrt(d*x^3 + c)) - sqrt(d*x
^3 + c)*(d*x^3 - 14*c))/d^3]
```

Sympy [A] (verification not implemented)

Time = 9.86 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \begin{cases} \frac{2\cdot\left(\frac{16\sqrt{3}c^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9}-\frac{5c\sqrt{c+dx^3}}{3}+\frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d^3} & \text{for } d\neq 0 \\ \frac{x^9}{36c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8/(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Piecewise((2*(16*sqrt(3)*c**(3/2)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)
))/9 - 5*c*sqrt(c + d*x**3)/3 + (c + d*x**3)**(3/2)/9)/d**3, Ne(d, 0)), (x
**9/(36*c**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.68

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\left(16\,\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)+(dx^3+c)^{\frac{3}{2}}-15\,\sqrt{dx^3+c}c\right)}{9\,d^3}$$

input
```
integrate(x^8/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
2/9*(16*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + (d*x
^3 + c)^(3/2) - 15*sqrt(d*x^3 + c)*c)/d^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\left(\frac{16\,\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{d}+\frac{(dx^3+c)^{\frac{3}{2}}d^2-15\,\sqrt{dx^3+c}cd^2}{d^3}\right)}{9\,d^2}$$

input
```
integrate(x^8/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
2/9*(16*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d + ((
d*x^3 + c)^(3/2)*d^2 - 15*sqrt(d*x^3 + c)*c*d^2)/d^3)/d^2
```

Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^2} - \frac{28\,c\,\sqrt{d\,x^3+c}}{9\,d^3} + \frac{\sqrt{3}\,c^{3/2}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,16\mathrm{i}}{9\,d^3}$$

input
```
int(x^8/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
(2*x^3*(c + d*x^3)^(1/2))/(9*d^2) - (28*c*(c + d*x^3)^(1/2))/(9*d^3) + (3^
(1/2)*c^(3/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*
x^3)/(4*c + d*x^3))*16i)/(9*d^3)
```

Reduce [F]

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{-\frac{4\sqrt{d\,x^3+c}\,c}{9} + \frac{2\sqrt{d\,x^3+c}\,d\,x^3}{9} - 4\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{d^2x^6+5cd\,x^3+4c^2}dx\right)c\,d^2}{d^3}$$

input
```
int(x^8/(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

output
```
(2*( - 2*sqrt(c + d*x**3)*c + sqrt(c + d*x**3)*d*x**3 - 18*int((sqrt(c + d
*x**3)*x**5)/(4*c**2 + 5*c*d*x**3 + d**2*x**6),x)*c*d**2))/(9*d**3)
```

3.446 $\int \frac{x^5}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{2\sqrt{c+dx^3}}{3d^2}-\frac{8\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^2}$$

output
```
2/3*(d*x^3+c)^(1/2)/d^2-8/9*c^(1/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(
1/2))*3^(1/2)/d^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{6\sqrt{c+dx^3}-8\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{9d^2}$$

input
```
Integrate[x^5/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(6*Sqrt[c + d*x^3] - 8*Sqrt[3]*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqr
t[c])])/(9*d^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {948, 90, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{\sqrt{dx^3+c}\,(dx^3+4c)}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{d^2}-\frac{4c\int \frac{1}{\sqrt{dx^3+c}(dx^3+4c)}dx^3}{d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{d^2}-\frac{8c\int \frac{1}{x^6+3c}d\sqrt{dx^3+c}}{d^2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{d^2}-\frac{8\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^2}\right)$$

input
```
Int[x^5/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
((2*Sqrt[c + d*x^3])/d^2 - (8*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt
[c])])/(Sqrt[3]*d^2))/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$\frac{-8\sqrt{c}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)+6\sqrt{d\,x^{3}+c}}{9d^{2}}$
default	$\frac{2\sqrt{d\,x^{3}+c}}{3d^{2}}-\frac{8\sqrt{c}\,\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{9d^{2}}$
risch	$\frac{2\sqrt{d\,x^{3}+c}}{3d^{2}}-\frac{8\sqrt{c}\,\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{9d^{2}}$
elliptic	$\frac{2\sqrt{d\,x^{3}+c}}{3d^{2}}+\frac{4i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right.}}}{}\right.}{}$

input
```
int(x^5/(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
1/9*(-8*c^(1/2)*3^(1/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))+6*(d*x
^3+c)^(1/2))/d^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.83

$$\int\frac{x^{5}}{\sqrt{c+dx^{3}}\,(4c+dx^{3})}\,dx$$
$$=\left[\frac{2\left(2\sqrt{\frac{1}{3}}\sqrt{-c}\log\left(\frac{dx^{3}-6\sqrt{\frac{1}{3}}\sqrt{dx^{3}+c}\sqrt{-c}-2\,c}{dx^{3}+4\,c}\right)+\sqrt{dx^{3}+c}\right)}{3\,d^{2}},\frac{2\left(4\sqrt{\frac{1}{3}}\sqrt{c}\arctan\left(\frac{3\sqrt{\frac{1}{3}}\sqrt{c}}{\sqrt{dx^{3}+c}}\right)+\sqrt{dx^{3}+c}\right)}{3\,d^{2}}\right.$$

input
```
integrate(x^5/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
[2/3*(2*sqrt(1/3)*sqrt(-c)*log((d*x^3 - 6*sqrt(1/3)*sqrt(d*x^3 + c)*sqrt(-
c) - 2*c)/(d*x^3 + 4*c)) + sqrt(d*x^3 + c))/d^2, 2/3*(4*sqrt(1/3)*sqrt(c)*
arctan(3*sqrt(1/3)*sqrt(c)/sqrt(d*x^3 + c)) + sqrt(d*x^3 + c))/d^2]
```

Sympy [A] (verification not implemented)

Time = 4.88 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \begin{cases} \frac{2\left(-\frac{4\sqrt{3}\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9}+\frac{\sqrt{c+dx^3}}{3}\right)}{d^2} & \text{for } d\neq 0 \\ \frac{x^6}{24c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Piecewise((2*(-4*sqrt(3)*sqrt(c)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c))
)/9 + sqrt(c + d*x**3)/3)/d**2, Ne(d, 0)), (x**6/(24*c**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{2\left(4\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-3\sqrt{dx^3+c}\right)}{9\,d^2}$$

input
```
integrate(x^5/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
-2/9*(4*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*sq
rt(d*x^3 + c))/d^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{2\left(\frac{4\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{d}-\frac{3\sqrt{dx^3+c}}{d}\right)}{9\,d}$$

input
```
integrate(x^5/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
-2/9*(4*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d - 3*
sqrt(d*x^3 + c)/d)/d
```

Mupad [B] (verification not implemented)

Time = 3.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,d^2}+\frac{\sqrt{3}\,\sqrt{c}\,\ln\left(\frac{2\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,4\mathrm{i}}{9\,d^2}$$

input
```
int(x^5/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*d^2) + (3^(1/2)*c^(1/2)*log((2*3^(1/2)*c + c^(1/2
)*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*4i)/(9*d^2)
```

Reduce [F]

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^5}{d^2x^6+5cd\,x^3+4c^2}dx$$

input
```
int(x^5/(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

output
```
int((sqrt(c + d*x**3)*x**5)/(4*c**2 + 5*c*d*x**3 + d**2*x**6),x)
```

3.447 $\int \frac{x^2}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 40

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}d}$$

output
```
2/9*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*3^(1/2)/c^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}d}$$

input
```
Integrate[x^2/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {946, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}dx$$
$$\Big\downarrow \text{946}$$
$$\frac{1}{3}\int \frac{1}{\sqrt{dx^3+c}\,(dx^3+4c)}dx^3$$
$$\Big\downarrow \text{73}$$
$$\frac{2\int \frac{1}{x^6+3c}d\sqrt{dx^3+c}}{3d}$$
$$\Big\downarrow \text{216}$$
$$\frac{2\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}d}$$

input
```
Int[x^2/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

method	result
default	$\frac{2\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{9\sqrt{c}\,d}$
pseudoelliptic	$\frac{2\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{9\sqrt{c}\,d}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x\right.}{}}}{}\right)}{}$

input
```
int(x^2/(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
2/9*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*3^(1/2)/c^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.15

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \left[-\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\,\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2\,c}{dx^3+4\,c}\right)}{9\,cd},\ -\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{c}}{\sqrt{dx^3+c}}\right)}{9\,\sqrt{c}d}\right]$$

input
```
integrate(x^2/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
[-1/9*sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2
*c)/(d*x^3 + 4*c))/(c*d), -2/9*sqrt(3)*arctan(sqrt(3)*sqrt(c)/sqrt(d*x^3 +
 c))/(sqrt(c)*d)]
```

Sympy [A] (verification not implemented)

Time = 3.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \begin{cases} \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d} & \text{for}\ d \neq 0 \\ \frac{x^3}{12c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Piecewise((2*sqrt(3)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/(9*sqrt(c)
*d), Ne(d, 0)), (x**3/(12*c**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

input
```
integrate(x^2/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/(sqrt(c)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

input
```
integrate(x^2/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/(sqrt(c)*d)
```

Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{\sqrt{3}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{2\,d\,x^3+8\,c}\right)\,1\mathrm{i}}{9\,\sqrt{c}\,d}$$

input
```
int(x^2/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
(3^(1/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)/
(8*c + 2*d*x^3))*1i)/(9*c^(1/2)*d)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^2}{d^2x^6+5cd\,x^3+4c^2}dx$$

input
```
int(x^2/(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

output
```
int((sqrt(c + d*x**3)*x**2)/(4*c**2 + 5*c*d*x**3 + d**2*x**6),x)
```

3.448 $\int \frac{1}{x\sqrt{c+dx^3}(4c+dx^3)} dx$

Optimal result

Integrand size = 26, antiderivative size = 65

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\, dx = -\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{6\sqrt{3}c^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}$$

output
```
-1/18*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*3^(1/2)/c^(3/2)-1/6*arct
anh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\, dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right) + 3\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{18c^{3/2}}$$

input
```
Integrate[1/(x*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
-1/18*(Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])] + 3*ArcTanh[Sqrt[
c + d*x^3]/Sqrt[c]])/c^(3/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {948, 97, 73, 216, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{1}{x^3\sqrt{dx^3+c}\,(dx^3+4c)}dx^3$$

$$\Big\downarrow \; 97$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{4c}-\frac{d\int \frac{1}{\sqrt{dx^3+c}(dx^3+4c)}dx^3}{4c}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2cd}-\frac{\int \frac{1}{x^6+3c}d\sqrt{dx^3+c}}{2c}\right)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2cd}-\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}c^{3/2}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(-\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}c^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2c^{3/2}}\right)$$

input
```
Int[1/(x*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(-1/2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(Sqrt[3]*c^(3/2)) - ArcTan
h[Sqrt[c + d*x^3]/Sqrt[c]]/(2*c^(3/2)))/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}+3\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{18c^{\frac{3}{2}}}$	45
default	$-\frac{\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{18c^{\frac{3}{2}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6c^{\frac{3}{2}}}$	47
elliptic	Expression too large to display	1508

input
```
int(1/x/(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
-1/18*(arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*3^(1/2)+3*arctanh((d*x^
3+c)^(1/2)/c^(1/2)))/c^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.22

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \left[\frac{2\,\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{c}}{\sqrt{dx^3+c}}\right)+3\,\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{36\,c^2},\right.$$
$$\left.-\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3+2\,\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2\,c}{dx^3+4\,c}\right)-6\,\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)}{36\,c^2}\right]$$

input
```
integrate(1/x/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
[1/36*(2*sqrt(3)*sqrt(c)*arctan(sqrt(3)*sqrt(c)/sqrt(d*x^3 + c)) + 3*sqrt(
c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c^2, -1/36*(sqrt(3)
*sqrt(-c)*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 +
4*c)) - 6*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)))/c^2]
```

Sympy [A] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \begin{cases} \frac{2\left(\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12c\sqrt{-c}} - \frac{\sqrt{3}d\,\mathrm{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{\frac{3}{2}}}\right)}{d} & \text{for } d \neq 0 \\ \frac{\log(x^3)}{12c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Piecewise((2*(d*atan(sqrt(c + d*x**3)/sqrt(-c))/(12*c*sqrt(-c)) - sqrt(3)*
d*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/(36*c**(3/2)))/d, Ne(d, 0)),
(log(x**3)/(12*c**(3/2)), True))
```

Maxima [F]

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+cx}}\,dx$$

input
```
integrate(1/x/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{18\,c^{\frac{3}{2}}} + \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6\sqrt{-c}c}$$

input
```
integrate(1/x/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
-1/18*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(3/2) + 1/6*ar
ctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c)
```

Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.45

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{12\,c^{3/2}} + \frac{\sqrt{3}\,\ln\left(\frac{2\,\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,1\mathrm{i}}{36\,c^{3/2}}$$

input
```
int(1/(x*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(
12*c^(3/2)) + (3^(1/2)*log((2*3^(1/2)*c + c^(1/2)*(c + d*x^3)^(1/2)*6i - 3
^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(36*c^(3/2))
```

Reduce [F]

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{\sqrt{c}\log\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)-\sqrt{c}\log\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)-3\left(\int \frac{\sqrt{d\,x^3+c}\,x^2}{d^2x^6+5cd\,x^3+4c^2}dx\right)cd}{12c^2}$$

input
```
int(1/x/(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

output
```
(sqrt(c)*log(sqrt(c + d*x**3) - sqrt(c)) - sqrt(c)*log(sqrt(c + d*x**3) +
sqrt(c)) - 3*int((sqrt(c + d*x**3)*x**2)/(4*c**2 + 5*c*d*x**3 + d**2*x**6)
,x)*c*d)/(12*c**2)
```

3.449 $\int \frac{1}{x^4\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=-\frac{\sqrt{c+dx^3}}{12c^2x^3}+\frac{d\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}c^{5/2}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}$$

output
```
-1/12*(d*x^3+c)^(1/2)/c^2/x^3+1/72*d*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^
(1/2))*3^(1/2)/c^(5/2)+1/8*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=-\frac{\sqrt{c+dx^3}}{12c^2x^3}+\frac{d\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}c^{5/2}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}$$

input
```
Integrate[1/(x^4*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
-1/12*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[
c])])/(24*Sqrt[3]*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(8*c^(5/
2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {948, 114, 27, 174, 73, 216, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^6\sqrt{dx^3+c}\,(dx^3+4c)}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int \frac{d(dx^3+6c)}{2x^3\sqrt{dx^3+c}(dx^3+4c)}dx^3}{4c^2}-\frac{\sqrt{c+dx^3}}{4c^2x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\int \frac{dx^3+6c}{x^3\sqrt{dx^3+c}(dx^3+4c)}dx^3}{8c^2}-\frac{\sqrt{c+dx^3}}{4c^2x^3}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{3}{2}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3-\frac{1}{2}d\int \frac{1}{\sqrt{dx^3+c}(dx^3+4c)}dx^3\right)}{8c^2}-\frac{\sqrt{c+dx^3}}{4c^2x^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{3\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{d}-\int \frac{1}{x^6+3c}d\sqrt{dx^3+c}\right)}{8c^2}-\frac{\sqrt{c+dx^3}}{4c^2x^3}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{3\int\frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{d}-\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\sqrt{c}}\right)}{8c^2}-\frac{\sqrt{c+dx^3}}{4c^2x^3}\right)$$

$$\downarrow 221$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\sqrt{c}}-\frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{8c^2}-\frac{\sqrt{c+dx^3}}{4c^2x^3}\right)$$

input
```
Int[1/(x^4*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(-1/4*Sqrt[c + d*x^3]/(c^2*x^3) - (d*(-(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sq
rt[c])]/(Sqrt[3]*Sqrt[c])) - (3*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c])
)/(8*c^2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{\sqrt{d\,x^{3}+c}}{12c^{2}x^{3}}+\frac{d\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{72c^{\frac{5}{2}}}+\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{8c^{\frac{5}{2}}}$	66
pseudoelliptic	$-\frac{\sqrt{d\,x^{3}+c}}{12c^{2}x^{3}}+\frac{d\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{72c^{\frac{5}{2}}}+\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{8c^{\frac{5}{2}}}$	66
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3c\,x^{3}}+\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{4c}+\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{24c^{\frac{5}{2}}}+\frac{d\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{72c^{\frac{5}{2}}}$	92
elliptic	Expression too large to display	1523

input
```
int(1/x^4/(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(d*x^3+c)^(1/2)/c^2/x^3+1/72*d*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^
(1/2))*3^(1/2)/c^(5/2)+1/8*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.16

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$
$$=\left[-\frac{2\,\sqrt{3}\sqrt{c}dx^3\arctan\left(\frac{\sqrt{3}\sqrt{c}}{\sqrt{dx^3+c}}\right)-9\,\sqrt{c}dx^3\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)+12\,\sqrt{dx^3+c}c}{144\,c^3x^3},\right.$$
$$\left.-\frac{\sqrt{3}\sqrt{-c}dx^3\log\left(\frac{dx^3-2\,\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2\,c}{dx^3+4\,c}\right)+18\,\sqrt{-c}dx^3\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)+12\,\sqrt{dx^3+c}c}{144\,c^3x^3}\right]$$

input
```
integrate(1/x^4/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
[-1/144*(2*sqrt(3)*sqrt(c)*d*x^3*arctan(sqrt(3)*sqrt(c)/sqrt(d*x^3 + c)) -
 9*sqrt(c)*d*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 12*s
qrt(d*x^3 + c)*c)/(c^3*x^3), -1/144*(sqrt(3)*sqrt(-c)*d*x^3*log((d*x^3 - 2
*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) + 18*sqrt(-c)*d*x^
3*arctan(sqrt(-c)/sqrt(d*x^3 + c)) + 12*sqrt(d*x^3 + c)*c)/(c^3*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{x^4\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

input
```
integrate(1/x**4/(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Integral(1/(x**4*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}x^4}\,dx$$

input
```
integrate(1/x^4/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{\sqrt{3}d\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{72\,c^{\frac{5}{2}}} - \frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{8\sqrt{-c}c^2} - \frac{\sqrt{dx^3+c}}{12\,c^2x^3}$$

input
```
integrate(1/x^4/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
1/72*sqrt(3)*d*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(5/2) - 1/8*d
*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/12*sqrt(d*x^3 + c)/(c
^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{d\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)^3}{x^6}\right)}{16\,c^{5/2}} - \frac{\sqrt{d\,x^3+c}}{12\,c^2\,x^3} + \frac{\sqrt{3}\,d\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,1\mathrm{i}}{144\,c^{5/2}}$$

input
```
int(1/(x^4*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
(d*log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6
))/(16*c^(5/2)) - (c + d*x^3)^(1/2)/(12*c^2*x^3) + (3^(1/2)*d*log((c^(1/2)
*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(1
44*c^(5/2))
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$
$$= \frac{-16\sqrt{d\,x^3+c}\,c+2\sqrt{d\,x^3+c}\,d\,x^3-12\sqrt{c}\log\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)d\,x^3+12\sqrt{c}\log\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)d\,x^3 -}{192c^3x^3}$$

input
```
int(1/x^4/(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

output
```
( - 16*sqrt(c + d*x**3)*c + 2*sqrt(c + d*x**3)*d*x**3 - 12*sqrt(c)*log(sqr
t(c + d*x**3) - sqrt(c))*d*x**3 + 12*sqrt(c)*log(sqrt(c + d*x**3) + sqrt(c
))*d*x**3 - 3*int((sqrt(c + d*x**3)*x**5)/(4*c**2 + 5*c*d*x**3 + d**2*x**6
),x)*d**3*x**3)/(192*c**3*x**3)
```

3.450 $\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 667

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{2\sqrt[3]{2}\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}} - \frac{2\sqrt[3]{2}\sqrt[6]{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^{5/3}} + \frac{2\sqrt[3]{2}\sqrt[6]{c}\,\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3d^{5/3}} - \frac{2\sqrt[3]{2}\sqrt[6]{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9d^{5/3}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
2*(d*x^3+c)^(1/2)/d^(5/3)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)+2/9*2^(1/3)*c^(1
/6)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*3^
(1/2)/d^(5/3)-2/9*2^(1/3)*c^(1/6)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/
2))*3^(1/2)/d^(5/3)+2/3*2^(1/3)*c^(1/6)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d
^(1/3)*x)/(d*x^3+c)^(1/2))/d^(5/3)-2/9*2^(1/3)*c^(1/6)*arctanh((d*x^3+c)^(
1/2)/c^(1/2))/d^(5/3)-3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*c^(1/3)*(c^(1/3)+d
^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^
(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c
^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/d^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+
3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+2/3*2^(1/2)*c^(1/3)*(
c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c
^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3
^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/d^(5/3)/(c^(1/3)*(c^(1/3
)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.10

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{x^5\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{20c\sqrt{c+dx^3}}$$

input
```
Integrate[x^4/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*
x^3)/c])/(20*c*Sqrt[c + d*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.13 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {983, 832, 759, 986, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$\Big\downarrow \ 983$$

$$\frac{\int \frac{x}{\sqrt{dx^3+c}}dx}{d}-\frac{4c\int \frac{x}{\sqrt{dx^3+c}(dx^3+4c)}dx}{d}$$

$$\Big\downarrow \ 832$$

$$\frac{\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}\int \frac{1}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}}{d}-\frac{4c\int \frac{x}{\sqrt{dx^3+c}(dx^3+4c)}dx}{d}$$

$$\Big\downarrow \ 759$$

$$\frac{\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{\cdots}}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{d}$$
$$\frac{4c\int \frac{x}{\sqrt{dx^3+c}(dx^3+4c)}dx}{d}$$

$$\Big\downarrow \ 986$$

$$\frac{\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{\cdots}\right.}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{d}$$

$$\frac{4c\left(-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}\right)}{d}$$

$\downarrow$ 2416

$$\frac{\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\cdots}{\cdots}}{d}$$

$$\frac{4c\left(-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}\right)}{d}$$

input
```
Int[x^4/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(-4*c*(-1/3*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c
+ d*x^3]]/(2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTan[Sqrt[c + d*x^3]/(Sqrt
[3]*Sqrt[c])]/(3*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) - ArcTanh[(c^(1/6)*(c^(1
/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]]/(3*2^(2/3)*c^(5/6)*d^(2/3)) + A
rcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(9*2^(2/3)*c^(5/6)*d^(2/3))))/d + (((2*Sqr
t[c + d*x^3])/(d^(1/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (3^(1/4)*Sqr
t[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/
3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(d^(1/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/d^(1/3) - (2*(1 - Sqrt[3])
*Sqrt[2 + Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[A
rcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x
))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/d
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 986
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.29 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.27

method	result	size
default	Expression too large to display	848
elliptic	Expression too large to display	848

input
```
int(x^4/(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^
(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+4/9*I/d^4*2
^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3
*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^
2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alp
ha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2268 vs. $2(470) = 940$.

Time = 2.06 (sec) , antiderivative size = 2268, normalized size of antiderivative = 3.40

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
1/18*(2*(4/27)^(1/6)*d^2*(-c/d^10)^(1/6)*log(32*(9*(4/27)^(5/6)*(d^11*x^9
- 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c^3*d^8)*(-c/d^10)^(5/6) - 96*sqrt(1
/3)*(c*d^7*x^7 - c^2*d^6*x^4 - 2*c^3*d^5*x)*sqrt(-c/d^10) + 4*(9*4^(2/3)*c
*d^8*x^5*(-c/d^10)^(2/3) + 2*c*d^2*x^7 - 32*c^2*d*x^4 - 16*c^3*x + 4^(1/3)
*(5*c*d^5*x^6 - 20*c^2*d^4*x^3 - 16*c^3*d^3)*(-c/d^10)^(1/3))*sqrt(d*x^3 +
 c) - 24*(4/27)^(1/6)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2)*(-c/d^10
)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 2*(4/27)^(1/6
)*d^2*(-c/d^10)^(1/6)*log(-32*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6 -
72*c^2*d^9*x^3 - 32*c^3*d^8)*(-c/d^10)^(5/6) - 96*sqrt(1/3)*(c*d^7*x^7 - c
^2*d^6*x^4 - 2*c^3*d^5*x)*sqrt(-c/d^10) - 4*(9*4^(2/3)*c*d^8*x^5*(-c/d^10)
^(2/3) + 2*c*d^2*x^7 - 32*c^2*d*x^4 - 16*c^3*x + 4^(1/3)*(5*c*d^5*x^6 - 20
*c^2*d^4*x^3 - 16*c^3*d^3)*(-c/d^10)^(1/3))*sqrt(d*x^3 + c) - 24*(4/27)^(1
/6)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2)*(-c/d^10)^(1/6))/(d^3*x^9
+ 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - (4/27)^(1/6)*(sqrt(-3)*d^2 - d^
2)*(-c/d^10)^(1/6)*log(32*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c
^2*d^9*x^3 - 32*c^3*d^8 + sqrt(-3)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*
x^3 - 32*c^3*d^8))*(-c/d^10)^(5/6) + 192*sqrt(1/3)*(c*d^7*x^7 - c^2*d^6*x^
4 - 2*c^3*d^5*x)*sqrt(-c/d^10) + 4*(4*c*d^2*x^7 - 64*c^2*d*x^4 - 32*c^3*x
+ 9*4^(2/3)*(sqrt(-3)*c*d^8*x^5 - c*d^8*x^5)*(-c/d^10)^(2/3) - 4^(1/3)*(5*
c*d^5*x^6 - 20*c^2*d^4*x^3 - 16*c^3*d^3 + sqrt(-3)*(5*c*d^5*x^6 - 20*c^...
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^4}{\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

input
```
integrate(x**4/(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Integral(x**4/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^4}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^4/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(x^4/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^4}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^4/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
integrate(x^4/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^4}{\sqrt{d\,x^3+c}\,(d\,x^3+4\,c)}\,dx$$

input
```
int(x^4/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
int(x^4/((c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^4}{d^2x^6+5cd\,x^3+4c^2}dx$$

input
```
int(x^4/(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

output
```
int((sqrt(c + d*x**3)*x**4)/(4*c**2 + 5*c*d*x**3 + d**2*x**6),x)
```

3.451 $\int \frac{x}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 206

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}$$

output

```
-1/18*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*
2^(1/3)*3^(1/2)/c^(5/6)/d^(2/3)+1/18*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^
(1/2))*2^(1/3)*3^(1/2)/c^(5/6)/d^(2/3)-1/6*arctanh(c^(1/6)*(c^(1/3)-2^(1/3
)*d^(1/3)*x)/(d*x^3+c)^(1/2))*2^(1/3)/c^(5/6)/d^(2/3)+1/18*arctanh((d*x^3+
c)^(1/2)/c^(1/2))*2^(1/3)/c^(5/6)/d^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.33

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{x^2\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{8c\sqrt{c+dx^3}}$$

input
```
Integrate[x/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -1/4*(d*
x^3)/c])/(8*c*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {986}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$\Big\downarrow \; 986$$

$$-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}$$

input
```
Int[x/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output

```
-1/3*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3
]]/(2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqr
t[c])]/(3*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) - ArcTanh[(c^(1/6)*(c^(1/3) - 2
^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]]/(3*2^(2/3)*c^(5/6)*d^(2/3)) + ArcTanh[
Sqrt[c + d*x^3]/Sqrt[c]]/(9*2^(2/3)*c^(5/6)*d^(2/3))
```

Defintions of rubi rules used

rule 986

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.11 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.02

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-\cdots\right.}{\cdots}\right)}{2\left(\cdots\right.}}\cdots}{\cdots}\right.}{\cdots}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-\cdots\right.}{\cdots}\right)}{2\left(\cdots\right.}}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x/(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
-1/9*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d
^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)
+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d
^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2289 vs. $2(141) = 282$.

Time = 0.75 (sec) , antiderivative size = 2289, normalized size of antiderivative = 11.11

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
-1/36*(1/432)^(1/6)*(sqrt(-3) + 1)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 66*
c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 - 24*(1/2)^(2/3)*(c^4*d^5*x^7 - c^5*d^4*
x^4 - 2*c^6*d^3*x + sqrt(-3)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x))*(-
1/(c^5*d^4))^(2/3) - 6*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^
2*x^2 - sqrt(-3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2))*(-1/(c^5*d
^4))^(1/3) + 6*sqrt(d*x^3 + c)*(648*(1/432)^(5/6)*(sqrt(-3)*c^5*d^5*x^5 -
c^5*d^5*x^5)*(-1/(c^5*d^4))^(5/6) + sqrt(1/3)*(5*c^3*d^4*x^6 - 20*c^4*d^3*
x^3 - 16*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d
^2*x^4 - 8*c^3*d*x + sqrt(-3)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x))*(-
1/(c^5*d^4))^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 1
/36*(1/432)^(1/6)*(sqrt(-3) + 1)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 66*c*
d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 - 24*(1/2)^(2/3)*(c^4*d^5*x^7 - c^5*d^4*x^
4 - 2*c^6*d^3*x + sqrt(-3)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x))*(-1/
(c^5*d^4))^(2/3) - 6*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*
x^2 - sqrt(-3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2))*(-1/(c^5*d^4
))^(1/3) - 6*sqrt(d*x^3 + c)*(648*(1/432)^(5/6)*(sqrt(-3)*c^5*d^5*x^5 - c^
5*d^5*x^5)*(-1/(c^5*d^4))^(5/6) + sqrt(1/3)*(5*c^3*d^4*x^6 - 20*c^4*d^3*x^
3 - 16*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2
*x^4 - 8*c^3*d*x + sqrt(-3)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x))*(-1/
(c^5*d^4))^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - ...
```

Sympy [F]

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x}{\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

input
```
integrate(x/(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Integral(x/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(x/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
integrate(x/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Mupad [B] (verification not implemented)

Time = 20.07 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.20

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$= \frac{\sqrt{3}\,314928^{1/3}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}+\sqrt{3}\,\sqrt{-c}-2^{1/3}\,\sqrt{3}\,(-c)^{1/6}\,d^{1/3}\,x\right)^3\left(54\,\sqrt{d\,x^3+c}-54\,\sqrt{3}\,\sqrt{-c}+54\,2^{1/3}\,\sqrt{3}\,(-c)^{1/6}\,d^{1/3}\,x\right)}{\left(d^{1/3}\,x-2^{2/3}\,(-c)^{1/3}\right)^6}\right)}{2916\,(-c)^{5/6}\,d^{2/3}}$$

$$+ \frac{\sqrt{3}\,314928^{1/3}\,\ln\left(\frac{\left(2\,\sqrt{3}\,\sqrt{-c}-2\,\sqrt{d\,x^3+c}+2^{1/3}\,\sqrt{3}\,(-c)^{1/6}\,d^{1/3}\,x+2^{1/3}\,(-c)^{1/6}\,d^{1/3}\,x\,3\mathrm{i}\right)^3\left(108\,\sqrt{d\,x^3+c}+108\,\sqrt{3}\,\sqrt{-c}+54\,2^{1/3}\,\sqrt{\cdots}\right.}{\left(2\,d^{1/3}\,x+2^{2/3}\,(-c)^{1/3}-2^{2/3}\,\sqrt{3}\,(-c)^{1/3}\,1\mathrm{i}\right)^6}\right.}{2916\,(-c)^{5/6}\,d^{2/3}}$$

$$+ \frac{\sqrt{3}\,314928^{1/3}\,\ln\left(\frac{\left(2\,\sqrt{d\,x^3+c}+2\,\sqrt{3}\,\sqrt{-c}+2^{1/3}\,\sqrt{3}\,(-c)^{1/6}\,d^{1/3}\,x-2^{1/3}\,(-c)^{1/6}\,d^{1/3}\,x\,3\mathrm{i}\right)^3\left(108\,\sqrt{d\,x^3+c}-108\,\sqrt{3}\,\sqrt{-c}-54\,2^{1/3}\,\sqrt{\cdots}\right.}{\left(2\,d^{1/3}\,x+2^{2/3}\,(-c)^{1/3}+2^{2/3}\,\sqrt{3}\,(-c)^{1/3}\,1\mathrm{i}\right)^6}\right.}{2916\,(-c)^{5/6}\,d^{2/3}}$$

input
```
int(x/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
(3^(1/2)*314928^(1/3)*log((((c + d*x^3)^(1/2) + 3^(1/2)*(-c)^(1/2) - 2^(1/
3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(54*(c + d*x^3)^(1/2) - 54*3^(1/2)*(-c)
^(1/2) + 54*2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x))/(d^(1/3)*x - 2^(2/3)*(-
c)^(1/3))^6))/(2916*(-c)^(5/6)*d^(2/3)) + (3^(1/2)*314928^(1/3)*log(((2*3^
(1/2)*(-c)^(1/2) - 2*(c + d*x^3)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*3i +
 2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(108*(c + d*x^3)^(1/2) + 108*3^(1
/2)*(-c)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*162i + 54*2^(1/3)*3^(1/2)*(-
c)^(1/6)*d^(1/3)*x))/(2*d^(1/3)*x + 2^(2/3)*(-c)^(1/3) - 2^(2/3)*3^(1/2)*(
-c)^(1/3)*1i)^6)*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(2916*(-c)^(5/6)*d^(2/3)) +
 (3^(1/2)*314928^(1/3)*log(((2*(c + d*x^3)^(1/2) + 2*3^(1/2)*(-c)^(1/2) -
2^(1/3)*(-c)^(1/6)*d^(1/3)*x*3i + 2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*
(108*(c + d*x^3)^(1/2) - 108*3^(1/2)*(-c)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/
3)*x*162i - 54*2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x))/(2*d^(1/3)*x + 2^(2/
3)*(-c)^(1/3) + 2^(2/3)*3^(1/2)*(-c)^(1/3)*1i)^6)*((3^(1/2)*1i)/2 + 1/2)^(
1/2)*1i)/(2916*(-c)^(5/6)*d^(2/3))
```

Reduce [F]

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\int \frac{\sqrt{d\,x^3+c}\,x}{d^2x^6+5cd\,x^3+4c^2}dx$$

input
```
int(x/(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

output
```
int((sqrt(c + d*x**3)*x)/(4*c**2 + 5*c*d*x**3 + d**2*x**6),x)
```

3.452 $\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 697

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$= -\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{4c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12\ 2^{2/3}\sqrt{3}c^{11/6}}$$
$$-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{12\ 2^{2/3}\sqrt{3}c^{11/6}}+\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12\ 2^{2/3}c^{11/6}}-\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{36\ 2^{2/3}c^{11/6}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{8c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{2\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-1/4*(d*x^3+c)^(1/2)/c^2/x+1/4*d^(1/3)*(d*x^3+c)^(1/2)/c^2/((1+3^(1/2))*c^
(1/3)+d^(1/3)*x)+1/72*d^(1/3)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+2^(1/3)*d^(1
/3)*x)/(d*x^3+c)^(1/2))*2^(1/3)*3^(1/2)/c^(11/6)-1/72*d^(1/3)*arctan(1/3*(
d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*2^(1/3)*3^(1/2)/c^(11/6)+1/24*d^(1/3)*arct
anh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*2^(1/3)/c^(11/6)-
1/72*d^(1/3)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*2^(1/3)/c^(11/6)-1/8*3^(1/4)
*(1/2*6^(1/2)-1/2*2^(1/2))*d^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d
^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE((
(1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2
*I)/c^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2
)^(1/2)/(d*x^3+c)^(1/2)+1/12*d^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)
*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF
(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)
+2*I)*2^(1/2)*3^(3/4)/c^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^
(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{-40c(c+dx^3)+5cdx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+d^2x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3}\right.}{160c^3x\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^2*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(-40*c*(c + d*x^3) + 5*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), -1/4*(d*x^3)/c] + d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5
/3, 1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(160*c^3*x*Sqrt[c + d*x^3]
)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {980, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{c+dx^3}\left(4c+dx^3\right)}dx$$

$$\Big\downarrow \text{ 980}$$

$$\frac{\int \frac{dx\left(dx^3+2c\right)}{2\sqrt{dx^3+c}\left(dx^3+4c\right)}dx}{4c^2}-\frac{\sqrt{c+dx^3}}{4c^2x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\int \frac{x\left(dx^3+2c\right)}{\sqrt{dx^3+c}\left(dx^3+4c\right)}dx}{8c^2}-\frac{\sqrt{c+dx^3}}{4c^2x}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{d\int \left(\frac{x}{\sqrt{dx^3+c}}-\frac{2cx}{\sqrt{dx^3+c}\left(dx^3+4c\right)}\right)dx}{8c^2}-\frac{\sqrt{c+dx^3}}{4c^2x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\left(\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\ldots}{\ldots}\right)}{\ldots}$$

$$\frac{\sqrt{c+dx^3}}{4c^2x}$$

input
```
Int[1/(x^2*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
-1/4*Sqrt[c + d*x^3]/(c^2*x) + (d*((2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)) + (2^(1/3)*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(
1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sqrt[3]*d^(2/3)) - (2^(1/3
)*c^(1/6)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*d^(2/3)) +
 (2^(1/3)*c^(1/6)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c +
 d*x^3]])/(3*d^(2/3)) - (2^(1/3)*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(9*d^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1
/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
+ (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)
*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
-7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(8*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.95 (sec) , antiderivative size = 868, normalized size of antiderivative = 1.25

method	result	size
elliptic	Expression too large to display	868
risch	Expression too large to display	872
default	Expression too large to display	874

input
```
int(1/x^2/(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(d*x^3+c)^(1/2)/c^2/x-1/12*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)))+1/36*I/c^2/d^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*
d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))
^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d
^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1
/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*_alpha^2
*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2293 vs. $2(490) = 980$.

Time = 0.86 (sec) , antiderivative size = 2293, normalized size of antiderivative = 3.29

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
-1/144*(2*(1/432)^(1/6)*c^2*x*(-d^2/c^11)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^
6 - 72*c^2*d^2*x^3 - 32*c^3*d + 48*(1/2)^(2/3)*(c^8*d^2*x^7 - c^9*d*x^4 -
2*c^10*x)*(-d^2/c^11)^(2/3) + 12*(1/2)^(1/3)*(c^4*d^3*x^8 - 7*c^5*d^2*x^5
- 8*c^6*d*x^2)*(-d^2/c^11)^(1/3) + 6*(1296*(1/432)^(5/6)*c^10*d*x^5*(-d^2/
c^11)^(5/6) + sqrt(1/3)*(5*c^6*d^2*x^6 - 20*c^7*d*x^3 - 16*c^8)*sqrt(-d^2/
c^11) + 2*(1/432)^(1/6)*(c^2*d^3*x^7 - 16*c^3*d^2*x^4 - 8*c^4*d*x)*(-d^2/c
^11)^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c
^3)) - 2*(1/432)^(1/6)*c^2*x*(-d^2/c^11)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6
 - 72*c^2*d^2*x^3 - 32*c^3*d + 48*(1/2)^(2/3)*(c^8*d^2*x^7 - c^9*d*x^4 - 2
*c^10*x)*(-d^2/c^11)^(2/3) + 12*(1/2)^(1/3)*(c^4*d^3*x^8 - 7*c^5*d^2*x^5 -
 8*c^6*d*x^2)*(-d^2/c^11)^(1/3) - 6*(1296*(1/432)^(5/6)*c^10*d*x^5*(-d^2/c
^11)^(5/6) + sqrt(1/3)*(5*c^6*d^2*x^6 - 20*c^7*d*x^3 - 16*c^8)*sqrt(-d^2/c
^11) + 2*(1/432)^(1/6)*(c^2*d^3*x^7 - 16*c^3*d^2*x^4 - 8*c^4*d*x)*(-d^2/c^
11)^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^
3)) + 36*sqrt(d)*x*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/
d, x)) - (1/432)^(1/6)*(sqrt(-3)*c^2*x + c^2*x)*(-d^2/c^11)^(1/6)*log((d^4
*x^9 - 66*c*d^3*x^6 - 72*c^2*d^2*x^3 - 32*c^3*d - 24*(1/2)^(2/3)*(c^8*d^2*
x^7 - c^9*d*x^4 - 2*c^10*x + sqrt(-3)*(c^8*d^2*x^7 - c^9*d*x^4 - 2*c^10*x)
)*(-d^2/c^11)^(2/3) - 6*(1/2)^(1/3)*(c^4*d^3*x^8 - 7*c^5*d^2*x^5 - 8*c^6*d
*x^2 - sqrt(-3)*(c^4*d^3*x^8 - 7*c^5*d^2*x^5 - 8*c^6*d*x^2))*(-d^2/c^11...
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{x^2\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

input
```
integrate(1/x**2/(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Integral(1/(x**2*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}x^2}\,dx$$

input
```
integrate(1/x^2/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}x^2}\,dx$$

input
```
integrate(1/x^2/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{x^2\,\sqrt{d\,x^3+c}\,\left(d\,x^3+4\,c\right)}\,dx$$

input
```
int(1/(x^2*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
int(1/(x^2*(c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}}{d^2x^8+5cd\,x^5+4c^2x^2}dx$$

input
```
int(1/x^2/(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

output
```
int(sqrt(c + d*x**3)/(4*c**2*x**2 + 5*c*d*x**5 + d**2*x**8),x)
```

3.453 $\int \frac{x^3}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{1}{2},\frac{7}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{c+dx^3}}$$

output
```
1/16*x^4*(1+d*x^3/c)^(1/2)*AppellF1(4/3,1/2,1,7/3,-d*x^3/c,-1/4*d*x^3/c)/c
/(d*x^3+c)^(1/2)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{x^4\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{16c\sqrt{c+dx^3}}$$

input
```
Integrate[x^3/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(x^4*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -1/4*(d*
x^3)/c])/(16*c*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^3}{(dx^3+4c)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{1}{2},\frac{7}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{c+dx^3}}$$

input
```
Int[x^3/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, -1/4*(d*x^3)/c, -((d*x
^3)/c)])/(16*c*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.52 (sec) , antiderivative size = 696, normalized size of antiderivative = 10.55

method	result
default	$-\frac{2i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{3d^{2}\sqrt{d\,x^{3}+c}}$
elliptic	$-\frac{2i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{3d^{2}\sqrt{d\,x^{3}+c}}$

input
```
int(x^3/(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d
*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2))+4/9*I/d^4*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(
-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/
c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2300 vs. $2(52) = 104$.

Time = 0.59 (sec) , antiderivative size = 2300, normalized size of antiderivative = 34.85

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
1/36*(2*(16/27)^(1/6)*d^2*(-1/(c*d^8))^(1/6)*log((4*d^3*x^9 - 264*c*d^2*x^
6 - 288*c^2*d*x^3 - 128*c^3 - 24*2^(2/3)*(c*d^8*x^8 - 7*c^2*d^7*x^5 - 8*c^
3*d^6*x^2)*(-1/(c*d^8))^(2/3) - 96*2^(1/3)*(c*d^5*x^7 - c^2*d^4*x^4 - 2*c^
3*d^3*x)*(-1/(c*d^8))^(1/3) + 3*(72*(16/27)^(1/6)*c*d^3*x^5*(-1/(c*d^8))^(
1/6) + 9*(16/27)^(5/6)*(c*d^9*x^7 - 16*c^2*d^8*x^4 - 8*c^3*d^7*x)*(-1/(c*d
^8))^(5/6) + 8*sqrt(1/3)*(5*c*d^6*x^6 - 20*c^2*d^5*x^3 - 16*c^3*d^4)*sqrt(
-1/(c*d^8)))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*
c^3)) - 2*(16/27)^(1/6)*d^2*(-1/(c*d^8))^(1/6)*log((4*d^3*x^9 - 264*c*d^2*
x^6 - 288*c^2*d*x^3 - 128*c^3 - 24*2^(2/3)*(c*d^8*x^8 - 7*c^2*d^7*x^5 - 8*
c^3*d^6*x^2)*(-1/(c*d^8))^(2/3) - 96*2^(1/3)*(c*d^5*x^7 - c^2*d^4*x^4 - 2*
c^3*d^3*x)*(-1/(c*d^8))^(1/3) - 3*(72*(16/27)^(1/6)*c*d^3*x^5*(-1/(c*d^8))
^(1/6) + 9*(16/27)^(5/6)*(c*d^9*x^7 - 16*c^2*d^8*x^4 - 8*c^3*d^7*x)*(-1/(c
*d^8))^(5/6) + 8*sqrt(1/3)*(5*c*d^6*x^6 - 20*c^2*d^5*x^3 - 16*c^3*d^4)*sqr
t(-1/(c*d^8)))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 6
4*c^3)) + (16/27)^(1/6)*(sqrt(-3)*d^2 + d^2)*(-1/(c*d^8))^(1/6)*log((8*d^3
*x^9 - 528*c*d^2*x^6 - 576*c^2*d*x^3 - 256*c^3 + 24*2^(2/3)*(c*d^8*x^8 - 7
*c^2*d^7*x^5 - 8*c^3*d^6*x^2 + sqrt(-3)*(c*d^8*x^8 - 7*c^2*d^7*x^5 - 8*c^3
*d^6*x^2))*(-1/(c*d^8))^(2/3) + 96*2^(1/3)*(c*d^5*x^7 - c^2*d^4*x^4 - 2*c^
3*d^3*x - sqrt(-3)*(c*d^5*x^7 - c^2*d^4*x^4 - 2*c^3*d^3*x))*(-1/(c*d^8))^(
1/3) + 3*sqrt(d*x^3 + c)*(9*(16/27)^(5/6)*(c*d^9*x^7 - 16*c^2*d^8*x^4 -...
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\int \frac{x^3}{\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

input
```
integrate(x**3/(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Integral(x**3/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^3}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^3/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(x^3/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^3}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^3/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
integrate(x^3/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^3}{\sqrt{d\,x^3+c}\,(d\,x^3+4\,c)}\,dx$$

input
```
int(x^3/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
int(x^3/((c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^3}{d^2x^6+5cd\,x^3+4c^2}dx$$

input
```
int(x^3/(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

output
```
int((sqrt(c + d*x**3)*x**3)/(4*c**2 + 5*c*d*x**3 + d**2*x**6),x)
```

3.454 $\int \frac{1}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{c+dx^3}}$$

output
```
1/4*x*(1+d*x^3/c)^(1/2)*AppellF1(1/3,1/2,1,4/3,-d*x^3/c,-1/4*d*x^3/c)/c/(d
*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 165 vs. 2(64) = 128.

Time = 10.06 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.58

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$=\frac{16cx\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{\sqrt{c+dx^3}\,(4c+dx^3)\left(16c\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-3dx^3\left(\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+2\right.\right.}$$

input
```
Integrate[1/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(16*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(Sqrt[c
+ d*x^3]*(4*c + d*x^3)*(16*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -1/4
*(d*x^3)/c] - 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -1/4*(d*x^
3)/c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -1/4*(d*x^3)/c])))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{(dx^3+4c)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{c+dx^3}}$$

input
```
Int[1/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, -1/4*(d*x^3)/c, -((d*x^3
)/c)])/(4*c*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.05 (sec) , antiderivative size = 416, normalized size of antiderivative = 6.50

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-\right.}{}\right.}{2\left(\right.}}}{}\right.}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-\right.}{}\right.}{2\left(\right.}}}{}\right.}{}$

input
```
int(1/(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
-1/9*I/d^3/c*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c
*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d
*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/
3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c
*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,
(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2347 vs. $2(50) = 100$.

Time = 0.66 (sec) , antiderivative size = 2347, normalized size of antiderivative = 36.67

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
-1/72*(2*(1/108)^(1/6)*c*d*(-1/(c^7*d^2))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^
6 - 72*c^2*d*x^3 - 32*c^3 - 24*(1/4)^(2/3)*(c^5*d^4*x^8 - 7*c^6*d^3*x^5 -
8*c^7*d^2*x^2)*(-1/(c^7*d^2))^(2/3) - 48*(1/4)^(1/3)*(c^3*d^3*x^7 - c^4*d^
2*x^4 - 2*c^5*d*x)*(-1/(c^7*d^2))^(1/3) + 6*(18*(1/108)^(1/6)*c^2*d^2*x^5*
(-1/(c^7*d^2))^(1/6) + 36*(1/108)^(5/6)*(c^6*d^4*x^7 - 16*c^7*d^3*x^4 - 8*
c^8*d^2*x)*(-1/(c^7*d^2))^(5/6) + sqrt(1/3)*(5*c^4*d^3*x^6 - 20*c^5*d^2*x^
3 - 16*c^6*d)*sqrt(-1/(c^7*d^2)))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6
 + 48*c^2*d*x^3 + 64*c^3)) - 2*(1/108)^(1/6)*c*d*(-1/(c^7*d^2))^(1/6)*log(
(d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 - 24*(1/4)^(2/3)*(c^5*d^4*
x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2)*(-1/(c^7*d^2))^(2/3) - 48*(1/4)^(1/3)
*(c^3*d^3*x^7 - c^4*d^2*x^4 - 2*c^5*d*x)*(-1/(c^7*d^2))^(1/3) - 6*(18*(1/1
08)^(1/6)*c^2*d^2*x^5*(-1/(c^7*d^2))^(1/6) + 36*(1/108)^(5/6)*(c^6*d^4*x^7
 - 16*c^7*d^3*x^4 - 8*c^8*d^2*x)*(-1/(c^7*d^2))^(5/6) + sqrt(1/3)*(5*c^4*d
^3*x^6 - 20*c^5*d^2*x^3 - 16*c^6*d)*sqrt(-1/(c^7*d^2)))*sqrt(d*x^3 + c))/(
d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + (1/108)^(1/6)*(sqrt(-3)
*c*d + c*d)*(-1/(c^7*d^2))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^
3 - 32*c^3 + 12*(1/4)^(2/3)*(c^5*d^4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2 +
 sqrt(-3)*(c^5*d^4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2))*(-1/(c^7*d^2))^(2
/3) + 24*(1/4)^(1/3)*(c^3*d^3*x^7 - c^4*d^2*x^4 - 2*c^5*d*x - sqrt(-3)*(c^
3*d^3*x^7 - c^4*d^2*x^4 - 2*c^5*d*x))*(-1/(c^7*d^2))^(1/3) + 6*sqrt(d*x...
```

Sympy [F]

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

input
```
integrate(1/(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Integral(1/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{\sqrt{d\,x^3+c}\,(d\,x^3+4\,c)}\,dx$$

input
```
int(1/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
int(1/((c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}}{d^2x^6+5cd\,x^3+4c^2}dx$$

input
```
int(1/(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

output
```
int(sqrt(c + d*x**3)/(4*c**2 + 5*c*d*x**3 + d**2*x**6),x)
```

3.455 $\int \frac{1}{x^3\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{c+dx^3}}$$

output
```
-1/8*(1+d*x^3/c)^(1/2)*AppellF1(-2/3,1/2,1,1/3,-d*x^3/c,-1/4*d*x^3/c)/c/x^
2/(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 243 vs. 2(66) = 132.

Time = 11.20 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.68

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$= \frac{-\frac{32(c+dx^3)}{c^2} - \frac{d^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{c^3} + \frac{2048dx^3\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\right.}{(4c+dx^3)\left(-16c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+3dx^3\left(\text{AppellF1}\left(\frac{4}{3}\right.\right.\right.}}{256x^2\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^3*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
((-32*(c + d*x^3))/c^2 - (d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1
, 7/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/c^3 + (2048*d*x^3*AppellF1[1/3, 1/2,
 1, 4/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/((4*c + d*x^3)*(-16*c*AppellF1[1/3
, 1/2, 1, 4/3, -((d*x^3)/c), -1/4*(d*x^3)/c] + 3*d*x^3*(AppellF1[4/3, 1/2,
 2, 7/3, -((d*x^3)/c), -1/4*(d*x^3)/c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d
*x^3)/c), -1/4*(d*x^3)/c]))))/(256*x^2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^3(dx^3+4c)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{c+dx^3}}$$

input
```
Int[1/(x^3*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
-1/8*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -1/4*(d*x^3)/c, -((d
*x^3)/c)])/(c*x^2*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.84 (sec) , antiderivative size = 716, normalized size of antiderivative = 10.85

method	result	size
elliptic	Expression too large to display	716
risch	Expression too large to display	720
default	Expression too large to display	722

input
```
int(1/x^3/(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(d*x^3+c)^(1/2)/c^2/x^2+1/24*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/36*I/c^2/d^2*2^(1/2)*sum(1/_alpha^2*(-
c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))
/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1
/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_al
pha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alph
a*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c
*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c
*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+
4*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2381 vs. $2(52) = 104$.

Time = 1.84 (sec) , antiderivative size = 2381, normalized size of antiderivative = 36.08

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
1/288*(2*(1/108)^(1/6)*c^2*x^2*(-d^4/c^13)^(1/6)*log((d^6*x^9 - 66*c*d^5*x
^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 - 24*(1/4)^(2/3)*(c^9*d^3*x^8 - 7*c^10*d^
2*x^5 - 8*c^11*d*x^2)*(-d^4/c^13)^(2/3) - 48*(1/4)^(1/3)*(c^5*d^4*x^7 - c^
6*d^3*x^4 - 2*c^7*d^2*x)*(-d^4/c^13)^(1/3) + 6*(18*(1/108)^(1/6)*c^3*d^4*x
^5*(-d^4/c^13)^(1/6) + 36*(1/108)^(5/6)*(c^11*d^2*x^7 - 16*c^12*d*x^4 - 8*
c^13*x)*(-d^4/c^13)^(5/6) + sqrt(1/3)*(5*c^7*d^3*x^6 - 20*c^8*d^2*x^3 - 16
*c^9*d)*sqrt(-d^4/c^13))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2
*d*x^3 + 64*c^3)) - 2*(1/108)^(1/6)*c^2*x^2*(-d^4/c^13)^(1/6)*log((d^6*x^9
 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 - 24*(1/4)^(2/3)*(c^9*d^3*x^
8 - 7*c^10*d^2*x^5 - 8*c^11*d*x^2)*(-d^4/c^13)^(2/3) - 48*(1/4)^(1/3)*(c^5
*d^4*x^7 - c^6*d^3*x^4 - 2*c^7*d^2*x)*(-d^4/c^13)^(1/3) - 6*(18*(1/108)^(1
/6)*c^3*d^4*x^5*(-d^4/c^13)^(1/6) + 36*(1/108)^(5/6)*(c^11*d^2*x^7 - 16*c^
12*d*x^4 - 8*c^13*x)*(-d^4/c^13)^(5/6) + sqrt(1/3)*(5*c^7*d^3*x^6 - 20*c^8
*d^2*x^3 - 16*c^9*d)*sqrt(-d^4/c^13))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2
*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 60*sqrt(d)*x^2*weierstrassPInverse(0, -4*
c/d, x) + (1/108)^(1/6)*(sqrt(-3)*c^2*x^2 + c^2*x^2)*(-d^4/c^13)^(1/6)*log
((d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 + 12*(1/4)^(2/3)*(c
^9*d^3*x^8 - 7*c^10*d^2*x^5 - 8*c^11*d*x^2 + sqrt(-3)*(c^9*d^3*x^8 - 7*c^1
0*d^2*x^5 - 8*c^11*d*x^2))*(-d^4/c^13)^(2/3) + 24*(1/4)^(1/3)*(c^5*d^4*x^7
 - c^6*d^3*x^4 - 2*c^7*d^2*x - sqrt(-3)*(c^5*d^4*x^7 - c^6*d^3*x^4 - 2*...
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\int \frac{1}{x^3\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

input
```
integrate(1/x**3/(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Integral(1/(x**3*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}x^3}\,dx$$

input
```
integrate(1/x^3/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}x^3}\,dx$$

input
```
integrate(1/x^3/(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{x^3\,\sqrt{d\,x^3+c}\,\left(d\,x^3+4\,c\right)}\,dx$$

input
```
int(1/(x^3*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
int(1/(x^3*(c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}}{d^2x^9+5cd\,x^6+4c^2x^3}dx$$

input
```
int(1/x^3/(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

output
```
int(sqrt(c + d*x**3)/(4*c**2*x**3 + 5*c*d*x**6 + d**2*x**9),x)
```

3.456 $\int \frac{x}{\sqrt{1-x^3}(4-x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{1+\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\operatorname{arctanh}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

output
```
-1/18*arctan(3^(1/2)*(1-2^(1/3)*x)/(-x^3+1)^(1/2))*2^(1/3)*3^(1/2)+1/18*ar
ctan(1/3*(-x^3+1)^(1/2)*3^(1/2))*2^(1/3)*3^(1/2)-1/6*arctanh((1+2^(1/3)*x)
/(-x^3+1)^(1/2))*2^(1/3)+1/18*arctanh((-x^3+1)^(1/2))*2^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.22

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \frac{1}{8}x^2\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{4}\right)$$

input
```
Integrate[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

output
```
(x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/4])/8
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {986}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{1-x^3}\left(4-x^3\right)} \, dx$$

$$\Big\downarrow \text{986}$$

$$-\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\text{arctanh}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\text{arctanh}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

input
```
Int[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

output
```
-1/3*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(2^(2/3)*Sqrt[3]) + A
rcTan[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)
/Sqrt[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))
```

Defintions of rubi rules used

rule 986
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 14.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.29

method	result
default	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{3}-4\right)}\frac{\_\alpha^{2}\sqrt{2}\,\sqrt{i\left(-i\sqrt{3}+2x+1\right)}\,\sqrt{\frac{-1+x}{i\sqrt{3}-3}}\,\sqrt{-\frac{i\left(i\sqrt{3}+2x+1\right)}{2}}\,\left(-2\_\alpha^{2}+\_\alpha+1+i\sqrt{3}\,(1-\_\alpha)\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}\right.}}{3}\right.}{2\sqrt{-x^{3}+1}}\right.}{36}$
elliptic	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{3}-4\right)}\frac{\_\alpha^{2}\sqrt{2}\,\sqrt{i\left(-i\sqrt{3}+2x+1\right)}\,\sqrt{\frac{-1+x}{i\sqrt{3}-3}}\,\sqrt{-\frac{i\left(i\sqrt{3}+2x+1\right)}{2}}\,\left(-2\_\alpha^{2}+\_\alpha+1+i\sqrt{3}\,(1-\_\alpha)\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}\right.}}{3}\right.}{2\sqrt{-x^{3}+1}}\right.}{36}$
trager	Expression too large to display

input
```
int(x/(-x^3+1)^(1/2)/(-x^3+4),x,method=_RETURNVERBOSE)
```

output
```
1/36*I*2^(1/2)*sum(_alpha^2*(1/2*I*(-I*3^(1/2)+2*x+1))^(1/2)*((-1+x)/(I*3^
(1/2)-3))^(1/2)*(-1/2*I*(I*3^(1/2)+2*x+1))^(1/2)/(-x^3+1)^(1/2)*(-2*_alpha
^2+_alpha+1+I*3^(1/2)*(1-_alpha))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2),1/2*_alpha-1/3*I*_alpha^2*3^(1/2)-1/2+1/6*I*_alpha*
3^(1/2)+1/6*I*3^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)),_alpha=RootO
f(_Z^3-4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1019 vs. $2(92) = 184$.

Time = 0.15 (sec) , antiderivative size = 1019, normalized size of antiderivative = 8.02

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-x^3+1)^(1/2)/(-x^3+4),x, algorithm="fricas")
```

output
```
-1/36*(-1/432)^(1/6)*(sqrt(-3) + 1)*log(-(x^9 + 66*x^6 - 72*x^3 - 24*(-1/2
)^(2/3)*(x^7 + x^4 + sqrt(-3)*(x^7 + x^4 - 2*x) - 2*x) + 6*sqrt(-x^3 + 1)*
(648*(-1/432)^(5/6)*(sqrt(-3)*x^5 - x^5) - sqrt(-1/3)*(5*x^6 + 20*x^3 - 16
) - (-1/432)^(1/6)*(x^7 + 16*x^4 + sqrt(-3)*(x^7 + 16*x^4 - 8*x) - 8*x)) +
 6*(-1/2)^(1/3)*(x^8 + 7*x^5 - 8*x^2 - sqrt(-3)*(x^8 + 7*x^5 - 8*x^2)) + 3
2)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/36*(-1/432)^(1/6)*(sqrt(-3) + 1)*log(
-(x^9 + 66*x^6 - 72*x^3 - 24*(-1/2)^(2/3)*(x^7 + x^4 + sqrt(-3)*(x^7 + x^4
 - 2*x) - 2*x) - 6*sqrt(-x^3 + 1)*(648*(-1/432)^(5/6)*(sqrt(-3)*x^5 - x^5)
 - sqrt(-1/3)*(5*x^6 + 20*x^3 - 16) - (-1/432)^(1/6)*(x^7 + 16*x^4 + sqrt(
-3)*(x^7 + 16*x^4 - 8*x) - 8*x)) + 6*(-1/2)^(1/3)*(x^8 + 7*x^5 - 8*x^2 - s
qrt(-3)*(x^8 + 7*x^5 - 8*x^2)) + 32)/(x^9 - 12*x^6 + 48*x^3 - 64)) - 1/36*
(-1/432)^(1/6)*(sqrt(-3) - 1)*log(-(x^9 + 66*x^6 - 72*x^3 - 24*(-1/2)^(2/3
)*(x^7 + x^4 - sqrt(-3)*(x^7 + x^4 - 2*x) - 2*x) + 6*sqrt(-x^3 + 1)*(648*(
-1/432)^(5/6)*(sqrt(-3)*x^5 + x^5) + sqrt(-1/3)*(5*x^6 + 20*x^3 - 16) + (-
1/432)^(1/6)*(x^7 + 16*x^4 - sqrt(-3)*(x^7 + 16*x^4 - 8*x) - 8*x)) + 6*(-1
/2)^(1/3)*(x^8 + 7*x^5 - 8*x^2 + sqrt(-3)*(x^8 + 7*x^5 - 8*x^2)) + 32)/(x^
9 - 12*x^6 + 48*x^3 - 64)) + 1/36*(-1/432)^(1/6)*(sqrt(-3) - 1)*log(-(x^9
+ 66*x^6 - 72*x^3 - 24*(-1/2)^(2/3)*(x^7 + x^4 - sqrt(-3)*(x^7 + x^4 - 2*x
) - 2*x) - 6*sqrt(-x^3 + 1)*(648*(-1/432)^(5/6)*(sqrt(-3)*x^5 + x^5) + sqr
t(-1/3)*(5*x^6 + 20*x^3 - 16) + (-1/432)^(1/6)*(x^7 + 16*x^4 - sqrt(-3)...
```

Sympy [F]

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = -\int \frac{x}{x^3\sqrt{1-x^3}-4\sqrt{1-x^3}}\,dx$$

input
```
integrate(x/(-x**3+1)**(1/2)/(-x**3+4),x)
```

output
```
-Integral(x/(x**3*sqrt(1 - x**3) - 4*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \int -\frac{x}{(x^3-4)\sqrt{-x^3+1}}\,dx$$

input
```
integrate(x/(-x^3+1)^(1/2)/(-x^3+4),x, algorithm="maxima")
```

output
```
-integrate(x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \int -\frac{x}{(x^3-4)\sqrt{-x^3+1}}\,dx$$

input
```
integrate(x/(-x^3+1)^(1/2)/(-x^3+4),x, algorithm="giac")
```

output
```
integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 653, normalized size of antiderivative = 5.14

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \text{Too large to display}$$

input
```
int(-x/((1 - x^3)^(1/2)*(x^3 - 4)),x)
```

output
```
- (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 +
1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3
^(1/2)*1i)/2 + 3/2)/(2^(2/3) - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(1 - x^3)^(1/2
)*(2^(2/3) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^
(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*
1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3
/2)/(2^(2/3)*((3^(1/2)*1i)/2 + 1/2) + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/
2)*1i)/2 + 1/2)*(1 - x^3)^(1/2)*(2^(2/3)*((3^(1/2)*1i)/2 + 1/2) + 1)*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x
^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*(
(x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(2/3)*((3^(1/
2)*1i)/2 - 1/2) - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 - 1/2)*(...
```

Reduce [F]

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \int \frac{\sqrt{-x^3+1}\,x}{x^6-5x^3+4}dx$$

input
```
int(x/(-x^3+1)^(1/2)/(-x^3+4),x)
```

output
```
int((sqrt( - x**3 + 1)*x)/(x**6 - 5*x**3 + 4),x)
```

3.457 $\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 111

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{1024c^3\sqrt{c+dx^3}}{3d^4} - \frac{38c^2(c+dx^3)^{3/2}}{3d^4} - \frac{4c(c+dx^3)^{5/2}}{5d^4} - \frac{2(c+dx^3)^{7/2}}{21d^4} + \frac{1024c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

output
```
-1024/3*c^3*(d*x^3+c)^(1/2)/d^4-38/3*c^2*(d*x^3+c)^(3/2)/d^4-4/5*c*(d*x^3+
c)^(5/2)/d^4-2/21*(d*x^3+c)^(7/2)/d^4+1024*c^(7/2)*arctanh(1/3*(d*x^3+c)^(
1/2)/c^(1/2))/d^4
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.74

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\sqrt{c+dx^3}(18632c^3+764c^2dx^3+57cd^2x^6+5d^3x^9)}{105d^4} + \frac{1024c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

input
```
Integrate[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
(-2*Sqrt[c + d*x^3]*(18632*c^3 + 764*c^2*d*x^3 + 57*c*d^2*x^6 + 5*d^3*x^9)
)/(105*d^4) + (1024*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^9\sqrt{dx^3+c}}{8c-dx^3}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{512\sqrt{dx^3+c}c^3}{d^3\left(8c-dx^3\right)}-\frac{57\sqrt{dx^3+c}c^2}{d^3}-\frac{6\left(dx^3+c\right)^{3/2}c}{d^3}-\frac{\left(dx^3+c\right)^{5/2}}{d^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3072c^{7/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{1024c^3\sqrt{c+dx^3}}{d^4}-\frac{38c^2\left(c+dx^3\right)^{3/2}}{d^4}-\frac{12c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{7d^4}\right)$$

input
```
Int[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
((-1024*c^3*Sqrt[c + d*x^3])/d^4 - (38*c^2*(c + d*x^3)^(3/2))/d^4 - (12*c*
(c + d*x^3)^(5/2))/(5*d^4) - (2*(c + d*x^3)^(7/2))/(7*d^4) + (3072*c^(7/2)
*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4)/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{1024c^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)-\frac{2\sqrt{d\,x^3+c}\,\left(5d^3x^9+57c\,d^2x^6+764c^2d\,x^3+18632c^3\right)}{105}}{d^4}$
risch	$-\frac{2\left(5d^3x^9+57c\,d^2x^6+764c^2d\,x^3+18632c^3\right)\sqrt{d\,x^3+c}}{105d^4}+\frac{1024c^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d^4}$
default	$-\frac{\frac{2x^9\sqrt{d\,x^3+c}}{21}+\frac{2c\,x^6\sqrt{d\,x^3+c}}{105d}-\frac{8c^2x^3\sqrt{d\,x^3+c}}{315d^2}+\frac{16c^3\sqrt{d\,x^3+c}}{315d^3}}{d}-\frac{128c^2\left(d\,x^3+c\right)^{\frac{3}{2}}}{9d^4}-\frac{8c\left(\frac{2x^6\sqrt{d\,x^3+c}}{15}+\frac{2c\,x^3\sqrt{d\,x^3+c}}{45d}-4\right.}{d^2}$
elliptic	$-\frac{2x^9\sqrt{d\,x^3+c}}{21d}-\frac{38c\,x^6\sqrt{d\,x^3+c}}{35d^2}-\frac{1528c^2x^3\sqrt{d\,x^3+c}}{105d^3}-\frac{37264c^3\sqrt{d\,x^3+c}}{105d^4}-\frac{512ic^3\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)}\right.}{}$

input
```
int(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
2/105*(53760*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-(d*x^3+c)^(1/2)*
(5*d^3*x^9+57*c*d^2*x^6+764*c^2*d*x^3+18632*c^3))/d^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.50

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx$$
$$=\left[\frac{2\left(26880\,c^{\frac{7}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(5\,d^3x^9+57\,cd^2x^6+764\,c^2dx^3+18632\,c^3)\sqrt{dx^3+c}\right)}{105\,d^4},\right.$$
$$\left.-\frac{2\left(53760\,\sqrt{-c}c^3\arctan\left(\frac{3\,\sqrt{-c}}{\sqrt{dx^3+c}}\right)+(5\,d^3x^9+57\,cd^2x^6+764\,c^2dx^3+18632\,c^3)\sqrt{dx^3+c}\right)}{105\,d^4}\right]$$

input
```
integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[2/105*(26880*c^(7/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^
3 - 8*c)) - (5*d^3*x^9 + 57*c*d^2*x^6 + 764*c^2*d*x^3 + 18632*c^3)*sqrt(d*
x^3 + c))/d^4, -2/105*(53760*sqrt(-c)*c^3*arctan(3*sqrt(-c)/sqrt(d*x^3 + c
)) + (5*d^3*x^9 + 57*c*d^2*x^6 + 764*c^2*d*x^3 + 18632*c^3)*sqrt(d*x^3 + c
))/d^4]
```

Sympy [A] (verification not implemented)

Time = 20.71 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx$$
$$=\begin{cases}\frac{2\left(-\frac{512c^4\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{512c^3\sqrt{c+dx^3}}{3}-\frac{19c^2\left(c+dx^3\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+dx^3\right)^{\frac{5}{2}}}{5}-\frac{\left(c+dx^3\right)^{\frac{7}{2}}}{21}\right)}{d^4} & \text{for } d\neq 0\\ \frac{x^{12}}{96\sqrt{c}} & \text{otherwise}\end{cases}$$

input
```
integrate(x**11*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-512*c**4*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 512
*c**3*sqrt(c + d*x**3)/3 - 19*c**2*(c + d*x**3)**(3/2)/3 - 2*c*(c + d*x**3
)**(5/2)/5 - (c + d*x**3)**(7/2)/21)/d**4, Ne(d, 0)), (x**12/(96*sqrt(c)),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\left(26880\,c^{\frac{7}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+5\left(dx^3+c\right)^{\frac{7}{2}}+42\left(dx^3+c\right)^{\frac{5}{2}}c+665\left(dx^3+c\right)^{\frac{3}{2}}c^2+17920\sqrt{dx^3+c}c\right.}{105\,d^4}$$

input
```
integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-2/105*(26880*c^(7/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) +
 3*sqrt(c))) + 5*(d*x^3 + c)^(7/2) + 42*(d*x^3 + c)^(5/2)*c + 665*(d*x^3 +
 c)^(3/2)*c^2 + 17920*sqrt(d*x^3 + c)*c^3)/d^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{1024\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^4} - \frac{2\left(5\left(dx^3+c\right)^{\frac{7}{2}}d^{24}+42\left(dx^3+c\right)^{\frac{5}{2}}cd^{24}+665\left(dx^3+c\right)^{\frac{3}{2}}c^2d^{24}+17920\sqrt{dx^3+c}c^3d^{24}\right)}{105\,d^{28}}$$

input
```
integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
-1024*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/105*(5*(
d*x^3 + c)^(7/2)*d^24 + 42*(d*x^3 + c)^(5/2)*c*d^24 + 665*(d*x^3 + c)^(3/2
)*c^2*d^24 + 17920*sqrt(d*x^3 + c)*c^3*d^24)/d^28
```

Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{512\,c^{7/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^4} - \frac{37264\,c^3\,\sqrt{d\,x^3+c}}{105\,d^4} - \frac{2\,x^9\,\sqrt{d\,x^3+c}}{21\,d} - \frac{38\,c\,x^6\,\sqrt{d\,x^3+c}}{35\,d^2} - \frac{1528\,c^2\,x^3\,\sqrt{d\,x^3+c}}{105\,d^3}$$

input
```
int((x^11*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

output
```
(512*c^(7/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/d^4 - (37264*c^3*(c + d*x^3)^(1/2))/(105*d^4) - (2*x^9*(c + d*x^3)^(1/
2))/(21*d) - (38*c*x^6*(c + d*x^3)^(1/2))/(35*d^2) - (1528*c^2*x^3*(c + d*
x^3)^(1/2))/(105*d^3)
```

Reduce [F]

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{\frac{3056\sqrt{d\,x^3+c}\,c^3}{105} - \frac{1528\sqrt{d\,x^3+c}\,c^2 d\,x^3}{105} - \frac{38\sqrt{d\,x^3+c}\,c\,d^2x^6}{35} - \frac{2\sqrt{d\,x^3+c}\,d^3x^9}{21} + 576\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{-d^2x^6+7cd\,x^3+8c^2}dx\right)c^3d^2}{d^4}$$

input
```
int(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

output
```
(2*(1528*sqrt(c + d*x**3)*c**3 - 764*sqrt(c + d*x**3)*c**2*d*x**3 - 57*sqr
t(c + d*x**3)*c*d**2*x**6 - 5*sqrt(c + d*x**3)*d**3*x**9 + 30240*int((sqrt
(c + d*x**3)*x**5)/(8*c**2 + 7*c*d*x**3 - d**2*x**6),x)*c**3*d**2))/(105*d
**4)
```

3.458 $\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{128c^2\sqrt{c+dx^3}}{3d^3}-\frac{14c(c+dx^3)^{3/2}}{9d^3}-\frac{2(c+dx^3)^{5/2}}{15d^3}+\frac{128c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

output
```
-128/3*c^2*(d*x^3+c)^(1/2)/d^3-14/9*c*(d*x^3+c)^(3/2)/d^3-2/15*(d*x^3+c)^(
5/2)/d^3+128*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.79

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\sqrt{c+dx^3}(998c^2+41cdx^3+3d^2x^6)}{45d^3}+\frac{128c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

input
```
Integrate[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
(-2*Sqrt[c + d*x^3]*(998*c^2 + 41*c*d*x^3 + 3*d^2*x^6))/(45*d^3) + (128*c^
(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6\sqrt{dx^3+c}}{8c-dx^3}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{64\sqrt{dx^3+c}c^2}{d^2\left(8c-dx^3\right)}-\frac{7\sqrt{dx^3+c}c}{d^2}-\frac{\left(dx^3+c\right)^{3/2}}{d^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{384c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{128c^2\sqrt{c+dx^3}}{d^3}-\frac{14c\left(c+dx^3\right)^{3/2}}{3d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{5d^3}\right)$$

input
```
Int[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
((-128*c^2*Sqrt[c + d*x^3])/d^3 - (14*c*(c + d*x^3)^(3/2))/(3*d^3) - (2*(c
 + d*x^3)^(5/2))/(5*d^3) + (384*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)])/d^3)/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{\frac{2\left(-3d^2x^6-41cd\,x^3-998c^2\right)\sqrt{d\,x^3+c}}{45}+128c^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d^3}$
risch	$-\frac{2(3d^2x^6+41cd\,x^3+998c^2)\sqrt{d\,x^3+c}}{45d^3}+\frac{128c^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d^3}$
default	$-\frac{\frac{2x^6\sqrt{d\,x^3+c}}{15}+\frac{2c\,x^3\sqrt{d\,x^3+c}}{45d}-\frac{4c^2\sqrt{d\,x^3+c}}{45d^2}}{d}-\frac{16c(d\,x^3+c)^{\frac{3}{2}}}{9d^3}+\frac{64c^2\left(-2\sqrt{d\,x^3+c}+6\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\right)}{3d^3}$
elliptic	$-\frac{2x^6\sqrt{d\,x^3+c}}{15d}-\frac{82c\,x^3\sqrt{d\,x^3+c}}{45d^2}-\frac{1996c^2\sqrt{d\,x^3+c}}{45d^3}-\frac{64ic^2\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{\cdots}}{\cdots}\right)}{\cdots}}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
2/45*((-3*d^2*x^6-41*c*d*x^3-998*c^2)*(d*x^3+c)^(1/2)+2880*c^(5/2)*arctanh
(1/3*(d*x^3+c)^(1/2)/c^(1/2)))/d^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.60

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = \left[\frac{2\left(1440\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(3\,d^2x^6+41\,cdx^3+998\,c^2)\sqrt{dx^3+c}\right)}{45\,d^3}, -\frac{2\left(2880\,\sqrt{-c}c^2\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)+(3\,d^2x^6+41\,cdx^3+998\,c^2)\sqrt{dx^3+c}\right)}{45\,d^3}\right]$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[2/45*(1440*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
- 8*c)) - (3*d^2*x^6 + 41*c*d*x^3 + 998*c^2)*sqrt(d*x^3 + c))/d^3, -2/45*(
2880*sqrt(-c)*c^2*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) + (3*d^2*x^6 + 41*c*d
*x^3 + 998*c^2)*sqrt(d*x^3 + c))/d^3]
```

Sympy [A] (verification not implemented)

Time = 8.83 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{64c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{64c^2\sqrt{c+dx^3}}{3}-\frac{7c\left(c+dx^3\right)^{\frac{3}{2}}}{9}-\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15}\right)}{d^3} & \text{for } d\neq 0 \\ \frac{x^9}{72\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-64*c**3*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 64*c
**2*sqrt(c + d*x**3)/3 - 7*c*(c + d*x**3)**(3/2)/9 - (c + d*x**3)**(5/2)/1
5)/d**3, Ne(d, 0)), (x**9/(72*sqrt(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\left(1440\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+3\,(dx^3+c)^{\frac{5}{2}}+35\,(dx^3+c)^{\frac{3}{2}}c+960\,\sqrt{dx^3+c}c^2\right)}{45\,d^3}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-2/45*(1440*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3
*sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 35*(d*x^3 + c)^(3/2)*c + 960*sqrt(d*x^3
 + c)*c^2)/d^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{128\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d^3} - \frac{2\left(3\,(dx^3+c)^{\frac{5}{2}}d^{12}+35\,(dx^3+c)^{\frac{3}{2}}cd^{12}+960\,\sqrt{dx^3+c}c^2d^{12}\right)}{45\,d^{15}}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
-128*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/45*(3*(d*
x^3 + c)^(5/2)*d^12 + 35*(d*x^3 + c)^(3/2)*c*d^12 + 960*sqrt(d*x^3 + c)*c^
2*d^12)/d^15
```

Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{64\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^3} - \frac{1996\,c^2\,\sqrt{d\,x^3+c}}{45\,d^3} - \frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d} - \frac{82\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d^2}$$

input
```
int((x^8*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

output
```
(64*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/d^3 - (1996*c^2*(c + d*x^3)^(1/2))/(45*d^3) - (2*x^6*(c + d*x^3)^(1/2))
/(15*d) - (82*c*x^3*(c + d*x^3)^(1/2))/(45*d^2)
```

Reduce [F]

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{\frac{164\sqrt{d\,x^3+c}\,c^2}{45} - \frac{82\sqrt{d\,x^3+c}\,cd\,x^3}{45} - \frac{2\sqrt{d\,x^3+c}\,d^2x^6}{15} + 72\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{-d^2x^6+7cd\,x^3+8c^2}dx\right)c^2d^2}{d^3}$$

input
```
int(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

output
```
(2*(82*sqrt(c + d*x**3)*c**2 - 41*sqrt(c + d*x**3)*c*d*x**3 - 3*sqrt(c + d
*x**3)*d**2*x**6 + 1620*int((sqrt(c + d*x**3)*x**5)/(8*c**2 + 7*c*d*x**3 -
 d**2*x**6),x)*c**2*d**2))/(45*d**3)
```

3.459 $\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 69

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx=-\frac{16c\sqrt{c+dx^3}}{3d^2}-\frac{2(c+dx^3)^{3/2}}{9d^2}+\frac{16c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

output
```
-16/3*c*(d*x^3+c)^(1/2)/d^2-2/9*(d*x^3+c)^(3/2)/d^2+16*c^(3/2)*arctanh(1/3
*(d*x^3+c)^(1/2)/c^(1/2))/d^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx=-\frac{2\sqrt{c+dx^3}(25c+dx^3)}{9d^2}+\frac{16c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

input
```
Integrate[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
(-2*Sqrt[c + d*x^3]*(25*c + d*x^3))/(9*d^2) + (16*c^(3/2)*ArcTanh[Sqrt[c +
 d*x^3]/(3*Sqrt[c])])/d^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {948, 90, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3\sqrt{dx^3+c}}{8c-dx^3}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(\frac{8c\int \frac{\sqrt{dx^3+c}}{8c-dx^3}dx^3}{d}-\frac{2(c+dx^3)^{3/2}}{3d^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{8c\left(9c\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3-\frac{2\sqrt{c+dx^3}}{d}\right)}{d}-\frac{2(c+dx^3)^{3/2}}{3d^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{8c\left(\frac{18c\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)}{d}-\frac{2(c+dx^3)^{3/2}}{3d^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{8c\left(\frac{6\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)}{d}-\frac{2(c+dx^3)^{3/2}}{3d^2}\right)$$

input
```
Int[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
((-2*(c + d*x^3)^(3/2))/(3*d^2) + (8*c*((-2*Sqrt[c + d*x^3])/d + (6*Sqrt[c
]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d))/d)/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{16c^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)-\frac{2\left(d\,x^{3}+25c\right)\sqrt{d\,x^{3}+c}}{9}}{d^{2}}$
risch	$-\frac{2(d\,x^{3}+25c)\sqrt{d\,x^{3}+c}}{9d^{2}}+\frac{16c^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{d^{2}}$
default	$-\frac{2(d\,x^{3}+c)^{\frac{3}{2}}}{9d^{2}}+\frac{8c\left(-2\sqrt{d\,x^{3}+c}+6\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\right)}{3d^{2}}$
elliptic	$-\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d}-\frac{50c\sqrt{d\,x^{3}+c}}{9d^{2}}-\frac{8ic\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\,\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\cdots}}{\cdots}\right)}{\cdots}$

input
```
int(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
2/9*(72*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-(d*x^3+25*c)*(d*x^3+c
)^(1/2))/d^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.71

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx = \left[\frac{2\left(36\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(dx^3+25\,c)\sqrt{dx^3+c}\right)}{9\,d^2},\ -\frac{2\left(72\,\sqrt{-c}c\arctan\left(\frac{3\,\sqrt{-c}}{\sqrt{dx^3+c}}\right)+(dx^3+25\,c)\sqrt{dx^3+c}\right)}{9\,d^2}\right]$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[2/9*(36*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8
*c)) - (d*x^3 + 25*c)*sqrt(d*x^3 + c))/d^2, -2/9*(72*sqrt(-c)*c*arctan(3*s
qrt(-c)/sqrt(d*x^3 + c)) + (d*x^3 + 25*c)*sqrt(d*x^3 + c))/d^2]
```

Sympy [A] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{8c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{8c\sqrt{c+dx^3}}{3}-\frac{(c+dx^3)^{\frac{3}{2}}}{9}\right)}{d^2} & \text{for } d\neq 0 \\ \frac{x^6}{48\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-8*c**2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 8*c*s
qrt(c + d*x**3)/3 - (c + d*x**3)**(3/2)/9)/d**2, Ne(d, 0)), (x**6/(48*sqrt
(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\left(36\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+(dx^3+c)^{\frac{3}{2}}+24\,\sqrt{dx^3+c}c\right)}{9\,d^2}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-2/9*(36*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sq
rt(c))) + (d*x^3 + c)^(3/2) + 24*sqrt(d*x^3 + c)*c)/d^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{16\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d^2} - \frac{2\left((dx^3+c)^{\frac{3}{2}}d^4+24\,\sqrt{dx^3+c}cd^4\right)}{9\,d^6}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
-16*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 2/9*((d*x^3
+ c)^(3/2)*d^4 + 24*sqrt(d*x^3 + c)*c*d^4)/d^6
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{8\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^2} - \frac{50\,c\,\sqrt{d\,x^3+c}}{9\,d^2} - \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d}$$

input
```
int((x^5*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

output
```
(8*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/d^2 - (50*c*(c + d*x^3)^(1/2))/(9*d^2) - (2*x^3*(c + d*x^3)^(1/2))/(9*d)
```

Reduce [F]

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{4\sqrt{d\,x^3+c}\,c-2\sqrt{d\,x^3+c}\,d\,x^3+81\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{-d^2x^6+7cd\,x^3+8c^2}dx\right)c\,d^2}{9d^2}$$

input
```
int(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

output
```
(4*sqrt(c + d*x**3)*c - 2*sqrt(c + d*x**3)*d*x**3 + 81*int((sqrt(c + d*x**
3)*x**5)/(8*c**2 + 7*c*d*x**3 - d**2*x**6),x)*c*d**2)/(9*d**2)
```

3.460 $\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 50

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\sqrt{c+dx^3}}{3d}+\frac{2\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

output
```
-2/3*(d*x^3+c)^(1/2)/d+2*c^(1/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\left(\sqrt{c+dx^3}-3\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{3d}$$

input
```
Integrate[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
(-2*(Sqrt[c + d*x^3] - 3*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(3
*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {946, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{8c-dx^3}dx^3$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(9c\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3-\frac{2\sqrt{c+dx^3}}{d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{18c\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{6\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)$$

input
```
Int[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
((-2*Sqrt[c + d*x^3])/d + (6*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])
/d)/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

method	result
default	$\frac{-2\sqrt{d\,x^{3}+c}+6\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3d}$
pseudoelliptic	$\frac{-2\sqrt{d\,x^{3}+c}+6\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3d}$
risch	$-\frac{2\sqrt{d\,x^{3}+c}}{3d}+\frac{2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{d}$
elliptic	$-\frac{2\sqrt{d\,x^{3}+c}}{3d}-\frac{\left(i\sqrt{2}\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c \cdots\right.}}\cdots}{\cdots}\right)}{\cdots}$

input
```
int(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(-2*(d*x^3+c)^(1/2)+6*c^(1/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.96

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = \left[\frac{3\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-2\sqrt{dx^3+c}}{3\,d},\ -\frac{2\left(3\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)+\sqrt{dx^3+c}\right)}{3\,d}\right]$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[1/3*(3*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - 2*sqrt(d*x^3 + c))/d, -2/3*(3*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3
+ c)) + sqrt(d*x^3 + c))/d]
```

Sympy [A] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{c+dx^3}}{3}\right)}{d} & \text{for}\ d\neq 0\\ \frac{x^3}{24\sqrt{c}} & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - sqrt(c + d
*x**3)/3)/d, Ne(d, 0)), (x**3/(24*sqrt(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{3\,\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+2\,\sqrt{dx^3+c}}{3\,d}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + 2*sqrt(d*x^3 + c))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d} - \frac{2\,\sqrt{dx^3+c}}{3\,d}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
-2*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 2/3*sqrt(d*x^3 +
c)/d
```

Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d} - \frac{2\,\sqrt{d\,x^3+c}}{3\,d}$$

input
```
int((x^2*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

output
```
(c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/
d - (2*(c + d*x^3)^(1/2))/(3*d)
```

Reduce [F]

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{2\sqrt{d\,x^3+c}+27\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{-d^2x^6+7cd\,x^3+8c^2}dx\right)d^2}{24d}$$

input
```
int(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

output
```
(2*sqrt(c + d*x**3) + 27*int((sqrt(c + d*x**3)*x**5)/(8*c**2 + 7*c*d*x**3
- d**2*x**6),x)*d**2)/(24*d)
```

3.461 $\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)} dx$

Optimal result

Integrand size = 27, antiderivative size = 58

$$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

output
```
1/4*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-1/12*arctanh((d*x^3+c)^(1
/2)/c^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)} dx = \frac{3\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - \operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)),x]
```

output
```
(3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]
)/(12*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {948, 94, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^3(8c-dx^3)}dx^3$$

$$\Big\downarrow \text{94}$$

$$\frac{1}{3}\left(\frac{1}{8}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{9}{8}d\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{9}{4}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{4d}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{4d}+\frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{4\sqrt{c}}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)),x]
```

output
```
((3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(4*Sqrt[c]) - ArcTanh[Sqrt[c + d
*x^3]/Sqrt[c]]/(4*Sqrt[c]))/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$-\frac{-3 \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{12 \sqrt{c}}$	38
default	$\frac{\frac{2 \sqrt{d\,x^{3}+c}}{3}-\frac{2 \sqrt{c}\, \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}}{8 c}+\frac{-2 \sqrt{d\,x^{3}+c}+6 \sqrt{c}\, \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{24 c}$	75
elliptic	Expression too large to display	1502

input
```
int((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))+arctanh((d*x^3+c)^(1/2)/c^(
1/2)))/c^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.29

$$\int \frac{\sqrt{c+dx^3}}{x\,(8c-dx^3)}\,dx = \left[\frac{3\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{24\,c},\ -\frac{3\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)-\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)}{12\,c}\right]$$

input
```
integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8
*c)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c, -1/1
2*(3*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) - sqrt(-c)*arctan(sqrt(-c
)/sqrt(d*x^3 + c)))/c]
```

Sympy [A] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{c+dx^3}}{x\,(8c-dx^3)}\,dx = \begin{cases} \frac{2\left(-\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{8\sqrt{-c}}+\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{24\sqrt{-c}}\right)}{d} & \text{for}\ d\neq 0\\ \frac{\log\left(x^3\right)}{24\sqrt{c}} & \text{otherwise}\end{cases}$$

input
```
integrate((d*x**3+c)**(1/2)/x/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-d*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(8*sqrt(-c)) + d*atan
(sqrt(c + d*x**3)/sqrt(-c))/(24*sqrt(-c)))/d, Ne(d, 0)), (log(x**3)/(24*sq
rt(c)), True))
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x\,(8c-dx^3)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{c+dx^3}}{x\,(8c-dx^3)}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}} - \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{4\sqrt{-c}}$$

input
```
integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 1/4*arctan(1/3*sqrt(d*x^3
 + c)/sqrt(-c))/sqrt(-c)
```

Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.16

$$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\left(6\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}\right)^3\left(24\,c^2-24\,c^{3/2}\,\sqrt{d\,x^3+c}+d^2\,x^6-20\,c\,d\,x^3\right)^3}{x^{15}\,\left(8\,c-d\,x^3\right)^3\,\left(24\,c-d\,x^3\right)^3}\right)}{24\,\sqrt{c}}$$

input
```
int((c + d*x^3)^(1/2)/(x*(8*c - d*x^3)),x)
```

output
```
log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))*(6*c +
d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))^3*(24*c^2 - 24*c^(3/2)*(c + d*x^3)^(1
/2) + d^2*x^6 - 20*c*d*x^3)^3)/(x^15*(8*c - d*x^3)^3*(24*c - d*x^3)^3))/(2
4*c^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)}\,dx = \frac{\sqrt{c}\log\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)-\sqrt{c}\log\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)+27\left(\int \frac{\sqrt{d\,x^3+c}\,x^2}{-d^2x^6+7cd\,x^3+8c^2}dx\right)cd}{24c}$$

input
```
int((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x)
```

output
```
(sqrt(c)*log(sqrt(c + d*x**3) - sqrt(c)) - sqrt(c)*log(sqrt(c + d*x**3) +
sqrt(c)) + 27*int((sqrt(c + d*x**3)*x**2)/(8*c**2 + 7*c*d*x**3 - d**2*x**6
),x)*c*d)/(24*c)
```

3.462 $\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 81

$$\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{24cx^3}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}}-\frac{5d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}$$

output
```
-1/24*(d*x^3+c)^(1/2)/c/x^3+1/32*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^
(3/2)-5/96*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{24cx^3}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}}-\frac{5d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)),x]
```

output
```
-1/24*Sqrt[c + d*x^3]/(c*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(
32*c^(3/2)) - (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*c^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {948, 110, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^6\left(8c-dx^3\right)}dx^3$$

$$\Big\downarrow \text{ 110}$$

$$\frac{1}{3}\left(\frac{\int \frac{d(dx^3+10c)}{2x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{8c}-\frac{\sqrt{c+dx^3}}{8cx^3}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{d\int \frac{dx^3+10c}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3}\right)$$

$$\Big\downarrow \text{ 174}$$

$$\frac{1}{3}\left(\frac{d\left(\frac{5}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{9}{4}d\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3\right)}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(\frac{d\left(\frac{9}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{5\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}\right)}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{d\left(\frac{5\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(\frac{d\left(\frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}-\frac{5\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)),x]
```

output
```
(-1/8*Sqrt[c + d*x^3]/(c*x^3) + (d*((3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/(2*Sqrt[c]) - (5*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c])))/(16*c)
)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}}{24c\,x^{3}}+\frac{d\left(-\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6\sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)}{16c}$
pseudoelliptic	$\frac{-5d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)x^{3}+3\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)d\,x^{3}-4\sqrt{d\,x^{3}+c}\,\sqrt{c}}{96c^{\frac{3}{2}}\,x^{3}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3x^{3}}-\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}}{8c}+\frac{d\left(\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}\right)}{64c^{2}}+\frac{d\left(-2\sqrt{d\,x^{3}+c}+6\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+}}{3\sqrt{c}}\right.\right.}{192c^{2}}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(d*x^3+c)^(1/2)/c/x^3+1/16/c*d*(-5/6*arctanh((d*x^3+c)^(1/2)/c^(1/2)
)/c^(1/2)+1/2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{c+dx^3}}{x^4\,(8c-dx^3)}\,dx$$
$$=\left[\frac{3\,\sqrt{c}dx^{3}\log\left(\frac{dx^{3}+6\,\sqrt{dx^{3}+c}\sqrt{c}+10\,c}{dx^{3}-8\,c}\right)+5\,\sqrt{c}dx^{3}\log\left(\frac{dx^{3}-2\,\sqrt{dx^{3}+c}\sqrt{c}+2\,c}{x^{3}}\right)-8\,\sqrt{dx^{3}+c}c}{192\,c^{2}x^{3}},\right.$$
$$\left.-\frac{3\,\sqrt{-c}dx^{3}\arctan\left(\frac{3\,\sqrt{-c}}{\sqrt{dx^{3}+c}}\right)-5\,\sqrt{-c}dx^{3}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{3}+c}}\right)+4\,\sqrt{dx^{3}+c}c}{96\,c^{2}x^{3}}\right]$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[1/192*(3*sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*
x^3 - 8*c)) + 5*sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c
)/x^3) - 8*sqrt(d*x^3 + c)*c)/(c^2*x^3), -1/96*(3*sqrt(-c)*d*x^3*arctan(3*
sqrt(-c)/sqrt(d*x^3 + c)) - 5*sqrt(-c)*d*x^3*arctan(sqrt(-c)/sqrt(d*x^3 +
c)) + 4*sqrt(d*x^3 + c)*c)/(c^2*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)}\,dx = -\int \frac{\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**4/(-d*x**3+8*c),x)
```

output
```
-Integral(sqrt(c + d*x**3)/(-8*c*x**4 + d*x**7), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^4}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)}\,dx=\frac{5\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\,\sqrt{-c}c}-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{32\,\sqrt{-c}c}-\frac{\sqrt{dx^3+c}}{24\,cx^3}$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
5/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/32*d*arctan(1/3*s
qrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/24*sqrt(d*x^3 + c)/(c*x^3)
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)}\,dx=\frac{d\,\mathrm{atanh}\left(\frac{c\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{32\,\sqrt{c^3}}-\frac{5\,d\,\mathrm{atanh}\left(\frac{c\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)}{96\,\sqrt{c^3}}-\frac{\sqrt{d\,x^3+c}}{24\,c\,x^3}$$

input
```
int((c + d*x^3)^(1/2)/(x^4*(8*c - d*x^3)),x)
```

output
```
(d*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2))))/(32*(c^3)^(1/2)) - (5*d*a
tanh((c*(c + d*x^3)^(1/2))/(c^3)^(1/2)))/(96*(c^3)^(1/2)) - (c + d*x^3)^(1
/2)/(24*c*x^3)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)}\,dx$$

$$=\frac{-256\sqrt{d\,x^3+c}\,c+18\sqrt{d\,x^3+c}\,d\,x^3+160\sqrt{c}\log\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)d\,x^3-160\sqrt{c}\log\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)d}{6144c^2x^3}$$

input
```
int((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x)
```

output

```
( - 256*sqrt(c + d*x**3)*c + 18*sqrt(c + d*x**3)*d*x**3 + 160*sqrt(c)*log(
sqrt(c + d*x**3) - sqrt(c))*d*x**3 - 160*sqrt(c)*log(sqrt(c + d*x**3) + sq
rt(c))*d*x**3 + 27*int((sqrt(c + d*x**3)*x**5)/(8*c**2 + 7*c*d*x**3 - d**2
*x**6),x)*d**3*x**3 + 648*int((sqrt(c + d*x**3)*x**2)/(8*c**2 + 7*c*d*x**3
 - d**2*x**6),x)*c*d**2*x**3)/(6144*c**2*x**3)
```

3.463 $\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 107

$$\int \frac{\sqrt{c+dx^3}}{x^7\,(8c-dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} + \frac{d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}} + \frac{d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}$$

output
```
-1/48*(d*x^3+c)^(1/2)/c/x^6-1/64*d*(d*x^3+c)^(1/2)/c^2/x^3+1/256*d^2*arcta
nh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/256*d^2*arctanh((d*x^3+c)^(1/2)/
c^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{c+dx^3}}{x^7\,(8c-dx^3)}\,dx = \frac{(-4c-3dx^3)\sqrt{c+dx^3}}{192c^2x^6} + \frac{d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}} + \frac{d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)),x]
```

output
```
((-4*c - 3*d*x^3)*Sqrt[c + d*x^3])/(192*c^2*x^6) + (d^2*ArcTanh[Sqrt[c + d
*x^3]/(3*Sqrt[c])])/(256*c^(5/2)) + (d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(256*c^(5/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 110, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^9\left(8c-dx^3\right)}dx^3$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}\left(\frac{\int \frac{3d\left(dx^3+4c\right)}{2x^6\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{16c}-\frac{\sqrt{c+dx^3}}{16cx^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{3d\int \frac{dx^3+4c}{x^6\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(\frac{3d\left(-\frac{\int \frac{2cd\left(2c-dx^3\right)}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{8c^2}-\frac{\sqrt{c+dx^3}}{2cx^3}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{3d\left(-\frac{d\int \frac{2c-dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{4c}-\frac{\sqrt{c+dx^3}}{2cx^3}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6}\right)$$

$\Big\downarrow$ 174

$$\frac{1}{3}\left(\frac{3d\left(-\frac{d\left(\frac{1}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3-\frac{3}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3\right)}{4c}-\frac{\sqrt{c+dx^3}}{2cx^3}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6}\right)$$

$\Big\downarrow$ 73

$$\frac{1}{3}\left(\frac{3d\left(-\frac{d\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{3}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}\right)}{4c}-\frac{\sqrt{c+dx^3}}{2cx^3}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6}\right)$$

$\Big\downarrow$ 219

$$\frac{1}{3}\left(\frac{3d\left(-\frac{d\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)}{4c}-\frac{\sqrt{c+dx^3}}{2cx^3}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6}\right)$$

$\Big\downarrow$ 221

$$\frac{1}{3}\left(\frac{3d\left(-\frac{d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)}{4c}-\frac{\sqrt{c+dx^3}}{2cx^3}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)),x]
```

output
```
(-1/16*Sqrt[c + d*x^3]/(c*x^6) + (3*d*(-1/2*Sqrt[c + d*x^3]/(c*x^3) - (d*(
-1/2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/Sqrt[c] - ArcTanh[Sqrt[c + d*x^3
]/Sqrt[c]]/(2*Sqrt[c])))/(4*c)))/(32*c))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}\,(3d\,x^{3}+4c)}{192c^{2}x^{6}}-\frac{3d^{2}\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6\sqrt{c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{6\sqrt{c}}\right)}{128c^{2}}$
pseudoelliptic	$\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d^{2}x^{6}+3\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)d^{2}x^{6}-12d\,x^{3}\sqrt{d\,x^{3}+c}\,\sqrt{c}-16\sqrt{d\,x^{3}+c}\,c^{\frac{3}{2}}}{768c^{\frac{5}{2}}x^{6}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{6x^{6}}-\frac{d\sqrt{d\,x^{3}+c}}{12c\,x^{3}}+\frac{d^{2}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{12c^{\frac{3}{2}}}}{8c}+\frac{d\left(-\frac{\sqrt{d\,x^{3}+c}}{3x^{3}}-\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}\right)}{64c^{2}}+\frac{d^{2}\left(\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\sqrt{c}\,\operatorname{arctanh}\cdots}{3\cdots}\right.}{512c^{3}}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(d*x^3+c)^(1/2)*(3*d*x^3+4*c)/c^2/x^6-3/128*d^2/c^2*(-1/6*arctanh((
d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-1/6*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{c+dx^3}}{x^7\,(8c-dx^3)}\,dx = \left[\frac{3\,\sqrt{c}d^2x^6\log\left(\frac{d^2x^6+24\,cdx^3+8\,(dx^3+4\,c)\sqrt{dx^3+c}\sqrt{c}+32\,c^2}{dx^6-8\,cx^3}\right)-8\,(3\,cdx^3+4\,c^2)\sqrt{dx^3+c}}{1536\,c^3x^6},\ -\frac{3\,\sqrt{-c}d^2x^6\arctan\left(\frac{(dx^3+4\,c)\sqrt{dx^3+c}\sqrt{-c}}{4\,(cdx^3+c^2)}\right)+4\,(3\,cdx^3+4\,c^2)\sqrt{dx^3+c}}{768\,c^3x^6}\right]$$

input
```
integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[1/1536*(3*sqrt(c)*d^2*x^6*log((d^2*x^6 + 24*c*d*x^3 + 8*(d*x^3 + 4*c)*sqr
t(d*x^3 + c)*sqrt(c) + 32*c^2)/(d*x^6 - 8*c*x^3)) - 8*(3*c*d*x^3 + 4*c^2)*
sqrt(d*x^3 + c))/(c^3*x^6), -1/768*(3*sqrt(-c)*d^2*x^6*arctan(1/4*(d*x^3 +
 4*c)*sqrt(d*x^3 + c)*sqrt(-c)/(c*d*x^3 + c^2)) + 4*(3*c*d*x^3 + 4*c^2)*sq
rt(d*x^3 + c))/(c^3*x^6)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}\,dx = -\int \frac{\sqrt{c+dx^3}}{-8cx^7+dx^{10}}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**7/(-d*x**3+8*c),x)
```

output
```
-Integral(sqrt(c + d*x**3)/(-8*c*x**7 + d*x**10), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^7}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^7), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)}\,dx = -\frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{256\sqrt{-c}c^2} - \frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{256\sqrt{-c}c^2} - \frac{3\left(dx^3+c\right)^{\frac{3}{2}}d^2+\sqrt{dx^3+c}cd^2}{192\,c^2d^2x^6}$$

input
```
integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
-1/256*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/256*d^2*arc
tan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/192*(3*(d*x^3 + c)^(3
/2)*d^2 + sqrt(d*x^3 + c)*c*d^2)/(c^2*d^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)}\,dx = \frac{d^2\,\mathrm{atanh}\left(\frac{d^4\sqrt{d\,x^3+c}}{2048\,c^{7/2}\left(\frac{d^4}{2048\,c^3}+\frac{d^5\,x^3}{8192\,c^4}\right)}\right)}{256\,c^{5/2}} - \frac{\sqrt{d\,x^3+c}}{192\,c\,x^6} - \frac{\left(d\,x^3+c\right)^{3/2}}{64\,c^2\,x^6}$$

input
```
int((c + d*x^3)^(1/2)/(x^7*(8*c - d*x^3)),x)
```

output
```
(d^2*atanh((d^4*(c + d*x^3)^(1/2))/(2048*c^(7/2)*(d^4/(2048*c^3) + (d^5*x^
3)/(8192*c^4)))))/(256*c^(5/2)) - (c + d*x^3)^(1/2)/(192*c*x^6) - (c + d*x
^3)^(3/2)/(64*c^2*x^6)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^7\,(8c-dx^3)}\,dx$$
$$= \frac{-10816\sqrt{d\,x^3+c}\,c^2-6960\sqrt{d\,x^3+c}\,cd\,x^3+630\sqrt{d\,x^3+c}\,d^2x^6-960\sqrt{c}\log\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)d^2x^6+9}{}$$

input
```
int((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x)
```

output
```
( - 10816*sqrt(c + d*x**3)*c**2 - 6960*sqrt(c + d*x**3)*c*d*x**3 + 630*sqr
t(c + d*x**3)*d**2*x**6 - 960*sqrt(c)*log(sqrt(c + d*x**3) - sqrt(c))*d**2
*x**6 + 960*sqrt(c)*log(sqrt(c + d*x**3) + sqrt(c))*d**2*x**6 - 27648*int(
sqrt(c + d*x**3)/(8*c**2*x**7 + 7*c*d*x**10 - d**2*x**13),x)*c**4*x**6 + 9
072*int(sqrt(c + d*x**3)/(8*c**2*x + 7*c*d*x**4 - d**2*x**7),x)*c**2*d**2*
x**6 + 945*int((sqrt(c + d*x**3)*x**5)/(8*c**2 + 7*c*d*x**3 - d**2*x**6),x
)*d**4*x**6)/(491520*c**3*x**6)
```

3.464 $\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 648

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{214cx^2\sqrt{c+dx^3}}{91d^2}-\frac{2x^5\sqrt{c+dx^3}}{13d}-\frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{32\sqrt{3}c^{13/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}+\frac{32c^{13/6}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}-\frac{32c^{13/6}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}}+\frac{6124\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{91d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{12248\sqrt{2}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{91\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-214/91*c*x^2*(d*x^3+c)^(1/2)/d^2-2/13*x^5*(d*x^3+c)^(1/2)/d-12248/91*c^2*
(d*x^3+c)^(1/2)/d^(8/3)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)-32*3^(1/2)*c^(13/6
)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))/d^(8/3)+32*c
^(13/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(8/3)
-32*c^(13/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)+6124/91*3^(1/4)*
(1/2*6^(1/2)-1/2*2^(1/2))*c^(7/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^
(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((
1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*
I)/d^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)
^(1/2)/(d*x^3+c)^(1/2)-12248/273*2^(1/2)*c^(7/3)*(c^(1/3)+d^(1/3)*x)*((c^(
2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/
2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*
x),I*3^(1/2)+2*I)*3^(3/4)/d^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2)
)*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.38 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.23

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{-20(107c^2x^2+114cdx^5+7d^2x^8)+2140c^2x^2\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+1531cdx^5\sqrt{1+\cdots}}{910d^2\sqrt{c+dx^3}}$$

input
```
Integrate[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
(-20*(107*c^2*x^2 + 114*c*d*x^5 + 7*d^2*x^8) + 2140*c^2*x^2*Sqrt[1 + (d*x^
3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 1531*c*d*x
^5*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8
*c)])/(910*d^2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {978, 27, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}dx$$

$$\Big\downarrow \text{ 978}$$

$$\frac{2\int \frac{cx^4(107dx^3+80c)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{13d}-\frac{2x^5\sqrt{c+dx^3}}{13d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c\int \frac{x^4(107dx^3+80c)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{13d}-\frac{2x^5\sqrt{c+dx^3}}{13d}$$

$$\Big\downarrow \text{ 1052}$$

$$\frac{c\left(\frac{2\int \frac{2cdx\left(1531dx^3+856c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d^2}-\frac{214x^2\sqrt{c+dx^3}}{7d}\right)}{13d}-\frac{2x^5\sqrt{c+dx^3}}{13d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c\left(\frac{4c\int \frac{x\left(1531dx^3+856c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d}-\frac{214x^2\sqrt{c+dx^3}}{7d}\right)}{13d}-\frac{2x^5\sqrt{c+dx^3}}{13d}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{c\left(\frac{4c\int \left(\frac{13104cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}-\frac{1531x}{\sqrt{dx^3+c}}\right)dx}{7d}-\frac{214x^2\sqrt{c+dx^3}}{7d}\right)}{13d}-\frac{2x^5\sqrt{c+dx^3}}{13d}$$

$$\Big\downarrow \text{ 2009}$$

$$c\left(\frac{4c\left(-\frac{3062\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{1531\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{2x^5\sqrt{c+dx^3}}{13d}$$

input
```
Int[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
(-2*x^5*Sqrt[c + d*x^3])/(13*d) + (c*((-214*x^2*Sqrt[c + d*x^3])/(7*d) + (
4*c*((-3062*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x))
 - (728*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqr
t[c + d*x^3]])/d^(2/3) + (728*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c
^(1/6)*Sqrt[c + d*x^3])])/d^(2/3) - (728*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(
3*Sqrt[c])])/d^(2/3) + (1531*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) +
d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)
*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(
c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3]) - (3062*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) -
 c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) +
 d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(7*
d))/(13*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.27 (sec) , antiderivative size = 884, normalized size of antiderivative = 1.36

method	result	size
risch	Expression too large to display	884
elliptic	Expression too large to display	889
default	Expression too large to display	1788

input
```
int(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-2/91*x^2*(7*d*x^3+107*c)*(d*x^3+c)^(1/2)/d^2-4/91/d^2*c^2*(-3062/3*I*3^(1
/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3
+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Elliptic
E(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1456/3*I/d^3*2^(1/2)*sum
(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^
(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^
2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)
^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1
/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2442 vs. $2(462) = 924$.

Time = 15.99 (sec) , antiderivative size = 2442, normalized size of antiderivative = 3.77

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
2/273*(728*d^3*(c^13/d^16)^(1/6)*log(33554432*((d^16*x^9 + 318*c*d^15*x^6
+ 1200*c^2*d^14*x^3 + 640*c^3*d^13)*(c^13/d^16)^(5/6) + 6*(c^11*d^2*x^7 +
80*c^12*d*x^4 + 160*c^13*x + 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2)*(c^13/d^
16)^(2/3) + (7*c^7*d^7*x^6 + 152*c^8*d^6*x^3 + 64*c^9*d^5)*(c^13/d^16)^(1/
3))*sqrt(d*x^3 + c) + 18*(5*c^5*d^10*x^7 + 64*c^6*d^9*x^4 + 32*c^7*d^8*x)*
sqrt(c^13/d^16) + 18*(c^9*d^5*x^8 + 38*c^10*d^4*x^5 + 64*c^11*d^3*x^2)*(c^
13/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 728*
d^3*(c^13/d^16)^(1/6)*log(-33554432*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2
*d^14*x^3 + 640*c^3*d^13)*(c^13/d^16)^(5/6) - 6*(c^11*d^2*x^7 + 80*c^12*d*
x^4 + 160*c^13*x + 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2)*(c^13/d^16)^(2/3)
+ (7*c^7*d^7*x^6 + 152*c^8*d^6*x^3 + 64*c^9*d^5)*(c^13/d^16)^(1/3))*sqrt(d
*x^3 + c) + 18*(5*c^5*d^10*x^7 + 64*c^6*d^9*x^4 + 32*c^7*d^8*x)*sqrt(c^13/
d^16) + 18*(c^9*d^5*x^8 + 38*c^10*d^4*x^5 + 64*c^11*d^3*x^2)*(c^13/d^16)^(
1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 18372*c^2*sqrt
(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 364*(s
qrt(-3)*d^3 - d^3)*(c^13/d^16)^(1/6)*log(33554432*((d^16*x^9 + 318*c*d^15*
x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x
^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c^13/d^16)^(5/6) + 6*(2*c^11*d^2*
x^7 + 160*c^12*d*x^4 + 320*c^13*x - 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2 -
sqrt(-3)*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2))*(c^13/d^16)^(2/3) - (7*c^7...
```

Sympy [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\int \frac{x^7\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

input
```
integrate(x**7*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

output
```
-Integral(x**7*sqrt(c + d*x**3)/(-8*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx = \int -\frac{\sqrt{dx^3+c}x^7}{dx^3-8\,c}\,dx$$

input
```
integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c), x)
```

Giac [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx = \int -\frac{\sqrt{dx^3+c}x^7}{dx^3-8\,c}\,dx$$

input
```
integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx=\int \frac{x^7\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\,dx$$

input
```
int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

output
```
int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3), x)
```

Reduce [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{-\frac{214\sqrt{d\,x^3+c}\,c\,x^2}{91}-\frac{2\sqrt{d\,x^3+c}\,d\,x^5}{13}+\frac{6124\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{-d^2x^6+7cd\,x^3+8c^2}dx\right)c^2d}{91}+\frac{3424\left(\int \frac{\sqrt{d\,x^3+c}\,x}{-d^2x^6+7cd\,x^3+8c^2}dx\right)c^3}{91}}{d^2}$$

input
```
int(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

output
```
(2*( - 107*sqrt(c + d*x**3)*c*x**2 - 7*sqrt(c + d*x**3)*d*x**5 + 3062*int(
(sqrt(c + d*x**3)*x**4)/(8*c**2 + 7*c*d*x**3 - d**2*x**6),x)*c**2*d + 1712
*int((sqrt(c + d*x**3)*x)/(8*c**2 + 7*c*d*x**3 - d**2*x**6),x)*c**3))/(91*
d**2)
```

3.465 $\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 624

$$\begin{aligned}\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx =& -\frac{2x^2\sqrt{c+dx^3}}{7d}-\frac{118c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\ & -\frac{4\sqrt{3}c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} \\ & +\frac{4c^{7/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}}-\frac{4c^{7/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}} \\ & +\frac{59\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\ & -\frac{118\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\end{aligned}$$

output
```
-2/7*x^2*(d*x^3+c)^(1/2)/d-118/7*c*(d*x^3+c)^(1/2)/d^(5/3)/((1+3^(1/2))*c^
(1/3)+d^(1/3)*x)-4*3^(1/2)*c^(7/6)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)
*x)/(d*x^3+c)^(1/2))/d^(5/3)+4*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c
^(1/6)/(d*x^3+c)^(1/2))/d^(5/3)-4*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1
/2))/d^(5/3)+59/7*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*c^(4/3)*(c^(1/3)+d^(1/
3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3
)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/
3)+d^(1/3)*x),I*3^(1/2)+2*I)/d^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1
/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)-118/21*2^(1/2)*c^(4/3)*(c
^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^
(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^
(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/d^(5/3)/(c^(1/3)*(c^(1/3)
+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.47 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.21

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{x^2\left(-80(c+dx^3)+80c\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+59dx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3}\right.\right.}{280d\sqrt{c+dx^3}}$$

input
```
Integrate[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
(x^2*(-80*(c + d*x^3) + 80*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3
, -((d*x^3)/c), (d*x^3)/(8*c)] + 59*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3
, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(280*d*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {978, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}dx$$

$$\Big\downarrow \text{978}$$

$$\frac{2\int \frac{cx(59dx^3+32c)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{7d}-\frac{2x^2\sqrt{c+dx^3}}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{x(59dx^3+32c)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{7d}-\frac{2x^2\sqrt{c+dx^3}}{7d}$$

$$\Big\downarrow \text{1054}$$

$$\frac{c\int \left(\frac{504cx}{(8c-dx^3)\sqrt{dx^3+c}}-\frac{59x}{\sqrt{dx^3+c}}\right)dx}{7d}-\frac{2x^2\sqrt{c+dx^3}}{7d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\left(-\frac{118\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{59\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\cdots\right.}{\cdots}\right.}{\cdots}$$

$$\frac{2x^2\sqrt{c+dx^3}}{7d}$$

input
```
Int[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
(-2*x^2*Sqrt[c + d*x^3])/(7*d) + (c*((-118*Sqrt[c + d*x^3])/(d^(2/3)*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (28*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1
/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(2/3) + (28*c^(1/6)*ArcTanh
[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(2/3) - (28*c^(1/
6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(2/3) + (59*3^(1/4)*Sqrt[2 - Sq
rt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4
*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (118*Sqrt[2]*c^(1/3)*(c^(1/3) + d
^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*
x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)
*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^
2]*Sqrt[c + d*x^3])))/(7*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.79 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.39

method	result	size
elliptic	Expression too large to display	867
risch	Expression too large to display	872
default	Expression too large to display	1310

input
```
int(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-2/7*x^2*(d*x^3+c)^(1/2)/d+118/21*I*c/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)))-8/3*I*c/d^4*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I
*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1
/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))
)^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(
1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))
*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alph
a^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2428 vs. $2(442) = 884$.

Time = 4.32 (sec) , antiderivative size = 2428, normalized size of antiderivative = 3.89

$$\int \frac{x^4\sqrt{c + dx^3}}{8c - dx^3}\,dx = \text{Too large to display}$$

input

```
integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output

```
-1/21*(6*sqrt(d*x^3 + c)*d*x^2 - 14*d^2*(c^7/d^10)^(1/6)*log(1024*((d^11*x
^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8)*(c^7/d^10)^(5/6) + 6
*(c^6*d^2*x^7 + 80*c^7*d*x^4 + 160*c^8*x + 6*(5*c^2*d^8*x^5 + 32*c^3*d^7*x
^2)*(c^7/d^10)^(2/3) + (7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3)*(c^7
/d^10)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c^3*d^7*x^7 + 64*c^4*d^6*x^4 + 32*c^
5*d^5*x)*sqrt(c^7/d^10) + 18*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^
2)*(c^7/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) +
 14*d^2*(c^7/d^10)^(1/6)*log(-1024*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*
d^9*x^3 + 640*c^3*d^8)*(c^7/d^10)^(5/6) - 6*(c^6*d^2*x^7 + 80*c^7*d*x^4 +
160*c^8*x + 6*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2)*(c^7/d^10)^(2/3) + (7*c^4*d
^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3)*(c^7/d^10)^(1/3))*sqrt(d*x^3 + c) +
 18*(5*c^3*d^7*x^7 + 64*c^4*d^6*x^4 + 32*c^5*d^5*x)*sqrt(c^7/d^10) + 18*(c
^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2)*(c^7/d^10)^(1/6))/(d^3*x^9 -
 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 354*c*sqrt(d)*weierstrassZeta(
0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 7*(sqrt(-3)*d^2 - d^2)*(c^
7/d^10)^(1/6)*log(1024*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 64
0*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c
^3*d^8))*(c^7/d^10)^(5/6) + 6*(2*c^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x -
 6*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2 - sqrt(-3)*(5*c^2*d^8*x^5 + 32*c^3*d^7*
x^2))*(c^7/d^10)^(2/3) - (7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3 ...
```

Sympy [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\int \frac{x^4\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

input
```
integrate(x**4*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

output
```
-Integral(x**4*sqrt(c + d*x**3)/(-8*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx = \int -\frac{\sqrt{dx^3+c}x^4}{dx^3-8\,c}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx = \int -\frac{\sqrt{dx^3+c}x^4}{dx^3-8\,c}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx=\int \frac{x^4\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\,dx$$

input
```
int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

output
```
int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx$$
$$=\frac{-2\sqrt{d\,x^3+c}\,x^2+59\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{-d^2x^6+7cd\,x^3+8c^2}dx\right)cd+32\left(\int \frac{\sqrt{d\,x^3+c}\,x}{-d^2x^6+7cd\,x^3+8c^2}dx\right)c^2}{7d}$$

input
```
int(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

output
```
( - 2*sqrt(c + d*x**3)*x**2 + 59*int((sqrt(c + d*x**3)*x**4)/(8*c**2 + 7*c
*d*x**3 - d**2*x**6),x)*c*d + 32*int((sqrt(c + d*x**3)*x)/(8*c**2 + 7*c*d*
x**3 - d**2*x**6),x)*c**2)/(7*d)
```

3.466 $\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 601

$$\begin{aligned}\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx =& -\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt{3}\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}}\\
&+\frac{\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{2d^{2/3}}-\frac{\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2d^{2/3}}\\
&+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\\
&-\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\end{aligned}$$

output
```
-2*(d*x^3+c)^(1/2)/d^(2/3)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)-1/2*3^(1/2)*c^(
1/6)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))/d^(2/3)+1
/2*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(2
/3)-1/2*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(2/3)+3^(1/4)*(1/2*
6^(1/2)-1/2*2^(1/2))*c^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)
*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(
1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/d^
(2/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2
)/(d*x^3+c)^(1/2)-2/3*2^(1/2)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3
)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*Elliptic
F(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2
)+2*I)*3^(3/4)/d^(2/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d
^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.90 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.10

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{x^2\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{16\sqrt{c+dx^3}}$$

input
```
Integrate[(x*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
(x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, -1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)
/(8*c)])/(16*Sqrt[c + d*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 2.95 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {984, 832, 759, 988, 946, 73, 219, 2416, 2563, 219, 2570, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx$$

$$\Big\downarrow \; 984$$

$$9c\int \frac{x}{(8c-dx^3)\sqrt{dx^3+c}}dx-\int \frac{x}{\sqrt{dx^3+c}}dx$$

$$\Big\downarrow \; 832$$

$$\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}\int \frac{1}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}+9c\int \frac{x}{(8c-dx^3)\sqrt{dx^3+c}}dx$$

$$\Big\downarrow \; 759$$

$$-\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}+9c\int \frac{x}{(8c-dx^3)\sqrt{dx^3+c}}dx+\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 988$$

$$9c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\sqrt[3]{d}\int \frac{x^2}{(8c-dx^3)\sqrt{dx^3+c}}dx}{4\sqrt[3]{c}}\right)-\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}+\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 946$$

$$9c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\sqrt[3]{d}\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{12\sqrt[3]{c}}\right)-$$

$$\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}+$$

$$\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 73$$

$$9c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{6\sqrt[3]{c}d^{2/3}}\right)-$$

$$\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}+$$

$$\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 219$$

$$9c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)-$$

$$\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}+\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$\Big\downarrow$ 2416

$$9c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)+$$

$$\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}$$

$\Big\downarrow$ 2563

$$9c\left(\frac{\int \frac{1}{9-\frac{\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)^4}{\sqrt[3]{c}(dx^3+c)}}d\frac{\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)^2}{c^{2/3}\sqrt{dx^3+c}}}{6\sqrt[3]{c}d^{2/3}}-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)+$$

$$\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}$$

$$\Big\downarrow \text{219}$$

$$9c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)+$$

$$\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}$$

$\big\downarrow$ 2570

$$9c\left(\frac{d^{4/3}\int \frac{1}{-\frac{2d^2}{c}-\frac{6\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)^2 d^2}{c^{2/3}(dx^3+c)}}d\frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\sqrt[3]{c}\sqrt{dx^3+c}}}{3c^{4/3}}+\frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)+\frac{\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\;}$$

$$\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}$$

$\big\downarrow$ 218

$$\frac{\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{}$$

$$\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}+$$

$$9c\left(-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}}+\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)$$

input

```
Int[(x*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output

```
9*c*(-1/6*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]]/
(Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqr
t[c + d*x^3])]/(18*c^(5/6)*d^(2/3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]
/(18*c^(5/6)*d^(2/3))) - ((2*Sqrt[c + d*x^3])/(d^(1/3)*((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*
x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(1/3)*Sqrt[(c^(1/3)*
(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x
^3]))/d^(1/3) + (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1
/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sq
rt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 988
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[d*(q/(4*b))   Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x
^3]), x], x] + (-Simp[q^2/(12*b)   Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]
), x], x] + Simp[1/(12*b*c)   Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x +
 q^2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[8*b*c + a*d, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2570
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-2*g*h   Subst[Int[1/(2*e*h -
 (b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8
*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.04 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.41

method	result	size
default	Expression too large to display	848
elliptic	Expression too large to display	848

input

```
int(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output

```
2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/3*I/d^3*2^(1
/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-
c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I
*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(
-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alph
a*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2194 vs. $2(424) = 848$.

Time = 1.17 (sec) , antiderivative size = 2194, normalized size of antiderivative = 3.65

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
-1/24*((sqrt(-3)*d - d)*(c/d^4)^(1/6)*log(1/4*((d^6*x^9 + 318*c*d^5*x^6 +
1200*c^2*d^4*x^3 + 640*c^3*d^3 + sqrt(-3)*(d^6*x^9 + 318*c*d^5*x^6 + 1200*
c^2*d^4*x^3 + 640*c^3*d^3))*(c/d^4)^(5/6) + 6*(2*c*d^2*x^7 + 160*c^2*d*x^4
 + 320*c^3*x - 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2 - sqrt(-3)*(5*c*d^4*x^5 + 3
2*c^2*d^3*x^2))*(c/d^4)^(2/3) - (7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d
+ sqrt(-3)*(7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d))*(c/d^4)^(1/3))*sqrt
(d*x^3 + c) - 36*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 + 32*c^3*d^2*x)*sqrt(c/d^4)
 + 18*(c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2 - sqrt(-3)*(c*d^3*x^8 + 3
8*c^2*d^2*x^5 + 64*c^3*d*x^2))*(c/d^4)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 19
2*c^2*d*x^3 - 512*c^3)) - (sqrt(-3)*d - d)*(c/d^4)^(1/6)*log(-1/4*((d^6*x^
9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + sqrt(-3)*(d^6*x^9 + 3
18*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3))*(c/d^4)^(5/6) - 6*(2*c*d^2
*x^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2 - sqrt(
-3)*(5*c*d^4*x^5 + 32*c^2*d^3*x^2))*(c/d^4)^(2/3) - (7*c*d^3*x^6 + 152*c^2
*d^2*x^3 + 64*c^3*d + sqrt(-3)*(7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d))
*(c/d^4)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 + 32*c^
3*d^2*x)*sqrt(c/d^4) + 18*(c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2 - sqr
t(-3)*(c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2))*(c/d^4)^(1/6))/(d^3*x^9
 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (sqrt(-3)*d + d)*(c/d^4)^(1/
6)*log(1/4*((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 -...
```

Sympy [F]

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\int \frac{x\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

input
```
integrate(x*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

output
```
-Integral(x*sqrt(c + d*x**3)/(-8*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = \int -\frac{\sqrt{dx^3+c}x}{dx^3-8\,c}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(d*x^3 + c)*x/(d*x^3 - 8*c), x)
```

Giac [F]

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = \int -\frac{\sqrt{dx^3+c}x}{dx^3-8\,c}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-sqrt(d*x^3 + c)*x/(d*x^3 - 8*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = \int \frac{x\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\,dx$$

input
```
int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

output
```
int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3), x)
```

Reduce [F]

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x}{-d\,x^3+8c}dx$$

input
```
int(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

output
```
int((sqrt(c + d*x**3)*x)/(8*c - d*x**3),x)
```

3.467 $\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 632

$$\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)}\,dx$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt{3}\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{16c^{5/6}}$$
$$+ \frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{16c^{5/6}} - \frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{16c^{5/6}}$$
$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+ \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-1/8*(d*x^3+c)^(1/2)/c/x+1/8*d^(1/3)*(d*x^3+c)^(1/2)/c/((1+3^(1/2))*c^(1/3
)+d^(1/3)*x)-1/16*3^(1/2)*d^(1/3)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*
x)/(d*x^3+c)^(1/2))/c^(5/6)+1/16*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2
/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)-1/16*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)
/c^(1/2))/c^(5/6)-1/16*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*d^(1/3)*(c^(1/3)+
d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d
^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*
c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/c^(2/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1
+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+1/24*d^(1/3)*(c^(1/3
)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)
+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2)
)*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/c^(2/3)/(c^(1/3)*(c^(1
/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{c+dx^3}}{x^2\,(8c-dx^3)}\,dx$$
$$= \frac{-80c(c+dx^3)+65cdx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-d^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},\right.}{640c^2x\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)),x]
```

output
```
(-80*c*(c + d*x^3) + 65*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1,
5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5
/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(640*c^2*x*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 625, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {975, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^2 \left(8c-dx^3\right)} \, dx$$

$$\Big\downarrow \text{ 975}$$

$$\frac{\int \frac{dx\left(26c-dx^3\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c}-\frac{\sqrt{c+dx^3}}{8cx}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\int \frac{x\left(26c-dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{16c}-\frac{\sqrt{c+dx^3}}{8cx}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{d\int \left(\frac{18cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{x}{\sqrt{dx^3+c}}\right)dx}{16c}-\frac{\sqrt{c+dx^3}}{8cx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\left(\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\cdots}{\cdots}\right)}{\cdots}$$

$$\frac{\sqrt{c+dx^3}}{8cx}$$

input
```
Int[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)),x]
```

output
```
-1/8*Sqrt[c + d*x^3]/(c*x) + (d*((2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)) - (Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/
3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(2/3) + (c^(1/6)*ArcTanh[(c^(1/3) + d
^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(2/3) - (c^(1/6)*ArcTanh[Sqrt[
c + d*x^3]/(3*Sqrt[c])])/d^(2/3) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(
1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)
*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^
2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/
3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^
2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/
3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))
/(16*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.80 (sec) , antiderivative size = 868, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	868
risch	Expression too large to display	872
default	Expression too large to display	1306

input

```
int((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output

```
-1/8*(d*x^3+c)^(1/2)/c/x-1/24*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*
((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1
/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2)))-1/24*I/d^2/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*
(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)
*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-
c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(
1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2219 vs. $2(444) = 888$.

Time = 0.46 (sec) , antiderivative size = 2219, normalized size of antiderivative = 3.51

$$\int \frac{\sqrt{c+dx^3}}{x^2 (8c-dx^3)} dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
1/192*(2*c*x*(d^2/c^5)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x
^3 + 640*c^3*d + 18*(5*c^4*d^2*x^7 + 64*c^5*d*x^4 + 32*c^6*x)*(d^2/c^5)^(2
/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2)*(d^2/c^5)^(5/6) + (7
*c^3*d^2*x^6 + 152*c^4*d*x^3 + 64*c^5)*sqrt(d^2/c^5) + (c*d^3*x^7 + 80*c^2
*d^2*x^4 + 160*c^3*d*x)*(d^2/c^5)^(1/6)) + 18*(c^2*d^3*x^8 + 38*c^3*d^2*x^
5 + 64*c^4*d*x^2)*(d^2/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3
 - 512*c^3)) - 2*c*x*(d^2/c^5)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c
^2*d^2*x^3 + 640*c^3*d + 18*(5*c^4*d^2*x^7 + 64*c^5*d*x^4 + 32*c^6*x)*(d^2
/c^5)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2)*(d^2/c^5)^(5
/6) + (7*c^3*d^2*x^6 + 152*c^4*d*x^3 + 64*c^5)*sqrt(d^2/c^5) + (c*d^3*x^7
+ 80*c^2*d^2*x^4 + 160*c^3*d*x)*(d^2/c^5)^(1/6)) + 18*(c^2*d^3*x^8 + 38*c^
3*d^2*x^5 + 64*c^4*d*x^2)*(d^2/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c
^2*d*x^3 - 512*c^3)) - 24*sqrt(d)*x*weierstrassZeta(0, -4*c/d, weierstrass
PInverse(0, -4*c/d, x)) + (sqrt(-3)*c*x + c*x)*(d^2/c^5)^(1/6)*log((d^4*x^
9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^4*d^2*x^7 + 64*c
^5*d*x^4 + 32*c^6*x + sqrt(-3)*(5*c^4*d^2*x^7 + 64*c^5*d*x^4 + 32*c^6*x))*
(d^2/c^5)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2 - sqrt(-3
)*(5*c^5*d*x^5 + 32*c^6*x^2))*(d^2/c^5)^(5/6) - 2*(7*c^3*d^2*x^6 + 152*c^4
*d*x^3 + 64*c^5)*sqrt(d^2/c^5) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x
 + sqrt(-3)*(c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x))*(d^2/c^5)^(1/6)...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)}\,dx = -\int \frac{\sqrt{c+dx^3}}{-8cx^2+dx^5}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**2/(-d*x**3+8*c),x)
```

output
```
-Integral(sqrt(c + d*x**3)/(-8*c*x**2 + d*x**5), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^2\,(8c-dx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}}{x^2\,\left(8\,c-d\,x^3\right)}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\,(8c-dx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}}{-d\,x^5+8c\,x^2}dx$$

input
```
int((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x)
```

output
```
int(sqrt(c + d*x**3)/(8*c*x**2 - d*x**5),x)
```

3.468 $\int \frac{\sqrt{c+dx^3}}{x^5(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 654

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)}\,dx =& -\frac{\sqrt{c+dx^3}}{32cx^4}-\frac{d\sqrt{c+dx^3}}{16c^2x} \\
&+\frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt{3}d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{11/6}} \\
&+\frac{d^{4/3}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{11/6}}-\frac{d^{4/3}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{11/6}} \\
&-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

output
```
-1/32*(d*x^3+c)^(1/2)/c/x^4-1/16*d*(d*x^3+c)^(1/2)/c^2/x+1/16*d^(4/3)*(d*x
^3+c)^(1/2)/c^2/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)-1/128*3^(1/2)*d^(4/3)*arct
an(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))/c^(11/6)+1/128*d^(
4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)-1
/128*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-1/32*3^(1/4)*(1
/2*6^(1/2)-1/2*2^(1/2))*d^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1
/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-
3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)
/c^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(
1/2)/(d*x^3+c)^(1/2)+1/48*d^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^
(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((
1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*
I)*2^(1/2)*3^(3/4)/c^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/
3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{c+dx^3}}{x^5(8c-dx^3)}\,dx$$
$$= \frac{125cd^2x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4\left(40c(c^2+3cdx^3+2d^2x^6)+d^3x^9\sqrt{1+\frac{dx^3}{c}}\operatorname{Appe}\right.}{5120c^3x^4\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)),x]
```

output
```
(125*c*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c)
, (d*x^3)/(8*c)] - 4*(40*c*(c^2 + 3*c*d*x^3 + 2*d^2*x^6) + d^3*x^9*Sqrt[1
+ (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(51
20*c^3*x^4*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {975, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{d\left(5dx^3+32c\right)}{2x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c}-\frac{\sqrt{c+dx^3}}{32cx^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{5dx^3+32c}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{64c}-\frac{\sqrt{c+dx^3}}{32cx^4}$$

$$\Big\downarrow \text{1053}$$

$$\frac{d\left(-\frac{\int -\frac{8cdx\left(25c-2dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{4\sqrt{c+dx^3}}{cx}\right)}{64c}-\frac{\sqrt{c+dx^3}}{32cx^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(\frac{d\int \frac{x\left(25c-2dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{4\sqrt{c+dx^3}}{cx}\right)}{64c}-\frac{\sqrt{c+dx^3}}{32cx^4}$$

$$\Big\downarrow \text{1054}$$

$$\frac{d\left(\frac{d\int \left(\frac{9cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{2x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{4\sqrt{c+dx^3}}{cx}\right)}{64c}-\frac{\sqrt{c+dx^3}}{32cx^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(d\left(\frac{4\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{2\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\cdots}}{d^{2/3}\cdots}\cdots\right)\cdots\right)}{\cdots}$$

$$\frac{\sqrt{c+dx^3}}{32cx^4}$$

input
```
Int[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)),x]
```

output
```
-1/32*Sqrt[c + d*x^3]/(c*x^4) + (d*((-4*Sqrt[c + d*x^3])/(c*x) + (d*((4*Sq
rt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (Sqrt[3]*c^
(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*
d^(2/3)) + (c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*
x^3])])/(2*d^(2/3)) - (c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*d^
(2/3)) - (2*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3)
+ d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (4
*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 -
 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/c))/(64*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.40 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.35

method	result	size
risch	Expression too large to display	882
elliptic	Expression too large to display	887
default	Expression too large to display	1782

input

```
int((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output

```
-1/32*(d*x^3+c)^(1/2)*(2*d*x^3+c)/c^2/x^4+1/64/c^2*d^2*(-4/3*I*3^(1/2)/d*(
-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/3*I/d^3*2^(1/2)*sum(1/_alpha*
(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^
(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_
alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_al
pha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I
*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2401 vs. $2(462) = 924$.

Time = 0.75 (sec) , antiderivative size = 2401, normalized size of antiderivative = 3.67

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
1/1536*(2*c^2*x^4*(d^8/c^11)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2
*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x)
*(d^8/c^11)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^8
/c^11)^(5/6) + (7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^1
1) + (c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d^5*x)*(d^8/c^11)^(1/6)) + 18
*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2)*(d^8/c^11)^(1/3))/(d^3*x^
9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*c^2*x^4*(d^8/c^11)^(1/6)*
log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^8*
d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x)*(d^8/c^11)^(2/3) - 6*sqrt(d*x^3 +
c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^8/c^11)^(5/6) + (7*c^6*d^4*x^6 + 152
*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 + 80*c^3*d^6*x^4
+ 160*c^4*d^5*x)*(d^8/c^11)^(1/6)) + 18*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64
*c^6*d^4*x^2)*(d^8/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
512*c^3)) - 96*d^(3/2)*x^4*weierstrassZeta(0, -4*c/d, weierstrassPInverse(
0, -4*c/d, x)) + (sqrt(-3)*c^2*x^4 + c^2*x^4)*(d^8/c^11)^(1/6)*log((d^9*x^
9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^8*d^3*x^7 + 64
*c^9*d^2*x^4 + 32*c^10*d*x + sqrt(-3)*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32
*c^10*d*x))*(d^8/c^11)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^1
1*x^2 - sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^8/c^11)^(5/6) - 2*(7*c^6
*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 ...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^5 (8c-dx^3)}\,dx = -\int \frac{\sqrt{c+dx^3}}{-8cx^5+dx^8}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**5/(-d*x**3+8*c),x)
```

output
```
-Integral(sqrt(c + d*x**3)/(-8*c*x**5 + d*x**8), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^5 (8c-dx^3)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^5}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^5), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^5 (8c-dx^3)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^5}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^5\,(8c-dx^3)}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^5\,\left(8\,c-d\,x^3\right)}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^5\,(8c-dx^3)}\,dx=\int \frac{\sqrt{d\,x^3+c}}{-d\,x^8+8c\,x^5}dx$$

input
```
int((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x)
```

output
```
int(sqrt(c + d*x**3)/(8*c*x**5 - d*x**8),x)
```

3.469 $\int \frac{\sqrt{c+dx^3}}{x^8(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 678

$$\int \frac{\sqrt{c+dx^3}}{x^8(8c-dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{56cx^7}-\frac{19d\sqrt{c+dx^3}}{1792c^2x^4}+\frac{d^2\sqrt{c+dx^3}}{112c^3x}-\frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt{3}d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{17/6}}+\frac{d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{17/6}}-\frac{d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{17/6}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{224c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{56\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-1/56*(d*x^3+c)^(1/2)/c/x^7-19/1792*d*(d*x^3+c)^(1/2)/c^2/x^4+1/112*d^2*(d
*x^3+c)^(1/2)/c^3/x-1/112*d^(7/3)*(d*x^3+c)^(1/2)/c^3/((1+3^(1/2))*c^(1/3)
+d^(1/3)*x)-1/1024*3^(1/2)*d^(7/3)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)
*x)/(d*x^3+c)^(1/2))/c^(17/6)+1/1024*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*
x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(17/6)-1/1024*d^(7/3)*arctanh(1/3*(d*x^3+c
)^(1/2)/c^(1/2))/c^(17/6)+1/224*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*d^(7/3)*
(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*
c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+
3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/c^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/
3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)-1/336*d^(7/
3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2
))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/(
(1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/c^(8/3)/(c^(
1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c
)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{c+dx^3}}{x^8\,(8c-dx^3)}\,dx$$
$$= \frac{-160c(32c^3+51c^2dx^3+3cd^2x^6-16d^3x^9)-325cd^3x^9\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+32d^4}{286720c^4x^7\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)),x]
```

output
```
(-160*c*(32*c^3 + 51*c^2*d*x^3 + 3*c*d^2*x^6 - 16*d^3*x^9) - 325*c*d^3*x^9
*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c
)] + 32*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/
c), (d*x^3)/(8*c)])/(286720*c^4*x^7*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {975, 27, 1053, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^8 (8c-dx^3)} dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{d(11dx^3+38c)}{2x^5(8c-dx^3)\sqrt{dx^3+c}}dx}{56c} - \frac{\sqrt{c+dx^3}}{56cx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{11dx^3+38c}{x^5(8c-dx^3)\sqrt{dx^3+c}}dx}{112c} - \frac{\sqrt{c+dx^3}}{56cx^7}$$

$$\Big\downarrow \text{1053}$$

$$\frac{d\left(-\frac{\int \frac{cd(256c-95dx^3)}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c^2} - \frac{19\sqrt{c+dx^3}}{16cx^4}\right)}{112c} - \frac{\sqrt{c+dx^3}}{56cx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(-\frac{d\int \frac{256c-95dx^3}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c} - \frac{19\sqrt{c+dx^3}}{16cx^4}\right)}{112c} - \frac{\sqrt{c+dx^3}}{56cx^7}$$

$$\Big\downarrow \text{1053}$$

$$\frac{d\left(-\frac{d\left(-\frac{\int -\frac{8cdx(65c-16dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{8c^2} - \frac{32\sqrt{c+dx^3}}{cx}\right)}{32c} - \frac{19\sqrt{c+dx^3}}{16cx^4}\right)}{112c} - \frac{\sqrt{c+dx^3}}{56cx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(-\frac{d\left(\frac{d\int \frac{x\left(65c-16dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{32\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{19\sqrt{c+dx^3}}{16cx^4}\right)}{112c}-\frac{\sqrt{c+dx^3}}{56cx^7}$$

$$\Big\downarrow \text{1054}$$

$$\frac{d\left(-\frac{d\left(\frac{d\int \left(\frac{16x}{\sqrt{dx^3+c}}-\frac{63cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}\right)dx}{c}-\frac{32\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{19\sqrt{c+dx^3}}{16cx^4}\right)}{112c}-\frac{\sqrt{c+dx^3}}{56cx^7}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(-\frac{d\left(\frac{d\left(\frac{32\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{16\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\right.}{}\right.}{}\right.}{}\right.}{}$$

$$\frac{\sqrt{c+dx^3}}{56cx^7}$$

input
```
Int[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)),x]
```

output
```
-1/56*Sqrt[c + d*x^3]/(c*x^7) + (d*((-19*Sqrt[c + d*x^3])/(16*c*x^4) - (d*
((-32*Sqrt[c + d*x^3])/(c*x) + (d*((32*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)) + (7*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(
c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*d^(2/3)) - (7*c^(1/6)*ArcTanh[(
c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(2*d^(2/3)) + (7*c^(1
/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*d^(2/3)) - (16*3^(1/4)*Sqrt[2
 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*
x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[(
(1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (32*Sqrt[2]*c^(1/3)*(c^(1/3)
 + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1
/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(
2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*Sqrt[c + d*x^3])))/c))/(32*c)))/(112*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.35 (sec) , antiderivative size = 895, normalized size of antiderivative = 1.32

method	result	size
risch	Expression too large to display	895
elliptic	Expression too large to display	906
default	Expression too large to display	2280

input
```
int((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-1/1792*(d*x^3+c)^(1/2)*(-16*d^2*x^6+19*c*d*x^3+32*c^2)/c^3/x^7-1/3584*d^3
/c^3*(-32/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(
1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x
+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/
3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*
d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+7/3*I/
d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3)
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha
^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2436 vs. $2(482) = 964$.

Time = 2.45 (sec) , antiderivative size = 2436, normalized size of antiderivative = 3.59

$$\int \frac{\sqrt{c + dx^3}}{x^8 (8c - dx^3)} \, dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
1/86016*(14*c^3*x^7*(d^14/c^17)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 120
0*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*
c^14*d^2*x)*(d^14/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^
16*x^2)*(d^14/c^17)^(5/6) + (7*c^9*d^6*x^6 + 152*c^10*d^5*x^3 + 64*c^11*d^
4)*sqrt(d^14/c^17) + (c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x)*(d^1
4/c^17)^(1/6)) + 18*(c^6*d^9*x^8 + 38*c^7*d^8*x^5 + 64*c^8*d^7*x^2)*(d^14/
c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 14*c^3*
x^7*(d^14/c^17)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 +
 640*c^3*d^11 + 18*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x)*(d^1
4/c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^14/c^
17)^(5/6) + (7*c^9*d^6*x^6 + 152*c^10*d^5*x^3 + 64*c^11*d^4)*sqrt(d^14/c^1
7) + (c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x)*(d^14/c^17)^(1/6)) +
 18*(c^6*d^9*x^8 + 38*c^7*d^8*x^5 + 64*c^8*d^7*x^2)*(d^14/c^17)^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 768*d^(5/2)*x^7*weierst
rassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 7*(sqrt(-3)*c^3*x
^7 + c^3*x^7)*(d^14/c^17)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*
d^12*x^3 + 640*c^3*d^11 - 9*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^
2*x + sqrt(-3)*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x))*(d^14/c
^17)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2 - sqrt(-3)*(
5*c^15*d*x^5 + 32*c^16*x^2))*(d^14/c^17)^(5/6) - 2*(7*c^9*d^6*x^6 + 152...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)}\,dx=-\int \frac{\sqrt{c+dx^3}}{-8cx^8+dx^{11}}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**8/(-d*x**3+8*c),x)
```

output
```
-Integral(sqrt(c + d*x**3)/(-8*c*x**8 + d*x**11), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^8(8c-dx^3)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^8}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^8), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^8(8c-dx^3)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^8}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^8(8c-dx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}}{x^8\,(8\,c-d\,x^3)}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)}\,dx=\int \frac{\sqrt{d\,x^3+c}}{-d\,x^{11}+8c\,x^8}dx$$

input
```
int((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x)
```

output
```
int(sqrt(c + d*x**3)/(8*c*x**8 - d*x**11),x)
```

3.470 $\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3} dx$

Optimal result

Integrand size = 27, antiderivative size = 130

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3} dx = -\frac{3072c^4\sqrt{c+dx^3}}{d^4} - \frac{1024c^3(c+dx^3)^{3/2}}{9d^4} - \frac{38c^2(c+dx^3)^{5/2}}{5d^4} - \frac{4c(c+dx^3)^{7/2}}{7d^4} - \frac{2(c+dx^3)^{9/2}}{27d^4} + \frac{9216c^{9/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

output
```
-3072*c^4*(d*x^3+c)^(1/2)/d^4-1024/9*c^3*(d*x^3+c)^(3/2)/d^4-38/5*c^2*(d*x
^3+c)^(5/2)/d^4-4/7*c*(d*x^3+c)^(7/2)/d^4-2/27*(d*x^3+c)^(9/2)/d^4+9216*c^
(9/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.72

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3} dx = -\frac{2\sqrt{c+dx^3}(1509176c^4+61892c^3dx^3+4611c^2d^2x^6+410cd^3x^9+35d^4x^{12})}{945d^4} + \frac{9216c^{9/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

input
```
Integrate[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(-2*Sqrt[c + d*x^3]*(1509176*c^4 + 61892*c^3*d*x^3 + 4611*c^2*d^2*x^6 + 41
0*c*d^3*x^9 + 35*d^4*x^12))/(945*d^4) + (9216*c^(9/2)*ArcTanh[Sqrt[c + d*x
^3]/(3*Sqrt[c])])/d^4
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11} \left(c+d x^3\right)^{3/2}}{8 c-d x^3} \, dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3} \int \frac{x^9 \left(d x^3+c\right)^{3/2}}{8 c-d x^3}dx^3$$

$$\Big\downarrow \; 99$$

$$\frac{1}{3} \int \left(-\frac{\left(d x^3+c\right)^{7/2}}{d^3}-\frac{6 c \left(d x^3+c\right)^{5/2}}{d^3}+\frac{512 c^3 \left(d x^3+c\right)^{3/2}}{d^3 \left(8 c-d x^3\right)}-\frac{57 c^2 \left(d x^3+c\right)^{3/2}}{d^3}\right)dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3} \left(\frac{27648 c^{9/2} \text{arctanh}\left(\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right)}{d^4}-\frac{9216 c^4 \sqrt{c+d x^3}}{d^4}-\frac{1024 c^3 \left(c+d x^3\right)^{3/2}}{3 d^4}-\frac{114 c^2 \left(c+d x^3\right)^{5/2}}{5 d^4}-\frac{12 c \left(c+d x^3\right)^{7}}{7 d^4}\right.$$

input
```
Int[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
((-9216*c^4*Sqrt[c + d*x^3])/d^4 - (1024*c^3*(c + d*x^3)^(3/2))/(3*d^4) -
(114*c^2*(c + d*x^3)^(5/2))/(5*d^4) - (12*c*(c + d*x^3)^(7/2))/(7*d^4) - (
2*(c + d*x^3)^(9/2))/(9*d^4) + (27648*c^(9/2)*ArcTanh[Sqrt[c + d*x^3]/(3*S
qrt[c])])/d^4)/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$\frac{\frac{2\left(-35d^4x^{12}-410c\,d^3x^9-4611c^2x^6d^2-61892c^3d\,x^3-1509176c^4\right)\sqrt{d\,x^3+c}}{945}+9216c^{\frac{9}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d^4}$
risch	$-\frac{2(35d^4x^{12}+410c\,d^3x^9+4611c^2x^6d^2+61892c^3d\,x^3+1509176c^4)\sqrt{d\,x^3+c}}{945d^4}+\frac{9216c^{\frac{9}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d^4}$
default	$-\frac{\frac{2d\,x^{12}\sqrt{d\,x^3+c}}{27}+\frac{20c\,x^9\sqrt{d\,x^3+c}}{189}+\frac{2c^2x^6\sqrt{d\,x^3+c}}{315d}-\frac{8c^3x^3\sqrt{d\,x^3+c}}{945d^2}+\frac{16c^4\sqrt{d\,x^3+c}}{945d^3}}{d}-\frac{128c^2(d\,x^3+c)^{\frac{5}{2}}}{15d^4}-\frac{8c\left(\frac{2d\,x^9\sqrt{d\,x^3}}{21}\right.}{}$ … $1536ic$ …
elliptic	$-\frac{2x^{12}\sqrt{d\,x^3+c}}{27}-\frac{164c\,x^9\sqrt{d\,x^3+c}}{189d}-\frac{3074c^2x^6\sqrt{d\,x^3+c}}{315d^2}-\frac{123784c^3x^3\sqrt{d\,x^3+c}}{945d^3}-\frac{3018352c^4\sqrt{d\,x^3+c}}{945d^4}-$ …

input
```
int(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
2/945*((-35*d^4*x^12-410*c*d^3*x^9-4611*c^2*d^2*x^6-61892*c^3*d*x^3-150917
6*c^4)*(d*x^3+c)^(1/2)+4354560*c^(9/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2)
))/d^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.45

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \left[\frac{2\left(2177280\,c^{\frac{9}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(35\,d^4x^{12}+410\,cd^3x^9+4611\,c^2d^2x^6+\right.}{945\,d^4}\right.$$

$$-\frac{2\left(4354560\sqrt{-c}c^4\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)+(35\,d^4x^{12}+410\,cd^3x^9+4611\,c^2d^2x^6+61892\,c^3dx^3+1509176\,c^4)\right.}{945\,d^4}$$

input
```
integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[2/945*(2177280*c^(9/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*
x^3 - 8*c)) - (35*d^4*x^12 + 410*c*d^3*x^9 + 4611*c^2*d^2*x^6 + 61892*c^3*
d*x^3 + 1509176*c^4)*sqrt(d*x^3 + c))/d^4, -2/945*(4354560*sqrt(-c)*c^4*ar
ctan(3*sqrt(-c)/sqrt(d*x^3 + c)) + (35*d^4*x^12 + 410*c*d^3*x^9 + 4611*c^2
*d^2*x^6 + 61892*c^3*d*x^3 + 1509176*c^4)*sqrt(d*x^3 + c))/d^4]
```

Sympy [A] (verification not implemented)

Time = 55.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{4608c^5\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d^3\sqrt{-c}} - \frac{1536c^4\sqrt{c+dx^3}}{d^3} - \frac{512c^3(c+dx^3)^{\frac{3}{2}}}{9d^3} - \frac{19c^2(c+dx^3)^{\frac{5}{2}}}{5d^3} - \frac{2c(c+dx^3)^{\frac{7}{2}}}{7d^3} - \frac{(c+dx^3)^{\frac{9}{2}}}{27d^3}\right)}{d} \\ \frac{\sqrt{c}x^{12}}{96} \end{cases}$$

input
```
integrate(x**11*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-4608*c**5*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d**3*sqrt(-c
)) - 1536*c**4*sqrt(c + d*x**3)/d**3 - 512*c**3*(c + d*x**3)**(3/2)/(9*d**
3) - 19*c**2*(c + d*x**3)**(5/2)/(5*d**3) - 2*c*(c + d*x**3)**(7/2)/(7*d**
3) - (c + d*x**3)**(9/2)/(27*d**3))/d, Ne(d, 0)), (sqrt(c)*x**12/96, True)
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx =$$

$$-\frac{2\left(2177280\,c^{\frac{9}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right) + 35\,(dx^3+c)^{\frac{9}{2}} + 270\,(dx^3+c)^{\frac{7}{2}}c + 3591\,(dx^3+c)^{\frac{5}{2}}c^2 + 53760\,(dx^3+\right.}{945\,d^4}$$

input
```
integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-2/945*(2177280*c^(9/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c)
 + 3*sqrt(c))) + 35*(d*x^3 + c)^(9/2) + 270*(d*x^3 + c)^(7/2)*c + 3591*(d*
x^3 + c)^(5/2)*c^2 + 53760*(d*x^3 + c)^(3/2)*c^3 + 1451520*sqrt(d*x^3 + c)
*c^4)/d^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{9216\,c^5 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^4} - \frac{2\left(35\,(dx^3+c)^{\frac{9}{2}}d^{32} + 270\,(dx^3+c)^{\frac{7}{2}}cd^{32} + 3591\,(dx^3+c)^{\frac{5}{2}}c^2d^{32} + 53760\,(dx^3+c)^{\frac{3}{2}}c^3d^{32} + 1451520\,\sqrt{d}}{945\,d^{36}}$$

input
```
integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
-9216*c^5*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/945*(35*
(d*x^3 + c)^(9/2)*d^32 + 270*(d*x^3 + c)^(7/2)*c*d^32 + 3591*(d*x^3 + c)^(
5/2)*c^2*d^32 + 53760*(d*x^3 + c)^(3/2)*c^3*d^32 + 1451520*sqrt(d*x^3 + c)
*c^4*d^32)/d^36
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{4608\,c^{9/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^4} - \frac{2\,x^{12}\,\sqrt{d\,x^3+c}}{27} - \frac{3018352\,c^4\,\sqrt{d\,x^3+c}}{945\,d^4} - \frac{164\,c\,x^9\,\sqrt{d\,x^3+c}}{189\,d} - \frac{123784\,c^3\,x^3\,\sqrt{d\,x^3+c}}{945\,d^3} - \frac{3074\,c^2\,x^6\,\sqrt{d\,x^3+c}}{315\,d^2}$$

input
```
int((x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

output
```
(4608*c^(9/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^
3)))/d^4 - (2*x^12*(c + d*x^3)^(1/2))/27 - (3018352*c^4*(c + d*x^3)^(1/2))
/(945*d^4) - (164*c*x^9*(c + d*x^3)^(1/2))/(189*d) - (123784*c^3*x^3*(c +
d*x^3)^(1/2))/(945*d^3) - (3074*c^2*x^6*(c + d*x^3)^(1/2))/(315*d^2)
```

Reduce [F]

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{\frac{247568\sqrt{dx^3+c}\,c^4}{945} - \frac{123784\sqrt{dx^3+c}\,c^3dx^3}{945} - \frac{3074\sqrt{dx^3+c}\,c^2d^2x^6}{315} - \frac{164\sqrt{dx^3+c}\,cd^3x^9}{189} - \frac{2\sqrt{dx^3+c}\,d^4}{27}}{d^4}$$

input
```
int(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

output
```
(2*(123784*sqrt(c + d*x**3)*c**4 - 61892*sqrt(c + d*x**3)*c**3*d*x**3 - 46
11*sqrt(c + d*x**3)*c**2*d**2*x**6 - 410*sqrt(c + d*x**3)*c*d**3*x**9 - 35
*sqrt(c + d*x**3)*d**4*x**12 + 2449440*int((sqrt(c + d*x**3)*x**5)/(8*c**2
 + 7*c*d*x**3 - d**2*x**6),x)*c**4*d**2))/(945*d**4)
```

3.471 $\int \frac{x^8 (c+dx^3)^{3/2}}{8c-dx^3} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 109

$$\int \frac{x^8 (c+dx^3)^{3/2}}{8c-dx^3} \, dx = -\frac{384c^3\sqrt{c+dx^3}}{d^3} - \frac{128c^2(c+dx^3)^{3/2}}{9d^3} - \frac{14c(c+dx^3)^{5/2}}{15d^3} - \frac{2(c+dx^3)^{7/2}}{21d^3} + \frac{1152c^{7/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

output

```
-384*c^3*(d*x^3+c)^(1/2)/d^3-128/9*c^2*(d*x^3+c)^(3/2)/d^3-14/15*c*(d*x^3+
c)^(5/2)/d^3-2/21*(d*x^3+c)^(7/2)/d^3+1152*c^(7/2)*arctanh(1/3*(d*x^3+c)^(
1/2)/c^(1/2))/d^3
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\int \frac{x^8 (c+dx^3)^{3/2}}{8c-dx^3} \, dx = -\frac{2\sqrt{c+dx^3}(62882c^3+2579c^2dx^3+192cd^2x^6+15d^3x^9)}{315d^3} + \frac{1152c^{7/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

input
```
Integrate[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(-2*Sqrt[c + d*x^3]*(62882*c^3 + 2579*c^2*d*x^3 + 192*c*d^2*x^6 + 15*d^3*x
^9))/(315*d^3) + (1152*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 \left(c+dx^3\right)^{3/2}}{8c-dx^3} \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^6 \left(dx^3+c\right)^{3/2}}{8c-dx^3}dx^3$$

$$\Big\downarrow \text{ 99}$$

$$\frac{1}{3}\int \left(-\frac{\left(dx^3+c\right)^{5/2}}{d^2}+\frac{64c^2 \left(dx^3+c\right)^{3/2}}{d^2 \left(8c-dx^3\right)}-\frac{7c\left(dx^3+c\right)^{3/2}}{d^2}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{3456c^{7/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{1152c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2 \left(c+dx^3\right)^{3/2}}{3d^3}-\frac{14c\left(c+dx^3\right)^{5/2}}{5d^3}-\frac{2\left(c+dx^3\right)^{7/2}}{7d^3}\right)$$

input
```
Int[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
((-1152*c^3*Sqrt[c + d*x^3])/d^3 - (128*c^2*(c + d*x^3)^(3/2))/(3*d^3) - (
14*c*(c + d*x^3)^(5/2))/(5*d^3) - (2*(c + d*x^3)^(7/2))/(7*d^3) + (3456*c^
(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3)/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{\frac{2\left(-15d^3x^9-192c\,d^2x^6-2579c^2d\,x^3-62882c^3\right)\sqrt{d\,x^3+c}}{315}+1152c^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d^3}$
risch	$-\frac{2(15d^3x^9+192c\,d^2x^6+2579c^2d\,x^3+62882c^3)\sqrt{d\,x^3+c}}{315d^3}+\frac{1152c^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d^3}$
default	$-\frac{\frac{2d\,x^9\sqrt{d\,x^3+c}}{21}+\frac{16c\,x^6\sqrt{d\,x^3+c}}{105}+\frac{2c^2x^3\sqrt{d\,x^3+c}}{105d}-\frac{4c^3\sqrt{d\,x^3+c}}{105d^2}}{d}-\frac{16c(d\,x^3+c)^{\frac{5}{2}}}{15d^3}+\frac{128c^2\left(81c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)-(\right.}{9d^3}$
elliptic	$-\frac{2x^9\sqrt{d\,x^3+c}}{21}-\frac{128c\,x^6\sqrt{d\,x^3+c}}{105d}-\frac{5158c^2x^3\sqrt{d\,x^3+c}}{315d^2}-\frac{125764c^3\sqrt{d\,x^3+c}}{315d^3}-\frac{192ic^3\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)}\right.}{}$

input
```
int(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
2/315*((-15*d^3*x^9-192*c*d^2*x^6-2579*c^2*d*x^3-62882*c^3)*(d*x^3+c)^(1/2
)+181440*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2)))/d^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.52

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \left[\frac{2\left(90720\,c^{\frac{7}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(15\,d^3x^9+192\,cd^2x^6+2579\,c^2dx^3+6288\right.}{315\,d^3}\right.$$

$$\left.-\frac{2\left(181440\sqrt{-c}c^3\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)+(15\,d^3x^9+192\,cd^2x^6+2579\,c^2dx^3+62882\,c^3)\sqrt{dx^3+c}\right)}{315\,d^3}\right]$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[2/315*(90720*c^(7/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^
3 - 8*c)) - (15*d^3*x^9 + 192*c*d^2*x^6 + 2579*c^2*d*x^3 + 62882*c^3)*sqrt
(d*x^3 + c))/d^3, -2/315*(181440*sqrt(-c)*c^3*arctan(3*sqrt(-c)/sqrt(d*x^3
 + c)) + (15*d^3*x^9 + 192*c*d^2*x^6 + 2579*c^2*d*x^3 + 62882*c^3)*sqrt(d*
x^3 + c))/d^3]
```

Sympy [A] (verification not implemented)

Time = 30.91 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.12

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx=\begin{cases}\frac{2\left(-\frac{576c^4\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d^2\sqrt{-c}}-\frac{192c^3\sqrt{c+dx^3}}{d^2}-\frac{64c^2\left(c+dx^3\right)^{\frac{3}{2}}}{9d^2}-\frac{7c\left(c+dx^3\right)^{\frac{5}{2}}}{15d^2}-\frac{\left(c+dx^3\right)^{\frac{7}{2}}}{21d^2}\right)}{d} & \text{for } d\neq 0\\ \frac{\sqrt{c}x^9}{72} & \text{otherwise}\end{cases}$$

input
```
integrate(x**8*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-576*c**4*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d**2*sqrt(-c)
) - 192*c**3*sqrt(c + d*x**3)/d**2 - 64*c**2*(c + d*x**3)**(3/2)/(9*d**2)
- 7*c*(c + d*x**3)**(5/2)/(15*d**2) - (c + d*x**3)**(7/2)/(21*d**2))/d, Ne
(d, 0)), (sqrt(c)*x**9/72, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.88

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{2\left(90720\,c^{\frac{7}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+15\,(dx^3+c)^{\frac{7}{2}}+147\,(dx^3+c)^{\frac{5}{2}}c+2240\,(dx^3+c)^{\frac{3}{2}}c^2+60480\,\sqrt{dx^3+c}\right.}{315\,d^3}$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-2/315*(90720*c^(7/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) +
 3*sqrt(c))) + 15*(d*x^3 + c)^(7/2) + 147*(d*x^3 + c)^(5/2)*c + 2240*(d*x^
3 + c)^(3/2)*c^2 + 60480*sqrt(d*x^3 + c)*c^3)/d^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{1152\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^3} - \frac{2\left(15\,(dx^3+c)^{\frac{7}{2}}d^{18}+147\,(dx^3+c)^{\frac{5}{2}}cd^{18}+2240\,(dx^3+c)^{\frac{3}{2}}c^2d^{18}+60480\,\sqrt{dx^3+c}c^3d^{18}\right)}{315\,d^{21}}$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
-1152*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/315*(15*
(d*x^3 + c)^(7/2)*d^18 + 147*(d*x^3 + c)^(5/2)*c*d^18 + 2240*(d*x^3 + c)^(
3/2)*c^2*d^18 + 60480*sqrt(d*x^3 + c)*c^3*d^18)/d^21
```

Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{576\,c^{7/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^3} - \frac{2\,x^9\,\sqrt{d\,x^3+c}}{21} - \frac{125764\,c^3\,\sqrt{d\,x^3+c}}{315\,d^3} - \frac{128\,c\,x^6\,\sqrt{d\,x^3+c}}{105\,d} - \frac{5158\,c^2\,x^3\,\sqrt{d\,x^3+c}}{315\,d^2}$$

input
```
int((x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

output
```
(576*c^(7/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/d^3 - (2*x^9*(c + d*x^3)^(1/2))/21 - (125764*c^3*(c + d*x^3)^(1/2))/(3
15*d^3) - (128*c*x^6*(c + d*x^3)^(1/2))/(105*d) - (5158*c^2*x^3*(c + d*x^3
)^(1/2))/(315*d^2)
```

Reduce [F]

$$\int \frac{x^8 (c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{\frac{10316\sqrt{dx^3+c}\,c^3}{315} - \frac{5158\sqrt{dx^3+c}\,c^2dx^3}{315} - \frac{128\sqrt{dx^3+c}\,cd^2x^6}{105} - \frac{2\sqrt{dx^3+c}\,d^3x^9}{21} + 648\left(\int \frac{\sqrt{dx^3+c}}{-d^2x^6+7cdx}\right.}{d^3}$$

input
```
int(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

output
```
(2*(5158*sqrt(c + d*x**3)*c**3 - 2579*sqrt(c + d*x**3)*c**2*d*x**3 - 192*s
qrt(c + d*x**3)*c*d**2*x**6 - 15*sqrt(c + d*x**3)*d**3*x**9 + 102060*int((
sqrt(c + d*x**3)*x**5)/(8*c**2 + 7*c*d*x**3 - d**2*x**6),x)*c**3*d**2))/(3
15*d**3)
```

3.472 $\int \frac{x^5 (c+dx^3)^{3/2}}{8c-dx^3} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 88

$$\int \frac{x^5 (c+dx^3)^{3/2}}{8c-dx^3} \, dx = -\frac{48c^2\sqrt{c+dx^3}}{d^2} - \frac{16c(c+dx^3)^{3/2}}{9d^2} - \frac{2(c+dx^3)^{5/2}}{15d^2} + \frac{144c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

output
```
-48*c^2*(d*x^3+c)^(1/2)/d^2-16/9*c*(d*x^3+c)^(3/2)/d^2-2/15*(d*x^3+c)^(5/2
)/d^2+144*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int \frac{x^5 (c+dx^3)^{3/2}}{8c-dx^3} \, dx = -\frac{2\sqrt{c+dx^3}(1123c^2+46cdx^3+3d^2x^6)}{45d^2} + \frac{144c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

input
```
Integrate[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(-2*Sqrt[c + d*x^3]*(1123*c^2 + 46*c*d*x^3 + 3*d^2*x^6))/(45*d^2) + (144*c
^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {948, 90, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(c+d x^3\right)^{3/2}}{8 c-d x^3} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^3 \left(d x^3+c\right)^{3/2}}{8 c-d x^3}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3} \left(\frac{8 c \int \frac{\left(d x^3+c\right)^{3/2}}{8 c-d x^3}dx^3}{d}-\frac{2 \left(c+d x^3\right)^{5/2}}{5 d^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3} \left(\frac{8 c \left(9 c \int \frac{\sqrt{d x^3+c}}{8 c-d x^3}dx^3-\frac{2 \left(c+d x^3\right)^{3/2}}{3 d}\right)}{d}-\frac{2 \left(c+d x^3\right)^{5/2}}{5 d^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3} \left(\frac{8 c \left(9 c \left(9 c \int \frac{1}{\left(8 c-d x^3\right) \sqrt{d x^3+c}}dx^3-\frac{2 \sqrt{c+d x^3}}{d}\right)-\frac{2 \left(c+d x^3\right)^{3/2}}{3 d}\right)}{d}-\frac{2 \left(c+d x^3\right)^{5/2}}{5 d^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{8c\left(9c\left(\frac{18c\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{2(c+dx^3)^{3/2}}{3d}\right)}{d}-\frac{2(c+dx^3)^{5/2}}{5d^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{8c\left(9c\left(\frac{6\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{2(c+dx^3)^{3/2}}{3d}\right)}{d}-\frac{2(c+dx^3)^{5/2}}{5d^2}\right)$$

input
```
Int[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
((-2*(c + d*x^3)^(5/2))/(5*d^2) + (8*c*((-2*(c + d*x^3)^(3/2))/(3*d) + 9*c
*((-2*Sqrt[c + d*x^3])/d + (6*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]
)/d)))/d)/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{6480c^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)+(-6d^2x^6-92cd x^3-2246c^2)\sqrt{d x^3+c}}{45d^2}$
risch	$-\frac{2(3d^2x^6+46cd x^3+1123c^2)\sqrt{d x^3+c}}{45d^2}+\frac{144c^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{d^2}$
default	$-\frac{2(d x^3+c)^{\frac{5}{2}}}{15d^2}+\frac{16c\left(81c^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)-(d x^3+28c)\sqrt{d x^3+c}\right)}{9d^2}$
elliptic	$-\frac{2x^6\sqrt{d x^3+c}}{15}-\frac{92c x^3\sqrt{d x^3+c}}{45d}-\frac{2246c^2\sqrt{d x^3+c}}{45d^2}-\frac{24ic^2\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{}}{}\right)}{}}}{}\right)}{}$

input
```
int(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
1/45*(6480*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))+(-6*d^2*x^6-92*c*d
*x^3-2246*c^2)*(d*x^3+c)^(1/2))/d^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.64

$$\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \left[\frac{2\left(1620\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(3\,d^2x^6+46\,cdx^3+1123\,c^2)\sqrt{dx^3+c}\right)}{45\,d^2},\right.$$
$$\left.-\frac{2\left(3240\sqrt{-c}c^2\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)+(3\,d^2x^6+46\,cdx^3+1123\,c^2)\sqrt{dx^3+c}\right)}{45\,d^2}\right]$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[2/45*(1620*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
- 8*c)) - (3*d^2*x^6 + 46*c*d*x^3 + 1123*c^2)*sqrt(d*x^3 + c))/d^2, -2/45*
(3240*sqrt(-c)*c^2*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) + (3*d^2*x^6 + 46*c*
d*x^3 + 1123*c^2)*sqrt(d*x^3 + c))/d^2]
```

Sympy [A] (verification not implemented)

Time = 15.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{72c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d\sqrt{-c}} - \frac{24c^2\sqrt{c+dx^3}}{d} - \frac{8c\left(c+dx^3\right)^{\frac{3}{2}}}{9d} - \frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15d}\right)}{d} & \text{for } d \neq 0 \\ \frac{\sqrt{c}x^6}{48} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-72*c**3*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d*sqrt(-c)) -
24*c**2*sqrt(c + d*x**3)/d - 8*c*(c + d*x**3)**(3/2)/(9*d) - (c + d*x**3)*
*(5/2)/(15*d))/d, Ne(d, 0)), (sqrt(c)*x**6/48, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{2\left(1620\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right) + 3\,(dx^3+c)^{\frac{5}{2}} + 40\,(dx^3+c)^{\frac{3}{2}}c + 1080\,\sqrt{dx^3+c}c^2\right)}{45\,d^2}$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-2/45*(1620*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3
*sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 40*(d*x^3 + c)^(3/2)*c + 1080*sqrt(d*x^
3 + c)*c^2)/d^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{144\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^2} - \frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}d^8+40\left(dx^3+c\right)^{\frac{3}{2}}cd^8+1080\sqrt{dx^3+c}c^2d^8\right)}{45\,d^{10}}$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
-144*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 2/45*(3*(d*
x^3 + c)^(5/2)*d^8 + 40*(d*x^3 + c)^(3/2)*c*d^8 + 1080*sqrt(d*x^3 + c)*c^2
*d^8)/d^10
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{72\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^2} - \frac{2\,x^6\,\sqrt{d\,x^3+c}}{15} - \frac{2246\,c^2\,\sqrt{d\,x^3+c}}{45\,d^2} - \frac{92\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d}$$

input
```
int((x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

output
```
(72*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/d^2 - (2*x^6*(c + d*x^3)^(1/2))/15 - (2246*c^2*(c + d*x^3)^(1/2))/(45*d
^2) - (92*c*x^3*(c + d*x^3)^(1/2))/(45*d)
```

Reduce [F]

$$\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{184\sqrt{d\,x^3+c}\,c^2-92\sqrt{d\,x^3+c}\,cd\,x^3-6\sqrt{d\,x^3+c}\,d^2x^6+3645\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{-d^2x^6+7cd\,x^3+8c^2}d\right.}{45d^2}$$

input
```
int(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

output
```
(184*sqrt(c + d*x**3)*c**2 - 92*sqrt(c + d*x**3)*c*d*x**3 - 6*sqrt(c + d*x
**3)*d**2*x**6 + 3645*int((sqrt(c + d*x**3)*x**5)/(8*c**2 + 7*c*d*x**3 - d
**2*x**6),x)*c**2*d**2)/(45*d**2)
```

3.473 $\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 67

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{6c\sqrt{c+dx^3}}{d}-\frac{2(c+dx^3)^{3/2}}{9d}+\frac{18c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

output
```
-6*c*(d*x^3+c)^(1/2)/d-2/9*(d*x^3+c)^(3/2)/d+18*c^(3/2)*arctanh(1/3*(d*x^3
+c)^(1/2)/c^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{2\sqrt{c+dx^3}(28c+dx^3)}{9d}+\frac{18c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

input
```
Integrate[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(-2*Sqrt[c + d*x^3]*(28*c + d*x^3))/(9*d) + (18*c^(3/2)*ArcTanh[Sqrt[c + d
*x^3]/(3*Sqrt[c])])/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {946, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(c+dx^3\right)^{3/2}}{8c-dx^3}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{\left(dx^3+c\right)^{3/2}}{8c-dx^3}dx^3$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(9c\int \frac{\sqrt{dx^3+c}}{8c-dx^3}dx^3-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(9c\left(9c\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(9c\left(\frac{18c\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(9c\left(\frac{6\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)$$

input
```
Int[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
((-2*(c + d*x^3)^(3/2))/(3*d) + 9*c*((-2*Sqrt[c + d*x^3])/d + (6*Sqrt[c]*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

method	result
default	$\frac{18c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)-\frac{2\left(d\,x^{3}+28c\right)\sqrt{d\,x^{3}+c}}{9}}{d}$
pseudoelliptic	$\frac{18c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)-\frac{2\left(d\,x^{3}+28c\right)\sqrt{d\,x^{3}+c}}{9}}{d}$
risch	$-\frac{2\left(d\,x^{3}+28c\right)\sqrt{d\,x^{3}+c}}{9d}+\frac{18c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{d}$
elliptic	$-\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9}-\frac{56c\sqrt{d\,x^{3}+c}}{9d}-\frac{3ic\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\cdots}}{\cdots}\right.}{\cdots}$

input
```
int(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
2/9*(81*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-(d*x^3+28*c)*(d*x^3+c
)^(1/2))/d
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx=\left[\frac{81\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-2\,(dx^3+28\,c)\sqrt{dx^3+c}}{9\,d},\right.$$
$$\left.-\frac{2\left(81\,\sqrt{-c}c\arctan\left(\frac{3\,\sqrt{-c}}{\sqrt{dx^3+c}}\right)+(dx^3+28\,c)\sqrt{dx^3+c}\right)}{9\,d}\right]$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[1/9*(81*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8
*c)) - 2*(d*x^3 + 28*c)*sqrt(d*x^3 + c))/d, -2/9*(81*sqrt(-c)*c*arctan(3*s
qrt(-c)/sqrt(d*x^3 + c)) + (d*x^3 + 28*c)*sqrt(d*x^3 + c))/d]
```

Sympy [A] (verification not implemented)

Time = 7.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{9c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-3c\sqrt{c+dx^3}-\frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d} & \text{for } d\neq 0 \\ \frac{\sqrt{c}x^3}{24} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-9*c**2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 3*c*s
qrt(c + d*x**3) - (c + d*x**3)**(3/2)/9)/d, Ne(d, 0)), (sqrt(c)*x**3/24, T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{81\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+2\,(dx^3+c)^{\frac{3}{2}}+54\,\sqrt{dx^3+c}c}{9\,d}$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-1/9*(81*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sq
rt(c))) + 2*(d*x^3 + c)^(3/2) + 54*sqrt(d*x^3 + c)*c)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{18\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d} - \frac{2\left((dx^3+c)^{\frac{3}{2}}d^2+27\sqrt{dx^3+c}cd^2\right)}{9\,d^3}$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
-18*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 2/9*((d*x^3 +
c)^(3/2)*d^2 + 27*sqrt(d*x^3 + c)*c*d^2)/d^3
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{9\,c^{3/2}\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d} - \frac{56\,c\,\sqrt{d\,x^3+c}}{9\,d} - \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9}$$

input
```
int((x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

output
```
(9*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/d - (56*c*(c + d*x^3)^(1/2))/(9*d) - (2*x^3*(c + d*x^3)^(1/2))/9
```

Reduce [F]

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{38\sqrt{d\,x^3+c}\,c-16\sqrt{d\,x^3+c}\,d\,x^3+729\left(\int\frac{\sqrt{d\,x^3+c}\,x^5}{-d^2x^6+7cd\,x^3+8c^2}dx\right)c\,d^2}{72d}$$

input
```
int(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

output
```
(38*sqrt(c + d*x**3)*c - 16*sqrt(c + d*x**3)*d*x**3 + 729*int((sqrt(c + d*
x**3)*x**5)/(8*c**2 + 7*c*d*x**3 - d**2*x**6),x)*c*d**2)/(72*d)
```

3.474 $\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 73

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx = -\frac{2}{3}\sqrt{c+dx^3} + \frac{9}{4}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - \frac{1}{12}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

output
```
-2/3*(d*x^3+c)^(1/2)+9/4*c^(1/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-1/12
*c^(1/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx = -\frac{2}{3}\sqrt{c+dx^3} + \frac{9}{4}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - \frac{1}{12}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x]
```

output
```
(-2*Sqrt[c + d*x^3])/3 + (9*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
4 - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/12
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {948, 95, 25, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(dx^3+c)^{3/2}}{x^3(8c-dx^3)}dx^3$$

$$\Big\downarrow \text{95}$$

$$\frac{1}{3}\left(-\frac{\int -\frac{cd(10dx^3+c)}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{d}-2\sqrt{c+dx^3}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{\int \frac{cd(10dx^3+c)}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{d}-2\sqrt{c+dx^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(c\int \frac{10dx^3+c}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3-2\sqrt{c+dx^3}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(c\left(\frac{1}{8}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{81}{8}d\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3\right)-2\sqrt{c+dx^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(c\left(\frac{81}{4}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{4d}\right)-2\sqrt{c+dx^3}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(c\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{4d}+\frac{27\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}\right)-2\sqrt{c+dx^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(c\left(\frac{27\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{4\sqrt{c}}\right)-2\sqrt{c+dx^3}\right)$$

input
```
Int[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x]
```

output
```
(-2*Sqrt[c + d*x^3] + c*((27*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(4*Sqrt
[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(4*Sqrt[c])))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$-\frac{2\sqrt{d\,x^{3}+c}}{3}+\frac{9\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{4}-\frac{\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{12}$	52
default	$\frac{\frac{2d\,x^{3}\sqrt{d\,x^{3}+c}}{9}+\frac{8c\sqrt{d\,x^{3}+c}}{9}-\frac{2c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}}{8c}+\frac{81c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)-\left(d\,x^{3}+28c\right)\sqrt{d\,x^{3}+c}}{36c}$	100
elliptic	Expression too large to display	1506

input
```
int((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(d*x^3+c)^(1/2)+9/4*c^(1/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-1/12
*c^(1/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.00

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(8c-dx^3\right)}\,dx = \left[\frac{9}{8}\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+\frac{1}{24}\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-\frac{2}{3}\sqrt{dx^3+c}, -\frac{9}{4}\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)+\frac{1}{12}\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)-\frac{2}{3}\sqrt{dx^3+c}\right]$$

input
```
integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[9/8*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c))
 + 1/24*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2/3*s
qrt(d*x^3 + c), -9/4*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) + 1/12*sq
rt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)) - 2/3*sqrt(d*x^3 + c)]
```

Sympy [A] (verification not implemented)

Time = 4.71 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(8c-dx^3\right)}\,dx = \begin{cases} \frac{2\left(-\frac{9cd\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{8\sqrt{-c}}+\frac{cd\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{24\sqrt{-c}}-\frac{d\sqrt{c+dx^3}}{3}\right)}{d} & \text{for}\ d\neq 0 \\ \frac{\sqrt{c}\log\left(x^3\right)}{24} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**3+c)**(3/2)/x/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-9*c*d*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(8*sqrt(-c)) + c*
d*atan(sqrt(c + d*x**3)/sqrt(-c))/(24*sqrt(-c)) - d*sqrt(c + d*x**3)/3)/d,
 Ne(d, 0)), (sqrt(c)*log(x**3)/24, True))
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx = \frac{c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}} - \frac{9\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{4\sqrt{-c}} - \frac{2}{3}\sqrt{dx^3+c}$$

input
```
integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
1/12*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 9/4*c*arctan(1/3*sqrt(d
*x^3 + c)/sqrt(-c))/sqrt(-c) - 2/3*sqrt(d*x^3 + c)
```

Mupad [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx = \frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\left(10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}\right)^{27}}{x^6\,(8\,c-d\,x^3)^{27}}\right)}{24} - \frac{2\,\sqrt{d\,x^3+c}}{3}$$

input
```
int((c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x)
```

output
```
(c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)
)*(10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))^27)/(x^6*(8*c - d*x^3)^27))
)/24 - (2*(c + d*x^3)^(1/2))/3
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx = \frac{17\sqrt{d\,x^3+c}}{96} + \frac{\sqrt{c}\log\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}{24} - \frac{\sqrt{c}\log\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{24} + \frac{81\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{-d^2x^6+7cd\,x^3+8c^2}dx\right)d^2}{64}$$

input
```
int((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x)
```

output
```
(34*sqrt(c + d*x**3) + 8*sqrt(c)*log(sqrt(c + d*x**3) - sqrt(c)) - 8*sqrt(
c)*log(sqrt(c + d*x**3) + sqrt(c)) + 243*int((sqrt(c + d*x**3)*x**5)/(8*c*
*2 + 7*c*d*x**3 - d**2*x**6),x)*d**2)/192
```

3.475 $\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{24x^3}+\frac{9d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{13d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

output
```
-1/24*(d*x^3+c)^(1/2)/x^3+9/32*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1
/2)-13/96*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{24x^3}+\frac{9d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{13d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x]
```

output
```
-1/24*Sqrt[c + d*x^3]/x^3 + (9*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32
*Sqrt[c]) - (13*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {948, 109, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(dx^3+c)^{3/2}}{x^6(8c-dx^3)}dx^3$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{3}\left(-\frac{\int -\frac{cd(17dx^3+26c)}{2x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{8c}-\frac{\sqrt{c+dx^3}}{8x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{16}d\int \frac{17dx^3+26c}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3-\frac{\sqrt{c+dx^3}}{8x^3}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{13}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{81}{4}d\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3\right)-\frac{\sqrt{c+dx^3}}{8x^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{81}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{13\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}\right)-\frac{\sqrt{c+dx^3}}{8x^3}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{13\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{27\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)-\frac{\sqrt{c+dx^3}}{8x^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{27\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}-\frac{13\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)-\frac{\sqrt{c+dx^3}}{8x^3}\right)$$

input
```
Int[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x]
```

output
```
(-1/8*Sqrt[c + d*x^3]/x^3 + (d*((27*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(2*Sqrt[c]) - (13*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c])))/16)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}}{24 x^{3}}+\frac{d \left(-\frac{13 \,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6 \sqrt{c}}+\frac{9 \,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{2 \sqrt{c}}\right)}{16}$
pseudoelliptic	$\frac{-13 d \,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right) x^{3}+27 \,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right) d\,x^{3}-4 \sqrt{d\,x^{3}+c}\,\sqrt{c}}{96 x^{3} \sqrt{c}}$
default	$\frac{-\frac{c \sqrt{d\,x^{3}+c}}{3 x^{3}}+\frac{2 d \sqrt{d\,x^{3}+c}}{3}-\sqrt{c}\, d \,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{8 c}+\frac{d \left(\frac{2 d\,x^{3} \sqrt{d\,x^{3}+c}}{9}+\frac{8 c \sqrt{d\,x^{3}+c}}{9}-\frac{2 c^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}\right)}{64 c^{2}}+\frac{d \left(8\right.}{}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(d*x^3+c)^(1/2)/x^3+1/16*d*(-13/6*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c
^(1/2)+9/2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.31

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx = \left[\frac{27\sqrt{c}dx^3\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+13\sqrt{c}dx^3\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-8\sqrt{dx^3+c}\,c}{192\,cx^3}, -\frac{27\sqrt{-c}dx^3\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)-13\sqrt{-c}dx^3\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)+4\sqrt{dx^3+c}\,c}{96\,cx^3}\right]$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[1/192*(27*sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d
*x^3 - 8*c)) + 13*sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2
*c)/x^3) - 8*sqrt(d*x^3 + c)*c)/(c*x^3), -1/96*(27*sqrt(-c)*d*x^3*arctan(3
*sqrt(-c)/sqrt(d*x^3 + c)) - 13*sqrt(-c)*d*x^3*arctan(sqrt(-c)/sqrt(d*x^3
+ c)) + 4*sqrt(d*x^3 + c)*c)/(c*x^3)]
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx = -\int \frac{c\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx - \int \frac{dx^3\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**4/(-d*x**3+8*c),x)
```

output
```
-Integral(c*sqrt(c + d*x**3)/(-8*c*x**4 + d*x**7), x) - Integral(d*x**3*sq
rt(c + d*x**3)/(-8*c*x**4 + d*x**7), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^4}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx = \frac{13\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\,\sqrt{-c}} - \frac{9\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{32\,\sqrt{-c}} - \frac{\sqrt{dx^3+c}}{24\,x^3}$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
13/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 9/32*d*arctan(1/3*sqrt
(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 1/24*sqrt(d*x^3 + c)/x^3
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.72

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx = \frac{9\,d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^3+c}}{3\,\sqrt{c}}\right)}{32\,\sqrt{c}} - \frac{13\,d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{96\,\sqrt{c}} - \frac{\sqrt{d\,x^3+c}}{24\,x^3}$$

input
```
int((c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x)
```

output
```
(9*d*atanh((c + d*x^3)^(1/2)/(3*c^(1/2))))/(32*c^(1/2)) - (13*d*atanh((c +
 d*x^3)^(1/2)/c^(1/2)))/(96*c^(1/2)) - (c + d*x^3)^(1/2)/(24*x^3)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx = \frac{-256\sqrt{d\,x^3+c}\,c+162\sqrt{d\,x^3+c}\,d\,x^3+416\sqrt{c}\log\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)d\,x^3-416\sqrt{c}\,\mathrm{lo}}{}$$

input

```
int((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x)
```

output

```
( - 256*sqrt(c + d*x**3)*c + 162*sqrt(c + d*x**3)*d*x**3 + 416*sqrt(c)*log
(sqrt(c + d*x**3) - sqrt(c))*d*x**3 - 416*sqrt(c)*log(sqrt(c + d*x**3) + s
qrt(c))*d*x**3 + 243*int((sqrt(c + d*x**3)*x**5)/(8*c**2 + 7*c*d*x**3 - d*
*2*x**6),x)*d**3*x**3 + 5832*int((sqrt(c + d*x**3)*x**2)/(8*c**2 + 7*c*d*x
**3 - d**2*x**6),x)*c*d**2*x**3)/(6144*c*x**3)
```

3.476 $\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 104

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{48x^6}-\frac{11d\sqrt{c+dx^3}}{192cx^3}+\frac{9d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}}-\frac{37d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}$$

output
```
-1/48*(d*x^3+c)^(1/2)/x^6-11/192*d*(d*x^3+c)^(1/2)/c/x^3+9/256*d^2*arctanh
(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-37/768*d^2*arctanh((d*x^3+c)^(1/2)/c
^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx = \frac{(-4c-11dx^3)\sqrt{c+dx^3}}{192cx^6}+\frac{9d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}}-\frac{37d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x]
```

output
```
((-4*c - 11*d*x^3)*Sqrt[c + d*x^3])/(192*c*x^6) + (9*d^2*ArcTanh[Sqrt[c +
d*x^3]/(3*Sqrt[c])])/(256*c^(3/2)) - (37*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[
c]])/(768*c^(3/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {948, 109, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{(dx^3+c)^{3/2}}{x^9(8c-dx^3)}dx^3$$

$$\Big\downarrow \text{ 109}$$

$$\frac{1}{3}\left(-\frac{\int -\frac{cd(35dx^3+44c)}{2x^6(8c-dx^3)\sqrt{dx^3+c}}dx^3}{16c}-\frac{\sqrt{c+dx^3}}{16x^6}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{1}{32}d\int \frac{35dx^3+44c}{x^6(8c-dx^3)\sqrt{dx^3+c}}dx^3-\frac{\sqrt{c+dx^3}}{16x^6}\right)$$

$$\Big\downarrow \text{ 168}$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(-\frac{\int -\frac{2cd(11dx^3+74c)}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{8c^2}-\frac{11\sqrt{c+dx^3}}{2cx^3}\right)-\frac{\sqrt{c+dx^3}}{16x^6}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{d\int \frac{11dx^3+74c}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{4c}-\frac{11\sqrt{c+dx^3}}{2cx^3}\right)-\frac{\sqrt{c+dx^3}}{16x^6}\right)$$

$\Big\downarrow$ 174

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{d\left(\frac{37}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{81}{4}d\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3\right)}{4c}-\frac{11\sqrt{c+dx^3}}{2cx^3}\right)-\frac{\sqrt{c+dx^3}}{16x^6}\right)$$

$\Big\downarrow$ 73

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{d\left(\frac{81}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{37\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}\right)}{4c}-\frac{11\sqrt{c+dx^3}}{2cx^3}\right)-\frac{\sqrt{c+dx^3}}{16x^6}\right)$$

$\Big\downarrow$ 219

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{d\left(\frac{37\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{27\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)}{4c}-\frac{11\sqrt{c+dx^3}}{2cx^3}\right)-\frac{\sqrt{c+dx^3}}{16x^6}\right)$$

$\Big\downarrow$ 221

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{d\left(\frac{27\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}-\frac{37\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)}{4c}-\frac{11\sqrt{c+dx^3}}{2cx^3}\right)-\frac{\sqrt{c+dx^3}}{16x^6}\right)$$

input
```
Int[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x]
```

output
```
(-1/16*Sqrt[c + d*x^3]/x^6 + (d*((-11*Sqrt[c + d*x^3])/(2*c*x^3) + (d*((27
*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*Sqrt[c]) - (37*ArcTanh[Sqrt[c +
d*x^3]/Sqrt[c]])/(2*Sqrt[c])))/(4*c)))/32)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}\,(11d\,x^{3}+4c)}{192x^{6}c}+\frac{d^{2}\left(-\frac{37\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6\sqrt{c}}+\frac{9\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)}{128c}$
pseudoelliptic	$\frac{-37\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d^{2}x^{6}+27\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)d^{2}x^{6}-44d\,x^{3}\sqrt{d\,x^{3}+c}\,\sqrt{c}-16\sqrt{d\,x^{3}+c}\,c^{\frac{3}{2}}}{768c^{\frac{3}{2}}x^{6}}$
default	$\frac{-\frac{c\sqrt{d\,x^{3}+c}}{6x^{6}}-\frac{5d\sqrt{d\,x^{3}+c}}{12x^{3}}-\frac{d^{2}\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{4\sqrt{c}}}{8c}+\frac{d\left(-\frac{c\sqrt{d\,x^{3}+c}}{3x^{3}}+\frac{2d\sqrt{d\,x^{3}+c}}{3}-\sqrt{c}\,d\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)\right)}{64c^{2}}+\frac{d^{2}\left(2d\,x\cdots\right.}{\cdots}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(d*x^3+c)^(1/2)*(11*d*x^3+4*c)/x^6/c+1/128/c*d^2*(-37/6*arctanh((d*
x^3+c)^(1/2)/c^(1/2))/c^(1/2)+9/2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.04

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx = \left[\frac{27\sqrt{c}d^2x^6\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+37\sqrt{c}d^2x^6\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-8\,(11\,cd\right.}{1536\,c^2x^6}$$

$$\left.-\frac{27\sqrt{-c}d^2x^6\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)-37\sqrt{-c}d^2x^6\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)+4\,(11\,cdx^3+4\,c^2)\sqrt{dx^3+c}}{768\,c^2x^6}\right]$$

input
```
integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[1/1536*(27*sqrt(c)*d^2*x^6*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)
/(d*x^3 - 8*c)) + 37*sqrt(c)*d^2*x^6*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c
) + 2*c)/x^3) - 8*(11*c*d*x^3 + 4*c^2)*sqrt(d*x^3 + c))/(c^2*x^6), -1/768*
(27*sqrt(-c)*d^2*x^6*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) - 37*sqrt(-c)*d^2*
x^6*arctan(sqrt(-c)/sqrt(d*x^3 + c)) + 4*(11*c*d*x^3 + 4*c^2)*sqrt(d*x^3 +
 c))/(c^2*x^6)]
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx = -\int \frac{c\sqrt{c+dx^3}}{-8cx^7+dx^{10}}\,dx - \int \frac{dx^3\sqrt{c+dx^3}}{-8cx^7+dx^{10}}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**7/(-d*x**3+8*c),x)
```

output
```
-Integral(c*sqrt(c + d*x**3)/(-8*c*x**7 + d*x**10), x) - Integral(d*x**3*s
qrt(c + d*x**3)/(-8*c*x**7 + d*x**10), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^7}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^7), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx = \frac{37\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{768\,\sqrt{-c}c} - \frac{9\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{256\,\sqrt{-c}c} - \frac{11\,(dx^3+c)^{\frac{3}{2}}d^2 - 7\,\sqrt{dx^3+c}cd^2}{192\,cd^2x^6}$$

input
```
integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
37/768*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 9/256*d^2*arcta
n(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/192*(11*(d*x^3 + c)^(3/2)
*d^2 - 7*sqrt(d*x^3 + c)*c*d^2)/(c*d^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx = \frac{7\,\sqrt{d\,x^3+c}}{192\,x^6} - \frac{37\,d^2\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)}{768\,\sqrt{c^3}} + \frac{9\,d^2\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{256\,\sqrt{c^3}} - \frac{11\,(d\,x^3+c)^{3/2}}{192\,c\,x^6}$$

input
```
int((c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x)
```

output
```
(7*(c + d*x^3)^(1/2))/(192*x^6) - (37*d^2*atanh((c*(c + d*x^3)^(1/2))/(c^3
)^(1/2)))/(768*(c^3)^(1/2)) + (9*d^2*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^
(1/2))))/(256*(c^3)^(1/2)) - (11*(c + d*x^3)^(3/2))/(192*c*x^6)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx = \frac{-15424\sqrt{d\,x^3+c}\,c^2-21680\sqrt{d\,x^3+c}\,cd\,x^3+5670\sqrt{d\,x^3+c}\,d^2x^6+11840\sqrt{c}\log\left(\sqrt{\right.}}{}$$

input
```
int((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x)
```

output
```
( - 15424*sqrt(c + d*x**3)*c**2 - 21680*sqrt(c + d*x**3)*c*d*x**3 + 5670*s
qrt(c + d*x**3)*d**2*x**6 + 11840*sqrt(c)*log(sqrt(c + d*x**3) - sqrt(c))*
d**2*x**6 - 11840*sqrt(c)*log(sqrt(c + d*x**3) + sqrt(c))*d**2*x**6 - 2488
32*int(sqrt(c + d*x**3)/(8*c**2*x**7 + 7*c*d*x**10 - d**2*x**13),x)*c**4*x
**6 + 81648*int(sqrt(c + d*x**3)/(8*c**2*x + 7*c*d*x**4 - d**2*x**7),x)*c*
*2*d**2*x**6 + 8505*int((sqrt(c + d*x**3)*x**5)/(8*c**2 + 7*c*d*x**3 - d**
2*x**6),x)*d**4*x**6)/(491520*c**2*x**6)
```

3.477 $\int \frac{x^7 (c+dx^3)^{3/2}}{8c-dx^3} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 669

$$\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2}-\frac{348cx^5\sqrt{c+dx^3}}{247d}-\frac{2}{19}x^8\sqrt{c+dx^3}$$
$$-\frac{2094648c^3\sqrt{c+dx^3}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{288\sqrt{3}c^{19/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}$$
$$+\frac{288c^{19/6}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}-\frac{288c^{19/6}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}}$$
$$+\frac{1047324\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{10/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{1729d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{698216\sqrt{2}3^{3/4}c^{10/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{1729d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-36534/1729*c^2*x^2*(d*x^3+c)^(1/2)/d^2-348/247*c*x^5*(d*x^3+c)^(1/2)/d-2/
19*x^8*(d*x^3+c)^(1/2)-2094648/1729*c^3*(d*x^3+c)^(1/2)/d^(8/3)/((1+3^(1/2
))*c^(1/3)+d^(1/3)*x)-288*3^(1/2)*c^(19/6)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)
+d^(1/3)*x)/(d*x^3+c)^(1/2))/d^(8/3)+288*c^(19/6)*arctanh(1/3*(c^(1/3)+d^(
1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(8/3)-288*c^(19/6)*arctanh(1/3*(d*x^3
+c)^(1/2)/c^(1/2))/d^(8/3)+1047324/1729*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*
c^(10/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+
3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3
)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/d^(8/3)/(c^(1/3)*(c^(1
/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)-69
8216/1729*2^(1/2)*3^(3/4)*c^(10/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d
^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF((
(1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2
*I)/d^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2
)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.24

$$\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{-20x^2(18267c^3+19485c^2dx^3+1309cd^2x^6+91d^3x^9)+365340c^3x^2\sqrt{1+\frac{dx^3}{c}}\,\mathrm{Appe}}{17290d^2}$$

input
```
Integrate[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(-20*x^2*(18267*c^3 + 19485*c^2*d*x^3 + 1309*c*d^2*x^6 + 91*d^3*x^9) + 365
340*c^3*x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (
d*x^3)/(8*c)] + 261831*c^2*d*x^5*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1,
 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(17290*d^2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {977, 27, 1052, 27, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \left(c+dx^3\right)^{3/2}}{8c-dx^3} \, dx$$

$$\Big\downarrow \text{977}$$

$$-\frac{2 \int -\frac{3cdx^7 \left(58dx^3+49c\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{19d}-\frac{2}{19}x^8\sqrt{c+dx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{19}c\int \frac{x^7\left(58dx^3+49c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx-\frac{2}{19}x^8\sqrt{c+dx^3}$$

$$\Big\downarrow \text{1052}$$

$$\frac{3}{19}c\left(\frac{2\int \frac{cdx^4\left(6089dx^3+4640c\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{13d^2}-\frac{116x^5\sqrt{c+dx^3}}{13d}\right)-\frac{2}{19}x^8\sqrt{c+dx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{19}c\left(\frac{c\int \frac{x^4\left(6089dx^3+4640c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{13d}-\frac{116x^5\sqrt{c+dx^3}}{13d}\right)-\frac{2}{19}x^8\sqrt{c+dx^3}$$

$$\Big\downarrow \text{1052}$$

$$\frac{3}{19}c\left(\frac{c\left(\frac{2\int \frac{2cdx\left(87277dx^3+48712c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d^2}-\frac{12178x^2\sqrt{c+dx^3}}{7d}\right)}{13d}-\frac{116x^5\sqrt{c+dx^3}}{13d}\right)-\frac{2}{19}x^8\sqrt{c+dx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{19}c\left(\frac{c\left(\frac{4c\int \frac{x\left(87277dx^3+48712c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d}-\frac{12178x^2\sqrt{c+dx^3}}{7d}\right)}{13d}-\frac{116x^5\sqrt{c+dx^3}}{13d}\right)-\frac{2}{19}x^8\sqrt{c+dx^3}$$

$$\Big\downarrow \text{1054}$$

$$\frac{3}{19}c\left(\frac{c\left(\frac{4c\int \left(\frac{746928cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}-\frac{87277x}{\sqrt{dx^3+c}}\right)dx}{7d}-\frac{12178x^2\sqrt{c+dx^3}}{7d}\right)}{13d}-\frac{116x^5\sqrt{c+dx^3}}{13d}\right)-\frac{2}{19}x^8\sqrt{c+dx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{19}c\left(\frac{c\left(4c\left(-\frac{174554\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{87277\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\cdots}{\cdots}\cdots\right)}{\cdots}\cdots}{\cdots}\cdots\right)$$

$$\frac{2}{19}x^8\sqrt{c+dx^3}$$

input
```
Int[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(-2*x^8*Sqrt[c + d*x^3])/19 + (3*c*((-116*x^5*Sqrt[c + d*x^3])/(13*d) + (c
*((-12178*x^2*Sqrt[c + d*x^3])/(7*d) + (4*c*((-174554*Sqrt[c + d*x^3])/(d^
(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (41496*Sqrt[3]*c^(1/6)*ArcTan
[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(2/3) + (4149
6*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/d^
(2/3) - (41496*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]])/d^(2/3) + (87
277*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
- c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3
)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (174554*Sq
rt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^
(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*
Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(7*d)))/(13*d)))/19
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 977
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c -
a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p,
q, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.23 (sec) , antiderivative size = 895, normalized size of antiderivative = 1.34

method	result	size
risch	Expression too large to display	895
elliptic	Expression too large to display	906
default	Expression too large to display	1840

input
```
int(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-2/1729*x^2*(91*d^2*x^6+1218*c*d*x^3+18267*c^2)/d^2*(d*x^3+c)^(1/2)-12/172
9*c^3/d^2*(-174554/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(
-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c
*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))
+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-
c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
))+27664*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c
*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d
*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/
3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2453 vs. $2(479) = 958$.

Time = 30.40 (sec) , antiderivative size = 2453, normalized size of antiderivative = 3.67

$$\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
2/1729*(1047324*c^3*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse
(0, -4*c/d, x)) + 41496*(c^19/d^16)^(1/6)*d^3*log(1981355655168*((d^16*x^9
 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13)*(c^19/d^16)^(5/6) +
6*(c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x + 6*(5*c^4*d^12*x^5 + 32*c^5*
d^11*x^2)*(c^19/d^16)^(2/3) + (7*c^10*d^7*x^6 + 152*c^11*d^6*x^3 + 64*c^12
*d^5)*(c^19/d^16)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c^7*d^10*x^7 + 64*c^8*d^9
*x^4 + 32*c^9*d^8*x)*sqrt(c^19/d^16) + 18*(c^13*d^5*x^8 + 38*c^14*d^4*x^5
+ 64*c^15*d^3*x^2)*(c^19/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*
x^3 - 512*c^3)) - 41496*(c^19/d^16)^(1/6)*d^3*log(-1981355655168*((d^16*x^
9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13)*(c^19/d^16)^(5/6) -
 6*(c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x + 6*(5*c^4*d^12*x^5 + 32*c^5
*d^11*x^2)*(c^19/d^16)^(2/3) + (7*c^10*d^7*x^6 + 152*c^11*d^6*x^3 + 64*c^1
2*d^5)*(c^19/d^16)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c^7*d^10*x^7 + 64*c^8*d^
9*x^4 + 32*c^9*d^8*x)*sqrt(c^19/d^16) + 18*(c^13*d^5*x^8 + 38*c^14*d^4*x^5
 + 64*c^15*d^3*x^2)*(c^19/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d
*x^3 - 512*c^3)) - 20748*(sqrt(-3)*d^3 - d^3)*(c^19/d^16)^(1/6)*log(198135
5655168*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + s
qrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c
^19/d^16)^(5/6) + 6*(2*c^16*d^2*x^7 + 160*c^17*d*x^4 + 320*c^18*x - 6*(5*c
^4*d^12*x^5 + 32*c^5*d^11*x^2 - sqrt(-3)*(5*c^4*d^12*x^5 + 32*c^5*d^11*...
```

Sympy [F]

$$\int \frac{x^7 (c + dx^3)^{3/2}}{8c - dx^3}\,dx = -\int \frac{cx^7\sqrt{c + dx^3}}{-8c + dx^3}\,dx - \int \frac{dx^{10}\sqrt{c + dx^3}}{-8c + dx^3}\,dx$$

input
```
integrate(x**7*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

output
```
-Integral(c*x**7*sqrt(c + d*x**3)/(-8*c + d*x**3), x) - Integral(d*x**10*s
qrt(c + d*x**3)/(-8*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^7 (c + dx^3)^{3/2}}{8c - dx^3} \, dx = \int -\frac{(dx^3 + c)^{\frac{3}{2}} x^7}{dx^3 - 8\,c} \, dx$$

input
```
integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate((d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c), x)
```

Giac [F]

$$\int \frac{x^7 (c + dx^3)^{3/2}}{8c - dx^3} \, dx = \int -\frac{(dx^3 + c)^{\frac{3}{2}} x^7}{dx^3 - 8\,c} \, dx$$

input
```
integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-(d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7 (c + dx^3)^{3/2}}{8c - dx^3} \, dx = \int \frac{x^7\,{(d\,x^3 + c)}^{3/2}}{8\,c - d\,x^3} \, dx$$

input
```
int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

output
```
int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3), x)
```

Reduce [F]

$$\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{-\frac{36534\sqrt{d\,x^3+c}\,c^2x^2}{1729} - \frac{348\sqrt{d\,x^3+c}\,cd\,x^5}{247} - \frac{2\sqrt{d\,x^3+c}\,d^2x^8}{19} + \frac{1047324\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{-d^2x^6+7cd\,x^3+8c^2}dx\right)c^3d}{1729} + \cdots}{d^2}$$

input
```
int(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

output
```
(2*( - 18267*sqrt(c + d*x**3)*c**2*x**2 - 1218*sqrt(c + d*x**3)*c*d*x**5 -
 91*sqrt(c + d*x**3)*d**2*x**8 + 523662*int((sqrt(c + d*x**3)*x**4)/(8*c**
2 + 7*c*d*x**3 - d**2*x**6),x)*c**3*d + 292272*int((sqrt(c + d*x**3)*x)/(8
*c**2 + 7*c*d*x**3 - d**2*x**6),x)*c**4))/(1729*d**2)
```

3.478 $\int \frac{x^4 (c+dx^3)^{3/2}}{8c-dx^3} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 645

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{240cx^2\sqrt{c+dx^3}}{91d}-\frac{2}{13}x^5\sqrt{c+dx^3}-\frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{36\sqrt{3}c^{13/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{36c^{13/6}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}}-\frac{36c^{13/6}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}}+\frac{6891\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{91d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{4594\sqrt{2}3^{3/4}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{91d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-240/91*c*x^2*(d*x^3+c)^(1/2)/d-2/13*x^5*(d*x^3+c)^(1/2)-13782/91*c^2*(d*x
^3+c)^(1/2)/d^(5/3)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)-36*3^(1/2)*c^(13/6)*ar
ctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))/d^(5/3)+36*c^(13
/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(5/3)-36*
c^(13/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)+6891/91*3^(1/4)*(1/2
*6^(1/2)-1/2*2^(1/2))*c^(7/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3
)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^
(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/d
^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/
2)/(d*x^3+c)^(1/2)-4594/91*2^(1/2)*3^(3/4)*c^(7/3)*(c^(1/3)+d^(1/3)*x)*((c
^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(
1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3
)*x),I*3^(1/2)+2*I)/d^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1
/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.97 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.23

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{-80(120c^2x^2+127cdx^5+7d^2x^8)+9600c^2x^2\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\right.}{3640d\sqrt{c+dx^3}}$$

input
```
Integrate[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(-80*(120*c^2*x^2 + 127*c*d*x^5 + 7*d^2*x^8) + 9600*c^2*x^2*Sqrt[1 + (d*x^
3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 6891*c*d*x
^5*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8
*c)])/(3640*d*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {977, 27, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(c+dx^3\right)^{3/2}}{8c-dx^3} \, dx$$

$$\Big\downarrow \text{977}$$

$$-\frac{2\int -\frac{3cdx^4(40dx^3+31c)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{13d}-\frac{2}{13}x^5\sqrt{c+dx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{13}c\int \frac{x^4\left(40dx^3+31c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx-\frac{2}{13}x^5\sqrt{c+dx^3}$$

$$\Big\downarrow \text{1052}$$

$$\frac{3}{13}c\left(\frac{2\int \frac{cdx(2297dx^3+1280c)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{7d^2}-\frac{80x^2\sqrt{c+dx^3}}{7d}\right)-\frac{2}{13}x^5\sqrt{c+dx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{13}c\left(\frac{c\int \frac{x(2297dx^3+1280c)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{7d}-\frac{80x^2\sqrt{c+dx^3}}{7d}\right)-\frac{2}{13}x^5\sqrt{c+dx^3}$$

$$\Big\downarrow \text{1054}$$

$$\frac{3}{13}c\left(\frac{c\int \left(\frac{19656cx}{(8c-dx^3)\sqrt{dx^3+c}}-\frac{2297x}{\sqrt{dx^3+c}}\right)dx}{7d}-\frac{80x^2\sqrt{c+dx^3}}{7d}\right)-\frac{2}{13}x^5\sqrt{c+dx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{13}c\left(\frac{c\left(-\frac{4594\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{2297\sqrt[4]{3}\sqrt{2-\sqrt{3}}\cdots}{\cdots}\right.}{\cdots}\right.$$

$$\frac{2}{13}x^5\sqrt{c+dx^3}$$

input
```
Int[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(-2*x^5*Sqrt[c + d*x^3])/13 + (3*c*((-80*x^2*Sqrt[c + d*x^3])/(7*d) + (c*(
(-4594*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (1
092*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c
+ d*x^3]])/d^(2/3) + (1092*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1
/6)*Sqrt[c + d*x^3])])/d^(2/3) - (1092*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*
Sqrt[c])])/d^(2/3) + (2297*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^
(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x
)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^
(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c
 + d*x^3]) - (4594*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c
^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d
^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(7*d)
))/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 977
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c -
a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p,
q, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.78 (sec) , antiderivative size = 884, normalized size of antiderivative = 1.37

method	result	size
risch	Expression too large to display	884
elliptic	Expression too large to display	886
default	Expression too large to display	1344

input
```
int(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-2/91*x^2*(7*d*x^3+120*c)/d*(d*x^3+c)^(1/2)-3/91*c^2/d*(-4594/3*I*3^(1/2)/
d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+728*I/d^3*2^(1/2)*sum(1/_alp
ha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I
*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*
_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(
2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2442 vs. $2(459) = 918$.

Time = 10.16 (sec) , antiderivative size = 2442, normalized size of antiderivative = 3.79

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
1/91*(13782*c^2*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0,
-4*c/d, x)) + 546*(c^13/d^10)^(1/6)*d^2*log(60466176*((d^11*x^9 + 318*c*d^
10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8)*(c^13/d^10)^(5/6) + 6*(c^11*d^2*x
^7 + 80*c^12*d*x^4 + 160*c^13*x + 6*(5*c^3*d^8*x^5 + 32*c^4*d^7*x^2)*(c^13
/d^10)^(2/3) + (7*c^7*d^5*x^6 + 152*c^8*d^4*x^3 + 64*c^9*d^3)*(c^13/d^10)^
(1/3))*sqrt(d*x^3 + c) + 18*(5*c^5*d^7*x^7 + 64*c^6*d^6*x^4 + 32*c^7*d^5*x
)*sqrt(c^13/d^10) + 18*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2)*(
c^13/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 54
6*(c^13/d^10)^(1/6)*d^2*log(-60466176*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c
^2*d^9*x^3 + 640*c^3*d^8)*(c^13/d^10)^(5/6) - 6*(c^11*d^2*x^7 + 80*c^12*d*
x^4 + 160*c^13*x + 6*(5*c^3*d^8*x^5 + 32*c^4*d^7*x^2)*(c^13/d^10)^(2/3) +
(7*c^7*d^5*x^6 + 152*c^8*d^4*x^3 + 64*c^9*d^3)*(c^13/d^10)^(1/3))*sqrt(d*x
^3 + c) + 18*(5*c^5*d^7*x^7 + 64*c^6*d^6*x^4 + 32*c^7*d^5*x)*sqrt(c^13/d^1
0) + 18*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2)*(c^13/d^10)^(1/6
))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 273*(c^13/d^10)^(
1/6)*(sqrt(-3)*d^2 - d^2)*log(60466176*((d^11*x^9 + 318*c*d^10*x^6 + 1200*
c^2*d^9*x^3 + 640*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2
*d^9*x^3 + 640*c^3*d^8))*(c^13/d^10)^(5/6) + 6*(2*c^11*d^2*x^7 + 160*c^12*
d*x^4 + 320*c^13*x - 6*(5*c^3*d^8*x^5 + 32*c^4*d^7*x^2 - sqrt(-3)*(5*c^3*d
^8*x^5 + 32*c^4*d^7*x^2))*(c^13/d^10)^(2/3) - (7*c^7*d^5*x^6 + 152*c^8*...
```

Sympy [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\int \frac{cx^4\sqrt{c+dx^3}}{-8c+dx^3}\,dx - \int \frac{dx^7\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

input
```
integrate(x**4*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

output
```
-Integral(c*x**4*sqrt(c + d*x**3)/(-8*c + d*x**3), x) - Integral(d*x**7*sq
rt(c + d*x**3)/(-8*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}x^4}{dx^3-8\,c}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate((d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c), x)
```

Giac [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}x^4}{dx^3-8\,c}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-(d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int \frac{x^4\,(d\,x^3+c)^{3/2}}{8\,c-d\,x^3}\,dx$$

input
```
int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

output
```
int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3), x)
```

Reduce [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{-240\sqrt{d\,x^3+c}\,c\,x^2-14\sqrt{d\,x^3+c}\,d\,x^5+6891\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{-d^2x^6+7cd\,x^3+8c^2}dx\right)c^2d+3840\left(\int\right.}{91d}$$

input
```
int(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

output
```
( - 240*sqrt(c + d*x**3)*c*x**2 - 14*sqrt(c + d*x**3)*d*x**5 + 6891*int((s
qrt(c + d*x**3)*x**4)/(8*c**2 + 7*c*d*x**3 - d**2*x**6),x)*c**2*d + 3840*i
nt((sqrt(c + d*x**3)*x)/(8*c**2 + 7*c*d*x**3 - d**2*x**6),x)*c**3)/(91*d)
```

3.479 $\int \frac{x\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 627

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{132c\sqrt{c+dx^3}}{7d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{9\sqrt{3}c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{9c^{7/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{2d^{2/3}} - \frac{9c^{7/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2d^{2/3}} + \frac{66\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{44\sqrt{2}3^{3/4}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-2/7*x^2*(d*x^3+c)^(1/2)-132/7*c*(d*x^3+c)^(1/2)/d^(2/3)/((1+3^(1/2))*c^(1
/3)+d^(1/3)*x)-9/2*3^(1/2)*c^(7/6)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)
*x)/(d*x^3+c)^(1/2))/d^(2/3)+9/2*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2
/c^(1/6)/(d*x^3+c)^(1/2))/d^(2/3)-9/2*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/
c^(1/2))/d^(2/3)+66/7*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*c^(4/3)*(c^(1/3)+d
^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^
(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c
^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/d^(2/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+
3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)-44/7*2^(1/2)*3^(3/4)*
c^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3
^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)
*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/d^(2/3)/(c^(1/3)*(c^(1/
3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.20

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{x^2\left(-160(c+dx^3)+195c\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+132dx^3\sqrt{1+\right.}{560\sqrt{c+dx^3}}$$

input
```
Integrate[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(x^2*(-160*(c + d*x^3) + 195*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 132*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[
5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(560*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {977, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}dx$$

$$\Big\downarrow \text{977}$$

$$-\frac{2\int -\frac{3cdx(22dx^3+13c)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{7d}-\frac{2}{7}x^2\sqrt{c+dx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{7}c\int \frac{x(22dx^3+13c)}{(8c-dx^3)\sqrt{dx^3+c}}dx-\frac{2}{7}x^2\sqrt{c+dx^3}$$

$$\Big\downarrow \text{1054}$$

$$\frac{3}{7}c\int \left(\frac{189cx}{(8c-dx^3)\sqrt{dx^3+c}}-\frac{22x}{\sqrt{dx^3+c}}\right)dx-\frac{2}{7}x^2\sqrt{c+dx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{7}c\left(-\frac{44\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{22\sqrt[4]{3}\cdots}{\cdots}\right.$$
$$\frac{2}{7}x^2\sqrt{c+dx^3}$$

input
```
Int[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(-2*x^2*Sqrt[c + d*x^3])/7 + (3*c*((-44*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)) - (21*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)
*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*d^(2/3)) + (21*c^(1/6)*ArcTan
h[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(2*d^(2/3)) - (21*
c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*d^(2/3)) + (22*3^(1/4)*Sq
rt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)
], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (44*Sqrt[2]*c^(1/3)*(c^(
1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) +
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)
*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*Sqrt[c + d*x^3])))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 977
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c -
a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p,
q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.68 (sec) , antiderivative size = 864, normalized size of antiderivative = 1.38

method	result	size
default	Expression too large to display	864
elliptic	Expression too large to display	864
risch	Expression too large to display	866

input
```
int(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-2/7*x^2*(d*x^3+c)^(1/2)+44/7*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^
(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)))-3*I*c/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+
1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(
x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*
(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c
*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipti
cPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1
/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2368 vs. $2(439) = 878$.

Time = 2.54 (sec) , antiderivative size = 2368, normalized size of antiderivative = 3.78

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \text{Too large to display}$$

input

```
integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output

```
-1/56*(16*sqrt(d*x^3 + c)*d*x^2 - 1056*c*sqrt(d)*weierstrassZeta(0, -4*c/d
, weierstrassPInverse(0, -4*c/d, x)) + 21*(c^7/d^4)^(1/6)*(sqrt(-3)*d - d)
*log(59049/4*((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 +
sqrt(-3)*(d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3))*(c^7/
d^4)^(5/6) + 6*(2*c^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^4*x
^5 + 32*c^3*d^3*x^2 - sqrt(-3)*(5*c^2*d^4*x^5 + 32*c^3*d^3*x^2))*(c^7/d^4)
^(2/3) - (7*c^4*d^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d + sqrt(-3)*(7*c^4*d^3
*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d))*(c^7/d^4)^(1/3))*sqrt(d*x^3 + c) - 36*
(5*c^3*d^4*x^7 + 64*c^4*d^3*x^4 + 32*c^5*d^2*x)*sqrt(c^7/d^4) + 18*(c^5*d^
3*x^8 + 38*c^6*d^2*x^5 + 64*c^7*d*x^2 - sqrt(-3)*(c^5*d^3*x^8 + 38*c^6*d^2
*x^5 + 64*c^7*d*x^2))*(c^7/d^4)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d
*x^3 - 512*c^3)) - 21*(c^7/d^4)^(1/6)*(sqrt(-3)*d - d)*log(-59049/4*((d^6*
x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + sqrt(-3)*(d^6*x^9 +
 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3))*(c^7/d^4)^(5/6) - 6*(2*c
^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^4*x^5 + 32*c^3*d^3*x^2
 - sqrt(-3)*(5*c^2*d^4*x^5 + 32*c^3*d^3*x^2))*(c^7/d^4)^(2/3) - (7*c^4*d^3
*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d + sqrt(-3)*(7*c^4*d^3*x^6 + 152*c^5*d^2*
x^3 + 64*c^6*d))*(c^7/d^4)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^4*x^7 + 64
*c^4*d^3*x^4 + 32*c^5*d^2*x)*sqrt(c^7/d^4) + 18*(c^5*d^3*x^8 + 38*c^6*d^2*
x^5 + 64*c^7*d*x^2 - sqrt(-3)*(c^5*d^3*x^8 + 38*c^6*d^2*x^5 + 64*c^7*d*...
```

Sympy [F]

$$\int \frac{x(c + dx^3)^{3/2}}{8c - dx^3}\,dx = -\int \frac{cx\sqrt{c + dx^3}}{-8c + dx^3}\,dx - \int \frac{dx^4\sqrt{c + dx^3}}{-8c + dx^3}\,dx$$

input
```
integrate(x*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

output
```
-Integral(c*x*sqrt(c + d*x**3)/(-8*c + d*x**3), x) - Integral(d*x**4*sqrt(
c + d*x**3)/(-8*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x(c + dx^3)^{3/2}}{8c - dx^3}\,dx = \int -\frac{(dx^3 + c)^{\frac{3}{2}}x}{dx^3 - 8\,c}\,dx$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate((d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c), x)
```

Giac [F]

$$\int \frac{x(c + dx^3)^{3/2}}{8c - dx^3}\,dx = \int -\frac{(dx^3 + c)^{\frac{3}{2}}x}{dx^3 - 8\,c}\,dx$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-(d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx=\int \frac{x\,(d\,x^3+c)^{3/2}}{8\,c-d\,x^3}\,dx$$

input
```
int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

output
```
int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3), x)
```

Reduce [F]

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx=-\frac{2\sqrt{d\,x^3+c}\,x^2}{7}+\frac{66\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{-d^2x^6+7cd\,x^3+8c^2}dx\right)cd}{7}+\frac{39\left(\int \frac{\sqrt{d\,x^3+c}\,x}{-d^2x^6+7cd\,x^3+8c^2}dx\right)c^2}{7}$$

input
```
int(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

output
```
( - 2*sqrt(c + d*x**3)*x**2 + 66*int((sqrt(c + d*x**3)*x**4)/(8*c**2 + 7*c
*d*x**3 - d**2*x**6),x)*c*d + 39*int((sqrt(c + d*x**3)*x)/(8*c**2 + 7*c*d*
x**3 - d**2*x**6),x)*c**2)/7
```

3.480 $\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 626

$$\begin{aligned}\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx &= -\frac{\sqrt{c+dx^3}}{8x}-\frac{15\sqrt[3]{d}\sqrt{c+dx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}\\
&-\frac{9}{16}\sqrt{3}\sqrt[6]{c}\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)\\
&+\frac{9}{16}\sqrt[6]{c}\sqrt[3]{d}\,\mathrm{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)-\frac{9}{16}\sqrt[6]{c}\sqrt[3]{d}\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\\
&+\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\\
&-\frac{5\ 3^{3/4}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\end{aligned}$$

output
```
-1/8*(d*x^3+c)^(1/2)/x-15*d^(1/3)*(d*x^3+c)^(1/2)/(8*(1+3^(1/2))*c^(1/3)+8
*d^(1/3)*x)-9/16*3^(1/2)*c^(1/6)*d^(1/3)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d
^(1/3)*x)/(d*x^3+c)^(1/2))+9/16*c^(1/6)*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/
3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))-9/16*c^(1/6)*d^(1/3)*arctanh(1/3*(d*x^3+c
)^(1/2)/c^(1/2))+15/16*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*c^(1/3)*d^(1/3)*(
c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c
^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3
^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1
+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)-5/8*3^(3/4)*c^(1/3)*
d^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3
^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)
*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)/(c^(1/3)*(c^(1/
3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.22

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = \frac{-16c(c+dx^3)+21cdx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3d^2x^6\sqrt{1+\frac{dx^3}{c}}}{128cx\sqrt{c+dx^3}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)),x]
```

output
```
(-16*c*(c + d*x^3) + 21*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1,
5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1
[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(128*c*x*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 620, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {974, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}dx$$

$$\Big\downarrow \text{974}$$

$$\frac{\int \frac{3cdx(5dx^3+14c)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{8c}-\frac{\sqrt{c+dx^3}}{8x}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{16}d\int \frac{x(5dx^3+14c)}{(8c-dx^3)\sqrt{dx^3+c}}dx-\frac{\sqrt{c+dx^3}}{8x}$$

$$\Big\downarrow \text{1054}$$

$$\frac{3}{16}d\int \left(\frac{54cx}{(8c-dx^3)\sqrt{dx^3+c}}-\frac{5x}{\sqrt{dx^3+c}}\right)dx-\frac{\sqrt{c+dx^3}}{8x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{16}d\left(-\frac{10\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{5\sqrt[4]{\ }}{}\right.$$

$$\frac{\sqrt{c+dx^3}}{8x}$$

input
```
Int[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)),x]
```

output
```
-1/8*Sqrt[c + d*x^3]/x + (3*d*((-10*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)) - (3*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(
1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(2/3) + (3*c^(1/6)*ArcTanh[(c^(1/3)
 + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(2/3) - (3*c^(1/6)*ArcTanh
[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(2/3) + (5*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1
/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)
/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^
(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/
(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*Sqrt[c + d*x^3]) - (10*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/
3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
d*x^3])))/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 974
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
+ b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.86 (sec) , antiderivative size = 859, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	859
risch	Expression too large to display	866
default	Expression too large to display	1339

input
```
int((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(d*x^3+c)^(1/2)/x+5/8*I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1
/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d
*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/
d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(
1/2)))-3/8*I/d^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I
*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = -\int \frac{c\sqrt{c+dx^3}}{-8cx^2+dx^5}\,dx - \int \frac{dx^3\sqrt{c+dx^3}}{-8cx^2+dx^5}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**2/(-d*x**3+8*c),x)
```

output
```
-Integral(c*sqrt(c + d*x**3)/(-8*c*x**2 + d*x**5), x) - Integral(d*x**3*sq
rt(c + d*x**3)/(-8*c*x**2 + d*x**5), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^2), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx=\int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-(d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx=\int \frac{(d\,x^3+c)^{3/2}}{x^2\,(8\,c-d\,x^3)}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)),x)
```

output
```
int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx=\frac{2\sqrt{d\,x^3+c}+21\left(\int \frac{\sqrt{d\,x^3+c}}{-d^2x^8+7cd\,x^5+8c^2x^2}dx\right)c^2x+6\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{-d^2x^6+7cd\,x^3+8c^2}dx\right)d^2x}{5x}$$

input
```
int((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x)
```

output
```
(2*sqrt(c + d*x**3) + 21*int(sqrt(c + d*x**3)/(8*c**2*x**2 + 7*c*d*x**5 -
d**2*x**8),x)*c**2*x + 6*int((sqrt(c + d*x**3)*x**4)/(8*c**2 + 7*c*d*x**3
- d**2*x**6),x)*d**2*x)/(5*x)
```

3.481 $\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 651

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{32x^4} - \frac{3d\sqrt{c+dx^3}}{16cx} + \frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{9\sqrt{3}d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{5/6}} + \frac{9d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{5/6}} - \frac{9d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{5/6}} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{3^{3/4}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{8\sqrt{2}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-1/32*(d*x^3+c)^(1/2)/x^4-3/16*d*(d*x^3+c)^(1/2)/c/x+3/16*d^(4/3)*(d*x^3+c
)^(1/2)/c/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)-9/128*3^(1/2)*d^(4/3)*arctan(3^(
1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))/c^(5/6)+9/128*d^(4/3)*ar
ctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)-9/128*d^(
4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)-3/32*3^(1/4)*(1/2*6^(1/2
)-1/2*2^(1/2))*d^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(
2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*
c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/c^(2/3)/
(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x
^3+c)^(1/2)+1/16*3^(3/4)*d^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(
1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1
-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I
)*2^(1/2)/c^(2/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3
)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.24

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = \frac{645cd^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4\Bigg(40c(c^2+7cdx^3+6d^2x^6)+}{5120c^2x^4\sqrt{c+dx^3}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)),x]
```

output
```
(645*c*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c)
, (d*x^3)/(8*c)] - 4*(40*c*(c^2 + 7*c*d*x^3 + 6*d^2*x^6) + 3*d^3*x^9*Sqrt[
1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(
5120*c^2*x^4*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {974, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}dx$$

$$\Big\downarrow \text{974}$$

$$\frac{\int \frac{3cd(23dx^3+32c)}{2x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c}-\frac{\sqrt{c+dx^3}}{32x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{64}d\int \frac{23dx^3+32c}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx-\frac{\sqrt{c+dx^3}}{32x^4}$$

$$\Big\downarrow \text{1053}$$

$$\frac{3}{64}d\left(-\frac{\int -\frac{8cdx(43c-2dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{8c^2}-\frac{4\sqrt{c+dx^3}}{cx}\right)-\frac{\sqrt{c+dx^3}}{32x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{64}d\left(\frac{d\int \frac{x(43c-2dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{c}-\frac{4\sqrt{c+dx^3}}{cx}\right)-\frac{\sqrt{c+dx^3}}{32x^4}$$

$$\Big\downarrow \text{1054}$$

$$\frac{3}{64}d\left(\frac{d\int \left(\frac{27cx}{(8c-dx^3)\sqrt{dx^3+c}}+\frac{2x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{4\sqrt{c+dx^3}}{cx}\right)-\frac{\sqrt{c+dx^3}}{32x^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{64}d\left(\frac{d\left(\frac{4\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{2\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{\cdots}\right.}{\cdots}\right)}{\cdots}\right.$$

$$\frac{\sqrt{c+dx^3}}{32x^4}$$

input

```
Int[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)),x]
```

output

```
-1/32*Sqrt[c + d*x^3]/x^4 + (3*d*((-4*Sqrt[c + d*x^3])/(c*x) + (d*((4*Sqrt
[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (3*Sqrt[3]*c^
(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*
d^(2/3)) + (3*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c +
d*x^3])])/(2*d^(2/3)) - (3*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(
2*d^(2/3)) - (2*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1
/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
+ (4*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)
*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
-7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/c))/64
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 974
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
+ b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.39 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.35

method	result	size
risch	Expression too large to display	882
elliptic	Expression too large to display	884
default	Expression too large to display	1810

input
```
int((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(d*x^3+c)^(1/2)*(6*d*x^3+c)/x^4/c+3/64/c*d^2*(-4/3*I*3^(1/2)/d*(-c*d
^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/
2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)
^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d
^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^
(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-
c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2
)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2369 vs. $2(459) = 918$.

Time = 0.66 (sec) , antiderivative size = 2369, normalized size of antiderivative = 3.64

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = \text{Too large to display}$$

input

```
integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="fricas")
```

output

```
-1/512*(96*d^(3/2)*x^4*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -
4*c/d, x)) - 6*(d^8/c^5)^(1/6)*c*x^4*log(6561*(d^9*x^9 + 318*c*d^8*x^6 + 1
200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^4*d^3*x^7 + 64*c^5*d^2*x^4 + 32*c^
6*d*x)*(d^8/c^5)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2)*(
d^8/c^5)^(5/6) + (7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(d^8/c
^5) + (c*d^7*x^7 + 80*c^2*d^6*x^4 + 160*c^3*d^5*x)*(d^8/c^5)^(1/6)) + 18*(
c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2)*(d^8/c^5)^(1/3))/(d^3*x^9 -
 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 6*(d^8/c^5)^(1/6)*c*x^4*log(65
61*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^4*d
^3*x^7 + 64*c^5*d^2*x^4 + 32*c^6*d*x)*(d^8/c^5)^(2/3) - 6*sqrt(d*x^3 + c)*
(6*(5*c^5*d*x^5 + 32*c^6*x^2)*(d^8/c^5)^(5/6) + (7*c^3*d^4*x^6 + 152*c^4*d
^3*x^3 + 64*c^5*d^2)*sqrt(d^8/c^5) + (c*d^7*x^7 + 80*c^2*d^6*x^4 + 160*c^3
*d^5*x)*(d^8/c^5)^(1/6)) + 18*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x
^2)*(d^8/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) -
 3*(sqrt(-3)*c*x^4 + c*x^4)*(d^8/c^5)^(1/6)*log(6561*(d^9*x^9 + 318*c*d^8*
x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^4*d^3*x^7 + 64*c^5*d^2*x^4 +
 32*c^6*d*x + sqrt(-3)*(5*c^4*d^3*x^7 + 64*c^5*d^2*x^4 + 32*c^6*d*x))*(d^8
/c^5)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2 - sqrt(-3)*(5
*c^5*d*x^5 + 32*c^6*x^2))*(d^8/c^5)^(5/6) - 2*(7*c^3*d^4*x^6 + 152*c^4*d^3
*x^3 + 64*c^5*d^2)*sqrt(d^8/c^5) + (c*d^7*x^7 + 80*c^2*d^6*x^4 + 160*c^...
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = -\int \frac{c\sqrt{c+dx^3}}{-8cx^5+dx^8}\,dx - \int \frac{dx^3\sqrt{c+dx^3}}{-8cx^5+dx^8}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**5/(-d*x**3+8*c),x)
```

output
```
-Integral(c*sqrt(c + d*x**3)/(-8*c*x**5 + d*x**8), x) - Integral(d*x**3*sq
rt(c + d*x**3)/(-8*c*x**5 + d*x**8), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^5}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^5), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^5}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-(d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^5\,(8\,c-d\,x^3)}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)),x)
```

output
```
int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = \frac{-2\sqrt{d\,x^3+c}+96\left(\int \frac{\sqrt{d\,x^3+c}}{-d^2x^8+7cd\,x^5+8c^2x^2}dx\right)cd\,x^4+69\left(\int \frac{\sqrt{d\,x^3+c}\,x}{-d^2x^6+7cd\,x^3+8c^2}dx\right)d^2x^4}{64x^4}$$

input
```
int((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x)
```

output
```
( - 2*sqrt(c + d*x**3) + 96*int(sqrt(c + d*x**3)/(8*c**2*x**2 + 7*c*d*x**5
 - d**2*x**8),x)*c*d*x**4 + 69*int((sqrt(c + d*x**3)*x)/(8*c**2 + 7*c*d*x*
*3 - d**2*x**6),x)*d**2*x**4)/(64*x**4)
```

3.482 $\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 675

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{9\sqrt{3}d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}}+\frac{9d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}}-\frac{9d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{11/6}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{112c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{28\sqrt{2}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-1/56*(d*x^3+c)^(1/2)/x^7-75/1792*d*(d*x^3+c)^(1/2)/c/x^4-3/56*d^2*(d*x^3+
c)^(1/2)/c^2/x+3/56*d^(7/3)*(d*x^3+c)^(1/2)/c^2/((1+3^(1/2))*c^(1/3)+d^(1/
3)*x)-9/1024*3^(1/2)*d^(7/3)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d
*x^3+c)^(1/2))/c^(11/6)+9/1024*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c
^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)-9/1024*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2
)/c^(1/2))/c^(11/6)-3/112*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*d^(7/3)*(c^(1/
3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3
)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2
))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/c^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/
((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+1/56*3^(3/4)*d^(7
/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/
2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/
((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)/c^(5/3)/(c^(1/3)*(c
^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.25

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx = \frac{6675cd^3x^9\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-32\Big(5c(32c^3+107c^2dx^3+17}{286720c^3x^7\sqrt{c+}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)),x]
```

output
```
(6675*c*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c
), (d*x^3)/(8*c)] - 32*(5*c*(32*c^3 + 107*c^2*d*x^3 + 171*c*d^2*x^6 + 96*d
^3*x^9) + 6*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x
^3)/c), (d*x^3)/(8*c)]))/(286720*c^3*x^7*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.86 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {974, 27, 1053, 25, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}dx$$

$$\Big\downarrow \text{974}$$

$$\frac{\int \frac{3cd(41dx^3+50c)}{2x^5(8c-dx^3)\sqrt{dx^3+c}}dx}{56c}-\frac{\sqrt{c+dx^3}}{56x^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{112}d\int \frac{41dx^3+50c}{x^5(8c-dx^3)\sqrt{dx^3+c}}dx-\frac{\sqrt{c+dx^3}}{56x^7}$$

$$\Big\downarrow \text{1053}$$

$$\frac{3}{112}d\left(-\frac{\int -\frac{cd(125dx^3+512c)}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c^2}-\frac{25\sqrt{c+dx^3}}{16cx^4}\right)-\frac{\sqrt{c+dx^3}}{56x^7}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{112}d\left(\frac{\int \frac{cd(125dx^3+512c)}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c^2}-\frac{25\sqrt{c+dx^3}}{16cx^4}\right)-\frac{\sqrt{c+dx^3}}{56x^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{112}d\left(\frac{d\int \frac{125dx^3+512c}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c}-\frac{25\sqrt{c+dx^3}}{16cx^4}\right)-\frac{\sqrt{c+dx^3}}{56x^7}$$

$$\Big\downarrow \text{1053}$$

$$\frac{3}{112}d\left(\frac{d\left(-\frac{\int -\frac{8cdx\left(445c-32dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{64\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{25\sqrt{c+dx^3}}{16cx^4}\right)-\frac{\sqrt{c+dx^3}}{56x^7}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{112}d\left(\frac{d\left(\frac{d\int \frac{x\left(445c-32dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{64\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{25\sqrt{c+dx^3}}{16cx^4}\right)-\frac{\sqrt{c+dx^3}}{56x^7}$$

$$\Big\downarrow \; 1054$$

$$\frac{3}{112}d\left(\frac{d\left(\frac{d\int \left(\frac{189cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{32x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{64\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{25\sqrt{c+dx^3}}{16cx^4}\right)-\frac{\sqrt{c+dx^3}}{56x^7}$$

$$\Big\downarrow \; 2009$$

$$\frac{3}{112}d\left(\frac{d\left(\frac{d\left(\frac{64\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{32\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\right.}{}\right.}{}\right.}{}\right.$$

$$\frac{\sqrt{c+dx^3}}{56x^7}$$

input
```
Int[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)),x]
```

output
```
-1/56*Sqrt[c + d*x^3]/x^7 + (3*d*((-25*Sqrt[c + d*x^3])/(16*c*x^4) + (d*((
-64*Sqrt[c + d*x^3])/(c*x) + (d*((64*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)) - (21*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c
^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*d^(2/3)) + (21*c^(1/6)*ArcTanh[(
c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(2*d^(2/3)) - (21*c^(
1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*d^(2/3)) - (32*3^(1/4)*Sqrt[
2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)
*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
-7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (64*Sqrt[2]*c^(1/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(
1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^
(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*Sqrt[c + d*x^3])))/c))/(32*c)))/112
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 974
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
+ b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.32 (sec) , antiderivative size = 895, normalized size of antiderivative = 1.33

method	result	size
risch	Expression too large to display	895
elliptic	Expression too large to display	903
default	Expression too large to display	2306

input
```
int((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-1/1792*(d*x^3+c)^(1/2)*(96*d^2*x^6+75*c*d*x^3+32*c^2)/x^7/c^2+3/3584*d^3/
c^2*(-64/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1
/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+
1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d
^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/
3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-7*I/d^3
*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(
-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*
d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2442 vs. $2(479) = 958$.

Time = 1.76 (sec) , antiderivative size = 2442, normalized size of antiderivative = 3.62

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
-1/28672*(1536*d^(5/2)*x^7*weierstrassZeta(0, -4*c/d, weierstrassPInverse(
0, -4*c/d, x)) - 42*(d^14/c^11)^(1/6)*c^2*x^7*log(6561*(d^14*x^9 + 318*c*d
^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^8*d^4*x^7 + 64*c^9*d^
3*x^4 + 32*c^10*d^2*x)*(d^14/c^11)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*
x^5 + 32*c^11*x^2)*(d^14/c^11)^(5/6) + (7*c^6*d^6*x^6 + 152*c^7*d^5*x^3 +
64*c^8*d^4)*sqrt(d^14/c^11) + (c^2*d^11*x^7 + 80*c^3*d^10*x^4 + 160*c^4*d^
9*x)*(d^14/c^11)^(1/6)) + 18*(c^4*d^9*x^8 + 38*c^5*d^8*x^5 + 64*c^6*d^7*x^
2)*(d^14/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
+ 42*(d^14/c^11)^(1/6)*c^2*x^7*log(6561*(d^14*x^9 + 318*c*d^13*x^6 + 1200*
c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^8*d^4*x^7 + 64*c^9*d^3*x^4 + 32*c^10
*d^2*x)*(d^14/c^11)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x
^2)*(d^14/c^11)^(5/6) + (7*c^6*d^6*x^6 + 152*c^7*d^5*x^3 + 64*c^8*d^4)*sqr
t(d^14/c^11) + (c^2*d^11*x^7 + 80*c^3*d^10*x^4 + 160*c^4*d^9*x)*(d^14/c^11
)^(1/6)) + 18*(c^4*d^9*x^8 + 38*c^5*d^8*x^5 + 64*c^6*d^7*x^2)*(d^14/c^11)^
(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 21*(sqrt(-3)*
c^2*x^7 + c^2*x^7)*(d^14/c^11)^(1/6)*log(6561*(d^14*x^9 + 318*c*d^13*x^6 +
 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^8*d^4*x^7 + 64*c^9*d^3*x^4 + 32
*c^10*d^2*x + sqrt(-3)*(5*c^8*d^4*x^7 + 64*c^9*d^3*x^4 + 32*c^10*d^2*x))*(
d^14/c^11)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 - sqrt
(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^14/c^11)^(5/6) - 2*(7*c^6*d^6*x^6...
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx = -\int \frac{c\sqrt{c+dx^3}}{-8cx^8+dx^{11}}\,dx - \int \frac{dx^3\sqrt{c+dx^3}}{-8cx^8+dx^{11}}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**8/(-d*x**3+8*c),x)
```

output
```
-Integral(c*sqrt(c + d*x**3)/(-8*c*x**8 + d*x**11), x) - Integral(d*x**3*s
qrt(c + d*x**3)/(-8*c*x**8 + d*x**11), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^8}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^8), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^8}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-(d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^8\,(8\,c-d\,x^3)}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)),x)
```

output
```
int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx = \frac{-2\sqrt{d\,x^3+c}+150\left(\int \frac{\sqrt{d\,x^3+c}}{-d^2x^{11}+7cd\,x^8+8c^2x^5}dx\right)cd\,x^7+123\left(\int \frac{\sqrt{d\,x^3+c}}{-d^2x^8+7cd\,x^5+8c^2x^2}dx\right)d^2x}{112x^7}$$

input
```
int((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x)
```

output
```
( - 2*sqrt(c + d*x**3) + 150*int(sqrt(c + d*x**3)/(8*c**2*x**5 + 7*c*d*x**
8 - d**2*x**11),x)*c*d*x**7 + 123*int(sqrt(c + d*x**3)/(8*c**2*x**2 + 7*c*
d*x**5 - d**2*x**8),x)*d**2*x**7)/(112*x**7)
```

3.483 $\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{38c^2\sqrt{c+dx^3}}{d^4}-\frac{4c(c+dx^3)^{3/2}}{3d^4}-\frac{2(c+dx^3)^{5/2}}{15d^4}+\frac{1024c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}$$

output
```
-38*c^2*(d*x^3+c)^(1/2)/d^4-4/3*c*(d*x^3+c)^(3/2)/d^4-2/15*(d*x^3+c)^(5/2)
/d^4+1024/9*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{-6\sqrt{c+dx^3}(296c^2+12cdx^3+d^2x^6)+5120c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{45d^4}$$

input
```
Integrate[x^11/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-6*Sqrt[c + d*x^3]*(296*c^2 + 12*c*d*x^3 + d^2*x^6) + 5120*c^(5/2)*ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^4)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^9}{(8c-dx^3)\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{512c^3}{d^3(8c-dx^3)\sqrt{dx^3+c}}-\frac{57c^2}{d^3\sqrt{dx^3+c}}-\frac{6\sqrt{dx^3+c}c}{d^3}-\frac{(dx^3+c)^{3/2}}{d^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{1024c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3d^4}-\frac{114c^2\sqrt{c+dx^3}}{d^4}-\frac{4c(c+dx^3)^{3/2}}{d^4}-\frac{2(c+dx^3)^{5/2}}{5d^4}\right)$$

input
```
Int[x^11/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
((-114*c^2*Sqrt[c + d*x^3])/d^4 - (4*c*(c + d*x^3)^(3/2))/d^4 - (2*(c + d*
x^3)^(5/2))/(5*d^4) + (1024*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(3*d^4))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{5120 c^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)-6 \sqrt{d\,x^{3}+c}\,\left(d^{2} x^{6}+12 c d\,x^{3}+296 c^{2}\right)}{45 d^{4}}$
risch	$-\frac{2\left(d^{2} x^{6}+12 c d\,x^{3}+296 c^{2}\right) \sqrt{d\,x^{3}+c}}{15 d^{4}}+\frac{1024 c^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{9 d^{4}}$
default	$-\frac{\frac{2 x^{6} \sqrt{d\,x^{3}+c}}{15 d}-\frac{8 c\,x^{3} \sqrt{d\,x^{3}+c}}{45 d^{2}}+\frac{16 c^{2} \sqrt{d\,x^{3}+c}}{45 d^{3}}}{d}-\frac{128 c^{2} \sqrt{d\,x^{3}+c}}{3 d^{4}}-\frac{8 c\left(\frac{2 x^{3} \sqrt{d\,x^{3}+c}}{9 d}-\frac{4 c \sqrt{d\,x^{3}+c}}{9 d^{2}}\right)}{d^{2}}+\frac{1024 c^{\frac{5}{2}} \operatorname{arctanh}\left(\right.}{9 d^{4}}$
elliptic	$-\frac{2 x^{6} \sqrt{d\,x^{3}+c}}{15 d^{2}}-\frac{8 c\,x^{3} \sqrt{d\,x^{3}+c}}{5 d^{3}}-\frac{592 c^{2} \sqrt{d\,x^{3}+c}}{15 d^{4}}-\frac{512 i c^{2} \sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\,\_Z^{3}-8 c\right)} \frac{\left(-c\,d^{2}\right)^{\frac{1}{3}} \sqrt{2} \sqrt{\frac{i d\left(2 x+\frac{-i \sqrt{3}}{\cdots}\right)}{\cdots}}}{\cdots}\right.}{\cdots}$

input

```
int(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/45*(5120*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-6*(d*x^3+c)^(1/2)*
(d^2*x^6+12*c*d*x^3+296*c^2))/d^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.59

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \left[\frac{2\left(1280\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\,(d^2x^6+12\,cdx^3+296\,c^2)\sqrt{dx^3+c}\right)}{45\,d^4},\; -\frac{2\left(2560\sqrt{-c}c^2\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)+3\,(d^2x^6+12\,cdx^3+296\,c^2)\sqrt{dx^3+c}\right)}{45\,d^4}\right]$$

input
```
integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[2/45*(1280*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
- 8*c)) - 3*(d^2*x^6 + 12*c*d*x^3 + 296*c^2)*sqrt(d*x^3 + c))/d^4, -2/45*(
2560*sqrt(-c)*c^2*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) + 3*(d^2*x^6 + 12*c*d
*x^3 + 296*c^2)*sqrt(d*x^3 + c))/d^4]
```

Sympy [A] (verification not implemented)

Time = 20.44 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \begin{cases} \frac{2\left(-\frac{512c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9\sqrt{-c}}-19c^2\sqrt{c+dx^3}-\frac{2c\left(c+dx^3\right)^{\frac{3}{2}}}{3}-\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15}\right)}{d^4} & \text{for } d\neq 0 \\ \frac{x^{12}}{96c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
Piecewise((2*(-512*c**3*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*sqrt(-c)) -
 19*c**2*sqrt(c + d*x**3) - 2*c*(c + d*x**3)**(3/2)/3 - (c + d*x**3)**(5/2
)/15)/d**4, Ne(d, 0)), (x**12/(96*c**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(1280\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\,(dx^3+c)^{\frac{5}{2}}+30\,(dx^3+c)^{\frac{3}{2}}c+855\,\sqrt{dx^3+c}c^2\right)}{45\,d^4}$$

input
```
integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-2/45*(1280*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3
*sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 30*(d*x^3 + c)^(3/2)*c + 855*sqrt(d*x^3
 + c)*c^2)/d^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{1024\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\,\sqrt{-c}d^4} - \frac{2\left((dx^3+c)^{\frac{5}{2}}d^{16}+10\,(dx^3+c)^{\frac{3}{2}}cd^{16}+285\,\sqrt{dx^3+c}c^2d^{16}\right)}{15\,d^{20}}$$

input
```
integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-1024/9*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/15*((d
*x^3 + c)^(5/2)*d^16 + 10*(d*x^3 + c)^(3/2)*c*d^16 + 285*sqrt(d*x^3 + c)*c
^2*d^16)/d^20
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{512\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^4} - \frac{592\,c^2\,\sqrt{d\,x^3+c}}{15\,d^4} - \frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d^2} - \frac{8\,c\,x^3\,\sqrt{d\,x^3+c}}{5\,d^3}$$

input
```
int(x^11/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
(512*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/(9*d^4) - (592*c^2*(c + d*x^3)^(1/2))/(15*d^4) - (2*x^6*(c + d*x^3)^(1
/2))/(15*d^2) - (8*c*x^3*(c + d*x^3)^(1/2))/(5*d^3)
```

Reduce [F]

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{\frac{16\sqrt{d\,x^3+c}\,c^2}{5} - \frac{8\sqrt{d\,x^3+c}\,c\,d\,x^3}{5} - \frac{2\sqrt{d\,x^3+c}\,d^2x^6}{15} + 64\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{-d^2x^6+7cd\,x^3+8c^2}dx\right)c^2d^2}{d^4}$$

input
```
int(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

output
```
(2*(24*sqrt(c + d*x**3)*c**2 - 12*sqrt(c + d*x**3)*c*d*x**3 - sqrt(c + d*x
**3)*d**2*x**6 + 480*int((sqrt(c + d*x**3)*x**5)/(8*c**2 + 7*c*d*x**3 - d*
*2*x**6),x)*c**2*d**2))/(15*d**4)
```

3.484 $\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 71

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{128c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}$$

output
```
-14/3*c*(d*x^3+c)^(1/2)/d^3-2/9*(d*x^3+c)^(3/2)/d^3+128/9*c^(3/2)*arctanh(
1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{-2\sqrt{c+dx^3}(22c+dx^3)+128c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}$$

input
```
Integrate[x^8/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-2*Sqrt[c + d*x^3]*(22*c + d*x^3) + 128*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(
3*Sqrt[c])])/(9*d^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{(8c-dx^3)\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{64c^2}{d^2(8c-dx^3)\sqrt{dx^3+c}}-\frac{7c}{d^2\sqrt{dx^3+c}}-\frac{\sqrt{dx^3+c}}{d^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{128c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3d^3}-\frac{14c\sqrt{c+dx^3}}{d^3}-\frac{2(c+dx^3)^{3/2}}{3d^3}\right)$$

input
```
Int[x^8/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
((-14*c*Sqrt[c + d*x^3])/d^3 - (2*(c + d*x^3)^(3/2))/(3*d^3) + (128*c^(3/2
)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^3))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{\frac{128c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9}-\frac{2\left(d\,x^{3}+22c\right)\sqrt{d\,x^{3}+c}}{9}}{d^{3}}$
risch	$-\frac{2\left(d\,x^{3}+22c\right)\sqrt{d\,x^{3}+c}}{9d^{3}}+\frac{128c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d^{3}}$
default	$-\frac{\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d}-\frac{4c\sqrt{d\,x^{3}+c}}{9d^{2}}}{d}-\frac{16c\sqrt{d\,x^{3}+c}}{3d^{3}}+\frac{128c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d^{3}}$
elliptic	$-\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d^{2}}-\frac{44c\sqrt{d\,x^{3}+c}}{9d^{3}}-\frac{64ic\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\cdots}{\cdots}\right)}{\cdots}$

input
```
int(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(64*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-(d*x^3+22*c)*(d*x^3+c
)^(1/2))/d^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.66

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \left[\frac{2\left(32\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(dx^3+22\,c)\sqrt{dx^3+c}\right)}{9\,d^3},\ -\frac{2\left(64\,\sqrt{-c}c\arctan\left(\frac{3\,\sqrt{-c}}{\sqrt{dx^3+c}}\right)+(dx^3+22\,c)\sqrt{dx^3+c}\right)}{9\,d^3}\right]$$

input
```
integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[2/9*(32*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8
*c)) - (d*x^3 + 22*c)*sqrt(d*x^3 + c))/d^3, -2/9*(64*sqrt(-c)*c*arctan(3*s
qrt(-c)/sqrt(d*x^3 + c)) + (d*x^3 + 22*c)*sqrt(d*x^3 + c))/d^3]
```

Sympy [A] (verification not implemented)

Time = 10.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \begin{cases} \frac{2\left(-\frac{64c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9\sqrt{-c}}-\frac{7c\sqrt{c+dx^3}}{3}-\frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d^3} & \text{for } d\neq 0 \\ \frac{x^9}{72c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
Piecewise((2*(-64*c**2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*sqrt(-c)) -
7*c*sqrt(c + d*x**3)/3 - (c + d*x**3)**(3/2)/9)/d**3, Ne(d, 0)), (x**9/(72
*c**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(32\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+(dx^3+c)^{\frac{3}{2}}+21\sqrt{dx^3+c}c\right)}{9\,d^3}$$

input
```
integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-2/9*(32*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sq
rt(c))) + (d*x^3 + c)^(3/2) + 21*sqrt(d*x^3 + c)*c)/d^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(\frac{64\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d}+\frac{(dx^3+c)^{\frac{3}{2}}d^2+21\sqrt{dx^3+c}cd^2}{d^3}\right)}{9\,d^2}$$

input
```
integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-2/9*(64*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + ((d*x^3 +
 c)^(3/2)*d^2 + 21*sqrt(d*x^3 + c)*c*d^2)/d^3)/d^2
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{64\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\sqrt{c}\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^3} - \frac{44\,c\sqrt{d\,x^3+c}}{9\,d^3} - \frac{2\,x^3\sqrt{d\,x^3+c}}{9\,d^2}$$

input
```
int(x^8/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
(64*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/(9*d^3) - (44*c*(c + d*x^3)^(1/2))/(9*d^3) - (2*x^3*(c + d*x^3)^(1/2))/
(9*d^2)
```

Reduce [F]

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{\frac{4\sqrt{d\,x^3+c}\,c}{9}-\frac{2\sqrt{d\,x^3+c}\,d\,x^3}{9}+8\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{-d^2x^6+7cd\,x^3+8c^2}dx\right)c\,d^2}{d^3}$$

input
```
int(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

output
```
(2*(2*sqrt(c + d*x**3)*c - sqrt(c + d*x**3)*d*x**3 + 36*int((sqrt(c + d*x*
*3)*x**5)/(8*c**2 + 7*c*d*x**3 - d**2*x**6),x)*c*d**2))/(9*d**3)
```

3.485 $\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 52

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\sqrt{c+dx^3}}{3d^2}+\frac{16\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

output
```
-2/3*(d*x^3+c)^(1/2)/d^2+16/9*c^(1/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))
/d^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(3\sqrt{c+dx^3}-8\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{9d^2}$$

input
```
Integrate[x^5/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-2*(3*Sqrt[c + d*x^3] - 8*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/
(9*d^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {948, 90, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{(8c-dx^3)\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(\frac{8c\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{d}-\frac{2\sqrt{c+dx^3}}{d^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{16c\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d^2}-\frac{2\sqrt{c+dx^3}}{d^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{16\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3d^2}-\frac{2\sqrt{c+dx^3}}{d^2}\right)$$

input
```
Int[x^5/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
((-2*Sqrt[c + d*x^3])/d^2 + (16*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)])/(3*d^2))/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{-\frac{2\sqrt{d\,x^3+c}}{3}+\frac{16\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{9}}{d^2}$
default	$-\frac{2\sqrt{d\,x^3+c}}{3d^2}+\frac{16\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{9d^2}$
risch	$-\frac{2\sqrt{d\,x^3+c}}{3d^2}+\frac{16\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{9d^2}$
elliptic	$-\frac{2\sqrt{d\,x^3+c}}{3d^2}-\frac{8i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-\right.}}}{}\right.}{}$

input
```
int(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(8*c^(1/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-3*(d*x^3+c)^(1/2))/d^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.92

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \left[\frac{2\left(4\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\sqrt{dx^3+c}\right)}{9\,d^2},\; -\frac{2\left(8\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)+3\sqrt{dx^3+c}\right)}{9\,d^2}\right]$$

input
```
integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[2/9*(4*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - 3*sqrt(d*x^3 + c))/d^2, -2/9*(8*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^
3 + c)) + 3*sqrt(d*x^3 + c))/d^2]
```

Sympy [A] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \begin{cases} \frac{2\left(-\frac{8c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9\sqrt{-c}}-\frac{\sqrt{c+dx^3}}{3}\right)}{d^2} & \text{for } d\neq 0\\ \frac{x^6}{48c^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

input
```
integrate(x**5/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
Piecewise((2*(-8*c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*sqrt(-c)) - sqrt
(c + d*x**3)/3)/d**2, Ne(d, 0)), (x**6/(48*c**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(4\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\sqrt{dx^3+c}\right)}{9\,d^2}$$

input
```
integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-2/9*(4*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + 3*sqrt(d*x^3 + c))/d^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(\frac{8\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d}+\frac{3\sqrt{dx^3+c}}{d}\right)}{9\,d}$$

input
```
integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-2/9*(8*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + 3*sqrt(d*x^3
 + c)/d)/d
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{8\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^2}-\frac{2\sqrt{d\,x^3+c}}{3\,d^2}$$

input
```
int(x^5/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
(8*c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/(9*d^2) - (2*(c + d*x^3)^(1/2))/(3*d^2)
```

Reduce [F]

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^5}{-d^2x^6+7cd\,x^3+8c^2}dx$$

input
```
int(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**5)/(8*c**2 + 7*c*d*x**3 - d**2*x**6),x)
```

3.486 $\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 33

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

output
```
2/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

input
```
Integrate[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {946, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{3d}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

input
```
Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

method	result
default	$\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$
pseudoelliptic	$\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x\cdots\right.}{\cdots}}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.27

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \left[\frac{\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)}{9\sqrt{c}d}, -\frac{2\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)}{9\,cd}\right]$$

input
```
integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/9*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c))/(sqrt(c
)*d), -2/9*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c))/(c*d)]
```

Sympy [A] (verification not implemented)

Time = 3.57 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \begin{cases} -\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9d\sqrt{-c}} & \text{for } d \neq 0 \\ \frac{x^3}{24c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
Piecewise((-2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*d*sqrt(-c)), Ne(d, 0)
), (x**3/(24*c**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{9\sqrt{c}d}$$

input
```
integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/9*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/(sqr
t(c)*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d}$$

input
```
integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-2/9*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d)
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,\sqrt{c}\,d}$$

input
```
int(x^2/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(9*c^(1/2)
*d)
```

Reduce [F]

$$\int \frac{x^2}{(8c - dx^3)\sqrt{c + dx^3}}\,dx = \int \frac{\sqrt{d\,x^3 + c}\,x^2}{-d^2x^6 + 7cd\,x^3 + 8c^2}dx$$

input
```
int(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**2)/(8*c**2 + 7*c*d*x**3 - d**2*x**6),x)
```

3.487 $\int \frac{1}{x(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 58

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}$$

output
```
1/36*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/12*arctanh((d*x^3+c)^(
1/2)/c^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-3\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{36c^{3/2}}$$

input
```
Integrate[1/(x*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 3*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]
)/(36*c^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {948, 97, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(8c-dx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{97}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{8c}+\frac{d\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{8c}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{4c}+\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{4cd}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{4cd}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{12c^{3/2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{12c^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{4c^{3/2}}\right)$$

input
```
Int[1/(x*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(12*c^(3/2)) - ArcTanh[Sqrt[c + d*x^
3]/Sqrt[c]]/(4*c^(3/2)))/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)-3\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{36c^{\frac{3}{2}}}$	38
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{36c^{\frac{3}{2}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{12c^{\frac{3}{2}}}$	41
elliptic	Expression too large to display	1508

input
```
int(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/36*(arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-3*arctanh((d*x^3+c)^(1/2)/c^(1/
2)))/c^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.28

$$\int \frac{1}{x\,(8c-dx^3)\,\sqrt{c+dx^3}}\,dx = \left[\frac{\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\,\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{72\,c^2},\ -\frac{\sqrt{-c}\arctan\left(\frac{3\,\sqrt{-c}}{\sqrt{dx^3+c}}\right)-3\,\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)}{36\,c^2}\right]$$

input
```
integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/72*(sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c
)) + 3*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c^2, -1
/36*(sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) - 3*sqrt(-c)*arctan(sqrt(
-c)/sqrt(d*x^3 + c)))/c^2]
```

Sympy [A] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \begin{cases} \frac{2\left(-\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{72c\sqrt{-c}}+\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{24c\sqrt{-c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{\log\left(x^3\right)}{24c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
Piecewise((2*(-d*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(72*c*sqrt(-c)) + d*a
tan(sqrt(c + d*x**3)/sqrt(-c))/(24*c*sqrt(-c)))/d, Ne(d, 0)), (log(x**3)/(
24*c**(3/2)), True))
```

Maxima [F]

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x}\,dx$$

input
```
integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}c} - \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{36\sqrt{-c}c}$$

input
```
integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/36*arctan(1/3*sqrt(
d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c)
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=-\frac{3\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)-\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{36\,\sqrt{c^3}}$$

input
```
int(1/(x*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
-(3*atanh((c*(c + d*x^3)^(1/2))/(c^3)^(1/2)) - atanh((c*(c + d*x^3)^(1/2))
/(3*(c^3)^(1/2))))/(36*(c^3)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$$
$$=\frac{\sqrt{c}\log\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)-\sqrt{c}\log\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)+3\left(\int\frac{\sqrt{d\,x^3+c}\,x^2}{-d^2x^6+7cd\,x^3+8c^2}dx\right)cd}{24c^2}$$

input
```
int(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

output
```
(sqrt(c)*log(sqrt(c + d*x**3) - sqrt(c)) - sqrt(c)*log(sqrt(c + d*x**3) +
sqrt(c)) + 3*int((sqrt(c + d*x**3)*x**2)/(8*c**2 + 7*c*d*x**3 - d**2*x**6)
,x)*c*d)/(24*c**2)
```

3.488 $\int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 81

$$\int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{24c^2x^3} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}$$

output
```
-1/24*(d*x^3+c)^(1/2)/c^2/x^3+1/288*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))
/c^(5/2)+1/32*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{24c^2x^3} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}$$

input
```
Integrate[1/(x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
-1/24*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])
/(288*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(32*c^(5/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {948, 114, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (8c-dx^3) \sqrt{c+dx^3}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^6 (8c-dx^3) \sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int \frac{d(6c-dx^3)}{2x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{8c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\int \frac{6c-dx^3}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{3}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3-\frac{1}{4}d\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3\right)}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{3\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{1}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}\right)}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{3\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}\right)}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3}\right)$$

input
```
Int[1/(x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-1/8*Sqrt[c + d*x^3]/(c^2*x^3) - (d*(-1/6*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt
[c])]/Sqrt[c] - (3*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c])))/(16*c^2
))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{9d\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)x^3+\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)d\,x^3-12\sqrt{d\,x^3+c}\,\sqrt{c}}{288c^{\frac{5}{2}}x^3}$	64
risch	$-\frac{\sqrt{d\,x^3+c}}{24c^2x^3}-\frac{d\left(-\frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{2\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{18\sqrt{c}}\right)}{16c^2}$	65
default	$\frac{-\frac{\sqrt{d\,x^3+c}}{3c\,x^3}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{8c}-\frac{d\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{96c^{\frac{5}{2}}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{288c^{\frac{5}{2}}}$	86
elliptic	Expression too large to display	1523

input
```
int(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/288/c^(5/2)*(9*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))*x^3+arctanh(1/3*(d*x^3
+c)^(1/2)/c^(1/2))*d*x^3-12*(d*x^3+c)^(1/2)*c^(1/2))/x^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}}\,dx$$
$$=\left[\frac{\sqrt{c}dx^3\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+9\sqrt{c}dx^3\log\left(\frac{dx^3+2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\sqrt{dx^3+c}c}{576\,c^3x^3},\right.$$
$$\left.-\frac{\sqrt{-c}dx^3\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)+9\sqrt{-c}dx^3\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)+12\sqrt{dx^3+c}c}{288\,c^3x^3}\right]$$

input
```
integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/576*(sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^
3 - 8*c)) + 9*sqrt(c)*d*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/
x^3) - 24*sqrt(d*x^3 + c)*c)/(c^3*x^3), -1/288*(sqrt(-c)*d*x^3*arctan(3*sq
rt(-c)/sqrt(d*x^3 + c)) + 9*sqrt(-c)*d*x^3*arctan(sqrt(-c)/sqrt(d*x^3 + c)
) + 12*sqrt(d*x^3 + c)*c)/(c^3*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (8c - dx^3) \sqrt{c + dx^3}} \, dx = -\int \frac{1}{-8cx^4\sqrt{c + dx^3} + dx^7\sqrt{c + dx^3}} \, dx$$

input
```
integrate(1/x**4/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(1/(-8*c*x**4*sqrt(c + d*x**3) + d*x**7*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int -\frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)x^4} \, dx$$

input
```
integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{32\sqrt{-c}c^2} - \frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{288\sqrt{-c}c^2} - \frac{\sqrt{dx^3+c}}{24\,c^2x^3}$$

input
```
integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-1/32*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/288*d*arctan(1
/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/24*sqrt(d*x^3 + c)/(c^2*x^
3)
```

Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{d\,\mathrm{atanh}\left(\frac{c^2\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)}{32\sqrt{c^5}} + \frac{d\,\mathrm{atanh}\left(\frac{c^2\sqrt{d\,x^3+c}}{3\sqrt{c^5}}\right)}{288\sqrt{c^5}} - \frac{\sqrt{d\,x^3+c}}{24\,c^2\,x^3}$$

input
```
int(1/(x^4*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
(d*atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2)))/(32*(c^5)^(1/2)) + (d*atanh
((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2))))/(288*(c^5)^(1/2)) - (c + d*x^3)
^(1/2)/(24*c^2*x^3)
```

Reduce [F]

$$\int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}}\,dx$$
$$= \frac{-64\sqrt{d\,x^3+c}\,c+2\sqrt{d\,x^3+c}\,d\,x^3-24\sqrt{c}\log\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)d\,x^3+24\sqrt{c}\log\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)d\,x^3+}{1536c^3x^3}$$

input
```
int(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

output
```
( - 64*sqrt(c + d*x**3)*c + 2*sqrt(c + d*x**3)*d*x**3 - 24*sqrt(c)*log(sqr
t(c + d*x**3) - sqrt(c))*d*x**3 + 24*sqrt(c)*log(sqrt(c + d*x**3) + sqrt(c
))*d*x**3 + 3*int((sqrt(c + d*x**3)*x**5)/(8*c**2 + 7*c*d*x**3 - d**2*x**6
),x)*d**3*x**3)/(1536*c**3*x**3)
```

3.489 $\int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 107

$$\int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{48c^2x^6}+\frac{5d\sqrt{c+dx^3}}{192c^3x^3}+\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}}-\frac{7d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}$$

output
```
-1/48*(d*x^3+c)^(1/2)/c^2/x^6+5/192*d*(d*x^3+c)^(1/2)/c^3/x^3+1/2304*d^2*a
rctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-7/256*d^2*arctanh((d*x^3+c)^(1
/2)/c^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{c+dx^3}(-4c+5dx^3)}{192c^3x^6}+\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}}-\frac{7d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}$$

input
```
Integrate[1/(x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(Sqrt[c + d*x^3]*(-4*c + 5*d*x^3))/(192*c^3*x^6) + (d^2*ArcTanh[Sqrt[c + d
*x^3]/(3*Sqrt[c])])/(2304*c^(7/2)) - (7*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c
]])/(256*c^(7/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {948, 114, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (8c - dx^3) \sqrt{c + dx^3}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{1}{x^9 (8c - dx^3) \sqrt{dx^3 + c}} dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3} \left(-\frac{\int \frac{d(20c - 3dx^3)}{2x^6 (8c - dx^3) \sqrt{dx^3 + c}} dx^3}{16c^2} - \frac{\sqrt{c + dx^3}}{16c^2 x^6} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(-\frac{d \int \frac{20c - 3dx^3}{x^6 (8c - dx^3) \sqrt{dx^3 + c}} dx^3}{32c^2} - \frac{\sqrt{c + dx^3}}{16c^2 x^6} \right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{\int \frac{2cd\left(42c-5dx^3\right)}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{8c^2}-\frac{5\sqrt{c+dx^3}}{2cx^3}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6}\right)$$

$\downarrow$ 27

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\int \frac{42c-5dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{4c}-\frac{5\sqrt{c+dx^3}}{2cx^3}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6}\right)$$

$\downarrow$ 174

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{21}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{1}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3\right)}{4c}-\frac{5\sqrt{c+dx^3}}{2cx^3}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6}\right)$$

$\downarrow$ 73

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{1}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{21\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}\right)}{4c}-\frac{5\sqrt{c+dx^3}}{2cx^3}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6}\right)$$

$\downarrow$ 219

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{21\int\frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}\right)}{4c}-\frac{5\sqrt{c+dx^3}}{2cx^3}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{21\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)}{4c}-\frac{5\sqrt{c+dx^3}}{2cx^3}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6}\right)$$

input
```
Int[1/(x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-1/16*Sqrt[c + d*x^3]/(c^2*x^6) - (d*((-5*Sqrt[c + d*x^3])/(2*c*x^3) - (d
*(ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(6*Sqrt[c]) - (21*ArcTanh[Sqrt[c +
d*x^3]/Sqrt[c]])/(2*Sqrt[c])))/(4*c)))/(32*c^2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}\,\left(-5 d\,x^{3}+4 c \right)}{192 c^{3} x^{6}}+\frac{d^{2} \left(-\frac{7 \,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{2 \sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{18 \sqrt{c}}\right)}{128 c^{3}}$
pseudoelliptic	$\frac{-63 \,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right) d^{2} x^{6}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right) d^{2} x^{6}+60 d\,x^{3} \sqrt{d\,x^{3}+c}\,\sqrt{c}-48 \sqrt{d\,x^{3}+c}\,c^{\frac{3}{2}}}{2304 c^{\frac{7}{2}} x^{6}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{6 c\,x^{6}}+\frac{d \sqrt{d\,x^{3}+c}}{4 c^{2} x^{3}}-\frac{d^{2} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{4 c^{\frac{5}{2}}}}{8 c}+\frac{d \left(-\frac{\sqrt{d\,x^{3}+c}}{3 c\,x^{3}}+\frac{d \,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3 c^{\frac{3}{2}}}\right)}{64 c^{2}}-\frac{d^{2} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{768 c^{\frac{7}{2}}}+$
elliptic	Expression too large to display

input
```
int(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(d*x^3+c)^(1/2)*(-5*d*x^3+4*c)/c^3/x^6+1/128*d^2/c^3*(-7/2*arctanh(
(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+1/18*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))
/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.96

$$\int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}}\,dx = \left[\frac{\sqrt{c}d^2x^6\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+63\sqrt{c}d^2x^6\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)+24\,(5\,cdx^3-4\,c^2)\sqrt{dx^3+c}}{4608\,c^4x^6}, -\frac{\sqrt{-c}d^2x^6\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)-63\sqrt{-c}d^2x^6\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)-12\,(5\,cdx^3-4\,c^2)\sqrt{dx^3+c}}{2304\,c^4x^6}\right]$$

input
```
integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4608*(sqrt(c)*d^2*x^6*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d
*x^3 - 8*c)) + 63*sqrt(c)*d^2*x^6*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) +
 2*c)/x^3) + 24*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^4*x^6), -1/2304*(s
qrt(-c)*d^2*x^6*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) - 63*sqrt(-c)*d^2*x^6*a
rctan(sqrt(-c)/sqrt(d*x^3 + c)) - 12*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/
(c^4*x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\int \frac{1}{-8cx^7\sqrt{c+dx^3}+dx^{10}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**7/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(1/(-8*c*x**7*sqrt(c + d*x**3) + d*x**10*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^7 (8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x^7}\,dx$$

input
```
integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^7), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^7 (8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{7\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{256\,\sqrt{-c}c^3} - \frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{2304\,\sqrt{-c}c^3} + \frac{5\,(dx^3+c)^{\frac{3}{2}}d^2 - 9\,\sqrt{dx^3+c}cd^2}{192\,c^3d^2x^6}$$

input
```
integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
7/256*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/2304*d^2*arc
tan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + 1/192*(5*(d*x^3 + c)^(3
/2)*d^2 - 9*sqrt(d*x^3 + c)*c*d^2)/(c^3*d^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^7 (8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{d^2\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)}{2304\,\sqrt{c^7}} - \frac{7\,d^2\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)}{256\,\sqrt{c^7}} - \frac{3\,\sqrt{d\,x^3+c}}{64\,c^2\,x^6} + \frac{5\,{\left(d\,x^3+c\right)}^{3/2}}{192\,c^3\,x^6}$$

input
```
int(1/(x^7*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
(d^2*atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2))))/(2304*(c^7)^(1/2)) -
(7*d^2*atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2)))/(256*(c^7)^(1/2)) - (3*
(c + d*x^3)^(1/2))/(64*c^2*x^6) + (5*(c + d*x^3)^(3/2))/(192*c^3*x^6)
```

Reduce [F]

$$\int \frac{1}{x^7 (8c - dx^3) \sqrt{c + dx^3}} \, dx$$
$$= \frac{-1064\sqrt{d\,x^3+c}\,c^2 + 1330\sqrt{d\,x^3+c}\,cd\,x^3 + 700\sqrt{c}\log\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)d^2x^6 - 700\sqrt{c}\log\left(\sqrt{d\,x^3+c}+\right.}{}$$

input
```
int(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

output
```
( - 1064*sqrt(c + d*x**3)*c**2 + 1330*sqrt(c + d*x**3)*c*d*x**3 + 700*sqrt
(c)*log(sqrt(c + d*x**3) - sqrt(c))*d**2*x**6 - 700*sqrt(c)*log(sqrt(c + d
*x**3) + sqrt(c))*d**2*x**6 + 128*int(sqrt(c + d*x**3)/(8*c**2*x**7 + 7*c*
d*x**10 - d**2*x**13),x)*c**4*x**6 - 42*int(sqrt(c + d*x**3)/(8*c**2*x + 7
*c*d*x**4 - d**2*x**7),x)*c**2*d**2*x**6 + 105*int((sqrt(c + d*x**3)*x**2)
/(8*c**2 + 7*c*d*x**3 - d**2*x**6),x)*c*d**3*x**6)/(51200*c**4*x**6)
```

3.490 $\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 630

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{32c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}}$$
$$+ \frac{32c^{7/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}} - \frac{32c^{7/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{8/3}}$$
$$+ \frac{52\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$- \frac{104\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-2/7*x^2*(d*x^3+c)^(1/2)/d^2-104/7*c*(d*x^3+c)^(1/2)/d^(8/3)/((1+3^(1/2))*
c^(1/3)+d^(1/3)*x)-32/9*c^(7/6)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)
/(d*x^3+c)^(1/2))*3^(1/2)/d^(8/3)+32/9*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3
)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(8/3)-32/9*c^(7/6)*arctanh(1/3*(d*x^3+c)
^(1/2)/c^(1/2))/d^(8/3)+52/7*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*c^(4/3)*(c^
(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(
1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(
1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/d^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*
x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)-104/21*2^(1/2)
*c^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+
3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3
)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/d^(8/3)/(c^(1/
3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^
(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.21

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$$
$$= \frac{x^2\left(-20(c+dx^3)+20c\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+13dx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3}\right.\right.}{70d^2\sqrt{c+dx^3}}$$

input
```
Integrate[x^7/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x^2*(-20*(c + d*x^3) + 20*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3
, -((d*x^3)/c), (d*x^3)/(8*c)] + 13*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3
, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(70*d^2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {979, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{979}$$

$$\frac{2\int \frac{2cx(13dx^3+8c)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{7d^2}-\frac{2x^2\sqrt{c+dx^3}}{7d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{4c\int \frac{x(13dx^3+8c)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{7d^2}-\frac{2x^2\sqrt{c+dx^3}}{7d^2}$$

$$\Big\downarrow \text{1054}$$

$$\frac{4c\int \left(\frac{112cx}{(8c-dx^3)\sqrt{dx^3+c}}-\frac{13x}{\sqrt{dx^3+c}}\right)dx}{7d^2}-\frac{2x^2\sqrt{c+dx^3}}{7d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4c\left(-\frac{26\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{13\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\cdots\right.}{\cdots}\right.}{\cdots}$$

$$\frac{2x^2\sqrt{c+dx^3}}{7d^2}$$

input
```
Int[x^7/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-2*x^2*Sqrt[c + d*x^3])/(7*d^2) + (4*c*((-26*Sqrt[c + d*x^3])/(d^(2/3)*((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (56*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(
c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sqrt[3]*d^(2/3)) + (56*c^(1/6)*
ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9*d^(2/3))
- (56*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^(2/3)) + (13*3^(1
/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3
)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (26*Sqrt[2]*c^(1/3
)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1
/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3
^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(7*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.29 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.38

method	result	size
elliptic	Expression too large to display	867
risch	Expression too large to display	872
default	Expression too large to display	1308

input
```
int(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*x^2*(d*x^3+c)^(1/2)/d^2+104/21*I/d^3*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)))-64/27*I*c/d^5*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1
/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/
(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I
*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2
/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_
alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2428 vs. $2(442) = 884$.

Time = 10.19 (sec) , antiderivative size = 2428, normalized size of antiderivative = 3.85

$$\int \frac{x^7}{(8c - dx^3)\sqrt{c + dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
2/189*(56*d^3*(c^7/d^16)^(1/6)*log(33554432/3*((d^16*x^9 + 318*c*d^15*x^6
+ 1200*c^2*d^14*x^3 + 640*c^3*d^13)*(c^7/d^16)^(5/6) + 6*(c^6*d^2*x^7 + 80
*c^7*d*x^4 + 160*c^8*x + 6*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2)*(c^7/d^16)^(
2/3) + (7*c^4*d^7*x^6 + 152*c^5*d^6*x^3 + 64*c^6*d^5)*(c^7/d^16)^(1/3))*sq
rt(d*x^3 + c) + 18*(5*c^3*d^10*x^7 + 64*c^4*d^9*x^4 + 32*c^5*d^8*x)*sqrt(c
^7/d^16) + 18*(c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 64*c^7*d^3*x^2)*(c^7/d^16)^(
1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 56*d^3*(c^7/d^
16)^(1/6)*log(-33554432/3*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3
+ 640*c^3*d^13)*(c^7/d^16)^(5/6) - 6*(c^6*d^2*x^7 + 80*c^7*d*x^4 + 160*c^8
*x + 6*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2)*(c^7/d^16)^(2/3) + (7*c^4*d^7*x^
6 + 152*c^5*d^6*x^3 + 64*c^6*d^5)*(c^7/d^16)^(1/3))*sqrt(d*x^3 + c) + 18*(
5*c^3*d^10*x^7 + 64*c^4*d^9*x^4 + 32*c^5*d^8*x)*sqrt(c^7/d^16) + 18*(c^5*d
^5*x^8 + 38*c^6*d^4*x^5 + 64*c^7*d^3*x^2)*(c^7/d^16)^(1/6))/(d^3*x^9 - 24*
c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 27*sqrt(d*x^3 + c)*d*x^2 + 1404*c*
sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 28
*(sqrt(-3)*d^3 - d^3)*(c^7/d^16)^(1/6)*log(33554432/3*((d^16*x^9 + 318*c*d
^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^
15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c^7/d^16)^(5/6) + 6*(2*c^6*d^
2*x^7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2 -
sqrt(-3)*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2))*(c^7/d^16)^(2/3) - (7*c^4*...
```

Sympy [F]

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\int \frac{x^7}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**7/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(x**7/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{x^7}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{x^7}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^7}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)}\,dx$$

input
```
int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{-\frac{2\sqrt{d\,x^3+c}\,x^2}{7}+\frac{52\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{-d^2x^6+7cd\,x^3+8c^2}dx\right)cd}{7}+\frac{32\left(\int \frac{\sqrt{d\,x^3+c}\,x}{-d^2x^6+7cd\,x^3+8c^2}dx\right)c^2}{7}}{d^2}$$

input
```
int(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

output
```
(2*( - sqrt(c + d*x**3)*x**2 + 26*int((sqrt(c + d*x**3)*x**4)/(8*c**2 + 7*
c*d*x**3 - d**2*x**6),x)*c*d + 16*int((sqrt(c + d*x**3)*x)/(8*c**2 + 7*c*d
*x**3 - d**2*x**6),x)*c**2))/(7*d**2)
```

3.491 $\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 601

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{4\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}}$$

$$+ \frac{4\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{5/3}} - \frac{4\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{5/3}}$$

$$+ \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-2*(d*x^3+c)^(1/2)/d^(5/3)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)-4/9*c^(1/6)*arc
tan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/d^(5/3)+4
/9*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(5
/3)-4/9*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)+3^(1/4)*(1/2*
6^(1/2)-1/2*2^(1/2))*c^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)
*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(
1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/d^
(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2
)/(d*x^3+c)^(1/2)-2/3*2^(1/2)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3
)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*Elliptic
F(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2
)+2*I)*3^(3/4)/d^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d
^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.11

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{x^5\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{40c\sqrt{c+dx^3}}$$

input
```
Integrate[x^4/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/
(8*c)])/(40*c*Sqrt[c + d*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 3.02 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {983, 832, 759, 988, 946, 73, 219, 2416, 2563, 219, 2570, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$\downarrow$ 983

$$\frac{8c\int \frac{x}{(8c-dx^3)\sqrt{dx^3+c}}dx}{d}-\frac{\int \frac{x}{\sqrt{dx^3+c}}dx}{d}$$

$\downarrow$ 832

$$\frac{8c\int \frac{x}{(8c-dx^3)\sqrt{dx^3+c}}dx}{d}-\frac{\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}\int \frac{1}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}}{d}$$

$\downarrow$ 759

$$\frac{8c\int \frac{x}{(8c-dx^3)\sqrt{dx^3+c}}dx}{d}-\frac{\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\surd\right.}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{d}$$

$\downarrow$ 988

$$\frac{8c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\sqrt[3]{d}\int \frac{x^2}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{4\sqrt[3]{c}}\right)}{d}-$$

$$\frac{\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\surd\right.}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{d}$$

$\downarrow$ 946

$$\frac{8c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\sqrt[3]{d}\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{12\sqrt[3]{c}}\right)}{d}-\frac{\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{}\right.}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{d}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{8c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{6\sqrt[3]{c}d^{2/3}}\right)}{d}-\frac{\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{}\right.}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{8c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)}{d}-$$

$$\frac{\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{\right.}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{d}$$

$$\Big\downarrow \; 2416$$

$$\frac{8c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)}{d}-$$

$$\frac{\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\cdots}{\cdots}}{d}$$

$$\Big\downarrow \; 2563$$

$$\frac{8c\left(\frac{\int \frac{1}{9-\frac{\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)^4}{\sqrt[3]{c}\left(dx^3+c\right)}}d\frac{\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)^2}{c^{2/3}\sqrt{dx^3+c}}}{6\sqrt[3]{c}d^{2/3}}-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)}{d}-\frac{\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\cdots}{\cdots}}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{8c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)}{d}-\frac{\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\cdots}{\cdots}}{d}$$

$$\Big\downarrow \; 2570$$

$$\frac{8c\left(\frac{d^{4/3}\int \frac{1}{-\frac{2d^2}{c}-\frac{6\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)^2 d^2}{c^{2/3}\left(dx^3+c\right)}}d\frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\sqrt[3]{c}\sqrt{dx^3+c}}}{3c^{4/3}}+\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)}{d}-\frac{\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\cdots}{\cdots}}{d}$$

$$\Big\downarrow \; 218$$

$$\frac{8c\left(-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}}+\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)}{d}-\frac{\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\cdots}{\cdots}}{d}$$

input

```
Int[x^4/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(8*c*(-1/6*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]]
/(Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sq
rt[c + d*x^3])]/(18*c^(5/6)*d^(2/3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
]/(18*c^(5/6)*d^(2/3))))/d - (((2*Sqrt[c + d*x^3])/(d^(1/3)*((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(
1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)
/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(1/3)*Sqrt[(c^(
1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c
+ d*x^3]))/d^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*c^(1/3)*(c^(1/3) +
 d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3
)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/
3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x
)^2]*Sqrt[c + d*x^3]))/d
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 988
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[d*(q/(4*b))   Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x
^3]), x], x] + (-Simp[q^2/(12*b)   Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]
), x], x] + Simp[1/(12*b*c)   Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x +
 q^2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[8*b*c + a*d, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2570
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-2*g*h   Subst[Int[1/(2*e*h -
 (b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8
*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.27 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.41

method	result	size
default	Expression too large to display	848
elliptic	Expression too large to display	848

input
```
int(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-8/27*I/d^4*2
^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3
*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^
2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_a
lpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2254 vs. $2(424) = 848$.

Time = 1.89 (sec) , antiderivative size = 2254, normalized size of antiderivative = 3.75

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/27*(2*d^2*(c/d^10)^(1/6)*log(1024/3*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c
^2*d^9*x^3 + 640*c^3*d^8)*(c/d^10)^(5/6) + 6*(c*d^2*x^7 + 80*c^2*d*x^4 + 1
60*c^3*x + 6*(5*c*d^8*x^5 + 32*c^2*d^7*x^2)*(c/d^10)^(2/3) + (7*c*d^5*x^6
+ 152*c^2*d^4*x^3 + 64*c^3*d^3)*(c/d^10)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c*
d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x)*sqrt(c/d^10) + 18*(c*d^4*x^8 + 38
*c^2*d^3*x^5 + 64*c^3*d^2*x^2)*(c/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 1
92*c^2*d*x^3 - 512*c^3)) - 2*d^2*(c/d^10)^(1/6)*log(-1024/3*((d^11*x^9 + 3
18*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8)*(c/d^10)^(5/6) - 6*(c*d^2*
x^7 + 80*c^2*d*x^4 + 160*c^3*x + 6*(5*c*d^8*x^5 + 32*c^2*d^7*x^2)*(c/d^10)
^(2/3) + (7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3)*(c/d^10)^(1/3))*sqrt
(d*x^3 + c) + 18*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x)*sqrt(c/d^10
) + 18*(c*d^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2)*(c/d^10)^(1/6))/(d^3*
x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (sqrt(-3)*d^2 - d^2)*(c/d
^10)^(1/6)*log(1024/3*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640
*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^
3*d^8))*(c/d^10)^(5/6) + 6*(2*c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5
*c*d^8*x^5 + 32*c^2*d^7*x^2 - sqrt(-3)*(5*c*d^8*x^5 + 32*c^2*d^7*x^2))*(c/
d^10)^(2/3) - (7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3 + sqrt(-3)*(7*c*
d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3))*(c/d^10)^(1/3))*sqrt(d*x^3 + c) -
 36*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x)*sqrt(c/d^10) + 18*(c*...
```

Sympy [F]

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\int \frac{x^4}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**4/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(x**4/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{x^4}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{x^4}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)}\,dx$$

input
```
int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^4}{(8c - dx^3)\sqrt{c + dx^3}}\, dx = \int \frac{\sqrt{d\,x^3 + c}\, x^4}{-d^2x^6 + 7cd\,x^3 + 8c^2} dx$$

input
```
int(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**4)/(8*c**2 + 7*c*d*x**3 - d**2*x**6),x)
```

3.492 $\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}} + \frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

output
```
-1/18*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/
c^(5/6)/d^(2/3)+1/18*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(
1/2))/c^(5/6)/d^(2/3)-1/18*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^
(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48

$$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=\frac{x^2\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{16c\sqrt{c+dx^3}}$$

input
```
Integrate[x/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/
(8*c)])/(16*c*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {988, 946, 73, 219, 2563, 219, 2570, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{988}$$

$$-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\sqrt[3]{d}\int \frac{x^2}{(8c-dx^3)\sqrt{dx^3+c}}dx}{4\sqrt[3]{c}}$$

$$\Big\downarrow \text{946}$$

$$-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\sqrt[3]{d}\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{12\sqrt[3]{c}}$$

$\Big\downarrow$ 73

$$-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{6\sqrt[3]{c}d^{2/3}}$$

$\Big\downarrow$ 219

$$-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

$\Big\downarrow$ 2563

$$\frac{\int \frac{1}{9-\frac{\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)^4}{\sqrt[3]{c}\left(dx^3+c\right)}}d\frac{\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)^2}{c^{2/3}\sqrt{dx^3+c}}}{6\sqrt[3]{c}d^{2/3}}-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

$\Big\downarrow$ 219

$$-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

$\Big\downarrow$ 2570

$$\frac{d^{4/3}\int \frac{1}{-\frac{2d^2}{c}-\frac{6\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)^2d^2}{c^{2/3}\left(dx^3+c\right)}}d\frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\sqrt[3]{c}\sqrt{dx^3+c}}}{3c^{4/3}}+\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

$\Big\downarrow$ 218

$$-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}}+\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

input
```
Int[x/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
-1/6*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]]/(Sqrt
[3]*c^(5/6)*d^(2/3)) + ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c +
 d*x^3])]/(18*c^(5/6)*d^(2/3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(18*
c^(5/6)*d^(2/3))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 988
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[d*(q/(4*b))   Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x
^3]), x], x] + (-Simp[q^2/(12*b)   Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]
), x], x] + Simp[1/(12*b*c)   Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x +
 q^2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[8*b*c + a*d, 0]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2570
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-2*g*h   Subst[Int[1/(2*e*h -
 (b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8
*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.07 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.95

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-\right.}{}\right)}{2\left(\right.}}}{}\right.}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-\right.}{}\right)}{2\left(\right.}}}{}\right.}{}$

input
```
int(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/27*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*
d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*
(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3
)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-
c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2285 vs. $2(95) = 190$.

Time = 0.49 (sec) , antiderivative size = 2285, normalized size of antiderivative = 16.21

$$\int \frac{x}{(8c - dx^3)\sqrt{c + dx^3}} \, dx = \text{Too large to display}$$

input
```
integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/216*(sqrt(-3) + 1)*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 12
00*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x
+ sqrt(-3)*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x))*(1/(c^5*d^4))^
(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2 - sqrt(-3)*(5
*c^5*d^5*x^5 + 32*c^6*d^4*x^2))*(1/(c^5*d^4))^(5/6) - 2*(7*c^3*d^4*x^6 + 1
52*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7 + 80*c^2*d^2*x
^4 + 160*c^3*d*x + sqrt(-3)*(c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x))*(1
/(c^5*d^4))^(1/6)) - 9*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2 - sq
rt(-3)*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2))*(1/(c^5*d^4))^(1/3
))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 1/216*(sqrt(-3) +
 1)*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 64
0*c^3 - 9*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x + sqrt(-3)*(5*c^4
*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x))*(1/(c^5*d^4))^(2/3) - 3*sqrt(d*
x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2 - sqrt(-3)*(5*c^5*d^5*x^5 + 32
*c^6*d^4*x^2))*(1/(c^5*d^4))^(5/6) - 2*(7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 +
64*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x
+ sqrt(-3)*(c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x))*(1/(c^5*d^4))^(1/6)
) - 9*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2 - sqrt(-3)*(c^2*d^4*x
^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2))*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*
c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 1/216*(sqrt(-3) - 1)*(1/(c^5*d^...
```

Sympy [F]

$$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\int \frac{x}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(x/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{x}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{x}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Mupad [B] (verification not implemented)

Time = 40.04 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.93

$$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\,{\left(\sqrt{d\,x^3+c}-\sqrt{c}+2\,c^{1/6}\,d^{1/3}\,x\right)}^3}{x^3\,{\left(d^{1/3}\,x-2\,c^{1/3}\right)}^3}\right)}{54\,c^{5/6}\,d^{2/3}} + \frac{\sqrt{2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\,{\left(-\sqrt{3}\,c^{1/6}\,d^{1/3}\,x+\sqrt{d\,x^3+c}\,1\mathrm{i}+\sqrt{c}\,1\mathrm{i}+c^{1/6}\,d^{1/3}\,x\,1\mathrm{i}\right)}^3}{x^3\,{\left(d^{1/3}\,x+c^{1/3}-\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)}^3}\right)\,\sqrt{-1+\sqrt{3}\,1\mathrm{i}}}{108\,c^{5/6}\,d^{2/3}} + \frac{\sqrt{2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\,{\left(\sqrt{3}\,c^{1/6}\,d^{1/3}\,x-\sqrt{d\,x^3+c}\,1\mathrm{i}+\sqrt{c}\,1\mathrm{i}+c^{1/6}\,d^{1/3}\,x\,1\mathrm{i}\right)}^3}{x^3\,{\left(d^{1/3}\,x+c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)}^3}\right)\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{108\,c^{5/6}\,d^{2/3}}$$

input
```
int(x/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
log((((c + d*x^3)^(1/2) + c^(1/2))*((c + d*x^3)^(1/2) - c^(1/2) + 2*c^(1/6
)*d^(1/3)*x)^3)/(x^3*(d^(1/3)*x - 2*c^(1/3))^3))/(54*c^(5/6)*d^(2/3)) + (2
^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2)*1i + c^(1/2)*
1i + c^(1/6)*d^(1/3)*x*1i - 3^(1/2)*c^(1/6)*d^(1/3)*x)^3)/(x^3*(d^(1/3)*x
- 3^(1/2)*c^(1/3)*1i + c^(1/3))^3))*(3^(1/2)*1i - 1)^(1/2))/(108*c^(5/6)*d
^(2/3)) + (2^(1/2)*log((((c + d*x^3)^(1/2) + c^(1/2))*(c^(1/2)*1i - (c + d
*x^3)^(1/2)*1i + c^(1/6)*d^(1/3)*x*1i + 3^(1/2)*c^(1/6)*d^(1/3)*x)^3)/(x^3
*(3^(1/2)*c^(1/3)*1i + d^(1/3)*x + c^(1/3))^3))*(3^(1/2)*1i + 1)^(1/2)*1i)
/(108*c^(5/6)*d^(2/3))
```

Reduce [F]

$$\int \frac{x}{(8c - dx^3)\sqrt{c + dx^3}}\, dx = \int \frac{\sqrt{d\,x^3 + c}\,x}{-d^2x^6 + 7cd\,x^3 + 8c^2} dx$$

input
```
int(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x)/(8*c**2 + 7*c*d*x**3 - d**2*x**6),x)
```

3.493 $\int \frac{1}{x^2 (8c-dx^3) \sqrt{c+dx^3}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 632

$$\int \frac{1}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{11/6}}$$
$$+\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{11/6}}-\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{11/6}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-1/8*(d*x^3+c)^(1/2)/c^2/x+1/8*d^(1/3)*(d*x^3+c)^(1/2)/c^2/((1+3^(1/2))*c^
(1/3)+d^(1/3)*x)-1/144*d^(1/3)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/
(d*x^3+c)^(1/2))*3^(1/2)/c^(11/6)+1/144*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/
3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)-1/144*d^(1/3)*arctanh(1/3*(d*x^3
+c)^(1/2)/c^(1/2))/c^(11/6)-1/16*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*d^(1/3)
*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))
*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1
+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/c^(5/3)/(c^(1/3)*(c^(1/3)+d^(1
/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+1/24*d^(1/
3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2
))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/(
(1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/c^(5/3)/(c^(
1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c
)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \frac{-80c(c+dx^3)+25cdx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-d^2x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},\right.}{640c^3x\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^2*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-80*c*(c + d*x^3) + 25*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1,
5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5
/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(640*c^3*x*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {980, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (8c-dx^3) \sqrt{c+dx^3}} \, dx$$

$$\Big\downarrow \; 980$$

$$\frac{\int \frac{dx(10c-dx^3)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{8c^2} - \frac{\sqrt{c+dx^3}}{8c^2 x}$$

$$\Big\downarrow \; 27$$

$$\frac{d\int \frac{x(10c-dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{16c^2} - \frac{\sqrt{c+dx^3}}{8c^2 x}$$

$$\Big\downarrow \; 1054$$

$$\frac{d\int \left(\frac{2cx}{(8c-dx^3)\sqrt{dx^3+c}} + \frac{x}{\sqrt{dx^3+c}}\right)dx}{16c^2} - \frac{\sqrt{c+dx^3}}{8c^2 x}$$

$$\Big\downarrow \; 2009$$

$$\frac{d\left(\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right) \cdots}{\cdots}\right.}{\cdots}$$

$$\frac{\sqrt{c+dx^3}}{8c^2 x}$$

input
```
Int[1/(x^2*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
-1/8*Sqrt[c + d*x^3]/(c^2*x) + (d*((2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)) - (c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d
^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sqrt[3]*d^(2/3)) + (c^(1/6)*ArcTanh[(c^(1/
3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9*d^(2/3)) - (c^(1/6)*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]
]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3
)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[
3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt
[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*
x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(
c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])))/(16*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.85 (sec) , antiderivative size = 868, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	868
risch	Expression too large to display	872
default	Expression too large to display	874

input
```
int(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(d*x^3+c)^(1/2)/c^2/x-1/24*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)))-1/216*I/c^2/d^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I
*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1
/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))
)^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(
1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))
*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alph
a^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2259 vs. $2(444) = 888$.

Time = 0.52 (sec) , antiderivative size = 2259, normalized size of antiderivative = 3.57

$$\int \frac{1}{x^2 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/1728*(2*c^2*x*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d
^2*x^3 + 640*c^3*d + 18*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x)*(d^2/c^
11)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^2/c^11)^(
5/6) + (7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11) + (c^2*d^3*
x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x)*(d^2/c^11)^(1/6)) + 18*(c^4*d^3*x^8 +
38*c^5*d^2*x^5 + 64*c^6*d*x^2)*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
 192*c^2*d*x^3 - 512*c^3)) - 2*c^2*x*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c
*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d + 18*(5*c^8*d^2*x^7 + 64*c^9*d*x^4
 + 32*c^10*x)*(d^2/c^11)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c
^11*x^2)*(d^2/c^11)^(5/6) + (7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(
d^2/c^11) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x)*(d^2/c^11)^(1/6))
 + 18*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2)*(d^2/c^11)^(1/3))/(d^3
*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 216*sqrt(d)*x*weierstras
sZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + (sqrt(-3)*c^2*x + c^
2*x)*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 64
0*c^3*d - 9*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x + sqrt(-3)*(5*c^8*d^
2*x^7 + 64*c^9*d*x^4 + 32*c^10*x))*(d^2/c^11)^(2/3) + 3*sqrt(d*x^3 + c)*(6
*(5*c^10*d*x^5 + 32*c^11*x^2 - sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^2
/c^11)^(5/6) - 2*(7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11) +
 (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 + ...
```

Sympy [F]

$$\int \frac{1}{x^2 (8c - dx^3) \sqrt{c + dx^3}} \, dx = - \int \frac{1}{-8cx^2 \sqrt{c + dx^3} + dx^5 \sqrt{c + dx^3}} \, dx$$

input
```
integrate(1/x**2/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(1/(-8*c*x**2*sqrt(c + d*x**3) + d*x**5*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int -\frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)x^2} \, dx$$

input
```
integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int -\frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)x^2} \, dx$$

input
```
integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (8c-dx^3) \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^2 \sqrt{d\,x^3+c}\,(8\,c-d\,x^3)}\,dx$$

input
```
int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (8c-dx^3) \sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}}{-d^2x^8+7cd\,x^5+8c^2x^2}dx$$

input
```
int(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**3)/(8*c**2*x**2 + 7*c*d*x**5 - d**2*x**8),x)
```

3.494 $\int \frac{1}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 654

$$\int \frac{1}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{32c^2x^4}+\frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{384\sqrt{3}c^{17/6}} + \frac{d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1152c^{17/6}} - \frac{d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{17/6}} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-1/32*(d*x^3+c)^(1/2)/c^2/x^4+1/16*d*(d*x^3+c)^(1/2)/c^3/x-1/16*d^(4/3)*(d
*x^3+c)^(1/2)/c^3/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)-1/1152*d^(4/3)*arctan(3^
(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/c^(17/6)+1/1152
*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(17/
6)-1/1152*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)+1/32*3^(1/
4)*(1/2*6^(1/2)-1/2*2^(1/2))*d^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)
*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE
(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)
+2*I)/c^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)
^2)^(1/2)/(d*x^3+c)^(1/2)-1/48*d^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/
3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*Ellipti
cF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/
2)+2*I)*2^(1/2)*3^(3/4)/c^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*
c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^5 (8c-dx^3)\sqrt{c+dx^3}}\,dx$$
$$= \frac{160c(-c^2+cdx^3+2d^2x^6)-75cd^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+4d^3x^9\sqrt{1+\frac{dx^3}{c}}\,\text{Appel}}{5120c^4x^4\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^5*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(160*c*(-c^2 + c*d*x^3 + 2*d^2*x^6) - 75*c*d^2*x^6*Sqrt[1 + (d*x^3)/c]*App
ellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 4*d^3*x^9*Sqrt[1 +
(d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(5120*
c^4*x^4*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {980, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (8c-dx^3) \sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{980}$$

$$\frac{\int -\frac{d(32c-5dx^3)}{2x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c^2}-\frac{\sqrt{c+dx^3}}{32c^2x^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \frac{32c-5dx^3}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{64c^2}-\frac{\sqrt{c+dx^3}}{32c^2x^4}$$

$$\Big\downarrow \text{1053}$$

$$-\frac{d\left(-\frac{\int -\frac{8cdx\left(15c-2dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{4\sqrt{c+dx^3}}{cx}\right)}{64c^2}-\frac{\sqrt{c+dx^3}}{32c^2x^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\left(\frac{d\int \frac{x\left(15c-2dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{4\sqrt{c+dx^3}}{cx}\right)}{64c^2}-\frac{\sqrt{c+dx^3}}{32c^2x^4}$$

$$\Big\downarrow \text{1054}$$

$$-\frac{d\left(\frac{d\int \left(\frac{2x}{\sqrt{dx^3+c}}-\frac{cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}\right)dx}{c}-\frac{4\sqrt{c+dx^3}}{cx}\right)}{64c^2}-\frac{\sqrt{c+dx^3}}{32c^2x^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\left(d\left(\frac{4\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{2\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\cdots}}{d^{2/\cdots}}\cdots\right)\cdots\right)}{\cdots}$$

$$\frac{\sqrt{c+dx^3}}{32c^2x^4}$$

input
```
Int[1/(x^5*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
-1/32*Sqrt[c + d*x^3]/(c^2*x^4) - (d*((-4*Sqrt[c + d*x^3])/(c*x) + (d*((4*
Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (c^(1/6)*
ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(6*Sqrt[3
]*d^(2/3)) - (c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c +
d*x^3])])/(18*d^(2/3)) + (c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1
8*d^(2/3)) - (2*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1
/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
 + (4*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)
*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
-7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/c))/(64*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.42 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.35

method	result	size
risch	Expression too large to display	882
elliptic	Expression too large to display	887
default	Expression too large to display	1351

input
```
int(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(d*x^3+c)^(1/2)*(-2*d*x^3+c)/c^3/x^4-1/64/c^3*d^2*(1/27*I/d^3*2^(1/2
)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*
d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(
-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c
*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*
3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^
2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_a
lpha=RootOf(_Z^3*d-8*c))-4/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*(
(x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3
)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/
2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
)^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2403 vs. $2(462) = 924$.

Time = 1.12 (sec) , antiderivative size = 2403, normalized size of antiderivative = 3.67

$$\int \frac{1}{x^5 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/13824*(2*c^3*x^4*(d^8/c^17)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^
2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^12*d^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d
*x)*(d^8/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(
d^8/c^17)^(5/6) + (7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^
8/c^17) + (c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 160*c^5*d^5*x)*(d^8/c^17)^(1/6))
 + 18*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4*x^2)*(d^8/c^17)^(1/3))/(d
^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*c^3*x^4*(d^8/c^17)^(
1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5
*c^12*d^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x)*(d^8/c^17)^(2/3) - 6*sqrt(d
*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^8/c^17)^(5/6) + (7*c^9*d^4*x^
6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17) + (c^3*d^7*x^7 + 80*c^4
*d^6*x^4 + 160*c^5*d^5*x)*(d^8/c^17)^(1/6)) + 18*(c^6*d^6*x^8 + 38*c^7*d^5
*x^5 + 64*c^8*d^4*x^2)*(d^8/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2
*d*x^3 - 512*c^3)) + 864*d^(3/2)*x^4*weierstrassZeta(0, -4*c/d, weierstras
sPInverse(0, -4*c/d, x)) + (sqrt(-3)*c^3*x^4 + c^3*x^4)*(d^8/c^17)^(1/6)*l
og((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^12*d
^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x + sqrt(-3)*(5*c^12*d^3*x^7 + 64*c^1
3*d^2*x^4 + 32*c^14*d*x))*(d^8/c^17)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^15*
d*x^5 + 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^8/c^17)^(5
/6) - 2*(7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17)...
```

Sympy [F]

$$\int \frac{1}{x^5 (8c - dx^3) \sqrt{c + dx^3}} \, dx = - \int \frac{1}{-8cx^5\sqrt{c + dx^3} + dx^8\sqrt{c + dx^3}} \, dx$$

input
```
integrate(1/x**5/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(1/(-8*c*x**5*sqrt(c + d*x**3) + d*x**8*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int -\frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)x^5} \, dx$$

input
```
integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int -\frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)x^5} \, dx$$

input
```
integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^5 \sqrt{d x^3 + c} \, (8 c - d x^3)} \, dx$$

input
```
int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^5 (8c - dx^3) \sqrt{c + dx^3}} \, dx$$
$$= \frac{-2\sqrt{d x^3 + c} - 32\left(\int \frac{\sqrt{d x^3+c}}{-d^2x^8+7cd x^5+8c^2x^2} dx\right) cd x^4 + 5\left(\int \frac{\sqrt{d x^3+c}\, x}{-d^2x^6+7cd x^3+8c^2} dx\right) d^2x^4}{64c^2x^4}$$

input
```
int(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**3) - 32*int(sqrt(c + d*x**3)/(8*c**2*x**2 + 7*c*d*x**5
 - d**2*x**8),x)*c*d*x**4 + 5*int((sqrt(c + d*x**3)*x)/(8*c**2 + 7*c*d*x**
3 - d**2*x**6),x)*d**2*x**4)/(64*c**2*x**4)
```

3.495 $\int \frac{1}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 678

$$\int \frac{1}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}$$
$$+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}c^{23/6}}$$
$$+\frac{d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{23/6}}-\frac{d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9216c^{23/6}}$$
$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{112c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{28\sqrt{2}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-1/56*(d*x^3+c)^(1/2)/c^2/x^7+37/1792*d*(d*x^3+c)^(1/2)/c^3/x^4-3/56*d^2*(
d*x^3+c)^(1/2)/c^4/x+3/56*d^(7/3)*(d*x^3+c)^(1/2)/c^4/((1+3^(1/2))*c^(1/3)
+d^(1/3)*x)-1/9216*d^(7/3)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x
^3+c)^(1/2))*3^(1/2)/c^(23/6)+1/9216*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*
x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(23/6)-1/9216*d^(7/3)*arctanh(1/3*(d*x^3+c
)^(1/2)/c^(1/2))/c^(23/6)-3/112*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*d^(7/3)*
(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*
c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+
3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/c^(11/3)/(c^(1/3)*(c^(1/3)+d^(1
/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+1/56*3^(3/
4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((
1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1
/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)/c^(11/3)/(c^
(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+
c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \frac{3875cd^3x^9\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-32\bigg(5c(32c^3-5c^2dx^3+59cd^2x^6+96d^3x^9)+6d^4}{286720c^5x^7\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^8*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(3875*c*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c
), (d*x^3)/(8*c)] - 32*(5*c*(32*c^3 - 5*c^2*d*x^3 + 59*c*d^2*x^6 + 96*d^3*
x^9) + 6*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)
/c), (d*x^3)/(8*c)]))/(286720*c^5*x^7*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {980, 27, 1053, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 (8c - dx^3) \sqrt{c + dx^3}} \, dx$$

$$\Big\downarrow \text{980}$$

$$\frac{\int -\frac{d(74c - 11dx^3)}{2x^5(8c - dx^3)\sqrt{dx^3 + c}} dx}{56c^2} - \frac{\sqrt{c + dx^3}}{56c^2 x^7}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d \int \frac{74c - 11dx^3}{x^5(8c - dx^3)\sqrt{dx^3 + c}} dx}{112c^2} - \frac{\sqrt{c + dx^3}}{56c^2 x^7}$$

$$\Big\downarrow \text{1053}$$

$$-\frac{d \left(-\frac{\int \frac{cd(1536c - 185dx^3)}{x^2(8c - dx^3)\sqrt{dx^3 + c}} dx}{32c^2} - \frac{37\sqrt{c + dx^3}}{16cx^4} \right)}{112c^2} - \frac{\sqrt{c + dx^3}}{56c^2 x^7}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d \left(-\frac{d \int \frac{1536c - 185dx^3}{x^2(8c - dx^3)\sqrt{dx^3 + c}} dx}{32c} - \frac{37\sqrt{c + dx^3}}{16cx^4} \right)}{112c^2} - \frac{\sqrt{c + dx^3}}{56c^2 x^7}$$

$$\Big\downarrow \text{1053}$$

$$-\frac{d \left(-\frac{d \left(-\frac{\int -\frac{8cdx(775c - 96dx^3)}{(8c - dx^3)\sqrt{dx^3 + c}} dx}{8c^2} - \frac{192\sqrt{c + dx^3}}{cx} \right)}{32c} - \frac{37\sqrt{c + dx^3}}{16cx^4} \right)}{112c^2} - \frac{\sqrt{c + dx^3}}{56c^2 x^7}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\left(-\frac{d\left(\frac{d\int \frac{x\left(775c-96dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{192\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{37\sqrt{c+dx^3}}{16cx^4}\right)}{112c^2}-\frac{\sqrt{c+dx^3}}{56c^2x^7}$$

$$\Big\downarrow \text{1054}$$

$$-\frac{d\left(-\frac{d\left(\frac{d\int \left(\frac{7cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{96x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{192\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{37\sqrt{c+dx^3}}{16cx^4}\right)}{112c^2}-\frac{\sqrt{c+dx^3}}{56c^2x^7}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\left(-\frac{d\left(\frac{d\left(\frac{64\sqrt{2}3^{3/4}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{96\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}\cdots\right.}{\cdots}\right.}{d\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{\sqrt{c+dx^3}}{56c^2x^7}$$

input
```
Int[1/(x^8*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
-1/56*Sqrt[c + d*x^3]/(c^2*x^7) - (d*((-37*Sqrt[c + d*x^3])/(16*c*x^4) - (
d*((-192*Sqrt[c + d*x^3])/(c*x) + (d*((192*Sqrt[c + d*x^3])/(d^(2/3)*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (7*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1
/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(6*Sqrt[3]*d^(2/3)) + (7*c^(1/6)*ArcTa
nh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(18*d^(2/3)) - (7
*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(18*d^(2/3)) - (96*3^(1/4)*
Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^
(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[Ar
cSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (64*Sqrt[2]*3^(3/4)*c^
(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^
2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]]
)/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d
^(1/3)*x)^2]*Sqrt[c + d*x^3])))/c))/(32*c)))/(112*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.36 (sec) , antiderivative size = 895, normalized size of antiderivative = 1.32

method	result	size
risch	Expression too large to display	895
elliptic	Expression too large to display	906
default	Expression too large to display	1849

input
```
int(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1792*(d*x^3+c)^(1/2)*(96*d^2*x^6-37*c*d*x^3+32*c^2)/c^4/x^7+1/3584*d^3/
c^4*(-64*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3
))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2
)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-7/27*I/d^
3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2
*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2436 vs. $2(482) = 964$.

Time = 3.05 (sec) , antiderivative size = 2436, normalized size of antiderivative = 3.59

$$\int \frac{1}{x^8 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/774144*(14*c^4*x^7*(d^14/c^23)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 12
00*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32
*c^18*d^2*x)*(d^14/c^23)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c
^21*x^2)*(d^14/c^23)^(5/6) + (7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*c^14*
d^4)*sqrt(d^14/c^23) + (c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x)*(d
^14/c^23)^(1/6)) + 18*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2)*(d^
14/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 14*c
^4*x^7*(d^14/c^23)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^
3 + 640*c^3*d^11 + 18*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x)*(
d^14/c^23)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2)*(d^14
/c^23)^(5/6) + (7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*c^14*d^4)*sqrt(d^14
/c^23) + (c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x)*(d^14/c^23)^(1/6
)) + 18*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2)*(d^14/c^23)^(1/3)
)/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 41472*d^(5/2)*x^7*
weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 7*(sqrt(-3
)*c^4*x^7 + c^4*x^7)*(d^14/c^23)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 12
00*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*
c^18*d^2*x + sqrt(-3)*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x))*
(d^14/c^23)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 - sqr
t(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^14/c^23)^(5/6) - 2*(7*c^12*d^6*x...
```

Sympy [F]

$$\int \frac{1}{x^8 (8c - dx^3) \sqrt{c + dx^3}} \, dx = -\int \frac{1}{-8cx^8\sqrt{c + dx^3} + dx^{11}\sqrt{c + dx^3}} \, dx$$

input
```
integrate(1/x**8/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(1/(-8*c*x**8*sqrt(c + d*x**3) + d*x**11*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^8 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int -\frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)x^8} \, dx$$

input
```
integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int -\frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)x^8} \, dx$$

input
```
integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^8\,\sqrt{d\,x^3 + c}\,\left(8\,c - d\,x^3\right)} \, dx$$

input
```
int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^8 (8c - dx^3) \sqrt{c + dx^3}} \, dx$$
$$= \frac{-64\sqrt{d\,x^3+c}\,c + 74\sqrt{d\,x^3+c}\,d\,x^3 + 1536\left(\int \frac{\sqrt{d\,x^3+c}}{-d^2x^8+7cd\,x^5+8c^2x^2}dx\right) c\,d^2x^7 - 185\left(\int \frac{\sqrt{d\,x^3+c}\,x}{-d^2x^6+7cd\,x^3+8c^2}dx\right) d}{3584c^3x^7}$$

input
```
int(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

output
```
( - 64*sqrt(c + d*x**3)*c + 74*sqrt(c + d*x**3)*d*x**3 + 1536*int(sqrt(c +
 d*x**3)/(8*c**2*x**2 + 7*c*d*x**5 - d**2*x**8),x)*c*d**2*x**7 - 185*int((
sqrt(c + d*x**3)*x)/(8*c**2 + 7*c*d*x**3 - d**2*x**6),x)*d**3*x**7)/(3584*
c**3*x**7)
```

3.496 $\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=\frac{x^4\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},1,\frac{1}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c\sqrt{c+dx^3}}$$

output
```
1/32*x^4*(1+d*x^3/c)^(1/2)*AppellF1(4/3,1/2,1,7/3,-d*x^3/c,1/8*d*x^3/c)/c/
(d*x^3+c)^(1/2)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=\frac{x^4\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{32c\sqrt{c+dx^3}}$$

input
```
Integrate[x^3/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x^4*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/
(8*c)])/(32*c*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^3}{(8c-dx^3)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{1}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c\sqrt{c+dx^3}}$$

input
```
Int[x^3/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, (d*x^3)/(8*c), -((d*x^
3)/c)])/(32*c*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.16 (sec) , antiderivative size = 696, normalized size of antiderivative = 10.55

method	result
default	$\frac{2i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\mathrm{El}}{3d^{2}\sqrt{d\,x^{3}+c}}$
elliptic	$\frac{2i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\mathrm{El}}{3d^{2}\sqrt{d\,x^{3}+c}}$

input
```
int(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*
(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2))-8/27*I/d^4*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I
*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d
)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2284 vs. $2(52) = 104$.

Time = 0.38 (sec) , antiderivative size = 2284, normalized size of antiderivative = 34.61

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/54*(2*d^2*(1/(c*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^
3 + 640*c^3 + 18*(c*d^8*x^8 + 38*c^2*d^7*x^5 + 64*c^3*d^6*x^2)*(1/(c*d^8))
^(2/3) + 6*sqrt(d*x^3 + c)*((c*d^9*x^7 + 80*c^2*d^8*x^4 + 160*c^3*d^7*x)*(
1/(c*d^8))^(5/6) + (7*c*d^6*x^6 + 152*c^2*d^5*x^3 + 64*c^3*d^4)*sqrt(1/(c*
d^8)) + 6*(5*c*d^3*x^5 + 32*c^2*d^2*x^2)*(1/(c*d^8))^(1/6)) + 18*(5*c*d^5*
x^7 + 64*c^2*d^4*x^4 + 32*c^3*d^3*x)*(1/(c*d^8))^(1/3))/(d^3*x^9 - 24*c*d^
2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*d^2*(1/(c*d^8))^(1/6)*log((d^3*x^9 +
 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c*d^8*x^8 + 38*c^2*d^7*x^5
 + 64*c^3*d^6*x^2)*(1/(c*d^8))^(2/3) - 6*sqrt(d*x^3 + c)*((c*d^9*x^7 + 80*
c^2*d^8*x^4 + 160*c^3*d^7*x)*(1/(c*d^8))^(5/6) + (7*c*d^6*x^6 + 152*c^2*d^
5*x^3 + 64*c^3*d^4)*sqrt(1/(c*d^8)) + 6*(5*c*d^3*x^5 + 32*c^2*d^2*x^2)*(1/
(c*d^8))^(1/6)) + 18*(5*c*d^5*x^7 + 64*c^2*d^4*x^4 + 32*c^3*d^3*x)*(1/(c*d
^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (sqrt(-3
)*d^2 + d^2)*(1/(c*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x
^3 + 640*c^3 - 9*(c*d^8*x^8 + 38*c^2*d^7*x^5 + 64*c^3*d^6*x^2 + sqrt(-3)*(
c*d^8*x^8 + 38*c^2*d^7*x^5 + 64*c^3*d^6*x^2))*(1/(c*d^8))^(2/3) + 3*sqrt(d
*x^3 + c)*((c*d^9*x^7 + 80*c^2*d^8*x^4 + 160*c^3*d^7*x - sqrt(-3)*(c*d^9*x
^7 + 80*c^2*d^8*x^4 + 160*c^3*d^7*x))*(1/(c*d^8))^(5/6) - 2*(7*c*d^6*x^6 +
 152*c^2*d^5*x^3 + 64*c^3*d^4)*sqrt(1/(c*d^8)) + 6*(5*c*d^3*x^5 + 32*c^2*d
^2*x^2 + sqrt(-3)*(5*c*d^3*x^5 + 32*c^2*d^2*x^2))*(1/(c*d^8))^(1/6)) - ...
```

Sympy [F]

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\int \frac{x^3}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**3/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(x**3/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{x^3}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{x^3}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)}\,dx$$

input
```
int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^3}{(8c - dx^3)\sqrt{c + dx^3}}\,dx = \int \frac{\sqrt{d\,x^3 + c}\,x^3}{-d^2x^6 + 7cd\,x^3 + 8c^2}dx$$

input
```
int(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**3)/(8*c**2 + 7*c*d*x**3 - d**2*x**6),x)
```

3.497 $\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c\sqrt{c+dx^3}}$$

output
```
1/8*x*(1+d*x^3/c)^(1/2)*AppellF1(1/3,1/2,1,4/3,-d*x^3/c,1/8*d*x^3/c)/c/(d*
x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 166 vs. 2(64) = 128.

Time = 10.17 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.59

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$$=\frac{32cx\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{(8c-dx^3)\sqrt{c+dx^3}\left(32c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4\,\text{Ap}\right.\right.}$$

input
```
Integrate[1/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(32*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d
*x^3)*Sqrt[c + d*x^3]*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^
3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c
)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{(8c-dx^3)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c\sqrt{c+dx^3}}$$

input
```
Int[1/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(8*c*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.07 (sec) , antiderivative size = 416, normalized size of antiderivative = 6.50

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-\cdots\right.}{\cdots}\right)}{2\left(\cdots\right.}}\cdots}{\cdots}\right.}{\cdots}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-\cdots\right.}{\cdots}\right)}{2\left(\cdots\right.}}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/27*I/d^3/c*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-
c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*
d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2
/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*
(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)
/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2319 vs. $2(50) = 100$.

Time = 0.58 (sec) , antiderivative size = 2319, normalized size of antiderivative = 36.23

$$\int \frac{1}{(8c - dx^3)\sqrt{c + dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/432*(2*c*d*(1/(c^7*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d
*x^3 + 640*c^3 + 18*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2)*(1/(c^
7*d^2))^(2/3) + 6*sqrt(d*x^3 + c)*((c^6*d^4*x^7 + 80*c^7*d^3*x^4 + 160*c^8
*d^2*x)*(1/(c^7*d^2))^(5/6) + (7*c^4*d^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d)
*sqrt(1/(c^7*d^2)) + 6*(5*c^2*d^2*x^5 + 32*c^3*d*x^2)*(1/(c^7*d^2))^(1/6))
 + 18*(5*c^3*d^3*x^7 + 64*c^4*d^2*x^4 + 32*c^5*d*x)*(1/(c^7*d^2))^(1/3))/(
d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*c*d*(1/(c^7*d^2))^(
1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c^5*d^4
*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2)*(1/(c^7*d^2))^(2/3) - 6*sqrt(d*x^3
 + c)*((c^6*d^4*x^7 + 80*c^7*d^3*x^4 + 160*c^8*d^2*x)*(1/(c^7*d^2))^(5/6)
+ (7*c^4*d^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d)*sqrt(1/(c^7*d^2)) + 6*(5*c^
2*d^2*x^5 + 32*c^3*d*x^2)*(1/(c^7*d^2))^(1/6)) + 18*(5*c^3*d^3*x^7 + 64*c^
4*d^2*x^4 + 32*c^5*d*x)*(1/(c^7*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192
*c^2*d*x^3 - 512*c^3)) + (sqrt(-3)*c*d + c*d)*(1/(c^7*d^2))^(1/6)*log((d^3
*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^5*d^4*x^8 + 38*c^6*
d^3*x^5 + 64*c^7*d^2*x^2 + sqrt(-3)*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7
*d^2*x^2))*(1/(c^7*d^2))^(2/3) + 3*sqrt(d*x^3 + c)*((c^6*d^4*x^7 + 80*c^7*
d^3*x^4 + 160*c^8*d^2*x - sqrt(-3)*(c^6*d^4*x^7 + 80*c^7*d^3*x^4 + 160*c^8
*d^2*x))*(1/(c^7*d^2))^(5/6) - 2*(7*c^4*d^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6
*d)*sqrt(1/(c^7*d^2)) + 6*(5*c^2*d^2*x^5 + 32*c^3*d*x^2 + sqrt(-3)*(5*c...
```

Sympy [F]

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\int \frac{1}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(1/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)}\,dx$$

input
```
int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{(8c - dx^3)\sqrt{c + dx^3}}\, dx = \int \frac{\sqrt{d\,x^3 + c}}{-d^2x^6 + 7cd\,x^3 + 8c^2} dx$$

input
```
int(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**3)/(8*c**2 + 7*c*d*x**3 - d**2*x**6),x)
```

3.498 $\int \frac{1}{x^3(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^3 (8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16cx^2\sqrt{c+dx^3}}$$

output
```
-1/16*(1+d*x^3/c)^(1/2)*AppellF1(-2/3,1/2,1,1/3,-d*x^3/c,1/8*d*x^3/c)/c/x^
2/(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 242 vs. $2(66) = 132$.

Time = 11.18 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.67

$$\int \frac{1}{x^3 (8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{-\frac{64(c+dx^3)}{c^2}+\frac{d^2x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+\frac{4096dx^3\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\right.}{(-8c+dx^3)\left(32c\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},2\right.\right.\right.}}{1024x^2\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
((-64*(c + d*x^3))/c^2 + (d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1
, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])/c^3 + (4096*d*x^3*AppellF1[1/3, 1/2,
1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((-8*c + d*x^3)*(32*c*AppellF1[1/3,
1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2,
 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3
)/c), (d*x^3)/(8*c)]))))/(1024*x^2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (8c - dx^3) \sqrt{c + dx^3}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{1}{x^3(8c-dx^3)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1} \operatorname{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16cx^2\sqrt{c+dx^3}}$$

input
```
Int[1/(x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
-1/16*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, (d*x^3)/(8*c), -((d
*x^3)/c)])/(c*x^2*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 2.39 (sec) , antiderivative size = 716, normalized size of antiderivative = 10.85

method	result	size
elliptic	Expression too large to display	716
risch	Expression too large to display	720
default	Expression too large to display	722

input
```
int(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(d*x^3+c)^(1/2)/c^2/x^2+1/48*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/216*I/c^2/d^2*2^(1/2)*sum(1/_alpha^2*
(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^
(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_
alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_al
pha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I
*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/
2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^
3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2373 vs. $2(52) = 104$.

Time = 0.97 (sec) , antiderivative size = 2373, normalized size of antiderivative = 35.95

$$\int \frac{1}{x^3 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3456*(2*c^2*x^2*(d^4/c^13)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2
*d^4*x^3 + 640*c^3*d^3 + 18*(c^9*d^3*x^8 + 38*c^10*d^2*x^5 + 64*c^11*d*x^2
)*(d^4/c^13)^(2/3) + 6*sqrt(d*x^3 + c)*((c^11*d^2*x^7 + 80*c^12*d*x^4 + 16
0*c^13*x)*(d^4/c^13)^(5/6) + (7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*
sqrt(d^4/c^13) + 6*(5*c^3*d^4*x^5 + 32*c^4*d^3*x^2)*(d^4/c^13)^(1/6)) + 18
*(5*c^5*d^4*x^7 + 64*c^6*d^3*x^4 + 32*c^7*d^2*x)*(d^4/c^13)^(1/3))/(d^3*x^
9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*c^2*x^2*(d^4/c^13)^(1/6)*
log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + 18*(c^9*d^
3*x^8 + 38*c^10*d^2*x^5 + 64*c^11*d*x^2)*(d^4/c^13)^(2/3) - 6*sqrt(d*x^3 +
 c)*((c^11*d^2*x^7 + 80*c^12*d*x^4 + 160*c^13*x)*(d^4/c^13)^(5/6) + (7*c^7
*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(d^4/c^13) + 6*(5*c^3*d^4*x^5 +
 32*c^4*d^3*x^2)*(d^4/c^13)^(1/6)) + 18*(5*c^5*d^4*x^7 + 64*c^6*d^3*x^4 +
32*c^7*d^2*x)*(d^4/c^13)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
512*c^3)) - 144*sqrt(d)*x^2*weierstrassPInverse(0, -4*c/d, x) + (sqrt(-3)*
c^2*x^2 + c^2*x^2)*(d^4/c^13)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^
2*d^4*x^3 + 640*c^3*d^3 - 9*(c^9*d^3*x^8 + 38*c^10*d^2*x^5 + 64*c^11*d*x^2
 + sqrt(-3)*(c^9*d^3*x^8 + 38*c^10*d^2*x^5 + 64*c^11*d*x^2))*(d^4/c^13)^(2
/3) + 3*sqrt(d*x^3 + c)*((c^11*d^2*x^7 + 80*c^12*d*x^4 + 160*c^13*x - sqrt
(-3)*(c^11*d^2*x^7 + 80*c^12*d*x^4 + 160*c^13*x))*(d^4/c^13)^(5/6) - 2*(7*
c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(d^4/c^13) + 6*(5*c^3*d^4...
```

Sympy [F]

$$\int \frac{1}{x^3 (8c - dx^3) \sqrt{c + dx^3}} \, dx = -\int \frac{1}{-8cx^3\sqrt{c + dx^3} + dx^6\sqrt{c + dx^3}} \, dx$$

input
```
integrate(1/x**3/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(1/(-8*c*x**3*sqrt(c + d*x**3) + d*x**6*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int -\frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)x^3} \, dx$$

input
```
integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int -\frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)x^3} \, dx$$

input
```
integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^3\,\sqrt{d\,x^3 + c}\,(8\,c - d\,x^3)} \, dx$$

input
```
int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int \frac{\sqrt{d\,x^3 + c}}{-d^2 x^9 + 7cd\,x^6 + 8c^2 x^3} dx$$

input
```
int(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**3)/(8*c**2*x**3 + 7*c*d*x**6 - d**2*x**9),x)
```

3.499 $\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2c^2}{27d^4\sqrt{c+dx^3}} - \frac{4c\sqrt{c+dx^3}}{d^4} - \frac{2(c+dx^3)^{3/2}}{9d^4} + \frac{1024c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}$$

output
```
2/27*c^2/d^4/(d*x^3+c)^(1/2)-4*c*(d*x^3+c)^(1/2)/d^4-2/9*(d*x^3+c)^(3/2)/d
^4+1024/81*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\left(-\frac{3(56c^2+60cdx^3+3d^2x^6)}{\sqrt{c+dx^3}} + 512c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81d^4}$$

input
```
Integrate[x^11/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2*((-3*(56*c^2 + 60*c*d*x^3 + 3*d^2*x^6))/Sqrt[c + d*x^3] + 512*c^(3/2)*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(81*d^4)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {948, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^9}{(8c-dx^3)\left(dx^3+c\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{ 98}$$

$$\frac{1}{3}\int \left(-\frac{x^3}{d^2\sqrt{dx^3+c}}+\frac{512c^2}{9d^3(8c-dx^3)\sqrt{dx^3+c}}-\frac{7c}{d^3\sqrt{dx^3+c}}-\frac{c^2}{9d^3\left(dx^3+c\right)^{3/2}}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{1024c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{27d^4}+\frac{2c^2}{9d^4\sqrt{c+dx^3}}-\frac{12c\sqrt{c+dx^3}}{d^4}-\frac{2\left(c+dx^3\right)^{3/2}}{3d^4}\right)$$

input
```
Int[x^11/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
((2*c^2)/(9*d^4*Sqrt[c + d*x^3]) - (12*c*Sqrt[c + d*x^3])/d^4 - (2*(c + d*
x^3)^(3/2))/(3*d^4) + (1024*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(27*d^4))/3
```

Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{-\frac{2d^2x^6}{9}+\frac{1024c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{d\,x^3+c}}{81}-\frac{40cd\,x^3}{9}-\frac{112c^2}{27}}{\sqrt{d\,x^3+c}\,d^4}$
risch	$-\frac{2(d\,x^3+19c)\sqrt{d\,x^3+c}}{9d^4}-\frac{c^2\left(-\frac{2}{27d\sqrt{d\,x^3+c}}-\frac{1024\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d}\right)}{d^3}$
default	$-\frac{-\frac{2c^2}{3d^3\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{2x^3\sqrt{d\,x^3+c}}{9d^2}-\frac{10c\sqrt{d\,x^3+c}}{9d^3}}{d}+\frac{128c^2}{3d^4\sqrt{d\,x^3+c}}-\frac{8c\left(\frac{2c}{3d^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{2\sqrt{d\,x^3+c}}{3d^2}\right)}{d^2}+\frac{1024c^3\left(-\frac{}{c\sqrt{d}}\right.}{}$
elliptic	$\frac{2c^2}{27d^4\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{2x^3\sqrt{d\,x^3+c}}{9d^3}-\frac{38c\sqrt{d\,x^3+c}}{9d^4}-\frac{512ic\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\right.}{}\right)}{\left(-c\right.}}}{}\right.}{}$

input

```
int(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/81*(-9*d^2*x^6+512*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*(d*x^3+c
)^(1/2)-180*c*d*x^3-168*c^2)/(d*x^3+c)^(1/2)/d^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.07

$$\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \left[\frac{2\left(256\,(cdx^3+c^2)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\,(3\,d^2x^6+60\,cdx^3+56\right.}{81\,(d^5x^3+cd^4)}\right.$$
$$\left.-\frac{2\left(512\,(cdx^3+c^2)\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)+3\,(3\,d^2x^6+60\,cdx^3+56\,c^2)\sqrt{dx^3+c}\right)}{81\,(d^5x^3+cd^4)}\right]$$

input
```
integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[2/81*(256*(c*d*x^3 + c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c)
+ 10*c)/(d*x^3 - 8*c)) - 3*(3*d^2*x^6 + 60*c*d*x^3 + 56*c^2)*sqrt(d*x^3 +
c))/(d^5*x^3 + c*d^4), -2/81*(512*(c*d*x^3 + c^2)*sqrt(-c)*arctan(3*sqrt(-
c)/sqrt(d*x^3 + c)) + 3*(3*d^2*x^6 + 60*c*d*x^3 + 56*c^2)*sqrt(d*x^3 + c))
/(d^5*x^3 + c*d^4)]
```

Sympy [A] (verification not implemented)

Time = 26.75 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{c^2}{27\sqrt{c+dx^3}}-\frac{512c^2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81\sqrt{-c}}-2c\sqrt{c+dx^3}-\frac{(c+dx^3)^{\frac{3}{2}}}{9}\right)}{d^4} & \text{for } d\neq 0 \\ \frac{x^{12}}{96c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
Piecewise((2*(c**2/(27*sqrt(c + d*x**3)) - 512*c**2*atan(sqrt(c + d*x**3)/
(3*sqrt(-c)))/(81*sqrt(-c)) - 2*c*sqrt(c + d*x**3) - (c + d*x**3)**(3/2)/9
)/d**4, Ne(d, 0)), (x**12/(96*c**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = -\frac{2\left(256\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+9\,(dx^3+c)^{\frac{3}{2}}+162\,\sqrt{dx^3+c}c-\frac{3\,c^2}{\sqrt{dx^3+c}}\right)}{81\,d^4}$$

input
```
integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-2/81*(256*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 9*(d*x^3 + c)^(3/2) + 162*sqrt(d*x^3 + c)*c - 3*c^2/sqrt(d*x^3
 + c))/d^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = -\frac{1024\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{81\,\sqrt{-c}d^4}+\frac{2\,c^2}{27\,\sqrt{dx^3+c}d^4}-\frac{2\left((dx^3+c)^{\frac{3}{2}}d^8+18\,\sqrt{dx^3+c}cd^8\right)}{9\,d^{12}}$$

input
```
integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-1024/81*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) + 2/27*c^
2/(sqrt(d*x^3 + c)*d^4) - 2/9*((d*x^3 + c)^(3/2)*d^8 + 18*sqrt(d*x^3 + c)*
c*d^8)/d^12
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{512\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^4} - \frac{38\,c\,\sqrt{d\,x^3+c}}{9\,d^4} + \frac{2\,c^2}{27\,d^4\,\sqrt{d\,x^3+c}} - \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^3}$$

input
```
int(x^11/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
(512*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/(81*d^4) - (38*c*(c + d*x^3)^(1/2))/(9*d^4) + (2*c^2)/(27*d^4*(c + d*x
^3)^(1/2)) - (2*x^3*(c + d*x^3)^(1/2))/(9*d^3)
```

Reduce [F]

$$\int \frac{x^{11}}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{-\frac{80\sqrt{d\,x^3+c}\,c^2}{9} - \frac{40\sqrt{d\,x^3+c}\,c\,d\,x^3}{9} - \frac{2\sqrt{d\,x^3+c}\,d^2x^6}{9} + 64\Big(\int \frac{\sqrt{d\,x^3+c}\,x^5}{-d^3x^9+6c\,d^2x^6+15c^2d\,x^3+8c^3}}{d^4\,(d\,x^3+c)}$$

input
```
int(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

output
```
(2*( - 40*sqrt(c + d*x**3)*c**2 - 20*sqrt(c + d*x**3)*c*d*x**3 - sqrt(c +
d*x**3)*d**2*x**6 + 288*int((sqrt(c + d*x**3)*x**5)/(8*c**3 + 15*c**2*d*x*
*3 + 6*c*d**2*x**6 - d**3*x**9),x)*c**3*d**2 + 288*int((sqrt(c + d*x**3)*x
**5)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**3*x**9),x)*c**2*d**3*x*
*3))/(9*d**4*(c + d*x**3))
```

3.500 $\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 71

$$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx=-\frac{2c}{27d^3\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{3d^3}+\frac{128\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

output
```
-2/27*c/d^3/(d*x^3+c)^(1/2)-2/3*(d*x^3+c)^(1/2)/d^3+128/81*c^(1/2)*arctanh
(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\frac{2\left(-\frac{3(10c+9dx^3)}{\sqrt{c+dx^3}}+64\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81d^3}$$

input
```
Integrate[x^8/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2*((-3*(10*c + 9*d*x^3))/Sqrt[c + d*x^3] + 64*Sqrt[c]*ArcTanh[Sqrt[c + d*
x^3]/(3*Sqrt[c])]))/(81*d^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {948, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$$
$$\Big\downarrow \text{948}$$
$$\frac{1}{3}\int \frac{x^6}{(8c-dx^3)(dx^3+c)^{3/2}}dx^3$$
$$\Big\downarrow \text{98}$$
$$\frac{1}{3}\int \left(\frac{64c}{9d^2(8c-dx^3)\sqrt{dx^3+c}}+\frac{c}{9d^2(dx^3+c)^{3/2}}-\frac{1}{d^2\sqrt{dx^3+c}}\right)dx^3$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3}\left(\frac{128\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{27d^3}-\frac{2c}{9d^3\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{d^3}\right)$$

input
```
Int[x^8/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
((-2*c)/(9*d^3*Sqrt[c + d*x^3]) - (2*Sqrt[c + d*x^3])/d^3 + (128*Sqrt[c]*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(27*d^3))/3
```

Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$-\frac{2\left(-64\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{d\,x^{3}+c}+27d\,x^{3}+30c\right)}{81\sqrt{d\,x^{3}+c}\,d^{3}}$
risch	$-\frac{2\sqrt{d\,x^{3}+c}}{3d^{3}}-\frac{c\left(\frac{2}{27d\sqrt{d\,x^{3}+c}}-\frac{128\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d}\right)}{d^{2}}$
default	$-\frac{\frac{2c}{3d^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{2\sqrt{d\,x^{3}+c}}{3d^{2}}}{d}+\frac{16c}{3d^{3}\sqrt{d\,x^{3}+c}}+\frac{128c^{2}\left(-\frac{1}{c\sqrt{d\,x^{3}+c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27d^{3}}$
elliptic	$-\frac{2c}{27d^{3}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{2\sqrt{d\,x^{3}+c}}{3d^{3}}-\frac{64i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{\ldots}\right.}{\ldots}$

input
```
int(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/81*(-64*c^(1/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*(d*x^3+c)^(1/2)+27
*d*x^3+30*c)/(d*x^3+c)^(1/2)/d^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.23

$$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \left[\frac{2\left(32\,(dx^3+c)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\,(9\,dx^3+10\,c)\sqrt{dx^3+c}\right)}{81\,(d^4x^3+cd^3)}, -\frac{2\left(64\,(dx^3+c)\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)+3\,(9\,dx^3+10\,c)\sqrt{dx^3+c}\right)}{81\,(d^4x^3+cd^3)}\right]$$

input
```
integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[2/81*(32*(d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*
c)/(d*x^3 - 8*c)) - 3*(9*d*x^3 + 10*c)*sqrt(d*x^3 + c))/(d^4*x^3 + c*d^3),
 -2/81*(64*(d*x^3 + c)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) + 3*(9*
d*x^3 + 10*c)*sqrt(d*x^3 + c))/(d^4*x^3 + c*d^3)]
```

Sympy [A] (verification not implemented)

Time = 15.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \begin{cases} \frac{2\left(-\frac{c}{27\sqrt{c+dx^3}}-\frac{64c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81\sqrt{-c}}-\frac{\sqrt{c+dx^3}}{3}\right)}{d^3} & \text{for } d\neq 0 \\ \frac{x^9}{72c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
Piecewise((2*(-c/(27*sqrt(c + d*x**3)) - 64*c*atan(sqrt(c + d*x**3)/(3*sqr
t(-c)))/(81*sqrt(-c)) - sqrt(c + d*x**3)/3)/d**3, Ne(d, 0)), (x**9/(72*c**
(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2\left(32\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+27\sqrt{dx^3+c}+\frac{3c}{\sqrt{dx^3+c}}\right)}{81\,d^3}$$

input
```
integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-2/81*(32*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 27*sqrt(d*x^3 + c) + 3*c/sqrt(d*x^3 + c))/d^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2\left(\frac{64\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d}+\frac{27\sqrt{dx^3+c}}{d}+\frac{3c}{\sqrt{dx^3+c}d}\right)}{81\,d^2}$$

input
```
integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-2/81*(64*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + 27*sqrt(d*
x^3 + c)/d + 3*c/(sqrt(d*x^3 + c)*d))/d^2
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{64\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^3} - \frac{2\,c}{27\,d^3\,\sqrt{d\,x^3+c}} - \frac{2\,\sqrt{d\,x^3+c}}{3\,d^3}$$

input
```
int(x^8/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
(64*c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/(81*d^3) - (2*c)/(27*d^3*(c + d*x^3)^(1/2)) - (2*(c + d*x^3)^(1/2))/(3*
d^3)
```

Reduce [F]

$$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{-\frac{4\sqrt{d\,x^3+c}\,c}{3}-\frac{2\sqrt{d\,x^3+c}\,d\,x^3}{3}+8\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{-d^3x^9+6c\,d^2x^6+15c^2d\,x^3+8c^3}dx\right)c^2d^2+8\left(\int \frac{}{-d}\right.}{d^3\left(d\,x^3+c\right)}$$

input
```
int(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

output
```
(2*( - 2*sqrt(c + d*x**3)*c - sqrt(c + d*x**3)*d*x**3 + 12*int((sqrt(c + d
*x**3)*x**5)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**3*x**9),x)*c**2
*d**2 + 12*int((sqrt(c + d*x**3)*x**5)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d**2
*x**6 - d**3*x**9),x)*c*d**3*x**3))/(3*d**3*(c + d*x**3))
```

3.501 $\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 52

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2}{27d^2\sqrt{c+dx^3}}+\frac{16\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}$$

output
```
2/27/d^2/(d*x^3+c)^(1/2)+16/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2
)/d^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\left(\frac{3}{\sqrt{c+dx^3}}+\frac{8\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{81d^2}$$

input
```
Integrate[x^5/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2*(3/Sqrt[c + d*x^3] + (8*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/Sqrt[c]))
/(81*d^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {948, 87, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{(8c-dx^3)(dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{8\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9d}+\frac{2}{9d^2\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{16\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{9d^2}+\frac{2}{9d^2\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{16\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{27\sqrt{c}d^2}+\frac{2}{9d^2\sqrt{c+dx^3}}\right)$$

input
```
Int[x^5/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2/(9*d^2*Sqrt[c + d*x^3]) + (16*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(27
*Sqrt[c]*d^2))/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{\frac{2}{27\sqrt{d\,x^{3}+c}}+\frac{16\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{81\sqrt{c}}}{d^{2}}$
default	$\frac{2}{3d^{2}\sqrt{d\,x^{3}+c}}+\frac{16c\left(-\frac{1}{c\sqrt{d\,x^{3}+c}}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27d^{2}}$
elliptic	$\frac{2}{27d^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{8i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\cdots\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\cdots}}\cdots}{\cdots}\right)}{\cdots}$

input
```
int(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/27*(1/(d*x^3+c)^(1/2)+8/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))/
d^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.81

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \left[\frac{2\left(4\,(dx^3+c)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\,\sqrt{dx^3+c}c\right)}{81\,(cd^3x^3+c^2d^2)},\right.$$
$$\left.-\frac{2\left(8\,(dx^3+c)\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)-3\,\sqrt{dx^3+c}c\right)}{81\,(cd^3x^3+c^2d^2)}\right]$$

input
```
integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[2/81*(4*(d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c
)/(d*x^3 - 8*c)) + 3*sqrt(d*x^3 + c)*c)/(c*d^3*x^3 + c^2*d^2), -2/81*(8*(d
*x^3 + c)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) - 3*sqrt(d*x^3 + c)*
c)/(c*d^3*x^3 + c^2*d^2)]
```

Sympy [A] (verification not implemented)

Time = 11.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \begin{cases} \frac{2\cdot\left(\frac{1}{27\sqrt{c+dx^3}} - \frac{8\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81\sqrt{-c}}\right)}{d^2} & \text{for } d \neq 0 \\ \frac{x^6}{48c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
Piecewise((2*(1/(27*sqrt(c + d*x**3)) - 8*atan(sqrt(c + d*x**3)/(3*sqrt(-c
)))/(81*sqrt(-c)))/d**2, Ne(d, 0)), (x**6/(48*c**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2\left(\frac{4\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}} - \frac{3}{\sqrt{dx^3+c}}\right)}{81\,d^2}$$

input
```
integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-2/81*(4*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/
sqrt(c) - 3/sqrt(d*x^3 + c))/d^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2\left(\frac{8\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d}-\frac{3}{\sqrt{dx^3+c}d}\right)}{81\,d}$$

input
```
integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-2/81*(8*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 3/(sqrt(d*x^3
 + c)*d))/d
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2}{27\,d^2\,\sqrt{d\,x^3+c}}+\frac{8\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,\sqrt{c}\,d^2}$$

input
```
int(x^5/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
2/(27*d^2*(c + d*x^3)^(1/2)) + (8*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3
)^(1/2))/(8*c - d*x^3)))/(81*c^(1/2)*d^2)
```

Reduce [F]

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^5}{-d^3x^9+6c\,d^2x^6+15c^2d\,x^3+8c^3}dx$$

input
```
int(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**5)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**
3*x**9),x)
```

3.502 $\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 55

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2}{27cd\sqrt{c+dx^3}}+\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

output
```
-2/27/c/d/(d*x^3+c)^(1/2)+2/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2
)/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\left(-\frac{3\sqrt{c}}{\sqrt{c+dx^3}}+\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81c^{3/2}d}$$

input
```
Integrate[x^2/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2*((-3*Sqrt[c])/Sqrt[c + d*x^3] + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/
(81*c^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {946, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{1}{(8c-dx^3)(dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9c}-\frac{2}{9cd\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{9cd}-\frac{2}{9cd\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{27c^{3/2}d}-\frac{2}{9cd\sqrt{c+dx^3}}\right)$$

input
```
Int[x^2/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(-2/(9*c*d*Sqrt[c + d*x^3]) + (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(27
*c^(3/2)*d))/3
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

method	result
default	$\frac{-\frac{2}{27 c \sqrt{d\,x^{3}+c}}+\frac{2 \,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{81 c^{\frac{3}{2}}}}{d}$
pseudoelliptic	$\frac{-\frac{2}{27 c \sqrt{d\,x^{3}+c}}+\frac{2 \,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{81 c^{\frac{3}{2}}}}{d}$
elliptic	$-\frac{2}{27 d c \sqrt{\left(x^{3}+\frac{c}{d}\right) d}}-\frac{i \sqrt{2}\left(\sum_{\_\alpha =\mathrm{RootOf}\left(d\,\_Z^{3}-8 c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}} \sqrt{2}\,\sqrt{\frac{i d \left(2 x+\frac{-i \sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d \left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3 \left(-c\,d^{2}\right)^{\frac{1}{3}}+i \sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}} \cdots}{\cdots}\right)}{\cdots}$

input
```
int(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/27*(-1/c/(d*x^3+c)^(1/2)+1/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2
))/d
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.62

$$\int \frac{x^2}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \left[\frac{(dx^3+c)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\sqrt{dx^3+c}c}{81\,(c^2d^2x^3+c^3d)},\ -\frac{2\left((dx^3+c)\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)+3\sqrt{dx^3+c}c\right)}{81\,(c^2d^2x^3+c^3d)}\right]$$

input
```
integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/81*((d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/
(d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c^2*d^2*x^3 + c^3*d), -2/81*((d*x^3
 + c)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) + 3*sqrt(d*x^3 + c)*c)/(
c^2*d^2*x^3 + c^3*d)]
```

Sympy [A] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \begin{cases} \frac{2\left(-\frac{1}{27c\sqrt{c+dx^3}}-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81c\sqrt{-c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{x^3}{24c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
Piecewise((2*(-1/(27*c*sqrt(c + d*x**3)) - atan(sqrt(c + d*x**3)/(3*sqrt(-
c)))/(81*c*sqrt(-c)))/d, Ne(d, 0)), (x**3/(24*c**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{6}{\sqrt{dx^3+c}c}}{81\,d}$$

input
```
integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/81*(log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^
(3/2) + 6/(sqrt(d*x^3 + c)*c))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}cd} - \frac{2}{27\sqrt{dx^3+c}cd}$$

input
```
integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-2/81*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) - 2/27/(sqrt(d*x
^3 + c)*c*d)
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,c^{3/2}\,d} - \frac{2}{27\,c\,d\,\sqrt{d\,x^3+c}}$$

input
```
int(x^2/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(81*c^(3/2
)*d) - 2/(27*c*d*(c + d*x^3)^(1/2))
```

Reduce [F]

$$\int \frac{x^2}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^2}{-d^3x^9+6c\,d^2x^6+15c^2d\,x^3+8c^3}dx$$

input
```
int(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**2)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**
3*x**9),x)
```

3.503 $\int \frac{1}{x(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 76

$$\int \frac{1}{x(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\frac{2}{27c^2\sqrt{c+dx^3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}}$$

output
```
2/27/c^2/(d*x^3+c)^(1/2)+1/324*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2
)-1/12*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \frac{1}{x(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\frac{\frac{24\sqrt{c}}{\sqrt{c+dx^3}}+\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-27\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{324c^{5/2}}$$

input
```
Integrate[1/(x*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
((24*Sqrt[c])/Sqrt[c + d*x^3] + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 27*
ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(324*c^(5/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {948, 96, 25, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(8c-dx^3)(c+dx^3)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^3(8c-dx^3)(dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{96}$$

$$\frac{1}{3}\left(\frac{2}{9c^2\sqrt{c+dx^3}}-\frac{\int -\frac{d(9c-dx^3)}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9c^2d}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{\int \frac{d(9c-dx^3)}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9c^2d}+\frac{2}{9c^2\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{9c-dx^3}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9c^2}+\frac{2}{9c^2\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{\frac{9}{8}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{1}{8}d\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9c^2}+\frac{2}{9c^2\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\frac{1}{4}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{9\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{4d}}{9c^2}+\frac{2}{9c^2\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\frac{9\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{4d}+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{12\sqrt{c}}}{9c^2}+\frac{2}{9c^2\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{12\sqrt{c}}-\frac{9\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{4\sqrt{c}}}{9c^2}+\frac{2}{9c^2\sqrt{c+dx^3}}\right)$$

input
```
Int[1/(x*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2/(9*c^2*Sqrt[c + d*x^3]) + (ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(12*Sqr
t[c]) - (9*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(4*Sqrt[c]))/(9*c^2))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.72

method	result	size
pseudoelliptic	$\frac{2}{27c^{2}\sqrt{d\,x^{3}+c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{324c^{\frac{5}{2}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{12c^{\frac{5}{2}}}$	55
default	$\frac{\frac{2}{3c\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{8c}+\frac{-\frac{1}{c\sqrt{d\,x^{3}+c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{108c}$	85
elliptic	Expression too large to display	1526

input
```
int(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/27/c^2/(d*x^3+c)^(1/2)+1/324*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2
)-1/12*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.71

$$\int \frac{1}{x\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \left[\frac{(dx^3+c)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+27\,(dx^3+c)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+}}{x^3}\right.}{648\,(c^3dx^3+c^4)}\right.$$
$$\left.-\frac{(dx^3+c)\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)-27\,(dx^3+c)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)-24\sqrt{dx^3+c}c}{324\,(c^3dx^3+c^4)}\right]$$

input
```
integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/648*((d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)
/(d*x^3 - 8*c)) + 27*(d*x^3 + c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sq
rt(c) + 2*c)/x^3) + 48*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 + c^4), -1/324*((d*x^
3 + c)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) - 27*(d*x^3 + c)*sqrt(-
c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)) - 24*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 + c
^4)]
```

Sympy [A] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.28

$$\int \frac{1}{x\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{d}{27c^2\sqrt{c+dx^3}}-\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{648c^2\sqrt{-c}}+\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{24c^2\sqrt{-c}}\right)}{d} & \text{for}\ d\neq 0 \\ \frac{\log\left(x^3\right)}{24c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
Piecewise((2*(d/(27*c**2*sqrt(c + d*x**3)) - d*atan(sqrt(c + d*x**3)/(3*sq
rt(-c)))/(648*c**2*sqrt(-c)) + d*atan(sqrt(c + d*x**3)/sqrt(-c))/(24*c**2*
sqrt(-c)))/d, Ne(d, 0)), (log(x**3)/(24*c**(5/2)), True))
```

Maxima [F]

$$\int \frac{1}{x\,(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x}\,dx$$

input
```
integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\,(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}c^2} - \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{324\sqrt{-c}c^2} + \frac{2}{27\sqrt{dx^3+c}c^2}$$

input
```
integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/324*arctan(1/3*sq
rt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) + 2/27/(sqrt(d*x^3 + c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2}{27\,c^2\,\sqrt{d\,x^3+c}}-\frac{\operatorname{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)}{12\,\sqrt{c^5}}+\frac{\operatorname{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)}{324\,\sqrt{c^5}}$$

input
```
int(1/(x*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
2/(27*c^2*(c + d*x^3)^(1/2)) - atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2))/
(12*(c^5)^(1/2)) + atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))/(324*(c^
5)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}}{-d^3x^{10}+6c\,d^2x^7+15c^2d\,x^4+8c^3x}dx$$

input
```
int(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

output
```
int(sqrt(c + d*x**3)/(8*c**3*x + 15*c**2*d*x**4 + 6*c*d**2*x**7 - d**3*x**
10),x)
```

3.504 $\int \frac{1}{x^4(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 100

$$\int \frac{1}{x^4(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}} + \frac{11d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

output
```
-25/216*d/c^3/(d*x^3+c)^(1/2)-1/24/c^2/x^3/(d*x^3+c)^(1/2)+1/2592*d*arctan
h(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)+11/96*d*arctanh((d*x^3+c)^(1/2)/c^(
1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{-\frac{12\sqrt{c}(9c+25dx^3)}{x^3\sqrt{c+dx^3}} + d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + 297d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2592c^{7/2}}$$

input
```
Integrate[1/(x^4*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
((-12*Sqrt[c]*(9*c + 25*d*x^3))/(x^3*Sqrt[c + d*x^3]) + d*ArcTanh[Sqrt[c +
 d*x^3]/(3*Sqrt[c])] + 297*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2592*c^(7/
2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 114, 27, 169, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (8c - dx^3) (c + dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{1}{x^6 (8c - dx^3) (dx^3 + c)^{3/2}} dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3} \left(-\frac{\int \frac{d(22c - 3dx^3)}{2x^3 (8c - dx^3)(dx^3 + c)^{3/2}} dx^3}{8c^2} - \frac{1}{8c^2 x^3 \sqrt{c + dx^3}} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(-\frac{d \int \frac{22c - 3dx^3}{x^3 (8c - dx^3)(dx^3 + c)^{3/2}} dx^3}{16c^2} - \frac{1}{8c^2 x^3 \sqrt{c + dx^3}} \right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{3} \left(-\frac{d \left(\frac{2 \int \frac{cd\left(198c - 25dx^3\right)}{2x^3 \left(8c - dx^3\right) \sqrt{dx^3 + c}} dx^3}{9c^2 d} + \frac{50}{9c\sqrt{c + dx^3}} \right)}{16c^2} - \frac{1}{8c^2 x^3 \sqrt{c + dx^3}} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{\int \frac{198c-25dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c}+\frac{50}{9c\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{\frac{99}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3-\frac{1}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c}+\frac{50}{9c\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{\frac{99\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{1}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{9c}+\frac{50}{9c\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{\frac{99\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{9c}+\frac{50}{9c\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{99\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{9c}+\frac{50}{9c\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\sqrt{c+dx^3}}\right)$$

input
```
Int[1/(x^4*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(-1/8*1/(c^2*x^3*Sqrt[c + d*x^3]) - (d*(50/(9*c*Sqrt[c + d*x^3]) + (-1/6*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/Sqrt[c] - (99*ArcTanh[Sqrt[c + d*x^3]/
Sqrt[c]])/(2*Sqrt[c]))/(9*c)))/(16*c^2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\sqrt{d x^{3}+c}}{24 c^{3} x^{3}}-\frac{d \left(-\frac{11 \,\operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{\sqrt{c}}\right)}{6 \sqrt{c}}+\frac{32}{27 \sqrt{d x^{3}+c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{3 \sqrt{c}}\right)}{162 \sqrt{c}}\right)}{16 c^{3}}$
pseudoelliptic	$-\frac{d \left(\frac{\frac{\sqrt{d x^{3}+c}}{d x^{3}}-\frac{11 \,\operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{\sqrt{c}}\right)}{4 \sqrt{c}}}{8 c^{3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{3 \sqrt{c}}\right)}{864 c^{\frac{7}{2}}}+\frac{2}{9 c^{3} \sqrt{d x^{3}+c}}\right)}{3}$
default	$\frac{-\frac{\sqrt{d x^{3}+c}}{3 c^{2} x^{3}}-\frac{2 d}{3 c^{2} \sqrt{\left(x^{3}+\frac{c}{d}\right) d}}+\frac{d \,\operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}}{8 c}+\frac{d \left(\frac{2}{3 c \sqrt{\left(x^{3}+\frac{c}{d}\right) d}}-\frac{2 \,\operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{\sqrt{c}}\right)}{3 c^{\frac{3}{2}}}\right)}{64 c^{2}}+\frac{d \left(-\frac{1}{c \sqrt{d x^{3}+c}}+\text{arcta}\cdots\right)}{864 c\cdots}$
elliptic	Expression too large to display

input
```
int(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24/c^3*(d*x^3+c)^(1/2)/x^3-1/16/c^3*d*(-11/6*arctanh((d*x^3+c)^(1/2)/c^
(1/2))/c^(1/2)+32/27/(d*x^3+c)^(1/2)-1/162*arctanh(1/3*(d*x^3+c)^(1/2)/c^(
1/2))/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.66

$$\int \frac{1}{x^4\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \left[\frac{(d^2x^6+cdx^3)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+297\,(d^2x^6+cdx^3)\sqrt{c}\log\left(\right.}{5184\,(c^4dx^6+c^5x^3)}\right.$$
$$-\frac{(d^2x^6+cdx^3)\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)+297\,(d^2x^6+cdx^3)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)+12\,(25\,cdx^3+9\,c^2)\sqrt{dx}}{2592\,(c^4dx^6+c^5x^3)}$$

input
```
integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/5184*((d^2*x^6 + c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c
) + 10*c)/(d*x^3 - 8*c)) + 297*(d^2*x^6 + c*d*x^3)*sqrt(c)*log((d*x^3 + 2*
sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(25*c*d*x^3 + 9*c^2)*sqrt(d*x^3 +
 c))/(c^4*d*x^6 + c^5*x^3), -1/2592*((d^2*x^6 + c*d*x^3)*sqrt(-c)*arctan(3
*sqrt(-c)/sqrt(d*x^3 + c)) + 297*(d^2*x^6 + c*d*x^3)*sqrt(-c)*arctan(sqrt(
-c)/sqrt(d*x^3 + c)) + 12*(25*c*d*x^3 + 9*c^2)*sqrt(d*x^3 + c))/(c^4*d*x^6
 + c^5*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = -\int \frac{1}{-8c^2x^4\sqrt{c+dx^3}-7cdx^7\sqrt{c+dx^3}+d^2x^{10}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**4/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
-Integral(1/(-8*c**2*x***4*sqrt(c + d*x***3) - 7*c*d*x***7*sqrt(c + d*x***3) +
d**2*x***10*sqrt(c + d*x***3)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int -\frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c)x^4} \, dx$$

input
```
integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = -\frac{11\,d \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\,\sqrt{-c}c^3} - \frac{d \arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{2592\,\sqrt{-c}c^3} - \frac{25\,(dx^3 + c)d - 16\,cd}{216\,\left((dx^3 + c)^{\frac{3}{2}} - \sqrt{dx^3 + c}c\right)c^3}$$

input
```
integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-11/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/2592*d*arctan
(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/216*(25*(d*x^3 + c)*d -
16*c*d)/(((d*x^3 + c)^(3/2) - sqrt(d*x^3 + c)*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4 (8c-dx^3) (c+dx^3)^{3/2}}\,dx = \frac{11\,d\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)}{96\,\sqrt{c^7}} - \frac{25\,d}{216\,c^3\,\sqrt{d\,x^3+c}} + \frac{d\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)}{2592\,\sqrt{c^7}} - \frac{1}{24\,c^2\,x^3\,\sqrt{d\,x^3+c}}$$

input
```
int(1/(x^4*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
(11*d*atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2)))/(96*(c^7)^(1/2)) - (25*d
)/(216*c^3*(c + d*x^3)^(1/2)) + (d*atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^
(1/2))))/(2592*(c^7)^(1/2)) - 1/(24*c^2*x^3*(c + d*x^3)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^4 (8c-dx^3) (c+dx^3)^{3/2}}\,dx = \frac{-2\sqrt{d\,x^3+c} - 66\left(\int \frac{\sqrt{d\,x^3+c}}{-d^3x^{10}+6c\,d^2x^7+15c^2d\,x^4+8c^3x}dx\right)c^2d\,x^3 - 66\left(\int \frac{}{-d^3x^{10}}\right.}{}$$

input
```
int(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

output
```
( - 2*sqrt(c + d*x**3) - 66*int(sqrt(c + d*x**3)/(8*c**3*x + 15*c**2*d*x**
4 + 6*c*d**2*x**7 - d**3*x**10),x)*c**2*d*x**3 - 66*int(sqrt(c + d*x**3)/(
8*c**3*x + 15*c**2*d*x**4 + 6*c*d**2*x**7 - d**3*x**10),x)*c*d**2*x**6 + 9
*int((sqrt(c + d*x**3)*x**2)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d*
*3*x**9),x)*c*d**2*x**3 + 9*int((sqrt(c + d*x**3)*x**2)/(8*c**3 + 15*c**2*
d*x**3 + 6*c*d**2*x**6 - d**3*x**9),x)*d**3*x**6)/(48*c**2*x**3*(c + d*x**
3))
```

3.505 $\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 128

$$\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{20736c^{9/2}} - \frac{109d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{9/2}}$$

output
```
245/1728*d^2/c^4/(d*x^3+c)^(1/2)-1/48/c^2/x^6/(d*x^3+c)^(1/2)+3/64*d/c^3/x
^3/(d*x^3+c)^(1/2)+1/20736*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(9/2
)-109/768*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{\frac{12\sqrt{c}(-36c^2+81cdx^3+245d^2x^6)}{x^6\sqrt{c+dx^3}} + d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 2943d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{20736c^{9/2}}$$

input
```
Integrate[1/(x^7*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
((12*Sqrt[c]*(-36*c^2 + 81*c*d*x^3 + 245*d^2*x^6))/(x^6*Sqrt[c + d*x^3]) +
 d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 2943*d^2*ArcTanh[Sqrt[c + d*x^
3]/Sqrt[c]])/(20736*c^(9/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {948, 114, 27, 168, 27, 169, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (8c-dx^3) (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^9 (8c-dx^3) (dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int \frac{d(36c-5dx^3)}{2x^6(8c-dx^3)(dx^3+c)^{3/2}}dx^3}{16c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\int \frac{36c-5dx^3}{x^6(8c-dx^3)(dx^3+c)^{3/2}}dx^3}{32c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{\int \frac{2cd(218c-27dx^3)}{x^3(8c-dx^3)(dx^3+c)^{3/2}}dx^3}{8c^2}-\frac{9}{2cx^3\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\int \frac{218c-27dx^3}{x^3\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx^3}{4c}-\frac{9}{2cx^3\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 169$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{2\int \frac{cd\left(1962c-245dx^3\right)}{2x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c^2d}+\frac{490}{9c\sqrt{c+dx^3}}\right)}{4c}-\frac{9}{2cx^3\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\int \frac{1962c-245dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c}+\frac{490}{9c\sqrt{c+dx^3}}\right)}{4c}-\frac{9}{2cx^3\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{981}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{1}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c}+\frac{490}{9c\sqrt{c+dx^3}}\right)}{4c}-\frac{9}{2cx^3\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{1}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{981\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{9c}+\frac{490}{9c\sqrt{c+dx^3}}\right)}{4c}-\frac{9}{2cx^3\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{981\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{9c}+\frac{490}{9c\sqrt{c+dx^3}}\right)}{4c}-\frac{9}{2cx^3\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{981\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{9c}+\frac{490}{9c\sqrt{c+dx^3}}\right)}{4c}-\frac{9}{2cx^3\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

input
```
Int[1/(x^7*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(-1/16*1/(c^2*x^6*Sqrt[c + d*x^3]) - (d*(-9/(2*c*x^3*Sqrt[c + d*x^3]) - (d
*(490/(9*c*Sqrt[c + d*x^3]) + (ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(6*Sqr
t[c]) - (981*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c]))/(9*c)))/(4*c))
)/(32*c^2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}\,\left(-13 d\,x^{3}+4 c\right)}{192 c^{4} x^{6}}+\frac{d^{2}\left(-\frac{109\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6\sqrt{c}}+\frac{256}{27\sqrt{d\,x^{3}+c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{162\sqrt{c}}\right)}{128 c^{4}}$
pseudoelliptic	$-\frac{d^{2}\left(\frac{\frac{\sqrt{d\,x^{3}+c}\,\left(-13 d\,x^{3}+4 c\right)}{d^{2} x^{6}}+\frac{109\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{4\sqrt{c}}}{64 c^{4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{6912 c^{\frac{9}{2}}}-\frac{2}{9 c^{4}\sqrt{d\,x^{3}+c}}\right)}{3}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{6 c^{2} x^{6}}+\frac{7 d\sqrt{d\,x^{3}+c}}{12 c^{3} x^{3}}+\frac{2 d^{2}}{3 c^{3}\sqrt{\left(x^{3}+\frac{c}{d}\right) d}}-\frac{5 d^{2}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{4 c^{\frac{7}{2}}}}{8 c}+\frac{d\left(-\frac{\sqrt{d\,x^{3}+c}}{3 c^{2} x^{3}}-\frac{2 d}{3 c^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right) d}}+\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right.}{64 c^{2}}$
elliptic	Expression too large to display

input
```
int(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(d*x^3+c)^(1/2)*(-13*d*x^3+4*c)/c^4/x^6+1/128*d^2/c^4*(-109/6*arcta
nh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+256/27/(d*x^3+c)^(1/2)+1/162*arctanh(1
/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.31

$$\int \frac{1}{x^7\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \left[\frac{\left(d^3x^9+cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+2943\left(d^3x^9+cd^2x^6\right)\sqrt{c}\,\mathrm{lo}}{41472\left(c^5dx\right.}\right.$$

$$-\frac{\left(d^3x^9+cd^2x^6\right)\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)-2943\left(d^3x^9+cd^2x^6\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)-12\left(245\,cd^2x^6+81\,c\right.}{20736\left(c^5dx^9+c^6x^6\right)}$$

input
```
integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/41472*((d^3*x^9 + c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqr
t(c) + 10*c)/(d*x^3 - 8*c)) + 2943*(d^3*x^9 + c*d^2*x^6)*sqrt(c)*log((d*x^
3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(245*c*d^2*x^6 + 81*c^2*d*x
^3 - 36*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 + c^6*x^6), -1/20736*((d^3*x^9 +
c*d^2*x^6)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) - 2943*(d^3*x^9 + c
*d^2*x^6)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)) - 12*(245*c*d^2*x^6 +
81*c^2*d*x^3 - 36*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 + c^6*x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = -\int \frac{1}{-8c^2x^7\sqrt{c+dx^3}-7cdx^{10}\sqrt{c+dx^3}+d^2x^{13}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**7/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
-Integral(1/(-8*c**2*x**7*sqrt(c + d*x**3) - 7*c*d*x**10*sqrt(c + d*x**3)
+ d**2*x**13*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^7 (8c-dx^3) (c+dx^3)^{3/2}}\,dx = \int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x^7}\,dx$$

input
```
integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^7), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^7 (8c-dx^3) (c+dx^3)^{3/2}}\,dx = \frac{109\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{768\,\sqrt{-c}c^4} - \frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{20736\,\sqrt{-c}c^4} + \frac{2\,d^2}{27\,\sqrt{dx^3+c}c^4} + \frac{13\,(dx^3+c)^{\frac{3}{2}}d^2 - 17\,\sqrt{dx^3+c}cd^2}{192\,c^4d^2x^6}$$

input
```
integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
109/768*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/20736*d^2*
arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) + 2/27*d^2/(sqrt(d*x^3
 + c)*c^4) + 1/192*(13*(d*x^3 + c)^(3/2)*d^2 - 17*sqrt(d*x^3 + c)*c*d^2)/(
c^4*d^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^7 (8c-dx^3) (c+dx^3)^{3/2}} \, dx = \frac{245\,d^2}{1728\,c^4\,\sqrt{d\,x^3+c}} - \frac{109\,d^2\,\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{\sqrt{c^9}}\right)}{768\,\sqrt{c^9}} + \frac{d^2\,\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^9}}\right)}{20736\,\sqrt{c^9}} - \frac{1}{48\,c^2\,x^6\,\sqrt{d\,x^3+c}} + \frac{3\,d}{64\,c^3\,x^3\,\sqrt{d\,x^3+c}}$$

input
```
int(1/(x^7*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
(245*d^2)/(1728*c^4*(c + d*x^3)^(1/2)) - (109*d^2*atanh((c^4*(c + d*x^3)^(
1/2))/(c^9)^(1/2)))/(768*(c^9)^(1/2)) + (d^2*atanh((c^4*(c + d*x^3)^(1/2))
/(3*(c^9)^(1/2))))/(20736*(c^9)^(1/2)) - 1/(48*c^2*x^6*(c + d*x^3)^(1/2))
+ (3*d)/(64*c^3*x^3*(c + d*x^3)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^7 (8c-dx^3) (c+dx^3)^{3/2}} \, dx = \frac{-8\sqrt{d\,x^3+c}\,c + 18\sqrt{d\,x^3+c}\,d\,x^3 + 654\left(\int \frac{\sqrt{d\,x^3+c}}{-d^3x^{10}+6c\,d^2x^7+15c^2d\,x^4+8c^3x}dx$$

input
```
int(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

output
```
( - 8*sqrt(c + d*x**3)*c + 18*sqrt(c + d*x**3)*d*x**3 + 654*int(sqrt(c + d
*x**3)/(8*c**3*x + 15*c**2*d*x**4 + 6*c*d**2*x**7 - d**3*x**10),x)*c**2*d*
*2*x**6 + 654*int(sqrt(c + d*x**3)/(8*c**3*x + 15*c**2*d*x**4 + 6*c*d**2*x
**7 - d**3*x**10),x)*c*d**3*x**9 - 81*int((sqrt(c + d*x**3)*x**2)/(8*c**3
+ 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**3*x**9),x)*c*d**3*x**6 - 81*int((sqr
t(c + d*x**3)*x**2)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**3*x**9),
x)*d**4*x**9)/(384*c**3*x**6*(c + d*x**3))
```

3.506 $\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 629

$$\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{56\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{32\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}d^{8/3}} + \frac{32\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81d^{8/3}} - \frac{32\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^{8/3}} + \frac{28\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{56\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
2/27*x^2/d^2/(d*x^3+c)^(1/2)-56/27*(d*x^3+c)^(1/2)/d^(8/3)/((1+3^(1/2))*c^
(1/3)+d^(1/3)*x)-32/81*c^(1/6)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/
(d*x^3+c)^(1/2))*3^(1/2)/d^(8/3)+32/81*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3
)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(8/3)-32/81*c^(1/6)*arctanh(1/3*(d*x^3+c
)^(1/2)/c^(1/2))/d^(8/3)+28/27*(1/2*6^(1/2)-1/2*2^(1/2))*c^(1/3)*(c^(1/3)+
d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d
^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*
c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/d^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/3
)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)-56/81*2^(1/2
)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1
+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/
3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/d^(8/3)/(c^(1
/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)
^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.85 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.20

$$\int \frac{x^7}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{x^2\left(20c-20c\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+7dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{A}\cdots\right.}{270cd^2\sqrt{c+dx^3}}$$

input
```
Integrate[x^7/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x^2*(20*c - 20*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)
/c), (d*x^3)/(8*c)] + 7*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/
3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(270*c*d^2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {970, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}dx$$

$$\Big\downarrow \text{ 970}$$

$$\frac{2x^2}{27d^2\sqrt{c+dx^3}}-\frac{2\int \frac{2cx(8c-7dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{27cd^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2x^2}{27d^2\sqrt{c+dx^3}}-\frac{4\int \frac{x(8c-7dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{27d^2}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{2x^2}{27d^2\sqrt{c+dx^3}}-\frac{4\int \left(\frac{7x}{\sqrt{dx^3+c}}-\frac{48cx}{(8c-dx^3)\sqrt{dx^3+c}}\right)dx}{27d^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2x^2}{27d^2\sqrt{c+dx^3}}-$$

$$4\left(\frac{14\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{7\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\right.}{}\right.$$

input
```
Int[x^7/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output

```
(2*x^2)/(27*d^2*Sqrt[c + d*x^3]) - (4*((14*Sqrt[c + d*x^3])/(d^(2/3)*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (8*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1
/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) - (8*c^(1/6)*ArcTanh
[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3*d^(2/3)) + (8*c^
(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^(2/3)) - (7*3^(1/4)*Sqrt[
2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)
*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
-7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (14*Sqrt[2]*c^(1/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(
1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^
(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*Sqrt[c + d*x^3])))/(27*d^2)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 970

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.67 (sec) , antiderivative size = 869, normalized size of antiderivative = 1.38

method	result	size
elliptic	Expression too large to display	869
default	Expression too large to display	1810

input
```
int(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/27/d^2*x^2/((x^3+c/d)*d)^(1/2)+56/81*I/d^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)))-64/243*I/d^5*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1
/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/
(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I
*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2
/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_
alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2384 vs. $2(441) = 882$.

Time = 4.52 (sec) , antiderivative size = 2384, normalized size of antiderivative = 3.79

$$\int \frac{x^7}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
2/243*(9*sqrt(d*x^3 + c)*d*x^2 + 252*(d*x^3 + c)*sqrt(d)*weierstrassZeta(0
, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 4*(d^4*x^3 + c*d^3 - sqrt(-
3)*(d^4*x^3 + c*d^3))*(c/d^16)^(1/6)*log(33554432/3*((d^16*x^9 + 318*c*d^1
5*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15
*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c/d^16)^(5/6) + 6*(2*c*d^2*x^7
+ 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^12*x^5 + 32*c^2*d^11*x^2 - sqrt(-3)
*(5*c*d^12*x^5 + 32*c^2*d^11*x^2))*(c/d^16)^(2/3) - (7*c*d^7*x^6 + 152*c^2
*d^6*x^3 + 64*c^3*d^5 + sqrt(-3)*(7*c*d^7*x^6 + 152*c^2*d^6*x^3 + 64*c^3*d
^5))*(c/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^10*x^7 + 64*c^2*d^9*x^4 +
 32*c^3*d^8*x)*sqrt(c/d^16) + 18*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*
x^2 - sqrt(-3)*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*x^2))*(c/d^16)^(1/
6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 4*(d^4*x^3 + c*d
^3 - sqrt(-3)*(d^4*x^3 + c*d^3))*(c/d^16)^(1/6)*log(-33554432/3*((d^16*x^9
 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9
+ 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c/d^16)^(5/6) - 6*(
2*c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^12*x^5 + 32*c^2*d^11*x^
2 - sqrt(-3)*(5*c*d^12*x^5 + 32*c^2*d^11*x^2))*(c/d^16)^(2/3) - (7*c*d^7*x
^6 + 152*c^2*d^6*x^3 + 64*c^3*d^5 + sqrt(-3)*(7*c*d^7*x^6 + 152*c^2*d^6*x^
3 + 64*c^3*d^5))*(c/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^10*x^7 + 64*c
^2*d^9*x^4 + 32*c^3*d^8*x)*sqrt(c/d^16) + 18*(c*d^5*x^8 + 38*c^2*d^4*x^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(8c - dx^3)(c + dx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**7/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^7}{(8c - dx^3)(c + dx^3)^{3/2}}\,dx = \int -\frac{x^7}{(dx^3 + c)^{\frac{3}{2}}(dx^3 - 8\,c)}\,dx$$

input
```
integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{x^7}{(8c - dx^3)(c + dx^3)^{3/2}}\,dx = \int -\frac{x^7}{(dx^3 + c)^{\frac{3}{2}}(dx^3 - 8\,c)}\,dx$$

input
```
integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{x^7}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)}\,dx$$

input
```
int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^7}{-d^3x^9+6c\,d^2x^6+15c^2d\,x^3+8c^3}dx$$

input
```
int(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**7)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**
3*x**9),x)
```

3.507 $\int \frac{x^4}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 635

$$\int \frac{x^4}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{4\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}} + \frac{4\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}} - \frac{4\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{5/6}d^{5/3}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\,3^{3/4}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-2/27*x^2/c/d/(d*x^3+c)^(1/2)+2/27*(d*x^3+c)^(1/2)/c/d^(5/3)/((1+3^(1/2))*
c^(1/3)+d^(1/3)*x)-4/81*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+
c)^(1/2))*3^(1/2)/c^(5/6)/d^(5/3)+4/81*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c
^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d^(5/3)-4/81*arctanh(1/3*(d*x^3+c)^(1/2)/c
^(1/2))/c^(5/6)/d^(5/3)-1/27*(1/2*6^(1/2)-1/2*2^(1/2))*(c^(1/3)+d^(1/3)*x)
*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^
2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^
(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/c^(2/3)/d^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*
x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+2/81*2^(1/2)*(
c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c
^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3
^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/c^(2/3)/d^(5/3)/(c^(1/3)
*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1
/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.65 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.20

$$\int \frac{x^4}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{x^2\left(80c-80c\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{1080c^2d\sqrt{c+dx^3}}$$

input
```
Integrate[x^4/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
-1/1080*(x^2*(80*c - 80*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -
((d*x^3)/c), (d*x^3)/(8*c)] + d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2,
 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(c^2*d*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {971, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{971}$$

$$\frac{2\int \frac{x(32c-dx^3)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{27cd}-\frac{2x^2}{27cd\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x(32c-dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{27cd}-\frac{2x^2}{27cd\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1054}$$

$$\frac{\int \left(\frac{24cx}{(8c-dx^3)\sqrt{dx^3+c}}+\frac{x}{\sqrt{dx^3+c}}\right)dx}{27cd}-\frac{2x^2}{27cd\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\cdots}}{d^{2}\cdots}\cdots}{\cdots}-\frac{2x^2}{27cd\sqrt{c+dx^3}}$$

input
```
Int[x^4/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(-2*x^2)/(27*c*d*Sqrt[c + d*x^3]) + ((2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)) - (4*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3)
 + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) + (4*c^(1/6)*ArcTanh[(c
^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3*d^(2/3)) - (4*c^(1/
6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^(2/3)) - (3^(1/4)*Sqrt[2 - S
qrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
 Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 -
4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^
(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x
)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*
Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2
]*Sqrt[c + d*x^3]))/(27*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.22 (sec) , antiderivative size = 878, normalized size of antiderivative = 1.38

method	result	size
elliptic	Expression too large to display	878
default	Expression too large to display	1346

input
```
int(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/27/d*x^2/c/((x^3+c/d)*d)^(1/2)-2/81*I/c/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)))-8/243*I/d^4/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)
*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)
^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*
d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3
)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2525 vs. $2(447) = 894$.

Time = 0.80 (sec) , antiderivative size = 2525, normalized size of antiderivative = 3.98

$$\int \frac{x^4}{(8c - dx^3)(c + dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/243*(18*sqrt(d*x^3 + c)*d*x^2 + 18*(d*x^3 + c)*sqrt(d)*weierstrassZeta(
0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - (c*d^3*x^3 + c^2*d^2 + sqr
t(-3)*(c*d^3*x^3 + c^2*d^2))*(1/(c^5*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2
*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c
^6*d^7*x + sqrt(-3)*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x))*(1/(c
^5*d^10))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2 -
sqrt(-3)*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2))*(1/(c^5*d^10))^(5/6) - 2*(7*c^
3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqrt(1/(c^5*d^10)) + (c*d^4*x^7
+ 80*c^2*d^3*x^4 + 160*c^3*d^2*x + sqrt(-3)*(c*d^4*x^7 + 80*c^2*d^3*x^4 +
160*c^3*d^2*x))*(1/(c^5*d^10))^(1/6)) - 9*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 +
64*c^4*d^4*x^2 - sqrt(-3)*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2))
*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
 + (c*d^3*x^3 + c^2*d^2 + sqrt(-3)*(c*d^3*x^3 + c^2*d^2))*(1/(c^5*d^10))^(
1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^
9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x + sqrt(-3)*(5*c^4*d^9*x^7 + 64*c^5*d
^8*x^4 + 32*c^6*d^7*x))*(1/(c^5*d^10))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^5
*d^10*x^5 + 32*c^6*d^9*x^2 - sqrt(-3)*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2))*(
1/(c^5*d^10))^(5/6) - 2*(7*c^3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqr
t(1/(c^5*d^10)) + (c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x + sqrt(-3)*(
c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x))*(1/(c^5*d^10))^(1/6)) - 9*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(8c - dx^3)(c + dx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4}{(8c - dx^3)(c + dx^3)^{3/2}}\,dx = \int -\frac{x^4}{(dx^3 + c)^{\frac{3}{2}}(dx^3 - 8\,c)}\,dx$$

input
```
integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{x^4}{(8c - dx^3)(c + dx^3)^{3/2}}\,dx = \int -\frac{x^4}{(dx^3 + c)^{\frac{3}{2}}(dx^3 - 8\,c)}\,dx$$

input
```
integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^4}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)}\,dx$$

input
```
int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^4}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^4}{-d^3x^9+6c\,d^2x^6+15c^2d\,x^3+8c^3}dx$$

input
```
int(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**4)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**
3*x**9),x)
```

3.508 $\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 632

$$\begin{aligned}
\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx &= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&- \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{54\sqrt{3}c^{11/6}d^{2/3}} + \frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{162c^{11/6}d^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{162c^{11/6}d^{2/3}} \\
&+ \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&- \frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
2/27*x^2/c^2/(d*x^3+c)^(1/2)-2/27*(d*x^3+c)^(1/2)/c^2/d^(2/3)/((1+3^(1/2))
*c^(1/3)+d^(1/3)*x)-1/162*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^
3+c)^(1/2))*3^(1/2)/c^(11/6)/d^(2/3)+1/162*arctanh(1/3*(c^(1/3)+d^(1/3)*x)
^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)/d^(2/3)-1/162*arctanh(1/3*(d*x^3+c)^(
1/2)/c^(1/2))/c^(11/6)/d^(2/3)+1/27*(1/2*6^(1/2)-1/2*2^(1/2))*(c^(1/3)+d^(
1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1
/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(
1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/c^(5/3)/d^(2/3)/(c^(1/3)*(c^(1/3)+d
^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)-2/81*2^
(1/2)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(
1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x
)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/c^(5/3)/d^(2/3)/(
c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^
3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.20

$$\int \frac{x}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{x^2\left(160c-25c\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+2dx^3\sqrt{1+\frac{dx^3}{c}}\right.}{2160c^3\sqrt{c+dx^3}}$$

input
```
Integrate[x/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x^2*(160*c - 25*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3
)/c), (d*x^3)/(8*c)] + 2*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8
/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(2160*c^3*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {972, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 972}$$

$$\frac{2x^2}{27c^2\sqrt{c+dx^3}}-\frac{2\int \frac{dx(5c-dx^3)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{27c^2d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2x^2}{27c^2\sqrt{c+dx^3}}-\frac{\int \frac{x(5c-dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{27c^2}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{2x^2}{27c^2\sqrt{c+dx^3}}-\frac{\int \left(\frac{x}{\sqrt{dx^3+c}}-\frac{3cx}{(8c-dx^3)\sqrt{dx^3+c}}\right)dx}{27c^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2x^2}{27c^2\sqrt{c+dx^3}}-$$

$$\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\cdots}}{d^{2\cdots}}$$

input
```
Int[x/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2*x^2)/(27*c^2*Sqrt[c + d*x^3]) - ((2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)) + (c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) +
d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*Sqrt[3]*d^(2/3)) - (c^(1/6)*ArcTanh[(c^(1
/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(6*d^(2/3)) + (c^(1/6)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*d^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3
]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt
[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqr
t[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)
*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqr
t[c + d*x^3]))/(27*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.05 (sec) , antiderivative size = 875, normalized size of antiderivative = 1.38

method	result	size
default	Expression too large to display	875
elliptic	Expression too large to display	875

input
```
int(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/27*x^2/c^2/((x^3+c/d)*d)^(1/2)+2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)))-1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3
)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(
1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2
)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)
*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2
)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/
3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2525 vs. $2(444) = 888$.

Time = 0.58 (sec) , antiderivative size = 2525, normalized size of antiderivative = 4.00

$$\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
1/1944*(144*sqrt(d*x^3 + c)*d*x^2 + 144*(d*x^3 + c)*sqrt(d)*weierstrassZet
a(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + (c^2*d^2*x^3 + c^3*d + s
qrt(-3)*(c^2*d^2*x^3 + c^3*d))*(1/(c^11*d^4))^(1/6)*log((d^3*x^9 + 318*c*d
^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32
*c^10*d^3*x + sqrt(-3)*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x))*(
1/(c^11*d^4))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d^5*x^5 + 32*c^11*d^4*x
^2 - sqrt(-3)*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2))*(1/(c^11*d^4))^(5/6) - 2
*(7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(1/(c^11*d^4)) + (c^2*
d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^
2*x^4 + 160*c^4*d*x))*(1/(c^11*d^4))^(1/6)) - 9*(c^4*d^4*x^8 + 38*c^5*d^3*
x^5 + 64*c^6*d^2*x^2 - sqrt(-3)*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64*c^6*d^2
*x^2))*(1/(c^11*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512
*c^3)) - (c^2*d^2*x^3 + c^3*d + sqrt(-3)*(c^2*d^2*x^3 + c^3*d))*(1/(c^11*d
^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*
c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x + sqrt(-3)*(5*c^8*d^5*x^7 + 6
4*c^9*d^4*x^4 + 32*c^10*d^3*x))*(1/(c^11*d^4))^(2/3) - 3*sqrt(d*x^3 + c)*(
6*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2 - sqrt(-3)*(5*c^10*d^5*x^5 + 32*c^11*d
^4*x^2))*(1/(c^11*d^4))^(5/6) - 2*(7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^
8*d^2)*sqrt(1/(c^11*d^4)) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x +
sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x))*(1/(c^11*d^4))^(...
```

Sympy [F]

$$\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\int \frac{x}{-8c^2\sqrt{c+dx^3}-7cdx^3\sqrt{c+dx^3}+d^2x^6\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
-Integral(x/(-8*c**2*sqrt(c + d*x**3) - 7*c*d*x**3*sqrt(c + d*x**3) + d**2
*x**6*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \int -\frac{x}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

input
```
integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \int -\frac{x}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

input
```
integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(8c - dx^3)(c + dx^3)^{3/2}} dx = \int \frac{x}{(d x^3 + c)^{3/2} (8 c - d x^3)} dx$$

input
```
int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{x}{(8c - dx^3)(c + dx^3)^{3/2}} dx = \int \frac{\sqrt{d x^3 + c}\, x}{-d^3x^9 + 6c\, d^2x^6 + 15c^2d\, x^3 + 8c^3} dx$$

input
```
int(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**3*x
**9),x)
```

3.509 $\int \frac{1}{x^2(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 653

$$\int \frac{1}{x^2(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{43\sqrt[3]{d}\sqrt{c+dx^3}}{216c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}c^{17/6}} + \frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1296c^{17/6}} - \frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1296c^{17/6}} - \frac{43\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{144\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{43\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
2/27/c^2/x/(d*x^3+c)^(1/2)-43/216*(d*x^3+c)^(1/2)/c^3/x+43/216*d^(1/3)*(d*
x^3+c)^(1/2)/c^3/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)-1/1296*d^(1/3)*arctan(3^(
1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/c^(17/6)+1/1296*
d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(17/6
)-1/1296*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-43/432*(1/2
*6^(1/2)-1/2*2^(1/2))*d^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3
)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^
(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3
^(1/4)/c^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x
)^2)^(1/2)/(d*x^3+c)^(1/2)+43/648*d^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^
(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*Elli
pticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^
(1/2)+2*I)*2^(1/2)*3^(3/4)/c^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2
))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^2 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \frac{-80c(27c + 43dx^3) + 875cdx^3 \sqrt{1 + \frac{dx^3}{c}} \operatorname{AppellF1}\left(\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{dx^3}{c}, \frac{dx^3}{8c}\right)}{17280c^4 x \sqrt{c + dx^3}}$$

input
```
Integrate[1/(x^2*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(-80*c*(27*c + 43*d*x^3) + 875*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1
/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 43*d^2*x^6*Sqrt[1 + (d*x^3)/c]*
AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(17280*c^4*x*Sqrt
[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 659, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {972, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (8c-dx^3) (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{972}$$

$$\frac{2}{27c^2x\sqrt{c+dx^3}}-\frac{2\int -\frac{d(43c-5dx^3)}{2x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{27c^2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{43c-5dx^3}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{27c^2}+\frac{2}{27c^2x\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int -\frac{cdx\left(350c-43dx^3\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{43\sqrt{c+dx^3}}{8cx}}{27c^2}+\frac{2}{27c^2x\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{d\int \frac{x\left(350c-43dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{16c}-\frac{43\sqrt{c+dx^3}}{8cx}}{27c^2}+\frac{2}{27c^2x\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1054}$$

$$\frac{\frac{d\int \left(\frac{6cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{43x}{\sqrt{dx^3+c}}\right)dx}{16c}-\frac{43\sqrt{c+dx^3}}{8cx}}{27c^2}+\frac{2}{27c^2x\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{2009}$$

$$d\left(\frac{86\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{43\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}\cdots}{\left(\left(1\cdots\right.\right.}}}{d^{2/3}\sqrt{\cdots}}\right.\cdots$$

$$\frac{2}{27c^2x\sqrt{c+dx^3}}$$

input
```
Int[1/(x^2*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
2/(27*c^2*x*Sqrt[c + d*x^3]) + ((-43*Sqrt[c + d*x^3])/(8*c*x) + (d*((86*Sq
rt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (c^(1/6)*Ar
cTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^
(2/3)) + (c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^
3])])/(3*d^(2/3)) - (c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^(2
/3)) - (43*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) +
 d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (86
*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 -
 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))))/(16*c))/(27*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.45 (sec) , antiderivative size = 890, normalized size of antiderivative = 1.36

method	result	size
elliptic	Expression too large to display	890
risch	Expression too large to display	1334
default	Expression too large to display	1361

input
```
int(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(d*x^3+c)^(1/2)/c^3/x-2/27*d/c^3*x^2/((x^3+c/d)*d)^(1/2)-43/648*I/c^3
*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/1944*I/c^3/d^2*2^(1
/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-
c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I
*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(
-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2379 vs. $2(461) = 922$.

Time = 0.73 (sec) , antiderivative size = 2379, normalized size of antiderivative = 3.64

$$\int \frac{1}{x^2 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/15552*(3096*(d*x^4 + c*x)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstras
sPInverse(0, -4*c/d, x)) - (c^3*d*x^4 + c^4*x + sqrt(-3)*(c^3*d*x^4 + c^4*
x))*(d^2/c^17)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640
*c^3*d - 9*(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x + sqrt(-3)*(5*c^12*
d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x))*(d^2/c^17)^(2/3) + 3*sqrt(d*x^3 + c)
*(6*(5*c^15*d*x^5 + 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(
d^2/c^17)^(5/6) - 2*(7*c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sqrt(d^2/c^
17) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x + sqrt(-3)*(c^3*d^3*x^7
+ 80*c^4*d^2*x^4 + 160*c^5*d*x))*(d^2/c^17)^(1/6)) - 9*(c^6*d^3*x^8 + 38*c
^7*d^2*x^5 + 64*c^8*d*x^2 - sqrt(-3)*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^
8*d*x^2))*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*
c^3)) + (c^3*d*x^4 + c^4*x + sqrt(-3)*(c^3*d*x^4 + c^4*x))*(d^2/c^17)^(1/6
)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^12*
d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x + sqrt(-3)*(5*c^12*d^2*x^7 + 64*c^13*d
*x^4 + 32*c^14*x))*(d^2/c^17)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 +
 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^2/c^17)^(5/6) - 2
*(7*c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sqrt(d^2/c^17) + (c^3*d^3*x^7
+ 80*c^4*d^2*x^4 + 160*c^5*d*x + sqrt(-3)*(c^3*d^3*x^7 + 80*c^4*d^2*x^4 +
160*c^5*d*x))*(d^2/c^17)^(1/6)) - 9*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8
*d*x^2 - sqrt(-3)*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^2))*(d^2/c...
```

Sympy [F]

$$\int \frac{1}{x^2\,(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = -\int \frac{1}{-8c^2x^2\sqrt{c+dx^3}-7cdx^5\sqrt{c+dx^3}+d^2x^8\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**2/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
-Integral(1/(-8*c**2*x**2*sqrt(c + d*x**3) - 7*c*d*x**5*sqrt(c + d*x**3) +
 d**2*x**8*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\,(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x^2}\,dx$$

input
```
integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\,(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x^2}\,dx$$

input
```
integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (8c-dx^3) (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^2 \left(d\,x^3+c\right)^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

input
```
int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (8c-dx^3) (c+dx^3)^{3/2}}\,dx = \frac{-2\sqrt{d\,x^3+c}+5\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{-d^3x^9+6c\,d^2x^6+15c^2d\,x^3+8c^3}dx\right)c\,d^2x+5\left(\int \frac{\sqrt{d\,x}}{-d^3x^9+6c\,d^2x}\right.}{}$$

input
```
int(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

output
```
( - 2*sqrt(c + d*x**3) + 5*int((sqrt(c + d*x**3)*x**4)/(8*c**3 + 15*c**2*d
*x**3 + 6*c*d**2*x**6 - d**3*x**9),x)*c*d**2*x + 5*int((sqrt(c + d*x**3)*x
**4)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**3*x**9),x)*d**3*x**4 -
38*int((sqrt(c + d*x**3)*x)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**
3*x**9),x)*c**2*d*x - 38*int((sqrt(c + d*x**3)*x)/(8*c**3 + 15*c**2*d*x**3
 + 6*c*d**2*x**6 - d**3*x**9),x)*c*d**2*x**4)/(16*c**2*x*(c + d*x**3))
```

3.510 $\int \frac{1}{x^5 (8c-dx^3) (c+dx^3)^{3/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 675

$$\int \frac{1}{x^5 (8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{113d^{4/3}\sqrt{c+dx^3}}{432c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3456\sqrt{3}c^{23/6}} + \frac{d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{10368c^{23/6}} - \frac{d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{23/6}} + \frac{113\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{288\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{113d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
2/27/c^2/x^4/(d*x^3+c)^(1/2)-91/864*(d*x^3+c)^(1/2)/c^3/x^4+113/432*d*(d*x
^3+c)^(1/2)/c^4/x-113/432*d^(4/3)*(d*x^3+c)^(1/2)/c^4/((1+3^(1/2))*c^(1/3)
+d^(1/3)*x)-1/10368*d^(4/3)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*
x^3+c)^(1/2))*3^(1/2)/c^(23/6)+1/10368*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3
)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(23/6)-1/10368*d^(4/3)*arctanh(1/3*(d*x^
3+c)^(1/2)/c^(1/2))/c^(23/6)+113/864*(1/2*6^(1/2)-1/2*2^(1/2))*d^(4/3)*(c^
(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(
1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(
1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/c^(11/3)/(c^(1/3)*(c^(1/3)
+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)-113/1
296*d^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(
(1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(
1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/c^(
11/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2
)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^5 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \frac{160c(-27c^2 + 135cdx^3 + 226d^2x^6) - 9025cd^2x^6 \sqrt{1 + \frac{dx^3}{c}} \operatorname{AppellF1}\left(\frac{2}{3}, \right.}{138240c^5x^4}$$

input
```
Integrate[1/(x^5*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(160*c*(-27*c^2 + 135*c*d*x^3 + 226*d^2*x^6) - 9025*c*d^2*x^6*Sqrt[1 + (d*
x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 452*d^3*
x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/(138240*c^5*x^4*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {972, 27, 1053, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (8c-dx^3) (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \textbf{972}$$

$$\frac{2}{27c^2x^4\sqrt{c+dx^3}}-\frac{2\int -\frac{d(91c-11dx^3)}{2x^5(8c-dx^3)\sqrt{dx^3+c}}dx}{27c^2d}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{91c-11dx^3}{x^5(8c-dx^3)\sqrt{dx^3+c}}dx}{27c^2}+\frac{2}{27c^2x^4\sqrt{c+dx^3}}$$

$$\Big\downarrow \textbf{1053}$$

$$\frac{-\frac{\int \frac{cd\left(3616c-455dx^3\right)}{2x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2}-\frac{91\sqrt{c+dx^3}}{32cx^4}}{27c^2}+\frac{2}{27c^2x^4\sqrt{c+dx^3}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{-\frac{d\int \frac{3616c-455dx^3}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{64c}-\frac{91\sqrt{c+dx^3}}{32cx^4}}{27c^2}+\frac{2}{27c^2x^4\sqrt{c+dx^3}}$$

$$\Big\downarrow \textbf{1053}$$

$$\frac{-\frac{d\left(-\frac{\int -\frac{8cdx\left(1805c-226dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{452\sqrt{c+dx^3}}{cx}\right)}{64c}-\frac{91\sqrt{c+dx^3}}{32cx^4}}{27c^2}+\frac{2}{27c^2x^4\sqrt{c+dx^3}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{-\frac{d\left(\frac{d\int \frac{x\left(1805c-226dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{452\sqrt{c+dx^3}}{cx}\right)}{64c}-\frac{91\sqrt{c+dx^3}}{32cx^4}}{27c^2}+\frac{2}{27c^2x^4\sqrt{c+dx^3}}$$

$\downarrow$ 1054

$$\frac{-\frac{d\left(\frac{d\int \left(\frac{226x}{\sqrt{dx^3+c}}-\frac{3cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}\right)dx}{c}-\frac{452\sqrt{c+dx^3}}{cx}\right)}{64c}-\frac{91\sqrt{c+dx^3}}{32cx^4}}{27c^2}+\frac{2}{27c^2x^4\sqrt{c+dx^3}}$$

$\downarrow$ 2009

$$-\frac{d\left(d\left(\frac{452\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{226\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\cdots}{d\cdots}\cdots$$

$$\frac{2}{27c^2x^4\sqrt{c+dx^3}}$$

input
```
Int[1/(x^5*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
2/(27*c^2*x^4*Sqrt[c + d*x^3]) + ((-91*Sqrt[c + d*x^3])/(32*c*x^4) - (d*((
-452*Sqrt[c + d*x^3])/(c*x) + (d*((452*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)) + (c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) +
d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*Sqrt[3]*d^(2/3)) - (c^(1/6)*ArcTanh[(c^(1
/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(6*d^(2/3)) + (c^(1/6)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*d^(2/3)) - (226*3^(1/4)*Sqrt[2 - Sq
rt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4
*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (452*Sqrt[2]*c^(1/3)*(c^(1/3) + d
^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*
x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)
*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^
2]*Sqrt[c + d*x^3])))/c))/(64*c))/(27*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.96 (sec) , antiderivative size = 911, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	911
risch	Expression too large to display	1344
default	Expression too large to display	1864

input
```
int(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(d*x^3+c)^(1/2)/c^3/x^4+3/16*d*(d*x^3+c)^(1/2)/c^4/x+2/27*d^2/c^4*x^
2/((x^3+c/d)*d)^(1/2)+113/1296*I*d/c^4*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)))-1/15552*I/d/c^4*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*
I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(
1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)
))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c
*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^
(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2529 vs. $2(479) = 958$.

Time = 2.54 (sec) , antiderivative size = 2529, normalized size of antiderivative = 3.75

$$\int \frac{1}{x^5 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
1/124416*(32544*(d^2*x^7 + c*d*x^4)*sqrt(d)*weierstrassZeta(0, -4*c/d, wei
erstrassPInverse(0, -4*c/d, x)) + (c^4*d*x^7 + c^5*x^4 + sqrt(-3)*(c^4*d*x
^7 + c^5*x^4))*(d^8/c^23)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^
7*x^3 + 640*c^3*d^6 - 9*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x +
sqrt(-3)*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x))*(d^8/c^23)^(2/3
) + 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 - sqrt(-3)*(5*c^20*d*
x^5 + 32*c^21*x^2))*(d^8/c^23)^(5/6) - 2*(7*c^12*d^4*x^6 + 152*c^13*d^3*x^
3 + 64*c^14*d^2)*sqrt(d^8/c^23) + (c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*
d^5*x + sqrt(-3)*(c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x))*(d^8/c^23
)^(1/6)) - 9*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2 - sqrt(-3)*(c
^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2))*(d^8/c^23)^(1/3))/(d^3*x^9
 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^4*d*x^7 + c^5*x^4 + sqrt(
-3)*(c^4*d*x^7 + c^5*x^4))*(d^8/c^23)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 +
 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32
*c^18*d*x + sqrt(-3)*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x))*(d^
8/c^23)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 - sqrt(-3
)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^8/c^23)^(5/6) - 2*(7*c^12*d^4*x^6 + 152
*c^13*d^3*x^3 + 64*c^14*d^2)*sqrt(d^8/c^23) + (c^4*d^7*x^7 + 80*c^5*d^6*x^
4 + 160*c^6*d^5*x + sqrt(-3)*(c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x
))*(d^8/c^23)^(1/6)) - 9*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^...
```

Sympy [F]

$$\int \frac{1}{x^5\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = -\int \frac{1}{-8c^2x^5\sqrt{c+dx^3}-7cdx^8\sqrt{c+dx^3}+d^2x^{11}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**5/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
-Integral(1/(-8*c**2*x**5*sqrt(c + d*x**3) - 7*c*d*x**8*sqrt(c + d*x**3) +
 d**2*x**11*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int -\frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\, c) x^5} \, dx$$

input
```
integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int -\frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\, c) x^5} \, dx$$

input
```
integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int \frac{1}{x^5 \, (d\, x^3 + c)^{3/2} \, (8\, c - d\, x^3)} \, dx$$

input
```
int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^5 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \frac{-2\sqrt{d\,x^3 + c}\,c + 10\sqrt{d\,x^3 + c}\,d\,x^3 - 25\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{-d^3x^9+6c\,d^2x^6+15c^2d\,x^3+8c^3}dx\right) c$$

input
```
int(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

output
```
( - 2*sqrt(c + d*x**3)*c + 10*sqrt(c + d*x**3)*d*x**3 - 25*int((sqrt(c + d
*x**3)*x**4)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**3*x**9),x)*c*d*
*3*x**4 - 25*int((sqrt(c + d*x**3)*x**4)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d*
*2*x**6 - d**3*x**9),x)*d**4*x**7 + 201*int((sqrt(c + d*x**3)*x)/(8*c**3 +
 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**3*x**9),x)*c**2*d**2*x**4 + 201*int((
sqrt(c + d*x**3)*x)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**3*x**9),
x)*c*d**3*x**7)/(64*c**3*x**4*(c + d*x**3))
```

3.511 $\int \frac{1}{x^8 (8c-dx^3) (c+dx^3)^{3/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 699

$$\int \frac{1}{x^8 (8c-dx^3) (c+dx^3)^{3/2}}\,dx = \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \frac{953d^{7/3}\sqrt{c+dx^3}}{3024c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}c^{29/6}} + \frac{d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{29/6}} - \frac{d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{82944c^{29/6}} - \frac{953\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{2016\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{953d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{1512\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
2/27/c^2/x^7/(d*x^3+c)^(1/2)-139/1512*(d*x^3+c)^(1/2)/c^3/x^7+6095/48384*d
*(d*x^3+c)^(1/2)/c^4/x^4-953/3024*d^2*(d*x^3+c)^(1/2)/c^5/x+953/3024*d^(7/
3)*(d*x^3+c)^(1/2)/c^5/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)-1/82944*d^(7/3)*arc
tan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/c^(29/6)+
1/82944*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))
/c^(29/6)-1/82944*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(29/6)-95
3/6048*(1/2*6^(1/2)-1/2*2^(1/2))*d^(7/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(
1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*Ellip
ticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(
1/2)+2*I)*3^(1/4)/c^(14/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/
3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+953/9072*d^(7/3)*(c^(1/3)+d^(1/3)*x
)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)
^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d
^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/c^(14/3)/(c^(1/3)*(c^(1/3)+d^(1/3
)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^8 (8c-dx^3) (c+dx^3)^{3/2}} \, dx = \frac{610025cd^3x^9\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-32\Big(5c(864c^3-$$

input
```
Integrate[1/(x^8*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(610025*c*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)
/c), (d*x^3)/(8*c)] - 32*(5*c*(864*c^3 - 1647*c^2*d*x^3 + 9153*c*d^2*x^6 +
 15248*d^3*x^9) + 953*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8
/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(7741440*c^6*x^7*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {972, 27, 1053, 27, 1053, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 (8c - dx^3) (c + dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \; 972$$

$$\frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{2\int -\frac{d(139c-17dx^3)}{2x^8(8c-dx^3)\sqrt{dx^3+c}}dx}{27c^2d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{139c-17dx^3}{x^8(8c-dx^3)\sqrt{dx^3+c}}dx}{27c^2} + \frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 1053$$

$$\frac{-\frac{\int \frac{cd\left(12190c-1529dx^3\right)}{2x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{56c^2} - \frac{139\sqrt{c+dx^3}}{56cx^7}}{27c^2} + \frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{d\int \frac{12190c-1529dx^3}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{112c} - \frac{139\sqrt{c+dx^3}}{56cx^7}}{27c^2} + \frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 1053$$

$$\frac{-\frac{d\left(-\frac{\int \frac{cd\left(243968c-30475dx^3\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2} - \frac{6095\sqrt{c+dx^3}}{16cx^4}\right)}{112c} - \frac{139\sqrt{c+dx^3}}{56cx^7}}{27c^2} + \frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{d\left(-\frac{d\int \frac{243968c-30475dx^3}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c}-\frac{6095\sqrt{c+dx^3}}{16cx^4}\right)}{112c}-\frac{139\sqrt{c+dx^3}}{56cx^7}}{27c^2}+\frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{-\frac{d\left(-\frac{d\left(-\frac{\int -\frac{8cdx\left(122005c-15248dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{30496\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{6095\sqrt{c+dx^3}}{16cx^4}\right)}{112c}-\frac{139\sqrt{c+dx^3}}{56cx^7}}{27c^2}+\frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{d\left(-\frac{d\left(\frac{d\int \frac{x\left(122005c-15248dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{30496\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{6095\sqrt{c+dx^3}}{16cx^4}\right)}{112c}-\frac{139\sqrt{c+dx^3}}{56cx^7}}{27c^2}+\frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{-\frac{d\left(-\frac{d\left(\frac{d\int \left(\frac{21cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{15248x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{30496\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{6095\sqrt{c+dx^3}}{16cx^4}\right)}{112c}-\frac{139\sqrt{c+dx^3}}{56cx^7}}{27c^2}+\frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{d\left(-\frac{d\left(\frac{d\left(\frac{30496\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{15248\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}\cdots\right.}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}}{\frac{2}{27c^2x^7\sqrt{c+dx^3}}}$$

input `Int[1/(x^8*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]`

output
```
2/(27*c^2*x^7*Sqrt[c + d*x^3]) + ((-139*Sqrt[c + d*x^3])/(56*c*x^7) - (d*(
(-6095*Sqrt[c + d*x^3])/(16*c*x^4) - (d*((-30496*Sqrt[c + d*x^3])/(c*x) +
(d*((30496*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x))
- (7*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3
]])/(2*Sqrt[3]*d^(2/3)) + (7*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^
(1/6)*Sqrt[c + d*x^3])])/(6*d^(2/3)) - (7*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(6*d^(2/3)) - (15248*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1
/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d
^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*
Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2
]*Sqrt[c + d*x^3]) + (30496*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^
(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^
(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]
)))/c))/(32*c)))/(112*c))/(27*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.87 (sec) , antiderivative size = 930, normalized size of antiderivative = 1.33

method	result	size
elliptic	Expression too large to display	930
risch	Expression too large to display	1357
default	Expression too large to display	2389

input
```
int(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/56*(d*x^3+c)^(1/2)/c^3/x^7+93/1792*d*(d*x^3+c)^(1/2)/c^4/x^4-27/112*d^2
*(d*x^3+c)^(1/2)/c^5/x-2/27*d^3/c^5*x^2/((x^3+c/d)*d)^(1/2)-953/9072*I*d^2
/c^5*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/124416*I/c^5*2^
(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2
-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2564 vs. $2(499) = 998$.

Time = 4.66 (sec) , antiderivative size = 2564, normalized size of antiderivative = 3.67

$$\int \frac{1}{x^8 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input

```
integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/6967296*(2195712*(d^3*x^10 + c*d^2*x^7)*sqrt(d)*weierstrassZeta(0, -4*c
/d, weierstrassPInverse(0, -4*c/d, x)) - 7*(c^5*d*x^10 + c^6*x^7 + sqrt(-3
)*(c^5*d*x^10 + c^6*x^7))*(d^14/c^29)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6
 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4
+ 32*c^22*d^2*x + sqrt(-3)*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2
*x))*(d^14/c^29)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2
- sqrt(-3)*(5*c^25*d*x^5 + 32*c^26*x^2))*(d^14/c^29)^(5/6) - 2*(7*c^15*d^6
*x^6 + 152*c^16*d^5*x^3 + 64*c^17*d^4)*sqrt(d^14/c^29) + (c^5*d^11*x^7 + 8
0*c^6*d^10*x^4 + 160*c^7*d^9*x + sqrt(-3)*(c^5*d^11*x^7 + 80*c^6*d^10*x^4
+ 160*c^7*d^9*x))*(d^14/c^29)^(1/6)) - 9*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 +
 64*c^12*d^7*x^2 - sqrt(-3)*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 + 64*c^12*d^7*
x^2))*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3
)) + 7*(c^5*d*x^10 + c^6*x^7 + sqrt(-3)*(c^5*d*x^10 + c^6*x^7))*(d^14/c^29
)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11
- 9*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2*x + sqrt(-3)*(5*c^20*d
^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2*x))*(d^14/c^29)^(2/3) - 3*sqrt(d*x^
3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2 - sqrt(-3)*(5*c^25*d*x^5 + 32*c^26*x
^2))*(d^14/c^29)^(5/6) - 2*(7*c^15*d^6*x^6 + 152*c^16*d^5*x^3 + 64*c^17*d^
4)*sqrt(d^14/c^29) + (c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 160*c^7*d^9*x + sqr
t(-3)*(c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 160*c^7*d^9*x))*(d^14/c^29)^(1/...
```

Sympy [F]

$$\int \frac{1}{x^8 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = -\int \frac{1}{-8c^2x^8\sqrt{c + dx^3} - 7cdx^{11}\sqrt{c + dx^3} + d^2x^{14}\sqrt{c + dx^3}} \, dx$$

input
```
integrate(1/x**8/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
-Integral(1/(-8*c**2*x**8*sqrt(c + d*x**3) - 7*c*d*x**11*sqrt(c + d*x**3)
+ d**2*x**14*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^8 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int -\frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c) x^8} \, dx$$

input
```
integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int -\frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c) x^8} \, dx$$

input
```
integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int \frac{1}{x^8 \left(d\,x^3 + c\right)^{3/2} \left(8\,c - d\,x^3\right)} \, dx$$

input
```
int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^8 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \frac{-64\sqrt{d\,x^3 + c}\,c^2 + 122\sqrt{d\,x^3 + c}\,cd\,x^3 - 68\sqrt{d\,x^3 + c}\,d^2 x^6 + 4880\Big(\int \frac{}{-d}$$

input
```
int(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

output
```
( - 64*sqrt(c + d*x**3)*c**2 + 122*sqrt(c + d*x**3)*c*d*x**3 - 68*sqrt(c +
 d*x**3)*d**2*x**6 + 4880*int(sqrt(c + d*x**3)/(8*c**3*x**2 + 15*c**2*d*x*
*5 + 6*c*d**2*x**8 - d**3*x**11),x)*c**3*d**2*x**7 + 4880*int(sqrt(c + d*x
**3)/(8*c**3*x**2 + 15*c**2*d*x**5 + 6*c*d**2*x**8 - d**3*x**11),x)*c**2*d
**3*x**10 + 170*int((sqrt(c + d*x**3)*x**4)/(8*c**3 + 15*c**2*d*x**3 + 6*c
*d**2*x**6 - d**3*x**9),x)*c*d**4*x**7 + 170*int((sqrt(c + d*x**3)*x**4)/(
8*c**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**3*x**9),x)*d**5*x**10 - 1963*
int((sqrt(c + d*x**3)*x)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**3*x
**9),x)*c**2*d**3*x**7 - 1963*int((sqrt(c + d*x**3)*x)/(8*c**3 + 15*c**2*d
*x**3 + 6*c*d**2*x**6 - d**3*x**9),x)*c*d**4*x**10)/(3584*c**4*x**7*(c + d
*x**3))
```

3.512 $\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{3}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c^2\sqrt{c+dx^3}}$$

output
```
1/32*x^4*(1+d*x^3/c)^(1/2)*AppellF1(4/3,3/2,1,7/3,-d*x^3/c,1/8*d*x^3/c)/c^
2/(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 233 vs. $2(66) = 132$.

Time = 8.77 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.53

$$\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{x\left(x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+\frac{64c\left(-1+\frac{\cdots}{(8c-dx^3)\left(32c\,\text{AppellF1}\cdots\right.}\right.}{\cdots}\right.}{864c^2\sqrt{c\cdots}}$$

input
```
Integrate[x^3/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x*(x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^
3)/(8*c)] + (64*c*(-1 + (256*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
(d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/
c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*
x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))
))/d))/(864*c^2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^3}{(8c-dx^3)\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{3}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c^2\sqrt{c+dx^3}}$$

input
```
Int[x^3/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 3/2, 7/3, (d*x^3)/(8*c), -((d*x^
3)/c)])/(32*c^2*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.16 (sec) , antiderivative size = 724, normalized size of antiderivative = 10.97

method	result	size
elliptic	Expression too large to display	724
default	Expression too large to display	1038

input
```
int(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/27/d*x/c/((x^3+c/d)*d)^(1/2)+2/81*I/c/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-8/243*I/d^4/c*2^(1/2)*sum(1/_alpha^
2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*
3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)
*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_
alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2
*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(
1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_
Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2535 vs. $2(52) = 104$.

Time = 0.63 (sec) , antiderivative size = 2535, normalized size of antiderivative = 38.41

$$\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/486*(36*sqrt(d*x^3 + c)*d*x - 36*(d*x^3 + c)*sqrt(d)*weierstrassPInvers
e(0, -4*c/d, x) - (c*d^3*x^3 + c^2*d^2 + sqrt(-3)*(c*d^3*x^3 + c^2*d^2))*(
1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3
 - 9*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2 + sqrt(-3)*(c^5*d^8*x^
8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2))*(1/(c^7*d^8))^(2/3) + 3*sqrt(d*x^3 +
 c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x - sqrt(-3)*(c^6*d^9*x^7
 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x))*(1/(c^7*d^8))^(5/6) - 2*(7*c^4*d^6*x^6
 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt(1/(c^7*d^8)) + 6*(5*c^2*d^3*x^5 + 32
*c^3*d^2*x^2 + sqrt(-3)*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2))*(1/(c^7*d^8))^(1
/6)) - 9*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x - sqrt(-3)*(5*c^3*
d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x))*(1/(c^7*d^8))^(1/3))/(d^3*x^9 -
24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c*d^3*x^3 + c^2*d^2 + sqrt(-3)
*(c*d^3*x^3 + c^2*d^2))*(1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 +
 1200*c^2*d*x^3 + 640*c^3 - 9*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x
^2 + sqrt(-3)*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2))*(1/(c^7*d^8
))^(2/3) - 3*sqrt(d*x^3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*
x - sqrt(-3)*(c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x))*(1/(c^7*d^8))
^(5/6) - 2*(7*c^4*d^6*x^6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt(1/(c^7*d^8)
) + 6*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2 + sqrt(-3)*(5*c^2*d^3*x^5 + 32*c^3*d
^2*x^2))*(1/(c^7*d^8))^(1/6)) - 9*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int -\frac{x^3}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{x^3}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int -\frac{x^3}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)}\,dx$$

input
```
int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^3}{(8c - dx^3)(c + dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3 + c}\,x^3}{-d^3x^9 + 6c\,d^2x^6 + 15c^2d\,x^3 + 8c^3}dx$$

input
```
int(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**3)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**
3*x**9),x)
```

3.513 $\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{1}{3},1,\frac{3}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c^2\sqrt{c+dx^3}}$$

output
```
1/8*x*(1+d*x^3/c)^(1/2)*AppellF1(1/3,3/2,1,4/3,-d*x^3/c,1/8*d*x^3/c)/c^2/(
d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 230 vs. 2(64) = 128.

Time = 10.16 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.59

$$\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\frac{x\left(-\frac{dx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+64\left(\frac{1}{c^2}+\frac{\cdots}{(8c-dx^3)\left(32c\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\cdots\right.\right.}\right.\right.}{864\sqrt{c+\cdots}}$$

input
```
Integrate[1/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x*(-((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
(d*x^3)/(8*c)])/c^3) + 64*(c^(-2) + (176*AppellF1[1/3, 1/2, 1, 4/3, -((d*x
^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -(
(d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)
/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(
8*c)]))))))/(864*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{(8c-dx^3)\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{3}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c^2\sqrt{c+dx^3}}$$

input
```
Int[1/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 3/2, 4/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(8*c^2*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.03 (sec) , antiderivative size = 721, normalized size of antiderivative = 11.27

method	result	size
default	Expression too large to display	721
elliptic	Expression too large to display	721

input
```
int(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/27*x/c^2/((x^3+c/d)*d)^(1/2)-2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha
^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I
*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*
_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(
2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^
(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2483 vs. $2(50) = 100$.

Time = 0.52 (sec) , antiderivative size = 2483, normalized size of antiderivative = 38.80

$$\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3888*(288*sqrt(d*x^3 + c)*d*x + 360*(d*x^3 + c)*sqrt(d)*weierstrassPInve
rse(0, -4*c/d, x) + (c^2*d^2*x^3 + c^3*d + sqrt(-3)*(c^2*d^2*x^3 + c^3*d))
*(1/(c^13*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*
c^3 - 9*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2 + sqrt(-3)*(c^9*d
^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2))*(1/(c^13*d^2))^(2/3) + 3*sqrt
(d*x^3 + c)*((c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x - sqrt(-3)*(
c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x))*(1/(c^13*d^2))^(5/6) - 2
*(7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(1/(c^13*d^2)) + 6*(5*c^
3*d^2*x^5 + 32*c^4*d*x^2 + sqrt(-3)*(5*c^3*d^2*x^5 + 32*c^4*d*x^2))*(1/(c^
13*d^2))^(1/6)) - 9*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x - sqrt(-3
)*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x))*(1/(c^13*d^2))^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^2*d^2*x^3 + c^3*d +
sqrt(-3)*(c^2*d^2*x^3 + c^3*d))*(1/(c^13*d^2))^(1/6)*log((d^3*x^9 + 318*c*
d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64
*c^11*d^2*x^2 + sqrt(-3)*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2)
)*(1/(c^13*d^2))^(2/3) - 3*sqrt(d*x^3 + c)*((c^11*d^4*x^7 + 80*c^12*d^3*x^
4 + 160*c^13*d^2*x - sqrt(-3)*(c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d
^2*x))*(1/(c^13*d^2))^(5/6) - 2*(7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*
d)*sqrt(1/(c^13*d^2)) + 6*(5*c^3*d^2*x^5 + 32*c^4*d*x^2 + sqrt(-3)*(5*c^3*
d^2*x^5 + 32*c^4*d*x^2))*(1/(c^13*d^2))^(1/6)) - 9*(5*c^5*d^3*x^7 + 64*...
```

Sympy [F]

$$\int \frac{1}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx=-\int \frac{1}{-8c^2\sqrt{c+dx^3}-7cdx^3\sqrt{c+dx^3}+d^2x^6\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
-Integral(1/(-8*c**2*sqrt(c + d*x**3) - 7*c*d*x**3*sqrt(c + d*x**3) + d**2
*x**6*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(8c - dx^3)\,(c + dx^3)^{3/2}}\,dx = \int -\frac{1}{(dx^3 + c)^{\frac{3}{2}}(dx^3 - 8\,c)}\,dx$$

input
```
integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{1}{(8c - dx^3)\,(c + dx^3)^{3/2}}\,dx = \int -\frac{1}{(dx^3 + c)^{\frac{3}{2}}(dx^3 - 8\,c)}\,dx$$

input
```
integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8c - dx^3)\,(c + dx^3)^{3/2}}\,dx = \int \frac{1}{(d\,x^3 + c)^{3/2}\,(8\,c - d\,x^3)}\,dx$$

input
```
int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}}{-d^3x^9+6c\,d^2x^6+15c^2d\,x^3+8c^3}dx$$

input
```
int(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

output
```
int(sqrt(c + d*x**3)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**3*x**9)
,x)
```

3.514 $\int \frac{1}{x^3(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^3(8c-dx^3)(c+dx^3)^{3/2}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(-\frac{2}{3},1,\frac{3}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16c^2x^2\sqrt{c+dx^3}}$$

output
```
-1/16*(1+d*x^3/c)^(1/2)*AppellF1(-2/3,3/2,1,1/3,-d*x^3/c,1/8*d*x^3/c)/c^2/
x^2/(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 248 vs. 2(66) = 132.

Time = 11.20 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.76

$$\int \frac{1}{x^3(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\frac{59d^2x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+64c\Big(-27c-59dx^3}{}$$

input
```
Integrate[1/(x^3*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(59*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (
d*x^3)/(8*c)] + 64*c*(-27*c - 59*d*x^3 - (7360*c^2*d*x^3*AppellF1[1/3, 1/2
, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*AppellF1[1/3,
 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2
, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^
3)/c), (d*x^3)/(8*c)])))))/(27648*c^4*x^2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(8c-dx^3)(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^3(8c-dx^3)\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{3}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16c^2x^2\sqrt{c+dx^3}}$$

input
```
Int[1/(x^3*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
-1/16*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 3/2, 1/3, (d*x^3)/(8*c), -((d
*x^3)/c)])/(c^2*x^2*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 2.39 (sec) , antiderivative size = 736, normalized size of antiderivative = 11.15

method	result	size
elliptic	Expression too large to display	736
risch	Expression too large to display	1028
default	Expression too large to display	1053

input
```
int(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(d*x^3+c)^(1/2)/c^3/x^2-2/27*d/c^3*x/((x^3+c/d)*d)^(1/2)+59/1296*I/c
^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/1
944*I/c^3/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-
c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*
d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2
/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*
(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)
/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2495 vs. $2(52) = 104$.

Time = 2.00 (sec) , antiderivative size = 2495, normalized size of antiderivative = 37.80

$$\int \frac{1}{x^3 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/31104*(4176*(d*x^5 + c*x^2)*sqrt(d)*weierstrassPInverse(0, -4*c/d, x) -
 (c^3*d*x^5 + c^4*x^2 + sqrt(-3)*(c^3*d*x^5 + c^4*x^2))*(d^4/c^19)^(1/6)*l
og((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 - 9*(c^13*d^3
*x^8 + 38*c^14*d^2*x^5 + 64*c^15*d*x^2 + sqrt(-3)*(c^13*d^3*x^8 + 38*c^14*
d^2*x^5 + 64*c^15*d*x^2))*(d^4/c^19)^(2/3) + 3*sqrt(d*x^3 + c)*((c^16*d^2*
x^7 + 80*c^17*d*x^4 + 160*c^18*x - sqrt(-3)*(c^16*d^2*x^7 + 80*c^17*d*x^4
+ 160*c^18*x))*(d^4/c^19)^(5/6) - 2*(7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 6
4*c^12*d)*sqrt(d^4/c^19) + 6*(5*c^4*d^4*x^5 + 32*c^5*d^3*x^2 + sqrt(-3)*(5
*c^4*d^4*x^5 + 32*c^5*d^3*x^2))*(d^4/c^19)^(1/6)) - 9*(5*c^7*d^4*x^7 + 64*
c^8*d^3*x^4 + 32*c^9*d^2*x - sqrt(-3)*(5*c^7*d^4*x^7 + 64*c^8*d^3*x^4 + 32
*c^9*d^2*x))*(d^4/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 5
12*c^3)) + (c^3*d*x^5 + c^4*x^2 + sqrt(-3)*(c^3*d*x^5 + c^4*x^2))*(d^4/c^1
9)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 - 9
*(c^13*d^3*x^8 + 38*c^14*d^2*x^5 + 64*c^15*d*x^2 + sqrt(-3)*(c^13*d^3*x^8
+ 38*c^14*d^2*x^5 + 64*c^15*d*x^2))*(d^4/c^19)^(2/3) - 3*sqrt(d*x^3 + c)*(
(c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x - sqrt(-3)*(c^16*d^2*x^7 + 80*c
^17*d*x^4 + 160*c^18*x))*(d^4/c^19)^(5/6) - 2*(7*c^10*d^3*x^6 + 152*c^11*d
^2*x^3 + 64*c^12*d)*sqrt(d^4/c^19) + 6*(5*c^4*d^4*x^5 + 32*c^5*d^3*x^2 + s
qrt(-3)*(5*c^4*d^4*x^5 + 32*c^5*d^3*x^2))*(d^4/c^19)^(1/6)) - 9*(5*c^7*d^4
*x^7 + 64*c^8*d^3*x^4 + 32*c^9*d^2*x - sqrt(-3)*(5*c^7*d^4*x^7 + 64*c^8...
```

Sympy [F]

$$\int \frac{1}{x^3 (8c - dx^3) (c + dx^3)^{3/2}}\,dx = -\int \frac{1}{-8c^2x^3\sqrt{c+dx^3} - 7cdx^6\sqrt{c+dx^3} + d^2x^9\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**3/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
-Integral(1/(-8*c**2*x**3*sqrt(c + d*x**3) - 7*c*d*x**6*sqrt(c + d*x**3) +
 d**2*x**9*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int -\frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c) x^3} \, dx$$

input
```
integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int -\frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c) x^3} \, dx$$

input
```
integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int \frac{1}{x^3 \left(d\,x^3 + c\right)^{3/2} \left(8\,c - d\,x^3\right)} \, dx$$

input
```
int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^3(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{-2\sqrt{d\,x^3+c}-52\left(\int \frac{\sqrt{d\,x^3+c}}{-d^3x^9+6c\,d^2x^6+15c^2d\,x^3+8c^3}dx\right)c^2d\,x^2-52\left(\int \frac{}{-d^3x^9+6}\right.}{}$$

input
```
int(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

output
```
( - 2*sqrt(c + d*x**3) - 52*int(sqrt(c + d*x**3)/(8*c**3 + 15*c**2*d*x**3
+ 6*c*d**2*x**6 - d**3*x**9),x)*c**2*d*x**2 - 52*int(sqrt(c + d*x**3)/(8*c
**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**3*x**9),x)*c*d**2*x**5 + 7*int((
sqrt(c + d*x**3)*x**3)/(8*c**3 + 15*c**2*d*x**3 + 6*c*d**2*x**6 - d**3*x**
9),x)*c*d**2*x**2 + 7*int((sqrt(c + d*x**3)*x**3)/(8*c**3 + 15*c**2*d*x**3
 + 6*c*d**2*x**6 - d**3*x**9),x)*d**3*x**5)/(32*c**2*x**2*(c + d*x**3))
```

3.515 $\int \frac{x\sqrt{a+bx^3}}{2(5+3\sqrt{3})a+bx^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 737

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$$

$$= \frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{3^{3/4}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$

$$+ \frac{\sqrt[6]{a}\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$

$$+ \frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2*(b*x^3+a)^(1/2)/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+1/4*3^(3/4)*a^(1
/6)*arctan(1/2*3^(1/4)*(1+3^(1/2))*a^(1/6)*(a^(1/3)+b^(1/3)*x)*2^(1/2)/(b*
x^3+a)^(1/2))*2^(1/2)/b^(2/3)+1/6*a^(1/6)*arctan(1/6*(1-3^(1/2))*(b*x^3+a)
^(1/2)*2^(1/2)*3^(1/4)/a^(1/2))*2^(1/2)*3^(3/4)/b^(2/3)+1/2*3^(1/4)*a^(1/6
)*arctanh(1/2*3^(1/4)*a^(1/6)*((1+3^(1/2))*a^(1/3)-2*b^(1/3)*x)*2^(1/2)/(b
*x^3+a)^(1/2))*2^(1/2)/b^(2/3)+1/4*3^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*(1-
3^(1/2))*a^(1/6)*(a^(1/3)+b^(1/3)*x)*2^(1/2)/(b*x^3+a)^(1/2))*2^(1/2)/b^(2
/3)-3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*
EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),
I*3^(1/2)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b
^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+2/3*2^(1/2)*a^(1/3)*(a^(1/3)+b^(1/3)*x)
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^
2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^
(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3
^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.11

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20+12\sqrt{3}\right)\sqrt{a+bx^3}}$$

input
```
Integrate[(x*Sqrt[a + b*x^3])/(2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

output
```
(x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x^
3)/(10*a + 6*Sqrt[3]*a))])/((20 + 12*Sqrt[3])*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.44 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {984, 832, 759, 989, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}dx$$

$$\Big\downarrow \text{ 984}$$

$$\int \frac{x}{\sqrt{bx^3+a}}dx-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\sqrt{bx^3+a}\left(bx^3+2\left(5+3\sqrt{3}\right)a\right)}dx$$

$$\Big\downarrow \text{ 832}$$

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\sqrt{bx^3+a}\left(bx^3+2\left(5+3\sqrt{3}\right)a\right)}dx$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\sqrt{bx^3+a}\left(bx^3+2\left(5+3\sqrt{3}\right)a\right)}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 989}$$

$$\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}} - \frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$3\left(3+2\sqrt{3}\right)a\left(-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2-\sqrt{3})\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2-\cdots}{\cdots}\right.$$

$\downarrow$ 2416

$$-3\left(3+2\sqrt{3}\right)a\left(-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt{2}\sqrt{bx^3+a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2-\sqrt{3})\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{bx^3+a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2-\cdots}{\cdots}\right.$$

$$\frac{\frac{2\sqrt{bx^3+a}}{\sqrt[3]{b}\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{bx^3+a}}}{\sqrt[3]{b}} -$$

$$\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{bx^3+a}}$$

input

```
Int[(x*Sqrt[a + b*x^3])/(2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

output
```
-3*(3 + 2*Sqrt[3])*a*(-1/2*((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^
(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(Sqrt[2]*3^(3/4)*
a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a + b*x^3])/(
Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqr
t[3])*ArcTanh[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqr
t[2]*Sqrt[a + b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3
])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))) + ((2*Sqrt[a + b*x
^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqr
t[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 +
 Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 989
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.47 (sec) , antiderivative size = 977, normalized size of antiderivative = 1.33

method	result	size
elliptic	Expression too large to display	977
default	Expression too large to display	995

input
```
int(x*(b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a+b*x^3),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1
/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+1/9*I/b^3*2^(
1/2)*sum(1/_alpha*(3+2*3^(1/2))*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)
^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)
^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x
+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*(-3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3
*I*3^(1/2)*(-a*b^2)^(2/3)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*(-a*b^2)^(1/3)*_al
pha*3^(1/2)*b-6*_alpha^2*b^2-6*I*(-a*b^2)^(2/3)-2*3^(1/2)*(-a*b^2)^(2/3)+3
*(-a*b^2)^(1/3)*_alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4847 vs. $2(520) = 1040$.

Time = 4.35 (sec) , antiderivative size = 4847, normalized size of antiderivative = 6.58

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a+b*x^3),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{x\sqrt{a+bx^3}}{10a+6\sqrt{3}a+bx^3}\,dx$$

input
```
integrate(x*(b*x**3+a)**(1/2)/(2*(5+3*3**(1/2))*a+b*x**3),x)
```

output
```
Integral(x*sqrt(a + b*x**3)/(10*a + 6*sqrt(3)*a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{\sqrt{bx^3+a}x}{bx^3+2\,a(3\,\sqrt{3}+5)}\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a+b*x^3),x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

Giac [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{\sqrt{bx^3+a}x}{bx^3+2\,a(3\,\sqrt{3}+5)}\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a+b*x^3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{x\,\sqrt{b\,x^3+a}}{b\,x^3+2\,a\,\left(3\,\sqrt{3}+5\right)}\,dx$$

input
```
int((x*(a + b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)),x)
```

output
```
int((x*(a + b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)), x)
```

Reduce [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=6\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{-b^2x^6-20ab\,x^3+8a^2}dx\right)a-\left(\int \frac{\sqrt{b\,x^3+a}\,x^4}{-b^2x^6-20ab\,x^3+8a^2}dx\right)b-10\left(\int \frac{\sqrt{b\,x^3+a}\,x}{-b^2x^6-20ab\,x^3+8a^2}dx\right)a$$

input
```
int(x*(b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a+b*x^3),x)
```

output
```
6*sqrt(3)*int((sqrt(a + b*x**3)*x)/(8*a**2 - 20*a*b*x**3 - b**2*x**6),x)*a
 - int((sqrt(a + b*x**3)*x**4)/(8*a**2 - 20*a*b*x**3 - b**2*x**6),x)*b - 1
0*int((sqrt(a + b*x**3)*x)/(8*a**2 - 20*a*b*x**3 - b**2*x**6),x)*a
```

3.516 $\int \frac{x\sqrt{a-bx^3}}{2(5+3\sqrt{3})a-bx^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 757

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$$
$$=\frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}+\frac{3^{3/4}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[6]{a}\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}+\frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$
$$+\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
2*(-b*x^3+a)^(1/2)/b^(2/3)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)+1/4*3^(3/4)*a^(
1/6)*arctan(1/2*3^(1/4)*(1+3^(1/2))*a^(1/6)*(a^(1/3)-b^(1/3)*x)*2^(1/2)/(-
b*x^3+a)^(1/2))*2^(1/2)/b^(2/3)+1/6*a^(1/6)*arctan(1/6*(1-3^(1/2))*(-b*x^3
+a)^(1/2)*2^(1/2)*3^(1/4)/a^(1/2))*2^(1/2)*3^(3/4)/b^(2/3)+1/4*3^(1/4)*a^(
1/6)*arctanh(1/2*3^(1/4)*(1-3^(1/2))*a^(1/6)*(a^(1/3)-b^(1/3)*x)*2^(1/2)/(
-b*x^3+a)^(1/2))*2^(1/2)/b^(2/3)+1/2*3^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a
^(1/6)*((1+3^(1/2))*a^(1/3)+2*b^(1/3)*x)*2^(1/2)/(-b*x^3+a)^(1/2))*2^(1/2)
/b^(2/3)-3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/3)*(a^(1/3)-b^(1/3)*x)*((a
^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(
1/2)*EllipticE(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3
)*x),I*3^(1/2)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1
/3)-b^(1/3)*x)^2)^(1/2)/(-b*x^3+a)^(1/2)+2/3*2^(1/2)*a^(1/3)*(a^(1/3)-b^(1
/3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)-b^(1/
3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1
/3)-b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(2/3)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)
/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(-b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.11

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx=\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20+12\sqrt{3}\right)\sqrt{a-bx^3}}$$

input
```
Integrate[(x*Sqrt[a - b*x^3])/(2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

output
```
(x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(1
0*a + 6*Sqrt[3]*a)])/((20 + 12*Sqrt[3])*Sqrt[a - b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.46 (sec) , antiderivative size = 834, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {984, 832, 759, 989, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}dx$$

$$\Big\downarrow \text{ 984}$$

$$\int \frac{x}{\sqrt{a-bx^3}}dx-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}dx$$

$$\Big\downarrow \text{ 832}$$

$$\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}dx$$

$$\Big\downarrow \text{ 759}$$

$$-\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{ 989}$$

$$-\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-$$

$$\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$3\left(3+2\sqrt{3}\right)a\left(-\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2-}{}\right.$$

$$\Big\downarrow \text{2416}$$

$$-3\left(3+2\sqrt{3}\right)a\left(-\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2-}{}\right.$$

$$\frac{\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}}{\sqrt[3]{b}}-$$

$$\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

input
```
Int[(x*Sqrt[a - b*x^3])/(2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

output
```
-3*(3 + 2*Sqrt[3])*a*(-1/2*((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^
(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(Sqrt[2]*3^(3/4)*
a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a - b*x^3])/(
Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqr
t[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[
2]*Sqrt[a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])
*ArcTanh[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*
Sqrt[a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))) - ((-2*Sqrt[a - b*
x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1/4)*Sqrt[2 - Sq
rt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2
+ Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 989
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.19 (sec) , antiderivative size = 924, normalized size of antiderivative = 1.22

method	result	size
elliptic	Expression too large to display	924
default	Expression too large to display	942

input
```
int(x*(-b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a-b*x^3),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^
3+a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE
(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(
1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)
*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(1/2)*sum(1/_alpha*(3+
2*3^(1/2))*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a*b^2)^(1/3)+(a*b^
2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(-3*(a*b^2)^(1/3)
-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(a*b^2)^(1/
3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*(3*I*(a*b^2)^(1/3
)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^2)^(2/3)*3^(1/2)-2*(a*b
^2)^(1/3)*_alpha*3^(1/2)*b-6*I*(a*b^2)^(1/3)*_alpha*b-6*_alpha^2*b^2-2*(a*
b^2)^(2/3)*3^(1/2)+6*I*(a*b^2)^(2/3)+3*(a*b^2)^(1/3)*_alpha*b+3*(a*b^2)^(2
/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*
b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*(a*b^2)^(1/3)*_a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4855 vs. $2(543) = 1086$.

Time = 4.25 (sec) , antiderivative size = 4855, normalized size of antiderivative = 6.41

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(-b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a-b*x^3),x, algorithm="frica
s")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = -\int \frac{x\sqrt{a-bx^3}}{-6\sqrt{3}a-10a+bx^3}\,dx$$

input
```
integrate(x*(-b*x**3+a)**(1/2)/(2*(5+3*3**(1/2))*a-b*x**3),x)
```

output
```
-Integral(x*sqrt(a - b*x**3)/(-6*sqrt(3)*a - 10*a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \int -\frac{\sqrt{-bx^3+a}x}{bx^3-2\,a\left(3\sqrt{3}+5\right)}\,dx$$

input
```
integrate(x*(-b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a-b*x^3),x, algorithm="maxim
a")
```

output
```
-integrate(sqrt(-b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

Giac [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \int -\frac{\sqrt{-bx^3+a}x}{bx^3-2\,a\left(3\sqrt{3}+5\right)}\,dx$$

input
```
integrate(x*(-b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a-b*x^3),x, algorithm="giac"
)
```

output
```
integrate(-sqrt(-b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = -\int \frac{x\,\sqrt{a-b\,x^3}}{b\,x^3-2\,a\,\left(3\,\sqrt{3}+5\right)}\,dx$$

input
```
int(-(x*(a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)),x)
```

output
```
-int((x*(a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)), x)
```

Reduce [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = 6\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{-b^2x^6+20ab\,x^3+8a^2}dx\right)a + \left(\int \frac{\sqrt{-b\,x^3+a}\,x^4}{-b^2x^6+20ab\,x^3+8a^2}dx\right)b - 10\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{-b^2x^6+20ab\,x^3+8a^2}dx\right)a$$

input
```
int(x*(-b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a-b*x^3),x)
```

output
```
6*sqrt(3)*int((sqrt(a - b*x**3)*x)/(8*a**2 + 20*a*b*x**3 - b**2*x**6),x)*a
 + int((sqrt(a - b*x**3)*x**4)/(8*a**2 + 20*a*b*x**3 - b**2*x**6),x)*b - 1
0*int((sqrt(a - b*x**3)*x)/(8*a**2 + 20*a*b*x**3 - b**2*x**6),x)*a
```

3.517 $\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 774

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$$

$$= -\frac{2\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$+\frac{3^{3/4}\sqrt[6]{a}\,\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}} - \frac{\sqrt[6]{a}\,\text{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$
$$-\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
-2*(b*x^3-a)^(1/2)/b^(2/3)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)+1/4*3^(1/4)*a^(
1/6)*arctan(1/2*3^(1/4)*(1-3^(1/2))*a^(1/6)*(a^(1/3)-b^(1/3)*x)*2^(1/2)/(b
*x^3-a)^(1/2))*2^(1/2)/b^(2/3)+1/2*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1
/6)*((1+3^(1/2))*a^(1/3)+2*b^(1/3)*x)*2^(1/2)/(b*x^3-a)^(1/2))*2^(1/2)/b^(
2/3)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*(1+3^(1/2))*a^(1/6)*(a^(1/3)-
b^(1/3)*x)*2^(1/2)/(b*x^3-a)^(1/2))*2^(1/2)/b^(2/3)-1/6*a^(1/6)*arctanh(1/
6*(1-3^(1/2))*(b*x^3-a)^(1/2)*2^(1/2)*3^(1/4)/a^(1/2))*2^(1/2)*3^(3/4)/b^(
2/3)+3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(1/3)*(a^(1/3)-b^(1/3)*x)*((a^(2/
3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)
*EllipticE(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)
,2*I-I*3^(1/2))/b^(2/3)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)
-b^(1/3)*x)^2)^(1/2)/(b*x^3-a)^(1/2)-2/3*2^(1/2)*a^(1/3)*(a^(1/3)-b^(1/3)*
x)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x
)^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-
b^(1/3)*x),2*I-I*3^(1/2))*3^(3/4)/b^(2/3)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/((
1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(b*x^3-a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.11

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=-\frac{x^2\sqrt{-a+bx^3}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)}{4\left(5+3\sqrt{3}\right)a\sqrt{1-\frac{bx^3}{a}}}$$

input
```
Integrate[(x*Sqrt[-a + b*x^3])/(-2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

output
```
-1/4*(x^2*Sqrt[-a + b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/
(10*a + 6*Sqrt[3]*a)])/((5 + 3*Sqrt[3])*a*Sqrt[1 - (b*x^3)/a])
```

Rubi [A] (warning: unable to verify)

Time = 1.42 (sec) , antiderivative size = 852, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {984, 25, 833, 760, 990, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{bx^3-a}}{bx^3-2\left(5+3\sqrt{3}\right)a}dx$$

$$\Big\downarrow \text{ 984}$$

$$\int \frac{x}{\sqrt{bx^3-a}}dx+3\left(3+2\sqrt{3}\right)a\int -\frac{x}{\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{x}{\sqrt{bx^3-a}}dx-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{ 833}$$

$$\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}-\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{ 760}$$

$$-\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\sqrt{bx^3-a}}dx-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \text{ 990}$$

$$-\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$3\left(3+2\sqrt{3}\right)a\left(-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}(1-\sqrt{3})\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}\right.}{\sqrt{2}\sqrt{bx^3-a}}\right.}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}\right.$$

$\downarrow$ 2418

$$-3\left(3+2\sqrt{3}\right)a\left(-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}(1-\sqrt{3})\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(2\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\right.}{\sqrt{2}\sqrt{bx^3-a}}\right.}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}\right.$$

$$\frac{\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}}{\sqrt[3]{b}}-$$

$$\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

input
```
Int[(x*Sqrt[-a + b*x^3])/(-2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

output
```
-3*(3 + 2*Sqrt[3])*a*(-1/6*((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^
(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(Sqrt[2]*3^(1/4)
*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*
a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^
(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^
(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6
)*b^(2/3)) + ((2 - Sqrt[3])*ArcTanh[((1 - Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt
[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3))) - ((2*Sqrt[-a
+ b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/4)*Sqrt[2
+ Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((
1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7
 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[
3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]))/b^(1/3) - (2*Sqrt[2 - Sqrt
[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[A
rcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)
*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)
*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 990
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.03 (sec) , antiderivative size = 926, normalized size of antiderivative = 1.20

method	result	size
elliptic	Expression too large to display	926
default	Expression too large to display	944

input
```
int(x*(b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a+b*x^3),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3
-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(
1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1
/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*
3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*
b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(1/2)*sum(1/_alpha*(3+2
*3^(1/2))*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2
)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(-3*(a*b^2)^(1/3)-
I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(a*b^2)^(1/3
)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*(3*I*(a*b^2)^(1/3)*
_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^2)^(2/3)*3^(1/2)-2*(a*b^2
)^(1/3)*_alpha*3^(1/2)*b-6*I*(a*b^2)^(1/3)*_alpha*b-6*_alpha^2*b^2-2*(a*b^
2)^(2/3)*3^(1/2)+6*I*(a*b^2)^(2/3)+3*(a*b^2)^(1/3)*_alpha*b+3*(a*b^2)^(2/3
))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^
2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*(a*b^2)^(1/3)*_alp...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4963 vs. $2(541) = 1082$.

Time = 4.25 (sec) , antiderivative size = 4963, normalized size of antiderivative = 6.41

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a+b*x^3),x, algorithm="frica
s")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = \int \frac{x\sqrt{-a+bx^3}}{-6\sqrt{3}a-10a+bx^3}\,dx$$

input
```
integrate(x*(b*x**3-a)**(1/2)/(-2*(5+3*3**(1/2))*a+b*x**3),x)
```

output
```
Integral(x*sqrt(-a + b*x**3)/(-6*sqrt(3)*a - 10*a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = \int \frac{\sqrt{bx^3-a}\,x}{bx^3-2\,a\left(3\sqrt{3}+5\right)}\,dx$$

input
```
integrate(x*(b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a+b*x^3),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

Giac [F]

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = \int \frac{\sqrt{bx^3-a}\,x}{bx^3-2\,a\left(3\sqrt{3}+5\right)}\,dx$$

input
```
integrate(x*(b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a+b*x^3),x, algorithm="giac"
)
```

output
```
integrate(sqrt(b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = \int \frac{x\,\sqrt{b\,x^3-a}}{b\,x^3-2\,a\,\left(3\,\sqrt{3}+5\right)}\,dx$$

input
```
int((x*(b*x^3 - a)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)),x)
```

output
```
int((x*(b*x^3 - a)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)), x)
```

Reduce [F]

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = -6\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}\,x}{-b^2x^6+20ab\,x^3+8a^2}dx\right)a - \left(\int \frac{\sqrt{b\,x^3-a}\,x^4}{-b^2x^6+20ab\,x^3+8a^2}dx\right)b + 10\left(\int \frac{\sqrt{b\,x^3-a}\,x}{-b^2x^6+20ab\,x^3+8a^2}dx\right)a$$

input
```
int(x*(b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a+b*x^3),x)
```

output
```
 - 6*sqrt(3)*int((sqrt( - a + b*x**3)*x)/(8*a**2 + 20*a*b*x**3 - b**2*x**6
),x)*a - int((sqrt( - a + b*x**3)*x**4)/(8*a**2 + 20*a*b*x**3 - b**2*x**6)
,x)*b + 10*int((sqrt( - a + b*x**3)*x)/(8*a**2 + 20*a*b*x**3 - b**2*x**6),
x)*a
```

3.518 $\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 768

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$$

$$= -\frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{\sqrt{2}b^{2/3}}$$

$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$

$$+\frac{3^{3/4}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}-\frac{\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$

$$+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$-\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
-2*(-b*x^3-a)^(1/2)/b^(2/3)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)+1/2*3^(1/4)*a^
(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*((1+3^(1/2))*a^(1/3)-2*b^(1/3)*x)*2^(1/2)
/(-b*x^3-a)^(1/2))*2^(1/2)/b^(2/3)+1/4*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*
(1-3^(1/2))*a^(1/6)*(a^(1/3)+b^(1/3)*x)*2^(1/2)/(-b*x^3-a)^(1/2))*2^(1/2)/
b^(2/3)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*(1+3^(1/2))*a^(1/6)*(a^(1/
3)+b^(1/3)*x)*2^(1/2)/(-b*x^3-a)^(1/2))*2^(1/2)/b^(2/3)-1/6*a^(1/6)*arctan
h(1/6*(1-3^(1/2))*(-b*x^3-a)^(1/2)*2^(1/2)*3^(1/4)/a^(1/2))*2^(1/2)*3^(3/4
)/b^(2/3)+3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(1/3)*(a^(1/3)+b^(1/3)*x)*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^
(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/
3)*x),2*I-I*3^(1/2))/b^(2/3)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^
(1/3)+b^(1/3)*x)^2)^(1/2)/(-b*x^3-a)^(1/2)-2/3*2^(1/2)*a^(1/3)*(a^(1/3)+b^
(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)+b^(
1/3)*x)^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^
(1/3)+b^(1/3)*x),2*I-I*3^(1/2))*3^(3/4)/b^(2/3)/(-a^(1/3)*(a^(1/3)+b^(1/3)
*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(-b*x^3-a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.12

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx=-\frac{x^2\sqrt{-a-bx^3}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)}{4\left(5+3\sqrt{3}\right)a\sqrt{\frac{a+bx^3}{a}}}$$

input
```
Integrate[(x*Sqrt[-a - b*x^3])/(-2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

output
```
-1/4*(x^2*Sqrt[-a - b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*
x^3)/(10*a + 6*Sqrt[3]*a))])/((5 + 3*Sqrt[3])*a*Sqrt[(a + b*x^3)/a])
```

Rubi [A] (warning: unable to verify)

Time = 1.45 (sec) , antiderivative size = 844, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {984, 25, 833, 760, 990, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}dx$$

$$\Big\downarrow \text{984}$$

$$\int \frac{x}{\sqrt{-bx^3-a}}dx+3\left(3+2\sqrt{3}\right)a\int -\frac{x}{\sqrt{-bx^3-a}\left(bx^3+2\left(5+3\sqrt{3}\right)a\right)}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{x}{\sqrt{-bx^3-a}}dx-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\sqrt{-bx^3-a}\left(bx^3+2\left(5+3\sqrt{3}\right)a\right)}dx$$

$$\Big\downarrow \text{833}$$

$$-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}+\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\sqrt{-bx^3-a}\left(bx^3+2\left(5+3\sqrt{3}\right)a\right)}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\sqrt{-bx^3-a}\left(bx^3+2\left(5+3\sqrt{3}\right)a\right)}dx-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \text{990}$$

$$\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}} - \frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$3\left(3+2\sqrt{3}\right)a\left(-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{\sqrt{2}\sqrt{-a-bx^3}}\right.}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}\right.$$

$\downarrow$ 2418

$$-3\left(3+2\sqrt{3}\right)a\left(-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{b}x+\right.}{\sqrt{2}\sqrt{-bx^3-a}}\right.}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}\right.$$

$$\frac{\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{-bx^3-a}} - \frac{2\sqrt{-bx^3-a}}{\sqrt[3]{b}\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)}}{\sqrt[3]{b}} -$$

$$\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{-bx^3-a}}$$

input
```
Int[(x*Sqrt[-a - b*x^3])/(-2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

output
```
-3*(3 + 2*Sqrt[3])*a*(-1/3*((2 - Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sq
rt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(Sqrt[2]*3^(1/
4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)
*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^
(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^
(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6
)*b^(2/3)) + ((2 - Sqrt[3])*ArcTanh[((1 - Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt
[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3))) + ((-2*Sqrt[-a
 - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(1/4)*Sqrt[2
 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]))/b^(1/3) - (2*Sqrt[2 - Sqr
t[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[
ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 990
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.00 (sec) , antiderivative size = 983, normalized size of antiderivative = 1.28

method	result	size
elliptic	Expression too large to display	983
default	Expression too large to display	1001

input
```
int(x*(-b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a-b*x^3),x,method=_RETURNVERBOSE)
```

output

```
-2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+1/9*I/b^3*2^
(1/2)*sum(1/_alpha*(3+2*3^(1/2))*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2
)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2
)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*
x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(-b
*x^3-a)^(1/2)*(-3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)
+3*I*3^(1/2)*(-a*b^2)^(2/3)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*(-a*b^2)^(1/3)*_
alpha*3^(1/2)*b-6*_alpha^2*b^2-6*I*(-a*b^2)^(2/3)-2*3^(1/2)*(-a*b^2)^(2/3)
+3*(-a*b^2)^(1/3)*_alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4981 vs. $2(546) = 1092$.

Time = 4.21 (sec) , antiderivative size = 4981, normalized size of antiderivative = 6.49

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \text{Too large to display}$$

input

```
integrate(x*(-b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a-b*x^3),x, algorithm="fric
as")
```

output

```
Too large to include
```

Sympy [F]

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = -\int \frac{x\sqrt{-a-bx^3}}{10a+6\sqrt{3}a+bx^3}\,dx$$

input
```
integrate(x*(-b*x**3-a)**(1/2)/(-2*(5+3*3**(1/2))*a-b*x**3),x)
```

output
```
-Integral(x*sqrt(-a - b*x**3)/(10*a + 6*sqrt(3)*a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \int -\frac{\sqrt{-bx^3-a}\,x}{bx^3+2\,a\left(3\sqrt{3}+5\right)}\,dx$$

input
```
integrate(x*(-b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a-b*x^3),x, algorithm="maxi
ma")
```

output
```
-integrate(sqrt(-b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

Giac [F]

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \int -\frac{\sqrt{-bx^3-a}\,x}{bx^3+2\,a\left(3\sqrt{3}+5\right)}\,dx$$

input
```
integrate(x*(-b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a-b*x^3),x, algorithm="giac
")
```

output
```
integrate(-sqrt(-b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \int -\frac{x\sqrt{-b\,x^3-a}}{b\,x^3+2\,a\,\left(3\,\sqrt{3}+5\right)}\,dx$$

input
```
int(-(x*(- a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)),x)
```

output
```
int(-(x*(- a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)), x)
```

Reduce [F]

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = i\left(-6\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{-b^2x^6-20ab\,x^3+8a^2}dx\right)a + \left(\int \frac{\sqrt{b\,x^3+a}\,x^4}{-b^2x^6-20ab\,x^3+8a^2}dx\right)b + 10\left(\int \frac{\sqrt{b\,x^3+a}\,x}{-b^2x^6-20ab\,x^3+8a^2}dx\right)a\right)$$

input
```
int(x*(-b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a-b*x^3),x)
```

output
```
i*( - 6*sqrt(3)*int((sqrt(a + b*x**3)*x)/(8*a**2 - 20*a*b*x**3 - b**2*x**6
),x)*a + int((sqrt(a + b*x**3)*x**4)/(8*a**2 - 20*a*b*x**3 - b**2*x**6),x)
*b + 10*int((sqrt(a + b*x**3)*x)/(8*a**2 - 20*a*b*x**3 - b**2*x**6),x)*a)
```

3.519 $\int \frac{x\sqrt{a+bx^3}}{2(5-3\sqrt{3})a+bx^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 738

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$$
$$= \frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$- \frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+ \frac{3^{3/4}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$
$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2*(b*x^3+a)^(1/2)/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)-1/2*3^(1/4)*a^(1
/6)*arctan(1/2*3^(1/4)*a^(1/6)*((1-3^(1/2))*a^(1/3)-2*b^(1/3)*x)*2^(1/2)/(
b*x^3+a)^(1/2))*2^(1/2)/b^(2/3)-1/4*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*(1+
3^(1/2))*a^(1/6)*(a^(1/3)+b^(1/3)*x)*2^(1/2)/(b*x^3+a)^(1/2))*2^(1/2)/b^(2
/3)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*(1-3^(1/2))*a^(1/6)*(a^(1/3)+b
^(1/3)*x)*2^(1/2)/(b*x^3+a)^(1/2))*2^(1/2)/b^(2/3)+1/6*a^(1/6)*arctanh(1/6
*(1+3^(1/2))*(b*x^3+a)^(1/2)*2^(1/2)*3^(1/4)/a^(1/2))*2^(1/2)*3^(3/4)/b^(2
/3)-3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*
EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),
I*3^(1/2)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b
^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+2/3*2^(1/2)*a^(1/3)*(a^(1/3)+b^(1/3)*x)
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^
2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^
(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3
^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.11

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20-12\sqrt{3}\right)\sqrt{a+bx^3}}$$

input
```
Integrate[(x*Sqrt[a + b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

output
```
(x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x^
3)/(10*a - 6*Sqrt[3]*a))])/((20 - 12*Sqrt[3])*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.36 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {984, 832, 759, 989, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}dx$$

$$\Big\downarrow \text{ 984}$$

$$\int \frac{x}{\sqrt{bx^3+a}}dx-3\left(3-2\sqrt{3}\right)a\int \frac{x}{\sqrt{bx^3+a}\left(bx^3+2\left(5-3\sqrt{3}\right)a\right)}dx$$

$$\Big\downarrow \text{ 832}$$

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-3\left(3-2\sqrt{3}\right)a\int \frac{x}{\sqrt{bx^3+a}\left(bx^3+2\left(5-3\sqrt{3}\right)a\right)}dx$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-3\left(3-2\sqrt{3}\right)a\int \frac{x}{\sqrt{bx^3+a}\left(bx^3+2\left(5-3\sqrt{3}\right)a\right)}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 989}$$

$$\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}} - \frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$3\left(3-2\sqrt{3}\right)a\left(-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\ldots\right.}{\sqrt{2}\sqrt{a+bx^3}}\right.}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}\right.$$

$$\Big\downarrow \text{2416}$$

$$-3\left(3-2\sqrt{3}\right)a\left(-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3+a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{b}x+\ldots\right.}{\sqrt{2}\sqrt{bx^3+a}}\right.}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}\right.$$

$$\frac{\frac{2\sqrt{bx^3+a}}{\sqrt[3]{b}\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{bx^3+a}}}{\sqrt[3]{b}} - \frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{bx^3+a}}$$

input

```
Int[(x*Sqrt[a + b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

output
```
-3*(3 - 2*Sqrt[3])*a*(-1/3*((2 + Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sq
rt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(Sqrt[2]*3^(1/4
)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*
(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5
/6)*b^(2/3)) + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1
/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b
^(2/3)) + ((2 + Sqrt[3])*ArcTanh[((1 + Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*
3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3))) + ((2*Sqrt[a + b*x
^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqr
t[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 +
 Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 989
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.62 (sec) , antiderivative size = 977, normalized size of antiderivative = 1.32

method	result	size
elliptic	Expression too large to display	977
default	Expression too large to display	995

input
```
int(x*(b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1
/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+1/9*I/b^3*2^(
1/2)*sum(1/_alpha*(-3+2*3^(1/2))*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2
)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2
)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*
x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*(3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3
*I*3^(1/2)*(-a*b^2)^(2/3)-2*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+6*I*(-a*b^2)^(
1/3)*_alpha*b+6*_alpha^2*b^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)-3
*(-a*b^2)^(1/3)*_alpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4931 vs. $2(519) = 1038$.

Time = 4.36 (sec) , antiderivative size = 4931, normalized size of antiderivative = 6.68

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{x\sqrt{a+bx^3}}{-6\sqrt{3}a+10a+bx^3}\,dx$$

input
```
integrate(x*(b*x**3+a)**(1/2)/(2*(5-3*3**(1/2))*a+b*x**3),x)
```

output
```
Integral(x*sqrt(a + b*x**3)/(-6*sqrt(3)*a + 10*a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{\sqrt{bx^3+a}x}{bx^3-2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

Giac [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{\sqrt{bx^3+a}x}{bx^3-2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{x\,\sqrt{b\,x^3+a}}{b\,x^3-2\,a\,\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
int((x*(a + b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)),x)
```

output
```
int((x*(a + b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)), x)
```

Reduce [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=-6\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{-b^2x^6-20ab\,x^3+8a^2}dx\right)a-\left(\int \frac{\sqrt{b\,x^3+a}\,x^4}{-b^2x^6-20ab\,x^3+8a^2}dx\right)b-10\left(\int \frac{\sqrt{b\,x^3+a}\,x}{-b^2x^6-20ab\,x^3+8a^2}dx\right)a$$

input
```
int(x*(b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x)
```

output
```
- 6*sqrt(3)*int((sqrt(a + b*x**3)*x)/(8*a**2 - 20*a*b*x**3 - b**2*x**6),x
)*a - int((sqrt(a + b*x**3)*x**4)/(8*a**2 - 20*a*b*x**3 - b**2*x**6),x)*b
- 10*int((sqrt(a + b*x**3)*x)/(8*a**2 - 20*a*b*x**3 - b**2*x**6),x)*a
```

3.520 $\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 758

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$$
$$= \frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$- \frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$+ \frac{3^{3/4}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$
$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$
$$+ \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
2*(-b*x^3+a)^(1/2)/b^(2/3)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)-1/4*3^(1/4)*a^(
1/6)*arctan(1/2*3^(1/4)*(1+3^(1/2))*a^(1/6)*(a^(1/3)-b^(1/3)*x)*2^(1/2)/(-
b*x^3+a)^(1/2))*2^(1/2)/b^(2/3)-1/2*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(
1/6)*((1-3^(1/2))*a^(1/3)+2*b^(1/3)*x)*2^(1/2)/(-b*x^3+a)^(1/2))*2^(1/2)/b
^(2/3)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*(1-3^(1/2))*a^(1/6)*(a^(1/3
)-b^(1/3)*x)*2^(1/2)/(-b*x^3+a)^(1/2))*2^(1/2)/b^(2/3)+1/6*a^(1/6)*arctanh
(1/6*(1+3^(1/2))*(-b*x^3+a)^(1/2)*2^(1/2)*3^(1/4)/a^(1/2))*2^(1/2)*3^(3/4)
/b^(2/3)-3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/3)*(a^(1/3)-b^(1/3)*x)*((a
^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(
1/2)*EllipticE(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3
)*x),I*3^(1/2)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1
/3)-b^(1/3)*x)^2)^(1/2)/(-b*x^3+a)^(1/2)+2/3*2^(1/2)*a^(1/3)*(a^(1/3)-b^(1
/3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)-b^(1/
3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1
/3)-b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(2/3)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)
/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(-b*x^3+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.11

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20-12\sqrt{3}\right)\sqrt{a-bx^3}}$$

input
```
Integrate[(x*Sqrt[a - b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

output
```
(x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(1
0*a - 6*Sqrt[3]*a)])/((20 - 12*Sqrt[3])*Sqrt[a - b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.43 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {984, 832, 759, 989, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}dx$$

$$\Big\downarrow \text{984}$$

$$\int \frac{x}{\sqrt{a-bx^3}}dx-3\left(3-2\sqrt{3}\right)a\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}dx$$

$$\Big\downarrow \text{832}$$

$$\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-3\left(3-2\sqrt{3}\right)a\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}dx$$

$$\Big\downarrow \text{759}$$

$$-\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-3\left(3-2\sqrt{3}\right)a\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{989}$$

$$-\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$3\left(3-2\sqrt{3}\right)a\left(-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}\right.}{\sqrt{2}\sqrt{a-bx^3}}\right.}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}\right.$$

$$\Big\downarrow \text{2416}$$

$$-3\left(3-2\sqrt{3}\right)a\left(-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(2\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\right.}{\sqrt{2}\sqrt{a-bx^3}}\right.}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}\right.$$

$$\frac{\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}}{\sqrt[3]{b}}-$$

$$\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

input
```
Int[(x*Sqrt[a - b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

output
```
-3*(3 - 2*Sqrt[3])*a*(-1/6*((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^
(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(Sqrt[2]*3^(1/4)*
a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a
^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5
/6)*b^(2/3)) + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1
/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b
^(2/3)) + ((2 + Sqrt[3])*ArcTanh[((1 + Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*
3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3))) - ((-2*Sqrt[a - b*
x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1/4)*Sqrt[2 - Sq
rt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2
+ Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 989
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.70 (sec) , antiderivative size = 924, normalized size of antiderivative = 1.22

method	result	size
elliptic	Expression too large to display	924
default	Expression too large to display	942

input
```
int(x*(-b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a-b*x^3),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^
3+a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE
(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(
1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)
*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(1/2)*sum(1/_alpha*(-3
+2*3^(1/2))*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a*b^2)^(1/3)+(a*b
^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(-3*(a*b^2)^(1/3
)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(a*b^2)^(1
/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*(-3*I*(a*b^2)^(1
/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*b^2)^(2/3)*3^(1/2)-6*I*
(a*b^2)^(1/3)*_alpha*b-2*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+6*_alpha^2*b^2+6*I
*(a*b^2)^(2/3)-2*(a*b^2)^(2/3)*3^(1/2)-3*(a*b^2)^(1/3)*_alpha*b-3*(a*b^2)^
(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*(a*b^2)^(1/3)*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4953 vs. $2(544) = 1088$.

Time = 4.30 (sec) , antiderivative size = 4953, normalized size of antiderivative = 6.53

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(-b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a-b*x^3),x, algorithm="frica
s")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = -\int \frac{x\sqrt{a-bx^3}}{-10a+6\sqrt{3}a+bx^3}\,dx$$

input
```
integrate(x*(-b*x**3+a)**(1/2)/(2*(5-3*3**(1/2))*a-b*x**3),x)
```

output
```
-Integral(x*sqrt(a - b*x**3)/(-10*a + 6*sqrt(3)*a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = \int -\frac{\sqrt{-bx^3+a}x}{bx^3+2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
integrate(x*(-b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a-b*x^3),x, algorithm="maxim
a")
```

output
```
-integrate(sqrt(-b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

Giac [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = \int -\frac{\sqrt{-bx^3+a}x}{bx^3+2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
integrate(x*(-b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a-b*x^3),x, algorithm="giac"
)
```

output
```
integrate(-sqrt(-b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=\int -\frac{x\,\sqrt{a-b\,x^3}}{b\,x^3+2\,a\,\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
int(-(x*(a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)),x)
```

output
```
int(-(x*(a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)), x)
```

Reduce [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=-6\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{-b^2x^6+20ab\,x^3+8a^2}dx\right)a+\left(\int \frac{\sqrt{-b\,x^3+a}\,x^4}{-b^2x^6+20ab\,x^3+8a^2}dx\right)b-10\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{-b^2x^6+20ab\,x^3+8a^2}dx\right)a$$

input
```
int(x*(-b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a-b*x^3),x)
```

output
```
- 6*sqrt(3)*int((sqrt(a - b*x**3)*x)/(8*a**2 + 20*a*b*x**3 - b**2*x**6),x
)*a + int((sqrt(a - b*x**3)*x**4)/(8*a**2 + 20*a*b*x**3 - b**2*x**6),x)*b
- 10*int((sqrt(a - b*x**3)*x)/(8*a**2 + 20*a*b*x**3 - b**2*x**6),x)*a
```

3.521 $\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 774

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$$

$$= \frac{2\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{3^{3/4}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+ \frac{\sqrt[6]{a}\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+ \frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$- \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$
$$+ \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
2*(b*x^3-a)^(1/2)/b^(2/3)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)-1/4*3^(3/4)*a^(1
/6)*arctan(1/2*3^(1/4)*(1-3^(1/2))*a^(1/6)*(a^(1/3)-b^(1/3)*x)*2^(1/2)/(b*
x^3-a)^(1/2))*2^(1/2)/b^(2/3)+1/6*a^(1/6)*arctan(1/6*(1+3^(1/2))*(b*x^3-a)
^(1/2)*2^(1/2)*3^(1/4)/a^(1/2))*2^(1/2)*3^(3/4)/b^(2/3)+1/4*3^(1/4)*a^(1/6
)*arctanh(1/2*3^(1/4)*(1+3^(1/2))*a^(1/6)*(a^(1/3)-b^(1/3)*x)*2^(1/2)/(b*x
^3-a)^(1/2))*2^(1/2)/b^(2/3)+1/2*3^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/
6)*((1-3^(1/2))*a^(1/3)+2*b^(1/3)*x)*2^(1/2)/(b*x^3-a)^(1/2))*2^(1/2)/b^(2
/3)-3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(1/3)*(a^(1/3)-b^(1/3)*x)*((a^(2/3
)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)*
EllipticE(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x),
2*I-I*3^(1/2))/b^(2/3)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-
b^(1/3)*x)^2)^(1/2)/(b*x^3-a)^(1/2)+2/3*2^(1/2)*a^(1/3)*(a^(1/3)-b^(1/3)*x
)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)
^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b
^(1/3)*x),2*I-I*3^(1/2))*3^(3/4)/b^(2/3)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1
-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(b*x^3-a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.11

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=-\frac{x^2\sqrt{-a+bx^3}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},-\frac{bx^3}{-10a+6\sqrt{3}a}\right)}{4\left(-5+3\sqrt{3}\right)a\sqrt{\frac{a-bx^3}{a}}}$$

input
```
Integrate[(x*Sqrt[-a + b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

output
```
-1/4*(x^2*Sqrt[-a + b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, -((b*x^3
)/(-10*a + 6*Sqrt[3]*a))])/((-5 + 3*Sqrt[3])*a*Sqrt[(a - b*x^3)/a])
```

Rubi [A] (warning: unable to verify)

Time = 1.46 (sec) , antiderivative size = 843, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {984, 833, 760, 990, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{bx^3-a}}{2\left(5-3\sqrt{3}\right)a-bx^3}dx$$

$$\Big\downarrow \ 984$$

$$3\left(3-2\sqrt{3}\right)a\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{bx^3-a}}dx-\int \frac{x}{\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \ 833$$

$$-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}+\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}+3\left(3-2\sqrt{3}\right)a\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \ 760$$

$$\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}+3\left(3-2\sqrt{3}\right)a\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{bx^3-a}}dx+\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \ 990$$

$$\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}+$$

$$\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$3\left(3-2\sqrt{3}\right)a\left(\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{3}}{}\right.$$

$\downarrow$ 2418

$$3\left(3-2\sqrt{3}\right)a\left(\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{3}}{}\right.$$

$$\frac{\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}}{\sqrt[3]{b}}+$$

$$\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

input
```
Int[(x*Sqrt[-a + b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

output
```
3*(3 - 2*Sqrt[3])*a*(((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*
(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(
5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a + b*x^3])/(Sq
rt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[
3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]
*Sqrt[-a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*
ArcTanh[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*S
qrt[-a + b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))) + ((2*Sqrt[-a + b*
x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/4)*Sqrt[2 + Sq
rt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 +
Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4
*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*
a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]))/b^(1/3) + (2*Sqrt[2 - Sqrt[3]]
*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)]
, -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))
/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 990
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.97 (sec) , antiderivative size = 926, normalized size of antiderivative = 1.20

method	result	size
elliptic	Expression too large to display	926
default	Expression too large to display	944

input
```
int(x*(b*x^3-a)^(1/2)/(2*(5-3*3^(1/2))*a-b*x^3),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2
/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^
3-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE
(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(
1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)
*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+1/9*I/b^3*2^(1/2)*sum(1/_alpha*(-3
+2*3^(1/2))*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a*b^2)^(1/3)+(a*b
^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(-3*(a*b^2)^(1/3
)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(a*b^2)^(1
/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*(-3*I*(a*b^2)^(1/
3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*b^2)^(2/3)*3^(1/2)-6*I*(
a*b^2)^(1/3)*_alpha*b-2*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+6*_alpha^2*b^2+6*I*
(a*b^2)^(2/3)-2*(a*b^2)^(2/3)*3^(1/2)-3*(a*b^2)^(1/3)*_alpha*b-3*(a*b^2)^(
2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*(a*b^2)^(1/3)*_...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4867 vs. $2(542) = 1084$.

Time = 4.35 (sec) , antiderivative size = 4867, normalized size of antiderivative = 6.29

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x^3-a)^(1/2)/(2*(5-3*3^(1/2))*a-b*x^3),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = -\int \frac{x\sqrt{-a+bx^3}}{-10a+6\sqrt{3}a+bx^3}\,dx$$

input
```
integrate(x*(b*x**3-a)**(1/2)/(2*(5-3*3**(1/2))*a-b*x**3),x)
```

output
```
-Integral(x*sqrt(-a + b*x**3)/(-10*a + 6*sqrt(3)*a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = \int -\frac{\sqrt{bx^3-a}\,x}{bx^3+2\,a\left(3\sqrt{3}-5\right)}\,dx$$

input
```
integrate(x*(b*x^3-a)^(1/2)/(2*(5-3*3^(1/2))*a-b*x^3),x, algorithm="maxima
")
```

output
```
-integrate(sqrt(b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

Giac [F]

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = \int -\frac{\sqrt{bx^3-a}\,x}{bx^3+2\,a\left(3\sqrt{3}-5\right)}\,dx$$

input
```
integrate(x*(b*x^3-a)^(1/2)/(2*(5-3*3^(1/2))*a-b*x^3),x, algorithm="giac")
```

output
```
integrate(-sqrt(b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=\int -\frac{x\,\sqrt{b\,x^3-a}}{b\,x^3+2\,a\,\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
int(-(x*(b*x^3 - a)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)),x)
```

output
```
int(-(x*(b*x^3 - a)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)), x)
```

Reduce [F]

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=-6\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}\,x}{-b^2x^6+20ab\,x^3+8a^2}dx\right)a+\left(\int \frac{\sqrt{b\,x^3-a}\,x^4}{-b^2x^6+20ab\,x^3+8a^2}dx\right)b-10\left(\int \frac{\sqrt{b\,x^3-a}\,x}{-b^2x^6+20ab\,x^3+8a^2}dx\right)a$$

input
```
int(x*(b*x^3-a)^(1/2)/(2*(5-3*3^(1/2))*a-b*x^3),x)
```

output
```
- 6*sqrt(3)*int((sqrt( - a + b*x**3)*x)/(8*a**2 + 20*a*b*x**3 - b**2*x**6
),x)*a + int((sqrt( - a + b*x**3)*x**4)/(8*a**2 + 20*a*b*x**3 - b**2*x**6)
,x)*b - 10*int((sqrt( - a + b*x**3)*x)/(8*a**2 + 20*a*b*x**3 - b**2*x**6),
x)*a
```

3.522 $\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 768

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$$

$$= \frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{3^{3/4}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+ \frac{\sqrt[6]{a}\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\,\text{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$+ \frac{\sqrt[4]{3}\sqrt[6]{a}\,\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$- \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$
$$+ \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
2*(-b*x^3-a)^(1/2)/b^(2/3)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)-1/4*3^(3/4)*a^(
1/6)*arctan(1/2*3^(1/4)*(1-3^(1/2))*a^(1/6)*(a^(1/3)+b^(1/3)*x)*2^(1/2)/(-
b*x^3-a)^(1/2))*2^(1/2)/b^(2/3)+1/6*a^(1/6)*arctan(1/6*(1+3^(1/2))*(-b*x^3
-a)^(1/2)*2^(1/2)*3^(1/4)/a^(1/2))*2^(1/2)*3^(3/4)/b^(2/3)+1/2*3^(1/4)*a^(
1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*((1-3^(1/2))*a^(1/3)-2*b^(1/3)*x)*2^(1/2)
/(-b*x^3-a)^(1/2))*2^(1/2)/b^(2/3)+1/4*3^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)
*(1+3^(1/2))*a^(1/6)*(a^(1/3)+b^(1/3)*x)*2^(1/2)/(-b*x^3-a)^(1/2))*2^(1/2)
/b^(2/3)-3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(1/3)*(a^(1/3)+b^(1/3)*x)*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(
1/2)*EllipticE(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3
)*x),2*I-I*3^(1/2))/b^(2/3)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(
1/3)+b^(1/3)*x)^2)^(1/2)/(-b*x^3-a)^(1/2)+2/3*2^(1/2)*a^(1/3)*(a^(1/3)+b^(
1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)+b^(1
/3)*x)^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(
1/3)+b^(1/3)*x),2*I-I*3^(1/2))*3^(3/4)/b^(2/3)/(-a^(1/3)*(a^(1/3)+b^(1/3)*
x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(-b*x^3-a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.12

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=-\frac{x^2\sqrt{-a-bx^3}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{4\left(-5+3\sqrt{3}\right)a\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(x*Sqrt[-a - b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

output
```
-1/4*(x^2*Sqrt[-a - b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*
x^3)/(10*a - 6*Sqrt[3]*a))])/((-5 + 3*Sqrt[3])*a*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (warning: unable to verify)

Time = 1.43 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {984, 833, 760, 990, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}dx$$

$$\Big\downarrow \text{984}$$

$$3\left(3-2\sqrt{3}\right)a\int \frac{x}{\sqrt{-bx^3-a}\left(bx^3+2\left(5-3\sqrt{3}\right)a\right)}dx-\int \frac{x}{\sqrt{-bx^3-a}}dx$$

$$\Big\downarrow \text{833}$$

$$\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}-\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}+3\left(3-2\sqrt{3}\right)a\int \frac{x}{\sqrt{-bx^3-a}\left(bx^3+2\left(5-3\sqrt{3}\right)a\right)}dx$$

$$\Big\downarrow \text{760}$$

$$-\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}+3\left(3-2\sqrt{3}\right)a\int \frac{x}{\sqrt{-bx^3-a}\left(bx^3+2\left(5-3\sqrt{3}\right)a\right)}dx+\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \text{990}$$

$$-\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}+$$

$$\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$3\left(3-2\sqrt{3}\right)a\left(\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{3})\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{}}{}\right.$$

$$\Big\downarrow \text{2418}$$

$$3\left(3-2\sqrt{3}\right)a\left(\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt{2}\sqrt{-bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{3})\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-bx^3-a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{}}{}\right.$$

$$\frac{\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{-bx^3-a}}-\frac{2\sqrt{-bx^3-a}}{\sqrt[3]{b}\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)}}{\sqrt[3]{b}}+$$

$$\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{-bx^3-a}}$$

input
```
Int[(x*Sqrt[-a - b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

output
```
3*(3 - 2*Sqrt[3])*a*(((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*
(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])]])/(2*Sqrt[2]*3^(3/4)*a^(
5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a - b*x^3])/(Sq
rt[2]*3^(3/4)*Sqrt[a])]])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[
3])*ArcTanh[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[
2]*Sqrt[-a - b*x^3])]])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3]
)*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*S
qrt[-a - b*x^3])]])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))) - ((-2*Sqrt[-a - b
*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(1/4)*Sqrt[2 + S
qrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 +
4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]))/b^(1/3) + (2*Sqrt[2 - Sqrt[3]
]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 990
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.02 (sec) , antiderivative size = 983, normalized size of antiderivative = 1.28

method	result	size
elliptic	Expression too large to display	983
default	Expression too large to display	1001

input
```
int(x*(-b*x^3-a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1
/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(
1/2)*sum(1/_alpha*(-3+2*3^(1/2))*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2
)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2
)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*
x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(-b
*x^3-a)^(1/2)*(3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-
3*I*3^(1/2)*(-a*b^2)^(2/3)-2*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+6*I*(-a*b^2)^
(1/3)*_alpha*b+6*_alpha^2*b^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)-
3*(-a*b^2)^(1/3)*_alpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4875 vs. 2(545) = 1090.

Time = 4.46 (sec) , antiderivative size = 4875, normalized size of antiderivative = 6.35

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(-b*x^3-a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x, algorithm="frica
s")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = \int \frac{x\sqrt{-a-bx^3}}{-6\sqrt{3}a+10a+bx^3}\,dx$$

input
```
integrate(x*(-b*x**3-a)**(1/2)/(2*(5-3*3**(1/2))*a+b*x**3),x)
```

output
```
Integral(x*sqrt(-a - b*x**3)/(-6*sqrt(3)*a + 10*a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = \int \frac{\sqrt{-bx^3-a}x}{bx^3-2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
integrate(x*(-b*x^3-a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

Giac [F]

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = \int \frac{\sqrt{-bx^3-a}x}{bx^3-2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
integrate(x*(-b*x^3-a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x, algorithm="giac"
)
```

output
```
integrate(sqrt(-b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{x\,\sqrt{-b\,x^3-a}}{b\,x^3-2\,a\,\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
int((x*(- a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)),x)
```

output
```
int((x*(- a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)), x)
```

Reduce [F]

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=i\left(-6\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{-b^2x^6-20ab\,x^3+8a^2}dx\right)a-\left(\int \frac{\sqrt{b\,x^3+a}\,x^4}{-b^2x^6-20ab\,x^3+8a^2}dx\right)b-10\left(\int \frac{\sqrt{b\,x^3+a}\,x}{-b^2x^6-20ab\,x^3+8a^2}dx\right)a\right)$$

input
```
int(x*(-b*x^3-a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x)
```

output
```
i*( - 6*sqrt(3)*int((sqrt(a + b*x**3)*x)/(8*a**2 - 20*a*b*x**3 - b**2*x**6
),x)*a - int((sqrt(a + b*x**3)*x**4)/(8*a**2 - 20*a*b*x**3 - b**2*x**6),x)
*b - 10*int((sqrt(a + b*x**3)*x)/(8*a**2 - 20*a*b*x**3 - b**2*x**6),x)*a)
```

3.523 $\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 318

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx &= -\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} \\ &\quad - \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} \\ &\quad - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} \\ &\quad - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}\end{aligned}$$

output
```
-1/12*(2-3^(1/2))*arctan(1/2*3^(1/4)*(1+3^(1/2))*a^(1/6)*(a^(1/3)+b^(1/3)*
x)*2^(1/2)/(b*x^3+a)^(1/2))*2^(1/2)*3^(1/4)/a^(5/6)/b^(2/3)-1/18*(2-3^(1/2
))*arctan(1/6*(1-3^(1/2))*(b*x^3+a)^(1/2)*2^(1/2)*3^(1/4)/a^(1/2))*2^(1/2)
*3^(1/4)/a^(5/6)/b^(2/3)-1/18*(2-3^(1/2))*arctanh(1/2*3^(1/4)*a^(1/6)*((1+
3^(1/2))*a^(1/3)-2*b^(1/3)*x)*2^(1/2)/(b*x^3+a)^(1/2))*2^(1/2)*3^(3/4)/a^(
5/6)/b^(2/3)-1/36*(2-3^(1/2))*arctanh(1/2*3^(1/4)*(1-3^(1/2))*a^(1/6)*(a^(
1/3)+b^(1/3)*x)*2^(1/2)/(b*x^3+a)^(1/2))*2^(1/2)*3^(3/4)/a^(5/6)/b^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.26

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \frac{x^2\sqrt{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20a+12\sqrt{3}a\right)\sqrt{a+bx^3}}$$

input
```
Integrate[x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

output
```
(x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3
)/(10*a + 6*Sqrt[3]*a))])/((20*a + 12*Sqrt[3]*a)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$$

$$\Big\downarrow \text{989}$$

$$-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}(1+\sqrt{3})\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2-\sqrt{3})\arctan\left(\frac{(1-\sqrt{3})\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2-\sqrt{3})\text{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left((1+\sqrt{3})\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{(2-\sqrt{3})\text{arctanh}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

input
```
Int[x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

output
```
-1/2*((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/
3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]])/(Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2
 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a
])]])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)
*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]
)/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1
 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]])/(6*
Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))
```

Defintions of rubi rules used

rule 989
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.95 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.69

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{ib\left(2x+\cdots\right.}{\cdots}}\cdots}{\cdots}\right)}{\cdots}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{ib\left(2x+\cdots\right.}{\cdots}}\cdots}{\cdots}\right)}{\cdots}$

input
```
int(x/(b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a+b*x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b
^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b
^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(
2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(
b*x^3+a)^(1/2)*(-3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2
)+3*I*3^(1/2)*(-a*b^2)^(2/3)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*(-a*b^2)^(1/3)*
_alpha*3^(1/2)*b-6*_alpha^2*b^2-6*I*(-a*b^2)^(2/3)-2*3^(1/2)*(-a*b^2)^(2/3
)+3*(-a*b^2)^(1/3)*_alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),-1/6/b*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)
*3^(1/2)*_alpha-4*I*(-a*b^2)^(1/3)*_alpha^2*b+2*I*(-a*b^2)^(2/3)*_alpha+2*
(-a*b^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-2*I*a*
b+2*3^(1/2)*a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a
+10*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5563 vs. $2(211) = 422$.

Time = 4.33 (sec) , antiderivative size = 5563, normalized size of antiderivative = 17.49

$$\int \frac{x}{\sqrt{a+bx^3}\,\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a+b*x^3),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx=\int \frac{x}{\sqrt{a+bx^3}\cdot\left(10a+6\sqrt{3}a+bx^3\right)}\,dx$$

input
```
integrate(x/(b*x**3+a)**(1/2)/(2*(5+3*3**(1/2))*a+b*x**3),x)
```

output
```
Integral(x/(sqrt(a + b*x**3)*(10*a + 6*sqrt(3)*a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx=\int \frac{x}{\left(bx^3+2\,a\left(3\,\sqrt{3}+5\right)\right)\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a+b*x^3),x, algorithm="maxima
")
```

output
```
integrate(x/((b*x^3 + 2*a*(3*sqrt(3) + 5))*sqrt(b*x^3 + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x/(b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a+b*x^3),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\sqrt{b\,x^3+a}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}+5\right)\right)}\,dx$$

input
```
int(x/((a + b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))),x)
```

output
```
int(x/((a + b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))), x)
```

Reduce [F]

$$\begin{aligned}&\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx\\&=6\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{-b^3x^9-21a\,b^2x^6-12a^2b\,x^3+8a^3}dx\right)a\\&\quad-\left(\int \frac{\sqrt{b\,x^3+a}\,x^4}{-b^3x^9-21a\,b^2x^6-12a^2b\,x^3+8a^3}dx\right)b\\&\quad-10\left(\int \frac{\sqrt{b\,x^3+a}\,x}{-b^3x^9-21a\,b^2x^6-12a^2b\,x^3+8a^3}dx\right)a\end{aligned}$$

input
```
int(x/(b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a+b*x^3),x)
```

output
```
6*sqrt(3)*int((sqrt(a + b*x**3)*x)/(8*a**3 - 12*a**2*b*x**3 - 21*a*b**2*x*
*6 - b**3*x**9),x)*a - int((sqrt(a + b*x**3)*x**4)/(8*a**3 - 12*a**2*b*x**
3 - 21*a*b**2*x**6 - b**3*x**9),x)*b - 10*int((sqrt(a + b*x**3)*x)/(8*a**3
 - 12*a**2*b*x**3 - 21*a*b**2*x**6 - b**3*x**9),x)*a
```

3.524 $\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 324

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = -\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

output
```
-1/12*(2-3^(1/2))*arctan(1/2*3^(1/4)*(1+3^(1/2))*a^(1/6)*(a^(1/3)-b^(1/3)*
x)*2^(1/2)/(-b*x^3+a)^(1/2))*2^(1/2)*3^(1/4)/a^(5/6)/b^(2/3)-1/18*(2-3^(1/
2))*arctan(1/6*(1-3^(1/2))*(-b*x^3+a)^(1/2)*2^(1/2)*3^(1/4)/a^(1/2))*2^(1/
2)*3^(1/4)/a^(5/6)/b^(2/3)-1/36*(2-3^(1/2))*arctanh(1/2*3^(1/4)*(1-3^(1/2)
)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*2^(1/2)/(-b*x^3+a)^(1/2))*2^(1/2)*3^(3/4)/a^
(5/6)/b^(2/3)-1/18*(2-3^(1/2))*arctanh(1/2*3^(1/4)*a^(1/6)*((1+3^(1/2))*a^
(1/3)+2*b^(1/3)*x)*2^(1/2)/(-b*x^3+a)^(1/2))*2^(1/2)*3^(3/4)/a^(5/6)/b^(2/
3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.26

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx=\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20a+12\sqrt{3}a\right)\sqrt{a-bx^3}}$$

input
```
Integrate[x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

output
```
(x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10
*a + 6*Sqrt[3]*a)])/((20*a + 12*Sqrt[3]*a)*Sqrt[a - b*x^3])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$$

$$\Big\downarrow \text{ 989}$$

$$-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}(1+\sqrt{3})\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2-\sqrt{3})\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2-\sqrt{3})\operatorname{arctanh}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{(2-\sqrt{3})\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

input
```
Int[x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

output
```
-1/2*((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/
3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])]])/(Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2
 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a
])]])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)
*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])]])/
(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*a^(1
/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])]])/(3*
Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))
```

Defintions of rubi rules used

rule 989
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.96 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.57

method	result
default	$i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\left(a\,b^{2}\right.}{}\right.}{\left(a\,b^{2}\right)}}}{}\right.$
elliptic	$i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\left(a\,b^{2}\right.}{}\right.}{\left(a\,b^{2}\right)}}}{}\right.$

input
```
int(x/(-b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a-b*x^3),x,method=_RETURNVERBOSE)
```

output
```
1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1
/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(
1/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*
(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^
(1/2)*(3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^
2)^(2/3)*3^(1/2)-2*(a*b^2)^(1/3)*_alpha*3^(1/2)*b-6*I*(a*b^2)^(1/3)*_alpha
*b-6*_alpha^2*b^2-2*(a*b^2)^(2/3)*3^(1/2)+6*I*(a*b^2)^(2/3)+3*(a*b^2)^(1/3
)*_alpha*b+3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-
2*I*(a*b^2)^(1/3)*_alpha^2*3^(1/2)*b+I*(a*b^2)^(2/3)*_alpha*3^(1/2)+4*I*(a
*b^2)^(1/3)*_alpha^2*b+2*(a*b^2)^(2/3)*_alpha*3^(1/2)-2*I*(a*b^2)^(2/3)*_a
lpha+I*3^(1/2)*a*b-3*(a*b^2)^(2/3)*_alpha-2*3^(1/2)*a*b-2*I*a*b+3*a*b)/a,(
-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(
1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*3^(1/2)*a-10*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5587 vs. $2(218) = 436$.

Time = 4.52 (sec) , antiderivative size = 5587, normalized size of antiderivative = 17.24

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a-b*x^3),x, algorithm="frica
s")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = -\int \frac{x}{-6\sqrt{3}a\sqrt{a-bx^3}-10a\sqrt{a-bx^3}+bx^3\sqrt{a-bx^3}}\,dx$$

input
```
integrate(x/(-b*x**3+a)**(1/2)/(2*(5+3*3**(1/2))*a-b*x**3),x)
```

output
```
-Integral(x/(-6*sqrt(3)*a*sqrt(a - b*x**3) - 10*a*sqrt(a - b*x**3) + b*x**
3*sqrt(a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx=\int -\frac{x}{\left(bx^3-2\,a\left(3\,\sqrt{3}+5\right)\right)\sqrt{-bx^3+a}}\,dx$$

input
```
integrate(x/(-b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a-b*x^3),x, algorithm="maxim
a")
```

output
```
-integrate(x/((b*x^3 - 2*a*(3*sqrt(3) + 5))*sqrt(-b*x^3 + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x/(-b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a-b*x^3),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = -\int \frac{x}{\sqrt{a-b\,x^3}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}+5\right)\right)}\,dx$$

input
```
int(-x/((a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))),x)
```

output
```
-int(x/((a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))), x)
```

Reduce [F]

$$\begin{aligned}&\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx\\&=6\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{b^3x^9-21a\,b^2x^6+12a^2b\,x^3+8a^3}dx\right)a\\&\quad+\left(\int \frac{\sqrt{-b\,x^3+a}\,x^4}{b^3x^9-21a\,b^2x^6+12a^2b\,x^3+8a^3}dx\right)b\\&\quad-10\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{b^3x^9-21a\,b^2x^6+12a^2b\,x^3+8a^3}dx\right)a\end{aligned}$$

input
```
int(x/(-b*x^3+a)^(1/2)/(2*(5+3*3^(1/2))*a-b*x^3),x)
```

output
```
6*sqrt(3)*int((sqrt(a - b*x**3)*x)/(8*a**3 + 12*a**2*b*x**3 - 21*a*b**2*x*
*6 + b**3*x**9),x)*a + int((sqrt(a - b*x**3)*x**4)/(8*a**3 + 12*a**2*b*x**
3 - 21*a*b**2*x**6 + b**3*x**9),x)*b - 10*int((sqrt(a - b*x**3)*x)/(8*a**3
 + 12*a**2*b*x**3 - 21*a*b**2*x**6 + b**3*x**9),x)*a
```

3.525 $\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 328

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

output
```
1/36*(2-3^(1/2))*arctan(1/2*3^(1/4)*(1-3^(1/2))*a^(1/6)*(a^(1/3)-b^(1/3)*x
)*2^(1/2)/(b*x^3-a)^(1/2))*2^(1/2)*3^(3/4)/a^(5/6)/b^(2/3)+1/18*(2-3^(1/2)
)*arctan(1/2*3^(1/4)*a^(1/6)*((1+3^(1/2))*a^(1/3)+2*b^(1/3)*x)*2^(1/2)/(b*
x^3-a)^(1/2))*2^(1/2)*3^(3/4)/a^(5/6)/b^(2/3)+1/12*(2-3^(1/2))*arctanh(1/2
*3^(1/4)*(1+3^(1/2))*a^(1/6)*(a^(1/3)-b^(1/3)*x)*2^(1/2)/(b*x^3-a)^(1/2))*
2^(1/2)*3^(1/4)/a^(5/6)/b^(2/3)-1/18*(2-3^(1/2))*arctanh(1/6*(1-3^(1/2))*(
b*x^3-a)^(1/2)*2^(1/2)*3^(1/4)/a^(1/2))*2^(1/2)*3^(1/4)/a^(5/6)/b^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.26

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = -\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20a+12\sqrt{3}a\right)\sqrt{-a+bx^3}}$$

input
```
Integrate[x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

output
```
-((x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(
10*a + 6*Sqrt[3]*a)])/((20*a + 12*Sqrt[3]*a)*Sqrt[-a + b*x^3]))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {990}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{bx^3-a}\left(bx^3-2\left(5+3\sqrt{3}\right)a\right)}\,dx$$

$$\Big\downarrow 990$$

$$\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

input
```
Int[x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

output
```
((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x)
)/(Sqrt[2]*Sqrt[-a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 -
 Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(
Sqrt[2]*Sqrt[-a + b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sq
rt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt
[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3
])*ArcTanh[((1 - Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3
*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3))
```

Defintions of rubi rules used

rule 990
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.92 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.55

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\left(a \ldots\right.}{\ldots}\right.}{\left(a\,b \ldots\right.}}\ldots}{\ldots}\right.}{\ldots}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\left(a \ldots\right.}{\ldots}\right.}{\left(a\,b \ldots\right.}}\ldots}{\ldots}\right.}{\ldots}$

input
```
int(x/(b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a+b*x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(
1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^
(1/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b
*(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^
(1/2)*(3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^
2)^(2/3)*3^(1/2)-2*(a*b^2)^(1/3)*_alpha*3^(1/2)*b-6*I*(a*b^2)^(1/3)*_alpha
*b-6*_alpha^2*b^2-2*(a*b^2)^(2/3)*3^(1/2)+6*I*(a*b^2)^(2/3)+3*(a*b^2)^(1/3
)*_alpha*b+3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-
2*I*(a*b^2)^(1/3)*_alpha^2*3^(1/2)*b+I*(a*b^2)^(2/3)*_alpha*3^(1/2)+4*I*(a
*b^2)^(1/3)*_alpha^2*b+2*(a*b^2)^(2/3)*_alpha*3^(1/2)-2*I*(a*b^2)^(2/3)*_a
lpha+I*3^(1/2)*a*b-3*(a*b^2)^(2/3)*_alpha-2*3^(1/2)*a*b-2*I*a*b+3*a*b)/a,(
-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(
1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*3^(1/2)*a-10*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5667 vs. $2(222) = 444$.

Time = 4.38 (sec) , antiderivative size = 5667, normalized size of antiderivative = 17.28

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a+b*x^3),x, algorithm="frica
s")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\sqrt{-a+bx^3}\left(-6\sqrt{3}a-10a+bx^3\right)}\,dx$$

input
```
integrate(x/(b*x**3-a)**(1/2)/(-2*(5+3*3**(1/2))*a+b*x**3),x)
```

output
```
Integral(x/(sqrt(-a + b*x**3)*(-6*sqrt(3)*a - 10*a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx=\int \frac{x}{\left(bx^3-2\,a\left(3\,\sqrt{3}+5\right)\right)\sqrt{bx^3-a}}\,dx$$

input
```
integrate(x/(b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a+b*x^3),x, algorithm="maxim
a")
```

output
```
integrate(x/((b*x^3 - 2*a*(3*sqrt(3) + 5))*sqrt(b*x^3 - a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x/(b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a+b*x^3),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\sqrt{b\,x^3-a}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}+5\right)\right)}\,dx$$

input
```
int(x/((b*x^3 - a)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))),x)
```

output
```
int(x/((b*x^3 - a)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))), x)
```

Reduce [F]

$$\begin{aligned}
&\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx \\
&= 6\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}\,x}{b^3x^9-21a\,b^2x^6+12a^2b\,x^3+8a^3}dx\right)a \\
&\quad + \left(\int \frac{\sqrt{b\,x^3-a}\,x^4}{b^3x^9-21a\,b^2x^6+12a^2b\,x^3+8a^3}dx\right)b \\
&\quad - 10\left(\int \frac{\sqrt{b\,x^3-a}\,x}{b^3x^9-21a\,b^2x^6+12a^2b\,x^3+8a^3}dx\right)a
\end{aligned}$$

input
```
int(x/(b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a+b*x^3),x)
```

output
```
6*sqrt(3)*int((sqrt( - a + b*x**3)*x)/(8*a**3 + 12*a**2*b*x**3 - 21*a*b**2
*x**6 + b**3*x**9),x)*a + int((sqrt( - a + b*x**3)*x**4)/(8*a**3 + 12*a**2
*b*x**3 - 21*a*b**2*x**6 + b**3*x**9),x)*b - 10*int((sqrt( - a + b*x**3)*x
)/(8*a**3 + 12*a**2*b*x**3 - 21*a*b**2*x**6 + b**3*x**9),x)*a
```

3.526 $\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 330

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

output
```
1/18*(2-3^(1/2))*arctan(1/2*3^(1/4)*a^(1/6)*((1+3^(1/2))*a^(1/3)-2*b^(1/3)
*x)*2^(1/2)/(-b*x^3-a)^(1/2))*2^(1/2)*3^(3/4)/a^(5/6)/b^(2/3)+1/36*(2-3^(1
/2))*arctan(1/2*3^(1/4)*(1-3^(1/2))*a^(1/6)*(a^(1/3)+b^(1/3)*x)*2^(1/2)/(-
b*x^3-a)^(1/2))*2^(1/2)*3^(3/4)/a^(5/6)/b^(2/3)+1/12*(2-3^(1/2))*arctanh(1
/2*3^(1/4)*(1+3^(1/2))*a^(1/6)*(a^(1/3)+b^(1/3)*x)*2^(1/2)/(-b*x^3-a)^(1/2
))*2^(1/2)*3^(1/4)/a^(5/6)/b^(2/3)-1/18*(2-3^(1/2))*arctanh(1/6*(1-3^(1/2)
)*(-b*x^3-a)^(1/2)*2^(1/2)*3^(1/4)/a^(1/2))*2^(1/2)*3^(1/4)/a^(5/6)/b^(2/3
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.26

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = -\frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20a+12\sqrt{3}a\right)\sqrt{-a-bx^3}}$$

input
```
Integrate[x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

output
```
-((x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x
^3)/(10*a + 6*Sqrt[3]*a))])/((20*a + 12*Sqrt[3]*a)*Sqrt[-a - b*x^3]))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = {990}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$$

$$\Big\downarrow \ 990$$

$$\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

input
```
Int[x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

output
```
((2 - Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*
x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2
 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(
Sqrt[2]*Sqrt[-a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sq
rt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt
[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3
])*ArcTanh[((1 - Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3
*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3))
```

Defintions of rubi rules used

rule 990
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.94 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.64

method	result
default	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{ib\left(2x+\frac{(-}{}\right.}{}}}{}\right.}{}$
elliptic	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{ib\left(2x+\frac{(-}{}\right.}{}}}{}\right.}{}$

input
```
int(x/(-b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a-b*x^3),x,method=_RETURNVERBOSE)
```

output
```
1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^
2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^
2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2
*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(-
b*x^3-a)^(1/2)*(-3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2
)+3*I*3^(1/2)*(-a*b^2)^(2/3)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*(-a*b^2)^(1/3)*
_alpha*3^(1/2)*b-6*_alpha^2*b^2-6*I*(-a*b^2)^(2/3)-2*3^(1/2)*(-a*b^2)^(2/3
)+3*(-a*b^2)^(1/3)*_alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),-1/6/b*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)
*3^(1/2)*_alpha-4*I*(-a*b^2)^(1/3)*_alpha^2*b+2*I*(-a*b^2)^(2/3)*_alpha+2*
(-a*b^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-2*I*a*
b+2*3^(1/2)*a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a
+10*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5679 vs. $2(223) = 446$.

Time = 4.16 (sec) , antiderivative size = 5679, normalized size of antiderivative = 17.21

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a-b*x^3),x, algorithm="fric
as")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = -\int \frac{x}{10a\sqrt{-a-bx^3}+6\sqrt{3}a\sqrt{-a-bx^3}+bx^3\sqrt{-a-bx^3}}\,dx$$

input
```
integrate(x/(-b*x**3-a)**(1/2)/(-2*(5+3*3**(1/2))*a-b*x**3),x)
```

output
```
-Integral(x/(10*a*sqrt(-a - b*x**3) + 6*sqrt(3)*a*sqrt(-a - b*x**3) + b*x*
*3*sqrt(-a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \int -\frac{x}{\left(bx^3+2\,a\left(3\,\sqrt{3}+5\right)\right)\sqrt{-bx^3-a}}\,dx$$

input
```
integrate(x/(-b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a-b*x^3),x, algorithm="maxi
ma")
```

output
```
-integrate(x/((b*x^3 + 2*a*(3*sqrt(3) + 5))*sqrt(-b*x^3 - a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(-b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a-b*x^3),x, algorithm="giac
")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx=\int -\frac{x}{\sqrt{-b\,x^3-a}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}+5\right)\right)}\,dx$$

input
```
int(-x/((- a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))),x)
```

output
```
int(-x/((- a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))), x)
```

Reduce [F]

$$\begin{aligned}&\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx\\&=i\left(6\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{-b^3x^9-21a\,b^2x^6-12a^2b\,x^3+8a^3}dx\right)a\right.\\&\quad-\left(\int \frac{\sqrt{b\,x^3+a}\,x^4}{-b^3x^9-21a\,b^2x^6-12a^2b\,x^3+8a^3}dx\right)b\\&\quad\left.-10\left(\int \frac{\sqrt{b\,x^3+a}\,x}{-b^3x^9-21a\,b^2x^6-12a^2b\,x^3+8a^3}dx\right)a\right)\end{aligned}$$

input
```
int(x/(-b*x^3-a)^(1/2)/(-2*(5+3*3^(1/2))*a-b*x^3),x)
```

output
```
i*(6*sqrt(3)*int((sqrt(a + b*x**3)*x)/(8*a**3 - 12*a**2*b*x**3 - 21*a*b**2
*x**6 - b**3*x**9),x)*a - int((sqrt(a + b*x**3)*x**4)/(8*a**3 - 12*a**2*b*
x**3 - 21*a*b**2*x**6 - b**3*x**9),x)*b - 10*int((sqrt(a + b*x**3)*x)/(8*a
**3 - 12*a**2*b*x**3 - 21*a*b**2*x**6 - b**3*x**9),x)*a)
```

3.527 $\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 310

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} + \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

output
```
-1/18*(2+3^(1/2))*arctan(1/2*3^(1/4)*a^(1/6)*((1-3^(1/2))*a^(1/3)-2*b^(1/3
)*x)*2^(1/2)/(b*x^3+a)^(1/2))*2^(1/2)*3^(3/4)/a^(5/6)/b^(2/3)-1/36*(2+3^(1
/2))*arctan(1/2*3^(1/4)*(1+3^(1/2))*a^(1/6)*(a^(1/3)+b^(1/3)*x)*2^(1/2)/(b
*x^3+a)^(1/2))*2^(1/2)*3^(3/4)/a^(5/6)/b^(2/3)+1/12*(2+3^(1/2))*arctanh(1/
2*3^(1/4)*(1-3^(1/2))*a^(1/6)*(a^(1/3)+b^(1/3)*x)*2^(1/2)/(b*x^3+a)^(1/2))
*2^(1/2)*3^(1/4)/a^(5/6)/b^(2/3)+1/18*(2+3^(1/2))*arctanh(1/6*(1+3^(1/2))*
(b*x^3+a)^(1/2)*2^(1/2)*3^(1/4)/a^(1/2))*2^(1/2)*3^(1/4)/a^(5/6)/b^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.27

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{a+bx^3}}$$

input
```
Integrate[x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

output
```
(x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3
)/(10*a - 6*Sqrt[3]*a))])/((20*a - 12*Sqrt[3]*a)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$$

$$\Big\downarrow \text{989}$$

$$-\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{(2+\sqrt{3})\,\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}+\frac{(2+\sqrt{3})\,\text{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

input
```
Int[x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

output
```
-1/3*((2 + Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(
1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]])/(Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - (
(2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))
/(Sqrt[2]*Sqrt[a + b*x^3])]])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 + S
qrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqr
t[2]*Sqrt[a + b*x^3])]])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3
])*ArcTanh[((1 + Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])]])/(3*
Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3))
```

Defintions of rubi rules used

rule 989
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.95 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.74

method	result
default	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{ib\left(2x+\frac{(-}{}\right.}{}}}{}\right)}{}$
elliptic	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{ib\left(2x+\frac{(-}{}\right.}{}}}{}\right)}{}$

input
```
int(x/(b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x,method=_RETURNVERBOSE)
```

output
```
1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^
2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^
2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2
*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*(3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-
3*I*3^(1/2)*(-a*b^2)^(2/3)-2*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+6*I*(-a*b^2)^
(1/3)*_alpha*b+6*_alpha^2*b^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)-
3*(-a*b^2)^(1/3)*_alpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),-1/6/b*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3
^(1/2)*_alpha+4*I*(-a*b^2)^(1/3)*_alpha^2*b-2*(-a*b^2)^(2/3)*_alpha*3^(1/2
)+I*3^(1/2)*a*b-2*I*(-a*b^2)^(2/3)*_alpha-2*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_
alpha+2*I*a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*3^(1/2)*a+1
0*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5631 vs. $2(210) = 420$.

Time = 4.97 (sec) , antiderivative size = 5631, normalized size of antiderivative = 18.16

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\sqrt{a+bx^3}\left(-6\sqrt{3}a+10a+bx^3\right)}\,dx$$

input
```
integrate(x/(b*x**3+a)**(1/2)/(2*(5-3*3**(1/2))*a+b*x**3),x)
```

output
```
Integral(x/(sqrt(a + b*x**3)*(-6*sqrt(3)*a + 10*a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\left(bx^3-2\,a\left(3\,\sqrt{3}-5\right)\right)\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x, algorithm="maxima
")
```

output
```
integrate(x/((b*x^3 - 2*a*(3*sqrt(3) - 5))*sqrt(b*x^3 + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx=\int \frac{x}{\sqrt{b\,x^3+a}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}-5\right)\right)}\,dx$$

input
```
int(x/((a + b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))),x)
```

output
```
int(x/((a + b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))), x)
```

Reduce [F]

$$\begin{aligned}&\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx\\&=-6\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{-b^3x^9-21a\,b^2x^6-12a^2b\,x^3+8a^3}dx\right)a\\&\quad-\left(\int \frac{\sqrt{b\,x^3+a}\,x^4}{-b^3x^9-21a\,b^2x^6-12a^2b\,x^3+8a^3}dx\right)b\\&\quad-10\left(\int \frac{\sqrt{b\,x^3+a}\,x}{-b^3x^9-21a\,b^2x^6-12a^2b\,x^3+8a^3}dx\right)a\end{aligned}$$

input
```
int(x/(b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x)
```

output
```
- 6*sqrt(3)*int((sqrt(a + b*x**3)*x)/(8*a**3 - 12*a**2*b*x**3 - 21*a*b**2
*x**6 - b**3*x**9),x)*a - int((sqrt(a + b*x**3)*x**4)/(8*a**3 - 12*a**2*b*
x**3 - 21*a*b**2*x**6 - b**3*x**9),x)*b - 10*int((sqrt(a + b*x**3)*x)/(8*a
**3 - 12*a**2*b*x**3 - 21*a*b**2*x**6 - b**3*x**9),x)*a
```

3.528 $\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 316

$$\begin{aligned}
&\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx \\
&= -\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} \\
&\quad-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} \\
&\quad+\frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} \\
&\quad+\frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}
\end{aligned}$$

output
```
-1/36*(2+3^(1/2))*arctan(1/2*3^(1/4)*(1+3^(1/2))*a^(1/6)*(a^(1/3)-b^(1/3)*
x)*2^(1/2)/(-b*x^3+a)^(1/2))*2^(1/2)*3^(3/4)/a^(5/6)/b^(2/3)-1/18*(2+3^(1/
2))*arctan(1/2*3^(1/4)*a^(1/6)*((1-3^(1/2))*a^(1/3)+2*b^(1/3)*x)*2^(1/2)/(
-b*x^3+a)^(1/2))*2^(1/2)*3^(3/4)/a^(5/6)/b^(2/3)+1/12*(2+3^(1/2))*arctanh(
1/2*3^(1/4)*(1-3^(1/2))*a^(1/6)*(a^(1/3)-b^(1/3)*x)*2^(1/2)/(-b*x^3+a)^(1/
2))*2^(1/2)*3^(1/4)/a^(5/6)/b^(2/3)+1/18*(2+3^(1/2))*arctanh(1/6*(1+3^(1/2
))*(-b*x^3+a)^(1/2)*2^(1/2)*3^(1/4)/a^(1/2))*2^(1/2)*3^(1/4)/a^(5/6)/b^(2/
3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.26

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx=\frac{x^2\sqrt{1-\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{a-bx^3}}$$

input
```
Integrate[x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)),x]
```

output
```
(x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10
*a - 6*Sqrt[3]*a)])/((20*a - 12*Sqrt[3]*a)*Sqrt[a - b*x^3])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx$$

$$\Big\downarrow \text{989}$$

$$-\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}(1+\sqrt{3})\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left((1-\sqrt{3})\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{(2+\sqrt{3})\,\text{arctanh}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}+\frac{(2+\sqrt{3})\,\text{arctanh}\left(\frac{(1+\sqrt{3})\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

input
```
Int[x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)),x]
```

output
```
-1/6*((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/
3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2
 + Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))
/(Sqrt[2]*Sqrt[a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 + S
qrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqr
t[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3
])*ArcTanh[((1 + Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*
Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3))
```

Defintions of rubi rules used

rule 989
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.94 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.61

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^2\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}+\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\left(a \cdots\right.}{\cdots}\right)}{\left(a\,b \cdots\right.}}\cdots}{\cdots}\right)}{\cdots}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^2\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}+\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\left(a \cdots\right.}{\cdots}\right)}{\left(a\,b \cdots\right.}}\cdots}{\cdots}\right)}{\cdots}$

input
```
int(x/(-b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a-b*x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(
1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^
(1/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b
*(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)
^(1/2)*(-3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*
b^2)^(2/3)*3^(1/2)-6*I*(a*b^2)^(1/3)*_alpha*b-2*(a*b^2)^(1/3)*_alpha*3^(1/
2)*b+6*_alpha^2*b^2+6*I*(a*b^2)^(2/3)-2*(a*b^2)^(2/3)*3^(1/2)-3*(a*b^2)^(1
/3)*_alpha*b-3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*
(-2*I*(a*b^2)^(1/3)*_alpha^2*3^(1/2)*b+I*(a*b^2)^(2/3)*_alpha*3^(1/2)-4*I*
(a*b^2)^(1/3)*_alpha^2*b+2*I*(a*b^2)^(2/3)*_alpha-2*(a*b^2)^(2/3)*_alpha*3
^(1/2)+I*3^(1/2)*a*b-3*(a*b^2)^(2/3)*_alpha+2*I*a*b+2*3^(1/2)*a*b+3*a*b)/a
,(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)
^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a-10*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5667 vs. $2(219) = 438$.

Time = 4.54 (sec) , antiderivative size = 5667, normalized size of antiderivative = 17.93

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a-b*x^3),x, algorithm="frica
s")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx = -\int \frac{x}{-10a\sqrt{a-bx^3}+6\sqrt{3}a\sqrt{a-bx^3}+bx^3\sqrt{a-bx^3}}\,dx$$

input
```
integrate(x/(-b*x**3+a)**(1/2)/(2*(5-3*3**(1/2))*a-b*x**3),x)
```

output
```
-Integral(x/(-10*a*sqrt(a - b*x**3) + 6*sqrt(3)*a*sqrt(a - b*x**3) + b*x**
3*sqrt(a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx=\int -\frac{x}{\left(bx^3+2\,a\left(3\,\sqrt{3}-5\right)\right)\sqrt{-bx^3+a}}\,dx$$

input
```
integrate(x/(-b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a-b*x^3),x, algorithm="maxim
a")
```

output
```
-integrate(x/((b*x^3 + 2*a*(3*sqrt(3) - 5))*sqrt(-b*x^3 + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x/(-b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a-b*x^3),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx = \int -\frac{x}{\sqrt{a-b\,x^3}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}-5\right)\right)}\,dx$$

input
```
int(-x/((a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))),x)
```

output
```
int(-x/((a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))), x)
```

Reduce [F]

$$\begin{aligned}
&\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx \\
&= -6\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{b^3x^9-21a\,b^2x^6+12a^2b\,x^3+8a^3}dx\right)a \\
&\quad+\left(\int \frac{\sqrt{-b\,x^3+a}\,x^4}{b^3x^9-21a\,b^2x^6+12a^2b\,x^3+8a^3}dx\right)b \\
&\quad-10\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{b^3x^9-21a\,b^2x^6+12a^2b\,x^3+8a^3}dx\right)a
\end{aligned}$$

input
```
int(x/(-b*x^3+a)^(1/2)/(2*(5-3*3^(1/2))*a-b*x^3),x)
```

output
```
- 6*sqrt(3)*int((sqrt(a - b*x**3)*x)/(8*a**3 + 12*a**2*b*x**3 - 21*a*b**2
*x**6 + b**3*x**9),x)*a + int((sqrt(a - b*x**3)*x**4)/(8*a**3 + 12*a**2*b*
x**3 - 21*a*b**2*x**6 + b**3*x**9),x)*b - 10*int((sqrt(a - b*x**3)*x)/(8*a
**3 + 12*a**2*b*x**3 - 21*a*b**2*x**6 + b**3*x**9),x)*a
```

3.529 $\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 320

$$\begin{aligned}
&\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx \\
&= \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} \\
&\quad - \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} \\
&\quad - \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}
\end{aligned}$$

output
```
1/12*(2+3^(1/2))*arctan(1/2*3^(1/4)*(1-3^(1/2))*a^(1/6)*(a^(1/3)-b^(1/3)*x
)*2^(1/2)/(b*x^3-a)^(1/2))*2^(1/2)*3^(1/4)/a^(5/6)/b^(2/3)-1/18*(2+3^(1/2)
)*arctan(1/6*(1+3^(1/2))*(b*x^3-a)^(1/2)*2^(1/2)*3^(1/4)/a^(1/2))*2^(1/2)*
3^(1/4)/a^(5/6)/b^(2/3)-1/36*(2+3^(1/2))*arctanh(1/2*3^(1/4)*(1+3^(1/2))*a
^(1/6)*(a^(1/3)-b^(1/3)*x)*2^(1/2)/(b*x^3-a)^(1/2))*2^(1/2)*3^(3/4)/a^(5/6
)/b^(2/3)-1/18*(2+3^(1/2))*arctanh(1/2*3^(1/4)*a^(1/6)*((1-3^(1/2))*a^(1/3
)+2*b^(1/3)*x)*2^(1/2)/(b*x^3-a)^(1/2))*2^(1/2)*3^(3/4)/a^(5/6)/b^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.26

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx = \frac{x^2\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{-a+bx^3}}$$

input
```
Integrate[x/((2*(5 - 3*Sqrt[3])*a - b*x^3)*Sqrt[-a + b*x^3]),x]
```

output
```
(x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10
*a - 6*Sqrt[3]*a)])/((20*a - 12*Sqrt[3]*a)*Sqrt[-a + b*x^3])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {990}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{bx^3-a}}\,dx$$

$$\Big\downarrow 990$$

$$\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}(1-\sqrt{3})\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{3})\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{3})\operatorname{arctanh}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{3})\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

input
```
Int[x/((2*(5 - 3*Sqrt[3])*a - b*x^3)*Sqrt[-a + b*x^3]),x]
```

output
```
((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x)
)/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 +
 Sqrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a]
)])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*
(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/
(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*a^(1
/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(3
*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))
```

Defintions of rubi rules used

rule 990
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.89 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.59

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\,\left(a \right.}{}\right)}{\left(a\,b\right.}}}{}\right.}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\,\left(a \right.}{}\right)}{\left(a\,b\right.}}}{}\right.}{}$

input
```
int(x/(2*(5-3*3^(1/2))*a-b*x^3)/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(
1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^
(1/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b
*(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^
(1/2)*(-3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*b
^2)^(2/3)*3^(1/2)-6*I*(a*b^2)^(1/3)*_alpha*b-2*(a*b^2)^(1/3)*_alpha*3^(1/2
)*b+6*_alpha^2*b^2+6*I*(a*b^2)^(2/3)-2*(a*b^2)^(2/3)*3^(1/2)-3*(a*b^2)^(1/
3)*_alpha*b-3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(
-2*I*(a*b^2)^(1/3)*_alpha^2*3^(1/2)*b+I*(a*b^2)^(2/3)*_alpha*3^(1/2)-4*I*(
a*b^2)^(1/3)*_alpha^2*b+2*I*(a*b^2)^(2/3)*_alpha-2*(a*b^2)^(2/3)*_alpha*3^
(1/2)+I*3^(1/2)*a*b-3*(a*b^2)^(2/3)*_alpha+2*I*a*b+2*3^(1/2)*a*b+3*a*b)/a,
(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^
(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a-10*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5599 vs. $2(223) = 446$.

Time = 4.25 (sec) , antiderivative size = 5599, normalized size of antiderivative = 17.50

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(2*(5-3*3^(1/2))*a-b*x^3)/(b*x^3-a)^(1/2),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx$$
$$= -\int \frac{x}{-10a\sqrt{-a+bx^3}+6\sqrt{3}a\sqrt{-a+bx^3}+bx^3\sqrt{-a+bx^3}}\,dx$$

input
```
integrate(x/(2*(5-3*3**(1/2))*a-b*x**3)/(b*x**3-a)**(1/2),x)
```

output
```
-Integral(x/(-10*a*sqrt(-a + b*x**3) + 6*sqrt(3)*a*sqrt(-a + b*x**3) + b*x
**3*sqrt(-a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx=\int -\frac{x}{\left(bx^3+2\,a\left(3\,\sqrt{3}-5\right)\right)\sqrt{bx^3-a}}\,dx$$

input
```
integrate(x/(2*(5-3*3^(1/2))*a-b*x^3)/(b*x^3-a)^(1/2),x, algorithm="maxima
")
```

output
```
-integrate(x/((b*x^3 + 2*a*(3*sqrt(3) - 5))*sqrt(b*x^3 - a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x/(2*(5-3*3^(1/2))*a-b*x^3)/(b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx=\int -\frac{x}{\sqrt{b\,x^3-a}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}-5\right)\right)}\,dx$$

input
```
int(-x/((b*x^3 - a)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))),x)
```

output
```
int(-x/((b*x^3 - a)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))), x)
```

Reduce [F]

$$\begin{aligned}&\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx\\&=6\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}\,x}{b^3x^9-21a\,b^2x^6+12a^2b\,x^3+8a^3}dx\right)a\\&\quad-\left(\int \frac{\sqrt{b\,x^3-a}\,x^4}{b^3x^9-21a\,b^2x^6+12a^2b\,x^3+8a^3}dx\right)b\\&\quad+10\left(\int \frac{\sqrt{b\,x^3-a}\,x}{b^3x^9-21a\,b^2x^6+12a^2b\,x^3+8a^3}dx\right)a\end{aligned}$$

input
```
int(x/(2*(5-3*3^(1/2))*a-b*x^3)/(b*x^3-a)^(1/2),x)
```

output
```
6*sqrt(3)*int((sqrt( - a + b*x**3)*x)/(8*a**3 + 12*a**2*b*x**3 - 21*a*b**2
*x**6 + b**3*x**9),x)*a - int((sqrt( - a + b*x**3)*x**4)/(8*a**3 + 12*a**2
*b*x**3 - 21*a*b**2*x**6 + b**3*x**9),x)*b + 10*int((sqrt( - a + b*x**3)*x
)/(8*a**3 + 12*a**2*b*x**3 - 21*a*b**2*x**6 + b**3*x**9),x)*a
```

3.530 $\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 322

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

output
```
1/12*(2+3^(1/2))*arctan(1/2*3^(1/4)*(1-3^(1/2))*a^(1/6)*(a^(1/3)+b^(1/3)*x
)*2^(1/2)/(-b*x^3-a)^(1/2))*2^(1/2)*3^(1/4)/a^(5/6)/b^(2/3)-1/18*(2+3^(1/2
))*arctan(1/6*(1+3^(1/2))*(-b*x^3-a)^(1/2)*2^(1/2)*3^(1/4)/a^(1/2))*2^(1/2
)*3^(1/4)/a^(5/6)/b^(2/3)-1/18*(2+3^(1/2))*arctanh(1/2*3^(1/4)*a^(1/6)*((1
-3^(1/2))*a^(1/3)-2*b^(1/3)*x)*2^(1/2)/(-b*x^3-a)^(1/2))*2^(1/2)*3^(3/4)/a
^(5/6)/b^(2/3)-1/36*(2+3^(1/2))*arctanh(1/2*3^(1/4)*(1+3^(1/2))*a^(1/6)*(a
^(1/3)+b^(1/3)*x)*2^(1/2)/(-b*x^3-a)^(1/2))*2^(1/2)*3^(3/4)/a^(5/6)/b^(2/3
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.27

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{-a-bx^3}}$$

input
```
Integrate[x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

output
```
(x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3
)/(10*a - 6*Sqrt[3]*a))])/((20*a - 12*Sqrt[3]*a)*Sqrt[-a - b*x^3])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {990}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$$

$$\Big\downarrow \; 990$$

$$\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

input
```
Int[x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

output
```
((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x)
)/(Sqrt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 +
 Sqrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a]
)])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*
a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])]
)/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1
 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(6
*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))
```

Defintions of rubi rules used

rule 990
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.93 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.68

method	result
default	$i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3-6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{ib\left(2x+\frac{\left(-\right.}{}\right.}{}}}{}\right.$
elliptic	$i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3-6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{ib\left(2x+\frac{\left(-\right.}{}\right.}{}}}{}\right.$

input
```
int(x/(-b*x^3-a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x,method=_RETURNVERBOSE)
```

output
```
1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^
2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^
2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2
*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(-
b*x^3-a)^(1/2)*(3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)
-3*I*3^(1/2)*(-a*b^2)^(2/3)-2*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+6*I*(-a*b^2)
^(1/3)*_alpha*b+6*_alpha^2*b^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)
-3*(-a*b^2)^(1/3)*_alpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),-1/6/b*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*
3^(1/2)*_alpha+4*I*(-a*b^2)^(1/3)*_alpha^2*b-2*(-a*b^2)^(2/3)*_alpha*3^(1/
2)+I*3^(1/2)*a*b-2*I*(-a*b^2)^(2/3)*_alpha-2*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*
_alpha+2*I*a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*3^(1/2)*a+
10*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5599 vs. $2(222) = 444$.

Time = 4.38 (sec) , antiderivative size = 5599, normalized size of antiderivative = 17.39

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-b*x^3-a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x, algorithm="frica
s")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx=\int \frac{x}{\sqrt{-a-bx^3}\left(-6\sqrt{3}a+10a+bx^3\right)}\,dx$$

input
```
integrate(x/(-b*x**3-a)**(1/2)/(2*(5-3*3**(1/2))*a+b*x**3),x)
```

output
```
Integral(x/(sqrt(-a - b*x**3)*(-6*sqrt(3)*a + 10*a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx=\int \frac{x}{\left(bx^3-2\,a\left(3\,\sqrt{3}-5\right)\right)\sqrt{-bx^3-a}}\,dx$$

input
```
integrate(x/(-b*x^3-a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x, algorithm="maxim
a")
```

output
```
integrate(x/((b*x^3 - 2*a*(3*sqrt(3) - 5))*sqrt(-b*x^3 - a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x/(-b*x^3-a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\sqrt{-b\,x^3-a}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}-5\right)\right)}\,dx$$

input
```
int(x/((- a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))),x)
```

output
```
int(x/((- a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))), x)
```

Reduce [F]

$$\begin{aligned}
&\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx \\
&= i\left(6\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{-b^3x^9-21a\,b^2x^6-12a^2b\,x^3+8a^3}dx\right)a\right. \\
&\qquad + \left(\int \frac{\sqrt{b\,x^3+a}\,x^4}{-b^3x^9-21a\,b^2x^6-12a^2b\,x^3+8a^3}dx\right)b \\
&\qquad \left. + 10\left(\int \frac{\sqrt{b\,x^3+a}\,x}{-b^3x^9-21a\,b^2x^6-12a^2b\,x^3+8a^3}dx\right)a\right)
\end{aligned}$$

input
```
int(x/(-b*x^3-a)^(1/2)/(2*(5-3*3^(1/2))*a+b*x^3),x)
```

output
```
i*(6*sqrt(3)*int((sqrt(a + b*x**3)*x)/(8*a**3 - 12*a**2*b*x**3 - 21*a*b**2
*x**6 - b**3*x**9),x)*a + int((sqrt(a + b*x**3)*x**4)/(8*a**3 - 12*a**2*b*
x**3 - 21*a*b**2*x**6 - b**3*x**9),x)*b + 10*int((sqrt(a + b*x**3)*x)/(8*a
**3 - 12*a**2*b*x**3 - 21*a*b**2*x**6 - b**3*x**9),x)*a)
```

3.531 $\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 125

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{2a^2\sqrt{c+dx^3}}{3b^3} - \frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2} + \frac{2(c+dx^3)^{5/2}}{15bd^2} - \frac{2a^2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

output
```
2/3*a^2*(d*x^3+c)^(1/2)/b^3-2/9*(a*d+b*c)*(d*x^3+c)^(3/2)/b^2/d^2+2/15*(d*
x^3+c)^(5/2)/b/d^2-2/3*a^2*(-a*d+b*c)^(1/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2
)/(-a*d+b*c)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{2\sqrt{c+dx^3}(15a^2d^2-5abd(c+dx^3)+b^2(-2c^2+cdx^3+3d^2x^6))}{45b^3d^2} - \frac{2a^2\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{7/2}}$$

input
```
Integrate[(x^8*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
(2*Sqrt[c + d*x^3]*(15*a^2*d^2 - 5*a*b*d*(c + d*x^3) + b^2*(-2*c^2 + c*d*x
^3 + 3*d^2*x^6)))/(45*b^3*d^2) - (2*a^2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]
*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(7/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^6\sqrt{dx^3+c}}{bx^3+a}dx^3$$

$$\Big\downarrow \text{ 99}$$

$$\frac{1}{3}\int \left(\frac{\sqrt{dx^3+c}a^2}{b^2\left(bx^3+a\right)}+\frac{\left(dx^3+c\right)^{3/2}}{bd}+\frac{(-bc-ad)\sqrt{dx^3+c}}{b^2d}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{2a^2\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{2a^2\sqrt{c+dx^3}}{b^3}-\frac{2\left(c+dx^3\right)^{3/2}(ad+bc)}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{5bd^2}\right)$$

input
```
Int[(x^8*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
((2*a^2*Sqrt[c + d*x^3])/b^3 - (2*(b*c + a*d)*(c + d*x^3)^(3/2))/(3*b^2*d^
2) + (2*(c + d*x^3)^(5/2))/(5*b*d^2) - (2*a^2*Sqrt[b*c - a*d]*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(7/2))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.99

method	result
risch	$\frac{2\left(3b^{2}d^{2}x^{6}-5x^{3}ab\,d^{2}+x^{3}b^{2}cd+15a^{2}d^{2}-5abcd-2b^{2}c^{2}\right)\sqrt{d\,x^{3}+c}}{45d^{2}b^{3}}-\frac{2a^{2}(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3b^{3}\sqrt{(ad-bc)b}}$
pseudoelliptic	$-\frac{2\left(-\left(-\frac{2\left(-\frac{3d\,x^{3}}{2}+c\right)\left(d\,x^{3}+c\right)b^{2}}{15}-\frac{\left(d\,x^{3}+c\right)abd}{3}+a^{2}d^{2}\right)\sqrt{(ad-bc)b}\,\sqrt{d\,x^{3}+c}+a^{2}d^{2}(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)\right)}{3\sqrt{(ad-bc)b}\,d^{2}b^{3}}$
default	$\frac{\frac{2x^{6}\sqrt{d\,x^{3}+c}}{15}+\frac{2c\,x^{3}\sqrt{d\,x^{3}+c}}{45d}-\frac{4c^{2}\sqrt{d\,x^{3}+c}}{45d^{2}}}{b}+\frac{2a^{2}\left(\sqrt{d\,x^{3}+c}-\frac{(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3b^{3}}-\frac{2a\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9b^{2}d}$
elliptic	$\frac{2x^{6}\sqrt{d\,x^{3}+c}}{15b}+\frac{2\left(-\frac{ad-bc}{b^{2}}-\frac{4c}{5b}\right)x^{3}\sqrt{d\,x^{3}+c}}{9d}+\frac{2\left(\frac{(ad-bc)a}{b^{3}}-\frac{2\left(-\frac{ad-bc}{b^{2}}-\frac{4c}{5b}\right)c}{3d}\right)\sqrt{d\,x^{3}+c}}{3d}+\frac{i a^{2}\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_\right.}\right.}{}$

input
```
int(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/45*(3*b^2*d^2*x^6-5*a*b*d^2*x^3+b^2*c*d*x^3+15*a^2*d^2-5*a*b*c*d-2*b^2*c
^2)*(d*x^3+c)^(1/2)/d^2/b^3-2/3*a^2*(a*d-b*c)/b^3/((a*d-b*c)*b)^(1/2)*arct
an(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.24

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx$$
$$= \left[\frac{15\,a^2d^2\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\,(3\,b^2d^2x^6-2\,b^2c^2-5\,abcd+15\,a^2d^2+(b^2cd-5\right.}{45\,b^3d^2}$$
$$-\frac{2\left(15\,a^2d^2\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(3\,b^2d^2x^6-2\,b^2c^2-5\,abcd+15\,a^2d^2+(b^2cd-5\,a\right.}{45\,b^3d^2}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/45*(15*a^2*d^2*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(
d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(3*b^2*d^2*x^6 - 2*b^2*
c^2 - 5*a*b*c*d + 15*a^2*d^2 + (b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d*x^3 + c))
/(b^3*d^2), -2/45*(15*a^2*d^2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)
*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (3*b^2*d^2*x^6 - 2*b^2*c^2 - 5*a*b*
c*d + 15*a^2*d^2 + (b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b^3*d^2)]
```

Sympy [A] (verification not implemented)

Time = 11.47 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.38

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx = \begin{cases} \frac{2\left(\frac{a^2d^3\sqrt{c+dx^3}}{3b^3}-\frac{a^2d^3(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^4\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^3\right)^{\frac{5}{2}}}{15b}+\frac{\left(c+dx^3\right)^{\frac{3}{2}}\left(-ad^2-bcd\right)}{9b^2}\right)}{d^3} & \text{for } d\neq 0 \\ \sqrt{c}\left(\frac{a^2\left(\begin{cases}\frac{x^3}{a} & \text{for } b=0\\ \frac{\log\left(a+bx^3\right)}{b} & \text{otherwise}\end{cases}\right)}{3b^2}-\frac{ax^3}{3b^2}+\frac{x^6}{6b}\right) & \text{otherwise} \end{cases}$$

input

```
integrate(x**8*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

output

```
Piecewise((2*(a**2*d**3*sqrt(c + d*x**3)/(3*b**3) - a**2*d**3*(a*d - b*c)*
atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**4*sqrt((a*d - b*c)/b)) +
d*(c + d*x**3)**(5/2)/(15*b) + (c + d*x**3)**(3/2)*(-a*d**2 - b*c*d)/(9*b*
*2))/d**3, Ne(d, 0)), (sqrt(c)*(a**2*Piecewise((x**3/a, Eq(b, 0)), (log(a
+ b*x**3)/b, True))/(3*b**2) - a*x**3/(3*b**2) + x**6/(6*b)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.11

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{2\left(a^2bc-a^3d\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^3} + \frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}b^4d^8-5\left(dx^3+c\right)^{\frac{3}{2}}b^4cd^8-5\left(dx^3+c\right)^{\frac{3}{2}}ab^3d^9+15\sqrt{dx^3+c}a^2b^2d^{10}\right)}{45\,b^5d^{10}}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*(a^2*b*c - a^3*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt
(-b^2*c + a*b*d)*b^3) + 2/45*(3*(d*x^3 + c)^(5/2)*b^4*d^8 - 5*(d*x^3 + c)^
(3/2)*b^4*c*d^8 - 5*(d*x^3 + c)^(3/2)*a*b^3*d^9 + 15*sqrt(d*x^3 + c)*a^2*b
^2*d^10)/(b^5*d^10)
```

Mupad [B] (verification not implemented)

Time = 4.74 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.41

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{2\,a^2\,\sqrt{d\,x^3+c}}{3\,b^3} + \frac{2\,{\left(d\,x^3+c\right)}^{5/2}}{15\,b\,d^2} - \frac{2\,a\,{\left(d\,x^3+c\right)}^{3/2}}{9\,b^2\,d} - \frac{2\,c\,{\left(d\,x^3+c\right)}^{3/2}}{9\,b\,d^2} + \frac{a^2\,\ln\left(\frac{a^2\,d^2\,1{}\mathrm{i}+b^2\,c^2\,2{}\mathrm{i}-2\,\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}-a\,b\,d^2\,x^3\,1{}\mathrm{i}+b^2\,c\,d\,x^3\,1{}\mathrm{i}-a\,b\,c\,d\,3{}\mathrm{i}}{2\,b\,x^3+2\,a}\right)\,\sqrt{a\,d-b\,c}\,1{}\mathrm{i}}{3\,b^{7/2}}$$

input
```
int((x^8*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

output
```
(2*a^2*(c + d*x^3)^(1/2))/(3*b^3) + (2*(c + d*x^3)^(5/2))/(15*b*d^2) - (2*
a*(c + d*x^3)^(3/2))/(9*b^2*d) - (2*c*(c + d*x^3)^(3/2))/(9*b*d^2) + (a^2*
log((a^2*d^2*1i + b^2*c^2*2i - 2*b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/
2) - a*b*d^2*x^3*1i + b^2*c*d*x^3*1i - a*b*c*d*3i)/(2*a + 2*b*x^3))*(a*d -
 b*c)^(1/2)*1i)/(3*b^(7/2))
```

Reduce [F]

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{20\sqrt{d\,x^3+c}\,acd-10\sqrt{d\,x^3+c}\,a\,d^2x^3-4\sqrt{d\,x^3+c}\,b\,c^2+2\sqrt{d\,x^3+c}\,bcd\,x^3+6\sqrt{d\,x^3+c}\,b\,d^2x^6+45\Big(\int}{45b^2d^2}$$

input
```
int(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x)
```

output
```
(20*sqrt(c + d*x**3)*a*c*d - 10*sqrt(c + d*x**3)*a*d**2*x**3 - 4*sqrt(c +
d*x**3)*b*c**2 + 2*sqrt(c + d*x**3)*b*c*d*x**3 + 6*sqrt(c + d*x**3)*b*d**2
*x**6 + 45*int((sqrt(c + d*x**3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x*
*6),x)*a**2*d**3 - 45*int((sqrt(c + d*x**3)*x**5)/(a*c + a*d*x**3 + b*c*x*
*3 + b*d*x**6),x)*a*b*c*d**2)/(45*b**2*d**2)
```

3.532 $\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx=-\frac{2a\sqrt{c+dx^3}}{3b^2}+\frac{2(c+dx^3)^{3/2}}{9bd}+\frac{2a\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}$$

output
```
-2/3*a*(d*x^3+c)^(1/2)/b^2+2/9*(d*x^3+c)^(3/2)/b/d+2/3*a*(-a*d+b*c)^(1/2)*
arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx=\frac{2\sqrt{c+dx^3}(-3ad+b(c+dx^3))}{9b^2d}+\frac{2a\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{5/2}}$$

input
```
Integrate[(x^5*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
(2*Sqrt[c + d*x^3]*(-3*a*d + b*(c + d*x^3)))/(9*b^2*d) + (2*a*Sqrt[-(b*c)
+ a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(5/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {948, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{x^3\sqrt{dx^3+c}}{bx^3+a}dx^3$$

$$\Big\downarrow \; 90$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{3/2}}{3bd}-\frac{a\int \frac{\sqrt{dx^3+c}}{bx^3+a}dx^3}{b}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{3/2}}{3bd}-\frac{a\left(\frac{(bc-ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{b}+\frac{2\sqrt{c+dx^3}}{b}\right)}{b}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{3/2}}{3bd}-\frac{a\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd}+\frac{2\sqrt{c+dx^3}}{b}\right)}{b}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{3/2}}{3bd}-\frac{a\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}\right)$$

input
```
Int[(x^5*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
((2*(c + d*x^3)^(3/2))/(3*b*d) - (a*((2*Sqrt[c + d*x^3])/b - (2*Sqrt[b*c -
 a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2)))/b)/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

method	result
default	$\frac{2(dx^3+c)^{\frac{3}{2}}}{9bd}-\frac{2a\left(\sqrt{dx^3+c}-\frac{(ad-bc)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3b^2}$
pseudoelliptic	$\frac{-\frac{2\sqrt{dx^3+c}\,(-bdx^3+3ad-bc)}{9}+\frac{2ad(ad-bc)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}}{d\,b^2}$
risch	$-\frac{2(-bdx^3+3ad-bc)\sqrt{dx^3+c}}{9d\,b^2}+\frac{2(ad-bc)a\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b^2\sqrt{(ad-bc)b}}$
elliptic	$\frac{2x^3\sqrt{dx^3+c}}{9b}+\frac{2\left(-\frac{ad-bc}{b^2}-\frac{2c}{3b}\right)\sqrt{dx^3+c}}{3d}-\frac{ia\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d\cdots\right.}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\cdots}{\cdots}\right)}{\cdots}$

input
```
int(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/9*(d*x^3+c)^(3/2)/b/d-2/3*a/b^2*((d*x^3+c)^(1/2)-(a*d-b*c)*arctan(b*(d*x
^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.10

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx = \left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\,(bdx^3+bc-3\,ad)\sqrt{dx^3+c}}{9\,b^2d}, \frac{2\left(3\,ad\sqrt{-\frac{bc-ad}{b}}\,\mathrm{arctan}\right.}{}\right.$$

input

```
integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

output

```
[1/9*(3*a*d*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3
+ c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(b*d*x^3 + b*c - 3*a*d)*sqrt(
d*x^3 + c))/(b^2*d), 2/9*(3*a*d*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 +
c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b*d*x^3 + b*c - 3*a*d)*sqrt(d*x^
3 + c))/(b^2*d)]
```

Sympy [A] (verification not implemented)

Time = 4.92 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.38

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx = \begin{cases} \frac{2\left(-\frac{ad^2\sqrt{c+dx^3}}{3b^2}+\frac{ad^2(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^3\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^3\right)^{\frac{3}{2}}}{9b}\right)}{d^2} & \text{for } d\neq 0 \\ \sqrt{c}\left(-\frac{a\left(\begin{cases}\frac{x^3}{a} & \text{for } b=0\\ \frac{\log\left(a+bx^3\right)}{b} & \text{otherwise}\end{cases}\right)}{3b}+\frac{x^3}{3b}\right) & \text{otherwise} \end{cases}$$

input

```
integrate(x**5*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

output
```
Piecewise((2*(-a*d**2*sqrt(c + d*x**3)/(3*b**2) + a*d**2*(a*d - b*c)*atan(
sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**3*sqrt((a*d - b*c)/b)) + d*(c
+ d*x**3)**(3/2)/(9*b))/d**2, Ne(d, 0)), (sqrt(c)*(-a*Piecewise((x**3/a, E
q(b, 0)), (log(a + b*x**3)/b, True))/(3*b) + x**3/(3*b)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.03

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx = -\frac{2\,(abc-a^2d)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^2} + \frac{2\left((dx^3+c)^{\frac{3}{2}}b^2d^2-3\,\sqrt{dx^3+c}abd^3\right)}{9\,b^3d^3}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
-2/3*(a*b*c - a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(
-b^2*c + a*b*d)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2*d^2 - 3*sqrt(d*x^3 + c)*
a*b*d^3)/(b^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 4.68 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.46

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{2\,{\left(d\,x^3+c\right)}^{3/2}}{9\,b\,d}-\frac{2\,a\,\sqrt{d\,x^3+c}}{3\,b^2}+\frac{a\,\ln\left(\frac{a^2\,d^2\,1\mathrm{i}+b^2\,c^2\,2\mathrm{i}+2\,\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}-a\,b\,d^2\,x^3\,1\mathrm{i}+b^2\,c\,d\,x^3\,1\mathrm{i}-a\,b\,c\,d\,3\mathrm{i}}{2\,b\,x^3+2\,a}\right)\,\sqrt{a\,d-b\,c}\,1\mathrm{i}}{3\,b^{5/2}}$$

input
```
int((x^5*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

output
```
(2*(c + d*x^3)^(3/2))/(9*b*d) - (2*a*(c + d*x^3)^(1/2))/(3*b^2) + (a*log((
a^2*d^2*1i + b^2*c^2*2i + 2*b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2) -
a*b*d^2*x^3*1i + b^2*c*d*x^3*1i - a*b*c*d*3i)/(2*a + 2*b*x^3))*(a*d - b*c)
^(1/2)*1i)/(3*b^(5/2))
```

Reduce [F]

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{-4\sqrt{d\,x^3+c}\,c+2\sqrt{d\,x^3+c}\,d\,x^3-9\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)a\,d^2+9\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)bcd}{9bd}$$

input
```
int(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x)
```

output
```
( - 4*sqrt(c + d*x**3)*c + 2*sqrt(c + d*x**3)*d*x**3 - 9*int((sqrt(c + d*x
**3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*d**2 + 9*int((sqrt(
c + d*x**3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b*c*d)/(9*b*d)
```

3.533 $\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx=\frac{2\sqrt{c+dx^3}}{3b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}$$

output
```
2/3*(d*x^3+c)^(1/2)/b-2/3*(-a*d+b*c)^(1/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)
/(-a*d+b*c)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx=\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{b^{3/2}}\right)$$

input
```
Integrate[(x^2*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
((2*Sqrt[c + d*x^3])/b - (2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[-(b*c) + a*d]])/b^(3/2))/3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {946, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{bx^3+a}dx^3$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{(bc-ad)\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{b}+\frac{2\sqrt{c+dx^3}}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd}+\frac{2\sqrt{c+dx^3}}{b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)$$

input
```
Int[(x^2*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
((2*Sqrt[c + d*x^3])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^
3])/Sqrt[b*c - a*d]])/b^(3/2))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

method	result
default	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}}{b}$
pseudoelliptic	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}}{b}$
risch	$\frac{2\sqrt{d\,x^{3}+c}}{3b}-\frac{2(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3b\sqrt{(ad-bc)b}}$
elliptic	$\frac{2\sqrt{d\,x^{3}+c}}{3b}+\frac{\left(i\sqrt{2}\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)}}}{}\right.}{}$

input

```
int(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output

```
2/3/b*((d*x^3+c)^(1/2)-(a*d-b*c)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1
/2))/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.23

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx = \left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}}{3\,b}, -\frac{2\left(\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx^3+c}\right)}{3\,b}\right]$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/3*(sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b
*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c))/b, -2/3*(sqrt(-(b*
c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) -
sqrt(d*x^3 + c))/b]
```

Sympy [A] (verification not implemented)

Time = 2.80 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx = \begin{cases} \frac{2\left(\frac{d\sqrt{c+dx^3}}{3b}-\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for } d\neq 0 \\ \sqrt{c}\left(\begin{cases}\frac{x^3}{3a} & \text{for } b=0\\ \frac{\log\left(3a+3bx^3\right)}{3b} & \text{otherwise}\end{cases}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

output
```
Piecewise((2*(d*sqrt(c + d*x**3)/(3*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**
3)/sqrt((a*d - b*c)/b))/(3*b**2*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (sqrt(
c)*Piecewise((x**3/(3*a), Eq(b, 0)), (log(3*a + 3*b*x**3)/(3*b), True)), T
rue))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{2\,(bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b} + \frac{2\,\sqrt{dx^3+c}}{3\,b}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*(b*c - a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*b) + 2/3*sqrt(d*x^3 + c)/b
```

Mupad [B] (verification not implemented)

Time = 4.78 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx=\frac{2\,\sqrt{d\,x^3+c}}{3\,b}+\frac{\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,1{}\mathrm{i}}{3\,b^{3/2}}$$

input
```
int((x^2*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*b) + (log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2
)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1i)/(3*b^
(3/2))
```

Reduce [F]

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx=\frac{2\sqrt{d\,x^3+c}\,c+3\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)a\,d^2-3\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)bcd}{3ad}$$

input
```
int(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x)
```

output
```
(2*sqrt(c + d*x**3)*c + 3*int((sqrt(c + d*x**3)*x**5)/(a*c + a*d*x**3 + b*
c*x**3 + b*d*x**6),x)*a*d**2 - 3*int((sqrt(c + d*x**3)*x**5)/(a*c + a*d*x*
*3 + b*c*x**3 + b*d*x**6),x)*b*c*d)/(3*a*d)
```

3.534 $\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx = -\frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}+\frac{2\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}}$$

output
```
-2/3*c^(1/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a+2/3*(-a*d+b*c)^(1/2)*arcta
nh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx = \frac{2\left(\frac{\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}}-\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)\right)}{3a}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x*(a + b*x^3)),x]
```

output
```
(2*((Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d
]])/Sqrt[b] - Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]))/(3*a)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {948, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^3\left(bx^3+a\right)}dx^3$$

$$\Big\downarrow \text{94}$$

$$\frac{1}{3}\left(\frac{c\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{(bc-ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2c\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2(bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x*(a + b*x^3)),x]
```

output
```
((-2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/a + (2*Sqrt[b*c - a*d]*ArcT
anh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b]))/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{-\frac{2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}+\frac{2(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}}{a}$	71
default	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}}{a}-\frac{2\left(\sqrt{d\,x^{3}+c}-\frac{(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3a}$	98
elliptic	Expression too large to display	1543

input
```
int((d*x^3+c)^(1/2)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/3/a*(-c^(1/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2))+(a*d-b*c)*arctan(b*(d*x^3
+c)^(1/2)/((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 377, normalized size of antiderivative = 4.44

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)}\,dx = \left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{3\,a}, \frac{2\,\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b}{bc-\right.}}{}\right.$$

input
```
integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/3*(sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b
*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c
)*sqrt(c) + 2*c)/x^3))/a, 1/3*(2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 +
 c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^
3 + c)*sqrt(c) + 2*c)/x^3))/a, 1/3*(2*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3
+ c)) + sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)
*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)))/a, 2/3*(sqrt(-(b*c - a*d)/b)*arctan(
-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(-c)*arctan(sqr
t(-c)/sqrt(d*x^3 + c)))/a]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(73) = 146$.

Time = 5.38 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx = \begin{cases} \frac{2\left(\frac{cd\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}+\frac{d(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3ab\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for } d\neq 0 \\ \sqrt{c}\left(-\frac{2b\left(\begin{cases}\frac{\frac{a}{2b}+x^3}{a} & \text{for } b=0\\ -\frac{\log\left(a-2b\left(\frac{a}{2b}+x^3\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{3a}-\frac{2b\left(\begin{cases}\frac{\frac{a}{2b}+x^3}{a} & \text{for } b=0\\ \frac{\log\left(a+2b\left(\frac{a}{2b}+x^3\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{3a}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((d*x**3+c)**(1/2)/x/(b*x**3+a),x)
```

output
```
Piecewise((2*(c*d*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sqrt(-c)) + d*(a*d
- b*c)*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*b*sqrt((a*d - b*c)/
b)))/d, Ne(d, 0)), (sqrt(c)*(-2*b*Piecewise(((a/(2*b) + x**3)/a, Eq(b, 0))
, (-log(a - 2*b*(a/(2*b) + x**3))/(2*b), True))/(3*a) - 2*b*Piecewise(((a/
(2*b) + x**3)/a, Eq(b, 0)), (log(a + 2*b*(a/(2*b) + x**3))/(2*b), True))/(
3*a)), True))
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx = \int \frac{\sqrt{dx^3+c}}{(bx^3+a)x}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx = -\frac{2(bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a} + \frac{2c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}$$

input
```
integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="giac")
```

output
```
-2/3*(b*c - a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2
*c + a*b*d)*a) + 2/3*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 6.70 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx = \frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a} + \frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,1\mathrm{i}}{3\,a\,\sqrt{b}}$$

input
```
int((c + d*x^3)^(1/2)/(x*(a + b*x^3)),x)
```

output
```
(c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)
))/x^6))/(3*a) + (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)
^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1i)/(3*a*b^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}}{b\,x^4+ax}dx$$

input
```
int((d*x^3+c)^(1/2)/x/(b*x^3+a),x)
```

output
```
int(sqrt(c + d*x**3)/(a*x + b*x**4),x)
```

3.535 $\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{3ax^3}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} - \frac{2\sqrt{b}\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2}$$

output
```
-1/3*(d*x^3+c)^(1/2)/a/x^3+1/3*(-a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2
))/a^2/c^(1/2)-2/3*b^(1/2)*(-a*d+b*c)^(1/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2
)/(-a*d+b*c)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)}\,dx = \frac{-\frac{a\sqrt{c+dx^3}}{x^3}-2\sqrt{b}\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^2}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)),x]
```

output
```
(-((a*Sqrt[c + d*x^3])/x^3) - 2*Sqrt[b]*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]
*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]] + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*
x^3]/Sqrt[c]])/Sqrt[c])/(3*a^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 110, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^4 (a+bx^3)} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^6 (bx^3+a)}dx^3$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}\left(\frac{\int -\frac{bdx^3+2bc-ad}{2x^3(bx^3+a)\sqrt{dx^3+c}}dx^3}{a}-\frac{\sqrt{c+dx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{bdx^3+2bc-ad}{x^3(bx^3+a)\sqrt{dx^3+c}}dx^3}{2a}-\frac{\sqrt{c+dx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(-\frac{\frac{(2bc-ad)\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{2b(bc-ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{2a}-\frac{\sqrt{c+dx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{\frac{2(2bc-ad)\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{4b(bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{2a}-\frac{\sqrt{c+dx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{\frac{4\sqrt{b}\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a}-\frac{2(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2a}-\frac{\sqrt{c+dx^3}}{ax^3}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)),x]
```

output
```
(-(Sqrt[c + d*x^3]/(a*x^3)) - ((-2*(2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/S
qrt[c]])/(a*Sqrt[c]) + (4*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x^3])/Sqrt[b*c - a*d]])/a)/(2*a))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{-\frac{a\sqrt{d\,x^{3}+c}}{x^{3}}-\frac{(ad-2bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{2b(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{3a^{2}}$
risch	$-\frac{\sqrt{d\,x^{3}+c}}{3a\,x^{3}}-\frac{\frac{2(ad-2bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{4b(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3a\sqrt{(ad-bc)b}}}{2a}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3x^{3}}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}}{a}+\frac{2b\left(\sqrt{d\,x^{3}+c}-\frac{(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3a^{2}}-\frac{b\left(\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}}}{\sqrt{c}}\right.}{3}\right.}{a^{2}}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/a^2*(-a*(d*x^3+c)^(1/2)/x^3-(a*d-2*b*c)/c^(1/2)*arctanh((d*x^3+c)^(1/2
)/c^(1/2))-2*b*(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*
d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 507, normalized size of antiderivative = 4.41

$$\int \frac{\sqrt{c+dx^3}}{x^4 (a+bx^3)}\,dx$$
$$= \left[\frac{2\sqrt{b^2c-abd}cx^3\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-(2bc-ad)\sqrt{c}x^3\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-2\sqrt{dx}}{6a^2cx^3}\right.$$
$$-\frac{(2bc-ad)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)-\sqrt{b^2c-abd}cx^3\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+\sqrt{dx^3+cac}}{3a^2cx^3}$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(2*sqrt(b^2*c - a*b*d)*c*x^3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^
3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - (2*b*c - a*d)*sqrt(c)*x^3*log((
d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3 + c)*a*c)/(a^
2*c*x^3), 1/6*(4*sqrt(-b^2*c + a*b*d)*c*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^3 + b*c)) - (2*b*c - a*d)*sqrt(c)*x^3*log((d*x^3 - 2*
sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3),
-1/3*((2*b*c - a*d)*sqrt(-c)*x^3*arctan(sqrt(-c)/sqrt(d*x^3 + c)) - sqrt(b
^2*c - a*b*d)*c*x^3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^
2*c - a*b*d))/(b*x^3 + a)) + sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3), 1/3*(2*sqrt
(-b^2*c + a*b*d)*c*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^
3 + b*c)) - (2*b*c - a*d)*sqrt(-c)*x^3*arctan(sqrt(-c)/sqrt(d*x^3 + c)) -
sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4 (a+bx^3)}\,dx = \int \frac{\sqrt{c+dx^3}}{x^4 (a+bx^3)}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**4/(b*x**3+a),x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**4*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)}\,dx=\int \frac{\sqrt{dx^3+c}}{(bx^3+a)x^4}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)}\,dx=\frac{2\,(b^2c-abd)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^2}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}-\frac{\sqrt{dx^3+c}}{3\,ax^3}$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*(b^2*c - a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-
b^2*c + a*b*d)*a^2) - 1/3*(2*b*c - a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(
a^2*sqrt(-c)) - 1/3*sqrt(d*x^3 + c)/(a*x^3)
```

Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)}\,dx=\frac{\ln\left(\frac{a\,d-2\,b\,c+2\,\sqrt{d\,x^3+c}\,\sqrt{b^2\,c-a\,b\,d}-b\,d\,x^3}{b\,x^3+a}\right)\sqrt{b^2\,c-a\,b\,d}}{3\,a^2}-\frac{\sqrt{d\,x^3+c}}{3\,a\,x^3}+\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)(a\,d-2\,b\,c)}{6\,a^2\,\sqrt{c}}$$

input
```
int((c + d*x^3)^(1/2)/(x^4*(a + b*x^3)),x)
```

output
```
(log((a*d - 2*b*c + 2*(c + d*x^3)^(1/2)*(b^2*c - a*b*d)^(1/2) - b*d*x^3)/(
a + b*x^3))*(b^2*c - a*b*d)^(1/2))/(3*a^2) - (c + d*x^3)^(1/2)/(3*a*x^3) +
 (log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)
*(a*d - 2*b*c))/(6*a^2*c^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)}\,dx=\int \frac{\sqrt{d\,x^3+c}}{b\,x^7+a\,x^4}dx$$

input
```
int((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x)
```

output
```
int(sqrt(c + d*x**3)/(a*x**4 + b*x**7),x)
```

3.536 $\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx=\frac{x^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},1,-\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/4*x^4*(d*x^3+c)^(1/2)*AppellF1(4/3,1,-1/2,7/3,-b*x^3/a,-d*x^3/c)/a/(1+d*
x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 241 vs. 2(64) = 128.

Time = 7.29 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.77

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx$$
$$=\frac{x\left(\frac{(3bc-5ad)x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+8\left(c+dx^3+\frac{8a^2c^2\,\text{Appe}}{(a+bx^3)\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bc\right.\right.}\right.\right.}{20b\sqrt{c+dx^3}}$$

input
```
Integrate[(x^3*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
(x*(((3*b*c - 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -(
(d*x^3)/c), -((b*x^3)/a)])/a + 8*(c + d*x^3 + (8*a^2*c^2*AppellF1[1/3, 1/2
, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)])/((a + b*x^3)*(-8*a*c*AppellF1[1/3,
1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2,
 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d
*x^3)/c), -((b*x^3)/a)]))))))/(20*b*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{c+dx^3}\int \frac{x^3\sqrt{\frac{dx^3}{c}+1}}{bx^3+a}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},1,-\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(x^3*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
(x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)])/(4*a*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 2.26 (sec) , antiderivative size = 741, normalized size of antiderivative = 11.58

method	result	size
elliptic	Expression too large to display	741
risch	Expression too large to display	757
default	Expression too large to display	1012

input
```
int(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/5*x/b*(d*x^3+c)^(1/2)-2/3*I*(-(a*d-b*c)/b^2-2/5*c/b)*3^(1/2)/d*(-c*d^2)^
(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/3*I*a/b^2/d^2*2^(1/2)
*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c
*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*
(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-
c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha
*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)
/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx$$

input
```
integrate(x**3*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

output
```
Integral(x**3*sqrt(c + d*x**3)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{dx^3+c}x^3}{bx^3+a}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{dx^3+c}x^3}{bx^3+a}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{x^3\,\sqrt{d\,x^3+c}}{b\,x^3+a}\,dx$$

input
```
int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

output
```
int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{2\sqrt{d\,x^3+c}\,x-2\left(\int \frac{\sqrt{d\,x^3+c}}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)ac-5\left(\int \frac{\sqrt{d\,x^3+c}\,x^3}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)ad+3\left(\int \frac{\sqrt{d\,x^3+c}\,x^3}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)}{5b}$$

input
```
int(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x)
```

output
```
(2*sqrt(c + d*x**3)*x - 2*int(sqrt(c + d*x**3)/(a*c + a*d*x**3 + b*c*x**3
+ b*d*x**6),x)*a*c - 5*int((sqrt(c + d*x**3)*x**3)/(a*c + a*d*x**3 + b*c*x
**3 + b*d*x**6),x)*a*d + 3*int((sqrt(c + d*x**3)*x**3)/(a*c + a*d*x**3 + b
*c*x**3 + b*d*x**6),x)*b*c)/(5*b)
```

3.537 $\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{x^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},1,-\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/2*x^2*(d*x^3+c)^(1/2)*AppellF1(2/3,1,-1/2,5/3,-b*x^3/a,-d*x^3/c)/a/(1+d*
x^3/c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 9.54 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{x^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{2a\sqrt{\frac{c+dx^3}{c}}}$$

input
```
Integrate[(x*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
(x^2*Sqrt[c + d*x^3]*AppellF1[2/3, -1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a
)])/(2*a*Sqrt[(c + d*x^3)/c])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{c+dx^3}\int \frac{x\sqrt{\frac{dx^3}{c}+1}}{bx^3+a}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},1,-\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(x*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
(x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 1, -1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c
)])/(2*a*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.19 (sec) , antiderivative size = 857, normalized size of antiderivative = 13.39

method	result	size
default	Expression too large to display	857
elliptic	Expression too large to display	857

input
```
int(x*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1
/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/b/d^2*2
^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3
*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^
2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_a
lpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx$$

input
```
integrate(x*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

output
```
Integral(x*sqrt(c + d*x**3)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{dx^3+c}x}{bx^3+a}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx=\int \frac{\sqrt{dx^3+c}x}{bx^3+a}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx=\int \frac{x\,\sqrt{d\,x^3+c}}{b\,x^3+a}\,dx$$

input
```
int((x*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

output
```
int((x*(c + d*x^3)^(1/2))/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx=\int \frac{\sqrt{d\,x^3+c\,x}}{b\,x^3+a}dx$$

input
```
int(x*(d*x^3+c)^(1/2)/(b*x^3+a),x)
```

output
```
int((sqrt(c + d*x**3)*x)/(a + b*x**3),x)
```

3.538 $\int \frac{\sqrt{c+dx^3}}{a+bx^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3} \, dx = \frac{x\sqrt{c+dx^3} \operatorname{AppellF1}\left(\frac{1}{3},1,-\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{1+\frac{dx^3}{c}}}$$

output
```
x*(d*x^3+c)^(1/2)*AppellF1(1/3,1,-1/2,4/3,-b*x^3/a,-d*x^3/c)/a/(1+d*x^3/c)
^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.

Time = 10.18 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3} \, dx$$
$$= \frac{8acx\sqrt{c+dx^3} \operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\left(8ac \operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(-2bc \operatorname{AppellF1}\left(\frac{4}{3},-\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+ad\, A\right.\right.}$$

input
```
Integrate[Sqrt[c + d*x^3]/(a + b*x^3),x]
```

output
```
(8*a*c*x*Sqrt[c + d*x^3]*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -((b*x^
3)/a)])/((a + b*x^3)*(8*a*c*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -((b
*x^3)/a)] + 3*x^3*(-2*b*c*AppellF1[4/3, -1/2, 2, 7/3, -((d*x^3)/c), -((b*x
^3)/a)] + a*d*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt{c+dx^3}\int \frac{\sqrt{\frac{dx^3}{c}+1}}{bx^3+a}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},1,-\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^3]/(a + b*x^3),x]
```

output
```
(x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(a*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.05 (sec) , antiderivative size = 705, normalized size of antiderivative = 11.95

method	result	size
default	Expression too large to display	705
elliptic	Expression too large to display	705

input
```
int((d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(
-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
))+1/3*I/b/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I
*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d
)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx=\text{Timed out}$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/(b*x**3+a),x)
```

output
```
Integral(sqrt(c + d*x**3)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{dx^3+c}}{bx^3+a}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{dx^3+c}}{bx^3+a}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx=\int \frac{\sqrt{d\,x^3+c}}{b\,x^3+a}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(a + b*x^3),x)
```

output
```
int((c + d*x^3)^(1/2)/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx=\int \frac{\sqrt{d\,x^3+c}}{b\,x^3+a}dx$$

input
```
int((d*x^3+c)^(1/2)/(b*x^3+a),x)
```

output
```
int(sqrt(c + d*x**3)/(a + b*x**3),x)
```

3.539 $\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)}\,dx=-\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},1,-\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{1+\frac{dx^3}{c}}}$$

output
```
-(d*x^3+c)^(1/2)*AppellF1(-1/3,1,-1/2,2/3,-b*x^3/a,-d*x^3/c)/a/x/(1+d*x^3/
c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 139 vs. 2(62) = 124.

Time = 10.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.24

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)}\,dx$$
$$=\frac{-20a(c+dx^3)+5(-2bc+3ad)x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bdx^6\sqrt{1+\frac{dx^3}{c}}\,\text{Appell}}{20a^2x\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)),x]
```

output
```
(-20*a*(c + d*x^3) + 5*(-2*b*c + 3*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2
/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*x^6*Sqrt[1 + (d*x^3)/
c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(20*a^2*x*Sqrt[
c + d*x^3])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{c+dx^3}\int \frac{\sqrt{\frac{dx^3}{c}+1}}{x^2(bx^3+a)}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},1,-\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)),x]
```

output
```
-((Sqrt[c + d*x^3]*AppellF1[-1/3, 1, -1/2, 2/3, -((b*x^3)/a), -((d*x^3)/c)
])/(a*x*Sqrt[1 + (d*x^3)/c]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 2.34 (sec) , antiderivative size = 892, normalized size of antiderivative = 14.39

method	result	size
elliptic	Expression too large to display	892
risch	Expression too large to display	893
default	Expression too large to display	1314

input
```
int((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/a*(d*x^3+c)^(1/2)/x-1/3*I/a*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x
-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)
/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2
)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)))+1/3*I/a/d^2*2^(1/2)*sum((-a*d+b*c)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3
)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(
1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2
)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)
*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2
)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/
3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(...
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   Not
integrable (provided residues have no relations)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\,(a+bx^3)}\,dx=\int \frac{\sqrt{c+dx^3}}{x^2\,(a+bx^3)}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**2/(b*x**3+a),x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\,(a+bx^3)}\,dx=\int \frac{\sqrt{dx^3+c}}{(bx^3+a)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\,(a+bx^3)}\,dx=\int \frac{\sqrt{dx^3+c}}{(bx^3+a)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^2\,(a+bx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}}{x^2\,\left(b\,x^3+a\right)}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\,(a+bx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}}{b\,x^5+a\,x^2}dx$$

input
```
int((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x)
```

output
```
int(sqrt(c + d*x**3)/(a*x**2 + b*x**5),x)
```

3.540 $\int \frac{\sqrt{c+dx^3}}{x^3(a+bx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},1,-\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
-1/2*(d*x^3+c)^(1/2)*AppellF1(-2/3,1,-1/2,1/3,-b*x^3/a,-d*x^3/c)/a/x^2/(1+
d*x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 335 vs. 2(64) = 128.

Time = 10.29 (sec) , antiderivative size = 335, normalized size of antiderivative = 5.23

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx = \frac{-bdx^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a\left(32ac\left(2ac+6bcx^3-adx^3+2bdx^6\right)\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-\right.}{\left(a+bx^3\right)\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\right.}}{16a^2x^2\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)),x]
```

output
```
(-(b*d*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -(
(b*x^3)/a)]) + (a*(32*a*c*(2*a*c + 6*b*c*x^3 - a*d*x^3 + 2*b*d*x^6)*Appell
F1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*x^3*(a + b*x^3)*(c +
 d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*
d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(
-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b
*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4
/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(16*a^2*x^2*Sqrt[c + d*x^
3])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{c+dx^3}\int \frac{\sqrt{\frac{dx^3}{c}+1}}{x^3(bx^3+a)}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},1,-\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)),x]
```

output
```
-1/2*(Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -1/2, 1/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*x^2*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 2.41 (sec) , antiderivative size = 740, normalized size of antiderivative = 11.56

method	result	size
elliptic	Expression too large to display	740
risch	Expression too large to display	741
default	Expression too large to display	1010

input
```
int((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2*(d*x^3+c)^(1/2)+1/6*I/a*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*I/a/d^2*2^(1/2)*sum((-a*d+b*c)/_alpha^2/(
a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(
1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2
)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^
(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2
*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2
)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),
_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   Not
integrable (provided residues have no relations)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3(a+bx^3)}\,dx=\int \frac{\sqrt{c+dx^3}}{x^3(a+bx^3)}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**3/(b*x**3+a),x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**3*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3(a+bx^3)}\,dx=\int \frac{\sqrt{dx^3+c}}{(bx^3+a)x^3}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^3), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3(a+bx^3)}\,dx=\int \frac{\sqrt{dx^3+c}}{(bx^3+a)x^3}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^3\,\left(b\,x^3+a\right)}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx=\int \frac{\sqrt{d\,x^3+c}}{b\,x^6+a\,x^3}dx$$

input
```
int((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x)
```

output
```
int(sqrt(c + d*x**3)/(a*x**3 + b*x**6),x)
```

3.541 $\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 154

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4}+\frac{2a^2(c+dx^3)^{3/2}}{9b^3} - \frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2}+\frac{2(c+dx^3)^{7/2}}{21bd^2}-\frac{2a^2(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}$$

output
```
2/3*a^2*(-a*d+b*c)*(d*x^3+c)^(1/2)/b^4+2/9*a^2*(d*x^3+c)^(3/2)/b^3-2/15*(a
*d+b*c)*(d*x^3+c)^(5/2)/b^2/d^2+2/21*(d*x^3+c)^(7/2)/b/d^2-2/3*a^2*(-a*d+b
*c)^(3/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.93

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2\sqrt{c+dx^3}\left(-105a^3d^3-21ab^2d(c+dx^3)^2-3b^3(2c-5dx^3)\left(c+dx^3\right)^2+35a^2bd^2(4\right.}{315b^4d^2} + \frac{2a^2(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{9/2}}$$

input
```
Integrate[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output
```
(2*Sqrt[c + d*x^3]*(-105*a^3*d^3 - 21*a*b^2*d*(c + d*x^3)^2 - 3*b^3*(2*c -
 5*d*x^3)*(c + d*x^3)^2 + 35*a^2*b*d^2*(4*c + d*x^3)))/(315*b^4*d^2) + (2*
a^2*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*
d]])/(3*b^(9/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 \left(c+dx^3\right)^{3/2}}{a+bx^3} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^6 \left(dx^3+c\right)^{3/2}}{bx^3+a}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3} \int \left(\frac{\left(dx^3+c\right)^{5/2}}{bd}+\frac{(-bc-ad)\left(dx^3+c\right)^{3/2}}{b^2d}+\frac{a^2\left(dx^3+c\right)^{3/2}}{b^2\left(bx^3+a\right)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{2a^2(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{9/2}}+\frac{2a^2\sqrt{c+dx^3}(bc-ad)}{b^4}+\frac{2a^2\left(c+dx^3\right)^{3/2}}{3b^3}-\frac{2\left(c+dx^3\right)^{5/2}(ad+bc}{5b^2d^2}\right.$$

input
```
Int[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output
```
((2*a^2*(b*c - a*d)*Sqrt[c + d*x^3])/b^4 + (2*a^2*(c + d*x^3)^(3/2))/(3*b^
3) - (2*(b*c + a*d)*(c + d*x^3)^(5/2))/(5*b^2*d^2) + (2*(c + d*x^3)^(7/2))
/(7*b*d^2) - (2*a^2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sq
rt[b*c - a*d]])/b^(9/2))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$-\frac{2\left(\sqrt{(ad-bc)b}\left(\frac{2\left(d\,x^{3}+c\right)^{2}\left(-\frac{5d\,x^{3}}{2}+c\right)b^{3}}{35}+\frac{ad\left(d\,x^{3}+c\right)^{2}b^{2}}{5}-\frac{4a^{2}d^{2}\left(\frac{d\,x^{3}}{4}+c\right)b}{3}+a^{3}d^{3}\right)\sqrt{d\,x^{3}+c}-a^{2}d^{2}(ad-bc)^{2}\arctan}{3\sqrt{(ad-bc)b}\,d^{2}b^{4}}$
default	$\frac{\frac{2d\,x^{9}\sqrt{d\,x^{3}+c}}{21}+\frac{16c\,x^{6}\sqrt{d\,x^{3}+c}}{105}+\frac{2c^{2}x^{3}\sqrt{d\,x^{3}+c}}{105d}-\frac{4c^{3}\sqrt{d\,x^{3}+c}}{105d^{2}}}{b}-\frac{2a^{2}\left(-(ad-bc)^{2}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)+\left(\frac{\left(-d\,x^{3}-4c\right)b}{3}+a\right.\right.}{3b^{4}\sqrt{(ad-bc)b}}$
risch	$-\frac{2(-15b^{3}d^{3}x^{9}+21a\,b^{2}d^{3}x^{6}-24b^{3}c\,d^{2}x^{6}-35a^{2}b\,d^{3}x^{3}+42a\,b^{2}c\,d^{2}x^{3}-3b^{3}c^{2}d\,x^{3}+105a^{3}d^{3}-140a^{2}bc\,d^{2}+21a\,b^{2}c^{2}d+6b^{3}c^{3}}{315d^{2}b^{4}}$
elliptic	$\frac{2d\,x^{9}\sqrt{d\,x^{3}+c}}{21b}+\frac{2\left(-\frac{d(ad-2bc)}{b^{2}}-\frac{6cd}{7b}\right)x^{6}\sqrt{d\,x^{3}+c}}{15d}+\frac{2\left(\frac{a^{2}d^{2}-2abcd+b^{2}c^{2}}{b^{3}}-\frac{4\left(-\frac{d(ad-2bc)}{b^{2}}-\frac{6cd}{7b}\right)c}{5d}\right)x^{3}\sqrt{d\,x^{3}+c}}{9d}+-$ 2

input

```
int(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output

```
-2/3/((a*d-b*c)*b)^(1/2)*(((a*d-b*c)*b)^(1/2)*(2/35*(d*x^3+c)^2*(-5/2*d*x^
3+c)*b^3+1/5*a*d*(d*x^3+c)^2*b^2-4/3*a^2*d^2*(1/4*d*x^3+c)*b+a^3*d^3)*(d*x
^3+c)^(1/2)-a^2*d^2*(a*d-b*c)^2*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/
2)))/d^2/b^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.66

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \left[-\frac{105\,(a^2bcd^2-a^3d^3)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\,(15\,b^3d^3x^9+3\right.$$

$$-\frac{2\left(105\,(a^2bcd^2-a^3d^3)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(15\,b^3d^3x^9+3\,(8\,b^3cd^2-7\,ab^2d^3)x^6-6\,b\right.}{315\,b^4d^2}$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/315*(105*(a^2*b*c*d^2 - a^3*d^3)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*
b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(15*
b^3*d^3*x^9 + 3*(8*b^3*c*d^2 - 7*a*b^2*d^3)*x^6 - 6*b^3*c^3 - 21*a*b^2*c^2
*d + 140*a^2*b*c*d^2 - 105*a^3*d^3 + (3*b^3*c^2*d - 42*a*b^2*c*d^2 + 35*a^
2*b*d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*d^2), -2/315*(105*(a^2*b*c*d^2 - a^3*d
^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b
*c - a*d)) - (15*b^3*d^3*x^9 + 3*(8*b^3*c*d^2 - 7*a*b^2*d^3)*x^6 - 6*b^3*c
^3 - 21*a*b^2*c^2*d + 140*a^2*b*c*d^2 - 105*a^3*d^3 + (3*b^3*c^2*d - 42*a*
b^2*c*d^2 + 35*a^2*b*d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*d^2)]
```

Sympy [A] (verification not implemented)

Time = 41.59 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.31

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \begin{cases} \frac{2\left(\frac{a^2d(c+dx^3)^{\frac{3}{2}}}{9b^3}+\frac{a^2d(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^5\sqrt{\frac{ad-bc}{b}}}+\frac{(c+dx^3)^{\frac{7}{2}}}{21bd}+\frac{(c+dx^3)^{\frac{5}{2}}(-ad-bc)}{15b^2d}+\frac{\sqrt{c+dx^3}(-a^3d^2+a^2bcd)}{3b^4}\right)}{d} \\ c^{\frac{3}{2}}\left(\frac{a^2\left(\begin{cases}\frac{x^3}{a} & \text{for } b=0\\ \frac{\log(a+bx^3)}{b} & \text{otherwise}\end{cases}\right)}{3b^2}-\frac{ax^3}{3b^2}+\frac{x^6}{6b}\right) \end{cases}$$

input
```
integrate(x**8*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

output
```
Piecewise((2*(a**2*d*(c + d*x**3)**(3/2)/(9*b**3) + a**2*d*(a*d - b*c)**2*
atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**5*sqrt((a*d - b*c)/b)) +
(c + d*x**3)**(7/2)/(21*b*d) + (c + d*x**3)**(5/2)*(-a*d - b*c)/(15*b**2*d
) + sqrt(c + d*x**3)*(-a**3*d**2 + a**2*b*c*d)/(3*b**4))/d, Ne(d, 0)), (c*
*(3/2)*(a**2*Piecewise((x**3/a, Eq(b, 0)), (log(a + b*x**3)/b, True))/(3*b
**2) - a*x**3/(3*b**2) + x**6/(6*b)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.25

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2\,(a^2b^2c^2-2\,a^3bcd+a^4d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^4} + \frac{2\left(15\,(dx^3+c)^{\frac{7}{2}}b^6d^{12}-21\,(dx^3+c)^{\frac{5}{2}}b^6cd^{12}-21\,(dx^3+c)^{\frac{5}{2}}ab^5d^{13}+35\,(dx^3+c)^{\frac{3}{2}}a^2b^4d^{14}+105\,\sqrt{dx^3+}\right.}{315\,b^7d^{14}}$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b
^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) + 2/315*(15*(d*x^3 + c)^(7/2)*b^
6*d^12 - 21*(d*x^3 + c)^(5/2)*b^6*c*d^12 - 21*(d*x^3 + c)^(5/2)*a*b^5*d^13
 + 35*(d*x^3 + c)^(3/2)*a^2*b^4*d^14 + 105*sqrt(d*x^3 + c)*a^2*b^4*c*d^14
- 105*sqrt(d*x^3 + c)*a^3*b^3*d^15)/(b^7*d^14)
```

Mupad [B] (verification not implemented)

Time = 4.61 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.14

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2\,d\,x^9\,\sqrt{d\,x^3+c}}{21\,b} - \frac{\left(\frac{2\,a\left(\frac{c^2}{b}+\frac{a\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}\right)}{b}+\frac{2\,c\left(\frac{2\,c^2}{b}+\frac{2\,a\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}+\frac{4\,c\left(\frac{2\,a\,d^2}{b^2}-\frac{16\,c\,d}{7\,b}\right)}{5\,d}\right)}{3\,d}\right)\sqrt{d\,x^3+c}}{3\,d} + \frac{x^3\,\sqrt{d\,x^3+c}\left(\frac{2\,c^2}{b}+\frac{2\,a\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}+\frac{4\,c\left(\frac{2\,a\,d^2}{b^2}-\frac{16\,c\,d}{7\,b}\right)}{5\,d}\right)}{9\,d} - \frac{x^6\,\sqrt{d\,x^3+c}\left(\frac{2\,a\,d^2}{b^2}-\frac{16\,c\,d}{7\,b}\right)}{15\,d} + \frac{a^2\,\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d-\sqrt{b}\,\sqrt{d\,x^3+c}\,(a\,d-b\,c)^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)(a\,d-b\,c)^{3/2}\,1{}\mathrm{i}}{3\,b^{9/2}}$$

input
```
int((x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

output
```
(2*d*x^9*(c + d*x^3)^(1/2))/(21*b) - (((2*a*(c^2/b + (a*((a*d^2)/b^2 - (2*
c*d)/b))/b))/b + (2*c*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b + (4*
c*((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(5*d)))/(3*d))*(c + d*x^3)^(1/2))/(3*d
) + (x^3*(c + d*x^3)^(1/2)*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b
+ (4*c*((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(5*d)))/(9*d) - (x^6*(c + d*x^3)^
(1/2)*((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(15*d) + (a^2*log((a^2*d^2 + 2*b^2
*c^2 - b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*b*d^2*x^3 + b^2*
c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*b^(9/2))
```

Reduce [F]

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{-140\sqrt{d\,x^3+c}\,a^2c\,d^2+70\sqrt{d\,x^3+c}\,a^2d^3x^3+168\sqrt{d\,x^3+c}\,ab\,c^2d-84\sqrt{d\,x^3+c}\,a}{}$$

input
```
int(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x)
```

output
```
( - 140*sqrt(c + d*x**3)*a**2*c*d**2 + 70*sqrt(c + d*x**3)*a**2*d**3*x**3
+ 168*sqrt(c + d*x**3)*a*b*c**2*d - 84*sqrt(c + d*x**3)*a*b*c*d**2*x**3 -
42*sqrt(c + d*x**3)*a*b*d**3*x**6 - 12*sqrt(c + d*x**3)*b**2*c**3 + 6*sqrt
(c + d*x**3)*b**2*c**2*d*x**3 + 48*sqrt(c + d*x**3)*b**2*c*d**2*x**6 + 30*
sqrt(c + d*x**3)*b**2*d**3*x**9 - 315*int((sqrt(c + d*x**3)*x**5)/(a*c + a
*d*x**3 + b*c*x**3 + b*d*x**6),x)*a**3*d**4 + 630*int((sqrt(c + d*x**3)*x*
*5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a**2*b*c*d**3 - 315*int((sqr
t(c + d*x**3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b**2*c**2*
d**2)/(315*b**3*d**2)
```

3.542 $\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx = -\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3} - \frac{2a(c+dx^3)^{3/2}}{9b^2} + \frac{2(c+dx^3)^{5/2}}{15bd} + \frac{2a(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

output
```
-2/3*a*(-a*d+b*c)*(d*x^3+c)^(1/2)/b^3-2/9*a*(d*x^3+c)^(3/2)/b^2+2/15*(d*x^
3+c)^(5/2)/b/d+2/3*a*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*
d+b*c)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2\sqrt{c+dx^3}\left(15a^2d^2+3b^2(c+dx^3)^2-5abd(4c+dx^3)\right)}{45b^3d} - \frac{2a(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{7/2}}$$

input
```
Integrate[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output
```
(2*Sqrt[c + d*x^3]*(15*a^2*d^2 + 3*b^2*(c + d*x^3)^2 - 5*a*b*d*(4*c + d*x^
3)))/(45*b^3*d) - (2*a*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3
])/Sqrt[-(b*c) + a*d]])/(3*b^(7/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(c+dx^3\right)^{3/2}}{a+bx^3} \, dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3} \int \frac{x^3 \left(dx^3+c\right)^{3/2}}{bx^3+a}dx^3$$

$$\Big\downarrow \; 90$$

$$\frac{1}{3} \left(\frac{2 \left(c+dx^3\right)^{5/2}}{5bd}-\frac{a \int \frac{\left(dx^3+c\right)^{3/2}}{bx^3+a}dx^3}{b}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3} \left(\frac{2 \left(c+dx^3\right)^{5/2}}{5bd}-\frac{a \left(\frac{(bc-ad) \int \frac{\sqrt{dx^3+c}}{bx^3+a}dx^3}{b}+\frac{2 \left(c+dx^3\right)^{3/2}}{3b}\right)}{b}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{5/2}}{5bd}-\frac{a\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{b}+\frac{2\sqrt{c+dx^3}}{b}\right)}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)}{b}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{5/2}}{5bd}-\frac{a\left(\frac{(bc-ad)\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd}+\frac{2\sqrt{c+dx^3}}{b}\right)}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)}{b}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{5/2}}{5bd}-\frac{a\left(\frac{(bc-ad)\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)}{b}\right)$$

input
```
Int[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output
```
((2*(c + d*x^3)^(5/2))/(5*b*d) - (a*((2*(c + d*x^3)^(3/2))/(3*b) + ((b*c -
 a*d)*((2*Sqrt[c + d*x^3]])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x^3])/Sqrt[b*c - a*d]])/b^(3/2)))/b))/b)/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.94

method	result
default	$\frac{2(d x^3+c)^{\frac{5}{2}}}{15bd}+\frac{2a\left(-(ad-bc)^2 \arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(\frac{\left(-d x^3-4c\right)b}{3}+ad\right)\sqrt{d x^3+c}\,\sqrt{(ad-bc)b}\right)}{3b^3\sqrt{(ad-bc)b}}$
pseudoelliptic	$-\frac{2\left(-\left(\frac{b^2\left(d x^3+c\right)^2}{5}-\frac{4ad\left(\frac{d x^3}{4}+c\right)b}{3}+a^2d^2\right)\sqrt{(ad-bc)b}\,\sqrt{d x^3+c}+ad(ad-bc)^2 \arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)\right)}{3\sqrt{(ad-bc)b}\,d\,b^3}$
risch	$\frac{2(3b^2d^2x^6-5x^3ab\,d^2+6x^3b^2cd+15a^2d^2-20abcd+3b^2c^2)\sqrt{d x^3+c}}{45d\,b^3}-\frac{2a(a^2d^2-2abcd+b^2c^2) \arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b^3\sqrt{(ad-bc)b}}$
elliptic	$\frac{2d\,x^6\sqrt{d x^3+c}}{15b}+\frac{2\left(-\frac{d(ad-2bc)}{b^2}-\frac{4cd}{5b}\right)x^3\sqrt{d x^3+c}}{9d}+\frac{2\left(\frac{a^2d^2-2abcd+b^2c^2}{b^3}-\frac{2\left(-\frac{d(ad-2bc)}{b^2}-\frac{4cd}{5b}\right)c}{3d}\right)\sqrt{d x^3+c}}{3d}+$ … $ia\sqrt{2}$ …

input
```
int(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/15*(d*x^3+c)^(5/2)/b/d+2/3*a/b^3*(-(a*d-b*c)^2*arctan(b*(d*x^3+c)^(1/2)/
((a*d-b*c)*b)^(1/2))+(1/3*(-d*x^3-4*c)*b+a*d)*(d*x^3+c)^(1/2)*((a*d-b*c)*b
)^(1/2))/((a*d-b*c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.48

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx = \left[-\frac{15\,(abcd-a^2d^2)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\,(3\,b^2d^2x^6+3\,b^2c^2}{45\,b^3d}\right.$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/45*(15*(a*b*c*d - a^2*d^2)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c -
a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(3*b^2*d^2
*x^6 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^3)*
sqrt(d*x^3 + c))/(b^3*d), 2/45*(15*(a*b*c*d - a^2*d^2)*sqrt(-(b*c - a*d)/b
)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (3*b^2*d^2
*x^6 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^3)*
sqrt(d*x^3 + c))/(b^3*d)]
```

Sympy [A] (verification not implemented)

Time = 20.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.28

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx = \begin{cases} \frac{2\left(-\frac{ad(c+dx^3)^{\frac{3}{2}}}{9b^2}-\frac{ad(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^4\sqrt{\frac{ad-bc}{b}}}+\frac{(c+dx^3)^{\frac{5}{2}}}{15b}+\frac{\sqrt{c+dx^3}(a^2d^2-abcd)}{3b^3}\right)}{d} & \text{for } d\neq 0 \\ c^{\frac{3}{2}}\left(-\frac{a\left(\begin{cases}\frac{x^3}{a} & \text{for } b=0\\ \frac{\log(a+bx^3)}{b} & \text{otherwise}\end{cases}\right)}{3b}+\frac{x^3}{3b}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(x**5*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

output
```
Piecewise((2*(-a*d*(c + d*x**3)**(3/2)/(9*b**2) - a*d*(a*d - b*c)**2*atan(
sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**4*sqrt((a*d - b*c)/b)) + (c +
d*x**3)**(5/2)/(15*b) + sqrt(c + d*x**3)*(a**2*d**2 - a*b*c*d)/(3*b**3))/d
, Ne(d, 0)), (c**(3/2)*(-a*Piecewise((x**3/a, Eq(b, 0)), (log(a + b*x**3)/
b, True))/(3*b) + x**3/(3*b)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.26

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx = -\frac{2\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^3} + \frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}b^4d^4-5\left(dx^3+c\right)^{\frac{3}{2}}ab^3d^5-15\sqrt{dx^3+c}ab^3cd^5+15\sqrt{dx^3+c}a^2b^2d^6\right)}{45\,b^5d^5}$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
-2/3*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^
2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 2/45*(3*(d*x^3 + c)^(5/2)*b^4*d
^4 - 5*(d*x^3 + c)^(3/2)*a*b^3*d^5 - 15*sqrt(d*x^3 + c)*a*b^3*c*d^5 + 15*s
qrt(d*x^3 + c)*a^2*b^2*d^6)/(b^5*d^5)
```

Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.79

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,c^2}{b}+\frac{2\,a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}+\frac{2\,c\,\left(\frac{2\,a\,d^2}{b^2}-\frac{12\,c\,d}{5\,b}\right)}{3\,d}\right)}{3\,d} + \frac{2\,d\,x^6\,\sqrt{d\,x^3+c}}{15\,b} - \frac{x^3\,\sqrt{d\,x^3+c}\,\left(\frac{2\,a\,d^2}{b^2}-\frac{12\,c\,d}{5\,b}\right)}{9\,d} + \frac{a\,\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d+\sqrt{b}\,\sqrt{d\,x^3+c}\,(a\,d-b\,c)^{3/2}\,2\mathrm{i}}{b\,x^3+a}\right)\,(a\,d-b\,c)^{3/2}\,1\mathrm{i}}{3\,b^{7/2}}$$

input
```
int((x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

output
```
((c + d*x^3)^(1/2)*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b + (2*c*(
(2*a*d^2)/b^2 - (12*c*d)/(5*b)))/(3*d)))/(3*d) + (2*d*x^6*(c + d*x^3)^(1/2
))/(15*b) - (x^3*(c + d*x^3)^(1/2)*((2*a*d^2)/b^2 - (12*c*d)/(5*b)))/(9*d)
 + (a*log((a^2*d^2 + 2*b^2*c^2 + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/
2)*2i - a*b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3
/2)*1i)/(3*b^(7/2))
```

Reduce [F]

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{20\sqrt{d\,x^3+c}\,acd-10\sqrt{d\,x^3+c}\,a\,d^2x^3-24\sqrt{d\,x^3+c}\,b\,c^2+12\sqrt{d\,x^3+c}\,bcd\,x^3+6\cdots}{\cdots}$$

input
```
int(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x)
```

output
```
(20*sqrt(c + d*x**3)*a*c*d - 10*sqrt(c + d*x**3)*a*d**2*x**3 - 24*sqrt(c +
 d*x**3)*b*c**2 + 12*sqrt(c + d*x**3)*b*c*d*x**3 + 6*sqrt(c + d*x**3)*b*d*
*2*x**6 + 45*int((sqrt(c + d*x**3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*
x**6),x)*a**2*d**3 - 90*int((sqrt(c + d*x**3)*x**5)/(a*c + a*d*x**3 + b*c*
x**3 + b*d*x**6),x)*a*b*c*d**2 + 45*int((sqrt(c + d*x**3)*x**5)/(a*c + a*d
*x**3 + b*c*x**3 + b*d*x**6),x)*b**2*c**2*d)/(45*b**2*d)
```

3.543 $\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 96

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3} dx = \frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2} + \frac{2(c+dx^3)^{3/2}}{9b} - \frac{2(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}$$

output
```
2/3*(-a*d+b*c)*(d*x^3+c)^(1/2)/b^2+2/9*(d*x^3+c)^(3/2)/b-2/3*(-a*d+b*c)^(3
/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3} dx = \frac{2\sqrt{c+dx^3}(4bc-3ad+bdx^3)}{9b^2} + \frac{2(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{5/2}}$$

input
```
Integrate[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output
```
(2*Sqrt[c + d*x^3]*(4*b*c - 3*a*d + b*d*x^3))/(9*b^2) + (2*(-(b*c) + a*d)^
(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(5/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {946, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (c+dx^3)^{3/2}}{a+bx^3} \, dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{3} \int \frac{(dx^3+c)^{3/2}}{bx^3+a} dx^3$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3} \left(\frac{(bc-ad) \int \frac{\sqrt{dx^3+c}}{bx^3+a} dx^3}{b} + \frac{2(c+dx^3)^{3/2}}{3b} \right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3} \left(\frac{(bc-ad) \left(\frac{(bc-ad) \int \frac{1}{(bx^3+a)\sqrt{dx^3+c}} dx^3}{b} + \frac{2\sqrt{c+dx^3}}{b} \right)}{b} + \frac{2(c+dx^3)^{3/2}}{3b} \right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3} \left(\frac{(bc-ad) \left(\frac{2(bc-ad) \int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}} d\sqrt{dx^3+c}}{bd} + \frac{2\sqrt{c+dx^3}}{b} \right)}{b} + \frac{2(c+dx^3)^{3/2}}{3b} \right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(\frac{(bc-ad)\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}+\frac{2(c+dx^3)^{3/2}}{3b}\right)$$

input
```
Int[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output
```
((2*(c + d*x^3)^(3/2))/(3*b) + ((b*c - a*d)*((2*Sqrt[c + d*x^3])/b - (2*Sq
rt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2))
)/b)/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98

method	result
default	$-\frac{2\left(-(ad-bc)^2\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(\frac{(-dx^3-4c)b}{3}+ad\right)\sqrt{dx^3+c}\,\sqrt{(ad-bc)b}\right)}{3\sqrt{(ad-bc)b}\,b^2}$
risch	$-\frac{2\sqrt{dx^3+c}\,(-bdx^3+3ad-4bc)}{9b^2}+\frac{2(a^2d^2-2abcd+b^2c^2)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b^2\sqrt{(ad-bc)b}}$
pseudoelliptic	$-\frac{2\left(-(ad-bc)^2\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(\frac{(-dx^3-4c)b}{3}+ad\right)\sqrt{dx^3+c}\,\sqrt{(ad-bc)b}\right)}{3\sqrt{(ad-bc)b}\,b^2}$
elliptic	$\frac{2dx^3\sqrt{dx^3+c}}{9b}+\frac{2\left(-\frac{d(ad-2bc)}{b^2}-\frac{2cd}{3b}\right)\sqrt{dx^3+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\left(-cd^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(\cdots\right.}{\cdots}}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(a*d-b*c)^2*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(1/3*(-d*
x^3-4*c)*b+a*d)*(d*x^3+c)^(1/2)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/b
^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.12

$$\int \frac{x^2 (c+dx^3)^{3/2}}{a+bx^3} \, dx = \left[-\frac{3\,(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right) - 2\,(bdx^3+4\,bc-3\,ad)\sqrt{c}}{9\,b^2}\right.$$

$$\left. -\frac{2\left(3\,(bc-ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right) - (bdx^3+4\,bc-3\,ad)\sqrt{dx^3+c}\right)}{9\,b^2}\right]$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/9*(3*(b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sq
rt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(b*d*x^3 + 4*b*c - 3
*a*d)*sqrt(d*x^3 + c))/b^2, -2/9*(3*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arcta
n(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (b*d*x^3 + 4*b*c
- 3*a*d)*sqrt(d*x^3 + c))/b^2]
```

Sympy [A] (verification not implemented)

Time = 9.97 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.29

$$\int \frac{x^2 (c+dx^3)^{3/2}}{a+bx^3} \, dx = \begin{cases} \frac{2\left(\frac{d(c+dx^3)^{\frac{3}{2}}}{9b} + \frac{\sqrt{c+dx^3}\left(-ad^2+bcd\right)}{3b^2} + \frac{d(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^3\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for } d \neq 0 \\ c^{\frac{3}{2}}\left(\begin{cases} \frac{x^3}{3a} & \text{for } b = 0 \\ \frac{\log(3a+3bx^3)}{3b} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

output
```
Piecewise((2*(d*(c + d*x**3)**(3/2)/(9*b) + sqrt(c + d*x**3)*(-a*d**2 + b*
c*d)/(3*b**2) + d*(a*d - b*c)**2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b)
)/(3*b**3*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (c**(3/2)*Piecewise((x**3/(3
*a), Eq(b, 0)), (log(3*a + 3*b*x**3)/(3*b), True)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^2} + \frac{2\,\left((dx^3+c)^{\frac{3}{2}}b^2+3\,\sqrt{dx^3+c}b^2c-3\,\sqrt{dx^3+c}abd\right)}{9\,b^3}$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c +
 a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2 + 3*sqrt(
d*x^3 + c)*b^2*c - 3*sqrt(d*x^3 + c)*a*b*d)/b^3
```

Mupad [B] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.49

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2\,d\,x^3\,\sqrt{d\,x^3+c}}{9\,b} - \frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,a\,d^2}{b^2}-\frac{8\,c\,d}{3\,b}\right)}{3\,d} + \frac{\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d-\sqrt{b}\,\sqrt{d\,x^3+c}\,(a\,d-b\,c)^{3/2}\,2\mathrm{i}}{b\,x^3+a}\right)\,(a\,d-b\,c)^{3/2}\,1\mathrm{i}}{3\,b^{5/2}}$$

input
```
int((x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

output
```
(log((a^2*d^2 + 2*b^2*c^2 - b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i
 - a*b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1
i)/(3*b^(5/2)) - ((c + d*x^3)^(1/2)*((2*a*d^2)/b^2 - (8*c*d)/(3*b)))/(3*d)
 + (2*d*x^3*(c + d*x^3)^(1/2))/(9*b)
```

Reduce [F]

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{-4\sqrt{d\,x^3+c}\,acd+2\sqrt{d\,x^3+c}\,a\,d^2x^3+6\sqrt{d\,x^3+c}\,b\,c^2-9\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)}{9abd}$$

input
```
int(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x)
```

output
```
( - 4*sqrt(c + d*x**3)*a*c*d + 2*sqrt(c + d*x**3)*a*d**2*x**3 + 6*sqrt(c +
 d*x**3)*b*c**2 - 9*int((sqrt(c + d*x**3)*x**5)/(a*c + a*d*x**3 + b*c*x**3
 + b*d*x**6),x)*a**2*d**3 + 18*int((sqrt(c + d*x**3)*x**5)/(a*c + a*d*x**3
 + b*c*x**3 + b*d*x**6),x)*a*b*c*d**2 - 9*int((sqrt(c + d*x**3)*x**5)/(a*c
 + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**2*c**2*d)/(9*a*b*d)
```

3.544 $\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx = \frac{2d\sqrt{c+dx^3}}{3b} - \frac{2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a} + \frac{2(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}}$$

output
```
2/3*d*(d*x^3+c)^(1/2)/b-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a+2/3
*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a/b^(3
/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx = \frac{2\left(a\sqrt{b}d\sqrt{c+dx^3} - (-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right) - b^{3/2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)\right)}{3ab^{3/2}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x*(a + b*x^3)),x]
```

output
```
(2*(a*Sqrt[b]*d*Sqrt[c + d*x^3] - (-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqr
t[c + d*x^3])/Sqrt[-(b*c) + a*d]] - b^(3/2)*c^(3/2)*ArcTanh[Sqrt[c + d*x^3
]/Sqrt[c]]))/(3*a*b^(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {948, 95, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\left(dx^3+c\right)^{3/2}}{x^3\left(bx^3+a\right)}dx^3$$

$$\Big\downarrow \text{95}$$

$$\frac{1}{3}\left(\frac{\int \frac{d(2bc-ad)x^3+bc^2}{x^3(bx^3+a)\sqrt{dx^3+c}}dx^3}{b}+\frac{2d\sqrt{c+dx^3}}{b}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{\frac{bc^2\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{(bc-ad)^2\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{b}+\frac{2d\sqrt{c+dx^3}}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\frac{2bc^2\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2(bc-ad)^2\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{b}+\frac{2d\sqrt{c+dx^3}}{b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{\frac{2(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{2bc^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a}}{b}+\frac{2d\sqrt{c+dx^3}}{b}\right)$$

input
```
Int[(c + d*x^3)^(3/2)/(x*(a + b*x^3)),x]
```

output
```
((2*d*Sqrt[c + d*x^3])/b + ((-2*b*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]
)/a + (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*
d]])/(a*Sqrt[b]))/b)/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{-\frac{2(ad-bc)^2\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3}+\frac{2\left(ad\sqrt{dx^3+c}-c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)b\right)\sqrt{(ad-bc)b}}{3}}{ab\sqrt{(ad-bc)b}}$
default	$\frac{\frac{2dx^3\sqrt{dx^3+c}}{9}+\frac{8c\sqrt{dx^3+c}}{9}-\frac{2c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{3}}{a}+\frac{-\frac{2(ad-bc)^2\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3}+\frac{2\left(\frac{(-dx^3-4c)b}{3}+ad\right)\sqrt{dx^3+c}\sqrt{(}}{3}}{ab\sqrt{(ad-bc)b}}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/3/((a*d-b*c)*b)^(1/2)*(-(a*d-b*c)^2*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*
b)^(1/2))+(a*d*(d*x^3+c)^(1/2)-c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*b)
*((a*d-b*c)*b)^(1/2))/b/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 480, normalized size of antiderivative = 4.62

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx = \left[\frac{bc^{\frac{3}{2}}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+2\sqrt{dx^3+c}ad-(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{}}{bx^3+}\right.}{3ab}\right.$$

input
```
integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/3*(b*c^(3/2)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqr
t(d*x^3 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c -
a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)))/(a*b), 1/3*(b
*c^(3/2)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3
 + c)*a*d + 2*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*s
qrt(-(b*c - a*d)/b)/(b*c - a*d)))/(a*b), 1/3*(2*b*sqrt(-c)*c*arctan(sqrt(-
c)/sqrt(d*x^3 + c)) + 2*sqrt(d*x^3 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)
/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/
(b*x^3 + a)))/(a*b), 2/3*(b*sqrt(-c)*c*arctan(sqrt(-c)/sqrt(d*x^3 + c)) +
sqrt(d*x^3 + c)*a*d + (b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3
+ c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)))/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(90) = 180$.

Time = 7.82 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.82

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx = \begin{cases} \frac{2\left(\frac{d^2\sqrt{c+dx^3}}{3b}+\frac{c^2 d \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}-\frac{d(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3ab^2\sqrt{\frac{ad-bc}{b}}}\right)}{d} \\ c^{\frac{3}{2}}\left(-\frac{2b\left(\begin{cases}\frac{\frac{a}{2b}+x^3}{a} & \text{for } b=0 \\ -\frac{\log\left(a-2b\left(\frac{a}{2b}+x^3\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{3a}-\frac{2b\left(\begin{cases}\frac{\frac{a}{2b}+x^3}{a} & \text{for } b=0 \\ \frac{\log\left(a+2b\left(\frac{a}{2b}+x^3\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{3a}\right) \end{cases}$$

input
```
integrate((d*x**3+c)**(3/2)/x/(b*x**3+a),x)
```

output
```
Piecewise((2*(d**2*sqrt(c + d*x**3)/(3*b) + c**2*d*atan(sqrt(c + d*x**3)/s
qrt(-c))/(3*a*sqrt(-c)) - d*(a*d - b*c)**2*atan(sqrt(c + d*x**3)/sqrt((a*d
 - b*c)/b))/(3*a*b**2*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (c**(3/2)*(-2*b*
Piecewise(((a/(2*b) + x**3)/a, Eq(b, 0)), (-log(a - 2*b*(a/(2*b) + x**3))/
(2*b), True))/(3*a) - 2*b*Piecewise(((a/(2*b) + x**3)/a, Eq(b, 0)), (log(a
 + 2*b*(a/(2*b) + x**3))/(2*b), True))/(3*a)), True))
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x\,(a+bx^3)}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx^3)^{3/2}}{x\,(a+bx^3)}\,dx = \frac{2\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}} + \frac{2\,\sqrt{dx^3+c}d}{3\,b} - \frac{2\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}ab}$$

input
```
integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*c^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c)) + 2/3*sqrt(d*x^3 + c
)*d/b - 2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(
-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a*b)
```

Mupad [B] (verification not implemented)

Time = 6.39 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.49

$$\int \frac{(c+dx^3)^{3/2}}{x\,(a+bx^3)}\,dx = \frac{c^{3/2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a} + \frac{2\,d\,\sqrt{d\,x^3+c}}{3\,b} + \frac{\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d+\sqrt{b}\,\sqrt{d\,x^3+c}\,(a\,d-b\,c)^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,(a\,d-b\,c)^{3/2}\,1{}\mathrm{i}}{3\,a\,b^{3/2}}$$

input
```
int((c + d*x^3)^(3/2)/(x*(a + b*x^3)),x)
```

output
```
(c^(3/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)
))/x^6))/(3*a) + (2*d*(c + d*x^3)^(1/2))/(3*b) + (log((a^2*d^2 + 2*b^2*c^2
 + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*b*d^2*x^3 + b^2*c*d*
x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*a*b^(3/2))
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x\,(a+bx^3)}\,dx = \frac{4\sqrt{d\,x^3+c}\,c+3\left(\int \frac{\sqrt{d\,x^3+c}}{bd\,x^7+ad\,x^4+bc\,x^4+acx}dx\right)a\,c^2+3\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)a\,d^2-6\left(\right.}{3a}$$

input
```
int((d*x^3+c)^(3/2)/x/(b*x^3+a),x)
```

output
```
(4*sqrt(c + d*x**3)*c + 3*int(sqrt(c + d*x**3)/(a*c*x + a*d*x**4 + b*c*x**
4 + b*d*x**7),x)*a*c**2 + 3*int((sqrt(c + d*x**3)*x**5)/(a*c + a*d*x**3 +
b*c*x**3 + b*d*x**6),x)*a*d**2 - 6*int((sqrt(c + d*x**3)*x**5)/(a*c + a*d*
x**3 + b*c*x**3 + b*d*x**6),x)*b*c*d)/(3*a)
```

3.545 $\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx = -\frac{c\sqrt{c+dx^3}}{3ax^3}+\frac{\sqrt{c}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}-\frac{2(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}}$$

output
```
-1/3*c*(d*x^3+c)^(1/2)/a/x^3+1/3*c^(1/2)*(-3*a*d+2*b*c)*arctanh((d*x^3+c)^
(1/2)/c^(1/2))/a^2-2/3*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-
a*d+b*c)^(1/2))/a^2/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx = \frac{-\frac{ac\sqrt{c+dx^3}}{x^3}+\frac{2(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}}+\sqrt{c}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x]
```

output
```
(-((a*c*Sqrt[c + d*x^3])/x^3) + (2*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sq
rt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/Sqrt[b] + Sqrt[c]*(2*b*c - 3*a*d)*ArcT
anh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(dx^3+c)^{3/2}}{x^6(bx^3+a)}dx^3$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{3}\left(-\frac{\int \frac{d(bc-2ad)x^3+c(2bc-3ad)}{2x^3(bx^3+a)\sqrt{dx^3+c}}dx^3}{a}-\frac{c\sqrt{c+dx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{d(bc-2ad)x^3+c(2bc-3ad)}{x^3(bx^3+a)\sqrt{dx^3+c}}dx^3}{2a}-\frac{c\sqrt{c+dx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(-\frac{\frac{c(2bc-3ad)\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{2(bc-ad)^2\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{a}}{2a}-\frac{c\sqrt{c+dx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{\frac{2c(2bc-3ad)\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{4(bc-ad)^2\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{2a}-\frac{c\sqrt{c+dx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{\frac{4(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{2\sqrt{c}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a}}{2a}-\frac{c\sqrt{c+dx^3}}{ax^3}\right)$$

input
```
Int[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x]
```

output
```
(-((c*Sqrt[c + d*x^3])/(a*x^3)) - ((-2*Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqr
t[c + d*x^3]/Sqrt[c]])/a + (4*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c +
d*x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b]))/(2*a))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{c\left(-\frac{a\sqrt{d\,x^3+c}}{x^3}-\frac{(3ad-2bc)\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)+\frac{2(ad-bc)^2\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{3a^2}$
risch	$-\frac{c\sqrt{d\,x^3+c}}{3a\,x^3}+\frac{-\frac{2\sqrt{c}\,(3ad-2bc)\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3a}+\frac{4\left(a^2d^2-2abcd+b^2c^2\right)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3a\sqrt{(ad-bc)b}}}{2a}$
default	$\frac{-\frac{c\sqrt{d\,x^3+c}}{3x^3}+\frac{2d\sqrt{d\,x^3+c}}{3}-\sqrt{c}\,d\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{a}-\frac{2\left(-(ad-bc)^2\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(\frac{\left(-d\,x^3-4c\right)b}{3}+ad\right)\sqrt{d\,x^3+}\right.}{3a^2\sqrt{(ad-bc)b}}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/a^2*(c*(-a*(d*x^3+c)^(1/2)/x^3-(3*a*d-2*b*c)/c^(1/2)*arctanh((d*x^3+c)
^(1/2)/c^(1/2)))+2*(a*d-b*c)^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2
)/((a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 532, normalized size of antiderivative = 4.59

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx = \left[-\frac{2\,(bc-ad)x^3\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+(2\,bc-3\,ad)\sqrt{c}x^3\log\left(\frac{}{}\right.}{6\,a^2x^3}\right.$$

$$-\frac{4\,(bc-ad)x^3\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+(2\,bc-3\,ad)\sqrt{c}x^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)+2\,\sqrt{dx}}{6\,a^2x^3}$$

$$-\frac{(2\,bc-3\,ad)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)+(bc-ad)x^3\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+\sqrt{dx^3+}}{3\,a^2x^3}$$

$$-\frac{2\,(bc-ad)x^3\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+(2\,bc-3\,ad)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)+\sqrt{dx^3+c}ac}{3\,a^2x^3}$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/6*(2*(b*c - a*d)*x^3*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d +
2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + (2*b*c - 3*a*d)*sq
rt(c)*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^
3 + c)*a*c)/(a^2*x^3), -1/6*(4*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/b)*arctan
(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (2*b*c - 3*a*d)*sq
rt(c)*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^
3 + c)*a*c)/(a^2*x^3), -1/3*((2*b*c - 3*a*d)*sqrt(-c)*x^3*arctan(sqrt(-c)/
sqrt(d*x^3 + c)) + (b*c - a*d)*x^3*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*
c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + sqrt(d*x
^3 + c)*a*c)/(a^2*x^3), -1/3*(2*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/b)*arcta
n(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (2*b*c - 3*a*d)*s
qrt(-c)*x^3*arctan(sqrt(-c)/sqrt(d*x^3 + c)) + sqrt(d*x^3 + c)*a*c)/(a^2*x
^3)]
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{x^4(a+bx^3)}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**4/(b*x**3+a),x)
```

output
```
Integral((c + d*x**3)**(3/2)/(x**4*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x^4}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx = \frac{2\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}a^2} - \frac{(2\,bc^2-3\,acd)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}} - \frac{\sqrt{dx^3+c}c}{3\,ax^3}$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c +
 a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2) - 1/3*(2*b*c^2 - 3*a*c*d)*arctan(sqrt(
d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/3*sqrt(d*x^3 + c)*c/(a*x^3)
```

Mupad [B] (verification not implemented)

Time = 7.98 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.44

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx = \frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)(3\,a\,d-2\,b\,c)}{6\,a^2} - \frac{c\,\sqrt{d\,x^3+c}}{3\,a\,x^3} + \frac{\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d-\sqrt{b}\,\sqrt{d\,x^3+c}\,(a\,d-b\,c)^{3/2}\,2\mathrm{i}}{b\,x^3+a}\right)(a\,d-b\,c)^{3/2}\,1\mathrm{i}}{3\,a^2\,\sqrt{b}}$$

input
```
int((c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x)
```

output
```
(c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)
))/x^6)*(3*a*d - 2*b*c))/(6*a^2) - (c*(c + d*x^3)^(1/2))/(3*a*x^3) + (log(
(a^2*d^2 + 2*b^2*c^2 - b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*
b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3
*a^2*b^(1/2))
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx = \frac{-2\sqrt{dx^3+c}\,acd + 4\sqrt{dx^3+c}\,a\,d^2x^3 - 4\sqrt{dx^3+c}\,b\,c^2 + 9\left(\int \frac{\sqrt{d}}{ab\,d^2x^7+2b^2cd\,x^7+a^2d^2x^4+3a}\right.}{}$$

input
```
int((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x)
```

output
```
( - 2*sqrt(c + d*x**3)*a*c*d + 4*sqrt(c + d*x**3)*a*d**2*x**3 - 4*sqrt(c +
 d*x**3)*b*c**2 + 9*int(sqrt(c + d*x**3)/(a**2*c*d*x + a**2*d**2*x**4 + 2*
a*b*c**2*x + 3*a*b*c*d*x**4 + a*b*d**2*x**7 + 2*b**2*c**2*x**4 + 2*b**2*c*
d*x**7),x)*a**3*c*d**3*x**3 + 30*int(sqrt(c + d*x**3)/(a**2*c*d*x + a**2*d
**2*x**4 + 2*a*b*c**2*x + 3*a*b*c*d*x**4 + a*b*d**2*x**7 + 2*b**2*c**2*x**
4 + 2*b**2*c*d*x**7),x)*a**2*b*c**2*d**2*x**3 + 12*int(sqrt(c + d*x**3)/(a
**2*c*d*x + a**2*d**2*x**4 + 2*a*b*c**2*x + 3*a*b*c*d*x**4 + a*b*d**2*x**7
 + 2*b**2*c**2*x**4 + 2*b**2*c*d*x**7),x)*a*b**2*c**3*d*x**3 - 24*int(sqrt
(c + d*x**3)/(a**2*c*d*x + a**2*d**2*x**4 + 2*a*b*c**2*x + 3*a*b*c*d*x**4
+ a*b*d**2*x**7 + 2*b**2*c**2*x**4 + 2*b**2*c*d*x**7),x)*b**3*c**4*x**3 -
6*int((sqrt(c + d*x**3)*x**5)/(a**2*c*d + a**2*d**2*x**3 + 2*a*b*c**2 + 3*
a*b*c*d*x**3 + a*b*d**2*x**6 + 2*b**2*c**2*x**3 + 2*b**2*c*d*x**6),x)*a**2
*b*d**4*x**3 - 12*int((sqrt(c + d*x**3)*x**5)/(a**2*c*d + a**2*d**2*x**3 +
 2*a*b*c**2 + 3*a*b*c*d*x**3 + a*b*d**2*x**6 + 2*b**2*c**2*x**3 + 2*b**2*c
*d*x**6),x)*a*b**2*c*d**3*x**3 + 9*int((sqrt(c + d*x**3)*x**2)/(a**2*c*d +
 a**2*d**2*x**3 + 2*a*b*c**2 + 3*a*b*c*d*x**3 + a*b*d**2*x**6 + 2*b**2*c**
2*x**3 + 2*b**2*c*d*x**6),x)*a**2*b*c*d**3*x**3 + 12*int((sqrt(c + d*x**3)
*x**2)/(a**2*c*d + a**2*d**2*x**3 + 2*a*b*c**2 + 3*a*b*c*d*x**3 + a*b*d**2
*x**6 + 2*b**2*c**2*x**3 + 2*b**2*c*d*x**6),x)*a*b**2*c**2*d**2*x**3 - 12*
int((sqrt(c + d*x**3)*x**2)/(a**2*c*d + a**2*d**2*x**3 + 2*a*b*c**2 + 3...
```

3.546 $\int \frac{x^3 \left(c+dx^3\right)^{3/2}}{a+bx^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{x^3 (c+dx^3)^{3/2}}{a+bx^3} \, dx = \frac{cx^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},1,-\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/4*c*x^4*(d*x^3+c)^(1/2)*AppellF1(4/3,1,-3/2,7/3,-b*x^3/a,-d*x^3/c)/a/(1+
d*x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 280 vs. 2(65) = 130.

Time = 8.94 (sec) , antiderivative size = 280, normalized size of antiderivative = 4.31

$$\int \frac{x^3 (c+dx^3)^{3/2}}{a+bx^3} \, dx = \frac{x\left(8(c+dx^3)\left(14bc-11ad+5bdx^3\right)+\frac{(27b^2c^2-88abcd+55a^2d^2)x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},\right.}{a}\right.}{}$$

input
```
Integrate[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output
```
(x*(8*(c + d*x^3)*(14*b*c - 11*a*d + 5*b*d*x^3) + ((27*b^2*c^2 - 88*a*b*c*
d + 55*a^2*d^2)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^
3)/c), -((b*x^3)/a)])/a - (64*a^2*c^2*(-14*b*c + 11*a*d)*AppellF1[1/3, 1/2
, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)])/((a + b*x^3)*(-8*a*c*AppellF1[1/3,
1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2,
 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d
*x^3)/c), -((b*x^3)/a)])))))/(220*b^2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (c+dx^3)^{3/2}}{a+bx^3} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{c\sqrt{c+dx^3} \int \frac{x^3 \left(\frac{dx^3}{c}+1\right)^{3/2}}{bx^3+a}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{cx^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},1,-\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output
```
(c*x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -3/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 2.38 (sec) , antiderivative size = 800, normalized size of antiderivative = 12.31

method	result	size
risch	Expression too large to display	800
elliptic	Expression too large to display	846
default	Expression too large to display	1101

input
```
int(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-2/55*x*(-5*b*d*x^3+11*a*d-14*b*c)*(d*x^3+c)^(1/2)/b^2+1/55/b^2*(-2/3*I*(5
5*a^2*d^2-88*a*b*c*d+27*b^2*c^2)/b*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2))+55/3*I*a*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/d^2*
2^(1/2)*sum(1/(a*d-b*c)/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^
2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2
*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+
2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*
d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*
d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{x^3(c+dx^3)^{\frac{3}{2}}}{a+bx^3}\,dx$$

input
```
integrate(x**3*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

output
```
Integral(x**3*(c + d*x**3)**(3/2)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x^3}{bx^3+a}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x^3}{bx^3+a}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx=\int \frac{x^3\,(d\,x^3+c)^{3/2}}{b\,x^3+a}\,dx$$

input
```
int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

output
```
int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx=\frac{-22\sqrt{d\,x^3+c}\,adx+28\sqrt{d\,x^3+c}\,bcx+10\sqrt{d\,x^3+c}\,bd\,x^4+22\left(\int \frac{\sqrt{d\,x^3+c}}{bd\,x^6+ad\,x^3+bc\,x^3+a}\right.}{}$$

input
```
int(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x)
```

output
```
( - 22*sqrt(c + d*x**3)*a*d*x + 28*sqrt(c + d*x**3)*b*c*x + 10*sqrt(c + d*
x**3)*b*d*x**4 + 22*int(sqrt(c + d*x**3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*
x**6),x)*a**2*c*d - 28*int(sqrt(c + d*x**3)/(a*c + a*d*x**3 + b*c*x**3 + b
*d*x**6),x)*a*b*c**2 + 55*int((sqrt(c + d*x**3)*x**3)/(a*c + a*d*x**3 + b*
c*x**3 + b*d*x**6),x)*a**2*d**2 - 88*int((sqrt(c + d*x**3)*x**3)/(a*c + a*
d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b*c*d + 27*int((sqrt(c + d*x**3)*x**3)/
(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**2*c**2)/(55*b**2)
```

3.547 $\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 65

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{cx^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},1,-\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/2*c*x^2*(d*x^3+c)^(1/2)*AppellF1(2/3,1,-3/2,5/3,-b*x^3/a,-d*x^3/c)/a/(1+
d*x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 149 vs. $2(65) = 130$.

Time = 10.17 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.29

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{x^2\left(20ad(c+dx^3)+5c(7bc-4ad)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2d\right.}{70ab\sqrt{c+dx^3}}$$

input
```
Integrate[(x*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output
```
(x^2*(20*a*d*(c + d*x^3) + 5*c*(7*b*c - 4*a*d)*Sqrt[1 + (d*x^3)/c]*AppellF
1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*d*(10*b*c - 7*a*d)*x^3
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)
]))/(70*a*b*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{c\sqrt{c+dx^3}\int \frac{x\left(\frac{dx^3}{c}+1\right)^{3/2}}{bx^3+a}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{cx^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},1,-\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(x*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output
```
(c*x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 1, -3/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 2.55 (sec) , antiderivative size = 921, normalized size of antiderivative = 14.17

method	result	size
risch	Expression too large to display	921
default	Expression too large to display	930
elliptic	Expression too large to display	930

input
```
int(x*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/7*d/b*x^2*(d*x^3+c)^(1/2)-1/7/b*(-2/3*I*(7*a*d-10*b*c)/b*3^(1/2)*(-c*d^2
)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+7/3*I*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/d
^2*2^(1/2)*sum(1/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d
^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)
+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{x(c+dx^3)^{\frac{3}{2}}}{a+bx^3}\,dx$$

input
```
integrate(x*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

output
```
Integral(x*(c + d*x**3)**(3/2)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x}{bx^3+a}\,dx$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x}{bx^3+a}\,dx$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{x\,(d\,x^3+c)^{3/2}}{b\,x^3+a}\,dx$$

input
```
int((x*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

output
```
int((x*(c + d*x^3)^(3/2))/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2\sqrt{d\,x^3+c}\,d\,x^2 - 7\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right) a\,d^2 + 10\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right) bcd -}{7b}$$

input
```
int(x*(d*x^3+c)^(3/2)/(b*x^3+a),x)
```

output
```
(2*sqrt(c + d*x**3)*d*x**2 - 7*int((sqrt(c + d*x**3)*x**4)/(a*c + a*d*x**3
 + b*c*x**3 + b*d*x**6),x)*a*d**2 + 10*int((sqrt(c + d*x**3)*x**4)/(a*c +
a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b*c*d - 4*int((sqrt(c + d*x**3)*x)/(a*c
 + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*c*d + 7*int((sqrt(c + d*x**3)*x)/(
a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b*c**2)/(7*b)
```

3.548 $\int \frac{(c+dx^3)^{3/2}}{a+bx^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3} \, dx = \frac{cx\sqrt{c+dx^3} \operatorname{AppellF1}\left(\frac{1}{3},1,-\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{1+\frac{dx^3}{c}}}$$

output
```
c*x*(d*x^3+c)^(1/2)*AppellF1(1/3,1,-3/2,4/3,-b*x^3/a,-d*x^3/c)/a/(1+d*x^3/
c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 351 vs. 2(60) = 120.

Time = 10.35 (sec) , antiderivative size = 351, normalized size of antiderivative = 5.85

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3} \, dx = \frac{x\left(\frac{d(8bc-5ad)x^3\sqrt{1+\frac{dx^3}{c}} \operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a} + \frac{8\left(-4ac\left(2ad^2x^3+b\left(5c^2+2cdx^3+2d^2x^6\right)\right) \operatorname{AppellF}}{(a+bx^3)\left(-8ac \operatorname{AppellF1}\left(\frac{1}{3}\right.\right.}\right.}{2}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(a + b*x^3),x]
```

output
```
(x*((d*(8*b*c - 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3,
-((d*x^3)/c), -((b*x^3)/a)])/a + (8*(-4*a*c*(2*a*d^2*x^3 + b*(5*c^2 + 2*c*
d*x^3 + 2*d^2*x^6))*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)]
 + 3*d*x^3*(a + b*x^3)*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*
x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b
*x^3)/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
 -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((
b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
))/(20*b*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{937, 936\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{a+bx^3} \, dx$$
$$\Big\downarrow \text{937}$$
$$\frac{c\sqrt{c+dx^3}\int \frac{\left(\frac{dx^3}{c}+1\right)^{3/2}}{bx^3+a}dx}{\sqrt{\frac{dx^3}{c}+1}}$$
$$\Big\downarrow \text{936}$$
$$\frac{cx\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},1,-\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(c + d*x^3)^(3/2)/(a + b*x^3),x]
```

output
```
(c*x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 2.15 (sec) , antiderivative size = 767, normalized size of antiderivative = 12.78

method	result	size
risch	Expression too large to display	767
default	Expression too large to display	776
elliptic	Expression too large to display	776

input
```
int((d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/5*d/b*x*(d*x^3+c)^(1/2)-1/5/b*(-2/3*I*(5*a*d-8*b*c)/b*3^(1/2)*(-c*d^2)^(
1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+5/3*I*(a^2*d^2-2*a*b*c*d
+b^2*c^2)/b/d^2*2^(1/2)*sum(1/(a*d-b*c)/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(
2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*
(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1
/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)
^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)
*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ell
ipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*
3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_
alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx=\text{Timed out}$$

input
```
integrate((d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(c + dx^3)^{3/2}}{a + bx^3} dx = \int \frac{(c + dx^3)^{\frac{3}{2}}}{a + bx^3} dx$$

input
```
integrate((d*x**3+c)**(3/2)/(b*x**3+a),x)
```

output
```
Integral((c + d*x**3)**(3/2)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{(c + dx^3)^{3/2}}{a + bx^3} dx = \int \frac{(dx^3 + c)^{\frac{3}{2}}}{bx^3 + a} dx$$

input
```
integrate((d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(c + dx^3)^{3/2}}{a + bx^3} dx = \int \frac{(dx^3 + c)^{\frac{3}{2}}}{bx^3 + a} dx$$

input
```
integrate((d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx=\int \frac{(d\,x^3+c)^{3/2}}{b\,x^3+a}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(a + b*x^3),x)
```

output
```
int((c + d*x^3)^(3/2)/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx=\frac{2\sqrt{d\,x^3+c}\,dx-2\left(\int \frac{\sqrt{d\,x^3+c}}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)acd+5\left(\int \frac{\sqrt{d\,x^3+c}}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)b\,c^2-5\left(\right.}{5b}$$

input
```
int((d*x^3+c)^(3/2)/(b*x^3+a),x)
```

output
```
(2*sqrt(c + d*x**3)*d*x - 2*int(sqrt(c + d*x**3)/(a*c + a*d*x**3 + b*c*x**
3 + b*d*x**6),x)*a*c*d + 5*int(sqrt(c + d*x**3)/(a*c + a*d*x**3 + b*c*x**3
 + b*d*x**6),x)*b*c**2 - 5*int((sqrt(c + d*x**3)*x**3)/(a*c + a*d*x**3 + b
*c*x**3 + b*d*x**6),x)*a*d**2 + 8*int((sqrt(c + d*x**3)*x**3)/(a*c + a*d*x
**3 + b*c*x**3 + b*d*x**6),x)*b*c*d)/(5*b)
```

3.549 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(a+bx^3\right)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(a+bx^3\right)}\,dx=-\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},1,-\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{1+\frac{dx^3}{c}}}$$

output
```
-c*(d*x^3+c)^(1/2)*AppellF1(-1/3,1,-3/2,2/3,-b*x^3/a,-d*x^3/c)/a/x/(1+d*x^
3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 148 vs. 2(63) = 126.

Time = 10.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.35

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(a+bx^3\right)}\,dx=\frac{-20ac\left(c+dx^3\right)+5c(-2bc+5ad)x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2a}{20a^2x\sqrt{c+dx^3}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)),x]
```

output
```
(-20*a*c*(c + d*x^3) + 5*c*(-2*b*c + 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*Appell
F1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*d*(b*c + 2*a*d)*x^6*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])
/(20*a^2*x*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{c\sqrt{c+dx^3}\int \frac{\left(\frac{dx^3}{c}+1\right)^{3/2}}{x^2(bx^3+a)}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},1,-\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)),x]
```

output
```
-((c*Sqrt[c + d*x^3]*AppellF1[-1/3, 1, -3/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*x*Sqrt[1 + (d*x^3)/c]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 2.90 (sec) , antiderivative size = 920, normalized size of antiderivative = 14.60

method	result	size
risch	Expression too large to display	920
elliptic	Expression too large to display	924
default	Expression too large to display	1404

input
```
int((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-c/a*(d*x^3+c)^(1/2)/x+1/2/a*(-2/3*I*(2*a*d+b*c)/b*3^(1/2)*(-c*d^2)^(1/3)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)))+2/3*I*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/d^2*2^(1/
2)*sum(1/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3
))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*
(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)
^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alph
a^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{x^2(a+bx^3)}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**2/(b*x**3+a),x)
```

output
```
Integral((c + d*x**3)**(3/2)/(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^2), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^2\,(b\,x^3+a)}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)),x)
```

output
```
int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx = \frac{4\sqrt{d\,x^3+c}\,cd+5\left(\int \frac{\sqrt{d\,x^3+c}}{ab\,d^2x^8-2b^2cd\,x^8+a^2d^2x^5-abcd\,x^5-2b^2c^2x^5+a^2cd\,x^2-2ab\,c^2x^2}dx\right)a^2c^2d^2x-1}{}$$

input
```
int((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x)
```

output
```
(4*sqrt(c + d*x**3)*c*d + 5*int(sqrt(c + d*x**3)/(a**2*c*d*x**2 + a**2*d**
2*x**5 - 2*a*b*c**2*x**2 - a*b*c*d*x**5 + a*b*d**2*x**8 - 2*b**2*c**2*x**5
 - 2*b**2*c*d*x**8),x)*a**2*c**2*d**2*x - 12*int(sqrt(c + d*x**3)/(a**2*c*
d*x**2 + a**2*d**2*x**5 - 2*a*b*c**2*x**2 - a*b*c*d*x**5 + a*b*d**2*x**8 -
 2*b**2*c**2*x**5 - 2*b**2*c*d*x**8),x)*a*b*c**3*d*x + 4*int(sqrt(c + d*x*
*3)/(a**2*c*d*x**2 + a**2*d**2*x**5 - 2*a*b*c**2*x**2 - a*b*c*d*x**5 + a*b
*d**2*x**8 - 2*b**2*c**2*x**5 - 2*b**2*c*d*x**8),x)*b**2*c**4*x + int((sqr
t(c + d*x**3)*x**4)/(a**2*c*d + a**2*d**2*x**3 - 2*a*b*c**2 - a*b*c*d*x**3
 + a*b*d**2*x**6 - 2*b**2*c**2*x**3 - 2*b**2*c*d*x**6),x)*a**2*d**4*x - 6*
int((sqrt(c + d*x**3)*x**4)/(a**2*c*d + a**2*d**2*x**3 - 2*a*b*c**2 - a*b*
c*d*x**3 + a*b*d**2*x**6 - 2*b**2*c**2*x**3 - 2*b**2*c*d*x**6),x)*a*b*c*d*
*3*x + 8*int((sqrt(c + d*x**3)*x**4)/(a**2*c*d + a**2*d**2*x**3 - 2*a*b*c*
*2 - a*b*c*d*x**3 + a*b*d**2*x**6 - 2*b**2*c**2*x**3 - 2*b**2*c*d*x**6),x)
*b**2*c**2*d**2*x)/(x*(a*d - 2*b*c))
```

3.550 $\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)} \, dx = -\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},1,-\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{1+\frac{dx^3}{c}}}$$

output

```
-1/2*c*(d*x^3+c)^(1/2)*AppellF1(-2/3,1,-3/2,1/3,-b*x^3/a,-d*x^3/c)/a/x^2/(
1+d*x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 343 vs. 2(65) = 130.

Time = 10.39 (sec) , antiderivative size = 343, normalized size of antiderivative = 5.28

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)} \, dx =$$

$$-\frac{d(bc-4ad)x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{8ac\left(-4ac(2ac+6bcx^3-5adx^3+2bdx^6)\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\right.\right.}{(a+bx^3)\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx}{c}\right.\right.}}{16a^2x^2\sqrt{c+dx^3}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)),x]
```

output
```
-1/16*(d*(b*c - 4*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3,
-((d*x^3)/c), -((b*x^3)/a)] + (8*a*c*(-4*a*c*(2*a*c + 6*b*c*x^3 - 5*a*d*x^
3 + 2*b*d*x^6)*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*
x^3*(a + b*x^3)*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c)
, -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a
)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x
^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/
a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(a^2*
x^2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^3\left(a+bx^3\right)}dx$$

$$\Big\downarrow \; 1013$$

$$\frac{c\sqrt{c+dx^3}\int \frac{\left(\frac{dx^3}{c}+1\right)^{3/2}}{x^3(bx^3+a)}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \; 1012$$

$$-\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},1,-\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)),x]
```

output
```
-1/2*(c*Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -3/2, 1/3, -((b*x^3)/a), -((d*x^
3)/c)])/(a*x^2*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 2.48 (sec) , antiderivative size = 769, normalized size of antiderivative = 11.83

method	result	size
risch	Expression too large to display	769
elliptic	Expression too large to display	772
default	Expression too large to display	1096

input
```
int((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c/a*(d*x^3+c)^(1/2)/x^2+1/4/a*(-2/3*I*(4*a*d-b*c)/b*3^(1/2)*(-c*d^2)^
(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+4/3*I*(a^2*d^2-2*a*b*c*
d+b^2*c^2)/b/d^2*2^(1/2)*sum(1/(a*d-b*c)/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*
(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(
1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2
)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*El
lipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2
*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*
_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{x^3(a+bx^3)}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**3/(b*x**3+a),x)
```

output
```
Integral((c + d*x**3)**(3/2)/(x**3*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x^3}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^3), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x^3}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)}\,dx=\int \frac{(d\,x^3+c)^{3/2}}{x^3\,(b\,x^3+a)}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)),x)
```

output
```
int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)}\,dx=\frac{-4\sqrt{d\,x^3+c}\,cd-7\left(\int \frac{\sqrt{d\,x^3+c}}{ab\,d^2x^9+4b^2cd\,x^9+a^2d^2x^6+5abcd\,x^6+4b^2c^2x^6+a^2cd\,x^3+4ab\,c^2x^3}dx\right)a^2c^2d^2x^2}{}$$

input
```
int((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x)
```

output
```
( - 4*sqrt(c + d*x**3)*c*d - 7*int(sqrt(c + d*x**3)/(a**2*c*d*x**3 + a**2*
d**2*x**6 + 4*a*b*c**2*x**3 + 5*a*b*c*d*x**6 + a*b*d**2*x**9 + 4*b**2*c**2
*x**6 + 4*b**2*c*d*x**9),x)*a**2*c**2*d**2*x**2 - 24*int(sqrt(c + d*x**3)/
(a**2*c*d*x**3 + a**2*d**2*x**6 + 4*a*b*c**2*x**3 + 5*a*b*c*d*x**6 + a*b*d
**2*x**9 + 4*b**2*c**2*x**6 + 4*b**2*c*d*x**9),x)*a*b*c**3*d*x**2 + 16*int
(sqrt(c + d*x**3)/(a**2*c*d*x**3 + a**2*d**2*x**6 + 4*a*b*c**2*x**3 + 5*a*
b*c*d*x**6 + a*b*d**2*x**9 + 4*b**2*c**2*x**6 + 4*b**2*c*d*x**9),x)*b**2*c
**4*x**2 + int((sqrt(c + d*x**3)*x**3)/(a**2*c*d + a**2*d**2*x**3 + 4*a*b*
c**2 + 5*a*b*c*d*x**3 + a*b*d**2*x**6 + 4*b**2*c**2*x**3 + 4*b**2*c*d*x**6
),x)*a**2*d**4*x**2 + 6*int((sqrt(c + d*x**3)*x**3)/(a**2*c*d + a**2*d**2*
x**3 + 4*a*b*c**2 + 5*a*b*c*d*x**3 + a*b*d**2*x**6 + 4*b**2*c**2*x**3 + 4*
b**2*c*d*x**6),x)*a*b*c*d**3*x**2 + 8*int((sqrt(c + d*x**3)*x**3)/(a**2*c*
d + a**2*d**2*x**3 + 4*a*b*c**2 + 5*a*b*c*d*x**3 + a*b*d**2*x**6 + 4*b**2*
c**2*x**3 + 4*b**2*c*d*x**6),x)*b**2*c**2*d**2*x**2)/(x**2*(a*d + 4*b*c))
```

3.551 $\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx = -\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2} + \frac{2(c+dx^3)^{3/2}}{9bd^2} - \frac{2a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

output
```
-2/3*(a*d+b*c)*(d*x^3+c)^(1/2)/b^2/d^2+2/9*(d*x^3+c)^(3/2)/b/d^2-2/3*a^2*a
rctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\sqrt{c+dx^3}(-2bc-3ad+bdx^3)}{9b^2d^2} + \frac{2a^2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{5/2}\sqrt{-bc+ad}}$$

input
```
Integrate[x^8/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(2*Sqrt[c + d*x^3]*(-2*b*c - 3*a*d + b*d*x^3))/(9*b^2*d^2) + (2*a^2*ArcTan
[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(5/2)*Sqrt[-(b*c) + a
*d])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{(bx^3+a)\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{a^2}{b^2(bx^3+a)\sqrt{dx^3+c}}+\frac{\sqrt{dx^3+c}}{bd}+\frac{-bc-ad}{b^2d\sqrt{dx^3+c}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}-\frac{2\sqrt{c+dx^3}(ad+bc)}{b^2d^2}+\frac{2(c+dx^3)^{3/2}}{3bd^2}\right)$$

input
```
Int[x^8/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
((-2*(b*c + a*d)*Sqrt[c + d*x^3])/(b^2*d^2) + (2*(c + d*x^3)^(3/2))/(3*b*d
^2) - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b^(5/2)*
Sqrt[b*c - a*d]))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{2(-bd\,x^3+3ad+2bc)\sqrt{d\,x^3+c}}{9d^2b^2}+\frac{2a^2\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b^2\sqrt{(ad-bc)b}}$
pseudoelliptic	$-\frac{2\left(-\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)a^2d^2+\sqrt{(ad-bc)b}\left(\frac{\left(-d\,x^3+2c\right)b}{3}+ad\right)\sqrt{d\,x^3+c}\right)}{3\sqrt{(ad-bc)b}\,b^2d^2}$
default	$\frac{\frac{2x^3\sqrt{d\,x^3+c}}{9d}-\frac{4c\sqrt{d\,x^3+c}}{9d^2}}{b}+\frac{2a^2\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b^2\sqrt{(ad-bc)b}}-\frac{2a\sqrt{d\,x^3+c}}{3b^2d}$
elliptic	$\frac{2x^3\sqrt{d\,x^3+c}}{9db}+\frac{2\left(-\frac{a}{b^2}-\frac{2c}{3db}\right)\sqrt{d\,x^3+c}}{3d}-\frac{ia^2\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)\cdots}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(-b*d*x^3+3*a*d+2*b*c)*(d*x^3+c)^(1/2)/d^2/b^2+2/3*a^2/b^2/((a*d-b*c)
*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.78

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$
$$=\left[\frac{3\sqrt{b^2c-abd}a^2d^2\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\left(2\,b^3c^2+ab^2cd-3\,a^2bd^2-(b^3cd-ab^2d^2)x^3\right)\cdots}{9\left(b^4cd^2-ab^3d^3\right)}\right.$$

input
```
integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/9*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*
x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*(2*b^3*c^2 + a*b^2*c*d - 3*
a^2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(b^4*c*d^2 - a*b^3
*d^3), 2/9*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^3 + c)*sqrt(-b^
2*c + a*b*d)/(b*d*x^3 + b*c)) - (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^
3*c*d - a*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**8/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**8/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\left(\frac{3\,a^2d^2\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2}+\frac{(dx^3+c)^{\frac{3}{2}}b^2-3\sqrt{dx^3+c}b^2c-3\sqrt{dx^3+c}abd}{b^3}\right)}{9\,d^2}$$

input
```
integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
2/9*(3*a^2*d^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*b^2) + ((d*x^3 + c)^(3/2)*b^2 - 3*sqrt(d*x^3 + c)*b^2*c - 3*sqrt
(d*x^3 + c)*a*b*d)/b^3)/d^2
```

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.16

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,b\,d}-\frac{\left(\frac{2\,a}{b^2}+\frac{4\,c}{3\,b\,d}\right)\,\sqrt{d\,x^3+c}}{3\,d} + \frac{a^2\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int(x^8/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output
```
(2*x^3*(c + d*x^3)^(1/2))/(9*b*d) - (((2*a)/b^2 + (4*c)/(3*b*d))*(c + d*x^
3)^(1/2))/(3*d) + (a^2*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d -
 b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(3*b^(5/2)*(a*d - b*c)^(1/2))
```

Reduce [F]

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$
$$= \frac{-4\sqrt{d\,x^3+c}\,c+2\sqrt{d\,x^3+c}\,d\,x^3-9\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)a\,d^2}{9b\,d^2}$$

input
```
int(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

output
```
( - 4*sqrt(c + d*x**3)*c + 2*sqrt(c + d*x**3)*d*x**3 - 9*int((sqrt(c + d*x
**3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*d**2)/(9*b*d**2)
```

3.552 $\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\sqrt{c+dx^3}}{3bd}+\frac{2a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

output
```
2/3*(d*x^3+c)^(1/2)/b/d+2/3*a*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(
1/2))/b^(3/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{d}-\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}\right)}{3b^{3/2}}$$

input
```
Integrate[x^5/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(2*((Sqrt[b]*Sqrt[c + d*x^3])/d - (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt
[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d]))/(3*b^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{(bx^3+a)\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{bd}-\frac{a\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{bd}-\frac{2a\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{2a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^3}}{bd}\right)$$

input
```
Int[x^5/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
((2*Sqrt[c + d*x^3])/(b*d) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b
*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d]))/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{\frac{2\sqrt{d\,x^3+c}}{3d}-\frac{2a\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}}{b}$
default	$\frac{2\sqrt{d\,x^3+c}}{3bd}-\frac{2a\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b\sqrt{(ad-bc)b}}$
risch	$\frac{2\sqrt{d\,x^3+c}}{3bd}-\frac{2a\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b\sqrt{(ad-bc)b}}$
elliptic	$\frac{2\sqrt{d\,x^3+c}}{3bd}+\frac{ia\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)}}}{}\right)}{}$

input
```
int(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b*((d*x^3+c)^(1/2)/d-a/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((
a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.77

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \left[\frac{\sqrt{b^2c-abd}ad\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}(b^2c-abd)}{3\,(b^3cd-ab^2d^2)}, -\frac{2\left(\sqrt{-b^2c+abd}ad\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-\sqrt{dx^3+c}(b^2c-abd)\right)}{3\,(b^3cd-ab^2d^2)}\right]$$

input
```
integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 +
c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(b^2*c - a*b*d))/
(b^3*c*d - a*b^2*d^2), -2/3*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^3 +
c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - sqrt(d*x^3 + c)*(b^2*c - a*b*d)
)/(b^3*c*d - a*b^2*d^2)]
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**5/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**5/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(\frac{ad\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b}-\frac{\sqrt{dx^3+c}}{b}\right)}{3\,d}$$

input
```
integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-2/3*(a*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*
b*d)*b) - sqrt(d*x^3 + c)/b)/d
```

Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,b\,d} + \frac{a\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int(x^5/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*b*d) + (a*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^
(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*b^(3/2)*(a*d - b
*c)^(1/2))
```

Reduce [F]

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^5}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx$$

input
```
int(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)
```

3.553 $\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

output
```
-2/3*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3\sqrt{b}\sqrt{-bc+ad}}$$

input
```
Integrate[x^2/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*Sqrt[b]*Sqrt[-
(b*c) + a*d])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {946, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$
$$\Big\downarrow \text{946}$$
$$\frac{1}{3}\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3$$
$$\Big\downarrow \text{73}$$
$$\frac{2\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{3d}$$
$$\Big\downarrow \text{221}$$
$$-\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

input
```
Int[x^2/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*Sqrt[b*
c - a*d])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result
default	$\frac{2\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}$
pseudoelliptic	$\frac{2\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\cdots\right.}{}}}{}\right.}{}$

input
```
int(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.55

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \left[\frac{\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)}{3\,\sqrt{b^2c-abd}}, \frac{2\,\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)}{3\,(b^2c-abd)}\right]$$

input
```
integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(
b*x^3 + a))/sqrt(b^2*c - a*b*d), 2/3*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^
3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c))/(b^2*c - a*b*d)]
```

Sympy [A] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.73

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \begin{cases} \frac{2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b\sqrt{\frac{ad-bc}{b}}} & \text{for } d\neq 0 \\ \begin{cases} \frac{x^3}{3a\sqrt{c}} & \text{for } b=0 \\ \tilde{\infty}x^3 & \text{for } \sqrt{c}=0 \\ \frac{\log(3a\sqrt{c}+3b\sqrt{c}x^3)}{3b\sqrt{c}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Piecewise((2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b*sqrt((a*d - b
*c)/b)), Ne(d, 0)), (Piecewise((x**3/(3*a*sqrt(c)), Eq(b, 0)), (zoo*x**3,
Eq(sqrt(c), 0)), (log(3*a*sqrt(c) + 3*b*sqrt(c)*x**3)/(3*b*sqrt(c)), True)
), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}}$$

input
```
integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
2/3*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

Mupad [B] (verification not implemented)

Time = 4.62 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{\ln\left(\frac{a\,d\,1\mathrm{i}-b\,c\,2\mathrm{i}+2\,\sqrt{d\,x^3+c}\,\sqrt{a\,b\,d-b^2\,c}-b\,d\,x^3\,1\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,\sqrt{a\,b\,d-b^2\,c}}$$

input
```
int(x^2/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output
```
(log((a*d*1i - b*c*2i + 2*(c + d*x^3)^(1/2)*(a*b*d - b^2*c)^(1/2) - b*d*x^
3*1i)/(a + b*x^3))*1i)/(3*(a*b*d - b^2*c)^(1/2))
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^2}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx$$

input
```
int(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**2)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)
```

3.554 $\int \frac{1}{x(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{1}{x(a+bx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}} + \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}$$

output
```
-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a/c^(1/2)+2/3*b^(1/2)*arctanh(b^(1/2
)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(a+bx^3)\sqrt{c+dx^3}}\,dx = -\frac{\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a}$$

input
```
Integrate[1/(x*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
-1/3*((2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/Sqr
t[-(b*c) + a*d] + (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c])/a
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{1}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \ 97$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{b\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{3}\left(\frac{2\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2b\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{3}\left(\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}\right)$$

input
```
Int[1/(x*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
((-2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(a*Sqrt[c]) + (2*Sqrt[b]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}-\frac{2b\,\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(a d-b c\right) b}}\right)}{3\sqrt{\left(a d-b c\right) b}}}{a}$	65
default	$-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}-\frac{2b\,\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(a d-b c\right) b}}\right)}{3a\sqrt{\left(a d-b c\right) b}}$	66
elliptic	Expression too large to display	1598

input
```
int(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/a*(-arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-b/((a*d-b*c)*b)^(1/2)*arc
tan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 385, normalized size of antiderivative = 4.53

$$\int \frac{1}{x(a+bx^3)\sqrt{c+dx^3}}\,dx$$
$$= \left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)+\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{3\,ac},\right.$$
$$-\frac{2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(\sqrt{dx^3+c}\sqrt{-\frac{b}{bc-ad}}\right)-\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{3\,ac}, \frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{\cdots}}{b\cdots}\right.}{}$$
$$\left.-\frac{2\left(c\sqrt{-\frac{b}{bc-ad}}\arctan\left(\sqrt{dx^3+c}\sqrt{-\frac{b}{bc-ad}}\right)-\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)\right)}{3\,ac}\right]$$

input
```
integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(c*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)
*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + sqrt(c)*log((d*x^3 - 2*sq
rt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*c), -1/3*(2*c*sqrt(-b/(b*c - a*d))*a
rctan(sqrt(d*x^3 + c)*sqrt(-b/(b*c - a*d))) - sqrt(c)*log((d*x^3 - 2*sqrt(
d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*c), 1/3*(c*sqrt(b/(b*c - a*d))*log((b*d
*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b
*x^3 + a)) + 2*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)))/(a*c), -2/3*(c*s
qrt(-b/(b*c - a*d))*arctan(sqrt(d*x^3 + c)*sqrt(-b/(b*c - a*d))) - sqrt(-c
)*arctan(sqrt(-c)/sqrt(d*x^3 + c)))/(a*c)]
```

Sympy [A] (verification not implemented)

Time = 6.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.40

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \begin{cases} \frac{2\left(-\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{2\,\mathrm{atan}\left(\frac{2\left(\frac{a}{2b}+x^3\right)}{\sqrt{-\frac{a^2}{b^2}}}\right)}{3b\sqrt{c}\sqrt{-\frac{a^2}{b^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Piecewise((2*(-d*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*sqrt((a*d
 - b*c)/b)) + d*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sqrt(-c)))/d, Ne(d, 0
)), (2*atan(2*(a/(2*b) + x**3)/sqrt(-a**2/b**2))/(3*b*sqrt(c)*sqrt(-a**2/b
**2)), True))
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)\sqrt{dx^3+c}x}\,dx$$

input
```
integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = -\frac{2\,b\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a} + \frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}}$$

input
```
integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-2/3*b*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 6.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a\,\sqrt{c}} + \frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,a\,\sqrt{a\,d-b\,c}}$$

input
```
int(1/(x*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output
```
log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(
3*a*c^(1/2)) + (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d
- b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*a*(a*d - b*c)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x\,(a+bx^3)\,\sqrt{c+dx^3}}\,dx=\int \frac{\sqrt{d\,x^3+c}}{bd\,x^7+ad\,x^4+bc\,x^4+acx}dx$$

input
```
int(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**3)/(a*c*x + a*d*x**4 + b*c*x**4 + b*d*x**7),x)
```

3.555 $\int \frac{1}{x^4(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{1}{x^4\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{3acx^3}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}} - \frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}$$

output
```
-1/3*(d*x^3+c)^(1/2)/a/c/x^3+1/3*(a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/
2))/a^2/c^(3/2)-2/3*b^(3/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/
2))/a^2/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \frac{-\frac{a\sqrt{c+dx^3}}{cx^3}+\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{c^{3/2}}}{3a^2}$$

input
```
Integrate[1/(x^4*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-((a*Sqrt[c + d*x^3])/(c*x^3)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
3])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d] + ((2*b*c + a*d)*ArcTanh[Sqrt[
c + d*x^3]/Sqrt[c]])/c^(3/2))/(3*a^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^3) \sqrt{c+dx^3}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{1}{x^6 (bx^3+a) \sqrt{dx^3+c}} dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3} \left(-\frac{\int \frac{bdx^3+2bc+ad}{2x^3(bx^3+a)\sqrt{dx^3+c}} dx^3}{ac} - \frac{\sqrt{c+dx^3}}{acx^3} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(-\frac{\int \frac{bdx^3+2bc+ad}{x^3(bx^3+a)\sqrt{dx^3+c}} dx^3}{2ac} - \frac{\sqrt{c+dx^3}}{acx^3} \right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3} \left(-\frac{\frac{(ad+2bc)\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a} - \frac{2b^2c\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{a}}{2ac} - \frac{\sqrt{c+dx^3}}{acx^3} \right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{\frac{2(ad+2bc)\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{4b^2c\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{2ac}-\frac{\sqrt{c+dx^3}}{acx^3}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{3}\left(-\frac{\frac{4b^{3/2}c\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(ad+2bc)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2ac}-\frac{\sqrt{c+dx^3}}{acx^3}\right)$$

input
```
Int[1/(x^4*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-(Sqrt[c + d*x^3]/(a*c*x^3)) - ((-2*(2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3]
/Sqrt[c]])/(a*Sqrt[c]) + (4*b^(3/2)*c*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sq
rt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(2*a*c))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{-\frac{a\sqrt{d\,x^{3}+c}}{c\,x^{3}}+\frac{(a d+2 b c)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2 b^{2} \arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(a d-b c) b}}\right)}{\sqrt{(a d-b c) b}}}{3 a^{2}}$	92
risch	$-\frac{\sqrt{d\,x^{3}+c}}{3 a c\,x^{3}}-\frac{-\frac{2(a d+2 b c)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3 a\sqrt{c}}-\frac{4 b^{2} c \arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(a d-b c) b}}\right)}{3 a\sqrt{(a d-b c) b}}}{2 a c}$	106
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3 c\,x^{3}}+\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3 c^{\frac{3}{2}}}}{a}+\frac{2 b^{2} \arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(a d-b c) b}}\right)}{3 a^{2}\sqrt{(a d-b c) b}}+\frac{2 b\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3 a^{2}\sqrt{c}}$	111
elliptic	Expression too large to display	1652

input
```
int(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a^2*(-a/c*(d*x^3+c)^(1/2)/x^3+(a*d+2*b*c)/c^(3/2)*arctanh((d*x^3+c)^(1
/2)/c^(1/2))+2*b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)
*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 518, normalized size of antiderivative = 4.43

$$\int \frac{1}{x^4 (a+bx^3) \sqrt{c+dx^3}} \, dx$$
$$= \left[\frac{2\,bc^2x^3\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)+(2\,bc+ad)\sqrt{c}x^3\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-2\,\sqrt{}}{6\,a^2c^2x^3} \right.$$

input
```
integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(2*b*c^2*x^3*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(
d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + (2*b*c + a*d)*s
qrt(c)*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x
^3 + c)*a*c)/(a^2*c^2*x^3), 1/6*(4*b*c^2*x^3*sqrt(-b/(b*c - a*d))*arctan(s
qrt(d*x^3 + c)*sqrt(-b/(b*c - a*d))) + (2*b*c + a*d)*sqrt(c)*x^3*log((d*x^
3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3 + c)*a*c)/(a^2*c^
2*x^3), 1/3*(b*c^2*x^3*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*
sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) - (2*b*c + a
*d)*sqrt(-c)*x^3*arctan(sqrt(-c)/sqrt(d*x^3 + c)) - sqrt(d*x^3 + c)*a*c)/(
a^2*c^2*x^3), 1/3*(2*b*c^2*x^3*sqrt(-b/(b*c - a*d))*arctan(sqrt(d*x^3 + c)
*sqrt(-b/(b*c - a*d))) - (2*b*c + a*d)*sqrt(-c)*x^3*arctan(sqrt(-c)/sqrt(d
*x^3 + c)) - sqrt(d*x^3 + c)*a*c)/(a^2*c^2*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (a+bx^3) \sqrt{c+dx^3}} \, dx = \int \frac{1}{x^4 (a+bx^3) \sqrt{c+dx^3}} \, dx$$

input
```
integrate(1/x**4/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^3) \sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)\sqrt{dx^3+cx^4}}\,dx$$

input
```
integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^4 (a+bx^3) \sqrt{c+dx^3}}\,dx = \frac{2\,b^2 \arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}a^2} - \frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}c} - \frac{\sqrt{dx^3+c}}{3\,acx^3}$$

input
```
integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
2/3*b^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*
d)*a^2) - 1/3*(2*b*c + a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)
*c) - 1/3*sqrt(d*x^3 + c)/(a*c*x^3)
```

Mupad [B] (verification not implemented)

Time = 7.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^4 (a+bx^3) \sqrt{c+dx^3}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)^3}{x^6}\right)(a\,d+2\,b\,c)}{6\,a^2\,c^{3/2}} - \frac{\sqrt{d\,x^3+c}}{3\,a\,c\,x^3} + \frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,a^2\,\sqrt{a\,d-b\,c}}$$

input
```
int(1/(x^4*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output
```
(log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*
(a*d + 2*b*c))/(6*a^2*c^(3/2)) - (c + d*x^3)^(1/2)/(3*a*c*x^3) + (b^(3/2)*
log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^
3)/(a + b*x^3))*1i)/(3*a^2*(a*d - b*c)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{-2\sqrt{d\,x^3+c}-3\left(\int \frac{\sqrt{d\,x^3+c}}{bd\,x^7+ad\,x^4+bc\,x^4+acx}dx\right)ad\,x^3-6\left(\int \frac{\sqrt{d\,x^3+c}}{bd\,x^7+ad\,x^4+bc\,x^4+acx}dx\right)bc\,x^3-3\left(\int \frac{\sqrt{d\,x^3+c}\,x^2}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)bd\,x^3}{6ac\,x^3}$$

input
```
int(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**3) - 3*int(sqrt(c + d*x**3)/(a*c*x + a*d*x**4 + b*c*x*
*4 + b*d*x**7),x)*a*d*x**3 - 6*int(sqrt(c + d*x**3)/(a*c*x + a*d*x**4 + b*
c*x**4 + b*d*x**7),x)*b*c*x**3 - 3*int((sqrt(c + d*x**3)*x**2)/(a*c + a*d*
x**3 + b*c*x**3 + b*d*x**6),x)*b*d*x**3)/(6*a*c*x**3)
```

3.556 $\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx=\frac{x^4\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},1,\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{c+dx^3}}$$

output
```
1/4*x^4*(1+d*x^3/c)^(1/2)*AppellF1(4/3,1,1/2,7/3,-b*x^3/a,-d*x^3/c)/a/(d*x
^3+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 10.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx=\frac{x^4\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{4a\sqrt{c+dx^3}}$$

input
```
Integrate[x^3/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x^4*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3
)/a)])/(4*a*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^3}{(bx^3+a)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{c+dx^3}}$$

input
```
Int[x^3/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*a*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.24 (sec) , antiderivative size = 719, normalized size of antiderivative = 11.23

method	result	size
default	Expression too large to display	719
elliptic	Expression too large to display	719

input
```
int(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I/b*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d
*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2))+1/3*I*a/b/d^2*2^(1/2)*sum(1/(a*d-b*c)/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*
d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))
^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d
^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1
/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha
^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3
)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**3/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{(b\,x^3+a)\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(x^3/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output
```
int(x^3/((a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^3}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx$$

input
```
int(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)
```

3.557 $\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},1,\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{c+dx^3}}$$

output
```
1/2*x^2*(1+d*x^3/c)^(1/2)*AppellF1(2/3,1,1/2,5/3,-b*x^3/a,-d*x^3/c)/a/(d*x
^3+c)^(1/2)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{2a\sqrt{c+dx^3}}$$

input
```
Integrate[x/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3
)/a)])/(2*a*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x}{(bx^3+a)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{2}{3},1,\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{c+dx^3}}$$

input
```
Int[x/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1, 1/2, 5/3, -((b*x^3)/a), -((d*x^3
)/c)])/(2*a*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.08 (sec) , antiderivative size = 429, normalized size of antiderivative = 6.70

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\right.}{}\right.}{2\left(-\right.}}}{}\right)}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\right.}{}\right.}{2\left(-\right.}}}{}\right)}{}$

input
```
int(x/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/d^2*2^(1/2)*sum(1/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d
*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1
/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1
/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^
2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)
*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3
*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x}{(b\,x^3+a)\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(x/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output
```
int(x/((a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx$$

input
```
int(x/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)
```

3.558 $\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{c+dx^3}}$$

output
```
x*(1+d*x^3/c)^(1/2)*AppellF1(1/3,1,1/2,4/3,-b*x^3/a,-d*x^3/c)/a/(d*x^3+c)^
(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. $2(59) = 118$.

Time = 10.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx=$$
$$-\frac{8acx\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\sqrt{c+dx^3}\left(-8ac\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bc\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right.\right.\right.}$$

input
```
Integrate[1/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-8*a*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)])/((a + b*
x^3)*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b
*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3
)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{(bx^3+a)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{c+dx^3}}$$

input
```
Int[1/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.07 (sec) , antiderivative size = 429, normalized size of antiderivative = 7.27

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{i d\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{i d\left(2x+\frac{i\sqrt{3}\left(-c\cdots\right.}{\cdots}\right)}{2\left(-\cdots\right.}}\cdots}{\cdots}\right.}{\cdots}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{i d\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{i d\left(2x+\frac{i\sqrt{3}\left(-c\cdots\right.}{\cdots}\right)}{2\left(-\cdots\right.}}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(1/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/d^2*2^(1/2)*sum(1/(a*d-b*c)/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1
/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x
-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(
-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*
d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/
2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha
-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{1}{(b\,x^3+a)\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(1/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output
```
int(1/((a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx$$

input
```
int(1/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)
```

3.559 $\int \frac{1}{x^2(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2(a+bx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(-\frac{1}{3},1,\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{c+dx^3}}$$

output
```
-(1+d*x^3/c)^(1/2)*AppellF1(-1/3,1,1/2,2/3,-b*x^3/a,-d*x^3/c)/a/x/(d*x^3+c
)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(62) = 124.

Time = 10.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^2(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{-20a(c+dx^3)+5(-2bc+ad)x^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bdx^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF}}{20a^2cx\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^2*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-20*a*(c + d*x^3) + 5*(-2*b*c + a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3
, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*x^6*Sqrt[1 + (d*x^3)/c]
*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(20*a^2*c*x*Sqrt[
c + d*x^3])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^3) \sqrt{c+dx^3}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{1}{x^2(bx^3+a)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\operatorname{AppellF1}\left(-\frac{1}{3},1,\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{c+dx^3}}$$

input
```
Int[1/(x^2*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
-((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1, 1/2, 2/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*x*Sqrt[c + d*x^3]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 2.04 (sec) , antiderivative size = 890, normalized size of antiderivative = 14.35

method	result	size
default	Expression too large to display	890
elliptic	Expression too large to display	891
risch	Expression too large to display	892

input
```
int(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*
((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1
/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2)))+1/3*I*b/a/d^2*2^(1/2)*sum(1/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(
1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3
))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(
1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))
/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-
I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(
2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*
_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx=\text{Timed out}$$

input
```
integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 (a + bx^3) \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^2 (a + bx^3) \sqrt{c + dx^3}} \, dx$$

input
```
integrate(1/x**2/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^3) \sqrt{c + dx^3}} \, dx = \int \frac{1}{(bx^3 + a)\sqrt{dx^3 + c}x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^3) \sqrt{c + dx^3}} \, dx = \int \frac{1}{(bx^3 + a)\sqrt{dx^3 + c}x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3) \sqrt{c+dx^3}} \, dx = \int \frac{1}{x^2 \left(b\,x^3+a\right) \sqrt{d\,x^3+c}} \, dx$$

input
```
int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^3) \sqrt{c+dx^3}} \, dx = \int \frac{\sqrt{d\,x^3+c}}{bd\,x^8+ad\,x^5+bc\,x^5+ac\,x^2} dx$$

input
```
int(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**3)/(a*c*x**2 + a*d*x**5 + b*c*x**5 + b*d*x**8),x)
```

3.560 $\int \frac{1}{x^3(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^3\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{c+dx^3}}$$

output
```
-1/2*(1+d*x^3/c)^(1/2)*AppellF1(-2/3,1,1/2,1/3,-b*x^3/a,-d*x^3/c)/a/x^2/(d
*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 339 vs. 2(64) = 128.

Time = 10.22 (sec) , antiderivative size = 339, normalized size of antiderivative = 5.30

$$\int \frac{1}{x^3\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \frac{-bdx^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{8a\left(-4ac(2ac+6bcx^3+3adx^3+2bdx^6)\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right.\right.}{(a+bx^3)\left(8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-\right.}}{16a^2cx^2\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^3*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-(b*d*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -(
(b*x^3)/a)]) + (8*a*(-4*a*c*(2*a*c + 6*b*c*x^3 + 3*a*d*x^3 + 2*b*d*x^6)*Ap
pellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(a + b*x^3)*(
c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] +
 a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3
)*(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 3*x^3*(2
*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1
[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(16*a^2*c*x^2*Sqrt[c +
d*x^3])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^3) \sqrt{c+dx^3}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{1}{x^3 (bx^3+a) \sqrt{\frac{dx^3}{c}+1}} dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1} \operatorname{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{c+dx^3}}$$

input
```
Int[1/(x^3*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
-1/2*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -((b*x^3)/a), -((d*x
^3)/c)])/(a*x^2*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 2.02 (sec) , antiderivative size = 738, normalized size of antiderivative = 11.53

method	result	size
default	Expression too large to display	738
elliptic	Expression too large to display	739
risch	Expression too large to display	740

input
```
int(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/2/c/x^2*(d*x^3+c)^(1/2)+1/6*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I*b/a/d^2*2^(1/2)*sum(1/(a*d-b*c)/_
alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1
/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)
^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(
1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*
b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)
+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_
alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^3) \sqrt{c + dx^3}} \, dx = \text{Timed out}$$

input
```
integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 (a+bx^3) \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^3 (a+bx^3) \sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**3/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^3) \sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)\sqrt{dx^3+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^3) \sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)\sqrt{dx^3+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^3) \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^3 \, (b \, x^3 + a) \, \sqrt{d \, x^3 + c}} \, dx$$

input
```
int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a + bx^3) \sqrt{c + dx^3}} \, dx = \int \frac{\sqrt{d \, x^3 + c}}{bd \, x^9 + ad \, x^6 + bc \, x^6 + ac \, x^3} dx$$

input
```
int(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**3)/(a*c*x**3 + a*d*x**6 + b*c*x**6 + b*d*x**9),x)
```

3.561 $\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{3bd^2} - \frac{2a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

output
```
2/3*c^2/d^2/(-a*d+b*c)/(d*x^3+c)^(1/2)+2/3*(d*x^3+c)^(1/2)/b/d^2-2/3*a^2*a
rctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{b}(ad(c+dx^3)-bc(2c+dx^3))}{d^2(-bc+ad)\sqrt{c+dx^3}} - \frac{a^2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}\right)}{3b^{3/2}}$$

input
```
Integrate[x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2*((Sqrt[b]*(a*d*(c + d*x^3) - b*c*(2*c + d*x^3)))/(d^2*(-(b*c) + a*d)*Sq
rt[c + d*x^3]) - (a^2*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]]
)/(-(b*c) + a*d)^(3/2)))/(3*b^(3/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{(bx^3+a)(dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{98}$$

$$\frac{1}{3}\int \left(\frac{a^2}{b(bc-ad)(bx^3+a)\sqrt{dx^3+c}}+\frac{1}{bd\sqrt{dx^3+c}}+\frac{c^2}{d(ad-bc)(dx^3+c)^{3/2}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}(bc-ad)^{3/2}}+\frac{2c^2}{d^2\sqrt{c+dx^3}(bc-ad)}+\frac{2\sqrt{c+dx^3}}{bd^2}\right)$$

input
```
Int[x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
((2*c^2)/(d^2*(b*c - a*d)*Sqrt[c + d*x^3]) + (2*Sqrt[c + d*x^3])/(b*d^2) -
 (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b^(3/2)*(b*c
- a*d)^(3/2)))/3
```

Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

method	result
risch	$\frac{2\sqrt{d\,x^{3}+c}}{3b\,d^{2}}-\frac{2a^{2}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3b(ad-bc)\sqrt{(ad-bc)b}}-\frac{2c^{2}}{3d^{2}(ad-bc)\sqrt{d\,x^{3}+c}}$
pseudoelliptic	$\frac{2\sqrt{d\,x^{3}+c}}{3b\,d^{2}}-\frac{2a^{2}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3b(ad-bc)\sqrt{(ad-bc)b}}-\frac{2c^{2}}{3d^{2}(ad-bc)\sqrt{d\,x^{3}+c}}$
default	$\frac{\frac{2c}{3d^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{2\sqrt{d\,x^{3}+c}}{3d^{2}}}{b}+\frac{2a^{2}\left(-\frac{b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{1}{\sqrt{d\,x^{3}+c}}\right)}{3b^{2}(ad-bc)}+\frac{2a}{3b^{2}d\sqrt{d\,x^{3}+c}}$
elliptic	$-\frac{2c^{2}}{3d^{2}(ad-bc)\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{2\sqrt{d\,x^{3}+c}}{3b\,d^{2}}+\frac{ia^{2}\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right.}{d}\right.}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$

input
```
int(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(d*x^3+c)^(1/2)/b/d^2-2/3/b*a^2/(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan(b
*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))-2/3/d^2*c^2/(a*d-b*c)/(d*x^3+c)^(1/2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(87) = 174$.

Time = 0.11 (sec) , antiderivative size = 440, normalized size of antiderivative = 4.11

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \left[-\frac{(a^2d^3x^3+a^2cd^2)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,(2\,b^3c}{3\,(b^4c^3d^2-2\,ab^3c^2d^3+a^2b^2cd^4+(b^4}\right.$$

input
```
integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/3*((a^2*d^3*x^3 + a^2*c*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c
- a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*(2*b^3*c^3
 - 3*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x
^3)*sqrt(d*x^3 + c))/(b^4*c^3*d^2 - 2*a*b^3*c^2*d^3 + a^2*b^2*c*d^4 + (b^4
*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^3), 2/3*((a^2*d^3*x^3 + a^2*c*d^
2)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x
^3 + b*c)) + (2*b^3*c^3 - 3*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b
^2*c*d^2 + a^2*b*d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*c^3*d^2 - 2*a*b^3*c^2*d^3
 + a^2*b^2*c*d^4 + (b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^3)]
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^3)\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{x^8}{(a+bx^3)\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**8/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**8/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx^3)\left(c+dx^3\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\left(\frac{a^2d^2\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}+\frac{c^2}{\sqrt{dx^3+c}(bc-ad)}+\frac{\sqrt{dx^3+c}}{b}\right)}{3\,d^2}$$

input
```
integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
2/3*(a^2*d^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c - a*b*
d)*sqrt(-b^2*c + a*b*d)) + c^2/(sqrt(d*x^3 + c)*(b*c - a*d)) + sqrt(d*x^3
+ c)/b)/d^2
```

Mupad [B] (verification not implemented)

Time = 5.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,b\,d^2}-\frac{2\,c^2}{3\,d^2\,\sqrt{d\,x^3+c}\,(a\,d-b\,c)} + \frac{a^2\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,b^{3/2}\,(a\,d-b\,c)^{3/2}}$$

input
```
int(x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*b*d^2) - (2*c^2)/(3*d^2*(c + d*x^3)^(1/2)*(a*d -
b*c)) + (a^2*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2
)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*b^(3/2)*(a*d - b*c)^(3/2))
```

Reduce [F]

$$\int \frac{x^8}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{4\sqrt{d\,x^3+c}\,c+2\sqrt{d\,x^3+c}\,d\,x^3-3\Big(\int \frac{\sqrt{d\,x^3+c}\,x^5}{b\,d^2x^9+a\,d^2x^6+2bcd\,x^6+2acd\,x^3+b\,c^2x^3+a\,c^2}\,dx}{3b\,d^2\,(d\,x^3+c)}$$

input
```
int(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

output
```
(4*sqrt(c + d*x**3)*c + 2*sqrt(c + d*x**3)*d*x**3 - 3*int((sqrt(c + d*x**3
)*x**5)/(a*c**2 + 2*a*c*d*x**3 + a*d**2*x**6 + b*c**2*x**3 + 2*b*c*d*x**6
+ b*d**2*x**9),x)*a*c*d**2 - 3*int((sqrt(c + d*x**3)*x**5)/(a*c**2 + 2*a*c
*d*x**3 + a*d**2*x**6 + b*c**2*x**3 + 2*b*c*d*x**6 + b*d**2*x**9),x)*a*d**
3*x**3)/(3*b*d**2*(c + d*x**3))
```

3.562 $\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}} + \frac{2a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}$$

output
```
-2/3*c/d/(-a*d+b*c)/(d*x^3+c)^(1/2)+2/3*a*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/
(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{2}{3}\left(\frac{c}{d(-bc+ad)\sqrt{c+dx^3}} + \frac{a\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

input
```
Integrate[x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2*(c/(d*(-(b*c) + a*d)*Sqrt[c + d*x^3]) + (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x
^3])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2))))/3
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{x^3}{(bx^3+a)(dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \ 87$$

$$\frac{1}{3}\left(-\frac{a\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{bc-ad}-\frac{2c}{d\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{3}\left(-\frac{2a\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{d(bc-ad)}-\frac{2c}{d\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{3}\left(\frac{2a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}-\frac{2c}{d\sqrt{c+dx^3}(bc-ad)}\right)$$

input
```
Int[x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
((-2*c)/(d*(b*c - a*d)*Sqrt[c + d*x^3]) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2)))/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{\frac{2c}{3d\sqrt{d\,x^{3}+c}}+\frac{2a\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(a d -b c \right) b}}\right)}{3\sqrt{\left(a d -b c \right) b}}}{a d -b c}$
default	$-\frac{2}{3 b d \sqrt{d\,x^{3}+c}}-\frac{2 a \left(-\frac{b \arctan\left(\frac{b \sqrt{d\,x^{3}+c}}{\sqrt{\left(a d -b c \right) b}}\right)}{\sqrt{\left(a d -b c \right) b}}-\frac{1}{\sqrt{d\,x^{3}+c}}\right)}{3 b \left(a d -b c \right)}$
elliptic	$\frac{2 c}{3 d \left(a d -b c \right) \sqrt{\left(x^{3}+\frac{c}{d}\right) d}}-\frac{i a \sqrt{2}\left(\sum_{\_\alpha =\operatorname{RootOf}\left(b \_Z^{3}+a \right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}} \sqrt{2}\, \sqrt{\frac{i d \left(2 x +\frac{-i \sqrt{3}\, \left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{d \left(x -\left(\ldots\right.\right.}{-3 \left(-c\,d^{2}\right)^{\frac{1}{3}}\ldots}}\ldots}{\ldots}\right.}{\ldots}$

input
```
int(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(a*d-b*c)*(1/d*c/(d*x^3+c)^(1/2)+a/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3
+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(66) = 132$.

Time = 0.12 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.98

$$\int \frac{x^5}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \left[-\frac{(ad^2x^3+acd)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,(b^2c^2-\ldots}{3\,(b^3c^3d-2\,ab^2c^2d^2+a^2bcd^3+(b^3c^2d^2-2\,ab^2cd^3+a^2bd^4)x\ldots}\right.$$

$$\left.-\frac{2\left((ad^2x^3+acd)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)+(b^2c^2-abcd)\sqrt{dx^3+c}\right)}{3\,(b^3c^3d-2\,ab^2c^2d^2+a^2bcd^3+(b^3c^2d^2-2\,ab^2cd^3+a^2bd^4)x^3)}\right]$$

input
```
integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/3*((a*d^2*x^3 + a*c*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d
- 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(b^2*c^2 - a*b*c
*d)*sqrt(d*x^3 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^3 + (b^3*c^2
*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3), -2/3*((a*d^2*x^3 + a*c*d)*sqrt(-b^
2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c))
+ (b^2*c^2 - a*b*c*d)*sqrt(d*x^3 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*
b*c*d^3 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)]
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{x^5}{(a+bx^3)(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**5/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2\left(\frac{ad\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}(bc-ad)}+\frac{c}{\sqrt{dx^3+c}(bc-ad)}\right)}{3d}$$

input
```
integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-2/3*(a*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*
b*d)*(b*c - a*d)) + c/(sqrt(d*x^3 + c)*(b*c - a*d)))/d
```

Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\,c}{3\,d\,\sqrt{d\,x^3+c}\,(a\,d-b\,c)} + \frac{a\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,\sqrt{b}\,(a\,d-b\,c)^{3/2}}$$

input
```
int(x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output
```
(2*c)/(3*d*(c + d*x^3)^(1/2)*(a*d - b*c)) + (a*log((2*b*c - a*d + b^(1/2)*
(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(3*b^(1
/2)*(a*d - b*c)^(3/2))
```

Reduce [F]

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^5}{b\,d^2x^9+a\,d^2x^6+2bcd\,x^6+2acd\,x^3+b\,c^2x^3+a\,c^2}dx$$

input
```
int(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**5)/(a*c**2 + 2*a*c*d*x**3 + a*d**2*x**6 + b*c**2*
x**3 + 2*b*c*d*x**6 + b*d**2*x**9),x)
```

3.563 $\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{2}{3(bc-ad)\sqrt{c+dx^3}} - \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}$$

output
```
2/3/(-a*d+b*c)/(d*x^3+c)^(1/2)-2/3*b^(1/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)
/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{2}{(3bc-3ad)\sqrt{c+dx^3}} - \frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3(-bc+ad)^{3/2}}$$

input
```
Integrate[x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
2/((3*b*c - 3*a*d)*Sqrt[c + d*x^3]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c +
d*x^3])/Sqrt[-(b*c) + a*d]])/(3*(-(b*c) + a*d)^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {946, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{1}{(bx^3+a)(dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(\frac{b\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{bc-ad}+\frac{2}{\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2b\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{d(bc-ad)}+\frac{2}{\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{2}{\sqrt{c+dx^3}(bc-ad)}-\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}\right)$$

input
```
Int[x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2/((b*c - a*d)*Sqrt[c + d*x^3]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/3
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

method	result
default	$\frac{-\frac{2b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}-\frac{2}{3\sqrt{d\,x^{3}+c}}}{ad-bc}$
pseudoelliptic	$\frac{-\frac{2b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}-\frac{2}{3\sqrt{d\,x^{3}+c}}}{ad-bc}$
elliptic	$-\frac{2}{3(ad-bc)\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{ib\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\right.}{-3\left(-c\,d^{2}\right)}}}{}\right)}{}$

input
```
int(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(a*d-b*c)*(-b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*
b)^(1/2))-1/(d*x^3+c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.77

$$\int \frac{x^2}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \left[-\frac{(dx^3+c)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)-2\,\sqrt{dx^3+c}}{3\,((bcd-ad^2)x^3+bc^2-acd)},\right.$$

input
```
integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/3*((d*x^3 + c)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt
(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) - 2*sqrt(d*x^3 +
 c))/((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d), 2/3*((d*x^3 + c)*sqrt(-b/(b*c
- a*d))*arctan(sqrt(d*x^3 + c)*sqrt(-b/(b*c - a*d))) + sqrt(d*x^3 + c))/((
b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)]
```

Sympy [A] (verification not implemented)

Time = 10.78 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.57

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \begin{cases} \frac{2\left(-\frac{d}{3\sqrt{c+dx^3}(ad-bc)} - \frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3\sqrt{\frac{ad-bc}{b}}(ad-bc)}\right)}{d} & \text{for } d \neq 0 \\ \begin{cases} \frac{x^3}{3ac^{\frac{3}{2}}} & \text{for } b = 0 \\ \tilde{\infty}x^3 & \text{for } c^{\frac{3}{2}} = 0 \\ \frac{\log\left(3ac^{\frac{3}{2}} + 3bc^{\frac{3}{2}}x^3\right)}{3bc^{\frac{3}{2}}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Piecewise((2*(-d/(3*sqrt(c + d*x**3)*(a*d - b*c)) - d*atan(sqrt(c + d*x**3
)/sqrt((a*d - b*c)/b))/(3*sqrt((a*d - b*c)/b)*(a*d - b*c)))/d, Ne(d, 0)),
(Piecewise((x**3/(3*a*c**(3/2)), Eq(b, 0)), (zoo*x**3, Eq(c**(3/2), 0)), (
log(3*a*c**(3/2) + 3*b*c**(3/2)*x**3)/(3*b*c**(3/2)), True)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{2\,b\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}(bc-ad)} + \frac{2}{3\,\sqrt{dx^3+c}(bc-ad)}$$

input
```
integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
2/3*b*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) + 2/3/(sqrt(d*x^3 + c)*(b*c - a*d))
```

Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16

$$\int \frac{x^2}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = -\frac{2}{3\,\sqrt{d\,x^3+c}\,(a\,d-b\,c)} + \frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,(a\,d-b\,c)^{3/2}}$$

input
```
int(x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output
```
(b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i
 - b*d*x^3)/(a + b*x^3))*1i)/(3*(a*d - b*c)^(3/2)) - 2/(3*(c + d*x^3)^(1/2
)*(a*d - b*c))
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^2}{b\,d^2x^9+a\,d^2x^6+2bcd\,x^6+2acd\,x^3+b\,c^2x^3+a\,c^2}dx$$

input

```
int(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

output

```
int((sqrt(c + d*x**3)*x**2)/(a*c**2 + 2*a*c*d*x**3 + a*d**2*x**6 + b*c**2*
x**3 + 2*b*c*d*x**6 + b*d**2*x**9),x)
```

3.564 $\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} - \frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}} + \frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a(bc-ad)^{3/2}}$$

output
```
-2/3*d/c/(-a*d+b*c)/(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a
/c^(3/2)+2/3*b^(3/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a/(
-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{2}{3}\left(\frac{d}{c(-bc+ad)\sqrt{c+dx^3}} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{a(-bc+ad)^{3/2}} - \frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{ac^{3/2}}\right)$$

input
```
Integrate[1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2*(d/(c*(-(b*c) + a*d)*Sqrt[c + d*x^3]) + (b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[-(b*c) + a*d]])/(a*(-(b*c) + a*d)^(3/2)) - ArcTanh[Sqrt[c
+ d*x^3]/Sqrt[c]]/(a*c^(3/2))))/3
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {948, 96, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+bx^3) (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{1}{x^3 (bx^3+a) (dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{96}$$

$$\frac{1}{3} \left(\frac{\int \frac{-bdx^3+bc-ad}{x^3(bx^3+a)\sqrt{dx^3+c}}dx^3}{c(bc-ad)}-\frac{2d}{c\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3} \left(\frac{\frac{(bc-ad)\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{b^2c\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{c(bc-ad)}-\frac{2d}{c\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3} \left(\frac{\frac{2(bc-ad)\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2b^2c\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{c(bc-ad)}-\frac{2d}{c\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{\frac{2b^{3/2}c\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)\mathrm{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}}{c(bc-ad)}-\frac{2d}{c\sqrt{c+dx^3}(bc-ad)}\right)$$

input
```
Int[1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
((-2*d)/(c*(b*c - a*d)*Sqrt[c + d*x^3]) + ((-2*(b*c - a*d)*ArcTanh[Sqrt[c
+ d*x^3]/Sqrt[c]])/(a*Sqrt[c]) + (2*b^(3/2)*c*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(c*(b*c - a*d)))/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

method	result	size
pseudoelliptic	$\frac{2d}{3(ad-bc)c\sqrt{d\,x^{3}+c}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3a\,c^{\frac{3}{2}}}+\frac{2b^{2}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3(ad-bc)a\sqrt{(ad-bc)b}}$	103
default	$\frac{\frac{2}{3c\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{a}-\frac{2b\left(-\frac{b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{1}{\sqrt{d\,x^{3}+c}}\right)}{3a(ad-bc)}$	111
elliptic	Expression too large to display	1637

input
```
int(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*d/(a*d-b*c)/c/(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a/c
^(3/2)+2/3/(a*d-b*c)*b^2/a/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((
a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 741, normalized size of antiderivative = 6.50

$$\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}}\,dx = \left[-\frac{2\sqrt{dx^3+c}acd+(bc^2dx^3+bc^3)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}(bc-ad)}{bx^3+a}\right.}{3(abc^4-a^2c^3d+(ab\right.$$

$$-\frac{2\sqrt{dx^3+c}acd+2(bc^2dx^3+bc^3)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\sqrt{dx^3+c}\sqrt{-\frac{b}{bc-ad}}\right)-((bcd-ad^2)x^3+bc^2-acd)\surd}{3(abc^4-a^2c^3d+(abc^3d-a^2c^2d^2)x^3)}$$

$$-\frac{2\sqrt{dx^3+c}acd-2((bcd-ad^2)x^3+bc^2-acd)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)+(bc^2dx^3+bc^3)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+}{}\right.}{3(abc^4-a^2c^3d+(abc^3d-a^2c^2d^2)x^3)}$$

$$-\frac{2\left(\sqrt{dx^3+c}acd+(bc^2dx^3+bc^3)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\sqrt{dx^3+c}\sqrt{-\frac{b}{bc-ad}}\right)-((bcd-ad^2)x^3+bc^2-acd)\surd\right.}{3(abc^4-a^2c^3d+(abc^3d-a^2c^2d^2)x^3)}$$

input
```
integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/3*(2*sqrt(d*x^3 + c)*a*c*d + (b*c^2*d*x^3 + b*c^3)*sqrt(b/(b*c - a*d))
*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c -
a*d)))/(b*x^3 + a)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*sqrt(c)*log((d
*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*b*c^4 - a^2*c^3*d + (a*b*
c^3*d - a^2*c^2*d^2)*x^3), -1/3*(2*sqrt(d*x^3 + c)*a*c*d + 2*(b*c^2*d*x^3
+ b*c^3)*sqrt(-b/(b*c - a*d))*arctan(sqrt(d*x^3 + c)*sqrt(-b/(b*c - a*d)))
 - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3
 + c)*sqrt(c) + 2*c)/x^3))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2
)*x^3), -1/3*(2*sqrt(d*x^3 + c)*a*c*d - 2*((b*c*d - a*d^2)*x^3 + b*c^2 - a
*c*d)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)) + (b*c^2*d*x^3 + b*c^3)*sq
rt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*
d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d -
a^2*c^2*d^2)*x^3), -2/3*(sqrt(d*x^3 + c)*a*c*d + (b*c^2*d*x^3 + b*c^3)*sqr
t(-b/(b*c - a*d))*arctan(sqrt(d*x^3 + c)*sqrt(-b/(b*c - a*d))) - ((b*c*d -
 a*d^2)*x^3 + b*c^2 - a*c*d)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)))/(a
*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^3)]
```

Sympy [A] (verification not implemented)

Time = 7.65 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.36

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{d^2}{3c\sqrt{c+dx^3}(ad-bc)}+\frac{bd\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}(ad-bc)}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3ac\sqrt{-c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{2\,\mathrm{atan}\left(\frac{2\left(\frac{a}{2b}+x^3\right)}{\sqrt{-\frac{a^2}{b^2}}}\right)}{3bc^{\frac{3}{2}}\sqrt{-\frac{a^2}{b^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Piecewise((2*(d**2/(3*c*sqrt(c + d*x**3)*(a*d - b*c)) + b*d*atan(sqrt(c +
d*x**3)/sqrt((a*d - b*c)/b))/(3*a*sqrt((a*d - b*c)/b)*(a*d - b*c)) + d*ata
n(sqrt(c + d*x**3)/sqrt(-c))/(3*a*c*sqrt(-c)))/d, Ne(d, 0)), (2*atan(2*(a/
(2*b) + x**3)/sqrt(-a**2/b**2))/(3*b*c**(3/2)*sqrt(-a**2/b**2)), True))
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2\,b^2\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,(abc-a^2d)\sqrt{-b^2c+abd}} - \frac{2\,d}{3\,\sqrt{dx^3+c}(bc^2-acd)} + \frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}c}$$

input
```
integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-2/3*b^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a*b*c - a^2*d)*s
qrt(-b^2*c + a*b*d)) - 2/3*d/(sqrt(d*x^3 + c)*(b*c^2 - a*c*d)) + 2/3*arcta
n(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c)*c)
```

Mupad [B] (verification not implemented)

Time = 7.00 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.22

$$\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a\,c^{3/2}} + \frac{2\,d}{3\,c\,\sqrt{d\,x^3+c}\,\left(a\,d-b\,c\right)} + \frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{3\,a\,\left(a\,d-b\,c\right)^{3/2}}$$

input
```
int(1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output
```
log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(
3*a*c^(3/2)) + (2*d)/(3*c*(c + d*x^3)^(1/2)*(a*d - b*c)) + (b^(3/2)*log((2
*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a
 + b*x^3))*1i)/(3*a*(a*d - b*c)^(3/2))
```

Reduce [F]

$$\int \frac{1}{x\left(a+bx^{3}\right)\left(c+dx^{3}\right)^{3/2}}\,dx=\int \frac{\sqrt{d\,x^{3}+c}}{b\,d^{2}x^{10}+a\,d^{2}x^{7}+2bcd\,x^{7}+2acd\,x^{4}+b\,c^{2}x^{4}+a\,c^{2}x}dx$$

input
```
int(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

output
```
int(sqrt(c + d*x**3)/(a*c**2*x + 2*a*c*d*x**4 + a*d**2*x**7 + b*c**2*x**4
+ 2*b*c*d*x**7 + b*d**2*x**10),x)
```

3.565 $\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{(2bc+3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}} - \frac{2b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}$$

output
```
-1/3*d*(-3*a*d+b*c)/a/c^2/(-a*d+b*c)/(d*x^3+c)^(1/2)-1/3/a/c/x^3/(d*x^3+c)
^(1/2)+1/3*(3*a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(5/2)-2/3*
b^(5/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(
3/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{\frac{a\left(-bc\left(c+dx^3\right)+ad\left(c+3dx^3\right)\right)}{c^2(bc-ad)x^3\sqrt{c+dx^3}} - \frac{2b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(2bc+3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{c^{5/2}}}{3a^2}$$

input
```
Integrate[1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
((a*(-(b*c*(c + d*x^3)) + a*d*(c + 3*d*x^3)))/(c^2*(b*c - a*d)*x^3*Sqrt[c
+ d*x^3]) - (2*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]
])/(-(b*c) + a*d)^(3/2) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]
])/c^(5/2))/(3*a^2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 114, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^3) (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{1}{x^6 (bx^3+a) (dx^3+c)^{3/2}} dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3} \left(-\frac{\int \frac{3bdx^3+2bc+3ad}{2x^3(bx^3+a)(dx^3+c)^{3/2}} dx^3}{ac} - \frac{1}{acx^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(-\frac{\int \frac{3bdx^3+2bc+3ad}{x^3(bx^3+a)(dx^3+c)^{3/2}} dx^3}{2ac} - \frac{1}{acx^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{3} \left(-\frac{\frac{2d(bc-3ad)}{c\sqrt{c+dx^3}(bc-ad)} - \frac{2\int -\frac{bd(bc-3ad)x^3+(bc-ad)(2bc+3ad)}{2x^3\left(bx^3+a\right)\sqrt{dx^3+c}} dx^3}{c(bc-ad)}}{2ac} - \frac{1}{acx^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\frac{\int \frac{bd(bc-3ad)x^3+(bc-ad)(2bc+3ad)}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{c(bc-ad)}+\frac{2d(bc-3ad)}{c\sqrt{c+dx^3}(bc-ad)}}{2ac}-\frac{1}{acx^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{(bc-ad)(3ad+2bc)\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{2b^3c^2\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{c(bc-ad)}+\frac{2d(bc-3ad)}{c\sqrt{c+dx^3}(bc-ad)}}{2ac}-\frac{1}{acx^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{2(bc-ad)(3ad+2bc)\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{4b^3c^2\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{c(bc-ad)}+\frac{2d(bc-3ad)}{c\sqrt{c+dx^3}(bc-ad)}}{2ac}-\frac{1}{acx^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{4b^{5/2}c^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)(3ad+2bc)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}}{c(bc-ad)}+\frac{2d(bc-3ad)}{c\sqrt{c+dx^3}(bc-ad)}}{2ac}-\frac{1}{acx^3\sqrt{c+dx^3}}\right)$$

input

```
Int[1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output

```
(-(1/(a*c*x^3*Sqrt[c + d*x^3])) - ((2*d*(b*c - 3*a*d))/(c*(b*c - a*d)*Sqrt
[c + d*x^3]) + ((-2*(b*c - a*d)*(2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sq
rt[c]])/(a*Sqrt[c]) + (4*b^(5/2)*c^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqr
t[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(c*(b*c - a*d)))/(2*a*c))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$\frac{d^{2}\left(\frac{-\frac{a\sqrt{d\,x^{3}+c}}{x^{3}}+\frac{(3ad+2bc)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{\sqrt{c}}}{a^{2}c^{2}d^{2}}-\frac{2b^{3}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)a^{2}d^{2}\sqrt{(ad-bc)b}}-\frac{2}{c^{2}(ad-bc)\sqrt{d\,x^{3}+c}}\right)}{3}$
risch	$-\frac{\sqrt{d\,x^{3}+c}}{3c^{2}a\,x^{3}}-\frac{-\frac{2(3ad+2bc)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{4a\,d^{2}}{3(ad-bc)\sqrt{d\,x^{3}+c}}+\frac{4b^{3}c^{2}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3(ad-bc)a\sqrt{(ad-bc)b}}}{2a\,c^{2}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3c^{2}x^{3}}-\frac{2d}{3c^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}}{a}+\frac{2b^{2}\left(-\frac{b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{1}{\sqrt{d\,x^{3}+c}}\right)}{3a^{2}(ad-bc)}-\frac{b\left(\frac{2}{3c\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{2\,a}{\ldots}\right.}{a^{2}\ldots}$
elliptic	Expression too large to display

input
```
int(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*d^2*((-a*(d*x^3+c)^(1/2)/x^3+(3*a*d+2*b*c)/c^(1/2)*arctanh((d*x^3+c)^(
1/2)/c^(1/2)))/a^2/c^2/d^2-2/(a*d-b*c)*b^3/a^2/d^2/((a*d-b*c)*b)^(1/2)*arc
tan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))-2/c^2/(a*d-b*c)/(d*x^3+c)^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1073, normalized size of antiderivative = 6.79

$$\int \frac{1}{x^4 (a + bx^3) (c + dx^3)^{3/2}} dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/6*(2*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 +
2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 +
a)) - ((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d
- 3*a^2*c*d^2)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/
x^3) + 2*(a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3
+ c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3), 1/6
*(4*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(-b/(b*c - a*d))*arctan(sqrt(d*x^3 +
 c)*sqrt(-b/(b*c - a*d))) + ((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (
2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^
3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c
*d^2)*x^3)*sqrt(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2*b*c^5
- a^3*c^4*d)*x^3), -1/3*(((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b
^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3
 + c)) + (b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 +
2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 +
a)) + (a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3 + c
))/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3), 1/3*(2
*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(-b/(b*c - a*d))*arctan(sqrt(d*x^3 + c)
*sqrt(-b/(b*c - a*d))) - ((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b
^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*...
```

Sympy [F]

$$\int \frac{1}{x^4 (a + bx^3) (c + dx^3)^{3/2}} dx = \int \frac{1}{x^4 (a + bx^3) (c + dx^3)^{\frac{3}{2}}} dx$$

input
```
integrate(1/x**4/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a + bx^3) (c + dx^3)^{3/2}} \, dx = \int \frac{1}{(bx^3 + a)(dx^3 + c)^{\frac{3}{2}} x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (a + bx^3) (c + dx^3)^{3/2}} \, dx = \frac{2 \, b^3 \arctan \left(\frac{\sqrt{dx^3 + c} b}{\sqrt{-b^2 c + abd}} \right)}{3 \, (a^2 bc - a^3 d) \sqrt{-b^2 c + abd}} - \frac{(dx^3 + c)bcd - 3 \, (dx^3 + c)ad^2 + 2 \, acd^2}{3 \, (abc^3 - a^2 c^2 d) \left((dx^3 + c)^{\frac{3}{2}} - \sqrt{dx^3 + c} c \right)} - \frac{(2 \, bc + 3 \, ad) \arctan \left(\frac{\sqrt{dx^3 + c}}{\sqrt{-c}} \right)}{3 \, a^2 \sqrt{-c} c^2}$$

input
```
integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
2/3*b^3*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b*c - a^3*d)*
sqrt(-b^2*c + a*b*d)) - 1/3*((d*x^3 + c)*b*c*d - 3*(d*x^3 + c)*a*d^2 + 2*a
*c*d^2)/((a*b*c^3 - a^2*c^2*d)*((d*x^3 + c)^(3/2) - sqrt(d*x^3 + c)*c)) -
1/3*(2*b*c + 3*a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 597, normalized size of antiderivative = 3.78

$$\int \frac{1}{x^4 (a+bx^3) (c+dx^3)^{3/2}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)^3}{x^6}\right)(3\,a\,d+2\,b\,c)}{6\,a^2\,c^{5/2}} - \frac{\sqrt{d\,x^3+c}}{3\,a\,c^2\,x^3}$$

$$- \frac{c\left(\frac{c\left(\frac{c\left(\frac{3\,a^2\,d^4+24\,a\,b\,c\,d^3+15\,b^2\,c^2\,d^2}{8\,a^3\,c^5}+\frac{c\left(\frac{c\left(\frac{3\,b^2\,d^4}{8\,a^3\,c^5}+\frac{b^2\,d^4\,(5\,a\,d-3\,b\,c)}{8\,a^3\,c^4\,\left(b\,c^2-a\,c\,d\right)}-\frac{b\,d^4\,(a\,d+2\,b\,c)\,(5\,a\,d-3\,b\,c)}{4\,a^3\,c^5\,\left(b\,c^2-a\,c\,d\right)}\right)}{d}-\frac{3\,b\,d^3\,(a\,d+2\,b\,c)}{4\,a^3\,c^5}+\frac{d\,(5\,a\,d-3\,b\,c)\left(3\,a^2\,d^4+24 \cdots\right.}{24\,a^3\,c^5\left(b\,c^{\cdots}\right.} \cdots}{d}\cdots\right)}{d}\cdots}{d}\cdots}{\cdots}$$

$$+ \frac{b^{5/2}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,a^2\,(a\,d-b\,c)^{3/2}}$$

input

```
int(1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output

```
(log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*
(3*a*d + 2*b*c))/(6*a^2*c^(5/2)) - (c + d*x^3)^(1/2)/(3*a*c^2*x^3) - ((c*(
(c*((c*((3*a^2*d^4 + 15*b^2*c^2*d^2 + 24*a*b*c*d^3)/(8*a^3*c^5) + (c*((c*(
(3*b^2*d^4)/(8*a^3*c^5) + (b^2*d^4*(5*a*d - 3*b*c))/(8*a^3*c^4*(b*c^2 - a*
c*d)) - (b*d^4*(a*d + 2*b*c)*(5*a*d - 3*b*c))/(4*a^3*c^5*(b*c^2 - a*c*d)))
)/d - (3*b*d^3*(a*d + 2*b*c))/(4*a^3*c^5) + (d*(5*a*d - 3*b*c)*(3*a^2*d^4
+ 15*b^2*c^2*d^2 + 24*a*b*c*d^3))/(24*a^3*c^5*(b*c^2 - a*c*d))))/d - (d^2*
(5*a*d - 3*b*c)*(6*a^2*d^2 + 3*b^2*c^2 + 14*a*b*c*d))/(12*a^3*c^4*(b*c^2 -
 a*c*d))))/d - (d*(6*a^2*d^2 + 3*b^2*c^2 + 14*a*b*c*d))/(4*a^3*c^4) + (d^2
*(5*a*d - 3*b*c)*(13*a*d + 18*b*c))/(24*a^2*c^3*(b*c^2 - a*c*d))))/d + (d*
(13*a*d + 18*b*c))/(8*a^2*c^3) - (d*(3*a*d + 2*b*c)*(5*a*d - 3*b*c))/(6*a^
2*c^2*(b*c^2 - a*c*d))))/d - (3*a*d + 2*b*c)/(2*a^2*c^2))/(c + d*x^3)^(1/2
) + (b^(5/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2
)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*a^2*(a*d - b*c)^(3/2))
```

Reduce [F]

$$\int \frac{1}{x^4 (a + bx^3) (c + dx^3)^{3/2}}\, dx = \frac{-2\sqrt{d\,x^3 + c} - 9\left(\int \frac{\sqrt{d\,x^3+c}}{b\,d^2x^{10}+a\,d^2x^7+2bcd\,x^7+2acd\,x^4+b\,c^2x^4+a\,c^2x}dx\right) acd\,x^3 - 9\left(\right.}{}$$

input

```
int(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

output

```
( - 2*sqrt(c + d*x**3) - 9*int(sqrt(c + d*x**3)/(a*c**2*x + 2*a*c*d*x**4 +
 a*d**2*x**7 + b*c**2*x**4 + 2*b*c*d*x**7 + b*d**2*x**10),x)*a*c*d*x**3 -
9*int(sqrt(c + d*x**3)/(a*c**2*x + 2*a*c*d*x**4 + a*d**2*x**7 + b*c**2*x**
4 + 2*b*c*d*x**7 + b*d**2*x**10),x)*a*d**2*x**6 - 6*int(sqrt(c + d*x**3)/(
a*c**2*x + 2*a*c*d*x**4 + a*d**2*x**7 + b*c**2*x**4 + 2*b*c*d*x**7 + b*d**
2*x**10),x)*b*c**2*x**3 - 6*int(sqrt(c + d*x**3)/(a*c**2*x + 2*a*c*d*x**4
+ a*d**2*x**7 + b*c**2*x**4 + 2*b*c*d*x**7 + b*d**2*x**10),x)*b*c*d*x**6 -
 9*int((sqrt(c + d*x**3)*x**2)/(a*c**2 + 2*a*c*d*x**3 + a*d**2*x**6 + b*c*
*2*x**3 + 2*b*c*d*x**6 + b*d**2*x**9),x)*b*c*d*x**3 - 9*int((sqrt(c + d*x*
*3)*x**2)/(a*c**2 + 2*a*c*d*x**3 + a*d**2*x**6 + b*c**2*x**3 + 2*b*c*d*x**
6 + b*d**2*x**9),x)*b*d**2*x**6)/(6*a*c*x**3*(c + d*x**3))
```

3.566 $\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac\sqrt{c+dx^3}}$$

output
```
1/4*x^4*(1+d*x^3/c)^(1/2)*AppellF1(4/3,1,3/2,7/3,-b*x^3/a,-d*x^3/c)/a/c/(d
*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 231 vs. $2(67) = 134$.

Time = 8.46 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.45

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{x\left(-8-\frac{bx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}-\frac{}{(a+bx^3)\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx}{c}\right.\right.}\right.}{12(-bc+ad)}$$

input
```
Integrate[x^3/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x*(-8 - (b*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c
), -((b*x^3)/a)])/a - (64*a^2*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -
((b*x^3)/a)])/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c)
, -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -(
(b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))
)))/(12*(-(b*c) + a*d)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{x^3}{(bx^3+a)\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac\sqrt{c+dx^3}}$$

input
```
Int[x^3/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 3/2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*a*c*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.53 (sec) , antiderivative size = 749, normalized size of antiderivative = 11.18

method	result	size
elliptic	Expression too large to display	749
default	Expression too large to display	1069

input
```
int(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x/(a*d-b*c)/((x^3+c/d)*d)^(1/2)+2/9*I/(a*d-b*c)*3^(1/2)/d*(-c*d^2)^(1
/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/3*I*a/d^2*2^(1/2)*sum(1
/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(
-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*
d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*
_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^3)\left(c+dx^3\right)^{3/2}}\,dx = \int \frac{x^3}{(a+bx^3)\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**3/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^3)\left(c+dx^3\right)^{3/2}}\,dx = \int \frac{x^3}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^3)\left(c+dx^3\right)^{3/2}}\,dx = \int \frac{x^3}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(b\,x^3+a)\,\left(d\,x^3+c\right)^{3/2}}\,dx$$

input
```
int(x^3/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output
```
int(x^3/((a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^3}{b\,d^2x^9+a\,d^2x^6+2bcd\,x^6+2acd\,x^3+b\,c^2x^3+a\,c^2}dx$$

input
```
int(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**3)/(a*c**2 + 2*a*c*d*x**3 + a*d**2*x**6 + b*c**2*
x**3 + 2*b*c*d*x**6 + b*d**2*x**9),x)
```

3.567 $\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{x^2\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},1,\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt{c+dx^3}}$$

output
```
1/2*x^2*(1+d*x^3/c)^(1/2)*AppellF1(2/3,1,3/2,5/3,-b*x^3/a,-d*x^3/c)/a/c/(d
*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 142 vs. 2(67) = 134.

Time = 10.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.12

$$\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{x^2\left(-20ad+5(3bc+ad)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bd\right.}{30ac(bc-ad)\sqrt{c+dx^3}}$$

input
```
Integrate[x/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x^2*(-20*a*d + 5*(3*b*c + a*d)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1,
5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[
5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)]))/(30*a*c*(b*c - a*d)*Sqrt[c
 + d*x^3])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x}{(bx^3+a)\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{2}{3},1,\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt{c+dx^3}}$$

input
```
Int[x/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1, 3/2, 5/3, -((b*x^3)/a), -((d*x^3
)/c)])/(2*a*c*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.05 (sec) , antiderivative size = 907, normalized size of antiderivative = 13.54

method	result	size
default	Expression too large to display	907
elliptic	Expression too large to display	907

input
```
int(x/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*d*x^2/c/(a*d-b*c)/((x^3+c/d)*d)^(1/2)+2/9*I/c/(a*d-b*c)*3^(1/2)*(-c*d^
2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/d^2*b*2^(1/2)*sum(1/(a*d-b*c)^
2/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(
1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2
)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^
(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2
*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{x}{(a+bx^3)\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{x}{(b\,x^3+a)\,\left(d\,x^3+c\right)^{3/2}}\,dx$$

input
```
int(x/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output
```
int(x/((a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x}{b\,d^2x^9+a\,d^2x^6+2bcd\,x^6+2acd\,x^3+b\,c^2x^3+a\,c^2}dx$$

input
```
int(x/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x)/(a*c**2 + 2*a*c*d*x**3 + a*d**2*x**6 + b*c**2*x**
3 + 2*b*c*d*x**6 + b*d**2*x**9),x)
```

3.568 $\int \frac{1}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{x\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{c+dx^3}}$$

output
```
x*(1+d*x^3/c)^(1/2)*AppellF1(1/3,1,3/2,4/3,-b*x^3/a,-d*x^3/c)/a/c/(d*x^3+c
)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 338 vs. $2(62) = 124$.

Time = 10.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.45

$$\int \frac{1}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{x\left(\frac{bdx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a(-bc+ad)} + \frac{32ac(-3bc+3ad+2bdx^3)\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\right.}{(bc-ad)(a+bx^3)\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1\right.\right.}\right.}{12c\sqrt{}}$$

input
```
Integrate[1/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x*((b*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
-((b*x^3)/a)])/(a*(-(b*c) + a*d)) + (32*a*c*(-3*b*c + 3*a*d + 2*b*d*x^3)*A
ppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*d*x^3*(a + b*x^
3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*App
ellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))/((b*c - a*d)*(a + b*
x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^
3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*Appe
llF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(12*c*Sqrt[c + d*x
^3])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{937, 936\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)(c+dx^3)^{3/2}}dx$$
$$\Big\downarrow \text{937}$$
$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{(bx^3+a)\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$
$$\Big\downarrow \text{936}$$
$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{c+dx^3}}$$

input
```
Int[1/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 3/2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*c*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.04 (sec) , antiderivative size = 753, normalized size of antiderivative = 12.15

method	result	size
default	Expression too large to display	753
elliptic	Expression too large to display	753

input
```
int(1/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*d*x/c/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-2/9*I/c/(a*d-b*c)*3^(1/2)*(-c*d^2)
^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*I/d^2*b*2^(1/2)*su
m(1/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3)
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha
^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/
3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{1}{(a+bx^3)\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{1}{(b\,x^3+a)\,\left(d\,x^3+c\right)^{3/2}}\,dx$$

input
```
int(1/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output
```
int(1/((a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}}{b\,d^2x^9+a\,d^2x^6+2bcd\,x^6+2acd\,x^3+b\,c^2x^3+a\,c^2}dx$$

input
```
int(1/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

output
```
int(sqrt(c + d*x**3)/(a*c**2 + 2*a*c*d*x**3 + a*d**2*x**6 + b*c**2*x**3 +
2*b*c*d*x**6 + b*d**2*x**9),x)
```

3.569 $\int \frac{1}{x^2(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{1}{x^2(a+bx^3)(c+dx^3)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\operatorname{AppellF1}\left(-\frac{1}{3},1,\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt{c+dx^3}}$$

output
```
-(1+d*x^3/c)^(1/2)*AppellF1(-1/3,1,3/2,2/3,-b*x^3/a,-d*x^3/c)/a/c/x/(d*x^3
+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 193 vs. 2(65) = 130.

Time = 10.17 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.97

$$\int \frac{1}{x^2(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{20a(-3bc(c+dx^3)+ad(3c+5dx^3))-5(6b^2c^2-3abcd+5a^2d^2)\,x^3\sqrt{1-}}{}$$

input
```
Integrate[1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(20*a*(-3*b*c*(c + d*x^3) + a*d*(3*c + 5*d*x^3)) - 5*(6*b^2*c^2 - 3*a*b*c*
d + 5*a^2*d^2)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3
)/c), -((b*x^3)/a)] + 2*b*d*(3*b*c - 5*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*Appell
F1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(60*a^2*c^2*(b*c - a*d)*
x*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^3) (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^2(bx^3+a)\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\operatorname{AppellF1}\left(-\frac{1}{3},1,\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt{c+dx^3}}$$

input
```
Int[1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
-((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1, 3/2, 2/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*c*x*Sqrt[c + d*x^3]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 3.10 (sec) , antiderivative size = 952, normalized size of antiderivative = 14.65

method	result	size
elliptic	Expression too large to display	952
risch	Expression too large to display	1382
default	Expression too large to display	1392

input
```
int(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/c^2/a*(d*x^3+c)^(1/2)/x-2/3*d^2*x^2/c^2/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-2
/3*I*(1/2/a/c^2*d+1/3*d^2/c^2/(a*d-b*c))*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)))-1/3*I*b^2/a/d^2*2^(1/2)*sum(1/(a*d-b*c)^2/_alpha*(-c*
d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(
-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2
)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alph
a*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*
d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3) (c+dx^3)^{3/2}} dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 (a + bx^3) (c + dx^3)^{3/2}} \, dx = \int \frac{1}{x^2 (a + bx^3) (c + dx^3)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**2/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^3) (c + dx^3)^{3/2}} \, dx = \int \frac{1}{(bx^3 + a)(dx^3 + c)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^3) (c + dx^3)^{3/2}} \, dx = \int \frac{1}{(bx^3 + a)(dx^3 + c)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3) (c+dx^3)^{3/2}} \, dx = \int \frac{1}{x^2 \, (b\,x^3+a) \, (d\,x^3+c)^{3/2}} \, dx$$

input
```
int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^3) (c+dx^3)^{3/2}} \, dx = \frac{-2\sqrt{d\,x^3+c} - 5\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{b\,d^2x^9+a\,d^2x^6+2bcd\,x^6+2acd\,x^3+b\,c^2x^3+a\,c^2}dx\right) bcdx - 5\left(\int \frac{}{b}\right.}{}$$

input
```
int(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

output
```
( - 2*sqrt(c + d*x**3) - 5*int((sqrt(c + d*x**3)*x**4)/(a*c**2 + 2*a*c*d*x
**3 + a*d**2*x**6 + b*c**2*x**3 + 2*b*c*d*x**6 + b*d**2*x**9),x)*b*c*d*x -
 5*int((sqrt(c + d*x**3)*x**4)/(a*c**2 + 2*a*c*d*x**3 + a*d**2*x**6 + b*c*
*2*x**3 + 2*b*c*d*x**6 + b*d**2*x**9),x)*b*d**2*x**4 - 5*int((sqrt(c + d*x
**3)*x)/(a*c**2 + 2*a*c*d*x**3 + a*d**2*x**6 + b*c**2*x**3 + 2*b*c*d*x**6
+ b*d**2*x**9),x)*a*c*d*x - 5*int((sqrt(c + d*x**3)*x)/(a*c**2 + 2*a*c*d*x
**3 + a*d**2*x**6 + b*c**2*x**3 + 2*b*c*d*x**6 + b*d**2*x**9),x)*a*d**2*x*
*4 - 2*int((sqrt(c + d*x**3)*x)/(a*c**2 + 2*a*c*d*x**3 + a*d**2*x**6 + b*c
**2*x**3 + 2*b*c*d*x**6 + b*d**2*x**9),x)*b*c**2*x - 2*int((sqrt(c + d*x**
3)*x)/(a*c**2 + 2*a*c*d*x**3 + a*d**2*x**6 + b*c**2*x**3 + 2*b*c*d*x**6 +
b*d**2*x**9),x)*b*c*d*x**4)/(2*a*c*x*(c + d*x**3))
```

3.570 $\int \frac{1}{x^3(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{1}{x^3(a+bx^3)(c+dx^3)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2acx^2\sqrt{c+dx^3}}$$

output
```
-1/2*(1+d*x^3/c)^(1/2)*AppellF1(-2/3,1,3/2,1/3,-b*x^3/a,-d*x^3/c)/a/c/x^2/
(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 408 vs. $2(67) = 134$.

Time = 10.42 (sec) , antiderivative size = 408, normalized size of antiderivative = 6.09

$$\int \frac{1}{x^3(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{bd(3bc-7ad)x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{8a(-4ac(-6b\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(b*d*(3*b*c - 7*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -(
(d*x^3)/c), -((b*x^3)/a)] + (8*a*(-4*a*c*(-6*b^2*c*x^3*(3*c + d*x^3) + 3*a
^2*d*(2*c + 7*d*x^3) + a*b*(-6*c^2 - 3*c*d*x^3 + 14*d^2*x^6))*AppellF1[1/3
, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(a + b*x^3)*(-3*b*c*(c
+ d*x^3) + a*d*(3*c + 7*d*x^3))*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3
)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^
3)/a)])))/((a + b*x^3)*(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((
b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^
3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(4
8*a^2*c^2*(-(b*c) + a*d)*x^2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{1}{x^3 (bx^3+a) \left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1} \operatorname{AppellF1}\left(-\frac{2}{3},1,\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2acx^2\sqrt{c+dx^3}}$$

input
```
Int[1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
-1/2*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 3/2, 1/3, -((b*x^3)/a), -((d*x
^3)/c)])/(a*c*x^2*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 3.21 (sec) , antiderivative size = 798, normalized size of antiderivative = 11.91

method	result	size
elliptic	Expression too large to display	798
risch	Expression too large to display	1076
default	Expression too large to display	1084

input
```
int(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^2/a*(d*x^3+c)^(1/2)/x^2-2/3*d^2*x/c^2/(a*d-b*c)/((x^3+c/d)*d)^(1/2)
-2/3*I*(-1/4/a/c^2*d-1/3*d^2/c^2/(a*d-b*c))*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/3*I*b^2/a/d^2*2^(1/2)*sum(1/(a*d
-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-
c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I
*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(
-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alph
a*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2
)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)^{3/2}} \, dx = \int \frac{1}{x^3 (a+bx^3) (c+dx^3)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**3/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)^{3/2}} \, dx = \int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}} x^3} \, dx$$

input
```
integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)^{3/2}} \, dx = \int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}} x^3} \, dx$$

input
```
integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)^{3/2}} \, dx = \int \frac{1}{x^3 \left(b\,x^3+a\right) \left(d\,x^3+c\right)^{3/2}} \,d x$$

input
```
int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output
```
int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)^{3/2}} \, dx = \frac{-2\sqrt{d\,x^3+c}-7\left(\int \frac{\sqrt{d\,x^3+c}}{b\,d^2x^9+a\,d^2x^6+2bcd\,x^6+2acd\,x^3+b\,c^2x^3+a\,c^2}dx\right) acd\,x^2-7\left(\int\right.}{}$$

input
```
int(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

output
```
( - 2*sqrt(c + d*x**3) - 7*int(sqrt(c + d*x**3)/(a*c**2 + 2*a*c*d*x**3 + a
*d**2*x**6 + b*c**2*x**3 + 2*b*c*d*x**6 + b*d**2*x**9),x)*a*c*d*x**2 - 7*i
nt(sqrt(c + d*x**3)/(a*c**2 + 2*a*c*d*x**3 + a*d**2*x**6 + b*c**2*x**3 + 2
*b*c*d*x**6 + b*d**2*x**9),x)*a*d**2*x**5 - 4*int(sqrt(c + d*x**3)/(a*c**2
 + 2*a*c*d*x**3 + a*d**2*x**6 + b*c**2*x**3 + 2*b*c*d*x**6 + b*d**2*x**9),
x)*b*c**2*x**2 - 4*int(sqrt(c + d*x**3)/(a*c**2 + 2*a*c*d*x**3 + a*d**2*x*
*6 + b*c**2*x**3 + 2*b*c*d*x**6 + b*d**2*x**9),x)*b*c*d*x**5 - 7*int((sqrt
(c + d*x**3)*x**3)/(a*c**2 + 2*a*c*d*x**3 + a*d**2*x**6 + b*c**2*x**3 + 2*
b*c*d*x**6 + b*d**2*x**9),x)*b*c*d*x**2 - 7*int((sqrt(c + d*x**3)*x**3)/(a
*c**2 + 2*a*c*d*x**3 + a*d**2*x**6 + b*c**2*x**3 + 2*b*c*d*x**6 + b*d**2*x
**9),x)*b*d**2*x**5)/(4*a*c*x**2*(c + d*x**3))
```

3.571 $\int \frac{x^{11}\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 123

$$\int \frac{x^{11}\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx=\frac{128c^2\sqrt{c+dx^3}}{d^4}+\frac{512c^3\sqrt{c+dx^3}}{3d^4\left(8c-dx^3\right)}+\frac{10c\left(c+dx^3\right)^{3/2}}{3d^4}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^4}-\frac{3968c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}$$

output
```
128*c^2*(d*x^3+c)^(1/2)/d^4+512/3*c^3*(d*x^3+c)^(1/2)/d^4/(-d*x^3+8*c)+10/
3*c*(d*x^3+c)^(3/2)/d^4+2/15*(d*x^3+c)^(5/2)/d^4-3968/9*c^(5/2)*arctanh(1/
3*(d*x^3+c)^(1/2)/c^(1/2))/d^4
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.74

$$\int \frac{x^{11}\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx=\frac{2\left(\frac{3\sqrt{c+dx^3}\left(-9168c^3+770c^2dx^3+19cd^2x^6+d^3x^9\right)}{-8c+dx^3}-9920c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{45d^4}$$

input
```
Integrate[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
(2*((3*Sqrt[c + d*x^3]*(-9168*c^3 + 770*c^2*d*x^3 + 19*c*d^2*x^6 + d^3*x^9
))/(-8*c + d*x^3) - 9920*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]])))/(4
5*d^4)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {948, 108, 27, 170, 25, 27, 164, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{x^9\sqrt{dx^3+c}}{(8c-dx^3)^2}dx^3$$

$$\Big\downarrow \ 108$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{c+dx^3}}{d(8c-dx^3)}-\frac{\int \frac{x^6(7dx^3+6c)}{2(8c-dx^3)\sqrt{dx^3+c}}dx^3}{d}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{c+dx^3}}{d(8c-dx^3)}-\frac{\int \frac{x^6(7dx^3+6c)}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{2d}\right)$$

$$\Big\downarrow \ 170$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{c+dx^3}}{d(8c-dx^3)}-\frac{-\frac{2\int -\frac{cdx^3\left(141dx^3+112c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{5d^2}-\frac{14x^6\sqrt{c+dx^3}}{5d}}{2d}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{c+dx^3}}{d\,(8c-dx^3)}-\frac{\frac{2\int \frac{cdx^3\left(141dx^3+112c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{5d^2}-\frac{14x^6\sqrt{c+dx^3}}{5d}}{2d}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{c+dx^3}}{d\,(8c-dx^3)}-\frac{\frac{2c\int \frac{x^3\left(141dx^3+112c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{5d}-\frac{14x^6\sqrt{c+dx^3}}{5d}}{2d}\right)$$

$$\Big\downarrow \ 164$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{c+dx^3}}{d\,(8c-dx^3)}-\frac{\frac{2c\left(\frac{9920c^2\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{d}-\frac{2\sqrt{c+dx^3}\left(1146c+47dx^3\right)}{d^2}\right)}{5d}-\frac{14x^6\sqrt{c+dx^3}}{5d}}{2d}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{c+dx^3}}{d\,(8c-dx^3)}-\frac{\frac{2c\left(\frac{19840c^2\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d^2}-\frac{2\sqrt{c+dx^3}\left(1146c+47dx^3\right)}{d^2}\right)}{5d}-\frac{14x^6\sqrt{c+dx^3}}{5d}}{2d}\right)$$

$$\Big\downarrow \ 219$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{c+dx^3}}{d\,(8c-dx^3)}-\frac{\frac{2c\left(\frac{19840c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3d^2}-\frac{2\sqrt{c+dx^3}\left(1146c+47dx^3\right)}{d^2}\right)}{5d}-\frac{14x^6\sqrt{c+dx^3}}{5d}}{2d}\right)$$

input
```
Int[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
((x^9*Sqrt[c + d*x^3])/(d*(8*c - d*x^3)) - ((-14*x^6*Sqrt[c + d*x^3])/(5*d
) + (2*c*((-2*Sqrt[c + d*x^3]*(1146*c + 47*d*x^3))/d^2 + (19840*c^(3/2)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^2)))/(5*d))/(2*d))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{158720\left(c^{3}\left(-\frac{d x^{3}}{8}+c\right) \operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{3 \sqrt{c}}\right)+\frac{3 \sqrt{d x^{3}+c}\left(\sqrt{c}\, d^{3} x^{9}+19 c^{\frac{3}{2}} d^{2} x^{6}+770 c^{\frac{5}{2}} d x^{3}-9168 c^{\frac{7}{2}}\right)}{79360}\right)}{\sqrt{c}\left(-45 d^{5} x^{3}+360 c d^{4}\right)}$
risch	$\frac{2\left(d^{2} x^{6}+27 c d x^{3}+986 c^{2}\right) \sqrt{d x^{3}+c}}{15 d^{4}}+\frac{64 c^{3}\left(-\frac{70 \operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{3 \sqrt{c}}\right)}{9 \sqrt{c}\, d}+\frac{8 c\left(-\frac{\sqrt{d x^{3}+c}}{c\left(d x^{3}-8 c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{3 \sqrt{c}}\right)}{3 c^{\frac{3}{2}}}\right)}{3 d}\right)}{d^{3}}$
default	$\frac{d\left(\frac{2 x^{6} \sqrt{d x^{3}+c}}{15}+\frac{2 c x^{3} \sqrt{d x^{3}+c}}{45 d}-\frac{4 c^{2} \sqrt{d x^{3}+c}}{45 d^{2}}\right)+\frac{32 c\left(d x^{3}+c\right)^{\frac{3}{2}}}{9 d}}{d^{3}}-\frac{64 c^{2}\left(-2 \sqrt{d x^{3}+c}+6 \sqrt{c} \operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{3 \sqrt{c}}\right)\right)}{d^{4}}+\frac{512 \ldots}{\ldots}$
elliptic	$\frac{512 c^{3} \sqrt{d x^{3}+c}}{3 d^{4}\left(-d x^{3}+8 c\right)}+\frac{2 x^{6} \sqrt{d x^{3}+c}}{15 d^{2}}+\frac{18 c x^{3} \sqrt{d x^{3}+c}}{5 d^{3}}+\frac{1972 c^{2} \sqrt{d x^{3}+c}}{15 d^{4}}+\frac{1984 i c^{2} \sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d \_Z^{3}-8 c\right)} \frac{\left(-c d^{2}\right) \ldots}{\ldots}\right)}{\ldots}$

input
```
int(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
-158720/c^(1/2)*(c^3*(-1/8*d*x^3+c)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))+3
/79360*(d*x^3+c)^(1/2)*(c^(1/2)*d^3*x^9+19*c^(3/2)*d^2*x^6+770*c^(5/2)*d*x
^3-9168*c^(7/2)))/(-45*d^5*x^3+360*c*d^4)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.76

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$
$$= \left[\frac{2\left(4960\,(c^2dx^3-8\,c^3)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\,(d^3x^9+19\,cd^2x^6+770\,c^2dx^3-9168\,c^3)\sqrt{dx^3+}\right.}{45\,(d^5x^3-8\,cd^4)}\right.$$

input
```
integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[2/45*(4960*(c^2*d*x^3 - 8*c^3)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqr
t(c) + 10*c)/(d*x^3 - 8*c)) + 3*(d^3*x^9 + 19*c*d^2*x^6 + 770*c^2*d*x^3 -
9168*c^3)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4), 2/45*(9920*(c^2*d*x^3 - 8*
c^3)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) + 3*(d^3*x^9 + 19*c*d^2*x
^6 + 770*c^2*d*x^3 - 9168*c^3)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

Sympy [F]

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x^{11}\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

input
```
integrate(x**11*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x**11*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.87

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{2\left(4960\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\,(dx^3+c)^{\frac{5}{2}}+75\,(dx^3+c)^{\frac{3}{2}}c+2880\,\sqrt{dx^3+c}c^2-\frac{3840\,\sqrt{dx^3+c}c^3}{dx^3-8\,c}\right)}{45\,d^4}$$

input
```
integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
2/45*(4960*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 75*(d*x^3 + c)^(3/2)*c + 2880*sqrt(d*x^3
 + c)*c^2 - 3840*sqrt(d*x^3 + c)*c^3/(d*x^3 - 8*c))/d^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{3968\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d^4} - \frac{512\,\sqrt{dx^3+c}c^3}{3\,(dx^3-8\,c)d^4} + \frac{2\left((dx^3+c)^{\frac{5}{2}}d^{16}+25\,(dx^3+c)^{\frac{3}{2}}cd^{16}+960\,\sqrt{dx^3+c}c^2d^{16}\right)}{15\,d^{20}}$$

input
```
integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
3968/9*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 512/3*sqr
t(d*x^3 + c)*c^3/((d*x^3 - 8*c)*d^4) + 2/15*((d*x^3 + c)^(5/2)*d^16 + 25*(
d*x^3 + c)^(3/2)*c*d^16 + 960*sqrt(d*x^3 + c)*c^2*d^16)/d^20
```

Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{1984\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^4}+\frac{1972\,c^2\,\sqrt{d\,x^3+c}}{15\,d^4}+\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d^2}+\frac{18\,c\,x^3\,\sqrt{d\,x^3+c}}{5\,d^3}+\frac{512\,c^3\,\sqrt{d\,x^3+c}}{3\,d^4\,\left(8\,c-d\,x^3\right)}$$

input
```
int((x^11*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

output
```
(1984*c^(5/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^
3)))/(9*d^4) + (1972*c^2*(c + d*x^3)^(1/2))/(15*d^4) + (2*x^6*(c + d*x^3)^
(1/2))/(15*d^2) + (18*c*x^3*(c + d*x^3)^(1/2))/(5*d^3) + (512*c^3*(c + d*x
^3)^(1/2))/(3*d^4*(8*c - d*x^3))
```

Reduce [F]

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{\frac{2464\sqrt{d\,x^3+c}\,c^3}{15}-\frac{308\sqrt{d\,x^3+c}\,c^2d\,x^3}{3}-\frac{38\sqrt{d\,x^3+c}\,c\,d^2x^6}{15}-\frac{2\sqrt{d\,x^3+c}\,d^3x^9}{15}+\frac{142848\left(\int\frac{\sqrt{d\,x^3+c}\,x^5}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)c^4d^2}{5}-\frac{1}{}}{d^4\left(-d\,x^3+8c\right)}$$

input
```
int(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

output
```
(2*(1232*sqrt(c + d*x**3)*c**3 - 770*sqrt(c + d*x**3)*c**2*d*x**3 - 19*sqr
t(c + d*x**3)*c*d**2*x**6 - sqrt(c + d*x**3)*d**3*x**9 + 214272*int((sqrt(
c + d*x**3)*x**5)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),
x)*c**4*d**2 - 26784*int((sqrt(c + d*x**3)*x**5)/(64*c**3 + 48*c**2*d*x**3
 - 15*c*d**2*x**6 + d**3*x**9),x)*c**3*d**3*x**3))/(15*d**4*(8*c - d*x**3)
)
```

3.572 $\int \frac{x^8\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 104

$$\int \frac{x^8\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx=\frac{32c\sqrt{c+dx^3}}{3d^3}+\frac{64c^2\sqrt{c+dx^3}}{3d^3\left(8c-dx^3\right)}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{352c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}$$

output
```
32/3*c*(d*x^3+c)^(1/2)/d^3+64/3*c^2*(d*x^3+c)^(1/2)/d^3/(-d*x^3+8*c)+2/9*(
d*x^3+c)^(3/2)/d^3-352/9*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.76

$$\int \frac{x^8\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx=\frac{2\left(\frac{\sqrt{c+dx^3}\left(-488c^2+41cdx^3+d^2x^6\right)}{-8c+dx^3}-176c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{9d^3}$$

input
```
Integrate[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
(2*((Sqrt[c + d*x^3]*(-488*c^2 + 41*c*d*x^3 + d^2*x^6))/(-8*c + d*x^3) - 1
76*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(9*d^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {948, 100, 27, 90, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6\sqrt{dx^3+c}}{(8c-dx^3)^2}dx^3$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{3}\left(\frac{64c(c+dx^3)^{3/2}}{9d^3(8c-dx^3)}-\frac{\int \frac{cd\sqrt{dx^3+c}(9dx^3+104c)}{8c-dx^3}dx^3}{9cd^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{64c(c+dx^3)^{3/2}}{9d^3(8c-dx^3)}-\frac{\int \frac{\sqrt{dx^3+c}(9dx^3+104c)}{8c-dx^3}dx^3}{9d^2}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(\frac{64c(c+dx^3)^{3/2}}{9d^3(8c-dx^3)}-\frac{176c\int \frac{\sqrt{dx^3+c}}{8c-dx^3}dx^3-\frac{6(c+dx^3)^{3/2}}{d}}{9d^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{64c(c+dx^3)^{3/2}}{9d^3(8c-dx^3)}-\frac{176c\left(9c\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{6(c+dx^3)^{3/2}}{d}}{9d^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{64c(c+dx^3)^{3/2}}{9d^3(8c-dx^3)}-\frac{176c\left(\frac{18c\int\frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{6(c+dx^3)^{3/2}}{d}}{9d^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{64c(c+dx^3)^{3/2}}{9d^3(8c-dx^3)}-\frac{176c\left(\frac{6\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{6(c+dx^3)^{3/2}}{d}}{9d^2}\right)$$

input
```
Int[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
((64*c*(c + d*x^3)^(3/2))/(9*d^3*(8*c - d*x^3)) - ((-6*(c + d*x^3)^(3/2))/
d + 176*c*((-2*Sqrt[c + d*x^3])/d + (6*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*
Sqrt[c])])/d))/(9*d^2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{\frac{2\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9}+\frac{32c\sqrt{d\,x^{3}+c}}{3}+\frac{32c^{2}\left(\frac{2\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{11\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{3}}{d^{3}}$
default	$\frac{2(d\,x^{3}+c)^{\frac{3}{2}}}{9d^{3}}-\frac{16c\left(-2\sqrt{d\,x^{3}+c}+6\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\right)}{3d^{3}}+\frac{64c^{2}\left(\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{3d^{3}}$
risch	$\frac{2(d\,x^{3}+49c)\sqrt{d\,x^{3}+c}}{9d^{3}}+\frac{16c^{2}\left(-\frac{26\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}+\frac{4c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3d}\right)}{d^{2}}$
elliptic	$\frac{64c^{2}\sqrt{d\,x^{3}+c}}{3d^{3}(-d\,x^{3}+8c)}+\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d^{2}}+\frac{98c\sqrt{d\,x^{3}+c}}{9d^{3}}+\frac{176ic\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d\right.}{}\right.}{\left(-c\,d\right.}}}{}\right.}{}$

input
```
int(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(1/3*(d*x^3+c)^(3/2)+16*c*(d*x^3+c)^(1/2)+16*c^2*(2*(d*x^3+c)^(1/2)/(-
d*x^3+8*c)-11/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)))/d^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.81

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$
$$= \left[\frac{2\left(88\,(cdx^3-8\,c^2)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+(d^2x^6+41\,cdx^3-488\,c^2)\sqrt{dx^3+c}\right)}{9\,(d^4x^3-8\,cd^3)},\frac{2\left(176\,(cdx^3-\right.}{}\right.$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[2/9*(88*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c)
+ 10*c)/(d*x^3 - 8*c)) + (d^2*x^6 + 41*c*d*x^3 - 488*c^2)*sqrt(d*x^3 + c))
/(d^4*x^3 - 8*c*d^3), 2/9*(176*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(3*sqrt(-c
)/sqrt(d*x^3 + c)) + (d^2*x^6 + 41*c*d*x^3 - 488*c^2)*sqrt(d*x^3 + c))/(d^
4*x^3 - 8*c*d^3)]
```

Sympy [F]

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x^8\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

input
```
integrate(x**8*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x**8*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{2\left(88\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+(dx^3+c)^{\frac{3}{2}}+48\sqrt{dx^3+c}c-\frac{96\sqrt{dx^3+c}c^2}{dx^3-8c}\right)}{9\,d^3}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
2/9*(88*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + (d*x^3 + c)^(3/2) + 48*sqrt(d*x^3 + c)*c - 96*sqrt(d*x^3 + c)*c^2
/(d*x^3 - 8*c))/d^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{352\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d^3}-\frac{64\sqrt{dx^3+c}c^2}{3\,(dx^3-8\,c)d^3}+\frac{2\left((dx^3+c)^{\frac{3}{2}}d^6+48\sqrt{dx^3+c}cd^6\right)}{9\,d^9}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
352/9*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 64/3*sqrt(
d*x^3 + c)*c^2/((d*x^3 - 8*c)*d^3) + 2/9*((d*x^3 + c)^(3/2)*d^6 + 48*sqrt(
d*x^3 + c)*c*d^6)/d^9
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{98\,c\,\sqrt{d\,x^3+c}}{9\,d^3}+\frac{176\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^3}+\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^2}+\frac{64\,c^2\,\sqrt{d\,x^3+c}}{3\,d^3\,\left(8\,c-d\,x^3\right)}$$

input
```
int((x^8*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

output
```
(98*c*(c + d*x^3)^(1/2))/(9*d^3) + (176*c^(3/2)*log((10*c + d*x^3 - 6*c^(1
/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(9*d^3) + (2*x^3*(c + d*x^3)^(1/2))
/(9*d^2) + (64*c^2*(c + d*x^3)^(1/2))/(3*d^3*(8*c - d*x^3))
```

Reduce [F]

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{\frac{656\sqrt{d\,x^3+c}\,c^2}{45}-\frac{82\sqrt{d\,x^3+c}\,cd\,x^3}{9}-\frac{2\sqrt{d\,x^3+c}\,d^2x^6}{9}+\frac{12672\left(\int\frac{\sqrt{d\,x^3+c}\,x^5}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)c^3d^2}{5}-\frac{1584\left(\int\frac{\sqrt{d\,x^3+c}\,x^5}{d^3x^9-15c\,d^2x^6+48c^2}\right.}{5}}{d^3\left(-d\,x^3+8c\right)}$$

input
```
int(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

output
```
(2*(328*sqrt(c + d*x**3)*c**2 - 205*sqrt(c + d*x**3)*c*d*x**3 - 5*sqrt(c +
 d*x**3)*d**2*x**6 + 57024*int((sqrt(c + d*x**3)*x**5)/(64*c**3 + 48*c**2*
d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**3*d**2 - 7128*int((sqrt(c + d*x
**3)*x**5)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**2
*d**3*x**3))/(45*d**3*(8*c - d*x**3))
```

3.573 $\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 83

$$\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{2\sqrt{c+dx^3}}{3d^2}+\frac{8c\sqrt{c+dx^3}}{3d^2(8c-dx^3)}-\frac{26\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

output
```
2/3*(d*x^3+c)^(1/2)/d^2+8/3*c*(d*x^3+c)^(1/2)/d^2/(-d*x^3+8*c)-26/9*c^(1/2
)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{2\left(\frac{3(-12c+dx^3)\sqrt{c+dx^3}}{-8c+dx^3}-13\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{9d^2}$$

input
```
Integrate[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
(2*((3*(-12*c + d*x^3)*Sqrt[c + d*x^3])/(-8*c + d*x^3) - 13*Sqrt[c]*ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(9*d^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {948, 87, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3\sqrt{dx^3+c}}{\left(8c-dx^3\right)^2}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{8\left(c+dx^3\right)^{3/2}}{9d^2\left(8c-dx^3\right)}-\frac{13\int \frac{\sqrt{dx^3+c}}{8c-dx^3}dx^3}{9d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{8\left(c+dx^3\right)^{3/2}}{9d^2\left(8c-dx^3\right)}-\frac{13\left(9c\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3-\frac{2\sqrt{c+dx^3}}{d}\right)}{9d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{8\left(c+dx^3\right)^{3/2}}{9d^2\left(8c-dx^3\right)}-\frac{13\left(\frac{18c\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)}{9d}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{8\left(c+dx^3\right)^{3/2}}{9d^2\left(8c-dx^3\right)}-\frac{13\left(\frac{6\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)}{9d}\right)$$

input
```
Int[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
((8*(c + d*x^3)^(3/2))/(9*d^2*(8*c - d*x^3)) - (13*((-2*Sqrt[c + d*x^3])/d
 + (6*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d))/(9*d))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$\frac{\frac{2\sqrt{d\,x^3+c}}{3}+\frac{2c\left(\frac{4\sqrt{d\,x^3+c}}{-d\,x^3+8c}-\frac{13\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{3}}{d^2}$
default	$-\frac{-2\sqrt{d\,x^3+c}+6\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{3d^2}+\frac{8c\left(\frac{\sqrt{d\,x^3+c}}{-d\,x^3+8c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{3d^2}$
risch	$\frac{2\sqrt{d\,x^3+c}}{3d^2}+\frac{c\left(-\frac{34\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}+\frac{8c\left(-\frac{\sqrt{d\,x^3+c}}{c\left(d\,x^3-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3d}\right)}{d}$
elliptic	$\frac{8c\sqrt{d\,x^3+c}}{3d^2(-d\,x^3+8c)}+\frac{2\sqrt{d\,x^3+c}}{3d^2}+\frac{13i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\cdots}}{\cdots}\right)}{\cdots}$

input
```
int(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*((d*x^3+c)^(1/2)+c*(4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-13/3*arctanh(1/3*(d
*x^3+c)^(1/2)/c^(1/2))/c^(1/2)))/d^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.95

$$\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$
$$= \left[\frac{13\,(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+6\,\sqrt{dx^3+c}(dx^3-12\,c)}{9\,(d^3x^3-8\,cd^2)}, \frac{2\left(13\,(dx^3-8\,c)\sqrt{-c}\arctan\left(\frac{3}{\sqrt{d}}\right.\right.}{9\,(d^3x^3}\right.$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/9*(13*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) + 6*sqrt(d*x^3 + c)*(d*x^3 - 12*c))/(d^3*x^3 - 8*c*d^2)
, 2/9*(13*(d*x^3 - 8*c)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) + 3*sq
rt(d*x^3 + c)*(d*x^3 - 12*c))/(d^3*x^3 - 8*c*d^2)]
```

Sympy [F]

$$\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x^5\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

input
```
integrate(x**5*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x**5*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{13\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+6\sqrt{dx^3+c}-\frac{24\sqrt{dx^3+c}c}{dx^3-8c}}{9d^2}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
1/9*(13*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + 6*sqrt(d*x^3 + c) - 24*sqrt(d*x^3 + c)*c/(d*x^3 - 8*c))/d^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{26\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d^2}+\frac{2\sqrt{dx^3+c}}{3d^2}-\frac{8\sqrt{dx^3+c}c}{3(dx^3-8c)d^2}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
26/9*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) + 2/3*sqrt(d*x^
3 + c)/d^2 - 8/3*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d^2)
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,d^2}+\frac{13\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\sqrt{c}\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^2}+\frac{8\,c\sqrt{d\,x^3+c}}{3\,d^2\,(8\,c-d\,x^3)}$$

input
```
int((x^5*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*d^2) + (13*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*
(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(9*d^2) + (8*c*(c + d*x^3)^(1/2))/(3*d^
2*(8*c - d*x^3))
```

Reduce [F]

$$\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$
$$= \frac{16\sqrt{d\,x^3+c}\,c-10\sqrt{d\,x^3+c}\,d\,x^3+2808\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)c^2d^2-351\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{d^3x^9-15c\,d^2x^6+48c^2d}\right.}{15d^2\left(-d\,x^3+8c\right)}$$

input
```
int(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

output
```
(16*sqrt(c + d*x**3)*c - 10*sqrt(c + d*x**3)*d*x**3 + 2808*int((sqrt(c + d
*x**3)*x**5)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c*
*2*d**2 - 351*int((sqrt(c + d*x**3)*x**5)/(64*c**3 + 48*c**2*d*x**3 - 15*c
*d**2*x**6 + d**3*x**9),x)*c*d**3*x**3)/(15*d**2*(8*c - d*x**3))
```

3.574 $\int \frac{x^2\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 64

$$\int \frac{x^2\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx=\frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

output
```
1/3*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)-1/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2)
)/c^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{x^2\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx=\frac{\frac{3\sqrt{c+dx^3}}{8c-dx^3}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}}{9d}$$

input
```
Integrate[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
((3*Sqrt[c + d*x^3])/(8*c - d*x^3) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/
Sqrt[c])/(9*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {946, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{(8c-dx^3)^2}dx^3$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}\left(\frac{\sqrt{c+dx^3}}{d(8c-dx^3)}-\frac{1}{2}\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\sqrt{c+dx^3}}{d(8c-dx^3)}-\frac{\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\sqrt{c+dx^3}}{d(8c-dx^3)}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3\sqrt{c}d}\right)$$

input
```
Int[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
(Sqrt[c + d*x^3]/(d*(8*c - d*x^3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/
(3*Sqrt[c]*d))/3
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

method	result
default	$\frac{\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}}{3d}$
pseudoelliptic	$\frac{\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}}{3d}$
elliptic	$\frac{\sqrt{d\,x^{3}+c}}{3d(-d\,x^{3}+8c)}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\ldots\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-\ldots\right)}}\ldots}{\ldots}\right)}{\ldots}$

input
```
int(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*((d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))
/c^(1/2))/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.28

$$\int \frac{x^2\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$$
$$=\left[\frac{(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,\sqrt{dx^3+c}c}{18\,(cd^2x^3-8\,c^2d)},\frac{(dx^3-8\,c)\sqrt{-c}\arctan\left(\frac{3\,\sqrt{-c}}{\sqrt{dx^3+c}}\right)-3\,\sqrt{dx^3+c}\ldots}{9\,(cd^2x^3-8\,c^2d)}\right.$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/18*((d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c
)/(d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c*d^2*x^3 - 8*c^2*d), 1/9*((d*x^3
 - 8*c)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) - 3*sqrt(d*x^3 + c)*c)
/(c*d^2*x^3 - 8*c^2*d)]
```

Sympy [F]

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x^2\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

input
```
integrate(x**2*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x**2*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}} - \frac{6\sqrt{dx^3+c}}{dx^3-8c}}{18d}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
1/18*(log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/sqr
t(c) - 6*sqrt(d*x^3 + c)/(d*x^3 - 8*c))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d} - \frac{\sqrt{dx^3+c}}{3\,(dx^3-8\,c)d}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
1/9*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 1/3*sqrt(d*x^3 + c
)/((d*x^3 - 8*c)*d)
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{18\,\sqrt{c}\,d} + \frac{\sqrt{d\,x^3+c}}{3\,d\,\left(8\,c-d\,x^3\right)}$$

input
```
int((x^2*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

output
```
log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(18*c^(1/2
)*d) + (c + d*x^3)^(1/2)/(3*d*(8*c - d*x^3))
```

Reduce [F]

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$
$$= \frac{2\sqrt{d\,x^3+c}+216\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)c\,d^2-27\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)d^3x^3}{30d\left(-d\,x^3+8c\right)}$$

input
```
int(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

output
```
(2*sqrt(c + d*x**3) + 216*int((sqrt(c + d*x**3)*x**5)/(64*c**3 + 48*c**2*d
*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c*d**2 - 27*int((sqrt(c + d*x**3)*x
**5)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*d**3*x**3)
/(30*d*(8*c - d*x**3))
```

3.575 $\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 88

$$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)^2} \, dx = \frac{\sqrt{c+dx^3}}{24c(8c-dx^3)} + \frac{5\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}$$

output
```
1/24*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)+5/288*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1
/2))/c^(3/2)-1/96*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)^2} \, dx = \frac{\frac{12\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3} + 5\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 3\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{288c^{3/2}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)^2),x]
```

output
```
((12*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + 5*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])] - 3*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(288*c^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {948, 110, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x (8c-dx^3)^2} \, dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3} \int \frac{\sqrt{dx^3+c}}{x^3 (8c-dx^3)^2} dx^3$$

$$\Big\downarrow \ 110$$

$$\frac{1}{3} \left(\frac{\sqrt{c+dx^3}}{8c (8c-dx^3)} - \frac{\int -\frac{dx^3+2c}{2x^3 (8c-dx^3)\sqrt{dx^3+c}} dx^3}{8c} \right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3} \left(\frac{\int \frac{dx^3+2c}{x^3 (8c-dx^3)\sqrt{dx^3+c}} dx^3}{16c} + \frac{\sqrt{c+dx^3}}{8c (8c-dx^3)} \right)$$

$$\Big\downarrow \ 174$$

$$\frac{1}{3} \left(\frac{\frac{1}{4} \int \frac{1}{x^3 \sqrt{dx^3+c}} dx^3 + \frac{5}{4} d \int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}} dx^3}{16c} + \frac{\sqrt{c+dx^3}}{8c (8c-dx^3)} \right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{3} \left(\frac{\frac{5}{2} \int \frac{1}{9c-x^6} d\sqrt{dx^3+c} + \frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}} d\sqrt{dx^3+c}}{2d}}{16c} + \frac{\sqrt{c+dx^3}}{8c (8c-dx^3)} \right)$$

$$\Big\downarrow \ 219$$

$$\frac{1}{3} \left(\frac{\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}} d\sqrt{dx^3+c}}{2d} + \frac{5\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{16c} + \frac{\sqrt{c+dx^3}}{8c (8c-dx^3)} \right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{3}\left(\frac{\frac{5\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{16c}+\frac{\sqrt{c+dx^3}}{8c\left(8c-dx^3\right)}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)^2),x]
```

output
```
(Sqrt[c + d*x^3]/(8*c*(8*c - d*x^3)) + ((5*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt
[c])])/(6*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(2*Sqrt[c]))/(16*c))
/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

method	result	size
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{96 c^{\frac{3}{2}}}+\frac{\frac{5 \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right) \left(d\,x^{3}-8 c\right)}{\sqrt{c}}-12 \sqrt{d\,x^{3}+c}}{288 \left(d\,x^{3}-8 c\right) c}$	78
default	$\frac{\frac{2 \sqrt{d\,x^{3}+c}}{3}-\frac{2 \sqrt{c}\, \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}}{64 c^{2}}+\frac{\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8 c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{3 \sqrt{c}}}{24 c}+\frac{-2 \sqrt{d\,x^{3}+c}+6 \sqrt{c}\, \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{192 c^{2}}$	123
elliptic	Expression too large to display	153

input
```
int((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/96*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)+1/288*(5*arctanh(1/3*(d*x^3
+c)^(1/2)/c^(1/2))/c^(1/2)*(d*x^3-8*c)-12*(d*x^3+c)^(1/2))/(d*x^3-8*c)/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.50

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx$$
$$= \left[\frac{5\left(dx^3-8\,c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\left(dx^3-8\,c\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\sqrt{dx^3+c}c}{576\left(c^2dx^3-8\,c^3\right)},\right.$$
$$\left.-\frac{5\left(dx^3-8\,c\right)\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)-3\left(dx^3-8\,c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)+12\sqrt{dx^3+c}c}{288\left(c^2dx^3-8\,c^3\right)}\right]$$

input
```
integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/576*(5*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 1
0*c)/(d*x^3 - 8*c)) + 3*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 +
c)*sqrt(c) + 2*c)/x^3) - 24*sqrt(d*x^3 + c)*c)/(c^2*d*x^3 - 8*c^3), -1/288
*(5*(d*x^3 - 8*c)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) - 3*(d*x^3 -
 8*c)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)) + 12*sqrt(d*x^3 + c)*c)/(c
^2*d*x^3 - 8*c^3)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx = \int \frac{\sqrt{c+dx^3}}{x\left(-8c+dx^3\right)^2}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x/(-d*x**3+8*c)**2,x)
```

output
```
Integral(sqrt(c + d*x**3)/(x*(-8*c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{(dx^3-8\,c)^2x}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}c} - \frac{5\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{288\sqrt{-c}c} - \frac{\sqrt{dx^3+c}}{24\,(dx^3-8\,c)c}$$

input
```
integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 5/288*arctan(1/3*sqrt
(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/24*sqrt(d*x^3 + c)/((d*x^3 - 8*c)*c
)
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)^2}\,dx = \frac{5\,\mathrm{atanh}\left(\frac{c\sqrt{d\,x^3+c}}{3\sqrt{c^3}}\right)}{288\sqrt{c^3}} - \frac{\mathrm{atanh}\left(\frac{c\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)}{96\sqrt{c^3}} + \frac{\sqrt{d\,x^3+c}}{8\,c\,(24\,c-3\,d\,x^3)}$$

input
```
int((c + d*x^3)^(1/2)/(x*(8*c - d*x^3)^2),x)
```

output
```
(5*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2))))/(288*(c^3)^(1/2)) - atanh
((c*(c + d*x^3)^(1/2))/(c^3)^(1/2))/(96*(c^3)^(1/2)) + (c + d*x^3)^(1/2)/(
8*c*(24*c - 3*d*x^3))
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{d\,x^3+c}}{d^2x^7-16cd\,x^4+64c^2x}dx$$

input
```
int((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x)
```

output
```
int(sqrt(c + d*x**3)/(64*c**2*x - 16*c*d*x**4 + d**2*x**7),x)
```

3.576 $\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx=\frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}+\frac{7d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}}$$

output
```
1/96*d*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)-1/24*(d*x^3+c)^(1/2)/c/x^3/(-d*x^3
+8*c)+7/1152*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-1/128*d*arctan
h((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx=\frac{\frac{12\sqrt{c}\left(4c-dx^3\right)\sqrt{c+dx^3}}{-8cx^3+dx^6}+7d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-9d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{1152c^{5/2}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)^2),x]
```

output
```
((12*Sqrt[c]*(4*c - d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 7*d*ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 9*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(1
152*c^(5/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {948, 110, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^4 \left(8c-dx^3\right)^2} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{\sqrt{dx^3+c}}{x^6 \left(8c-dx^3\right)^2}dx^3$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3} \left(\frac{\int \frac{3d(dx^3+4c)}{2x^3(8c-dx^3)^2\sqrt{dx^3+c}}dx^3}{8c}-\frac{\sqrt{c+dx^3}}{8cx^3 \left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(\frac{3d\int \frac{dx^3+4c}{x^3(8c-dx^3)^2\sqrt{dx^3+c}}dx^3}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3 \left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3} \left(\frac{3d\left(\frac{\sqrt{c+dx^3}}{6c(8c-dx^3)}-\frac{\int -\frac{6cd\left(dx^3+6c\right)}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{72c^2d}\right)}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3 \left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{3d\left(\frac{\int \frac{dx^3+6c}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{12c}+\frac{\sqrt{c+dx^3}}{6c(8c-dx^3)}\right)}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3(8c-dx^3)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{3d\left(\frac{\frac{3}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{7}{4}d\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{12c}+\frac{\sqrt{c+dx^3}}{6c(8c-dx^3)}\right)}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3(8c-dx^3)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{3d\left(\frac{\frac{7}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{3\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{12c}+\frac{\sqrt{c+dx^3}}{6c(8c-dx^3)}\right)}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3(8c-dx^3)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{3d\left(\frac{\frac{3\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{7\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{12c}+\frac{\sqrt{c+dx^3}}{6c(8c-dx^3)}\right)}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3(8c-dx^3)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{3d\left(\frac{\frac{7\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{12c}+\frac{\sqrt{c+dx^3}}{6c(8c-dx^3)}\right)}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3(8c-dx^3)}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)^2),x]
```

output
```
(-1/8*Sqrt[c + d*x^3]/(c*x^3*(8*c - d*x^3)) + (3*d*(Sqrt[c + d*x^3]/(6*c*(
8*c - d*x^3)) + ((7*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*Sqrt[c]) - (3
*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c]))/(12*c)))/(16*c))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{d\left(-\frac{\sqrt{d\,x^{3}+c}}{d\,x^{3}}-\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}+\frac{7\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{6\sqrt{c}}\right)}{192c^{2}}$
risch	$-\frac{\sqrt{d\,x^{3}+c}}{192c^{2}x^{3}}-\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9\sqrt{c}}-\frac{2c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3}\right)}{128c^{2}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3x^{3}}-\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}}{64c^{2}}+\frac{d\left(\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}\right)}{256c^{3}}+\frac{d\left(\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{192c^{2}}+$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/192*d/c^2*(-(d*x^3+c)^(1/2)/d/x^3-3/2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c
^(1/2)+(d*x^3+c)^(1/2)/(-d*x^3+8*c)+7/6*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2
))/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.19

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx$$
$$= \left[\frac{7\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+9\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\,(cdx}{2304\left(c^3dx^6-8\,c^4x^3\right)}\right.$$
$$-\frac{7\left(d^2x^6-8\,cdx^3\right)\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)-9\left(d^2x^6-8\,cdx^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)+12\left(cdx^3-4\,c^2\right)\sqrt{}}{1152\left(c^3dx^6-8\,c^4x^3\right)}$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/2304*(7*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sq
rt(c) + 10*c)/(d*x^3 - 8*c)) + 9*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3
- 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(c*d*x^3 - 4*c^2)*sqrt(d*x^3
+ c))/(c^3*d*x^6 - 8*c^4*x^3), -1/1152*(7*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*a
rctan(3*sqrt(-c)/sqrt(d*x^3 + c)) - 9*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arcta
n(sqrt(-c)/sqrt(d*x^3 + c)) + 12*(c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^3*d
*x^6 - 8*c^4*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{c+dx^3}}{x^4\left(-8c+dx^3\right)^2}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**4/(-d*x**3+8*c)**2,x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**4*(-8*c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^4}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx=\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{128\sqrt{-c}c^2}-\frac{7\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{1152\sqrt{-c}c^2}-\frac{\left(dx^3+c\right)^{\frac{3}{2}}d-5\sqrt{dx^3+c}cd}{96\left(\left(dx^3+c\right)^2-10\left(dx^3+c\right)c+9\,c^2\right)c^2}$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
1/128*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 7/1152*d*arctan(
1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/96*((d*x^3 + c)^(3/2)*d -
 5*sqrt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c^2)
```

Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{c+dx^3}}{x^4 (8c-dx^3)^2} \, dx = \frac{\frac{5 d \sqrt{d x^3+c}}{32 c} - \frac{d (d x^3+c)^{3/2}}{32 c^2}}{3 (d x^3+c)^2 - 30 c (d x^3+c) + 27 c^2} + \frac{d \left(\operatorname{atanh}\left(\frac{c^2 \sqrt{d x^3+c}}{\sqrt{c^5}}\right) 1i - \frac{\operatorname{atanh}\left(\frac{c^2 \sqrt{d x^3+c}}{3 \sqrt{c^5}}\right) 7i}{9}\right) 1i}{128 \sqrt{c^5}}$$

input
```
int((c + d*x^3)^(1/2)/(x^4*(8*c - d*x^3)^2),x)
```

output
```
((5*d*(c + d*x^3)^(1/2))/(32*c) - (d*(c + d*x^3)^(3/2))/(32*c^2))/(3*(c +
d*x^3)^2 - 30*c*(c + d*x^3) + 27*c^2) + (d*(atanh((c^2*(c + d*x^3)^(1/2))/
(c^5)^(1/2))*1i - (atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))*7i)/9)*1
i)/(128*(c^5)^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4 (8c-dx^3)^2} \, dx = \int \frac{\sqrt{d x^3+c}}{d^2 x^{10} - 16 c d x^7 + 64 c^2 x^4} dx$$

input
```
int((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x)
```

output
```
int(sqrt(c + d*x**3)/(64*c**2*x**4 - 16*c*d*x**7 + d**2*x**10),x)
```

3.577 $\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 164

$$\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)^2}\,dx = \frac{5d^2\sqrt{c+dx^3}}{1536c^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48cx^6(8c-dx^3)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3(8c-dx^3)} + \frac{23d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}} - \frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{7/2}}$$

output
```
5/1536*d^2*(d*x^3+c)^(1/2)/c^3/(-d*x^3+8*c)-1/48*(d*x^3+c)^(1/2)/c/x^6/(-d
*x^3+8*c)-7/384*d*(d*x^3+c)^(1/2)/c^2/x^3/(-d*x^3+8*c)+23/18432*d^2*arctan
h(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-1/2048*d^2*arctanh((d*x^3+c)^(1/2)/
c^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)^2}\,dx = \frac{\frac{12\sqrt{c}\sqrt{c+dx^3}(32c^2+28cdx^3-5d^2x^6)}{-8cx^6+dx^9} + 23d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 9d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{18432c^{7/2}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)^2),x]
```

output
```
((12*Sqrt[c]*Sqrt[c + d*x^3]*(32*c^2 + 28*c*d*x^3 - 5*d^2*x^6))/(-8*c*x^6
+ d*x^9) + 23*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 9*d^2*ArcTanh[Sqr
t[c + d*x^3]/Sqrt[c]])/(18432*c^(7/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {948, 110, 27, 168, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^7 (8c-dx^3)^2} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{\sqrt{dx^3+c}}{x^9 (8c-dx^3)^2} dx^3$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3} \left(\frac{\int \frac{d(5dx^3+14c)}{2x^6 (8c-dx^3)^2 \sqrt{dx^3+c}} dx^3}{16c} - \frac{\sqrt{c+dx^3}}{16cx^6 (8c-dx^3)} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(\frac{d \int \frac{5dx^3+14c}{x^6 (8c-dx^3)^2 \sqrt{dx^3+c}} dx^3}{32c} - \frac{\sqrt{c+dx^3}}{16cx^6 (8c-dx^3)} \right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(\frac{d\left(-\frac{\int -\frac{3cd\left(7dx^3+4c\right)}{x^3\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{8c^2}-\frac{7\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{d\left(\frac{3d\int \frac{7dx^3+4c}{x^3\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{8c}-\frac{7\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \; 168$$

$$\frac{1}{3}\left(\frac{d\left(\frac{3d\left(\frac{5\sqrt{c+dx^3}}{6c\left(8c-dx^3\right)}-\frac{\int -\frac{6cd\left(5dx^3+6c\right)}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{72c^2d}\right)}{8c}-\frac{7\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{d\left(\frac{3d\left(\frac{\int \frac{5dx^3+6c}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{12c}+\frac{5\sqrt{c+dx^3}}{6c\left(8c-dx^3\right)}\right)}{8c}-\frac{7\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{ 174}$$

$$\frac{1}{3}\left(\frac{d\left(\frac{3d\left(\frac{\frac{3}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{23}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{12c}+\frac{5\sqrt{c+dx^3}}{6c\left(8c-dx^3\right)}\right)}{8c}-\frac{7\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(\frac{d\left(\frac{3d\left(\frac{\frac{23}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{3\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{12c}+\frac{5\sqrt{c+dx^3}}{6c\left(8c-dx^3\right)}\right)}{8c}-\frac{7\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{d\left(\frac{3d\left(\frac{\frac{3\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{23\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{12c}+\frac{5\sqrt{c+dx^3}}{6c\left(8c-dx^3\right)}\right)}{8c}-\frac{7\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{d\left(\frac{3d\left(\frac{\frac{23\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{12c}+\frac{5\sqrt{c+dx^3}}{6c\left(8c-dx^3\right)}\right)}{8c}-\frac{7\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6\left(8c-dx^3\right)}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)^2),x]
```

output
```
(-1/16*Sqrt[c + d*x^3]/(c*x^6*(8*c - d*x^3)) + (d*((-7*Sqrt[c + d*x^3])/(4
*c*x^3*(8*c - d*x^3)) + (3*d*((5*Sqrt[c + d*x^3])/(6*c*(8*c - d*x^3)) + ((
23*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*Sqrt[c]) - (3*ArcTanh[Sqrt[c +
 d*x^3]/Sqrt[c]])/(2*Sqrt[c]))/(12*c)))/(8*c)))/(32*c))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.54

method	result
pseudoelliptic	$\frac{d^{2}\left(-\frac{\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{d^{2}x^{6}}-\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{16\sqrt{c}}+\frac{\sqrt{d\,x^{3}+c}}{-4d\,x^{3}+32c}+\frac{23\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{48\sqrt{c}}\right)}{384c^{3}}$
risch	$-\frac{\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{384c^{3}x^{6}}-\frac{d^{2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{8\sqrt{c}}-\frac{19\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{72\sqrt{c}}-\frac{c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{6}\right)}{256c^{3}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{6x^{6}}-\frac{d\sqrt{d\,x^{3}+c}}{12c\,x^{3}}+\frac{d^{2}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{12c^{\frac{3}{2}}}}{64c^{2}}+\frac{d\left(-\frac{\sqrt{d\,x^{3}+c}}{3x^{3}}-\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}\right)}{256c^{3}}+\frac{3d^{2}\left(\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\sqrt{c}\,\operatorname{arctan}}{}\right.}{4096c^{4}}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/384*d^2/c^3*(-(d*x^3+c)^(3/2)/d^2/x^6-3/16*arctanh((d*x^3+c)^(1/2)/c^(1/
2))/c^(1/2)+1/4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+23/48*arctanh(1/3*(d*x^3+c)^(
1/2)/c^(1/2))/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{c+dx^3}}{x^7 (8c-dx^3)^2}\,dx$$
$$= \left[\frac{23\,(d^3x^9-8\,cd^2x^6)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+9\,(d^3x^9-8\,cd^2x^6)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\,(5}{36864\,(c^4dx^9-8\,c^5x^6)}\right.$$
$$-\frac{23\,(d^3x^9-8\,cd^2x^6)\sqrt{-c}\arctan\left(\frac{3\,\sqrt{-c}}{\sqrt{dx^3+c}}\right)-9\,(d^3x^9-8\,cd^2x^6)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)+12\,(5\,cd^2x^6-2}{18432\,(c^4dx^9-8\,c^5x^6)}$$

input
```
integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/36864*(23*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c
)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 9*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((
d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(5*c*d^2*x^6 - 28*c^2*d
*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^4*d*x^9 - 8*c^5*x^6), -1/18432*(23*(d^3
*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) - 9*(d^3*x
^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)) + 12*(5*c*d^2*
x^6 - 28*c^2*d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^4*d*x^9 - 8*c^5*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^7 (8c-dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**(1/2)/x**7/(-d*x**3+8*c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^7}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^7), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx=\frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\sqrt{-c}c^3}-\frac{23\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{18432\sqrt{-c}c^3}-\frac{\sqrt{dx^3+c}d^2}{1536\left(dx^3-8\,c\right)c^3}-\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{384\,c^3x^6}$$

input
```
integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
1/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 23/18432*d^2*
arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/1536*sqrt(d*x^3 +
c)*d^2/((d*x^3 - 8*c)*c^3) - 1/384*(d*x^3 + c)^(3/2)/(c^3*x^6)
```

Mupad [B] (verification not implemented)

Time = 2.96 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx=\frac{\frac{d^2\sqrt{d\,x^3+c}}{512\,c}-\frac{19\,d^2\left(d\,x^3+c\right)^{3/2}}{256\,c^2}+\frac{5\,d^2\left(d\,x^3+c\right)^{5/2}}{512\,c^3}}{33\,c\left(d\,x^3+c\right)^2-57\,c^2\left(d\,x^3+c\right)-3\left(d\,x^3+c\right)^3+27\,c^3}+\frac{d^2\left(\mathrm{atanh}\left(\frac{c^3\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)\,1\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^3\sqrt{d\,x^3+c}}{3\sqrt{c^7}}\right)23\mathrm{i}}{9}\right)1\mathrm{i}}{2048\sqrt{c^7}}$$

input
```
int((c + d*x^3)^(1/2)/(x^7*(8*c - d*x^3)^2),x)
```

output
```
((d^2*(c + d*x^3)^(1/2))/(512*c) - (19*d^2*(c + d*x^3)^(3/2))/(256*c^2) +
(5*d^2*(c + d*x^3)^(5/2))/(512*c^3))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*x
^3) - 3*(c + d*x^3)^3 + 27*c^3) + (d^2*(atanh((c^3*(c + d*x^3)^(1/2))/(c^7
)^(1/2))*1i - (atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2)))*23i)/9)*1i)/
(2048*(c^7)^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)^2}\,dx=\int \frac{\sqrt{d\,x^3+c}}{d^2x^{13}-16cd\,x^{10}+64c^2x^7}dx$$

input
```
int((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x)
```

output
```
int(sqrt(c + d*x**3)/(64*c**2*x**7 - 16*c*d*x**10 + d**2*x**13),x)
```

3.578 $\int \frac{x^7\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 663

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{13x^2\sqrt{c+dx^3}}{21d^2}+\frac{746c\sqrt{c+dx^3}}{21d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^5\sqrt{c+dx^3}}{3d\,(8c-dx^3)}+\frac{76c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}}-\frac{76c^{7/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}}+\frac{76c^{7/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{8/3}}-\frac{373\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{746\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{21\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
13/21*x^2*(d*x^3+c)^(1/2)/d^2+746/21*c*(d*x^3+c)^(1/2)/d^(8/3)/((1+3^(1/2)
)*c^(1/3)+d^(1/3)*x)+1/3*x^5*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)+76/9*c^(7/6)*a
rctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/d^(8/3)
-76/9*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d
^(8/3)+76/9*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)-373/21*3^
(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*c^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1
/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*Ellipt
icE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1
/2)+2*I)/d^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)
*x)^2)^(1/2)/(d*x^3+c)^(1/2)+746/63*2^(1/2)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*((
c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^
(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/
3)*x),I*3^(1/2)+2*I)*3^(3/4)/d^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1
/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 5.89 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.27

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx =$$

$$-\frac{80(52c^2x^2+49cdx^5-3d^2x^8)+520cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+373d}{840d^2(-8c+dx^3)\sqrt{c+dx^3}}$$

input
```
Integrate[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
-1/840*(80*(52*c^2*x^2 + 49*c*d*x^5 - 3*d^2*x^8) + 520*c*x^2*(-8*c + d*x^3
)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*
c)] + 373*d*x^5*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8
/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(d^2*(-8*c + d*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {967, 27, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}dx$$

$$\Big\downarrow \ 967$$

$$\frac{x^5\sqrt{c+dx^3}}{3d(8c-dx^3)}-\frac{\int \frac{x^4(13dx^3+10c)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{3d}$$

$$\Big\downarrow \ 27$$

$$\frac{x^5\sqrt{c+dx^3}}{3d(8c-dx^3)}-\frac{\int \frac{x^4(13dx^3+10c)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{6d}$$

$$\Big\downarrow \ 1052$$

$$\frac{x^5\sqrt{c+dx^3}}{3d(8c-dx^3)}-\frac{\frac{2\int \frac{cdx\left(373dx^3+208c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d^2}-\frac{26x^2\sqrt{c+dx^3}}{7d}}{6d}$$

$$\Big\downarrow \ 27$$

$$\frac{x^5\sqrt{c+dx^3}}{3d(8c-dx^3)}-\frac{\frac{2c\int \frac{x\left(373dx^3+208c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d}-\frac{26x^2\sqrt{c+dx^3}}{7d}}{6d}$$

$$\Big\downarrow \ 1054$$

$$\frac{x^5\sqrt{c+dx^3}}{3d(8c-dx^3)}-\frac{\frac{2c\int \left(\frac{3192cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}-\frac{373x}{\sqrt{dx^3+c}}\right)dx}{7d}-\frac{26x^2\sqrt{c+dx^3}}{7d}}{6d}$$

$$\Big\downarrow \ 2009$$

$$\frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-$$

$$2c\left(-\frac{746\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{373\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\cdots}}{d^{2/3}\cdots}\right.$$

input

```
Int[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output

```
(x^5*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)) - ((-26*x^2*Sqrt[c + d*x^3])/(7*
d) + (2*c*((-746*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)) - (532*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c
 + d*x^3]])/(Sqrt[3]*d^(2/3)) + (532*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)
^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3*d^(2/3)) - (532*c^(1/6)*ArcTanh[Sqrt[c
 + d*x^3]/(3*Sqrt[c])])/(3*d^(2/3)) + (373*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/
3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(
1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(
d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*Sqrt[c + d*x^3]) - (746*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/
3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
d*x^3])))/(7*d))/(6*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 967
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Simp[e^n/(b*n*(p + 1))   Int[(e*x)^(m -
 n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(
q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d,
0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBino
mialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.29 (sec) , antiderivative size = 897, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	897
risch	Expression too large to display	1758
default	Expression too large to display	2199

input
```
int(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
8/3*x^2*c/d^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/7*x^2*(d*x^3+c)^(1/2)/d^2-746
/63*I/d^3*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3)
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+152/27*I*c
/d^5*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3
))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*
(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)
^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alph
a^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2568 vs. $2(470) = 940$.

Time = 8.16 (sec) , antiderivative size = 2568, normalized size of antiderivative = 3.87

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/189*(6714*(c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstr
assPInverse(0, -4*c/d, x)) + 133*(d^4*x^3 - 8*c*d^3 - sqrt(-3)*(d^4*x^3 -
8*c*d^3))*(c^7/d^16)^(1/6)*log(2535525376/3*((d^16*x^9 + 318*c*d^15*x^6 +
1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1
200*c^2*d^14*x^3 + 640*c^3*d^13))*(c^7/d^16)^(5/6) + 6*(2*c^6*d^2*x^7 + 16
0*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2 - sqrt(-3)*(
5*c^2*d^12*x^5 + 32*c^3*d^11*x^2))*(c^7/d^16)^(2/3) - (7*c^4*d^7*x^6 + 152
*c^5*d^6*x^3 + 64*c^6*d^5 + sqrt(-3)*(7*c^4*d^7*x^6 + 152*c^5*d^6*x^3 + 64
*c^6*d^5))*(c^7/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^10*x^7 + 64*c^4
*d^9*x^4 + 32*c^5*d^8*x)*sqrt(c^7/d^16) + 18*(c^5*d^5*x^8 + 38*c^6*d^4*x^5
 + 64*c^7*d^3*x^2 - sqrt(-3)*(c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 64*c^7*d^3*x^
2))*(c^7/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
- 133*(d^4*x^3 - 8*c*d^3 - sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(c^7/d^16)^(1/6)*
log(-2535525376/3*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^
3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3
*d^13))*(c^7/d^16)^(5/6) - 6*(2*c^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x -
6*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2 - sqrt(-3)*(5*c^2*d^12*x^5 + 32*c^3*d^
11*x^2))*(c^7/d^16)^(2/3) - (7*c^4*d^7*x^6 + 152*c^5*d^6*x^3 + 64*c^6*d^5
+ sqrt(-3)*(7*c^4*d^7*x^6 + 152*c^5*d^6*x^3 + 64*c^6*d^5))*(c^7/d^16)^(1/3
))*sqrt(d*x^3 + c) - 36*(5*c^3*d^10*x^7 + 64*c^4*d^9*x^4 + 32*c^5*d^8*x...
```

Sympy [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x^7\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

input
```
integrate(x**7*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x**7*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x^7}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c)^2, x)
```

Giac [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x^7}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x^7\,\sqrt{d\,x^3+c}}{(8\,c-d\,x^3)^2}\,dx$$

input
```
int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

output
```
int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{80\sqrt{d\,x^3+c}\,c\,x^2-58\sqrt{d\,x^3+c}\,d\,x^5+25144\left(\int \frac{\sqrt{d\,x^3+c}\,x^7}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)c^2d^2-3143\left(\int \frac{\sqrt{d\,x^3}}{d^3x^9-15c\,d^2x^6+}\right.}{203d^2}$$

input
```
int(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

output
```
(80*sqrt(c + d*x**3)*c*x**2 - 58*sqrt(c + d*x**3)*d*x**5 + 25144*int((sqrt
(c + d*x**3)*x**7)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9)
,x)*c**2*d**2 - 3143*int((sqrt(c + d*x**3)*x**7)/(64*c**3 + 48*c**2*d*x**3
 - 15*c*d**2*x**6 + d**3*x**9),x)*c*d**3*x**3 - 10240*int((sqrt(c + d*x**3
)*x)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**4 + 128
0*int((sqrt(c + d*x**3)*x)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d*
*3*x**9),x)*c**3*d*x**3)/(203*d**2*(8*c - d*x**3))
```

3.579 $\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 641

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$

$$= \frac{7\sqrt{c+dx^3}}{3d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{3d\,(8c-dx^3)}+\frac{5\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}}$$
$$-\frac{5\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{5/3}}+\frac{5\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{5/3}}$$
$$-\frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{2\ 3^{3/4}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{7\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output

```
7/3*(d*x^3+c)^(1/2)/d^(5/3)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)+1/3*x^2*(d*x^3
+c)^(1/2)/d/(-d*x^3+8*c)+5/9*c^(1/6)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/
3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/d^(5/3)-5/9*c^(1/6)*arctanh(1/3*(c^(1/3)+d^
(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(5/3)+5/9*c^(1/6)*arctanh(1/3*(d*x^3
+c)^(1/2)/c^(1/2))/d^(5/3)-7/6*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*c^(1/3)*(
c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c
^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3
^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/d^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3
)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+7/9*2^(1/2)*
c^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3
^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)
*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/d^(5/3)/(c^(1/3
)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(
1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.59 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.26

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$
$$= \frac{80cx^2(c+dx^3)+10cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+7dx^5(-8c+dx^3)\sqrt{1+}}{240cd\,(8c-dx^3)\sqrt{c+dx^3}}$$

input

```
Integrate[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output

```
(80*c*x^2*(c + d*x^3) + 10*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Appell
F1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 7*d*x^5*(-8*c + d*x^3)
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c
)])/(240*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {967, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}dx$$

$$\Big\downarrow \ 967$$

$$\frac{x^2\sqrt{c+dx^3}}{3d\,(8c-dx^3)}-\frac{\int \frac{x(7dx^3+4c)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{3d}$$

$$\Big\downarrow \ 27$$

$$\frac{x^2\sqrt{c+dx^3}}{3d\,(8c-dx^3)}-\frac{\int \frac{x(7dx^3+4c)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{6d}$$

$$\Big\downarrow \ 1054$$

$$\frac{x^2\sqrt{c+dx^3}}{3d\,(8c-dx^3)}-\frac{\int \left(\frac{60cx}{(8c-dx^3)\sqrt{dx^3+c}}-\frac{7x}{\sqrt{dx^3+c}}\right)dx}{6d}$$

$$\Big\downarrow \ 2009$$

$$\frac{x^2\sqrt{c+dx^3}}{3d\,(8c-dx^3)}-\frac{-\frac{14\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{7\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right.}{}}{}$$

input
```
Int[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
(x^2*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)) - ((-14*Sqrt[c + d*x^3])/(d^(2/3
)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (10*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/
6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) + (10*c^(1/6
)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3*d^(2/3)
) - (10*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^(2/3)) + (7*3^(
1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipti
cE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (14*Sqrt[2]*c^(1/
3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(
1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(
3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/(6*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 967
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Simp[e^n/(b*n*(p + 1))   Int[(e*x)^(m -
 n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(
q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d,
0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBino
mialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.67 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	877
default	Expression too large to display	1742

input
```
int(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^2*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)-7/9*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)))+10/27*I/d^4*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)
*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)
^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*
d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3
)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2397 vs. $2(452) = 904$.

Time = 1.66 (sec) , antiderivative size = 2397, normalized size of antiderivative = 3.74

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/108*(36*sqrt(d*x^3 + c)*d*x^2 + 252*(d*x^3 - 8*c)*sqrt(d)*weierstrassZe
ta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 5*(d^3*x^3 - 8*c*d^2 -
sqrt(-3)*(d^3*x^3 - 8*c*d^2))*(c/d^10)^(1/6)*log(3125/3*((d^11*x^9 + 318*c
*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c*d^
10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8))*(c/d^10)^(5/6) + 6*(2*c*d^2*x^7
+ 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^8*x^5 + 32*c^2*d^7*x^2 - sqrt(-3)*(
5*c*d^8*x^5 + 32*c^2*d^7*x^2))*(c/d^10)^(2/3) - (7*c*d^5*x^6 + 152*c^2*d^4
*x^3 + 64*c^3*d^3 + sqrt(-3)*(7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3))
*(c/d^10)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c
^3*d^5*x)*sqrt(c/d^10) + 18*(c*d^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2 -
 sqrt(-3)*(c*d^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2))*(c/d^10)^(1/6))/(
d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 5*(d^3*x^3 - 8*c*d^2
- sqrt(-3)*(d^3*x^3 - 8*c*d^2))*(c/d^10)^(1/6)*log(-3125/3*((d^11*x^9 + 31
8*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c
*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8))*(c/d^10)^(5/6) - 6*(2*c*d^2*x
^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^8*x^5 + 32*c^2*d^7*x^2 - sqrt(-3
)*(5*c*d^8*x^5 + 32*c^2*d^7*x^2))*(c/d^10)^(2/3) - (7*c*d^5*x^6 + 152*c^2*
d^4*x^3 + 64*c^3*d^3 + sqrt(-3)*(7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^
3))*(c/d^10)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 3
2*c^3*d^5*x)*sqrt(c/d^10) + 18*(c*d^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2...
```

Sympy [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x^4\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

input
```
integrate(x**4*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x**4*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x^4}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c)^2, x)
```

Giac [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x^4}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x^4\,\sqrt{d\,x^3+c}}{{\left(8\,c-d\,x^3\right)}^2}\,dx$$

input
```
int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

output
```
int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$
$$= \frac{2\sqrt{d\,x^3+c}\,x^2+472\left(\int \frac{\sqrt{d\,x^3+c}\,x^7}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)c\,d^2-59\left(\int \frac{\sqrt{d\,x^3+c}\,x^7}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)d^3x^3-256}{58d\left(-d\,x^3+8c\right)}$$

input
```
int(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

output
```
(2*sqrt(c + d*x**3)*x**2 + 472*int((sqrt(c + d*x**3)*x**7)/(64*c**3 + 48*c
**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c*d**2 - 59*int((sqrt(c + d*x*
*3)*x**7)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*d**3*
x**3 - 256*int((sqrt(c + d*x**3)*x)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*
x**6 + d**3*x**9),x)*c**3 + 32*int((sqrt(c + d*x**3)*x)/(64*c**3 + 48*c**2
*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**2*d*x**3)/(58*d*(8*c - d*x**3)
)
```

3.580 $\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 644

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{\sqrt{c+dx^3}}{24cd^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{24c\,(8c-dx^3)}$$
$$+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{5/6}d^{2/3}}-\frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{5/6}d^{2/3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{5/6}d^{2/3}}$$
$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)\middle|-7-4\sqrt{3}\right)}{16\,3^{3/4}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
1/24*(d*x^3+c)^(1/2)/c/d^(2/3)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)+1/24*x^2*(d
*x^3+c)^(1/2)/c/(-d*x^3+8*c)+1/144*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)
*x)/(d*x^3+c)^(1/2))*3^(1/2)/c^(5/6)/d^(2/3)-1/144*arctanh(1/3*(c^(1/3)+d^
(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d^(2/3)+1/144*arctanh(1/3*(d*x
^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(2/3)-1/48*(1/2*6^(1/2)-1/2*2^(1/2))*(c^(1/
3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3
)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2
))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/c^(2/3)/d^(2/3)/(c^(1/3)*(c^(
1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+1
/72*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/
2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/
((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/c^(2/3)/d^(
2/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)
/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.25

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$
$$= \frac{80cx^2(c+dx^3)+5cx^2(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^5(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}}{1920c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

input
```
Integrate[(x*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
(80*c*x^2*(c + d*x^3) + 5*c*x^2*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1
[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/
(1920*c^2*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 640, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {969, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}dx$$

$$\Big\downarrow \text{969}$$

$$\frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)}-\frac{\int -\frac{x(2c-dx^3)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{24c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x(2c-dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{48c}+\frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)}$$

$$\Big\downarrow \text{1054}$$

$$\frac{\int \left(\frac{x}{\sqrt{dx^3+c}}-\frac{6cx}{(8c-dx^3)\sqrt{dx^3+c}}\right)dx}{48c}+\frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\cdots}}{d^{2\cdots}}\cdots}{\cdots}$$

$$\frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)}$$

input
```
Int[(x*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
(x^2*Sqrt[c + d*x^3])/(24*c*(8*c - d*x^3)) + ((2*Sqrt[c + d*x^3])/(d^(2/3)
*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(
c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) - (c^(1/6)*ArcTa
nh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3*d^(2/3)) + (c^
(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^(2/3)) - (3^(1/4)*Sqrt[2
- Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x
 + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((
1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7
 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) +
 d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3
)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/
3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x
)^2]*Sqrt[c + d*x^3]))/(48*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 969
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[(e*x)^m*(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
 m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.12 (sec) , antiderivative size = 883, normalized size of antiderivative = 1.37

method	result	size
default	Expression too large to display	883
elliptic	Expression too large to display	883

input
```
int(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/24*x^2*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)-1/72*I/c*3^(1/2)/d*(-c*d^2)^(1/3)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)))+1/216*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(
1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2
)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*
d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1
/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*
d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^
(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2496 vs. $2(455) = 910$.

Time = 0.44 (sec) , antiderivative size = 2496, normalized size of antiderivative = 3.88

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/1728*(72*sqrt(d*x^3 + c)*d*x^2 + 72*(d*x^3 - 8*c)*sqrt(d)*weierstrassZe
ta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + (c*d^2*x^3 - 8*c^2*d +
sqrt(-3)*(c*d^2*x^3 - 8*c^2*d))*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d
^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32
*c^6*d^3*x + sqrt(-3)*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x))*(1/
(c^5*d^4))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2 -
sqrt(-3)*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2))*(1/(c^5*d^4))^(5/6) - 2*(7*c^3*
d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7 + 8
0*c^2*d^2*x^4 + 160*c^3*d*x + sqrt(-3)*(c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c
^3*d*x))*(1/(c^5*d^4))^(1/6)) - 9*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d
^2*x^2 - sqrt(-3)*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2))*(1/(c^5
*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c*d^2
*x^3 - 8*c^2*d + sqrt(-3)*(c*d^2*x^3 - 8*c^2*d))*(1/(c^5*d^4))^(1/6)*log((
d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^5*x^7 + 64
*c^5*d^4*x^4 + 32*c^6*d^3*x + sqrt(-3)*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 3
2*c^6*d^3*x))*(1/(c^5*d^4))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 +
32*c^6*d^4*x^2 - sqrt(-3)*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2))*(1/(c^5*d^4))^
(5/6) - 2*(7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4))
 + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x + sqrt(-3)*(c*d^3*x^7 + 80*c^
2*d^2*x^4 + 160*c^3*d*x))*(1/(c^5*d^4))^(1/6)) - 9*(c^2*d^4*x^8 + 38*c^...
```

Sympy [F]

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

input
```
integrate(x*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x/(d*x^3 - 8*c)^2, x)
```

Giac [F]

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x/(d*x^3 - 8*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx=\int \frac{x\,\sqrt{d\,x^3+c}}{{\left(8\,c-d\,x^3\right)}^2}\,dx$$

input
```
int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

output
```
int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx=\int \frac{\sqrt{d\,x^3+c}\,x}{d^2x^6-16cd\,x^3+64c^2}dx$$

input
```
int(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

output
```
int((sqrt(c + d*x**3)*x)/(64*c**2 - 16*c*d*x**3 + d**2*x**6),x)
```

3.581 $\int \frac{\sqrt{c+dx^3}}{x^2 (8c-dx^3)^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 665

$$\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)^2}\,dx = -\frac{\sqrt{c+dx^3}}{48c^2x}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{48c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{24cx(8c-dx^3)}-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{11/6}}+\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{11/6}}-\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{11/6}}-\frac{\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32\ 3^{3/4}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{24\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-1/48*(d*x^3+c)^(1/2)/c^2/x+1/48*d^(1/3)*(d*x^3+c)^(1/2)/c^2/((1+3^(1/2))*
c^(1/3)+d^(1/3)*x)+1/24*(d*x^3+c)^(1/2)/c/x/(-d*x^3+8*c)-1/144*d^(1/3)*arc
tan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/c^(11/6)+
1/144*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(11/6)-1/144*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-1/96*3
^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*d^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(
1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*Ellip
ticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(
1/2)+2*I)/c^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3
)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+1/144*d^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-
c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*El
lipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*
3^(1/2)+2*I)*2^(1/2)*3^(3/4)/c^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1
/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx$$
$$= \frac{-80c(6c^2+5cdx^3-d^2x^6)+50cdx^3(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+d^2x^6(-8c+}{3840c^3\sqrt{c+dx^3}\left(8cx-dx^4\right)}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)^2),x]
```

output
```
(-80*c*(6*c^2 + 5*c*d*x^3 - d^2*x^6) + 50*c*d*x^3*(8*c - d*x^3)*Sqrt[1 + (
d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d^2*x^
6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/
c), (d*x^3)/(8*c)])/(3840*c^3*Sqrt[c + d*x^3]*(8*c*x - d*x^4))
```

Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {969, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^2 (8c-dx^3)^2} \, dx$$

$$\Big\downarrow \text{ 969}$$

$$\frac{\sqrt{c+dx^3}}{24cx (8c-dx^3)}-\frac{\int -\frac{5dx^3+8c}{2x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{24c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{5dx^3+8c}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{48c}+\frac{\sqrt{c+dx^3}}{24cx (8c-dx^3)}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{-\frac{\int -\frac{4cdx\left(20c-dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{\sqrt{c+dx^3}}{cx}}{48c}+\frac{\sqrt{c+dx^3}}{24cx (8c-dx^3)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{d\int \frac{x\left(20c-dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{2c}-\frac{\sqrt{c+dx^3}}{cx}}{48c}+\frac{\sqrt{c+dx^3}}{24cx (8c-dx^3)}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{\frac{d\int \left(\frac{12cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{x}{\sqrt{dx^3+c}}\right)dx}{2c}-\frac{\sqrt{c+dx^3}}{cx}}{48c}+\frac{\sqrt{c+dx^3}}{24cx (8c-dx^3)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\left(\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{\ldots}}{\left(\left(1+\sqrt{\ldots}\right.\right.}}\ldots}{d^{2/3}\sqrt{\frac{\sqrt[3]{\ldots}}{\left(\left(1\ldots\right.\right.}}\ldots}\ldots\right.}{\ldots}$$

$$\frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)}$$

input

```
Int[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)^2),x]
```

output

```
Sqrt[c + d*x^3]/(24*c*x*(8*c - d*x^3)) + (-(Sqrt[c + d*x^3]/(c*x)) + (d*((
2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (2*c^(1
/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt
[3]*d^(2/3)) + (2*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[
c + d*x^3])])/(3*d^(2/3)) - (2*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/(3*d^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*
Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^
(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]
) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/
3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(2*c))/(48*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 969
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[(e*x)^m*(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
 m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.17 (sec) , antiderivative size = 898, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	898
risch	Expression too large to display	1758
default	Expression too large to display	2194

input
```
int((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/192*x^2/c^2*d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/64*(d*x^3+c)^(1/2)/c^2/x-1/
144*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/216*I/c^2/
d^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3)
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha
^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2390 vs. $2(472) = 944$.

Time = 0.46 (sec) , antiderivative size = 2390, normalized size of antiderivative = 3.59

$$\int \frac{\sqrt{c+dx^3}}{x^2 (8c-dx^3)^2} \, dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/1728*(36*(d*x^4 - 8*c*x)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrass
PInverse(0, -4*c/d, x)) - (c^2*d*x^4 - 8*c^3*x + sqrt(-3)*(c^2*d*x^4 - 8*c
^3*x))*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 +
640*c^3*d - 9*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x + sqrt(-3)*(5*c^8*
d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x))*(d^2/c^11)^(2/3) + 3*sqrt(d*x^3 + c)*
(6*(5*c^10*d*x^5 + 32*c^11*x^2 - sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d
^2/c^11)^(5/6) - 2*(7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11)
 + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 + 8
0*c^3*d^2*x^4 + 160*c^4*d*x))*(d^2/c^11)^(1/6)) - 9*(c^4*d^3*x^8 + 38*c^5*
d^2*x^5 + 64*c^6*d*x^2 - sqrt(-3)*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d
*x^2))*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3
)) + (c^2*d*x^4 - 8*c^3*x + sqrt(-3)*(c^2*d*x^4 - 8*c^3*x))*(d^2/c^11)^(1/
6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^8*
d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x + sqrt(-3)*(5*c^8*d^2*x^7 + 64*c^9*d*x^
4 + 32*c^10*x))*(d^2/c^11)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32
*c^11*x^2 - sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^2/c^11)^(5/6) - 2*(7
*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11) + (c^2*d^3*x^7 + 80*
c^3*d^2*x^4 + 160*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c
^4*d*x))*(d^2/c^11)^(1/6)) - 9*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^
2 - sqrt(-3)*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2))*(d^2/c^11)^...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{c+dx^3}}{x^2\left(-8c+dx^3\right)^2}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**2/(-d*x**3+8*c)**2,x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**2*(-8*c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^2\,(8c-dx^3)^2}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^2\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)^2),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\,(8c-dx^3)^2}\,dx=\int \frac{\sqrt{d\,x^3+c}}{d^2x^8-16cd\,x^5+64c^2x^2}dx$$

input
```
int((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x)
```

output
```
int(sqrt(c + d*x**3)/(64*c**2*x**2 - 16*c*d*x**5 + d**2*x**8),x)
```

3.582 $\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 687

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^5(8c-dx^3)^2}\,dx = &-\frac{7\sqrt{c+dx^3}}{768c^2x^4}-\frac{d\sqrt{c+dx^3}}{96c^3x}+\frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&+\frac{\sqrt{c+dx^3}}{24cx^4(8c-dx^3)}-\frac{17d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}c^{17/6}} \\
&+\frac{17d^{4/3}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{17/6}}-\frac{17d^{4/3}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9216c^{17/6}} \\
&-\frac{\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{64\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{48\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

output
```
-7/768*(d*x^3+c)^(1/2)/c^2/x^4-1/96*d*(d*x^3+c)^(1/2)/c^3/x+1/96*d^(4/3)*(
d*x^3+c)^(1/2)/c^3/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)+1/24*(d*x^3+c)^(1/2)/c/
x^4/(-d*x^3+8*c)-17/9216*d^(4/3)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x
)/(d*x^3+c)^(1/2))*3^(1/2)/c^(17/6)+17/9216*d^(4/3)*arctanh(1/3*(c^(1/3)+d
^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(17/6)-17/9216*d^(4/3)*arctanh(1/3*
(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-1/192*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*
d^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3
^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)
*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/c^(8/3)/(c^(1/3)*(c^(1/
3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+1/2
88*d^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((
1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1
/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/c^(8
/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/
(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.29

$$\int \frac{\sqrt{c+dx^3}}{x^5 (8c-dx^3)^2}\,dx = \sqrt{c+dx^3}\left(-\frac{1}{256c^2x^4}-\frac{5d}{512c^3x}-\frac{d^2x^2}{1536c^3(-8c+dx^3)}\right) + \frac{115d^2x^2\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{24576c^3\sqrt{c+dx^3}} - \frac{d^3x^5\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{7680c^4\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)^2),x]
```

output
```
Sqrt[c + d*x^3]*(-1/256*1/(c^2*x^4) - (5*d)/(512*c^3*x) - (d^2*x^2)/(1536*
c^3*(-8*c + d*x^3))) + (115*d^2*x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2,
 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(24576*c^3*Sqrt[c + d*x^3]) - (d^3*
x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/(7680*c^4*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {969, 27, 1053, 25, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^5 (8c-dx^3)^2} \, dx$$

$$\Big\downarrow \text{ 969}$$

$$\frac{\sqrt{c+dx^3}}{24cx^4 (8c-dx^3)} - \frac{\int -\frac{11dx^3+14c}{2x^5(8c-dx^3)\sqrt{dx^3+c}}dx}{24c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{11dx^3+14c}{x^5(8c-dx^3)\sqrt{dx^3+c}}dx}{48c} + \frac{\sqrt{c+dx^3}}{24cx^4 (8c-dx^3)}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{-\frac{\int -\frac{cd\left(35dx^3+128c\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2} - \frac{7\sqrt{c+dx^3}}{16cx^4}}{48c} + \frac{\sqrt{c+dx^3}}{24cx^4 (8c-dx^3)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{cd\left(35dx^3+128c\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2} - \frac{7\sqrt{c+dx^3}}{16cx^4}}{48c} + \frac{\sqrt{c+dx^3}}{24cx^4 (8c-dx^3)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{d\int \frac{35dx^3+128c}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c} - \frac{7\sqrt{c+dx^3}}{16cx^4}}{48c} + \frac{\sqrt{c+dx^3}}{24cx^4 (8c-dx^3)}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{\frac{d\left(-\frac{\int -\frac{8cdx\left(115c-8dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{16\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{7\sqrt{c+dx^3}}{16cx^4}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}$$

$\big\downarrow$ 27

$$\frac{\frac{d\left(\frac{d\int \frac{x\left(115c-8dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{16\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{7\sqrt{c+dx^3}}{16cx^4}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}$$

$\big\downarrow$ 1054

$$\frac{\frac{d\left(\frac{d\int \left(\frac{51cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{8x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{16\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{7\sqrt{c+dx^3}}{16cx^4}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}$$

$\big\downarrow$ 2009

$$\frac{d\left(\frac{d\left(\frac{16\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{8\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/\cdots}}{\left(\cdots\right.}}\cdots}{d^{2/3}\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots$$

$$\frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}$$

input
```
Int[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)^2),x]
```

output
```
Sqrt[c + d*x^3]/(24*c*x^4*(8*c - d*x^3)) + ((-7*Sqrt[c + d*x^3])/(16*c*x^4
) + (d*((-16*Sqrt[c + d*x^3])/(c*x) + (d*((16*Sqrt[c + d*x^3])/(d^(2/3)*((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (17*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(
c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*Sqrt[3]*d^(2/3)) + (17*c^(1/6)*
ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(6*d^(2/3))
- (17*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*d^(2/3)) - (8*3^(1/
4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)
*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (16*Sqrt[2]*c^(1/3)
*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/
3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3)
+ d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/c))/(32*c))/(48*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 969
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[(e*x)^m*(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
 m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.97 (sec) , antiderivative size = 919, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	919
risch	Expression too large to display	1770
default	Expression too large to display	2672

input
```
int((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/1536*d^2*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)/c^3-1/256*(d*x^3+c)^(1/2)/c^2/
x^4-5/512*d*(d*x^3+c)^(1/2)/c^3/x-1/288*I*d/c^3*3^(1/2)*(-c*d^2)^(1/3)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)))-17/13824*I/d/c^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(
1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2
)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*
d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1
/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*
d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2549 vs. $2(490) = 980$.

Time = 1.51 (sec) , antiderivative size = 2549, normalized size of antiderivative = 3.71

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/110592*(1152*(d^2*x^7 - 8*c*d*x^4)*sqrt(d)*weierstrassZeta(0, -4*c/d, w
eierstrassPInverse(0, -4*c/d, x)) - 17*(c^3*d*x^7 - 8*c^4*x^4 + sqrt(-3)*(
c^3*d*x^7 - 8*c^4*x^4))*(d^8/c^17)^(1/6)*log(1419857*(d^9*x^9 + 318*c*d^8*
x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^12*d^3*x^7 + 64*c^13*d^2*x^4
 + 32*c^14*d*x + sqrt(-3)*(5*c^12*d^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x)
)*(d^8/c^17)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2 - sq
rt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^8/c^17)^(5/6) - 2*(7*c^9*d^4*x^6 +
 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17) + (c^3*d^7*x^7 + 80*c^4*d^
6*x^4 + 160*c^5*d^5*x + sqrt(-3)*(c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 160*c^5*d
^5*x))*(d^8/c^17)^(1/6)) - 9*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4*x^
2 - sqrt(-3)*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4*x^2))*(d^8/c^17)^(
1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 17*(c^3*d*x^7
 - 8*c^4*x^4 + sqrt(-3)*(c^3*d*x^7 - 8*c^4*x^4))*(d^8/c^17)^(1/6)*log(14198
57*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^12*d
^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x + sqrt(-3)*(5*c^12*d^3*x^7 + 64*c^1
3*d^2*x^4 + 32*c^14*d*x))*(d^8/c^17)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^15*
d*x^5 + 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^8/c^17)^(5
/6) - 2*(7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17) +
(c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 160*c^5*d^5*x + sqrt(-3)*(c^3*d^7*x^7 + 80
*c^4*d^6*x^4 + 160*c^5*d^5*x))*(d^8/c^17)^(1/6)) - 9*(c^6*d^6*x^8 + 38*...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^5 (8c-dx^3)^2}\,dx = \int \frac{\sqrt{c+dx^3}}{x^5 (-8c+dx^3)^2}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**5/(-d*x**3+8*c)**2,x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**5*(-8*c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^5}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^5), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^5}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^5\,\left(8\,c-d\,x^3\right)^2}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)^2),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^5 \left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{d\,x^3+c}}{d^2x^{11}-16cd\,x^8+64c^2x^5}dx$$

input
```
int((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x)
```

output
```
int(sqrt(c + d*x**3)/(64*c**2*x**5 - 16*c*d*x**8 + d**2*x**11),x)
```

3.583 $\int \frac{\sqrt{c+dx^3}}{x^8 (8c-dx^3)^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 711

$$\int \frac{\sqrt{c+dx^3}}{x^8(8c-dx^3)^2}\,dx = -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{24cx^7(8c-dx^3)} - \frac{13d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12288\sqrt{3}c^{23/6}} + \frac{13d^{7/3}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{36864c^{23/6}} - \frac{13d^{7/3}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36864c^{23/6}} - \frac{\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{3584\,3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{2688\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-5/672*(d*x^3+c)^(1/2)/c^2/x^7-53/21504*d*(d*x^3+c)^(1/2)/c^3/x^4-1/5376*d
^2*(d*x^3+c)^(1/2)/c^4/x+1/5376*d^(7/3)*(d*x^3+c)^(1/2)/c^4/((1+3^(1/2))*c
^(1/3)+d^(1/3)*x)+1/24*(d*x^3+c)^(1/2)/c/x^7/(-d*x^3+8*c)-13/36864*d^(7/3)
*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/c^(23
/6)+13/36864*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(
1/2))/c^(23/6)-13/36864*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23
/6)-1/10752*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*d^(7/3)*(c^(1/3)+d^(1/3)*x)*
((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2
)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(
1/3)*x),I*3^(1/2)+2*I)/c^(11/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*
c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+1/16128*3^(3/4)*d^(7/3)*(c^(1/
3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3
)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2
))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)/c^(11/3)/(c^(1/3)*(c^(1/3)+d^
(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.29

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx = \frac{1525cd^3x^9(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-8\Big(20c(384c^4+648c^3dx^3+243c^2d^2x^6}{3440640c^5x^7\left(8c-dx^3\right)\sqrt{c}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)^2),x]
```

output
```
(1525*c*d^3*x^9*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/
3, -((d*x^3)/c), (d*x^3)/(8*c)] - 8*(20*c*(384*c^4 + 648*c^3*d*x^3 + 243*c
^2*d^2*x^6 - 25*c*d^3*x^9 - 4*d^4*x^12) + d^4*x^12*(8*c - d*x^3)*Sqrt[1 +
(d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(3440
640*c^5*x^7*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 728, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {969, 27, 1053, 27, 1053, 25, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^8 (8c-dx^3)^2} dx$$

$$\Big\downarrow \text{969}$$

$$\frac{\sqrt{c+dx^3}}{24cx^7(8c-dx^3)} - \frac{\int -\frac{17dx^3+20c}{2x^8(8c-dx^3)\sqrt{dx^3+c}}dx}{24c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{17dx^3+20c}{x^8(8c-dx^3)\sqrt{dx^3+c}}dx}{48c} + \frac{\sqrt{c+dx^3}}{24cx^7(8c-dx^3)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int -\frac{2cd\left(55dx^3+106c\right)}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{56c^2} - \frac{5\sqrt{c+dx^3}}{14cx^7}}{48c} + \frac{\sqrt{c+dx^3}}{24cx^7(8c-dx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{d\int \frac{55dx^3+106c}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{28c} - \frac{5\sqrt{c+dx^3}}{14cx^7}}{48c} + \frac{\sqrt{c+dx^3}}{24cx^7(8c-dx^3)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{\frac{d\left(-\frac{\int -\frac{cd\left(265dx^3+64c\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2} - \frac{53\sqrt{c+dx^3}}{16cx^4}\right)}{28c} - \frac{5\sqrt{c+dx^3}}{14cx^7}}{48c} + \frac{\sqrt{c+dx^3}}{24cx^7(8c-dx^3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{d\left(\frac{\int \frac{cd(265dx^3+64c)}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c^2}-\frac{53\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{5\sqrt{c+dx^3}}{14cx^7}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^7(8c-dx^3)}$$

$\Big\downarrow$ 27

$$\frac{\frac{d\left(\frac{d\int \frac{265dx^3+64c}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c}-\frac{53\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{5\sqrt{c+dx^3}}{14cx^7}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^7(8c-dx^3)}$$

$\Big\downarrow$ 1053

$$\frac{\frac{d\left(\frac{d\left(-\frac{\int -\frac{8cdx(305c-4dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{8c^2}-\frac{8\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{53\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{5\sqrt{c+dx^3}}{14cx^7}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^7(8c-dx^3)}$$

$\Big\downarrow$ 27

$$\frac{\frac{d\left(\frac{d\left(\frac{d\int \frac{x(305c-4dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{c}-\frac{8\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{53\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{5\sqrt{c+dx^3}}{14cx^7}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^7(8c-dx^3)}$$

$\Big\downarrow$ 1054

$$\frac{\frac{d\left(\frac{d\left(\frac{d\int \left(\frac{273cx}{(8c-dx^3)\sqrt{dx^3+c}}+\frac{4x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{8\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{53\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{5\sqrt{c+dx^3}}{14cx^7}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^7(8c-dx^3)}$$

$\Big\downarrow$ 2009

$$d\left(d\left(d\left(\frac{8\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\ldots}}{d^{2/3}\ldots}\right.\right.\right.$$

$$\frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)}$$

input
```
Int[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)^2),x]
```

output
```
Sqrt[c + d*x^3]/(24*c*x^7*(8*c - d*x^3)) + ((-5*Sqrt[c + d*x^3])/(14*c*x^7
) + (d*((-53*Sqrt[c + d*x^3])/(16*c*x^4) + (d*((-8*Sqrt[c + d*x^3])/(c*x)
+ (d*((8*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) -
(91*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]
])/(2*Sqrt[3]*d^(2/3)) + (91*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^
(1/6)*Sqrt[c + d*x^3])])/(6*d^(2/3)) - (91*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]
/(3*Sqrt[c])])/(6*d^(2/3)) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3)
 + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1
/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqr
t[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*S
qrt[c + d*x^3]) + (8*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) -
 c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) +
 d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/c))
/(32*c)))/(28*c))/(48*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 969
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[(e*x)^m*(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
 m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.11 (sec) , antiderivative size = 938, normalized size of antiderivative = 1.32

method	result	size
elliptic	Expression too large to display	938
risch	Expression too large to display	1781
default	Expression too large to display	3170

input
```
int((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/448*(d*x^3+c)^(1/2)/c^2/x^7-13/7168*d*(d*x^3+c)^(1/2)/c^3/x^4-3/28672*d
^2*(d*x^3+c)^(1/2)/c^4/x+1/12288*x^2*d^3*(d*x^3+c)^(1/2)/(-d*x^3+8*c)/c^4-
1/16128*I*d^2/c^4*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)
^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*
(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(
1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-
c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^
(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-13/5
5296*I/c^4*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2
)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*
x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2
*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2582 vs. $2(510) = 1020$.

Time = 3.18 (sec) , antiderivative size = 2582, normalized size of antiderivative = 3.63

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/3096576*(576*(d^3*x^10 - 8*c*d^2*x^7)*sqrt(d)*weierstrassZeta(0, -4*c/d
, weierstrassPInverse(0, -4*c/d, x)) - 91*(c^4*d*x^10 - 8*c^5*x^7 + sqrt(-
3)*(c^4*d*x^10 - 8*c^5*x^7))*(d^14/c^23)^(1/6)*log(371293*(d^14*x^9 + 318*
c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^16*d^4*x^7 + 64*c^1
7*d^3*x^4 + 32*c^18*d^2*x + sqrt(-3)*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 3
2*c^18*d^2*x))*(d^14/c^23)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32
*c^21*x^2 - sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^14/c^23)^(5/6) - 2*(
7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*c^14*d^4)*sqrt(d^14/c^23) + (c^4*d^
11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x + sqrt(-3)*(c^4*d^11*x^7 + 80*c^5
*d^10*x^4 + 160*c^6*d^9*x))*(d^14/c^23)^(1/6)) - 9*(c^8*d^9*x^8 + 38*c^9*d
^8*x^5 + 64*c^10*d^7*x^2 - sqrt(-3)*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^1
0*d^7*x^2))*(d^14/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 5
12*c^3)) + 91*(c^4*d*x^10 - 8*c^5*x^7 + sqrt(-3)*(c^4*d*x^10 - 8*c^5*x^7))
*(d^14/c^23)^(1/6)*log(371293*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x
^3 + 640*c^3*d^11 - 9*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x +
sqrt(-3)*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x))*(d^14/c^23)^(
2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 - sqrt(-3)*(5*c^20
*d*x^5 + 32*c^21*x^2))*(d^14/c^23)^(5/6) - 2*(7*c^12*d^6*x^6 + 152*c^13*d^
5*x^3 + 64*c^14*d^4)*sqrt(d^14/c^23) + (c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 1
60*c^6*d^9*x + sqrt(-3)*(c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^8 (8c-dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**(1/2)/x**8/(-d*x**3+8*c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^8 (8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{(dx^3-8\,c)^2 x^8}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^8), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^8 (8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{(dx^3-8\,c)^2 x^8}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^8 (8c-dx^3)^2}\,dx = \int \frac{\sqrt{d\,x^3+c}}{x^8\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)^2),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{d\,x^3+c}}{d^2x^{14}-16cd\,x^{11}+64c^2x^8}dx$$

input
```
int((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x)
```

output
```
int(sqrt(c + d*x**3)/(64*c**2*x**8 - 16*c*d*x**11 + d**2*x**14),x)
```

3.584 $\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 140

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx = \frac{4480c^3\sqrt{c+dx^3}}{3d^4} + \frac{1536c^4\sqrt{c+dx^3}}{d^4(8c-dx^3)} + \frac{128c^2(c+dx^3)^{3/2}}{3d^4} + \frac{2c(c+dx^3)^{5/2}}{d^4} + \frac{2(c+dx^3)^{7/2}}{21d^4} - \frac{4992c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

output
```
4480/3*c^3*(d*x^3+c)^(1/2)/d^4+1536*c^4*(d*x^3+c)^(1/2)/d^4/(-d*x^3+8*c)+1
28/3*c^2*(d*x^3+c)^(3/2)/d^4+2*c*(d*x^3+c)^(5/2)/d^4+2/21*(d*x^3+c)^(7/2)/
d^4-4992*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.74

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx = \frac{2\sqrt{c+dx^3}(-145328c^4 + 12206c^3dx^3 + 301c^2d^2x^6 + 16cd^3x^9 + d^4x^{12})}{21d^4(-8c+dx^3)} - \frac{4992c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

input
```
Integrate[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
(2*Sqrt[c + d*x^3]*(-145328*c^4 + 12206*c^3*d*x^3 + 301*c^2*d^2*x^6 + 16*c
*d^3*x^9 + d^4*x^12))/(21*d^4*(-8*c + d*x^3)) - (4992*c^(7/2)*ArcTanh[Sqrt
[c + d*x^3]/(3*Sqrt[c])])/d^4
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {948, 108, 27, 170, 25, 27, 164, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11} \left(c+d x^3\right)^{3/2}}{\left(8 c-d x^3\right)^2} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^9 \left(d x^3+c\right)^{3/2}}{\left(8 c-d x^3\right)^2}dx^3$$

$$\Big\downarrow \text{108}$$

$$\frac{1}{3} \left(\frac{x^9 \left(c+d x^3\right)^{3/2}}{d \left(8 c-d x^3\right)}-\frac{\int \frac{3 x^6 \sqrt{d x^3+c} \left(3 d x^3+2 c\right)}{2 \left(8 c-d x^3\right)}dx^3}{d}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(\frac{x^9 \left(c+d x^3\right)^{3/2}}{d \left(8 c-d x^3\right)}-\frac{3 \int \frac{x^6 \sqrt{d x^3+c} \left(3 d x^3+2 c\right)}{8 c-d x^3}dx^3}{2 d}\right)$$

$$\Big\downarrow \text{170}$$

$$\frac{1}{3}\left(\frac{x^9\left(c+dx^3\right)^{3/2}}{d\left(8c-dx^3\right)}-\frac{3\left(-\frac{2\int -\frac{cdx^3\sqrt{dx^3+c}\left(85dx^3+48c\right)}{8c-dx^3}dx^3}{7d^2}-\frac{6x^6\left(c+dx^3\right)^{3/2}}{7d}\right)}{2d}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{3}\left(\frac{x^9\left(c+dx^3\right)^{3/2}}{d\left(8c-dx^3\right)}-\frac{3\left(\frac{2\int \frac{cdx^3\sqrt{dx^3+c}\left(85dx^3+48c\right)}{8c-dx^3}dx^3}{7d^2}-\frac{6x^6\left(c+dx^3\right)^{3/2}}{7d}\right)}{2d}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{x^9\left(c+dx^3\right)^{3/2}}{d\left(8c-dx^3\right)}-\frac{3\left(\frac{2c\int \frac{x^3\sqrt{dx^3+c}\left(85dx^3+48c\right)}{8c-dx^3}dx^3}{7d}-\frac{6x^6\left(c+dx^3\right)^{3/2}}{7d}\right)}{2d}\right)$$

$$\Big\downarrow \; 164$$

$$\frac{1}{3}\left(\frac{x^9\left(c+dx^3\right)^{3/2}}{d\left(8c-dx^3\right)}-\frac{3\left(\frac{2c\left(\frac{5824c^2\int \frac{\sqrt{dx^3+c}}{8c-dx^3}dx^3}{d}-\frac{2\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{3d^2}\right)}{7d}-\frac{6x^6\left(c+dx^3\right)^{3/2}}{7d}\right)}{2d}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3}\left(\frac{x^9\left(c+dx^3\right)^{3/2}}{d\left(8c-dx^3\right)}-\frac{3\left(\frac{2c\left(\frac{5824c^2\left(9c\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3-\frac{2\sqrt{c+dx^3}}{d}\right)}{d}-\frac{2\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{3d^2}\right)}{7d}-\frac{6x^6\left(c+dx^3\right)^{3/2}}{7d}\right)}{2d}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{x^9\left(c+dx^3\right)^{3/2}}{d\left(8c-dx^3\right)}-\frac{3\left(\frac{2c\left(\frac{5824c^2\left(\frac{18c\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)}{d}-\frac{2\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{3d^2}\right)}{7d}-\frac{6x^6\left(c+dx^3\right)^{3/2}}{7d}\right)}{2d}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(\frac{x^9\left(c+dx^3\right)^{3/2}}{d\left(8c-dx^3\right)}-\frac{3\left(\frac{2c\left(\frac{5824c^2\left(\frac{6\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)}{d}-\frac{2\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{3d^2}\right)}{7d}-\frac{6x^6\left(c+dx^3\right)^{3/2}}{7d}\right)}{2d}\right)$$

input
```
Int[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
((x^9*(c + d*x^3)^(3/2))/(d*(8*c - d*x^3)) - (3*((-6*x^6*(c + d*x^3)^(3/2)
)/(7*d) + (2*c*((-2*(c + d*x^3)^(3/2)*(694*c + 51*d*x^3))/(3*d^2) + (5824*
c^2*((-2*Sqrt[c + d*x^3])/d + (6*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c
])])/d))/d))/(7*d)))/(2*d))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$-\frac{39936\left(c^4\left(-\frac{d x^3}{8}+c\right)\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)+\frac{\left(\sqrt{c}\, d^4 x^{12}+16 c^{\frac{3}{2}} d^3 x^9+301 c^{\frac{5}{2}} d^2 x^6+12206 c^{\frac{7}{2}} d x^3-145328 c^{\frac{9}{2}}\right)\sqrt{d x^3+c}}{419328}\right)}{\sqrt{c}\,(-d^5 x^3+8 c d^4)}$
risch	$\frac{2(d^3x^9+24c d^2x^6+493c^2 d x^3+16150c^3)\sqrt{d x^3+c}}{21d^4}+\frac{576c^4\left(-\frac{86\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}+\frac{8c\left(-\frac{\sqrt{d x^3+c}}{c\left(d x^3-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+}}{3\sqrt{c}}\right.}{3c^{\frac{3}{2}}}\right.}{3d}\right.}{d^3}$
default	$\frac{d\left(\frac{2d x^9\sqrt{d x^3+c}}{21}+\frac{16c x^6\sqrt{d x^3+c}}{105}+\frac{2c^2x^3\sqrt{d x^3+c}}{105d}-\frac{4c^3\sqrt{d x^3+c}}{105d^2}\right)+\frac{32c\left(d x^3+c\right)^{\frac{5}{2}}}{15d}}{d^3}-\frac{128c^2\left(81c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)-(c\right.}{3d^4}$
	$832ic^3\sqrt{2}\left(\right.$ _α=Roo
elliptic	$\frac{1536c^4\sqrt{d x^3+c}}{d^4(-d x^3+8c)}+\frac{2x^9\sqrt{d x^3+c}}{21d}+\frac{16c x^6\sqrt{d x^3+c}}{7d^2}+\frac{986c^2x^3\sqrt{d x^3+c}}{21d^3}+\frac{32300c^3\sqrt{d x^3+c}}{21d^4}+$

input
```
int(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
-39936*(c^4*(-1/8*d*x^3+c)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))+1/419328*(
c^(1/2)*d^4*x^12+16*c^(3/2)*d^3*x^9+301*c^(5/2)*d^2*x^6+12206*c^(7/2)*d*x^
3-145328*c^(9/2))*(d*x^3+c)^(1/2))/c^(1/2)/(-d^5*x^3+8*c*d^4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.69

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \left[\frac{2\left(26208\,(c^3dx^3-8\,c^4)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+(d^4x^{12}+16\,cd^3x^9+301\,c^2\right.}{21\,(d^5x^3-8\,cd^4)}\right.$$

input
```
integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[2/21*(26208*(c^3*d*x^3 - 8*c^4)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sq
rt(c) + 10*c)/(d*x^3 - 8*c)) + (d^4*x^12 + 16*c*d^3*x^9 + 301*c^2*d^2*x^6
+ 12206*c^3*d*x^3 - 145328*c^4)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4), 2/21
*(52416*(c^3*d*x^3 - 8*c^4)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) +
(d^4*x^12 + 16*c*d^3*x^9 + 301*c^2*d^2*x^6 + 12206*c^3*d*x^3 - 145328*c^4)
*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**11*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{2\left(26208\,c^{\frac{7}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+(dx^3+c)^{\frac{7}{2}}+21\,(dx^3+c)^{\frac{5}{2}}c+448\,(dx^3+c)^{\frac{3}{2}}c^2+\right.}{21\,d^4}$$

input
```
integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
2/21*(26208*c^(7/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3
*sqrt(c))) + (d*x^3 + c)^(7/2) + 21*(d*x^3 + c)^(5/2)*c + 448*(d*x^3 + c)^
(3/2)*c^2 + 15680*sqrt(d*x^3 + c)*c^3 - 16128*sqrt(d*x^3 + c)*c^4/(d*x^3 -
 8*c))/d^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{4992\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^4} - \frac{1536\sqrt{dx^3+c}c^4}{(dx^3-8\,c)d^4} + \frac{2\left((dx^3+c)^{\frac{7}{2}}d^{24}+21\,(dx^3+c)^{\frac{5}{2}}cd^{24}+448\,(dx^3+c)^{\frac{3}{2}}c^2d^{24}+15680\sqrt{dx^3+c}c^3d^{24}\right)}{21\,d^{28}}$$

input
```
integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
4992*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 1536*sqrt(d
*x^3 + c)*c^4/((d*x^3 - 8*c)*d^4) + 2/21*((d*x^3 + c)^(7/2)*d^24 + 21*(d*x
^3 + c)^(5/2)*c*d^24 + 448*(d*x^3 + c)^(3/2)*c^2*d^24 + 15680*sqrt(d*x^3 +
 c)*c^3*d^24)/d^28
```

Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{2496\,c^{7/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^4}+\frac{32300\,c^3\,\sqrt{d\,x^3+c}}{21\,d^4}+\frac{2\,x^9\,\sqrt{d\,x^3+c}}{21\,d}+\frac{16\,c\,x^6\,\sqrt{d\,x^3+c}}{7\,d^2}+\frac{986\,c^2\,x^3\,\sqrt{d\,x^3+c}}{21\,d^3}+\frac{1536\,c^4\,\sqrt{d\,x^3+c}}{d^4\,\left(8\,c-d\,x^3\right)}$$

input
```
int((x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

output
```
(2496*c^(7/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^
3)))/d^4 + (32300*c^3*(c + d*x^3)^(1/2))/(21*d^4) + (2*x^9*(c + d*x^3)^(1/
2))/(21*d) + (16*c*x^6*(c + d*x^3)^(1/2))/(7*d^2) + (986*c^2*x^3*(c + d*x^
3)^(1/2))/(21*d^3) + (1536*c^4*(c + d*x^3)^(1/2))/(d^4*(8*c - d*x^3))
```

Reduce [F]

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{\frac{195296\sqrt{d\,x^3+c}\,c^4}{105}-\frac{24412\sqrt{d\,x^3+c}\,c^3d\,x^3}{21}-\frac{86\sqrt{d\,x^3+c}\,c^2d^2x^6}{3}-\frac{32\sqrt{d\,x^3+c}\,c\,d^3x^9}{21}-\frac{2\sqrt{d\,x^3+c}\,d^4x^{12}}{21}}{d^4\left(-d\,x\right.}$$

input
```
int(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

output
```
(2*(97648*sqrt(c + d*x**3)*c**4 - 61030*sqrt(c + d*x**3)*c**3*d*x**3 - 150
5*sqrt(c + d*x**3)*c**2*d**2*x**6 - 80*sqrt(c + d*x**3)*c*d**3*x**9 - 5*sq
rt(c + d*x**3)*d**4*x**12 + 16982784*int((sqrt(c + d*x**3)*x**5)/(64*c**3
+ 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**5*d**2 - 2122848*int(
(sqrt(c + d*x**3)*x**5)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*
x**9),x)*c**4*d**3*x**3))/(105*d**4*(8*c - d*x**3))
```

3.585 $\int \frac{x^8 (c+dx^3)^{3/2}}{(8c-dx^3)^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{x^8 (c+dx^3)^{3/2}}{(8c-dx^3)^2} \, dx = \frac{416c^2\sqrt{c+dx^3}}{3d^3} + \frac{192c^3\sqrt{c+dx^3}}{d^3(8c-dx^3)} + \frac{32c(c+dx^3)^{3/2}}{9d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3} - \frac{480c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

output
```
416/3*c^2*(d*x^3+c)^(1/2)/d^3+192*c^3*(d*x^3+c)^(1/2)/d^3/(-d*x^3+8*c)+32/
9*c*(d*x^3+c)^(3/2)/d^3+2/15*(d*x^3+c)^(5/2)/d^3-480*c^(5/2)*arctanh(1/3*(
d*x^3+c)^(1/2)/c^(1/2))/d^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.77

$$\int \frac{x^8 (c+dx^3)^{3/2}}{(8c-dx^3)^2} \, dx = \frac{2\sqrt{c+dx^3}(-29944c^3 + 2515c^2dx^3 + 62cd^2x^6 + 3d^3x^9)}{45d^3(-8c+dx^3)} - \frac{480c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

input
```
Integrate[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
(2*Sqrt[c + d*x^3]*(-29944*c^3 + 2515*c^2*d*x^3 + 62*c*d^2*x^6 + 3*d^3*x^9
))/(45*d^3*(-8*c + d*x^3)) - (480*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/d^3
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {948, 100, 27, 90, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 \left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^6 \left(dx^3+c\right)^{3/2}}{\left(8c-dx^3\right)^2}dx^3$$

$$\Big\downarrow \text{ 100}$$

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{5/2}}{9d^3\left(8c-dx^3\right)}-\frac{\int \frac{3cd\left(dx^3+c\right)^{3/2}\left(3dx^3+56c\right)}{8c-dx^3}dx^3}{9cd^3}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{5/2}}{9d^3\left(8c-dx^3\right)}-\frac{\int \frac{\left(dx^3+c\right)^{3/2}\left(3dx^3+56c\right)}{8c-dx^3}dx^3}{3d^2}\right)$$

$$\Big\downarrow \text{ 90}$$

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{5/2}}{9d^3\left(8c-dx^3\right)}-\frac{80c\int \frac{\left(dx^3+c\right)^{3/2}}{8c-dx^3}dx^3-\frac{6\left(c+dx^3\right)^{5/2}}{5d}}{3d^2}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{5/2}}{9d^3\left(8c-dx^3\right)}-\frac{80c\left(9c\int\frac{\sqrt{dx^3+c}}{8c-dx^3}dx^3-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)-\frac{6\left(c+dx^3\right)^{5/2}}{5d}}{3d^2}\right)$$

$\Big\downarrow$ 60

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{5/2}}{9d^3\left(8c-dx^3\right)}-\frac{80c\left(9c\left(9c\int\frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)-\frac{6\left(c+dx^3\right)^{5/2}}{5d}}{3d^2}\right)$$

$\Big\downarrow$ 73

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{5/2}}{9d^3\left(8c-dx^3\right)}-\frac{80c\left(9c\left(\frac{18c\int\frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)-\frac{6\left(c+dx^3\right)^{5/2}}{5d}}{3d^2}\right)$$

$\Big\downarrow$ 219

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{5/2}}{9d^3\left(8c-dx^3\right)}-\frac{80c\left(9c\left(\frac{6\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)-\frac{6\left(c+dx^3\right)^{5/2}}{5d}}{3d^2}\right)$$

input
```
Int[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
((64*c*(c + d*x^3)^(5/2))/(9*d^3*(8*c - d*x^3)) - ((-6*(c + d*x^3)^(5/2))/
(5*d) + 80*c*((-2*(c + d*x^3)^(3/2))/(3*d) + 9*c*((-2*Sqrt[c + d*x^3])/d +
 (6*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d)))/(3*d^2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$-\frac{3840\left(c^{3}\left(-\frac{d\,x^{3}}{8}+c\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)+\frac{\sqrt{d\,x^{3}+c}\left(\sqrt{c}\,d^{3}x^{9}+\frac{62c^{\frac{3}{2}}d^{2}x^{6}}{3}+\frac{2515c^{\frac{5}{2}}d\,x^{3}}{3}-\frac{29944c^{\frac{7}{2}}}{3}\right)}{28800}\right)}{\sqrt{c}\left(-d^{4}x^{3}+8c\,d^{3}\right)}$
risch	$\frac{2\left(3d^{2}x^{6}+86cd\,x^{3}+3203c^{2}\right)\sqrt{d\,x^{3}+c}}{45d^{3}}+\frac{144c^{3}\left(-\frac{34\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}+\frac{4c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3d}\right)}{d^{2}}$
default	$\frac{2\left(d\,x^{3}+c\right)^{\frac{5}{2}}}{15d^{3}}-\frac{32c\left(81c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)-\left(d\,x^{3}+28c\right)\sqrt{d\,x^{3}+c}\right)}{9d^{3}}+\frac{64c^{2}\left(\frac{2\sqrt{d\,x^{3}+c}}{3}-3c\left(-\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}+\frac{\operatorname{arctanh}(\ldots}{\ldots}\right.\right.}{d^{3}}$
elliptic	$\frac{192c^{3}\sqrt{d\,x^{3}+c}}{d^{3}\left(-d\,x^{3}+8c\right)}+\frac{2x^{6}\sqrt{d\,x^{3}+c}}{15d}+\frac{172c\,x^{3}\sqrt{d\,x^{3}+c}}{45d^{2}}+\frac{6406c^{2}\sqrt{d\,x^{3}+c}}{45d^{3}}+\frac{80ic^{2}\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\ldots}{\ldots}\right.}{\ldots}$

input
```
int(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
-3840/c^(1/2)*(c^3*(-1/8*d*x^3+c)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))+1/2
8800*(d*x^3+c)^(1/2)*(c^(1/2)*d^3*x^9+62/3*c^(3/2)*d^2*x^6+2515/3*c^(5/2)*
d*x^3-29944/3*c^(7/2)))/(-d^4*x^3+8*c*d^3)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.79

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \left[\frac{2\left(5400\,(c^2dx^3-8\,c^3)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+(3\,d^3x^9+62\,cd^2x^6+2515\,c^2\right.}{45\,(d^4x^3-8\,cd^3)}\right.$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[2/45*(5400*(c^2*d*x^3 - 8*c^3)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqr
t(c) + 10*c)/(d*x^3 - 8*c)) + (3*d^3*x^9 + 62*c*d^2*x^6 + 2515*c^2*d*x^3 -
 29944*c^3)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3), 2/45*(10800*(c^2*d*x^3 -
 8*c^3)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) + (3*d^3*x^9 + 62*c*d^
2*x^6 + 2515*c^2*d*x^3 - 29944*c^3)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**8*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{2\left(5400\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\,(dx^3+c)^{\frac{5}{2}}+80\,(dx^3+c)^{\frac{3}{2}}c+3120\sqrt{dx^3+c}c^2-\right.}{45\,d^3}$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
2/45*(5400*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 80*(d*x^3 + c)^(3/2)*c + 3120*sqrt(d*x^3
 + c)*c^2 - 4320*sqrt(d*x^3 + c)*c^3/(d*x^3 - 8*c))/d^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{480\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^3} - \frac{192\sqrt{dx^3+c}c^3}{(dx^3-8\,c)d^3} + \frac{2\left(3\,(dx^3+c)^{\frac{5}{2}}d^{12}+80\,(dx^3+c)^{\frac{3}{2}}cd^{12}+3120\sqrt{dx^3+c}c^2d^{12}\right)}{45\,d^{15}}$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
480*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 192*sqrt(d*x
^3 + c)*c^3/((d*x^3 - 8*c)*d^3) + 2/45*(3*(d*x^3 + c)^(5/2)*d^12 + 80*(d*x
^3 + c)^(3/2)*c*d^12 + 3120*sqrt(d*x^3 + c)*c^2*d^12)/d^15
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{240\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^3}+\frac{6406\,c^2\,\sqrt{d\,x^3+c}}{45\,d^3}+\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d}+\frac{172\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d^2}+\frac{192\,c^3\,\sqrt{d\,x^3+c}}{d^3\,\left(8\,c-d\,x^3\right)}$$

input
```
int((x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

output
```
(240*c^(5/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/d^3 + (6406*c^2*(c + d*x^3)^(1/2))/(45*d^3) + (2*x^6*(c + d*x^3)^(1/2)
)/(15*d) + (172*c*x^3*(c + d*x^3)^(1/2))/(45*d^2) + (192*c^3*(c + d*x^3)^(
1/2))/(d^3*(8*c - d*x^3))
```

Reduce [F]

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{\frac{8048\sqrt{d\,x^3+c}\,c^3}{45}-\frac{1006\sqrt{d\,x^3+c}\,c^2 d\,x^3}{9}-\frac{124\sqrt{d\,x^3+c}\,c\,d^2x^6}{45}-\frac{2\sqrt{d\,x^3+c}\,d^3x^9}{15}+31104\left(\int \frac{\cdots}{d^3x^9-15c\cdots}\right.}{d^3\left(-d\,x^3+8c\cdots\right.}$$

input
```
int(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

output
```
(2*(4024*sqrt(c + d*x**3)*c**3 - 2515*sqrt(c + d*x**3)*c**2*d*x**3 - 62*sq
rt(c + d*x**3)*c*d**2*x**6 - 3*sqrt(c + d*x**3)*d**3*x**9 + 699840*int((sq
rt(c + d*x**3)*x**5)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**
9),x)*c**4*d**2 - 87480*int((sqrt(c + d*x**3)*x**5)/(64*c**3 + 48*c**2*d*x
**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**3*d**3*x**3))/(45*d**3*(8*c - d*x*
*3))
```

3.586 $\int \frac{x^5 (c+dx^3)^{3/2}}{(8c-dx^3)^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 100

$$\int \frac{x^5 (c+dx^3)^{3/2}}{(8c-dx^3)^2} \, dx = \frac{34c\sqrt{c+dx^3}}{3d^2} + \frac{24c^2\sqrt{c+dx^3}}{d^2(8c-dx^3)} + \frac{2(c+dx^3)^{3/2}}{9d^2} - \frac{42c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

output
```
34/3*c*(d*x^3+c)^(1/2)/d^2+24*c^2*(d*x^3+c)^(1/2)/d^2/(-d*x^3+8*c)+2/9*(d*
x^3+c)^(3/2)/d^2-42*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.81

$$\int \frac{x^5 (c+dx^3)^{3/2}}{(8c-dx^3)^2} \, dx = \frac{2\sqrt{c+dx^3}(-524c^2+44cdx^3+d^2x^6)}{9d^2(-8c+dx^3)} - \frac{42c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

input
```
Integrate[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
(2*Sqrt[c + d*x^3]*(-524*c^2 + 44*c*d*x^3 + d^2*x^6))/(9*d^2*(-8*c + d*x^3
)) - (42*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {948, 87, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^3 \left(dx^3+c\right)^{3/2}}{\left(8c-dx^3\right)^2}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3} \left(\frac{8\left(c+dx^3\right)^{5/2}}{9d^2\left(8c-dx^3\right)}-\frac{7\int \frac{\left(dx^3+c\right)^{3/2}}{8c-dx^3}dx^3}{3d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3} \left(\frac{8\left(c+dx^3\right)^{5/2}}{9d^2\left(8c-dx^3\right)}-\frac{7\left(9c\int \frac{\sqrt{dx^3+c}}{8c-dx^3}dx^3-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)}{3d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3} \left(\frac{8\left(c+dx^3\right)^{5/2}}{9d^2\left(8c-dx^3\right)}-\frac{7\left(9c\left(9c\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)}{3d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{8\left(c+dx^3\right)^{5/2}}{9d^2\left(8c-dx^3\right)}-\frac{7\left(9c\left(\frac{18c\int\frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)}{3d}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{8\left(c+dx^3\right)^{5/2}}{9d^2\left(8c-dx^3\right)}-\frac{7\left(9c\left(\frac{6\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)}{3d}\right)$$

input
```
Int[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
((8*(c + d*x^3)^(5/2))/(9*d^2*(8*c - d*x^3)) - (7*((-2*(c + d*x^3)^(3/2))/
(3*d) + 9*c*((-2*Sqrt[c + d*x^3])/d + (6*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(
3*Sqrt[c])])/d)))/(3*d))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{\frac{2\left(d\,x^3+c\right)^{\frac{3}{2}}}{9}+\frac{34c\sqrt{d\,x^3+c}}{3}+6c^2\left(\frac{4\sqrt{d\,x^3+c}}{-d\,x^3+8c}-\frac{7\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{d^2}$
risch	$\frac{2\left(d\,x^3+52c\right)\sqrt{d\,x^3+c}}{9d^2}+\frac{9c^2\left(-\frac{50\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}+\frac{8c\left(-\frac{\sqrt{d\,x^3+c}}{c\left(d\,x^3-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3d}\right)}{d}$
default	$-\frac{2\left(81c^{\frac{3}{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)-\left(d\,x^3+28c\right)\sqrt{d\,x^3+c}\right)}{9d^2}+\frac{8c\left(\frac{2\sqrt{d\,x^3+c}}{3}-3c\left(-\frac{\sqrt{d\,x^3+c}}{-d\,x^3+8c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)\right)}{d^2}$
elliptic	$\frac{24c^2\sqrt{d\,x^3+c}}{d^2\left(-d\,x^3+8c\right)}+\frac{2x^3\sqrt{d\,x^3+c}}{9d}+\frac{104c\sqrt{d\,x^3+c}}{9d^2}+\frac{7ic\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)\ldots}{\ldots}\right)}{\left(-c\,d^2\right)\ldots}}\ldots}{\ldots}\right)}{\ldots}$

input
```
int(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(1/9*(d*x^3+c)^(3/2)+17/3*c*(d*x^3+c)^(1/2)+3*c^2*(4*(d*x^3+c)^(1/2)/(-d
*x^3+8*c)-7*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)))/d^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.89

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \left[\frac{189\,(cdx^3-8\,c^2)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+2\,(d^2x^6+44\,cdx^3-524\,c^2)\sqrt{dx^3+}}{9\,(d^3x^3-8\,cd^2)}\right.$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/9*(189*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c)
 + 10*c)/(d*x^3 - 8*c)) + 2*(d^2*x^6 + 44*c*d*x^3 - 524*c^2)*sqrt(d*x^3 +
c))/(d^3*x^3 - 8*c*d^2), 2/9*(189*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(3*sqrt
(-c)/sqrt(d*x^3 + c)) + (d^2*x^6 + 44*c*d*x^3 - 524*c^2)*sqrt(d*x^3 + c))/
(d^3*x^3 - 8*c*d^2)]
```

Sympy [F]

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x^5(c+dx^3)^{\frac{3}{2}}}{(-8c+dx^3)^2}\,dx$$

input
```
integrate(x**5*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x**5*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.93

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{189\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+2\,(dx^3+c)^{\frac{3}{2}}+102\,\sqrt{dx^3+c}c-\frac{216\,\sqrt{dx^3+c}c^2}{dx^3-8\,c}}{9\,d^2}$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
1/9*(189*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sq
rt(c))) + 2*(d*x^3 + c)^(3/2) + 102*sqrt(d*x^3 + c)*c - 216*sqrt(d*x^3 + c
)*c^2/(d*x^3 - 8*c))/d^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.93

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{42\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d^2} - \frac{24\,\sqrt{dx^3+c}c^2}{(dx^3-8\,c)d^2} + \frac{2\left((dx^3+c)^{\frac{3}{2}}d^4+51\,\sqrt{dx^3+c}cd^4\right)}{9\,d^6}$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
42*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 24*sqrt(d*x^3
 + c)*c^2/((d*x^3 - 8*c)*d^2) + 2/9*((d*x^3 + c)^(3/2)*d^4 + 51*sqrt(d*x^3
 + c)*c*d^4)/d^6
```

Mupad [B] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{104\,c\,\sqrt{d\,x^3+c}}{9\,d^2} + \frac{21\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^2} + \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d} + \frac{24\,c^2\,\sqrt{d\,x^3+c}}{d^2\,\left(8\,c-d\,x^3\right)}$$

input
```
int((x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

output
```
(104*c*(c + d*x^3)^(1/2))/(9*d^2) + (21*c^(3/2)*log((10*c + d*x^3 - 6*c^(1
/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/d^2 + (2*x^3*(c + d*x^3)^(1/2))/(9*
d) + (24*c^2*(c + d*x^3)^(1/2))/(d^2*(8*c - d*x^3))
```

Reduce [F]

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{704\sqrt{d\,x^3+c}\,c^2-440\sqrt{d\,x^3+c}\,cd\,x^3-10\sqrt{d\,x^3+c}\,d^2x^6+122472\left(\int \frac{\sqrt{d\,x^3}}{d^3x^9-15c\,d^2x^6}\right.}{45d^2\left(-d\,x^3+8c\right.}$$

input
```
int(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

output
```
(704*sqrt(c + d*x**3)*c**2 - 440*sqrt(c + d*x**3)*c*d*x**3 - 10*sqrt(c + d
*x**3)*d**2*x**6 + 122472*int((sqrt(c + d*x**3)*x**5)/(64*c**3 + 48*c**2*d
*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**3*d**2 - 15309*int((sqrt(c + d*x
**3)*x**5)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**2
*d**3*x**3)/(45*d**2*(8*c - d*x**3))
```

3.587 $\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 77

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx=\frac{\sqrt{c+dx^3}}{d}+\frac{(c+dx^3)^{3/2}}{3d(8c-dx^3)}-\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

output
```
(d*x^3+c)^(1/2)/d+1/3*(d*x^3+c)^(3/2)/d/(-d*x^3+8*c)-3*c^(1/2)*arctanh(1/3
*(d*x^3+c)^(1/2)/c^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx=\frac{(25c-2dx^3)\sqrt{c+dx^3}}{3d(8c-dx^3)}-\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

input
```
Integrate[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
((25*c - 2*d*x^3)*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)) - (3*Sqrt[c]*ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {946, 51, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (c+dx^3)^{3/2}}{(8c-dx^3)^2} \, dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{3} \int \frac{(dx^3+c)^{3/2}}{(8c-dx^3)^2} dx^3$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{3} \left(\frac{(c+dx^3)^{3/2}}{d(8c-dx^3)} - \frac{3}{2} \int \frac{\sqrt{dx^3+c}}{8c-dx^3} dx^3\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{3} \left(\frac{(c+dx^3)^{3/2}}{d(8c-dx^3)} - \frac{3}{2} \left(9c \int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}} dx^3 - \frac{2\sqrt{c+dx^3}}{d}\right)\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3} \left(\frac{(c+dx^3)^{3/2}}{d(8c-dx^3)} - \frac{3}{2} \left(\frac{18c \int \frac{1}{9c-x^6} d\sqrt{dx^3+c}}{d} - \frac{2\sqrt{c+dx^3}}{d}\right)\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3} \left(\frac{(c+dx^3)^{3/2}}{d(8c-dx^3)} - \frac{3}{2} \left(\frac{6\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d} - \frac{2\sqrt{c+dx^3}}{d}\right)\right)$$

input
```
Int[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
((c + d*x^3)^(3/2)/(d*(8*c - d*x^3)) - (3*((-2*Sqrt[c + d*x^3])/d + (6*Sqr
t[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d))/2)/3
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

method	result
default	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}-3c\left(-\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{d}$
pseudoelliptic	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}+3c\left(\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{d}$
risch	$\frac{2\sqrt{d\,x^{3}+c}}{3d}+9c\left(-\frac{4\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}+\frac{c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3d}\right)$
elliptic	$\frac{3c\sqrt{d\,x^{3}+c}}{d(-d\,x^{3}+8c)}+\frac{2\sqrt{d\,x^{3}+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{i d\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d \cdots}{-3\left(-c \cdots\right.}} \cdots}{\cdots}\right.}{\cdots}$

input
```
int(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
(2/3*(d*x^3+c)^(1/2)-3*c*(-(d*x^3+c)^(1/2)/(-d*x^3+8*c)+arctanh(1/3*(d*x^3
+c)^(1/2)/c^(1/2))/c^(1/2)))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.06

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \left[\frac{9\,(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+2\,(2\,dx^3-25\,c)\sqrt{dx^3+c}}{6\,(d^2x^3-8\,cd)}, \frac{9\,(dx^3-8\,c\right.$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/6*(9*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*
c)/(d*x^3 - 8*c)) + 2*(2*d*x^3 - 25*c)*sqrt(d*x^3 + c))/(d^2*x^3 - 8*c*d),
 1/3*(9*(d*x^3 - 8*c)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) + (2*d*x
^3 - 25*c)*sqrt(d*x^3 + c))/(d^2*x^3 - 8*c*d)]
```

Sympy [F]

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x^2(c+dx^3)^{\frac{3}{2}}}{(-8c+dx^3)^2}\,dx$$

input
```
integrate(x**2*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x**2*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{9\,\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+4\,\sqrt{dx^3+c}-\frac{18\,\sqrt{dx^3+c}c}{dx^3-8\,c}}{6\,d}$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
1/6*(9*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt
(c))) + 4*sqrt(d*x^3 + c) - 18*sqrt(d*x^3 + c)*c/(d*x^3 - 8*c))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{3\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d} + \frac{2\,\sqrt{dx^3+c}}{3\,d} - \frac{3\,\sqrt{dx^3+c}c}{(dx^3-8\,c)d}$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
3*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + 2/3*sqrt(d*x^3 + c
)/d - 3*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d)
```

Mupad [B] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{2\,\sqrt{d\,x^3+c}}{3\,d} + \frac{3\,\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{2\,d} + \frac{3\,c\,\sqrt{d\,x^3+c}}{d\,(8\,c-d\,x^3)}$$

input
```
int((x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*d) + (3*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c
+ d*x^3)^(1/2))/(8*c - d*x^3)))/(2*d) + (3*c*(c + d*x^3)^(1/2))/(d*(8*c -
d*x^3))
```

Reduce [F]

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{34\sqrt{d\,x^3+c}\,c-20\sqrt{d\,x^3+c}\,d\,x^3+5832\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)c^2d^2-729}{30d\,(-d\,x^3+8c)}$$

input
```
int(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

output
```
(34*sqrt(c + d*x**3)*c - 20*sqrt(c + d*x**3)*d*x**3 + 5832*int((sqrt(c + d
*x**3)*x**5)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c*
*2*d**2 - 729*int((sqrt(c + d*x**3)*x**5)/(64*c**3 + 48*c**2*d*x**3 - 15*c
*d**2*x**6 + d**3*x**9),x)*c*d**3*x**3)/(30*d*(8*c - d*x**3))
```

3.588 $\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 85

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx=\frac{3\sqrt{c+dx^3}}{8(8c-dx^3)}-\frac{3\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

output
```
3*(d*x^3+c)^(1/2)/(-8*d*x^3+64*c)-3/32*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2)
)/c^(1/2)-1/96*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx=\frac{3\sqrt{c+dx^3}}{8(8c-dx^3)}-\frac{3\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x]
```

output
```
(3*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) - (3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt
[c])])/(32*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {948, 109, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{(dx^3+c)^{3/2}}{x^3(8c-dx^3)^2}dx^3$$

$$\Big\downarrow \text{ 109}$$

$$\frac{1}{3}\left(\frac{9\sqrt{c+dx^3}}{8(8c-dx^3)}-\frac{\int -\frac{cd(2c-7dx^3)}{2x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{8cd}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{1}{16}\int \frac{2c-7dx^3}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3+\frac{9\sqrt{c+dx^3}}{8(8c-dx^3)}\right)$$

$$\Big\downarrow \text{ 174}$$

$$\frac{1}{3}\left(\frac{1}{16}\left(\frac{1}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3-\frac{27}{4}d\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3\right)+\frac{9\sqrt{c+dx^3}}{8(8c-dx^3)}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(\frac{1}{16}\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{27}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}\right)+\frac{9\sqrt{c+dx^3}}{8(8c-dx^3)}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{1}{16}\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{9\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)+\frac{9\sqrt{c+dx^3}}{8(8c-dx^3)}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{3}\left(\frac{1}{16}\left(-\frac{9\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)+\frac{9\sqrt{c+dx^3}}{8\,(8c-dx^3)}\right)$$

input
```
Int[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x]
```

output
```
((9*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) + ((-9*ArcTanh[Sqrt[c + d*x^3]/(3*S
qrt[c])])/(2*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(2*Sqrt[c]))/16)/
3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$\frac{3\sqrt{d\,x^{3}+c}}{-8d\,x^{3}+64c}-\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{96\sqrt{c}}$
default	$\frac{\frac{2d\,x^{3}\sqrt{d\,x^{3}+c}}{9}+\frac{8c\sqrt{d\,x^{3}+c}}{9}-\frac{2c^{\frac{3}{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}}{64c^{2}}+\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}-3c\left(-\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{8c}+\frac{81c^{\frac{3}{2}}\,\operatorname{arcta}}{}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
3*(d*x^3+c)^(1/2)/(-8*d*x^3+64*c)-3/32*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2)
)/c^(1/2)-1/96*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.52

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx = \left[\frac{9\,(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{192\,(cdx^3-8\,c^2)}\right.$$

input
```
integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/192*(9*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 1
0*c)/(d*x^3 - 8*c)) + (d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)
*sqrt(c) + 2*c)/x^3) - 72*sqrt(d*x^3 + c)*c)/(c*d*x^3 - 8*c^2), 1/96*(9*(d
*x^3 - 8*c)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) + (d*x^3 - 8*c)*sq
rt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)) - 36*sqrt(d*x^3 + c)*c)/(c*d*x^3 -
 8*c^2)]
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{x(-8c+dx^3)^2}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x/(-d*x**3+8*c)**2,x)
```

output
```
Integral((c + d*x**3)**(3/2)/(x*(-8*c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}} + \frac{3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{32\sqrt{-c}} - \frac{3\sqrt{dx^3+c}}{8\,(dx^3-8\,c)}$$

input
```
integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) + 3/32*arctan(1/3*sqrt(d*x^
3 + c)/sqrt(-c))/sqrt(-c) - 3/8*sqrt(d*x^3 + c)/(d*x^3 - 8*c)
```

Mupad [B] (verification not implemented)

Time = 3.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx = \frac{3\sqrt{d\,x^3+c}}{8\,(8\,c-d\,x^3)} + \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\left(10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}\right)^9}{x^6\,(8\,c-d\,x^3)^9}\right)}{192\,\sqrt{c}}$$

input
```
int((c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x)
```

output
```
(3*(c + d*x^3)^(1/2))/(8*(8*c - d*x^3)) + log((((c + d*x^3)^(1/2) - c^(1/2
))^3*((c + d*x^3)^(1/2) + c^(1/2))*(10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(
1/2))^9)/(x^6*(8*c - d*x^3)^9))/(192*c^(1/2))
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx = \frac{2\sqrt{d\,x^3+c}+120\left(\int \frac{\sqrt{d\,x^3+c}}{d^3x^{10}-15c\,d^2x^7+48c^2d\,x^4+64c^3x}dx\right)c^3-15\left(\int \frac{\sqrt{d\,x^3+c}}{d^3x^{10}-15c\,d^2x^7+48c^2d\,x^4+64}\right.}{-}$$

input
```
int((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x)
```

output
```
(2*sqrt(c + d*x**3) + 120*int(sqrt(c + d*x**3)/(64*c**3*x + 48*c**2*d*x**4
 - 15*c*d**2*x**7 + d**3*x**10),x)*c**3 - 15*int(sqrt(c + d*x**3)/(64*c**3
*x + 48*c**2*d*x**4 - 15*c*d**2*x**7 + d**3*x**10),x)*c**2*d*x**3 + 96*int
((sqrt(c + d*x**3)*x**5)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3
*x**9),x)*c*d**2 - 12*int((sqrt(c + d*x**3)*x**5)/(64*c**3 + 48*c**2*d*x**
3 - 15*c*d**2*x**6 + d**3*x**9),x)*d**3*x**3)/(15*(8*c - d*x**3))
```

3.589 $\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2} \, dx = \frac{5d\sqrt{c+dx^3}}{96c(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24x^3(8c-dx^3)} + \frac{3d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}} - \frac{7d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}$$

output
```
5/96*d*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)-1/24*(d*x^3+c)^(1/2)/x^3/(-d*x^3+8*c
)+3/128*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-7/384*d*arctanh((d*
x^3+c)^(1/2)/c^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.80

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2} \, dx = \frac{\frac{4\sqrt{c}(4c-5dx^3)\sqrt{c+dx^3}}{-8cx^3+dx^6} + 9d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 7d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x]
```

output
```
((4*Sqrt[c]*(4*c - 5*d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 9*d*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 7*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(
384*c^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 109, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(dx^3+c)^{3/2}}{x^6(8c-dx^3)^2}dx^3$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{3}\left(-\frac{\int -\frac{cd(19dx^3+28c)}{2x^3(8c-dx^3)^2\sqrt{dx^3+c}}dx^3}{8c}-\frac{\sqrt{c+dx^3}}{8x^3(8c-dx^3)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{16}d\int \frac{19dx^3+28c}{x^3(8c-dx^3)^2\sqrt{dx^3+c}}dx^3-\frac{\sqrt{c+dx^3}}{8x^3(8c-dx^3)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{5\sqrt{c+dx^3}}{2c(8c-dx^3)}-\frac{\int -\frac{18cd(5dx^3+14c)}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{72c^2d}\right)-\frac{\sqrt{c+dx^3}}{8x^3(8c-dx^3)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{\int \frac{5dx^3+14c}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{4c}+\frac{5\sqrt{c+dx^3}}{2c(8c-dx^3)}\right)-\frac{\sqrt{c+dx^3}}{8x^3(8c-dx^3)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{\frac{7}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{27}{4}d\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{4c}+\frac{5\sqrt{c+dx^3}}{2c\,(8c-dx^3)}\right)-\frac{\sqrt{c+dx^3}}{8x^3\,(8c-dx^3)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{\frac{27}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{7\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{4c}+\frac{5\sqrt{c+dx^3}}{2c\,(8c-dx^3)}\right)-\frac{\sqrt{c+dx^3}}{8x^3\,(8c-dx^3)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{\frac{7\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{9\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}}{4c}+\frac{5\sqrt{c+dx^3}}{2c\,(8c-dx^3)}\right)-\frac{\sqrt{c+dx^3}}{8x^3\,(8c-dx^3)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{\frac{9\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}-\frac{7\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{4c}+\frac{5\sqrt{c+dx^3}}{2c\,(8c-dx^3)}\right)-\frac{\sqrt{c+dx^3}}{8x^3\,(8c-dx^3)}\right)$$

input
```
Int[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x]
```

output
```
(-1/8*Sqrt[c + d*x^3]/(x^3*(8*c - d*x^3)) + (d*((5*Sqrt[c + d*x^3])/(2*c*(
8*c - d*x^3)) + ((9*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*Sqrt[c]) - (7
*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c]))/(4*c)))/16)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{d\left(-\frac{\sqrt{d\,x^3+c}}{d\,x^3}-\frac{7\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{9\sqrt{d\,x^3+c}}{-d\,x^3+8c}+\frac{9\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)}{192c}$
risch	$-\frac{\sqrt{d\,x^3+c}}{192c\,x^3}-\frac{d\left(\frac{7\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3\sqrt{c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{\sqrt{c}}-6c\left(-\frac{\sqrt{d\,x^3+c}}{c\left(d\,x^3-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)\right)}{128c}$
default	$\frac{-\frac{c\sqrt{d\,x^3+c}}{3x^3}+\frac{2d\sqrt{d\,x^3+c}}{3}-\sqrt{c}\,d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{64c^2}+\frac{d\left(\frac{2d\,x^3\sqrt{d\,x^3+c}}{9}+\frac{8c\sqrt{d\,x^3+c}}{9}-\frac{2c^{\frac{3}{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3}\right)}{256c^3}+\frac{d\left(2\right.}{}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/192*d/c*(-(d*x^3+c)^(1/2)/d/x^3-7/2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(
1/2)+9*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+9/2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2
))/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.26

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx = \left[\frac{9\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+7\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx}}{x}\right.}{768\left(c^2dx^6-8\,c^3x^3\right)}\right.$$

$$-\frac{9\left(d^2x^6-8\,cdx^3\right)\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)-7\left(d^2x^6-8\,cdx^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)+4\left(5\,cdx^3-4\,c^2\right)\sqrt{dx}}{384\left(c^2dx^6-8\,c^3x^3\right)}$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/768*(9*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqr
t(c) + 10*c)/(d*x^3 - 8*c)) + 7*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 -
 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3
+ c))/(c^2*d*x^6 - 8*c^3*x^3), -1/384*(9*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*ar
ctan(3*sqrt(-c)/sqrt(d*x^3 + c)) - 7*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan
(sqrt(-c)/sqrt(d*x^3 + c)) + 4*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^2*d
*x^6 - 8*c^3*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**(3/2)/x**4/(-d*x**3+8*c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x^4}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx = \frac{7\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\,\sqrt{-c}c} - \frac{3\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{128\,\sqrt{-c}c} - \frac{5\,(dx^3+c)^{\frac{3}{2}}d - 9\,\sqrt{dx^3+c}cd}{96\,\left((dx^3+c)^2 - 10\,(dx^3+c)c + 9\,c^2\right)c}$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
7/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 3/128*d*arctan(1/3
*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/96*(5*(d*x^3 + c)^(3/2)*d - 9*
sqrt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c)
```

Mupad [B] (verification not implemented)

Time = 2.84 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx = \frac{\frac{9\,d\,\sqrt{d\,x^3+c}}{32} - \frac{5\,d\,(d\,x^3+c)^{3/2}}{32\,c}}{3\,(d\,x^3+c)^2 - 30\,c\,(d\,x^3+c) + 27\,c^2} + \frac{d\left(\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)\,1\mathrm{i} - \frac{\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)\,9\mathrm{i}}{7}\right)\,7\mathrm{i}}{384\,\sqrt{c^3}}$$

input
```
int((c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x)
```

output
```
((9*d*(c + d*x^3)^(1/2))/32 - (5*d*(c + d*x^3)^(3/2))/(32*c))/(3*(c + d*x^
3)^2 - 30*c*(c + d*x^3) + 27*c^2) + (d*(atanh((c*(c + d*x^3)^(1/2))/(c^3)^
(1/2))*1i - (atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2)))*9i)/7)*7i)/(384*
(c^3)^(1/2))
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4\,(8c-dx^3)^2}\,dx = \frac{-2\sqrt{d\,x^3+c}\,c+8\sqrt{d\,x^3+c}\,d\,x^3+672\left(\int \frac{\sqrt{d\,x^3+c}}{d^3x^{10}-15c\,d^2x^7+48c^2d\,x^4+64c^3x}dx\right)c^3d\,x^3-84$$

input
```
int((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x)
```

output
```
( - 2*sqrt(c + d*x**3)*c + 8*sqrt(c + d*x**3)*d*x**3 + 672*int(sqrt(c + d*
x**3)/(64*c**3*x + 48*c**2*d*x**4 - 15*c*d**2*x**7 + d**3*x**10),x)*c**3*d
*x**3 - 84*int(sqrt(c + d*x**3)/(64*c**3*x + 48*c**2*d*x**4 - 15*c*d**2*x*
*7 + d**3*x**10),x)*c**2*d**2*x**6 - 96*int((sqrt(c + d*x**3)*x**5)/(64*c*
*3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c*d**3*x**3 + 12*int(
(sqrt(c + d*x**3)*x**5)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*
x**9),x)*d**4*x**6 - 504*int((sqrt(c + d*x**3)*x**2)/(64*c**3 + 48*c**2*d*
x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**2*d**2*x**3 + 63*int((sqrt(c + d*
x**3)*x**2)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c*d
**3*x**6)/(48*c*x**3*(8*c - d*x**3))
```

3.590 $\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 161

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx = \frac{7d^2\sqrt{c+dx^3}}{512c^2(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48x^6(8c-dx^3)} - \frac{23d\sqrt{c+dx^3}}{384cx^3(8c-dx^3)} + \frac{15d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}} - \frac{17d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{5/2}}$$

output
```
7/512*d^2*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)-1/48*(d*x^3+c)^(1/2)/x^6/(-d*x^
3+8*c)-23/384*d*(d*x^3+c)^(1/2)/c/x^3/(-d*x^3+8*c)+15/2048*d^2*arctanh(1/3
*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-17/2048*d^2*arctanh((d*x^3+c)^(1/2)/c^(1
/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx = \frac{\frac{4\sqrt{c}\sqrt{c+dx^3}(32c^2+92cdx^3-21d^2x^6)}{-8cx^6+dx^9} + 45d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 51d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{5/2}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x]
```

output
```
((4*Sqrt[c]*Sqrt[c + d*x^3]*(32*c^2 + 92*c*d*x^3 - 21*d^2*x^6))/(-8*c*x^6
+ d*x^9) + 45*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 51*d^2*ArcTanh[Sq
rt[c + d*x^3]/Sqrt[c]])/(6144*c^(5/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {948, 109, 27, 168, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(dx^3+c)^{3/2}}{x^9(8c-dx^3)^2}dx^3$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{3}\left(-\frac{\int -\frac{cd(37dx^3+46c)}{2x^6(8c-dx^3)^2\sqrt{dx^3+c}}dx^3}{16c}-\frac{\sqrt{c+dx^3}}{16x^6(8c-dx^3)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{32}d\int \frac{37dx^3+46c}{x^6(8c-dx^3)^2\sqrt{dx^3+c}}dx^3-\frac{\sqrt{c+dx^3}}{16x^6(8c-dx^3)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(-\frac{\int -\frac{3cd(23dx^3+68c)}{x^3(8c-dx^3)^2\sqrt{dx^3+c}}dx^3}{8c^2}-\frac{23\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)-\frac{\sqrt{c+dx^3}}{16x^6(8c-dx^3)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{3d\int \frac{23dx^3+68c}{x^3(8c-dx^3)^2\sqrt{dx^3+c}}dx^3}{8c}-\frac{23\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)-\frac{\sqrt{c+dx^3}}{16x^6(8c-dx^3)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{3d\left(\frac{7\sqrt{c+dx^3}}{2c(8c-dx^3)}-\frac{\int -\frac{18cd\left(7dx^3+34c\right)}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{72c^2d}\right)}{8c}-\frac{23\sqrt{c+dx^3}}{4cx^3\left(8c-dx^3\right)}\right)-\frac{\sqrt{c+dx^3}}{16x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{3d\left(\frac{\int \frac{7dx^3+34c}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{4c}+\frac{7\sqrt{c+dx^3}}{2c(8c-dx^3)}\right)}{8c}-\frac{23\sqrt{c+dx^3}}{4cx^3\left(8c-dx^3\right)}\right)-\frac{\sqrt{c+dx^3}}{16x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{3d\left(\frac{\frac{17}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{45}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{4c}+\frac{7\sqrt{c+dx^3}}{2c(8c-dx^3)}\right)}{8c}-\frac{23\sqrt{c+dx^3}}{4cx^3\left(8c-dx^3\right)}\right)-\frac{\sqrt{c+dx^3}}{16x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{3d\left(\frac{\frac{45}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{17\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{4c}+\frac{7\sqrt{c+dx^3}}{2c(8c-dx^3)}\right)}{8c}-\frac{23\sqrt{c+dx^3}}{4cx^3\left(8c-dx^3\right)}\right)-\frac{\sqrt{c+dx^3}}{16x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{3d\left(\frac{\frac{17\int\frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{15\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}}{4c}+\frac{7\sqrt{c+dx^3}}{2c(8c-dx^3)}\right)}{8c}-\frac{23\sqrt{c+dx^3}}{4cx^3\left(8c-dx^3\right)}\right)-\frac{\sqrt{c+dx^3}}{16x^6\left(8c-dx^3\right)}\right)$$

$\Big\downarrow$ 221

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{3d\left(\frac{\frac{15\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}-\frac{17\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{4c}+\frac{7\sqrt{c+dx^3}}{2c(8c-dx^3)}\right)}{8c}-\frac{23\sqrt{c+dx^3}}{4cx^3\left(8c-dx^3\right)}\right)-\frac{\sqrt{c+dx^3}}{16x^6\left(8c-dx^3\right)}\right)$$

input
```
Int[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x]
```

output
```
(-1/16*Sqrt[c + d*x^3]/(x^6*(8*c - d*x^3)) + (d*((-23*Sqrt[c + d*x^3])/(4*
c*x^3*(8*c - d*x^3)) + (3*d*((7*Sqrt[c + d*x^3])/(2*c*(8*c - d*x^3)) + ((1
5*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*Sqrt[c]) - (17*ArcTanh[Sqrt[c +
 d*x^3]/Sqrt[c]])/(2*Sqrt[c]))/(4*c)))/(8*c)))/32)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.60

method	result
pseudoelliptic	$\frac{d^{2}\left(-\frac{\sqrt{d\,x^{3}+c}\,\left(3d\,x^{3}+c\right)}{d^{2}x^{6}}-\frac{51\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{16\sqrt{c}}+\frac{9\sqrt{d\,x^{3}+c}}{4\left(-d\,x^{3}+8c\right)}+\frac{45\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{16\sqrt{c}}\right)}{384c^{2}}$
risch	$-\frac{\sqrt{d\,x^{3}+c}\,\left(3d\,x^{3}+c\right)}{384c^{2}x^{6}}-\frac{3d^{2}\left(\frac{17\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{24\sqrt{c}}-\frac{11\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{24\sqrt{c}}-\frac{c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{2}\right)}{256c^{2}}$
default	$\frac{-\frac{c\sqrt{d\,x^{3}+c}}{6x^{6}}-\frac{5d\sqrt{d\,x^{3}+c}}{12x^{3}}-\frac{d^{2}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{4\sqrt{c}}}{64c^{2}}+\frac{d\left(-\frac{c\sqrt{d\,x^{3}+c}}{3x^{3}}+\frac{2d\sqrt{d\,x^{3}+c}}{3}-\sqrt{c}\,d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)\right)}{256c^{3}}+\frac{3d^{2}\left(2d\right.}{}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/384*d^2/c^2*(-(d*x^3+c)^(1/2)*(3*d*x^3+c)/d^2/x^6-51/16*arctanh((d*x^3+c
)^(1/2)/c^(1/2))/c^(1/2)+9/4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+45/16*arctanh(1/
3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.89

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx=\left[\frac{45\,(d^3x^9-8\,cd^2x^6)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+51\,(d^3x^9-8\,cd^2x^6)\sqrt{c}\log\left(\frac{dx^3-}{}\right.}{12288\,(c^3dx^9-8\,c^4x^6)}\right.$$

$$-\frac{45\,(d^3x^9-8\,cd^2x^6)\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)-51\,(d^3x^9-8\,cd^2x^6)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)+4\,(21\,cd^2x^6-92}{6144\,(c^3dx^9-8\,c^4x^6)}$$

input
```
integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/12288*(45*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c
)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 51*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log(
(d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*(21*c*d^2*x^6 - 92*c^2*
d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^3*d*x^9 - 8*c^4*x^6), -1/6144*(45*(d^3
*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) - 51*(d^3*
x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)) + 4*(21*c*d^2
*x^6 - 92*c^2*d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^3*d*x^9 - 8*c^4*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^7\,(8c-dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**(3/2)/x**7/(-d*x**3+8*c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^7\,(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x^7}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^7), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.80

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx = \frac{17\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\,\sqrt{-c}c^2} - \frac{15\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{2048\,\sqrt{-c}c^2} - \frac{3\,\sqrt{dx^3+c}d^2}{512\,(dx^3-8\,c)c^2} - \frac{3\,(dx^3+c)^{\frac{3}{2}}d^2-2\,\sqrt{dx^3+c}cd^2}{384\,c^2d^2x^6}$$

input
```
integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
17/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 15/2048*d^2*
arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 3/512*sqrt(d*x^3 + c
)*d^2/((d*x^3 - 8*c)*c^2) - 1/384*(3*(d*x^3 + c)^(3/2)*d^2 - 2*sqrt(d*x^3
+ c)*c*d^2)/(c^2*d^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 3.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx = \frac{\frac{81\,d^2\,\sqrt{d\,x^3+c}}{512} - \frac{67\,d^2\,(d\,x^3+c)^{3/2}}{256\,c} + \frac{21\,d^2\,(d\,x^3+c)^{5/2}}{512\,c^2}}{33\,c\,(d\,x^3+c)^2 - 57\,c^2\,(d\,x^3+c) - 3\,(d\,x^3+c)^3 + 27\,c^3} + \frac{d^2\left(\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)\,1\mathrm{i} - \frac{\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)\,15\mathrm{i}}{17}\right)\,17\mathrm{i}}{2048\,\sqrt{c^5}}$$

input
```
int((c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x)
```

output
```
((81*d^2*(c + d*x^3)^(1/2))/512 - (67*d^2*(c + d*x^3)^(3/2))/(256*c) + (21
*d^2*(c + d*x^3)^(5/2))/(512*c^2))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*x^3
) - 3*(c + d*x^3)^3 + 27*c^3) + (d^2*(atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^
(1/2))*1i - (atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))*15i)/17)*17i)/
(2048*(c^5)^(1/2))
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx = \frac{-2\sqrt{d\,x^3+c}+1104\left(\int \frac{\sqrt{d\,x^3+c}}{d^3x^{13}-15c\,d^2x^{10}+48c^2d\,x^7+64c^3x^4}dx\right)c^2d\,x^6-138\left(\int \frac{\sqrt{}}{d^3x^{13}-15c\,d^2x}\right.}{}$$

input
```
int((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x)
```

output
```
( - 2*sqrt(c + d*x**3) + 1104*int(sqrt(c + d*x**3)/(64*c**3*x**4 + 48*c**2
*d*x**7 - 15*c*d**2*x**10 + d**3*x**13),x)*c**2*d*x**6 - 138*int(sqrt(c +
d*x**3)/(64*c**3*x**4 + 48*c**2*d*x**7 - 15*c*d**2*x**10 + d**3*x**13),x)*
c*d**2*x**9 + 888*int(sqrt(c + d*x**3)/(64*c**3*x + 48*c**2*d*x**4 - 15*c*
d**2*x**7 + d**3*x**10),x)*c*d**2*x**6 - 111*int(sqrt(c + d*x**3)/(64*c**3
*x + 48*c**2*d*x**4 - 15*c*d**2*x**7 + d**3*x**10),x)*d**3*x**9)/(96*x**6*
(8*c - d*x**3))
```

3.591 $\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 681

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx=\frac{103cx^2\sqrt{c+dx^3}}{13d^2}+\frac{19x^5\sqrt{c+dx^3}}{39d}+\frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^5(c+dx^3)^{3/2}}{3d(8c-dx^3)}+\frac{108\sqrt{3}c^{13/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}-\frac{108c^{13/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}+\frac{108c^{13/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}}-\frac{2953\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{13d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{5906\sqrt{2}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{13\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
103/13*c*x^2*(d*x^3+c)^(1/2)/d^2+19/39*x^5*(d*x^3+c)^(1/2)/d+5906/13*c^2*(
d*x^3+c)^(1/2)/d^(8/3)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)+1/3*x^5*(d*x^3+c)^(
3/2)/d/(-d*x^3+8*c)+108*3^(1/2)*c^(13/6)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d
^(1/3)*x)/(d*x^3+c)^(1/2))/d^(8/3)-108*c^(13/6)*arctanh(1/3*(c^(1/3)+d^(1/
3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(8/3)+108*c^(13/6)*arctanh(1/3*(d*x^3+c
)^(1/2)/c^(1/2))/d^(8/3)-2953/13*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*c^(7/3)
*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))
*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1
+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/d^(8/3)/(c^(1/3)*(c^(1/3)+d^(1
/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+5906/39*2^
(1/2)*c^(7/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)
/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d
^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/d^(8/3)/(
c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^
3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.96 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.28

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{80x^2(-412c^3-388c^2dx^3+25cd^2x^6+d^3x^9)+4120c^2x^2(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\mathrm{Appel}}{520d^2\,(-8}$$

input
```
Integrate[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
(80*x^2*(-412*c^3 - 388*c^2*d*x^3 + 25*c*d^2*x^6 + d^3*x^9) + 4120*c^2*x^2
*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c)
, (d*x^3)/(8*c)] + 2953*c*d*x^5*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1
[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(520*d^2*(-8*c + d*x^3)*S
qrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {967, 27, 1051, 27, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2} \, dx$$

$$\Big\downarrow \text{ 967}$$

$$\frac{x^5 \left(c+dx^3\right)^{3/2}}{3d \left(8c-dx^3\right)}-\frac{\int \frac{x^4 \sqrt{dx^3+c} \left(19dx^3+10c\right)}{2 \left(8c-dx^3\right)}dx}{3d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^5 \left(c+dx^3\right)^{3/2}}{3d \left(8c-dx^3\right)}-\frac{\int \frac{x^4 \sqrt{dx^3+c} \left(19dx^3+10c\right)}{8c-dx^3}dx}{6d}$$

$$\Big\downarrow \text{ 1051}$$

$$\frac{x^5 \left(c+dx^3\right)^{3/2}}{3d \left(8c-dx^3\right)}-\frac{-\frac{2 \int -\frac{3cdx^4 \left(721dx^3+550c\right)}{2 \left(8c-dx^3\right) \sqrt{dx^3+c}}dx}{13d}-\frac{38}{13} x^5 \sqrt{c+dx^3}}{6d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^5 \left(c+dx^3\right)^{3/2}}{3d \left(8c-dx^3\right)}-\frac{\frac{3}{13} c \int \frac{x^4 \left(721dx^3+550c\right)}{\left(8c-dx^3\right) \sqrt{dx^3+c}}dx-\frac{38}{13} x^5 \sqrt{c+dx^3}}{6d}$$

$$\Big\downarrow \text{ 1052}$$

$$\frac{x^5 \left(c+dx^3\right)^{3/2}}{3d \left(8c-dx^3\right)}-\frac{\frac{3}{13} c \left(\frac{2 \int \frac{7cdx \left(2953dx^3+1648c\right)}{\left(8c-dx^3\right) \sqrt{dx^3+c}}dx}{7d^2}-\frac{206x^2 \sqrt{c+dx^3}}{d}\right)-\frac{38}{13} x^5 \sqrt{c+dx^3}}{6d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^5\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{\frac{3}{13}c\left(\frac{2c\int \frac{x\left(2953dx^3+1648c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{d}-\frac{206x^2\sqrt{c+dx^3}}{d}\right)-\frac{38}{13}x^5\sqrt{c+dx^3}}{6d}$$

$\Big\downarrow$ 1054

$$\frac{x^5\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{\frac{3}{13}c\left(\frac{2c\int \left(\frac{25272cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}-\frac{2953x}{\sqrt{dx^3+c}}\right)dx}{d}-\frac{206x^2\sqrt{c+dx^3}}{d}\right)-\frac{38}{13}x^5\sqrt{c+dx^3}}{6d}$$

$\Big\downarrow$ 2009

$$\frac{x^5\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-$$

$$\frac{3}{13}c\left(2c\left(-\frac{5906\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{2953\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\right.}{}\right.\right.$$

input
```
Int[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
(x^5*(c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)) - ((-38*x^5*Sqrt[c + d*x^3])/1
3 + (3*c*((-206*x^2*Sqrt[c + d*x^3])/d + (2*c*((-5906*Sqrt[c + d*x^3])/(d^
(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (1404*Sqrt[3]*c^(1/6)*ArcTan[
(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(2/3) + (1404*
c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(2
/3) - (1404*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(2/3) + (2953*
3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^
(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elli
pticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x)
)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (5906*Sqrt[2]*
c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*
x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3]
)*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3
]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/d))/13)/(6*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 967
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Simp[e^n/(b*n*(p + 1))   Int[(e*x)^(m -
 n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(
q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d,
0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBino
mialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1051
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Simp[1/(
b*(m + n*(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*
Simp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
 f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simpl
erQ[e + f*x^n, c + d*x^n])
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.23 (sec) , antiderivative size = 921, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	921
risch	Expression too large to display	1769
default	Expression too large to display	2224

input
```
int(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
24*c^2/d^2*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/13*x^5*(d*x^3+c)^(1/2)/d+64/
13*c*x^2*(d*x^3+c)^(1/2)/d^2-5906/39*I*c^2/d^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)))+72*I*c^2/d^5*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*
(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/
3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^
(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
)/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d
-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^
(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2580 vs. $2(490) = 980$.

Time = 16.00 (sec) , antiderivative size = 2580, normalized size of antiderivative = 3.79

$$\int \frac{x^7 (c + dx^3)^{3/2}}{(8c - dx^3)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/13*(5906*(c^2*d*x^3 - 8*c^3)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierst
rassPInverse(0, -4*c/d, x)) + 117*(d^4*x^3 - 8*c*d^3 - sqrt(-3)*(d^4*x^3 -
 8*c*d^3))*(c^13/d^16)^(1/6)*log(14693280768*((d^16*x^9 + 318*c*d^15*x^6 +
 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 +
1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c^13/d^16)^(5/6) + 6*(2*c^11*d^2*x^7 +
 160*c^12*d*x^4 + 320*c^13*x - 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2 - sqrt(
-3)*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2))*(c^13/d^16)^(2/3) - (7*c^7*d^7*x^6
 + 152*c^8*d^6*x^3 + 64*c^9*d^5 + sqrt(-3)*(7*c^7*d^7*x^6 + 152*c^8*d^6*x^
3 + 64*c^9*d^5))*(c^13/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^5*d^10*x^7 +
 64*c^6*d^9*x^4 + 32*c^7*d^8*x)*sqrt(c^13/d^16) + 18*(c^9*d^5*x^8 + 38*c^1
0*d^4*x^5 + 64*c^11*d^3*x^2 - sqrt(-3)*(c^9*d^5*x^8 + 38*c^10*d^4*x^5 + 64
*c^11*d^3*x^2))*(c^13/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3
 - 512*c^3)) - 117*(d^4*x^3 - 8*c*d^3 - sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(c^1
3/d^16)^(1/6)*log(-14693280768*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14
*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*
x^3 + 640*c^3*d^13))*(c^13/d^16)^(5/6) - 6*(2*c^11*d^2*x^7 + 160*c^12*d*x^
4 + 320*c^13*x - 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2 - sqrt(-3)*(5*c^3*d^1
2*x^5 + 32*c^4*d^11*x^2))*(c^13/d^16)^(2/3) - (7*c^7*d^7*x^6 + 152*c^8*d^6
*x^3 + 64*c^9*d^5 + sqrt(-3)*(7*c^7*d^7*x^6 + 152*c^8*d^6*x^3 + 64*c^9*d^5
))*(c^13/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^5*d^10*x^7 + 64*c^6*d^9...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**7*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x^7}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c)^2, x)
```

Giac [F]

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x^7}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x^7\,(d\,x^3+c)^{3/2}}{(8\,c-d\,x^3)^2}\,dx$$

input
```
int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

output
```
int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{1920\sqrt{d\,x^3+c}\,c^2x^2 - 1392\sqrt{d\,x^3+c}\,cd\,x^5 - 58\sqrt{d\,x^3+c}\,d^2x^8 + 597192\left(\int \frac{}{d^3x^9-15c}\right.}{}$$

input
```
int(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

output
```
(1920*sqrt(c + d*x**3)*c**2*x**2 - 1392*sqrt(c + d*x**3)*c*d*x**5 - 58*sqr
t(c + d*x**3)*d**2*x**8 + 597192*int((sqrt(c + d*x**3)*x**7)/(64*c**3 + 48
*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**3*d**2 - 74649*int((sqrt(
c + d*x**3)*x**7)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),
x)*c**2*d**3*x**3 - 245760*int((sqrt(c + d*x**3)*x)/(64*c**3 + 48*c**2*d*x
**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**5 + 30720*int((sqrt(c + d*x**3)*x)
/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**4*d*x**3)/(
377*d**2*(8*c - d*x**3))
```

3.592 $\int \frac{x^4 (c+dx^3)^{3/2}}{(8c-dx^3)^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 657

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{265c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{9\sqrt{3}c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} - \frac{9c^{7/6}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}} + \frac{9c^{7/6}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}} - \frac{265\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{14d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{265\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
13/21*x^2*(d*x^3+c)^(1/2)/d+265/7*c*(d*x^3+c)^(1/2)/d^(5/3)/((1+3^(1/2))*c
^(1/3)+d^(1/3)*x)+1/3*x^2*(d*x^3+c)^(3/2)/d/(-d*x^3+8*c)+9*3^(1/2)*c^(7/6)
*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))/d^(5/3)-9*c^(
7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(5/3)+9*
c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)-265/14*3^(1/4)*(1/2*6
^(1/2)-1/2*2^(1/2))*c^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*
x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1
/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/d^(
5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)
/(d*x^3+c)^(1/2)+265/21*2^(1/2)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1
/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*Ellipt
icF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1
/2)+2*I)*3^(3/4)/d^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)
+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 9.65 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.27

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx =$$

$$-\frac{16x^2(37c^2+35cdx^3-2d^2x^6)+74cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+53dx^5(-8}{112d\,(-8c+dx^3)\sqrt{c+dx^3}}$$

input
```
Integrate[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
-1/112*(16*x^2*(37*c^2 + 35*c*d*x^3 - 2*d^2*x^6) + 74*c*x^2*(-8*c + d*x^3)
*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c
)] + 53*d*x^5*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3
, -((d*x^3)/c), (d*x^3)/(8*c)])/(d*(-8*c + d*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 661, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {967, 27, 1051, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}dx$$

$$\Big\downarrow \text{ 967}$$

$$\frac{x^2 \left(c+dx^3\right)^{3/2}}{3d \left(8c-dx^3\right)}-\frac{\int \frac{x\sqrt{dx^3+c}\left(13dx^3+4c\right)}{2(8c-dx^3)}dx}{3d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^2 \left(c+dx^3\right)^{3/2}}{3d \left(8c-dx^3\right)}-\frac{\int \frac{x\sqrt{dx^3+c}\left(13dx^3+4c\right)}{8c-dx^3}dx}{6d}$$

$$\Big\downarrow \text{ 1051}$$

$$\frac{x^2 \left(c+dx^3\right)^{3/2}}{3d \left(8c-dx^3\right)}-\frac{-\frac{2\int -\frac{3cdx\left(265dx^3+148c\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d}-\frac{26}{7}x^2\sqrt{c+dx^3}}{6d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^2 \left(c+dx^3\right)^{3/2}}{3d \left(8c-dx^3\right)}-\frac{\frac{3}{7}c\int \frac{x\left(265dx^3+148c\right)}{(8c-dx^3)\sqrt{dx^3+c}}dx-\frac{26}{7}x^2\sqrt{c+dx^3}}{6d}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{x^2 \left(c+dx^3\right)^{3/2}}{3d \left(8c-dx^3\right)}-\frac{\frac{3}{7}c\int \left(\frac{2268cx}{(8c-dx^3)\sqrt{dx^3+c}}-\frac{265x}{\sqrt{dx^3+c}}\right)dx-\frac{26}{7}x^2\sqrt{c+dx^3}}{6d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} - \frac{\frac{3}{7}c\left(-\frac{530\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{265\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{\ldots}\right.}{\ldots}\right.}{\ldots}$$

input
```
Int[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
(x^2*(c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)) - ((-26*x^2*Sqrt[c + d*x^3])/7
 + (3*c*((-530*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)) - (126*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/
Sqrt[c + d*x^3]])/d^(2/3) + (126*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(
3*c^(1/6)*Sqrt[c + d*x^3])])/d^(2/3) - (126*c^(1/6)*ArcTanh[Sqrt[c + d*x^3
]/(3*Sqrt[c])])/d^(2/3) + (265*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3)
+ d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt
[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sq
rt[c + d*x^3]) - (530*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
- c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3)
+ d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/7)
/(6*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 967
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Simp[e^n/(b*n*(p + 1))   Int[(e*x)^(m -
 n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(
q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d,
0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBino
mialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1051
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Simp[1/(
b*(m + n*(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*
Simp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
 f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simpl
erQ[e + f*x^n, c + d*x^n])
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.97 (sec) , antiderivative size = 897, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	897
default	Expression too large to display	1749
risch	Expression too large to display	1758

input
```
int(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
3*c/d*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/7*x^2*(d*x^3+c)^(1/2)/d-265/21*I*
c/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+6*I*c/d^4*2^(1/2
)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*
d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(
-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c
*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alp...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2568 vs. $2(470) = 940$.

Time = 3.53 (sec) , antiderivative size = 2568, normalized size of antiderivative = 3.91

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/28*(1060*(c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstra
ssPInverse(0, -4*c/d, x)) + 21*(d^3*x^3 - 8*c*d^2 - sqrt(-3)*(d^3*x^3 - 8*
c*d^2))*(c^7/d^10)^(1/6)*log(59049*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*
d^9*x^3 + 640*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9
*x^3 + 640*c^3*d^8))*(c^7/d^10)^(5/6) + 6*(2*c^6*d^2*x^7 + 160*c^7*d*x^4 +
 320*c^8*x - 6*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2 - sqrt(-3)*(5*c^2*d^8*x^5 +
 32*c^3*d^7*x^2))*(c^7/d^10)^(2/3) - (7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64
*c^6*d^3 + sqrt(-3)*(7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3))*(c^7/d
^10)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^7*x^7 + 64*c^4*d^6*x^4 + 32*c^5*
d^5*x)*sqrt(c^7/d^10) + 18*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2
- sqrt(-3)*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2))*(c^7/d^10)^(1/
6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 21*(d^3*x^3 - 8*
c*d^2 - sqrt(-3)*(d^3*x^3 - 8*c*d^2))*(c^7/d^10)^(1/6)*log(-59049*((d^11*x
^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + sqrt(-3)*(d^11*x^9
+ 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8))*(c^7/d^10)^(5/6) - 6*(
2*c^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^8*x^5 + 32*c^3*d^7*
x^2 - sqrt(-3)*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2))*(c^7/d^10)^(2/3) - (7*c^4
*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3 + sqrt(-3)*(7*c^4*d^5*x^6 + 152*c^
5*d^4*x^3 + 64*c^6*d^3))*(c^7/d^10)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^7
*x^7 + 64*c^4*d^6*x^4 + 32*c^5*d^5*x)*sqrt(c^7/d^10) + 18*(c^5*d^4*x^8 ...
```

Sympy [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x^4(c+dx^3)^{\frac{3}{2}}}{(-8c+dx^3)^2}\,dx$$

input
```
integrate(x**4*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x**4*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x^4}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c)^2, x)
```

Giac [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x^4}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x^4\,(d\,x^3+c)^{3/2}}{(8\,c-d\,x^3)^2}\,dx$$

input
```
int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

output
```
int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{6\sqrt{d\,x^3+c}\,c\,x^2-4\sqrt{d\,x^3+c}\,d\,x^5+1848\left(\int \frac{\sqrt{d\,x^3+c}\,x^7}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)c^2d^2-231$$

input
```
int(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

output
```
(6*sqrt(c + d*x**3)*c*x**2 - 4*sqrt(c + d*x**3)*d*x**5 + 1848*int((sqrt(c
+ d*x**3)*x**7)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)
*c**2*d**2 - 231*int((sqrt(c + d*x**3)*x**7)/(64*c**3 + 48*c**2*d*x**3 - 1
5*c*d**2*x**6 + d**3*x**9),x)*c*d**3*x**3 - 768*int((sqrt(c + d*x**3)*x)/(
64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**4 + 96*int((s
qrt(c + d*x**3)*x)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9)
,x)*c**3*d*x**3)/(14*d*(8*c - d*x**3))
```

3.593 $\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 638

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{19\sqrt{c+dx^3}}{8d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} + \frac{9\sqrt{3}\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{16d^{2/3}} - \frac{9\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{16d^{2/3}} + \frac{9\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{16d^{2/3}} - \frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{19\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
19/8*(d*x^3+c)^(1/2)/d^(2/3)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)+3*x^2*(d*x^3+
c)^(1/2)/(-8*d*x^3+64*c)+9/16*3^(1/2)*c^(1/6)*arctan(3^(1/2)*c^(1/6)*(c^(1
/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))/d^(2/3)-9/16*c^(1/6)*arctanh(1/3*(c^(1/3)+
d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(2/3)+9/16*c^(1/6)*arctanh(1/3*(d*
x^3+c)^(1/2)/c^(1/2))/d^(2/3)-19/16*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*c^(1
/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/
2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/
((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/d^(2/3)/(c^(1/3)*(c^(1/3)+d
^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+19/24*2
^(1/2)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+
d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/d^(2/3)/
(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x
^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.22

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{x^2\left(\frac{240(c+dx^3)}{8c-dx^3} - 25\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right) - \frac{19dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{\ldots}\right.}{c}\right.}{640\sqrt{c+dx^3}}$$

input
```
Integrate[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
(x^2*((240*(c + d*x^3))/(8*c - d*x^3) - 25*Sqrt[1 + (d*x^3)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - (19*d*x^3*Sqrt[1 + (d*x^3)/
c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/c))/(640*Sqrt[
c + d*x^3])
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 631, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {968, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}dx$$

$$\Big\downarrow \ 968$$

$$\frac{\int -\frac{3cdx(19dx^3+10c)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{24cd}+\frac{3x^2\sqrt{c+dx^3}}{8(8c-dx^3)}$$

$$\Big\downarrow \ 27$$

$$\frac{3x^2\sqrt{c+dx^3}}{8(8c-dx^3)}-\frac{1}{16}\int \frac{x(19dx^3+10c)}{(8c-dx^3)\sqrt{dx^3+c}}dx$$

$$\Big\downarrow \ 1054$$

$$\frac{3x^2\sqrt{c+dx^3}}{8(8c-dx^3)}-\frac{1}{16}\int \left(\frac{162cx}{(8c-dx^3)\sqrt{dx^3+c}}-\frac{19x}{\sqrt{dx^3+c}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{16}\left(\frac{38\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{19\sqrt[4]{3}\cdots}{\cdots}\right.$$

$$\frac{3x^2\sqrt{c+dx^3}}{8(8c-dx^3)}$$

input
```
Int[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
(3*x^2*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) + ((38*Sqrt[c + d*x^3])/(d^(2/3)
*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (9*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]
*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(2/3) - (9*c^(1/6)*Arc
Tanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(2/3) + (9*c^
(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(2/3) - (19*3^(1/4)*Sqrt[2 -
 Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x
+ d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1
 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7
- 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (38*Sqrt[2]*c^(1/3)*(c^(1/3) +
 d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3
)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/
3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x
)^2]*Sqrt[c + d*x^3]))/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.13 (sec) , antiderivative size = 874, normalized size of antiderivative = 1.37

method	result	size
default	Expression too large to display	874
elliptic	Expression too large to display	874

input
```
int(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
3/8*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-19/24*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)))+3/8*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/
2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))
^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/
3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(
-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*
3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_a
lpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2335 vs. $2(449) = 898$.

Time = 1.06 (sec) , antiderivative size = 2335, normalized size of antiderivative = 3.66

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/64*(24*sqrt(d*x^3 + c)*d*x^2 + 152*(d*x^3 - 8*c)*sqrt(d)*weierstrassZet
a(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 3*(d^2*x^3 - 8*c*d - sqr
t(-3)*(d^2*x^3 - 8*c*d))*(c/d^4)^(1/6)*log(59049/4*((d^6*x^9 + 318*c*d^5*x
^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + sqrt(-3)*(d^6*x^9 + 318*c*d^5*x^6 +
1200*c^2*d^4*x^3 + 640*c^3*d^3))*(c/d^4)^(5/6) + 6*(2*c*d^2*x^7 + 160*c^2*
d*x^4 + 320*c^3*x - 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2 - sqrt(-3)*(5*c*d^4*x^
5 + 32*c^2*d^3*x^2))*(c/d^4)^(2/3) - (7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c
^3*d + sqrt(-3)*(7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d))*(c/d^4)^(1/3))
*sqrt(d*x^3 + c) - 36*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 + 32*c^3*d^2*x)*sqrt(c
/d^4) + 18*(c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2 - sqrt(-3)*(c*d^3*x^
8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2))*(c/d^4)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6
 + 192*c^2*d*x^3 - 512*c^3)) - 3*(d^2*x^3 - 8*c*d - sqrt(-3)*(d^2*x^3 - 8*
c*d))*(c/d^4)^(1/6)*log(-59049/4*((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*
x^3 + 640*c^3*d^3 + sqrt(-3)*(d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 +
 640*c^3*d^3))*(c/d^4)^(5/6) - 6*(2*c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3*x
- 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2 - sqrt(-3)*(5*c*d^4*x^5 + 32*c^2*d^3*x^2
))*(c/d^4)^(2/3) - (7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d + sqrt(-3)*(7
*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d))*(c/d^4)^(1/3))*sqrt(d*x^3 + c) -
 36*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 + 32*c^3*d^2*x)*sqrt(c/d^4) + 18*(c*d^3*
x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2 - sqrt(-3)*(c*d^3*x^8 + 38*c^2*d^2*...
```

Sympy [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x(c+dx^3)^{\frac{3}{2}}}{(-8c+dx^3)^2}\,dx$$

input
```
integrate(x*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c)^2, x)
```

Giac [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x\,(d\,x^3+c)^{3/2}}{(8\,c-d\,x^3)^2}\,dx$$

input
```
int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

output
```
int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{2\sqrt{d\,x^3+c}\,x^2+240\left(\int \frac{\sqrt{d\,x^3+c}\,x^7}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)c\,d^2-30\left(\int \frac{\sqrt{d\,x^3+c}\,x^7}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3-}\right.}{-2}$$

input
```
int(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

output
```
(2*sqrt(c + d*x**3)*x**2 + 240*int((sqrt(c + d*x**3)*x**7)/(64*c**3 + 48*c
**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c*d**2 - 30*int((sqrt(c + d*x*
*3)*x**7)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*d**3*
x**3 - 24*int((sqrt(c + d*x**3)*x)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x
**6 + d**3*x**9),x)*c**3 + 3*int((sqrt(c + d*x**3)*x)/(64*c**3 + 48*c**2*d
*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**2*d*x**3)/(29*(8*c - d*x**3))
```

3.594 $\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 522

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx = -\frac{\sqrt{c+dx^3}}{16cx}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{3\sqrt{c+dx^3}}{8x(8c-dx^3)}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-1/16*(d*x^3+c)^(1/2)/c/x+1/16*d^(1/3)*(d*x^3+c)^(1/2)/c/((1+3^(1/2))*c^(1
/3)+d^(1/3)*x)+3/8*(d*x^3+c)^(1/2)/x/(-d*x^3+8*c)-1/32*3^(1/4)*(1/2*6^(1/2
)-1/2*2^(1/2))*d^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(
2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*
c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/c^(2/3)/
(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x
^3+c)^(1/2)+1/48*d^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d
^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2)
)*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2
)*3^(3/4)/c^(2/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3
)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.50 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.46

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx = \frac{(2c-dx^3)\sqrt{c+dx^3}}{16cx(-8c+dx^3)}$$

$$-\frac{\sqrt[6]{-1}\sqrt[3]{-d}\sqrt{(-1)^{5/6}\left(-1+\frac{\sqrt[3]{-d}x}{\sqrt[3]{c}}\right)}\sqrt{1+\frac{\sqrt[3]{-d}x}{\sqrt[3]{c}}+\frac{(-d)^{2/3}x^2}{c^{2/3}}}\left(-i\sqrt{3}E\left(\arcsin\left(\frac{\sqrt{-(-1)^{5/6}-\frac{i\sqrt[3]{-d}x}{\sqrt[3]{c}}}}{\sqrt[4]{3}}\right)|\right.\right.}{16\sqrt[4]{3}\sqrt[3]{c}\sqrt{c+dx^3}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2),x]
```

output
```
((2*c - d*x^3)*Sqrt[c + d*x^3])/(16*c*x*(-8*c + d*x^3)) - ((-1)^(1/6)*(-d)
^(1/3)*Sqrt[(-1)^(5/6)*(-1 + ((-d)^(1/3)*x)/c^(1/3))]*Sqrt[1 + ((-d)^(1/3)
*x)/c^(1/3) + ((-d)^(2/3)*x^2)/c^(2/3)]*((-I)*Sqrt[3]*EllipticE[ArcSin[Sqr
t[-(-1)^(5/6) - (I*(-d)^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)^(1/3)] + (-1)^(1/
3)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-d)^(1/3)*x)/c^(1/3)]/3^(1/4)],
 (-1)^(1/3)]))/(16*3^(1/4)*c^(1/3)*Sqrt[c + d*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.02 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {968, 27, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2 \left(8c-dx^3\right)^2} \, dx$$

$$\Big\downarrow \text{968}$$

$$\frac{\int \frac{3cd}{2x^2\sqrt{dx^3+c}}dx}{24cd}+\frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{1}{x^2\sqrt{dx^3+c}}dx+\frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{847}$$

$$\frac{1}{16}\left(\frac{d\int \frac{x}{\sqrt{dx^3+c}}dx}{2c}-\frac{\sqrt{c+dx^3}}{cx}\right)+\frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{832}$$

$$\frac{1}{16}\left(\frac{d\left(\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}\int \frac{1}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}\right)}{2c}-\frac{\sqrt{c+dx^3}}{cx}\right)+\frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{759}$$

$$\frac{1}{16}\left(\frac{d\left(\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right.\right.}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\right.}{2c}\right.$$

$$\frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)}$$

$\downarrow$ 2416

$$\frac{1}{16}\left(\frac{d\left(\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}-\right.}{2c}\right.$$

$$\frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)}$$

input
```
Int[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2),x]
```

output
```
(3*Sqrt[c + d*x^3])/(8*x*(8*c - d*x^3)) + (-(Sqrt[c + d*x^3]/(c*x)) + (d*(
((2*Sqrt[c + d*x^3])/(d^(1/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (3^(1
/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3
)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)], -7 - 4*Sqrt[3]])/(d^(1/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/d^(1/3) - (2*(1 - S
qrt[3])*Sqrt[2 + Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^
(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^
(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(2*c))
/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.93

method	result
	$i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{(}{}\right.}{}}$
elliptic	$\frac{3d\,x^2\sqrt{d\,x^3+c}}{64c(-d\,x^3+8c)}-\frac{\sqrt{d\,x^3+c}}{64cx}-$
risch	Expression too large to display
default	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
3/64*d*x^2/c*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/64*(d*x^3+c)^(1/2)/c/x-1/48*I/
c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Elli
pticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.13

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx = -\frac{(dx^4-8\,cx)\sqrt{d}\text{weierstrassZeta}\left(0,-\frac{4\,c}{d},\text{weierstrassPInverse}\left(0,-\frac{4\,c}{d},x\right)\right)+\sqrt{dx^3+c}(dx^3-2\,c)}{16\,(cdx^4-8\,c^2x)}$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/16*((d*x^4 - 8*c*x)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInve
rse(0, -4*c/d, x)) + sqrt(d*x^3 + c)*(d*x^3 - 2*c))/(c*d*x^4 - 8*c^2*x)
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{x^2(-8c+dx^3)^2}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**2/(-d*x**3+8*c)**2,x)
```

output
```
Integral((c + d*x**3)**(3/2)/(x**2*(-8*c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^2), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2 (8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2 x^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2 (8c-dx^3)^2}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^2\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2),x)
```

output
```
int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2 (8c-dx^3)^2}\,dx = \frac{-2\sqrt{d\,x^3+c}+168\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)c\,d^2x-21\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{d^3x^9-15c\,d^2x^6+48c^2d\,x}\right.}{}$$

input
```
int((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x)
```

output
```
( - 2*sqrt(c + d*x**3) + 168*int((sqrt(c + d*x**3)*x**4)/(64*c**3 + 48*c**
2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c*d**2*x - 21*int((sqrt(c + d*x*
*3)*x**4)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*d**3*
x**4 + 384*int((sqrt(c + d*x**3)*x)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*
x**6 + d**3*x**9),x)*c**2*d*x - 48*int((sqrt(c + d*x**3)*x)/(64*c**3 + 48*
c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c*d**2*x**4)/(16*x*(8*c - d*x
**3))
```

3.595 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^5\left(8c-dx^3\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 684

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx = -\frac{13\sqrt{c+dx^3}}{256cx^4}-\frac{d\sqrt{c+dx^3}}{32c^2x}+\frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$
$$+\frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)}-\frac{9\sqrt{3}d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}}$$
$$+\frac{9d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}}-\frac{9d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{11/6}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{64c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{16\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-13/256*(d*x^3+c)^(1/2)/c/x^4-1/32*d*(d*x^3+c)^(1/2)/c^2/x+1/32*d^(4/3)*(d
*x^3+c)^(1/2)/c^2/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)+3/8*(d*x^3+c)^(1/2)/x^4/
(-d*x^3+8*c)-9/1024*3^(1/2)*d^(4/3)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3
)*x)/(d*x^3+c)^(1/2))/c^(11/6)+9/1024*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)
*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)-9/1024*d^(4/3)*arctanh(1/3*(d*x^3+
c)^(1/2)/c^(1/2))/c^(11/6)-1/64*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*d^(4/3)*
(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*
c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+
3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/c^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/
3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+1/96*d^(4/3
)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2)
)*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((
1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/c^(5/3)/(c^(1
/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)
^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.29

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx = \sqrt{c+dx^3}\left(-\frac{1}{256cx^4}-\frac{13d}{512c^2x}-\frac{3d^2x^2}{512c^2(-8c+dx^3)}\right) + \frac{145d^2x^2\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{8192c^2\sqrt{c+dx^3}} - \frac{d^3x^5\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{2560c^3\sqrt{c+dx^3}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2),x]
```

output
```
Sqrt[c + d*x^3]*(-1/256*1/(c*x^4) - (13*d)/(512*c^2*x) - (3*d^2*x^2)/(512*
c^2*(-8*c + d*x^3))) + (145*d^2*x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2,
 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(8192*c^2*Sqrt[c + d*x^3]) - (d^3*x
^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8
*c)])/(2560*c^3*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.96 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {968, 27, 1053, 25, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{3/2}}{x^5 (8c-dx^3)^2} \, dx$$

$$\Big\downarrow \text{968}$$

$$\frac{\int \frac{3cd(17dx^3+26c)}{2x^5(8c-dx^3)\sqrt{dx^3+c}}dx}{24cd}+\frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{17dx^3+26c}{x^5(8c-dx^3)\sqrt{dx^3+c}}dx+\frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{16}\left(-\frac{\int -\frac{cd(65dx^3+128c)}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c^2}-\frac{13\sqrt{c+dx^3}}{16cx^4}\right)+\frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{16}\left(\frac{\int \frac{cd(65dx^3+128c)}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c^2}-\frac{13\sqrt{c+dx^3}}{16cx^4}\right)+\frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{d\int \frac{65dx^3+128c}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c}-\frac{13\sqrt{c+dx^3}}{16cx^4}\right)+\frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{16}\left(\frac{d\left(-\frac{\int -\frac{8cdx(145c-8dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{8c^2}-\frac{16\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{13\sqrt{c+dx^3}}{16cx^4}\right)+\frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{16}\left(\frac{d\left(\frac{d\int \frac{x(145c-8dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{c}-\frac{16\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{13\sqrt{c+dx^3}}{16cx^4}\right)+\frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)}$$

$$\Big\downarrow \; 1054$$

$$\frac{1}{16}\left(\frac{d\left(\frac{d\int \left(\frac{81cx}{(8c-dx^3)\sqrt{dx^3+c}}+\frac{8x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{16\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{13\sqrt{c+dx^3}}{16cx^4}\right)+\frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{16}\left(\frac{d\left(\frac{d\left(\frac{16\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{8\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\right.}{}\right.}{}\right.}{}\right.$$

$$\frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)}$$

input
```
Int[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2),x]
```

output
```
(3*Sqrt[c + d*x^3])/(8*x^4*(8*c - d*x^3)) + ((-13*Sqrt[c + d*x^3])/(16*c*x
^4) + (d*((-16*Sqrt[c + d*x^3])/(c*x) + (d*((16*Sqrt[c + d*x^3])/(d^(2/3)*
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (9*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*
c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*d^(2/3)) + (9*c^(1/6)*
ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(2*d^(2/3))
- (9*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*d^(2/3)) - (8*3^(1/4
)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (16*Sqrt[2]*c^(1/3)*
(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3
) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(
1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/c))/(32*c))/16
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.90 (sec) , antiderivative size = 919, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	919
risch	Expression too large to display	1770
default	Expression too large to display	2691

input
```
int((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
3/512/c^2*x^2*d^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/256*(d*x^3+c)^(1/2)/c/x^4
-13/512*d*(d*x^3+c)^(1/2)/c^2/x-1/96*I*d/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)))-3/512*I/d/c^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(
1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3
))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(
1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))
/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-
I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(
2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2549 vs. $2(487) = 974$.

Time = 0.86 (sec) , antiderivative size = 2549, normalized size of antiderivative = 3.73

$$\int \frac{(c+dx^3)^{3/2}}{x^5 (8c-dx^3)^2} \, dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/4096*(128*(d^2*x^7 - 8*c*d*x^4)*sqrt(d)*weierstrassZeta(0, -4*c/d, weie
rstrassPInverse(0, -4*c/d, x)) - 3*(c^2*d*x^7 - 8*c^3*x^4 + sqrt(-3)*(c^2*
d*x^7 - 8*c^3*x^4))*(d^8/c^11)^(1/6)*log(6561*(d^9*x^9 + 318*c*d^8*x^6 + 1
200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^1
0*d*x + sqrt(-3)*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x))*(d^8/c^11
)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 - sqrt(-3)*(5*c
^10*d*x^5 + 32*c^11*x^2))*(d^8/c^11)^(5/6) - 2*(7*c^6*d^4*x^6 + 152*c^7*d^
3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c
^4*d^5*x + sqrt(-3)*(c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d^5*x))*(d^8/c
^11)^(1/6)) - 9*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2 - sqrt(-3)*
(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2))*(d^8/c^11)^(1/3))/(d^3*x^
9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 3*(c^2*d*x^7 - 8*c^3*x^4 +
sqrt(-3)*(c^2*d*x^7 - 8*c^3*x^4))*(d^8/c^11)^(1/6)*log(6561*(d^9*x^9 + 318
*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^8*d^3*x^7 + 64*c^9*d^
2*x^4 + 32*c^10*d*x + sqrt(-3)*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d
*x))*(d^8/c^11)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 -
 sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^8/c^11)^(5/6) - 2*(7*c^6*d^4*x^
6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 + 80*c^3*d
^6*x^4 + 160*c^4*d^5*x + sqrt(-3)*(c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*
d^5*x))*(d^8/c^11)^(1/6)) - 9*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2} dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**(3/2)/x**5/(-d*x**3+8*c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x^5}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^5), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x^5}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^5\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2),x)
```

output
```
int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^5\,(8c-dx^3)^2}\,dx = \frac{-2\sqrt{d\,x^3+c}+816\left(\int \frac{\sqrt{d\,x^3+c}}{d^3x^{11}-15c\,d^2x^8+48c^2d\,x^5+64c^3x^2}dx\right)c^2d\,x^4-102\left(\int \frac{\sqrt{d\,x}}{d^3x^{11}-15c\,d^2x^8+}\right.}{}$$

input
```
int((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x)
```

output
```
( - 2*sqrt(c + d*x**3) + 816*int(sqrt(c + d*x**3)/(64*c**3*x**2 + 48*c**2*
d*x**5 - 15*c*d**2*x**8 + d**3*x**11),x)*c**2*d*x**4 - 102*int(sqrt(c + d*
x**3)/(64*c**3*x**2 + 48*c**2*d*x**5 - 15*c*d**2*x**8 + d**3*x**11),x)*c*d
**2*x**7 + 600*int((sqrt(c + d*x**3)*x)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d
**2*x**6 + d**3*x**9),x)*c*d**2*x**4 - 75*int((sqrt(c + d*x**3)*x)/(64*c**
3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*d**3*x**7)/(64*x**4*(8
*c - d*x**3))
```

3.596 $\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 708

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)^2}\,dx = -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3\sqrt{c+dx^3}}{8x^7(8c-dx^3)} - \frac{9\sqrt{3}d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4096c^{17/6}} + \frac{9d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{4096c^{17/6}} - \frac{9d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4096c^{17/6}} - \frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{3584c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{19d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{896\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-11/224*(d*x^3+c)^(1/2)/c/x^7-83/7168*d*(d*x^3+c)^(1/2)/c^2/x^4-19/1792*d^
2*(d*x^3+c)^(1/2)/c^3/x+19/1792*d^(7/3)*(d*x^3+c)^(1/2)/c^3/((1+3^(1/2))*c
^(1/3)+d^(1/3)*x)+3/8*(d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)-9/4096*3^(1/2)*d^(7
/3)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))/c^(17/6)+9
/4096*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(17/6)-9/4096*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-19/35
84*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*d^(7/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)
-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*E
llipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I
*3^(1/2)+2*I)/c^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^
(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+19/5376*d^(7/3)*(c^(1/3)+d^(1/3)*x)*((c^
(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1
/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)
*x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/c^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((
1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.30

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)^2}\,dx = \sqrt{c+dx^3}\left(-\frac{1}{448cx^7}-\frac{41d}{7168c^2x^4}-\frac{283d^2}{28672c^3x}-\frac{3d^3x^2}{4096c^3(-8c+dx^3)}\right)+\frac{1175d^3x^2\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{229376c^3\sqrt{c+dx^3}}-\frac{19d^4x^5\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{143360c^4\sqrt{c+dx^3}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2),x]
```

output
```
Sqrt[c + d*x^3]*(-1/448*1/(c*x^7) - (41*d)/(7168*c^2*x^4) - (283*d^2)/(286
72*c^3*x) - (3*d^3*x^2)/(4096*c^3*(-8*c + d*x^3))) + (1175*d^3*x^2*Sqrt[(c
 + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(229
376*c^3*Sqrt[c + d*x^3]) - (19*d^4*x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1
/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(143360*c^4*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 722, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {968, 27, 1053, 27, 1053, 25, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)^2}dx$$

$$\Big\downarrow \text{968}$$

$$\frac{\int \frac{3cd(35dx^3+44c)}{2x^8(8c-dx^3)\sqrt{dx^3+c}}dx}{24cd}+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{35dx^3+44c}{x^8\left(8c-dx^3\right)\sqrt{dx^3+c}}dx+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{16}\left(-\frac{\int -\frac{2cd(121dx^3+166c)}{x^5(8c-dx^3)\sqrt{dx^3+c}}dx}{56c^2}-\frac{11\sqrt{c+dx^3}}{14cx^7}\right)+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{d\int \frac{121dx^3+166c}{x^5(8c-dx^3)\sqrt{dx^3+c}}dx}{28c}-\frac{11\sqrt{c+dx^3}}{14cx^7}\right)+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{16}\left(\frac{d\left(-\frac{\int -\frac{cd\left(415dx^3+1216c\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2}-\frac{83\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{11\sqrt{c+dx^3}}{14cx^7}\right)+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{16}\left(\frac{d\left(\frac{\int \frac{cd\left(415dx^3+1216c\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2}-\frac{83\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{11\sqrt{c+dx^3}}{14cx^7}\right)+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{16}\left(\frac{d\left(\frac{d\int \frac{415dx^3+1216c}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c}-\frac{83\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{11\sqrt{c+dx^3}}{14cx^7}\right)+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{1}{16}\left(\frac{d\left(\frac{d\left(-\frac{\int -\frac{8cdx\left(1175c-76dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{152\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{83\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{11\sqrt{c+dx^3}}{14cx^7}\right)+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{16}\left(\frac{d\left(\frac{d\left(\frac{d\int \frac{x\left(1175c-76dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{152\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{83\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{11\sqrt{c+dx^3}}{14cx^7}\right)+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$\downarrow$ 1054

$$\frac{1}{16}\left(\frac{d\left(\frac{d\left(\frac{d\int \left(\frac{567cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{76x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{152\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{83\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{11\sqrt{c+dx^3}}{14cx^7}\right)+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$\downarrow$ 2009

$$\frac{1}{16}\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{152\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{76\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2),x]
```

output
```
(3*Sqrt[c + d*x^3])/(8*x^7*(8*c - d*x^3)) + ((-11*Sqrt[c + d*x^3])/(14*c*x
^7) + (d*((-83*Sqrt[c + d*x^3])/(16*c*x^4) + (d*((-152*Sqrt[c + d*x^3])/(c
*x) + (d*((152*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)) - (63*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/S
qrt[c + d*x^3]])/(2*d^(2/3)) + (63*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2
/(3*c^(1/6)*Sqrt[c + d*x^3])])/(2*d^(2/3)) - (63*c^(1/6)*ArcTanh[Sqrt[c +
d*x^3]/(3*Sqrt[c])])/(2*d^(2/3)) - (76*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3)
 + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2
/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*Sqrt[c + d*x^3]) + (152*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(
c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^
3]))/c))/(32*c)))/(28*c))/16
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.08 (sec) , antiderivative size = 938, normalized size of antiderivative = 1.32

method	result	size
elliptic	Expression too large to display	938
risch	Expression too large to display	1781
default	Expression too large to display	3187

input
```
int((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/448*(d*x^3+c)^(1/2)/c/x^7-41/7168*d*(d*x^3+c)^(1/2)/c^2/x^4-283/28672*d
^2*(d*x^3+c)^(1/2)/c^3/x+3/4096/c^3*x^2*d^3*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1
9/5376*I/c^3*d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^
(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(
x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c
*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(
1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-3/204
8*I/c^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(
1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1
/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3
+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_a
lpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2582 vs. $2(507) = 1014$.

Time = 1.87 (sec) , antiderivative size = 2582, normalized size of antiderivative = 3.65

$$\int \frac{(c+dx^3)^{3/2}}{x^8 (8c-dx^3)^2} \, dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/114688*(1216*(d^3*x^10 - 8*c*d^2*x^7)*sqrt(d)*weierstrassZeta(0, -4*c/d
, weierstrassPInverse(0, -4*c/d, x)) - 21*(c^3*d*x^10 - 8*c^4*x^7 + sqrt(-
3)*(c^3*d*x^10 - 8*c^4*x^7))*(d^14/c^17)^(1/6)*log(6561*(d^14*x^9 + 318*c*
d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^12*d^4*x^7 + 64*c^13*
d^3*x^4 + 32*c^14*d^2*x + sqrt(-3)*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*
c^14*d^2*x))*(d^14/c^17)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c
^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^14/c^17)^(5/6) - 2*(7*
c^9*d^6*x^6 + 152*c^10*d^5*x^3 + 64*c^11*d^4)*sqrt(d^14/c^17) + (c^3*d^11*
x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x + sqrt(-3)*(c^3*d^11*x^7 + 80*c^4*d^
10*x^4 + 160*c^5*d^9*x))*(d^14/c^17)^(1/6)) - 9*(c^6*d^9*x^8 + 38*c^7*d^8*
x^5 + 64*c^8*d^7*x^2 - sqrt(-3)*(c^6*d^9*x^8 + 38*c^7*d^8*x^5 + 64*c^8*d^7
*x^2))*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^
3)) + 21*(c^3*d*x^10 - 8*c^4*x^7 + sqrt(-3)*(c^3*d*x^10 - 8*c^4*x^7))*(d^1
4/c^17)^(1/6)*log(6561*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 64
0*c^3*d^11 - 9*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x + sqrt(-3
)*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x))*(d^14/c^17)^(2/3) -
3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5
+ 32*c^16*x^2))*(d^14/c^17)^(5/6) - 2*(7*c^9*d^6*x^6 + 152*c^10*d^5*x^3 +
64*c^11*d^4)*sqrt(d^14/c^17) + (c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d
^9*x + sqrt(-3)*(c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x))*(d^14...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^8 (8c-dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**(3/2)/x**8/(-d*x**3+8*c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^8 (8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2 x^8}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^8), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^8 (8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2 x^8}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^8 (8c-dx^3)^2}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^8\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2),x)
```

output
```
int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)^2}\,dx = \frac{-4\sqrt{d\,x^3+c}\,c^2-42\sqrt{d\,x^3+c}\,d^2x^6+2496\left(\int \frac{\sqrt{d\,x^3+c}}{d^3x^{14}-15c\,d^2x^{11}+48c^2d\,x^8+64c^3x^5}dx\right)c^4d\,x$$

input
```
int((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x)
```

output
```
( - 4*sqrt(c + d*x**3)*c**2 - 42*sqrt(c + d*x**3)*d**2*x**6 + 2496*int(sqr
t(c + d*x**3)/(64*c**3*x**5 + 48*c**2*d*x**8 - 15*c*d**2*x**11 + d**3*x**1
4),x)*c**4*d*x**7 - 312*int(sqrt(c + d*x**3)/(64*c**3*x**5 + 48*c**2*d*x**
8 - 15*c*d**2*x**11 + d**3*x**14),x)*c**3*d**2*x**10 - 624*int(sqrt(c + d*
x**3)/(64*c**3*x**2 + 48*c**2*d*x**5 - 15*c*d**2*x**8 + d**3*x**11),x)*c**
3*d**2*x**7 + 78*int(sqrt(c + d*x**3)/(64*c**3*x**2 + 48*c**2*d*x**5 - 15*
c*d**2*x**8 + d**3*x**11),x)*c**2*d**3*x**10 + 840*int((sqrt(c + d*x**3)*x
**4)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c*d**4*x**
7 - 105*int((sqrt(c + d*x**3)*x**4)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*
x**6 + d**3*x**9),x)*d**5*x**10 + 2688*int((sqrt(c + d*x**3)*x)/(64*c**3 +
 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**2*d**3*x**7 - 336*int(
(sqrt(c + d*x**3)*x)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**
9),x)*c*d**4*x**10)/(224*c**2*x**7*(8*c - d*x**3))
```

3.597 $\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 102

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{10c\sqrt{c+dx^3}}{d^4}+\frac{512c^2\sqrt{c+dx^3}}{27d^4(8c-dx^3)}+\frac{2(c+dx^3)^{3/2}}{9d^4}-\frac{2944c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}$$

output
```
10*c*(d*x^3+c)^(1/2)/d^4+512/27*c^2*(d*x^3+c)^(1/2)/d^4/(-d*x^3+8*c)+2/9*(
d*x^3+c)^(3/2)/d^4-2944/81*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^
4
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\left(\frac{3\sqrt{c+dx^3}(-1360c^2+114cdx^3+3d^2x^6)}{-8c+dx^3}-1472c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81d^4}$$

input
```
Integrate[x^11/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(2*((3*Sqrt[c + d*x^3]*(-1360*c^2 + 114*c*d*x^3 + 3*d^2*x^6))/(-8*c + d*x^
3) - 1472*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(81*d^4)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {948, 109, 27, 164, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^9}{(8c-dx^3)^2\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{ 109}$$

$$\frac{1}{3}\left(\frac{8x^6\sqrt{c+dx^3}}{9d^2(8c-dx^3)}-\frac{\int \frac{cx^3(21dx^3+16c)}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9cd^2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{8x^6\sqrt{c+dx^3}}{9d^2(8c-dx^3)}-\frac{\int \frac{x^3(21dx^3+16c)}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9d^2}\right)$$

$$\Big\downarrow \text{ 164}$$

$$\frac{1}{3}\left(\frac{8x^6\sqrt{c+dx^3}}{9d^2(8c-dx^3)}-\frac{\frac{1472c^2\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{d}-\frac{2\sqrt{c+dx^3}(170c+7dx^3)}{d^2}}{9d^2}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(\frac{8x^6\sqrt{c+dx^3}}{9d^2(8c-dx^3)}-\frac{\frac{2944c^2\int\frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d^2}-\frac{2\sqrt{c+dx^3}(170c+7dx^3)}{d^2}}{9d^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{8x^6\sqrt{c+dx^3}}{9d^2(8c-dx^3)}-\frac{\frac{2944c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3d^2}-\frac{2\sqrt{c+dx^3}(170c+7dx^3)}{d^2}}{9d^2}\right)$$

input
```
Int[x^11/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((8*x^6*Sqrt[c + d*x^3])/(9*d^2*(8*c - d*x^3)) - ((-2*Sqrt[c + d*x^3]*(170
*c + 7*d*x^3))/d^2 + (2944*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(
3*d^2))/(9*d^2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$\frac{\frac{2\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9}+10c\sqrt{d\,x^{3}+c}+\frac{128c^{2}\left(\frac{4\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{23\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{27}}{d^{4}}$
risch	$\frac{2\left(d\,x^{3}+46c\right)\sqrt{d\,x^{3}+c}}{9d^{4}}+\frac{64c^{2}\left(-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}\,d}+\frac{8c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27d}\right)}{d^{3}}$
default	$\frac{d\left(\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d}-\frac{4c\sqrt{d\,x^{3}+c}}{9d^{2}}\right)+\frac{32c\sqrt{d\,x^{3}+c}}{3d}}{d^{3}}-\frac{128c^{\frac{3}{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3d^{4}}+\frac{512c^{3}\left(\frac{\sqrt{d\,x^{3}+c}}{c\left(-d\,x^{3}+8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right.}{27d^{4}}$
elliptic	$\frac{512c^{2}\sqrt{d\,x^{3}+c}}{27d^{4}\left(-d\,x^{3}+8c\right)}+\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d^{3}}+\frac{92c\sqrt{d\,x^{3}+c}}{9d^{4}}+\frac{1472ic\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-\right.}{}\right.}{\left(-\right.}}}{}\right.}{}$

input
```
int(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1/9*(d*x^3+c)^(3/2)+5*c*(d*x^3+c)^(1/2)+64/27*c^2*(4*(d*x^3+c)^(1/2)/(-
d*x^3+8*c)-23/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)))/d^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.88

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \left[\frac{2\left(736\,(cdx^3-8\,c^2)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\,(3\,d^2x^6+114\,cdx^3-1360\,c^2)\sqrt{dx^3+c}\right)}{81\,(d^5x^3-8\,cd^4)}, \frac{2\left(1472\right.}{}\right.$$

input
```
integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[2/81*(736*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c
) + 10*c)/(d*x^3 - 8*c)) + 3*(3*d^2*x^6 + 114*c*d*x^3 - 1360*c^2)*sqrt(d*x
^3 + c))/(d^5*x^3 - 8*c*d^4), 2/81*(1472*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan
(3*sqrt(-c)/sqrt(d*x^3 + c)) + 3*(3*d^2*x^6 + 114*c*d*x^3 - 1360*c^2)*sqrt
(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

Sympy [F]

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^{11}}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**11/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**11/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\left(736\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+9\,(dx^3+c)^{\frac{3}{2}}+405\sqrt{dx^3+c}c-\frac{768\sqrt{dx^3+c}c^2}{dx^3-8\,c}\right)}{81\,d^4}$$

input
```
integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
2/81*(736*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 9*(d*x^3 + c)^(3/2) + 405*sqrt(d*x^3 + c)*c - 768*sqrt(d*x^3 +
c)*c^2/(d*x^3 - 8*c))/d^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2944\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}d^4}-\frac{512\sqrt{dx^3+c}c^2}{27\,(dx^3-8\,c)d^4}+\frac{2\left((dx^3+c)^{\frac{3}{2}}d^8+45\sqrt{dx^3+c}cd^8\right)}{9\,d^{12}}$$

input
```
integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
2944/81*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 512/27*s
qrt(d*x^3 + c)*c^2/((d*x^3 - 8*c)*d^4) + 2/9*((d*x^3 + c)^(3/2)*d^8 + 45*s
qrt(d*x^3 + c)*c*d^8)/d^12
```

Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.05

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{92\,c\,\sqrt{d\,x^3+c}}{9\,d^4}+\frac{1472\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^4}+\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^3}+\frac{512\,c^2\,\sqrt{d\,x^3+c}}{27\,d^4\,\left(8\,c-d\,x^3\right)}$$

input

```
int(x^11/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output

```
(92*c*(c + d*x^3)^(1/2))/(9*d^4) + (1472*c^(3/2)*log((10*c + d*x^3 - 6*c^(
1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*d^4) + (2*x^3*(c + d*x^3)^(1/2
))/(9*d^3) + (512*c^2*(c + d*x^3)^(1/2))/(27*d^4*(8*c - d*x^3))
```

Reduce [F]

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\frac{608\sqrt{d\,x^3+c}\,c^2}{45}-\frac{76\sqrt{d\,x^3+c}\,cd\,x^3}{9}-\frac{2\sqrt{d\,x^3+c}\,d^2x^6}{9}+\frac{11776\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)c^3d^2}{5}-\frac{1472\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{d^3x^9-15c\,d^2x^6+48c^2}\right.}{5}}{d^4\left(-d\,x^3+8c\right)}$$

input

```
int(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

output

```
(2*(304*sqrt(c + d*x**3)*c**2 - 190*sqrt(c + d*x**3)*c*d*x**3 - 5*sqrt(c +
 d*x**3)*d**2*x**6 + 52992*int((sqrt(c + d*x**3)*x**5)/(64*c**3 + 48*c**2*
d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**3*d**2 - 6624*int((sqrt(c + d*x
**3)*x**5)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**2
*d**3*x**3))/(45*d**4*(8*c - d*x**3))
```

3.598 $\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 83

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\sqrt{c+dx^3}}{3d^3}+\frac{64c\sqrt{c+dx^3}}{27d^3(8c-dx^3)}-\frac{224\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

output
```
2/3*(d*x^3+c)^(1/2)/d^3+64/27*c*(d*x^3+c)^(1/2)/d^3/(-d*x^3+8*c)-224/81*c^
(1/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\left(\frac{3\sqrt{c+dx^3}(-104c+9dx^3)}{-8c+dx^3}-112\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81d^3}$$

input
```
Integrate[x^8/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(2*((3*Sqrt[c + d*x^3]*(-104*c + 9*d*x^3))/(-8*c + d*x^3) - 112*Sqrt[c]*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(81*d^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {948, 100, 27, 90, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{(8c-dx^3)^2\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{3}\left(\frac{64c\sqrt{c+dx^3}}{9d^3(8c-dx^3)}-\frac{\int \frac{cd(9dx^3+40c)}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9cd^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{64c\sqrt{c+dx^3}}{9d^3(8c-dx^3)}-\frac{\int \frac{9dx^3+40c}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9d^2}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(\frac{64c\sqrt{c+dx^3}}{9d^3(8c-dx^3)}-\frac{112c\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3-\frac{18\sqrt{c+dx^3}}{d}}{9d^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{64c\sqrt{c+dx^3}}{9d^3(8c-dx^3)}-\frac{\frac{224c\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{18\sqrt{c+dx^3}}{d}}{9d^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{64c\sqrt{c+dx^3}}{9d^3(8c-dx^3)}-\frac{\frac{224\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3d}-\frac{18\sqrt{c+dx^3}}{d}}{9d^2}\right)$$

input
```
Int[x^8/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((64*c*Sqrt[c + d*x^3])/(9*d^3*(8*c - d*x^3)) - ((-18*Sqrt[c + d*x^3])/d +
 (224*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d))/(9*d^2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}+\frac{32c\left(\frac{2\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{7\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{27}}{d^{3}}$
default	$\frac{2\sqrt{d\,x^{3}+c}}{3d^{3}}-\frac{32\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d^{3}}+\frac{64c^{2}\left(\frac{\sqrt{d\,x^{3}+c}}{c\left(-d\,x^{3}+8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27d^{3}}$
risch	$\frac{2\sqrt{d\,x^{3}+c}}{3d^{3}}+\frac{16c\left(-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}+\frac{4c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27d}\right)}{d^{2}}$
elliptic	$\frac{64c\sqrt{d\,x^{3}+c}}{27d^{3}\left(-d\,x^{3}+8c\right)}+\frac{2\sqrt{d\,x^{3}+c}}{3d^{3}}+\frac{112i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\cdots}}{\cdots}\right)}{\cdots}$

input
```
int(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*((d*x^3+c)^(1/2)+16/9*c*(2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-7/3*arctanh(1/
3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)))/d^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.98

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \left[\frac{2\left(56\,(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\,(9\,dx^3-104\,c)\sqrt{dx^3+c}\right)}{81\,(d^4x^3-8\,cd^3)}, \frac{2\left(112\,(dx^3-8\,c)\sqrt{-c}\,\mathrm{arc}\right.}{}\right.$$

input
```
integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[2/81*(56*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 1
0*c)/(d*x^3 - 8*c)) + 3*(9*d*x^3 - 104*c)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*
d^3), 2/81*(112*(d*x^3 - 8*c)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c))
+ 3*(9*d*x^3 - 104*c)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3)]
```

Sympy [F]

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^8}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**8/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\left(56\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+27\sqrt{dx^3+c}-\frac{96\sqrt{dx^3+c}c}{dx^3-8c}\right)}{81d^3}$$

input
```
integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
2/81*(56*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sq
rt(c))) + 27*sqrt(d*x^3 + c) - 96*sqrt(d*x^3 + c)*c/(d*x^3 - 8*c))/d^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\left(\frac{112c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d}+\frac{27\sqrt{dx^3+c}}{d}-\frac{96\sqrt{dx^3+c}c}{(dx^3-8c)d}\right)}{81d^2}$$

input
```
integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
2/81*(112*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + 27*sqrt(d*
x^3 + c)/d - 96*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d))/d^2
```

Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,d^3}+\frac{112\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\sqrt{c}\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^3}+\frac{64\,c\sqrt{d\,x^3+c}}{27\,d^3\,\left(8\,c-d\,x^3\right)}$$

input
```
int(x^8/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*d^3) + (112*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)
*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*d^3) + (64*c*(c + d*x^3)^(1/2))/(2
7*d^3*(8*c - d*x^3))
```

Reduce [F]

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$

$$= \frac{\frac{16\sqrt{d\,x^3+c}\,c}{15}-\frac{2\sqrt{d\,x^3+c}\,d\,x^3}{3}+\frac{896\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)c^2d^2}{5}-\frac{112\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)c\,d^3x^3}{5}}{d^3\left(-d\,x^3+8c\right)}$$

input
```
int(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

output
```
(2*(8*sqrt(c + d*x**3)*c - 5*sqrt(c + d*x**3)*d*x**3 + 1344*int((sqrt(c +
d*x**3)*x**5)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c
**2*d**2 - 168*int((sqrt(c + d*x**3)*x**5)/(64*c**3 + 48*c**2*d*x**3 - 15*
c*d**2*x**6 + d**3*x**9),x)*c*d**3*x**3))/(15*d**3*(8*c - d*x**3))
```

3.599 $\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 64

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{8\sqrt{c+dx^3}}{27d^2(8c-dx^3)} - \frac{10\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}$$

output
```
8/27*(d*x^3+c)^(1/2)/d^2/(-d*x^3+8*c)-10/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^
(1/2))/c^(1/2)/d^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{8\sqrt{c+dx^3}}{27d^2(-8c+dx^3)} - \frac{10\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}$$

input
```
Integrate[x^5/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(-8*Sqrt[c + d*x^3])/(27*d^2*(-8*c + d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]
/(3*Sqrt[c])])/(81*Sqrt[c]*d^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {948, 87, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{x^3}{(8c-dx^3)^2\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \ 87$$

$$\frac{1}{3}\left(\frac{8\sqrt{c+dx^3}}{9d^2(8c-dx^3)}-\frac{5\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9d}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{3}\left(\frac{8\sqrt{c+dx^3}}{9d^2(8c-dx^3)}-\frac{10\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{9d^2}\right)$$

$$\Big\downarrow \ 219$$

$$\frac{1}{3}\left(\frac{8\sqrt{c+dx^3}}{9d^2(8c-dx^3)}-\frac{10\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{27\sqrt{c}d^2}\right)$$

input
```
Int[x^5/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((8*Sqrt[c + d*x^3])/(9*d^2*(8*c - d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]/(
3*Sqrt[c])])/(27*Sqrt[c]*d^2))/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\frac{8\sqrt{d\,x^{3}+c}}{27\left(-d\,x^{3}+8c\right)}-\frac{10\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{81\sqrt{c}}}{d^{2}}$
default	$-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d^{2}}+\frac{8c\left(\frac{\sqrt{d\,x^{3}+c}}{c\left(-d\,x^{3}+8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27d^{2}}$
elliptic	$\frac{8\sqrt{d\,x^{3}+c}}{27d^{2}\left(-d\,x^{3}+8c\right)}+\frac{5i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right.}{d}\right.}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\ldots}}}{\ldots}\right.}{\ldots}$

input
```
int(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/27*(4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-5/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/
2))/c^(1/2))/d^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.38

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$
$$=\left[\frac{5\left(dx^3-8\,c\right)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-24\,\sqrt{dx^3+c}c}{81\left(cd^3x^3-8\,c^2d^2\right)},\frac{2\left(5\left(dx^3-8\,c\right)\sqrt{-c}\arctan\left(\frac{3\,\sqrt{-c}}{\sqrt{dx^3+c}}\right)-12\,\sqrt{\ldots}\right.}{81\left(cd^3x^3-8\,c^2d^2\right)}\right.$$

input
```
integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/81*(5*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) - 24*sqrt(d*x^3 + c)*c)/(c*d^3*x^3 - 8*c^2*d^2), 2/81*(
5*(d*x^3 - 8*c)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) - 12*sqrt(d*x^
3 + c)*c)/(c*d^3*x^3 - 8*c^2*d^2)]
```

Sympy [F]

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^5}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**5/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\frac{5\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}} - \frac{24\sqrt{dx^3+c}}{dx^3-8c}}{81\,d^2}$$

input
```
integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
1/81*(5*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/s
qrt(c) - 24*sqrt(d*x^3 + c)/(d*x^3 - 8*c))/d^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\left(\frac{5\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d} - \frac{12\sqrt{dx^3+c}}{(dx^3-8c)d}\right)}{81\,d}$$

input
```
integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
2/81*(5*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 12*sqrt(d*x^3
+ c)/((d*x^3 - 8*c)*d))/d
```

Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{5\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,\sqrt{c}\,d^2} + \frac{8\,\sqrt{d\,x^3+c}}{27\,d^2\,(8\,c-d\,x^3)}$$

input
```
int(x^5/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
(5*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*c^
(1/2)*d^2) + (8*(c + d*x^3)^(1/2))/(27*d^2*(8*c - d*x^3))
```

Reduce [F]

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^5}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx$$

input
```
int(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**5)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d
**3*x**9),x)
```

3.600 $\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 67

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{c+dx^3}}{27cd\,(8c-dx^3)} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

output
```
1/27*(d*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)+1/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(
1/2))/c^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\frac{3\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3} + \operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

input
```
Integrate[x^2/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((3*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])])/(81*c^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {946, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \ 946$$

$$\frac{1}{3}\int \frac{1}{(8c-dx^3)^2\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \ 52$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{18c}+\frac{\sqrt{c+dx^3}}{9cd(8c-dx^3)}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{9cd}+\frac{\sqrt{c+dx^3}}{9cd(8c-dx^3)}\right)$$

$$\Big\downarrow \ 219$$

$$\frac{1}{3}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{27c^{3/2}d}+\frac{\sqrt{c+dx^3}}{9cd(8c-dx^3)}\right)$$

input
```
Int[x^2/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(Sqrt[c + d*x^3]/(9*c*d*(8*c - d*x^3)) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c
])]/(27*c^(3/2)*d))/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

method	result
default	$\frac{\frac{\sqrt{d\,x^{3}+c}}{c\left(-d\,x^{3}+8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{27d}$
pseudoelliptic	$\frac{\frac{\sqrt{d\,x^{3}+c}}{c\left(-d\,x^{3}+8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{27d}$
elliptic	$\frac{\sqrt{d\,x^{3}+c}}{27cd(-d\,x^{3}+8c)}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)}{d}\right.}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}}}}{}\right.}{}$

input
```
int(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/27*((d*x^3+c)^(1/2)/c/(-d*x^3+8*c)+1/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/
2))/c^(3/2))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.24

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \left[\frac{(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,\sqrt{dx^3+c}c}{162\,(c^2d^2x^3-8\,c^3d)},\; -\frac{(dx^3-8\,c)\sqrt{-c}\arctan\left(\frac{3\,\sqrt{-c}}{\sqrt{dx^3+c}}\right)+3\,\sqrt{dx^3+c}c}{81\,(c^2d^2x^3-8\,c^3d)}\right]$$

input
```
integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/162*((d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*
c)/(d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c^2*d^2*x^3 - 8*c^3*d), -1/81*((
d*x^3 - 8*c)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) + 3*sqrt(d*x^3 +
c)*c)/(c^2*d^2*x^3 - 8*c^3*d)]
```

Sympy [F]

$$\int \frac{x^2}{(8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{x^2}{(-8c + dx^3)^2 \sqrt{c + dx^3}} \, dx$$

input
```
integrate(x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**2/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = -\frac{\frac{6\sqrt{dx^3+c}}{(dx^3+c)c-9\,c^2} + \frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{162\,d}$$

input
```
integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/162*(6*sqrt(d*x^3 + c)/((d*x^3 + c)*c - 9*c^2) + log((sqrt(d*x^3 + c) -
 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(3/2))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}cd} - \frac{\sqrt{dx^3+c}}{27\,(dx^3-8\,c)cd}$$

input
```
integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-1/81*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) - 1/27*sqrt(d*x^
3 + c)/((d*x^3 - 8*c)*c*d)
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{162\,c^{3/2}\,d} + \frac{\sqrt{d\,x^3+c}}{27\,c\,d\,\left(8\,c-d\,x^3\right)}$$

input
```
int(x^2/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(162*c^(3/
2)*d) + (c + d*x^3)^(1/2)/(27*c*d*(8*c - d*x^3))
```

Reduce [F]

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^2}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx$$

input
```
int(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**2)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d
**3*x**9),x)
```

3.601 $\int \frac{1}{x(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 88

$$\int \frac{1}{x(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{c+dx^3}}{216c^2(8c-dx^3)} + \frac{13\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}$$

output
```
1/216*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)+13/2592*arctanh(1/3*(d*x^3+c)^(1/2)
/c^(1/2))/c^(5/2)-1/96*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\frac{12\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3} + 13\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 27\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2592c^{5/2}}$$

input
```
Integrate[1/(x*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((12*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + 13*ArcTanh[Sqrt[c + d*x^3]/(
3*Sqrt[c])] - 27*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2592*c^(5/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {948, 114, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(8c-dx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^3(8c-dx^3)^2\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(\frac{\sqrt{c+dx^3}}{72c^2(8c-dx^3)}-\frac{\int -\frac{d(dx^3+18c)}{2x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{72c^2d}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{dx^3+18c}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{144c^2}+\frac{\sqrt{c+dx^3}}{72c^2(8c-dx^3)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{\frac{9}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{13}{4}d\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{144c^2}+\frac{\sqrt{c+dx^3}}{72c^2(8c-dx^3)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\frac{13}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{9\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{144c^2}+\frac{\sqrt{c+dx^3}}{72c^2(8c-dx^3)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\frac{9\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{13\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{144c^2}+\frac{\sqrt{c+dx^3}}{72c^2\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{\frac{13\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{9\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{144c^2}+\frac{\sqrt{c+dx^3}}{72c^2\left(8c-dx^3\right)}\right)$$

input
```
Int[1/(x*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(Sqrt[c + d*x^3]/(72*c^2*(8*c - d*x^3)) + ((13*ArcTanh[Sqrt[c + d*x^3]/(3*
Sqrt[c])])/(6*Sqrt[c]) - (9*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c]))
/(144*c^2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

method	result	size
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{96 c^{\frac{5}{2}}}+\frac{\frac{13 \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)\left(d\,x^{3}-8 c\right)}{\sqrt{c}}-12 \sqrt{d\,x^{3}+c}}{2592\left(d\,x^{3}-8 c\right) c^{2}}$	78
default	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{96 c^{\frac{5}{2}}}+\frac{\frac{\sqrt{d\,x^{3}+c}}{c\left(-d\,x^{3}+8 c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{3 c^{\frac{3}{2}}}}{216 c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{288 c^{\frac{5}{2}}}$	92
elliptic	Expression too large to display	1534

input
```
int(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/2592*(13*arctanh(1/3*(d*x
^3+c)^(1/2)/c^(1/2))/c^(1/2)*(d*x^3-8*c)-12*(d*x^3+c)^(1/2))/(d*x^3-8*c)/c
^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.50

$$\int \frac{1}{x\left(8c-dx^{3}\right)^{2}\sqrt{c+dx^{3}}}\,dx = \left[\frac{13\,(dx^{3}-8\,c)\sqrt{c}\log\left(\frac{dx^{3}+6\,\sqrt{dx^{3}+c}\sqrt{c}+10\,c}{dx^{3}-8\,c}\right)+27\,(dx^{3}-8\,c)\sqrt{c}\log\left(\frac{dx^{3}-2\,\sqrt{dx^{3}+c}\sqrt{c}+2\,c}{x^{3}}\right)-24\,\sqrt{dx^{3}+c}c}{5184\,(c^{3}dx^{3}-8\,c^{4})}, -\frac{13\,(dx^{3}-8\,c)\sqrt{-c}\arctan\left(\frac{3\,\sqrt{-c}}{\sqrt{dx^{3}+c}}\right)-27\,(dx^{3}-8\,c)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{3}+c}}\right)+12\,\sqrt{dx^{3}+c}c}{2592\,(c^{3}dx^{3}-8\,c^{4})}\right]$$

input
```
integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/5184*(13*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) +
 10*c)/(d*x^3 - 8*c)) + 27*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3
 + c)*sqrt(c) + 2*c)/x^3) - 24*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 - 8*c^4), -1/
2592*(13*(d*x^3 - 8*c)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) - 27*(d
*x^3 - 8*c)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)) + 12*sqrt(d*x^3 + c)
*c)/(c^3*d*x^3 - 8*c^4)]
```

Sympy [F]

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{x\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x}\,dx$$

input
```
integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}c^2} - \frac{13\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{2592\sqrt{-c}c^2} - \frac{\sqrt{dx^3+c}}{216\,(dx^3-8\,c)c^2}$$

input
```
integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 13/2592*arctan(1/3*
sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/216*sqrt(d*x^3 + c)/((d*x^3 -
 8*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{13\,\mathrm{atanh}\left(\frac{c^2\sqrt{d\,x^3+c}}{3\sqrt{c^5}}\right)}{2592\sqrt{c^5}} - \frac{\mathrm{atanh}\left(\frac{c^2\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)}{96\sqrt{c^5}} + \frac{\sqrt{d\,x^3+c}}{72\,c^2\,\left(24\,c-3\,d\,x^3\right)}$$

input
```
int(1/(x*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
(13*atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2))))/(2592*(c^5)^(1/2)) - a
tanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2))/(96*(c^5)^(1/2)) + (c + d*x^3)^(
1/2)/(72*c^2*(24*c - 3*d*x^3))
```

Reduce [F]

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{\sqrt{d\,x^3+c}}{d^3x^{10}-15c\,d^2x^7+48c^2d\,x^4+64c^3x}dx$$

input
```
int(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**3)/(64*c**3*x + 48*c**2*d*x**4 - 15*c*d**2*x**7 + d**3*x
**10),x)
```

3.602 $\int \frac{1}{x^4(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{1}{x^4 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{5d\sqrt{c+dx^3}}{864c^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24c^2x^3(8c-dx^3)} + \frac{11d\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}} + \frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{7/2}}$$

output
```
5/864*d*(d*x^3+c)^(1/2)/c^3/(-d*x^3+8*c)-1/24*(d*x^3+c)^(1/2)/c^2/x^3/(-d*
x^3+8*c)+11/10368*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)+1/384*d*a
rctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{\frac{12\sqrt{c}(36c-5dx^3)\sqrt{c+dx^3}}{-8cx^3+dx^6} + 11d\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + 27d\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{10368c^{7/2}}$$

input
```
Integrate[1/(x^4*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((12*Sqrt[c]*(36*c - 5*d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 11*d*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] + 27*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]
])/(10368*c^(7/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 114, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (8c-dx^3)^2 \sqrt{c+dx^3}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^6 (8c-dx^3)^2 \sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int \frac{d(4c-3dx^3)}{2x^3(8c-dx^3)^2\sqrt{dx^3+c}}dx^3}{8c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3(8c-dx^3)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\int \frac{4c-3dx^3}{x^3(8c-dx^3)^2\sqrt{dx^3+c}}dx^3}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3(8c-dx^3)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{\int -\frac{2cd(18c-5dx^3)}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{72c^2d}-\frac{5\sqrt{c+dx^3}}{18c(8c-dx^3)}\right)}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3(8c-dx^3)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{\int \frac{18c-5dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{36c}-\frac{5\sqrt{c+dx^3}}{18c(8c-dx^3)}\right)}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3\left(8c-dx^3\right)}\right)$$

$\Big\downarrow$ 174

$$\frac{1}{3}\left(-\frac{d\left(\frac{\frac{9}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3-\frac{11}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{36c}-\frac{5\sqrt{c+dx^3}}{18c(8c-dx^3)}\right)}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3\left(8c-dx^3\right)}\right)$$

$\Big\downarrow$ 73

$$\frac{1}{3}\left(-\frac{d\left(\frac{\frac{9\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{11}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{36c}-\frac{5\sqrt{c+dx^3}}{18c(8c-dx^3)}\right)}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3\left(8c-dx^3\right)}\right)$$

$\Big\downarrow$ 219

$$\frac{1}{3}\left(-\frac{d\left(\frac{\frac{9\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{11\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{36c}-\frac{5\sqrt{c+dx^3}}{18c(8c-dx^3)}\right)}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3\left(8c-dx^3\right)}\right)$$

$\Big\downarrow$ 221

$$\frac{1}{3}\left(-\frac{d\left(\frac{-\frac{11\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{9\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{36c}-\frac{5\sqrt{c+dx^3}}{18c(8c-dx^3)}\right)}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3\left(8c-dx^3\right)}\right)$$

input
```
Int[1/(x^4*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(-1/8*Sqrt[c + d*x^3]/(c^2*x^3*(8*c - d*x^3)) - (d*((-5*Sqrt[c + d*x^3])/(
18*c*(8*c - d*x^3)) + ((-11*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*Sqrt[
c]) - (9*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c]))/(36*c)))/(16*c^2))
/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{d\left(-\frac{\sqrt{d\,x^{3}+c}}{d\,x^{3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\sqrt{d\,x^{3}+c}}{-9d\,x^{3}+72c}+\frac{11\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{54\sqrt{c}}\right)}{192c^{3}}$
risch	$-\frac{\sqrt{d\,x^{3}+c}}{192c^{3}x^{3}}-\frac{d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9\sqrt{c}}-\frac{2c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27}\right)}{128c^{3}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3c\,x^{3}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{64c^{2}}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{384c^{\frac{7}{2}}}+\frac{d\left(\frac{\sqrt{d\,x^{3}+c}}{c\left(-d\,x^{3}+8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{1728c^{2}}+\frac{d\operatorname{arctanh}\left(\sqrt{}\right.}{1152c}$
elliptic	Expression too large to display

input
```
int(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*d/c^3*(-(d*x^3+c)^(1/2)/d/x^3+1/2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c
^(1/2)+1/9*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+11/54*arctanh(1/3*(d*x^3+c)^(1/2)/
c^(1/2))/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.21

$$\int \frac{1}{x^4 (8c-dx^3)^2 \sqrt{c+dx^3}} \, dx$$
$$= \left[\frac{11 \left(d^2x^6 - 8\,cdx^3\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right) + 27 \left(d^2x^6 - 8\,cdx^3\right)\sqrt{c}\log\left(\frac{dx^3+2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right) - 24 \left(5\right.}{20736 \left(c^4dx^6 - 8\,c^5x^3\right)} \right.$$
$$- \frac{11 \left(d^2x^6 - 8\,cdx^3\right)\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right) + 27 \left(d^2x^6 - 8\,cdx^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right) + 12 \left(5\,cdx^3 - 36\right.}{10368 \left(c^4dx^6 - 8\,c^5x^3\right)}$$

input
```
integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/20736*(11*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*
sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 27*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x
^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(5*c*d*x^3 - 36*c^2)*sqrt(
d*x^3 + c))/(c^4*d*x^6 - 8*c^5*x^3), -1/10368*(11*(d^2*x^6 - 8*c*d*x^3)*sq
rt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) + 27*(d^2*x^6 - 8*c*d*x^3)*sqrt(
-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)) + 12*(5*c*d*x^3 - 36*c^2)*sqrt(d*x^3
+ c))/(c^4*d*x^6 - 8*c^5*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (8c - dx^3)^2 \sqrt{c + dx^3}}\, dx = \int \frac{1}{x^4 (-8c + dx^3)^2 \sqrt{c + dx^3}}\, dx$$

input
```
integrate(1/x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (8c - dx^3)^2 \sqrt{c + dx^3}}\, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^4}\, dx$$

input
```
integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (8c - dx^3)^2 \sqrt{c + dx^3}}\, dx = -\frac{d \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384 \sqrt{-c}c^3} - \frac{11\, d \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{10368 \sqrt{-c}c^3} - \frac{5\,(dx^3 + c)^{\frac{3}{2}} d - 41\sqrt{dx^3 + c}cd}{864\left((dx^3 + c)^2 - 10\,(dx^3 + c)c + 9\,c^2\right)c^3}$$

input
```
integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-1/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 11/10368*d*arct
an(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/864*(5*(d*x^3 + c)^(3/
2)*d - 41*sqrt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)
*c^3)
```

Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{\frac{41\,d\,\sqrt{d\,x^3+c}}{288\,c^2}-\frac{5\,d\,\left(d\,x^3+c\right)^{3/2}}{288\,c^3}}{3\,\left(d\,x^3+c\right)^2-30\,c\,\left(d\,x^3+c\right)+27\,c^2} - \frac{d\,\left(\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)\,1\mathrm{i}+\frac{\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)\,11\mathrm{i}}{27}\right)\,1\mathrm{i}}{384\,\sqrt{c^7}}$$

input
```
int(1/(x^4*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
((41*d*(c + d*x^3)^(1/2))/(288*c^2) - (5*d*(c + d*x^3)^(3/2))/(288*c^3))/(
3*(c + d*x^3)^2 - 30*c*(c + d*x^3) + 27*c^2) - (d*(atanh((c^3*(c + d*x^3)^
(1/2))/(c^7)^(1/2))*1i + (atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2)))*1
1i)/27)*1i)/(384*(c^7)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^4 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{-2\sqrt{d\,x^3+c}-96\left(\int \frac{\sqrt{d\,x^3+c}}{d^3x^{10}-15c\,d^2x^7+48c^2d\,x^4+64c^3x}dx\right)c^2d\,x^3+12\left(\int \frac{\sqrt{d\,x^3+c}}{d^3x^{10}-15c\,d^2x^7+48c^2d\,x^4+64c^3x}dx\right)c\,d^2x^6 +}{48c^2x^3\left(-d\,x^3+8c\right)}$$

input
```
int(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

output

```
( - 2*sqrt(c + d*x**3) - 96*int(sqrt(c + d*x**3)/(64*c**3*x + 48*c**2*d*x*
*4 - 15*c*d**2*x**7 + d**3*x**10),x)*c**2*d*x**3 + 12*int(sqrt(c + d*x**3)
/(64*c**3*x + 48*c**2*d*x**4 - 15*c*d**2*x**7 + d**3*x**10),x)*c*d**2*x**6
 + 72*int((sqrt(c + d*x**3)*x**2)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x*
*6 + d**3*x**9),x)*c*d**2*x**3 - 9*int((sqrt(c + d*x**3)*x**2)/(64*c**3 +
48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*d**3*x**6)/(48*c**2*x**3*(
8*c - d*x**3))
```

3.603 $\int \frac{1}{x^7(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 164

$$\int \frac{1}{x^7(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{35d^2\sqrt{c+dx^3}}{13824c^4(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48c^2x^6(8c-dx^3)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3(8c-dx^3)} + \frac{31d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{165888c^{9/2}} - \frac{19d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{9/2}}$$

output
```
-35/13824*d^2*(d*x^3+c)^(1/2)/c^4/(-d*x^3+8*c)-1/48*(d*x^3+c)^(1/2)/c^2/x^
6/(-d*x^3+8*c)+3/128*d*(d*x^3+c)^(1/2)/c^3/x^3/(-d*x^3+8*c)+31/165888*d^2*
arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)-19/6144*d^2*arctanh((d*x^3+c)
^(1/2)/c^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^7 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{\frac{12\sqrt{c}\sqrt{c+dx^3}(288c^2-324cdx^3+35d^2x^6)}{-8cx^6+dx^9}+31d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-513d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{165888c^{9/2}}$$

input
```
Integrate[1/(x^7*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((12*Sqrt[c]*Sqrt[c + d*x^3]*(288*c^2 - 324*c*d*x^3 + 35*d^2*x^6))/(-8*c*x
^6 + d*x^9) + 31*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 513*d^2*ArcTan
h[Sqrt[c + d*x^3]/Sqrt[c]])/(165888*c^(9/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {948, 114, 27, 168, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^9 (8c-dx^3)^2 \sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int \frac{d(18c-5dx^3)}{2x^6(8c-dx^3)^2\sqrt{dx^3+c}}dx^3}{16c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6(8c-dx^3)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\int \frac{18c-5dx^3}{x^6(8c-dx^3)^2\sqrt{dx^3+c}}dx^3}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6(8c-dx^3)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{\int \frac{cd\left(76c-27dx^3\right)}{x^3\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{8c^2}-\frac{9\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6(8c-dx^3)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\int \frac{76c-27dx^3}{x^3\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{8c}-\frac{9\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6(8c-dx^3)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(-\frac{\int -\frac{2cd\left(342c-35dx^3\right)}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{72c^2d}-\frac{35\sqrt{c+dx^3}}{18c\left(8c-dx^3\right)}\right)}{8c}-\frac{9\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6(8c-dx^3)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\int \frac{342c-35dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{36c}-\frac{35\sqrt{c+dx^3}}{18c\left(8c-dx^3\right)}\right)}{8c}-\frac{9\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{171}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{31}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{36c}-\frac{35\sqrt{c+dx^3}}{18c\left(8c-dx^3\right)}\right)}{8c}-\frac{9\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{31}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{171\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{36c}-\frac{35\sqrt{c+dx^3}}{18c\left(8c-dx^3\right)}\right)}{8c}-\frac{9\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{171\int\frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{31\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{36c}-\frac{35\sqrt{c+dx^3}}{18c\left(8c-dx^3\right)}\right)}{8c}-\frac{9\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6\left(8c-dx^3\right)}\right)$$

$\downarrow$ 221

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{31\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{171\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{36c}-\frac{35\sqrt{c+dx^3}}{18c\left(8c-dx^3\right)}\right)}{8c}-\frac{9\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6\left(8c-dx^3\right)}\right)$$

input
```
Int[1/(x^7*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(-1/16*Sqrt[c + d*x^3]/(c^2*x^6*(8*c - d*x^3)) - (d*((-9*Sqrt[c + d*x^3])/
(4*c*x^3*(8*c - d*x^3)) - (d*((-35*Sqrt[c + d*x^3])/(18*c*(8*c - d*x^3)) +
 ((31*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*Sqrt[c]) - (171*ArcTanh[Sqr
t[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c]))/(36*c)))/(8*c)))/(32*c^2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.59

method	result
pseudoelliptic	$\frac{d^{2}\left(-\frac{\sqrt{d\,x^{3}+c}\,\left(-d\,x^{3}+c\right)}{d^{2} x^{6}}-\frac{19\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{16\sqrt{c}}+\frac{\sqrt{d\,x^{3}+c}}{-36 d\,x^{3}+288 c}+\frac{31\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{432\sqrt{c}}\right)}{384 c^{4}}$
risch	$-\frac{\sqrt{d\,x^{3}+c}\,\left(-d\,x^{3}+c\right)}{384 c^{4} x^{6}}+\frac{d^{2}\left(-\frac{19\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{24\sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{24\sqrt{c}}+\frac{c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8 c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3 c^{\frac{3}{2}}}\right)}{54}\right)}{256 c^{4}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{6 c\,x^{6}}+\frac{d\sqrt{d\,x^{3}+c}}{4 c^{2} x^{3}}-\frac{d^{2} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{4 c^{\frac{5}{2}}}}{64 c^{2}}+\frac{d\left(-\frac{\sqrt{d\,x^{3}+c}}{3 c\,x^{3}}+\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3 c^{\frac{3}{2}}}\right)}{256 c^{3}}-\frac{d^{2} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{2048 c^{\frac{9}{2}}}+$
elliptic	Expression too large to display

input
```
int(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/384*d^2/c^4*(-(d*x^3+c)^(1/2)*(-d*x^3+c)/d^2/x^6-19/16*arctanh((d*x^3+c)
^(1/2)/c^(1/2))/c^(1/2)+1/36*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+31/432*arctanh(1
/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.85

$$\int \frac{1}{x^7 \left(8c - dx^3\right)^2 \sqrt{c + dx^3}}\, dx$$
$$= \left[\frac{31\,(d^3x^9 - 8\,cd^2x^6)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right) + 513\,(d^3x^9 - 8\,cd^2x^6)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right) + 24}{331776\,(c^5dx^9 - 8\,c^6x^6)} \right.$$
$$- \frac{31\,(d^3x^9 - 8\,cd^2x^6)\sqrt{-c}\arctan\left(\frac{3\,\sqrt{-c}}{\sqrt{dx^3+c}}\right) - 513\,(d^3x^9 - 8\,cd^2x^6)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right) - 12\,(35\,cd^2x^6}{165888\,(c^5dx^9 - 8\,c^6x^6)}$$

input
```
integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/331776*(31*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 +
c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 513*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*lo
g((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(35*c*d^2*x^6 - 324*
c^2*d*x^3 + 288*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 - 8*c^6*x^6), -1/165888*(
31*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) - 5
13*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)) - 12*
(35*c*d^2*x^6 - 324*c^2*d*x^3 + 288*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 - 8*c
^6*x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7 \left(8c - dx^3\right)^2 \sqrt{c + dx^3}}\, dx = \int \frac{1}{x^7 \left(-8c + dx^3\right)^2 \sqrt{c + dx^3}}\, dx$$

input
```
integrate(1/x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**7*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^7 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^7} \, dx$$

input
```
integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^7), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^7 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \frac{19\, d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6144\, \sqrt{-c}c^4} - \frac{31\, d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{165888\, \sqrt{-c}c^4} - \frac{\sqrt{dx^3 + c}d^2}{13824\, (dx^3 - 8\,c)c^4} + \frac{(dx^3 + c)^{\frac{3}{2}} d^2 - 2\, \sqrt{dx^3 + c}cd^2}{384\, c^4 d^2 x^6}$$

input
```
integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
19/6144*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 31/165888*d^
2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/13824*sqrt(d*x^3
 + c)*d^2/((d*x^3 - 8*c)*c^4) + 1/384*((d*x^3 + c)^(3/2)*d^2 - 2*sqrt(d*x^
3 + c)*c*d^2)/(c^4*d^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^7 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = -\frac{\frac{647\,d^2\,\sqrt{d\,x^3+c}}{4608\,c^2}-\frac{197\,d^2\,{\left(d\,x^3+c\right)}^{3/2}}{2304\,c^3}+\frac{35\,d^2\,{\left(d\,x^3+c\right)}^{5/2}}{4608\,c^4}}{33\,c\,{\left(d\,x^3+c\right)}^2-57\,c^2\,\left(d\,x^3+c\right)-3\,{\left(d\,x^3+c\right)}^3+27\,c^3} + \frac{d^2\,\left(\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{\sqrt{c^9}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^9}}\right)\,31{}\mathrm{i}}{513}\right)\,19{}\mathrm{i}}{6144\,\sqrt{c^9}}$$

input
```
int(1/(x^7*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
(d^2*(atanh((c^4*(c + d*x^3)^(1/2))/(c^9)^(1/2))*1i - (atanh((c^4*(c + d*x
^3)^(1/2))/(3*(c^9)^(1/2)))*31i)/513)*19i)/(6144*(c^9)^(1/2)) - ((647*d^2*
(c + d*x^3)^(1/2))/(4608*c^2) - (197*d^2*(c + d*x^3)^(3/2))/(2304*c^3) + (
35*d^2*(c + d*x^3)^(5/2))/(4608*c^4))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*
x^3) - 3*(c + d*x^3)^3 + 27*c^3)
```

Reduce [F]

$$\int \frac{1}{x^7 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{-16\sqrt{d\,x^3+c}\,c+18\sqrt{d\,x^3+c}\,d\,x^3+1824\left(\int \frac{\sqrt{d\,x^3+c}}{d^3x^{10}-15c\,d^2x^7+48c^2d\,x^4+64c^3x}dx\right)c^2d^2x^6-228\left(\int \frac{\sqrt{\cdots}}{d^3x^{10}-15c\,d^2x\cdots}\right.}{768c^3x^6\cdots}$$

input
```
int(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

output

```
( - 16*sqrt(c + d*x**3)*c + 18*sqrt(c + d*x**3)*d*x**3 + 1824*int(sqrt(c +
 d*x**3)/(64*c**3*x + 48*c**2*d*x**4 - 15*c*d**2*x**7 + d**3*x**10),x)*c**
2*d**2*x**6 - 228*int(sqrt(c + d*x**3)/(64*c**3*x + 48*c**2*d*x**4 - 15*c*
d**2*x**7 + d**3*x**10),x)*c*d**3*x**9 - 648*int((sqrt(c + d*x**3)*x**2)/(
64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c*d**3*x**6 + 81
*int((sqrt(c + d*x**3)*x**2)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 +
d**3*x**9),x)*d**4*x**9)/(768*c**3*x**6*(8*c - d*x**3))
```

3.604 $\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 641

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$

$$= \frac{62\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}+\frac{44\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}d^{8/3}}$$
$$-\frac{44\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81d^{8/3}}+\frac{44\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^{8/3}}$$
$$-\frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{62\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
62/27*(d*x^3+c)^(1/2)/d^(8/3)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)+8/27*x^2*(d*
x^3+c)^(1/2)/d^2/(-d*x^3+8*c)+44/81*c^(1/6)*arctan(3^(1/2)*c^(1/6)*(c^(1/3
)+d^(1/3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/d^(8/3)-44/81*c^(1/6)*arctanh(1/3*(c
^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(8/3)+44/81*c^(1/6)*arctanh
(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)-31/27*(1/2*6^(1/2)-1/2*2^(1/2))*c^(1
/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/
2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/
((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/d^(8/3)/(c^(1/3)*(c
^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
+62/81*2^(1/2)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(
2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*
c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/
d^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1
/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.26

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{320cx^2(c+dx^3)+40cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+31dx^5(-8c+dx^3)\sqrt{1}}{1080cd^2\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

input
```
Integrate[x^7/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(320*c*x^2*(c + d*x^3) + 40*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Appel
lF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 31*d*x^5*(-8*c + d*x^
3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8
*c)])/(1080*c*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {970, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{970}$$

$$\frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)}-\frac{\int \frac{cx(31dx^3+16c)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{27cd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)}-\frac{\int \frac{x(31dx^3+16c)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{27d^2}$$

$$\Big\downarrow \text{1054}$$

$$\frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)}-\frac{\int \left(\frac{264cx}{(8c-dx^3)\sqrt{dx^3+c}}-\frac{31x}{\sqrt{dx^3+c}}\right)dx}{27d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)}-$$

$$-\frac{62\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{31\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\right.}{}$$

input
```
Int[x^7/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(8*x^2*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) - ((-62*Sqrt[c + d*x^3])/(d
^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (44*c^(1/6)*ArcTan[(Sqrt[3]*
c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) + (44*c
^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3*d^
(2/3)) - (44*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^(2/3)) + (
31*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) -
 c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*E
llipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)
*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (62*Sqrt[2]
*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3
])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[
3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/(27*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.84 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	877
default	Expression too large to display	1738

input
```
int(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
8/27*x^2*(d*x^3+c)^(1/2)/d^2/(-d*x^3+8*c)-62/81*I/d^3*3^(1/2)*(-c*d^2)^(1/
3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+88/243*I/d^5*2^(1/2)*sum(1/_alpha*(-c*d^2)
^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d
^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^
(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-
c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2
)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2397 vs. $2(452) = 904$.

Time = 3.40 (sec) , antiderivative size = 2397, normalized size of antiderivative = 3.74

$$\int \frac{x^7}{(8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \text{Too large to display}$$

input
```
integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/243*(72*sqrt(d*x^3 + c)*d*x^2 + 558*(d*x^3 - 8*c)*sqrt(d)*weierstrassZe
ta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 11*(d^4*x^3 - 8*c*d^3 -
 sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(c/d^16)^(1/6)*log(164916224/3*((d^16*x^9 +
 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 +
318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c/d^16)^(5/6) + 6*(2*
c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^12*x^5 + 32*c^2*d^11*x^2
- sqrt(-3)*(5*c*d^12*x^5 + 32*c^2*d^11*x^2))*(c/d^16)^(2/3) - (7*c*d^7*x^6
 + 152*c^2*d^6*x^3 + 64*c^3*d^5 + sqrt(-3)*(7*c*d^7*x^6 + 152*c^2*d^6*x^3
+ 64*c^3*d^5))*(c/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^10*x^7 + 64*c^2
*d^9*x^4 + 32*c^3*d^8*x)*sqrt(c/d^16) + 18*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 6
4*c^3*d^3*x^2 - sqrt(-3)*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*x^2))*(c
/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 11*(d^
4*x^3 - 8*c*d^3 - sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(c/d^16)^(1/6)*log(-164916
224/3*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqr
t(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c/d
^16)^(5/6) - 6*(2*c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^12*x^5
+ 32*c^2*d^11*x^2 - sqrt(-3)*(5*c*d^12*x^5 + 32*c^2*d^11*x^2))*(c/d^16)^(2
/3) - (7*c*d^7*x^6 + 152*c^2*d^6*x^3 + 64*c^3*d^5 + sqrt(-3)*(7*c*d^7*x^6
+ 152*c^2*d^6*x^3 + 64*c^3*d^5))*(c/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c
*d^10*x^7 + 64*c^2*d^9*x^4 + 32*c^3*d^8*x)*sqrt(c/d^16) + 18*(c*d^5*x^8...
```

Sympy [F]

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^7}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**7/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^7}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^7}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^7}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^7}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx$$

input
```
int(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**7)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d
**3*x**9),x)
```

3.605 $\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 647

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}$$
$$+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}}-\frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{5/6}d^{5/3}}$$
$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{18\,3^{3/4}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
1/27*(d*x^3+c)^(1/2)/c/d^(5/3)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)+1/27*x^2*(d
*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)+1/81*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3
)*x)/(d*x^3+c)^(1/2))*3^(1/2)/c^(5/6)/d^(5/3)-1/81*arctanh(1/3*(c^(1/3)+d^
(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d^(5/3)+1/81*arctanh(1/3*(d*x^
3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(5/3)-1/54*(1/2*6^(1/2)-1/2*2^(1/2))*(c^(1/3
)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)
+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2)
)*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/c^(2/3)/d^(5/3)/(c^(1/3)*(c^(1
/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+1/
81*2^(1/2)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((
1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1
/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/c^(2/3)/d^(5
/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/
(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.26

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{80cx^2(c+dx^3)+10cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^5(-8c+dx^3)\sqrt{1+}}{2160c^2d\,(8c-dx^3)\sqrt{c+dx^3}}$$

input
```
Integrate[x^4/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(80*c*x^2*(c + d*x^3) + 10*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Appell
F1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]
)/(2160*c^2*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {971, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{ 971}$$

$$\frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}-\frac{\int \frac{x(dx^3+4c)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{27cd}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}-\frac{\int \frac{x(dx^3+4c)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{54cd}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}-\frac{\int \left(\frac{12cx}{(8c-dx^3)\sqrt{dx^3+c}}-\frac{x}{\sqrt{dx^3+c}}\right)dx}{54cd}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}-$$

$$-\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{}$$

input
```
Int[x^4/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x^2*Sqrt[c + d*x^3])/(27*c*d*(8*c - d*x^3)) - ((-2*Sqrt[c + d*x^3])/(d^(2
/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (2*c^(1/6)*ArcTan[(Sqrt[3]*c^(1
/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) + (2*c^(1/6
)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3*d^(2/3)
) - (2*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^(2/3)) + (3^(1/4
)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (2*Sqrt[2]*c^(1/3)*(
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3)
 + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1
/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/(54*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.59 (sec) , antiderivative size = 886, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	886
default	Expression too large to display	1305

input
```
int(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/27*x^2*(d*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)-1/81*I/c/d^2*3^(1/2)*(-c*d^2)^(1
/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+2/243*I/d^4/c*2^(1/2)*sum(1/_alpha*(-c*d^
2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c
*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*
(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*
3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-
(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d
^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2538 vs. $2(458) = 916$.

Time = 0.39 (sec) , antiderivative size = 2538, normalized size of antiderivative = 3.92

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/972*(36*sqrt(d*x^3 + c)*d*x^2 + 36*(d*x^3 - 8*c)*sqrt(d)*weierstrassZet
a(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + (c*d^3*x^3 - 8*c^2*d^2 +
 sqrt(-3)*(c*d^3*x^3 - 8*c^2*d^2))*(1/(c^5*d^10))^(1/6)*log((d^3*x^9 + 318
*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4
+ 32*c^6*d^7*x + sqrt(-3)*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x))
*(1/(c^5*d^10))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*
x^2 - sqrt(-3)*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2))*(1/(c^5*d^10))^(5/6) - 2
*(7*c^3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqrt(1/(c^5*d^10)) + (c*d^
4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x + sqrt(-3)*(c*d^4*x^7 + 80*c^2*d^3*
x^4 + 160*c^3*d^2*x))*(1/(c^5*d^10))^(1/6)) - 9*(c^2*d^6*x^8 + 38*c^3*d^5*
x^5 + 64*c^4*d^4*x^2 - sqrt(-3)*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4
*x^2))*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512
*c^3)) - (c*d^3*x^3 - 8*c^2*d^2 + sqrt(-3)*(c*d^3*x^3 - 8*c^2*d^2))*(1/(c^
5*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9
*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x + sqrt(-3)*(5*c^4*d^9*x^7
+ 64*c^5*d^8*x^4 + 32*c^6*d^7*x))*(1/(c^5*d^10))^(2/3) - 3*sqrt(d*x^3 + c)
*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2 - sqrt(-3)*(5*c^5*d^10*x^5 + 32*c^6*d
^9*x^2))*(1/(c^5*d^10))^(5/6) - 2*(7*c^3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^
5*d^5)*sqrt(1/(c^5*d^10)) + (c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x +
sqrt(-3)*(c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x))*(1/(c^5*d^10))^(...
```

Sympy [F]

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**4/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^4}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx$$

input
```
int(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**4)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d
**3*x**9),x)
```

3.606 $\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 644

$$\begin{aligned}\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)}\\
&-\frac{7\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}c^{11/6}d^{2/3}}+\frac{7\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1296c^{11/6}d^{2/3}}-\frac{7\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1296c^{11/6}d^{2/3}}\\
&-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{144\ 3^{3/4}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\\
&+\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\end{aligned}$$

output
```
1/216*(d*x^3+c)^(1/2)/c^2/d^(2/3)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)+1/216*x^
2*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)-7/1296*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+
d^(1/3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/c^(11/6)/d^(2/3)+7/1296*arctanh(1/3*(c
^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)/d^(2/3)-7/1296*arcta
nh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)/d^(2/3)-1/432*(1/2*6^(1/2)-1/2*2^
(1/2))*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^
(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*
x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/c^(5/3)/d^(2/3)/
(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x
^3+c)^(1/2)+1/648*2^(1/2)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+
d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2
))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(3/
4)/c^(5/3)/d^(2/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/
3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.25

$$\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{80cx^2(c+dx^3)+125cx^2(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^5(-8c+dx^3)\sqrt{1+\cdots}}{17280c^3\,(8c-dx^3)\sqrt{c+dx^3}}$$

input
```
Integrate[x/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(80*c*x^2*(c + d*x^3) + 125*c*x^2*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*Appell
F1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]
)/(17280*c^3*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {972, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \ 972$$

$$\frac{\int \frac{dx(50c-dx^3)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{216c^2d}+\frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{x(50c-dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{432c^2}+\frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)}$$

$$\Big\downarrow \ 1054$$

$$\frac{\int \left(\frac{42cx}{(8c-dx^3)\sqrt{dx^3+c}}+\frac{x}{\sqrt{dx^3+c}}\right)dx}{432c^2}+\frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\cdots}}{d^{2}\cdots}\cdots}{\cdots}$$

$$\frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)}$$

input
```
Int[x/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x^2*Sqrt[c + d*x^3])/(216*c^2*(8*c - d*x^3)) + ((2*Sqrt[c + d*x^3])/(d^(2
/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (7*c^(1/6)*ArcTan[(Sqrt[3]*c^(1
/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) + (7*c^(1/6
)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3*d^(2/3)
) - (7*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^(2/3)) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3)
 + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1
/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/(432*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.26 (sec) , antiderivative size = 883, normalized size of antiderivative = 1.37

method	result	size
default	Expression too large to display	883
elliptic	Expression too large to display	883

input
```
int(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/216*x^2*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)-1/648*I/c^2*3^(1/2)/d*(-c*d^2)^
(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-7/1944*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(
-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
)/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_a
lpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alp
ha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*
(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2540 vs. $2(455) = 910$.

Time = 0.48 (sec) , antiderivative size = 2540, normalized size of antiderivative = 3.94

$$\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/15552*(72*sqrt(d*x^3 + c)*d*x^2 + 72*(d*x^3 - 8*c)*sqrt(d)*weierstrassZ
eta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 7*(c^2*d^2*x^3 - 8*c^3
*d + sqrt(-3)*(c^2*d^2*x^3 - 8*c^3*d))*(1/(c^11*d^4))^(1/6)*log((d^3*x^9 +
 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^8*d^5*x^7 + 64*c^9*d^4*
x^4 + 32*c^10*d^3*x + sqrt(-3)*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d
^3*x))*(1/(c^11*d^4))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d^5*x^5 + 32*c^
11*d^4*x^2 - sqrt(-3)*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2))*(1/(c^11*d^4))^(
5/6) - 2*(7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(1/(c^11*d^4))
 + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 + 8
0*c^3*d^2*x^4 + 160*c^4*d*x))*(1/(c^11*d^4))^(1/6)) - 9*(c^4*d^4*x^8 + 38*
c^5*d^3*x^5 + 64*c^6*d^2*x^2 - sqrt(-3)*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64
*c^6*d^2*x^2))*(1/(c^11*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x
^3 - 512*c^3)) + 7*(c^2*d^2*x^3 - 8*c^3*d + sqrt(-3)*(c^2*d^2*x^3 - 8*c^3*
d))*(1/(c^11*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 6
40*c^3 - 9*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x + sqrt(-3)*(5*c
^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x))*(1/(c^11*d^4))^(2/3) - 3*sqr
t(d*x^3 + c)*(6*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2 - sqrt(-3)*(5*c^10*d^5*x
^5 + 32*c^11*d^4*x^2))*(1/(c^11*d^4))^(5/6) - 2*(7*c^6*d^4*x^6 + 152*c^7*d
^3*x^3 + 64*c^8*d^2)*sqrt(1/(c^11*d^4)) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 +
160*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x))*(1...
```

Sympy [F]

$$\int \frac{x}{(8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{x}{(-8c + dx^3)^2 \sqrt{c + dx^3}} \, dx$$

input
```
integrate(x/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{(8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{x}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2} \, dx$$

input
```
integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x}{(8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{x}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2} \, dx$$

input
```
integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(x/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx$$

input
```
int(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3
*x**9),x)
```

3.607 $\int \frac{1}{x^2 (8c-dx^3)^2 \sqrt{c+dx^3}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 665

$$\int \frac{1}{x^2(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{7\sqrt{c+dx^3}}{432c^3x}+\frac{7\sqrt[3]{d}\sqrt{c+dx^3}}{432c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{216c^2x(8c-dx^3)}-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{216\sqrt{3}c^{17/6}}+\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{648c^{17/6}}-\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{648c^{17/6}}-\frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{288\,3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{7\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-7/432*(d*x^3+c)^(1/2)/c^3/x+7/432*d^(1/3)*(d*x^3+c)^(1/2)/c^3/((1+3^(1/2)
)*c^(1/3)+d^(1/3)*x)+1/216*(d*x^3+c)^(1/2)/c^2/x/(-d*x^3+8*c)-1/648*d^(1/3
)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/c^(1
7/6)+1/648*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/
2))/c^(17/6)-1/648*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-7
/864*(1/2*6^(1/2)-1/2*2^(1/2))*d^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/
3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*Ellipti
cE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/
2)+2*I)*3^(1/4)/c^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+
d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+7/1296*d^(1/3)*(c^(1/3)+d^(1/3)*x)*((c
^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(
1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3
)*x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/c^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(
(1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.17 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.27

$$\int \frac{1}{x^2 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx$$
$$= \frac{-80c(54c^2+47cdx^3-7d^2x^6)+200cdx^3(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+7d^2x^6(-}{34560c^4\sqrt{c+dx^3}\,(8cx-dx^4)}$$

input
```
Integrate[1/(x^2*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(-80*c*(54*c^2 + 47*c*d*x^3 - 7*d^2*x^6) + 200*c*d*x^3*(8*c - d*x^3)*Sqrt[
1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 7
*d^2*x^6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((
d*x^3)/c), (d*x^3)/(8*c)])/(34560*c^4*Sqrt[c + d*x^3]*(8*c*x - d*x^4))
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {972, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (8c-dx^3)^2 \sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{\int \frac{d(5dx^3+56c)}{2x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{216c^2d}+\frac{\sqrt{c+dx^3}}{216c^2x(8c-dx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5dx^3+56c}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x(8c-dx^3)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int -\frac{4cdx(80c-7dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{8c^2}-\frac{7\sqrt{c+dx^3}}{cx}}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x(8c-dx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{d\int \frac{x(80c-7dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{2c}-\frac{7\sqrt{c+dx^3}}{cx}}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x(8c-dx^3)}$$

$$\Big\downarrow \text{1054}$$

$$\frac{\frac{d\int \left(\frac{24cx}{(8c-dx^3)\sqrt{dx^3+c}}+\frac{7x}{\sqrt{dx^3+c}}\right)dx}{2c}-\frac{7\sqrt{c+dx^3}}{cx}}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x(8c-dx^3)}$$

$$\Big\downarrow \text{2009}$$

$$d\left(\frac{14\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{7\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\cdots}{\left(\left(1+\cdots\right.\right.}}}{d^{2/3}\sqrt{\cdots}}\right.$$

$$\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}$$

input
```
Int[1/(x^2*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
Sqrt[c + d*x^3]/(216*c^2*x*(8*c - d*x^3)) + ((-7*Sqrt[c + d*x^3])/(c*x) +
(d*((14*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (
4*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])
/(Sqrt[3]*d^(2/3)) + (4*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)
*Sqrt[c + d*x^3])])/(3*d^(2/3)) - (4*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])])/(3*d^(2/3)) - (7*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(
1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)
/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(
1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c
+ d*x^3]) + (14*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1
/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1
/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(2*c))/(
432*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.11 (sec) , antiderivative size = 898, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	898
risch	Expression too large to display	1758
default	Expression too large to display	1762

input
```
int(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1728/c^3*x^2*d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/64*(d*x^3+c)^(1/2)/c^3/x-7
/1296*I/c^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3)
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/972*I/c^
3/d^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/
3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d
*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alp
ha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2391 vs. $2(472) = 944$.

Time = 0.60 (sec) , antiderivative size = 2391, normalized size of antiderivative = 3.60

$$\int \frac{1}{x^2 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/7776*(126*(d*x^4 - 8*c*x)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstras
sPInverse(0, -4*c/d, x)) - (c^3*d*x^4 - 8*c^4*x + sqrt(-3)*(c^3*d*x^4 - 8*
c^4*x))*(d^2/c^17)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 +
 640*c^3*d - 9*(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x + sqrt(-3)*(5*c
^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x))*(d^2/c^17)^(2/3) + 3*sqrt(d*x^3
+ c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2
))*(d^2/c^17)^(5/6) - 2*(7*c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sqrt(d^
2/c^17) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x + sqrt(-3)*(c^3*d^3*
x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x))*(d^2/c^17)^(1/6)) - 9*(c^6*d^3*x^8 +
38*c^7*d^2*x^5 + 64*c^8*d*x^2 - sqrt(-3)*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 6
4*c^8*d*x^2))*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
512*c^3)) + (c^3*d*x^4 - 8*c^4*x + sqrt(-3)*(c^3*d*x^4 - 8*c^4*x))*(d^2/c^
17)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*
(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x + sqrt(-3)*(5*c^12*d^2*x^7 + 6
4*c^13*d*x^4 + 32*c^14*x))*(d^2/c^17)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^15
*d*x^5 + 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^2/c^17)^(
5/6) - 2*(7*c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sqrt(d^2/c^17) + (c^3*
d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x + sqrt(-3)*(c^3*d^3*x^7 + 80*c^4*d^
2*x^4 + 160*c^5*d*x))*(d^2/c^17)^(1/6)) - 9*(c^6*d^3*x^8 + 38*c^7*d^2*x^5
+ 64*c^8*d*x^2 - sqrt(-3)*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^2)...
```

Sympy [F]

$$\int \frac{1}{x^2 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^2 (-8c+dx^3)^2 \sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^2 \sqrt{d\,x^3 + c}\,(8\,c - d\,x^3)^2} \, dx$$

input
```
int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx$$
$$= \frac{-2\sqrt{d\,x^3+c}+40\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)c\,d^2x-5\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)d^3x^4+128\left(\int \right.}{16c^2x\left(-d\,x^3+8c\right)}$$

input
```
int(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**3) + 40*int((sqrt(c + d*x**3)*x**4)/(64*c**3 + 48*c**2
*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c*d**2*x - 5*int((sqrt(c + d*x**3
)*x**4)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*d**3*x*
*4 + 128*int((sqrt(c + d*x**3)*x)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x*
*6 + d**3*x**9),x)*c**2*d*x - 16*int((sqrt(c + d*x**3)*x)/(64*c**3 + 48*c*
*2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c*d**2*x**4)/(16*c**2*x*(8*c -
d*x**3))
```

3.608 $\int \frac{1}{x^5(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 687

$$\int \frac{1}{x^5 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx$$
$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} - \frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$
$$+ \frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)} - \frac{25d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}c^{23/6}}$$
$$+ \frac{25d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{23/6}} - \frac{25d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{82944c^{23/6}}$$
$$+ \frac{5\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{576\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$- \frac{5d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{432\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-31/6912*(d*x^3+c)^(1/2)/c^3/x^4+5/864*d*(d*x^3+c)^(1/2)/c^4/x-5/864*d^(4/
3)*(d*x^3+c)^(1/2)/c^4/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)+1/216*(d*x^3+c)^(1/
2)/c^2/x^4/(-d*x^3+8*c)-25/82944*d^(4/3)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d
^(1/3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/c^(23/6)+25/82944*d^(4/3)*arctanh(1/3*(
c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(23/6)-25/82944*d^(4/3)*ar
ctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)+5/1728*(1/2*6^(1/2)-1/2*2^(1/2
))*d^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((
1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1
/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/c^(11/3)/(c^
(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+
c)^(1/2)-5/2592*d^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^
(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))
*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)
*3^(3/4)/c^(11/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3
)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^5 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx$$
$$= \frac{245cd^2x^6(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-16\Big(2c(216c^3-135c^2dx^3-311cd^2x^6}{221184c^5x^4(8c-dx^3)\sqrt{c+dx}}$$

input
```
Integrate[1/(x^5*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(245*c*d^2*x^6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/
3, -((d*x^3)/c), (d*x^3)/(8*c)] - 16*(2*c*(216*c^3 - 135*c^2*d*x^3 - 311*c
*d^2*x^6 + 40*d^3*x^9) + d^3*x^9*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Appell
F1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(221184*c^5*x^4*(8*c -
 d*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {972, 27, 1053, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (8c-dx^3)^2 \sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{\int \frac{d(11dx^3+62c)}{2x^5(8c-dx^3)\sqrt{dx^3+c}}dx}{216c^2d}+\frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{11dx^3+62c}{x^5(8c-dx^3)\sqrt{dx^3+c}}dx}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int \frac{5cd(128c-31dx^3)}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c^2}-\frac{31\sqrt{c+dx^3}}{16cx^4}}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{5d\int \frac{128c-31dx^3}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c}-\frac{31\sqrt{c+dx^3}}{16cx^4}}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{5d\left(-\frac{\int -\frac{8cdx(49c-8dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{8c^2}-\frac{16\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{31\sqrt{c+dx^3}}{16cx^4}}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{5d\left(\frac{d\int \frac{x\left(49c-8dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{16\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{31\sqrt{c+dx^3}}{16cx^4}}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{-\frac{5d\left(\frac{d\int \left(\frac{8x}{\sqrt{dx^3+c}}-\frac{15cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}\right)dx}{c}-\frac{16\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{31\sqrt{c+dx^3}}{16cx^4}}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{5d\left(d\left(\frac{16\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{8\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\cdots}{d^2\cdots}\cdots\right)\cdots\right)}{\cdots}\cdots$$

$$\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}$$

input
```
Int[1/(x^5*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
Sqrt[c + d*x^3]/(216*c^2*x^4*(8*c - d*x^3)) + ((-31*Sqrt[c + d*x^3])/(16*c
*x^4) - (5*d*((-16*Sqrt[c + d*x^3])/(c*x) + (d*((16*Sqrt[c + d*x^3])/(d^(2
/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (5*c^(1/6)*ArcTan[(Sqrt[3]*c^(1
/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*Sqrt[3]*d^(2/3)) - (5*c^(1
/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(6*d^(2/
3)) + (5*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*d^(2/3)) - (8*3^
(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1
/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (16*Sqrt[2]*c^(1
/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)
/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^
(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/
(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/c))/(32*c))/(432*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.78 (sec) , antiderivative size = 919, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	919
risch	Expression too large to display	1770
default	Expression too large to display	2241

input
```
int(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/13824*x^2*d^2/c^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/256*(d*x^3+c)^(1/2)/c^3
/x^4+3/512*d*(d*x^3+c)^(1/2)/c^4/x+5/2592*I*d/c^4*3^(1/2)*(-c*d^2)^(1/3)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)))-25/124416*I/d/c^4*2^(1/2)*sum(1/_alpha*(-c*d^2
)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(
-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3
^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(
-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2549 vs. $2(490) = 980$.

Time = 1.94 (sec) , antiderivative size = 2549, normalized size of antiderivative = 3.71

$$\int \frac{1}{x^5 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/995328*(5760*(d^2*x^7 - 8*c*d*x^4)*sqrt(d)*weierstrassZeta(0, -4*c/d, we
ierstrassPInverse(0, -4*c/d, x)) + 25*(c^4*d*x^7 - 8*c^5*x^4 + sqrt(-3)*(c
^4*d*x^7 - 8*c^5*x^4))*(d^8/c^23)^(1/6)*log(9765625*(d^9*x^9 + 318*c*d^8*x
^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4
+ 32*c^18*d*x + sqrt(-3)*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x))
*(d^8/c^23)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 - sqr
t(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^8/c^23)^(5/6) - 2*(7*c^12*d^4*x^6 +
 152*c^13*d^3*x^3 + 64*c^14*d^2)*sqrt(d^8/c^23) + (c^4*d^7*x^7 + 80*c^5*d^
6*x^4 + 160*c^6*d^5*x + sqrt(-3)*(c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d
^5*x))*(d^8/c^23)^(1/6)) - 9*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x
^2 - sqrt(-3)*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2))*(d^8/c^23)
^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 25*(c^4*d*x^
7 - 8*c^5*x^4 + sqrt(-3)*(c^4*d*x^7 - 8*c^5*x^4))*(d^8/c^23)^(1/6)*log(976
5625*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^16
*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x + sqrt(-3)*(5*c^16*d^3*x^7 + 64*c
^17*d^2*x^4 + 32*c^18*d*x))*(d^8/c^23)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^2
0*d*x^5 + 32*c^21*x^2 - sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^8/c^23)^
(5/6) - 2*(7*c^12*d^4*x^6 + 152*c^13*d^3*x^3 + 64*c^14*d^2)*sqrt(d^8/c^23)
 + (c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x + sqrt(-3)*(c^4*d^7*x^7 +
 80*c^5*d^6*x^4 + 160*c^6*d^5*x))*(d^8/c^23)^(1/6)) - 9*(c^8*d^6*x^8 + ...
```

Sympy [F]

$$\int \frac{1}{x^5 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^5 (-8c + dx^3)^2 \sqrt{c + dx^3}} \, dx$$

input
```
integrate(1/x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**5*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (8c - dx^3)^2 \sqrt{c + dx^3}}\, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^5}\, dx$$

input
```
integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 (8c - dx^3)^2 \sqrt{c + dx^3}}\, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^5}\, dx$$

input
```
integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (8c - dx^3)^2 \sqrt{c + dx^3}}\, dx = \int \frac{1}{x^5\,\sqrt{d\,x^3 + c}\,(8\,c - d\,x^3)^2}\, dx$$

input
```
int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^5 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx$$
$$= \frac{-16\sqrt{d\,x^3+c}\,c+26\sqrt{d\,x^3+c}\,d\,x^3-520\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)c\,d^3x^4+65\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{d^3x^9-15c\,d^2x^6+48c^2}\right.}{512c^3x^4\,(}$$

input
```
int(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

output
```
( - 16*sqrt(c + d*x**3)*c + 26*sqrt(c + d*x**3)*d*x**3 - 520*int((sqrt(c +
 d*x**3)*x**4)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*
c*d**3*x**4 + 65*int((sqrt(c + d*x**3)*x**4)/(64*c**3 + 48*c**2*d*x**3 - 1
5*c*d**2*x**6 + d**3*x**9),x)*d**4*x**7 - 960*int((sqrt(c + d*x**3)*x)/(64
*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**2*d**2*x**4 + 1
20*int((sqrt(c + d*x**3)*x)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d
**3*x**9),x)*c*d**3*x**7)/(512*c**3*x**4*(8*c - d*x**3))
```

3.609 $\int \frac{1}{x^8 (8c-dx^3)^2 \sqrt{c+dx^3}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 711

$$\int \frac{1}{x^8 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = -\frac{17\sqrt{c+dx^3}}{6048c^3x^7}+\frac{391d\sqrt{c+dx^3}}{193536c^4x^4}-\frac{289d^2\sqrt{c+dx^3}}{48384c^5x}+\frac{289d^{7/3}\sqrt{c+dx^3}}{48384c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)}-\frac{17d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{110592\sqrt{3}c^{29/6}}+\frac{17d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{331776c^{29/6}}-\frac{17d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{331776c^{29/6}}-\frac{289\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32256\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{289d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{24192\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-17/6048*(d*x^3+c)^(1/2)/c^3/x^7+391/193536*d*(d*x^3+c)^(1/2)/c^4/x^4-289/
48384*d^2*(d*x^3+c)^(1/2)/c^5/x+289/48384*d^(7/3)*(d*x^3+c)^(1/2)/c^5/((1+
3^(1/2))*c^(1/3)+d^(1/3)*x)+1/216*(d*x^3+c)^(1/2)/c^2/x^7/(-d*x^3+8*c)-17/
331776*d^(7/3)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))
*3^(1/2)/c^(29/6)+17/331776*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1
/6)/(d*x^3+c)^(1/2))/c^(29/6)-17/331776*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2
)/c^(1/2))/c^(29/6)-289/96768*(1/2*6^(1/2)-1/2*2^(1/2))*d^(7/3)*(c^(1/3)+d
^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^
(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c
^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/c^(14/3)/(c^(1/3)*(c^(1/3)+d^(1/3
)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+289/145152*d
^(7/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^
(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*
x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/c^(14/3)
/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*
x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^8 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \sqrt{c+dx^3}\left(-\frac{1}{448c^3x^7}+\frac{15d}{7168c^4x^4}-\frac{171d^2}{28672c^5x} - \frac{d^3x^2}{110592c^5(-8c+dx^3)}\right) + \frac{9605d^3x^2\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{6193152c^5\sqrt{c+dx^3}} - \frac{289d^4x^5\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3870720c^6\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^8*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
Sqrt[c + d*x^3]*(-1/448*1/(c^3*x^7) + (15*d)/(7168*c^4*x^4) - (171*d^2)/(2
8672*c^5*x) - (d^3*x^2)/(110592*c^5*(-8*c + d*x^3))) + (9605*d^3*x^2*Sqrt[
(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(6
193152*c^5*Sqrt[c + d*x^3]) - (289*d^4*x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/
3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(3870720*c^6*Sqrt[c + d*x^3]
)
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 728, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {972, 27, 1053, 27, 1053, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 (8c-dx^3)^2 \sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{\int \frac{17d(dx^3+4c)}{2x^8(8c-dx^3)\sqrt{dx^3+c}}dx}{216c^2d}+\frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{17\int \frac{dx^3+4c}{x^8(8c-dx^3)\sqrt{dx^3+c}}dx}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{17\left(-\frac{\int \frac{2cd(46c-11dx^3)}{x^5(8c-dx^3)\sqrt{dx^3+c}}dx}{56c^2}-\frac{\sqrt{c+dx^3}}{14cx^7}\right)}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{17\left(-\frac{d\int \frac{46c-11dx^3}{x^5(8c-dx^3)\sqrt{dx^3+c}}dx}{28c}-\frac{\sqrt{c+dx^3}}{14cx^7}\right)}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{17\left(-\frac{d\left(-\frac{\int \frac{cd(1088c-115dx^3)}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c^2}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{\sqrt{c+dx^3}}{14cx^7}\right)}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{17\left(-\frac{d\left(-\frac{d\int \frac{1088c-115dx^3}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{\sqrt{c+dx^3}}{14cx^7}\right)}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{17\left(-\frac{d\left(-\frac{d\left(-\frac{\int -\frac{8cdx(565c-68dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{8c^2}-\frac{136\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{\sqrt{c+dx^3}}{14cx^7}\right)}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{17\left(-\frac{d\left(-\frac{d\left(\frac{d\int \frac{x\left(565c-68dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{136\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{\sqrt{c+dx^3}}{14cx^7}\right)}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{17\left(-\frac{d\left(-\frac{d\left(\frac{d\int \left(\frac{21cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{68x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{136\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{\sqrt{c+dx^3}}{14cx^7}\right)}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$17\left(-\frac{d\left(-\frac{d\left(\frac{d\left(\frac{136\sqrt{2}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{68\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[1/(x^8*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
Sqrt[c + d*x^3]/(216*c^2*x^7*(8*c - d*x^3)) + (17*(-1/14*Sqrt[c + d*x^3]/(
c*x^7) - (d*((-23*Sqrt[c + d*x^3])/(16*c*x^4) - (d*((-136*Sqrt[c + d*x^3])
/(c*x) + (d*((136*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)) - (7*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c
+ d*x^3]])/(2*Sqrt[3]*d^(2/3)) + (7*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^
2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(6*d^(2/3)) - (7*c^(1/6)*ArcTanh[Sqrt[c +
d*x^3]/(3*Sqrt[c])])/(6*d^(2/3)) - (68*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3)
 + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2
/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*Sqrt[c + d*x^3]) + (136*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(
c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^
3]))/c))/(32*c)))/(28*c)))/(432*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.02 (sec) , antiderivative size = 938, normalized size of antiderivative = 1.32

method	result	size
elliptic	Expression too large to display	938
risch	Expression too large to display	1781
default	Expression too large to display	2739

input
```
int(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/448*(d*x^3+c)^(1/2)/c^3/x^7+15/7168*d*(d*x^3+c)^(1/2)/c^4/x^4-171/28672
*d^2*(d*x^3+c)^(1/2)/c^5/x+1/110592*d^3*x^2/c^5*(d*x^3+c)^(1/2)/(-d*x^3+8*
c)-289/145152*I*d^2/c^5*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-
c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+
1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c
*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))
)-17/497664*I/c^5*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*
(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*
I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^
(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2582 vs. $2(510) = 1020$.

Time = 4.44 (sec) , antiderivative size = 2582, normalized size of antiderivative = 3.63

$$\int \frac{1}{x^8 \left(8c - dx^3\right)^2 \sqrt{c + dx^3}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/27869184*(166464*(d^3*x^10 - 8*c*d^2*x^7)*sqrt(d)*weierstrassZeta(0, -4
*c/d, weierstrassPInverse(0, -4*c/d, x)) - 119*(c^5*d*x^10 - 8*c^6*x^7 + s
qrt(-3)*(c^5*d*x^10 - 8*c^6*x^7))*(d^14/c^29)^(1/6)*log(1419857*(d^14*x^9
+ 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^20*d^4*x^7 +
64*c^21*d^3*x^4 + 32*c^22*d^2*x + sqrt(-3)*(5*c^20*d^4*x^7 + 64*c^21*d^3*x
^4 + 32*c^22*d^2*x))*(d^14/c^29)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^
5 + 32*c^26*x^2 - sqrt(-3)*(5*c^25*d*x^5 + 32*c^26*x^2))*(d^14/c^29)^(5/6)
 - 2*(7*c^15*d^6*x^6 + 152*c^16*d^5*x^3 + 64*c^17*d^4)*sqrt(d^14/c^29) + (
c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 160*c^7*d^9*x + sqrt(-3)*(c^5*d^11*x^7 +
80*c^6*d^10*x^4 + 160*c^7*d^9*x))*(d^14/c^29)^(1/6)) - 9*(c^10*d^9*x^8 + 3
8*c^11*d^8*x^5 + 64*c^12*d^7*x^2 - sqrt(-3)*(c^10*d^9*x^8 + 38*c^11*d^8*x^
5 + 64*c^12*d^7*x^2))*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2
*d*x^3 - 512*c^3)) + 119*(c^5*d*x^10 - 8*c^6*x^7 + sqrt(-3)*(c^5*d*x^10 -
8*c^6*x^7))*(d^14/c^29)^(1/6)*log(1419857*(d^14*x^9 + 318*c*d^13*x^6 + 120
0*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*c
^22*d^2*x + sqrt(-3)*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2*x))*(
d^14/c^29)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2 - sqrt
(-3)*(5*c^25*d*x^5 + 32*c^26*x^2))*(d^14/c^29)^(5/6) - 2*(7*c^15*d^6*x^6 +
 152*c^16*d^5*x^3 + 64*c^17*d^4)*sqrt(d^14/c^29) + (c^5*d^11*x^7 + 80*c^6*
d^10*x^4 + 160*c^7*d^9*x + sqrt(-3)*(c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 1...
```

Sympy [F]

$$\int \frac{1}{x^8 \left(8c - dx^3\right)^2 \sqrt{c + dx^3}}\, dx = \int \frac{1}{x^8 \left(-8c + dx^3\right)^2 \sqrt{c + dx^3}}\, dx$$

input
```
integrate(1/x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**8*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^8 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^8} \, dx$$

input
```
integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^8} \, dx$$

input
```
integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^8 \sqrt{d\,x^3 + c}\,(8\,c - d\,x^3)^2} \, dx$$

input
```
int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^8 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx$$
$$= \frac{-32\sqrt{d\,x^3+c}\,c^2+34\sqrt{d\,x^3+c}\,cd\,x^3-34\sqrt{d\,x^3+c}\,d^2x^6+3536\left(\int \frac{\sqrt{d\,x^3+c}}{d^3x^{11}-15c\,d^2x^8+48c^2d\,x^5+64c^3x^2}dx\right)c^3d^2x$$

input
```
int(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

output
```
( - 32*sqrt(c + d*x**3)*c**2 + 34*sqrt(c + d*x**3)*c*d*x**3 - 34*sqrt(c +
d*x**3)*d**2*x**6 + 3536*int(sqrt(c + d*x**3)/(64*c**3*x**2 + 48*c**2*d*x*
*5 - 15*c*d**2*x**8 + d**3*x**11),x)*c**3*d**2*x**7 - 442*int(sqrt(c + d*x
**3)/(64*c**3*x**2 + 48*c**2*d*x**5 - 15*c*d**2*x**8 + d**3*x**11),x)*c**2
*d**3*x**10 + 680*int((sqrt(c + d*x**3)*x**4)/(64*c**3 + 48*c**2*d*x**3 -
15*c*d**2*x**6 + d**3*x**9),x)*c*d**4*x**7 - 85*int((sqrt(c + d*x**3)*x**4
)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*d**5*x**10 +
680*int((sqrt(c + d*x**3)*x)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 +
d**3*x**9),x)*c**2*d**3*x**7 - 85*int((sqrt(c + d*x**3)*x)/(64*c**3 + 48*c
**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c*d**4*x**10)/(1792*c**4*x**7*
(8*c - d*x**3))
```

3.610 $\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x^7\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{7}{3},2,\frac{1}{2},\frac{10}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^2\sqrt{c+dx^3}}$$

output
```
1/448*x^7*(1+d*x^3/c)^(1/2)*AppellF1(7/3,1/2,2,10/3,-d*x^3/c,1/8*d*x^3/c)/
c^2/(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 239 vs. 2(66) = 132.

Time = 10.36 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.62

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{x\left(-\frac{23dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c}+\frac{256\left(c+dx^3-\frac{32c^2\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{32c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right.\right.}\right.}{8c-dx^3}\right.}{864d^2\sqrt{c+dx^3}}$$

input
```
Integrate[x^6/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x*((-23*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c)
, (d*x^3)/(8*c)])/c + (256*(c + d*x^3 - (32*c^2*AppellF1[1/3, 1/2, 1, 4/3,
 -((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/
c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*
x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))
)/(8*c - d*x^3)))/(864*d^2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^6}{(8c-dx^3)^2\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^7\sqrt{\frac{dx^3}{c}+1}\operatorname{AppellF1}\left(\frac{7}{3},2,\frac{1}{2},\frac{10}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^2\sqrt{c+dx^3}}$$

input
```
Int[x^6/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x^7*Sqrt[1 + (d*x^3)/c]*AppellF1[7/3, 2, 1/2, 10/3, (d*x^3)/(8*c), -((d*x
^3)/c)])/(448*c^2*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.74 (sec) , antiderivative size = 723, normalized size of antiderivative = 10.95

method	result	size
elliptic	Expression too large to display	723
default	Expression too large to display	1432

input
```
int(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
8/27*x*(d*x^3+c)^(1/2)/d^2/(-d*x^3+8*c)-46/81*I/d^3*3^(1/2)*(-c*d^2)^(1/3)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+64/243*I/d^5*2^(1/2)*sum(1/_
alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1
/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)
^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(
1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/1
8/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)
+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=Ro
otOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2425 vs. $2(52) = 104$.

Time = 0.50 (sec) , antiderivative size = 2425, normalized size of antiderivative = 36.74

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/243*(36*sqrt(d*x^3 + c)*d*x - 63*(d*x^3 - 8*c)*sqrt(d)*weierstrassPInve
rse(0, -4*c/d, x) + 2*(d^4*x^3 - 8*c*d^3 + sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(
1/(c*d^14))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3
- 9*(c*d^12*x^8 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2 + sqrt(-3)*(c*d^12*x^8
 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2))*(1/(c*d^14))^(2/3) + 3*sqrt(d*x^3 +
 c)*((c*d^14*x^7 + 80*c^2*d^13*x^4 + 160*c^3*d^12*x - sqrt(-3)*(c*d^14*x^7
 + 80*c^2*d^13*x^4 + 160*c^3*d^12*x))*(1/(c*d^14))^(5/6) - 2*(7*c*d^9*x^6
+ 152*c^2*d^8*x^3 + 64*c^3*d^7)*sqrt(1/(c*d^14)) + 6*(5*c*d^4*x^5 + 32*c^2
*d^3*x^2 + sqrt(-3)*(5*c*d^4*x^5 + 32*c^2*d^3*x^2))*(1/(c*d^14))^(1/6)) -
9*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x - sqrt(-3)*(5*c*d^7*x^7 + 6
4*c^2*d^6*x^4 + 32*c^3*d^5*x))*(1/(c*d^14))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
 + 192*c^2*d*x^3 - 512*c^3)) - 2*(d^4*x^3 - 8*c*d^3 + sqrt(-3)*(d^4*x^3 -
8*c*d^3))*(1/(c*d^14))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3
 + 640*c^3 - 9*(c*d^12*x^8 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2 + sqrt(-3)*
(c*d^12*x^8 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2))*(1/(c*d^14))^(2/3) - 3*s
qrt(d*x^3 + c)*((c*d^14*x^7 + 80*c^2*d^13*x^4 + 160*c^3*d^12*x - sqrt(-3)*
(c*d^14*x^7 + 80*c^2*d^13*x^4 + 160*c^3*d^12*x))*(1/(c*d^14))^(5/6) - 2*(7
*c*d^9*x^6 + 152*c^2*d^8*x^3 + 64*c^3*d^7)*sqrt(1/(c*d^14)) + 6*(5*c*d^4*x
^5 + 32*c^2*d^3*x^2 + sqrt(-3)*(5*c*d^4*x^5 + 32*c^2*d^3*x^2))*(1/(c*d^14)
)^(1/6)) - 9*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x - sqrt(-3)*(5...
```

Sympy [F]

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^6}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**6/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^6}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^6}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^6}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(x^6/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x^6/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^6}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx$$

input
```
int(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**6)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d
**3*x**9),x)
```

3.611 $\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},2,\frac{1}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^2\sqrt{c+dx^3}}$$

output
```
1/256*x^4*(1+d*x^3/c)^(1/2)*AppellF1(4/3,1/2,2,7/3,-d*x^3/c,1/8*d*x^3/c)/c
^2/(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 237 vs. 2(66) = 132.

Time = 10.24 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.59

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x\left(x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-\frac{64c\left(c+dx^3-\frac{32c^2\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\cdots\right.}{32c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},\cdots\right.\right.}\right.}{d(-8c+dx^3)}\right.}{1728c^2\sqrt{c+dx^3}}$$

input
```
Integrate[x^3/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x*(x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^
3)/(8*c)] - (64*c*(c + d*x^3 - (32*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3
)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^
3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c
)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(d*(-8*
c + d*x^3))))/(1728*c^2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^3}{(8c-dx^3)^2\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},2,\frac{1}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^2\sqrt{c+dx^3}}$$

input
```
Int[x^3/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 1/2, 7/3, (d*x^3)/(8*c), -((d*x^
3)/c)])/(256*c^2*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.96 (sec) , antiderivative size = 732, normalized size of antiderivative = 11.09

method	result	size
elliptic	Expression too large to display	732
default	Expression too large to display	1151

input
```
int(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/27*x*(d*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)+1/81*I/c/d^2*3^(1/2)*(-c*d^2)^(1/3
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/243*I/d^4/c*2^(1/2)*sum(1
/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^
(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^
2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)
^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1
/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/
2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=
RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2548 vs. $2(52) = 104$.

Time = 0.57 (sec) , antiderivative size = 2548, normalized size of antiderivative = 38.61

$$\int \frac{x^3}{(8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \text{Too large to display}$$

input

```
integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/3888*(144*sqrt(d*x^3 + c)*d*x + 72*(d*x^3 - 8*c)*sqrt(d)*weierstrassPIn
verse(0, -4*c/d, x) - (c*d^3*x^3 - 8*c^2*d^2 + sqrt(-3)*(c*d^3*x^3 - 8*c^2
*d^2))*(1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 +
 640*c^3 - 9*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2 + sqrt(-3)*(c^
5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2))*(1/(c^7*d^8))^(2/3) + 3*sqrt
(d*x^3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x - sqrt(-3)*(c^6
*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x))*(1/(c^7*d^8))^(5/6) - 2*(7*c^4
*d^6*x^6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt(1/(c^7*d^8)) + 6*(5*c^2*d^3*
x^5 + 32*c^3*d^2*x^2 + sqrt(-3)*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2))*(1/(c^7*
d^8))^(1/6)) - 9*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x - sqrt(-3)
*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x))*(1/(c^7*d^8))^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c*d^3*x^3 - 8*c^2*d^2
+ sqrt(-3)*(c*d^3*x^3 - 8*c^2*d^2))*(1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 318
*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 +
64*c^7*d^6*x^2 + sqrt(-3)*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2))
*(1/(c^7*d^8))^(2/3) - 3*sqrt(d*x^3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 +
160*c^8*d^7*x - sqrt(-3)*(c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x))*(
1/(c^7*d^8))^(5/6) - 2*(7*c^4*d^6*x^6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt
(1/(c^7*d^8)) + 6*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2 + sqrt(-3)*(5*c^2*d^3*x^
5 + 32*c^3*d^2*x^2))*(1/(c^7*d^8))^(1/6)) - 9*(5*c^3*d^5*x^7 + 64*c^4*d...
```

Sympy [F]

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^3}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**3/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^3}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^3}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^3}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{x^3}{(8c - dx^3)^2 \sqrt{c + dx^3}}\, dx = \int \frac{\sqrt{d\, x^3 + c}\, x^3}{d^3 x^9 - 15c\, d^2 x^6 + 48c^2 d\, x^3 + 64c^3} dx$$

input
```
int(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**3)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d
**3*x**9),x)
```

3.612 $\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{1}{3},2,\frac{1}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^2\sqrt{c+dx^3}}$$

output
```
1/64*x*(1+d*x^3/c)^(1/2)*AppellF1(1/3,1/2,2,4/3,-d*x^3/c,1/8*d*x^3/c)/c^2/
(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 237 vs. 2(64) = 128.

Time = 10.20 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.70

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x\left(\frac{dx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+\frac{64\left(\frac{c+dx^3}{c^2}+\frac{832\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{32c\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4\,\mathrm{A}\right.}\right.}{8c-dx^3}\right.}{13824\sqrt{c+dx^3}}$$

input
```
Integrate[1/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x*((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d
*x^3)/(8*c)])/c^3 + (64*((c + d*x^3)/c^2 + (832*AppellF1[1/3, 1/2, 1, 4/3,
 -((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/
c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*
x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))
)/(8*c - d*x^3)))/(13824*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{937, 936\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{(8c-dx^3)^2\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{1}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^2\sqrt{c+dx^3}}$$

input
```
Int[1/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 1/2, 4/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(64*c^2*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.11 (sec) , antiderivative size = 729, normalized size of antiderivative = 11.39

method	result	size
default	Expression too large to display	729
elliptic	Expression too large to display	729

input
```
int(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/216*x*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)+1/648*I/c^2*3^(1/2)/d*(-c*d^2)^(1
/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-5/972*I/c^2/d^3*2^(1/2)*s
um(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d
^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-
c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*
d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3
^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_al
pha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2498 vs. $2(50) = 100$.

Time = 0.64 (sec) , antiderivative size = 2498, normalized size of antiderivative = 39.03

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/15552*(72*sqrt(d*x^3 + c)*d*x - 288*(d*x^3 - 8*c)*sqrt(d)*weierstrassPI
nverse(0, -4*c/d, x) - 5*(c^2*d^2*x^3 - 8*c^3*d + sqrt(-3)*(c^2*d^2*x^3 -
8*c^3*d))*(1/(c^13*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x
^3 + 640*c^3 - 9*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2 + sqrt(-
3)*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2))*(1/(c^13*d^2))^(2/3)
 + 3*sqrt(d*x^3 + c)*((c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x - s
qrt(-3)*(c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x))*(1/(c^13*d^2))^
(5/6) - 2*(7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(1/(c^13*d^2))
+ 6*(5*c^3*d^2*x^5 + 32*c^4*d*x^2 + sqrt(-3)*(5*c^3*d^2*x^5 + 32*c^4*d*x^2
))*(1/(c^13*d^2))^(1/6)) - 9*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x
- sqrt(-3)*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x))*(1/(c^13*d^2))^(
1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 5*(c^2*d^2*x^3
 - 8*c^3*d + sqrt(-3)*(c^2*d^2*x^3 - 8*c^3*d))*(1/(c^13*d^2))^(1/6)*log((d
^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^9*d^4*x^8 + 38*c^
10*d^3*x^5 + 64*c^11*d^2*x^2 + sqrt(-3)*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 6
4*c^11*d^2*x^2))*(1/(c^13*d^2))^(2/3) - 3*sqrt(d*x^3 + c)*((c^11*d^4*x^7 +
 80*c^12*d^3*x^4 + 160*c^13*d^2*x - sqrt(-3)*(c^11*d^4*x^7 + 80*c^12*d^3*x
^4 + 160*c^13*d^2*x))*(1/(c^13*d^2))^(5/6) - 2*(7*c^7*d^3*x^6 + 152*c^8*d^
2*x^3 + 64*c^9*d)*sqrt(1/(c^13*d^2)) + 6*(5*c^3*d^2*x^5 + 32*c^4*d*x^2 + s
qrt(-3)*(5*c^3*d^2*x^5 + 32*c^4*d*x^2))*(1/(c^13*d^2))^(1/6)) - 9*(5*c^...
```

Sympy [F]

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx$$

input
```
int(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**3)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**
9),x)
```

3.613 $\int \frac{1}{x^3(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^3(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^2x^2\sqrt{c+dx^3}}$$

output
```
-1/128*(1+d*x^3/c)^(1/2)*AppellF1(-2/3,1/2,2,1/3,-d*x^3/c,1/8*d*x^3/c)/c^2
/x^2/(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 266 vs. 2(66) = 132.

Time = 10.23 (sec) , antiderivative size = 266, normalized size of antiderivative = 4.03

$$\int \frac{1}{x^3(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{-\frac{64(c+dx^3)(-216c+29dx^3)}{c^3x^2(-8c+dx^3)} + \frac{29d^2x^4\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^4} - \frac{4096dx\operatorname{App}}{c(8c-dx^3)\left(32c\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\right.}}{221184\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^3*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((-64*(c + d*x^3)*(-216*c + 29*d*x^3))/(c^3*x^2*(-8*c + d*x^3)) + (29*d^2*
x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/c^4 - (4096*d*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8
*c)])/(c*(8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x
^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*
c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(22118
4*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (8c-dx^3)^2 \sqrt{c+dx^3}} \, dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^3(8c-dx^3)^2\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\operatorname{AppellF1}\left(-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^2x^2\sqrt{c+dx^3}}$$

input
```
Int[1/(x^3*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
-1/128*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 1/2, 1/3, (d*x^3)/(8*c), -((
d*x^3)/c)])/(c^2*x^2*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 3.26 (sec) , antiderivative size = 744, normalized size of antiderivative = 11.27

method	result	size
elliptic	Expression too large to display	744
default	Expression too large to display	1456
risch	Expression too large to display	1457

input
```
int(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/128*(d*x^3+c)^(1/2)/c^3/x^2+1/1728/c^3*x*d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)
+29/10368*I/c^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(
1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x
+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1
/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2))-19/15552*I/c^3/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*
d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))
^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d
^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1
/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha
^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3
)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2515 vs. $2(52) = 104$.

Time = 1.61 (sec) , antiderivative size = 2515, normalized size of antiderivative = 38.11

$$\int \frac{1}{x^3 (8c - dx^3)^2 \sqrt{c + dx^3}}\, dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/248832*(720*(d*x^5 - 8*c*x^2)*sqrt(d)*weierstrassPInverse(0, -4*c/d, x)
 - 19*(c^3*d*x^5 - 8*c^4*x^2 + sqrt(-3)*(c^3*d*x^5 - 8*c^4*x^2))*(d^4/c^19
)^(1/6)*log(2476099*(d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*
d^3 - 9*(c^13*d^3*x^8 + 38*c^14*d^2*x^5 + 64*c^15*d*x^2 + sqrt(-3)*(c^13*d
^3*x^8 + 38*c^14*d^2*x^5 + 64*c^15*d*x^2))*(d^4/c^19)^(2/3) + 3*sqrt(d*x^3
 + c)*((c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x - sqrt(-3)*(c^16*d^2*x^7
 + 80*c^17*d*x^4 + 160*c^18*x))*(d^4/c^19)^(5/6) - 2*(7*c^10*d^3*x^6 + 152
*c^11*d^2*x^3 + 64*c^12*d)*sqrt(d^4/c^19) + 6*(5*c^4*d^4*x^5 + 32*c^5*d^3*
x^2 + sqrt(-3)*(5*c^4*d^4*x^5 + 32*c^5*d^3*x^2))*(d^4/c^19)^(1/6)) - 9*(5*
c^7*d^4*x^7 + 64*c^8*d^3*x^4 + 32*c^9*d^2*x - sqrt(-3)*(5*c^7*d^4*x^7 + 64
*c^8*d^3*x^4 + 32*c^9*d^2*x))*(d^4/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
192*c^2*d*x^3 - 512*c^3)) + 19*(c^3*d*x^5 - 8*c^4*x^2 + sqrt(-3)*(c^3*d*x^
5 - 8*c^4*x^2))*(d^4/c^19)^(1/6)*log(2476099*(d^6*x^9 + 318*c*d^5*x^6 + 12
00*c^2*d^4*x^3 + 640*c^3*d^3 - 9*(c^13*d^3*x^8 + 38*c^14*d^2*x^5 + 64*c^15
*d*x^2 + sqrt(-3)*(c^13*d^3*x^8 + 38*c^14*d^2*x^5 + 64*c^15*d*x^2))*(d^4/c
^19)^(2/3) - 3*sqrt(d*x^3 + c)*((c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x
 - sqrt(-3)*(c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x))*(d^4/c^19)^(5/6)
- 2*(7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 64*c^12*d)*sqrt(d^4/c^19) + 6*(5*
c^4*d^4*x^5 + 32*c^5*d^3*x^2 + sqrt(-3)*(5*c^4*d^4*x^5 + 32*c^5*d^3*x^2))*
(d^4/c^19)^(1/6)) - 9*(5*c^7*d^4*x^7 + 64*c^8*d^3*x^4 + 32*c^9*d^2*x - ...
```

Sympy [F]

$$\int \frac{1}{x^3 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^3 (-8c+dx^3)^2 \sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^3} \, dx$$

input
```
integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^3} \, dx$$

input
```
integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^3 \sqrt{d\,x^3 + c}\,(8\,c - d\,x^3)^2} \, dx$$

input
```
int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (8c-dx^3)^2 \sqrt{c+dx^3}} \, dx$$
$$= \frac{-2\sqrt{d\,x^3+c}+16\left(\int \frac{\sqrt{d\,x^3+c}}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right) c^2d\,x^2-2\left(\int \frac{\sqrt{d\,x^3+c}}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right) c\,d^2x^5+56\left(\right.}{32c^2x^2\,(-d\,x^3+8c)}$$

input
```
int(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**3) + 16*int(sqrt(c + d*x**3)/(64*c**3 + 48*c**2*d*x**3
 - 15*c*d**2*x**6 + d**3*x**9),x)*c**2*d*x**2 - 2*int(sqrt(c + d*x**3)/(64
*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c*d**2*x**5 + 56*i
nt((sqrt(c + d*x**3)*x**3)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d*
*3*x**9),x)*c*d**2*x**2 - 7*int((sqrt(c + d*x**3)*x**3)/(64*c**3 + 48*c**2
*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*d**3*x**5)/(32*c**2*x**2*(8*c - d
*x**3))
```

3.614 $\int \frac{1}{x^6(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^6\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(-\frac{5}{3},2,\frac{1}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^2x^5\sqrt{c+dx^3}}$$

output
```
-1/320*(1+d*x^3/c)^(1/2)*AppellF1(-5/3,1/2,2,-2/3,-d*x^3/c,1/8*d*x^3/c)/c^
2/x^5/(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 279 vs. $2(66) = 132$.

Time = 10.31 (sec) , antiderivative size = 279, normalized size of antiderivative = 4.23

$$\int \frac{1}{x^6\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\frac{64(c+dx^3)(864c^2-1080cdx^3+119d^2x^6)}{c^4x^5(-8c+dx^3)}-\frac{119d^3x^4\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^5}+\frac{14\cdots}{c^2(8c-dx^3)\left(32c\,\mathrm{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c}\cdots\right.\right.}}{2211840\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^6*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((64*(c + d*x^3)*(864*c^2 - 1080*c*d*x^3 + 119*d^2*x^6))/(c^4*x^5*(-8*c +
d*x^3)) - (119*d^3*x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d
*x^3)/c), (d*x^3)/(8*c)])/c^5 + (1404928*d^2*x*AppellF1[1/3, 1/2, 1, 4/3,
-((d*x^3)/c), (d*x^3)/(8*c)])/(c^2*(8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2,
1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3,
 -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)]))))/(2211840*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (8c-dx^3)^2 \sqrt{c+dx^3}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{1}{x^6 (8c-dx^3)^2 \sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1} \operatorname{AppellF1}\left(-\frac{5}{3},2,\frac{1}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^2x^5\sqrt{c+dx^3}}$$

input
```
Int[1/(x^6*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
-1/320*(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 2, 1/2, -2/3, (d*x^3)/(8*c), -(
(d*x^3)/c)])/(c^2*x^5*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.14 (sec) , antiderivative size = 765, normalized size of antiderivative = 11.59

method	result	size
elliptic	Expression too large to display	765
risch	Expression too large to display	1464
default	Expression too large to display	1783

input
```
int(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/320*(d*x^3+c)^(1/2)/c^3/x^5+9/2560*d*(d*x^3+c)^(1/2)/c^4/x^2+1/13824*x*
d^2/c^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-119/103680*I*d/c^4*3^(1/2)*(-c*d^2)^(
1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-7/31104*I/d/c^4*2^(1/2)*
sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*
d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(
-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c
*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*
3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^
2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_a
lpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2561 vs. $2(52) = 104$.

Time = 3.55 (sec) , antiderivative size = 2561, normalized size of antiderivative = 38.80

$$\int \frac{1}{x^6 (8c-dx^3)^2 \sqrt{c+dx^3}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/2488320*(11088*(d^2*x^8 - 8*c*d*x^5)*sqrt(d)*weierstrassPInverse(0, -4*c
/d, x) + 35*(c^4*d*x^8 - 8*c^5*x^5 + sqrt(-3)*(c^4*d*x^8 - 8*c^5*x^5))*(d^
10/c^25)^(1/6)*log(16807*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 6
40*c^3*d^8 - 9*(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2 + sqrt(-3
)*(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2))*(d^10/c^25)^(2/3) +
3*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x - sqrt(-3)*(
c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x))*(d^10/c^25)^(5/6) - 2*(7*c^13*
d^5*x^6 + 152*c^14*d^4*x^3 + 64*c^15*d^3)*sqrt(d^10/c^25) + 6*(5*c^5*d^8*x
^5 + 32*c^6*d^7*x^2 + sqrt(-3)*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2))*(d^10/c^2
5)^(1/6)) - 9*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x - sqrt(-3)*
(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x))*(d^10/c^25)^(1/3))/(d^3
*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 35*(c^4*d*x^8 - 8*c^5*x^
5 + sqrt(-3)*(c^4*d*x^8 - 8*c^5*x^5))*(d^10/c^25)^(1/6)*log(16807*(d^11*x^
9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - 9*(c^17*d^4*x^8 + 38
*c^18*d^3*x^5 + 64*c^19*d^2*x^2 + sqrt(-3)*(c^17*d^4*x^8 + 38*c^18*d^3*x^5
 + 64*c^19*d^2*x^2))*(d^10/c^25)^(2/3) - 3*sqrt(d*x^3 + c)*((c^21*d^2*x^7
+ 80*c^22*d*x^4 + 160*c^23*x - sqrt(-3)*(c^21*d^2*x^7 + 80*c^22*d*x^4 + 16
0*c^23*x))*(d^10/c^25)^(5/6) - 2*(7*c^13*d^5*x^6 + 152*c^14*d^4*x^3 + 64*c
^15*d^3)*sqrt(d^10/c^25) + 6*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2 + sqrt(-3)*(5
*c^5*d^8*x^5 + 32*c^6*d^7*x^2))*(d^10/c^25)^(1/6)) - 9*(5*c^9*d^7*x^7 +...
```

Sympy [F]

$$\int \frac{1}{x^6 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^6 (-8c+dx^3)^2 \sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**6*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^6 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^6} \, dx$$

input
```
integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^6} \, dx$$

input
```
integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^6\,\sqrt{d\,x^3 + c}\,(8\,c - d\,x^3)^2} \, dx$$

input
```
int(1/(x^6*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^6*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx$$
$$= \frac{-8\sqrt{d\,x^3 + c}\,c + 10\sqrt{d\,x^3 + c}\,d\,x^3 + 336\left(\int \frac{\sqrt{d\,x^3+c}}{d^3x^9-15c\,d^2x^6+48c^2d\,x^3+64c^3}dx\right)c^2d^2x^5 - 42\left(\int \frac{\sqrt{d\,x^3+c}}{d^3x^9-15c\,d^2x^6+48c^2 d}\right.}{320c^3x^5\,(-}$$

input
```
int(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

output
```
( - 8*sqrt(c + d*x**3)*c + 10*sqrt(c + d*x**3)*d*x**3 + 336*int(sqrt(c + d
*x**3)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c**2*d**
2*x**5 - 42*int(sqrt(c + d*x**3)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**
6 + d**3*x**9),x)*c*d**3*x**8 - 280*int((sqrt(c + d*x**3)*x**3)/(64*c**3 +
 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9),x)*c*d**3*x**5 + 35*int((sqr
t(c + d*x**3)*x**3)/(64*c**3 + 48*c**2*d*x**3 - 15*c*d**2*x**6 + d**3*x**9
),x)*d**4*x**8)/(320*c**3*x**5*(8*c - d*x**3))
```

3.615 $\int \frac{x^{11}}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 102

$$\int \frac{x^{11}}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \frac{2c}{243d^4\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{3d^4} + \frac{512c\sqrt{c+dx^3}}{243d^4(8c-dx^3)} - \frac{640\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243d^4}$$

output
```
2/243*c/d^4/(d*x^3+c)^(1/2)+2/3*(d*x^3+c)^(1/2)/d^4+512/243*c*(d*x^3+c)^(1
/2)/d^4/(-d*x^3+8*c)-640/243*c^(1/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/
d^4
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = -\frac{2\left(912c^2+822cdx^3-81d^2x^6-320\sqrt{c}(8c-dx^3)\sqrt{c+dx^3}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{243d^4(-8c+dx^3)\sqrt{c+dx^3}}$$

input
```
Integrate[x^11/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-2*(912*c^2 + 822*c*d*x^3 - 81*d^2*x^6 - 320*Sqrt[c]*(8*c - d*x^3)*Sqrt[c
 + d*x^3]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(243*d^4*(-8*c + d*x^3)*S
qrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {948, 109, 27, 163, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^9}{(8c-dx^3)^2 (dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{3}\left(\frac{8x^6}{9d^2(8c-dx^3)\sqrt{c+dx^3}}-\frac{\int \frac{cx^3(13dx^3+16c)}{(8c-dx^3)(dx^3+c)^{3/2}}dx^3}{9cd^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{8x^6}{9d^2(8c-dx^3)\sqrt{c+dx^3}}-\frac{\int \frac{x^3(13dx^3+16c)}{(8c-dx^3)(dx^3+c)^{3/2}}dx^3}{9d^2}\right)$$

$$\Big\downarrow \text{163}$$

$$\frac{1}{3}\left(\frac{8x^6}{9d^2(8c-dx^3)\sqrt{c+dx^3}}-\frac{\frac{320c\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{3d}-\frac{2(38c+39dx^3)}{3d^2\sqrt{c+dx^3}}}{9d^2}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{8x^6}{9d^2\,(8c-dx^3)\sqrt{c+dx^3}}-\frac{\frac{640c\int\frac{1}{9c-x^6}d\sqrt{dx^3+c}}{3d^2}-\frac{2(38c+39dx^3)}{3d^2\sqrt{c+dx^3}}}{9d^2}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(\frac{8x^6}{9d^2\,(8c-dx^3)\sqrt{c+dx^3}}-\frac{\frac{640\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}-\frac{2(38c+39dx^3)}{3d^2\sqrt{c+dx^3}}}{9d^2}\right)$$

input
```
Int[x^11/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
((8*x^6)/(9*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) - ((-2*(38*c + 39*d*x^3))/(
3*d^2*Sqrt[c + d*x^3]) + (640*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]
)/(9*d^2))/(9*d^2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}+\frac{128c\left(\frac{4\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{243}+\frac{2c}{243\sqrt{d\,x^{3}+c}}}{d^{4}}$
risch	$\frac{2\sqrt{d\,x^{3}+c}}{3d^{4}}+\frac{c\left(\frac{2}{243d\sqrt{d\,x^{3}+c}}-\frac{2432\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{729\sqrt{c}\,d}+\frac{512c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{243d}\right)}{d^{3}}$
default	$\frac{d\left(\frac{2c}{3d^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{2\sqrt{d\,x^{3}+c}}{3d^{2}}\right)-\frac{32c}{3d\sqrt{d\,x^{3}+c}}}{d^{3}}-\frac{128c^{2}\left(-\frac{1}{c\sqrt{d\,x^{3}+c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{9d^{4}}+\frac{512c\left(-\frac{2}{\sqrt{d\,x^{3}+c}}+\frac{\sqrt{d\,x^{3}}}{-d\,x^{3}\cdots}\cdots\right.}{24\cdots}$
elliptic	$\frac{2c}{243d^{4}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{512c\sqrt{d\,x^{3}+c}}{243d^{4}\left(-d\,x^{3}+8c\right)}+\frac{2\sqrt{d\,x^{3}+c}}{3d^{4}}+\frac{320i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,(\cdots}{\cdots}\right)}{(\cdots}}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*((d*x^3+c)^(1/2)+64/81*c*(4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-5*arctanh(1/3
*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))+1/81*c/(d*x^3+c)^(1/2))/d^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.25

$$\int \frac{x^{11}}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \left[\frac{2\left(160\,(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\,(27\,d^2x^6-\right.}{243\,(d^6x^6-7\,cd^5x^3-8\,c^2d^4)}\right.$$

input
```
integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[2/243*(160*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^
3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 3*(27*d^2*x^6 - 274*c*d*x^3 - 304*
c^2)*sqrt(d*x^3 + c))/(d^6*x^6 - 7*c*d^5*x^3 - 8*c^2*d^4), 2/243*(320*(d^2
*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) + 3*
(27*d^2*x^6 - 274*c*d*x^3 - 304*c^2)*sqrt(d*x^3 + c))/(d^6*x^6 - 7*c*d^5*x
^3 - 8*c^2*d^4)]
```

Sympy [F]

$$\int \frac{x^{11}}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^{11}}{(-8c+dx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**11/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**11/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.96

$$\int \frac{x^{11}}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{2\left(160\,\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+81\,\sqrt{dx^3+c}-\frac{3\,(85\,(dx^3+c)c+3\,c^2)}{(dx^3+c)^{\frac{3}{2}}-9\,\sqrt{dx^3+c}c}\right)}{243\,d^4}$$

input
```
integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
2/243*(160*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 81*sqrt(d*x^3 + c) - 3*(85*(d*x^3 + c)*c + 3*c^2)/((d*x^3 + c)
^(3/2) - 9*sqrt(d*x^3 + c)*c))/d^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.86

$$\int \frac{x^{11}}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{640\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{243\sqrt{-c}d^4} + \frac{2\sqrt{dx^3+c}}{3\,d^4} - \frac{2\,(85\,(dx^3+c)c+3\,c^2)}{81\left((dx^3+c)^{\frac{3}{2}}-9\sqrt{dx^3+c}c\right)d^4}$$

input
```
integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
640/243*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) + 2/3*sqrt(d
*x^3 + c)/d^4 - 2/81*(85*(d*x^3 + c)*c + 3*c^2)/(((d*x^3 + c)^(3/2) - 9*sq
rt(d*x^3 + c)*c)*d^4)
```

Mupad [B] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

$$\int \frac{x^{11}}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,d^4} + \frac{320\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\sqrt{c}\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{243\,d^4} + \frac{\sqrt{d\,x^3+c}\left(\frac{176\,c^2}{81\,d^4}+\frac{170\,c\,x^3}{81\,d^3}\right)}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}$$

input
```
int(x^11/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*d^4) + (320*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)
*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(243*d^4) + ((c + d*x^3)^(1/2)*((176*c
^2)/(81*d^4) + (170*c*x^3)/(81*d^3)))/(8*c^2 - d^2*x^6 + 7*c*d*x^3)
```

Reduce [F]

$$\int \frac{x^{11}}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{\frac{352\sqrt{d\,x^3+c}\,c^2}{9} + \frac{44\sqrt{d\,x^3+c}\,c\,d\,x^3}{3} - \frac{2\sqrt{d\,x^3+c}\,d^2x^6}{3} - 2560\Big(\int \frac{\sqrt{d\,x^3+c}\,x^5}{d^4x^{12}-14c\,d^3x^9+33c^2d^2x^6 \cdots}}{}$$

input
```
int(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

output
```
(2*(176*sqrt(c + d*x**3)*c**2 + 66*sqrt(c + d*x**3)*c*d*x**3 - 3*sqrt(c +
d*x**3)*d**2*x**6 - 11520*int((sqrt(c + d*x**3)*x**5)/(64*c**4 + 112*c**3*
d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*c**4*d**2 - 1
0080*int((sqrt(c + d*x**3)*x**5)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2
*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*c**3*d**3*x**3 + 1440*int((sqrt(c
+ d*x**3)*x**5)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3
*x**9 + d**4*x**12),x)*c**2*d**4*x**6))/(9*d**4*(8*c**2 + 7*c*d*x**3 - d**
2*x**6))
```

3.616 $\int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}} dx$

Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}} dx = -\frac{2}{243d^3\sqrt{c+dx^3}} + \frac{64\sqrt{c+dx^3}}{243d^3(8c-dx^3)} - \frac{32\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}d^3}$$

output
```
-2/243/d^3/(d*x^3+c)^(1/2)+64/243*(d*x^3+c)^(1/2)/d^3/(-d*x^3+8*c)-32/243*
arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)/d^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}} dx = \frac{2\left(\frac{3(8c+11dx^3)}{(8c-dx^3)\sqrt{c+dx^3}} - \frac{16\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{243d^3}$$

input
```
Integrate[x^8/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(2*((3*(8*c + 11*d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) - (16*ArcTanh[Sqr
t[c + d*x^3]/(3*Sqrt[c])])/Sqrt[c]))/(243*d^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {948, 100, 27, 87, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^6}{(8c-dx^3)^2 (dx^3+c)^{3/2}} dx^3$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{3} \left(\frac{64c}{9d^3 (8c-dx^3) \sqrt{c+dx^3}} - \frac{\int -\frac{3cd(8c-3dx^3)}{(8c-dx^3)(dx^3+c)^{3/2}} dx^3}{9cd^3} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(\frac{\int \frac{8c-3dx^3}{(8c-dx^3)(dx^3+c)^{3/2}} dx^3}{3d^2} + \frac{64c}{9d^3 (8c-dx^3) \sqrt{c+dx^3}} \right)$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3} \left(\frac{-\frac{16}{9} \int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}} dx^3 - \frac{22}{9d\sqrt{c+dx^3}}}{3d^2} + \frac{64c}{9d^3 (8c-dx^3) \sqrt{c+dx^3}} \right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3} \left(\frac{-\frac{32 \int \frac{1}{9c-x^6} d\sqrt{dx^3+c}}{9d} - \frac{22}{9d\sqrt{c+dx^3}}}{3d^2} + \frac{64c}{9d^3 (8c-dx^3) \sqrt{c+dx^3}} \right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(\frac{-\frac{32\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{27\sqrt{c}d}-\frac{22}{9d\sqrt{c+dx^3}}}{3d^2}+\frac{64c}{9d^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

input
```
Int[x^8/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
((64*c)/(9*d^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (-22/(9*d*Sqrt[c + d*x^3])
 - (32*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(27*Sqrt[c]*d))/(3*d^2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{\frac{64\sqrt{d\,x^{3}+c}}{243\left(-d\,x^{3}+8c\right)}-\frac{32\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{243\sqrt{c}}-\frac{2}{243\sqrt{d\,x^{3}+c}}}{d^{3}}$
default	$-\frac{2}{3d^{3}\sqrt{d\,x^{3}+c}}-\frac{32c\left(-\frac{1}{c\sqrt{d\,x^{3}+c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27d^{3}}+\frac{-\frac{128}{243\sqrt{d\,x^{3}+c}}+\frac{64\sqrt{d\,x^{3}+c}}{243\left(-d\,x^{3}+8c\right)}+\frac{64\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{243\sqrt{c}}}{d^{3}}$
elliptic	$-\frac{2}{243d^{3}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{64\sqrt{d\,x^{3}+c}}{243d^{3}(-d\,x^{3}+8c)}+\frac{16i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\right.}{d}\right.}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$

input
```
int(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/243*(32*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-16*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1
/2))/c^(1/2)-1/(d*x^3+c)^(1/2))/d^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.68

$$\int \frac{x^8}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx=\left[\frac{2\left(8\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\,(11\,cdx^3+8\right.}{243\,(cd^5x^6-7\,c^2d^4x^3-8\,c^3d^3)}\right.$$

input
```
integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[2/243*(8*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3
+ c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 3*(11*c*d*x^3 + 8*c^2)*sqrt(d*x^3 +
c))/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3), 2/243*(16*(d^2*x^6 - 7*c*d*x^
3 - 8*c^2)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) - 3*(11*c*d*x^3 + 8
*c^2)*sqrt(d*x^3 + c))/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3)]
```

Sympy [F]

$$\int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{x^8}{(-8c+dx^3)^2(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**8/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99

$$\int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{2\left(\frac{8\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}} - \frac{3(11\,dx^3+8\,c)}{(dx^3+c)^{\frac{3}{2}}-9\sqrt{dx^3+c}c}\right)}{243\,d^3}$$

input
```
integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
2/243*(8*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/
sqrt(c) - 3*(11*d*x^3 + 8*c)/((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c))/d^
3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \frac{x^8}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\, dx = \frac{2\left(\frac{16 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d} - \frac{3\,(11\,dx^3+8\,c)}{\left((dx^3+c)^{\frac{3}{2}}-9\,\sqrt{dx^3+c}c\right)d}\right)}{243\,d^2}$$

input
```
integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
2/243*(16*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 3*(11*d*x^3
+ 8*c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*d))/d^2
```

Mupad [B] (verification not implemented)

Time = 3.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

$$\int \frac{x^8}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\, dx = \frac{16\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{243\,\sqrt{c}\,d^3} + \frac{\sqrt{d\,x^3+c}\,\left(\frac{16\,c}{81\,d^3}+\frac{22\,x^3}{81\,d^2}\right)}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}$$

input
```
int(x^8/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
(16*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(243*
c^(1/2)*d^3) + ((c + d*x^3)^(1/2)*((16*c)/(81*d^3) + (22*x^3)/(81*d^2)))/(
8*c^2 - d^2*x^6 + 7*c*d*x^3)
```

Reduce [F]

$$\int \frac{x^8}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\, dx = \frac{\frac{16\sqrt{d\,x^3+c}\,c}{9} + \frac{2\sqrt{d\,x^3+c}\,d\,x^3}{3} - 128\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{d^4x^{12}-14c\,d^3x^9+33c^2d^2x^6+112c^3d\,x^3+64c^4}dx\right) c^3}{}$$

input
```
int(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

output
```
(2*(8*sqrt(c + d*x**3)*c + 3*sqrt(c + d*x**3)*d*x**3 - 576*int((sqrt(c + d
*x**3)*x**5)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x*
*9 + d**4*x**12),x)*c**3*d**2 - 504*int((sqrt(c + d*x**3)*x**5)/(64*c**4 +
 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*c**
2*d**3*x**3 + 72*int((sqrt(c + d*x**3)*x**5)/(64*c**4 + 112*c**3*d*x**3 +
33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*c*d**4*x**6))/(9*d**3*
(8*c**2 + 7*c*d*x**3 - d**2*x**6))
```

3.617 $\int \frac{x^5}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 88

$$\int \frac{x^5}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx=\frac{2}{243cd^2\sqrt{c+dx^3}}+\frac{8\sqrt{c+dx^3}}{243cd^2(8c-dx^3)}+\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2}$$

output
```
2/243/c/d^2/(d*x^3+c)^(1/2)+8/243*(d*x^3+c)^(1/2)/c/d^2/(-d*x^3+8*c)+2/243
*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)/d^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx=\frac{2\left(\frac{3\sqrt{c}(4c+dx^3)}{(8c-dx^3)\sqrt{c+dx^3}}+\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{243c^{3/2}d^2}$$

input
```
Integrate[x^5/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(2*((3*Sqrt[c]*(4*c + d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) + ArcTanh[Sq
rt[c + d*x^3]/(3*Sqrt[c])]))/(243*c^(3/2)*d^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {948, 87, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{(8c-dx^3)^2 (dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{(8c-dx^3)(dx^3+c)^{3/2}}dx^3}{3d}+\frac{8}{9d^2 (8c-dx^3) \sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(\frac{\frac{\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9c}-\frac{2}{9cd\sqrt{c+dx^3}}}{3d}+\frac{8}{9d^2 (8c-dx^3) \sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\frac{2\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{9cd}-\frac{2}{9cd\sqrt{c+dx^3}}}{3d}+\frac{8}{9d^2 (8c-dx^3) \sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{27c^{3/2}d}-\frac{2}{9cd\sqrt{c+dx^3}}}{3d}+\frac{8}{9d^2 (8c-dx^3) \sqrt{c+dx^3}}\right)$$

input
```
Int[x^5/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(8/(9*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (-2/(9*c*d*Sqrt[c + d*x^3]) + (
2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(27*c^(3/2)*d))/(3*d))/3
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{\frac{8\sqrt{d\,x^{3}+c}}{243\left(-d\,x^{3}+8c\right)}+\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{243\sqrt{c}}+\frac{2}{243\sqrt{d\,x^{3}+c}}}{c\,d^{2}}$
default	$-\frac{2\left(-\frac{1}{c\sqrt{d\,x^{3}+c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27d^{2}}+\frac{-\frac{16}{243\sqrt{d\,x^{3}+c}}+\frac{8\sqrt{d\,x^{3}+c}}{243\left(-d\,x^{3}+8c\right)}+\frac{8\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{243\sqrt{c}}}{c\,d^{2}}$
elliptic	$\frac{2}{243d^{2}c\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{8\sqrt{d\,x^{3}+c}}{243c\,d^{2}(-d\,x^{3}+8c)}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right.}{d}\right.}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}\right)}{}$

input
```
int(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/243/c*(4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2
))/c^(1/2)+1/(d*x^3+c)^(1/2))/d^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.50

$$\int \frac{x^5}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \left[\frac{(d^2x^6 - 7\,cdx^3 - 8\,c^2)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right) - 6\,(cdx^3 + 4\,c^2)\sqrt{dx}}{243\,(c^2d^4x^6 - 7\,c^3d^3x^3 - 8\,c^4d^2)}\right.$$

$$\left. -\frac{2\left((d^2x^6 - 7\,cdx^3 - 8\,c^2)\sqrt{-c}\arctan\left(\frac{3\,\sqrt{-c}}{\sqrt{dx^3+c}}\right) + 3\,(cdx^3 + 4\,c^2)\sqrt{dx^3+c}\right)}{243\,(c^2d^4x^6 - 7\,c^3d^3x^3 - 8\,c^4d^2)}\right]$$

input
```
integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 +
c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*(c*d*x^3 + 4*c^2)*sqrt(d*x^3 + c))/(
c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2), -2/243*((d^2*x^6 - 7*c*d*x^3 - 8
*c^2)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) + 3*(c*d*x^3 + 4*c^2)*sq
rt(d*x^3 + c))/(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)]
```

Sympy [F]

$$\int \frac{x^5}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^5}{(-8c+dx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**5/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{\frac{6\,(dx^3+4\,c)}{(dx^3+c)^{\frac{3}{2}}c-9\,\sqrt{dx^3+c}c^2}+\frac{\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{243\,d^2}$$

input
```
integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/243*(6*(d*x^3 + 4*c)/((d*x^3 + c)^(3/2)*c - 9*sqrt(d*x^3 + c)*c^2) + lo
g((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(3/2))/d^
2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{2\left(\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}cd}+\frac{3\,(dx^3+4\,c)}{\left((dx^3+c)^{\frac{3}{2}}-9\,\sqrt{dx^3+c}c\right)cd}\right)}{243\,d}$$

input
```
integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-2/243*(arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) + 3*(d*x^3 + 4
*c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c*d))/d
```

Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09

$$\int \frac{x^5}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \frac{\left(\frac{8}{81\,d^2}+\frac{2\,x^3}{81\,c\,d}\right)\,\sqrt{d\,x^3+c}}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}+\frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{243\,c^{3/2}\,d^2}$$

input
```
int(x^5/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
((8/(81*d^2) + (2*x^3)/(81*c*d))*(c + d*x^3)^(1/2))/(8*c^2 - d^2*x^6 + 7*c
*d*x^3) + log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/
(243*c^(3/2)*d^2)
```

Reduce [F]

$$\int \frac{x^5}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \int \frac{\sqrt{d\,x^3+c}\,x^5}{d^4x^{12}-14c\,d^3x^9+33c^2d^2x^6+112c^3d\,x^3+64c^4}dx$$

input
```
int(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**5)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6
 - 14*c*d**3*x**9 + d**4*x**12),x)
```

3.618 $\int \frac{x^2}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 88

$$\int \frac{x^2}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd(8c-dx^3)\sqrt{c+dx^3}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}$$

output
```
-1/81/c^2/d/(d*x^3+c)^(1/2)+1/27/c/d/(-d*x^3+8*c)/(d*x^3+c)^(1/2)+1/243*ar
ctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{\frac{3\sqrt{c}(-5c+dx^3)}{(8c-dx^3)\sqrt{c+dx^3}} + \operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}$$

input
```
Integrate[x^2/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
((3*Sqrt[c]*(-5*c + d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) + ArcTanh[Sqrt
[c + d*x^3]/(3*Sqrt[c])])/(243*c^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {946, 52, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(8c-dx^3)^2(c+dx^3)^{3/2}}dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{3}\int \frac{1}{(8c-dx^3)^2(dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \; 52$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{(8c-dx^3)(dx^3+c)^{3/2}}dx^3}{6c}+\frac{1}{9cd(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{3}\left(\frac{\frac{\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9c}-\frac{2}{9cd\sqrt{c+dx^3}}}{6c}+\frac{1}{9cd(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{\frac{2\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{9cd}-\frac{2}{9cd\sqrt{c+dx^3}}}{6c}+\frac{1}{9cd(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{27c^{3/2}d}-\frac{2}{9cd\sqrt{c+dx^3}}}{6c}+\frac{1}{9cd\,(8c-dx^3)\,\sqrt{c+dx^3}}\right)$$

input
```
Int[x^2/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(1/(9*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (-2/(9*c*d*Sqrt[c + d*x^3]) + (
2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(27*c^(3/2)*d))/(6*c))/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

method	result
default	$\frac{-\frac{2}{\sqrt{d\,x^{3}+c}}+\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{\sqrt{c}}}{243c^{2}d}$
pseudoelliptic	$\frac{-\frac{2}{\sqrt{d\,x^{3}+c}}+\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{\sqrt{c}}}{243c^{2}d}$
elliptic	$-\frac{2}{243d\,c^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{\sqrt{d\,x^{3}+c}}{243d\,c^{2}\left(-d\,x^{3}+8c\right)}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{i d\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-\right.}{d}\right.}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$

input
```
int(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/243/c^2*(-2/(d*x^3+c)^(1/2)+(d*x^3+c)^(1/2)/(-d*x^3+8*c)+arctanh(1/3*(d*
x^3+c)^(1/2)/c^(1/2))/c^(1/2))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.45

$$\int \frac{x^2}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \left[\frac{(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,(cdx^3-5\,c^2)\sqrt{dx^3+c}}{486\,(c^3d^3x^6-7\,c^4d^2x^3-8\,c^5d)}, -\frac{(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)+3\,(cdx^3-5\,c^2)\sqrt{dx^3+c}}{243\,(c^3d^3x^6-7\,c^4d^2x^3-8\,c^5d)}\right]$$

input
```
integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/486*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 +
c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*(c*d*x^3 - 5*c^2)*sqrt(d*x^3 + c))/(
c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d), -1/243*((d^2*x^6 - 7*c*d*x^3 - 8*c
^2)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) + 3*(c*d*x^3 - 5*c^2)*sqrt
(d*x^3 + c))/(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)]
```

Sympy [F]

$$\int \frac{x^2}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^2}{(-8c+dx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**2/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = -\frac{\frac{6\,(dx^3-5\,c)}{(dx^3+c)^{\frac{3}{2}}c^2-9\,\sqrt{dx^3+c}c^3}+\frac{\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)}{c^{\frac{5}{2}}}}{486\,d}$$

input
```
integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/486*(6*(d*x^3 - 5*c)/((d*x^3 + c)^(3/2)*c^2 - 9*sqrt(d*x^3 + c)*c^3) +
log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(5/2))/
d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{243\,\sqrt{-c}c^2d} - \frac{dx^3-5\,c}{81\,\left((dx^3+c)^{\frac{3}{2}}-9\,\sqrt{dx^3+c}c\right)c^2d}$$

input
```
integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-1/243*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2*d) - 1/81*(d*x^3
 - 5*c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^2*d)
```

Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{486\,c^{5/2}\,d} - \frac{\left(\frac{5}{81\,c\,d}-\frac{x^3}{81\,c^2}\right)\sqrt{d\,x^3+c}}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}$$

input
```
int(x^2/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(486*c^(5/
2)*d) - ((5/(81*c*d) - x^3/(81*c^2))*(c + d*x^3)^(1/2))/(8*c^2 - d^2*x^6 +
 7*c*d*x^3)
```

Reduce [F]

$$\int \frac{x^2}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^2}{d^4x^{12}-14c\,d^3x^9+33c^2d^2x^6+112c^3d\,x^3+64c^4}dx$$

input
```
int(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**2)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6
 - 14*c*d**3*x**9 + d**4*x**12),x)
```

3.619 $\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 106

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{7\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

output
```
5/648/c^3/(d*x^3+c)^(1/2)+1/216/c^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2)+7/7776*ar
ctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-1/96*arctanh((d*x^3+c)^(1/2)/c^
(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{\frac{12\sqrt{c}\left(43c-5dx^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}} + 7\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 81\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{7776c^{7/2}}$$

input
```
Integrate[1/(x*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
((12*Sqrt[c]*(43*c - 5*d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) + 7*ArcTanh
[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 81*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(7776
*c^(7/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 114, 27, 169, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{1}{x^3 (8c-dx^3)^2 (dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3} \left(\frac{1}{72c^2 (8c-dx^3) \sqrt{c+dx^3}}-\frac{\int -\frac{3d(dx^3+6c)}{2x^3(8c-dx^3)(dx^3+c)^{3/2}}dx^3}{72c^2d}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(\frac{\int \frac{dx^3+6c}{x^3(8c-dx^3)(dx^3+c)^{3/2}}dx^3}{48c^2}+\frac{1}{72c^2 (8c-dx^3) \sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{3} \left(\frac{\frac{2\int \frac{cd\left(54c-5dx^3\right)}{2x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c^2d}+\frac{10}{9c\sqrt{c+dx^3}}}{48c^2}+\frac{1}{72c^2 (8c-dx^3) \sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\frac{\int \frac{54c-5dx^3}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9c}+\frac{10}{9c\sqrt{c+dx^3}}}{48c^2}+\frac{1}{72c^2(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$\Big\downarrow$ 174

$$\frac{1}{3}\left(\frac{\frac{\frac{27}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{7}{4}d\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9c}+\frac{10}{9c\sqrt{c+dx^3}}}{48c^2}+\frac{1}{72c^2(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$\Big\downarrow$ 73

$$\frac{1}{3}\left(\frac{\frac{\frac{7}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{27\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{9c}+\frac{10}{9c\sqrt{c+dx^3}}}{48c^2}+\frac{1}{72c^2(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$\Big\downarrow$ 219

$$\frac{1}{3}\left(\frac{\frac{\frac{27\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{7\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{9c}+\frac{10}{9c\sqrt{c+dx^3}}}{48c^2}+\frac{1}{72c^2(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$\Big\downarrow$ 221

$$\frac{1}{3}\left(\frac{\frac{\frac{7\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{27\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{9c}+\frac{10}{9c\sqrt{c+dx^3}}}{48c^2}+\frac{1}{72c^2(8c-dx^3)\sqrt{c+dx^3}}\right)$$

input

```
Int[1/(x*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output

```
(1/(72*c^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (10/(9*c*Sqrt[c + d*x^3]) + ((
7*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*Sqrt[c]) - (27*ArcTanh[Sqrt[c +
 d*x^3]/Sqrt[c]])/(2*Sqrt[c]))/(9*c))/(48*c^2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{96 c^{\frac{7}{2}}}+\frac{\frac{7 \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3 \sqrt{c}}\right)\left(d x^3-8 c\right)}{\sqrt{c}}-4 \sqrt{d x^3+c}}{7776\left(d x^3-8 c\right) c^3}+\frac{2}{243 c^3 \sqrt{d x^3+c}}$
default	$\frac{\frac{2}{3 c \sqrt{\left(x^3+\frac{c}{d}\right) d}}-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{3 c^{\frac{3}{2}}}}{64 c^2}+\frac{-\frac{2}{\sqrt{d x^3+c}}+\frac{\sqrt{d x^3+c}}{-d x^3+8 c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3 \sqrt{c}}\right)}{\sqrt{c}}}{1944 c^3}+\frac{-\frac{1}{c \sqrt{d x^3+c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3 \sqrt{c}}\right)}{3 c^{\frac{3}{2}}}}{864 c^2}$
elliptic	Expression too large to display

input
```
int(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)+1/7776*(7*arctanh(1/3*(d*x^
3+c)^(1/2)/c^(1/2))/c^(1/2)*(d*x^3-8*c)-4*(d*x^3+c)^(1/2))/(d*x^3-8*c)/c^3
+2/243/c^3/(d*x^3+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.92

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \left[\frac{7\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+81\left(d^2x^6-7\,cdx\right.}{15552\left(c^4d^2x^6-\right.}\right.$$

$$-\frac{7\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)-81\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right)-12\left(5\,c\right.}{7776\left(c^4d^2x^6-7\,c^5dx^3-8\,c^6\right)}$$

input
```
integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/15552*(7*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^
3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 81*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*s
qrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(5*c*d*x^3
- 43*c^2)*sqrt(d*x^3 + c))/(c^4*d^2*x^6 - 7*c^5*d*x^3 - 8*c^6), -1/7776*(7
*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c))
 - 81*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 +
c)) - 12*(5*c*d*x^3 - 43*c^2)*sqrt(d*x^3 + c))/(c^4*d^2*x^6 - 7*c^5*d*x^3
- 8*c^6)]
```

Sympy [F]

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \int \frac{1}{x\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2 x}\,dx$$

input
```
integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}c^3} - \frac{7\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{7776\sqrt{-c}c^3} + \frac{5\,dx^3-43\,c}{648\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}c\right)c^3}$$

input
```
integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 7/7776*arctan(1/3*s
qrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + 1/648*(5*d*x^3 - 43*c)/(((d*x^3
+ c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 3.01 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = -\frac{\frac{5\,\left(d\,x^3+c\right)}{216\,c^3}-\frac{2}{9\,c^2}}{27\,c\,\sqrt{d\,x^3+c}-3\,\left(d\,x^3+c\right)^{3/2}} + \frac{\left(\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)\,7{}\mathrm{i}}{81}\right)\,1{}\mathrm{i}}{96\,\sqrt{c^7}}$$

input
```
int(1/(x*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
((atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2))*1i - (atanh((c^3*(c + d*x^3)^
(1/2))/(3*(c^7)^(1/2)))*7i)/81)*1i)/(96*(c^7)^(1/2)) - ((5*(c + d*x^3))/(2
16*c^3) - 2/(9*c^2))/(27*c*(c + d*x^3)^(1/2) - 3*(c + d*x^3)^(3/2))
```

Reduce [F]

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{32\sqrt{d\,x^3+c}\,c^2-6\sqrt{d\,x^3+c}\,cd\,x^3+24\sqrt{c}\log\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)c^2+21}{}$$

input
```
int(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

output
```
(32*sqrt(c + d*x**3)*c**2 - 6*sqrt(c + d*x**3)*c*d*x**3 + 24*sqrt(c)*log(s
qrt(c + d*x**3) - sqrt(c))*c**2 + 21*sqrt(c)*log(sqrt(c + d*x**3) - sqrt(c
))*c*d*x**3 - 3*sqrt(c)*log(sqrt(c + d*x**3) - sqrt(c))*d**2*x**6 - 24*sqr
t(c)*log(sqrt(c + d*x**3) + sqrt(c))*c**2 - 21*sqrt(c)*log(sqrt(c + d*x**3
) + sqrt(c))*c*d*x**3 + 3*sqrt(c)*log(sqrt(c + d*x**3) + sqrt(c))*d**2*x**
6 + 504*int((sqrt(c + d*x**3)*x**5)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d
**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*c**4*d**2 + 441*int((sqrt(c + d
*x**3)*x**5)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x*
*9 + d**4*x**12),x)*c**3*d**3*x**3 - 63*int((sqrt(c + d*x**3)*x**5)/(64*c*
*4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)
*c**2*d**4*x**6)/(576*c**4*(8*c**2 + 7*c*d*x**3 - d**2*x**6))
```

3.620 $\int \frac{1}{x^4(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{1}{x^4 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3(8c-dx^3)\sqrt{c+dx^3}} + \frac{5d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{31104c^{9/2}} + \frac{5d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{9/2}}$$

output
```
-35/2592*d/c^4/(d*x^3+c)^(1/2)+5/864*d/c^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2)-1/
24/c^2/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2)+5/31104*d*arctanh(1/3*(d*x^3+c)^(1
/2)/c^(1/2))/c^(9/2)+5/384*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^4 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{-\frac{12\sqrt{c}(108c^2+265cdx^3-35d^2x^6)}{x^3(8c-dx^3)\sqrt{c+dx^3}} + 5d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + 405d\operatorname{arctanh}\left(\cdots \right.}{31104c^{9/2}}$$

input
```
Integrate[1/(x^4*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
((-12*Sqrt[c]*(108*c^2 + 265*c*d*x^3 - 35*d^2*x^6))/(x^3*(8*c - d*x^3)*Sqr
t[c + d*x^3]) + 5*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] + 405*d*ArcTanh[S
qrt[c + d*x^3]/Sqrt[c]])/(31104*c^(9/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {948, 114, 27, 168, 27, 169, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{1}{x^6 (8c-dx^3)^2 (dx^3+c)^{3/2}} dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3} \left(-\frac{\int \frac{5d(4c-dx^3)}{2x^3(8c-dx^3)^2(dx^3+c)^{3/2}} dx^3}{8c^2} - \frac{1}{8c^2x^3 (8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(-\frac{5d \int \frac{4c-dx^3}{x^3(8c-dx^3)^2(dx^3+c)^{3/2}} dx^3}{16c^2} - \frac{1}{8c^2x^3 (8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3} \left(-\frac{5d \left(-\frac{\int -\frac{6cd(6c-dx^3)}{x^3(8c-dx^3)(dx^3+c)^{3/2}} dx^3}{72c^2d} - \frac{1}{18c(8c-dx^3)\sqrt{c+dx^3}}\right)}{16c^2} - \frac{1}{8c^2x^3 (8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{5d\left(\frac{\int \frac{6c-dx^3}{x^3\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx^3}{12c}-\frac{1}{18c(8c-dx^3)\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{3}\left(-\frac{5d\left(\frac{\frac{2\int \frac{cd\left(54c-7dx^3\right)}{2x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c^2d}+\frac{14}{9c\sqrt{c+dx^3}}}{12c}-\frac{1}{18c(8c-dx^3)\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{5d\left(\frac{\frac{\int \frac{54c-7dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c}+\frac{14}{9c\sqrt{c+dx^3}}}{12c}-\frac{1}{18c(8c-dx^3)\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(-\frac{5d\left(\frac{\frac{\frac{27}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3-\frac{1}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c}+\frac{14}{9c\sqrt{c+dx^3}}}{12c}-\frac{1}{18c(8c-dx^3)\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{5d\left(\frac{\frac{\frac{27\int\frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{1}{2}\int\frac{1}{9c-x^6}d\sqrt{dx^3+c}}{9c}+\frac{14}{9c\sqrt{c+dx^3}}}{12c}-\frac{1}{18c(8c-dx^3)\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(-\frac{5d\left(\frac{\frac{\frac{27\int\frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{9c}+\frac{14}{9c\sqrt{c+dx^3}}}{12c}-\frac{1}{18c(8c-dx^3)\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{5d\left(\frac{\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{27\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{9c}+\frac{14}{9c\sqrt{c+dx^3}}}{12c}-\frac{1}{18c(8c-dx^3)\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3(8c-dx^3)\sqrt{c+dx^3}}\right)$$

input
```
Int[1/(x^4*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-1/8*1/(c^2*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) - (5*d*(-1/18*1/(c*(8*c -
d*x^3)*Sqrt[c + d*x^3]) + (14/(9*c*Sqrt[c + d*x^3]) + (-1/6*ArcTanh[Sqrt[c
 + d*x^3]/(3*Sqrt[c])]/Sqrt[c] - (27*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*
Sqrt[c]))/(9*c))/(12*c)))/(16*c^2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{d\left(-\frac{\sqrt{d\,x^{3}+c}}{64d\,x^{3}}+\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{128\sqrt{c}}+\frac{\sqrt{d\,x^{3}+c}}{-5184d\,x^{3}+41472c}+\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{10368\sqrt{c}}-\frac{2}{81\sqrt{d\,x^{3}+c}}\right)}{3c^{4}}$
risch	$-\frac{\sqrt{d\,x^{3}+c}}{192c^{4}x^{3}}-\frac{d\left(-\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}+\frac{256}{243\sqrt{d\,x^{3}+c}}-\frac{13\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{729\sqrt{c}}-\frac{2c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{243}\right.}{128c^{4}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3c^{2}x^{3}}-\frac{2d}{3c^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}}{64c^{2}}+\frac{d\left(\frac{2}{3c\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{256c^{3}}+\frac{d\left(-\frac{2}{\sqrt{d\,x^{3}+c}}+\frac{\sqrt{d\,x}}{-d\,x^{3}}\right.}{1}$
elliptic	Expression too large to display

input
```
int(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*d/c^4*(-1/64*(d*x^3+c)^(1/2)/d/x^3+5/128*arctanh((d*x^3+c)^(1/2)/c^(1/
2))/c^(1/2)+1/5184*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+5/10368*arctanh(1/3*(d*x^3
+c)^(1/2)/c^(1/2))/c^(1/2)-2/81/(d*x^3+c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.53

$$\int \frac{1}{x^4\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\left[\frac{5\left(d^3x^9-7\,cd^2x^6-8\,c^2dx^3\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+405\left(d^3x^9\right.}{6220}\right.$$

$$-\frac{5\left(d^3x^9-7\,cd^2x^6-8\,c^2dx^3\right)\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right)+405\left(d^3x^9-7\,cd^2x^6-8\,c^2dx^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^3+c}}\right.}{31104\left(c^5d^2x^9-7\,c^6dx^6-8\,c^7x^3\right)}$$

input
```
integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/62208*(5*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*d*x^3)*sqrt(c)*log((d*x^3 + 6*s
qrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 405*(d^3*x^9 - 7*c*d^2*x^6
 - 8*c^2*d*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3)
 - 24*(35*c*d^2*x^6 - 265*c^2*d*x^3 - 108*c^3)*sqrt(d*x^3 + c))/(c^5*d^2*x
^9 - 7*c^6*d*x^6 - 8*c^7*x^3), -1/31104*(5*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*
d*x^3)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt(d*x^3 + c)) + 405*(d^3*x^9 - 7*c*d^
2*x^6 - 8*c^2*d*x^3)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)) + 12*(35*c*
d^2*x^6 - 265*c^2*d*x^3 - 108*c^3)*sqrt(d*x^3 + c))/(c^5*d^2*x^9 - 7*c^6*d
*x^6 - 8*c^7*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^4 (-8c+dx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**4*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}} (dx^3-8\,c)^2 x^4}\,dx$$

input
```
integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = -\frac{5\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\,\sqrt{-c}c^4} - \frac{5\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{31104\,\sqrt{-c}c^4} - \frac{35\,(dx^3+c)^2 d - 335\,(dx^3+c)cd + 192\,c^2 d}{2592\left((dx^3+c)^{\frac{5}{2}} - 10\,(dx^3+c)^{\frac{3}{2}}c + 9\,\sqrt{dx^3+c}c^2\right)c^4}$$

input
```
integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-5/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 5/31104*d*arcta
n(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/2592*(35*(d*x^3 + c)^2*
d - 335*(d*x^3 + c)*c*d + 192*c^2*d)/(((d*x^3 + c)^(5/2) - 10*(d*x^3 + c)^
(3/2)*c + 9*sqrt(d*x^3 + c)*c^2)*c^4)
```

Mupad [B] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = -\frac{\frac{2\,d}{9\,c^2} + \frac{35\,d\,(d\,x^3+c)^2}{864\,c^4} - \frac{335\,d\,(d\,x^3+c)}{864\,c^3}}{3\,(d\,x^3+c)^{5/2} - 30\,c\,(d\,x^3+c)^{3/2} + 27\,c^2\,\sqrt{d\,x^3+c}} - \frac{d\left(\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{\sqrt{c^9}}\right)\,1\mathrm{i} + \frac{\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^9}}\right)\,1\mathrm{i}}{81}\right)\,5\mathrm{i}}{384\,\sqrt{c^9}}$$

input
```
int(1/(x^4*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
- ((2*d)/(9*c^2) + (35*d*(c + d*x^3)^2)/(864*c^4) - (335*d*(c + d*x^3))/(8
64*c^3))/(3*(c + d*x^3)^(5/2) - 30*c*(c + d*x^3)^(3/2) + 27*c^2*(c + d*x^3
)^(1/2)) - (d*(atanh((c^4*(c + d*x^3)^(1/2))/(c^9)^(1/2))*1i + (atanh((c^4
*(c + d*x^3)^(1/2))/(3*(c^9)^(1/2)))*1i)/81)*5i)/(384*(c^9)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^4 (8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \frac{-32\sqrt{d\,x^3+c}\,c^3 - 80\sqrt{d\,x^3+c}\,c^2 d\,x^3 + 10\sqrt{d\,x^3+c}\,c\,d^2x^6 - 40\sqrt{c}\log}{}$$

input
```
int(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

output
```
( - 32*sqrt(c + d*x**3)*c**3 - 80*sqrt(c + d*x**3)*c**2*d*x**3 + 10*sqrt(c
 + d*x**3)*c*d**2*x**6 - 40*sqrt(c)*log(sqrt(c + d*x**3) - sqrt(c))*c**2*d
*x**3 - 35*sqrt(c)*log(sqrt(c + d*x**3) - sqrt(c))*c*d**2*x**6 + 5*sqrt(c)
*log(sqrt(c + d*x**3) - sqrt(c))*d**3*x**9 + 40*sqrt(c)*log(sqrt(c + d*x**
3) + sqrt(c))*c**2*d*x**3 + 35*sqrt(c)*log(sqrt(c + d*x**3) + sqrt(c))*c*d
**2*x**6 - 5*sqrt(c)*log(sqrt(c + d*x**3) + sqrt(c))*d**3*x**9 + 120*int((
sqrt(c + d*x**3)*x**5)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14
*c*d**3*x**9 + d**4*x**12),x)*c**4*d**3*x**3 + 105*int((sqrt(c + d*x**3)*x
**5)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**
4*x**12),x)*c**3*d**4*x**6 - 15*int((sqrt(c + d*x**3)*x**5)/(64*c**4 + 112
*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*c**2*d*
*5*x**9)/(768*c**5*x**3*(8*c**2 + 7*c*d*x**3 - d**2*x**6))
```

3.621 $\int \frac{1}{x^7(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 185

$$\int \frac{1}{x^7(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4(8c-dx^3)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6(8c-dx^3)\sqrt{c+dx^3}} + \frac{17d}{384c^3x^3(8c-dx^3)\sqrt{c+dx^3}} + \frac{13d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{497664c^{11/2}} - \frac{33d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{11/2}}$$

output
```
665/41472*d^2/c^5/(d*x^3+c)^(1/2)-71/13824*d^2/c^4/(-d*x^3+8*c)/(d*x^3+c)^
(1/2)-1/48/c^2/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2)+17/384*d/c^3/x^3/(-d*x^3+8
*c)/(d*x^3+c)^(1/2)+13/497664*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(
11/2)-33/2048*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^7 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{\frac{12\sqrt{c}(864c^3-1836c^2dx^3-5107cd^2x^6+665d^3x^9)}{x^6(-8c+dx^3)\sqrt{c+dx^3}}+13d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-8019d^2 \ldots}{497664c^{11/2}}$$

input

```
Integrate[1/(x^7*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output

```
((12*Sqrt[c]*(864*c^3 - 1836*c^2*d*x^3 - 5107*c*d^2*x^6 + 665*d^3*x^9))/(x
^6*(-8*c + d*x^3)*Sqrt[c + d*x^3]) + 13*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqr
t[c])] - 8019*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(497664*c^(11/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {948, 114, 27, 168, 27, 168, 27, 169, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (8c-dx^3)^2 (c+dx^3)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^9 (8c-dx^3)^2 (dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int \frac{d(34c-7dx^3)}{2x^6(8c-dx^3)^2(dx^3+c)^{3/2}}dx^3}{16c^2}-\frac{1}{16c^2x^6(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\int \frac{34c-7dx^3}{x^6(8c-dx^3)^2(dx^3+c)^{3/2}}dx^3}{32c^2}-\frac{1}{16c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$\big\downarrow$ 168

$$\frac{1}{3}\left(-\frac{d\left(-\frac{\int \frac{cd\left(396c-85dx^3\right)}{x^3\left(8c-dx^3\right)^2\left(dx^3+c\right)^{3/2}}dx^3}{8c^2}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$\big\downarrow$ 27

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\int \frac{396c-85dx^3}{x^3\left(8c-dx^3\right)^2\left(dx^3+c\right)^{3/2}}dx^3}{8c}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$\big\downarrow$ 168

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(-\frac{\int -\frac{6cd\left(594c-71dx^3\right)}{x^3\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx^3}{72c^2d}-\frac{71}{18c\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{8c}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$\big\downarrow$ 27

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\int \frac{594c-71dx^3}{x^3\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx^3}{12c}-\frac{71}{18c\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{8c}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$\Big\downarrow$ 169

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{2\int \frac{cd\left(5346c-665dx^3\right)}{2x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c^2d}+\frac{1330}{9c\sqrt{c+dx^3}}}{12c}-\frac{71}{18c\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{8c}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\left(8c-dx^3\right)\sqrt{c}}\right.$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{\int \frac{5346c-665dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c}+\frac{1330}{9c\sqrt{c+dx^3}}}{12c}-\frac{71}{18c\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{8c}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\left(8c-dx^3\right)\sqrt{c+\cdots}}\right.$$

$\Big\downarrow$ 174

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{\frac{2673}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{13}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c}+\frac{1330}{9c\sqrt{c+dx^3}}}{12c}-\frac{71}{18c\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{8c}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{\cdots}{16c^2\cdots}\right.$$

$\Big\downarrow$ 73

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{\frac{13}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{2673\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{9c}+\frac{1330}{9c\sqrt{c+dx^3}}}{12c}-\frac{71}{18c\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{8c}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{}{16c^2x^6\,(\ldots}\right.$$

$\downarrow$ 219

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{\frac{2673\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{13\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{9c}+\frac{1330}{9c\sqrt{c+dx^3}}}{12c}-\frac{71}{18c\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{8c}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{}{16c^2x\ldots}\right.$$

$\downarrow$ 221

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{\frac{13\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{2673\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{9c}+\frac{1330}{9c\sqrt{c+dx^3}}}{12c}-\frac{71}{18c\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{8c}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{}{16}\right.$$

input
```
Int[1/(x^7*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-1/16*1/(c^2*x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]) - (d*(-17/(4*c*x^3*(8*c -
 d*x^3)*Sqrt[c + d*x^3]) - (d*(-71/(18*c*(8*c - d*x^3)*Sqrt[c + d*x^3]) +
(1330/(9*c*Sqrt[c + d*x^3]) + ((13*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(
6*Sqrt[c]) - (2673*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c]))/(9*c))/(
12*c)))/(8*c)))/(32*c^2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.58

method	result
pseudoelliptic	$\frac{d^{2}\left(-\frac{\sqrt{d\,x^{3}+c}\,\left(-3d\,x^{3}+c\right)}{128d^{2}x^{6}}-\frac{99\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{2048\sqrt{c}}+\frac{\sqrt{d\,x^{3}+c}}{-41472d\,x^{3}+331776c}+\frac{13\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{165888\sqrt{c}}+\frac{2}{81\sqrt{d\,x^{3}+c}}\right)}{3c^{5}}$
risch	$-\frac{\sqrt{d\,x^{3}+c}\,\left(-3d\,x^{3}+c\right)}{384c^{5}x^{6}}+\frac{d^{2}\left(-\frac{33\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{8\sqrt{c}}+\frac{512}{243\sqrt{d\,x^{3}+c}}+\frac{35\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{5832\sqrt{c}}+\frac{c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\right.}{3}\right.}{486}\right.}{256c^{5}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{6c^{2}x^{6}}+\frac{7d\sqrt{d\,x^{3}+c}}{12c^{3}x^{3}}+\frac{2d^{2}}{3c^{3}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{5d^{2}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{4c^{\frac{7}{2}}}}{64c^{2}}+\frac{d\left(-\frac{\sqrt{d\,x^{3}+c}}{3c^{2}x^{3}}-\frac{2d}{3c^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right.}{256c^{3}}$
elliptic	Expression too large to display

input
```
int(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*d^2/c^5*(-1/128*(d*x^3+c)^(1/2)*(-3*d*x^3+c)/d^2/x^6-99/2048*arctanh((
d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+1/41472*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+13/16
5888*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+2/81/(d*x^3+c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.12

$$\int \frac{1}{x^7 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \left[\frac{13\,(d^4x^{12} - 7\,cd^3x^9 - 8\,c^2d^2x^6)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right) + 8019\,(d \cdots}{\cdots}\right.$$

$$-\frac{13\,(d^4x^{12} - 7\,cd^3x^9 - 8\,c^2d^2x^6)\sqrt{-c}\arctan\left(\frac{3\sqrt{-c}}{\sqrt{dx^3+c}}\right) - 8019\,(d^4x^{12} - 7\,cd^3x^9 - 8\,c^2d^2x^6)\sqrt{-c}\arctan\left(\cdots\right.}{497664\,(c^6d^2x^{12} - 7\,c^7dx^9 - 8\,c^8x^6 \cdots}$$

input
```
integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/995328*(13*(d^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(c)*log((d*x^3
+ 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 8019*(d^4*x^12 - 7*c*
d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) +
2*c)/x^3) + 24*(665*c*d^3*x^9 - 5107*c^2*d^2*x^6 - 1836*c^3*d*x^3 + 864*c^
4)*sqrt(d*x^3 + c))/(c^6*d^2*x^12 - 7*c^7*d*x^9 - 8*c^8*x^6), -1/497664*(1
3*(d^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(-c)*arctan(3*sqrt(-c)/sqrt
(d*x^3 + c)) - 8019*(d^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(-c)*arct
an(sqrt(-c)/sqrt(d*x^3 + c)) - 12*(665*c*d^3*x^9 - 5107*c^2*d^2*x^6 - 1836
*c^3*d*x^3 + 864*c^4)*sqrt(d*x^3 + c))/(c^6*d^2*x^12 - 7*c^7*d*x^9 - 8*c^8
*x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^7\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**7*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^7 (8c - dx^3)^2 (c + dx^3)^{3/2}}\, dx = \int \frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c)^2 x^7}\, dx$$

input
```
integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^7), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^7 (8c - dx^3)^2 (c + dx^3)^{3/2}}\, dx = \frac{33\, d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\,\sqrt{-c}c^5} - \frac{13\, d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{497664\,\sqrt{-c}c^5} + \frac{341\,(dx^3 + c)d^2 - 3072\,cd^2}{41472\left((dx^3 + c)^{\frac{3}{2}} - 9\,\sqrt{dx^3 + c}c\right)c^5} + \frac{3\,(dx^3 + c)^{\frac{3}{2}} d^2 - 4\,\sqrt{dx^3 + c}cd^2}{384\,c^5 d^2 x^6}$$

input
```
integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
33/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^5) - 13/497664*d^
2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^5) + 1/41472*(341*(d*x^
3 + c)*d^2 - 3072*c*d^2)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^5) +
 1/384*(3*(d*x^3 + c)^(3/2)*d^2 - 4*sqrt(d*x^3 + c)*c*d^2)/(c^5*d^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^7 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{\frac{2\,d^2}{9\,c^2}-\frac{10373\,d^2\,(d\,x^3+c)}{13824\,c^3}+\frac{3551\,d^2\,{(d\,x^3+c)}^2}{6912\,c^4}-\frac{665\,d^2\,{(d\,x^3+c)}^3}{13824\,c^5}}{33\,c\,{(d\,x^3+c)}^{5/2}-3\,{(d\,x^3+c)}^{7/2}+27\,c^3\,\sqrt{d\,x^3+c}-57\,c^2\,{(d\,x^3+c)}^{3/2}} + \frac{d^2\,\left(\mathrm{atanh}\left(\frac{c^5\,\sqrt{d\,x^3+c}}{\sqrt{c^{11}}}\right)\,1\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^5\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^{11}}}\right)\,13\mathrm{i}}{8019}\right)\,33\mathrm{i}}{2048\,\sqrt{c^{11}}}$$

input
```
int(1/(x^7*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
((2*d^2)/(9*c^2) - (10373*d^2*(c + d*x^3))/(13824*c^3) + (3551*d^2*(c + d*
x^3)^2)/(6912*c^4) - (665*d^2*(c + d*x^3)^3)/(13824*c^5))/(33*c*(c + d*x^3
)^(5/2) - 3*(c + d*x^3)^(7/2) + 27*c^3*(c + d*x^3)^(1/2) - 57*c^2*(c + d*x
^3)^(3/2)) + (d^2*(atanh((c^5*(c + d*x^3)^(1/2))/(c^11)^(1/2))*1i - (atanh
((c^5*(c + d*x^3)^(1/2))/(3*(c^11)^(1/2)))*13i)/8019)*33i)/(2048*(c^11)^(1
/2))
```

Reduce [F]

$$\int \frac{1}{x^7 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

output

```
( - 4930464*sqrt(c + d*x**3)*c**4 + 10477236*sqrt(c + d*x**3)*c**3*d*x**3
+ 29076630*sqrt(c + d*x**3)*c**2*d**2*x**6 - 3814800*sqrt(c + d*x**3)*c*d*
*3*x**9 + 15259200*sqrt(c)*log(sqrt(c + d*x**3) - sqrt(c))*c**2*d**2*x**6
+ 13351800*sqrt(c)*log(sqrt(c + d*x**3) - sqrt(c))*c*d**3*x**9 - 1907400*s
qrt(c)*log(sqrt(c + d*x**3) - sqrt(c))*d**4*x**12 - 15259200*sqrt(c)*log(s
qrt(c + d*x**3) + sqrt(c))*c**2*d**2*x**6 - 13351800*sqrt(c)*log(sqrt(c +
d*x**3) + sqrt(c))*c*d**3*x**9 + 1907400*sqrt(c)*log(sqrt(c + d*x**3) + sq
rt(c))*d**4*x**12 + 692224*int(sqrt(c + d*x**3)/(64*c**4*x**7 + 112*c**3*d
*x**10 + 33*c**2*d**2*x**13 - 14*c*d**3*x**16 + d**4*x**19),x)*c**8*x**6 +
 605696*int(sqrt(c + d*x**3)/(64*c**4*x**7 + 112*c**3*d*x**10 + 33*c**2*d*
*2*x**13 - 14*c*d**3*x**16 + d**4*x**19),x)*c**7*d*x**9 - 86528*int(sqrt(c
 + d*x**3)/(64*c**4*x**7 + 112*c**3*d*x**10 + 33*c**2*d**2*x**13 - 14*c*d*
*3*x**16 + d**4*x**19),x)*c**6*d**2*x**12 - 1070784*int(sqrt(c + d*x**3)/(
64*c**4*x + 112*c**3*d*x**4 + 33*c**2*d**2*x**7 - 14*c*d**3*x**10 + d**4*x
**13),x)*c**6*d**2*x**6 - 936936*int(sqrt(c + d*x**3)/(64*c**4*x + 112*c**
3*d*x**4 + 33*c**2*d**2*x**7 - 14*c*d**3*x**10 + d**4*x**13),x)*c**5*d**3*
x**9 + 133848*int(sqrt(c + d*x**3)/(64*c**4*x + 112*c**3*d*x**4 + 33*c**2*
d**2*x**7 - 14*c*d**3*x**10 + d**4*x**13),x)*c**4*d**4*x**12 + 6126120*int
((sqrt(c + d*x**3)*x**5)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 -
14*c*d**3*x**9 + d**4*x**12),x)*c**4*d**4*x**6 + 5360355*int((sqrt(c + ...
```

3.622 $\int \frac{x^7}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 668

$$\int \frac{x^7}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{2x^2}{81cd^2\sqrt{c+dx^3}}+\frac{8x^2}{27d^2(8c-dx^3)\sqrt{c+dx^3}}+\frac{2\sqrt{c+dx^3}}{81cd^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{4\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{5/6}d^{8/3}}-\frac{4\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{5/6}d^{8/3}}+\frac{4\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/6}d^{8/3}}-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\ 3^{3/4}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{81\sqrt[4]{3}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-2/81*x^2/c/d^2/(d*x^3+c)^(1/2)+8/27*x^2/d^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2)+
2/81*(d*x^3+c)^(1/2)/c/d^(8/3)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)+4/243*arcta
n(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/c^(5/6)/d^(
8/3)-4/243*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5
/6)/d^(8/3)+4/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(8/3)-1/8
1*(1/2*6^(1/2)-1/2*2^(1/2))*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*
x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1
/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(
1/4)/c^(2/3)/d^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(
1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+2/243*2^(1/2)*(c^(1/3)+d^(1/3)*x)*((c^(2/
3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)
*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)
,I*3^(1/2)+2*I)*3^(3/4)/c^(2/3)/d^(8/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3
^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.25

$$\int \frac{x^7}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{80cx^2(4c+dx^3)+40cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c}\right.}{3240c^2d^2\,(8c-dx}$$

input
```
Integrate[x^7/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(80*c*x^2*(4*c + d*x^3) + 40*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Appe
llF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c
)])/(3240*c^2*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {970, 27, 1049, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{970}$$

$$\frac{8x^2}{27d^2 (8c-dx^3) \sqrt{c+dx^3}} - \frac{\int \frac{cx(7dx^3+16c)}{(8c-dx^3)(dx^3+c)^{3/2}}dx}{27cd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{8x^2}{27d^2 (8c-dx^3) \sqrt{c+dx^3}} - \frac{\int \frac{x(7dx^3+16c)}{(8c-dx^3)(dx^3+c)^{3/2}}dx}{27d^2}$$

$$\Big\downarrow \text{1049}$$

$$\frac{8x^2}{27d^2 (8c-dx^3) \sqrt{c+dx^3}} - \frac{\frac{2x^2}{3c\sqrt{c+dx^3}} - \frac{2\int -\frac{9cdx\left(dx^3+16c\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{27c^2d}}{27d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{8x^2}{27d^2 (8c-dx^3) \sqrt{c+dx^3}} - \frac{\frac{\int \frac{x\left(dx^3+16c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{3c} + \frac{2x^2}{3c\sqrt{c+dx^3}}}{27d^2}$$

$$\Big\downarrow \text{1054}$$

$$\frac{8x^2}{27d^2 (8c-dx^3) \sqrt{c+dx^3}} - \frac{\frac{\int \left(\frac{24cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}} - \frac{x}{\sqrt{dx^3+c}}\right)dx}{3c} + \frac{2x^2}{3c\sqrt{c+dx^3}}}{27d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{-\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\cdots}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}\cdots}{d^{2/3}\sqrt{\frac{\sqrt[3]{\cdots}}{\left(\left(1+\cdots\right.\right.}}\cdots}}{\cdots}$$

input
```
Int[x^7/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(8*x^2)/(27*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) - ((2*x^2)/(3*c*Sqrt[c + d*
x^3]) + ((-2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)
) - (4*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x
^3]])/(Sqrt[3]*d^(2/3)) + (4*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^
(1/6)*Sqrt[c + d*x^3])])/(3*d^(2/3)) - (4*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(3*d^(2/3)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) +
d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)
*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(
c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3]) - (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^
(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^
(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/(3*c))/
(27*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.99 (sec) , antiderivative size = 910, normalized size of antiderivative = 1.36

method	result	size
elliptic	Expression too large to display	910
default	Expression too large to display	2256

input
```
int(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/243/d^2*x^2/c/((x^3+c/d)*d)^(1/2)+8/243*x^2/c/d^2*(d*x^3+c)^(1/2)/(-d*x^
3+8*c)-2/243*I/d^3/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d
^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(
-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d
*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^
2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+8
/729*I/d^5/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d
^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)
+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2681 vs. $2(475) = 950$.

Time = 0.67 (sec) , antiderivative size = 2681, normalized size of antiderivative = 4.01

$$\int \frac{x^7}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/729*(18*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassZeta(0, -4*c/d
, weierstrassPInverse(0, -4*c/d, x)) + (c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*
d^3 + sqrt(-3)*(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3))*(1/(c^5*d^16))^(1/
6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^13
*x^7 + 64*c^5*d^12*x^4 + 32*c^6*d^11*x + sqrt(-3)*(5*c^4*d^13*x^7 + 64*c^5
*d^12*x^4 + 32*c^6*d^11*x))*(1/(c^5*d^16))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5
*c^5*d^15*x^5 + 32*c^6*d^14*x^2 - sqrt(-3)*(5*c^5*d^15*x^5 + 32*c^6*d^14*x
^2))*(1/(c^5*d^16))^(5/6) - 2*(7*c^3*d^10*x^6 + 152*c^4*d^9*x^3 + 64*c^5*d
^8)*sqrt(1/(c^5*d^16)) + (c*d^5*x^7 + 80*c^2*d^4*x^4 + 160*c^3*d^3*x + sqr
t(-3)*(c*d^5*x^7 + 80*c^2*d^4*x^4 + 160*c^3*d^3*x))*(1/(c^5*d^16))^(1/6))
- 9*(c^2*d^8*x^8 + 38*c^3*d^7*x^5 + 64*c^4*d^6*x^2 - sqrt(-3)*(c^2*d^8*x^8
 + 38*c^3*d^7*x^5 + 64*c^4*d^6*x^2))*(1/(c^5*d^16))^(1/3))/(d^3*x^9 - 24*c
*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*
d^3 + sqrt(-3)*(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3))*(1/(c^5*d^16))^(1/
6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^13
*x^7 + 64*c^5*d^12*x^4 + 32*c^6*d^11*x + sqrt(-3)*(5*c^4*d^13*x^7 + 64*c^5
*d^12*x^4 + 32*c^6*d^11*x))*(1/(c^5*d^16))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5
*c^5*d^15*x^5 + 32*c^6*d^14*x^2 - sqrt(-3)*(5*c^5*d^15*x^5 + 32*c^6*d^14*x
^2))*(1/(c^5*d^16))^(5/6) - 2*(7*c^3*d^10*x^6 + 152*c^4*d^9*x^3 + 64*c^5*d
^8)*sqrt(1/(c^5*d^16)) + (c*d^5*x^7 + 80*c^2*d^4*x^4 + 160*c^3*d^3*x + ...
```

Sympy [F]

$$\int \frac{x^7}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^7}{(-8c+dx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**7/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^7}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^7}{(dx^3+c)^{\frac{3}{2}} (dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x^7}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^7}{(dx^3+c)^{\frac{3}{2}} (dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^7}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{x^7}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^7}{d^4x^{12}-14c\,d^3x^9+33c^2d^2x^6+112c^3d\,x^3+64c^4}dx$$

input
```
int(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**7)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6
 - 14*c*d**3*x**9 + d**4*x**12),x)
```

3.623 $\int \frac{x^4}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 671

$$\begin{aligned}
\int \frac{x^4}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx &= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} \\
&+ \frac{x^2}{27cd(8c-dx^3)\sqrt{c+dx^3}} + \frac{\sqrt{c+dx^3}}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&- \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{11/6}d^{5/3}} + \frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{11/6}d^{5/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{11/6}d^{5/3}} \\
&- \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)\middle|-7-4\sqrt{3}\right)}{54\ 3^{3/4}c^{5/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{81\sqrt[4]{3}c^{5/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
-1/81*x^2/c^2/d/(d*x^3+c)^(1/2)+1/27*x^2/c/d/(-d*x^3+8*c)/(d*x^3+c)^(1/2)+
1/81*(d*x^3+c)^(1/2)/c^2/d^(5/3)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)-1/243*arc
tan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/c^(11/6)/
d^(5/3)+1/243*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(11/6)/d^(5/3)-1/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)/d^(5/3
)-1/162*(1/2*6^(1/2)-1/2*2^(1/2))*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^
(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((
1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*
I)*3^(1/4)/c^(5/3)/d^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/
3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+1/243*2^(1/2)*(c^(1/3)+d^(1/3)*x)*(
(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)
^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1
/3)*x),I*3^(1/2)+2*I)*3^(3/4)/c^(5/3)/d^(5/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)
/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.25

$$\int \frac{x^4}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \frac{80cx^2(-5c+dx^3)+50cx^2(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c}\right.}{6480c^3d\,(8c-dx^3}$$

input
```
Integrate[x^4/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(80*c*x^2*(-5*c + d*x^3) + 50*c*x^2*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*Appe
llF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c
)])/(6480*c^3*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {971, 27, 1049, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(8c-dx^3)^2 (c+dx^3)^{3/2}} dx$$

$$\Big\downarrow \text{971}$$

$$\frac{x^2}{27cd(8c-dx^3)\sqrt{c+dx^3}}-\frac{\int \frac{x(4c-5dx^3)}{2(8c-dx^3)(dx^3+c)^{3/2}}dx}{27cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2}{27cd(8c-dx^3)\sqrt{c+dx^3}}-\frac{\int \frac{x(4c-5dx^3)}{(8c-dx^3)(dx^3+c)^{3/2}}dx}{54cd}$$

$$\Big\downarrow \text{1049}$$

$$\frac{x^2}{27cd(8c-dx^3)\sqrt{c+dx^3}}-\frac{\frac{2x^2}{3c\sqrt{c+dx^3}}-\frac{2\int \frac{9cdx(20c-dx^3)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{27c^2d}}{54cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2}{27cd(8c-dx^3)\sqrt{c+dx^3}}-\frac{\frac{2x^2}{3c\sqrt{c+dx^3}}-\frac{\int \frac{x(20c-dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{3c}}{54cd}$$

$$\Big\downarrow \text{1054}$$

$$\frac{x^2}{27cd(8c-dx^3)\sqrt{c+dx^3}}-\frac{\frac{2x^2}{3c\sqrt{c+dx^3}}-\frac{\int \left(\frac{12cx}{(8c-dx^3)\sqrt{dx^3+c}}+\frac{x}{\sqrt{dx^3+c}}\right)dx}{3c}}{54cd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} -$$

$$\frac{2x^2}{3c\sqrt{c+dx^3}} - \frac{\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right.}{}}{}$$

input

```
Int[x^4/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output

```
x^2/(27*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3]) - ((2*x^2)/(3*c*Sqrt[c + d*x^3]
) - ((2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (
2*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])
/(Sqrt[3]*d^(2/3)) + (2*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)
*Sqrt[c + d*x^3])])/(3*d^(2/3)) - (2*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])])/(3*d^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/
3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/
3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)
*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF
[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)
*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/(3*c))/(54*c
*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.68 (sec) , antiderivative size = 910, normalized size of antiderivative = 1.36

method	result	size
elliptic	Expression too large to display	910
default	Expression too large to display	1790

input

```
int(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/243*x^2/c^2/d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-2/243/d*x^2/c^2/((x^3+c/d)*d)
^(1/2)-1/243*I/d^2/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c
*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1
/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))
-2/729*I/c^2/d^4*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2723 vs. $2(478) = 956$.

Time = 0.57 (sec) , antiderivative size = 2723, normalized size of antiderivative = 4.06

$$\int \frac{x^4}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input

```
integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output

```
-1/2916*(36*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassZeta(0, -4*c/
d, weierstrassPInverse(0, -4*c/d, x)) - (c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c
^4*d^2 + sqrt(-3)*(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2))*(1/(c^11*d^10
))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^
8*d^9*x^7 + 64*c^9*d^8*x^4 + 32*c^10*d^7*x + sqrt(-3)*(5*c^8*d^9*x^7 + 64*
c^9*d^8*x^4 + 32*c^10*d^7*x))*(1/(c^11*d^10))^(2/3) + 3*sqrt(d*x^3 + c)*(6
*(5*c^10*d^10*x^5 + 32*c^11*d^9*x^2 - sqrt(-3)*(5*c^10*d^10*x^5 + 32*c^11*
d^9*x^2))*(1/(c^11*d^10))^(5/6) - 2*(7*c^6*d^7*x^6 + 152*c^7*d^6*x^3 + 64*
c^8*d^5)*sqrt(1/(c^11*d^10)) + (c^2*d^4*x^7 + 80*c^3*d^3*x^4 + 160*c^4*d^2
*x + sqrt(-3)*(c^2*d^4*x^7 + 80*c^3*d^3*x^4 + 160*c^4*d^2*x))*(1/(c^11*d^1
0))^(1/6)) - 9*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2 - sqrt(-3)*(
c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2))*(1/(c^11*d^10))^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^2*d^4*x^6 - 7*c^3*d^
3*x^3 - 8*c^4*d^2 + sqrt(-3)*(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2))*(1
/(c^11*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^
3 - 9*(5*c^8*d^9*x^7 + 64*c^9*d^8*x^4 + 32*c^10*d^7*x + sqrt(-3)*(5*c^8*d^
9*x^7 + 64*c^9*d^8*x^4 + 32*c^10*d^7*x))*(1/(c^11*d^10))^(2/3) - 3*sqrt(d*
x^3 + c)*(6*(5*c^10*d^10*x^5 + 32*c^11*d^9*x^2 - sqrt(-3)*(5*c^10*d^10*x^5
 + 32*c^11*d^9*x^2))*(1/(c^11*d^10))^(5/6) - 2*(7*c^6*d^7*x^6 + 152*c^7*d^
6*x^3 + 64*c^8*d^5)*sqrt(1/(c^11*d^10)) + (c^2*d^4*x^7 + 80*c^3*d^3*x^4...
```

Sympy [F]

$$\int \frac{x^4}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{x^4}{(-8c+dx^3)^2(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**4/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^4}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{x^4}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x^4}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{x^4}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^4}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{x^4}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^4}{d^4x^{12}-14c\,d^3x^9+33c^2d^2x^6+112c^3d\,x^3+64c^4}dx$$

input
```
int(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**4)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6
 - 14*c*d**3*x**9 + d**4*x**12),x)
```

3.624 $\int \frac{x}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 665

$$\int \frac{x}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{5x^2}{648c^3\sqrt{c+dx^3}}+\frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}}$$
$$-\frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{5\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1296\sqrt{3}c^{17/6}d^{2/3}}$$
$$+\frac{5\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{3888c^{17/6}d^{2/3}}-\frac{5\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3888c^{17/6}d^{2/3}}$$
$$+\frac{5\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{432\ 3^{3/4}c^{8/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{5\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{324\sqrt{2}\sqrt[4]{3}c^{8/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
5/648*x^2/c^3/(d*x^3+c)^(1/2)+1/216*x^2/c^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2)-5
/648*(d*x^3+c)^(1/2)/c^3/d^(2/3)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)-5/3888*ar
ctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/c^(17/6)
/d^(2/3)+5/3888*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))
/c^(17/6)/d^(2/3)-5/3888*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)/d^(
2/3)+5/1296*(1/2*6^(1/2)-1/2*2^(1/2))*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3
)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*Elliptic
E(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2
)+2*I)*3^(1/4)/c^(8/3)/d^(2/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c
^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)-5/1944*(c^(1/3)+d^(1/3)*x)*((c^
(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1
/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)
*x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/c^(8/3)/d^(2/3)/(c^(1/3)*(c^(1/3)+d^(1/
3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.25

$$\int \frac{x}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{16cx^2(43c-5dx^3)+5cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx}{c}\right.}{10368c^4\,(8c-dx^3}$$

input
```
Integrate[x/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(16*c*x^2*(43*c - 5*d*x^3) + 5*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Ap
pellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(8*c - d*x^3
)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*
c)])/(10368*c^4*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {972, 27, 1049, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(8c-dx^3)^2(c+dx^3)^{3/2}}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{\int \frac{5dx(dx^3+10c)}{2(8c-dx^3)(dx^3+c)^{3/2}}dx}{216c^2d}+\frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\int \frac{x(dx^3+10c)}{(8c-dx^3)(dx^3+c)^{3/2}}dx}{432c^2}+\frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1049}$$

$$\frac{5\left(\frac{2x^2}{3c\sqrt{c+dx^3}}-\frac{2\int \frac{9cdx\left(2c-dx^3\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{27c^2d}\right)}{432c^2}+\frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\left(\frac{2x^2}{3c\sqrt{c+dx^3}}-\frac{\int \frac{x\left(2c-dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{3c}\right)}{432c^2}+\frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1054}$$

$$\frac{5\left(\frac{2x^2}{3c\sqrt{c+dx^3}}-\frac{\int \left(\frac{x}{\sqrt{dx^3+c}}-\frac{6cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}\right)dx}{3c}\right)}{432c^2}+\frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{2009}$$

$$5\left(\frac{2x^2}{3c\sqrt{c+dx^3}}-\frac{\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\cdots\right.}{\cdots}}{\cdots}\right.$$

$$\frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

input
```
Int[x/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
x^2/(216*c^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (5*((2*x^2)/(3*c*Sqrt[c + d*
x^3]) - ((2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x))
 + (c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]
])/(Sqrt[3]*d^(2/3)) - (c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)
*Sqrt[c + d*x^3])])/(3*d^(2/3)) + (c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt
[c])])/(3*d^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)
*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)
*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*
x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[A
rcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x
))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/(3*c)))/(432*c
^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.11 (sec) , antiderivative size = 904, normalized size of antiderivative = 1.36

method	result	size
default	Expression too large to display	904
elliptic	Expression too large to display	904

input

```
int(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/243*x^2/c^3/((x^3+c/d)*d)^(1/2)+1/1944/c^3*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8
*c)+5/1944*I/c^3*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2
)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I
*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^
(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(
-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)
^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-5/5
832*I/c^3/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*
d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*
(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3
)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2684 vs. $2(472) = 944$.

Time = 0.76 (sec) , antiderivative size = 2684, normalized size of antiderivative = 4.04

$$\int \frac{x}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
1/46656*(360*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassZeta(0, -4*c
/d, weierstrassPInverse(0, -4*c/d, x)) + 5*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 -
8*c^5*d + sqrt(-3)*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d))*(1/(c^17*d^4))
^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^12
*d^5*x^7 + 64*c^13*d^4*x^4 + 32*c^14*d^3*x + sqrt(-3)*(5*c^12*d^5*x^7 + 64
*c^13*d^4*x^4 + 32*c^14*d^3*x))*(1/(c^17*d^4))^(2/3) + 3*sqrt(d*x^3 + c)*(
6*(5*c^15*d^5*x^5 + 32*c^16*d^4*x^2 - sqrt(-3)*(5*c^15*d^5*x^5 + 32*c^16*d
^4*x^2))*(1/(c^17*d^4))^(5/6) - 2*(7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c
^11*d^2)*sqrt(1/(c^17*d^4)) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x
+ sqrt(-3)*(c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x))*(1/(c^17*d^4))^(1
/6)) - 9*(c^6*d^4*x^8 + 38*c^7*d^3*x^5 + 64*c^8*d^2*x^2 - sqrt(-3)*(c^6*d^
4*x^8 + 38*c^7*d^3*x^5 + 64*c^8*d^2*x^2))*(1/(c^17*d^4))^(1/3))/(d^3*x^9 -
 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 5*(c^3*d^3*x^6 - 7*c^4*d^2*x^3
 - 8*c^5*d + sqrt(-3)*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d))*(1/(c^17*d^
4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c
^12*d^5*x^7 + 64*c^13*d^4*x^4 + 32*c^14*d^3*x + sqrt(-3)*(5*c^12*d^5*x^7 +
 64*c^13*d^4*x^4 + 32*c^14*d^3*x))*(1/(c^17*d^4))^(2/3) - 3*sqrt(d*x^3 + c
)*(6*(5*c^15*d^5*x^5 + 32*c^16*d^4*x^2 - sqrt(-3)*(5*c^15*d^5*x^5 + 32*c^1
6*d^4*x^2))*(1/(c^17*d^4))^(5/6) - 2*(7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 6
4*c^11*d^2)*sqrt(1/(c^17*d^4)) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^...
```

Sympy [F]

$$\int \frac{x}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \int \frac{x}{(-8c+dx^3)^2 (c+dx^3)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \int \frac{x}{(dx^3+c)^{\frac{3}{2}} (dx^3-8\,c)^2} \, dx$$

input
```
integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \int \frac{x}{(dx^3+c)^{\frac{3}{2}} (dx^3-8\,c)^2} \, dx$$

input
```
integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{x}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{x}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x}{d^4x^{12}-14c\,d^3x^9+33c^2d^2x^6+112c^3d\,x^3+64c^4}dx$$

input
```
int(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 -
14*c*d**3*x**9 + d**4*x**12),x)
```

3.625 $\int \frac{1}{x^2 (8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 686

$$\int \frac{1}{x^2 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{5}{648c^3x\sqrt{c+dx^3}}+\frac{1}{216c^2x(8c-dx^3)\sqrt{c+dx^3}}-\frac{31\sqrt{c+dx^3}}{1296c^4x}+\frac{31\sqrt[3]{d}\sqrt{c+dx^3}}{1296c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1296\sqrt{3}c^{23/6}}+\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{3888c^{23/6}}-\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3888c^{23/6}}-\frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{864\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{31\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{648\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
5/648/c^3/x/(d*x^3+c)^(1/2)+1/216/c^2/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2)-31/12
96*(d*x^3+c)^(1/2)/c^4/x+31/1296*d^(1/3)*(d*x^3+c)^(1/2)/c^4/((1+3^(1/2))*
c^(1/3)+d^(1/3)*x)-1/3888*d^(1/3)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*
x)/(d*x^3+c)^(1/2))*3^(1/2)/c^(23/6)+1/3888*d^(1/3)*arctanh(1/3*(c^(1/3)+d
^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(23/6)-1/3888*d^(1/3)*arctanh(1/3*(
d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-31/2592*(1/2*6^(1/2)-1/2*2^(1/2))*d^(1/3)
*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))
*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1
+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/c^(11/3)/(c^(1/3)*(c^(
1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+3
1/3888*d^(1/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+
d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/
c^(11/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(
1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.17 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^2 (8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \frac{-80c(162c^2+227cdx^3-31d^2x^6)+650cdx^3(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\text{Appe}}{10368}$$

input
```
Integrate[1/(x^2*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-80*c*(162*c^2 + 227*c*d*x^3 - 31*d^2*x^6) + 650*c*d*x^3*(8*c - d*x^3)*Sq
rt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)]
+ 31*d^2*x^6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3,
 -((d*x^3)/c), (d*x^3)/(8*c)])/(103680*c^5*Sqrt[c + d*x^3]*(8*c*x - d*x^4)
)
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {972, 27, 1049, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 972}$$

$$\frac{\int \frac{d(11dx^3+56c)}{2x^2(8c-dx^3)(dx^3+c)^{3/2}}dx}{216c^2d}+\frac{1}{216c^2x(8c-dx^3)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{11dx^3+56c}{x^2(8c-dx^3)(dx^3+c)^{3/2}}dx}{432c^2}+\frac{1}{216c^2x(8c-dx^3)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 1049}$$

$$\frac{\frac{10}{3cx\sqrt{c+dx^3}}-\frac{2\int -\frac{9cd(248c-25dx^3)}{2x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{27c^2d}}{432c^2}+\frac{1}{216c^2x(8c-dx^3)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{248c-25dx^3}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{3c}+\frac{10}{3cx\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x(8c-dx^3)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{\frac{-\frac{\int -\frac{4cdx(260c-31dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{8c^2}-\frac{31\sqrt{c+dx^3}}{cx}}{3c}+\frac{10}{3cx\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x(8c-dx^3)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{d\int \frac{x\left(260c-31dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{2c}-\frac{31\sqrt{c+dx^3}}{cx}}{3c}+\frac{10}{3cx\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 1054$$

$$\frac{\frac{\frac{d\int \left(\frac{12cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{31x}{\sqrt{dx^3+c}}\right)dx}{2c}-\frac{31\sqrt{c+dx^3}}{cx}}{3c}+\frac{10}{3cx\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 2009$$

$$d\left(\frac{62\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{31\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\cdots}{\left(\left(1+\cdots\right.\right.}}\cdots}{d^{2/3}\sqrt{\cdots}\cdots}\cdots\right.$$

$$\frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

input
```
Int[1/(x^2*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
1/(216*c^2*x*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (10/(3*c*x*Sqrt[c + d*x^3])
+ ((-31*Sqrt[c + d*x^3])/(c*x) + (d*((62*Sqrt[c + d*x^3])/(d^(2/3)*((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)) - (2*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3
) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) + (2*c^(1/6)*ArcTanh[(
c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3*d^(2/3)) - (2*c^(1
/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^(2/3)) - (31*3^(1/4)*Sqrt[2
 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*
x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[(
(1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (62*Sqrt[2]*c^(1/3)*(c^(1/3)
 + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1
/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(
2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*Sqrt[c + d*x^3])))/(2*c))/(3*c))/(432*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.93 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	920
risch	Expression too large to display	2220
default	Expression too large to display	2270

input
```
int(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/243*d*x^2/c^4/((x^3+c/d)*d)^(1/2)+1/15552*d*x^2/c^4*(d*x^3+c)^(1/2)/(-d
*x^3+8*c)-1/64*(d*x^3+c)^(1/2)/c^4/x-31/3888*I/c^4*3^(1/2)*(-c*d^2)^(1/3)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)))-1/5832*I/c^4/d^2*2^(1/2)*sum(1/_alpha*(-c*d^2
)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(
-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3
^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(
-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2534 vs. $2(489) = 978$.

Time = 0.98 (sec) , antiderivative size = 2534, normalized size of antiderivative = 3.69

$$\int \frac{1}{x^2 (8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/46656*(1116*(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)*sqrt(d)*weierstrassZeta(0,
-4*c/d, weierstrassPInverse(0, -4*c/d, x)) - (c^4*d^2*x^7 - 7*c^5*d*x^4 -
8*c^6*x + sqrt(-3)*(c^4*d^2*x^7 - 7*c^5*d*x^4 - 8*c^6*x))*(d^2/c^23)^(1/6)
*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^16*d
^2*x^7 + 64*c^17*d*x^4 + 32*c^18*x + sqrt(-3)*(5*c^16*d^2*x^7 + 64*c^17*d*
x^4 + 32*c^18*x))*(d^2/c^23)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 +
32*c^21*x^2 - sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^2/c^23)^(5/6) - 2*
(7*c^12*d^2*x^6 + 152*c^13*d*x^3 + 64*c^14)*sqrt(d^2/c^23) + (c^4*d^3*x^7
+ 80*c^5*d^2*x^4 + 160*c^6*d*x + sqrt(-3)*(c^4*d^3*x^7 + 80*c^5*d^2*x^4 +
160*c^6*d*x))*(d^2/c^23)^(1/6)) - 9*(c^8*d^3*x^8 + 38*c^9*d^2*x^5 + 64*c^1
0*d*x^2 - sqrt(-3)*(c^8*d^3*x^8 + 38*c^9*d^2*x^5 + 64*c^10*d*x^2))*(d^2/c^
23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^4*d^2*
x^7 - 7*c^5*d*x^4 - 8*c^6*x + sqrt(-3)*(c^4*d^2*x^7 - 7*c^5*d*x^4 - 8*c^6*
x))*(d^2/c^23)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640
*c^3*d - 9*(5*c^16*d^2*x^7 + 64*c^17*d*x^4 + 32*c^18*x + sqrt(-3)*(5*c^16*
d^2*x^7 + 64*c^17*d*x^4 + 32*c^18*x))*(d^2/c^23)^(2/3) - 3*sqrt(d*x^3 + c)
*(6*(5*c^20*d*x^5 + 32*c^21*x^2 - sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(
d^2/c^23)^(5/6) - 2*(7*c^12*d^2*x^6 + 152*c^13*d*x^3 + 64*c^14)*sqrt(d^2/c
^23) + (c^4*d^3*x^7 + 80*c^5*d^2*x^4 + 160*c^6*d*x + sqrt(-3)*(c^4*d^3*x^7
 + 80*c^5*d^2*x^4 + 160*c^6*d*x))*(d^2/c^23)^(1/6)) - 9*(c^8*d^3*x^8 + ...
```

Sympy [F]

$$\int \frac{1}{x^2 (8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \int \frac{1}{x^2 (-8c + dx^3)^2 (c + dx^3)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**2*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \int \frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8c)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \int \frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8c)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \int \frac{1}{x^2 (d x^3 + c)^{3/2} (8c - d x^3)^2} \, dx$$

input
```
int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \frac{-2\sqrt{d\,x^3+c} + 88\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{d^4x^{12}-14c\,d^3x^9+33c^2d^2x^6+112c^3d\,x^3+64c^4}dx\right)c^2d^2x + 77}{}$$

input
```
int(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

output
```
( - 2*sqrt(c + d*x**3) + 88*int((sqrt(c + d*x**3)*x**4)/(64*c**4 + 112*c**
3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*c**2*d**2*x
 + 77*int((sqrt(c + d*x**3)*x**4)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**
2*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*c*d**3*x**4 - 11*int((sqrt(c + d*
x**3)*x**4)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**
9 + d**4*x**12),x)*d**4*x**7 - 256*int((sqrt(c + d*x**3)*x)/(64*c**4 + 112
*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*c**3*d*
x - 224*int((sqrt(c + d*x**3)*x)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2
*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*c**2*d**2*x**4 + 32*int((sqrt(c +
d*x**3)*x)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9
 + d**4*x**12),x)*c*d**3*x**7)/(16*c**2*x*(8*c**2 + 7*c*d*x**3 - d**2*x**6
))
```

3.626 $\int \frac{1}{x^5 (8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 708

$$\int \frac{1}{x^5 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4(8c-dx^3)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} + \frac{77d\sqrt{c+dx^3}}{2592c^5x} - \frac{77d^{4/3}\sqrt{c+dx^3}}{2592c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{11d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{82944\sqrt{3}c^{29/6}} + \frac{11d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{248832c^{29/6}} - \frac{11d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{248832c^{29/6}} + \frac{77\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{1728\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{77d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{1296\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
5/648/c^3/x^4/(d*x^3+c)^(1/2)+1/216/c^2/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2)-2
53/20736*(d*x^3+c)^(1/2)/c^4/x^4+77/2592*d*(d*x^3+c)^(1/2)/c^5/x-77/2592*d
^(4/3)*(d*x^3+c)^(1/2)/c^5/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)-11/248832*d^(4/
3)*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/c^(
29/6)+11/248832*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c
)^(1/2))/c^(29/6)-11/248832*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c
^(29/6)+77/5184*(1/2*6^(1/2)-1/2*2^(1/2))*d^(4/3)*(c^(1/3)+d^(1/3)*x)*((c^
(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1
/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)
*x),I*3^(1/2)+2*I)*3^(1/4)/c^(14/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/
2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)-77/7776*d^(4/3)*(c^(1/3)+d
^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^
(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c
^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/c^(14/3)/(c^(1/3)*(c^(1/3
)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^5 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{-24475cd^2x^6(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-16}{}$$

input
```
Integrate[1/(x^5*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-24475*c*d^2*x^6*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1,
5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 16*(10*c*(648*c^3 - 2997*c^2*d*x^3 - 4
565*c*d^2*x^6 + 616*d^3*x^9) + 77*d^3*x^9*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/
c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(3317760*c^6*
x^4*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 2.07 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {972, 27, 1049, 27, 1053, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 972}$$

$$\frac{\int \frac{d\left(17dx^3+62c\right)}{2x^5\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx}{216c^2d}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{17dx^3+62c}{x^5\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx}{432c^2}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 1049}$$

$$\frac{\frac{10}{3cx^4\sqrt{c+dx^3}}-\frac{2\int -\frac{99cd\left(46c-5dx^3\right)}{2x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{27c^2d}}{432c^2}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{11\int \frac{46c-5dx^3}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{3c}+\frac{10}{3cx^4\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{\frac{11\left(-\frac{\int \frac{cd\left(896c-115dx^3\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{3c}+\frac{10}{3cx^4\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{11\left(-\frac{d\int \frac{896c-115dx^3}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{3c}+\frac{10}{3cx^4\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$\Big\downarrow$ 1053

$$\frac{\frac{11\left(-\frac{d\left(-\frac{\int -\frac{8cdx\left(445c-56dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{112\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{3c}+\frac{10}{3cx^4\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$\Big\downarrow$ 27

$$\frac{\frac{11\left(-\frac{d\left(\frac{d\int \frac{x\left(445c-56dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{112\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{3c}+\frac{10}{3cx^4\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$\Big\downarrow$ 1054

$$\frac{\frac{11\left(-\frac{d\left(\frac{d\int \left(\frac{56x}{\sqrt{dx^3+c}}-\frac{3cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}\right)dx}{c}-\frac{112\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{3c}+\frac{10}{3cx^4\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$\Big\downarrow$ 2009

$$11\left(-\frac{d\left(\frac{d\left(\frac{112\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{56\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\cdots\right.}{\cdots}\right.}{d\cdots}\right.}{\cdots}\right.$$

$$\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

input

```
Int[1/(x^5*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
1/(216*c^2*x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (10/(3*c*x^4*Sqrt[c + d*x^
3]) + (11*((-23*Sqrt[c + d*x^3])/(16*c*x^4) - (d*((-112*Sqrt[c + d*x^3])/(
c*x) + (d*((112*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)) + (c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*
x^3]])/(2*Sqrt[3]*d^(2/3)) - (c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c
^(1/6)*Sqrt[c + d*x^3])])/(6*d^(2/3)) + (c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(
3*Sqrt[c])])/(6*d^(2/3)) - (56*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3)
+ d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt
[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sq
rt[c + d*x^3]) + (112*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
- c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3)
+ d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/c)
)/(32*c)))/(3*c))/(432*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.80 (sec) , antiderivative size = 943, normalized size of antiderivative = 1.33

method	result	size
elliptic	Expression too large to display	943
risch	Expression too large to display	2232
default	Expression too large to display	2775

input
```
int(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/243*d^2*x^2/c^5/((x^3+c/d)*d)^(1/2)+1/124416*d^2*x^2/c^5*(d*x^3+c)^(1/2)
/(-d*x^3+8*c)-1/256*(d*x^3+c)^(1/2)/c^4/x^4+11/512*d*(d*x^3+c)^(1/2)/c^5/x
+77/7776*I*d/c^5*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^
(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(
x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c
*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(
1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-11/37
3248*I/c^5/d*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d
^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)
+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2692 vs. $2(507) = 1014$.

Time = 3.30 (sec) , antiderivative size = 2692, normalized size of antiderivative = 3.80

$$\int \frac{1}{x^5 (8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
1/2985984*(88704*(d^3*x^10 - 7*c*d^2*x^7 - 8*c^2*d*x^4)*sqrt(d)*weierstras
sZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 11*(c^5*d^2*x^10 - 7
*c^6*d*x^7 - 8*c^7*x^4 + sqrt(-3)*(c^5*d^2*x^10 - 7*c^6*d*x^7 - 8*c^7*x^4)
)*(d^8/c^29)^(1/6)*log(161051*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3
+ 640*c^3*d^6 - 9*(5*c^20*d^3*x^7 + 64*c^21*d^2*x^4 + 32*c^22*d*x + sqrt(-
3)*(5*c^20*d^3*x^7 + 64*c^21*d^2*x^4 + 32*c^22*d*x))*(d^8/c^29)^(2/3) + 3*
sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2 - sqrt(-3)*(5*c^25*d*x^5 +
32*c^26*x^2))*(d^8/c^29)^(5/6) - 2*(7*c^15*d^4*x^6 + 152*c^16*d^3*x^3 + 64
*c^17*d^2)*sqrt(d^8/c^29) + (c^5*d^7*x^7 + 80*c^6*d^6*x^4 + 160*c^7*d^5*x
+ sqrt(-3)*(c^5*d^7*x^7 + 80*c^6*d^6*x^4 + 160*c^7*d^5*x))*(d^8/c^29)^(1/6
)) - 9*(c^10*d^6*x^8 + 38*c^11*d^5*x^5 + 64*c^12*d^4*x^2 - sqrt(-3)*(c^10*
d^6*x^8 + 38*c^11*d^5*x^5 + 64*c^12*d^4*x^2))*(d^8/c^29)^(1/3))/(d^3*x^9 -
 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 11*(c^5*d^2*x^10 - 7*c^6*d*x^7
 - 8*c^7*x^4 + sqrt(-3)*(c^5*d^2*x^10 - 7*c^6*d*x^7 - 8*c^7*x^4))*(d^8/c^2
9)^(1/6)*log(161051*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*
d^6 - 9*(5*c^20*d^3*x^7 + 64*c^21*d^2*x^4 + 32*c^22*d*x + sqrt(-3)*(5*c^20
*d^3*x^7 + 64*c^21*d^2*x^4 + 32*c^22*d*x))*(d^8/c^29)^(2/3) - 3*sqrt(d*x^3
 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2 - sqrt(-3)*(5*c^25*d*x^5 + 32*c^26*x^
2))*(d^8/c^29)^(5/6) - 2*(7*c^15*d^4*x^6 + 152*c^16*d^3*x^3 + 64*c^17*d^2)
*sqrt(d^8/c^29) + (c^5*d^7*x^7 + 80*c^6*d^6*x^4 + 160*c^7*d^5*x + sqrt(...
```

Sympy [F]

$$\int \frac{1}{x^5 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^5 (-8c+dx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**5*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2x^5}\,dx$$

input
```
integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2x^5}\,dx$$

input
```
integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^5\,(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^5 (8c - dx^3)^2 (c + dx^3)^{3/2}}\,dx = \frac{-16\sqrt{d\,x^3+c}\,c + 74\sqrt{d\,x^3+c}\,d\,x^3 - 3256\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{d^4x^{12}-14c\,d^3x^9+33c^2d^2x^6+112c}\right.}{}$$

input
```
int(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

output
```
( - 16*sqrt(c + d*x**3)*c + 74*sqrt(c + d*x**3)*d*x**3 - 3256*int((sqrt(c
+ d*x**3)*x**4)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3
*x**9 + d**4*x**12),x)*c**2*d**3*x**4 - 2849*int((sqrt(c + d*x**3)*x**4)/(
64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**1
2),x)*c*d**4*x**7 + 407*int((sqrt(c + d*x**3)*x**4)/(64*c**4 + 112*c**3*d*
x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*d**5*x**10 + 10
560*int((sqrt(c + d*x**3)*x)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**
6 - 14*c*d**3*x**9 + d**4*x**12),x)*c**3*d**2*x**4 + 9240*int((sqrt(c + d*
x**3)*x)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 +
 d**4*x**12),x)*c**2*d**3*x**7 - 1320*int((sqrt(c + d*x**3)*x)/(64*c**4 +
112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*c*d*
*4*x**10)/(512*c**3*x**4*(8*c**2 + 7*c*d*x**3 - d**2*x**6))
```

3.627 $\int \frac{1}{x^8 (8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 732

$$\int \frac{1}{x^8 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7(8c-dx^3)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} - \frac{5179d^2\sqrt{c+dx^3}}{145152c^6x} + \frac{5179d^{7/3}\sqrt{c+dx^3}}{145152c^6\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{7d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{331776\sqrt{3}c^{35/6}} + \frac{7d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{995328c^{35/6}} - \frac{7d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{995328c^{35/6}} - \frac{5179\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{96768\ 3^{3/4}c^{17/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{5179d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{72576\sqrt{2}\sqrt[4]{3}c^{17/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
5/648/c^3/x^7/(d*x^3+c)^(1/2)+1/216/c^2/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2)-1
91/18144*(d*x^3+c)^(1/2)/c^4/x^7+8257/580608*d*(d*x^3+c)^(1/2)/c^5/x^4-517
9/145152*d^2*(d*x^3+c)^(1/2)/c^6/x+5179/145152*d^(7/3)*(d*x^3+c)^(1/2)/c^6
/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)-7/995328*d^(7/3)*arctan(3^(1/2)*c^(1/6)*(
c^(1/3)+d^(1/3)*x)/(d*x^3+c)^(1/2))*3^(1/2)/c^(35/6)+7/995328*d^(7/3)*arct
anh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(35/6)-7/995328*d
^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(35/6)-5179/290304*(1/2*6^(1
/2)-1/2*2^(1/2))*d^(7/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d
^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2)
)*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4
)/c^(17/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)
^(1/2)/(d*x^3+c)^(1/2)+5179/435456*d^(7/3)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c
^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*Ell
ipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3
^(1/2)+2*I)*2^(1/2)*3^(3/4)/c^(17/3)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1
/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.21 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^8 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{829375cd^3x^9(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-8\big(}{}$$

input
```
Integrate[1/(x^8*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(829375*c*d^3*x^9*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1,
5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 8*(20*c*(10368*c^4 - 18792*c^3*d*x^3 +
 101817*c^2*d^2*x^6 + 153269*c*d^3*x^9 - 20716*d^4*x^12) + 5179*d^4*x^12*(
8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c),
(d*x^3)/(8*c)]))/(92897280*c^7*x^7*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 757, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {972, 27, 1049, 27, 1053, 27, 1053, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 972}$$

$$\frac{\int \frac{d\left(23dx^3+68c\right)}{2x^8\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx}{216c^2d}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{23dx^3+68c}{x^8\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx}{432c^2}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 1049}$$

$$\frac{\frac{10}{3cx^7\sqrt{c+dx^3}}-\frac{2\int -\frac{9cd\left(764c-85dx^3\right)}{2x^8\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{27c^2d}}{432c^2}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{764c-85dx^3}{x^8\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{3c}+\frac{10}{3cx^7\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{\frac{-\frac{\int \frac{2cd\left(16514c-2101dx^3\right)}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{56c^2}-\frac{191\sqrt{c+dx^3}}{14cx^7}}{3c}+\frac{10}{3cx^7\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{-\frac{d\int \frac{16514c-2101dx^3}{x^5(8c-dx^3)\sqrt{dx^3+c}}dx}{28c}-\frac{191\sqrt{c+dx^3}}{14cx^7}}{3c}+\frac{10}{3cx^7\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^7(8c-dx^3)\sqrt{c+dx^3}}$$

$\downarrow$ 1053

$$\frac{\frac{-\frac{d\left(-\frac{\int \frac{cd(331456c-41285dx^3)}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c^2}-\frac{8257\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{191\sqrt{c+dx^3}}{14cx^7}}{3c}+\frac{10}{3cx^7\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^7(8c-dx^3)\sqrt{c+dx^3}}$$

$\downarrow$ 27

$$\frac{\frac{-\frac{d\left(-\frac{d\int \frac{331456c-41285dx^3}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c}-\frac{8257\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{191\sqrt{c+dx^3}}{14cx^7}}{3c}+\frac{10}{3cx^7\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^7(8c-dx^3)\sqrt{c+dx^3}}$$

$\downarrow$ 1053

$$\frac{\frac{-\frac{d\left(-\frac{d\left(-\frac{\int -\frac{8cdx(165875c-20716dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{8c^2}-\frac{41432\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{8257\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{191\sqrt{c+dx^3}}{14cx^7}}{3c}+\frac{10}{3cx^7\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^7(8c-dx^3)\sqrt{c+dx^3}}$$

$\downarrow$ 27

$$\frac{\frac{-\frac{d\left(-\frac{d\left(\frac{d\int \frac{x(165875c-20716dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{c}-\frac{41432\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{8257\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{191\sqrt{c+dx^3}}{14cx^7}}{3c}+\frac{10}{3cx^7\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^7(8c-dx^3)\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 1054$$

$$\frac{-\frac{d\left(-\frac{d\left(\frac{d\int\left(\frac{147cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{20716x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{41432\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{8257\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{191\sqrt{c+dx^3}}{14cx^7}}{3c}+\frac{10}{3cx^7\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{d\left(-\frac{d\left(\frac{d\left(\frac{41432\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{20716\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}\ldots\right.}{\ldots}\ldots\right)}{\ldots}\ldots\right)}{\ldots}\ldots\right)}{\ldots}$$

$$\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

input
```
Int[1/(x^8*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
1/(216*c^2*x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (10/(3*c*x^7*Sqrt[c + d*x^
3]) + ((-191*Sqrt[c + d*x^3])/(14*c*x^7) - (d*((-8257*Sqrt[c + d*x^3])/(16
*c*x^4) - (d*((-41432*Sqrt[c + d*x^3])/(c*x) + (d*((41432*Sqrt[c + d*x^3])
/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (49*c^(1/6)*ArcTan[(Sqrt[
3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*Sqrt[3]*d^(2/3)) +
(49*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/
(6*d^(2/3)) - (49*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*d^(2/3)
) - (20716*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) +
 d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (41
432*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*
x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/c))/(32*c)))/(28*c))
/(3*c))/(432*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.80 (sec) , antiderivative size = 962, normalized size of antiderivative = 1.31

method	result	size
elliptic	Expression too large to display	962
risch	Expression too large to display	2243
default	Expression too large to display	3300

input
```
int(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/448*(d*x^3+c)^(1/2)/c^4/x^7+43/7168*d*(d*x^3+c)^(1/2)/c^5/x^4-787/28672
*d^2*(d*x^3+c)^(1/2)/c^6/x-2/243*d^3/c^6*x^2/((x^3+c/d)*d)^(1/2)+1/995328*
d^3*x^2/c^6*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-5179/435456*I/c^6*d^2*3^(1/2)*(-c
*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-7/1492992*I/c^6*2^(1/2)*sum(1/_al
pha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+
I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/
3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)
*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2725 vs. $2(527) = 1054$.

Time = 5.80 (sec) , antiderivative size = 2725, normalized size of antiderivative = 3.72

$$\int \frac{1}{x^8 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/83607552*(2983104*(d^4*x^13 - 7*c*d^3*x^10 - 8*c^2*d^2*x^7)*sqrt(d)*wei
erstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 49*(c^6*d^2*x
^13 - 7*c^7*d*x^10 - 8*c^8*x^7 + sqrt(-3)*(c^6*d^2*x^13 - 7*c^7*d*x^10 - 8
*c^8*x^7))*(d^14/c^35)^(1/6)*log(16807*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c
^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4 + 32*c^26
*d^2*x + sqrt(-3)*(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4 + 32*c^26*d^2*x))*(d^1
4/c^35)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^30*d*x^5 + 32*c^31*x^2 - sqrt(-3
)*(5*c^30*d*x^5 + 32*c^31*x^2))*(d^14/c^35)^(5/6) - 2*(7*c^18*d^6*x^6 + 15
2*c^19*d^5*x^3 + 64*c^20*d^4)*sqrt(d^14/c^35) + (c^6*d^11*x^7 + 80*c^7*d^1
0*x^4 + 160*c^8*d^9*x + sqrt(-3)*(c^6*d^11*x^7 + 80*c^7*d^10*x^4 + 160*c^8
*d^9*x))*(d^14/c^35)^(1/6)) - 9*(c^12*d^9*x^8 + 38*c^13*d^8*x^5 + 64*c^14*
d^7*x^2 - sqrt(-3)*(c^12*d^9*x^8 + 38*c^13*d^8*x^5 + 64*c^14*d^7*x^2))*(d^
14/c^35)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 49*(
c^6*d^2*x^13 - 7*c^7*d*x^10 - 8*c^8*x^7 + sqrt(-3)*(c^6*d^2*x^13 - 7*c^7*d
*x^10 - 8*c^8*x^7))*(d^14/c^35)^(1/6)*log(16807*(d^14*x^9 + 318*c*d^13*x^6
 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4
+ 32*c^26*d^2*x + sqrt(-3)*(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4 + 32*c^26*d^2
*x))*(d^14/c^35)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^30*d*x^5 + 32*c^31*x^2
- sqrt(-3)*(5*c^30*d*x^5 + 32*c^31*x^2))*(d^14/c^35)^(5/6) - 2*(7*c^18*d^6
*x^6 + 152*c^19*d^5*x^3 + 64*c^20*d^4)*sqrt(d^14/c^35) + (c^6*d^11*x^7 ...
```

Sympy [F]

$$\int \frac{1}{x^8 (8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \int \frac{1}{x^8 (-8c + dx^3)^2 (c + dx^3)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**8*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^8 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2 x^8}\,dx$$

input
```
integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2 x^8}\,dx$$

input
```
integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^8\,(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^8 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{-32\sqrt{d\,x^3+c}\,c^2+58\sqrt{d\,x^3+c}\,cd\,x^3-46\sqrt{d\,x^3+c}\,d^2x^6+17168\left(\int \frac{}{d}\right.}{}$$

input
```
int(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

output
```
( - 32*sqrt(c + d*x**3)*c**2 + 58*sqrt(c + d*x**3)*c*d*x**3 - 46*sqrt(c +
d*x**3)*d**2*x**6 + 17168*int(sqrt(c + d*x**3)/(64*c**4*x**2 + 112*c**3*d*
x**5 + 33*c**2*d**2*x**8 - 14*c*d**3*x**11 + d**4*x**14),x)*c**4*d**2*x**7
 + 15022*int(sqrt(c + d*x**3)/(64*c**4*x**2 + 112*c**3*d*x**5 + 33*c**2*d*
*2*x**8 - 14*c*d**3*x**11 + d**4*x**14),x)*c**3*d**3*x**10 - 2146*int(sqrt
(c + d*x**3)/(64*c**4*x**2 + 112*c**3*d*x**5 + 33*c**2*d**2*x**8 - 14*c*d*
*3*x**11 + d**4*x**14),x)*c**2*d**4*x**13 + 2024*int((sqrt(c + d*x**3)*x**
4)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*
x**12),x)*c**2*d**4*x**7 + 1771*int((sqrt(c + d*x**3)*x**4)/(64*c**4 + 112
*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*c*d**5*
x**10 - 253*int((sqrt(c + d*x**3)*x**4)/(64*c**4 + 112*c**3*d*x**3 + 33*c*
*2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*d**6*x**13 - 9832*int((sqrt
(c + d*x**3)*x)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3
*x**9 + d**4*x**12),x)*c**3*d**3*x**7 - 8603*int((sqrt(c + d*x**3)*x)/(64*
c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),
x)*c**2*d**4*x**10 + 1229*int((sqrt(c + d*x**3)*x)/(64*c**4 + 112*c**3*d*x
**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*c*d**5*x**13)/(1
792*c**4*x**7*(8*c**2 + 7*c*d*x**3 - d**2*x**6))
```

3.628 $\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 256

$$\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{2x(4c+dx^3)}{81cd^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{81\sqrt[4]{3}cd^{7/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
2/81*x*(d*x^3+4*c)/c/d^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2)-2/243*(1/2*6^(1/2)+1
/2*2^(1/2))*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(
(1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(
1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/c/d^(7/3)/(
c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^
3+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.65 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.74

$$\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx =$$

$$-\frac{6\sqrt[3]{-d}x(4c+dx^3)+2i3^{3/4}\sqrt[3]{c}\sqrt{\frac{(-1)^{5/6}\left(-\sqrt[3]{c}+\sqrt[3]{-d}x\right)}{\sqrt[3]{c}}}\sqrt{1+\frac{\sqrt[3]{-d}x}{\sqrt[3]{c}}+\frac{(-d)^{2/3}x^2}{c^{2/3}}}(-8c+dx^3)\,\text{EllipticF}\Bigg(\text{ar}}{243c(-d)^{7/3}(-8c+dx^3)\sqrt{c+dx^3}}$$

input
```
Integrate[x^6/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
-1/243*(6*(-d)^(1/3)*x*(4*c + d*x^3) + (2*I)*3^(3/4)*c^(1/3)*Sqrt[((-1)^(5
/6)*(-c^(1/3) + (-d)^(1/3)*x))/c^(1/3)]*Sqrt[1 + ((-d)^(1/3)*x)/c^(1/3) +
((-d)^(2/3)*x^2)/c^(2/3)]*(-8*c + d*x^3)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6)
 - (I*(-d)^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)^(1/3)])/(c*(-d)^(7/3)*(-8*c +
d*x^3)*Sqrt[c + d*x^3])
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^6}{(8c-dx^3)^2\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 1012$$

$$\frac{x^7\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{7}{3},2,\frac{3}{2},\frac{10}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^3\sqrt{c+dx^3}}$$

input
```
Int[x^6/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(x^7*Sqrt[1 + (d*x^3)/c]*AppellF1[7/3, 2, 3/2, 10/3, (d*x^3)/(8*c), -((d*x
^3)/c)])/(448*c^3*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.32

method	result
elliptic	$\frac{2x}{243d^2c\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{8x\sqrt{d\,x^3+c}}{243c\,d^2\left(-d\,x^3+8c\right)}+\frac{2i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^2\right)}{2d}}}}{}$
default	Expression too large to display

input
```
int(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/243/d^2*x/c/((x^3+c/d)*d)^(1/2)+8/243*x/c/d^2*(d*x^3+c)^(1/2)/(-d*x^3+8*
c)+2/243*I/d^3/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^
(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(
x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.35

$$\int \frac{x^6}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = -\frac{2\left(\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{d}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,c}{d},x\right)+\left(d^2x^4+4\,cdx\right)\sqrt{dx^3+c}\right)}{81\left(cd^5x^6-7\,c^2d^4x^3-8\,c^3d^3\right)}$$

input
```
integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/81*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassPInverse(0, -4*c/d
, x) + (d^2*x^4 + 4*c*d*x)*sqrt(d*x^3 + c))/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8
*c^3*d^3)
```

Sympy [F]

$$\int \frac{x^6}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^6}{(-8c+dx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**6/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^6}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^6}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^6/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x^6}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^6}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^6/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^6}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(x^6/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x^6/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{x^6}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^6}{d^4x^{12}-14c\,d^3x^9+33c^2d^2x^6+112c^3d\,x^3+64c^4}dx$$

input
```
int(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**6)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6
 - 14*c*d**3*x**9 + d**4*x**12),x)
```

3.629 $\int \frac{x^3}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\, dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{x^3}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\, dx = \frac{x^4 \sqrt{1+\frac{dx^3}{c}}\, \text{AppellF1}\left(\frac{4}{3},2,\frac{3}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^3\sqrt{c+dx^3}}$$

output
```
1/256*x^4*(1+d*x^3/c)^(1/2)*AppellF1(4/3,3/2,2,7/3,-d*x^3/c,1/8*d*x^3/c)/c
^3/(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 242 vs. 2(66) = 132.

Time = 10.33 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.67

$$\int \frac{x^3}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\, dx = \frac{x\left(\frac{3x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3} + \frac{192\left(\frac{-5c+dx^3}{c^2} + \frac{\cdots}{32c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\cdots\right.}\right.}{\cdots}\right.}{15552\sqrt{c+dx^3}\cdots}$$

input
```
Integrate[x^3/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(x*((3*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d
*x^3)/(8*c)])/c^3 + (192*((-5*c + d*x^3)/c^2 + (160*AppellF1[1/3, 1/2, 1,
4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x
^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)
]))))/(d*(8*c - d*x^3))))/(15552*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{x^3}{(8c-dx^3)^2 \left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4 \sqrt{\frac{dx^3}{c}+1} \operatorname{AppellF1}\left(\frac{4}{3},2,\frac{3}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^3\sqrt{c+dx^3}}$$

input
```
Int[x^3/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 3/2, 7/3, (d*x^3)/(8*c), -((d*x^
3)/c)])/(256*c^3*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.70 (sec) , antiderivative size = 754, normalized size of antiderivative = 11.42

method	result	size
elliptic	Expression too large to display	754
default	Expression too large to display	1480

input
```
int(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/243/d*x/c^2/((x^3+c/d)*d)^(1/2)+1/243*x/c^2/d*(d*x^3+c)^(1/2)/(-d*x^3+8
*c)+1/243*I/d^2/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^
2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-
I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2))-1/243*I/c^2/d^4*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I
*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1
/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))
)^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(
1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))
*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alph
a^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/
3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2713 vs. $2(52) = 104$.

Time = 0.87 (sec) , antiderivative size = 2713, normalized size of antiderivative = 41.11

$$\int \frac{x^3}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3888*(24*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassPInverse(0, -4
*c/d, x) + (c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2 + sqrt(-3)*(c^2*d^4*x^
6 - 7*c^3*d^3*x^3 - 8*c^4*d^2))*(1/(c^13*d^8))^(1/6)*log((d^3*x^9 + 318*c*
d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^9*d^8*x^8 + 38*c^10*d^7*x^5 + 64
*c^11*d^6*x^2 + sqrt(-3)*(c^9*d^8*x^8 + 38*c^10*d^7*x^5 + 64*c^11*d^6*x^2)
)*(1/(c^13*d^8))^(2/3) + 3*sqrt(d*x^3 + c)*((c^11*d^9*x^7 + 80*c^12*d^8*x^
4 + 160*c^13*d^7*x - sqrt(-3)*(c^11*d^9*x^7 + 80*c^12*d^8*x^4 + 160*c^13*d
^7*x))*(1/(c^13*d^8))^(5/6) - 2*(7*c^7*d^6*x^6 + 152*c^8*d^5*x^3 + 64*c^9*
d^4)*sqrt(1/(c^13*d^8)) + 6*(5*c^3*d^3*x^5 + 32*c^4*d^2*x^2 + sqrt(-3)*(5*
c^3*d^3*x^5 + 32*c^4*d^2*x^2))*(1/(c^13*d^8))^(1/6)) - 9*(5*c^5*d^5*x^7 +
64*c^6*d^4*x^4 + 32*c^7*d^3*x - sqrt(-3)*(5*c^5*d^5*x^7 + 64*c^6*d^4*x^4 +
 32*c^7*d^3*x))*(1/(c^13*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*
x^3 - 512*c^3)) - (c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2 + sqrt(-3)*(c^2
*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2))*(1/(c^13*d^8))^(1/6)*log((d^3*x^9 +
 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^9*d^8*x^8 + 38*c^10*d^7*x
^5 + 64*c^11*d^6*x^2 + sqrt(-3)*(c^9*d^8*x^8 + 38*c^10*d^7*x^5 + 64*c^11*d
^6*x^2))*(1/(c^13*d^8))^(2/3) - 3*sqrt(d*x^3 + c)*((c^11*d^9*x^7 + 80*c^12
*d^8*x^4 + 160*c^13*d^7*x - sqrt(-3)*(c^11*d^9*x^7 + 80*c^12*d^8*x^4 + 160
*c^13*d^7*x))*(1/(c^13*d^8))^(5/6) - 2*(7*c^7*d^6*x^6 + 152*c^8*d^5*x^3 +
64*c^9*d^4)*sqrt(1/(c^13*d^8)) + 6*(5*c^3*d^3*x^5 + 32*c^4*d^2*x^2 + sq...
```

Sympy [F]

$$\int \frac{x^3}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx=\int \frac{x^3}{(-8c+dx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**3/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^3}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \int \frac{x^3}{(dx^3+c)^{\frac{3}{2}} (dx^3-8\,c)^2} \, dx$$

input
```
integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x^3}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \int \frac{x^3}{(dx^3+c)^{\frac{3}{2}} (dx^3-8\,c)^2} \, dx$$

input
```
integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \int \frac{x^3}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2} \, dx$$

input
```
int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{x^3}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^3}{d^4x^{12}-14c\,d^3x^9+33c^2d^2x^6+112c^3d\,x^3+64c^4}dx$$

input
```
int(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**3)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6
 - 14*c*d**3*x**9 + d**4*x**12),x)
```

3.630 $\int \frac{1}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{3}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^3\sqrt{c+dx^3}}$$

output
```
1/64*x*(1+d*x^3/c)^(1/2)*AppellF1(1/3,3/2,2,4/3,-d*x^3/c,1/8*d*x^3/c)/c^3/
(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 253 vs. 2(64) = 128.

Time = 10.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.95

$$\int \frac{1}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx=\frac{x\left(-15dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+192c\left(\frac{-43c+5dx^3}{-8c+dx^3}+\right.\right.}{}$$

input
```
Integrate[1/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(x*(-15*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] + 192*c*((-43*c + 5*d*x^3)/(-8*c + d*x^3) + (1216*c^2*Appe
llF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*
AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF
1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1,
 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))))/(124416*c^4*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{1}{(8c-dx^3)^2 \left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt{\frac{dx^3}{c}+1} \operatorname{AppellF1}\left(\frac{1}{3},2,\frac{3}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^3\sqrt{c+dx^3}}$$

input
```
Int[1/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 3/2, 4/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(64*c^3*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.10 (sec) , antiderivative size = 748, normalized size of antiderivative = 11.69

method	result	size
default	Expression too large to display	748
elliptic	Expression too large to display	748

input
```
int(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/243*x/c^3/((x^3+c/d)*d)^(1/2)+1/1944/c^3*x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-
5/1944*I/c^3*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1
/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+
1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3
)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/
2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
)^(1/2))-1/972*I/c^3/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2
*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(
d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/
2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*
(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3
^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_a
lpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2640 vs. $2(50) = 100$.

Time = 0.78 (sec) , antiderivative size = 2640, normalized size of antiderivative = 41.25

$$\int \frac{1}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
1/15552*(192*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassPInverse(0,
-4*c/d, x) + (c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d + sqrt(-3)*(c^3*d^3*x^
6 - 7*c^4*d^2*x^3 - 8*c^5*d))*(1/(c^19*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^
2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*
c^15*d^2*x^2 + sqrt(-3)*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2)
)*(1/(c^19*d^2))^(2/3) + 3*sqrt(d*x^3 + c)*((c^16*d^4*x^7 + 80*c^17*d^3*x^
4 + 160*c^18*d^2*x - sqrt(-3)*(c^16*d^4*x^7 + 80*c^17*d^3*x^4 + 160*c^18*d
^2*x))*(1/(c^19*d^2))^(5/6) - 2*(7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 64*c^
12*d)*sqrt(1/(c^19*d^2)) + 6*(5*c^4*d^2*x^5 + 32*c^5*d*x^2 + sqrt(-3)*(5*c
^4*d^2*x^5 + 32*c^5*d*x^2))*(1/(c^19*d^2))^(1/6)) - 9*(5*c^7*d^3*x^7 + 64*
c^8*d^2*x^4 + 32*c^9*d*x - sqrt(-3)*(5*c^7*d^3*x^7 + 64*c^8*d^2*x^4 + 32*c
^9*d*x))*(1/(c^19*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 5
12*c^3)) - (c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d + sqrt(-3)*(c^3*d^3*x^6
- 7*c^4*d^2*x^3 - 8*c^5*d))*(1/(c^19*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*
x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^
15*d^2*x^2 + sqrt(-3)*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2))*
(1/(c^19*d^2))^(2/3) - 3*sqrt(d*x^3 + c)*((c^16*d^4*x^7 + 80*c^17*d^3*x^4
+ 160*c^18*d^2*x - sqrt(-3)*(c^16*d^4*x^7 + 80*c^17*d^3*x^4 + 160*c^18*d^2
*x))*(1/(c^19*d^2))^(5/6) - 2*(7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 64*c^12
*d)*sqrt(1/(c^19*d^2)) + 6*(5*c^4*d^2*x^5 + 32*c^5*d*x^2 + sqrt(-3)*(5*...
```

Sympy [F]

$$\int \frac{1}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(-8c+dx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{1}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{1}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}}{d^4x^{12}-14c\,d^3x^9+33c^2d^2x^6+112c^3d\,x^3+64c^4}dx$$

input
```
int(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

output
```
int(sqrt(c + d*x**3)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c
*d**3*x**9 + d**4*x**12),x)
```

3.631 $\int \frac{1}{x^3 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^3 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},2,\frac{3}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^3x^2\sqrt{c+dx^3}}$$

output
```
-1/128*(1+d*x^3/c)^(1/2)*AppellF1(-2/3,3/2,2,1/3,-d*x^3/c,1/8*d*x^3/c)/c^3
/x^2/(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 259 vs. 2(66) = 132.

Time = 10.28 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.92

$$\int \frac{1}{x^3 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{167d^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+\frac{64c\left(-648c^2-1249cdx^3\cdots\right.}{\cdots}}{66\cdots}$$

input
```
Integrate[1/(x^3*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(167*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
(d*x^3)/(8*c)] + (64*c*(-648*c^2 - 1249*c*d*x^3 + 167*d^2*x^6 - (19648*c^2
*d*x^3*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*Appe
llF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/
3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3
, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(8*c - d*x^3))/(663552*c^5*x^2*Sqrt[c +
 d*x^3])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^3 (8c-dx^3)^2 \left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},2,\frac{3}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^3x^2\sqrt{c+dx^3}}$$

input
```
Int[1/(x^3*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
-1/128*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 3/2, 1/3, (d*x^3)/(8*c), -((
d*x^3)/c)])/(c^3*x^2*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.06 (sec) , antiderivative size = 764, normalized size of antiderivative = 11.58

method	result	size
elliptic	Expression too large to display	764
risch	Expression too large to display	1760
default	Expression too large to display	1806

input
```
int(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/128*(d*x^3+c)^(1/2)/c^4/x^2-2/243*d*x/c^4/((x^3+c/d)*d)^(1/2)+1/15552*d
*x/c^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+167/31104*I/c^4*3^(1/2)*(-c*d^2)^(1/3)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/5184*I/c^4/d^2*2^(1/2)*sum
(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2
)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*
d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^
2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),
-1/18/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(
1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^
(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alph
a=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2650 vs. $2(52) = 104$.

Time = 1.88 (sec) , antiderivative size = 2650, normalized size of antiderivative = 40.15

$$\int \frac{1}{x^3 (8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/82944*(1264*(d^2*x^8 - 7*c*d*x^5 - 8*c^2*x^2)*sqrt(d)*weierstrassPInver
se(0, -4*c/d, x) - (c^4*d^2*x^8 - 7*c^5*d*x^5 - 8*c^6*x^2 + sqrt(-3)*(c^4*
d^2*x^8 - 7*c^5*d*x^5 - 8*c^6*x^2))*(d^4/c^25)^(1/6)*log((d^6*x^9 + 318*c*
d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 - 9*(c^17*d^3*x^8 + 38*c^18*d^2*x
^5 + 64*c^19*d*x^2 + sqrt(-3)*(c^17*d^3*x^8 + 38*c^18*d^2*x^5 + 64*c^19*d*
x^2))*(d^4/c^25)^(2/3) + 3*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4
+ 160*c^23*x - sqrt(-3)*(c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x))*(d^4/
c^25)^(5/6) - 2*(7*c^13*d^3*x^6 + 152*c^14*d^2*x^3 + 64*c^15*d)*sqrt(d^4/c
^25) + 6*(5*c^5*d^4*x^5 + 32*c^6*d^3*x^2 + sqrt(-3)*(5*c^5*d^4*x^5 + 32*c^
6*d^3*x^2))*(d^4/c^25)^(1/6)) - 9*(5*c^9*d^4*x^7 + 64*c^10*d^3*x^4 + 32*c^
11*d^2*x - sqrt(-3)*(5*c^9*d^4*x^7 + 64*c^10*d^3*x^4 + 32*c^11*d^2*x))*(d^
4/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^4*
d^2*x^8 - 7*c^5*d*x^5 - 8*c^6*x^2 + sqrt(-3)*(c^4*d^2*x^8 - 7*c^5*d*x^5 -
8*c^6*x^2))*(d^4/c^25)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x
^3 + 640*c^3*d^3 - 9*(c^17*d^3*x^8 + 38*c^18*d^2*x^5 + 64*c^19*d*x^2 + sqr
t(-3)*(c^17*d^3*x^8 + 38*c^18*d^2*x^5 + 64*c^19*d*x^2))*(d^4/c^25)^(2/3) -
 3*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x - sqrt(-3)*
(c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x))*(d^4/c^25)^(5/6) - 2*(7*c^13*
d^3*x^6 + 152*c^14*d^2*x^3 + 64*c^15*d)*sqrt(d^4/c^25) + 6*(5*c^5*d^4*x^5
+ 32*c^6*d^3*x^2 + sqrt(-3)*(5*c^5*d^4*x^5 + 32*c^6*d^3*x^2))*(d^4/c^25...
```

Sympy [F]

$$\int \frac{1}{x^3 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^3 \left(-8c+dx^3\right)^2 \left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**3*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (8c - dx^3)^2 (c + dx^3)^{3/2}}\, dx = \int \frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c)^2 x^3}\, dx$$

input
```
integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (8c - dx^3)^2 (c + dx^3)^{3/2}}\, dx = \int \frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c)^2 x^3}\, dx$$

input
```
integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (8c - dx^3)^2 (c + dx^3)^{3/2}}\, dx = \int \frac{1}{x^3\,(d\,x^3 + c)^{3/2}\,(8\,c - d\,x^3)^2}\, dx$$

input
```
int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^3 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = \frac{-2\sqrt{d\,x^3+c}-368\left(\int \frac{\sqrt{d\,x^3+c}}{d^4x^{12}-14c\,d^3x^9+33c^2d^2x^6+112c^3d\,x^3+64c^4}dx\right)c^3d\,x^2-3}{}$$

input
```
int(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

output
```
( - 2*sqrt(c + d*x**3) - 368*int(sqrt(c + d*x**3)/(64*c**4 + 112*c**3*d*x*
*3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*c**3*d*x**2 - 322
*int(sqrt(c + d*x**3)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*
c*d**3*x**9 + d**4*x**12),x)*c**2*d**2*x**5 + 46*int(sqrt(c + d*x**3)/(64*
c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),
x)*c*d**3*x**8 + 104*int((sqrt(c + d*x**3)*x**3)/(64*c**4 + 112*c**3*d*x**
3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*c**2*d**2*x**2 + 9
1*int((sqrt(c + d*x**3)*x**3)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x*
*6 - 14*c*d**3*x**9 + d**4*x**12),x)*c*d**3*x**5 - 13*int((sqrt(c + d*x**3
)*x**3)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 +
d**4*x**12),x)*d**4*x**8)/(32*c**2*x**2*(8*c**2 + 7*c*d*x**3 - d**2*x**6))
```

3.632 $\int \frac{1}{x^6(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^6(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{5}{3},2,\frac{3}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^3x^5\sqrt{c+dx^3}}$$

output
```
-1/320*(1+d*x^3/c)^(1/2)*AppellF1(-5/3,3/2,2,-2/3,-d*x^3/c,1/8*d*x^3/c)/c^
3/x^5/(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 283 vs. 2(66) = 132.

Time = 10.28 (sec) , antiderivative size = 283, normalized size of antiderivative = 4.29

$$\int \frac{1}{x^6(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{\frac{64(2592c^3-7128c^2dx^3-15373cd^2x^6+2027d^3x^9)}{c^5x^5(-8c+dx^3)} - \frac{2027d^3x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx}{c}\right.}{c^6}}{}$$

input
```
Integrate[1/(x^6*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
((64*(2592*c^3 - 7128*c^2*d*x^3 - 15373*c*d^2*x^6 + 2027*d^3*x^9))/(c^5*x^
5*(-8*c + d*x^3)) - (2027*d^3*x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1
, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])/c^6 + (16789504*d^2*x*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(c^3*(8*c - d*x^3)*(32*c*AppellF
1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3,
1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -
((d*x^3)/c), (d*x^3)/(8*c)]))))/(6635520*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^6(8c-dx^3)^2\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{5}{3},2,\frac{3}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^3x^5\sqrt{c+dx^3}}$$

input
```
Int[1/(x^6*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
-1/320*(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 2, 3/2, -2/3, (d*x^3)/(8*c), -(
(d*x^3)/c)])/(c^3*x^5*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.78 (sec) , antiderivative size = 787, normalized size of antiderivative = 11.92

method	result	size
elliptic	Expression too large to display	787
risch	Expression too large to display	1772
default	Expression too large to display	2157

input
```
int(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/320*(d*x^3+c)^(1/2)/c^4/x^5+29/2560*d*(d*x^3+c)^(1/2)/c^5/x^2+2/243*d^2
*x/c^5/((x^3+c/d)*d)^(1/2)+1/124416*d^2*x/c^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)
-2027/311040*I*d/c^5*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d
^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(
-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2))-1/31104*I/c^5/d*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*
I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(
1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)
))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c
*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^
(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*_alp
ha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2
/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2698 vs. $2(52) = 104$.

Time = 5.78 (sec) , antiderivative size = 2698, normalized size of antiderivative = 40.88

$$\int \frac{1}{x^6 (8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
1/2488320*(49008*(d^3*x^11 - 7*c*d^2*x^8 - 8*c^2*d*x^5)*sqrt(d)*weierstras
sPInverse(0, -4*c/d, x) + 5*(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5 + sqrt
(-3)*(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5))*(d^10/c^31)^(1/6)*log((d^11
*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - 9*(c^21*d^4*x^8 +
 38*c^22*d^3*x^5 + 64*c^23*d^2*x^2 + sqrt(-3)*(c^21*d^4*x^8 + 38*c^22*d^3*
x^5 + 64*c^23*d^2*x^2))*(d^10/c^31)^(2/3) + 3*sqrt(d*x^3 + c)*((c^26*d^2*x
^7 + 80*c^27*d*x^4 + 160*c^28*x - sqrt(-3)*(c^26*d^2*x^7 + 80*c^27*d*x^4 +
 160*c^28*x))*(d^10/c^31)^(5/6) - 2*(7*c^16*d^5*x^6 + 152*c^17*d^4*x^3 + 6
4*c^18*d^3)*sqrt(d^10/c^31) + 6*(5*c^6*d^8*x^5 + 32*c^7*d^7*x^2 + sqrt(-3)
*(5*c^6*d^8*x^5 + 32*c^7*d^7*x^2))*(d^10/c^31)^(1/6)) - 9*(5*c^11*d^7*x^7
+ 64*c^12*d^6*x^4 + 32*c^13*d^5*x - sqrt(-3)*(5*c^11*d^7*x^7 + 64*c^12*d^6
*x^4 + 32*c^13*d^5*x))*(d^10/c^31)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^
2*d*x^3 - 512*c^3)) - 5*(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5 + sqrt(-3)
*(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5))*(d^10/c^31)^(1/6)*log((d^11*x^9
 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - 9*(c^21*d^4*x^8 + 38*
c^22*d^3*x^5 + 64*c^23*d^2*x^2 + sqrt(-3)*(c^21*d^4*x^8 + 38*c^22*d^3*x^5
+ 64*c^23*d^2*x^2))*(d^10/c^31)^(2/3) - 3*sqrt(d*x^3 + c)*((c^26*d^2*x^7 +
 80*c^27*d*x^4 + 160*c^28*x - sqrt(-3)*(c^26*d^2*x^7 + 80*c^27*d*x^4 + 160
*c^28*x))*(d^10/c^31)^(5/6) - 2*(7*c^16*d^5*x^6 + 152*c^17*d^4*x^3 + 64*c^
18*d^3)*sqrt(d^10/c^31) + 6*(5*c^6*d^8*x^5 + 32*c^7*d^7*x^2 + sqrt(-3)*...
```

Sympy [F]

$$\int \frac{1}{x^6 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx=\int \frac{1}{x^6 (-8c+dx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**6*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^6 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2x^6}\,dx$$

input
```
integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2x^6}\,dx$$

input
```
integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^6\,(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(1/(x^6*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^6*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (8c - dx^3)^2 (c + dx^3)^{3/2}}\, dx = \frac{-8\sqrt{d\,x^3 + c}\,c + 22\sqrt{d\,x^3 + c}\,d\,x^3 + 4656\left(\int \frac{\sqrt{d\,x^3+c}}{d^4x^{12}-14c\,d^3x^9+33c^2d^2x^6+112c^3}\right.}{}$$

input
```
int(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

output
```
( - 8*sqrt(c + d*x**3)*c + 22*sqrt(c + d*x**3)*d*x**3 + 4656*int(sqrt(c +
d*x**3)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 +
d**4*x**12),x)*c**3*d**2*x**5 + 4074*int(sqrt(c + d*x**3)/(64*c**4 + 112*c
**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*c**2*d**3
*x**8 - 582*int(sqrt(c + d*x**3)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2
*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*c*d**4*x**11 - 1144*int((sqrt(c +
d*x**3)*x**3)/(64*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x
**9 + d**4*x**12),x)*c**2*d**3*x**5 - 1001*int((sqrt(c + d*x**3)*x**3)/(64
*c**4 + 112*c**3*d*x**3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12)
,x)*c*d**4*x**8 + 143*int((sqrt(c + d*x**3)*x**3)/(64*c**4 + 112*c**3*d*x*
*3 + 33*c**2*d**2*x**6 - 14*c*d**3*x**9 + d**4*x**12),x)*d**5*x**11)/(320*
c**3*x**5*(8*c**2 + 7*c*d*x**3 - d**2*x**6))
```

3.633 $\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = -\frac{4a\sqrt{c+dx^3}}{3b^3} - \frac{a^2\sqrt{c+dx^3}}{3b^3(a+bx^3)} + \frac{2(c+dx^3)^{3/2}}{9b^2d} + \frac{a(4bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}\sqrt{bc-ad}}$$

output
```
-4/3*a*(d*x^3+c)^(1/2)/b^3-1/3*a^2*(d*x^3+c)^(1/2)/b^3/(b*x^3+a)+2/9*(d*x^
3+c)^(3/2)/b^2/d+1/3*a*(-5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*
d+b*c)^(1/2))/b^(7/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{\sqrt{c+dx^3}(-15a^2d+2ab(c-5dx^3)+2b^2x^3(c+dx^3))}{9b^3d(a+bx^3)} + \frac{a(-4bc+5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{7/2}\sqrt{-bc+ad}}$$

input
```
Integrate[(x^8*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
(Sqrt[c + d*x^3]*(-15*a^2*d + 2*a*b*(c - 5*d*x^3) + 2*b^2*x^3*(c + d*x^3))
)/(9*b^3*d*(a + b*x^3)) + (a*(-4*b*c + 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x
^3])/Sqrt[-(b*c) + a*d]])/(3*b^(7/2)*Sqrt[-(b*c) + a*d])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {948, 100, 27, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6\sqrt{dx^3+c}}{(bx^3+a)^2}dx^3$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{3}\left(\frac{\int -\frac{\sqrt{dx^3+c}(a(2bc-3ad)-2b(bc-ad)x^3)}{2(bx^3+a)}dx^3}{b^2(bc-ad)}-\frac{a^2(c+dx^3)^{3/2}}{b^2(a+bx^3)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{\sqrt{dx^3+c}(a(2bc-3ad)-2b(bc-ad)x^3)}{bx^3+a}dx^3}{2b^2(bc-ad)}-\frac{a^2(c+dx^3)^{3/2}}{b^2(a+bx^3)(bc-ad)}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(-\frac{a(4bc-5ad)\int \frac{\sqrt{dx^3+c}}{bx^3+a}dx^3-\frac{4(c+dx^3)^{3/2}(bc-ad)}{3d}}{2b^2(bc-ad)}-\frac{a^2(c+dx^3)^{3/2}}{b^2(a+bx^3)(bc-ad)}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(-\frac{a(4bc-5ad)\left(\frac{(bc-ad)\int\frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{b}+\frac{2\sqrt{c+dx^3}}{b}\right)-\frac{4(c+dx^3)^{3/2}(bc-ad)}{3d}}{2b^2(bc-ad)}-\frac{a^2(c+dx^3)^{3/2}}{b^2(a+bx^3)(bc-ad)}\right)$$

$$\Big\downarrow 73$$

$$\frac{1}{3}\left(-\frac{a(4bc-5ad)\left(\frac{2(bc-ad)\int\frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd}+\frac{2\sqrt{c+dx^3}}{b}\right)-\frac{4(c+dx^3)^{3/2}(bc-ad)}{3d}}{2b^2(bc-ad)}-\frac{a^2(c+dx^3)^{3/2}}{b^2(a+bx^3)(bc-ad)}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{3}\left(-\frac{a^2(c+dx^3)^{3/2}}{b^2(a+bx^3)(bc-ad)}-\frac{a(4bc-5ad)\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)-\frac{4(c+dx^3)^{3/2}(bc-ad)}{3d}}{2b^2(bc-ad)}\right)$$

input
```
Int[(x^8*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
(-((a^2*(c + d*x^3)^(3/2))/(b^2*(b*c - a*d)*(a + b*x^3))) - ((-4*(b*c - a*
d)*(c + d*x^3)^(3/2))/(3*d) + a*(4*b*c - 5*a*d)*((2*Sqrt[c + d*x^3])/b - (
2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(3
/2)))/(2*b^2*(b*c - a*d)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$-\frac{5\left(-a d\left(b\,x^{3}+a\right)\left(a d-\frac{4 b c}{5}\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(a d-b c\right)b}}\right)+\sqrt{d\,x^{3}+c}\left(-\frac{2 x^{3}\left(d\,x^{3}+c\right)b^{2}}{15}-\frac{2 a\left(-5 d\,x^{3}+c\right)b}{15}+d\,a^{2}\right)\sqrt{\left(a d-b c\right)b}\right)}{3\sqrt{\left(a d-b c\right)b}\,d\,b^{3}\left(b\,x^{3}+a\right)}$
default	$\frac{2\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9 b^{2} d}+\frac{a^{2}\left(-\frac{\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(a d-b c\right)b}}\right)}{\sqrt{\left(a d-b c\right)b}}\right)}{3 b^{3}}-\frac{4 a\left(\sqrt{d\,x^{3}+c}-\frac{\left(a d-b c\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(a d-b c\right)b}}\right)}{\sqrt{\left(a d-b c\right)b}}\right)}{3 b^{3}}$
risch	$-\frac{2\left(-b d\,x^{3}+6 a d-b c\right)\sqrt{d\,x^{3}+c}}{9 d\,b^{3}}+\frac{5 a^{2}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(a d-b c\right)b}}\right)d}{3 b^{3}\sqrt{\left(a d-b c\right)b}}-\frac{4 a\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(a d-b c\right)b}}\right)c}{3 b^{2}\sqrt{\left(a d-b c\right)b}}-\frac{a^{2}\sqrt{d\,x^{3}+c}}{3 b^{3}\left(b\,x^{3}+a\right)}$
elliptic	$-\frac{a^{2}\sqrt{d\,x^{3}+c}}{3 b^{3}\left(b\,x^{3}+a\right)}+\frac{2 x^{3}\sqrt{d\,x^{3}+c}}{9 b^{2}}+\frac{2\left(-\frac{2 a d-b c}{b^{3}}-\frac{2 c}{3 b^{2}}\right)\sqrt{d\,x^{3}+c}}{3 d}-\frac{i a\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(5 a d-4 b c\right)\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\cdots}{\cdots}\right)}{\cdots}$

input

```
int(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output

```
-5/3/((a*d-b*c)*b)^(1/2)*(-a*d*(b*x^3+a)*(a*d-4/5*b*c)*arctan(b*(d*x^3+c)^
(1/2)/((a*d-b*c)*b)^(1/2))+(d*x^3+c)^(1/2)*(-2/15*x^3*(d*x^3+c)*b^2-2/15*a
*(-5*d*x^3+c)*b+d*a^2)*((a*d-b*c)*b)^(1/2))/d/b^3/(b*x^3+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(108) = 216$.

Time = 0.10 (sec) , antiderivative size = 469, normalized size of antiderivative = 3.55

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$
$$= \left[-\frac{3\,(4\,a^2bcd-5\,a^3d^2+(4\,ab^2cd-5\,a^2bd^2)x^3)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,(2\,(b^4cd}{18\,(ab^5cd-a^2b^4d^2+(b^6c}\right.$$
$$-\frac{3\,(4\,a^2bcd-5\,a^3d^2+(4\,ab^2cd-5\,a^2bd^2)x^3)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-(2\,(b^4cd-ab^3d^2}{9\,(ab^5cd-a^2b^4d^2+(b^6cd-a}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/18*(3*(4*a^2*b*c*d - 5*a^3*d^2 + (4*a*b^2*c*d - 5*a^2*b*d^2)*x^3)*sqrt
(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c
- a*b*d))/(b*x^3 + a)) - 2*(2*(b^4*c*d - a*b^3*d^2)*x^6 + 2*a*b^3*c^2 - 17
*a^2*b^2*c*d + 15*a^3*b*d^2 + 2*(b^4*c^2 - 6*a*b^3*c*d + 5*a^2*b^2*d^2)*x^
3)*sqrt(d*x^3 + c))/(a*b^5*c*d - a^2*b^4*d^2 + (b^6*c*d - a*b^5*d^2)*x^3),
 -1/9*(3*(4*a^2*b*c*d - 5*a^3*d^2 + (4*a*b^2*c*d - 5*a^2*b*d^2)*x^3)*sqrt(
-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c
)) - (2*(b^4*c*d - a*b^3*d^2)*x^6 + 2*a*b^3*c^2 - 17*a^2*b^2*c*d + 15*a^3*
b*d^2 + 2*(b^4*c^2 - 6*a*b^3*c*d + 5*a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a
*b^5*c*d - a^2*b^4*d^2 + (b^6*c*d - a*b^5*d^2)*x^3)]
```

Sympy [F]

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

input
```
integrate(x**8*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

output
```
Integral(x**8*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.03

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = -\frac{\sqrt{dx^3+c}a^2d}{3\left((dx^3+c)b-bc+ad\right)b^3} - \frac{(4\,abc-5\,a^2d)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^3} + \frac{2\left((dx^3+c)^{\frac{3}{2}}b^4d^2-6\sqrt{dx^3+c}ab^3d^3\right)}{9\,b^6d^3}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/3*sqrt(d*x^3 + c)*a^2*d/(((d*x^3 + c)*b - b*c + a*d)*b^3) - 1/3*(4*a*b*
c - 5*a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c +
 a*b*d)*b^3) + 2/9*((d*x^3 + c)^(3/2)*b^4*d^2 - 6*sqrt(d*x^3 + c)*a*b^3*d^
3)/(b^6*d^3)
```

Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.53

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,b^2} - \frac{\sqrt{d\,x^3+c}\,\left(\frac{4\,c}{3\,b^2}-\frac{2\,b^2\,c-2\,a\,b\,d}{b^4}+\frac{2\,a\,d}{b^3}\right)}{3\,d} + \frac{a^2\,\left(\frac{2\,a\,d}{3\,(2\,b^2\,c-2\,a\,b\,d)}-\frac{2\,b\,c}{3\,(2\,b^2\,c-2\,a\,b\,d)}\right)\,\sqrt{d\,x^3+c}}{b^2\,\left(b\,x^3+a\right)} + \frac{a\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,(5\,a\,d-4\,b\,c)\,1{}\mathrm{i}}{6\,b^{7/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int((x^8*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

output
```
(2*x^3*(c + d*x^3)^(1/2))/(9*b^2) - ((c + d*x^3)^(1/2)*((4*c)/(3*b^2) - (2
*b^2*c - 2*a*b*d)/b^4 + (2*a*d)/b^3))/(3*d) + (a*log((2*b*c - a*d + b^(1/2
)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5*a*d -
4*b*c)*1i)/(6*b^(7/2)*(a*d - b*c)^(1/2)) + (a^2*((2*a*d)/(3*(2*b^2*c - 2*a
*b*d)) - (2*b*c)/(3*(2*b^2*c - 2*a*b*d)))*(c + d*x^3)^(1/2))/(b^2*(a + b*x
^3))
```

Reduce [F]

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

output

```
(20*sqrt(c + d*x**3)*a**2*c*d - 10*sqrt(c + d*x**3)*a**2*d**2*x**3 - 4*sqr
t(c + d*x**3)*a*b*c**2 + 22*sqrt(c + d*x**3)*a*b*c*d*x**3 + 2*sqrt(c + d*x
**3)*a*b*d**2*x**6 - 4*sqrt(c + d*x**3)*b**2*c**2*x**3 - 4*sqrt(c + d*x**3
)*b**2*c*d*x**6 + 45*int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**
3 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d
*x**6 + a*b**2*d**2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**5*d**
4 - 171*int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*
c**2 + 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**
2*d**2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**4*b*c*d**3 + 45*in
t((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a
**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**
9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**4*b*d**4*x**3 + 198*int((sqr
t(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*
d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2*
b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**3*b**2*c**2*d**2 - 171*int((sqrt(c
 + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*d**
2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2*b**
3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**3*b**2*c*d**3*x**3 - 72*int((sqrt(c +
 d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*d**2*
x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2*b*...
```

3.634 $\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{2\sqrt{c+dx^3}}{3b^2}+\frac{a\sqrt{c+dx^3}}{3b^2(a+bx^3)}-\frac{(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

output
```
2/3*(d*x^3+c)^(1/2)/b^2+1/3*a*(d*x^3+c)^(1/2)/b^2/(b*x^3+a)-1/3*(-3*a*d+2*
b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{\frac{\sqrt{b}(3a+2bx^3)\sqrt{c+dx^3}}{a+bx^3}+\frac{(2bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}}{3b^{5/2}}$$

input
```
Integrate[(x^5*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
((Sqrt[b]*(3*a + 2*b*x^3)*Sqrt[c + d*x^3])/(a + b*x^3) + ((2*b*c - 3*a*d)*
ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d])/
(3*b^(5/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {948, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3\sqrt{dx^3+c}}{(bx^3+a)^2}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{(2bc-3ad)\int \frac{\sqrt{dx^3+c}}{bx^3+a}dx^3}{2b(bc-ad)}+\frac{a(c+dx^3)^{3/2}}{b(a+bx^3)(bc-ad)}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{(2bc-3ad)\left(\frac{(bc-ad)\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{b}+\frac{2\sqrt{c+dx^3}}{b}\right)}{2b(bc-ad)}+\frac{a(c+dx^3)^{3/2}}{b(a+bx^3)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{(2bc-3ad)\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd}+\frac{2\sqrt{c+dx^3}}{b}\right)}{2b(bc-ad)}+\frac{a(c+dx^3)^{3/2}}{b(a+bx^3)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{(2bc-3ad)\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{2b(bc-ad)}+\frac{a(c+dx^3)^{3/2}}{b\,(a+bx^3)\,(bc-ad)}\right)$$

input
```
Int[(x^5*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
((a*(c + d*x^3)^(3/2))/(b*(b*c - a*d)*(a + b*x^3)) + ((2*b*c - 3*a*d)*((2*
Sqrt[c + d*x^3])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/
Sqrt[b*c - a*d]])/b^(3/2)))/(2*b*(b*c - a*d)))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$\frac{-\left(b\,x^{3}+a\right)\left(a d-\frac{2 b c}{3}\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(a d-b c\right)b}}\right)+\sqrt{\left(a d-b c\right)b}\,\left(\frac{2 b\,x^{3}}{3}+a\right)\sqrt{d\,x^{3}+c}}{\sqrt{\left(a d-b c\right)b}\,b^{2}\left(b\,x^{3}+a\right)}$
risch	$\frac{2\sqrt{d\,x^{3}+c}}{3 b^{2}}-\frac{\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(a d-b c\right)b}}\right)a d}{b^{2}\sqrt{\left(a d-b c\right)b}}+\frac{2\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(a d-b c\right)b}}\right)c}{3 b\sqrt{\left(a d-b c\right)b}}+\frac{a\sqrt{d\,x^{3}+c}}{3 b^{2}\left(b\,x^{3}+a\right)}$
default	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\left(a d-b c\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(a d-b c\right)b}}\right)}{3\sqrt{\left(a d-b c\right)b}}}{b^{2}}-\frac{a\left(-\frac{\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(a d-b c\right)b}}\right)}{\sqrt{\left(a d-b c\right)b}}\right)}{3 b^{2}}$
elliptic	$\frac{a\sqrt{d\,x^{3}+c}}{3 b^{2}\left(b\,x^{3}+a\right)}+\frac{2\sqrt{d\,x^{3}+c}}{3 b^{2}}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\,\_Z^{3}+a\right)}\frac{\left(3 a d-2 b c\right)\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{i d\left(2 x+\frac{-i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\ \cdots}{\cdots}\right)}{\cdots}$

input
```
int(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/((a*d-b*c)*b)^(1/2)*(-(b*x^3+a)*(a*d-2/3*b*c)*arctan(b*(d*x^3+c)^(1/2)/(
(a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(2/3*b*x^3+a)*(d*x^3+c)^(1/2))/b^2
/(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.12

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$
$$= \left[-\frac{((2\,b^2c-3\,abd)x^3+2\,abc-3\,a^2d)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,(3\,ab^2c-3\,a^2bd}{6\,(ab^4c-a^2b^3d+(b^5c-ab^4d)x^3)}\right.$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/6*(((2*b^2*c - 3*a*b*d)*x^3 + 2*a*b*c - 3*a^2*d)*sqrt(b^2*c - a*b*d)*l
og((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3
+ a)) - 2*(3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a*b^2*d)*x^3)*sqrt(d*x^3 + c
))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^3), 1/3*(((2*b^2*c - 3*a*b*d
)*x^3 + 2*a*b*c - 3*a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqr
t(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a
*b^2*d)*x^3)*sqrt(d*x^3 + c))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^3
)]
```

Sympy [F]

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

input
```
integrate(x**5*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

output
```
Integral(x**5*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.95

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{\sqrt{dx^3+c}ad}{3\left((dx^3+c)b-bc+ad\right)b^2} + \frac{(2\,bc-3\,ad)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^2} + \frac{2\sqrt{dx^3+c}}{3\,b^2}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*sqrt(d*x^3 + c)*a*d/(((d*x^3 + c)*b - b*c + a*d)*b^2) + 1/3*(2*b*c - 3
*a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*b^2) + 2/3*sqrt(d*x^3 + c)/b^2
```

Mupad [B] (verification not implemented)

Time = 5.06 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.42

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,b^2} - \frac{a\left(\frac{2\,a\,d}{3\,(2\,b^2\,c-2\,a\,b\,d)} - \frac{2\,b\,c}{3\,(2\,b^2\,c-2\,a\,b\,d)}\right)\sqrt{d\,x^3+c}}{b\,(b\,x^3+a)} + \frac{\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)(3\,a\,d-2\,b\,c)\,1\mathrm{i}}{6\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int((x^5*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*b^2) + (log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1
/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(3*a*d - 2*b*c)*1i)/(6*b^
(5/2)*(a*d - b*c)^(1/2)) - (a*((2*a*d)/(3*(2*b^2*c - 2*a*b*d)) - (2*b*c)/(
3*(2*b^2*c - 2*a*b*d)))*(c + d*x^3)^(1/2))/(b*(a + b*x^3))
```

Reduce [F]

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

output
```
( - 4*sqrt(c + d*x**3)*a*c + 2*sqrt(c + d*x**3)*a*d*x**3 - 4*sqrt(c + d*x*
*3)*b*c*x**3 - 9*int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 -
2*a**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**
6 + a*b**2*d**2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**4*d**3 +
33*int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2
+ 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**
2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**3*b*c*d**2 - 9*int((sqr
t(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*
d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2*
b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**3*b*d**3*x**3 - 36*int((sqrt(c + d
*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x*
*6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2*b**3*c*
*2*x**6 - 2*b**3*c*d*x**9),x)*a**2*b**2*c**2*d + 33*int((sqrt(c + d*x**3)*
x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*
a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2*b**3*c**2*x**6
 - 2*b**3*c*d*x**9),x)*a**2*b**2*c*d**2*x**3 + 12*int((sqrt(c + d*x**3)*x*
*5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*
b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2*b**3*c**2*x**6 -
 2*b**3*c*d*x**9),x)*a*b**3*c**3 - 36*int((sqrt(c + d*x**3)*x**5)/(a**3*c*
d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2...
```

3.635 $\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx=-\frac{\sqrt{c+dx^3}}{3b(a+bx^3)}-\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

output
```
-1/3*(d*x^3+c)^(1/2)/b/(b*x^3+a)-1/3*d*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a
*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx=-\frac{\sqrt{c+dx^3}}{3b(a+bx^3)}+\frac{d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{3/2}\sqrt{-bc+ad}}$$

input
```
Integrate[(x^2*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
-1/3*Sqrt[c + d*x^3]/(b*(a + b*x^3)) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])
/Sqrt[-(b*c) + a*d]])/(3*b^(3/2)*Sqrt[-(b*c) + a*d])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {946, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{(bx^3+a)^2}dx^3$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}\left(\frac{d\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{2b}-\frac{\sqrt{c+dx^3}}{b(a+bx^3)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{b}-\frac{\sqrt{c+dx^3}}{b(a+bx^3)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^3}}{b(a+bx^3)}\right)$$

input
```
Int[(x^2*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
(-(Sqrt[c + d*x^3]/(b*(a + b*x^3))) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])
/Sqrt[b*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d]))/3
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.81

method	result
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{3b}$
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{3b}$
elliptic	$-\frac{\sqrt{d\,x^{3}+c}}{3b(b\,x^{3}+a)}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\right.}}}{}\right.}{}$

input
```
int(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/b*(-(d*x^3+c)^(1/2)/(b*x^3+a)+d/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)
^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.19

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$
$$=\left[\frac{(bdx^3+ad)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,\sqrt{dx^3+c}(b^2c-abd)}{6\,(ab^3c-a^2b^2d+(b^4c-ab^3d)x^3)},\ \frac{(bdx^3+ad)\sqrt{-b^2c}}{}\right.$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/6*((b*d*x^3 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*s
qrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*sqrt(d*x^3 + c)*(b^2*
c - a*b*d))/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x^3), 1/3*((b*d*x^3 +
 a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*
d*x^3 + b*c)) - sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(a*b^3*c - a^2*b^2*d + (b
^4*c - a*b^3*d)*x^3)]
```

Sympy [F]

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

input
```
integrate(x**2*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

output
```
Integral(x**2*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{d\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b} - \frac{\sqrt{dx^3+c}d}{3\left((dx^3+c)b-bc+ad\right)b}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*b) - 1/3*sqrt(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*b)
```

Mupad [B] (verification not implemented)

Time = 4.62 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.56

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{\left(\frac{2\,a\,d}{3\,(2\,b^2\,c-2\,a\,b\,d)}-\frac{2\,b\,c}{3\,(2\,b^2\,c-2\,a\,b\,d)}\right)\sqrt{d\,x^3+c}}{b\,x^3+a} + \frac{d\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{6\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int((x^2*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

output
```
(((2*a*d)/(3*(2*b^2*c - 2*a*b*d)) - (2*b*c)/(3*(2*b^2*c - 2*a*b*d)))*(c +
d*x^3)^(1/2))/(a + b*x^3) + (d*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2
)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(6*b^(3/2)*(a*d - b*c)^
(1/2))
```

Reduce [F]

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

$$= \frac{2\sqrt{d\,x^3+c}\,c+3\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{a\,b^2d^2x^9-2b^3cd\,x^9+2a^2b\,d^2x^6-3a\,b^2cd\,x^6-2b^3c^2x^6+a^3d^2x^3-4a\,b^2c^2x^3+a^3cd-2a^2b\,c^2}dx\right)a^3d^3-9\left(\int \frac{}{a\,b^2 \cdots}\right.}{}$$

input
```
int(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

output
```
(2*sqrt(c + d*x**3)*c + 3*int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**
2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**
2*c*d*x**6 + a*b**2*d**2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**
3*d**3 - 9*int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2
*b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*
b**2*d**2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**2*b*c*d**2 + 3*
int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2
*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x
**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**2*b*d**3*x**3 + 6*int((sqr
t(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*
d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2*
b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a*b**2*c**2*d - 9*int((sqrt(c + d*x**
3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x**6 -
 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2*b**3*c**2*x
**6 - 2*b**3*c*d*x**9),x)*a*b**2*c*d**2*x**3 + 6*int((sqrt(c + d*x**3)*x**
5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*b
**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2*b**3*c**2*x**6 -
2*b**3*c*d*x**9),x)*b**3*c**2*d*x**3)/(3*(a**2*d - 2*a*b*c + a*b*d*x**3 -
2*b**2*c*x**3))
```

3.636 $\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 121

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)^2} dx = \frac{\sqrt{c+dx^3}}{3a(a+bx^3)} - \frac{2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{(2bc-ad)\text{arctan}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}\sqrt{bc-ad}}$$

output
```
1/3*(d*x^3+c)^(1/2)/a/(b*x^3+a)-2/3*c^(1/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2
))/a^2+1/3*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/
a^2/b^(1/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)^2} dx = \frac{\frac{a\sqrt{c+dx^3}}{a+bx^3} + \frac{(-2bc+ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\sqrt{-bc+ad}} - 2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x*(a + b*x^3)^2),x]
```

output
```
((a*Sqrt[c + d*x^3])/(a + b*x^3) + ((-2*b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c
+ d*x^3])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*Sqrt[-(b*c) + a*d]) - 2*Sqrt[c]*Ar
cTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 110, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^3\left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}\left(\frac{\sqrt{c+dx^3}}{a\left(a+bx^3\right)}-\frac{\int -\frac{dx^3+2c}{2x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{dx^3+2c}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{2a}+\frac{\sqrt{c+dx^3}}{a\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{\frac{2c\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{(2bc-ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{2a}+\frac{\sqrt{c+dx^3}}{a\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\frac{4c\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2(2bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{2a}+\frac{\sqrt{c+dx^3}}{a\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{\frac{2(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}\sqrt{bc-ad}}-\frac{4\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a}}{2a}+\frac{\sqrt{c+dx^3}}{a\left(a+bx^3\right)}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x*(a + b*x^3)^2),x]
```

output
```
(Sqrt[c + d*x^3]/(a*(a + b*x^3)) + ((-4*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sq
rt[c]])/a + (2*(2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c -
a*d]])/(a*Sqrt[b]*Sqrt[b*c - a*d]))/(2*a))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$\frac{(b x^3+a)(ad-2bc)\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)-2\sqrt{(ad-bc)b}\left(\sqrt{c}\,(b x^3+a)\,\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)-\frac{\sqrt{d x^3+c}\,a}{2}\right)}{3\sqrt{(ad-bc)b}\,a^2(b x^3+a)}$
default	$\frac{\frac{2\sqrt{d x^3+c}}{3}-\frac{2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{3}}{a^2}-\frac{2\left(\sqrt{d x^3+c}-\frac{(ad-bc)\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3a^2}-\frac{-\frac{\sqrt{d x^3+c}}{b x^3+a}+\frac{d\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{3a}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)*b)^(1/2)*((b*x^3+a)*(a*d-2*b*c)*arctan(b*(d*x^3+c)^(1/2)/((
a*d-b*c)*b)^(1/2))-2*((a*d-b*c)*b)^(1/2)*(c^(1/2)*(b*x^3+a)*arctanh((d*x^3
+c)^(1/2)/c^(1/2))-1/2*(d*x^3+c)^(1/2)*a))/a^2/(b*x^3+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(97) = 194$.

Time = 0.13 (sec) , antiderivative size = 850, normalized size of antiderivative = 7.02

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/6*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((
b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)
) - 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*s
qrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c)
)/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3), -1/3*(((2*b^2*c - a
*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*s
qrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - (a*b^2*c - a^2*b*d + (b^3*c - a*b^2
*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - (a*b
^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b
^2*d)*x^3), 1/6*(4*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-c)*ar
ctan(sqrt(-c)/sqrt(d*x^3 + c)) - ((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)
*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b
^2*c - a*b*d))/(b*x^3 + a)) + 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a^3*
b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3), -1/3*(((2*b^2*c - a*b*d)*x
^3 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^
2*c + a*b*d)/(b*d*x^3 + b*c)) - 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x
^3)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)) - (a*b^2*c - a^2*b*d)*sqrt(d
*x^3 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx = \int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x/(b*x**3+a)**2,x)
```

output
```
Integral(sqrt(c + d*x**3)/(x*(a + b*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2x}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx=\frac{\sqrt{dx^3+c}d}{3\left((dx^3+c)b-bc+ad\right)a}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^2}+\frac{2\,c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}$$

input
```
integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*sqrt(d*x^3 + c)*d/((((d*x^3 + c)*b - b*c + a*d)*a) - 1/3*(2*b*c - a*d)*
arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2)
+ 2/3*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 7.50 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)^2}\,dx = \frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2} - \frac{\left(\frac{b\,d}{3\,(b^2\,c-a\,b\,d)}-\frac{b^2\,c}{3\,a\,(b^2\,c-a\,b\,d)}\right)\sqrt{d\,x^3+c}}{b\,x^3+a} + \frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{d\,x^3+c}\,\sqrt{a\,b\,d-b^2\,c}\,2\mathrm{i}}{b\,x^3+a}\right)(a\,d-2\,b\,c)\,1\mathrm{i}}{6\,a^2\,\sqrt{a\,b\,d-b^2\,c}}$$

input
```
int((c + d*x^3)^(1/2)/(x*(a + b*x^3)^2),x)
```

output
```
(c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)
))/x^6))/(3*a^2) - (((b*d)/(3*(b^2*c - a*b*d)) - (b^2*c)/(3*a*(b^2*c - a*b
*d)))*(c + d*x^3)^(1/2))/(a + b*x^3) + (log((2*b*c - a*d + (c + d*x^3)^(1/
2)*(a*b*d - b^2*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - 2*b*c)*1i)/(6*a
^2*(a*b*d - b^2*c)^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)^2}\,dx = \int \frac{\sqrt{d\,x^3+c}}{b^2x^7+2ab\,x^4+a^2x}dx$$

input
```
int((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x)
```

output
```
int(sqrt(c + d*x**3)/(a**2*x + 2*a*b*x**4 + b**2*x**7),x)
```

3.637 $\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 161

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx = -\frac{2b\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3ax^3(a+bx^3)} + \frac{(4bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}} - \frac{\sqrt{b}(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{bc-ad}}$$

output
```
-2/3*b*(d*x^3+c)^(1/2)/a^2/(b*x^3+a)-1/3*(d*x^3+c)^(1/2)/a/x^3/(b*x^3+a)+1
/3*(-a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^3/c^(1/2)-1/3*b^(1/2)*(
-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+
b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx = \frac{-\frac{a(a+2bx^3)\sqrt{c+dx^3}}{x^3(a+bx^3)} + \frac{\sqrt{b}(4bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{(4bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^3}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)^2),x]
```

output
```
(-((a*(a + 2*b*x^3)*Sqrt[c + d*x^3])/(x^3*(a + b*x^3))) + (Sqrt[b]*(4*b*c
- 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c)
 + a*d] + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c])/(3*a^3
)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 110, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^4 (a+bx^3)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^6 (bx^3+a)^2}dx^3$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}\left(\frac{\int -\frac{3bdx^3+4bc-ad}{2x^3(bx^3+a)^2\sqrt{dx^3+c}}dx^3}{a}-\frac{\sqrt{c+dx^3}}{ax^3 (a+bx^3)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{3bdx^3+4bc-ad}{x^3(bx^3+a)^2\sqrt{dx^3+c}}dx^3}{2a}-\frac{\sqrt{c+dx^3}}{ax^3 (a+bx^3)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(-\frac{\frac{\int \frac{(bc-ad)\left(2bdx^3+4bc-ad\right)}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a(bc-ad)}+\frac{4b\sqrt{c+dx^3}}{a(a+bx^3)}}{2a}-\frac{\sqrt{c+dx^3}}{ax^3 (a+bx^3)}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(-\frac{\frac{\int \frac{2bdx^3+4bc-ad}{x^3(bx^3+a)\sqrt{dx^3+c}}dx^3}{a}+\frac{4b\sqrt{c+dx^3}}{a(a+bx^3)}}{2a}-\frac{\sqrt{c+dx^3}}{ax^3(a+bx^3)}\right)$$

$$\Big\downarrow \ 174$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{(4bc-ad)\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{b(4bc-3ad)\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{a}}{a}+\frac{4b\sqrt{c+dx^3}}{a(a+bx^3)}}{2a}-\frac{\sqrt{c+dx^3}}{ax^3(a+bx^3)}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{2(4bc-ad)\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2b(4bc-3ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{a}+\frac{4b\sqrt{c+dx^3}}{a(a+bx^3)}}{2a}-\frac{\sqrt{c+dx^3}}{ax^3(a+bx^3)}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{2\sqrt{b}(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(4bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}}{a}+\frac{4b\sqrt{c+dx^3}}{a(a+bx^3)}}{2a}-\frac{\sqrt{c+dx^3}}{ax^3(a+bx^3)}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)^2),x]
```

output
```
(-(Sqrt[c + d*x^3]/(a*x^3*(a + b*x^3))) - ((4*b*Sqrt[c + d*x^3])/(a*(a + b
*x^3)) + ((-2*(4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(a*Sqrt[c])
+ (2*Sqrt[b]*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c -
a*d]])/(a*Sqrt[b*c - a*d]))/a)/(2*a))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{\frac{4\sqrt{c}\left(b\,x^{3}+a\right)b\,x^{3}\left(bc-\frac{3ad}{4}\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3}-\frac{\sqrt{(ad-bc)b}\left(x^{3}\left(b\,x^{3}+a\right)(ad-4bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)+\sqrt{c}\,a\left(2b\,x^{3}+a\right)\sqrt{d\,x^{3}+c}\cdots}{3}}{x^{3}\sqrt{c}\,a^{3}(b\,x^{3}+a)\sqrt{(ad-bc)b}}$
risch	$-\frac{\sqrt{d\,x^{3}+c}}{3a^{2}x^{3}}-\frac{\frac{2(ad-4bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{2b\left(\frac{\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3}+\frac{4b(ad-2bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3a\sqrt{(ad-bc)b}}}{2a^{2}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3x^{3}}-\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}}{a^{2}}+\frac{b\left(-\frac{\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3a^{2}}-\frac{2b\left(\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}\cdots\right)}{a^{3}}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
4/3/((a*d-b*c)*b)^(1/2)/c^(1/2)*(c^(1/2)*(b*x^3+a)*b*x^3*(b*c-3/4*a*d)*arc
tan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))-1/4*((a*d-b*c)*b)^(1/2)*(x^3*(b
*x^3+a)*(a*d-4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))+c^(1/2)*a*(2*b*x^3+a)
*(d*x^3+c)^(1/2)))/x^3/a^3/(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 825, normalized size of antiderivative = 5.12

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/6*(((4*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(b/
(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sq
rt(b/(b*c - a*d)))/(b*x^3 + a)) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*
d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(2*
a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c*x^6 + a^4*c*x^3), 1/6*(2*((4*
b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(-b/(b*c - a*d
))*arctan(sqrt(d*x^3 + c)*sqrt(-b/(b*c - a*d))) - ((4*b^2*c - a*b*d)*x^6 +
 (4*a*b*c - a^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2
*c)/x^3) - 2*(2*a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c*x^6 + a^4*c*x
^3), -1/6*(2*((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(-c)*arct
an(sqrt(-c)/sqrt(d*x^3 + c)) + ((4*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 -
 3*a^2*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d
*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(2*a*b*c*x^3 +
 a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c*x^6 + a^4*c*x^3), 1/3*(((4*b^2*c^2 - 3*a
*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(sqr
t(d*x^3 + c)*sqrt(-b/(b*c - a*d))) - ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a
^2*d)*x^3)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)) - (2*a*b*c*x^3 + a^2*
c)*sqrt(d*x^3 + c))/(a^3*b*c*x^6 + a^4*c*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)^2}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**4/(b*x**3+a)**2,x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**4*(a + b*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{(bx^3+a)^2x^4}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx = \frac{(4\,b^2c-3\,abd)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}a^3} - \frac{(4\,bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}} - \frac{2\,(dx^3+c)^{\frac{3}{2}}bd-2\,\sqrt{dx^3+c}bcd+\sqrt{dx^3+c}ad^2}{3\,\left((dx^3+c)^2b-2\,(dx^3+c)bc+bc^2+(dx^3+c)ad-acd\right)a^2}$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*(4*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sq
rt(-b^2*c + a*b*d)*a^3) - 1/3*(4*b*c - a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c
))/(a^3*sqrt(-c)) - 1/3*(2*(d*x^3 + c)^(3/2)*b*d - 2*sqrt(d*x^3 + c)*b*c*d
 + sqrt(d*x^3 + c)*a*d^2)/(((d*x^3 + c)^2*b - 2*(d*x^3 + c)*b*c + b*c^2 +
(d*x^3 + c)*a*d - a*c*d)*a^2)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.72

$$\int \frac{\sqrt{c+dx^3}}{x^4 (a+bx^3)^2}\,dx$$

$$= \frac{\left(\frac{a\left(\frac{a\left(\frac{a\left(\frac{b^2 d^2}{2a^3c^2}-\frac{b^2d^2(3ad-4bc)}{6a^2c^2(a^2d-abc)}+\frac{b^2d(2ad-bc)(3ad-4bc)}{6a^3c^2(a^2d-abc)}\right)}{b}-\frac{bd(2ad-bc)}{2a^3c^2}+\frac{b(3ad-4bc)(-a^2d^2+2abcd+2b^2c^2)}{6a^3c^2(a^2d-abc)}\right)}{b}-\frac{-a^2d^2+2abcd+2\cdots}{2a^3c^2}\cdots}{b}\cdots\right)\cdots}{b\,x^3+a}$$

$$-\frac{\sqrt{d\,x^3+c}}{3\,a^2\,x^3}+\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)(a\,d-4\,b\,c)}{6\,a^3\,\sqrt{c}}$$

$$+\frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)(3\,a\,d-4\,b\,c)\,1\mathrm{i}}{6\,a^3\,\sqrt{a\,d-b\,c}}$$

input
```
int((c + d*x^3)^(1/2)/(x^4*(a + b*x^3)^2),x)
```

output
```
(((a*((a*((a*((b^2*d^2)/(2*a^3*c^2) - (b^2*d^2*(3*a*d - 4*b*c))/(6*a^2*c^2
*(a^2*d - a*b*c)) + (b^2*d*(2*a*d - b*c)*(3*a*d - 4*b*c))/(6*a^3*c^2*(a^2*
d - a*b*c))))/b - (b*d*(2*a*d - b*c))/(2*a^3*c^2) + (b*(3*a*d - 4*b*c)*(2*
b^2*c^2 - a^2*d^2 + 2*a*b*c*d))/(6*a^3*c^2*(a^2*d - a*b*c))))/b - (2*b^2*c
^2 - a^2*d^2 + 2*a*b*c*d)/(2*a^3*c^2) + (b*(a*d - 4*b*c)*(3*a*d - 4*b*c))/
(6*a^2*c*(a^2*d - a*b*c))))/b - (a*d - 4*b*c)/(2*a^2*c))*(c + d*x^3)^(1/2)
)/(a + b*x^3) - (c + d*x^3)^(1/2)/(3*a^2*x^3) + (log((((c + d*x^3)^(1/2) -
 c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)*(a*d - 4*b*c))/(6*a^3*c^(1
/2)) + (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(
1/2)*2i - b*d*x^3)/(a + b*x^3))*(3*a*d - 4*b*c)*1i)/(6*a^3*(a*d - b*c)^(1/
2))
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4 (a+bx^3)^2} \, dx = \int \frac{\sqrt{d\,x^3+c}}{b^2x^{10}+2ab\,x^7+a^2x^4} dx$$

input
```
int((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x)
```

output
```
int(sqrt(c + d*x**3)/(a**2*x**4 + 2*a*b*x**7 + b**2*x**10),x)
```

3.638 $\int \frac{x^3\sqrt{c+dx^3}}{\left(a+bx^3\right)^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3\sqrt{c+dx^3}}{\left(a+bx^3\right)^2} \, dx = \frac{x^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},2,-\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/4*x^4*(d*x^3+c)^(1/2)*AppellF1(4/3,2,-1/2,7/3,-b*x^3/a,-d*x^3/c)/a^2/(1+
d*x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 235 vs. 2(64) = 128.

Time = 10.25 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.67

$$\int \frac{x^3\sqrt{c+dx^3}}{\left(a+bx^3\right)^2} \, dx = \frac{x\left(\frac{5dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+\frac{8\left(-c-dx^3+\frac{8ac^2\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right.}{8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bc\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\right.\right.}\right.}{a+bx^3}\right.}{24b\sqrt{c+dx^3}}$$

input
```
Integrate[(x^3*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
(x*((5*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
-((b*x^3)/a)])/a + (8*(-c - d*x^3 + (8*a*c^2*AppellF1[1/3, 1/2, 1, 4/3, -(
(d*x^3)/c), -((b*x^3)/a)])/(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
 -((b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((
b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
)/(a + b*x^3)))/(24*b*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{c+dx^3}\int \frac{x^3\sqrt{\frac{dx^3}{c}+1}}{(bx^3+a)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},2,-\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(x^3*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
(x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 2, -1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)])/(4*a^2*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 2.08 (sec) , antiderivative size = 748, normalized size of antiderivative = 11.69

method	result	size
elliptic	Expression too large to display	748
default	Expression too large to display	1468

input
```
int(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*x/b*(d*x^3+c)^(1/2)/(b*x^3+a)-5/9*I/b^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/b^2/d^2*2^(1/2)*sum((5*a*d-2
*b*c)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3)
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha
^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/
3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

input
```
integrate(x**3*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

output
```
Integral(x**3*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x^3}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x^3}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{x^3\,\sqrt{d\,x^3+c}}{(b\,x^3+a)^2}\,dx$$

input
```
int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

output
```
int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

output
```
(2*sqrt(c + d*x**3)*c*x - 10*int(sqrt(c + d*x**3)/(5*a**3*c*d + 5*a**3*d**
2*x**3 - 4*a**2*b*c**2 + 6*a**2*b*c*d*x**3 + 10*a**2*b*d**2*x**6 - 8*a*b**
2*c**2*x**3 - 3*a*b**2*c*d*x**6 + 5*a*b**2*d**2*x**9 - 4*b**3*c**2*x**6 -
4*b**3*c*d*x**9),x)*a**3*c**2*d + 8*int(sqrt(c + d*x**3)/(5*a**3*c*d + 5*a
**3*d**2*x**3 - 4*a**2*b*c**2 + 6*a**2*b*c*d*x**3 + 10*a**2*b*d**2*x**6 -
8*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + 5*a*b**2*d**2*x**9 - 4*b**3*c**2*
x**6 - 4*b**3*c*d*x**9),x)*a**2*b*c**3 - 10*int(sqrt(c + d*x**3)/(5*a**3*c
*d + 5*a**3*d**2*x**3 - 4*a**2*b*c**2 + 6*a**2*b*c*d*x**3 + 10*a**2*b*d**2
*x**6 - 8*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + 5*a*b**2*d**2*x**9 - 4*b*
*3*c**2*x**6 - 4*b**3*c*d*x**9),x)*a**2*b*c**2*d*x**3 + 8*int(sqrt(c + d*x
**3)/(5*a**3*c*d + 5*a**3*d**2*x**3 - 4*a**2*b*c**2 + 6*a**2*b*c*d*x**3 +
10*a**2*b*d**2*x**6 - 8*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + 5*a*b**2*d*
*2*x**9 - 4*b**3*c**2*x**6 - 4*b**3*c*d*x**9),x)*a*b**2*c**3*x**3 + 25*int
((sqrt(c + d*x**3)*x**6)/(5*a**3*c*d + 5*a**3*d**2*x**3 - 4*a**2*b*c**2 +
6*a**2*b*c*d*x**3 + 10*a**2*b*d**2*x**6 - 8*a*b**2*c**2*x**3 - 3*a*b**2*c*
d*x**6 + 5*a*b**2*d**2*x**9 - 4*b**3*c**2*x**6 - 4*b**3*c*d*x**9),x)*a**3*
d**3 - 35*int((sqrt(c + d*x**3)*x**6)/(5*a**3*c*d + 5*a**3*d**2*x**3 - 4*a
**2*b*c**2 + 6*a**2*b*c*d*x**3 + 10*a**2*b*d**2*x**6 - 8*a*b**2*c**2*x**3
- 3*a*b**2*c*d*x**6 + 5*a*b**2*d**2*x**9 - 4*b**3*c**2*x**6 - 4*b**3*c*d*x
**9),x)*a**2*b*c*d**2 + 25*int((sqrt(c + d*x**3)*x**6)/(5*a**3*c*d + 5*...
```

3.639 $\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx=\frac{x^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},2,-\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/2*x^2*(d*x^3+c)^(1/2)*AppellF1(2/3,2,-1/2,5/3,-b*x^3/a,-d*x^3/c)/a^2/(1+
d*x^3/c)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 153 vs. 2(64) = 128.

Time = 10.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.39

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$
$$=\frac{10ax^2(c+dx^3)+5cx^2(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-dx^5(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{A}}{30a^2\left(a+bx^3\right)\sqrt{c+dx^3}}$$

input
```
Integrate[(x*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
(10*a*x^2*(c + d*x^3) + 5*c*x^2*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2
/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] - d*x^5*(a + b*x^3)*Sqrt[1 +
(d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(30*a^2
*(a + b*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{c+dx^3}\int \frac{x\sqrt{\frac{dx^3}{c}+1}}{(bx^3+a)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},2,-\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(x*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
(x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 2, -1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c
)])/(2*a^2*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.22 (sec) , antiderivative size = 908, normalized size of antiderivative = 14.19

method	result	size
default	Expression too large to display	908
elliptic	Expression too large to display	908

input
```
int(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/a*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)+1/9*I/a/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)))+1/18*I/a/b/d^2*2^(1/2)*sum((-a*d-2*b*c)/_alpha/(a*
d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/
3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)
^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^
(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1
/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b
/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

input
```
integrate(x*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

output
```
Integral(x*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Maxima [F]

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x}{(bx^3+a)^2}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x}{(bx^3+a)^2}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{x\,\sqrt{d\,x^3+c}}{(b\,x^3+a)^2}\,dx$$

input
```
int((x*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

output
```
int((x*(c + d*x^3)^(1/2))/(a + b*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x}{b^2x^6+2ab\,x^3+a^2}dx$$

input
```
int(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

output
```
int((sqrt(c + d*x**3)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
```

3.640 $\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^2} \, dx = \frac{x\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},2,-\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
x*(d*x^3+c)^(1/2)*AppellF1(1/3,2,-1/2,4/3,-b*x^3/a,-d*x^3/c)/a^2/(1+d*x^3/
c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(59) = 118.

Time = 10.32 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.93

$$\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^2} \, dx$$

$$= \frac{x\left(\frac{dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a^2}+\frac{8\left(\frac{c+dx^3}{a}+\frac{16c^2\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bc\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right.\right.}\right.}{a+bx^3}\right.}{24\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(a + b*x^3)^2,x]
```

output
```
(x*((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -(
(b*x^3)/a)])/a^2 + (8*((c + d*x^3)/a + (16*c^2*AppellF1[1/3, 1/2, 1, 4/3,
-((d*x^3)/c), -((b*x^3)/a)])/(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c
), -((b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -
((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])
)))/(a + b*x^3)))/(24*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2} \, dx$$

$$\Big\downarrow \; 937$$

$$\frac{\sqrt{c+dx^3} \int \frac{\sqrt{\frac{dx^3}{c}+1}}{(bx^3+a)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \; 936$$

$$\frac{x\sqrt{c+dx^3} \operatorname{AppellF1}\left(\frac{1}{3},2,-\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^3]/(a + b*x^3)^2,x]
```

output
```
(x*Sqrt[c + d*x^3]*AppellF1[1/3, 2, -1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(a^2*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.14 (sec) , antiderivative size = 753, normalized size of antiderivative = 12.76

method	result	size
default	Expression too large to display	753
elliptic	Expression too large to display	753

input
```
int((d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x/a*(d*x^3+c)^(1/2)/(b*x^3+a)-1/9*I/a/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a/b/d^2*2^(1/2)*sum((a*d-4*b*
c)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(
-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*
d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*
_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

output
```
Integral(sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{(bx^3+a)^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{(bx^3+a)^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{d\,x^3+c}}{(b\,x^3+a)^2}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(a + b*x^3)^2,x)
```

output
```
int((c + d*x^3)^(1/2)/(a + b*x^3)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{d\,x^3+c}}{b^2x^6+2ab\,x^3+a^2}dx$$

input
```
int((d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

output
```
int(sqrt(c + d*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
```

3.641 $\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2} \, dx = -\frac{\sqrt{c+dx^3}\operatorname{AppellF1}\left(-\frac{1}{3},2,-\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{1+\frac{dx^3}{c}}}$$

output
```
-(d*x^3+c)^(1/2)*AppellF1(-1/3,2,-1/2,2/3,-b*x^3/a,-d*x^3/c)/a^2/x/(1+d*x^
3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(62) = 124.

Time = 10.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.77

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2} \, dx$$
$$= \frac{-20a(3a+4bx^3)\left(c+dx^3\right)+5(-8bc+9ad)x^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+8}{60a^3x\left(a+bx^3\right)\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)^2),x]
```

output
```
(-20*a*(3*a + 4*b*x^3)*(c + d*x^3) + 5*(-8*b*c + 9*a*d)*x^3*(a + b*x^3)*Sq
rt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] +
 8*b*d*x^6*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d
*x^3)/c), -((b*x^3)/a)])/(60*a^3*x*(a + b*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^2 (a+bx^3)^2} \, dx$$
$$\Big\downarrow \text{1013}$$
$$\frac{\sqrt{c+dx^3}\int \frac{\sqrt{\frac{dx^3}{c}+1}}{x^2(bx^3+a)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$
$$\Big\downarrow \text{1012}$$
$$-\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},2,-\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2 x\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)^2),x]
```

output
```
-((Sqrt[c + d*x^3]*AppellF1[-1/3, 2, -1/2, 2/3, -((b*x^3)/a), -((d*x^3)/c)
])/(a^2*x*Sqrt[1 + (d*x^3)/c]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.78 (sec) , antiderivative size = 920, normalized size of antiderivative = 14.84

method	result	size
elliptic	Expression too large to display	920
risch	Expression too large to display	1819
default	Expression too large to display	2227

input
```
int((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b/a^2*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-1/a^2*(d*x^3+c)^(1/2)/x-4/9*I/a^2
*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/a^2/d^2*2^(1/2
)*sum((-5*a*d+8*b*c)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)^2}\,dx = \int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)^2}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**2/(b*x**3+a)**2,x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**2*(a + b*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{(bx^3+a)^2x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{(bx^3+a)^2x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^2\,{\left(b\,x^3+a\right)}^2}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)^2),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{d\,x^3+c}}{b^2x^8+2ab\,x^5+a^2x^2}dx$$

input
```
int((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x)
```

output
```
int(sqrt(c + d*x**3)/(a**2*x**2 + 2*a*b*x**5 + b**2*x**8),x)
```

3.642 $\int \frac{\sqrt{c+dx^3}}{x^3(a+bx^3)^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{\sqrt{c+dx^3}}{x^3(a+bx^3)^2} \, dx = -\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},2,-\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
-1/2*(d*x^3+c)^(1/2)*AppellF1(-2/3,2,-1/2,1/3,-b*x^3/a,-d*x^3/c)/a^2/x^2/(
1+d*x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 338 vs. 2(64) = 128.

Time = 10.25 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.28

$$\int \frac{\sqrt{c+dx^3}}{x^3(a+bx^3)^2} \, dx = \frac{-5bdx^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right) + \frac{a\left(32ac(6ac+30bcx^3-3adx^3+10bdx^6)\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{b}{}\right.\right.}{(a+bx^3)\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx}{a}\right.\right.}}{48a^3x^2\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)^2),x]
```

output
```
(-5*b*d*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -
((b*x^3)/a)] + (a*(32*a*c*(6*a*c + 30*b*c*x^3 - 3*a*d*x^3 + 10*b*d*x^6)*Ap
pellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*x^3*(3*a + 5*b*x
^3)*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/
a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a +
b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*
x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*Ap
pellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(48*a^3*x^2*Sqrt[
c + d*x^3])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^3 (a+bx^3)^2} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{c+dx^3}\int \frac{\sqrt{\frac{dx^3}{c}+1}}{x^3 (bx^3+a)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},2,-\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)^2),x]
```

output
```
-1/2*(Sqrt[c + d*x^3]*AppellF1[-2/3, 2, -1/2, 1/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*x^2*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.05 (sec) , antiderivative size = 766, normalized size of antiderivative = 11.97

method	result	size
elliptic	Expression too large to display	766
risch	Expression too large to display	1513
default	Expression too large to display	1768

input
```
int((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/x^2*(d*x^3+c)^(1/2)-1/3*b*x/a^2/(b*x^3+a)*(d*x^3+c)^(1/2)+5/18*I/
a^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^
2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/
18*I/a^2/d^2*2^(1/2)*sum((-7*a*d+10*b*c)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)
*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)
^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*
d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3
)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d
^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)
)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx = \int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**3/(b*x**3+a)**2,x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**3*(a + b*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2x^3}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^3), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2x^3}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^3 (a+bx^3)^2} \, dx = \int \frac{\sqrt{d\,x^3+c}}{x^3\,(b\,x^3+a)^2} \, dx$$

input
```
int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)^2),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3 (a+bx^3)^2} \, dx = \int \frac{\sqrt{d\,x^3+c}}{b^2x^9+2ab\,x^6+a^2x^3} dx$$

input
```
int((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x)
```

output
```
int(sqrt(c + d*x**3)/(a**2*x**3 + 2*a*b*x**6 + b**2*x**9),x)
```

3.643 $\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = -\frac{2a(2bc-3ad)\sqrt{c+dx^3}}{3b^4} - \frac{a^2(bc-ad)\sqrt{c+dx^3}}{3b^4(a+bx^3)} - \frac{4a(c+dx^3)^{3/2}}{9b^3} + \frac{2(c+dx^3)^{5/2}}{15b^2d} + \frac{a(4bc-7ad)\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}$$

output
```
-2/3*a*(-3*a*d+2*b*c)*(d*x^3+c)^(1/2)/b^4-1/3*a^2*(-a*d+b*c)*(d*x^3+c)^(1/
2)/b^4/(b*x^3+a)-4/9*a*(d*x^3+c)^(3/2)/b^3+2/15*(d*x^3+c)^(5/2)/b^2/d+1/3*
a*(-7*a*d+4*b*c)*(-a*d+b*c)^(1/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*
c)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.96

$$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{\sqrt{c+dx^3}\left(105a^3d^2 + 6b^3x^3(c+dx^3)^2 + 5a^2bd(-19c+14dx^3) + 2ab^2(3c^2 - 34cdx^3\right.}{45b^4d(a+bx^3)} + \frac{a(4bc-7ad)\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{9/2}}$$

input
```
Integrate[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
(Sqrt[c + d*x^3]*(105*a^3*d^2 + 6*b^3*x^3*(c + d*x^3)^2 + 5*a^2*b*d*(-19*c
 + 14*d*x^3) + 2*a*b^2*(3*c^2 - 34*c*d*x^3 - 7*d^2*x^6)))/(45*b^4*d*(a + b
*x^3)) + (a*(4*b*c - 7*a*d)*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(9/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 100, 27, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 \left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^6 \left(dx^3+c\right)^{3/2}}{\left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{3} \left(\frac{\int -\frac{\left(dx^3+c\right)^{3/2} \left(a(2bc-5ad)-2b(bc-ad)x^3\right)}{2\left(bx^3+a\right)}dx^3}{b^2(bc-ad)}-\frac{a^2 \left(c+dx^3\right)^{5/2}}{b^2 \left(a+bx^3\right) (bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(-\frac{\int \frac{\left(dx^3+c\right)^{3/2} \left(a(2bc-5ad)-2b(bc-ad)x^3\right)}{bx^3+a}dx^3}{2b^2(bc-ad)}-\frac{a^2 \left(c+dx^3\right)^{5/2}}{b^2 \left(a+bx^3\right) (bc-ad)}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3} \left(-\frac{a(4bc-7ad) \int \frac{\left(dx^3+c\right)^{3/2}}{bx^3+a}dx^3-\frac{4\left(c+dx^3\right)^{5/2}(bc-ad)}{5d}}{2b^2(bc-ad)}-\frac{a^2 \left(c+dx^3\right)^{5/2}}{b^2 \left(a+bx^3\right) (bc-ad)}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3}\left(-\frac{a(4bc-7ad)\left(\frac{(bc-ad)\int \frac{\sqrt{dx^3+c}}{bx^3+a}dx^3}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)-\frac{4\left(c+dx^3\right)^{5/2}(bc-ad)}{5d}}{2b^2(bc-ad)}-\frac{a^2\left(c+dx^3\right)^{5/2}}{b^2\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3}\left(-\frac{a(4bc-7ad)\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{b}+\frac{2\sqrt{c+dx^3}}{b}\right)}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)-\frac{4\left(c+dx^3\right)^{5/2}(bc-ad)}{5d}}{2b^2(bc-ad)}-\frac{a^2\left(c+\right.}{b^2\left(a+bx^3\right.}\right.$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(-\frac{a(4bc-7ad)\left(\frac{(bc-ad)\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd}+\frac{2\sqrt{c+dx^3}}{b}\right)}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)-\frac{4\left(c+dx^3\right)^{5/2}(bc-ad)}{5d}}{2b^2(bc-ad)}-\frac{a^2\left(c+\right.}{b^2\left(a+bx\right.}\right.$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(-\frac{a^2(c+dx^3)^{5/2}}{b^2(a+bx^3)(bc-ad)}-\frac{a(4bc-7ad)\left(\frac{(bc-ad)\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}+\frac{2(c+dx^3)^{3/2}}{3b}\right)-\frac{4(c+}{2b^2(bc-ad)}\right.$$

input
```
Int[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
(-((a^2*(c + d*x^3)^(5/2))/(b^2*(b*c - a*d)*(a + b*x^3))) - ((-4*(b*c - a*
d)*(c + d*x^3)^(5/2))/(5*d) + a*(4*b*c - 7*a*d)*((2*(c + d*x^3)^(3/2))/(3*
b) + ((b*c - a*d)*((2*Sqrt[c + d*x^3])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2)))/b))/(2*b^2*(b*c - a*d)))
/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{7\left(a(ad-bc)\left(ad-\frac{4bc}{7}\right)d(b\,x^3+a)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)-\sqrt{(ad-bc)b}\,\sqrt{d\,x^3+c}\left(\frac{2x^3(d\,x^3+c)^2b^3}{35}+\frac{2a\left(-\frac{7}{3}d^2x^6-\frac{34}{3}cd\,x^3+\cdots\right.}{35}\cdots\right.\right.}{3\sqrt{(ad-bc)b}\,b^4d(b\,x^3+a)}$
default	$\frac{2(d\,x^3+c)^{\frac{5}{2}}}{15b^2d}+\frac{a^2\left(-d(b\,x^3+a)(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\,\sqrt{d\,x^3+c}\left(\frac{(2d\,x^3-c)b}{3}+ad\right)\right)}{b^4\sqrt{(ad-bc)b}\,(b\,x^3+a)}+\frac{4a\left(-(ad\cdots\right.}{\cdots}$
risch	$\frac{2(3b^2d^2x^6-10x^3ab\,d^2+6x^3b^2cd+45a^2d^2-40abcd+3b^2c^2)\sqrt{d\,x^3+c}}{45d\,b^4}-\frac{a\left(\frac{2(4a^2d^2-6abcd+2b^2c^2)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}-\cdots\right.}{\cdots}$
elliptic	$\frac{a^2(ad-bc)\sqrt{d\,x^3+c}}{3b^4(b\,x^3+a)}+\frac{2d\,x^6\sqrt{d\,x^3+c}}{15b^2}+\frac{2\left(-\frac{2(ad-bc)d}{b^3}-\frac{4cd}{5b^2}\right)x^3\sqrt{d\,x^3+c}}{9d}+\frac{2\left(\frac{3a^2d^2-4abcd+b^2c^2}{b^4}-\frac{2\left(-\frac{2(ad-bc)d}{b^3}-\cdots\right.}{3d}\cdots\right.}{3d}$

input
```
int(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-7/3/((a*d-b*c)*b)^(1/2)*(a*(a*d-b*c)*(a*d-4/7*b*c)*d*(b*x^3+a)*arctan(b*(
d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))-((a*d-b*c)*b)^(1/2)*(d*x^3+c)^(1/2)*(2
/35*x^3*(d*x^3+c)^2*b^3+2/35*a*(-7/3*d^2*x^6-34/3*c*d*x^3+c^2)*b^2-19/21*a
^2*d*(-14/19*d*x^3+c)*b+a^3*d^2))/b^4/d/(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.64

$$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \left[-\frac{15\,(4\,a^2bcd - 7\,a^3d^2 + (4\,ab^2cd - 7\,a^2bd^2)x^3)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,b\sqrt{\cdots}}{bx^3+a}\right.}{\cdots}\right.$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/90*(15*(4*a^2*b*c*d - 7*a^3*d^2 + (4*a*b^2*c*d - 7*a^2*b*d^2)*x^3)*sqr
t((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*
c - a*d)/b))/(b*x^3 + a)) - 2*(6*b^3*d^2*x^9 + 2*(6*b^3*c*d - 7*a*b^2*d^2)
*x^6 + 6*a*b^2*c^2 - 95*a^2*b*c*d + 105*a^3*d^2 + 2*(3*b^3*c^2 - 34*a*b^2*
c*d + 35*a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b^5*d*x^3 + a*b^4*d), 1/45*(15*
(4*a^2*b*c*d - 7*a^3*d^2 + (4*a*b^2*c*d - 7*a^2*b*d^2)*x^3)*sqrt(-(b*c - a
*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (6*b^
3*d^2*x^9 + 2*(6*b^3*c*d - 7*a*b^2*d^2)*x^6 + 6*a*b^2*c^2 - 95*a^2*b*c*d +
 105*a^3*d^2 + 2*(3*b^3*c^2 - 34*a*b^2*c*d + 35*a^2*b*d^2)*x^3)*sqrt(d*x^3
 + c))/(b^5*d*x^3 + a*b^4*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**8*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.26

$$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = -\frac{(4\,ab^2c^2-11\,a^2bcd+7\,a^3d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^4} - \frac{\sqrt{dx^3+c}a^2bcd-\sqrt{dx^3+c}a^3d^2}{3\left((dx^3+c)b-bc+ad\right)b^4} + \frac{2\left(3\,(dx^3+c)^{\frac{5}{2}}b^8d^4-10\,(dx^3+c)^{\frac{3}{2}}ab^7d^5-30\sqrt{dx^3+c}ab^7cd^5+45\sqrt{dx^3+c}a^2b^6d^6\right)}{45\,b^{10}d^5}$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/3*(4*a*b^2*c^2 - 11*a^2*b*c*d + 7*a^3*d^2)*arctan(sqrt(d*x^3 + c)*b/sqr
t(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) - 1/3*(sqrt(d*x^3 + c)*a^2*b
*c*d - sqrt(d*x^3 + c)*a^3*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^4) + 2/45*(
3*(d*x^3 + c)^(5/2)*b^8*d^4 - 10*(d*x^3 + c)^(3/2)*a*b^7*d^5 - 30*sqrt(d*x
^3 + c)*a*b^7*c*d^5 + 45*sqrt(d*x^3 + c)*a^2*b^6*d^6)/(b^10*d^5)
```

Mupad [B] (verification not implemented)

Time = 6.52 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.97

$$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,(a\,d-b\,c)^2}{b^4}+\frac{2\,c\,\left(\frac{2\,d\,(a\,d-2\,b\,c)}{b^3}+\frac{2\,a\,d^2}{b^3}+\frac{8\,c\,d}{5\,b^2}\right)}{3\,d}+\frac{2\,a\,\left(\frac{d\,(a\,d-2\,b\,c)}{b^3}+\frac{a\,d^2}{b^3}\right)}{b}\right)}{3\,d} + \frac{2\,d\,x^6\,\sqrt{d\,x^3+c}}{15\,b^2} - \frac{x^3\,\sqrt{d\,x^3+c}\,\left(\frac{2\,d\,(a\,d-2\,b\,c)}{b^3}+\frac{2\,a\,d^2}{b^3}+\frac{8\,c\,d}{5\,b^2}\right)}{9\,d} - \frac{a^2\,\left(\frac{2\,b\,c^2}{3\,(2\,b^2\,c-2\,a\,b\,d)}+\frac{a\,\left(\frac{2\,a\,d^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{4\,b\,c\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)}{b}\right)\,\sqrt{d\,x^3+c}}{b^2\,\left(b\,x^3+a\right)} + \frac{a\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,(7\,a\,d-4\,b\,c)\,1{}\mathrm{i}}{6\,b^{9/2}}$$

input
```
int((x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

output
```
((c + d*x^3)^(1/2)*((2*(a*d - b*c)^2)/b^4 + (2*c*((2*d*(a*d - 2*b*c))/b^3
+ (2*a*d^2)/b^3 + (8*c*d)/(5*b^2)))/(3*d) + (2*a*((d*(a*d - 2*b*c))/b^3 +
(a*d^2)/b^3))/b))/(3*d) + (2*d*x^6*(c + d*x^3)^(1/2))/(15*b^2) - (x^3*(c +
 d*x^3)^(1/2)*((2*d*(a*d - 2*b*c))/b^3 + (2*a*d^2)/b^3 + (8*c*d)/(5*b^2)))
/(9*d) + (a*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)
*2i - b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*(7*a*d - 4*b*c)*1i)/(6*b^(9/
2)) - (a^2*((2*b*c^2)/(3*(2*b^2*c - 2*a*b*d)) + (a*((2*a*d^2)/(3*(2*b^2*c
- 2*a*b*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*a*b*d))))/b)*(c + d*x^3)^(1/2))/(b
^2*(a + b*x^3))
```

Reduce [F]

$$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

output
```
( - 140*sqrt(c + d*x**3)*a**3*c*d**2 + 70*sqrt(c + d*x**3)*a**3*d**3*x**3
+ 136*sqrt(c + d*x**3)*a**2*b*c**2*d - 208*sqrt(c + d*x**3)*a**2*b*c*d**2*
x**3 - 14*sqrt(c + d*x**3)*a**2*b*d**3*x**6 - 12*sqrt(c + d*x**3)*a*b**2*c
**3 + 142*sqrt(c + d*x**3)*a*b**2*c**2*d*x**3 + 40*sqrt(c + d*x**3)*a*b**2
*c*d**2*x**6 + 6*sqrt(c + d*x**3)*a*b**2*d**3*x**9 - 12*sqrt(c + d*x**3)*b
**3*c**3*x**3 - 24*sqrt(c + d*x**3)*b**3*c**2*d*x**6 - 12*sqrt(c + d*x**3)
*b**3*c*d**2*x**9 - 315*int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*
x**3 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*
c*d*x**6 + a*b**2*d**2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**6*
d**5 + 1440*int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**
2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a
*b**2*d**2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**5*b*c*d**4 - 3
15*int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2
+ 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**
2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**5*b*d**5*x**3 - 2295*in
t((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a
**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**
9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**4*b**2*c**2*d**3 + 1440*int(
(sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**
2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x*...
```

3.644 $\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{2(bc-2ad)\sqrt{c+dx^3}}{3b^3} + \frac{a(bc-ad)\sqrt{c+dx^3}}{3b^3(a+bx^3)} + \frac{2(c+dx^3)^{3/2}}{9b^2} - \frac{(2bc-5ad)\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

output
```
2/3*(-2*a*d+b*c)*(d*x^3+c)^(1/2)/b^3+1/3*a*(-a*d+b*c)*(d*x^3+c)^(1/2)/b^3/
(b*x^3+a)+2/9*(d*x^3+c)^(3/2)/b^2-1/3*(-5*a*d+2*b*c)*(-a*d+b*c)^(1/2)*arct
anh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89

$$\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{\sqrt{c+dx^3}(-15a^2d+ab(11c-10dx^3)+2b^2x^3(4c+dx^3))}{9b^3(a+bx^3)} - \frac{(2bc-5ad)\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{7/2}}$$

input
```
Integrate[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
(Sqrt[c + d*x^3]*(-15*a^2*d + a*b*(11*c - 10*d*x^3) + 2*b^2*x^3*(4*c + d*x
^3)))/(9*b^3*(a + b*x^3)) - ((2*b*c - 5*a*d)*Sqrt[-(b*c) + a*d]*ArcTan[(Sq
rt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(7/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(c+d x^3\right)^{3/2}}{\left(a+b x^3\right)^2} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^3 \left(d x^3+c\right)^{3/2}}{\left(b x^3+a\right)^2}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3} \left(\frac{(2 b c-5 a d) \int \frac{\left(d x^3+c\right)^{3/2}}{b x^3+a}dx^3}{2 b (b c-a d)}+\frac{a \left(c+d x^3\right)^{5/2}}{b \left(a+b x^3\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3} \left(\frac{(2 b c-5 a d) \left(\frac{(b c-a d) \int \frac{\sqrt{d x^3+c}}{b x^3+a}dx^3}{b}+\frac{2 \left(c+d x^3\right)^{3/2}}{3 b}\right)}{2 b (b c-a d)}+\frac{a \left(c+d x^3\right)^{5/2}}{b \left(a+b x^3\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{(2bc-5ad)\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{b}+\frac{2\sqrt{c+dx^3}}{b}\right)}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)}{2b(bc-ad)}+\frac{a\left(c+dx^3\right)^{5/2}}{b\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(\frac{(2bc-5ad)\left(\frac{(bc-ad)\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd}+\frac{2\sqrt{c+dx^3}}{b}\right)}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)}{2b(bc-ad)}+\frac{a\left(c+dx^3\right)^{5/2}}{b\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{3}\left(\frac{(2bc-5ad)\left(\frac{(bc-ad)\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)}{2b(bc-ad)}+\frac{a\left(c+dx^3\right)^{5/2}}{b\left(a+bx^3\right)(bc-ad)}\right)$$

input
```
Int[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
((a*(c + d*x^3)^(5/2))/(b*(b*c - a*d)*(a + b*x^3)) + ((2*b*c - 5*a*d)*((2*
(c + d*x^3)^(3/2))/(3*b) + ((b*c - a*d)*((2*Sqrt[c + d*x^3])/b - (2*Sqrt[b
*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2)))/b)
)/(2*b*(b*c - a*d)))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$-\frac{5\left(-(ad-bc)\left(ad-\frac{2bc}{5}\right)(bx^3+a)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(-\frac{8\left(\frac{dx^3}{4}+c\right)x^3b^2}{15}-\frac{11a\left(-\frac{10dx^3}{11}+c\right)b}{15}+da^2\right)\sqrt{(ad-bc)b}\,\sqrt{d}\cdots}{3\sqrt{(ad-bc)b}\,b^3(bx^3+a)}$
risch	$-\frac{2(-bdx^3+6ad-4bc)\sqrt{dx^3+c}}{9b^3}+\frac{\frac{2(3a^2d^2-4abcd+b^2c^2)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}-\frac{a(a^2d^2-2abcd+b^2c^2)\left(\frac{\sqrt{dx^3+c}}{bx^3+a}+\frac{d\arctan(\cdots}{\sqrt{(\cdots}}\right.}{3(ad-bc)}}{b^3}$
default	$-\frac{2\left(-(ad-bc)^2\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(\frac{(-dx^3-4c)b}{3}+ad\right)\sqrt{dx^3+c}\,\sqrt{(ad-bc)b}\right)}{3b^3\sqrt{(ad-bc)b}}-\frac{a\left(-d(bx^3+a)(ad-bc)\arctan\left(\cdots\right.\right.}{b\cdots}$
elliptic	$-\frac{(ad-bc)a\sqrt{dx^3+c}}{3b^3(bx^3+a)}+\frac{2dx^3\sqrt{dx^3+c}}{9b^2}+\frac{2\left(-\frac{2(ad-bc)d}{b^3}-\frac{2cd}{3b^2}\right)\sqrt{dx^3+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-5a^2d^2+7ab\cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/3*(-(a*d-b*c)*(a*d-2/5*b*c)*(b*x^3+a)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*
c)*b)^(1/2))+(-8/15*(1/4*d*x^3+c)*x^3*b^2-11/15*a*(-10/11*d*x^3+c)*b+d*a^2
)*((a*d-b*c)*b)^(1/2)*(d*x^3+c)^(1/2))/((a*d-b*c)*b)^(1/2)/b^3/(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.23

$$\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \left[-\frac{3\left((2\,b^2c-5\,abd)x^3+2\,abc-5\,a^2d\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-}{18\,(b^4x^3+ab^3)}\right.$$

$$-\frac{3\left((2\,b^2c-5\,abd)x^3+2\,abc-5\,a^2d\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(2\,b^2dx^6+2\,(4\,b^2c-5\,abd)x}{9\,(b^4x^3+ab^3)}$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/18*(3*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*sqrt((b*c - a*d)/b
)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b
*x^3 + a)) - 2*(2*b^2*d*x^6 + 2*(4*b^2*c - 5*a*b*d)*x^3 + 11*a*b*c - 15*a^
2*d)*sqrt(d*x^3 + c))/(b^4*x^3 + a*b^3), -1/9*(3*((2*b^2*c - 5*a*b*d)*x^3
+ 2*a*b*c - 5*a^2*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-
(b*c - a*d)/b)/(b*c - a*d)) - (2*b^2*d*x^6 + 2*(4*b^2*c - 5*a*b*d)*x^3 + 1
1*a*b*c - 15*a^2*d)*sqrt(d*x^3 + c))/(b^4*x^3 + a*b^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.23

$$\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{(2\,b^2c^2-7\,abcd+5\,a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^3} + \frac{\sqrt{dx^3+c}abcd-\sqrt{dx^3+c}a^2d^2}{3\,((dx^3+c)b-bc+ad)b^3} + \frac{2\left((dx^3+c)^{\frac{3}{2}}b^4+3\,\sqrt{dx^3+c}b^4c-6\,\sqrt{dx^3+c}ab^3d\right)}{9\,b^6}$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*(2*b^2*c^2 - 7*a*b*c*d + 5*a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2
*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/3*(sqrt(d*x^3 + c)*a*b*c*d - s
qrt(d*x^3 + c)*a^2*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^3) + 2/9*((d*x^3 +
c)^(3/2)*b^4 + 3*sqrt(d*x^3 + c)*b^4*c - 6*sqrt(d*x^3 + c)*a*b^3*d)/b^6
```

Mupad [B] (verification not implemented)

Time = 6.47 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.62

$$\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{2\,d\,x^3\,\sqrt{d\,x^3+c}}{9\,b^2} - \frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,d\,(a\,d-2\,b\,c)}{b^3}+\frac{2\,a\,d^2}{b^3}+\frac{4\,c\,d}{3\,b^2}\right)}{3\,d} + \frac{a\left(\frac{2\,b\,c^2}{3\,(2\,b^2\,c-2\,a\,b\,d)}+\frac{a\left(\frac{2\,a\,d^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{4\,b\,c\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)}{b}\right)\sqrt{d\,x^3+c}}{b\,\left(b\,x^3+a\right)} + \frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,(5\,a\,d-2\,b\,c)\,1\mathrm{i}}{6\,b^{7/2}}$$

input
```
int((x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

output
```
(2*d*x^3*(c + d*x^3)^(1/2))/(9*b^2) - ((c + d*x^3)^(1/2)*((2*d*(a*d - 2*b*
c))/b^3 + (2*a*d^2)/b^3 + (4*c*d)/(3*b^2)))/(3*d) + (log((2*b*c - a*d + b^
(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d
- b*c)^(1/2)*(5*a*d - 2*b*c)*1i)/(6*b^(7/2)) + (a*((2*b*c^2)/(3*(2*b^2*c -
 2*a*b*d)) + (a*((2*a*d^2)/(3*(2*b^2*c - 2*a*b*d)) - (4*b*c*d)/(3*(2*b^2*c
 - 2*a*b*d))))/b)*(c + d*x^3)^(1/2))/(b*(a + b*x^3))
```

Reduce [F]

$$\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

output
```
(20*sqrt(c + d*x**3)*a**2*c*d - 10*sqrt(c + d*x**3)*a**2*d**2*x**3 - 16*sq
rt(c + d*x**3)*a*b*c**2 + 28*sqrt(c + d*x**3)*a*b*c*d*x**3 + 2*sqrt(c + d*
x**3)*a*b*d**2*x**6 - 16*sqrt(c + d*x**3)*b**2*c**2*x**3 - 4*sqrt(c + d*x*
*3)*b**2*c*d*x**6 + 45*int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x
**3 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c
*d*x**6 + a*b**2*d**2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**5*d
**4 - 198*int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*
b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b
**2*d**2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**4*b*c*d**3 + 45*
int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2
*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x
**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**4*b*d**4*x**3 + 297*int((s
qrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*
b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 -
2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**3*b**2*c**2*d**2 - 198*int((sqrt
(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*d
**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2*b
**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**3*b**2*c*d**3*x**3 - 180*int((sqrt(
c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*d*
*2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2...
```

3.645 $\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2} dx = \frac{d\sqrt{c+dx^3}}{b^2} - \frac{(c+dx^3)^{3/2}}{3b(a+bx^3)} - \frac{d\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

output
```
d*(d*x^3+c)^(1/2)/b^2-1/3*(d*x^3+c)^(3/2)/b/(b*x^3+a)-d*(-a*d+b*c)^(1/2)*a
rctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2} dx = \frac{\sqrt{c+dx^3}(-bc+3ad+2bdx^3)}{3b^2(a+bx^3)} - \frac{d\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{b^{5/2}}$$

input
```
Integrate[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
(Sqrt[c + d*x^3]*(-(b*c) + 3*a*d + 2*b*d*x^3))/(3*b^2*(a + b*x^3)) - (d*Sq
rt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/b^(
5/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {946, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (c+dx^3)^{3/2}}{(a+bx^3)^2} \, dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{3}\int \frac{(dx^3+c)^{3/2}}{(bx^3+a)^2}dx^3$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{3}\left(\frac{3d\int \frac{\sqrt{dx^3+c}}{bx^3+a}dx^3}{2b}-\frac{(c+dx^3)^{3/2}}{b(a+bx^3)}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{3}\left(\frac{3d\left(\frac{(bc-ad)\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{b}+\frac{2\sqrt{c+dx^3}}{b}\right)}{2b}-\frac{(c+dx^3)^{3/2}}{b(a+bx^3)}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(\frac{3d\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd}+\frac{2\sqrt{c+dx^3}}{b}\right)}{2b}-\frac{(c+dx^3)^{3/2}}{b(a+bx^3)}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{3}\left(\frac{3d\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{2b}-\frac{\left(c+dx^3\right)^{3/2}}{b\left(a+bx^3\right)}\right)$$

input
```
Int[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
(-((c + d*x^3)^(3/2)/(b*(a + b*x^3))) + (3*d*((2*Sqrt[c + d*x^3])/b - (2*S
qrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2)
))/(2*b))/3
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

method	result
default	$\frac{-d(b\,x^3+a)(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\,\sqrt{d\,x^3+c}\left(\frac{(2d\,x^3-c)b}{3}+ad\right)}{\sqrt{(ad-bc)b}\,b^2(b\,x^3+a)}$
pseudoelliptic	$\frac{-d(b\,x^3+a)(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\,\sqrt{d\,x^3+c}\left(\frac{(2d\,x^3-c)b}{3}+ad\right)}{\sqrt{(ad-bc)b}\,b^2(b\,x^3+a)}$
risch	$\frac{2d\sqrt{d\,x^3+c}}{3b^2}-\frac{\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\left(\frac{\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3ad-3bc}+\frac{4(ad-bc)d\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}}{b^2}$
elliptic	$\frac{(ad-bc)\sqrt{d\,x^3+c}}{3b^2(b\,x^3+a)}+\frac{2d\sqrt{d\,x^3+c}}{3b^2}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\cdots}}{\cdots}\right.}{\cdots}$

input
```
int(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-d*(b*x^3+a)*(a*d-b*c)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*
d-b*c)*b)^(1/2)*(d*x^3+c)^(1/2)*(1/3*(2*d*x^3-c)*b+a*d))/((a*d-b*c)*b)^(1/
2)/b^2/(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.49

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \left[\frac{3\,(bdx^3+ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\,(2\,bdx^3-bc+3\,ad)\sqrt{}}{6\,(b^3x^3+ab^2)}\right.$$

$$\left.-\frac{3\,(bdx^3+ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(2\,bdx^3-bc+3\,ad)\sqrt{dx^3+c}}{3\,(b^3x^3+ab^2)}\right]$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/6*(3*(b*d*x^3 + a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2
*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(2*b*d*x^3 - b*c
+ 3*a*d)*sqrt(d*x^3 + c))/(b^3*x^3 + a*b^2), -1/3*(3*(b*d*x^3 + a*d)*sqrt(
-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)
) - (2*b*d*x^3 - b*c + 3*a*d)*sqrt(d*x^3 + c))/(b^3*x^3 + a*b^2)]
```

Sympy [F]

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{x^2(c+dx^3)^{\frac{3}{2}}}{(a+bx^3)^2}\,dx$$

input
```
integrate(x**2*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

output
```
Integral(x**2*(c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.30

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{2\sqrt{dx^3+c}d}{3\,b^2} + \frac{(bcd-ad^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2} - \frac{\sqrt{dx^3+c}bcd-\sqrt{dx^3+c}ad^2}{3\left((dx^3+c)b-bc+ad\right)b^2}$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
2/3*sqrt(d*x^3 + c)*d/b^2 + (b*c*d - a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(
-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) - 1/3*(sqrt(d*x^3 + c)*b*c*d -
 sqrt(d*x^3 + c)*a*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^2)
```

Mupad [B] (verification not implemented)

Time = 6.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.81

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{2\,d\,\sqrt{d\,x^3+c}}{3\,b^2} - \frac{\left(\frac{2\,b\,c^2}{3\,(2\,b^2\,c-2\,a\,b\,d)}+\frac{a\,\left(\frac{2\,a\,d^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{4\,b\,c\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)}{b}\right)\,\sqrt{d\,x^3+c}}{b\,x^3+a} + \frac{d\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,1{}\mathrm{i}}{2\,b^{5/2}}$$

input
```
int((x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

output
```
(2*d*(c + d*x^3)^(1/2))/(3*b^2) - (((2*b*c^2)/(3*(2*b^2*c - 2*a*b*d)) + (a
*((2*a*d^2)/(3*(2*b^2*c - 2*a*b*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*a*b*d))))/
b)*(c + d*x^3)^(1/2))/(a + b*x^3) + (d*log((a*d - 2*b*c + b^(1/2)*(c + d*x
^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1
i)/(2*b^(5/2))
```

Reduce [F]

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

output
```
( - 4*sqrt(c + d*x**3)*a*c*d + 2*sqrt(c + d*x**3)*a*d**2*x**3 + 2*sqrt(c +
 d*x**3)*b*c**2 - 4*sqrt(c + d*x**3)*b*c*d*x**3 - 9*int((sqrt(c + d*x**3)*
x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*
a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2*b**3*c**2*x**6
 - 2*b**3*c*d*x**9),x)*a**4*d**4 + 36*int((sqrt(c + d*x**3)*x**5)/(a**3*c*
d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x*
*3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*
x**9),x)*a**3*b*c*d**3 - 9*int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d*
*2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b*
*2*c*d*x**6 + a*b**2*d**2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a*
*3*b*d**4*x**3 - 45*int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3
 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*
x**6 + a*b**2*d**2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**2*b**2
*c**2*d**2 + 36*int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2
*a**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6
 + a*b**2*d**2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**2*b**2*c*d
**3*x**3 + 18*int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a
**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 +
 a*b**2*d**2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a*b**3*c**3*d -
 45*int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c...
```

3.646 $\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2} dx = \frac{(bc-ad)\sqrt{c+dx^3}}{3ab(a+bx^3)} - \frac{2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{\sqrt{bc-ad}(2bc+ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2b^{3/2}}$$

output
```
1/3*(-a*d+b*c)*(d*x^3+c)^(1/2)/a/b/(b*x^3+a)-2/3*c^(3/2)*arctanh((d*x^3+c)
^(1/2)/c^(1/2))/a^2+1/3*(-a*d+b*c)^(1/2)*(a*d+2*b*c)*arctanh(b^(1/2)*(d*x^
3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2} dx = \frac{\frac{a(bc-ad)\sqrt{c+dx^3}}{b(a+bx^3)} + \frac{\sqrt{-bc+ad}(2bc+ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{b^{3/2}} - 2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x]
```

output
```
((a*(b*c - a*d)*Sqrt[c + d*x^3])/(b*(a + b*x^3)) + (Sqrt[-(b*c) + a*d]*(2*
b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/b^(3/2) -
 2*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\left(dx^3+c\right)^{3/2}}{x^3\left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{3}\left(\frac{\int \frac{d(bc+ad)x^3+2bc^2}{2x^3(bx^3+a)\sqrt{dx^3+c}}dx^3}{ab}+\frac{\sqrt{c+dx^3}(bc-ad)}{ab\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{d(bc+ad)x^3+2bc^2}{x^3(bx^3+a)\sqrt{dx^3+c}}dx^3}{2ab}+\frac{\sqrt{c+dx^3}(bc-ad)}{ab\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{\frac{2bc^2\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{(bc-ad)(ad+2bc)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{2ab}+\frac{\sqrt{c+dx^3}(bc-ad)}{ab\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\frac{4bc^2\int\frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2(bc-ad)(ad+2bc)\int\frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{2ab}+\frac{\sqrt{c+dx^3}(bc-ad)}{ab\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{\frac{2\sqrt{bc-ad}(ad+2bc)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{4bc^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a}}{2ab}+\frac{\sqrt{c+dx^3}(bc-ad)}{ab\left(a+bx^3\right)}\right)$$

input
```
Int[(c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x]
```

output
```
(((b*c - a*d)*Sqrt[c + d*x^3])/(a*b*(a + b*x^3)) + ((-4*b*c^(3/2)*ArcTanh[
Sqrt[c + d*x^3]/Sqrt[c]])/a + (2*Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b]))/(2*a*b))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$-\frac{-\left(b\,x^{3}+a\right)\left(ad+2bc\right)\left(ad-bc\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)+\left(2b\,c^{\frac{3}{2}}\left(b\,x^{3}+a\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)+a\sqrt{d\,x^{3}+c}\,\left(ad-bc\right)\right)\sqrt{(ad-b}}{3\sqrt{\left(ad-bc\right)b}\,a^{2}b\left(b\,x^{3}+a\right)}$
default	$\frac{\frac{2d\,x^{3}\sqrt{d\,x^{3}+c}}{9}+\frac{8c\sqrt{d\,x^{3}+c}}{9}-\frac{2c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}}{a^{2}}+\frac{-\frac{2\left(ad-bc\right)^{2}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)}{3}+\frac{2\left(\frac{\left(-d\,x^{3}-4c\right)b}{3}+ad\right)\sqrt{d\,x^{3}+c}\,\sqrt{(}}{3}}{b\,a^{2}\sqrt{\left(ad-bc\right)b}}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/((a*d-b*c)*b)^(1/2)*(-(b*x^3+a)*(a*d+2*b*c)*(a*d-b*c)*arctan(b*(d*x^3
+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(2*b*c^(3/2)*(b*x^3+a)*arctanh((d*x^3+c)^(1
/2)/c^(1/2))+a*(d*x^3+c)^(1/2)*(a*d-b*c))*((a*d-b*c)*b)^(1/2))/a^2/b/(b*x^
3+a)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 680, normalized size of antiderivative = 5.19

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2}\,dx = \left[\frac{((2\,b^2c+abd)x^3+2\,abc+a^2d)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\,(b^2cx^3}{6\,(a^2b^2x^3+a^3b)}\right.$$

input
```
integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/6*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt((b*c - a*d)/b)*log((b
*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a
)) + 2*(b^2*c*x^3 + a*b*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c)
+ 2*c)/x^3) + 2*sqrt(d*x^3 + c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b), 1/
3*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt(-(b*c - a*d)/b)*arctan(-
sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b^2*c*x^3 + a*b*c)*
sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + sqrt(d*x^3 +
c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b), 1/6*(4*(b^2*c*x^3 + a*b*c)*sqrt
(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)) + ((2*b^2*c + a*b*d)*x^3 + 2*a*b*c +
 a^2*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)
*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(a*b*c - a^2*d))/
(a^2*b^2*x^3 + a^3*b), 1/3*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt
(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d
)) + 2*(b^2*c*x^3 + a*b*c)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)) + sqr
t(d*x^3 + c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b)]
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{x(a+bx^3)^2}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x/(b*x**3+a)**2,x)
```

output
```
Integral((c + d*x**3)**(3/2)/(x*(a + b*x**3)**2), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2x}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.18

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2}\,dx = \frac{2\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}} - \frac{(2\,b^2c^2-abcd-a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}a^2b} + \frac{\sqrt{dx^3+c}bcd-\sqrt{dx^3+c}ad^2}{3\,((dx^3+c)b-bc+ad)ab}$$

input
```
integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
2/3*c^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/3*(2*b^2*c^2 -
 a*b*c*d - a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-
b^2*c + a*b*d)*a^2*b) + 1/3*(sqrt(d*x^3 + c)*b*c*d - sqrt(d*x^3 + c)*a*d^2
)/(((d*x^3 + c)*b - b*c + a*d)*a*b)
```

Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.63

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2}\,dx = \frac{c^{3/2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2} + \frac{\sqrt{d\,x^3+c}\left(\frac{a\left(\frac{b\,d^2}{3\left(b^2\,c-a\,b\,d\right)}-\frac{2\,b^2\,c\,d}{3\,a\left(b^2\,c-a\,b\,d\right)}\right)}{b}+\frac{b^2\,c^2}{3\,a\,(b^2\,c-a\,b\,d)}\right)}{b\,x^3+a} + \frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,(a\,d+2\,b\,c)\,1\mathrm{i}}{6\,a^2\,b^{3/2}}$$

input
```
int((c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x)
```

output
```
(c^(3/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)
))/x^6))/(3*a^2) + ((c + d*x^3)^(1/2)*((a*((b*d^2)/(3*(b^2*c - a*b*d)) - (
2*b^2*c*d)/(3*a*(b^2*c - a*b*d))))/b + (b^2*c^2)/(3*a*(b^2*c - a*b*d))))/(
a + b*x^3) + (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/
2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*(a*d + 2*b*c)*1i)/(6*a^2*b
^(3/2))
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2}\,dx = \text{Too large to display}$$

input

```
int((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x)
```

output

```
(4*sqrt(c + d*x**3)*c*d + 3*int(sqrt(c + d*x**3)/(a**3*c*d*x + a**3*d**2*x
**4 - 2*a**2*b*c**2*x + 2*a**2*b*d**2*x**7 - 4*a*b**2*c**2*x**4 - 3*a*b**2
*c*d*x**7 + a*b**2*d**2*x**10 - 2*b**3*c**2*x**7 - 2*b**3*c*d*x**10),x)*a*
*3*c**2*d**2 - 12*int(sqrt(c + d*x**3)/(a**3*c*d*x + a**3*d**2*x**4 - 2*a*
*2*b*c**2*x + 2*a**2*b*d**2*x**7 - 4*a*b**2*c**2*x**4 - 3*a*b**2*c*d*x**7
+ a*b**2*d**2*x**10 - 2*b**3*c**2*x**7 - 2*b**3*c*d*x**10),x)*a**2*b*c**3*
d + 3*int(sqrt(c + d*x**3)/(a**3*c*d*x + a**3*d**2*x**4 - 2*a**2*b*c**2*x
+ 2*a**2*b*d**2*x**7 - 4*a*b**2*c**2*x**4 - 3*a*b**2*c*d*x**7 + a*b**2*d**
2*x**10 - 2*b**3*c**2*x**7 - 2*b**3*c*d*x**10),x)*a**2*b*c**2*d**2*x**3 +
12*int(sqrt(c + d*x**3)/(a**3*c*d*x + a**3*d**2*x**4 - 2*a**2*b*c**2*x + 2
*a**2*b*d**2*x**7 - 4*a*b**2*c**2*x**4 - 3*a*b**2*c*d*x**7 + a*b**2*d**2*x
**10 - 2*b**3*c**2*x**7 - 2*b**3*c*d*x**10),x)*a*b**2*c**4 - 12*int(sqrt(c
 + d*x**3)/(a**3*c*d*x + a**3*d**2*x**4 - 2*a**2*b*c**2*x + 2*a**2*b*d**2*
x**7 - 4*a*b**2*c**2*x**4 - 3*a*b**2*c*d*x**7 + a*b**2*d**2*x**10 - 2*b**3
*c**2*x**7 - 2*b**3*c*d*x**10),x)*a*b**2*c**3*d*x**3 + 12*int(sqrt(c + d*x
**3)/(a**3*c*d*x + a**3*d**2*x**4 - 2*a**2*b*c**2*x + 2*a**2*b*d**2*x**7 -
 4*a*b**2*c**2*x**4 - 3*a*b**2*c*d*x**7 + a*b**2*d**2*x**10 - 2*b**3*c**2*
x**7 - 2*b**3*c*d*x**10),x)*b**3*c**4*x**3 + 3*int((sqrt(c + d*x**3)*x**5)
/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*b**
2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2*b**3*c**2*x**6 -...
```

3.647 $\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 170

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2} dx = -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{c\sqrt{c+dx^3}}{3ax^3(a+bx^3)} + \frac{\sqrt{c}(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3} - \frac{\sqrt{bc-ad}(4bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{b}}$$

output
```
-1/3*(-a*d+2*b*c)*(d*x^3+c)^(1/2)/a^2/(b*x^3+a)-1/3*c*(d*x^3+c)^(1/2)/a/x^
3/(b*x^3+a)+1/3*c^(1/2)*(-3*a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^
3-1/3*(-a*d+b*c)^(1/2)*(-a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+
b*c)^(1/2))/a^3/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2} dx = \frac{\frac{a\sqrt{c+dx^3}(-ac-2bcx^3+adx^3)}{x^3(a+bx^3)} + \frac{(4b^2c^2-5abcd+a^2d^2)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\sqrt{-bc+ad}} + \sqrt{c}(4bc-3ad)\text{arctanh}\left(\right.}{3a^3}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x]
```

output
```
((a*Sqrt[c + d*x^3]*(-(a*c) - 2*b*c*x^3 + a*d*x^3))/(x^3*(a + b*x^3)) + ((
4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b
*c) + a*d]])/(Sqrt[b]*Sqrt[-(b*c) + a*d]) + Sqrt[c]*(4*b*c - 3*a*d)*ArcTan
h[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^3)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {948, 109, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4 \left(a+bx^3\right)^2} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{\left(dx^3+c\right)^{3/2}}{x^6 \left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{3} \left(-\frac{\int \frac{d(3bc-2ad)x^3+c(4bc-3ad)}{2x^3\left(bx^3+a\right)^2\sqrt{dx^3+c}}dx^3}{a}-\frac{c\sqrt{c+dx^3}}{ax^3\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(-\frac{\int \frac{d(3bc-2ad)x^3+c(4bc-3ad)}{x^3\left(bx^3+a\right)^2\sqrt{dx^3+c}}dx^3}{2a}-\frac{c\sqrt{c+dx^3}}{ax^3\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3} \left(-\frac{\frac{\int \frac{d(bc-ad)(2bc-ad)x^3+c(4bc-3ad)(bc-ad)}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a(bc-ad)}+\frac{2\sqrt{c+dx^3}(2bc-ad)}{a(a+bx^3)}}{2a}-\frac{c\sqrt{c+dx^3}}{ax^3\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{c(4bc-3ad)(bc-ad)\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{(bc-ad)^2(4bc-ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{a(bc-ad)}+\frac{2\sqrt{c+dx^3}(2bc-ad)}{a(a+bx^3)}}{2a}-\frac{c\sqrt{c+dx^3}}{ax^3\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{2c(4bc-3ad)(bc-ad)\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2(bc-ad)^2(4bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{a(bc-ad)}+\frac{2\sqrt{c+dx^3}(2bc-ad)}{a(a+bx^3)}}{2a}-\frac{c\sqrt{c+dx^3}}{ax^3\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{2(bc-ad)^{3/2}(4bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{2\sqrt{c}(4bc-3ad)(bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a}}{a(bc-ad)}+\frac{2\sqrt{c+dx^3}(2bc-ad)}{a(a+bx^3)}}{2a}-\frac{c\sqrt{c+dx^3}}{ax^3\left(a+bx^3\right)}\right)$$

input
```
Int[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x]
```

output
```
(-((c*Sqrt[c + d*x^3])/(a*x^3*(a + b*x^3))) - ((2*(2*b*c - a*d)*Sqrt[c + d
*x^3])/(a*(a + b*x^3)) + ((-2*Sqrt[c]*(4*b*c - 3*a*d)*(b*c - a*d)*ArcTanh[
Sqrt[c + d*x^3]/Sqrt[c]])/a + (2*(b*c - a*d)^(3/2)*(4*b*c - a*d)*ArcTanh[(
Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b]))/(a*(b*c - a*d)))/(
2*a))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{\frac{4\left(b\,x^{3}+a\right)\left(-ad+bc\right)\left(bc-\frac{ad}{4}\right)\sqrt{c}\,x^{3}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)}{3}-\frac{\sqrt{\left(ad-bc\right)b}\,\left(3c\left(ad-\frac{4bc}{3}\right)\left(b\,x^{3}+a\right)x^{3}\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)+\left(2x^{3}bc+a\left(-\cdots\right)\right)\right)}{3}}{a^{3}x^{3}\sqrt{c}\,\left(b\,x^{3}+a\right)\sqrt{\left(ad-bc\right)b}}$ …
risch	$-\frac{c\sqrt{d\,x^{3}+c}}{3a^{2}x^{3}}-\frac{\frac{\left(-2a^{2}d^{2}+4abcd-2b^{2}c^{2}\right)\left(\frac{\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)}{\sqrt{\left(ad-bc\right)b}}\right)}{3ad-3bc}+\frac{2\sqrt{c}\,\left(3ad-4bc\right)\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3a}+\frac{8bc(ad-bc\cdots}{\cdots}}{2a^{2}}$ …
default	$\frac{-\frac{c\sqrt{d\,x^{3}+c}}{3x^{3}}+\frac{2d\sqrt{d\,x^{3}+c}}{3}-\sqrt{c}\,d\ \mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{a^{2}}+\frac{-d\left(b\,x^{3}+a\right)\left(ad-bc\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)+\sqrt{\left(ad-bc\right)b}\,\sqrt{d\,x^{3}+c}\,\cdots}{a^{2}\sqrt{\left(ad-bc\right)b}\,\left(b\,x^{3}+a\right)}$ …
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
4/3*((b*x^3+a)*(-a*d+b*c)*(b*c-1/4*a*d)*c^(1/2)*x^3*arctan(b*(d*x^3+c)^(1/
2)/((a*d-b*c)*b)^(1/2))-1/4*((a*d-b*c)*b)^(1/2)*(3*c*(a*d-4/3*b*c)*(b*x^3+
a)*x^3*arctanh((d*x^3+c)^(1/2)/c^(1/2))+(2*x^3*b*c+a*(-d*x^3+c))*a*c^(1/2)
*(d*x^3+c)^(1/2)))/((a*d-b*c)*b)^(1/2)/c^(1/2)/a^3/x^3/(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 832, normalized size of antiderivative = 4.89

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(a+bx^3\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/6*(((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt((b*c - a*d)/b)
*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*
x^3 + a)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(c)*lo
g((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*((2*a*b*c - a^2*d)*x^
3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3), -1/6*(2*((4*b^2*c - a*b
*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 +
 c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*
b*c - 3*a^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/
x^3) + 2*((2*a*b*c - a^2*d)*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4
*x^3), -1/6*(2*((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(-c
)*arctan(sqrt(-c)/sqrt(d*x^3 + c)) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a
^2*d)*x^3)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 +
 c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*((2*a*b*c - a^2*d)*x^3 + a^2*c
)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3), -1/3*(((4*b^2*c - a*b*d)*x^6 + (
4*a*b*c - a^2*d)*x^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(
-(b*c - a*d)/b)/(b*c - a*d)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2
*d)*x^3)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)) + ((2*a*b*c - a^2*d)*x^
3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**(3/2)/x**4/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2x^4}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx=\frac{(4\,b^2c^2-5\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^3}-\frac{(4\,bc^2-3\,acd)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}}-\frac{2\,(dx^3+c)^{\frac{3}{2}}bcd-2\sqrt{dx^3+c}bc^2d-(dx^3+c)^{\frac{3}{2}}ad^2+2\sqrt{dx^3+c}acd^2}{3\left((dx^3+c)^2b-2\,(dx^3+c)bc+bc^2+(dx^3+c)ad-acd\right)a^2}$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*(4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c
 + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^3) - 1/3*(4*b*c^2 - 3*a*c*d)*arctan(sqr
t(d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)) - 1/3*(2*(d*x^3 + c)^(3/2)*b*c*d - 2
*sqrt(d*x^3 + c)*b*c^2*d - (d*x^3 + c)^(3/2)*a*d^2 + 2*sqrt(d*x^3 + c)*a*c
*d^2)/(((d*x^3 + c)^2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x^3 + c)*a*d - a*
c*d)*a^2)
```

Mupad [B] (verification not implemented)

Time = 10.13 (sec) , antiderivative size = 531, normalized size of antiderivative = 3.12

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx = \frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)(3\,a\,d-4\,b\,c)}{6\,a^3} - \frac{c\,\sqrt{d\,x^3+c}}{3\,a^2\,x^3}$$

$$-\frac{\sqrt{d\,x^3+c}\left(\frac{3\,a\,d-4\,b\,c}{2\,a^2}-\frac{a\left(\frac{a\left(\frac{a\left(\frac{b\,d^2\,(a\,d+b\,c)}{a^3\,c^2}-\frac{a\left(\frac{b^2\,d^3}{2\,a^3\,c^2}-\frac{b^2\,d^3\,(3\,a\,d-4\,b\,c)}{6\,a^2\,c^2\,(a^2\,d-a\,b\,c)}+\frac{b^2\,d^2\,(a\,d+b\,c)\,(3\,a\,d-4\,b\,c)}{3\,a^3\,c^2\,(a^2\,d-a\,b\,c)}\right)}{b}+\frac{b\,(3\,a\,d-4\,b\,c)\,(a^2\,d^3+4\,a\,b\,c\ldots}{6\,a^3\,c^2\,(a^2\,d-a\,b\,c\ldots}\right.}{b}}{b}\ldots}{\ldots}\right)}{b\,x^3+a\ldots}$$

$$+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\sqrt{a\,d-b\,c}\,(a\,d-4\,b\,c)\,1\mathrm{i}}{6\,a^3\,\sqrt{b}}$$

input
```
int((c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x)
```

output
```
(c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)
))/x^6)*(3*a*d - 4*b*c))/(6*a^3) - (c*(c + d*x^3)^(1/2))/(3*a^2*x^3) - ((c
 + d*x^3)^(1/2)*((3*a*d - 4*b*c)/(2*a^2) - (a*((a*((a*((b*d^2*(a*d + b*c))
/(a^3*c^2) - (a*((b^2*d^3)/(2*a^3*c^2) - (b^2*d^3*(3*a*d - 4*b*c))/(6*a^2*
c^2*(a^2*d - a*b*c)) + (b^2*d^2*(a*d + b*c)*(3*a*d - 4*b*c))/(3*a^3*c^2*(a
^2*d - a*b*c))))/b + (b*(3*a*d - 4*b*c)*(a^2*d^3 - b^2*c^2*d + 4*a*b*c*d^2
))/(6*a^3*c^2*(a^2*d - a*b*c))))/b - (a^2*d^3 - b^2*c^2*d + 4*a*b*c*d^2)/(
2*a^3*c^2) + (b*(3*a*d - 4*b*c)*(2*b^2*c^3 - 4*a^2*c*d^2 + 2*a*b*c^2*d))/(
6*a^3*c^2*(a^2*d - a*b*c))))/b - (2*b^2*c^3 - 4*a^2*c*d^2 + 2*a*b*c^2*d)/(
2*a^3*c^2) + (b*(3*a*d - 4*b*c)^2)/(6*a^2*(a^2*d - a*b*c))))/b))/(a + b*x^
3) + (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i +
b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*(a*d - 4*b*c)*1i)/(6*a^3*b^(1/2))
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4 (a+bx^3)^2} \, dx = \text{too large to display}$$

input
```
int((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x)
```

output
```
( - 2*sqrt(c + d*x**3)*a*c*d + 4*sqrt(c + d*x**3)*a*d**2*x**3 - 8*sqrt(c +
 d*x**3)*b*c**2 + 9*int(sqrt(c + d*x**3)/(a**3*c*d*x + a**3*d**2*x**4 + 4*
a**2*b*c**2*x + 6*a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**7 + 8*a*b**2*c**2*x**
4 + 9*a*b**2*c*d*x**7 + a*b**2*d**2*x**10 + 4*b**3*c**2*x**7 + 4*b**3*c*d*
x**10),x)*a**4*c*d**3*x**3 + 60*int(sqrt(c + d*x**3)/(a**3*c*d*x + a**3*d*
*2*x**4 + 4*a**2*b*c**2*x + 6*a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**7 + 8*a*b
**2*c**2*x**4 + 9*a*b**2*c*d*x**7 + a*b**2*d**2*x**10 + 4*b**3*c**2*x**7 +
 4*b**3*c*d*x**10),x)*a**3*b*c**2*d**2*x**3 + 9*int(sqrt(c + d*x**3)/(a**3
*c*d*x + a**3*d**2*x**4 + 4*a**2*b*c**2*x + 6*a**2*b*c*d*x**4 + 2*a**2*b*d
**2*x**7 + 8*a*b**2*c**2*x**4 + 9*a*b**2*c*d*x**7 + a*b**2*d**2*x**10 + 4*
b**3*c**2*x**7 + 4*b**3*c*d*x**10),x)*a**3*b*c*d**3*x**6 + 48*int(sqrt(c +
 d*x**3)/(a**3*c*d*x + a**3*d**2*x**4 + 4*a**2*b*c**2*x + 6*a**2*b*c*d*x**
4 + 2*a**2*b*d**2*x**7 + 8*a*b**2*c**2*x**4 + 9*a*b**2*c*d*x**7 + a*b**2*d
**2*x**10 + 4*b**3*c**2*x**7 + 4*b**3*c*d*x**10),x)*a**2*b**2*c**3*d*x**3
+ 60*int(sqrt(c + d*x**3)/(a**3*c*d*x + a**3*d**2*x**4 + 4*a**2*b*c**2*x +
 6*a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**7 + 8*a*b**2*c**2*x**4 + 9*a*b**2*c*
d*x**7 + a*b**2*d**2*x**10 + 4*b**3*c**2*x**7 + 4*b**3*c*d*x**10),x)*a**2*
b**2*c**2*d**2*x**6 - 192*int(sqrt(c + d*x**3)/(a**3*c*d*x + a**3*d**2*x**
4 + 4*a**2*b*c**2*x + 6*a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**7 + 8*a*b**2*c*
*2*x**4 + 9*a*b**2*c*d*x**7 + a*b**2*d**2*x**10 + 4*b**3*c**2*x**7 + 4*...
```

3.648 $\int \frac{x^3 (c+dx^3)^{3/2}}{(a+bx^3)^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{x^3 (c+dx^3)^{3/2}}{(a+bx^3)^2} \, dx = \frac{cx^4 \sqrt{c+dx^3} \operatorname{AppellF1}\left(\frac{4}{3}, 2, -\frac{3}{2}, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{4a^2 \sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/4*c*x^4*(d*x^3+c)^(1/2)*AppellF1(4/3,2,-3/2,7/3,-b*x^3/a,-d*x^3/c)/a^2/(
1+d*x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 338 vs. 2(65) = 130.

Time = 10.32 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.20

$$\int \frac{x^3 (c+dx^3)^{3/2}}{(a+bx^3)^2} \, dx = \frac{x^4 \left(\frac{d(43bc-55ad)\sqrt{1+\frac{dx^3}{c}} \operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a} + \frac{8\left(-8acd(11ad+b(c+6dx^3)) \operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1 \cdots\right.\right.}{(a+bx^3)\left(-8ac \operatorname{AppellF1}\left(\cdots\right.\right.} \cdots \right.}{12\cdots}$$

input
```
Integrate[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
(x^4*((d*(43*b*c - 55*a*d)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3,
-((d*x^3)/c), -((b*x^3)/a)])/a + (8*(-8*a*c*d*(11*a*d + b*(c + 6*d*x^3))*A
ppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 3*(c + d*x^3)*(5*b
*c - 11*a*d - 6*b*d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -
((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])
))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)
/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)]
 + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(120*b^
2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{c\sqrt{c+dx^3} \int \frac{x^3 \left(\frac{dx^3}{c}+1\right)^{3/2}}{\left(bx^3+a\right)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{cx^4\sqrt{c+dx^3} \,\text{AppellF1}\left(\frac{4}{3},2,-\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
(c*x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 2, -3/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a^2*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.67 (sec) , antiderivative size = 808, normalized size of antiderivative = 12.43

method	result	size
elliptic	Expression too large to display	808
risch	Expression too large to display	1563
default	Expression too large to display	1587

input
```
int(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*d-b*c)*x/b^2*(d*x^3+c)^(1/2)/(b*x^3+a)+2/5*x/b^2*d*(d*x^3+c)^(1/2)-
2/3*I*(-11/6*(a*d-b*c)*d/b^3-2/5*c*d/b^2)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/b^3/d^2*2^(1/2)*sum((-11*a^2*
d^2+13*a*b*c*d-2*b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+
1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(
x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*
(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c
*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipti
cPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1
/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alph
a-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3 (c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{x^3(c+dx^3)^{\frac{3}{2}}}{(a+bx^3)^2}\,dx$$

input
```
integrate(x**3*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

output
```
Integral(x**3*(c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

Maxima [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x^3}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x^3}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 (c + dx^3)^{3/2}}{(a + bx^3)^2} \, dx = \int \frac{x^3 \left(d\,x^3 + c\right)^{3/2}}{\left(b\,x^3 + a\right)^2} \, dx$$

input
```
int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

output
```
int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x^3 (c + dx^3)^{3/2}}{(a + bx^3)^2} \, dx = \text{too large to display}$$

input
```
int(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

output
```
( - 16*sqrt(c + d*x**3)*a*c*d*x + 10*sqrt(c + d*x**3)*a*d**2*x**4 + 10*sqr
t(c + d*x**3)*b*c**2*x - 8*sqrt(c + d*x**3)*b*c*d*x**4 + 80*int(sqrt(c + d
*x**3)/(5*a**3*c*d + 5*a**3*d**2*x**3 - 4*a**2*b*c**2 + 6*a**2*b*c*d*x**3
+ 10*a**2*b*d**2*x**6 - 8*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + 5*a*b**2*
d**2*x**9 - 4*b**3*c**2*x**6 - 4*b**3*c*d*x**9),x)*a**4*c**2*d**2 - 114*in
t(sqrt(c + d*x**3)/(5*a**3*c*d + 5*a**3*d**2*x**3 - 4*a**2*b*c**2 + 6*a**2
*b*c*d*x**3 + 10*a**2*b*d**2*x**6 - 8*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6
 + 5*a*b**2*d**2*x**9 - 4*b**3*c**2*x**6 - 4*b**3*c*d*x**9),x)*a**3*b*c**3
*d + 80*int(sqrt(c + d*x**3)/(5*a**3*c*d + 5*a**3*d**2*x**3 - 4*a**2*b*c**
2 + 6*a**2*b*c*d*x**3 + 10*a**2*b*d**2*x**6 - 8*a*b**2*c**2*x**3 - 3*a*b**
2*c*d*x**6 + 5*a*b**2*d**2*x**9 - 4*b**3*c**2*x**6 - 4*b**3*c*d*x**9),x)*a
**3*b*c**2*d**2*x**3 + 40*int(sqrt(c + d*x**3)/(5*a**3*c*d + 5*a**3*d**2*x
**3 - 4*a**2*b*c**2 + 6*a**2*b*c*d*x**3 + 10*a**2*b*d**2*x**6 - 8*a*b**2*c
**2*x**3 - 3*a*b**2*c*d*x**6 + 5*a*b**2*d**2*x**9 - 4*b**3*c**2*x**6 - 4*b
**3*c*d*x**9),x)*a**2*b**2*c**4 - 114*int(sqrt(c + d*x**3)/(5*a**3*c*d + 5
*a**3*d**2*x**3 - 4*a**2*b*c**2 + 6*a**2*b*c*d*x**3 + 10*a**2*b*d**2*x**6
- 8*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + 5*a*b**2*d**2*x**9 - 4*b**3*c**
2*x**6 - 4*b**3*c*d*x**9),x)*a**2*b**2*c**3*d*x**3 + 40*int(sqrt(c + d*x**
3)/(5*a**3*c*d + 5*a**3*d**2*x**3 - 4*a**2*b*c**2 + 6*a**2*b*c*d*x**3 + 10
*a**2*b*d**2*x**6 - 8*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + 5*a*b**2*d...
```

3.649 $\int \frac{x\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 65

$$\int \frac{x\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2} \, dx = \frac{cx^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},2,-\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/2*c*x^2*(d*x^3+c)^(1/2)*AppellF1(2/3,2,-3/2,5/3,-b*x^3/a,-d*x^3/c)/a^2/(
1+d*x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(65) = 130.

Time = 10.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.72

$$\int \frac{x\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2} \, dx = \frac{x^2\left(-10a(-bc+ad)\left(c+dx^3\right)+5c(bc+2ad)\left(a+bx^3\right)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1\right.\right.}{30a^2b\left(a+\right.}$$

input
```
Integrate[(x*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
(x^2*(-10*a*(-(b*c) + a*d)*(c + d*x^3) + 5*c*(b*c + 2*a*d)*(a + b*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] -
d*(b*c - 7*a*d)*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1,
8/3, -((d*x^3)/c), -((b*x^3)/a)]))/(30*a^2*b*(a + b*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{c\sqrt{c+dx^3}\int \frac{x\left(\frac{dx^3}{c}+1\right)^{3/2}}{(bx^3+a)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{cx^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},2,-\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(x*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
(c*x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 2, -3/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a^2*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.23 (sec) , antiderivative size = 955, normalized size of antiderivative = 14.69

method	result	size
default	Expression too large to display	955
elliptic	Expression too large to display	955

input
```
int(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a*d-b*c)/b/a*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(d^2/b^2+1/6/b^2*d*
(a*d-b*c)/a)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1
/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+
1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d
^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/
3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/
a/b^2/d^2*2^(1/2)*sum((7*a^2*d^2-5*a*b*c*d-2*b^2*c^2)/_alpha/(a*d-b*c)*(-c
*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/
(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/
2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alp
ha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha
*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{x(c+dx^3)^{\frac{3}{2}}}{(a+bx^3)^2}\,dx$$

input
```
integrate(x*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

output
```
Integral(x*(c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

Maxima [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x}{(bx^3+a)^2}\,dx$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x}{(bx^3+a)^2}\,dx$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{x\,(d\,x^3+c)^{3/2}}{(b\,x^3+a)^2}\,dx$$

input
```
int((x*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

output
```
int((x*(c + d*x^3)^(3/2))/(a + b*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

output
```
(4*sqrt(c + d*x**3)*c*d*x**2 + 49*int((sqrt(c + d*x**3)*x**7)/(7*a**3*c*d
+ 7*a**3*d**2*x**3 - 2*a**2*b*c**2 + 12*a**2*b*c*d*x**3 + 14*a**2*b*d**2*x
**6 - 4*a*b**2*c**2*x**3 + 3*a*b**2*c*d*x**6 + 7*a*b**2*d**2*x**9 - 2*b**3
*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**3*d**4 - 42*int((sqrt(c + d*x**3)*x**7
)/(7*a**3*c*d + 7*a**3*d**2*x**3 - 2*a**2*b*c**2 + 12*a**2*b*c*d*x**3 + 14
*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 + 3*a*b**2*c*d*x**6 + 7*a*b**2*d**2
*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**2*b*c*d**3 + 49*int((sqr
t(c + d*x**3)*x**7)/(7*a**3*c*d + 7*a**3*d**2*x**3 - 2*a**2*b*c**2 + 12*a*
*2*b*c*d*x**3 + 14*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 + 3*a*b**2*c*d*x*
*6 + 7*a*b**2*d**2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**2*b*d*
*4*x**3 + 8*int((sqrt(c + d*x**3)*x**7)/(7*a**3*c*d + 7*a**3*d**2*x**3 - 2
*a**2*b*c**2 + 12*a**2*b*c*d*x**3 + 14*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x*
*3 + 3*a*b**2*c*d*x**6 + 7*a*b**2*d**2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*
d*x**9),x)*a*b**2*c**2*d**2 - 42*int((sqrt(c + d*x**3)*x**7)/(7*a**3*c*d +
 7*a**3*d**2*x**3 - 2*a**2*b*c**2 + 12*a**2*b*c*d*x**3 + 14*a**2*b*d**2*x*
*6 - 4*a*b**2*c**2*x**3 + 3*a*b**2*c*d*x**6 + 7*a*b**2*d**2*x**9 - 2*b**3*
c**2*x**6 - 2*b**3*c*d*x**9),x)*a*b**2*c*d**3*x**3 + 8*int((sqrt(c + d*x**
3)*x**7)/(7*a**3*c*d + 7*a**3*d**2*x**3 - 2*a**2*b*c**2 + 12*a**2*b*c*d*x*
*3 + 14*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 + 3*a*b**2*c*d*x**6 + 7*a*b*
*2*d**2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*b**3*c**2*d**2*x*...
```

3.650 $\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx=\frac{cx\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},2,-\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
c*x*(d*x^3+c)^(1/2)*AppellF1(1/3,2,-3/2,4/3,-b*x^3/a,-d*x^3/c)/a^2/(1+d*x^
3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 339 vs. 2(60) = 120.

Time = 10.26 (sec) , antiderivative size = 339, normalized size of antiderivative = 5.65

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx=\frac{x\left(d(bc+5ad)x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a\left(-64ac\left(-ad^2x^3+bc(3c+dx\cdots\right.\right.}{(a+bx^3)\left(-\cdots\right.}}{\cdots}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(a + b*x^3)^2,x]
```

output
```
(x*(d*(b*c + 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)] + (a*(-64*a*c*(-(a*d^2*x^3) + b*c*(3*c + d*x^3))*
AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 24*(b*c - a*d)*x^
3*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)
] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*
x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^
3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*Appe
llF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(24*a^2*b*Sqrt[c +
 d*x^3])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{937, 936\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{c\sqrt{c+dx^3}\int \frac{\left(\frac{dx^3}{c}+1\right)^{3/2}}{\left(bx^3+a\right)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{cx\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},2,-\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(c + d*x^3)^(3/2)/(a + b*x^3)^2,x]
```

output
```
(c*x*Sqrt[c + d*x^3]*AppellF1[1/3, 2, -3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a^2*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.14 (sec) , antiderivative size = 801, normalized size of antiderivative = 13.35

method	result	size
default	Expression too large to display	801
elliptic	Expression too large to display	801

input
```
int((d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/3*(a*d-b*c)/b/a*x*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(d^2/b^2-1/6/b^2*d*(a
*d-b*c)/a)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3
))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)
/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2))+1/18*I/a/b^2/d^2*2^(1/2)*sum((5*a^2*d^2-a*b*c*d-4*b^2*c^2)/_alpha^2
/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d
^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2
)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1
/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1
/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-
c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input

```
integrate((d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output

```
Timed out
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{(a+bx^3)^2}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

output
```
Integral((c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx=\int \frac{(d\,x^3+c)^{3/2}}{(b\,x^3+a)^2}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(a + b*x^3)^2,x)
```

output
```
int((c + d*x^3)^(3/2)/(a + b*x^3)^2, x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx=\text{Too large to display}$$

input
```
int((d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

output
```
(4*sqrt(c + d*x**3)*c*d*x + 5*int(sqrt(c + d*x**3)/(5*a**3*c*d + 5*a**3*d*
*2*x**3 - 4*a**2*b*c**2 + 6*a**2*b*c*d*x**3 + 10*a**2*b*d**2*x**6 - 8*a*b*
*2*c**2*x**3 - 3*a*b**2*c*d*x**6 + 5*a*b**2*d**2*x**9 - 4*b**3*c**2*x**6 -
 4*b**3*c*d*x**9),x)*a**3*c**2*d**2 - 24*int(sqrt(c + d*x**3)/(5*a**3*c*d
+ 5*a**3*d**2*x**3 - 4*a**2*b*c**2 + 6*a**2*b*c*d*x**3 + 10*a**2*b*d**2*x*
*6 - 8*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + 5*a*b**2*d**2*x**9 - 4*b**3*
c**2*x**6 - 4*b**3*c*d*x**9),x)*a**2*b*c**3*d + 5*int(sqrt(c + d*x**3)/(5*
a**3*c*d + 5*a**3*d**2*x**3 - 4*a**2*b*c**2 + 6*a**2*b*c*d*x**3 + 10*a**2*
b*d**2*x**6 - 8*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + 5*a*b**2*d**2*x**9
- 4*b**3*c**2*x**6 - 4*b**3*c*d*x**9),x)*a**2*b*c**2*d**2*x**3 + 16*int(sq
rt(c + d*x**3)/(5*a**3*c*d + 5*a**3*d**2*x**3 - 4*a**2*b*c**2 + 6*a**2*b*c
*d*x**3 + 10*a**2*b*d**2*x**6 - 8*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + 5
*a*b**2*d**2*x**9 - 4*b**3*c**2*x**6 - 4*b**3*c*d*x**9),x)*a*b**2*c**4 - 2
4*int(sqrt(c + d*x**3)/(5*a**3*c*d + 5*a**3*d**2*x**3 - 4*a**2*b*c**2 + 6*
a**2*b*c*d*x**3 + 10*a**2*b*d**2*x**6 - 8*a*b**2*c**2*x**3 - 3*a*b**2*c*d*
x**6 + 5*a*b**2*d**2*x**9 - 4*b**3*c**2*x**6 - 4*b**3*c*d*x**9),x)*a*b**2*
c**3*d*x**3 + 16*int(sqrt(c + d*x**3)/(5*a**3*c*d + 5*a**3*d**2*x**3 - 4*a
**2*b*c**2 + 6*a**2*b*c*d*x**3 + 10*a**2*b*d**2*x**6 - 8*a*b**2*c**2*x**3
- 3*a*b**2*c*d*x**6 + 5*a*b**2*d**2*x**9 - 4*b**3*c**2*x**6 - 4*b**3*c*d*x
**9),x)*b**3*c**4*x**3 + 25*int((sqrt(c + d*x**3)*x**6)/(5*a**3*c*d + 5...
```

3.651 $\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx = -\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},2,-\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{1+\frac{dx^3}{c}}}$$

output
```
-c*(d*x^3+c)^(1/2)*AppellF1(-1/3,2,-3/2,2/3,-b*x^3/a,-d*x^3/c)/a^2/x/(1+d*
x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(63) = 126.

Time = 10.15 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.02

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx = \frac{-20a(c+dx^3)(3ac+4bcx^3-adx^3)+5c(-8bc+11ad)x^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{Appe}}{60a^3x}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2),x]
```

output
```
(-20*a*(c + d*x^3)*(3*a*c + 4*b*c*x^3 - a*d*x^3) + 5*c*(-8*b*c + 11*a*d)*x
^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c)
, -((b*x^3)/a)] + 2*d*(4*b*c - a*d)*x^6*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*Ap
pellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(60*a^3*x*(a + b*x^3
)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{c\sqrt{c+dx^3}\int \frac{\left(\frac{dx^3}{c}+1\right)^{3/2}}{x^2\left(bx^3+a\right)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},2,-\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2),x]
```

output
```
-((c*Sqrt[c + d*x^3]*AppellF1[-1/3, 2, -3/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a^2*x*Sqrt[1 + (d*x^3)/c]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.81 (sec) , antiderivative size = 970, normalized size of antiderivative = 15.40

method	result	size
elliptic	Expression too large to display	970
risch	Expression too large to display	1854
default	Expression too large to display	2364

input
```
int((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*d-b*c)/a^2*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-c/a^2*(d*x^3+c)^(1/2)/x-2/
3*I*(-1/6*d*(a*d-b*c)/a^2/b+1/2*d*c/a^2)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)))+1/18*I/a^2/b/d^2*2^(1/2)*sum((-a^2*d^2-7*a*b*c*d+8*b^
2*c^2)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))
/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^
2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**(3/2)/x**2/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2x^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^2), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2x^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^2\,(b\,x^3+a)^2}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2),x)
```

output
```
int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x)
```

output
```
( - 2*sqrt(c + d*x**3)*c + 2*int((sqrt(c + d*x**3)*x**4)/(a**3*c*d + a**3*
d**2*x**3 - 8*a**2*b*c**2 - 6*a**2*b*c*d*x**3 + 2*a**2*b*d**2*x**6 - 16*a*
b**2*c**2*x**3 - 15*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 8*b**3*c**2*x**6
- 8*b**3*c*d*x**9),x)*a**3*d**3*x - 21*int((sqrt(c + d*x**3)*x**4)/(a**3*c
*d + a**3*d**2*x**3 - 8*a**2*b*c**2 - 6*a**2*b*c*d*x**3 + 2*a**2*b*d**2*x*
*6 - 16*a*b**2*c**2*x**3 - 15*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 8*b**3*
c**2*x**6 - 8*b**3*c*d*x**9),x)*a**2*b*c*d**2*x + 2*int((sqrt(c + d*x**3)*
x**4)/(a**3*c*d + a**3*d**2*x**3 - 8*a**2*b*c**2 - 6*a**2*b*c*d*x**3 + 2*a
**2*b*d**2*x**6 - 16*a*b**2*c**2*x**3 - 15*a*b**2*c*d*x**6 + a*b**2*d**2*x
**9 - 8*b**3*c**2*x**6 - 8*b**3*c*d*x**9),x)*a**2*b*d**3*x**4 + 40*int((sq
rt(c + d*x**3)*x**4)/(a**3*c*d + a**3*d**2*x**3 - 8*a**2*b*c**2 - 6*a**2*b
*c*d*x**3 + 2*a**2*b*d**2*x**6 - 16*a*b**2*c**2*x**3 - 15*a*b**2*c*d*x**6
+ a*b**2*d**2*x**9 - 8*b**3*c**2*x**6 - 8*b**3*c*d*x**9),x)*a*b**2*c**2*d*
x - 21*int((sqrt(c + d*x**3)*x**4)/(a**3*c*d + a**3*d**2*x**3 - 8*a**2*b*c
**2 - 6*a**2*b*c*d*x**3 + 2*a**2*b*d**2*x**6 - 16*a*b**2*c**2*x**3 - 15*a*
b**2*c*d*x**6 + a*b**2*d**2*x**9 - 8*b**3*c**2*x**6 - 8*b**3*c*d*x**9),x)*
a*b**2*c*d**2*x**4 + 40*int((sqrt(c + d*x**3)*x**4)/(a**3*c*d + a**3*d**2*
x**3 - 8*a**2*b*c**2 - 6*a**2*b*c*d*x**3 + 2*a**2*b*d**2*x**6 - 16*a*b**2*
c**2*x**3 - 15*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 8*b**3*c**2*x**6 - 8*b
**3*c*d*x**9),x)*b**3*c**2*d*x**4 + 5*int((sqrt(c + d*x**3)*x)/(a**3*c*...
```

3.652 $\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx = -\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},2,-\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
-1/2*c*(d*x^3+c)^(1/2)*AppellF1(-2/3,2,-3/2,1/3,-b*x^3/a,-d*x^3/c)/a^2/x^2
/(1+d*x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 370 vs. 2(65) = 130.

Time = 10.29 (sec) , antiderivative size = 370, normalized size of antiderivative = 5.69

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx = \frac{-d(5bc-2ad)x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{8a\left(4ac(10bcx^3(3c+dx^3)+a\right.}{}}{}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2),x]
```

output
```
(-(d*(5*b*c - 2*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -(
(d*x^3)/c), -((b*x^3)/a)]) + (8*a*(4*a*c*(10*b*c*x^3*(3*c + d*x^3) + a*(6*
c^2 - 15*c*d*x^3 - 4*d^2*x^6))*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -(
(b*x^3)/a)] - 3*x^3*(c + d*x^3)*(3*a*c + 5*b*c*x^3 - 2*a*d*x^3)*(2*b*c*App
ellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/
2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/
3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1
/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -
((d*x^3)/c), -((b*x^3)/a)]))))/(48*a^3*x^2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}dx$$
$$\Big\downarrow \text{1013}$$
$$\frac{c\sqrt{c+dx^3}\int \frac{\left(\frac{dx^3}{c}+1\right)^{3/2}}{x^3(bx^3+a)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$
$$\Big\downarrow \text{1012}$$
$$-\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},2,-\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2),x]
```

output
```
-1/2*(c*Sqrt[c + d*x^3]*AppellF1[-2/3, 2, -3/2, 1/3, -((b*x^3)/a), -((d*x^
3)/c)])/(a^2*x^2*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.29 (sec) , antiderivative size = 815, normalized size of antiderivative = 12.54

method	result	size
elliptic	Expression too large to display	815
risch	Expression too large to display	1548
default	Expression too large to display	1902

input
```
int((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*c/a^2*(d*x^3+c)^(1/2)/x^2+1/3*(a*d-b*c)/a^2*x*(d*x^3+c)^(1/2)/(b*x^3+
a)-2/3*I*(-1/4*d*c/a^2+1/6*d*(a*d-b*c)/a^2/b)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a^2/b/d^2*2^(1/2)*sum((a^
2*d^2-11*a*b*c*d+10*b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2
*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(
d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/
2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*
(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3
^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_a
lpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**(3/2)/x**3/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2x^3}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^3), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2x^3}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^3\,(b\,x^3+a)^2}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2),x)
```

output
```
int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x)
```

output
```
( - 2*sqrt(c + d*x**3)*c + 7*int(sqrt(c + d*x**3)/(a**3*c*d + a**3*d**2*x*
*3 + 10*a**2*b*c**2 + 12*a**2*b*c*d*x**3 + 2*a**2*b*d**2*x**6 + 20*a*b**2*
c**2*x**3 + 21*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 + 10*b**3*c**2*x**6 + 10
*b**3*c*d*x**9),x)*a**3*c*d**2*x**2 + 60*int(sqrt(c + d*x**3)/(a**3*c*d +
a**3*d**2*x**3 + 10*a**2*b*c**2 + 12*a**2*b*c*d*x**3 + 2*a**2*b*d**2*x**6
+ 20*a*b**2*c**2*x**3 + 21*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 + 10*b**3*c*
*2*x**6 + 10*b**3*c*d*x**9),x)*a**2*b*c**2*d*x**2 + 7*int(sqrt(c + d*x**3)
/(a**3*c*d + a**3*d**2*x**3 + 10*a**2*b*c**2 + 12*a**2*b*c*d*x**3 + 2*a**2
*b*d**2*x**6 + 20*a*b**2*c**2*x**3 + 21*a*b**2*c*d*x**6 + a*b**2*d**2*x**9
 + 10*b**3*c**2*x**6 + 10*b**3*c*d*x**9),x)*a**2*b*c*d**2*x**5 - 100*int(s
qrt(c + d*x**3)/(a**3*c*d + a**3*d**2*x**3 + 10*a**2*b*c**2 + 12*a**2*b*c*
d*x**3 + 2*a**2*b*d**2*x**6 + 20*a*b**2*c**2*x**3 + 21*a*b**2*c*d*x**6 + a
*b**2*d**2*x**9 + 10*b**3*c**2*x**6 + 10*b**3*c*d*x**9),x)*a*b**2*c**3*x**
2 + 60*int(sqrt(c + d*x**3)/(a**3*c*d + a**3*d**2*x**3 + 10*a**2*b*c**2 +
12*a**2*b*c*d*x**3 + 2*a**2*b*d**2*x**6 + 20*a*b**2*c**2*x**3 + 21*a*b**2*
c*d*x**6 + a*b**2*d**2*x**9 + 10*b**3*c**2*x**6 + 10*b**3*c*d*x**9),x)*a*b
**2*c**2*d*x**5 - 100*int(sqrt(c + d*x**3)/(a**3*c*d + a**3*d**2*x**3 + 10
*a**2*b*c**2 + 12*a**2*b*c*d*x**3 + 2*a**2*b*d**2*x**6 + 20*a*b**2*c**2*x*
*3 + 21*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 + 10*b**3*c**2*x**6 + 10*b**3*c
*d*x**9),x)*b**3*c**3*x**5 + 4*int((sqrt(c + d*x**3)*x**3)/(a**3*c*d + ...
```

3.653 $\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\sqrt{c+dx^3}}{3b^2d} - \frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)(a+bx^3)} + \frac{a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}(bc-ad)^{3/2}}$$

output
```
2/3*(d*x^3+c)^(1/2)/b^2/d-1/3*a^2*(d*x^3+c)^(1/2)/b^2/(-a*d+b*c)/(b*x^3+a)
+1/3*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^
(5/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\frac{\sqrt{b}\sqrt{c+dx^3}(-3a^2d+2b^2cx^3+2ab(c-dx^3))}{d(bc-ad)(a+bx^3)} + \frac{a(4bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{3b^{5/2}}$$

input
```
Integrate[x^8/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((Sqrt[b]*Sqrt[c + d*x^3]*(-3*a^2*d + 2*b^2*c*x^3 + 2*a*b*(c - d*x^3)))/(d
*(b*c - a*d)*(a + b*x^3)) + (a*(4*b*c - 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(3*b^(5/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 100, 27, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{(bx^3+a)^2\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{3}\left(\frac{\int -\frac{a(2bc-ad)-2b(bc-ad)x^3}{2(bx^3+a)\sqrt{dx^3+c}}dx^3}{b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^3}}{b^2(a+bx^3)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{a(2bc-ad)-2b(bc-ad)x^3}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^3}}{b^2(a+bx^3)(bc-ad)}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(-\frac{a(4bc-3ad)\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3-\frac{4\sqrt{c+dx^3}(bc-ad)}{d}}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^3}}{b^2(a+bx^3)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{\frac{2a(4bc-3ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{d}-\frac{4\sqrt{c+dx^3}(bc-ad)}{d}}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^3}}{b^2(a+bx^3)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{3}\left(-\frac{a^2\sqrt{c+dx^3}}{b^2\left(a+bx^3\right)(bc-ad)}-\frac{-\frac{2a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}-\frac{4\sqrt{c+dx^3}(bc-ad)}{d}}{2b^2(bc-ad)}\right)$$

input
```
Int[x^8/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(-((a^2*Sqrt[c + d*x^3])/(b^2*(b*c - a*d)*(a + b*x^3))) - ((-4*(b*c - a*d)
*Sqrt[c + d*x^3])/d - (2*a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3
])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a*d]))/(2*b^2*(b*c - a*d)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{-ad\left(ad-\frac{4bc}{3}\right)\left(b\,x^{3}+a\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)+\left(-\frac{2b^{2}c\,x^{3}}{3}-\frac{2a\left(-d\,x^{3}+c\right)b}{3}+d\,a^{2}\right)\sqrt{(ad-bc)b}\,\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}\,d\,b^{2}(ad-bc)\left(b\,x^{3}+a\right)}$
risch	$\frac{2\sqrt{d\,x^{3}+c}}{3b^{2}d}-\frac{a\left(\frac{4\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}-\frac{a\left(\frac{\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3(ad-bc)}\right)}{b^{2}}$
default	$\frac{2\sqrt{d\,x^{3}+c}}{3b^{2}d}+\frac{a^{2}\left(\frac{\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3b^{2}(ad-bc)}-\frac{4a\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3b^{2}\sqrt{(ad-bc)b}}$
elliptic	$\frac{a^{2}\sqrt{d\,x^{3}+c}}{3(ad-bc)b^{2}\left(b\,x^{3}+a\right)}+\frac{2\sqrt{d\,x^{3}+c}}{3b^{2}d}+\frac{ia\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(3ad-4bc)\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\right.}{d}\right.}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$

input

```
int(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
(-a*d*(a*d-4/3*b*c)*(b*x^3+a)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2)
)+(-2/3*b^2*c*x^3-2/3*a*(-d*x^3+c)*b+d*a^2)*((a*d-b*c)*b)^(1/2)*(d*x^3+c)^
(1/2))/((a*d-b*c)*b)^(1/2)/d/b^2/(a*d-b*c)/(b*x^3+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(103) = 206.

Time = 0.27 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.86

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \left[\frac{(4\,a^2bcd-3\,a^3d^2+(4\,ab^2cd-3\,a^2bd^2)x^3)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,(2\,ab^3c^2-}{6\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+}\right.$$
$$-\frac{(4\,a^2bcd-3\,a^3d^2+(4\,ab^2cd-3\,a^2bd^2)x^3)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-(2\,ab^3c^2-5\,a^2b^2c}{3\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+a^2}$$

input
```
integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^3)*sqrt(b^2
*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*
b*d))/(b*x^3 + a)) + 2*(2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4
*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^2*d - 2*a
^2*b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^3
), -1/3*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^3)*sqrt(
-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c
)) - (2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^2 - 2*a*b^3*c*d
 + a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3
*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^3)]
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**8/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**8/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{\frac{\sqrt{dx^3+c}a^2d^3}{(b^3c-ab^2d)((dx^3+c)b-bc+ad)} + \frac{(4\,abcd^2-3\,a^2d^3)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^3c-ab^2d)\sqrt{-b^2c+abd}} - \frac{2\sqrt{dx^3+c}d}{b^2}}{3\,d^2}$$

input
```
integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-1/3*(sqrt(d*x^3 + c)*a^2*d^3/((b^3*c - a*b^2*d)*((d*x^3 + c)*b - b*c + a*
d)) + (4*a*b*c*d^2 - 3*a^2*d^3)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b
*d))/((b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) - 2*sqrt(d*x^3 + c)*d/b^2)/d
^2
```

Mupad [B] (verification not implemented)

Time = 6.49 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.30

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\sqrt{d\,x^3+c}\,(2\,b^2\,c-2\,a\,b\,d)}{3\,d\,(2\,b^4\,c-2\,a\,b^3\,d)} - \frac{2\,a^2\,\sqrt{d\,x^3+c}}{3\,b\,(b\,x^3+a)\,(2\,b^2\,c-2\,a\,b\,d)} + \frac{a\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,(3\,a\,d-4\,b\,c)\,1\mathrm{i}}{6\,b^{5/2}\,(a\,d-b\,c)^{3/2}}$$

input
```
int(x^8/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output
```
(2*(c + d*x^3)^(1/2)*(2*b^2*c - 2*a*b*d))/(3*d*(2*b^4*c - 2*a*b^3*d)) - (2
*a^2*(c + d*x^3)^(1/2))/(3*b*(a + b*x^3)*(2*b^2*c - 2*a*b*d)) + (a*log((a*
d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a +
 b*x^3))*(3*a*d - 4*b*c)*1i)/(6*b^(5/2)*(a*d - b*c)^(3/2))
```

Reduce [F]

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{-4\sqrt{d\,x^3+c}\,ac+2\sqrt{d\,x^3+c}\,ad\,x^3-4\sqrt{d\,x^3+c}\,bc\,x^3-9\left(\int \frac{\sqrt{d\,x^3+c}\,x^5}{a\,b^2d^2x^9-2b^3cd\,x^9+2a^2b\,d^2x^6-3a\,b^2cd\,x^6-2b^3c^2x^6+a^3d\cdots}\right.}{\cdots}$$

input
```
int(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

output
```
( - 4*sqrt(c + d*x**3)*a*c + 2*sqrt(c + d*x**3)*a*d*x**3 - 4*sqrt(c + d*x*
*3)*b*c*x**3 - 9*int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 -
2*a**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**
6 + a*b**2*d**2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**4*d**3 +
30*int((sqrt(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2
+ 2*a**2*b*d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**
2*x**9 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**3*b*c*d**2 - 9*int((sqr
t(c + d*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*
d**2*x**6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2*
b**3*c**2*x**6 - 2*b**3*c*d*x**9),x)*a**3*b*d**3*x**3 - 24*int((sqrt(c + d
*x**3)*x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x*
*6 - 4*a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2*b**3*c*
*2*x**6 - 2*b**3*c*d*x**9),x)*a**2*b**2*c**2*d + 30*int((sqrt(c + d*x**3)*
x**5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*
a*b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2*b**3*c**2*x**6
 - 2*b**3*c*d*x**9),x)*a**2*b**2*c*d**2*x**3 - 24*int((sqrt(c + d*x**3)*x*
*5)/(a**3*c*d + a**3*d**2*x**3 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x**6 - 4*a*
b**2*c**2*x**3 - 3*a*b**2*c*d*x**6 + a*b**2*d**2*x**9 - 2*b**3*c**2*x**6 -
 2*b**3*c*d*x**9),x)*a*b**3*c**2*d*x**3)/(3*b*d*(a**2*d - 2*a*b*c + a*b*d*
x**3 - 2*b**2*c*x**3))
```

3.654 $\int \frac{x^5}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{x^5}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{a\sqrt{c+dx^3}}{3b(bc-ad)\left(a+bx^3\right)}-\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

output
```
1/3*a*(d*x^3+c)^(1/2)/b/(-a*d+b*c)/(b*x^3+a)-1/3*(-a*d+2*b*c)*arctanh(b^(1
/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \frac{x^5}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{\frac{a\sqrt{b}\sqrt{c+dx^3}}{(bc-ad)(a+bx^3)}-\frac{(2bc-ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{3b^{3/2}}$$

input
```
Integrate[x^5/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((a*Sqrt[b]*Sqrt[c + d*x^3])/((b*c - a*d)*(a + b*x^3)) - ((2*b*c - a*d)*Ar
cTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/
(3*b^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{x^3}{(bx^3+a)^2\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \; 87$$

$$\frac{1}{3}\left(\frac{(2bc-ad)\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{2b(bc-ad)}+\frac{a\sqrt{c+dx^3}}{b(a+bx^3)(bc-ad)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{(2bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd(bc-ad)}+\frac{a\sqrt{c+dx^3}}{b(a+bx^3)(bc-ad)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(\frac{a\sqrt{c+dx^3}}{b(a+bx^3)(bc-ad)}-\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}(bc-ad)^{3/2}}\right)$$

input
```
Int[x^5/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((a*Sqrt[c + d*x^3])/(b*(b*c - a*d)*(a + b*x^3)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b^(3/2)*(b*c - a*d)^(3/2)))/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{-\frac{a\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{(ad-2bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{3(ad-bc)b}$
default	$\frac{2\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3b\sqrt{(ad-bc)b}}-\frac{a\left(\frac{\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3b(ad-bc)}$
elliptic	$-\frac{a\sqrt{d\,x^{3}+c}}{3(ad-bc)b(b\,x^{3}+a)}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(-ad+2bc)\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-3(\cdots}}}{\cdots}\right.}{\cdots}$

input
```
int(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(a*d-b*c)/b*(-a*(d*x^3+c)^(1/2)/(b*x^3+a)+(a*d-2*b*c)/((a*d-b*c)*b)^(1
/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.52

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \left[\frac{((2\,b^2c-abd)x^3+2\,abc-a^2d)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,(ab^2c-a^2bd)\sqrt{dx^3+}}{6\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^3)}\right.$$

input
```
integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a))
 + 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3
*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^3), 1/3*(((2*b^2*c - a*
b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sq
rt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))
/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b
^3*d^2)*x^3)]
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**5/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**5/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\frac{\sqrt{dx^3+c}ad^2}{(b^2c-abd)((dx^3+c)b-bc+ad)} + \frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{3\,d}$$

input
```
integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
1/3*(sqrt(d*x^3 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^3 + c)*b - b*c + a*d)) +
 (2*b*c*d - a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c
- a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.12

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\,a\,\sqrt{d\,x^3+c}}{3\,\left(b\,x^3+a\right)\,\left(2\,b^2\,c-2\,a\,b\,d\right)} + \frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{6\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

input
```
int(x^5/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output
```
(log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x
^3)/(a + b*x^3))*(a*d - 2*b*c)*1i)/(6*b^(3/2)*(a*d - b*c)^(3/2)) + (2*a*(c
 + d*x^3)^(1/2))/(3*(a + b*x^3)*(2*b^2*c - 2*a*b*d))
```

Reduce [F]

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^5}{b^2d\,x^9+2abd\,x^6+b^2c\,x^6+a^2d\,x^3+2abc\,x^3+a^2c}dx$$

input
```
int(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**5)/(a**2*c + a**2*d*x**3 + 2*a*b*c*x**3 + 2*a*b*d
*x**6 + b**2*c*x**6 + b**2*d*x**9),x)
```

3.655 $\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{3(bc-ad)(a+bx^3)} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}$$

output
```
-1/3*(d*x^3+c)^(1/2)/(-a*d+b*c)/(b*x^3+a)+1/3*d*arctanh(b^(1/2)*(d*x^3+c)^
(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\left(-\frac{\sqrt{c+dx^3}}{(bc-ad)(a+bx^3)} + \frac{d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

input
```
Integrate[x^2/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(-(Sqrt[c + d*x^3]/((b*c - a*d)*(a + b*x^3))) + (d*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/3
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {946, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{1}{(bx^3+a)^2\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(-\frac{d\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{2(bc-ad)}-\frac{\sqrt{c+dx^3}}{(a+bx^3)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bc-ad}-\frac{\sqrt{c+dx^3}}{(a+bx^3)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^3}}{(a+bx^3)(bc-ad)}\right)$$

input
```
Int[x^2/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(-(Sqrt[c + d*x^3]/((b*c - a*d)*(a + b*x^3))) + (d*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2)))/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

method	result
default	$\frac{\frac{\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{3ad-3bc}$
pseudoelliptic	$\frac{\frac{\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{3ad-3bc}$
elliptic	$\frac{\sqrt{d\,x^3+c}}{3(ad-bc)(b\,x^3+a)}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right.}{d}\right.}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}}}}{}\right.}{}$

input
```
int(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(a*d-b*c)*((d*x^3+c)^(1/2)/(b*x^3+a)+d/((a*d-b*c)*b)^(1/2)*arctan(b*(d
*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(71) = 142$.

Time = 0.13 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.47

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \left[-\frac{(bdx^3+ad)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}(b^2c-abd)}{6\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^3)}, -\frac{(bdx^3+ad)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)+\sqrt{dx^3+c}(b^2c-abd)}{3\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^3)}\right]$$

input
```
integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/6*((b*d*x^3 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*
sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(b^2
*c - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c
*d + a^2*b^2*d^2)*x^3), -1/3*((b*d*x^3 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(
sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + sqrt(d*x^3 + c)*(b
^2*c - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3
*c*d + a^2*b^2*d^2)*x^3)]
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**2/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**2/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{d\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}(bc-ad)} - \frac{\sqrt{dx^3+c}d}{3\left((dx^3+c)b-bc+ad\right)(bc-ad)}$$

input
```
integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-1/3*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*(b*c - a*d)) - 1/3*sqrt(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*(b*c -
 a*d))
```

Mupad [B] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{2\,b\,\sqrt{d\,x^3+c}}{3\,\left(b\,x^3+a\right)\,\left(2\,b^2\,c-2\,a\,b\,d\right)} + \frac{d\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{6\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

input

```
int(x^2/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output

```
(d*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d
*x^3)/(a + b*x^3))*1i)/(6*b^(1/2)*(a*d - b*c)^(3/2)) - (2*b*(c + d*x^3)^(1
/2))/(3*(a + b*x^3)*(2*b^2*c - 2*a*b*d))
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^2}{b^2d\,x^9+2abd\,x^6+b^2c\,x^6+a^2d\,x^3+2abc\,x^3+a^2c}dx$$

input

```
int(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

output

```
int((sqrt(c + d*x**3)*x**2)/(a**2*c + a**2*d*x**3 + 2*a*b*c*x**3 + 2*a*b*d
*x**6 + b**2*c*x**6 + b**2*d*x**9),x)
```

3.656 $\int \frac{1}{x(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{\sqrt{b}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}$$

output
```
1/3*b*(d*x^3+c)^(1/2)/a/(-a*d+b*c)/(b*x^3+a)-2/3*arctanh((d*x^3+c)^(1/2)/c
^(1/2))/a^2/c^(1/2)+1/3*b^(1/2)*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(
1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{-\frac{ab\sqrt{c+dx^3}}{(-bc+ad)(a+bx^3)}+\frac{\sqrt{b}(2bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^2}$$

input
```
Integrate[1/(x*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(-((a*b*Sqrt[c + d*x^3])/((-(b*c) + a*d)*(a + b*x^3))) + (Sqrt[b]*(2*b*c -
 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*
d)^(3/2) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c])/(3*a^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+bx^3)^2 \sqrt{c+dx^3}} \, dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3} \int \frac{1}{x^3 (bx^3+a)^2 \sqrt{dx^3+c}} dx^3$$

$$\Big\downarrow \; 114$$

$$\frac{1}{3} \left(\frac{\int \frac{bdx^3+2bc-2ad}{2x^3(bx^3+a)\sqrt{dx^3+c}}dx^3}{a(bc-ad)} + \frac{b\sqrt{c+dx^3}}{a(a+bx^3)(bc-ad)} \right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3} \left(\frac{\int \frac{bdx^3+2(bc-ad)}{x^3(bx^3+a)\sqrt{dx^3+c}}dx^3}{2a(bc-ad)} + \frac{b\sqrt{c+dx^3}}{a(a+bx^3)(bc-ad)} \right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{3} \left(\frac{\frac{2(bc-ad)\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a} - \frac{b(2bc-3ad)\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{a}}{2a(bc-ad)} + \frac{b\sqrt{c+dx^3}}{a(a+bx^3)(bc-ad)} \right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{\frac{4(bc-ad)\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2b(2bc-3ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^3}}{a\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(\frac{\frac{2\sqrt{b}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{4(bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^3}}{a\left(a+bx^3\right)(bc-ad)}\right)$$

input
```
Int[1/(x*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((b*Sqrt[c + d*x^3])/(a*(b*c - a*d)*(a + b*x^3)) + ((-4*(b*c - a*d)*ArcTan
h[Sqrt[c + d*x^3]/Sqrt[c]])/(a*Sqrt[c]) + (2*Sqrt[b]*(2*b*c - 3*a*d)*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(2*a*(
b*c - a*d)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{\sqrt{c}}\right)}{3 a^{2} \sqrt{c}}-\frac{2 b \arctan\left(\frac{b \sqrt{d x^{3}+c}}{\sqrt{\left(a d -b c \right) b}}\right)}{3 a^{2} \sqrt{\left(a d -b c \right) b}}-\frac{b \left(\frac{\sqrt{d x^{3}+c}}{b x^{3}+a}+\frac{d \arctan\left(\frac{b \sqrt{d x^{3}+c}}{\sqrt{\left(a d -b c \right) b}}\right)}{\sqrt{\left(a d -b c \right) b}}\right)}{3 a \left(a d -b c \right)}$	140
pseudoelliptic	$\frac{\frac{2 \sqrt{c}\, \left(b \,x^{3}+a \right) \left(b c -\frac{3 a d}{2}\right) b \arctan\left(\frac{b \sqrt{d x^{3}+c}}{\sqrt{\left(a d -b c \right) b}}\right)}{3}-\frac{\left(2 \left(a d -b c \right) \left(b \,x^{3}+a \right) \operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{\sqrt{c}}\right)+\sqrt{c}\, \sqrt{d x^{3}+c}\, a b \right) \sqrt{\left(a d -b c \right) b}}{3}}{a^{2} \sqrt{c}\, \left(a d -b c \right) \left(b \,x^{3}+a \right) \sqrt{\left(a d -b c \right) b}}$	146
elliptic	Expression too large to display	1677

input
```
int(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(1/2)-2/3*b/a^2/((a*d-b*c)*b)^
(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))-1/3*b/a/(a*d-b*c)*((d*
x^3+c)^(1/2)/(b*x^3+a)+d/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*
d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 816, normalized size of antiderivative = 6.18

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(2*sqrt(d*x^3 + c)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*
b*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3
+ c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*((b^2*c - a*b*d)*x^
3 + a*b*c - a^2*d)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x
^3))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3), 1/3*(sqrt(d*x^
3 + c)*a*b*c - (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(
-b/(b*c - a*d))*arctan(sqrt(d*x^3 + c)*sqrt(-b/(b*c - a*d))) + ((b^2*c - a
*b*d)*x^3 + a*b*c - a^2*d)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c)
+ 2*c)/x^3))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3), 1/6*(2
*sqrt(d*x^3 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*sqrt(-c)*
arctan(sqrt(-c)/sqrt(d*x^3 + c)) + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3
*a*b*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x
^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)))/(a^3*b*c^2 - a^4*c*
d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3), 1/3*(sqrt(d*x^3 + c)*a*b*c - (2*a*b*c^
2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(s
qrt(d*x^3 + c)*sqrt(-b/(b*c - a*d))) + 2*((b^2*c - a*b*d)*x^3 + a*b*c - a^
2*d)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^3 + c)))/(a^3*b*c^2 - a^4*c*d + (a^
2*b^2*c^2 - a^3*b*c*d)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+c}\,x}\,dx$$

input
```
integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{\sqrt{dx^3+c}bd}{3\left(abc-a^2d\right)\left((dx^3+c)b-bc+ad\right)}-\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\left(a^2bc-a^3d\right)\sqrt{-b^2c+abd}}+\frac{2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}$$

input
```
integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(d*x^3 + c)*b*d/((a*b*c - a^2*d)*((d*x^3 + c)*b - b*c + a*d)) - 1/
3*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2
*b*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))
/(a^2*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2\,\sqrt{c}} + \frac{b^2\,\sqrt{d\,x^3+c}}{3\,a\,\left(b\,x^3+a\right)\,\left(b^2\,c-a\,b\,d\right)} + \frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(3\,a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{6\,a^2\,{\left(a\,d-b\,c\right)}^{3/2}}$$

input
```
int(1/(x*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output
```
log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(
3*a^2*c^(1/2)) + (b^2*(c + d*x^3)^(1/2))/(3*a*(a + b*x^3)*(b^2*c - a*b*d))
 + (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)
*2i - b*d*x^3)/(a + b*x^3))*(3*a*d - 2*b*c)*1i)/(6*a^2*(a*d - b*c)^(3/2))
```

Reduce [F]

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}}{b^2d\,x^{10}+2abd\,x^7+b^2c\,x^7+a^2d\,x^4+2abc\,x^4+a^2cx}dx$$

input
```
int(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**3)/(a**2*c*x + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**7
 + b**2*c*x**7 + b**2*d*x**10),x)
```

3.657 $\int \frac{1}{x^4\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 185

$$\int \frac{1}{x^4\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)\left(a+bx^3\right)}-\frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)}+\frac{(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}}-\frac{b^{3/2}(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{3/2}}$$

output

```
-1/3*b*(-a*d+2*b*c)*(d*x^3+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^3+a)-1/3*(d*x^3+
c)^(1/2)/a/c/x^3/(b*x^3+a)+1/3*(a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2)
)/a^3/c^(3/2)-1/3*b^(3/2)*(-5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(
-a*d+b*c)^(1/2))/a^3/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{\frac{a\sqrt{c+dx^3}(-a^2d+2b^2cx^3+ab(c-dx^3))}{c(-bc+ad)x^3(a+bx^3)} - \frac{b^{3/2}(4bc-5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{c^{3/2}}}{3a^3}$$

input
```
Integrate[1/(x^4*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((a*Sqrt[c + d*x^3]*(-(a^2*d) + 2*b^2*c*x^3 + a*b*(c - d*x^3)))/(c*(-(b*c)
 + a*d)*x^3*(a + b*x^3)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2) + ((4*b*c + a*d)*ArcT
anh[Sqrt[c + d*x^3]/Sqrt[c]])/c^(3/2))/(3*a^3)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {948, 114, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^6 (bx^3+a)^2 \sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int \frac{3bdx^3+4bc+ad}{2x^3(bx^3+a)^2\sqrt{dx^3+c}}dx^3}{ac} - \frac{\sqrt{c+dx^3}}{acx^3(a+bx^3)}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(-\frac{\int \frac{3bdx^3+4bc+ad}{x^3(bx^3+a)^2\sqrt{dx^3+c}}dx^3}{2ac}-\frac{\sqrt{c+dx^3}}{acx^3\,(a+bx^3)}\right)$$

$$\Big\downarrow \; 168$$

$$\frac{1}{3}\left(-\frac{-\frac{\int \frac{bd(2bc-ad)x^3+(bc-ad)(4bc+ad)}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a(bc-ad)}+\frac{2b\sqrt{c+dx^3}(2bc-ad)}{a(a+bx^3)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^3}}{acx^3\,(a+bx^3)}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{3}\left(-\frac{-\frac{\frac{(bc-ad)(ad+4bc)\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{b^2c(4bc-5ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^3}(2bc-ad)}{a(a+bx^3)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^3}}{acx^3\,(a+bx^3)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(-\frac{-\frac{\frac{2(bc-ad)(ad+4bc)\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2b^2c(4bc-5ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^3}(2bc-ad)}{a(a+bx^3)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^3}}{acx^3\,(a+bx^3)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(-\frac{-\frac{\frac{2b^{3/2}c(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)(ad+4bc)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^3}(2bc-ad)}{a(a+bx^3)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^3}}{acx^3\,(a+bx^3)}\right)$$

input
```
Int[1/(x^4*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(-(Sqrt[c + d*x^3]/(a*c*x^3*(a + b*x^3))) - ((2*b*(2*b*c - a*d)*Sqrt[c + d
*x^3])/(a*(b*c - a*d)*(a + b*x^3)) + ((-2*(b*c - a*d)*(4*b*c + a*d)*ArcTan
h[Sqrt[c + d*x^3]/Sqrt[c]])/(a*Sqrt[c]) + (2*b^(3/2)*c*(4*b*c - 5*a*d)*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(a*(
b*c - a*d)))/(2*a*c))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}}{3 c\,a^{2} x^{3}}-\frac{-\frac{2 \left(a d +4 b c \right) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3 a \sqrt{c}}-\frac{2 b^{2} c \left(\frac{\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{d \arctan\left(\frac{b \sqrt{d\,x^{3}+c}}{\sqrt{\left(a d -b c \right) b}}\right)}{\sqrt{\left(a d -b c \right) b}}\right)}{3 \left(a d -b c \right)}-\frac{8 b^{2} c \arctan\left(\frac{b \sqrt{d\,x^{3}+c}}{\sqrt{\left(a d -b c \right) b}}\right)}{3 a \sqrt{\left(a d -b c \right) b}}}{2 a^{2} c}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3 c\,x^{3}}+\frac{d \,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3 c^{\frac{3}{2}}}}{a^{2}}+\frac{b^{2} \left(\frac{\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{d \arctan\left(\frac{b \sqrt{d\,x^{3}+c}}{\sqrt{\left(a d -b c \right) b}}\right)}{\sqrt{\left(a d -b c \right) b}}\right)}{3 a^{2} \left(a d -b c \right)}+\frac{4 b \,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3 a^{3} \sqrt{c}}+\frac{4 b^{2} \arctan\left(\cdots\right.}{3 a^{3} \sqrt{(a\cdots}}$
pseudoelliptic	$\frac{-4 c^{\frac{5}{2}} \left(b\,x^{3}+a \right) b^{2} \left(b c -\frac{5 a d}{4}\right) x^{3} \arctan\left(\frac{b \sqrt{d\,x^{3}+c}}{\sqrt{\left(a d -b c \right) b}}\right)+\left(c\,x^{3} \left(b\,x^{3}+a \right) \left(a d +4 b c \right) \left(a d -b c \right) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)+a\,c^{\frac{3}{2}} \left(2 b^{2} c\,x^{3}\cdots\right.}{3 \sqrt{\left(a d -b c \right) b}\, c^{\frac{5}{2}} x^{3} a^{3} \left(a d -b c \right) \left(b\,x^{3}+a \right)}$
elliptic	Expression too large to display

input
```
int(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/c/a^2*(d*x^3+c)^(1/2)/x^3-1/2/a^2/c*(-2/3/a*(a*d+4*b*c)*arctanh((d*x^
3+c)^(1/2)/c^(1/2))/c^(1/2)-2/3*b^2*c/(a*d-b*c)*((d*x^3+c)^(1/2)/(b*x^3+a)
+d/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2)))-8/3/
a*b^2*c/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 1189, normalized size of antiderivative = 6.43

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3
)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c
- a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2
*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(c)*log((d*x^
3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*
b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^
6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), 1/6*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6
+ (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(sqrt(d*x^
3 + c)*sqrt(-b/(b*c - a*d))) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^6
+ (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d
*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^
2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c
^3 - a^5*c^2*d)*x^3), -1/6*(2*((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^6 +
 (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(-c)*arctan(sqrt(-c)/sqrt(
d*x^3 + c)) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^
2*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 +
c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a^2*b*c^2 - a^3*c*d
+ (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a^4*b*c^
2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), 1/3*(((4*b^3*c^3 - 5*a*b^2*c^2*d)
*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(s...
```

Sympy [F]

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**4/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^2 \sqrt{dx^3+c}\,x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{(4\,b^3c - 5\,ab^2d)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,(a^3bc - a^4d)\sqrt{-b^2c+abd}} - \frac{2\,(dx^3+c)^{\frac{3}{2}}b^2cd - 2\,\sqrt{dx^3+c}b^2c^2d - (dx^3+c)^{\frac{3}{2}}abd^2 + 2\,\sqrt{dx^3+c}abcd^2 - \sqrt{dx^3+c}a^2d^3}{3\,(a^2bc^2 - a^3cd)\left((dx^3+c)^2b - 2\,(dx^3+c)bc + bc^2 + (dx^3+c)ad - acd\right)} - \frac{(4\,bc+ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}c}$$

input
```
integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
1/3*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^3 + c)^(3/2)*b^2*c*d
 - 2*sqrt(d*x^3 + c)*b^2*c^2*d - (d*x^3 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^3
+ c)*a*b*c*d^2 - sqrt(d*x^3 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^3 +
 c)^2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x^3 + c)*a*d - a*c*d)) - 1/3*(4*b
*c + a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

Mupad [B] (verification not implemented)

Time = 10.44 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.92

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{\sqrt{d\,x^3+c}\left(\frac{d\,a^2+4\,b\,c\,a}{2\,a^3\,c^2}-\frac{a\left(\frac{2\,c\,b^2+2\,a\,d\,b}{2\,a^3\,c^2}-\frac{a\left(\frac{b^2\,d}{2\,a^3\,c^2}+\frac{b\left(2\,c\,b^2+2\,a\,d\,b\right)\,\left(3\,a\,d-4\,b\,c\right)}{6\,a^3\,c^2\,\left(a^2\,d-a\,b\,c\right)}-\frac{b^2\,d\,\left(3\,a\,d-4\,b\,c\right)}{6\,a^2\,c^2\,\left(a^2\,d-a\,b\,c\right)}\right)}{b}+\frac{b\,\left(d\,a^2+4\,b\,c\,a\right)\,\left(3\,a\,d-4\,b\,c\right)}{6\,a^3\,c^2\,\left(a^2\,d-a\,b\,c\right)}\right)}{b}\right)}{b\,x^3+a} - \frac{\sqrt{d\,x^3+c}}{3\,a^2\,c\,x^3}+\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\,{\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}^3}{x^6}\right)\,\left(a\,d+4\,b\,c\right)}{6\,a^3\,c^{3/2}} + \frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(5\,a\,d-4\,b\,c\right)\,1{}\mathrm{i}}{6\,a^3\,{\left(a\,d-b\,c\right)}^{3/2}}$$

input
```
int(1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output
```
((c + d*x^3)^(1/2)*((a^2*d + 4*a*b*c)/(2*a^3*c^2) - (a*((2*b^2*c + 2*a*b*d
)/(2*a^3*c^2) - (a*((b^2*d)/(2*a^3*c^2) + (b*(2*b^2*c + 2*a*b*d)*(3*a*d -
4*b*c))/(6*a^3*c^2*(a^2*d - a*b*c)) - (b^2*d*(3*a*d - 4*b*c))/(6*a^2*c^2*(
a^2*d - a*b*c))))/b + (b*(a^2*d + 4*a*b*c)*(3*a*d - 4*b*c))/(6*a^3*c^2*(a^
2*d - a*b*c))))/b))/(a + b*x^3) - (c + d*x^3)^(1/2)/(3*a^2*c*x^3) + (log((
((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*(a*d +
 4*b*c))/(6*a^3*c^(3/2)) + (b^(3/2)*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)
^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5*a*d - 4*b*c)*1i)/(6
*a^3*(a*d - b*c)^(3/2))
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}} \, dx$$
$$= \frac{-2\sqrt{d\,x^3+c} - 3\left(\int \frac{\sqrt{d\,x^3+c}}{b^2d\,x^{10}+2abd\,x^7+b^2c\,x^7+a^2d\,x^4+2abc\,x^4+a^2cx}dx\right)a^2d\,x^3 - 12\left(\int \frac{\sqrt{d\,x^3+c}}{b^2d\,x^{10}+2abd\,x^7+b^2c\,x^7+a^2d\,x^4+2abc\ldots}\right.}{}$$

input
```
int(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**3) - 3*int(sqrt(c + d*x**3)/(a**2*c*x + a**2*d*x**4 +
2*a*b*c*x**4 + 2*a*b*d*x**7 + b**2*c*x**7 + b**2*d*x**10),x)*a**2*d*x**3 -
 12*int(sqrt(c + d*x**3)/(a**2*c*x + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*
x**7 + b**2*c*x**7 + b**2*d*x**10),x)*a*b*c*x**3 - 3*int(sqrt(c + d*x**3)/
(a**2*c*x + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**7 + b**2*c*x**7 + b**2
*d*x**10),x)*a*b*d*x**6 - 12*int(sqrt(c + d*x**3)/(a**2*c*x + a**2*d*x**4
+ 2*a*b*c*x**4 + 2*a*b*d*x**7 + b**2*c*x**7 + b**2*d*x**10),x)*b**2*c*x**6
 - 9*int((sqrt(c + d*x**3)*x**2)/(a**2*c + a**2*d*x**3 + 2*a*b*c*x**3 + 2*
a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**9),x)*a*b*d*x**3 - 9*int((sqrt(c + d*
x**3)*x**2)/(a**2*c + a**2*d*x**3 + 2*a*b*c*x**3 + 2*a*b*d*x**6 + b**2*c*x
**6 + b**2*d*x**9),x)*b**2*d*x**6)/(6*a*c*x**3*(a + b*x**3))
```

3.658 $\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},2,\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{c+dx^3}}$$

output
```
1/4*x^4*(1+d*x^3/c)^(1/2)*AppellF1(4/3,2,1/2,7/3,-b*x^3/a,-d*x^3/c)/a^2/(d
*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 238 vs. 2(64) = 128.

Time = 10.28 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.72

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x\left(\frac{dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+\frac{8\left(c+dx^3+\frac{8ac^2\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bc\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right) \cdots\right.}\right.}{a+bx^3}\right.}{24(-bc+ad)\sqrt{c+dx^3}}$$

input
```
Integrate[x^3/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x*((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -(
(b*x^3)/a)])/a + (8*(c + d*x^3 + (8*a*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d*
x^3)/c), -((b*x^3)/a)])/(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -
((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*
x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/
(a + b*x^3)))/(24*(-(b*c) + a*d)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^3}{(bx^3+a)^2\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\operatorname{AppellF1}\left(\frac{4}{3},2,\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{c+dx^3}}$$

input
```
Int[x^3/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*a^2*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 2.04 (sec) , antiderivative size = 764, normalized size of antiderivative = 11.94

method	result	size
elliptic	Expression too large to display	764
default	Expression too large to display	1207

input
```
int(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(a*d-b*c)*x*(d*x^3+c)^(1/2)/(b*x^3+a)-1/9*I/(a*d-b*c)/b*3^(1/2)*(-c*d^
2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/b/d^2*2^(1/2)
*sum((a*d+2*b*c)/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I
*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d
)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**3/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^3}{(bx^3+a)^2\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^3}{(bx^3+a)^2\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^3}{{\left(b\,x^3+a\right)}^2\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(x^3/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output
```
int(x^3/((a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{\sqrt{d\,x^3+c}\,x^3}{b^2d\,x^9+2abd\,x^6+b^2c\,x^6+a^2d\,x^3+2abc\,x^3+a^2c}dx$$

input
```
int(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**3)/(a**2*c + a**2*d*x**3 + 2*a*b*c*x**3 + 2*a*b*d
*x**6 + b**2*c*x**6 + b**2*d*x**9),x)
```

3.659 $\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},2,\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{c+dx^3}}$$

output
```
1/2*x^2*(1+d*x^3/c)^(1/2)*AppellF1(2/3,2,1/2,5/3,-b*x^3/a,-d*x^3/c)/a^2/(d
*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(64) = 128.

Time = 10.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.69

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{10abx^2(c+dx^3)+5(bc-3ad)x^2(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-bdx^5(a+bx^3)\ldots}{30a^2(bc-ad)\,(a+bx^3)\sqrt{c+dx^3}}$$

input
```
Integrate[x/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(10*a*b*x^2*(c + d*x^3) + 5*(b*c - 3*a*d)*x^2*(a + b*x^3)*Sqrt[1 + (d*x^3)
/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] - b*d*x^5*(a +
b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x
^3)/a)])/(30*a^2*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x}{(bx^3+a)^2\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{2}{3},2,\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{c+dx^3}}$$

input
```
Int[x/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 2, 1/2, 5/3, -((b*x^3)/a), -((d*x^3
)/c)])/(2*a^2*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.20 (sec) , antiderivative size = 923, normalized size of antiderivative = 14.42

method	result	size
default	Expression too large to display	923
elliptic	Expression too large to display	923

input
```
int(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b/(a*d-b*c)/a*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-1/9*I/(a*d-b*c)/a*3^(1/2)
*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/a/d^2*2^(1/2)*sum((-5*
a*d+2*b*c)/(a*d-b*c)^2/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^
(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+
1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_
alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x}{(bx^3+a)^2\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x}{(bx^3+a)^2\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{x}{(b\,x^3+a)^2\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(x/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output
```
int(x/((a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x}{b^2d\,x^9+2abd\,x^6+b^2c\,x^6+a^2d\,x^3+2abc\,x^3+a^2c}dx$$

input
```
int(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*x)/(a**2*c + a**2*d*x**3 + 2*a*b*c*x**3 + 2*a*b*d*x*
*6 + b**2*c*x**6 + b**2*d*x**9),x)
```

3.660 $\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{c+dx^3}}$$

output
```
x*(1+d*x^3/c)^(1/2)*AppellF1(1/3,2,1/2,4/3,-b*x^3/a,-d*x^3/c)/a^2/(d*x^3+c
)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 392 vs. 2(59) = 118.

Time = 10.21 (sec) , antiderivative size = 392, normalized size of antiderivative = 6.64

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{-8acx\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\left(8a(3bc-3ad+bdx^3)+bdx^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3}\right.\right.}{24a^2(bc-ad)\,(a+bx^3)\sqrt{c+dx^3}\,(-8a}$$

input
```
Integrate[1/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(-8*a*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)]*(8*a*(3*b
*c - 3*a*d + b*d*x^3) + b*d*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4
/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]) + 3*b*x^4*(8*a*(c + d*x^3) +
 d*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3
)/c), -((b*x^3)/a)])*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*
x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))/(2
4*a^2*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1
, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/
3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/
c), -((b*x^3)/a)])))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{(bx^3+a)^2\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{c+dx^3}}$$

input
```
Int[1/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a^2*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.14 (sec) , antiderivative size = 769, normalized size of antiderivative = 13.03

method	result	size
default	Expression too large to display	769
elliptic	Expression too large to display	769

input
```
int(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b/(a*d-b*c)/a*x*(d*x^3+c)^(1/2)/(b*x^3+a)+1/9*I/(a*d-b*c)/a*3^(1/2)*(
-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a/d^2*2^
(1/2)*sum((-7*a*d+4*b*c)/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*
(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)
*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-
c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(
1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alp
ha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^2\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^2\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{(b\,x^3+a)^2\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(1/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output
```
int(1/((a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}}{b^2d\,x^9+2abd\,x^6+b^2c\,x^6+a^2d\,x^3+2abc\,x^3+a^2c}dx$$

input
```
int(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**3)/(a**2*c + a**2*d*x**3 + 2*a*b*c*x**3 + 2*a*b*d*x**6 +
 b**2*c*x**6 + b**2*d*x**9),x)
```

3.661 $\int \frac{1}{x^2(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2(a+bx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{1}{3},2,\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{c+dx^3}}$$

output
```
-(1+d*x^3/c)^(1/2)*AppellF1(-1/3,2,1/2,2/3,-b*x^3/a,-d*x^3/c)/a^2/x/(d*x^3
+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 226 vs. 2(62) = 124.

Time = 10.20 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.65

$$\int \frac{1}{x^2(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{20a(c+dx^3)\left(3a^2d-4b^2cx^3-3ab(c-dx^3)\right)-5\left(8b^2c^2-15abcd+3a^2d^2\right)x^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{Appell}}{60a^3c(bc-ad)x\,(a}$$

input
```
Integrate[1/(x^2*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(20*a*(c + d*x^3)*(3*a^2*d - 4*b^2*c*x^3 - 3*a*b*(c - d*x^3)) - 5*(8*b^2*c
^2 - 15*a*b*c*d + 3*a^2*d^2)*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[
2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*(4*b*c - 3*a*d)*x^6*
(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -
((b*x^3)/a)])/(60*a^3*c*(b*c - a*d)*x*(a + b*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^3)^2 \sqrt{c+dx^3}} \, dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{1}{x^2 (bx^3+a)^2 \sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1} \operatorname{AppellF1}\left(-\frac{1}{3},2,\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2 x \sqrt{c+dx^3}}$$

input
```
Int[1/(x^2*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
-((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 2, 1/2, 2/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*x*Sqrt[c + d*x^3]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.49 (sec) , antiderivative size = 963, normalized size of antiderivative = 15.53

method	result	size
elliptic	Expression too large to display	963
default	Expression too large to display	1818
risch	Expression too large to display	1819

input
```
int(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(a*d-b*c)/a^2*b^2*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-1/c/a^2*(d*x^3+c)^(1/2
)/x-2/3*I*(-1/6*d*b/(a*d-b*c)/a^2+1/2*d/c/a^2)*3^(1/2)/d*(-c*d^2)^(1/3)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/a^2/d^2*b*2^(1/2)*sum((11*a*d-8*b*c)/(a*
d-b*c)^2/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c
*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*
(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-
c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a+bx^3\right)^2 \sqrt{c+dx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{x^2(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**2/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^2\sqrt{dx^3+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^2\sqrt{dx^3+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3)^2 \sqrt{c+dx^3}} \, dx = \int \frac{1}{x^2 (b\,x^3+a)^2 \sqrt{d\,x^3+c}} \, dx$$

input
```
int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^3)^2 \sqrt{c+dx^3}} \, dx$$
$$= \frac{-2\sqrt{d\,x^3+c} - 5\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{b^2 d\,x^9+2abd\,x^6+b^2 c\,x^6+a^2 d\,x^3+2abc\,x^3+a^2 c} dx\right) abdx - 5\left(\int \frac{\sqrt{d\,x^3+c}\,x^4}{b^2 d\,x^9+2abd\,x^6+b^2 c\,x^6+a^2 d\,x^3+2abc\,x^3+a}\right.}{}$$

input
```
int(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**3) - 5*int((sqrt(c + d*x**3)*x**4)/(a**2*c + a**2*d*x*
*3 + 2*a*b*c*x**3 + 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**9),x)*a*b*d*x -
 5*int((sqrt(c + d*x**3)*x**4)/(a**2*c + a**2*d*x**3 + 2*a*b*c*x**3 + 2*a*
b*d*x**6 + b**2*c*x**6 + b**2*d*x**9),x)*b**2*d*x**4 + int((sqrt(c + d*x**
3)*x)/(a**2*c + a**2*d*x**3 + 2*a*b*c*x**3 + 2*a*b*d*x**6 + b**2*c*x**6 +
b**2*d*x**9),x)*a**2*d*x - 8*int((sqrt(c + d*x**3)*x)/(a**2*c + a**2*d*x**
3 + 2*a*b*c*x**3 + 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**9),x)*a*b*c*x +
int((sqrt(c + d*x**3)*x)/(a**2*c + a**2*d*x**3 + 2*a*b*c*x**3 + 2*a*b*d*x*
*6 + b**2*c*x**6 + b**2*d*x**9),x)*a*b*d*x**4 - 8*int((sqrt(c + d*x**3)*x)
/(a**2*c + a**2*d*x**3 + 2*a*b*c*x**3 + 2*a*b*d*x**6 + b**2*c*x**6 + b**2*
d*x**9),x)*b**2*c*x**4)/(2*a*c*x*(a + b*x**3))
```

3.662 $\int \frac{1}{x^3(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^3(a+bx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{c+dx^3}}$$

output
```
-1/2*(1+d*x^3/c)^(1/2)*AppellF1(-2/3,2,1/2,1/3,-b*x^3/a,-d*x^3/c)/a^2/x^2/
(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 411 vs. 2(64) = 128.

Time = 10.78 (sec) , antiderivative size = 411, normalized size of antiderivative = 6.42

$$\int \frac{1}{x^3(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{bd(5bc-3ad)x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a\left(32ac(-10b^2cx^3(3c+dx^3)+3a^2d(2c+3dx^3)+3ab(-2c\cdots\right.}{(a+bx\cdots}}{4\cdots}$$

input
```
Integrate[1/(x^3*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(b*d*(5*b*c - 3*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -(
(d*x^3)/c), -((b*x^3)/a)] + (a*(32*a*c*(-10*b^2*c*x^3*(3*c + d*x^3) + 3*a^
2*d*(2*c + 3*d*x^3) + 3*a*b*(-2*c^2 + 7*c*d*x^3 + 2*d^2*x^6))*AppellF1[1/3
, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 24*x^3*(c + d*x^3)*(-3*a^2*d
+ 5*b^2*c*x^3 + 3*a*b*(c - d*x^3))*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*
x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b
*x^3)/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
 -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((
b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
)/(48*a^3*c*(-(b*c) + a*d)*x^2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^3)^2 \sqrt{c+dx^3}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{1}{x^3 (bx^3+a)^2 \sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1} \operatorname{AppellF1}\left(-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{c+dx^3}}$$

input
```
Int[1/(x^3*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
-1/2*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 1/2, 1/3, -((b*x^3)/a), -((d*x
^3)/c)])/(a^2*x^2*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.64 (sec) , antiderivative size = 809, normalized size of antiderivative = 12.64

method	result	size
elliptic	Expression too large to display	809
default	Expression too large to display	1512
risch	Expression too large to display	1513

input
```
int(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c/a^2*(d*x^3+c)^(1/2)/x^2+1/3/(a*d-b*c)/a^2*b^2*x*(d*x^3+c)^(1/2)/(b*
x^3+a)-2/3*I*(-1/4*d/c/a^2+1/6*d*b/(a*d-b*c)/a^2)*3^(1/2)/d*(-c*d^2)^(1/3)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a^2/d^2*b*2^(1/2)*sum
((13*a*d-10*b*c)/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I
*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d
)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 (a + bx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^3 (a + bx^3)^2 \sqrt{c + dx^3}} \, dx$$

input
```
integrate(1/x**3/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a + bx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{(bx^3 + a)^2 \sqrt{dx^3 + c}x^3} \, dx$$

input
```
integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a + bx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{(bx^3 + a)^2 \sqrt{dx^3 + c}x^3} \, dx$$

input
```
integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^3 \left(b\,x^3+a\right)^2 \sqrt{d\,x^3+c}}\,dx$$

input
```
int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx$$
$$= \frac{-2\sqrt{d\,x^3+c}-\left(\int \frac{\sqrt{d\,x^3+c}}{b^2d\,x^9+2abd\,x^6+b^2c\,x^6+a^2d\,x^3+2abc\,x^3+a^2c}dx\right)a^2d\,x^2-10\left(\int \frac{\sqrt{d\,x^3+c}}{b^2d\,x^9+2abd\,x^6+b^2c\,x^6+a^2d\,x^3+2abc\,x^3+}\right.}{}$$

input
```
int(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**3) - int(sqrt(c + d*x**3)/(a**2*c + a**2*d*x**3 + 2*a*
b*c*x**3 + 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**9),x)*a**2*d*x**2 - 10*i
nt(sqrt(c + d*x**3)/(a**2*c + a**2*d*x**3 + 2*a*b*c*x**3 + 2*a*b*d*x**6 +
b**2*c*x**6 + b**2*d*x**9),x)*a*b*c*x**2 - int(sqrt(c + d*x**3)/(a**2*c +
a**2*d*x**3 + 2*a*b*c*x**3 + 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**9),x)*
a*b*d*x**5 - 10*int(sqrt(c + d*x**3)/(a**2*c + a**2*d*x**3 + 2*a*b*c*x**3
+ 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**9),x)*b**2*c*x**5 - 7*int((sqrt(c
 + d*x**3)*x**3)/(a**2*c + a**2*d*x**3 + 2*a*b*c*x**3 + 2*a*b*d*x**6 + b**
2*c*x**6 + b**2*d*x**9),x)*a*b*d*x**2 - 7*int((sqrt(c + d*x**3)*x**3)/(a**
2*c + a**2*d*x**3 + 2*a*b*c*x**3 + 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**
9),x)*b**2*d*x**5)/(4*a*c*x**2*(a + b*x**3))
```

3.663 $\int \frac{x^8}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{x^8}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = -\frac{2c^2}{3d(bc-ad)^2\sqrt{c+dx^3}} - \frac{a^2\sqrt{c+dx^3}}{3b(bc-ad)^2\left(a+bx^3\right)} + \frac{a(4bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{5/2}}$$

output
```
-2/3*c^2/d/(-a*d+b*c)^2/(d*x^3+c)^(1/2)-1/3*a^2*(d*x^3+c)^(1/2)/b/(-a*d+b*
c)^2/(b*x^3+a)+1/3*a*(-a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*
c)^(1/2))/b^(3/2)/(-a*d+b*c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{-\frac{\sqrt{b}\left(2abc^2+2b^2c^2x^3+a^2d\left(c+dx^3\right)\right)}{d(bc-ad)^2\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{a(-4bc+ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}}{3b^{3/2}}$$

input
```
Integrate[x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-((Sqrt[b]*(2*a*b*c^2 + 2*b^2*c^2*x^3 + a^2*d*(c + d*x^3)))/(d*(b*c - a*d
)^2*(a + b*x^3)*Sqrt[c + d*x^3])) + (a*(-4*b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt
[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2))/(3*b^(3/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 100, 27, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3} \int \frac{x^6}{(bx^3+a)^2 (dx^3+c)^{3/2}} dx^3$$

$$\Big\downarrow \text{ 100}$$

$$\frac{1}{3} \left(\frac{\int -\frac{a(2bc+ad)-2b(bc-ad)x^3}{2(bx^3+a)(dx^3+c)^{3/2}} dx^3}{b^2(bc-ad)} - \frac{a^2}{b^2 (a+bx^3) \sqrt{c+dx^3}(bc-ad)} \right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3} \left(-\frac{\int \frac{a(2bc+ad)-2b(bc-ad)x^3}{(bx^3+a)(dx^3+c)^{3/2}} dx^3}{2b^2(bc-ad)} - \frac{a^2}{b^2 (a+bx^3) \sqrt{c+dx^3}(bc-ad)} \right)$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{3} \left(-\frac{\frac{ab(4bc-ad)\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}} dx^3}{bc-ad} + \frac{2(a^2d^2+2b^2c^2)}{d\sqrt{c+dx^3}(bc-ad)}}{2b^2(bc-ad)} - \frac{a^2}{b^2 (a+bx^3) \sqrt{c+dx^3}(bc-ad)} \right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(-\frac{\frac{2ab(4bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{d(bc-ad)}+\frac{2(a^2d^2+2b^2c^2)}{d\sqrt{c+dx^3}(bc-ad)}}{2b^2(bc-ad)}-\frac{a^2}{b^2(a+bx^3)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{\frac{2(a^2d^2+2b^2c^2)}{d\sqrt{c+dx^3}(bc-ad)}-\frac{2a\sqrt{b}(4bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{2b^2(bc-ad)}-\frac{a^2}{b^2(a+bx^3)\sqrt{c+dx^3}(bc-ad)}\right)$$

input
```
Int[x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-(a^2/(b^2*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3])) - ((2*(2*b^2*c^2 + a
^2*d^2))/(d*(b*c - a*d)*Sqrt[c + d*x^3]) - (2*a*Sqrt[b]*(4*b*c - a*d)*ArcT
anh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(2*b^2*
(b*c - a*d)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$-\frac{-ad\sqrt{d\,x^{3}+c}\,(b\,x^{3}+a)(ad-4bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)+(2b^{2}c^{2}x^{3}+2b\,c^{2}a+a^{2}d(d\,x^{3}+c))\sqrt{(ad-bc)b}}{3\sqrt{(ad-bc)b}\,\sqrt{d\,x^{3}+c}\,bd(b\,x^{3}+a)(ad-bc)^{2}}$
default	$-\frac{2}{3b^{2}d\sqrt{d\,x^{3}+c}}+\frac{a^{2}d\left(-\frac{b\sqrt{d\,x^{3}+c}}{d\left(b\,x^{3}+a\right)}-\frac{3b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{2}{\sqrt{d\,x^{3}+c}}\right)}{3b^{2}(ad-bc)^{2}}-\frac{4a\left(-\frac{b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{1}{\sqrt{d\,x^{3}+c}}\right)}{3b^{2}(ad-bc)}$
elliptic	$-\frac{2c^{2}}{3d(ad-bc)^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{a^{2}\sqrt{d\,x^{3}+c}}{3(ad-bc)^{2}b(b\,x^{3}+a)}-\frac{ia\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(ad-4bc)\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}}{\cdots}\right)}{\cdots}}\cdots}{\cdots}\right)}{\cdots}$

input
```
int(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-a*d*(d*x^3+c)^(1/2)*(b*x^3+a)*(a*d-4*b*c)*arctan(b*(d*x^3+c)^(1/2)/
((a*d-b*c)*b)^(1/2))+(2*b^2*c^2*x^3+2*b*c^2*a+a^2*d*(d*x^3+c))*((a*d-b*c)*
b)^(1/2))/((a*d-b*c)*b)^(1/2)/(d*x^3+c)^(1/2)/b/d/(b*x^3+a)/(a*d-b*c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(113) = 226$.

Time = 0.26 (sec) , antiderivative size = 746, normalized size of antiderivative = 5.61

$$\int \frac{x^8}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \left[-\frac{((4\,ab^2cd^2 - a^2bd^3)x^6 + 4\,a^2bc^2d - a^3cd^2 + (4\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)}{6\,(ab^5c^4d - 3\,a^2b^4c^3d^2 + 3\,a^3b^3c^2d^3 -} \right.$$

$$-\frac{((4\,ab^2cd^2 - a^2bd^3)x^6 + 4\,a^2bc^2d - a^3cd^2 + (4\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)x^3)\sqrt{-b^2c + abd}\arctan\left(\frac{\sqrt{dx^3+c}}{bdx}\right.}{3\,(ab^5c^4d - 3\,a^2b^4c^3d^2 + 3\,a^3b^3c^2d^3 - a^4b^2cd^4 + (b^6c^3d^2 - 3\,ab^5c^2d^3 + 3\,a^2b}$$

input
```
integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/6*(((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^2 + (4*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*sqrt(b^2*c - a*b*d)*log((b*d*x^
3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*
(2*a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + (2*b^4*c^3 - 2*a*b^3*c^2*d +
a^2*b^2*c*d^2 - a^3*b*d^3)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^4*d - 3*a^2*b^4*
c^3*d^2 + 3*a^3*b^3*c^2*d^3 - a^4*b^2*c*d^4 + (b^6*c^3*d^2 - 3*a*b^5*c^2*d
^3 + 3*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^6 + (b^6*c^4*d - 2*a*b^5*c^3*d^2 + 2
*a^3*b^3*c*d^4 - a^4*b^2*d^5)*x^3), -1/3*(((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6
 + 4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x
^3)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*
x^3 + b*c)) + (2*a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + (2*b^4*c^3 - 2*
a*b^3*c^2*d + a^2*b^2*c*d^2 - a^3*b*d^3)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^4*
d - 3*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - a^4*b^2*c*d^4 + (b^6*c^3*d^2 -
 3*a*b^5*c^2*d^3 + 3*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^6 + (b^6*c^4*d - 2*a*b
^5*c^3*d^2 + 2*a^3*b^3*c*d^4 - a^4*b^2*d^5)*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**8/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.53

$$\int \frac{x^8}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{\frac{(4\,abcd^2-a^2d^3)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^3c^2-2\,ab^2cd+a^2bd^2)\sqrt{-b^2c+abd}}+\frac{2\,(dx^3+c)b^2c^2d-2\,b^2c^3d+2\,abc^2d^2+(dx^3+c)a^2d^3}{(b^3c^2-2\,ab^2cd+a^2bd^2)\left((dx^3+c)^{\frac{3}{2}}b-\sqrt{dx^3+c}bc+\sqrt{dx^3+c}ad\right)}}{3\,d^2}$$

input
```
integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-1/3*((4*a*b*c*d^2 - a^2*d^3)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d
))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(-b^2*c + a*b*d)) + (2*(d*x^3
+ c)*b^2*c^2*d - 2*b^2*c^3*d + 2*a*b*c^2*d^2 + (d*x^3 + c)*a^2*d^3)/((b^3*
c^2 - 2*a*b^2*c*d + a^2*b*d^2)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c
+ sqrt(d*x^3 + c)*a*d))/d^2
```

Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.76

$$\int \frac{x^8}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{\sqrt{d\,x^3+c}\,\left(x^3\,\left(\frac{\left(\frac{3\,b\,d\,(a\,d+b\,c)-b\,d\,(a\,d+2\,b\,c)}{3\,\left(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}-\frac{b\,d\,(a\,d+b\,c)}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)\,(a\,d+b\,c)}{b\,d}+\frac{}{a^2\,b\,d^3}\right.\right.}{b\,d\,x^6+(a\,d+b\,c}$$
$$+\frac{a\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,(a\,d-4\,b\,c)\,1\mathrm{i}}{6\,b^{3/2}\,(a\,d-b\,c)^{5/2}}$$

input
```
int(x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output
```
((c + d*x^3)^(1/2)*(x^3*((((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2
*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*
c^2*d - 2*a*b^2*c*d^2))*(a*d + b*c))/(b*d) + (a*b*c*d)/(a^2*b*d^3 + b^3*c^
2*d - 2*a*b^2*c*d^2)) + (a*c*((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(
a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b
^3*c^2*d - 2*a*b^2*c*d^2)))/(b*d)))/(a*c + x^3*(a*d + b*c) + b*d*x^6) + (a
*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x
^3)/(a + b*x^3))*(a*d - 4*b*c)*1i)/(6*b^(3/2)*(a*d - b*c)^(5/2))
```

Reduce [F]

$$\int \frac{x^8}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = \text{too large to display}$$

input
```
int(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

output

```
( - 4*sqrt(c + d*x**3)*a*c - 2*sqrt(c + d*x**3)*a*d*x**3 - 4*sqrt(c + d*x*
*3)*b*c*x**3 + 3*int((sqrt(c + d*x**3)*x**5)/(a**3*c**2*d + 2*a**3*c*d**2*
x**3 + a**3*d**3*x**6 + 2*a**2*b*c**3 + 6*a**2*b*c**2*d*x**3 + 6*a**2*b*c*
d**2*x**6 + 2*a**2*b*d**3*x**9 + 4*a*b**2*c**3*x**3 + 9*a*b**2*c**2*d*x**6
 + 6*a*b**2*c*d**2*x**9 + a*b**2*d**3*x**12 + 2*b**3*c**3*x**6 + 4*b**3*c*
*2*d*x**9 + 2*b**3*c*d**2*x**12),x)*a**4*c*d**3 + 3*int((sqrt(c + d*x**3)*
x**5)/(a**3*c**2*d + 2*a**3*c*d**2*x**3 + a**3*d**3*x**6 + 2*a**2*b*c**3 +
 6*a**2*b*c**2*d*x**3 + 6*a**2*b*c*d**2*x**6 + 2*a**2*b*d**3*x**9 + 4*a*b*
*2*c**3*x**3 + 9*a*b**2*c**2*d*x**6 + 6*a*b**2*c*d**2*x**9 + a*b**2*d**3*x
**12 + 2*b**3*c**3*x**6 + 4*b**3*c**2*d*x**9 + 2*b**3*c*d**2*x**12),x)*a**
4*d**4*x**3 - 6*int((sqrt(c + d*x**3)*x**5)/(a**3*c**2*d + 2*a**3*c*d**2*x
**3 + a**3*d**3*x**6 + 2*a**2*b*c**3 + 6*a**2*b*c**2*d*x**3 + 6*a**2*b*c*d
**2*x**6 + 2*a**2*b*d**3*x**9 + 4*a*b**2*c**3*x**3 + 9*a*b**2*c**2*d*x**6
+ 6*a*b**2*c*d**2*x**9 + a*b**2*d**3*x**12 + 2*b**3*c**3*x**6 + 4*b**3*c**
2*d*x**9 + 2*b**3*c*d**2*x**12),x)*a**3*b*c**2*d**2 - 3*int((sqrt(c + d*x*
*3)*x**5)/(a**3*c**2*d + 2*a**3*c*d**2*x**3 + a**3*d**3*x**6 + 2*a**2*b*c*
*3 + 6*a**2*b*c**2*d*x**3 + 6*a**2*b*c*d**2*x**6 + 2*a**2*b*d**3*x**9 + 4*
a*b**2*c**3*x**3 + 9*a*b**2*c**2*d*x**6 + 6*a*b**2*c*d**2*x**9 + a*b**2*d*
*3*x**12 + 2*b**3*c**3*x**6 + 4*b**3*c**2*d*x**9 + 2*b**3*c*d**2*x**12),x)
*a**3*b*c*d**3*x**3 + 3*int((sqrt(c + d*x**3)*x**5)/(a**3*c**2*d + 2*a*...
```

3.664 $\int \frac{x^5}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 121

$$\int \frac{x^5}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{2c}{3(bc-ad)^2\sqrt{c+dx^3}} + \frac{a\sqrt{c+dx^3}}{3(bc-ad)^2(a+bx^3)} - \frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{5/2}}$$

output
```
2/3*c/(-a*d+b*c)^2/(d*x^3+c)^(1/2)+1/3*a*(d*x^3+c)^(1/2)/(-a*d+b*c)^2/(b*x
^3+a)-1/3*(a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^
(1/2)/(-a*d+b*c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.91

$$\int \frac{x^5}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{1}{3}\left(\frac{3ac+2bcx^3+adx^3}{(bc-ad)^2(a+bx^3)\sqrt{c+dx^3}} + \frac{(2bc+ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{5/2}}\right)$$

input
```
Integrate[x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
((3*a*c + 2*b*c*x^3 + a*d*x^3)/((b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3])
 + ((2*b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(S
qrt[b]*(-(b*c) + a*d)^(5/2)))/3
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {948, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^3}{(bx^3+a)^2 (dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3} \left(\frac{(ad+2bc) \int \frac{1}{(bx^3+a)(dx^3+c)^{3/2}}dx^3}{2b(bc-ad)}+\frac{a}{b(a+bx^3)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3} \left(\frac{(ad+2bc) \left(\frac{b\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{bc-ad}+\frac{2}{\sqrt{c+dx^3}(bc-ad)}\right)}{2b(bc-ad)}+\frac{a}{b(a+bx^3)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{(ad+2bc)\left(\frac{2b\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{d(bc-ad)}+\frac{2}{\sqrt{c+dx^3}(bc-ad)}\right)}{2b(bc-ad)}+\frac{a}{b\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(\frac{(ad+2bc)\left(\frac{2}{\sqrt{c+dx^3}(bc-ad)}-\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}\right)}{2b(bc-ad)}+\frac{a}{b\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}\right)$$

input
```
Int[x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(a/(b*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]) + ((2*b*c + a*d)*(2/((b*c -
 a*d)*Sqrt[c + d*x^3]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt
[b*c - a*d]])/(b*c - a*d)^(3/2)))/(2*b*(b*c - a*d)))/3
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{\frac{\sqrt{d\,x^{3}+c}\,\left(b\,x^{3}+a\right)(ad+2bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3}+\left(\frac{2x^{3}bc}{3}+a\left(\frac{d\,x^{3}}{3}+c\right)\right)\sqrt{(ad-bc)b}}{\sqrt{(ad-bc)b}\,\sqrt{d\,x^{3}+c}\,(b\,x^{3}+a)(ad-bc)^{2}}$
default	$\frac{-\frac{2b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}-\frac{2}{3\sqrt{d\,x^{3}+c}}}{(ad-bc)b}-\frac{ad\left(-\frac{b\sqrt{d\,x^{3}+c}}{d\left(b\,x^{3}+a\right)}-\frac{3b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{2}{\sqrt{d\,x^{3}+c}}\right)}{3b(ad-bc)^{2}}$
elliptic	$\frac{2c}{3(ad-bc)^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{a\sqrt{d\,x^{3}+c}}{3(ad-bc)^{2}(b\,x^{3}+a)}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(-ad-2bc)\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\right.}{}\right.}{\left(-c\right.}}}{}\right.}{}$

input
```
int(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(1/3*(d*x^3+c)^(1/2)*(b*x^3+a)*(a*d+2*b*c)*arctan(b*(d*x^3+c)^(1/2)/((a*d-
b*c)*b)^(1/2))+(2/3*x^3*b*c+a*(1/3*d*x^3+c))*((a*d-b*c)*b)^(1/2))/((a*d-b*
c)*b)^(1/2)/(d*x^3+c)^(1/2)/(b*x^3+a)/(a*d-b*c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(101) = 202$.

Time = 0.17 (sec) , antiderivative size = 630, normalized size of antiderivative = 5.21

$$\int \frac{x^5}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = \left[\frac{((2\,b^2cd+abd^2)x^6+2\,abc^2+a^2cd+(2\,b^2c^2+3\,abcd+a^2d^2)x^3)\sqrt{b^2c-a}}{6\,(ab^4c^4-3\,a^2b^3c^3d+3\,a^3b^2c^2d^2-a^4bcd^3+(b^5c^3d-}\right.$$

input
```
integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*(((2*b^2*c*d + a*b*d^2)*x^6 + 2*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*
b*c*d + a^2*d^2)*x^3)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*s
qrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(3*a*b^2*c^2 - 3*a^2*
b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^4*c
^4 - 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (b^5*c^3*d - 3*a*
b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (b^5*c^4 - 2*a*b^4*c^3*
d + 2*a^3*b^2*c*d^3 - a^4*b*d^4)*x^3), 1/3*(((2*b^2*c*d + a*b*d^2)*x^6 + 2
*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^3)*sqrt(-b^2*c +
a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (3*a
*b^2*c^2 - 3*a^2*b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^3)*sqrt(d*x
^3 + c))/(a*b^4*c^4 - 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 +
(b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (b^5*c
^4 - 2*a*b^4*c^3*d + 2*a^3*b^2*c*d^3 - a^4*b*d^4)*x^3)]
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^5}{(a+bx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**5/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.50

$$\int \frac{x^5}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{\frac{(2\,bcd+ad^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}} + \frac{2\,(dx^3+c)bcd-2\,bc^2d+(dx^3+c)ad^2+2\,acd^2}{(b^2c^2-2\,abcd+a^2d^2)\left((dx^3+c)^{\frac{3}{2}}b-\sqrt{dx^3+c}bc+\sqrt{dx^3+c}ad\right)}}{3\,d}$$

input
```
integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
1/3*((2*b*c*d + a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (2*(d*x^3 + c)*b*c*d
- 2*b*c^2*d + (d*x^3 + c)*a*d^2 + 2*a*c*d^2)/((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c + sqrt(d*x^3 + c)*a*d)))/d
```

Mupad [B] (verification not implemented)

Time = 6.60 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.04

$$\int \frac{x^5}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = -\frac{\sqrt{d\,x^3+c}\left(x^3\left(\frac{3\,b\,d\,(a\,d+b\,c)-b\,d\,(a\,d+2\,b\,c)}{3\,(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d)}-\frac{b\,d\,(a\,d+b\,c)}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)-\frac{a\,b\,c\,d}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)}{b\,d\,x^6+(a\,d+b\,c)\,x^3+a\,c} + \frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,(a\,d+2\,b\,c)\,1{}\mathrm{i}}{6\,\sqrt{b}\,(a\,d-b\,c)^{5/2}}$$

input
```
int(x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output
```
(log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x
^3)/(a + b*x^3))*(a*d + 2*b*c)*1i)/(6*b^(1/2)*(a*d - b*c)^(5/2)) - ((c + d
*x^3)^(1/2)*(x^3*((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2*b*d^3 +
b^3*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*c^2*d - 2
*a*b^2*c*d^2)) - (a*b*c*d)/(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)))/(a*c
+ x^3*(a*d + b*c) + b*d*x^6)
```

Reduce [F]

$$\int \frac{x^5}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^5}{b^2d^2x^{12}+2ab\,d^2x^9+2b^2cd\,x^9+a^2d^2x^6+4abcd\,x^6+b^2c^2x^6+2a^2cd\,x^3}$$

input
```
int(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**5)/(a**2*c**2 + 2*a**2*c*d*x**3 + a**2*d**2*x**6
+ 2*a*b*c**2*x**3 + 4*a*b*c*d*x**6 + 2*a*b*d**2*x**9 + b**2*c**2*x**6 + 2*
b**2*c*d*x**9 + b**2*d**2*x**12),x)
```

3.665 $\int \frac{x^2}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 108

$$\int \frac{x^2}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{d}{(bc-ad)^2\sqrt{c+dx^3}} - \frac{1}{3(bc-ad)(a+bx^3)\sqrt{c+dx^3}} + \frac{\sqrt{b}d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

output
```
-d/(-a*d+b*c)^2/(d*x^3+c)^(1/2)-1/3/(-a*d+b*c)/(b*x^3+a)/(d*x^3+c)^(1/2)+b
^(1/2)*d*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(5/2
)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{-2ad-b(c+3dx^3)}{3(bc-ad)^2(a+bx^3)\sqrt{c+dx^3}} - \frac{\sqrt{b}d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}$$

input
```
Integrate[x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-2*a*d - b*(c + 3*d*x^3))/(3*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3]) -
 (Sqrt[b]*d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c)
+ a*d)^(5/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {946, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{1}{\left(bx^3+a\right)^2 \left(dx^3+c\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(-\frac{3d\int \frac{1}{\left(bx^3+a\right)\left(dx^3+c\right)^{3/2}}dx^3}{2(bc-ad)}-\frac{1}{\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(-\frac{3d\left(\frac{b\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{bc-ad}+\frac{2}{\sqrt{c+dx^3}(bc-ad)}\right)}{2(bc-ad)}-\frac{1}{\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{3d\left(\frac{2b\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{d(bc-ad)}+\frac{2}{\sqrt{c+dx^3}(bc-ad)}\right)}{2(bc-ad)}-\frac{1}{\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{3d\left(\frac{2}{\sqrt{c+dx^3}(bc-ad)}-\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}\right)}{2(bc-ad)}-\frac{1}{(a+bx^3)\sqrt{c+dx^3}(bc-ad)}\right)$$

input
```
Int[x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-(1/((b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3])) - (3*d*(2/((b*c - a*d)*Sqr
t[c + d*x^3]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*
d]])/(b*c - a*d)^(3/2)))/(2*(b*c - a*d)))/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{-\frac{b\sqrt{d\,x^3+c}}{3\left(b\,x^3+a\right)}-\frac{db\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{2d}{3\sqrt{d\,x^3+c}}}{(ad-bc)^2}$
default	$\frac{d\left(-\frac{b\sqrt{d\,x^3+c}}{d\left(b\,x^3+a\right)}-\frac{3b\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{2}{\sqrt{d\,x^3+c}}\right)}{3(ad-bc)^2}$
elliptic	$-\frac{2d}{3(ad-bc)^2\sqrt{(x^3+\frac{c}{d})d}}-\frac{b\sqrt{d\,x^3+c}}{3(ad-bc)^2(b\,x^3+a)}+\frac{ib\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\cdots}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}\cdots}}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*d-b*c)^2*(-1/3*b*(d*x^3+c)^(1/2)/(b*x^3+a)-d*b/((a*d-b*c)*b)^(1/2)*ar
ctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))-2/3*d/(d*x^3+c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(92) = 184$.

Time = 0.17 (sec) , antiderivative size = 428, normalized size of antiderivative = 3.96

$$\int \frac{x^2}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \left[\frac{3\left(bd^2x^6+(bcd+ad^2)x^3+acd\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{\ldots}}}{bx^3+a}\right.}{6\left((b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^6+ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c\ldots\right.}\right.$$

$$\left. -\frac{3\left(bd^2x^6+(bcd+ad^2)x^3+acd\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\sqrt{dx^3+c}\sqrt{-\frac{b}{bc-ad}}\right)+(3\,bdx^3+bc+2\,ad)\sqrt{dx^3+c}}{3\left((b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^6+ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x^3\right)}\right]$$

input
```
integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(b*d^2*x^6 + (b*c*d + a*d^2)*x^3 + a*c*d)*sqrt(b/(b*c - a*d))*log(
(b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d))
)/(b*x^3 + a)) - 2*(3*b*d*x^3 + b*c + 2*a*d)*sqrt(d*x^3 + c))/((b^3*c^2*d
- 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 +
 (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^3), -1/3*(3*(b*d^2*x^6
+ (b*c*d + a*d^2)*x^3 + a*c*d)*sqrt(-b/(b*c - a*d))*arctan(sqrt(d*x^3 + c)
*sqrt(-b/(b*c - a*d))) + (3*b*d*x^3 + b*c + 2*a*d)*sqrt(d*x^3 + c))/((b^3*
c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c
*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^3)]
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^2}{(a+bx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**2/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.42

$$\int \frac{x^2}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = -\frac{bd \arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}} - \frac{3\,(dx^3+c)bd-2\,bcd+2\,ad^2}{3\,(b^2c^2-2\,abcd+a^2d^2)\left((dx^3+c)^{\frac{3}{2}}b-\sqrt{dx^3+c}bc+\sqrt{dx^3+c}ad\right)}$$

input
```
integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-b*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(3*(d*x^3 + c)*b*d - 2*b*c*d + 2*a*
d^2)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 +
c)*b*c + sqrt(d*x^3 + c)*a*d))
```

Mupad [B] (verification not implemented)

Time = 6.35 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.84

$$\int \frac{x^2}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = -\frac{\left(\frac{3\,b\,d\,(a\,d+b\,c)-b\,d\,(a\,d+2\,b\,c)}{3\,(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d)}+\frac{b^2\,d^2\,x^3}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)\sqrt{d\,x^3+c}}{b\,d\,x^6+(a\,d+b\,c)\,x^3+a\,c} + \frac{\sqrt{b}\,d\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{2\,(a\,d-b\,c)^{5/2}}$$

input
```
int(x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output
```
(b^(1/2)*d*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*
2i - b*d*x^3)/(a + b*x^3))*1i)/(2*(a*d - b*c)^(5/2)) - (((3*b*d*(a*d + b*c
) - b*d*(a*d + 2*b*c))/(3*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (b^2*
d^2*x^3)/(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2))*(c + d*x^3)^(1/2))/(a*c
+ x^3*(a*d + b*c) + b*d*x^6)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \int \frac{\sqrt{d\,x^3+c}\,x^2}{b^2d^2x^{12}+2ab\,d^2x^9+2b^2cd\,x^9+a^2d^2x^6+4abcd\,x^6+b^2c^2x^6+2a^2cd\,x^3 \cdots}$$

input
```
int(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**2)/(a**2*c**2 + 2*a**2*c*d*x**3 + a**2*d**2*x**6
+ 2*a*b*c**2*x**3 + 4*a*b*c*d*x**6 + 2*a*b*d**2*x**9 + b**2*c**2*x**6 + 2*
b**2*c*d*x**9 + b**2*d**2*x**12),x)
```

3.666 $\int \frac{1}{x(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 172

$$\int \frac{1}{x(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} + \frac{b}{3a(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}} + \frac{b^{3/2}(2bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{5/2}}$$

output
```
1/3*d*(2*a*d+b*c)/a/c/(-a*d+b*c)^2/(d*x^3+c)^(1/2)+1/3*b/a/(-a*d+b*c)/(b*x
^3+a)/(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(3/2)+1/3
*b^(3/2)*(-5*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/
a^2/(-a*d+b*c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{\frac{a\left(2a^2d^2+2abd^2x^3+b^2c\left(c+dx^3\right)\right)}{c(bc-ad)^2\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{b^{3/2}(2bc-5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}} - \frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{c^{3/2}}}{3a^2}$$

input
```
Integrate[1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
((a*(2*a^2*d^2 + 2*a*b*d^2*x^3 + b^2*c*(c + d*x^3)))/(c*(b*c - a*d)^2*(a +
 b*x^3)*Sqrt[c + d*x^3]) - (b^(3/2)*(2*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2) - (2*ArcTanh[Sqrt[c +
 d*x^3]/Sqrt[c]])/c^(3/2))/(3*a^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 114, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^3\left(bx^3+a\right)^2\left(dx^3+c\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(\frac{\int \frac{3bdx^3+2bc-2ad}{2x^3\left(bx^3+a\right)\left(dx^3+c\right)^{3/2}}dx^3}{a(bc-ad)}+\frac{b}{a\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{3bdx^3+2(bc-ad)}{x^3(bx^3+a)(dx^3+c)^{3/2}}dx^3}{2a(bc-ad)}+\frac{b}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)}\right)$$

$\big\downarrow$ 169

$$\frac{1}{3}\left(\frac{\frac{2d(2ad+bc)}{c\sqrt{c+dx^3}(bc-ad)}-\frac{2\int -\frac{bd(bc+2ad)x^3+2(bc-ad)^2}{2x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{c(bc-ad)}}{2a(bc-ad)}+\frac{b}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)}\right)$$

$\big\downarrow$ 27

$$\frac{1}{3}\left(\frac{\frac{\int \frac{bd(bc+2ad)x^3+2(bc-ad)^2}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{c(bc-ad)}+\frac{2d(2ad+bc)}{c\sqrt{c+dx^3}(bc-ad)}}{2a(bc-ad)}+\frac{b}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)}\right)$$

$\big\downarrow$ 174

$$\frac{1}{3}\left(\frac{\frac{\frac{2(bc-ad)^2\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{b^2c(2bc-5ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{c(bc-ad)}+\frac{2d(2ad+bc)}{c\sqrt{c+dx^3}(bc-ad)}}{2a(bc-ad)}+\frac{b}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)}\right)$$

$\big\downarrow$ 73

$$\frac{1}{3}\left(\frac{\frac{\frac{4(bc-ad)^2\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2b^2c(2bc-5ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{c(bc-ad)}+\frac{2d(2ad+bc)}{c\sqrt{c+dx^3}(bc-ad)}}{2a(bc-ad)}+\frac{b}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)}\right)$$

$\big\downarrow$ 221

$$\frac{1}{3}\left(\frac{\frac{\frac{2b^{3/2}c(2bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{4(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}}{c(bc-ad)}+\frac{2d(2ad+bc)}{c\sqrt{c+dx^3}(bc-ad)}}{2a(bc-ad)}+\frac{b}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)}\right.$$

input
```
Int[1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(b/(a*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]) + ((2*d*(b*c + 2*a*d))/(c*(
b*c - a*d)*Sqrt[c + d*x^3]) + ((-4*(b*c - a*d)^2*ArcTanh[Sqrt[c + d*x^3]/S
qrt[c]])/(a*Sqrt[c]) + (2*b^(3/2)*c*(2*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(c*(b*c - a*d)))/(2*a*(
b*c - a*d)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{-2\sqrt{d\,x^{3}+c}\,c^{\frac{5}{2}}\left(b\,x^{3}+a\right)\left(-\frac{5ad}{2}+bc\right)b^{2}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\,\left(-2c\sqrt{d\,x^{3}+c}\,\left(b\,x^{3}+a\right)(ad-bc)^{2}\operatorname{arctanh}\left(\frac{\sqrt{d\cdots}}{\sqrt{\cdots}}\right)\cdots\right)}{3\sqrt{(ad-bc)b}\,\sqrt{d\,x^{3}+c}\,c^{\frac{5}{2}}a^{2}\left(b\,x^{3}+a\right)(ad-bc)^{2}}$
default	$\frac{\frac{2}{3c\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{a^{2}}-\frac{2b\left(-\frac{b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{1}{\sqrt{d\,x^{3}+c}}\right)}{3a^{2}(ad-bc)}-\frac{bd\left(-\frac{b\sqrt{d\,x^{3}+c}}{d\left(b\,x^{3}+a\right)}-\frac{3b\arctan\left(\frac{b\sqrt{d\,x^{3}\cdots}}{\sqrt{(ad-b\cdots}}\right)}{\sqrt{(ad-bc)b}}\cdots\right)}{3a(ad-bc)^{2}}$
elliptic	Expression too large to display

input
```
int(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)*b)^(1/2)*(-2*(d*x^3+c)^(1/2)*c^(5/2)*(b*x^3+a)*(-5/2*a*d+b*
c)*b^2*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(
-2*c*(d*x^3+c)^(1/2)*(b*x^3+a)*(a*d-b*c)^2*arctanh((d*x^3+c)^(1/2)/c^(1/2)
)+a*c^(3/2)*(c*(d*x^3+c)*b^2+2*x^3*a*b*d^2+2*a^2*d^2)))/(d*x^3+c)^(1/2)/c^
(5/2)/a^2/(b*x^3+a)/(a*d-b*c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(144) = 288$.

Time = 0.39 (sec) , antiderivative size = 1774, normalized size of antiderivative = 10.31

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/6*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^6
+ (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(b/(b*c - a*d))*l
og((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*
d)))/(b*x^3 + a)) - 2*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2
*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 +
a^3*d^3)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) -
 2*(a*b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^
3 + c))/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^
3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*
c^3*d^2 + a^5*c^2*d^3)*x^3), -1/3*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c
^3*d - 5*a*b^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2
)*x^3)*sqrt(-b/(b*c - a*d))*arctan(sqrt(d*x^3 + c)*sqrt(-b/(b*c - a*d))) -
 ((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 - 2*a^2*b*c^2*d
+ a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^3)*sqrt(c)
*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - (a*b^2*c^3 + 2*a^3*c
*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))/(a^3*b^2*c^5 -
2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c
^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*
x^3), 1/6*(4*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 - 2*
a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^...
```

Sympy [F]

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.31

$$\begin{aligned}\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=&-\frac{\left(2\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\sqrt{-b^2c+abd}}\\&+\frac{(dx^3+c)b^2cd+2\,(dx^3+c)abd^2-2\,abcd^2+2\,a^2d^3}{3\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2\right)\left((dx^3+c)^{\frac{3}{2}}b-\sqrt{dx^3+c}bc+\sqrt{dx^3+c}ad\right)}\\&+\frac{2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}c}\end{aligned}$$

input
```
integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-1/3*(2*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/
((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b^2*c + a*b*d)) + 1/3*((d*x^3
 + c)*b^2*c*d + 2*(d*x^3 + c)*a*b*d^2 - 2*a*b*c*d^2 + 2*a^2*d^3)/((a*b^2*c
^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c
 + sqrt(d*x^3 + c)*a*d)) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(
-c)*c)
```

Mupad [B] (verification not implemented)

Time = 11.21 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2\,c^{3/2}} + \frac{\left(\frac{(2\,a\,d+b\,c)^4+(2\,a\,d+b\,c)^2\,((a\,d+2\,b\,c)\,(2\,a\,d+b\,c)-9\,a\,b\,c\,d)}{9\,a\,c\,(2\,a\,d+b\,c)^2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{b\,d\,x^3\,(2\,a\,d+b\,c)}{3\,a\,c\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)\sqrt{d\,x^3+c}}{b\,d\,x^6+(a\,d+b\,c)\,x^3+a\,c} + \frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,(5\,a\,d-2\,b\,c)\,1\mathrm{i}}{6\,a^2\,(a\,d-b\,c)^{5/2}}$$

input
```
int(1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output
```
log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(
3*a^2*c^(3/2)) + ((((2*a*d + b*c)^4 + (2*a*d + b*c)^2*((a*d + 2*b*c)*(2*a*
d + b*c) - 9*a*b*c*d))/(9*a*c*(2*a*d + b*c)^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c
*d)) + (b*d*x^3*(2*a*d + b*c))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(c
 + d*x^3)^(1/2))/(a*c + x^3*(a*d + b*c) + b*d*x^6) + (b^(3/2)*log((2*b*c -
 a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^
3))*(5*a*d - 2*b*c)*1i)/(6*a^2*(a*d - b*c)^(5/2))
```

Reduce [F]

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{\sqrt{d\,x^3+c}}{b^2d^2x^{13}+2ab\,d^2x^{10}+2b^2cd\,x^{10}+a^2d^2x^7+4abcd\,x^7+b^2c^2x^7+2a^2cd}$$

input
```
int(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

output
```
int(sqrt(c + d*x**3)/(a**2*c**2*x + 2*a**2*c*d*x**4 + a**2*d**2*x**7 + 2*a
*b*c**2*x**4 + 4*a*b*c*d*x**7 + 2*a*b*d**2*x**10 + b**2*c**2*x**7 + 2*b**2
*c*d*x**10 + b**2*d**2*x**13),x)
```

3.667 $\int \frac{1}{x^4(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 241

$$\int \frac{1}{x^4(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{d(2b^2c^2-2abcd+3a^2d^2)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{1}{3acx^3(a+bx^3)\sqrt{c+dx^3}} + \frac{(4bc+3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{5/2}} - \frac{b^{5/2}(4bc-7ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{5/2}}$$

output
```
-1/3*d*(3*a^2*d^2-2*a*b*c*d+2*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(d*x^3+c)^(1/2
)-1/3*b*(-a*d+2*b*c)/a^2/c/(-a*d+b*c)/(b*x^3+a)/(d*x^3+c)^(1/2)-1/3/a/c/x^
3/(b*x^3+a)/(d*x^3+c)^(1/2)+1/3*(3*a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1
/2))/a^3/c^(5/2)-1/3*b^(5/2)*(-7*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2
)/(-a*d+b*c)^(1/2))/a^3/(-a*d+b*c)^(5/2)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{-\frac{a\left(2b^3c^2x^3(c+dx^3)+a^3d^2(c+3dx^3)+ab^2c(c^2-cdx^3-2d^2x^6)+a^2bd(-2c^2-cdx^3+3d^2x^6)\right)}{c^2(bc-ad)^2x^3(a+bx^3)\sqrt{c+dx^3}}+ \cdots}{3a^3}$$

input
```
Integrate[1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-((a*(2*b^3*c^2*x^3*(c + d*x^3) + a^3*d^2*(c + 3*d*x^3) + a*b^2*c*(c^2 -
c*d*x^3 - 2*d^2*x^6) + a^2*b*d*(-2*c^2 - c*d*x^3 + 3*d^2*x^6)))/(c^2*(b*c
- a*d)^2*x^3*(a + b*x^3)*Sqrt[c + d*x^3])) + (b^(5/2)*(4*b*c - 7*a*d)*ArcT
an[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2) + (
(4*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/c^(5/2))/(3*a^3)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {948, 114, 27, 168, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^3)^2 (c+dx^3)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^6 (bx^3+a)^2 (dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int \frac{5bdx^3+4bc+3ad}{2x^3(bx^3+a)^2(dx^3+c)^{3/2}}dx^3}{ac}-\frac{1}{acx^3(a+bx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{5bdx^3+4bc+3ad}{x^3(bx^3+a)^2(dx^3+c)^{3/2}}dx^3}{2ac}-\frac{1}{acx^3\left(a+bx^3\right)\sqrt{c+dx^3}}\right)$$

$\downarrow$ 168

$$\frac{1}{3}\left(-\frac{\frac{\int \frac{3bd(2bc-ad)x^3+(bc-ad)(4bc+3ad)}{x^3\left(bx^3+a\right)\left(dx^3+c\right)^{3/2}}dx^3}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)}}{2ac}-\frac{1}{acx^3\left(a+bx^3\right)\sqrt{c+dx^3}}\right)$$

$\downarrow$ 169

$$\frac{1}{3}\left(-\frac{\frac{\frac{2d\left(3a^2d^2-2abcd+2b^2c^2\right)}{c\sqrt{c+dx^3}(bc-ad)}-\frac{2\int -\frac{bd\left(2b^2c^2-2abdc+3a^2d^2\right)x^3+(bc-ad)^2(4bc+3ad)}{2x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{c(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)}}{2ac}-\frac{1}{acx^3\left(a+bx^3\right)\sqrt{}}\right.$$

$\downarrow$ 27

$$\frac{1}{3}\left(-\frac{\frac{\frac{\int \frac{bd\left(2b^2c^2-2abdc+3a^2d^2\right)x^3+(bc-ad)^2(4bc+3ad)}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{c(bc-ad)}+\frac{2d\left(3a^2d^2-2abcd+2b^2c^2\right)}{c\sqrt{c+dx^3}(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)}}{2ac}-\frac{1}{acx^3\left(a+bx^3\right)\sqrt{c-}}\right.$$

$\downarrow$ 174

$$\frac{1}{3}\left(-\frac{\frac{\frac{\frac{(bc-ad)^2(3ad+4bc)\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{b^3c^2(4bc-7ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{c(bc-ad)}+\frac{2d\left(3a^2d^2-2abcd+2b^2c^2\right)}{c\sqrt{c+dx^3}(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)}}{2ac}-\frac{}{ac}\right.$$

$\downarrow$ 73

$$\frac{1}{3}\left(-\frac{\frac{\frac{\frac{2(bc-ad)^2(3ad+4bc)\int\frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2b^3c^2(4bc-7ad)\int\frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{c(bc-ad)}+\frac{2d\left(3a^2d^2-2abcd+2b^2c^2\right)}{c\sqrt{c+dx^3}(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)}}{2ac}-\right.$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{2d\left(3a^2d^2-2abcd+2b^2c^2\right)}{c\sqrt{c+dx^3}(bc-ad)}+\frac{\frac{2b^{5/2}c^2(4bc-7ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)^2(3ad+4bc)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}}{c(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^3)\sqrt{c+dx^3}(bc-\right.$$

input
```
Int[1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-(1/(a*c*x^3*(a + b*x^3)*Sqrt[c + d*x^3])) - ((2*b*(2*b*c - a*d))/(a*(b*c
 - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]) + ((2*d*(2*b^2*c^2 - 2*a*b*c*d + 3*a^
2*d^2))/(c*(b*c - a*d)*Sqrt[c + d*x^3]) + ((-2*(b*c - a*d)^2*(4*b*c + 3*a*
d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(a*Sqrt[c]) + (2*b^(5/2)*c^2*(4*b*c -
 7*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c -
a*d]))/(c*(b*c - a*d)))/(a*(b*c - a*d)))/(2*a*c))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{\sqrt{d x^3+c}}{3c^2a^2x^3}-\frac{-\frac{2(3ad+4bc)\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{4a^2d^3}{3(ad-bc)^2\sqrt{d x^3+c}}+\frac{2b^3c^2\left(\frac{\sqrt{d x^3+c}}{b x^3+a}+\frac{d\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3(ad-bc)^2}+\frac{4b^3c^2(3a\cdots}{3a\cdots}}{2a^2c^2}$
pseudoelliptic	$\frac{d^3\left(\frac{-\frac{a\sqrt{d x^3+c}}{x^3}+\frac{(3ad+4bc)\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{\sqrt{c}}}{a^3c^2d^3}-\frac{\sqrt{d x^3+c}\,b^3}{a^2d^3\left(b x^3+a\right)(ad-bc)^2}-\frac{7\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)b^3}{\sqrt{(ad-bc)b}\,a^2d^2(ad-bc)^2}+\frac{4\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)b^4}{\sqrt{(ad-bc)b}\,a^3d^3(ad-bc)^2}\cdots\right)}{3}$
default	$\frac{-\frac{\sqrt{d x^3+c}}{3c^2x^3}-\frac{2d}{3c^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}}{a^2}+\frac{b^2d\left(-\frac{b\sqrt{d x^3+c}}{d\left(b x^3+a\right)}-\frac{3b\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{2}{\sqrt{d x^3+c}}\right)}{3a^2(ad-bc)^2}-\frac{2b\left(\frac{\cdots}{3c\sqrt{\cdots}}\cdots\right)}{\cdots}$
elliptic	Expression too large to display

input
```
int(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/c^2/a^2*(d*x^3+c)^(1/2)/x^3-1/2/a^2/c^2*(-2/3/a*(3*a*d+4*b*c)*arctanh
((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+4/3*a^2*d^3/(a*d-b*c)^2/(d*x^3+c)^(1/2)+
2/3*b^3*c^2/(a*d-b*c)^2*((d*x^3+c)^(1/2)/(b*x^3+a)+d/((a*d-b*c)*b)^(1/2)*a
rctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2)))+4/3*b^3*c^2*(3*a*d-2*b*c)/a/
(a*d-b*c)^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/
2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(209) = 418$.

Time = 0.67 (sec) , antiderivative size = 2337, normalized size of antiderivative = 9.70

$$\int \frac{1}{x^4 (a + bx^3)^2 (c + dx^3)^{3/2}} dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/6*(((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3*c^4*d -
 7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt(b/(b*c
 - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b
/(b*c - a*d)))/(b*x^3 + a)) - ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*
c*d^3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 +
a^3*b*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^
2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) +
 2*c)/x^3) + 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d
 - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*c^4 - a^2*b^2*c^3*d -
 a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(d*x^3 + c))/((a^3*b^3*c^5*d - 2*a^
4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*
c^4*d^2 + a^6*c^3*d^3)*x^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x
^3), 1/6*(2*((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3*c^
4*d - 7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt(-
b/(b*c - a*d))*arctan(sqrt(d*x^3 + c)*sqrt(-b/(b*c - a*d))) + ((4*b^4*c^3*
d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*
b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^
4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(c)*log((d*x
^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*
d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c*d^3)*x...
```

Sympy [F]

$$\int \frac{1}{x^4 (a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \int \frac{1}{x^4 (a+bx^3)^2 (c+dx^3)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**4/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \int \frac{1}{(bx^3+a)^2 (dx^3+c)^{\frac{3}{2}} x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^4 (a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \frac{(4\,b^4c - 7\,ab^3d)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,(a^3b^2c^2 - 2\,a^4bcd + a^5d^2)\sqrt{-b^2c+abd}} - \frac{2\,(dx^3+c)^2b^3c^2d - 2\,(dx^3+c)b^3c^3d - 2\,(dx^3+c)^2ab^2cd^2 + 3\,(dx^3+c)ab^2c^2d^2 + 3\,(dx^3+c)^2a^2bd^3 - 7\,(}{3\,(a^2b^2c^4 - 2\,a^3bc^3d + a^4c^2d^2)\left((dx^3+c)^{\frac{5}{2}}b - 2\,(dx^3+c)^{\frac{3}{2}}bc + \sqrt{dx^3+c}bc^2 + (\right.} - \frac{(4\,bc+3\,ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}c^2}$$

input
```
integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
1/3*(4*b^4*c - 7*a*b^3*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^
3 + c)^2*b^3*c^2*d - 2*(d*x^3 + c)*b^3*c^3*d - 2*(d*x^3 + c)^2*a*b^2*c*d^2
 + 3*(d*x^3 + c)*a*b^2*c^2*d^2 + 3*(d*x^3 + c)^2*a^2*b*d^3 - 7*(d*x^3 + c)
*a^2*b*c*d^3 + 2*a^2*b*c^2*d^3 + 3*(d*x^3 + c)*a^3*d^4 - 2*a^3*c*d^4)/((a^
2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*((d*x^3 + c)^(5/2)*b - 2*(d*x^3 +
 c)^(3/2)*b*c + sqrt(d*x^3 + c)*b*c^2 + (d*x^3 + c)^(3/2)*a*d - sqrt(d*x^3
 + c)*a*c*d)) - 1/3*(4*b*c + 3*a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^3*
sqrt(-c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 18.79 (sec) , antiderivative size = 18847, normalized size of antiderivative = 78.20

$$\int \frac{1}{x^4 (a + bx^3)^2 (c + dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
int(1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output
```
(2*b*log(1/x^6))/(3*a^3*c^(3/2)) - (c + d*x^3)^(1/2)/(3*a^2*c^2*x^3) + (d*
log(1/x^6))/(2*a^2*c^(5/2)) + (2*b*log(c^(3/2)*(c + d*x^3)^(1/2) - c^(1/2)
*(c + d*x^3)^(3/2) + d^2*x^6 + 2*c*d*x^3 + 3*c^(1/2)*d*x^3*(c + d*x^3)^(1/
2)))/(3*a^3*c^(3/2)) + (d*log(c^(3/2)*(c + d*x^3)^(1/2) - c^(1/2)*(c + d*x
^3)^(3/2) + d^2*x^6 + 2*c*d*x^3 + 3*c^(1/2)*d*x^3*(c + d*x^3)^(1/2)))/(2*a
^2*c^(5/2)) - (b^7*c^9*x^4*(c + d*x^3)^(1/2))/(2*(2*a^9*c^6*d^5*x + 2*a^9*
c^5*d^6*x^4 + a^5*b^4*c^9*d^2*x^4 + a^6*b^3*c^8*d^3*x^4 - 3*a^7*b^2*c^7*d^
4*x^4 + a^5*b^4*c^8*d^3*x^7 - 3*a^7*b^2*c^6*d^5*x^7 - 3*a^8*b*c^7*d^4*x +
a^6*b^3*c^9*d^2*x - a^8*b*c^6*d^5*x^4 + 2*a^8*b*c^5*d^6*x^7)) - (5*a^9*d^7
*x^4*(c + d*x^3)^(1/2))/(4*(a^6*b^5*c^9*x + a^5*b^6*c^9*x^4 - 3*a^7*b^4*c^
7*d^2*x^4 - a^8*b^3*c^6*d^3*x^4 + 2*a^9*b^2*c^5*d^4*x^4 - 3*a^7*b^4*c^6*d^
3*x^7 + 2*a^8*b^3*c^5*d^4*x^7 - 3*a^8*b^3*c^7*d^2*x + 2*a^9*b^2*c^6*d^3*x
+ a^6*b^5*c^8*d*x^4 + a^5*b^6*c^8*d*x^7)) + (3*a^2*d^2*x*(c + d*x^3)^(1/2)
)/(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3*d*x + 3*a
^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7) + (2*b^2*c^2*x*(
c + d*x^3)^(1/2))/(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a
^4*c^3*d*x + 3*a^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7)
- (b^(7/2)*c*log((a^6*b^(15/2)*c^10*36i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d
 - b*c)^(1/2)) - (a^7*b^(13/2)*c^9*d*198i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a
*d - b*c)^(1/2)) + (a^12*b^(3/2)*c^4*d^6*18i)/(a*(a*d - b*c)^(1/2) + b*...
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
int(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

output

```
( - 2*sqrt(c + d*x**3) - 9*int(sqrt(c + d*x**3)/(a**2*c**2*x + 2*a**2*c*d*
x**4 + a**2*d**2*x**7 + 2*a*b*c**2*x**4 + 4*a*b*c*d*x**7 + 2*a*b*d**2*x**1
0 + b**2*c**2*x**7 + 2*b**2*c*d*x**10 + b**2*d**2*x**13),x)*a**2*c*d*x**3
- 9*int(sqrt(c + d*x**3)/(a**2*c**2*x + 2*a**2*c*d*x**4 + a**2*d**2*x**7 +
 2*a*b*c**2*x**4 + 4*a*b*c*d*x**7 + 2*a*b*d**2*x**10 + b**2*c**2*x**7 + 2*
b**2*c*d*x**10 + b**2*d**2*x**13),x)*a**2*d**2*x**6 - 12*int(sqrt(c + d*x*
*3)/(a**2*c**2*x + 2*a**2*c*d*x**4 + a**2*d**2*x**7 + 2*a*b*c**2*x**4 + 4*
a*b*c*d*x**7 + 2*a*b*d**2*x**10 + b**2*c**2*x**7 + 2*b**2*c*d*x**10 + b**2
*d**2*x**13),x)*a*b*c**2*x**3 - 21*int(sqrt(c + d*x**3)/(a**2*c**2*x + 2*a
**2*c*d*x**4 + a**2*d**2*x**7 + 2*a*b*c**2*x**4 + 4*a*b*c*d*x**7 + 2*a*b*d
**2*x**10 + b**2*c**2*x**7 + 2*b**2*c*d*x**10 + b**2*d**2*x**13),x)*a*b*c*
d*x**6 - 9*int(sqrt(c + d*x**3)/(a**2*c**2*x + 2*a**2*c*d*x**4 + a**2*d**2
*x**7 + 2*a*b*c**2*x**4 + 4*a*b*c*d*x**7 + 2*a*b*d**2*x**10 + b**2*c**2*x*
*7 + 2*b**2*c*d*x**10 + b**2*d**2*x**13),x)*a*b*d**2*x**9 - 12*int(sqrt(c
+ d*x**3)/(a**2*c**2*x + 2*a**2*c*d*x**4 + a**2*d**2*x**7 + 2*a*b*c**2*x**
4 + 4*a*b*c*d*x**7 + 2*a*b*d**2*x**10 + b**2*c**2*x**7 + 2*b**2*c*d*x**10
+ b**2*d**2*x**13),x)*b**2*c**2*x**6 - 12*int(sqrt(c + d*x**3)/(a**2*c**2*
x + 2*a**2*c*d*x**4 + a**2*d**2*x**7 + 2*a*b*c**2*x**4 + 4*a*b*c*d*x**7 +
2*a*b*d**2*x**10 + b**2*c**2*x**7 + 2*b**2*c*d*x**10 + b**2*d**2*x**13),x)
*b**2*c*d*x**9 - 15*int((sqrt(c + d*x**3)*x**2)/(a**2*c**2 + 2*a**2*c*d...
```

3.668 $\int \frac{x^3}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{x^3}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},2,\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2c\sqrt{c+dx^3}}$$

output
```
1/4*x^4*(1+d*x^3/c)^(1/2)*AppellF1(4/3,2,3/2,7/3,-b*x^3/a,-d*x^3/c)/a^2/c/
(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 381 vs. 2(67) = 134.

Time = 10.24 (sec) , antiderivative size = 381, normalized size of antiderivative = 5.69

$$\int \frac{x^3}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx =$$

$$-\frac{x^4\left(-8abcd\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\left(8a+(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right.\right.\right.}{8a(bc-ad)^2(a+bx^3)\sqrt{c+dx^3}\left(-8ac\,\text{Ap}\right.}$$

input
```
Integrate[x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
-1/8*(x^4*(-8*a*b*c*d*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a
)]*(8*a + (a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*
x^3)/c), -((b*x^3)/a)]) + (8*a*(2*a*d + b*(c + 3*d*x^3)) + 3*b*d*x^3*(a +
b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x
^3)/a)])*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a
*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/(a*(b*c - a*d
)^2*(a + b*x^3)*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^
3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c
), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/
a)])))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{x^3}{(bx^3+a)^2 \left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4 \sqrt{\frac{dx^3}{c}+1} \operatorname{AppellF1}\left(\frac{4}{3},2,\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2c\sqrt{c+dx^3}}$$

input
```
Int[x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*a^2*c*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 2.13 (sec) , antiderivative size = 787, normalized size of antiderivative = 11.75

method	result	size
elliptic	Expression too large to display	787
default	Expression too large to display	1593

input
```
int(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*d*x/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)-1/3*b/(a*d-b*c)^2*x*(d*x^3+c)^(1/
2)/(b*x^3+a)+1/3*I/(a*d-b*c)^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x
-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2))+1/18*I/d^2*2^(1/2)*sum((-7*a*d-2*b*c)/(a*d-b*c)^3
/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^
(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^
2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)
^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/
2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/
2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c
*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))
,_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(a+bx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**3/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(bx^3+a)^2(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(bx^3+a)^2(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(b\,x^3+a)^2\,(d\,x^3+c)^{3/2}}\,dx$$

input
```
int(x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output
```
int(x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x^3}{b^2d^2x^{12}+2ab\,d^2x^9+2b^2cd\,x^9+a^2d^2x^6+4abcd\,x^6+b^2c^2x^6+2a^2cd\,x^3+}$$

input
```
int(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x**3)/(a**2*c**2 + 2*a**2*c*d*x**3 + a**2*d**2*x**6
+ 2*a*b*c**2*x**3 + 4*a*b*c*d*x**6 + 2*a*b*d**2*x**9 + b**2*c**2*x**6 + 2*
b**2*c*d*x**9 + b**2*d**2*x**12),x)
```

3.669 $\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \frac{x^2 \sqrt{1+\frac{dx^3}{c}} \operatorname{AppellF1}\left(\frac{2}{3},2,\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2 c\sqrt{c+dx^3}}$$

output
```
1/2*x^2*(1+d*x^3/c)^(1/2)*AppellF1(2/3,2,3/2,5/3,-b*x^3/a,-d*x^3/c)/a^2/c/
(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 216 vs. 2(67) = 134.

Time = 10.20 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.22

$$\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx =$$

$$-\frac{x^2 \left(-10a(2a^2d^2+2abd^2x^3+b^2c(c+dx^3))+5(-b^2c^2+6abcd+a^2d^2)\,(a+bx^3)\,\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\right.\right.}{30a^2c(bc-ad)^2\,(a+bx}$$

input
```
Integrate[x/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
-1/30*(x^2*(-10*a*(2*a^2*d^2 + 2*a*b*d^2*x^3 + b^2*c*(c + d*x^3)) + 5*(-(b
^2*c^2) + 6*a*b*c*d + a^2*d^2)*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + b*d*(b*c + 2*a*d)*x^3*(a + b
*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^
3)/a)]))/(a^2*c*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx$$
$$\Big\downarrow \text{ 1013}$$
$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x}{(bx^3+a)^2\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$
$$\Big\downarrow \text{ 1012}$$
$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{2}{3},2,\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2c\sqrt{c+dx^3}}$$

input
```
Int[x/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 2, 3/2, 5/3, -((b*x^3)/a), -((d*x^3
)/c)])/(2*a^2*c*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.24 (sec) , antiderivative size = 986, normalized size of antiderivative = 14.72

method	result	size
default	Expression too large to display	986
elliptic	Expression too large to display	986

input
```
int(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*d^2*x^2/c/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)+1/3*b^2/(a*d-b*c)^2/a*x^2*(d
*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(-1/3*d^2/(a*d-b*c)^2/c-1/6*d*b/(a*d-b*c)^2/
a)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/a/d^2*b*2^
(1/2)*sum((11*a*d-2*b*c)/(a*d-b*c)^3/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1
/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x
-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(
-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*
d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x}{(a+bx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+a)^2(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+a)^2(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{x}{(b\,x^3+a)^2\,(d\,x^3+c)^{3/2}}\,dx$$

input
```
int(x/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output
```
int(x/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,x}{b^2d^2x^{12}+2ab\,d^2x^9+2b^2cd\,x^9+a^2d^2x^6+4abcd\,x^6+b^2c^2x^6+2a^2cd\,x^3+}$$

input
```
int(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*x)/(a**2*c**2 + 2*a**2*c*d*x**3 + a**2*d**2*x**6 + 2
*a*b*c**2*x**3 + 4*a*b*c*d*x**6 + 2*a*b*d**2*x**9 + b**2*c**2*x**6 + 2*b**
2*c*d*x**9 + b**2*d**2*x**12),x)
```

3.670 $\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \frac{x\sqrt{1+\frac{dx^3}{c}} \operatorname{AppellF1}\left(\frac{1}{3},2,\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2 c\sqrt{c+dx^3}}$$

output
```
x*(1+d*x^3/c)^(1/2)*AppellF1(1/3,2,3/2,4/3,-b*x^3/a,-d*x^3/c)/a^2/c/(d*x^3
+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 381 vs. 2(62) = 124.

Time = 10.41 (sec) , antiderivative size = 381, normalized size of antiderivative = 6.15

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \frac{x\left(bd(bc+2ad)x^3\sqrt{1+\frac{dx^3}{c}} \operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a\left(64ac(3a^2d\right.}{}\right.}{}$$

input
```
Integrate[1/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(x*(b*d*(b*c + 2*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -
((d*x^3)/c), -((b*x^3)/a)] + (a*(64*a*c*(3*a^2*d^2 + 2*a*b*d*(-3*c + d*x^3
) + b^2*c*(3*c + d*x^3))*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3
)/a)] - 24*x^3*(2*a^2*d^2 + 2*a*b*d^2*x^3 + b^2*c*(c + d*x^3))*(2*b*c*Appe
llF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2
, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(8*a*c*AppellF1[1/3,
 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2
, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)])))))/(24*a^2*c*(b*c - a*d)^2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{1}{(bx^3+a)^2 \left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{dx^3}{c}+1} \operatorname{AppellF1}\left(\frac{1}{3},2,\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2 c\sqrt{c+dx^3}}$$

input
```
Int[1/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 3/2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a^2*c*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.11 (sec) , antiderivative size = 830, normalized size of antiderivative = 13.39

method	result	size
default	Expression too large to display	830
elliptic	Expression too large to display	830

input
```
int(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*d^2*x/c/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)+1/3*b^2/(a*d-b*c)^2/a*x*(d*x^3
+c)^(1/2)/(b*x^3+a)-2/3*I*(1/3*d^2/(a*d-b*c)^2/c+1/6*d*b/(a*d-b*c)^2/a)*3^
(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(
1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I
/a/d^2*b*2^(1/2)*sum((13*a*d-4*b*c)/(a*d-b*c)^3/_alpha^2*(-c*d^2)^(1/3)*(1
/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/
(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I
*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2
/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_
alpha^2*3^(1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)
^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(a+bx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^2 (dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^2 (dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(b\,x^3+a)^2\,(d\,x^3+c)^{3/2}}\,dx$$

input
```
int(1/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output
```
int(1/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}}{b^2d^2x^{12}+2ab\,d^2x^9+2b^2cd\,x^9+a^2d^2x^6+4abcd\,x^6+b^2c^2x^6+2a^2cd\,x^3 \cdots}$$

input
```
int(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

output
```
int(sqrt(c + d*x**3)/(a**2*c**2 + 2*a**2*c*d*x**3 + a**2*d**2*x**6 + 2*a*b
*c**2*x**3 + 4*a*b*c*d*x**6 + 2*a*b*d**2*x**9 + b**2*c**2*x**6 + 2*b**2*c*
d*x**9 + b**2*d**2*x**12),x)
```

3.671 $\int \frac{1}{x^2(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{1}{x^2 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{1}{3},2,\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2cx\sqrt{c+dx^3}}$$

output
```
-(1+d*x^3/c)^(1/2)*AppellF1(-1/3,2,3/2,2/3,-b*x^3/a,-d*x^3/c)/a^2/c/x/(d*x
^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 308 vs. 2(65) = 130.

Time = 10.31 (sec) , antiderivative size = 308, normalized size of antiderivative = 4.74

$$\int \frac{1}{x^2 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{-20a(4b^3c^2x^3(c+dx^3)+a^3d^2(3c+5dx^3)+3ab^2c(c^2-cdx^3-2d^2x^6)-}{}$$

input
```
Integrate[1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-20*a*(4*b^3*c^2*x^3*(c + d*x^3) + a^3*d^2*(3*c + 5*d*x^3) + 3*a*b^2*c*(c
^2 - c*d*x^3 - 2*d^2*x^6) + a^2*b*d*(-6*c^2 - 3*c*d*x^3 + 5*d^2*x^6)) + 5*
(-8*b^3*c^3 + 21*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 5*a^3*d^3)*x^3*(a + b*x^3)*
Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)]
 + 2*b*d*(4*b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*x^6*(a + b*x^3)*Sqrt[1 + (d*x
^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(60*a^3*c^2
*(b*c - a*d)^2*x*(a + b*x^3)*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{1}{x^2(bx^3+a)^2 \left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\, \text{AppellF1}\left(-\frac{1}{3},2,\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2 c x \sqrt{c+dx^3}}$$

input
```
Int[1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
-((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 2, 3/2, 2/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*c*x*Sqrt[c + d*x^3]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.02 (sec) , antiderivative size = 1019, normalized size of antiderivative = 15.68

method	result	size
elliptic	Expression too large to display	1019
risch	Expression too large to display	2334
default	Expression too large to display	2383

input
```
int(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*d^3*x^2/c^2/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)-1/3/(a*d-b*c)^2/a^2*b^3*x
^2*(d*x^3+c)^(1/2)/(b*x^3+a)-1/c^2/a^2*(d*x^3+c)^(1/2)/x-2/3*I*(1/3/c^2*d^
3/(a*d-b*c)^2+1/6*b^2*d/a^2/(a*d-b*c)^2+1/2*d/c^2/a^2)*3^(1/2)/d*(-c*d^2)^
(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/18*I*b^2/a^2/d^2*2^(1/2)*sum((17*a*d-
8*b*c)/(a*d-b*c)^3/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3
))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*
(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)
^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alph
a^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 (a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \int \frac{1}{x^2 (a+bx^3)^2 (c+dx^3)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**2/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \int \frac{1}{(bx^3+a)^2 (dx^3+c)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \int \frac{1}{(bx^3+a)^2 (dx^3+c)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \int \frac{1}{x^2 \left(b\,x^3+a\right)^2 \left(d\,x^3+c\right)^{3/2}} \,dx$$

input
```
int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
int(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

output

```
( - 2*sqrt(c + d*x**3) - 11*int((sqrt(c + d*x**3)*x**4)/(a**2*c**2 + 2*a**
2*c*d*x**3 + a**2*d**2*x**6 + 2*a*b*c**2*x**3 + 4*a*b*c*d*x**6 + 2*a*b*d**
2*x**9 + b**2*c**2*x**6 + 2*b**2*c*d*x**9 + b**2*d**2*x**12),x)*a*b*c*d*x
 - 11*int((sqrt(c + d*x**3)*x**4)/(a**2*c**2 + 2*a**2*c*d*x**3 + a**2*d**2*
x**6 + 2*a*b*c**2*x**3 + 4*a*b*c*d*x**6 + 2*a*b*d**2*x**9 + b**2*c**2*x**6
 + 2*b**2*c*d*x**9 + b**2*d**2*x**12),x)*a*b*d**2*x**4 - 11*int((sqrt(c +
d*x**3)*x**4)/(a**2*c**2 + 2*a**2*c*d*x**3 + a**2*d**2*x**6 + 2*a*b*c**2*x
**3 + 4*a*b*c*d*x**6 + 2*a*b*d**2*x**9 + b**2*c**2*x**6 + 2*b**2*c*d*x**9
 + b**2*d**2*x**12),x)*b**2*c*d*x**4 - 11*int((sqrt(c + d*x**3)*x**4)/(a**2
*c**2 + 2*a**2*c*d*x**3 + a**2*d**2*x**6 + 2*a*b*c**2*x**3 + 4*a*b*c*d*x**
6 + 2*a*b*d**2*x**9 + b**2*c**2*x**6 + 2*b**2*c*d*x**9 + b**2*d**2*x**12),
x)*b**2*d**2*x**7 - 5*int((sqrt(c + d*x**3)*x)/(a**2*c**2 + 2*a**2*c*d*x**
3 + a**2*d**2*x**6 + 2*a*b*c**2*x**3 + 4*a*b*c*d*x**6 + 2*a*b*d**2*x**9 +
b**2*c**2*x**6 + 2*b**2*c*d*x**9 + b**2*d**2*x**12),x)*a**2*c*d*x - 5*int(
(sqrt(c + d*x**3)*x)/(a**2*c**2 + 2*a**2*c*d*x**3 + a**2*d**2*x**6 + 2*a*b
*c**2*x**3 + 4*a*b*c*d*x**6 + 2*a*b*d**2*x**9 + b**2*c**2*x**6 + 2*b**2*c*
d*x**9 + b**2*d**2*x**12),x)*a**2*d**2*x**4 - 8*int((sqrt(c + d*x**3)*x)/(
a**2*c**2 + 2*a**2*c*d*x**3 + a**2*d**2*x**6 + 2*a*b*c**2*x**3 + 4*a*b*c*d
*x**6 + 2*a*b*d**2*x**9 + b**2*c**2*x**6 + 2*b**2*c*d*x**9 + b**2*d**2*x**
12),x)*a*b*c**2*x - 13*int((sqrt(c + d*x**3)*x)/(a**2*c**2 + 2*a**2*c*d...
```

3.672 $\int \frac{1}{x^3(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{1}{x^3\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},2,\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2cx^2\sqrt{c+dx^3}}$$

output
```
-1/2*(1+d*x^3/c)^(1/2)*AppellF1(-2/3,2,3/2,1/3,-b*x^3/a,-d*x^3/c)/a^2/c/x^
2/(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 515 vs. 2(67) = 134.

Time = 10.72 (sec) , antiderivative size = 515, normalized size of antiderivative = 7.69

$$\int \frac{1}{x^3\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{-bd\left(5b^2c^2-6abcd+7a^2d^2\right)x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx}{a}\right.}{}$$

input
```
Integrate[1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-(b*d*(5*b^2*c^2 - 6*a*b*c*d + 7*a^2*d^2)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF
1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]) + (a*(32*a*c*(10*b^3*c^2*
x^3*(3*c + d*x^3) + 3*a^3*d^2*(2*c + 7*d*x^3) + 3*a*b^2*c*(2*c^2 - 13*c*d*
x^3 - 4*d^2*x^6) + 2*a^2*b*d*(-6*c^2 - 6*c*d*x^3 + 7*d^2*x^6))*AppellF1[1/
3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*x^3*(5*b^3*c^2*x^3*(c + d
*x^3) + a^3*d^2*(3*c + 7*d*x^3) + 3*a*b^2*c*(c^2 - c*d*x^3 - 2*d^2*x^6) +
a^2*b*d*(-6*c^2 - 3*c*d*x^3 + 7*d^2*x^6))*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3
, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c
), -((b*x^3)/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x
^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/
c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)
/a)]))))/(48*a^3*c^2*(b*c - a*d)^2*x^2*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^3)^2 (c+dx^3)^{3/2}} dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^3(bx^3+a)^2\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},2,\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2cx^2\sqrt{c+dx^3}}$$

input
```
Int[1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
-1/2*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 3/2, 1/3, -((b*x^3)/a), -((d*x
^3)/c)])/(a^2*c*x^2*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.11 (sec) , antiderivative size = 863, normalized size of antiderivative = 12.88

method	result	size
elliptic	Expression too large to display	863
risch	Expression too large to display	1874
default	Expression too large to display	1919

input
```
int(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^2/a^2*(d*x^3+c)^(1/2)/x^2-2/3*d^3*x/c^2/(a*d-b*c)^2/((x^3+c/d)*d)^(
1/2)-1/3/(a*d-b*c)^2/a^2*b^3*x*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(-1/4*d/c^2
/a^2-1/3/c^2*d^3/(a*d-b*c)^2-1/6*b^2*d/a^2/(a*d-b*c)^2)*3^(1/2)/d*(-c*d^2)
^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/18*I*b^2/a^2/d^2*2^(
1/2)*sum((19*a*d-10*b*c)/(a*d-b*c)^3/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*
(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)
*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-
c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*_alpha^2*3^(
1/2)*d-I*(-c*d^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alp
ha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^3 (a+bx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^2 (dx^3+c)^{\frac{3}{2}} x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^2 (dx^3+c)^{\frac{3}{2}} x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^3 (b\,x^3+a)^2 (d\,x^3+c)^{3/2}}\,dx$$

input
```
int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output
```
int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

output
```
( - 2*sqrt(c + d*x**3) - 7*int(sqrt(c + d*x**3)/(a**2*c**2 + 2*a**2*c*d*x*
*3 + a**2*d**2*x**6 + 2*a*b*c**2*x**3 + 4*a*b*c*d*x**6 + 2*a*b*d**2*x**9 +
 b**2*c**2*x**6 + 2*b**2*c*d*x**9 + b**2*d**2*x**12),x)*a**2*c*d*x**2 - 7*
int(sqrt(c + d*x**3)/(a**2*c**2 + 2*a**2*c*d*x**3 + a**2*d**2*x**6 + 2*a*b
*c**2*x**3 + 4*a*b*c*d*x**6 + 2*a*b*d**2*x**9 + b**2*c**2*x**6 + 2*b**2*c*
d*x**9 + b**2*d**2*x**12),x)*a**2*d**2*x**5 - 10*int(sqrt(c + d*x**3)/(a**
2*c**2 + 2*a**2*c*d*x**3 + a**2*d**2*x**6 + 2*a*b*c**2*x**3 + 4*a*b*c*d*x*
*6 + 2*a*b*d**2*x**9 + b**2*c**2*x**6 + 2*b**2*c*d*x**9 + b**2*d**2*x**12)
,x)*a*b*c**2*x**2 - 17*int(sqrt(c + d*x**3)/(a**2*c**2 + 2*a**2*c*d*x**3 +
 a**2*d**2*x**6 + 2*a*b*c**2*x**3 + 4*a*b*c*d*x**6 + 2*a*b*d**2*x**9 + b**
2*c**2*x**6 + 2*b**2*c*d*x**9 + b**2*d**2*x**12),x)*a*b*c*d*x**5 - 7*int(s
qrt(c + d*x**3)/(a**2*c**2 + 2*a**2*c*d*x**3 + a**2*d**2*x**6 + 2*a*b*c**2
*x**3 + 4*a*b*c*d*x**6 + 2*a*b*d**2*x**9 + b**2*c**2*x**6 + 2*b**2*c*d*x**
9 + b**2*d**2*x**12),x)*a*b*d**2*x**8 - 10*int(sqrt(c + d*x**3)/(a**2*c**2
 + 2*a**2*c*d*x**3 + a**2*d**2*x**6 + 2*a*b*c**2*x**3 + 4*a*b*c*d*x**6 + 2
*a*b*d**2*x**9 + b**2*c**2*x**6 + 2*b**2*c*d*x**9 + b**2*d**2*x**12),x)*b*
*2*c**2*x**5 - 10*int(sqrt(c + d*x**3)/(a**2*c**2 + 2*a**2*c*d*x**3 + a**2
*d**2*x**6 + 2*a*b*c**2*x**3 + 4*a*b*c*d*x**6 + 2*a*b*d**2*x**9 + b**2*c**
2*x**6 + 2*b**2*c*d*x**9 + b**2*d**2*x**12),x)*b**2*c*d*x**8 - 13*int((sqr
t(c + d*x**3)*x**3)/(a**2*c**2 + 2*a**2*c*d*x**3 + a**2*d**2*x**6 + 2*a...
```

3.673 $\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 264

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = -\frac{c^3\sqrt[3]{a+bx^3}}{d^4}+\frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{4/3}}{4b^3d^3} - \frac{(bc+2ad)\,(a+bx^3)^{7/3}}{7b^3d^2}+\frac{(a+bx^3)^{10/3}}{10b^3d} - \frac{c^3\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{13/3}} - \frac{c^3\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^{13/3}} + \frac{c^3\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{13/3}}$$

output

```
-c^3*(b*x^3+a)^(1/3)/d^4+1/4*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(4/3)/b^3
/d^3-1/7*(2*a*d+b*c)*(b*x^3+a)^(7/3)/b^3/d^2+1/10*(b*x^3+a)^(10/3)/b^3/d-1
/3*c^3*(-a*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)
^(1/3))*3^(1/2))*3^(1/2)/d^(13/3)-1/6*c^3*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^(
13/3)+1/2*c^3*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3)
)/d^(13/3)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$= \frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}(9a^3d^3-3a^2bd^2(-5c+dx^3)+ab^2d(35c^2-5cdx^3+2d^2x^6)+b^3(-140c^3+35c^2dx^3-20cd^2x^6+14d^3x^9))}{b^3}-140\sqrt{3}c^3\sqrt[3]{bc}}{}$$

input
```
Integrate[(x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
((3*d^(1/3)*(a + b*x^3)^(1/3)*(9*a^3*d^3 - 3*a^2*b*d^2*(-5*c + d*x^3) + a*
b^2*d*(35*c^2 - 5*c*d*x^3 + 2*d^2*x^6) + b^3*(-140*c^3 + 35*c^2*d*x^3 - 20
*c*d^2*x^6 + 14*d^3*x^9)))/b^3 - 140*Sqrt[3]*c^3*(b*c - a*d)^(1/3)*ArcTan[
(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 140*c^3*(
b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - 70*c
^3*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a
+ b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(420*d^(13/3))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^9\sqrt[3]{bx^3+a}}{dx^3+c}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(-\frac{\sqrt[3]{bx^3+a}c^3}{d^3(dx^3+c)}+\frac{(bx^3+a)^{7/3}}{b^2d}+\frac{(-bc-2ad)(bx^3+a)^{4/3}}{b^2d^2}+\frac{(b^2c^2+abdc+a^2d^2)\sqrt[3]{bx^3+a}}{b^2d^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3(a+bx^3)^{4/3}(a^2d^2+abcd+b^2c^2)}{4b^3d^3}-\frac{\sqrt{3}c^3\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{13/3}}-\frac{3(a+bx^3)^{7/3}(2ad+b}{7b^3d^2}\right.$$

input
```
Int[(x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
((-3*c^3*(a + b*x^3)^(1/3))/d^4 + (3*(b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*
x^3)^(4/3))/(4*b^3*d^3) - (3*(b*c + 2*a*d)*(a + b*x^3)^(7/3))/(7*b^3*d^2)
+ (3*(a + b*x^3)^(10/3))/(10*b^3*d) - (Sqrt[3]*c^3*(b*c - a*d)^(1/3)*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/d^(13/3)
 - (c^3*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(2*d^(13/3)) + (3*c^3*(b*c - a*d
)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(13/3)))/
3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.96 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{\frac{27\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}} d\left(\frac{\left(14d^3x^9-20c\,d^2x^6+35c^2d\,x^3-140c^3\right)b^3}{9}+\frac{35ad\left(\frac{2}{35}d^2x^6-\frac{1}{7}cd\,x^3+c^2\right)b^2}{9}+\frac{5\left(-\frac{d\,x^3}{5}+c\right)a^2d^2b}{3}+a^3d^3\right)\left(b\,x^3+a\right)^{\frac{1}{3}}}{70}}{}$

input
```
int(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/d)^(2/3)*(27/70*((a*d-b*c)/d)^(2/3)*d*(1/9*(14*d^3*x^9-20*c
*d^2*x^6+35*c^2*d*x^3-140*c^3)*b^3+35/9*a*d*(2/35*d^2*x^6-1/7*c*d*x^3+c^2)
*b^2+5/3*(-1/5*d*x^3+c)*a^2*d^2*b+a^3*d^3)*(b*x^3+a)^(1/3)+b^3*c^3*(a*d-b*
c)*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*c
)/d)^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)
+((a*d-b*c)/d)^(2/3))-2*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))))/d^5/b^3
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.23

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx =$$

$$-\frac{140\sqrt{3}b^3c^3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+70\,b^3c^3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}-(b\cdots\right.}{}$$

input
```
integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/420*(140*sqrt(3)*b^3*c^3*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(
b*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d
)) + 70*b^3*c^3*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^
(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 140*b^3*c^3*((b*c -
 a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)) - 3*(14*b^3*
d^3*x^9 - 2*(10*b^3*c*d^2 - a*b^2*d^3)*x^6 - 140*b^3*c^3 + 35*a*b^2*c^2*d
+ 15*a^2*b*c*d^2 + 9*a^3*d^3 + (35*b^3*c^2*d - 5*a*b^2*c*d^2 - 3*a^2*b*d^3
)*x^3)*(b*x^3 + a)^(1/3))/(b^3*d^4)
```

Sympy [F]

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate(x**11*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**11*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.44

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = -\frac{(b^{34}c^4d^6 - ab^{33}c^3d^7)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{34}cd^{10} - ab^{33}d^{11})} + \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^5} + \frac{(-bcd^2+ad^3)^{\frac{1}{3}}c^3\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^5} - \frac{140\,(bx^3+a)^{\frac{1}{3}}b^{30}c^3d^6 - 35\,(bx^3+a)^{\frac{4}{3}}b^{29}c^2d^7 + 20\,(bx^3+a)^{\frac{7}{3}}b^{28}cd^8 - 35\,(bx^3+a)^{\frac{4}{3}}ab^{28}cd^8 - 14\,(bx^3}{140\,b^{30}d^{10}}$$

input
```
integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*(b^34*c^4*d^6 - a*b^33*c^3*d^7)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3
 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^34*c*d^10 - a*b^33*d^11) + 1/3*s
qrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3
) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^5 + 1/6*(-b*c*d^2 +
a*d^3)^(1/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d
)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^5 - 1/140*(140*(b*x^3 + a)^(1/3)*b^30*
c^3*d^6 - 35*(b*x^3 + a)^(4/3)*b^29*c^2*d^7 + 20*(b*x^3 + a)^(7/3)*b^28*c*
d^8 - 35*(b*x^3 + a)^(4/3)*a*b^28*c*d^8 - 14*(b*x^3 + a)^(10/3)*b^27*d^9 +
 40*(b*x^3 + a)^(7/3)*a*b^27*d^9 - 35*(b*x^3 + a)^(4/3)*a^2*b^27*d^9)/(b^3
0*d^10)
```

Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.67

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \left(\frac{3\,a^2}{4\,b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{4\,b^3\,d}\right)\,\left(b\,x^3+a\right)^{4/3} - \left(\frac{3\,a}{7\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{7\,b^6\,d^2}\right)\,\left(b\,x^3+a\right)^{7/3} - \left(b\,x^3+a\right)^{1/3}\left(\frac{a^3}{b^3\,d}+\frac{\left(\frac{3\,a^2}{b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{b^3\,d}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{b^3\,d}\right)+\frac{\left(b\,x^3+a\right)^{10/3}}{10\,b^3\,d}-\frac{c^3\,\ln\left((a\,d\right.}{}$$

input
```
int((x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

output
```
((3*a^2)/(4*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
 - a*b^3*d))/(4*b^3*d))*(a + b*x^3)^(4/3) - ((3*a)/(7*b^3*d) + (b^4*c - a*
b^3*d)/(7*b^6*d^2))*(a + b*x^3)^(7/3) - (a + b*x^3)^(1/3)*(a^3/(b^3*d) + (
((3*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c -
 a*b^3*d))/(b^3*d))*(b^4*c - a*b^3*d))/(b^3*d)) + (a + b*x^3)^(10/3)/(10*b
^3*d) - (c^3*log((a*d - b*c)^(1/3) - d^(1/3)*(a + b*x^3)^(1/3))*(a*d - b*c
)^(1/3))/(3*d^(13/3)) - (c^3*log((3*(a + b*x^3)^(1/3)*(b*c^4 - a*c^3*d))/d
^2 + (3*c^3*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(4/3))/d^(7/3))*((3^(1/2)*1
i)/2 - 1/2)*(a*d - b*c)^(1/3))/(3*d^(13/3)) + (c^3*log((3*(a + b*x^3)^(1/3
)*(b*c^4 - a*c^3*d))/d^2 - (9*c^3*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(4/3)
)/d^(7/3))*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(1/3))/d^(13/3)
```

Reduce [F]

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{9(b\,x^3+a)^{\frac{1}{3}}\,a^3d^2+15(b\,x^3+a)^{\frac{1}{3}}\,a^2bcd-3(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,d^2x^3-105(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2c^2-5(b\,x^3+a)^{\frac{1}{3}}\,a}{}$$

input
```
int(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
(9*(a + b*x**3)**(1/3)*a**3*d**2 + 15*(a + b*x**3)**(1/3)*a**2*b*c*d - 3*(
a + b*x**3)**(1/3)*a**2*b*d**2*x**3 - 105*(a + b*x**3)**(1/3)*a*b**2*c**2
- 5*(a + b*x**3)**(1/3)*a*b**2*c*d*x**3 + 2*(a + b*x**3)**(1/3)*a*b**2*d**
2*x**6 + 35*(a + b*x**3)**(1/3)*b**3*c**2*x**3 - 20*(a + b*x**3)**(1/3)*b*
*3*c*d*x**6 + 14*(a + b*x**3)**(1/3)*b**3*d**2*x**9 + 140*int(((a + b*x**3
)**(1/3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b**3*c**2*d - 1
40*int(((a + b*x**3)**(1/3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x
)*b**4*c**3)/(140*b**3*d**3)
```

3.674 $\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 220

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \frac{c^2 \sqrt[3]{a+bx^3}}{d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^2d} + \frac{c^2 \sqrt[3]{bc-ad} \arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}} + \frac{c^2 \sqrt[3]{bc-ad} \log\left(c+dx^3\right)}{6d^{10/3}} - \frac{c^2 \sqrt[3]{bc-ad} \log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}$$

output
```
c^2*(b*x^3+a)^(1/3)/d^3-1/4*(a*d+b*c)*(b*x^3+a)^(4/3)/b^2/d^2+1/7*(b*x^3+a
)^(7/3)/b^2/d+1/3*c^2*(-a*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(
1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/d^(10/3)+1/6*c^2*(-a*d+b*c)^(1/3)*
ln(d*x^3+c)/d^(10/3)-1/2*c^2*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*
(b*x^3+a)^(1/3))/d^(10/3)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.20

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$
$$= \frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}(-3a^2d^2+abd(-7c+dx^3)+b^2(28c^2-7cdx^3+4d^2x^6))}{b^2}+28\sqrt{3}c^2\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-}{}$$

input

```
Integrate[(x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output

```
((3*d^(1/3)*(a + b*x^3)^(1/3)*(-3*a^2*d^2 + a*b*d*(-7*c + d*x^3) + b^2*(28
*c^2 - 7*c*d*x^3 + 4*d^2*x^6)))/b^2 + 28*Sqrt[3]*c^2*(b*c - a*d)^(1/3)*Arc
Tan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 28*c^
2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + 1
4*c^2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*
(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(84*d^(10/3))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$
$$\Big\downarrow \text{ 948}$$
$$\frac{1}{3}\int \frac{x^6\sqrt[3]{bx^3+a}}{dx^3+c}dx^3$$
$$\Big\downarrow \text{ 99}$$

$$\frac{1}{3}\int \left(\frac{\sqrt[3]{bx^3+a}c^2}{d^2\left(dx^3+c\right)}+\frac{\left(bx^3+a\right)^{4/3}}{bd}+\frac{(-bc-ad)\sqrt[3]{bx^3+a}}{bd^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}c^2\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{10/3}}-\frac{3\left(a+bx^3\right)^{4/3}(ad+bc)}{4b^2d^2}+\frac{3\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{bc-ad}\log\left(c\right.}{2d^{10/3}}\right.$$

input
```
Int[(x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
((3*c^2*(a + b*x^3)^(1/3))/d^3 - (3*(b*c + a*d)*(a + b*x^3)^(4/3))/(4*b^2*
d^2) + (3*(a + b*x^3)^(7/3))/(7*b^2*d) + (Sqrt[3]*c^2*(b*c - a*d)^(1/3)*Ar
cTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/d^(10
/3) + (c^2*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(2*d^(10/3)) - (3*c^2*(b*c -
a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/3)
))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$-\frac{\frac{9\left(\left(b\,x^{3}+a\right)\left(-\frac{4b\,x^{3}}{3}+a\right)d^{2}+\frac{7\left(b\,x^{3}+a\right)bcd}{3}-\frac{28b^{2}c^{2}}{3}\right)d\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{14}+b^{2}c^{2}(ad-bc)\left(2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\right.\right.}{3\left(\frac{ad-bc}{d}\right)}\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}b^{2}}$

input
```
int(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/((a*d-b*c)/d)^(2/3)*(9/14*((b*x^3+a)*(-4/3*b*x^3+a)*d^2+7/3*(b*x^3+a)
*b*c*d-28/3*b^2*c^2)*d*((a*d-b*c)/d)^(2/3)*(b*x^3+a)^(1/3)+b^2*c^2*(a*d-b*
c)*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*c
)/d)^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)
+((a*d-b*c)/d)^(2/3))-2*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))))/b^2/d^4
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.28

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{c+dx^3}\,dx =$$

$$-\frac{28\sqrt{3}b^{2}c^{2}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^{3}+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+14\,b^{2}c^{2}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^{3}+a)^{\frac{2}{3}}\right.}{}$$

input
```
integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/84*(28*sqrt(3)*b^2*c^2*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b
*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d
)) + 14*b^2*c^2*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)
^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 28*b^2*c^2*(-(b*
c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) - 3*(4*b
^2*d^2*x^6 + 28*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2 - (7*b^2*c*d - a*b*d^2)*x^
3)*(b*x^3 + a)^(1/3))/(b^2*d^3)
```

Sympy [F]

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate(x**8*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**8*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.45

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$
$$= \frac{(b^{17}c^3d^4 - ab^{16}c^2d^5)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{17}cd^7 - ab^{16}d^8)}$$
$$- \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^4}$$
$$- \frac{(-bcd^2+ad^3)^{\frac{1}{3}}c^2\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^4}$$
$$+ \frac{28\,(bx^3+a)^{\frac{1}{3}}b^{14}c^2d^4 - 7\,(bx^3+a)^{\frac{4}{3}}b^{13}cd^5 + 4\,(bx^3+a)^{\frac{7}{3}}b^{12}d^6 - 7\,(bx^3+a)^{\frac{4}{3}}ab^{12}d^6}{28\,b^{14}d^7}$$

input

```
integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output

```
1/3*(b^17*c^3*d^4 - a*b^16*c^2*d^5)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3
+ a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^17*c*d^7 - a*b^16*d^8) - 1/3*sqrt
(3)*(-b*c*d^2 + a*d^3)^(1/3)*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) +
 (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^4 - 1/6*(-b*c*d^2 + a*d
^3)^(1/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(
1/3) + (-(b*c - a*d)/d)^(2/3))/d^4 + 1/28*(28*(b*x^3 + a)^(1/3)*b^14*c^2*d
^4 - 7*(b*x^3 + a)^(4/3)*b^13*c*d^5 + 4*(b*x^3 + a)^(7/3)*b^12*d^6 - 7*(b*
x^3 + a)^(4/3)*a*b^12*d^6)/(b^14*d^7)
```

Mupad [B] (verification not implemented)

Time = 3.65 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.53

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$$
$$= \left(\frac{a^2}{b^2 d} + \frac{\left(\frac{2a}{b^2 d} + \frac{b^3 c - a b^2 d}{b^4 d^2}\right)(b^3 c - a b^2 d)}{b^2 d}\right)(b x^3 + a)^{1/3}$$
$$- \left(\frac{a}{2 b^2 d} + \frac{b^3 c - a b^2 d}{4 b^4 d^2}\right)(b x^3 + a)^{4/3} + \frac{(b x^3 + a)^{7/3}}{7 b^2 d}$$
$$+ \frac{c^2 \ln\left((a d - b c)^{1/3} - d^{1/3} (b x^3 + a)^{1/3}\right)(a d - b c)^{1/3}}{3 d^{10/3}}$$
$$- \frac{c^2 \ln\left(\frac{3 (b x^3+a)^{1/3} (b c^3 - a c^2 d)}{d} - \frac{3 c^2 \left(\frac{1}{2} + \frac{\sqrt{3}\,1i}{2}\right)(a d - b c)^{4/3}}{d^{4/3}}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1i}{2}\right)(a d - b c)^{1/3}}{3 d^{10/3}}$$
$$+ \frac{c^2 \ln\left(\frac{3 (b x^3+a)^{1/3} (b c^3 - a c^2 d)}{d} + \frac{9 c^2 \left(-\frac{1}{6} + \frac{\sqrt{3}\,1i}{6}\right)(a d - b c)^{4/3}}{d^{4/3}}\right)\left(-\frac{1}{6} + \frac{\sqrt{3}\,1i}{6}\right)(a d - b c)^{1/3}}{d^{10/3}}$$

input
```
int((x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

output
```
(a^2/(b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a*b
^2*d))/(b^2*d))*(a + b*x^3)^(1/3) - (a/(2*b^2*d) + (b^3*c - a*b^2*d)/(4*b^
4*d^2))*(a + b*x^3)^(4/3) + (a + b*x^3)^(7/3)/(7*b^2*d) + (c^2*log((a*d -
b*c)^(1/3) - d^(1/3)*(a + b*x^3)^(1/3))*(a*d - b*c)^(1/3))/(3*d^(10/3)) -
(c^2*log((3*(a + b*x^3)^(1/3)*(b*c^3 - a*c^2*d))/d - (3*c^2*((3^(1/2)*1i)/
2 + 1/2)*(a*d - b*c)^(4/3))/d^(4/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1
/3))/(3*d^(10/3)) + (c^2*log((3*(a + b*x^3)^(1/3)*(b*c^3 - a*c^2*d))/d + (
9*c^2*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(4/3))/d^(4/3))*((3^(1/2)*1i)/6 -
 1/6)*(a*d - b*c)^(1/3))/d^(10/3)
```

Reduce [F]

$$\int \frac{x^8 \sqrt[3]{a + bx^3}}{c + dx^3} \, dx$$

$$= \frac{-3(b\,x^3+a)^{\frac{1}{3}}\,a^2d+21(b\,x^3+a)^{\frac{1}{3}}\,abc+(b\,x^3+a)^{\frac{1}{3}}\,abd\,x^3-7(b\,x^3+a)^{\frac{1}{3}}\,b^2c\,x^3+4(b\,x^3+a)^{\frac{1}{3}}\,b^2d\,x^6-}{28b^2d^2}$$

input
```
int(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
( - 3*(a + b*x**3)**(1/3)*a**2*d + 21*(a + b*x**3)**(1/3)*a*b*c + (a + b*x
**3)**(1/3)*a*b*d*x**3 - 7*(a + b*x**3)**(1/3)*b**2*c*x**3 + 4*(a + b*x**3
)**(1/3)*b**2*d*x**6 - 28*int(((a + b*x**3)**(1/3)*x**5)/(a*c + a*d*x**3 +
 b*c*x**3 + b*d*x**6),x)*a*b**2*c*d + 28*int(((a + b*x**3)**(1/3)*x**5)/(a
*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**3*c**2)/(28*b**2*d**2)
```

3.675 $\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 186

$$\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = -\frac{c\sqrt[3]{a+bx^3}}{d^2}+\frac{(a+bx^3)^{4/3}}{4bd} - \frac{c\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}} - \frac{c\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^{7/3}} + \frac{c\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}$$

output
```
-c*(b*x^3+a)^(1/3)/d^2+1/4*(b*x^3+a)^(4/3)/b/d-1/3*c*(-a*d+b*c)^(1/3)*arct
an(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/d^(
7/3)-1/6*c*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^(7/3)+1/2*c*(-a*d+b*c)^(1/3)*ln(
(-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(7/3)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.22

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$= \frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}(-4bc+ad+bdx^3)}{b} - 4\sqrt{3}c\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + 4c\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}\right.}{12d^{7/3}}$$

input
```
Integrate[(x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
((3*d^(1/3)*(a + b*x^3)^(1/3)*(-4*b*c + a*d + b*d*x^3))/b - 4*Sqrt[3]*c*(b
*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3
))/Sqrt[3]] + 4*c*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b
*x^3)^(1/3)] - 2*c*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c
- a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(12*d^(7/3))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {948, 90, 60, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{x^3\sqrt[3]{bx^3+a}}{dx^3+c}dx^3$$

$$\Big\downarrow \; 90$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4bd}-\frac{c\int \frac{\sqrt[3]{bx^3+a}}{dx^3+c}dx^3}{d}\right)$$

$\Big\downarrow$ 60

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4bd}-\frac{c\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{d}\right)}{d}\right)$$

$\Big\downarrow$ 70

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4bd}-\frac{c\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{\cdots}}\cdots}{2\sqrt[3]{d\cdots}}\cdots\right.}{d}\cdots\right.}{d}\cdots\right.$$

$\Big\downarrow$ 16

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4bd}-\frac{c\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log \cdots}{\cdots}\cdots\right)}{d}\cdots\right)}{d}\cdots\right)$$

$\downarrow$ 1082

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4bd}-\frac{c\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{\cdots}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}\cdots\right)}{d}\cdots\right)}{d}\cdots\right)$$

$\downarrow$ 217

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4bd}-\frac{c\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{\cdots}\right.}{2\sqrt[3]{d}(bc-ad)^{2/\cdots}}\right.}{d}\right.}{d}\right.$$

input
```
Int[(x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
((3*(a + b*x^3)^(4/3))/(4*b*d) - (c*((3*(a + b*x^3)^(1/3))/d - ((b*c - a*d
)*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))
/Sqrt[3]])/(d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c -
 a*d)^(2/3)) + (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d
^(1/3)*(b*c - a*d)^(2/3))))/d))/d)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{-\frac{3d\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\left(d\left(b\,x^{3}+a\right)-4bc\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{2}+bc(ad-bc)\left(-2\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}+1\right)}{3}\right)\sqrt{3}+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\frac{ad}{}\right.\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}b\,d^{3}}$

input
```
int(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-3/2*d*((a*d-b*c)/d)^(2/3)*(d*(b*x^3+a)-4*b*c)*(b*x^3+a)^(1/3)+b*c*(
a*d-b*c)*(-2*arctan(1/3*3^(1/2)*(2/((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+1))
*3^(1/2)+2*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))-ln((b*x^3+a)^(2/3)+((a*
d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))))/((a*d-b*c)/d)^(2/3)
/b/d^3
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.19

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx =$$

$$-\frac{4\sqrt{3}bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+2\,bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}-\left(bx^3+a\right.\right.}{12\,b}$$

input
```
integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/12*(4*sqrt(3)*b*c*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 +
 a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 2*
b*c*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*((b*c
- a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 4*b*c*((b*c - a*d)/d)^(1/3)*log
((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)) - 3*(b*d*x^3 - 4*b*c + a*d)*(b
*x^3 + a)^(1/3))/(b*d^2)
```

Sympy [F]

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate(x**5*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**5*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.48

$$\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = -\frac{(b^6c^2d^2 - ab^5cd^3)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^6cd^4 - ab^5d^5)} + \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}c\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^3} + \frac{(-bcd^2+ad^3)^{\frac{1}{3}}c\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^3} - \frac{4\,(bx^3+a)^{\frac{1}{3}}b^4cd^2 - (bx^3+a)^{\frac{4}{3}}b^3d^3}{4\,b^4d^4}$$

input
```
integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*(b^6*c^2*d^2 - a*b^5*c*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a
)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^6*c*d^4 - a*b^5*d^5) + 1/3*sqrt(3)*(
-b*c*d^2 + a*d^3)^(1/3)*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c
 - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^3 + 1/6*(-b*c*d^2 + a*d^3)^(1/
3)*c*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-
(b*c - a*d)/d)^(2/3))/d^3 - 1/4*(4*(b*x^3 + a)^(1/3)*b^4*c*d^2 - (b*x^3 +
a)^(4/3)*b^3*d^3)/(b^4*d^4)
```

Mupad [B] (verification not implemented)

Time = 3.33 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.60

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{\left(b\,x^3+a\right)^{4/3}}{4\,b\,d} - \left(b\,x^3+a\right)^{1/3}\left(\frac{a}{b\,d}+\frac{b^2\,c-a\,b\,d}{b^2\,d^2}\right) - \frac{c\,\ln\left(\left(b\,x^3+a\right)^{1/3}\left(3\,b\,c^2-3\,a\,c\,d\right)+\frac{c\,(a\,d-b\,c)^{1/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}}\right)\,(a\,d-b\,c)^{1/3}}{3\,d^{7/3}} - \frac{c\,\ln\left(\left(b\,x^3+a\right)^{1/3}\left(3\,b\,c^2-3\,a\,c\,d\right)+\frac{c\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^{1/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^{1/3}}{3\,d^{7/3}} + \frac{c\,\ln\left(\left(b\,x^3+a\right)^{1/3}\left(3\,b\,c^2-3\,a\,c\,d\right)-\frac{c\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^{1/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^{1/3}}{3\,d^{7/3}}$$

input
```
int((x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

output
```
(a + b*x^3)^(4/3)/(4*b*d) - (a + b*x^3)^(1/3)*(a/(b*d) + (b^2*c - a*b*d)/(
b^2*d^2)) - (c*log((a + b*x^3)^(1/3)*(3*b*c^2 - 3*a*c*d) + (c*(a*d - b*c)^
(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*(a*d - b*c)^(1/3))/(3*d^(7/3)) -
 (c*log((a + b*x^3)^(1/3)*(3*b*c^2 - 3*a*c*d) + (c*((3^(1/2)*1i)/2 - 1/2)*
(a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*((3^(1/2)*1i)/2 - 1/
2)*(a*d - b*c)^(1/3))/(3*d^(7/3)) + (c*log((a + b*x^3)^(1/3)*(3*b*c^2 - 3*
a*c*d) - (c*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2)
)/(3*d^(7/3)))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3))/(3*d^(7/3))
```

Reduce [F]

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{-3(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3+4\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}\,x^5}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)abd-4\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}\,x^5}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)b^2c}{4bd}$$

input
```
int(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
( - 3*(a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3 + 4*int(((a + b*x
**3)**(1/3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b*d - 4*int(
((a + b*x**3)**(1/3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**2*
c)/(4*b*d)
```

3.676 $\int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 159

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad} \arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{4/3}} + \frac{\sqrt[3]{bc-ad} \log\left(c+dx^3\right)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad} \log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}}$$

output
```
(b*x^3+a)^(1/3)/d+1/3*(-a*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(
1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/d^(4/3)+1/6*(-a*d+b*c)^(1/3)*ln(d*
x^3+c)/d^(4/3)-1/2*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^
(1/3))/d^(4/3)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.29

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$
$$= \frac{6\sqrt[3]{d}\sqrt[3]{a+bx^3}+2\sqrt{3}\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right.}{6d^{4/3}}$$

input
```
Integrate[(x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(6*d^(1/3)*(a + b*x^3)^(1/3) + 2*Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*
d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/
3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*
Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2
/3)*(a + b*x^3)^(2/3)])/(6*d^(4/3))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {946, 60, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$
$$\Big\downarrow \ 946$$
$$\frac{1}{3}\int \frac{\sqrt[3]{bx^3+a}}{dx^3+c}dx^3$$
$$\Big\downarrow \ 60$$

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\int \frac{1}{(bx^3+a)^{2/3}(dx^3+c)}dx^3}{d}\right)$$

$\downarrow$ 70

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{bx}}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right.}{d}\right.$$

$\downarrow$ 16

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log(c+dx^3)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ac}\right.}{2\sqrt[3]{d}(}\right.}{d}\right.$$

$\downarrow$ 1082

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log(c+dx^3)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right)}{d}\right.$$

$\downarrow$ 217

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log(c+dx^3)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right.}{d}\right.$$

input
```
Int[(x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
((3*(a + b*x^3)^(1/3))/d - ((b*c - a*d)*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)
*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(1/3)*(b*c - a*d)^(2/3
))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c - a*d)^(2/3)) + (3*Log[(b*c - a*d)^(1
/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1/3)*(b*c - a*d)^(2/3))))/d)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.32

method	result
pseudoelliptic	$\frac{(b\,x^3+a)^{\frac{1}{3}}}{d}+\frac{\ln\left((b\,x^3+a)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)(ad-bc)}{3d^2\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left((b\,x^3+a)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}(b\,x^3+a)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)(ad-bc)}{6d^2\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}}$ -

input
```
int(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
(b*x^3+a)^(1/3)/d+1/3/d^2/((a*d-b*c)/d)^(2/3)*ln((b*x^3+a)^(1/3)-((a*d-b*c
)/d)^(1/3))*(a*d-b*c)-1/6/d^2/((a*d-b*c)/d)^(2/3)*ln((b*x^3+a)^(2/3)+((a*d
-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))*(a*d-b*c)-1/3/d^2/((a*
d-b*c)/d)^(2/3)*3^(1/2)*arctan(2/3*3^(1/2)/((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(
1/3)+1/3*3^(1/2))*(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.30

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = -\frac{2\sqrt{3}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)\cdots\right.}{6\,d}$$

input
```
integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^
(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + (-(b*
c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/
d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 2*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 +
 a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) - 6*(b*x^3 + a)^(1/3))/d
```

Sympy [F]

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate(x**2*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**2*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.40

$$\begin{aligned}
&\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx \\
&= \frac{(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bcd-ad^2)} \\
&\quad - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^2} + \frac{(bx^3+a)^{\frac{1}{3}}}{d} \\
&\quad - \frac{(-bcd^2+ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^2}
\end{aligned}$$

input
```
integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
- a*d)/d)^(1/3)))/(b*c*d - a*d^2) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*a
rctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
a*d)/d)^(1/3))/d^2 + (b*x^3 + a)^(1/3)/d - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*lo
g((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c -
a*d)/d)^(2/3))/d^2
```

Mupad [B] (verification not implemented)

Time = 3.14 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.57

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{\left(b\,x^3+a\right)^{1/3}}{d} + \frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(3\,a\,d^2-3\,b\,c\,d\right)-\frac{\left(a\,d-b\,c\right)^{1/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{4/3}}\right)\,\left(a\,d-b\,c\right)^{1/3}}{3\,d^{4/3}} - \frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(3\,a\,d^2-3\,b\,c\,d\right)+\frac{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{1/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{1/3}}{3\,d^{4/3}} + \frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(3\,a\,d^2-3\,b\,c\,d\right)-\frac{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,\left(a\,d-b\,c\right)^{1/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{d^{4/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,\left(a\,d-b\,c\right)^{1/3}}{d^{4/3}}$$

input
```
int((x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

output
```
(a + b*x^3)^(1/3)/d + (log((a + b*x^3)^(1/3)*(3*a*d^2 - 3*b*c*d) - ((a*d -
 b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(4/3)))*(a*d - b*c)^(1/3))/(3*d^(4
/3)) - (log((a + b*x^3)^(1/3)*(3*a*d^2 - 3*b*c*d) + (((3^(1/2)*1i)/2 + 1/2
)*(a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(4/3)))*((3^(1/2)*1i)/2 +
1/2)*(a*d - b*c)^(1/3))/(3*d^(4/3)) + (log((a + b*x^3)^(1/3)*(3*a*d^2 - 3*
b*c*d) - (((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/
d^(4/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(1/3))/d^(4/3)
```

Reduce [F]

$$\int \frac{x^2 \sqrt[3]{a + bx^3}}{c + dx^3} \, dx$$
$$= \frac{(b\,x^3 + a)^{\frac{1}{3}}\,a - \left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}x^5}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)abd + \left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}x^5}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)b^2c}{bc}$$

input
```
int(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
((a + b*x**3)**(1/3)*a - int(((a + b*x**3)**(1/3)*x**5)/(a*c + a*d*x**3 +
b*c*x**3 + b*d*x**6),x)*a*b*d + int(((a + b*x**3)**(1/3)*x**5)/(a*c + a*d*
x**3 + b*c*x**3 + b*d*x**6),x)*b**2*c)/(b*c)
```

3.677 $\int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 246

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(c+dx^3\right)}\,dx = -\frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c} - \frac{\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{d}} - \frac{\sqrt[3]{a}\log(x)}{2c} - \frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c\sqrt[3]{d}} + \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} + \frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{d}}$$

output

```
-1/3*a^(1/3)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/
2)/c-1/3*(-a*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*
c)^(1/3))*3^(1/2))*3^(1/2)/c/d^(1/3)-1/2*a^(1/3)*ln(x)/c-1/6*(-a*d+b*c)^(1
/3)*ln(d*x^3+c)/c/d^(1/3)+1/2*a^(1/3)*ln(a^(1/3)-(b*x^3+a)^(1/3))/c+1/2*(-
a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c/d^(1/3)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(c+dx^3\right)}\,dx =$$

$$-\frac{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{d}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt{3}\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-2\sqrt[3]{a}\sqrt[3]{d}\log\left(-\sqrt[3]{\right.}}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x*(c + d*x^3)),x]
```

output
```
-1/6*(2*Sqrt[3]*a^(1/3)*d^(1/3)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))
/Sqrt[3]] + 2*Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)
^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*a^(1/3)*d^(1/3)*Log[-a^(1/3) + (a
+ b*x^3)^(1/3)] - 2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a +
 b*x^3)^(1/3)] + a^(1/3)*d^(1/3)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) +
 (a + b*x^3)^(2/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b
*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(c*d^(1/3)
)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {948, 94, 69, 16, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt[3]{bx^3+a}}{x^3\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \text{94}$$

$$\frac{1}{3}\left(\frac{(bc-ad)\int \frac{1}{(bx^3+a)^{2/3}(dx^3+c)}dx^3}{c}+\frac{a\int \frac{1}{x^3(bx^3+a)^{2/3}}dx^3}{c}\right)$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{3}\left(\frac{a\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2a^{2/3}}\right)}{c}+\frac{(bc-ad)\int \frac{}{(bx^3+a)^{2}}}{c}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{a\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)}{c}+\frac{(bc-ad)\int \frac{1}{(bx^3+a)^{2/3}(dx^3+c)}}{c}\right.$$

$$\Big\downarrow \text{70}$$

$$\frac{1}{3}\left(\frac{a\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)}{c}+\frac{(bc-ad)\left(\frac{3\int \frac{}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-}}{}\right.}{}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{a\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}+\frac{(bc-ad)\left(\frac{3\int \frac{\cdots}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\cdots}\cdots}{\cdots}\right.\cdots}{\cdots}\right.$$

$\big\downarrow$ 1082

$$\frac{1}{3}\left(\frac{a\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}+\frac{(bc-ad)\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+\cdots}}{\sqrt[3]{bc-ad}}\cdots\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}\cdots\right.}{\cdots}\right.$$

$\big\downarrow$ 217

$$\frac{1}{3}\left(\frac{a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}+\frac{(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+b\cdots}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\cdots\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}\cdots\right.}{\cdots}\right.$$

input
```
Int[(a + b*x^3)^(1/3)/(x*(c + d*x^3)),x]
```

output
```
((a*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/
3)) - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/
3))))/c + ((b*c - a*d)*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3
))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x
^3]/(2*d^(1/3)*(b*c - a*d)^(2/3)) + (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a
+ b*x^3)^(1/3)])/(2*d^(1/3)*(b*c - a*d)^(2/3))))/c)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$\frac{\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}} d \left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)\right) a^{\frac{1}{3}} \cdots}{}$

input
```
int((b*x^3+a)^(1/3)/x/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/d)^(2/3)*(((a*d-b*c)/d)^(2/3)*d*(-2*3^(1/2)*arctan(1/3*3^(1
/2)*(2/a^(1/3)*(b*x^3+a)^(1/3)+1))+2*ln((b*x^3+a)^(1/3)-a^(1/3))-ln((b*x^3
+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3)))*a^(1/3)+(a*d-b*c)*(2*arctan(1/
3*3^(1/2)*(2/((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+1))*3^(1/2)+ln((b*x^3+a)^
(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))-2*ln((b*x^3
+a)^(1/3)-((a*d-b*c)/d)^(1/3))))/d/c
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)}\,dx = -\frac{2\sqrt{3}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+2\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+a^{\frac{1}{3}}$$

input
```
integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(
1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 2*sqrt(
3)*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*a^(2/3) + sqrt(3)*a)/a)
 + a^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) +
((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*((b*c - a*
d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 2*a^(1/3)*log((b*x^3 + a)^(1/3) - a
^(1/3)) - 2*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^
(1/3)))/c
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/x/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(x*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)x}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.26

$$\begin{aligned}
&\int \frac{\sqrt[3]{a+bx^3}}{x\,(c+dx^3)}\,dx \\
&= -\frac{(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} \\
&\quad -\frac{\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,c} \\
&\quad -\frac{a^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,c}+\frac{a^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,c} \\
&\quad +\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd} \\
&\quad +\frac{(-bcd^2+ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,cd}
\end{aligned}$$

input
```
integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b*c^2 - a*c*d) - 1/3*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3
)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/c - 1/6*a^(1/3)*log((b*x^3 + a)
^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/c + 1/3*a^(1/3)*log(abs((b*x
^3 + a)^(1/3) - a^(1/3)))/c + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*arctan(
1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d
)^(1/3))/(c*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x
^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 3.65 (sec) , antiderivative size = 1607, normalized size of antiderivative = 6.53

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^3)^(1/3)/(x*(c + d*x^3)),x)
```

output
```
log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4
+ 9*a^2*b^6*c^2*d^3) - (a/(27*c^3))^(1/3)*(((243*a*b^6*c^6*d^3 - 729*a^2*b
^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a/(27*c^3))^(1/3) - (a + b*x^3)^(1/3)*(
81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4))*(a/(27*c^3))^(2/3) - 9*a*b^7*c^4*d
^2 + 27*a^2*b^6*c^3*d^3 - 18*a^3*b^5*c^2*d^4))*(a/(27*c^3))^(1/3) + log((a
 + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 + 9*a^
2*b^6*c^2*d^3) - (((243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*
c^4*d^5)*(-(a*d - b*c)/(27*c^3*d))^(1/3) - (a + b*x^3)^(1/3)*(81*a*b^6*c^5
*d^3 - 81*a^2*b^5*c^4*d^4))*(-(a*d - b*c)/(27*c^3*d))^(2/3) - 9*a*b^7*c^4*
d^2 + 27*a^2*b^6*c^3*d^3 - 18*a^3*b^5*c^2*d^4)*(-(a*d - b*c)/(27*c^3*d))^(
1/3))*(-(a*d - b*c)/(27*c^3*d))^(1/3) + log((a + b*x^3)^(1/3)*(6*a^4*b^4*d
^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 + 9*a^2*b^6*c^2*d^3) + ((3^(1/2)*1
i)/2 - 1/2)*(-(a*d - b*c)/(27*c^3*d))^(1/3)*(((3^(1/2)*1i)/2 - 1/2)^2*((a
+ b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4) - ((3^(1/2)*1i)/2 -
 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a
*d - b*c)/(27*c^3*d))^(1/3))*(-(a*d - b*c)/(27*c^3*d))^(2/3) + 9*a*b^7*c^4
*d^2 - 27*a^2*b^6*c^3*d^3 + 18*a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 - 1/2)*(-
(a*d - b*c)/(27*c^3*d))^(1/3) - log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a
*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 + 9*a^2*b^6*c^2*d^3) - ((3^(1/2)*1i)/2 + 1
/2)*(-(a*d - b*c)/(27*c^3*d))^(1/3)*(((3^(1/2)*1i)/2 + 1/2)^2*((a + b*x...
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{\frac{1}{3}}}{d\,x^4+cx}dx$$

input
```
int((b*x^3+a)^(1/3)/x/(d*x^3+c),x)
```

output
```
int((a + b*x**3)**(1/3)/(c*x + d*x**4),x)
```

3.678 $\int \frac{\sqrt[3]{a+bx^3}}{x^4(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 317

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^4(c+dx^3)}\,dx &= \frac{b\sqrt[3]{a+bx^3}}{3ac}-\frac{(a+bx^3)^{4/3}}{3acx^3}-\frac{(bc-3ad)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}c^2}\\
&\quad+\frac{d^{2/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2}-\frac{(bc-3ad)\log(x)}{6a^{2/3}c^2}\\
&\quad+\frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^2}+\frac{(bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}c^2}\\
&\quad-\frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2}\end{aligned}$$

output
```
1/3*b*(b*x^3+a)^(1/3)/a/c-1/3*(b*x^3+a)^(4/3)/a/c/x^3-1/9*(-3*a*d+b*c)*arc
tan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)/c^2+1
/3*d^(2/3)*(-a*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+
b*c)^(1/3))*3^(1/2))*3^(1/2)/c^2-1/6*(-3*a*d+b*c)*ln(x)/a^(2/3)/c^2+1/6*d^
(2/3)*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^2+1/6*(-3*a*d+b*c)*ln(a^(1/3)-(b*x^3+
a)^(1/3))/a^(2/3)/c^2-1/2*d^(2/3)*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(
1/3)*(b*x^3+a)^(1/3))/c^2
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4 (c+dx^3)}\,dx$$

$$= \frac{-\frac{6c\sqrt[3]{a+bx^3}}{x^3} + \frac{2\sqrt{3}(-bc+3ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + 6\sqrt{3}d^{2/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + \frac{2(}{}}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x]
```

output
```
((-6*c*(a + b*x^3)^(1/3))/x^3 + (2*Sqrt[3]*(-(b*c) + 3*a*d)*ArcTan[(1 + (2
*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3) + 6*Sqrt[3]*d^(2/3)*(b*c -
a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sq
rt[3]] + (2*(b*c - 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(2/3) - 6*d
^(2/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)
] + ((-(b*c) + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3
)^(2/3)])/a^(2/3) + 3*d^(2/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^
(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(1
8*c^2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {948, 114, 27, 174, 60, 69, 16, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4 (c+dx^3)}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt[3]{bx^3+a}}{x^6\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int -\frac{\sqrt[3]{bx^3+a}(bdx^3+bc-3ad)}{3x^3(dx^3+c)}dx^3}{ac}-\frac{\left(a+bx^3\right)^{4/3}}{acx^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{\sqrt[3]{bx^3+a}(bdx^3+bc-3ad)}{x^3(dx^3+c)}dx^3}{3ac}-\frac{\left(a+bx^3\right)^{4/3}}{acx^3}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{\frac{3ad^2\int \frac{\sqrt[3]{bx^3+a}}{dx^3+c}dx^3}{c}+\frac{(bc-3ad)\int \frac{\sqrt[3]{bx^3+a}}{x^3}dx^3}{c}}{3ac}-\frac{\left(a+bx^3\right)^{4/3}}{acx^3}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{\frac{3ad^2\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{d}\right)}{c}+\frac{(bc-3ad)\left(a\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3+3\sqrt[3]{a+bx^3}\right)}{c}}{3ac}-\frac{\left(a+bx^3\right)^{4/3}}{acx^3}\right)$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{3}\left(\frac{\frac{(bc-3ad)\left(a\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{3a\cdots}{\cdots}}{3ac}\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(\frac{\frac{(bc-3ad)\left(a\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{3ad^2\left(\frac{3\sqrt[3]{a}}{}\right.}{}}{3ac}\right.$$

$$\Big\downarrow \; 70$$

$$\frac{1}{3}\left(\frac{\frac{(bc-3ad)\left(a\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{3ad^2\left(\frac{3\sqrt[3]{a}}{}\right.}{}}{}\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(\frac{\frac{(bc-3ad)\left(a\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{3ad^2\left(\frac{3\sqrt[3]{a}\cdots}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{\frac{(bc-3ad)\left(a\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{3ad^2\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-\cdots}{\cdots}\right.}{3ac}}{\cdots}\right.$$

$\downarrow$ 217

$$\frac{1}{3}\left(\frac{(bc-3ad)\left(a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{3ad^2\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-}{}\right.}{3ac}\right.$$

input `Int[(a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x]`

output
```
(-((a + b*x^3)^(4/3)/(a*c*x^3)) + (((b*c - 3*a*d)*(3*(a + b*x^3)^(1/3) + a
*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3))
 - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3))
)))/c + (3*a*d^2*((3*(a + b*x^3)^(1/3))/d - ((b*c - a*d)*(-((Sqrt[3]*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(1/3)
*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c - a*d)^(2/3)) + (3*L
og[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1/3)*(b*c - a*d)^
(2/3))))/d))/c)/(3*a*c))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.94 (sec), antiderivative size = 357, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$-\frac{-\frac{x^3\left(a^{\frac{2}{3}}bc-a^{\frac{5}{3}}d\right)\ln\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)}{2}-x^3\sqrt{3}\left(a^{\frac{2}{3}}bc-a^{\frac{5}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(\right.\right.}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right.}{}$

input
```
int((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-1/2*x^3*(a^(2/3)*b*c-a^(5/3)*d)*ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1
/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))-x^3*3^(1/2)*(a^(2/3)*b*c-a^(5/3)*
d)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*c)/d
)^(1/3))-1/2*(a*d-1/3*b*c)*((a*d-b*c)/d)^(2/3)*x^3*ln((b*x^3+a)^(2/3)+a^(1
/3)*(b*x^3+a)^(1/3)+a^(2/3))+x^3*(a^(2/3)*b*c-a^(5/3)*d)*ln((b*x^3+a)^(1/3
)-((a*d-b*c)/d)^(1/3))+(-(a*d-1/3*b*c)*3^(1/2)*x^3*arctan(1/3*(a^(1/3)+2*(
b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))+x^3*(a*d-1/3*b*c)*ln((b*x^3+a)^(1/3)-a^(1
/3))+a^(2/3)*(b*x^3+a)^(1/3)*c)*((a*d-b*c)/d)^(2/3))/((a*d-b*c)/d)^(2/3)/a
^(2/3)/c^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec), antiderivative size = 426, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx =$$

$$-\frac{6\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}a^2x^3\arctan\left(-\frac{2\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}-\sqrt{3}(bcd-ad^2)}{3\,(bcd-ad^2)}\right)+3\left(-bcd^2+ad^3\right)^{\frac{1}{3}}a^2x^3\log\left(\right.}{}$$

input
```
integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/18*(6*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*arctan(-1/3*(2*sqrt(3)*(
-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d^2))/(b*c*
d - a*d^2)) + 3*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*log((b*x^3 + a)^(2/3)*d^2
 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)
) - 6*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2
 + a*d^3)^(1/3)) + 6*sqrt(1/3)*(a*b*c - 3*a^2*d)*x^3*sqrt(-(-a^2)^(1/3))*a
rctan(-sqrt(1/3)*((-a^2)^(1/3)*a - 2*(b*x^3 + a)^(1/3)*(-a^2)^(2/3))*sqrt(
-(-a^2)^(1/3))/a^2) + (-a^2)^(2/3)*(b*c - 3*a*d)*x^3*log((b*x^3 + a)^(2/3)
*a - (-a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(-a^2)^(2/3)) - 2*(-a^2)^(2/3)*(b*
c - 3*a*d)*x^3*log((b*x^3 + a)^(1/3)*a - (-a^2)^(2/3)) + 6*(b*x^3 + a)^(1/
3)*a^2*c)/(a^2*c^2*x^3)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/x**4/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(x**4*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx=\int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)x^4}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4(c+dx^3)}\,dx = \frac{(bcd-ad^2)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^3-ac^2d)} - \frac{\sqrt{3}(bc-3\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{2}{3}}c^2} - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^2} - \frac{(bc-3\,ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{2}{3}}c^2} - \frac{(-bcd^2+ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^2} + \frac{(bc-3\,ad)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{2}{3}}c^2} - \frac{(bx^3+a)^{\frac{1}{3}}}{3\,cx^3}$$

input
```
integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*(b*c*d - a*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(
b*c - a*d)/d)^(1/3)))/(b*c^3 - a*c^2*d) - 1/9*sqrt(3)*(b*c - 3*a*d)*arctan
(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(2/3)*c^2) - 1/3*
sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) +
 (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/c^2 - 1/18*(b*c - 3*a*d)*
log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(2/3)*c^2)
 - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*
(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/c^2 + 1/9*(b*c - 3*a*d)*l
og(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/3)*c^2) - 1/3*(b*x^3 + a)^(1/3)
/(c*x^3)
```

Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 1917, normalized size of antiderivative = 6.05

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x)
```

output
```
log(- ((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*
d) - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d - b*c)^3/
(a^2*c^6))^(1/3))*(-(3*a*d - b*c)^3/(a^2*c^6))^(2/3))/81 - (b^5*d^4*(27*a^
3*d^3 + b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*(-(3*a*d - b*c)
^3/(a^2*c^6))^(1/3))/9 - (2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*
c^4 + 72*a^2*b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*(-(2
7*a^3*d^3 - b^3*c^3 + 9*a*b^2*c^2*d - 27*a^2*b*c*d^2)/(729*a^2*c^6))^(1/3)
 + log(- ((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b
*c*d) - 81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2*(a*d - b*
c))/c^6)^(1/3))*((d^2*(a*d - b*c))/c^6)^(2/3))/9 - (b^5*d^4*(27*a^3*d^3 +
b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d - b*c))/c^6)
^(1/3))/3 - (2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*
b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((a*d^3 - b*c*d^2
)/(27*c^6))^(1/3) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(
27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) - 81*a*
b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2
*(a*d - b*c))/c^6)^(1/3))*((d^2*(a*d - b*c))/c^6)^(2/3))/9 + (b^5*d^4*(27*
a^3*d^3 + b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d -
b*c))/c^6)^(1/3))/3 - (2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4
 + 72*a^2*b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((3^...
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{\frac{1}{3}}}{d\,x^7+c\,x^4}dx$$

input
```
int((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x)
```

output
```
int((a + b*x**3)**(1/3)/(c*x**4 + d*x**7),x)
```

3.679 $\int \frac{\sqrt[3]{a+bx^3}}{x^7(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 370

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^7(c+dx^3)}\,dx &= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3}-\frac{(a+bx^3)^{4/3}}{6acx^6}\\
&\quad+\frac{(b^2c^2+3abcd-9a^2d^2)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}c^3}\\
&\quad-\frac{d^{5/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^3}\\
&\quad+\frac{(b^2c^2+3abcd-9a^2d^2)\log(x)}{18a^{5/3}c^3}-\frac{d^{5/3}\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^3}\\
&\quad-\frac{(b^2c^2+3abcd-9a^2d^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{5/3}c^3}\\
&\quad+\frac{d^{5/3}\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3}\end{aligned}$$

output
```
1/9*(3*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x^3-1/6*(b*x^3+a)^(4/3)/a/c/x^6+1/27
*(-9*a^2*d^2+3*a*b*c*d+b^2*c^2)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(
1/2)/a^(1/3))*3^(1/2)/a^(5/3)/c^3-1/3*d^(5/3)*(-a*d+b*c)^(1/3)*arctan(1/3*
(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/c^3+1/18*(
-9*a^2*d^2+3*a*b*c*d+b^2*c^2)*ln(x)/a^(5/3)/c^3-1/6*d^(5/3)*(-a*d+b*c)^(1/
3)*ln(d*x^3+c)/c^3-1/18*(-9*a^2*d^2+3*a*b*c*d+b^2*c^2)*ln(a^(1/3)-(b*x^3+a
)^(1/3))/a^(5/3)/c^3+1/2*d^(5/3)*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1
/3)*(b*x^3+a)^(1/3))/c^3
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx = \frac{\frac{3c\sqrt[3]{a+bx^3}\left(-3ac-bcx^3+6adx^3\right)}{ax^6} + \frac{2\sqrt{3}\left(b^2c^2+3abcd-9a^2d^2\right)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} - 18\sqrt{3}d^{5/3}\sqrt[3]{bc-ad}\arctan\left(\cdots\right.}{\cdots}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x]
```

output
```
((3*c*(a + b*x^3)^(1/3)*(-3*a*c - b*c*x^3 + 6*a*d*x^3))/(a*x^6) + (2*Sqrt[
3]*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(
1/3))/Sqrt[3]])/a^(5/3) - 18*Sqrt[3]*d^(5/3)*(b*c - a*d)^(1/3)*ArcTan[(1 -
 (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - (2*(b^2*c^2 +
 3*a*b*c*d - 9*a^2*d^2)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(5/3) + 18*d^
(5/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]
 + ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1
/3) + (a + b*x^3)^(2/3)])/a^(5/3) - 9*d^(5/3)*(b*c - a*d)^(1/3)*Log[(b*c -
 a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b
*x^3)^(2/3)])/(54*c^3)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {948, 114, 27, 166, 27, 174, 69, 16, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7 (c+dx^3)} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt[3]{bx^3+a}}{x^9 (dx^3+c)}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int \frac{2\sqrt[3]{bx^3+a}(bdx^3+bc+3ad)}{3x^6(dx^3+c)}dx^3}{2ac}-\frac{(a+bx^3)^{4/3}}{2acx^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{\sqrt[3]{bx^3+a}(bdx^3+bc+3ad)}{x^6(dx^3+c)}dx^3}{3ac}-\frac{(a+bx^3)^{4/3}}{2acx^6}\right)$$

$$\Big\downarrow \text{166}$$

$$\frac{1}{3}\left(-\frac{\frac{\int \frac{bd(bc-6ad)x^3+b^2c^2-9a^2d^2+3abcd}{3x^3(bx^3+a)^{2/3}(dx^3+c)}dx^3}{c}-\frac{\sqrt[3]{a+bx^3}(3ad+bc)}{cx^3}}{3ac}-\frac{(a+bx^3)^{4/3}}{2acx^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\frac{\int \frac{bd(bc-6ad)x^3+b^2c^2-9a^2d^2+3abcd}{x^3(bx^3+a)^{2/3}(dx^3+c)}dx^3}{3c}-\frac{\sqrt[3]{a+bx^3}(3ad+bc)}{cx^3}}{3ac}-\frac{(a+bx^3)^{4/3}}{2acx^6}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\int\frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3}{c}-\frac{9ad^2(bc-ad)\int\frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{c}}{3c}-\frac{\sqrt[3]{a+bx^3}(3ad+bc)}{cx^3}}{3ac}-\frac{\left(a+bx^3\right)^{4/3}}{2acx^6}\right)$$

$$\Big\downarrow \ 69$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\left(-\frac{3\int\frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{9ad^2(bc-ad)\ldots}{\ldots}}{3c}\ldots}{3ac}\ldots\right.$$

$$\Big\downarrow \ 16$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\left(-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{9ad^2(bc-ad)\int\frac{\ldots}{\left(bx^3+a\right)^{2\ldots}}}{c}}{3c}\ldots}{3ac}\ldots\right.$$

$$\Big\downarrow \ 70$$

$$\frac{1}{3}\left(-\frac{\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{9ad^2(bc-ad)\left(\frac{3\int \frac{}{x^6+\frac{(b}{}}}{}\right.}{3c}}{}\right.$$

$\downarrow$ 16

$$\frac{1}{3}\left(-\frac{\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{9ad^2(bc-ad)\left(\frac{3\int \frac{}{x^6+\frac{(b}{}}}{}\right.}{3c}}{3}\right.$$

$\downarrow$ 1082

$$\frac{1}{3}\left(-\frac{\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{9ad^2(bc-ad)\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{}}{\sqrt[3]{bc}}\right)}{\sqrt[3]{d}(bc-ad)^{2/}}\right.}{3c}}{3ac}\right.$$

$\downarrow$ 217

$$\frac{1}{3}\left(-\frac{\frac{\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{9ad^2(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{\cdots}}{\sqrt[3]{bc\cdots}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/\cdots}}\cdots\right)}{\cdots}}{3c}}{3ac}\cdots\right.$$

input
```
Int[(a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x]
```

output
```
(-1/2*(a + b*x^3)^(4/3)/(a*c*x^6) - (-(((b*c + 3*a*d)*(a + b*x^3)^(1/3))/(
c*x^3)) + (((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(-((Sqrt[3]*ArcTan[(1 + (2*(
a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^3]/(2*a^(2/3)) + (3*
Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3))))/c - (9*a*d^2*(b*c - a*d)*(
-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sq
rt[3]])/(d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c - a*
d)^(2/3)) + (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1
/3)*(b*c - a*d)^(2/3))))/c)/(3*c))/(3*a*c))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$-\frac{-\frac{d\left(a^{\frac{11}{3}} d-a^{\frac{8}{3}} b c\right) x^{6} \ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{a d-b c}{d}\right)^{\frac{2}{3}}\right)}{2}-d\left(a^{\frac{11}{3}} d-a^{\frac{8}{3}} b c\right) \sqrt{3}\, x^{6} \arctan\left(\frac{2\sqrt{3}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{3\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}}}\right.}{}$

input
```
int((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/((a*d-b*c)/d)^(2/3)*(-1/2*d*(a^(11/3)*d-a^(8/3)*b*c)*x^6*ln((b*x^3+a)
^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))-d*(a^(11/3
)*d-a^(8/3)*b*c)*3^(1/2)*x^6*arctan(2/3*3^(1/2)/((a*d-b*c)/d)^(1/3)*(b*x^3
+a)^(1/3)+1/3*3^(1/2))+1/2*a*(a^2*d^2-1/3*a*b*c*d-1/9*b^2*c^2)*((a*d-b*c)/
d)^(2/3)*x^6*ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))+d*(a^(11/
3)*d-a^(8/3)*b*c)*x^6*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))-((a*d-b*c)/d
)^(2/3)*(-a*(a^2*d^2-1/3*a*b*c*d-1/9*b^2*c^2)*3^(1/2)*x^6*arctan(2/3*3^(1/
2)/a^(1/3)*(b*x^3+a)^(1/3)+1/3*3^(1/2))+a*(a^2*d^2-1/3*a*b*c*d-1/9*b^2*c^2
)*x^6*ln((b*x^3+a)^(1/3)-a^(1/3))-1/6*c*(b*x^3+a)^(1/3)*((-6*d*x^3+3*c)*a^
(8/3)+a^(5/3)*b*c*x^3)))/a^(8/3)/c^3/x^6
```

Fricas [A] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\,(c+dx^3)}\,dx =$$

$$-\frac{18\sqrt{3}(bcd^2-ad^3)^{\frac{1}{3}}a^3dx^6\arctan\left(-\frac{2\sqrt{3}(bcd^2-ad^3)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}-\sqrt{3}(bcd-ad^2)}{3\,(bcd-ad^2)}\right)+9\,(bcd^2-ad^3)^{\frac{1}{3}}a^3dx^6\log\left(\left(\right.\right.}{}$$

input
```
integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/54*(18*sqrt(3)*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*arctan(-1/3*(2*sqrt(3)
*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d^2))/(b*c
*d - a*d^2)) + 9*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*log((b*x^3 + a)^(2/3)*d
^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)
) - 18*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*log((b*x^3 + a)^(1/3)*d + (b*c*d^
2 - a*d^3)^(1/3)) - 6*sqrt(1/3)*(a*b^2*c^2 + 3*a^2*b*c*d - 9*a^3*d^2)*(a^2
)^(1/6)*x^6*arctan(sqrt(1/3)*(a^2)^(1/6)*((a^2)^(1/3)*a + 2*(b*x^3 + a)^(1
/3)*(a^2)^(2/3))/a^2) - (b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(a^2)^(2/3)*x^6*
log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) +
 2*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(a^2)^(2/3)*x^6*log((b*x^3 + a)^(1/3)
*a - (a^2)^(2/3)) + 3*(3*a^3*c^2 + (a^2*b*c^2 - 6*a^3*c*d)*x^3)*(b*x^3 + a
)^(1/3))/(a^3*c^3*x^6)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/x**7/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(x**7*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx=\int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)x^7}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^7), x)
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7(c+dx^3)}\,dx$$
$$= -\frac{(bcd^2-ad^3)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^4-ac^3d)}$$
$$+\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^3}$$
$$+\frac{(-bcd^2+ad^3)^{\frac{1}{3}}d\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^3}$$
$$+\frac{\sqrt{3}(b^2c^2+3\,abcd-9\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{5}{3}}c^3}$$
$$+\frac{(b^2c^2+3\,abcd-9\,a^2d^2)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{54\,a^{\frac{5}{3}}c^3}$$
$$-\frac{(b^2c^2+3\,abcd-9\,a^2d^2)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27\,a^{\frac{5}{3}}c^3}$$
$$-\frac{(bx^3+a)^{\frac{4}{3}}b^2c+2\,(bx^3+a)^{\frac{1}{3}}ab^2c-6\,(bx^3+a)^{\frac{4}{3}}abd+6\,(bx^3+a)^{\frac{1}{3}}a^2bd}{18\,ab^2c^2x^6}$$

input
```
integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*(b*c*d^2 - a*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) -
(-(b*c - a*d)/d)^(1/3)))/(b*c^4 - a*c^3*d) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3
)^(1/3)*d*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3)
)/(-(b*c - a*d)/d)^(1/3))/c^3 + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*d*log((b*x^3
+ a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(
2/3))/c^3 + 1/27*sqrt(3)*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*arctan(1/3*sqrt
(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(5/3)*c^3) + 1/54*(b^2*c^2
 + 3*a*b*c*d - 9*a^2*d^2)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3
) + a^(2/3))/(a^(5/3)*c^3) - 1/27*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*log(ab
s((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5/3)*c^3) - 1/18*((b*x^3 + a)^(4/3)*b^
2*c + 2*(b*x^3 + a)^(1/3)*a*b^2*c - 6*(b*x^3 + a)^(4/3)*a*b*d + 6*(b*x^3 +
 a)^(1/3)*a^2*b*d)/(a*b^2*c^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 11.99 (sec) , antiderivative size = 2767, normalized size of antiderivative = 7.48

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x)
```

output
```
log((((81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^5*(a*d - b
*c))/c^9)^(1/3) + (9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 +
 a*b^2*c^2*d - 14*a^2*b*c*d^2))/a)*(-(d^5*(a*d - b*c))/c^9)^(2/3))/9 - (b^
5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 + 8
64*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(d
^5*(a*d - b*c))/c^9)^(1/3))/3 - (b^4*d^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 +
 b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4
 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*a^3*c^8)
)*(-(a*d^6 - b*c*d^5)/(27*c^9))^(1/3) + log((((((9*b^5*c^2*d^3*(a + b*x^3)
^(1/3)*(12*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d - 14*a^2*b*c*d^2))/a + 9*a*b^4*
c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c
*d)^3/(a^5*c^9))^(1/3))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(
2/3))/729 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*
b^3*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81
*a^3*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))/27 - (b
^4*d^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 13
5*a^3*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*
c^6*d - 3888*a^6*b*c*d^6))/(243*a^3*c^8))*(-(b^6*c^6 - 729*a^6*d^6 - 135*a
^3*b^3*c^3*d^3 + 9*a*b^5*c^5*d + 729*a^5*b*c*d^5)/(19683*a^5*c^9))^(1/3) -
 (((a + b*x^3)^(1/3)*(b^2*c + 3*a*b*d))/(9*c^2) - (b*(a + b*x^3)^(4/3)*...
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\,(c+dx^3)}\,dx=\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{d\,x^{10}+c\,x^7}dx$$

input
```
int((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x)
```

output
```
int((a + b*x**3)**(1/3)/(c*x**7 + d*x**10),x)
```

3.680 $\int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 336

$$\begin{aligned}\int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = & -\frac{(6bc-ad)x^2\sqrt[3]{a+bx^3}}{18bd^2} + \frac{x^5\sqrt[3]{a+bx^3}}{6d} \\ & - \frac{(9b^2c^2-3abcd-a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{5/3}d^3} \\ & + \frac{c^{5/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^3} \\ & - \frac{c^{5/3}\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^3} \\ & - \frac{(9b^2c^2-3abcd-a^2d^2)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}d^3} \\ & + \frac{c^{5/3}\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^3}\end{aligned}$$

output
```
-1/18*(-a*d+6*b*c)*x^2*(b*x^3+a)^(1/3)/b/d^2+1/6*x^5*(b*x^3+a)^(1/3)/d-1/2
7*(-a^2*d^2-3*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3)
)*3^(1/2))*3^(1/2)/b^(5/3)/d^3+1/3*c^(5/3)*(-a*d+b*c)^(1/3)*arctan(1/3*(1+
2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d^3-1/6*c^(
5/3)*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^3-1/18*(-a^2*d^2-3*a*b*c*d+9*b^2*c^2)*
ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)/d^3+1/2*c^(5/3)*(-a*d+b*c)^(1/3)*ln(
(-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^3
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.41 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.57

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$= \frac{\frac{6dx^2\sqrt[3]{a+bx^3}(-6bc+ad+3bdx^3)}{b} - \frac{4\sqrt{3}(9b^2c^2-3abcd-a^2d^2)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{5/3}} - 18\sqrt{-6-6i\sqrt{3}}c^{5/3}\sqrt[3]{bc-\cdots}}{}$$

input
```
Integrate[(x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
((6*d*x^2*(a + b*x^3)^(1/3)*(-6*b*c + a*d + 3*b*d*x^3))/b - (4*Sqrt[3]*(9*
b^2*c^2 - 3*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(
a + b*x^3)^(1/3))])/b^(5/3) - 18*Sqrt[-6 - (6*I)*Sqrt[3]]*c^(5/3)*(b*c - a
*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3
*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (4*(-9*b^2*c^2 + 3*a*b*c*d + a
^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(5/3) + (18*I)*(I + Sqrt[
3])*c^(5/3)*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*
c^(1/3)*(a + b*x^3)^(1/3)] + (2*(9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*Log[b^(2
/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(5/3) + 9*(1
 - I*Sqrt[3])*c^(5/3)*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1
- I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3
])*c^(2/3)*(a + b*x^3)^(2/3)])/(108*d^3)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {978, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$$

$$\Big\downarrow \text{ 978}$$

$$\frac{x^5 \sqrt[3]{a+bx^3}}{6d}-\frac{\int \frac{x^4 \left((6bc-ad)x^3+5ac\right)}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{6d}$$

$$\Big\downarrow \text{ 1052}$$

$$\frac{x^5 \sqrt[3]{a+bx^3}}{6d}-\frac{\frac{x^2 \sqrt[3]{a+bx^3}(6bc-ad)}{3bd}-\frac{\int \frac{2x\left(\left(9b^2c^2-3abdc-a^2d^2\right)x^3+ac(6bc-ad)\right)}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{3bd}}{6d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^5 \sqrt[3]{a+bx^3}}{6d}-\frac{\frac{x^2 \sqrt[3]{a+bx^3}(6bc-ad)}{3bd}-\frac{2\int \frac{x\left(\left(9b^2c^2-3abdc-a^2d^2\right)x^3+ac(6bc-ad)\right)}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{3bd}}{6d}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{x^5 \sqrt[3]{a+bx^3}}{6d}-\frac{\frac{x^2 \sqrt[3]{a+bx^3}(6bc-ad)}{3bd}-\frac{2\int \left(\frac{\left(9b^2c^2-3abdc-a^2d^2\right)x}{d\left(bx^3+a\right)^{2/3}}+\frac{9\left(abc^2d-b^2c^3\right)x}{d\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}\right)dx}{3bd}}{6d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^5\sqrt[3]{a+bx^3}}{6d} - \frac{\frac{x^2\sqrt[3]{a+bx^3}(6bc-ad)}{3bd} - \frac{2\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\left(-a^2d^2-3abcd+9b^2c^2\right)}{\sqrt{3}b^{2/3}d} - \frac{\left(-a^2d^2-3abcd+9b^2c^2\right)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d} + \frac{3\sqrt{3}bc^{5/3}\cdots}{\cdots}\right.}{\cdots}}{6d}$$

input
```
Int[(x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(x^5*(a + b*x^3)^(1/3))/(6*d) - (((6*b*c - a*d)*x^2*(a + b*x^3)^(1/3))/(3*
b*d) - (2*(-(((9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*ArcTan[(1 + (2*b^(1/3)*x)/
(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)*d)) + (3*Sqrt[3]*b*c^(5/3)*(
b*c - a*d)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^
(1/3)))/Sqrt[3]])/d - (3*b*c^(5/3)*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(2*d)
 - ((9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/
(2*b^(2/3)*d) + (9*b*c^(5/3)*(b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c
^(1/3) - (a + b*x^3)^(1/3)])/(2*d)))/(3*b*d))/(6*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.27

method	result
pseudoelliptic	$\frac{-\frac{\left(b^{\frac{11}{3}} c - a\, b^{\frac{8}{3}} d\right) c \ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}-\frac{b\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} \left(a^{2} d^{2}+3abcd-9b^{2}c^{2}\right) \ln\left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}} \left(b\right.}{}\right.}{18}}{}$

input
```
int(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-1/2*(b^(11/3)*c-a*b^(8/3)*d)*c*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c
)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-1/18*b*((a*d-b*c)/c)^(2
/3)*(a^2*d^2+3*a*b*c*d-9*b^2*c^2)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*
x+(b*x^3+a)^(2/3))/x^2)-(b^(11/3)*c-a*b^(8/3)*d)*c*3^(1/2)*arctan(1/3*3^(1
/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)+(b^(11/3)*c-a*b^(8/3)*d)
*c*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+1/9*((a*d-b*c)/c)^(2/3)*(
-3^(1/2)*b*(a^2*d^2+3*a*b*c*d-9*b^2*c^2)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(
1/3)/b^(1/3)+x)/x)+b*(a^2*d^2+3*a*b*c*d-9*b^2*c^2)*ln((-b^(1/3)*x+(b*x^3+a
)^(1/3))/x)+3/2*d*(b*x^3+a)^(1/3)*((3*d*x^3-6*c)*b^(8/3)+a*b^(5/3)*d)*x^2)
)/((a*d-b*c)/c)^(2/3)/b^(8/3)/d^3
```

Fricas [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.46

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$= \frac{18\sqrt{3}(bc^3-ac^2d)^{\frac{1}{3}}b^3c\arctan\left(-\frac{\sqrt{3}(bc^2-acd)x+2\sqrt{3}(bc^3-ac^2d)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}}{3(bc^2-acd)x}\right)+18\left(bc^3-ac^2d\right)^{\frac{1}{3}}b^3c\log\left(\frac{(bx^3+a)}{}\right.}{}$$

input
```
integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/54*(18*sqrt(3)*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*arctan(-1/3*(sqrt(3)*(b*c^2
 - a*c*d)*x + 2*sqrt(3)*(b*c^3 - a*c^2*d)^(2/3)*(b*x^3 + a)^(1/3))/((b*c^2
 - a*c*d)*x)) + 18*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*log(((b*x^3 + a)^(1/3)*c
- (b*c^3 - a*c^2*d)^(1/3)*x)/x) - 9*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*log(((b*
x^3 + a)^(2/3)*c^2 + (b*c^3 - a*c^2*d)^(1/3)*(b*x^3 + a)^(1/3)*c*x + (b*c^
3 - a*c^2*d)^(2/3)*x^2)/x^2) + 6*sqrt(1/3)*(9*b^3*c^2 - 3*a*b^2*c*d - a^2*
b*d^2)*(b^2)^(1/6)*arctan(sqrt(1/3)*((b^2)^(1/3)*b*x + 2*(b*x^3 + a)^(1/3)
*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 2*(9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*(
b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (9*b^2*c^2 - 3*
a*b*c*d - a^2*d^2)*(b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*
(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2) + 3*(3*b^3*d^2*x^5 - (6*b^3*c*d
- a*b^2*d^2)*x^2)*(b*x^3 + a)^(1/3))/(b^3*d^3)
```

Sympy [F]

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate(x**7*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**7*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^7}{dx^3+c}\,dx$$

input
```
integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*x^7/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^7}{dx^3+c}\,dx$$

input
```
integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*x^7/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{x^7\,{\left(b\,x^3+a\right)}^{1/3}}{d\,x^3+c}\,dx$$

input
```
int((x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

output
```
int((x^7*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$
$$=\frac{(b\,x^3+a)^{\frac{1}{3}}\,a d\,x^2-6(b\,x^3+a)^{\frac{1}{3}}\,bc\,x^2+3(b\,x^3+a)^{\frac{1}{3}}\,bd\,x^5-2\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}x^4}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)a^2d^2-6\left(\int \frac{(}{bd\,x^6}\right.}{}$$

input
```
int(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
((a + b*x**3)**(1/3)*a*d*x**2 - 6*(a + b*x**3)**(1/3)*b*c*x**2 + 3*(a + b*
x**3)**(1/3)*b*d*x**5 - 2*int(((a + b*x**3)**(1/3)*x**4)/(a*c + a*d*x**3 +
 b*c*x**3 + b*d*x**6),x)*a**2*d**2 - 6*int(((a + b*x**3)**(1/3)*x**4)/(a*c
 + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b*c*d + 18*int(((a + b*x**3)**(1/3
)*x**4)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**2*c**2 - 2*int(((a +
b*x**3)**(1/3)*x)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a**2*c*d + 12*
int(((a + b*x**3)**(1/3)*x)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b*
c**2)/(18*b*d**2)
```

3.681 $\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 276

$$\begin{aligned}\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx &= \frac{x^2\sqrt[3]{a+bx^3}}{3d}+\frac{(3bc-ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}d^2}\\
&\quad-\frac{c^{2/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2}\\
&\quad+\frac{c^{2/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^2}+\frac{(3bc-ad)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}d^2}\\
&\quad-\frac{c^{2/3}\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2}\end{aligned}$$

output
```
1/3*x^2*(b*x^3+a)^(1/3)/d+1/9*(-a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^
3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)/d^2-1/3*c^(2/3)*(-a*d+b*c)^(1/3)*arct
an(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d
^2+1/6*c^(2/3)*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^2+1/6*(-a*d+3*b*c)*ln(b^(1/3
)*x-(b*x^3+a)^(1/3))/b^(2/3)/d^2-1/2*c^(2/3)*(-a*d+b*c)^(1/3)*ln((-a*d+b*c
)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.98 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.69

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$= \frac{12dx^2\sqrt[3]{a+bx^3} + \frac{4\sqrt{3}(3bc-ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{2/3}} + 6\sqrt{-6-6i\sqrt{3}}c^{2/3}\sqrt[3]{bc-ad}\arctan\left(\frac{}{\sqrt{3}\sqrt[3]{bc-a}}\right.}{}$$

input
```
Integrate[(x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(12*d*x^2*(a + b*x^3)^(1/3) + (4*Sqrt[3]*(3*b*c - a*d)*ArcTan[(Sqrt[3]*b^(
1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/b^(2/3) + 6*Sqrt[-6 - (6*I)*Sq
rt[3]]*c^(2/3)*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(
b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (4*(3*b
*c - a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(2/3) + 6*(1 - I*Sqrt[3
])*c^(2/3)*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c
^(1/3)*(a + b*x^3)^(1/3)] + (2*(-3*b*c + a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*
(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(2/3) + (3*I)*(I + Sqrt[3])*c^(2
/3)*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/
3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*
x^3)^(2/3)])/(36*d^2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {978, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}dx$$

$$\Big\downarrow \text{ 978}$$

$$\frac{x^2\sqrt[3]{a+bx^3}}{3d}-\frac{\int \frac{x\left((3bc-ad)x^3+2ac\right)}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{3d}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{x^2\sqrt[3]{a+bx^3}}{3d}-\frac{\int \left(\frac{(3bc-ad)x}{d\left(bx^3+a\right)^{2/3}}+\frac{3\left(acd-bc^2\right)x}{d\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}\right)dx}{3d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^2\sqrt[3]{a+bx^3}}{3d}-$$

$$\frac{-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(3bc-ad)}{\sqrt{3}b^{2/3}d}+\frac{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{d}-\frac{(3bc-ad)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}-\frac{c^{2/}\cdots}{}}{3d}$$

input
```
Int[(x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(x^2*(a + b*x^3)^(1/3))/(3*d) - (-(((3*b*c - a*d)*ArcTan[(1 + (2*b^(1/3)*x
)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)*d)) + (Sqrt[3]*c^(2/3)*(b*
c - a*d)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1
/3)))/Sqrt[3]])/d - (c^(2/3)*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(2*d) - ((3
*b*c - a*d)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*b^(2/3)*d) + (3*c^(2/3)
*(b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])
/(2*d))/(3*d)
```

Defintions of rubi rules used

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.37

method	result
pseudoelliptic	$\frac{\frac{\left(-a d\, b^{\frac{2}{3}}+b^{\frac{5}{3}} c\right) \ln\left(\frac{\left(\frac{a d-b c}{c}\right)^{\frac{2}{3}} x^{2}-\left(\frac{a d-b c}{c}\right)^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}-\sqrt{3}\left(a d\, b^{\frac{2}{3}}-b^{\frac{5}{3}} c\right) \arctan\left(\frac{\sqrt{3}\left(\left(\frac{a d-b c}{c}\right)^{\frac{1}{3}} x-2 \left(b\, x\right.\right.}{3\left(\frac{a d-b c}{c}\right)^{\frac{1}{3}} x}\right.}{}$

input
```
int(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(2/3)*(1/2*(-a*d*b^(2/3)+b^(5/3)*c)*ln((((a*d-b*c)/c)^(2
/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-3^(1/2
)*(a*d*b^(2/3)-b^(5/3)*c)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x
^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)+1/6*((a*d-b*c)/c)^(2/3)*(a*d-3*b*c)*ln
((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(a*d*b^(2/3)
-b^(5/3)*c)*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/3*(-3^(1/2)*(a
*d-3*b*c)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+(a*d
-3*b*c)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-3*b^(2/3)*(b*x^3+a)^(1/3)*d*x^2
)*((a*d-b*c)/c)^(2/3))/b^(2/3)/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(222) = 444$.

Time = 0.25 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.63

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$= \frac{6\,(bx^3+a)^{\frac{1}{3}}b^2dx^2 + 6\,\sqrt{3}(-bc^3+ac^2d)^{\frac{1}{3}}b^2\arctan\left(-\frac{\sqrt{3}(bc^2-acd)x+2\,\sqrt{3}(-bc^3+ac^2d)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}}{3\,(bc^2-acd)x}\right) + 6\,(-bc^3+}{}$$

input
```
integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/18*(6*(b*x^3 + a)^(1/3)*b^2*d*x^2 + 6*sqrt(3)*(-b*c^3 + a*c^2*d)^(1/3)*b
^2*arctan(-1/3*(sqrt(3)*(b*c^2 - a*c*d)*x + 2*sqrt(3)*(-b*c^3 + a*c^2*d)^(
2/3)*(b*x^3 + a)^(1/3))/((b*c^2 - a*c*d)*x)) + 6*(-b*c^3 + a*c^2*d)^(1/3)*
b^2*log(((b*x^3 + a)^(1/3)*c + (-b*c^3 + a*c^2*d)^(1/3)*x)/x) - 3*(-b*c^3
+ a*c^2*d)^(1/3)*b^2*log(((b*x^3 + a)^(2/3)*c^2 - (-b*c^3 + a*c^2*d)^(1/3)
*(b*x^3 + a)^(1/3)*c*x + (-b*c^3 + a*c^2*d)^(2/3)*x^2)/x^2) - 6*sqrt(1/3)*
(3*b^2*c - a*b*d)*sqrt(-(-b^2)^(1/3))*arctan(-sqrt(1/3)*((-b^2)^(1/3)*b*x
- 2*(b*x^3 + a)^(1/3)*(-b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) + 2*(-b^2
)^(2/3)*(3*b*c - a*d)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) - (-b
^2)^(2/3)*(3*b*c - a*d)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2
)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d^2)
```

Sympy [F]

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate(x**4*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**4*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^4}{dx^3+c}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*x^4/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^4}{dx^3+c}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*x^4/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^4\,{\left(b\,x^3+a\right)}^{1/3}}{d\,x^3+c}\,dx$$

input
```
int((x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

output
```
int((x^4*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{\left(b\,x^3+a\right)^{\frac{1}{3}} x^2+\left(\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}} x^4}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right) ad-3\left(\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}} x^4}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right) bc-2\left(\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}} x}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right) ac}{3d}$$

input
```
int(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
((a + b*x**3)**(1/3)*x**2 + int(((a + b*x**3)**(1/3)*x**4)/(a*c + a*d*x**3
 + b*c*x**3 + b*d*x**6),x)*a*d - 3*int(((a + b*x**3)**(1/3)*x**4)/(a*c + a
*d*x**3 + b*c*x**3 + b*d*x**6),x)*b*c - 2*int(((a + b*x**3)**(1/3)*x)/(a*c
 + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*c)/(3*d)
```

3.682 $\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 234

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = -\frac{\sqrt[3]{b}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d} + \frac{\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}d} - \frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6\sqrt[3]{c}d} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d} + \frac{\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}d}$$

output
```
-1/3*b^(1/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d
+1/3*(-a*d+b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)
^(1/3))*3^(1/2))*3^(1/2)/c^(1/3)/d-1/6*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(1/3
)/d-1/2*b^(1/3)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/d+1/2*(-a*d+b*c)^(1/3)*ln((-
a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(1/3)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.90 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.81

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$= \frac{-4\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - \frac{2\sqrt{-6-6i\sqrt{3}}\sqrt[3]{bc-ad}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{c}} -$$

input
```
Integrate[(x*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(-4*Sqrt[3]*b^(1/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^
(1/3))] - (2*Sqrt[-6 - (6*I)*Sqrt[3]]*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a
*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b
*x^3)^(1/3))])/c^(1/3) - 4*b^(1/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] +
 ((2*I)*(I + Sqrt[3])*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I
*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/c^(1/3) + 2*b^(1/3)*Log[b^(2/3)*x^2
+ b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + ((1 - I*Sqrt[3])*(b*c
 - a*d)^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c
- a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/
3)])/c^(1/3))/(12*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {984, 853, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$\Big\downarrow \ 984$$

$$\frac{b\int \frac{x}{(bx^3+a)^{2/3}}dx}{d}-\frac{(bc-ad)\int \frac{x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{d}$$

$$\Big\downarrow \ 853$$

$$\frac{b\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\right)}{d}-\frac{(bc-ad)\int \frac{x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{d}$$

$$\Big\downarrow \ 992$$

$$\frac{b\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\right)}{d}-\frac{(bc-ad)\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log(c+dx^3)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{d}$$

input
```
Int[(x*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(b*(-(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/
3))) - Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b^(2/3))))/d - ((b*c - a*d)*(
-(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]
]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[c + d*x^3]/(6*c^(1/3)*(b*c -
a*d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^
(1/3)*(b*c - a*d)^(2/3))))/d
```

Defintions of rubi rules used

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.39

method	result
pseudoelliptic	$\frac{\frac{\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right) (ad-bc)}{2}+\frac{b^{\frac{1}{3}} \ln\left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right) c\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}}{2}+(-}{}$

input
```
int(x*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(2/3)*(1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/
3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*(a*d-b*c)+1/2*b^(1/3)*ln((b^(2/
3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*c*((a*d-b*c)/c)^(2/
3)+(-a*d+b*c)*ln(((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+b^(1/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)/b^(1/3)+x)/x)*c*((a*d-b*c)/c)^(2/3
)-b^(1/3)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*c*((a*d-b*c)/c)^(2/3)+(a*d-b*
c)*3^(1/2)*arctan(1/3*3^(1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x
))/d/c
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.41

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$= \frac{2\sqrt{3}\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}}{3\,(bc-ad)x}\right)-2\sqrt{3}(-b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}bx+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}}{3\,bx}\right)}{}$$

input
```
integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*((b*c - a*d)/c)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x +
2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b*c - a*d)/c)^(2/3))/((b*c - a*d)*x)) - 2*
sqrt(3)*(-b)^(1/3)*arctan(1/3*(sqrt(3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(
-b)^(2/3))/(b*x)) + 2*(-b)^(1/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x)
 + 2*((b*c - a*d)/c)^(1/3)*log(-(x*((b*c - a*d)/c)^(1/3) - (b*x^3 + a)^(1/
3))/x) - (-b)^(1/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x +
 (b*x^3 + a)^(2/3))/x^2) - ((b*c - a*d)/c)^(1/3)*log((x^2*((b*c - a*d)/c)^
(2/3) + (b*x^3 + a)^(1/3)*x*((b*c - a*d)/c)^(1/3) + (b*x^3 + a)^(2/3))/x^2
))/d
```

Sympy [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate(x*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{1}{3}}x}{dx^3+c}\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*x/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x}{dx^3+c}\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*x/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x\,(b\,x^3+a)^{1/3}}{d\,x^3+c}\,dx$$

input
```
int((x*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

output
```
int((x*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^{\frac{1}{3}}\,x}{d\,x^3+c}dx$$

input
```
int(x*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(((a + b*x**3)**(1/3)*x)/(c + d*x**3),x)
```

3.683 $\int \frac{\sqrt[3]{a+bx^3}}{x^2(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2(c+dx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{cx} - \frac{\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}} + \frac{\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^{4/3}} - \frac{\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}}$$

output
```
-(b*x^3+a)^(1/3)/c/x-1/3*(-a*d+b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)
*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(4/3)+1/6*(-a*d+b*c)^(1/3)*
ln(d*x^3+c)/c^(4/3)-1/2*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*
x^3+a)^(1/3))/c^(4/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2(c+dx^3)}\,dx = \frac{-\frac{12\sqrt[3]{c}\sqrt[3]{a+bx^3}}{x}+2\sqrt{-6-6i\sqrt{3}}\sqrt[3]{bc-ad}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+2\left(1-i\sqrt{3}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x]
```

output
```
((-12*c^(1/3)*(a + b*x^3)^(1/3))/x + 2*Sqrt[-6 - (6*I)*Sqrt[3]]*(b*c - a*d
)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I
 + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 2*(1 - I*Sqrt[3])*(b*c - a*d)^(1
/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]
 + I*(I + Sqrt[3])*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I
*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*
c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(4/3))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {975, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2(c+dx^3)}\,dx$$

$$\Big\downarrow \text{ 975}$$

$$\frac{\int \frac{(bc-ad)x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{c}-\frac{\sqrt[3]{a+bx^3}}{cx}$$

$$\Big\downarrow 27$$

$$\frac{(bc-ad)\int \frac{x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{c}-\frac{\sqrt[3]{a+bx^3}}{cx}$$

$$\Big\downarrow 992$$

$$\frac{(bc-ad)\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log(c+dx^3)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{c}-\frac{\sqrt[3]{a+bx^3}}{cx}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x]
```

output
```
-((a + b*x^3)^(1/3)/(c*x)) + ((b*c - a*d)*(-(ArcTan[(1 + (2*(b*c - a*d)^(1
/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^
(2/3))) + Log[c + d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) - Log[((b*c - a*d)^
(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a*d)^(2/3))))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.21

method	result
pseudoelliptic	$-\frac{-2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)(ad-bc)x+6\left(b\,x^{3}+a\right)^{\frac{1}{3}}c\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}+(ad-bc)\left(2\arctan\left(\frac{\sqrt{3}\left(-\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}+x\right)}{3x}\right)\sqrt{}}{6\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x\,c^{2}}$

input
```
int((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*(a*d-b*c)*x+6*(b*x^
3+a)^(1/3)*c*((a*d-b*c)/c)^(2/3)+(a*d-b*c)*(2*arctan(1/3*3^(1/2)*(-2/((a*d
-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-(
(a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*x)/((a*d-b*c)/
c)^(2/3)/x/c^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/x**2/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(x**2*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{x^2\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^2*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2(c+dx^3)}\,dx = \frac{-(b\,x^3+a)^{\frac{1}{3}}\,b+\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{bd\,x^8+ad\,x^5+bc\,x^5+ac\,x^2}dx\right)a^2dx-\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{bd\,x^8+ad\,x^5+bc\,x^5+ac\,x^2}dx\right)abcx}{adx}$$

input
```
int((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x)
```

output
```
( - (a + b*x**3)**(1/3)*b + int((a + b*x**3)**(1/3)/(a*c*x**2 + a*d*x**5 +
 b*c*x**5 + b*d*x**8),x)*a**2*d*x - int((a + b*x**3)**(1/3)/(a*c*x**2 + a*
d*x**5 + b*c*x**5 + b*d*x**8),x)*a*b*c*x)/(a*d*x)
```

3.684 $\int \frac{\sqrt[3]{a+bx^3}}{x^5(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 204

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5(c+dx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{4cx^4} - \frac{(bc-4ad)\sqrt[3]{a+bx^3}}{4ac^2x} + \frac{d\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}} - \frac{d\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^{7/3}} + \frac{d\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{7/3}}$$

output
```
-1/4*(b*x^3+a)^(1/3)/c/x^4-1/4*(-4*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x+1/3*d*
(-a*d+b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3
))*3^(1/2))*3^(1/2)/c^(7/3)-1/6*d*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(7/3)+1/2
*d*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(7/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.63

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5(c+dx^3)}\,dx$$
$$= \frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}(-ac-bcx^3+4adx^3)}{ax^4} - 2\sqrt{-6-6i\sqrt{3}}d\sqrt[3]{bc-ad}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x]
```

output
```
((3*c^(1/3)*(a + b*x^3)^(1/3)*(-(a*c) - b*c*x^3 + 4*a*d*x^3))/(a*x^4) - 2*
Sqrt[-6 - (6*I)*Sqrt[3]]*d*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x
)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)
)] + (2*I)*(I + Sqrt[3])*d*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (
1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + (1 - I*Sqrt[3])*d*(b*c - a*d)^
(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(
1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12
*c^(7/3))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {975, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5(c+dx^3)}\,dx$$
$$\Big\downarrow \; 975$$

$$\frac{\int \frac{-3bdx^3+bc-4ad}{x^2(bx^3+a)^{2/3}(dx^3+c)}dx}{4c} - \frac{\sqrt[3]{a+bx^3}}{4cx^4}$$

$\Big\downarrow$ 1053

$$\frac{-\frac{\int \frac{4ad(bc-ad)x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{ac} - \frac{\sqrt[3]{a+bx^3}(bc-4ad)}{acx}}{4c} - \frac{\sqrt[3]{a+bx^3}}{4cx^4}$$

$\Big\downarrow$ 27

$$\frac{-\frac{4d(bc-ad)\int \frac{x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{c} - \frac{\sqrt[3]{a+bx^3}(bc-4ad)}{acx}}{4c} - \frac{\sqrt[3]{a+bx^3}}{4cx^4}$$

$\Big\downarrow$ 992

$$\frac{-\frac{4d(bc-ad)\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log(c+dx^3)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{c} - \frac{\sqrt[3]{a+bx^3}(bc-4ad)}{acx}}{4c} - \frac{\sqrt[3]{a+bx^3}}{4cx^4}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x]
```

output
```
-1/4*(a + b*x^3)^(1/3)/(c*x^4) + (-(((b*c - 4*a*d)*(a + b*x^3)^(1/3))/(a*c
*x)) - (4*d*(b*c - a*d)*(-(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a
 + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[c +
d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
 (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a*d)^(2/3))))/c)/(4*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$\frac{-\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a(ad-bc)d\,x^{4}-\frac{3\left((-4ad+bc)x^{3}+ac\right)c\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}}{4}+a(ad-bc)d\left(\arctan\left(\frac{\sqrt{3}\left(-\frac{2(}{(}\right.}{}\right.\right.}{3\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}c^{3}x^{4}a}$

input
```
int((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(2/3)*(-ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*a*
(a*d-b*c)*d*x^4-3/4*((-4*a*d+b*c)*x^3+a*c)*c*(b*x^3+a)^(1/3)*((a*d-b*c)/c)
^(2/3)+a*(a*d-b*c)*d*(arctan(1/3*3^(1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)
^(1/3)+x)/x)*3^(1/2)+1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(
b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*x^4)/c^3/x^4/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/x**5/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(x**5*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx = \int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^5), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx = \int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5 (c+dx^3)}\,dx = \int \frac{\left(b\,x^3+a\right)^{1/3}}{x^5\,\left(d\,x^3+c\right)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^5*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5 (c+dx^3)}\,dx$$
$$= \frac{-(b\,x^3+a)^{\frac{1}{3}}\,a+3(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3-4\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{bd\,x^8+ad\,x^5+bc\,x^5+ac\,x^2}dx\right)a^2d\,x^4+4\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{bd\,x^8+ad\,x^5+bc\,x^5+ac\,x^2}dx\right)}{4ac\,x^4}$$

input
```
int((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x)
```

output
```
( - (a + b*x**3)**(1/3)*a + 3*(a + b*x**3)**(1/3)*b*x**3 - 4*int((a + b*x*
*3)**(1/3)/(a*c*x**2 + a*d*x**5 + b*c*x**5 + b*d*x**8),x)*a**2*d*x**4 + 4*
int((a + b*x**3)**(1/3)/(a*c*x**2 + a*d*x**5 + b*c*x**5 + b*d*x**8),x)*a*b
*c*x**4)/(4*a*c*x**4)
```

3.685 $\int \frac{\sqrt[3]{a+bx^3}}{x^8(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 258

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^8(c+dx^3)}\,dx &= -\frac{\sqrt[3]{a+bx^3}}{7cx^7}-\frac{(bc-7ad)\sqrt[3]{a+bx^3}}{28ac^2x^4}\\
&\quad+\frac{(3b^2c^2+7abcd-28a^2d^2)\sqrt[3]{a+bx^3}}{28a^2c^3x}\\
&\quad-\frac{d^2\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{10/3}}\\
&\quad+\frac{d^2\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^{10/3}}\\
&\quad-\frac{d^2\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{10/3}}\end{aligned}$$

output
```
-1/7*(b*x^3+a)^(1/3)/c/x^7-1/28*(-7*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x^4+1/2
8*(-28*a^2*d^2+7*a*b*c*d+3*b^2*c^2)*(b*x^3+a)^(1/3)/a^2/c^3/x-1/3*d^2*(-a*
d+b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3
^(1/2))*3^(1/2)/c^(10/3)+1/6*d^2*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(10/3)-1/2
*d^2*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(10
/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\,(c+dx^3)}\,dx$$
$$= \frac{-\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}(-3b^2c^2x^6+abcx^3(c-7dx^3)+a^2(4c^2-7cdx^3+28d^2x^6))}{a^2x^7}+14\sqrt{-6-6i\sqrt{3}}d^2\sqrt[3]{bc-ad}\arctan\left(\frac{}{\sqrt{3}\sqrt[3]{bc}}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x]
```

output
```
((-3*c^(1/3)*(a + b*x^3)^(1/3)*(-3*b^2*c^2*x^6 + a*b*c*x^3*(c - 7*d*x^3) +
 a^2*(4*c^2 - 7*c*d*x^3 + 28*d^2*x^6)))/(a^2*x^7) + 14*Sqrt[-6 - (6*I)*Sqr
t[3]]*d^2*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
 a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 14*(1 - I*Sq
rt[3])*d^2*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c
^(1/3)*(a + b*x^3)^(1/3)] + (7*I)*(I + Sqrt[3])*d^2*(b*c - a*d)^(1/3)*Log[
2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a
+ b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(84*c^(10/3))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {975, 1053, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{-6bdx^3+bc-7ad}{x^5(bx^3+a)^{2/3}(dx^3+c)}dx}{7c}-\frac{\sqrt[3]{a+bx^3}}{7cx^7}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int \frac{3bd(bc-7ad)x^3+3b^2c^2-28a^2d^2+7abcd}{x^2\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{4ac}-\frac{\sqrt[3]{a+bx^3}(bc-7ad)}{4acx^4}}{7c}-\frac{\sqrt[3]{a+bx^3}}{7cx^7}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{-\frac{\int \frac{28a^2d^2(bc-ad)x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{3b^2c}{a}-\frac{28ad^2}{c}+7bd\right)}{x}}{4ac}-\frac{\sqrt[3]{a+bx^3}(bc-7ad)}{4acx^4}}{7c}-\frac{\sqrt[3]{a+bx^3}}{7cx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{-\frac{28ad^2(bc-ad)\int \frac{x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{c}-\frac{\sqrt[3]{a+bx^3}\left(\frac{3b^2c}{a}-\frac{28ad^2}{c}+7bd\right)}{x}}{4ac}-\frac{\sqrt[3]{a+bx^3}(bc-7ad)}{4acx^4}}{7c}-\frac{\sqrt[3]{a+bx^3}}{7cx^7}$$

$$\Big\downarrow \text{992}$$

$$
-\frac{-\frac{\frac{28ad^2(bc-ad)\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{c}-\frac{\sqrt[3]{a+bx^3}\left(\frac{3b^2c}{a}-\frac{28ad^2}{c}+7bd\right)}{x}}{4ac}}{7c}
$$

$$
\frac{\sqrt[3]{a+bx^3}}{7cx^7}
$$

input
```
Int[(a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x]
```

output
```
-1/7*(a + b*x^3)^(1/3)/(c*x^7) + (-1/4*((b*c - 7*a*d)*(a + b*x^3)^(1/3))/(
a*c*x^4) - (-((((3*b^2*c)/a + 7*b*d - (28*a*d^2)/c)*(a + b*x^3)^(1/3))/x)
- (28*a*d^2*(b*c - a*d)*(-(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a
 + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[c +
d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
 (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a*d)^(2/3))))/c)/(4*a*c))/(7*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{\frac{6\left(\left(b x^3+a\right)\left(-\frac{3b x^3}{4}+a\right)c^2-\frac{7\left(b x^3+a\right)a c d x^3}{4}+7a^2d^2x^6\right)c\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}\left(b x^3+a\right)^{\frac{1}{3}}}{7}+a^2d^2x^7(ad-bc)(2\sqrt{3}\arctan(\frac{\sqrt{3}((\frac{ad\cdots}{}}{}}{6\left(\frac{ad-bc}{c}\right)\cdots}$

input
```
int((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/((a*d-b*c)/c)^(2/3)*(6/7*((b*x^3+a)*(-3/4*b*x^3+a)*c^2-7/4*(b*x^3+a)*
a*c*d*x^3+7*a^2*d^2*x^6)*c*((a*d-b*c)/c)^(2/3)*(b*x^3+a)^(1/3)+a^2*d^2*x^7
*(a*d-b*c)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a
)^(1/3))/((a*d-b*c)/c)^(1/3)/x)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^
(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+
(b*x^3+a)^(1/3))/x)))/x^7/c^4/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/x**8/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(x**8*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^8}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^8), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^8}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx=\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^8\,\left(d\,x^3+c\right)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^8*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx$$
$$=\frac{-4\left(b\,x^3+a\right)^{\frac{1}{3}}a^2c+7\left(b\,x^3+a\right)^{\frac{1}{3}}a^2d\,x^3-\left(b\,x^3+a\right)^{\frac{1}{3}}abc\,x^3-21\left(b\,x^3+a\right)^{\frac{1}{3}}abd\,x^6+3\left(b\,x^3+a\right)^{\frac{1}{3}}b^2c\,x^6}{28a^2c^2x^7}$$

input
```
int((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x)
```

output
```
( - 4*(a + b*x**3)**(1/3)*a**2*c + 7*(a + b*x**3)**(1/3)*a**2*d*x**3 - (a
+ b*x**3)**(1/3)*a*b*c*x**3 - 21*(a + b*x**3)**(1/3)*a*b*d*x**6 + 3*(a + b
*x**3)**(1/3)*b**2*c*x**6 + 28*int((a + b*x**3)**(1/3)/(a*c*x**2 + a*d*x**
5 + b*c*x**5 + b*d*x**8),x)*a**3*d**2*x**7 - 28*int((a + b*x**3)**(1/3)/(a
*c*x**2 + a*d*x**5 + b*c*x**5 + b*d*x**8),x)*a**2*b*c*d*x**7)/(28*a**2*c**
2*x**7)
```

3.686 $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 318

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(c+dx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{10cx^{10}} - \frac{(bc-10ad)\sqrt[3]{a+bx^3}}{70ac^2x^7} + \frac{(3b^2c^2+5abcd-35a^2d^2)\sqrt[3]{a+bx^3}}{140a^2c^3x^4} - \frac{(9b^3c^3+15ab^2c^2d+35a^2bcd^2-140a^3d^3)\sqrt[3]{a+bx^3}}{140a^3c^4x} + \frac{d^3\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{13/3}} - \frac{d^3\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^{13/3}} + \frac{d^3\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{13/3}}$$

output
```
-1/10*(b*x^3+a)^(1/3)/c/x^10-1/70*(-10*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x^7+
1/140*(-35*a^2*d^2+5*a*b*c*d+3*b^2*c^2)*(b*x^3+a)^(1/3)/a^2/c^3/x^4-1/140*
(-140*a^3*d^3+35*a^2*b*c*d^2+15*a*b^2*c^2*d+9*b^3*c^3)*(b*x^3+a)^(1/3)/a^3
/c^4/x+1/3*d^3*(-a*d+b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)
/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(13/3)-1/6*d^3*(-a*d+b*c)^(1/3)*ln(d*
x^3+c)/c^(13/3)+1/2*d^3*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*
x^3+a)^(1/3))/c^(13/3)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.75 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\,(c+dx^3)}\,dx$$

$$= \frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}(-9b^3c^3x^9+3ab^2c^2x^6(c-5dx^3)+a^2bcx^3(-2c^2+5cdx^3-35d^2x^6)+a^3(-14c^3+20c^2dx^3-35cd^2x^6+140d^3x^9))}{a^3x^{10}} - 70\sqrt{-6}}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x]
```

output
```
((3*c^(1/3)*(a + b*x^3)^(1/3)*(-9*b^3*c^3*x^9 + 3*a*b^2*c^2*x^6*(c - 5*d*x
^3) + a^2*b*c*x^3*(-2*c^2 + 5*c*d*x^3 - 35*d^2*x^6) + a^3*(-14*c^3 + 20*c^
2*d*x^3 - 35*c*d^2*x^6 + 140*d^3*x^9)))/(a^3*x^10) - 70*Sqrt[-6 - (6*I)*Sq
rt[3]]*d^3*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c
- a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (70*I)*(I +
 Sqrt[3])*d^3*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3]
)*c^(1/3)*(a + b*x^3)^(1/3)] + 35*(1 - I*Sqrt[3])*d^3*(b*c - a*d)^(1/3)*Lo
g[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(
a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(420*c^(13/
3))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {975, 1053, 27, 1053, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{-9bdx^3+bc-10ad}{x^8(bx^3+a)^{2/3}(dx^3+c)}dx}{10c}-\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int \frac{2\left(3bd(bc-10ad)x^3+3b^2c^2-35a^2d^2+5abcd\right)}{x^5\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{7ac}-\frac{\sqrt[3]{a+bx^3}(bc-10ad)}{7acx^7}}{10c}-\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{2\int \frac{3bd(bc-10ad)x^3+3b^2c^2-35a^2d^2+5abcd}{x^5\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{7ac}-\frac{\sqrt[3]{a+bx^3}(bc-10ad)}{7acx^7}}{10c}-\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{2\left(-\frac{\int \frac{9b^3c^3+15ab^2dc^2+35a^2bd^2c-140a^3d^3+3bd\left(3b^2c^2+5abdc-35a^2d^2\right)x^3}{x^2\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{3b^2c}{a}-\frac{35ad^2}{c}+5bd\right)}{4x^4}\right)}{7ac}-\frac{\sqrt[3]{a+bx^3}(bc-10ad)}{7acx^7}}{10c}-\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \text{1053}$$

$$-\frac{-\frac{2\left(-\frac{-\frac{\int \frac{140a^3d^3(bc-ad)x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{ac}-\frac{\sqrt[3]{a+bx^3}\left(-140a^3d^3+35a^2bcd^2+15ab^2c^2d+9b^3c^3\right)}{acx}}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{3b^2c}{a}-\frac{35ad^2}{c}+5bd\right)}{4x^4}\right)}{7ac}-\frac{\sqrt[3]{a+bx^3}(\cdots}{7acx^7}}{10c}$$

$$\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{2\left(-\frac{-\frac{140a^2d^3(bc-ad)\int \frac{x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{c}-\frac{\sqrt[3]{a+bx^3}\left(-140a^3d^3+35a^2bcd^2+15ab^2c^2d+9b^3c^3\right)}{acx}}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{3b^2c}{a}-\frac{35ad^2}{c}+5bd\right)}{4x^4}\right)}{7ac}-\cdots}{10c}$$

$$\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \; 992$$

$$-\frac{-\frac{2\left(-\frac{-\frac{140a^2d^3(bc-ad)\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{c}-\frac{\sqrt[3]{a+bx^3}\left(-140a^3d^3+3\cdots\right)}{ac\cdots}}{4ac}\cdots\right)}{7ac}\cdots}{10c}$$

$$\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x]
```

output
```
-1/10*(a + b*x^3)^(1/3)/(c*x^10) + (-1/7*((b*c - 10*a*d)*(a + b*x^3)^(1/3)
)/(a*c*x^7) - (2*(-1/4*(((3*b^2*c)/a + 5*b*d - (35*a*d^2)/c)*(a + b*x^3)^(
1/3))/x^4 - (-(((9*b^3*c^3 + 15*a*b^2*c^2*d + 35*a^2*b*c*d^2 - 140*a^3*d^3
)*(a + b*x^3)^(1/3))/(a*c*x)) - (140*a^2*d^3*(b*c - a*d)*(-(ArcTan[(1 + (2
*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(1/
3)*(b*c - a*d)^(2/3))) + Log[c + d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) - Lo
g[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a*d
)^(2/3))))/c)/(4*a*c)))/(7*a*c))/(10*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{c\left(\left(\frac{9}{14}b^2x^6-\frac{6}{7}ab\,x^3+a^2\right)\left(b\,x^3+a\right)c^3-\frac{10ad\left(b\,x^3+a\right)\left(-\frac{3b\,x^3}{4}+a\right)x^3c^2}{7}+\frac{5\left(b\,x^3+a\right)a^2c\,d^2x^6}{2}-10a^3d^3x^9\right)\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}\left(b\,x^3\right.}{}$

input
```
int((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/10*(c*((9/14*b^2*x^6-6/7*a*b*x^3+a^2)*(b*x^3+a)*c^3-10/7*a*d*(b*x^3+a)*
(-3/4*b*x^3+a)*x^3*c^2+5/2*(b*x^3+a)*a^2*c*d^2*x^6-10*a^3*d^3*x^9)*((a*d-b
*c)/c)^(2/3)*(b*x^3+a)^(1/3)-5/3*a^3*d^3*x^10*(a*d-b*c)*(2*3^(1/2)*arctan(
1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/
x)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^
3+a)^(2/3))/x^2)-2*ln(((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)))/((a*d-b
*c)/c)^(2/3)/x^10/c^5/a^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/x**11/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(x**11*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^11), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^11), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx = \int \frac{\left(b\,x^3+a\right)^{1/3}}{x^{11}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^11*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx = \frac{-14(b\,x^3+a)^{\frac{1}{3}}\,a^3c^2+20(b\,x^3+a)^{\frac{1}{3}}\,a^3cd\,x^3-35(b\,x^3+a)^{\frac{1}{3}}\,a^3d^2x^6-2(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,c^2x^3+5(b\,x^3+a)}{}$$

input
```
int((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x)
```

output
```
( - 14*(a + b*x**3)**(1/3)*a**3*c**2 + 20*(a + b*x**3)**(1/3)*a**3*c*d*x**
3 - 35*(a + b*x**3)**(1/3)*a**3*d**2*x**6 - 2*(a + b*x**3)**(1/3)*a**2*b*c
**2*x**3 + 5*(a + b*x**3)**(1/3)*a**2*b*c*d*x**6 + 105*(a + b*x**3)**(1/3)
*a**2*b*d**2*x**9 + 3*(a + b*x**3)**(1/3)*a*b**2*c**2*x**6 - 15*(a + b*x**
3)**(1/3)*a*b**2*c*d*x**9 - 9*(a + b*x**3)**(1/3)*b**3*c**2*x**9 - 140*int
((a + b*x**3)**(1/3)/(a*c*x**2 + a*d*x**5 + b*c*x**5 + b*d*x**8),x)*a**4*d
**3*x**10 + 140*int((a + b*x**3)**(1/3)/(a*c*x**2 + a*d*x**5 + b*c*x**5 +
b*d*x**8),x)*a**3*b*c*d**2*x**10)/(140*a**3*c**3*x**10)
```

3.687 $\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \frac{x^7 \sqrt[3]{a+bx^3} \operatorname{AppellF1}\left(\frac{7}{3},-\frac{1}{3},1,\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/7*x^7*(b*x^3+a)^(1/3)*AppellF1(7/3,-1/3,1,10/3,-b*x^3/a,-d*x^3/c)/c/(1+b
*x^3/a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 281 vs. $2(64) = 128$.

Time = 6.84 (sec) , antiderivative size = 281, normalized size of antiderivative = 4.39

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \frac{x\left(4(a+bx^3)\left(-5bc+ad+2bdx^3\right) - \frac{(-10b^2c^2+5abcd+a^2d^2)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c} + \frac{\cdots}{(c+dx^3)(-\cdots}\right.}{40bd^2(a+bx^3)\cdots}$$

input
```
Integrate[(x^6*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(x*(4*(a + b*x^3)*(-5*b*c + a*d + 2*b*d*x^3) - ((-10*b^2*c^2 + 5*a*b*c*d +
 a^2*d^2)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a
), -((d*x^3)/c)])/c + (16*a^2*c^2*(-5*b*c + a*d)*AppellF1[1/3, 2/3, 1, 4/3
, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1,
4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -
((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a)
, -((d*x^3)/c)])))))/(40*b*d^2*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$$

$$\Big\downarrow \ 1013$$

$$\frac{\sqrt[3]{a+bx^3} \int \frac{x^6 \sqrt[3]{\frac{bx^3}{a}+1}}{dx^3+c}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \ 1012$$

$$\frac{x^7 \sqrt[3]{a+bx^3} \operatorname{AppellF1}\left(\frac{7}{3},-\frac{1}{3},1,\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(x^6*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(x^7*(a + b*x^3)^(1/3)*AppellF1[7/3, -1/3, 1, 10/3, -((b*x^3)/a), -((d*x^3
)/c)])/(7*c*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{6} \left(b \,x^{3}+a \right)^{\frac{1}{3}}}{d \,x^{3}+c}d x$$

input
```
int(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx=\text{Timed out}$$

input
```
integrate(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^6\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate(x**6*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**6*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^6}{dx^3+c}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*x^6/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^6}{dx^3+c}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*x^6/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^6\,{\left(b\,x^3+a\right)}^{1/3}}{d\,x^3+c}\,dx$$

input
```
int((x^6*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

output
```
int((x^6*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{(b\,x^3+a)^{\frac{1}{3}}\,adx - 5(b\,x^3+a)^{\frac{1}{3}}\,bcx + 2(b\,x^3+a)^{\frac{1}{3}}\,bd\,x^4 - \left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right) a^2cd + 5\left(\int \frac{(b\,x^3}{bd\,x^6+ad\,x}\right.}{}$$

input
```
int(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
((a + b*x**3)**(1/3)*a*d*x - 5*(a + b*x**3)**(1/3)*b*c*x + 2*(a + b*x**3)*
*(1/3)*b*d*x**4 - int((a + b*x**3)**(1/3)/(a*c + a*d*x**3 + b*c*x**3 + b*d
*x**6),x)*a**2*c*d + 5*int((a + b*x**3)**(1/3)/(a*c + a*d*x**3 + b*c*x**3
+ b*d*x**6),x)*a*b*c**2 - int(((a + b*x**3)**(1/3)*x**3)/(a*c + a*d*x**3 +
 b*c*x**3 + b*d*x**6),x)*a**2*d**2 - 5*int(((a + b*x**3)**(1/3)*x**3)/(a*c
 + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b*c*d + 10*int(((a + b*x**3)**(1/3
)*x**3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**2*c**2)/(10*b*d**2)
```

3.688 $\int \frac{x^3 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \frac{x^4 \sqrt[3]{a+bx^3} \operatorname{AppellF1}\left(\frac{4}{3},-\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{1+\frac{bx^3}{a}}}$$

output

```
1/4*x^4*(b*x^3+a)^(1/3)*AppellF1(4/3,-1/3,1,7/3,-b*x^3/a,-d*x^3/c)/c/(1+b*
x^3/a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 240 vs. 2(64) = 128.

Time = 6.27 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.75

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$$
$$= \frac{x\left(\frac{(-2bc+ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c} + 4\left(a+bx^3+\frac{4a^2c^2 \text{A}}{(c+dx^3)\left(-4ac\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(\right.\right.}\right.\right.}{8d\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(x^3*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(x*(((-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])/c + 4*(a + b*x^3 + (4*a^2*c^2*AppellF1[1/3, 2/
3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3,
 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3,
2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((
b*x^3)/a), -((d*x^3)/c)])))))/(8*d*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$$

$$\Big\downarrow \; 1013$$

$$\frac{\sqrt[3]{a+bx^3} \int \frac{x^3 \sqrt[3]{\frac{bx^3}{a}+1}}{dx^3+c}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \; 1012$$

$$\frac{x^4 \sqrt[3]{a+bx^3} \operatorname{AppellF1}\left(\frac{4}{3},-\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(x^3*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(x^4*(a + b*x^3)^(1/3)*AppellF1[4/3, -1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*c*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{3} \left(b \,x^{3}+a \right)^{\frac{1}{3}}}{d \,x^{3}+c}d x$$

input
```
int(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx=\text{Timed out}$$

input
```
integrate(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate(x**3*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**3*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^3}{dx^3+c}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*x^3/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^3}{dx^3+c}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*x^3/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{x^3\,{\left(b\,x^3+a\right)}^{1/3}}{d\,x^3+c}\,dx$$

input
```
int((x^3*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

output
```
int((x^3*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$
$$=\frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x-\left(\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)ac+\left(\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x^3}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)ad-2\left(\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x^3}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)}{2d}$$

input
```
int(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
((a + b*x**3)**(1/3)*x - int((a + b*x**3)**(1/3)/(a*c + a*d*x**3 + b*c*x**
3 + b*d*x**6),x)*a*c + int(((a + b*x**3)**(1/3)*x**3)/(a*c + a*d*x**3 + b*
c*x**3 + b*d*x**6),x)*a*d - 2*int(((a + b*x**3)**(1/3)*x**3)/(a*c + a*d*x*
*3 + b*c*x**3 + b*d*x**6),x)*b*c)/(2*d)
```

3.689 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a)
^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(59) = 118.

Time = 0.03 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$
$$= \frac{4acx\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(4ac\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(-3ad\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bc\,A\right.\right.}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

output
```
(4*a*c*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*
x^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -(
(d*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), -((d*x
^3)/c)] + b*c*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3} \, dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{\frac{bx^3}{a}+1}}{dx^3+c} dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x \sqrt[3]{a+bx^3} \text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c \sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(c*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}}{d\,x^{3}+c}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(1/3)/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{d\,x^{3}+c}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int((a + b*x**3)**(1/3)/(c + d*x**3),x)
```

3.690 $\int \frac{\sqrt[3]{a+bx^3}}{x^3(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx=-\frac{\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{1}{3},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-1/2*(b*x^3+a)^(1/3)*AppellF1(-2/3,-1/3,1,1/3,-b*x^3/a,-d*x^3/c)/c/x^2/(1+
b*x^3/a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 327 vs. 2(64) = 128.

Time = 10.23 (sec) , antiderivative size = 327, normalized size of antiderivative = 5.11

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx$$
$$=\frac{-bdx^6\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{c\left(16ac\left(bdx^6+a\left(c+3dx^3\right)\right)\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-4x^3\cdots\right.}{\left(c+dx^3\right)\left(-4ac\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\cdots\right.}}{8c^2x^2\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^3*(c + d*x^3)),x]
```

output
```
(-(b*d*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
-((d*x^3)/c)]) + (c*(16*a*c*(b*d*x^6 + a*(c + 3*d*x^3))*AppellF1[1/3, 2/3,
 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 4*x^3*(a + b*x^3)*(c + d*x^3)*(3*a*
d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[
4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(-4*a*c*Appe
llF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4
/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(8*c^2*x^2*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{\frac{bx^3}{a}+1}}{x^3(dx^3+c)}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{1}{3},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^3*(c + d*x^3)),x]
```

output
```
-1/2*((a + b*x^3)^(1/3)*AppellF1[-2/3, -1/3, 1, 1/3, -((b*x^3)/a), -((d*x^
3)/c)])/(c*x^2*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x^{3}\left(d\,x^{3}+c\right)}dx$$

input
```
int((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/x**3/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(x**3*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx = \int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx = \int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx=\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^3\,\left(d\,x^3+c\right)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^3*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^3*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx=\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{d\,x^6+c\,x^3}dx$$

input
```
int((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x)
```

output
```
int((a + b*x**3)**(1/3)/(c*x**3 + d*x**6),x)
```

3.691 $\int \frac{\sqrt[3]{a+bx^3}}{x^6(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6(c+dx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{5}{3},-\frac{1}{3},1,-\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-1/5*(b*x^3+a)^(1/3)*AppellF1(-5/3,-1/3,1,-2/3,-b*x^3/a,-d*x^3/c)/c/x^5/(1
+b*x^3/a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 289 vs. 2(64) = 128.

Time = 10.31 (sec) , antiderivative size = 289, normalized size of antiderivative = 4.52

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6(c+dx^3)}\,dx$$
$$= \frac{-\frac{4(a+bx^3)(2ac+bcx^3-5adx^3)}{ac^2x^5} + \frac{bd(-bc+5ad)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3} + \frac{1}{c(c+dx^3)\left(-4ac\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\right.\right.}}{40\,(a+bx^3)^{2/3}}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^6*(c + d*x^3)),x]
```

output
```
((-4*(a + b*x^3)*(2*a*c + b*c*x^3 - 5*a*d*x^3))/(a*c^2*x^5) + (b*d*(-(b*c)
 + 5*a*d)*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a
), -((d*x^3)/c)])/(a*c^3) + (16*(b^2*c^2 + 5*a*b*c*d - 10*a^2*d^2)*x*Appel
lF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*(c + d*x^3)*(-4*a*c*
AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*Appell
F1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3
, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(40*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6 (c+dx^3)} \, dx$$

$$\Big\downarrow \ 1013$$

$$\frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{\frac{bx^3}{a}+1}}{x^6 (dx^3+c)}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \ 1012$$

$$-\frac{\sqrt[3]{a+bx^3} \operatorname{AppellF1}\left(-\frac{5}{3},-\frac{1}{3},1,-\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5 \sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^6*(c + d*x^3)),x]
```

output
```
-1/5*((a + b*x^3)^(1/3)*AppellF1[-5/3, -1/3, 1, -2/3, -((b*x^3)/a), -((d*x
^3)/c)])/(c*x^5*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x^{6}\left(d\,x^{3}+c\right)}dx$$

input
```
int((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6(c+dx^3)}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{x^6(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/x**6/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(x**6*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(c+dx^3\right)}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{x^6\,\left(d\,x^3+c\right)} \,d x$$

input
```
int((a + b*x^3)^(1/3)/(x^6*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^6*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(c+dx^3\right)}\,dx=\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{d\,x^{9}+c\,x^{6}}d x$$

input
```
int((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x)
```

output
```
int((a + b*x**3)**(1/3)/(c*x**6 + d*x**9),x)
```

3.692 $\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 266

$$\begin{aligned}\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx &= -\frac{c^3(a+bx^3)^{2/3}}{2d^4} \\ &+ \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{8/3}}{8b^3d^2} \\ &+ \frac{(a+bx^3)^{11/3}}{11b^3d} - \frac{c^3(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{14/3}} \\ &+ \frac{c^3(bc-ad)^{2/3}\log(c+dx^3)}{6d^{14/3}} - \frac{c^3(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{14/3}}\end{aligned}$$

output
```
-1/2*c^3*(b*x^3+a)^(2/3)/d^4+1/5*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(5/3)
/b^3/d^3-1/8*(2*a*d+b*c)*(b*x^3+a)^(8/3)/b^3/d^2+1/11*(b*x^3+a)^(11/3)/b^3
/d-1/3*c^3*(-a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+
b*c)^(1/3))*3^(1/2))*3^(1/2)/d^(14/3)+1/6*c^3*(-a*d+b*c)^(2/3)*ln(d*x^3+c)
/d^(14/3)-1/2*c^3*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(
1/3))/d^(14/3)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{\frac{3d^{2/3}(a+bx^3)^{2/3}(18a^3d^3+3a^2bd^2(11c-4dx^3)+2ab^2d(44c^2-11cdx^3+5d^2x^6)+b^3(-220c^3+88c^2dx^3-55cd^2x^6+}{b^3}}{}$$

input
```
Integrate[(x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
((3*d^(2/3)*(a + b*x^3)^(2/3)*(18*a^3*d^3 + 3*a^2*b*d^2*(11*c - 4*d*x^3) +
 2*a*b^2*d*(44*c^2 - 11*c*d*x^3 + 5*d^2*x^6) + b^3*(-220*c^3 + 88*c^2*d*x^
3 - 55*c*d^2*x^6 + 40*d^3*x^9)))/b^3 - 440*Sqrt[3]*c^3*(b*c - a*d)^(2/3)*A
rcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 440
*c^3*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]
+ 220*c^3*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1
/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(1320*d^(14/3))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{x^9(bx^3+a)^{2/3}}{dx^3+c}dx^3$$

$$\Big\downarrow \; 99$$

$$\frac{1}{3}\int \left(-\frac{(bx^3+a)^{2/3}c^3}{d^3(dx^3+c)}+\frac{(bx^3+a)^{8/3}}{b^2d}+\frac{(-bc-2ad)(bx^3+a)^{5/3}}{b^2d^2}+\frac{(b^2c^2+abdc+a^2d^2)(bx^3+a)^{2/3}}{b^2d^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3(a+bx^3)^{5/3}(a^2d^2+abcd+b^2c^2)}{5b^3d^3}-\frac{\sqrt{3}c^3(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{14/3}}-\frac{3(a+bx^3)^{8/3}(2ad\right.$$

input
```
Int[(x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
((-3*c^3*(a + b*x^3)^(2/3))/(2*d^4) + (3*(b^2*c^2 + a*b*c*d + a^2*d^2)*(a
+ b*x^3)^(5/3))/(5*b^3*d^3) - (3*(b*c + 2*a*d)*(a + b*x^3)^(8/3))/(8*b^3*d
^2) + (3*(a + b*x^3)^(11/3))/(11*b^3*d) - (Sqrt[3]*c^3*(b*c - a*d)^(2/3)*A
rcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/d^(1
4/3) + (c^3*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(2*d^(14/3)) - (3*c^3*(b*c -
 a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(14/3
)))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{\frac{27\left(\left(\frac{20}{9}b^{2}x^{6}-\frac{5}{3}ab\,x^{3}+a^{2}\right)\left(b\,x^{3}+a\right)d^{3}+\frac{11c\left(b\,x^{3}+a\right)\left(-\frac{5b\,x^{3}}{3}+a\right)b\,d^{2}}{6}+\frac{44b^{2}c^{2}\left(b\,x^{3}+a\right)d}{9}-\frac{110b^{3}c^{3}}{9}\right)d\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}{110}+b^{3}\cdots}{\cdots}$

input
```
int(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(27/110*((20/9*b^2*x^6-5/3*a*b*x^3+a^2)*(b*x^3+a)*d^3+11/6*c*(b*x^3+a)
*(-5/3*b*x^3+a)*b*d^2+44/9*b^2*c^2*(b*x^3+a)*d-110/9*b^3*c^3)*d*(b*x^3+a)^
(2/3)*((a*d-b*c)/d)^(1/3)+b^3*c^3*(a*d-b*c)*(-2*arctan(1/3*3^(1/2)*(2*(b*x
^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*c)/d)^(1/3))*3^(1/2)+ln((b*x^3+a)
^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))-2*ln((b*x^
3+a)^(1/3)-((a*d-b*c)/d)^(1/3))))/((a*d-b*c)/d)^(1/3)/b^3/d^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. 2(219) = 438.

Time = 0.22 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.71

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx =$$

$$-\frac{440\sqrt{3}b^{3}c^{3}\left(-\frac{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}{d^{2}}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^{3}+a)^{\frac{1}{3}}d\left(-\frac{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}{d^{2}}\right)^{\frac{1}{3}}+\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+220\,b^{3}c^{3}\left(-\frac{b^{2}c^{2}-2\cdots}{\cdots}\right)\cdots}{\cdots}$$

input
```
integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/1320*(440*sqrt(3)*b^3*c^3*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*
arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^
2)/d^2)^(1/3) + sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 220*b^3*c^3*(-(b^2*c^2
 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d
)*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) - 440*b^3*c^3*(-(b^2*c^2 -
 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/
d^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)) - 3*(40*b^3*d^3*x^9 - 5*(11*b^
3*c*d^2 - 2*a*b^2*d^3)*x^6 - 220*b^3*c^3 + 88*a*b^2*c^2*d + 33*a^2*b*c*d^2
 + 18*a^3*d^3 + 2*(44*b^3*c^2*d - 11*a*b^2*c*d^2 - 6*a^2*b*d^3)*x^3)*(b*x^
3 + a)^(2/3))/(b^3*d^4)
```

Sympy [F]

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^{11}(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**11*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**11*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.54

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx =$$
$$-\frac{\left(b^{37}c^4d^7\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ab^{36}c^3d^8\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{37}cd^{11}-ab^{36}d^{12})}$$
$$-\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^6}$$
$$+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^6}$$
$$-\frac{220\,(bx^3+a)^{\frac{2}{3}}b^{33}c^3d^7-88\,(bx^3+a)^{\frac{5}{3}}b^{32}c^2d^8+55\,(bx^3+a)^{\frac{8}{3}}b^{31}cd^9-88\,(bx^3+a)^{\frac{5}{3}}ab^{31}cd^9-40\,(bx^3+a}{440\,b^{33}d^{11}}$$

input
```
integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*(b^37*c^4*d^7*(-(b*c - a*d)/d)^(1/3) - a*b^36*c^3*d^8*(-(b*c - a*d)/d
)^(1/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/
d)^(1/3)))/(b^37*c*d^11 - a*b^36*d^12) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2
/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/
(-(b*c - a*d)/d)^(1/3))/d^6 + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^3*log((b*x^3
+ a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(
2/3))/d^6 - 1/440*(220*(b*x^3 + a)^(2/3)*b^33*c^3*d^7 - 88*(b*x^3 + a)^(5/
3)*b^32*c^2*d^8 + 55*(b*x^3 + a)^(8/3)*b^31*c*d^9 - 88*(b*x^3 + a)^(5/3)*a
*b^31*c*d^9 - 40*(b*x^3 + a)^(11/3)*b^30*d^10 + 110*(b*x^3 + a)^(8/3)*a*b^
30*d^10 - 88*(b*x^3 + a)^(5/3)*a^2*b^30*d^10)/(b^33*d^11)
```

Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.84

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx = \left(\frac{3\,a^2}{5\,b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{5\,b^3\,d}\right)\,\left(b\,x^3+a\right)^{5/3} - \left(\frac{3\,a}{8\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{8\,b^6\,d^2}\right)\,\left(b\,x^3+a\right)^{8/3} - \left(b\,x^3+a\right)^{2/3}\left(\frac{a^3}{2\,b^3\,d}+\frac{\left(\frac{3\,a^2}{b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{b^3\,d}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{2\,b^3\,d}\right)+\frac{\left(b\,x^3+a\right)^{11/3}}{11\,b^3\,d}-\frac{c^3\,\ln\left(\frac{(b\,x^3+}{}\right.}{}$$

input
```
int((x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

output
```
((3*a^2)/(5*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
 - a*b^3*d))/(5*b^3*d))*(a + b*x^3)^(5/3) - ((3*a)/(8*b^3*d) + (b^4*c - a*
b^3*d)/(8*b^6*d^2))*(a + b*x^3)^(8/3) - (a + b*x^3)^(2/3)*(a^3/(2*b^3*d) +
 (((3*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
 - a*b^3*d))/(b^3*d))*(b^4*c - a*b^3*d))/(2*b^3*d)) + (a + b*x^3)^(11/3)/(
11*b^3*d) - (c^3*log(((a + b*x^3)^(1/3)*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7
*d))/d^7 - (c^6*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(28/3)))*(a*
d - b*c)^(2/3))/(3*d^(14/3)) - (c^3*log((c^6*((3^(1/2)*1i)/2 + 1/2)*(a*d -
 b*c)^(7/3))/d^(22/3) + (c^6*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^7)*((3^(1/
2)*1i)/2 - 1/2)*(a*d - b*c)^(2/3))/(3*d^(14/3)) + (c^3*log((c^6*(a + b*x^3
)^(1/3)*(a*d - b*c)^2)/d^7 - (c^6*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(7/3))/(4
*d^(22/3)))*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(2/3))/d^(14/3)
```

Reduce [F]

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{18(b\,x^3+a)^{\frac{2}{3}}\,a^3d^2+33(b\,x^3+a)^{\frac{2}{3}}\,a^2bcd-12(b\,x^3+a)^{\frac{2}{3}}\,a^2b\,d^2x^3-132(b\,x^3+a)^{\frac{2}{3}}}{}$$

input
```
int(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output

```
(18*(a + b*x**3)**(2/3)*a**3*d**2 + 33*(a + b*x**3)**(2/3)*a**2*b*c*d - 12
*(a + b*x**3)**(2/3)*a**2*b*d**2*x**3 - 132*(a + b*x**3)**(2/3)*a*b**2*c**
2 - 22*(a + b*x**3)**(2/3)*a*b**2*c*d*x**3 + 10*(a + b*x**3)**(2/3)*a*b**2
*d**2*x**6 + 88*(a + b*x**3)**(2/3)*b**3*c**2*x**3 - 55*(a + b*x**3)**(2/3
)*b**3*c*d*x**6 + 40*(a + b*x**3)**(2/3)*b**3*d**2*x**9 + 440*int(((a + b*
x**3)**(2/3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b**3*c**2*d
 - 440*int(((a + b*x**3)**(2/3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**
6),x)*b**4*c**3)/(440*b**3*d**3)
```

3.693 $\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 223

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{c^2(a+bx^3)^{2/3}}{2d^3} - \frac{(bc+ad)(a+bx^3)^{5/3}}{5b^2d^2} + \frac{(a+bx^3)^{8/3}}{8b^2d} + \frac{c^2(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}} - \frac{c^2(bc-ad)^{2/3}\log(c+dx^3)}{6d^{11/3}} + \frac{c^2(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}}$$

output
```
1/2*c^2*(b*x^3+a)^(2/3)/d^3-1/5*(a*d+b*c)*(b*x^3+a)^(5/3)/b^2/d^2+1/8*(b*x
^3+a)^(8/3)/b^2/d+1/3*c^2*(-a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+
a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/d^(11/3)-1/6*c^2*(-a*d+b*c)^(2
/3)*ln(d*x^3+c)/d^(11/3)+1/2*c^2*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1
/3)*(b*x^3+a)^(1/3))/d^(11/3)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.19

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{\frac{3d^{2/3}(a+bx^3)^{2/3}(-3a^2d^2+2abd(-4c+dx^3)+b^2(20c^2-8cdx^3+5d^2x^6))}{b^2}+40\sqrt{3}c^2(bc-ad)^{2/3}\arctan}{}$$

input
```
Integrate[(x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
((3*d^(2/3)*(a + b*x^3)^(2/3)*(-3*a^2*d^2 + 2*a*b*d*(-4*c + d*x^3) + b^2*(
20*c^2 - 8*c*d*x^3 + 5*d^2*x^6)))/b^2 + 40*Sqrt[3]*c^2*(b*c - a*d)^(2/3)*A
rcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 40*
c^2*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] -
 20*c^2*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3
)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(120*d^(11/3))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6(bx^3+a)^{2/3}}{dx^3+c}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{(bx^3+a)^{2/3}c^2}{d^2(dx^3+c)}+\frac{(bx^3+a)^{5/3}}{bd}+\frac{(-bc-ad)(bx^3+a)^{2/3}}{bd^2}\right)dx^3$$

$$\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}c^2(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{11/3}}-\frac{3\left(a+bx^3\right)^{5/3}(ad+bc)}{5b^2d^2}+\frac{3\left(a+bx^3\right)^{8/3}}{8b^2d}-\frac{c^2(bc-ad)^{2/3}}{2d^{11}}\right.$$

input
```
Int[(x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
((3*c^2*(a + b*x^3)^(2/3))/(2*d^3) - (3*(b*c + a*d)*(a + b*x^3)^(5/3))/(5*
b^2*d^2) + (3*(a + b*x^3)^(8/3))/(8*b^2*d) + (Sqrt[3]*c^2*(b*c - a*d)^(2/3
)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/d
^(11/3) - (c^2*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(2*d^(11/3)) + (3*c^2*(b*
c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1
1/3)))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$-\frac{\frac{9d\left(\left(b\,x^{3}+a\right)\left(-\frac{5b\,x^{3}}{3}+a\right)d^{2}+\frac{8\left(b\,x^{3}+a\right)bcd}{3}-\frac{20b^{2}c^{2}}{3}\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}{20}+b^{2}c^{2}(ad-bc)\left(-2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\cdots\right.}{3\left(\frac{ad-bc}{d}\right.}\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}d\cdots}$

input
```
int(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/((a*d-b*c)/d)^(1/3)*(9/20*d*((b*x^3+a)*(-5/3*b*x^3+a)*d^2+8/3*(b*x^3+
a)*b*c*d-20/3*b^2*c^2)*(b*x^3+a)^(2/3)*((a*d-b*c)/d)^(1/3)+b^2*c^2*(a*d-b*
c)*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*
c)/d)^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3
)+((a*d-b*c)/d)^(2/3))-2*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))))/d^4/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(181) = 362$.

Time = 0.21 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.78

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{40\sqrt{3}b^2c^2\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)-\cdots}{\cdots}$$

input
```
integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/120*(40*sqrt(3)*b^2*c^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arct
an(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^
2)^(1/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 20*b^2*c^2*((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) - (b*c - a*d)*((b^2*
c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) + 40*b^2*c^2*((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)/d^2)^(1/3)*log(-d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) -
 (b*x^3 + a)^(1/3)*(b*c - a*d)) + 3*(5*b^2*d^2*x^6 + 20*b^2*c^2 - 8*a*b*c*
d - 3*a^2*d^2 - 2*(4*b^2*c*d - a*b*d^2)*x^3)*(b*x^3 + a)^(2/3))/(b^2*d^3)
```

Sympy [F]

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^8(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**8*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**8*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.57

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{\left(b^{19}c^3d^5\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ab^{18}c^2d^6\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{19}cd^8 - ab^{18}d^9)}$$
$$+ \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^5}$$
$$- \frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^2\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^5}$$
$$+ \frac{20\,(bx^3+a)^{\frac{2}{3}}b^{16}c^2d^5 - 8\,(bx^3+a)^{\frac{5}{3}}b^{15}cd^6 + 5\,(bx^3+a)^{\frac{8}{3}}b^{14}d^7 - 8\,(bx^3+a)^{\frac{5}{3}}ab^{14}d^7}{40\,b^{16}d^8}$$

input
```
integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*(b^19*c^3*d^5*(-(b*c - a*d)/d)^(1/3) - a*b^18*c^2*d^6*(-(b*c - a*d)/d)
^(1/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b^19*c*d^8 - a*b^18*d^9) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)
*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(
b*c - a*d)/d)^(1/3))/d^5 - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^2*log((b*x^3 + a
)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3
))/d^5 + 1/40*(20*(b*x^3 + a)^(2/3)*b^16*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^1
5*c*d^6 + 5*(b*x^3 + a)^(8/3)*b^14*d^7 - 8*(b*x^3 + a)^(5/3)*a*b^14*d^7)/(
b^16*d^8)
```

Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.73

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx = \left(\frac{a^2}{2\,b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b^3\,c-a\,b^2\,d\right)}{2\,b^2\,d}\right)\,\left(b\,x^3+a\right)^{2/3} - \left(\frac{2\,a}{5\,b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{5\,b^4\,d^2}\right)\,\left(b\,x^3+a\right)^{5/3}+\frac{\left(b\,x^3+a\right)^{8/3}}{8\,b^2\,d} + \frac{c^2\,\ln\left(\frac{\left(b\,x^3+a\right)^{1/3}\,\left(a^2\,c^4\,d^2-2\,a\,b\,c^5\,d+b^2\,c^6\right)}{d^5}-\frac{c^4\,{\left(a\,d-b\,c\right)}^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{9\,d^{22/3}}\right)\,{\left(a\,d-b\,c\right)}^{2/3}}{3\,d^{11/3}} - \frac{c^2\,\ln\left(\frac{c^4\,{\left(b\,x^3+a\right)}^{1/3}\,{\left(a\,d-b\,c\right)}^2}{d^5}-\frac{c^4\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{7/3}}{d^{16/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{2/3}}{3\,d^{11/3}} + \frac{c^2\,\ln\left(\frac{c^4\,{\left(b\,x^3+a\right)}^{1/3}\,{\left(a\,d-b\,c\right)}^2}{d^5}-\frac{c^4\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(a\,d-b\,c\right)}^{7/3}}{4\,d^{16/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{2/3}}{d^{11/3}}$$

input
```
int((x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

output
```
(a^2/(2*b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a
*b^2*d))/(2*b^2*d))*(a + b*x^3)^(2/3) - ((2*a)/(5*b^2*d) + (b^3*c - a*b^2*
d)/(5*b^4*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^2*d) + (c^2*log
(((a + b*x^3)^(1/3)*(b^2*c^6 + a^2*c^4*d^2 - 2*a*b*c^5*d))/d^5 - (c^4*(a*d
 - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(22/3)))*(a*d - b*c)^(2/3))/(3*d
^(11/3)) - (c^2*log((c^4*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^5 - (c^4*((3^(
1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(16/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d
 - b*c)^(2/3))/(3*d^(11/3)) + (c^2*log((c^4*(a + b*x^3)^(1/3)*(a*d - b*c)^
2)/d^5 - (c^4*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(7/3))/(4*d^(16/3)))*((3^(1/2
)*1i)/6 - 1/6)*(a*d - b*c)^(2/3))/d^(11/3)
```

Reduce [F]

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{-3(b\,x^3+a)^{\frac{2}{3}}\,a^2d+12(b\,x^3+a)^{\frac{2}{3}}\,abc+2(b\,x^3+a)^{\frac{2}{3}}\,abd\,x^3-8(b\,x^3+a)^{\frac{2}{3}}\,b^2c\,x^3+}{}$$

input

```
int(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output

```
( - 3*(a + b*x**3)**(2/3)*a**2*d + 12*(a + b*x**3)**(2/3)*a*b*c + 2*(a + b
*x**3)**(2/3)*a*b*d*x**3 - 8*(a + b*x**3)**(2/3)*b**2*c*x**3 + 5*(a + b*x*
*3)**(2/3)*b**2*d*x**6 - 40*int(((a + b*x**3)**(2/3)*x**5)/(a*c + a*d*x**3
 + b*c*x**3 + b*d*x**6),x)*a*b**2*c*d + 40*int(((a + b*x**3)**(2/3)*x**5)/
(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**3*c**2)/(40*b**2*d**2)
```

3.694 $\int \frac{x^5\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 188

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx = -\frac{c(a+bx^3)^{2/3}}{2d^2}+\frac{(a+bx^3)^{5/3}}{5bd} - \frac{c(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}}+\frac{c(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{8/3}} - \frac{c(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}}$$

output

```
-1/2*c*(b*x^3+a)^(2/3)/d^2+1/5*(b*x^3+a)^(5/3)/b/d-1/3*c*(-a*d+b*c)^(2/3)*
arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)
/d^(8/3)+1/6*c*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^(8/3)-1/2*c*(-a*d+b*c)^(2/3)
*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(8/3)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.21

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{\frac{3d^{2/3}(a+bx^3)^{2/3}(-5bc+2ad+2bdx^3)}{b} - 10\sqrt{3}c(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) - 10\ldots}{\ldots}$$

input
```
Integrate[(x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
((3*d^(2/3)*(a + b*x^3)^(2/3)*(-5*b*c + 2*a*d + 2*b*d*x^3))/b - 10*Sqrt[3]
*c*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)
^(1/3))/Sqrt[3]] - 10*c*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*
(a + b*x^3)^(1/3)] + 5*c*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)
*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(30*d^(
8/3))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {948, 90, 60, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3(bx^3+a)^{2/3}}{dx^3+c}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{5/3}}{5bd}-\frac{c\int \frac{\left(bx^3+a\right)^{2/3}}{dx^3+c}dx^3}{d}\right)$$

$\downarrow$ 60

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{5/3}}{5bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx^3}{d}\right)}{d}\right)$$

$\downarrow$ 68

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{5/3}}{5bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\,\sqrt[3]{\cdots}}{\sqrt[3]{d}}}\cdots}{2d}\cdots\right)}{d}\cdots\right)}{d}\cdots\right)$$

$\downarrow$ 16

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{5/3}}{5bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\left(\frac{3\int\frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\cdots\right)}{d}\right)}{d}\cdots\right)$$

$\downarrow$ 1082

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{5/3}}{5bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\left(\frac{3\int\frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\cdots\right)}{2d^{2/3}\sqrt[3]{bc-\cdots}}\cdots\right)}{d}\right)}{d}\cdots\right)$$

$\downarrow$ 217

$$\frac{1}{3}\left(\frac{3(a+bx^3)^{5/3}}{5bd}-\frac{c\left(\frac{3(a+bx^3)^{2/3}}{2d}-\frac{(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{\cdots}\right.}{2d^{2/3}\sqrt[3]{bc-\cdots}}\right.}{d}\right.}{d}\right.$$

input
```
Int[(x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
((3*(a + b*x^3)^(5/3))/(5*b*d) - (c*((3*(a + b*x^3)^(2/3))/(2*d) - ((b*c -
 a*d)*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1
/3))/Sqrt[3]])/(d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b
*c - a*d)^(1/3)) - (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/
(2*d^(2/3)*(b*c - a*d)^(1/3))))/d))/d)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{\frac{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}} d\left(\left(d\,x^{3}-\frac{5c}{2}\right)b+ad\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{5}+bc(ad-bc)\left(-2\arctan\left(\frac{2\sqrt{3}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}} b\,d^{3}}$

input
```
int(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/d)^(1/3)*(6/5*((a*d-b*c)/d)^(1/3)*d*((d*x^3-5/2*c)*b+a*d)*(
b*x^3+a)^(2/3)+b*c*(a*d-b*c)*(-2*arctan(2/3*3^(1/2)/((a*d-b*c)/d)^(1/3)*(b
*x^3+a)^(1/3)+1/3*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*
(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))-2*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1
/3))))/b/d^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(149) = 298.

Time = 0.19 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.88

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx =$$

$$-\frac{10\sqrt{3}bc\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+5\,bc\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right.}{}$$

input
```
integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/30*(10*sqrt(3)*b*c*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(
-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)
^(1/3) + sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 5*b*c*(-(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(b^2*c^2
- 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) - 10*b*c*(-(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)/d^2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^
3 + a)^(1/3)*(b*c - a*d)) - 3*(2*b*d*x^3 - 5*b*c + 2*a*d)*(b*x^3 + a)^(2/3
))/(b*d^2)
```

Sympy [F]

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^5(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**5*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**5*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(149) = 298$.

Time = 0.16 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.63

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx = -\frac{\left(b^7c^2d^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ab^6cd^4\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^7cd^5 - ab^6d^6)} - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}c\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^4} + \frac{(-bcd^2+ad^3)^{\frac{2}{3}}c\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^4} - \frac{5\,(bx^3+a)^{\frac{2}{3}}b^5cd^3 - 2\,(bx^3+a)^{\frac{5}{3}}b^4d^4}{10\,b^5d^5}$$

input
```
integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*(b^7*c^2*d^3*(-(b*c - a*d)/d)^(1/3) - a*b^6*c*d^4*(-(b*c - a*d)/d)^(1
/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(
1/3)))/(b^7*c*d^5 - a*b^6*d^6) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*c*ar
ctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a
*d)/d)^(1/3))/d^4 + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c*log((b*x^3 + a)^(2/3) +
 (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^4 -
1/10*(5*(b*x^3 + a)^(2/3)*b^5*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^4*d^4)/(b^5*d^
5)
```

Mupad [B] (verification not implemented)

Time = 3.66 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.61

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{(b\,x^3+a)^{5/3}}{5\,b\,d} - (b\,x^3+a)^{2/3}\left(\frac{a}{2\,b\,d}+\frac{b^2\,c-a\,b\,d}{2\,b^2\,d^2}\right) - \frac{c\,\ln\left(\frac{(b\,x^3+a)^{1/3}\,(a^2\,c^2\,d^2-2\,a\,b\,c^3\,d+b^2\,c^4)}{d^3}-\frac{c^2\,(a\,d-b\,c)^{4/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{9\,d^{16/3}}\right)\,(a\,c}{3\,d^{8/3}}$$

input

```
int((x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

output

```
(a + b*x^3)^(5/3)/(5*b*d) - (a + b*x^3)^(2/3)*(a/(2*b*d) + (b^2*c - a*b*d)
/(2*b^2*d^2)) - (c*log(((a + b*x^3)^(1/3)*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c
^3*d))/d^3 - (c^2*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(16/3)))*(
a*d - b*c)^(2/3))/(3*d^(8/3)) - (c*log((c^2*((3^(1/2)*1i)/2 + 1/2)*(a*d -
b*c)^(7/3))/d^(10/3) + (c^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^3)*((3^(1/2
)*1i)/2 - 1/2)*(a*d - b*c)^(2/3))/(3*d^(8/3)) + (c*log((c^2*(a + b*x^3)^(1
/3)*(a*d - b*c)^2)/d^3 - (c^2*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^
(10/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(2/3))/(3*d^(8/3))
```

Reduce [F]

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{-3(b\,x^3+a)^{\frac{2}{3}}\,a+2(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3+10\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}x^5}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)abd-10\left(\int \frac{(}{bd\,x^6}\right.}{10bd}$$

input

```
int(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output

```
( - 3*(a + b*x**3)**(2/3)*a + 2*(a + b*x**3)**(2/3)*b*x**3 + 10*int(((a +
b*x**3)**(2/3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b*d - 10*
int(((a + b*x**3)**(2/3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b
**2*c)/(10*b*d)
```

3.695 $\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 162

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{(a+bx^3)^{2/3}}{2d} + \frac{(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}} - \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6d^{5/3}} + \frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}}$$

output
```
1/2*(b*x^3+a)^(2/3)/d+1/3*(-a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+
a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/d^(5/3)-1/6*(-a*d+b*c)^(2/3)*l
n(d*x^3+c)/d^(5/3)+1/2*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3
+a)^(1/3))/d^(5/3)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.27

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{3d^{2/3}(a+bx^3)^{2/3}+2\sqrt{3}(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)+2(bc-ad)^{2/3}\log\ldots}{\ldots}$$

input
```
Integrate[(x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(3*d^(2/3)*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*
d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 2*(b*c - a*d)^(2/
3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - (b*c - a*d)^(2/3)*
Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2
/3)*(a + b*x^3)^(2/3)])/(6*d^(5/3))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {946, 60, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{2/3}}{dx^3+c}dx^3$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3}\left(\frac{3(a+bx^3)^{2/3}}{2d}-\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{d}\right)$$

$\downarrow$ 68

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d}{2d}\right.}{d}\right.$$

$\downarrow$ 16

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-\right.}{2d^2}\right.}{d}\right.$$

$\downarrow$ 1082

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx\right.}{2d^{2/3}\sqrt[3]{bc-ad}}\right.}{d}\right.$$

$\downarrow$ 217

$$\frac{1}{3}\left(\frac{3(a+bx^3)^{2/3}}{2d}-\frac{(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{d}\right)$$

input
```
Int[(x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
((3*(a + b*x^3)^(2/3))/(2*d) - ((b*c - a*d)*(-((Sqrt[3]*ArcTan[(1 - (2*d^(
1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(2/3)*(b*c - a*d)^
(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b*c - a*d)^(1/3)) - (3*Log[(b*c - a*d
)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c - a*d)^(1/3))))/d)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.30

method	result
pseudoelliptic	$\frac{(b\,x^3+a)^{\frac{2}{3}}}{2d}+\frac{\ln\left((b\,x^3+a)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)(ad-bc)}{3d^2\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}-\frac{\ln\left((b\,x^3+a)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}(b\,x^3+a)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)(ad-bc)}{6d^2\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}-$

input
```
int(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x^3+a)^(2/3)/d+1/3/d^2/((a*d-b*c)/d)^(1/3)*ln((b*x^3+a)^(1/3)-((a*d
-b*c)/d)^(1/3))*(a*d-b*c)-1/6/d^2/((a*d-b*c)/d)^(1/3)*ln((b*x^3+a)^(2/3)+(
(a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))*(a*d-b*c)+1/3*3^(1
/2)/d^2/((a*d-b*c)/d)^(1/3)*arctan(2/3*3^(1/2)/((a*d-b*c)/d)^(1/3)*(b*x^3+
a)^(1/3)+1/3*3^(1/2))*(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(128) = 256$.

Time = 0.14 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.99

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{2\sqrt{3}\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)-\left(\frac{b^2c^2-\cdots}{}\right.}{}$$

input
```
integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-1/3*(2*
sqrt(3)*(b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3) -
sqrt(3)*(b*c - a*d))/(b*c - a*d)) - ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^
(1/3)*log((b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3)
- (b*x^3 + a)^(2/3)*(b*c - a*d) - (b*c - a*d)*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)/d^2)^(1/3)) + 2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*((
b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))
 + 3*(b*x^3 + a)^(2/3))/d
```

Sympy [F]

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^2(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**2*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**2*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(128) = 256.

Time = 0.15 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.60

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{\left(bcd\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ad^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bcd^2-ad^3)} + \frac{(bx^3+a)^{\frac{2}{3}}}{2\,d} + \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^3} - \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^3}$$

input
```
integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*(b*c*d*(-(b*c - a*d)/d)^(1/3) - a*d^2*(-(b*c - a*d)/d)^(1/3))*(-(b*c -
 a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c*d
^2 - a*d^3) + 1/2*(b*x^3 + a)^(2/3)/d + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/
3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*
c - a*d)/d)^(1/3))/d^3 - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3
) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^3
```

Mupad [B] (verification not implemented)

Time = 3.67 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.47

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{(b\,x^3+a)^{2/3}}{2\,d} + \frac{\ln\left(\frac{(b\,x^3+a)^{1/3}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{d}-\frac{(a\,d-b\,c)^{4/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{9\,d^{10/3}}\right)(a\,d-b\,c)^{2/3}}{3\,d^{5/3}} - \frac{\ln\left(\frac{(b\,x^3+a)^{1/3}\,(a\,d-b\,c)^2}{d}-\frac{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(a\,d-b\,c)^{7/3}}{d^{4/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(a\,d-b\,c)^{2/3}}{3\,d^{5/3}} + \frac{\ln\left(\frac{(b\,x^3+a)^{1/3}\,(a\,d-b\,c)^2}{d}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2(a\,d-b\,c)^{7/3}}{4\,d^{4/3}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)(a\,d-b\,c)^{2/3}}{d^{5/3}}$$

input
```
int((x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

output
```
(a + b*x^3)^(2/3)/(2*d) + (log(((a + b*x^3)^(1/3)*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d))/d - ((a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(10/3)))*(a*d
- b*c)^(2/3))/(3*d^(5/3)) - (log(((a + b*x^3)^(1/3)*(a*d - b*c)^2)/d - (((
3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(4/3))*((3^(1/2)*1i)/2 + 1/2)*(a
*d - b*c)^(2/3))/(3*d^(5/3)) + (log(((a + b*x^3)^(1/3)*(a*d - b*c)^2)/d -
((3^(1/2)*1i - 1)^2*(a*d - b*c)^(7/3))/(4*d^(4/3)))*((3^(1/2)*1i)/6 - 1/6)
*(a*d - b*c)^(2/3))/d^(5/3)
```

Reduce [F]

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{(b\,x^3+a)^{\frac{2}{3}}\,a-2\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}\,x^5}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)abd+2\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}\,x^5}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)b^2c}{2bc}$$

input
```
int(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
((a + b*x**3)**(2/3)*a - 2*int(((a + b*x**3)**(2/3)*x**5)/(a*c + a*d*x**3
+ b*c*x**3 + b*d*x**6),x)*a*b*d + 2*int(((a + b*x**3)**(2/3)*x**5)/(a*c +
a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**2*c)/(2*b*c)
```

3.696 $\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 245

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)} \, dx = \frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c} - \frac{(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}cd^{2/3}} - \frac{a^{2/3}\log(x)}{2c} + \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6cd^{2/3}} + \frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} - \frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2cd^{2/3}}$$

output

```
1/3*a^(2/3)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2
)/c-1/3*(-a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c
)^(1/3))*3^(1/2))*3^(1/2)/c/d^(2/3)-1/2*a^(2/3)*ln(x)/c+1/6*(-a*d+b*c)^(2/
3)*ln(d*x^3+c)/c/d^(2/3)+1/2*a^(2/3)*ln(a^(1/3)-(b*x^3+a)^(1/3))/c-1/2*(-a
*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c/d^(2/3)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx = \frac{2\sqrt{3}a^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\frac{-2\sqrt{3}(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)+2a^{2/3}d^{2/3}\log}{}}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x*(c + d*x^3)),x]
```

output
```
(2*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + (
-2*Sqrt[3]*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*
c - a*d)^(1/3))/Sqrt[3]] + 2*a^(2/3)*d^(2/3)*Log[-a^(1/3) + (a + b*x^3)^(1
/3)] - 2*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/
3)] - a^(2/3)*d^(2/3)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3
)^(2/3)] + (b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(
1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/d^(2/3))/(6*c)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {948, 94, 67, 16, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{2/3}}{x^3(dx^3+c)}dx^3$$

$$\Big\downarrow \; 94$$

$$\frac{1}{3}\left(\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{c}+\frac{a\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3}{c}\right)$$

$$\Big\downarrow \; 67$$

$$\frac{1}{3}\left(\frac{a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{c}+\frac{(bc-ad)\int \frac{}{\sqrt[3]{bx^3+}}}{c}\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(\frac{a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{c}+\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c}}{c}\right.$$

$$\Big\downarrow \; 68$$

$$\frac{1}{3}\left(\frac{a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{c}+\frac{(bc-ad)\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+}}{2d^{2/}}\right.}{}\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(\frac{a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{c}+\frac{(bc-ad)\left(\frac{3\int \frac{}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-}}{}\right.}{}\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{(bc-ad)\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{c}+\frac{a\left(-\frac{3\int}{}\right.}{}\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{c}+\frac{a\left(\frac{\sqrt{3}\,\text{arc}}{}\right.}{}\right.$$

input
```
Int[(a + b*x^3)^(2/3)/(x*(c + d*x^3)),x]
```

output
```
((a*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3)
 - Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3))
))/c + ((b*c - a*d)*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/
(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]
/(2*d^(2/3)*(b*c - a*d)^(1/3)) - (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b
*x^3)^(1/3)])/(2*d^(2/3)*(b*c - a*d)^(1/3))))/c)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{2\left(\arctan\left(\frac{2\sqrt{3}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{3a^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2}\right)a^{\frac{2}{3}}d\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}+(ad-}{6}$

input
```
int((b*x^3+a)^(2/3)/x/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/d)^(1/3)*(2*(arctan(2/3*3^(1/2)/a^(1/3)*(b*x^3+a)^(1/3)+1/3
*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(1/3)-a^(1/3))-1/2*ln((b*x^3+a)^(2/3)+a^(1/
3)*(b*x^3+a)^(1/3)+a^(2/3)))*a^(2/3)*d*((a*d-b*c)/d)^(1/3)+(a*d-b*c)*(-2*a
rctan(2/3*3^(1/2)/((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+1/3*3^(1/2))*3^(1/2)
+ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3
))-2*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))))/d/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(192) = 384.

Time = 0.18 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx =$$

$$-\frac{2\sqrt{3}\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)-2\sqrt{3}(a^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}a-\cdots}{\cdots}\right)\cdots}{\cdots}$$

input
```
integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-1/3*(
2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)
 + sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 2*sqrt(3)*(a^2)^(1/3)*arctan(1/3*(s
qrt(3)*a + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(a^2)^(1/3))/a) + (-(b^2*c^2 - 2*a*
b*c*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) + (a^2)^(1/3)*log((b*x^3 + a)^(2/
3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 2*(-(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)
^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)) - 2*(a^2)^(1/3)*log((b*x^3 + a)^(1
/3)*a - (a^2)^(2/3)))/c
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/x/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(x*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx = -\frac{\left(bc\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ad\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} + \frac{\sqrt{3}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,c} - \frac{a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{6\,c} + \frac{a^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{3\,c} - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd^2} + \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,cd^2}$$

input
```
integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*(b*c*(-(b*c - a*d)/d)^(1/3) - a*d*(-(b*c - a*d)/d)^(1/3))*(-(b*c - a*
d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^2 -
a*c*d) + 1/3*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(
1/3))/a^(1/3))/c - 1/6*a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a
^(1/3) + a^(2/3))/c + 1/3*a^(2/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/c
- 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(
1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c*d^2) + 1/6*(-b*c
*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d
)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c*d^2)
```

Mupad [B] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 1963, normalized size of antiderivative = 8.01

$$\int \frac{(a+bx^3)^{2/3}}{x\,(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^3)^(2/3)/(x*(c + d*x^3)),x)
```

output
```
log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4
*a^3*b^7*c^2*d^2) - (a^2/(27*c^3))^(2/3)*(((a + b*x^3)^(1/3)*(54*a^2*b^6*c
^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - (243*a*b^6*c^6*d^3 -
729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a^2/(27*c^3))^(2/3))*(a^2/(27*
c^3))^(1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4
 - 9*a*b^8*c^5*d))*(a^2/(27*c^3))^(1/3) + log((a + b*x^3)^(1/3)*(2*a^5*b^5
*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) - (-(a^2*d^2 +
 b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3)*(((a + b*x^3)^(1/3)*(54*a^2*b^6*
c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - (243*a*b^6*c^6*d^3 -
 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c
*d)/(27*c^3*d^2))^(2/3))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(
1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*a*
b^8*c^5*d))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) - log((a
 + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b
^7*c^2*d^2) + ((3^(1/2)*1i)/2 + 1/2)^2*(a^2/(27*c^3))^(2/3)*(((3^(1/2)*1i)
/2 + 1/2)*((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 5
4*a^4*b^4*c^2*d^5) - ((3^(1/2)*1i)/2 + 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2
*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a^2/(27*c^3))^(2/3))*(a^2/(27*c^3))^(
1/3) - 36*a^2*b^7*c^4*d^2 + 54*a^3*b^6*c^3*d^3 - 27*a^4*b^5*c^2*d^4 + 9*a*
b^8*c^5*d))*((3^(1/2)*1i)/2 + 1/2)*(a^2/(27*c^3))^(1/3) + log((a + b*x^...
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx=\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{d\,x^4+cx}dx$$

input
```
int((b*x^3+a)^(2/3)/x/(d*x^3+c),x)
```

output
```
int((a + b*x**3)**(2/3)/(c*x + d*x**4),x)
```

3.697 $\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 320

$$
\begin{aligned}
\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx &= \frac{b(a+bx^3)^{2/3}}{3ac}-\frac{(a+bx^3)^{5/3}}{3acx^3} \\
&+\frac{(2bc-3ad)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}c^2} \\
&+\frac{\sqrt[3]{d}(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2}-\frac{(2bc-3ad)\log(x)}{6\sqrt[3]{a}c^2} \\
&-\frac{\sqrt[3]{d}(bc-ad)^{2/3}\log(c+dx^3)}{6c^2}+\frac{(2bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{a}c^2} \\
&+\frac{\sqrt[3]{d}(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2}
\end{aligned}
$$

output
```
1/3*b*(b*x^3+a)^(2/3)/a/c-1/3*(b*x^3+a)^(5/3)/a/c/x^3+1/9*(-3*a*d+2*b*c)*a
rctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(1/3)/c^2
+1/3*d^(1/3)*(-a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*
d+b*c)^(1/3))*3^(1/2))*3^(1/2)/c^2-1/6*(-3*a*d+2*b*c)*ln(x)/a^(1/3)/c^2-1/
6*d^(1/3)*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^2+1/6*(-3*a*d+2*b*c)*ln(a^(1/3)-(
b*x^3+a)^(1/3))/a^(1/3)/c^2+1/2*d^(1/3)*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/
3)+d^(1/3)*(b*x^3+a)^(1/3))/c^2
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx = \frac{-\frac{6c(a+bx^3)^{2/3}}{x^3} + \frac{2\sqrt{3}(2bc-3ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + 6\sqrt{3}\sqrt[3]{d}(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\cdots}}{\cdots}}{\cdots}\right.\cdots}{\cdots}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x]
```

output
```
((-6*c*(a + b*x^3)^(2/3))/x^3 + (2*Sqrt[3]*(2*b*c - 3*a*d)*ArcTan[(1 + (2*
(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) + 6*Sqrt[3]*d^(1/3)*(b*c - a
*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqr
t[3]] + (2*(2*b*c - 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(1/3) + 6*
d^(1/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3
)] + ((-2*b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^
3)^(2/3)])/a^(1/3) - 3*d^(1/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d
^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(
18*c^2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {948, 114, 27, 174, 60, 67, 16, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{2/3}}{x^6(dx^3+c)}dx^3$$

$$\Big\downarrow \text{ 114}$$

$$\frac{1}{3}\left(-\frac{\int -\frac{(bx^3+a)^{2/3}(2bdx^3+2bc-3ad)}{3x^3(dx^3+c)}dx^3}{ac}-\frac{(a+bx^3)^{5/3}}{acx^3}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{\int \frac{(bx^3+a)^{2/3}(2bdx^3+2bc-3ad)}{x^3(dx^3+c)}dx^3}{3ac}-\frac{(a+bx^3)^{5/3}}{acx^3}\right)$$

$$\Big\downarrow \text{ 174}$$

$$\frac{1}{3}\left(\frac{\frac{3ad^2\int \frac{(bx^3+a)^{2/3}}{dx^3+c}dx^3}{c}+\frac{(2bc-3ad)\int \frac{(bx^3+a)^{2/3}}{x^3}dx^3}{c}}{3ac}-\frac{(a+bx^3)^{5/3}}{acx^3}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{3}\left(\frac{\frac{3ad^2\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx^3}{d}\right)}{c}+\frac{(2bc-3ad)\left(a\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)}{c}}{3ac}-\frac{\left(a+bx^3\right)^{5/3}}{acx^3}\right.$$

$\Big\downarrow$ 67

$$\frac{1}{3}\left(\frac{\frac{(2bc-3ad)\left(a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)}{c}+\frac{3ad\ldots}{\ldots}}{3ac}\right.$$

$\Big\downarrow$ 16

$$\frac{1}{3}\left(\frac{\frac{(2bc-3ad)\left(a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)}{c}+\frac{3ad^2\left(\frac{3\left(a+bx\ldots\right.}{2d\ldots}\right.}{\ldots}}{3ac}\right.$$

$\Big\downarrow$ 68

$$\frac{1}{3}\left(\frac{\frac{(2bc-3ad)\left(a\left(\frac{3}{2}\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)}{c}+\frac{3ad^2\left(\frac{3\left(a+bx\right.}{2c}\right.}{}}{}\right.$$

$\downarrow$ 16

$$\frac{1}{3}\left(\frac{\frac{(2bc-3ad)\left(a\left(\frac{3}{2}\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)}{c}+\frac{3ad^2\left(\frac{3\left(a+bx\right.}{2d}\right.}{}}{}\right.$$

$\downarrow$ 1082

$$\frac{1}{3}\left(\frac{\frac{3ad^2\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\left(\frac{3\int\frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{d}\right)}{c}+\frac{(\cdots}{}}{3ac}\right.$$

$\downarrow$ 217

$$\frac{1}{3}\left(\frac{3ad^2\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{d}\right)}{c}}{3ac}+ \cdots\right.$$

(2

input

```
Int[(a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x]
```

output
```
(-((a + b*x^3)^(5/3)/(a*c*x^3)) + (((2*b*c - 3*a*d)*((3*(a + b*x^3)^(2/3))
/2 + a*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1
/3) - Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/
3)))))/c + (3*a*d^2*((3*(a + b*x^3)^(2/3))/(2*d) - ((b*c - a*d)*(-((Sqrt[3
]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(
d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b*c - a*d)^(1/3))
 - (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c
- a*d)^(1/3))))/d))/c)/(3*a*c))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$-\frac{\left(2a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{2}{3}}c-\left(ad-\frac{2bc}{3}\right)\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\right.\right.\right.}{}$

input
```
int((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/((a*d-b*c)/d)^(1/3)*((2*a^(1/3)*(b*x^3+a)^(2/3)*c-(a*d-2/3*b*c)*(-2*a
rctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)+ln((b*x^3+a
)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*x^
3)*((a*d-b*c)/d)^(1/3)+(d*a^(4/3)-a^(1/3)*b*c)*(-2*arctan(1/3*3^(1/2)*(2*(
b*x^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*c)/d)^(1/3))*3^(1/2)+ln((b*x^3
+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))-2*ln((b
*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3)))*x^3)/a^(1/3)/c^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1030, normalized size of antiderivative = 3.22

$$\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/18*(3*sqrt(1/3)*(2*a*b*c - 3*a^2*d)*x^3*sqrt((-a)^(1/3)/a)*log((2*b*x^
3 - 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-
a)^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3
) - 6*sqrt(3)*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*arctan(1/3*(
sqrt(3)*(b*c - a*d) - 2*sqrt(3)*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*
(b*x^3 + a)^(1/3))/(b*c - a*d)) + (2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^
3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(2*b*c - 3*a
*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) + 3*(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) -
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b
*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 6*(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)^(1/3)*a*x^3*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)^(2/3)) + 6*(b*x^3 + a)^(2/3)*a*c)/(a*c^2*x^3), 1/1
8*(6*sqrt(1/3)*(2*a*b*c - 3*a^2*d)*x^3*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3
)*(2*(b*x^3 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 6*sqrt(3)*(b^2
*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*arctan(1/3*(sqrt(3)*(b*c - a*d
) - 2*sqrt(3)*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3))
/(b*c - a*d)) - (2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(2/3) - (b*
x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(2*b*c - 3*a*d)*(-a)^(2/3)*x^3
*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2...
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x^4(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/x**4/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(x**4*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^4}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx = \frac{\left(bcd\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ad^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^3-ac^2d)} + \frac{\sqrt{3}(2\,bc-3\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{1}{3}}c^2} - \frac{(2\,bc-3\,ad)\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{18\,a^{\frac{1}{3}}c^2} + \frac{\left(2\,a^{\frac{1}{3}}bc - 3\,a^{\frac{4}{3}}d\right)\log\left(\left|(bx^3+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{2}{3}}c^2} + \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^2d} - \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^2d} - \frac{(bx^3+a)^{\frac{2}{3}}}{3\,cx^3}$$

input
```
integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*(b*c*d*(-(b*c - a*d)/d)^(1/3) - a*d^2*(-(b*c - a*d)/d)^(1/3))*(-(b*c -
 a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^3
 - a*c^2*d) + 1/9*sqrt(3)*(2*b*c - 3*a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a
)^(1/3) + a^(1/3))/a^(1/3))/(a^(1/3)*c^2) - 1/18*(2*b*c - 3*a*d)*log((b*x^
3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(1/3)*c^2) + 1/9*(2
*a^(1/3)*b*c - 3*a^(4/3)*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/3)
*c^2) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3
+ a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c^2*d) - 1/6
*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c
 - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c^2*d) - 1/3*(b*x^3 + a)^(2/3)
/(c*x^3)
```

Mupad [B] (verification not implemented)

Time = 9.77 (sec) , antiderivative size = 1908, normalized size of antiderivative = 5.96

$$\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x)
```

output
```
log(- ((((6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2
 - 6*a*b*c*d) - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d*(a*
d - b*c)^2)/c^6)^(2/3))*((d*(a*d - b*c)^2)/c^6)^(1/3))/3 - (a*b^5*d^4*(27*
a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*((d*(a*d -
 b*c)^2)/c^6)^(2/3))/9 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(3*a^2
*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((a^2*d^3 + b^2*c^2*d - 2*a*b*c
*d^2)/(27*c^6))^(1/3) + log(- ((((6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^
2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) - 3*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c
^2 - 3*a*b*c*d)*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))*(-(3*a*d - 2*b*c)^3/(a
*c^6))^(1/3))/9 - (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 7
2*a^2*b*c*d^2))/(3*c))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))/81 - (b^5*d^4*(
a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(2
7*c^5))*(-(27*a^3*d^3 - 8*b^3*c^3 + 36*a*b^2*c^2*d - 54*a^2*b*c*d^2)/(729*
a*c^6))^(1/3) - log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(6*b^
4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d)
- 3*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)
*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(1/3))/9
 + (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))
/(3*c))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))/81 - (b^5*d^4*(a + b*x^3)^(1/3
)*(4*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((3^...
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^4\,(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{\frac{2}{3}}}{d\,x^7+c\,x^4}dx$$

input
```
int((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x)
```

output
```
int((a + b*x**3)**(2/3)/(c*x**4 + d*x**7),x)
```

3.698 $\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 370

$$\begin{aligned}\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx &= \frac{(bc+6ad)\,(a+bx^3)^{2/3}}{18ac^2x^3} - \frac{(a+bx^3)^{5/3}}{6acx^6} \\ &\quad - \frac{(b^2c^2+6abcd-9a^2d^2)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}c^3} \\ &\quad - \frac{d^{4/3}(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^3} + \frac{(b^2c^2+6abcd-9a^2d^2)\log(x)}{18a^{4/3}c^3} \\ &\quad + \frac{d^{4/3}(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^3} - \frac{(b^2c^2+6abcd-9a^2d^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{4/3}c^3} \\ &\quad - \frac{d^{4/3}(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3}\end{aligned}$$

output
```
1/18*(6*a*d+b*c)*(b*x^3+a)^(2/3)/a/c^2/x^3-1/6*(b*x^3+a)^(5/3)/a/c/x^6-1/2
7*(-9*a^2*d^2+6*a*b*c*d+b^2*c^2)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^
(1/2)/a^(1/3))*3^(1/2)/a^(4/3)/c^3-1/3*d^(4/3)*(-a*d+b*c)^(2/3)*arctan(1/3
*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/c^3+1/18*
(-9*a^2*d^2+6*a*b*c*d+b^2*c^2)*ln(x)/a^(4/3)/c^3+1/6*d^(4/3)*(-a*d+b*c)^(2
/3)*ln(d*x^3+c)/c^3-1/18*(-9*a^2*d^2+6*a*b*c*d+b^2*c^2)*ln(a^(1/3)-(b*x^3+
a)^(1/3))/a^(4/3)/c^3-1/2*d^(4/3)*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(
1/3)*(b*x^3+a)^(1/3))/c^3
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx = \frac{\frac{3c(a+bx^3)^{2/3}(-3ac-2bcx^3+6adx^3)}{ax^6} - \frac{2\sqrt{3}(b^2c^2+6abcd-9a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} - 18\sqrt{3}d^{4/3}(b}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x]
```

output
```
((3*c*(a + b*x^3)^(2/3)*(-3*a*c - 2*b*c*x^3 + 6*a*d*x^3))/(a*x^6) - (2*Sqr
t[3]*(b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a
^(1/3))/Sqrt[3]])/a^(4/3) - 18*Sqrt[3]*d^(4/3)*(b*c - a*d)^(2/3)*ArcTan[(1
 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - (2*(b^2*c^2
 + 6*a*b*c*d - 9*a^2*d^2)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(4/3) - 18*
d^(4/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3
)] + ((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^
(1/3) + (a + b*x^3)^(2/3)])/a^(4/3) + 9*d^(4/3)*(b*c - a*d)^(2/3)*Log[(b*c
 - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a +
 b*x^3)^(2/3)])/(54*c^3)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {948, 114, 27, 166, 27, 174, 67, 16, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^7 (c+dx^3)} \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3} \int \frac{(bx^3+a)^{2/3}}{x^9 (dx^3+c)} dx^3$$

$$\Big\downarrow \text{ 114}$$

$$\frac{1}{3} \left(-\frac{\int \frac{(bx^3+a)^{2/3}(bdx^3+bc+6ad)}{3x^6(dx^3+c)}dx^3}{2ac} - \frac{(a+bx^3)^{5/3}}{2acx^6} \right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3} \left(-\frac{\int \frac{(bx^3+a)^{2/3}(bdx^3+bc+6ad)}{x^6(dx^3+c)}dx^3}{6ac} - \frac{(a+bx^3)^{5/3}}{2acx^6} \right)$$

$$\Big\downarrow \text{ 166}$$

$$\frac{1}{3} \left(-\frac{\frac{\int \frac{2\left(bd(bc-3ad)x^3+b^2c^2-9a^2d^2+6abcd\right)}{3x^3\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{c} - \frac{(a+bx^3)^{2/3}(6ad+bc)}{cx^3}}{6ac} - \frac{(a+bx^3)^{5/3}}{2acx^6} \right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3} \left(-\frac{\frac{2\int \frac{bd(bc-3ad)x^3+b^2c^2-9a^2d^2+6abcd}{x^3\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{3c} - \frac{(a+bx^3)^{2/3}(6ad+bc)}{cx^3}}{6ac} - \frac{(a+bx^3)^{5/3}}{2acx^6} \right)$$

$$\Big\downarrow \text{ 174}$$

$$\frac{1}{3}\left(-\frac{\frac{2\left(\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3}{c}-\frac{9ad^2(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx^3}{c}\right)}{3c}-\frac{\left(a+bx^3\right)^{2/3}(6ad+bc)}{cx^3}}{6ac}-\frac{\left(a+bx^3\right)^{5/3}}{2acx^6}\right.$$

$$\Big\downarrow \; 67$$

$$\frac{1}{3}\left(-\frac{\frac{2\left(\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{9ad^2(bc-ad}{}\right.}{3c}}{6ac}\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(-\frac{\frac{2\left(\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{9ad^2(bc-ad)\int \frac{}{\sqrt[3]{bx^3}}}{c}\right.}{3c}}{6ac}\right.$$

$\downarrow$ 68

$$\frac{1}{3}\left(-\frac{2\left(\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\left(\frac{3}{2}\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{9ad^2(bc-ad)\left(-\frac{3\int\frac{\cdots}{\sqrt[3]{\cdots}}}{\cdots}\cdots\right)}{\cdots}\right)}{3c}\cdots\right.$$

$\downarrow$ 16

$$\frac{1}{3}\left(-\frac{2\left(\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\left(\frac{3}{2}\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{9ad^2(bc-ad)\left(\frac{3\int\frac{\quad}{x^6+\cdots}}{\quad}\right.}{\quad}\right.}{3c}\right.$$

$\downarrow$ 1082

$$\frac{1}{3}\left(-\frac{2\left(\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\left(-\frac{3\int\frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{9ad^2(bc-ad)\left(\frac{3\int\frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{\cdots}}{\cdots}\right)}{d^{2/3}\sqrt[3]{bc\cdots}}\right.}{\quad}\right.}{3c}}{6ac}\right.$$

$\downarrow$ 217

$$\frac{1}{3}\left(-\frac{\frac{2\left(\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{9ad^2(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{\cdots}}{\sqrt[3]{bc\cdots}}}{\sqrt{\cdots}}\right)}{d^{2/3}\sqrt[3]{bc-\cdots}}\cdots\right.}{\cdots}\right)}{3c}}{6ac}\right.$$

input
```
Int[(a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x]
```

output
```
(-1/2*(a + b*x^3)^(5/3)/(a*c*x^6) - (-(((b*c + 6*a*d)*(a + b*x^3)^(2/3))/(
c*x^3)) + (2*(((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*((Sqrt[3]*ArcTan[(1 + (2*
(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^3]/(2*a^(1/3)) + (3*
Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3))))/c - (9*a*d^2*(b*c - a*d)*(
-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sq
rt[3]])/(d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b*c - a*
d)^(1/3)) - (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2
/3)*(b*c - a*d)^(1/3))))/c))/(3*c))/(6*a*c))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$\frac{\left(-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} c\left(-6ad\,x^{3}+2x^{3}bc+3ac\right)a^{\frac{4}{3}}}{6}+a\left(a^{2}d^{2}-\frac{2}{3}abcd-\frac{1}{9}b^{2}c^{2}\right)\left(\arctan\left(\frac{2\sqrt{3}\,\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{3a^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a\right.\right.\right.}{}$

input
```
int((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/d)^(1/3)*((-1/6*(b*x^3+a)^(2/3)*c*(-6*a*d*x^3+2*b*c*x^3+3*a
*c)*a^(4/3)+a*(a^2*d^2-2/3*a*b*c*d-1/9*b^2*c^2)*(arctan(2/3*3^(1/2)/a^(1/3
)*(b*x^3+a)^(1/3)+1/3*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(1/3)-a^(1/3))-1/2*ln(
(b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3)))*x^6)*((a*d-b*c)/d)^(1/3)
+1/2*d*(-2*arctan(2/3*3^(1/2)/((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+1/3*3^(1
/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-
b*c)/d)^(2/3))-2*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3)))*(a*d-b*c)*a^(7/3
)*x^6)/c^3/x^6/a^(7/3)
```

Fricas [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 1151, normalized size of antiderivative = 3.11

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(c+dx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/54*(18*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*a^2*d*x^6*ar
ctan(-1/3*(sqrt(3)*(b*c - a*d) + 2*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2
*d^3)^(1/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) + 9*(-b^2*c^2*d + 2*a*b*c*d^2
- a^2*d^3)^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*
c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d
^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 18*(-b^2*c^2*d + 2*a*b*c*d^2 - a^
2*d^3)^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (-b^2*c^2*
d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) + 3*sqrt(1/3)*(a*b^2*c^2 + 6*a^2*b*c*d -
 9*a^3*d^2)*x^6*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)
^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^
3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) - (b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*a^(2/
3)*x^6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b
^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log((b*x^3 + a)^(1/3) - a^(1/3
)) + 3*(3*a^2*c^2 + 2*(a*b*c^2 - 3*a^2*c*d)*x^3)*(b*x^3 + a)^(2/3))/(a^2*c
^3*x^6), -1/54*(18*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*a^2*
d*x^6*arctan(-1/3*(sqrt(3)*(b*c - a*d) + 2*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d
^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) + 9*(-b^2*c^2*d + 2*a*
b*c*d^2 - a^2*d^3)^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2)
+ (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2
*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 18*(-b^2*c^2*d + 2*a*b...
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x^7(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/x**7/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(x**7*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^7}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^7), x)
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx = -\frac{\left(bcd^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ad^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^4-ac^3d)} - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^3} + \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^3} - \frac{\sqrt{3}(b^2c^2+6\,abcd-9\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{4}{3}}c^3} + \frac{(b^2c^2+6\,abcd-9\,a^2d^2)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{54\,a^{\frac{4}{3}}c^3} - \frac{\left(a^{\frac{1}{3}}b^2c^2+6\,a^{\frac{4}{3}}bcd-9\,a^{\frac{7}{3}}d^2\right)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27\,a^{\frac{5}{3}}c^3} - \frac{2\,(bx^3+a)^{\frac{5}{3}}b^2c+(bx^3+a)^{\frac{2}{3}}ab^2c-6\,(bx^3+a)^{\frac{5}{3}}abd+6\,(bx^3+a)^{\frac{2}{3}}a^2bd}{18\,ab^2c^2x^6}$$

input
```
integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*(b*c*d^2*(-(b*c - a*d)/d)^(1/3) - a*d^3*(-(b*c - a*d)/d)^(1/3))*(-(b*
c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*
c^4 - a*c^3*d) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(
2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/c^3
+ 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(
-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/c^3 - 1/27*sqrt(3)*(b^2*c^
2 + 6*a*b*c*d - 9*a^2*d^2)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/
3))/a^(1/3))/(a^(4/3)*c^3) + 1/54*(b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*log((b
*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(4/3)*c^3) - 1/2
7*(a^(1/3)*b^2*c^2 + 6*a^(4/3)*b*c*d - 9*a^(7/3)*d^2)*log(abs((b*x^3 + a)^
(1/3) - a^(1/3)))/(a^(5/3)*c^3) - 1/18*(2*(b*x^3 + a)^(5/3)*b^2*c + (b*x^3
 + a)^(2/3)*a*b^2*c - 6*(b*x^3 + a)^(5/3)*a*b*d + 6*(b*x^3 + a)^(2/3)*a^2*
b*d)/(a*b^2*c^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 15.02 (sec) , antiderivative size = 2788, normalized size of antiderivative = 7.54

$$\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x)
```

output
```
log((((27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^4*(a*d - b
*c)^2)/c^9)^(2/3) - (b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4
 + b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2
*c^2))*(-(d^4*(a*d - b*c)^2)/c^9)^(1/3))/3 - (b^5*d^4*(729*a^6*d^6 + b^6*c
^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*
a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(d^4*(a*d - b*c)^2)/c^9)^
(2/3))/9 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c
^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*(-(a^2*d^6 + b^2*c
^2*d^4 - 2*a*b*c*d^5)/(27*c^9))^(1/3) + log((((((a*b^4*c^4*d^3*(2*a^2*d^2
 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2
/3))/3 - (b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 +
 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2))*(-(b
^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(1/3))/27 - (b^5*d^4*(729*a^6
*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c
^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(b^2*c^2 - 9*
a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2/3))/729 + (2*b^5*d^7*(a + b*x^3)^(1/3
)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2
)/(729*a^2*c^10))*(-(b^6*c^6 - 729*a^6*d^6 + 81*a^2*b^4*c^4*d^2 - 108*a^3*
b^3*c^3*d^3 - 729*a^4*b^2*c^2*d^4 + 18*a*b^5*c^5*d + 1458*a^5*b*c*d^5)/(19
683*a^4*c^9))^(1/3) - (((a + b*x^3)^(2/3)*(b^2*c + 6*a*b*d))/(18*c^2) -...
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^7\left(c+dx^3\right)}\,dx=\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{d\,x^{10}+c\,x^7}dx$$

input
```
int((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x)
```

output
```
int((a + b*x**3)**(2/3)/(c*x**7 + d*x**10),x)
```

3.699 $\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 334

$$\begin{aligned}\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = &-\frac{(3bc-ad)x(a+bx^3)^{2/3}}{9bd^2} \\ &+\frac{x^4(a+bx^3)^{2/3}}{6d}+\frac{(9b^2c^2-6abcd-a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{4/3}d^3} \\ &-\frac{c^{4/3}(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^3}-\frac{c^{4/3}(bc-ad)^{2/3}\log(c+dx^3)}{6d^3} \\ &+\frac{c^{4/3}(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^3} \\ &-\frac{(9b^2c^2-6abcd-a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{4/3}d^3}\end{aligned}$$

output
```
-1/9*(-a*d+3*b*c)*x*(b*x^3+a)^(2/3)/b/d^2+1/6*x^4*(b*x^3+a)^(2/3)/d+1/27*(
-a^2*d^2-6*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3
^(1/2))*3^(1/2)/b^(4/3)/d^3-1/3*c^(4/3)*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(
-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d^3-1/6*c^(4/3
)*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^3+1/2*c^(4/3)*(-a*d+b*c)^(2/3)*ln((-a*d+b
*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^3-1/18*(-a^2*d^2-6*a*b*c*d+9*b^2*c^
2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)/d^3
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.82 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.58

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{\frac{6d(a+bx^3)^{2/3}(-6bcx+2adx+3bdx^4)}{b} + \frac{4\sqrt{3}(9b^2c^2-6abcd-a^2d^2)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{4/3}} + 18\sqrt{-}}{}$$

input
```
Integrate[(x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
((6*d*(a + b*x^3)^(2/3)*(-6*b*c*x + 2*a*d*x + 3*b*d*x^4))/b + (4*Sqrt[3]*(
9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2
*(a + b*x^3)^(1/3))])/b^(4/3) + 18*Sqrt[-6 + (6*I)*Sqrt[3]]*c^(4/3)*(b*c -
 a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x -
(3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (4*(-9*b^2*c^2 + 6*a*b*c*d +
 a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(4/3) - (18*I)*(-I + Sq
rt[3])*c^(4/3)*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3
])*c^(1/3)*(a + b*x^3)^(1/3)] + (2*(9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*Log[b
^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(4/3) + 9
*(1 + I*Sqrt[3])*c^(4/3)*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (
-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqr
t[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(108*d^3)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {978, 27, 1052, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(a+bx^3\right)^{2/3}}{c+dx^3}dx$$

$$\Big\downarrow \text{978}$$

$$\frac{x^4\left(a+bx^3\right)^{2/3}}{6d}-\frac{\int \frac{2x^3\left((3bc-ad)x^3+2ac\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4\left(a+bx^3\right)^{2/3}}{6d}-\frac{\int \frac{x^3\left((3bc-ad)x^3+2ac\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3d}$$

$$\Big\downarrow \text{1052}$$

$$\frac{x^4\left(a+bx^3\right)^{2/3}}{6d}-\frac{\frac{x\left(a+bx^3\right)^{2/3}(3bc-ad)}{3bd}-\frac{\int \frac{\left(9b^2c^2-6abdc-a^2d^2\right)x^3+ac(3bc-ad)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3bd}}{3d}$$

$$\Big\downarrow \text{1026}$$

$$\frac{x^4\left(a+bx^3\right)^{2/3}}{6d}-\frac{\frac{x\left(a+bx^3\right)^{2/3}(3bc-ad)}{3bd}-\frac{\frac{\left(-a^2d^2-6abcd+9b^2c^2\right)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{9bc^2(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{d}}{3bd}}{3d}$$

$$\Big\downarrow \text{769}$$

$$\frac{\frac{x^4(a+bx^3)^{2/3}}{6d} - \frac{\frac{x(a+bx^3)^{2/3}(3bc-ad)}{3bd} - \frac{\frac{(-a^2d^2-6abcd+9b^2c^2)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d} - \frac{9bc^2(bc-ad)\int\frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}}{d}}{3bd}}{3d}}{}$$

$\downarrow$ 901

$$\frac{\frac{x^4(a+bx^3)^{2/3}}{6d} - \frac{\frac{x(a+bx^3)^{2/3}(3bc-ad)}{3bd} - \frac{\frac{(-a^2d^2-6abcd+9b^2c^2)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d} - \frac{9bc^2(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-}}{\sqrt[3]{c}\sqrt[3]{a+}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-}}\right.}{}}{3bd}}{3d}}{}$$

input
```
Int[(x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(x^4*(a + b*x^3)^(2/3))/(6*d) - (((3*b*c - a*d)*x*(a + b*x^3)^(2/3))/(3*b*
d) - ((-9*b*c^2*(b*c - a*d)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*
(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c +
 d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3)
- (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/d + ((9*b^2*c^2 - 6*a
*b*c*d - a^2*d^2)*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(
Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/d)/
(3*b*d))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{\left(-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} d\left(\left(\frac{3d\,x^{3}}{2}-3c\right)b^{\frac{7}{3}}+b^{\frac{4}{3}}ad\right)x}{3}-\frac{\left(a^{2}d^{2}+6abcd-9b^{2}c^{2}\right)\left(\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{b^{\frac{1}{3}}}+x\right)}{3x}\right)\sqrt{3}+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right.}{x}\right.\right.}{9}\right.}{}$

input
```
int(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/((a*d-b*c)/c)^(1/3)*((-1/3*(b*x^3+a)^(2/3)*d*((3/2*d*x^3-3*c)*b^(7/3)
+b^(4/3)*a*d)*x-1/9*(a^2*d^2+6*a*b*c*d-9*b^2*c^2)*(arctan(1/3*3^(1/2)*(2*(
b*x^3+a)^(1/3)/b^(1/3)+x)/x)*3^(1/2)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/
2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*b)*((a*
d-b*c)/c)^(1/3)+c*(b^(10/3)*c-b^(7/3)*a*d)*(arctan(1/3*3^(1/2)*(-2/((a*d-b
*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^
3+a)^(1/3))/x)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+
a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))/b^(7/3)/d^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(276) = 552.

Time = 1.21 (sec) , antiderivative size = 1164, normalized size of antiderivative = 3.49

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/54*(18*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*b^2*c*arctan(
-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2
)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) - 18*(b^2*c^3 - 2*a*b*c^2*d +
a^2*c*d^2)^(1/3)*b^2*c*log(((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*x -
(b*x^3 + a)^(1/3)*(b*c^2 - a*c*d))/x) + 9*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d
^2)^(1/3)*b^2*c*log(-((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*(b*c - a*d
)*x^2 + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x + (b
*x^3 + a)^(2/3)*(b*c^2 - a*c*d))/x^2) + 3*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c
*d - a^2*b*d^2)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)
*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^
(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(9*b^2*c^2 - 6*a*b*c*d - a^2*
d^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*
c*d - a^2*d^2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b
*x^3 + a)^(2/3))/x^2) - 3*(3*b^2*d^2*x^4 - 2*(3*b^2*c*d - a*b*d^2)*x)*(b*x
^3 + a)^(2/3))/(b^2*d^3), -1/54*(18*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c
*d^2)^(1/3)*b^2*c*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b^2*c^3
- 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) - 18*
(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*b^2*c*log(((b^2*c^3 - 2*a*b*c^2*
d + a^2*c*d^2)^(2/3)*x - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d))/x) + 9*(b^2*c^
3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*b^2*c*log(-((b^2*c^3 - 2*a*b*c^2*d +...
```

Sympy [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^6(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**6*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**6*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}x^6}{dx^3+c}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^6/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}x^6}{dx^3+c}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^6/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^6\,(b\,x^3+a)^{2/3}}{d\,x^3+c}\,dx$$

input
```
int((x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

output
```
int((x^6*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{2(b\,x^3+a)^{\frac{2}{3}}\,adx - 6(b\,x^3+a)^{\frac{2}{3}}\,bcx + 3(b\,x^3+a)^{\frac{2}{3}}\,bd\,x^4 - 2\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right.}{}$$

input
```
int(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
(2*(a + b*x**3)**(2/3)*a*d*x - 6*(a + b*x**3)**(2/3)*b*c*x + 3*(a + b*x**3
)**(2/3)*b*d*x**4 - 2*int((a + b*x**3)**(2/3)/(a*c + a*d*x**3 + b*c*x**3 +
 b*d*x**6),x)*a**2*c*d + 6*int((a + b*x**3)**(2/3)/(a*c + a*d*x**3 + b*c*x
**3 + b*d*x**6),x)*a*b*c**2 - 2*int(((a + b*x**3)**(2/3)*x**3)/(a*c + a*d*
x**3 + b*c*x**3 + b*d*x**6),x)*a**2*d**2 - 12*int(((a + b*x**3)**(2/3)*x**
3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b*c*d + 18*int(((a + b*x**3
)**(2/3)*x**3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**2*c**2)/(18*b*
d**2)
```

3.700 $\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 272

$$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{x(a+bx^3)^{2/3}}{3d} - \frac{(3bc-2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}d^2} + \frac{\sqrt[3]{c}(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{\sqrt[3]{c}(bc-ad)^{2/3}\log(c+dx^3)}{6d^2} - \frac{\sqrt[3]{c}(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2d^2} + \frac{(3bc-2ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{b}d^2}$$

output
```
1/3*x*(b*x^3+a)^(2/3)/d-1/9*(-2*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^
3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(1/3)/d^2+1/3*c^(1/3)*(-a*d+b*c)^(2/3)*arct
an(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d
^2+1/6*c^(1/3)*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^2-1/2*c^(1/3)*(-a*d+b*c)^(2/
3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2+1/6*(-2*a*d+3*b*c)*l
n(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)/d^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.03 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.71

$$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{12dx(a+bx^3)^{2/3} - \frac{4\sqrt{3}(3bc-2ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}} - 6\sqrt{-6+6i\sqrt{3}}\sqrt[3]{c}(bc-a$$

input
```
Integrate[(x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(12*d*x*(a + b*x^3)^(2/3) - (4*Sqrt[3]*(3*b*c - 2*a*d)*ArcTan[(Sqrt[3]*b^(
1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/b^(1/3) - 6*Sqrt[-6 + (6*I)*Sq
rt[3]]*c^(1/3)*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(
b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (4*(3*b
*c - 2*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(1/3) + 6*(1 + I*Sqrt
[3])*c^(1/3)*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])
*c^(1/3)*(a + b*x^3)^(1/3)] + (2*(-3*b*c + 2*a*d)*Log[b^(2/3)*x^2 + b^(1/3
)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(1/3) - (3*I)*(-I + Sqrt[3])
*c^(1/3)*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*
c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a
 + b*x^3)^(2/3)])/(36*d^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {978, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+b x^3\right)^{2/3}}{c+d x^3} \, dx$$

$$\Big\downarrow \ 978$$

$$\frac{x \left(a+b x^3\right)^{2/3}}{3 d}-\frac{\int \frac{(3bc-2ad)x^3+ac}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3d}$$

$$\Big\downarrow \ 1026$$

$$\frac{x \left(a+b x^3\right)^{2/3}}{3 d}-\frac{\frac{(3bc-2ad)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{3c(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{d}}{3d}$$

$$\Big\downarrow \ 769$$

$$\frac{x \left(a+b x^3\right)^{2/3}}{3 d}-\frac{\frac{(3bc-2ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{3c(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{d}}{3d}$$

$$\Big\downarrow \ 901$$

$$\frac{x\left(a+bx^3\right)^{2/3}}{3d}-\frac{\frac{(3bc-2ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{3c(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\cdots\right.}{\cdots}\right.}{d}}{3d}$$

input
```
Int[(x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^(2/3))/(3*d) - ((-3*c*(b*c - a*d)*(ArcTan[(1 + (2*(b*c - a*
d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c -
a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a
*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/
d + ((3*b*c - 2*a*d)*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]
]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/
d)/(3*d)
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.18

method	result
pseudoelliptic	$-\frac{\left(-dx\left(b\,x^{3}+a\right)^{\frac{2}{3}}b^{\frac{1}{3}}+\frac{2\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}\right.}{x^{2}}\right.}{2}\right.}{3}\right.}{}$

input
```
int(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((-d*x*(b*x^3+a)^(2/3)*b^(1/3)+2/3*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/
3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*
ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(a*d-3/2*
b*c))*((a*d-b*c)/c)^(1/3)+(3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)
*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)+ln((((a*d-b*c)/c)^(1/3)*x+(b*
x^3+a)^(1/3))/x)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^
3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(b^(1/3)*a*d-b^(4/3)*c))/((a*d-b*c)/c)
^(1/3)/b^(1/3)/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(219) = 438$.

Time = 0.29 (sec) , antiderivative size = 1091, normalized size of antiderivative = 4.01

$$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c - 2*a*b*d)*sqrt((-
b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)
*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2
/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 6*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2*d - a^2
*c*d^2)^(1/3)*b*arctan(1/3*(sqrt(3)*(b*c - a*d)*x - 2*sqrt(3)*(-b^2*c^3 +
2*a*b*c^2*d - a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) + 2*(3*
b*c - 2*a*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (3*b*c
 - 2*a*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x
+ (b*x^3 + a)^(2/3))/x^2) + 6*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b
*log(((-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(2/3)*x - (b*x^3 + a)^(1/3)*(b*
c^2 - a*c*d))/x) - 3*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b
^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*(b*c - a*d)*x^2 - (-b^2*c^3 + 2*a*
b*c^2*d - a^2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x - (b*x^3 + a)^(2/3)*(b*c^2
- a*c*d))/x^2))/(b*d^2), 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x + 6*sqrt(1/3)*(3*
b^2*c - 2*a*b*d)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(
b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 6*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2
*d - a^2*c*d^2)^(1/3)*b*arctan(1/3*(sqrt(3)*(b*c - a*d)*x - 2*sqrt(3)*(-b^
2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x))
 + 2*(3*b*c - 2*a*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x)
- (3*b*c - 2*a*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-...
```

Sympy [F]

$$\int \frac{x^3 (a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^3 (a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**3*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**3*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}x^3}{dx^3+c}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^3/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}x^3}{dx^3+c}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^3/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{x^3\,(b\,x^3+a)^{2/3}}{d\,x^3+c}\,dx$$

input
```
int((x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

output
```
int((x^3*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{(b\,x^3+a)^{\frac{2}{3}}\,x-\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)ac+2\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}x^3}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)ad-3\left(\int\right.}{3d}$$

input
```
int(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
((a + b*x**3)**(2/3)*x - int((a + b*x**3)**(2/3)/(a*c + a*d*x**3 + b*c*x**
3 + b*d*x**6),x)*a*c + 2*int(((a + b*x**3)**(2/3)*x**3)/(a*c + a*d*x**3 +
b*c*x**3 + b*d*x**6),x)*a*d - 3*int(((a + b*x**3)**(2/3)*x**3)/(a*c + a*d*
x**3 + b*c*x**3 + b*d*x**6),x)*b*c)/(3*d)
```

3.701 $\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 233

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d} - \frac{(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d} - \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6c^{2/3}d} + \frac{(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}d} - \frac{b^{2/3}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2d}$$

output

```
1/3*b^(2/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d-
1/3*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^
(1/3))*3^(1/2))*3^(1/2)/c^(2/3)/d-1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(2/3)
/d+1/2*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(
2/3)/d-1/2*b^(2/3)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{4\sqrt{3}b^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+\frac{2\sqrt{-6+6i\sqrt{3}}(bc-ad)^{2/3}\arctan\left(\frac{3\sqrt[3]{bc-ad}\ldots}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{\ldots}}\right)}{c^{2/3}}\ldots}{\ldots}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

output
```
(4*Sqrt[3]*b^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(
1/3))] + (2*Sqrt[-6 + (6*I)*Sqrt[3]]*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*
d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*
x^3)^(1/3))]])/c^(2/3) - 4*b^(2/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] -
((2*I)*(-I + Sqrt[3])*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I
*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]])/c^(2/3) + 2*b^(2/3)*Log[b^(2/3)*x^2
+ b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + ((1 + I*Sqrt[3])*(b*c
 - a*d)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c
- a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/
3)]])/c^(2/3))/(12*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {916, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx$$

$$\Big\downarrow \text{916}$$

$$\frac{b\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}$$

$$\Big\downarrow \text{769}$$

$$\frac{b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}$$

$$\Big\downarrow \text{901}$$

$$\frac{b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{d}$$

input
```
Int[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

output
```
-(((b*c - a*d)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(
1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^
(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)
^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/d) + (b*(ArcTan[(1 + (2*b^(1/3)*x)
/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b
*x^3)^(1/3)]/(2*b^(1/3))))/d
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 916
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[b/d   Int[(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*
x^n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c -
a*d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.40

method	result
pseudoelliptic	$\frac{\frac{(-ad+bc)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}+\frac{b^{\frac{2}{3}}\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)c\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}{2}+\cdots}{\cdots}$

input
```
int((b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(1/3)*(1/2*(-a*d+b*c)*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-
b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+1/2*b^(2/3)*ln((b^(2
/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*c*((a*d-b*c)/c)^(1
/3)+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*(a*d-b*c)-b^(2/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)/b^(1/3)+x)/x)*c*((a*d-b*c)/c)^(1/3
)-b^(2/3)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*c*((a*d-b*c)/c)^(1/3)+(a*d-b*
c)*3^(1/2)*arctan(1/3*3^(1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x
))/d/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(186) = 372$.

Time = 0.19 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.01

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx =$$

$$-\frac{2\sqrt{3}\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3\,(bc-ad)x}\right)+2\sqrt{3}(-b^2)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(s
qrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(-b^2)^(1/3)*arctan(-
1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((
b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*
log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*log(-((-b^2)
^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2
) + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2
- 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2))/d
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{2/3}}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(2/3)/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{d\,x^{3}+c}dx$$

input
```
int((b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int((a + b*x**3)**(2/3)/(c + d*x**3),x)
```

3.702 $\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 169

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{2cx^2} + \frac{(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}} + \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6c^{5/3}} - \frac{(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{5/3}}$$

output
```
-1/2*(b*x^3+a)^(2/3)/c/x^2+1/3*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)
^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(5/3)+1/6*(-a*d+b*c)^
(2/3)*ln(d*x^3+c)/c^(5/3)-1/2*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/
3)-(b*x^3+a)^(1/3))/c^(5/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.85 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx = \frac{-\frac{6c^{2/3}(a+bx^3)^{2/3}}{x^2} - 2\sqrt{-6+6i\sqrt{3}}(bc-ad)^{2/3}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+}}\right.}{}$$

input

```
Integrate[(a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x]
```

output

```
((-6*c^(2/3)*(a + b*x^3)^(2/3))/x^2 - 2*Sqrt[-6 + (6*I)*Sqrt[3]]*(b*c - a*
d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*
I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 2*(1 + I*Sqrt[3])*(b*c - a*d)^(
2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)
] - I*(-I + Sqrt[3])*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 -
 I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3]
)*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(5/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {975, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx$$

$$\Big\downarrow \ 975$$

$$\frac{\int \frac{2(bc-ad)}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{2c} - \frac{(a+bx^3)^{2/3}}{2cx^2}$$

$$\Big\downarrow \ 27$$

$$\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{c}-\frac{\left(a+bx^3\right)^{2/3}}{2cx^2}$$

$$\Big\downarrow \textcolor{blue}{901}$$

$$\frac{(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{\left(a+bx^3\right)^{2/3}}{2cx^2}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x]
```

output
```
-1/2*(a + b*x^3)^(2/3)/(c*x^2) + ((b*c - a*d)*(ArcTan[(1 + (2*(b*c - a*d)^
(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d
)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)
^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$\frac{-2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)(ad-bc)x^{2}-3\left(b\,x^{3}+a\right)^{\frac{2}{3}}c\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}+(ad-bc)\left(-2\arctan\left(\frac{\sqrt{3}\left(-\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}+x\right)}{3x}\right)\sqrt{}\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}c^{2}x^{2}}$

input
```
int((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/c)^(1/3)*(-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*
(a*d-b*c)*x^2-3*(b*x^3+a)^(2/3)*c*((a*d-b*c)/c)^(1/3)+(a*d-b*c)*(-2*arctan
(1/3*3^(1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*3^(1/2)+ln((((a
*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3)
)/x^2))*x^2)/c^2/x^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^3\,(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x^3(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/x**3/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(x**3*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^3\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^3*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx = \frac{-(b\,x^3+a)^{\frac{2}{3}}\,b+2\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{bd\,x^9+ad\,x^6+bc\,x^6+ac\,x^3}dx\right)a^2d\,x^2-2\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{bd\,x^9+ad\,x^6+bc\,x^6+ac\,x^3}dx\right)a}{2ad\,x^2}$$

input
```
int((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x)
```

output
```
( - (a + b*x**3)**(2/3)*b + 2*int((a + b*x**3)**(2/3)/(a*c*x**3 + a*d*x**6
 + b*c*x**6 + b*d*x**9),x)*a**2*d*x**2 - 2*int((a + b*x**3)**(2/3)/(a*c*x*
*3 + a*d*x**6 + b*c*x**6 + b*d*x**9),x)*a*b*c*x**2)/(2*a*d*x**2)
```

3.703 $\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 206

$$\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{5cx^5} - \frac{(2bc-5ad)(a+bx^3)^{2/3}}{10ac^2x^2} - \frac{d(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{8/3}} - \frac{d(bc-ad)^{2/3}\log(c+dx^3)}{6c^{8/3}} + \frac{d(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{8/3}}$$

output
```
-1/5*(b*x^3+a)^(2/3)/c/x^5-1/10*(-5*a*d+2*b*c)*(b*x^3+a)^(2/3)/a/c^2/x^2-1
/3*d*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)
^(1/3))*3^(1/2))*3^(1/2)/c^(8/3)-1/6*d*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(8/3
)+1/2*d*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^
(8/3)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.02 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx = \frac{\frac{6c^{2/3}(a+bx^3)^{2/3}(-2ac-2bcx^3+5adx^3)}{ax^5} + 10\sqrt{-6+6i\sqrt{3}}\,d(bc-ad)^{2/3}\arctan\left(\frac{3\sqrt[3]{\cdots}}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x]
```

output
```
((6*c^(2/3)*(a + b*x^3)^(2/3)*(-2*a*c - 2*b*c*x^3 + 5*a*d*x^3))/(a*x^5) +
10*Sqrt[-6 + (6*I)*Sqrt[3]]*d*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3
)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1
/3))] - (10*I)*(-I + Sqrt[3])*d*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*
x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + 5*(1 + I*Sqrt[3])*d*(b*c
- a*d)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c -
 a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3
)])/(60*c^(8/3))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {975, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx$$

$$\Big\downarrow \; 975$$

$$\frac{\int \frac{-3bdx^3+2bc-5ad}{x^3\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{5c} - \frac{(a+bx^3)^{2/3}}{5cx^5}$$

$$\Big\downarrow \; 1053$$

$$\frac{-\frac{\int \frac{10ad(bc-ad)}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{2ac}-\frac{(a+bx^3)^{2/3}(2bc-5ad)}{2acx^2}}{5c}-\frac{(a+bx^3)^{2/3}}{5cx^5}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{5d(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{c}-\frac{(a+bx^3)^{2/3}(2bc-5ad)}{2acx^2}}{5c}-\frac{(a+bx^3)^{2/3}}{5cx^5}$$

$$\Big\downarrow \; 901$$

$$\frac{-\frac{5d(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{(a+bx^3)^{2/3}(2bc-5ad)}{2acx^2}}{5c}-\frac{(a+bx^3)^{2/3}}{5cx^5}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x]
```

output
```
-1/5*(a + b*x^3)^(2/3)/(c*x^5) + (-1/2*((2*b*c - 5*a*d)*(a + b*x^3)^(2/3))
/(a*c*x^2) - (5*d*(b*c - a*d)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3
)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c
 + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3
) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/c)/(5*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a(ad-bc)d\,x^{5}-\frac{3c\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(\left(-\frac{5ad}{2}+bc\right)x^{3}+ac\right)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}{5}+a(ad-bc)\left(\arctan\left(\frac{\sqrt{3}\left(-\frac{2(b\cdots}{(ad\cdots}\right.}{3\cdots}\right.\right.}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}c^{3}x^{5}a}$

input
```
int((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(1/3)*(ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*a*(
a*d-b*c)*d*x^5-3/5*c*(b*x^3+a)^(2/3)*((-5/2*a*d+b*c)*x^3+a*c)*((a*d-b*c)/c
)^(1/3)+a*(a*d-b*c)*(arctan(1/3*3^(1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^
(1/3)+x)/x)*3^(1/2)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b
*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*d*x^5)/c^3/x^5/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^6\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x^6(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/x**6/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(x**6*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^6), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^6\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^6*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx = \frac{-2(b\,x^3+a)^{\frac{2}{3}}\,a+3(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3-10\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{bd\,x^9+ad\,x^6+bc\,x^6+ac\,x^3}dx\right)a^2d\,x^5+10\left(\int \frac{}{bd}\right.}{10ac\,x^5}$$

input
```
int((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x)
```

output
```
( - 2*(a + b*x**3)**(2/3)*a + 3*(a + b*x**3)**(2/3)*b*x**3 - 10*int((a + b
*x**3)**(2/3)/(a*c*x**3 + a*d*x**6 + b*c*x**6 + b*d*x**9),x)*a**2*d*x**5 +
 10*int((a + b*x**3)**(2/3)/(a*c*x**3 + a*d*x**6 + b*c*x**6 + b*d*x**9),x)
*a*b*c*x**5)/(10*a*c*x**5)
```

3.704 $\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 257

$$\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{8cx^8} - \frac{(bc-4ad)(a+bx^3)^{2/3}}{20ac^2x^5} + \frac{(3b^2c^2+8abcd-20a^2d^2)(a+bx^3)^{2/3}}{40a^2c^3x^2} + \frac{d^2(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{11/3}} + \frac{d^2(bc-ad)^{2/3}\log(c+dx^3)}{6c^{11/3}} - \frac{d^2(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{11/3}}$$

output
```
-1/8*(b*x^3+a)^(2/3)/c/x^8-1/20*(-4*a*d+b*c)*(b*x^3+a)^(2/3)/a/c^2/x^5+1/4
0*(-20*a^2*d^2+8*a*b*c*d+3*b^2*c^2)*(b*x^3+a)^(2/3)/a^2/c^3/x^2+1/3*d^2*(-
a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))
*3^(1/2))*3^(1/2)/c^(11/3)+1/6*d^2*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(11/3)-1
/2*d^2*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(
11/3)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.28 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx = \frac{-\frac{3c^{2/3}(a+bx^3)^{2/3}(-3b^2c^2x^6+2abcx^3(c-4dx^3)+a^2(5c^2-8cdx^3+20d^2x^6))}{a^2x^8} - 20\sqrt{-6+6i\sqrt{3}}d^2(bc-ad\cdots}{\cdots}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x]
```

output
```
((-3*c^(2/3)*(a + b*x^3)^(2/3)*(-3*b^2*c^2*x^6 + 2*a*b*c*x^3*(c - 4*d*x^3)
 + a^2*(5*c^2 - 8*c*d*x^3 + 20*d^2*x^6)))/(a^2*x^8) - 20*Sqrt[-6 + (6*I)*S
qrt[3]]*d^2*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c
 - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 20*(1 + I*
Sqrt[3])*d^2*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])
*c^(1/3)*(a + b*x^3)^(1/3)] - (10*I)*(-I + Sqrt[3])*d^2*(b*c - a*d)^(2/3)*
Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x
*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(120*c^(1
1/3))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {975, 27, 1053, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{2(-3bdx^3+bc-4ad)}{x^6\sqrt[3]{bx^3+a}(dx^3+c)}dx}{8c} - \frac{(a+bx^3)^{2/3}}{8cx^8}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{-3bdx^3+bc-4ad}{x^6\sqrt[3]{bx^3+a}(dx^3+c)}dx}{4c}-\frac{\left(a+bx^3\right)^{2/3}}{8cx^8}$$

$\Big\downarrow$ 1053

$$\frac{-\frac{\int \frac{3bd(bc-4ad)x^3+3b^2c^2-20a^2d^2+8abcd}{x^3\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{5ac}-\frac{\left(a+bx^3\right)^{2/3}(bc-4ad)}{5acx^5}}{4c}-\frac{\left(a+bx^3\right)^{2/3}}{8cx^8}$$

$\Big\downarrow$ 1053

$$\frac{-\frac{-\frac{\int \frac{40a^2d^2(bc-ad)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{2ac}-\frac{\left(a+bx^3\right)^{2/3}\left(\frac{3b^2c}{a}-\frac{20ad^2}{c}+8bd\right)}{2x^2}}{5ac}-\frac{\left(a+bx^3\right)^{2/3}(bc-4ad)}{5acx^5}}{4c}-\frac{\left(a+bx^3\right)^{2/3}}{8cx^8}$$

$\Big\downarrow$ 27

$$\frac{-\frac{-\frac{20ad^2(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{c}-\frac{\left(a+bx^3\right)^{2/3}\left(\frac{3b^2c}{a}-\frac{20ad^2}{c}+8bd\right)}{2x^2}}{5ac}-\frac{\left(a+bx^3\right)^{2/3}(bc-4ad)}{5acx^5}}{4c}-\frac{\left(a+bx^3\right)^{2/3}}{8cx^8}$$

$\Big\downarrow$ 901

$$\frac{-\frac{-\frac{20ad^2(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{\left(a+bx^3\right)^{2/3}\left(\frac{3b^2c}{a}-\frac{20ad^2}{c}+8bd\right)}{2x^2}}{5ac}}{4c}$$

$$\frac{\left(a+bx^3\right)^{2/3}}{8cx^8}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x]
```

output
```
-1/8*(a + b*x^3)^(2/3)/(c*x^8) + (-1/5*((b*c - 4*a*d)*(a + b*x^3)^(2/3))/(
a*c*x^5) - (-1/2*(((3*b^2*c)/a + 8*b*d - (20*a*d^2)/c)*(a + b*x^3)^(2/3))/
x^2 - (20*a*d^2*(b*c - a*d)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*
(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c +
 d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3)
- (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/c)/(5*a*c))/(4*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{-3c\left(\left(4a^2d^2-\frac{8}{5}abcd-\frac{3}{5}b^2c^2\right)x^6+\frac{2\left(-4a^2cd+b\,c^2a\right)x^3}{5}+a^2c^2\right)\left(b\,x^3+a\right)^{\frac{2}{3}}\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}+4a^2d^2x^8(ad-bc)\left(-2\sqrt{3}\,\arctan\left(\sqrt{\ldots}\right.\right.}{24\left(\frac{ad-b\ldots}{c}\right.}$

input
```
int((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/24/((a*d-b*c)/c)^(1/3)*(-3*c*((4*a^2*d^2-8/5*a*b*c*d-3/5*b^2*c^2)*x^6+2/
5*(-4*a^2*c*d+a*b*c^2)*x^3+a^2*c^2)*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3)+4*
a^2*d^2*x^8*(a*d-b*c)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*
x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)+ln((((a*d-b*c)/c)^(2/3)*x^2-((
a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)
/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)))/x^8/c^4/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^9\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x^9(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/x**9/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(x**9*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^9}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^9), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^9}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^9\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^9*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx = \frac{-5(b\,x^3+a)^{\frac{2}{3}}\,a^2c+8(b\,x^3+a)^{\frac{2}{3}}\,a^2d\,x^3-2(b\,x^3+a)^{\frac{2}{3}}\,abc\,x^3-12(b\,x^3+a)^{\frac{2}{3}}\,abd\,x^6 -}{}$$

input
```
int((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x)
```

output
```
( - 5*(a + b*x**3)**(2/3)*a**2*c + 8*(a + b*x**3)**(2/3)*a**2*d*x**3 - 2*(
a + b*x**3)**(2/3)*a*b*c*x**3 - 12*(a + b*x**3)**(2/3)*a*b*d*x**6 + 3*(a +
 b*x**3)**(2/3)*b**2*c*x**6 + 40*int((a + b*x**3)**(2/3)/(a*c*x**3 + a*d*x
**6 + b*c*x**6 + b*d*x**9),x)*a**3*d**2*x**8 - 40*int((a + b*x**3)**(2/3)/
(a*c*x**3 + a*d*x**6 + b*c*x**6 + b*d*x**9),x)*a**2*b*c*d*x**8)/(40*a**2*c
**2*x**8)
```

3.705 $\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 320

$$
\begin{aligned}
\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx &= -\frac{(a+bx^3)^{2/3}}{11cx^{11}} - \frac{(2bc-11ad)\,(a+bx^3)^{2/3}}{88ac^2x^8} \\
&+ \frac{(6b^2c^2+11abcd-44a^2d^2)\,(a+bx^3)^{2/3}}{220a^2c^3x^5} \\
&- \frac{(18b^3c^3+33ab^2c^2d+88a^2bcd^2-220a^3d^3)\,(a+bx^3)^{2/3}}{440a^3c^4x^2} \\
&- \frac{d^3(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{14/3}} - \frac{d^3(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{14/3}} \\
&+ \frac{d^3(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{14/3}}
\end{aligned}
$$

output
```
-1/11*(b*x^3+a)^(2/3)/c/x^11-1/88*(-11*a*d+2*b*c)*(b*x^3+a)^(2/3)/a/c^2/x^
8+1/220*(-44*a^2*d^2+11*a*b*c*d+6*b^2*c^2)*(b*x^3+a)^(2/3)/a^2/c^3/x^5-1/4
40*(-220*a^3*d^3+88*a^2*b*c*d^2+33*a*b^2*c^2*d+18*b^3*c^3)*(b*x^3+a)^(2/3)
/a^3/c^4/x^2-1/3*d^3*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c
^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(14/3)-1/6*d^3*(-a*d+b*c)^(2/3)
*ln(d*x^3+c)/c^(14/3)+1/2*d^3*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/
3)-(b*x^3+a)^(1/3))/c^(14/3)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.80 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx = \frac{\frac{3c^{2/3}(a+bx^3)^{2/3}(-18b^3c^3x^9+3ab^2c^2x^6(4c-11dx^3)-2a^2bcx^3(5c^2-11cdx^3+44d^2x^6)+a^3(-40c^3+55c^2dx^3-88cd^2}{a^3x^{11}}}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x]
```

output
```
((3*c^(2/3)*(a + b*x^3)^(2/3)*(-18*b^3*c^3*x^9 + 3*a*b^2*c^2*x^6*(4*c - 11
*d*x^3) - 2*a^2*b*c*x^3*(5*c^2 - 11*c*d*x^3 + 44*d^2*x^6) + a^3*(-40*c^3 +
 55*c^2*d*x^3 - 88*c*d^2*x^6 + 220*d^3*x^9)))/(a^3*x^11) + 220*Sqrt[-6 + (
6*I)*Sqrt[3]]*d^3*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3
]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] - (220
*I)*(-I + Sqrt[3])*d^3*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 +
I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + 110*(1 + I*Sqrt[3])*d^3*(b*c - a*d
)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)
^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(
1320*c^(14/3))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {975, 1053, 27, 1053, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}dx$$

$$\Big\downarrow \; 975$$

$$\frac{\int \frac{-9bdx^3+2bc-11ad}{x^9\sqrt[3]{bx^3+a}(dx^3+c)}dx}{11c}-\frac{(a+bx^3)^{2/3}}{11cx^{11}}$$

$$\Big\downarrow \; 1053$$

$$\frac{-\frac{\int \frac{2\left(3bd(2bc-11ad)x^3+6b^2c^2-44a^2d^2+11abcd\right)}{x^6\sqrt[3]{bx^3+a}(dx^3+c)}dx}{8ac}-\frac{(a+bx^3)^{2/3}(2bc-11ad)}{8acx^8}}{11c}-\frac{(a+bx^3)^{2/3}}{11cx^{11}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{3bd(2bc-11ad)x^3+6b^2c^2-44a^2d^2+11abcd}{x^6\sqrt[3]{bx^3+a}(dx^3+c)}dx}{4ac}-\frac{(a+bx^3)^{2/3}(2bc-11ad)}{8acx^8}}{11c}-\frac{(a+bx^3)^{2/3}}{11cx^{11}}$$

$$\Big\downarrow \; 1053$$

$$\frac{-\frac{-\frac{\int \frac{18b^3c^3+33ab^2dc^2+88a^2bd^2c-220a^3d^3+3bd\left(6b^2c^2+11abdc-44a^2d^2\right)x^3}{x^3\sqrt[3]{bx^3+a}(dx^3+c)}dx}{5ac}-\frac{(a+bx^3)^{2/3}\left(\frac{6b^2c}{a}-\frac{44ad^2}{c}+11bd\right)}{5x^5}}{4ac}-\frac{(a+bx^3)^{2/3}(2bc-11ad)}{8acx^8}}{11c}-\frac{(a+bx^3)^{2/3}}{11cx^{11}}$$

$$\Big\downarrow \; 1053$$

$$-\frac{-\frac{-\frac{\int \frac{440a^3d^3(bc-ad)}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{2ac}-\frac{(a+bx^3)^{2/3}(-220a^3d^3+88a^2bcd^2+33ab^2c^2d+18b^3c^3)}{2acx^2}}{5ac}-\frac{(a+bx^3)^{2/3}\left(\frac{6b^2c}{a}-\frac{44ad^2}{c}+11bd\right)}{5x^5}}{4ac}-\frac{(a+bx^3)^{2/3}(2bc-1}{8acx^8}}{11c}$$

$$\frac{(a+bx^3)^{2/3}}{11cx^{11}}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{-\frac{220a^2d^3(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{c}-\frac{(a+bx^3)^{2/3}(-220a^3d^3+88a^2bcd^2+33ab^2c^2d+18b^3c^3)}{2acx^2}}{5ac}-\frac{(a+bx^3)^{2/3}\left(\frac{6b^2c}{a}-\frac{44ad^2}{c}+11bd\right)}{5x^5}}{4ac}-\frac{(a+}{}}{11c}$$

$$\frac{(a+bx^3)^{2/3}}{11cx^{11}}$$

$$\Big\downarrow \; 901$$

$$-\frac{-\frac{-\frac{220a^2d^3(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{(a+bx^3)^{2/3}(-220a^3d^3+88a^2bc}{2acx^2}}{5ac}}{4ac}}{11c}$$

$$\frac{(a+bx^3)^{2/3}}{11cx^{11}}$$

input

```
Int[(a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x]
```

output
```
-1/11*(a + b*x^3)^(2/3)/(c*x^11) + (-1/8*((2*b*c - 11*a*d)*(a + b*x^3)^(2/
3))/(a*c*x^8) - (-1/5*(((6*b^2*c)/a + 11*b*d - (44*a*d^2)/c)*(a + b*x^3)^(
2/3))/x^5 - (-1/2*((18*b^3*c^3 + 33*a*b^2*c^2*d + 88*a^2*b*c*d^2 - 220*a^3
*d^3)*(a + b*x^3)^(2/3))/(a*c*x^2) - (220*a^2*d^3*(b*c - a*d)*(ArcTan[(1 +
 (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^
(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) -
Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a
*d)^(1/3))))/c)/(5*a*c))/(4*a*c))/(11*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{6c\left(\left(\frac{9}{20}b^2x^6-\frac{3}{4}ab\,x^3+a^2\right)\left(b\,x^3+a\right)c^3-\frac{11a\left(-\frac{3b\,x^3}{5}+a\right)d\left(b\,x^3+a\right)x^3c^2}{8}+\frac{11\left(b\,x^3+a\right)a^2c\,d^2x^6}{5}-\frac{11a^3d^3x^9}{2}\right)\left(b\,x^3+a\right)^{\frac{2}{3}}\left(\cdots\right.}{}$

input
```
int((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/66/((a*d-b*c)/c)^(1/3)*(6*c*((9/20*b^2*x^6-3/4*a*b*x^3+a^2)*(b*x^3+a)*c
^3-11/8*a*(-3/5*b*x^3+a)*d*(b*x^3+a)*x^3*c^2+11/5*(b*x^3+a)*a^2*c*d^2*x^6-
11/2*a^3*d^3*x^9)*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3)+11*a^3*d^3*x^11*(a*d
-b*c)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1
/3))/((a*d-b*c)/c)^(1/3)/x)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3
)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln(((((a*d-b*c)/c)^(1/3)*x+(b*x
^3+a)^(1/3))/x)))/x^11/c^5/a^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x^{12}(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/x**12/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(x**12*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^{12}}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^12), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^{12}}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^{12}\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^12*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx = \frac{-40(b\,x^3+a)^{\frac{2}{3}}\,a^3c^2+55(b\,x^3+a)^{\frac{2}{3}}\,a^3cd\,x^3-88(b\,x^3+a)^{\frac{2}{3}}\,a^3d^2x^6-10(b\,x^3+a)^{\frac{2}{3}}\,a}{}$$

input
```
int((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x)
```

output
```
( - 40*(a + b*x**3)**(2/3)*a**3*c**2 + 55*(a + b*x**3)**(2/3)*a**3*c*d*x**
3 - 88*(a + b*x**3)**(2/3)*a**3*d**2*x**6 - 10*(a + b*x**3)**(2/3)*a**2*b*
c**2*x**3 + 22*(a + b*x**3)**(2/3)*a**2*b*c*d*x**6 + 132*(a + b*x**3)**(2/
3)*a**2*b*d**2*x**9 + 12*(a + b*x**3)**(2/3)*a*b**2*c**2*x**6 - 33*(a + b*
x**3)**(2/3)*a*b**2*c*d*x**9 - 18*(a + b*x**3)**(2/3)*b**3*c**2*x**9 - 440
*int((a + b*x**3)**(2/3)/(a*c*x**3 + a*d*x**6 + b*c*x**6 + b*d*x**9),x)*a*
*4*d**3*x**11 + 440*int((a + b*x**3)**(2/3)/(a*c*x**3 + a*d*x**6 + b*c*x**
6 + b*d*x**9),x)*a**3*b*c*d**2*x**11)/(440*a**3*c**3*x**11)
```

3.706 $\int \frac{x^7 \left(a+bx^3\right)^{2/3}}{c+dx^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^7 (a+bx^3)^{2/3}}{c+dx^3} \, dx = \frac{x^8 (a+bx^3)^{2/3} \operatorname{AppellF1}\left(\frac{8}{3},-\frac{2}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c \left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
1/8*x^8*(b*x^3+a)^(2/3)*AppellF1(8/3,-2/3,1,11/3,-b*x^3/a,-d*x^3/c)/c/(1+b
*x^3/a)^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(64) = 128.

Time = 7.62 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.83

$$\int \frac{x^7 (a+bx^3)^{2/3}}{c+dx^3} \, dx = \frac{x^2 \left(5c(a+bx^3)\left(-7bc+2ad+4bdx^3\right)+5ac(7bc-2ad)\sqrt[3]{1+\frac{bx^3}{a}} \operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{3},\right.\right.}{1}$$

input
```
Integrate[(x^7*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(x^2*(5*c*(a + b*x^3)*(-7*b*c + 2*a*d + 4*b*d*x^3) + 5*a*c*(7*b*c - 2*a*d)
*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/
c)] - 2*(-14*b^2*c^2 + 7*a*b*c*d + 2*a^2*d^2)*x^3*(1 + (b*x^3)/a)^(1/3)*Ap
pellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]))/(140*b*c*d^2*(a + b
*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \left(a+bx^3\right)^{2/3}}{c+dx^3} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\left(a+bx^3\right)^{2/3} \int \frac{x^7 \left(\frac{bx^3}{a}+1\right)^{2/3}}{dx^3+c}dx}{\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^8 \left(a+bx^3\right)^{2/3} \text{AppellF1}\left(\frac{8}{3},-\frac{2}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c \left(\frac{bx^3}{a}+1\right)^{2/3}}$$

input
```
Int[(x^7*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(x^8*(a + b*x^3)^(2/3)*AppellF1[8/3, -2/3, 1, 11/3, -((b*x^3)/a), -((d*x^3
)/c)])/(8*c*(1 + (b*x^3)/a)^(2/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{7} \left(b\,x^{3}+a\right)^{\frac{2}{3}}}{d\,x^{3}+c}dx$$

input
```
int(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3}\,dx=\text{Timed out}$$

input
```
integrate(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{x^7(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**7*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**7*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}x^7}{dx^3+c}\,dx$$

input
```
integrate(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^7/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}x^7}{dx^3+c}\,dx$$

input
```
integrate(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^7/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7 (a+bx^3)^{2/3}}{c+dx^3} \, dx = \int \frac{x^7 \left(b x^3+a\right)^{2/3}}{d x^3+c} \, dx$$

input
```
int((x^7*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

output
```
int((x^7*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{x^7 (a+bx^3)^{2/3}}{c+dx^3} \, dx = \frac{2(b x^3+a)^{\frac{2}{3}} a d x^2 - 7(b x^3+a)^{\frac{2}{3}} b c x^2 + 4(b x^3+a)^{\frac{2}{3}} b d x^5 - 4\left(\int \frac{(b x^3+a)^{\frac{2}{3}} x^4}{b d x^6+a d x^3+b c x^3+a}\right.}{}$$

input
```
int(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
(2*(a + b*x**3)**(2/3)*a*d*x**2 - 7*(a + b*x**3)**(2/3)*b*c*x**2 + 4*(a +
b*x**3)**(2/3)*b*d*x**5 - 4*int(((a + b*x**3)**(2/3)*x**4)/(a*c + a*d*x**3
 + b*c*x**3 + b*d*x**6),x)*a**2*d**2 - 14*int(((a + b*x**3)**(2/3)*x**4)/(
a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b*c*d + 28*int(((a + b*x**3)**(
2/3)*x**4)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**2*c**2 - 4*int(((a
 + b*x**3)**(2/3)*x)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a**2*c*d +
14*int(((a + b*x**3)**(2/3)*x)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a
*b*c**2)/(28*b*d**2)
```

3.707 $\int \frac{x^4\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{x^5(a+bx^3)^{2/3}\operatorname{AppellF1}\left(\frac{5}{3},-\frac{2}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
1/5*x^5*(b*x^3+a)^(2/3)*AppellF1(5/3,-2/3,1,8/3,-b*x^3/a,-d*x^3/c)/c/(1+b*
x^3/a)^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 141 vs. $2(64) = 128$.

Time = 6.99 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.20

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{5cx^2(a+bx^3)-5acx^2\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2(-2bc+ad)}{20cd\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(x^4*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(5*c*x^2*(a + b*x^3) - 5*a*c*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3,
1, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*(-2*b*c + a*d)*x^5*(1 + (b*x^3)/a)
^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*c*d*(a
+ b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (a+bx^3)^{2/3}}{c+dx^3} \, dx$$

$$\Big\downarrow \ 1013$$

$$\frac{(a+bx^3)^{2/3} \int \frac{x^4 \left(\frac{bx^3}{a}+1\right)^{2/3}}{dx^3+c}dx}{\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

$$\Big\downarrow \ 1012$$

$$\frac{x^5 (a+bx^3)^{2/3} \operatorname{AppellF1}\left(\frac{5}{3},-\frac{2}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

input
```
Int[(x^4*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(x^5*(a + b*x^3)^(2/3)*AppellF1[5/3, -2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)
/c)])/(5*c*(1 + (b*x^3)/a)^(2/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4(b\,x^3+a)^{\frac{2}{3}}}{d\,x^3+c}dx$$

input
```
int(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{x^4(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**4*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**4*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}x^4}{dx^3+c}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^4/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}x^4}{dx^3+c}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^4/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 (a+bx^3)^{2/3}}{c+dx^3} \, dx = \int \frac{x^4 \left(b x^3+a\right)^{2/3}}{d x^3+c} \, dx$$

input
```
int((x^4*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

output
```
int((x^4*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{x^4 (a+bx^3)^{2/3}}{c+dx^3} \, dx = \frac{(b x^3+a)^{\frac{2}{3}} x^2+2 \left(\int \frac{(b x^3+a)^{\frac{2}{3}} x^4}{bd x^6+ad x^3+bc x^3+ac}dx\right) ad-4 \left(\int \frac{(b x^3+a)^{\frac{2}{3}} x^4}{bd x^6+ad x^3+bc x^3+ac}dx\right) bc-2 (}{4d}$$

input
```
int(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
((a + b*x**3)**(2/3)*x**2 + 2*int(((a + b*x**3)**(2/3)*x**4)/(a*c + a*d*x*
*3 + b*c*x**3 + b*d*x**6),x)*a*d - 4*int(((a + b*x**3)**(2/3)*x**4)/(a*c +
 a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b*c - 2*int(((a + b*x**3)**(2/3)*x)/(a
*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*c)/(4*d)
```

3.708 $\int \frac{x\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{x^2(a+bx^3)^{2/3}\operatorname{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
1/2*x^2*(b*x^3+a)^(2/3)*AppellF1(2/3,-2/3,1,5/3,-b*x^3/a,-d*x^3/c)/c/(1+b*
x^3/a)^(2/3)
```

Mathematica [A] (verified)

Time = 9.63 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{x^2(a+bx^3)^{2/3}\operatorname{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(\frac{a+bx^3}{a}\right)^{2/3}}$$

input
```
Integrate[(x*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(x^2*(a + b*x^3)^(2/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*c*((a + b*x^3)/a)^(2/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{(a+bx^3)^{2/3}\int \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}}{dx^3+c}dx}{\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2(a+bx^3)^{2/3}\operatorname{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

input
```
Int[(x*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(x^2*(a + b*x^3)^(2/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*c*(1 + (b*x^3)/a)^(2/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x(b\,x^3+a)^{\frac{2}{3}}}{d\,x^3+c}dx$$

input
```
int(x*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}x}{dx^3+c}\,dx$$

input
```
integrate(x*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)*x/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}x}{dx^3+c}\,dx$$

input
```
integrate(x*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*x/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{x\,(b\,x^3+a)^{2/3}}{d\,x^3+c}\,dx$$

input
```
int((x*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

output
```
int((x*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^{\frac{2}{3}}\,x}{d\,x^3+c}dx$$

input
```
int(x*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(((a + b*x**3)**(2/3)*x)/(c + d*x**3),x)
```

3.709 $\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx = -\frac{(a+bx^3)^{2/3}\,\text{AppellF1}\left(-\frac{1}{3},-\frac{2}{3},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
-(b*x^3+a)^(2/3)*AppellF1(-1/3,-2/3,1,2/3,-b*x^3/a,-d*x^3/c)/c/x/(1+b*x^3/
a)^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 138 vs. 2(62) = 124.

Time = 10.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.23

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx = \frac{-10c(a+bx^3)-5(-2bc+ad)x^3\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bdx}{10c^2x\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^2*(c + d*x^3)),x]
```

output
```
(-10*c*(a + b*x^3) - 5*(-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2
/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*d*x^6*(1 + (b*x^3)/a)^(
1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(10*c^2*x*(a
+ b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{(a+bx^3)^{2/3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{2/3}}{x^2(dx^3+c)}dx}{\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{(a+bx^3)^{2/3}\text{AppellF1}\left(-\frac{1}{3},-\frac{2}{3},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^2*(c + d*x^3)),x]
```

output
```
-(((a + b*x^3)^(2/3)*AppellF1[-1/3, -2/3, 1, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c*x*(1 + (b*x^3)/a)^(2/3)))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{2}{3}}}{x^{2}\,(d\,x^{3}+c)}dx$$

input
```
int((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^2\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x^2(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/x**2/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(x**2*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^2\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^2*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^2*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{\frac{2}{3}}}{d\,x^5+c\,x^2}dx$$

input
```
int((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x)
```

output
```
int((a + b*x**3)**(2/3)/(c*x**2 + d*x**5),x)
```

3.710 $\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx = -\frac{(a+bx^3)^{2/3}\,\text{AppellF1}\left(-\frac{4}{3},-\frac{2}{3},1,-\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
-1/4*(b*x^3+a)^(2/3)*AppellF1(-4/3,-2/3,1,-1/3,-b*x^3/a,-d*x^3/c)/c/x^4/(1
+b*x^3/a)^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(64) = 128.

Time = 10.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.83

$$\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx = \frac{-5c(a+bx^3)\left(2bcx^3+a(c-4dx^3)\right)+5(b^2c^2-4abcd+2a^2d^2)\,x^6\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}}{20ac^3x^4}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^5*(c + d*x^3)),x]
```

output
```
(-5*c*(a + b*x^3)*(2*b*c*x^3 + a*(c - 4*d*x^3)) + 5*(b^2*c^2 - 4*a*b*c*d +
 2*a^2*d^2)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)
/a), -((d*x^3)/c)] + 2*b*d*(b*c - 2*a*d)*x^9*(1 + (b*x^3)/a)^(1/3)*AppellF
1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*a*c^3*x^4*(a + b*x^3)
^(1/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{(a+bx^3)^{2/3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{2/3}}{x^5(dx^3+c)}dx}{\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{(a+bx^3)^{2/3}\operatorname{AppellF1}\left(-\frac{4}{3},-\frac{2}{3},1,-\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^5*(c + d*x^3)),x]
```

output
```
-1/4*((a + b*x^3)^(2/3)*AppellF1[-4/3, -2/3, 1, -1/3, -((b*x^3)/a), -((d*x
^3)/c)])/(c*x^4*(1 + (b*x^3)/a)^(2/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{2}{3}}}{x^{5}\,(d\,x^{3}+c)}dx$$

input
```
int((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x^5(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/x**5/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(x**5*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^5), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^5\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^5*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^5*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{\frac{2}{3}}}{d\,x^8+c\,x^5}dx$$

input
```
int((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x)
```

output
```
int((a + b*x**3)**(2/3)/(c*x**5 + d*x**8),x)
```

3.711 $\int \frac{x^8 (a+bx^3)^{4/3}}{c+dx^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 251

$$\int \frac{x^8 (a+bx^3)^{4/3}}{c+dx^3} \, dx = -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4} + \frac{c^2(a+bx^3)^{4/3}}{4d^3} - \frac{(bc+ad)(a+bx^3)^{7/3}}{7b^2d^2} + \frac{(a+bx^3)^{10/3}}{10b^2d} - \frac{c^2(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{13/3}} - \frac{c^2(bc-ad)^{4/3}\log(c+dx^3)}{6d^{13/3}} + \frac{c^2(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{13/3}}$$

output
```
-c^2*(-a*d+b*c)*(b*x^3+a)^(1/3)/d^4+1/4*c^2*(b*x^3+a)^(4/3)/d^3-1/7*(a*d+b
*c)*(b*x^3+a)^(7/3)/b^2/d^2+1/10*(b*x^3+a)^(10/3)/b^2/d-1/3*c^2*(-a*d+b*c)
^(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*
3^(1/2)/d^(13/3)-1/6*c^2*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/d^(13/3)+1/2*c^2*(-a
*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(13/3)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.23

$$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}(-6a^3d^3+2a^2bd^2(-10c+dx^3)+ab^2d(175c^2-40cdx^3+22d^2x^6)+b^3(-140c^3+35c^2dx^3-20cd^2x \cdots}{b^2}}{\cdots}$$

input
```
Integrate[(x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
((3*d^(1/3)*(a + b*x^3)^(1/3)*(-6*a^3*d^3 + 2*a^2*b*d^2*(-10*c + d*x^3) +
a*b^2*d*(175*c^2 - 40*c*d*x^3 + 22*d^2*x^6) + b^3*(-140*c^3 + 35*c^2*d*x^3
 - 20*c*d^2*x^6 + 14*d^3*x^9)))/b^2 - 140*Sqrt[3]*c^2*(b*c - a*d)^(4/3)*Ar
cTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 140*
c^2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] -
 70*c^2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3
)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(420*d^(13/3))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{948, 99, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\left(a+bx^3\right)^{4/3}}{c+dx^3}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^6\left(bx^3+a\right)^{4/3}}{dx^3+c}dx^3$$

$$\Big\downarrow \text{ 99}$$

$$\frac{1}{3}\int \left(\frac{\left(bx^3+a\right)^{7/3}}{bd}+\frac{(-bc-ad)\left(bx^3+a\right)^{4/3}}{bd^2}+\frac{c^2\left(bx^3+a\right)^{4/3}}{d^2\left(dx^3+c\right)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}c^2(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{13/3}}-\frac{3\left(a+bx^3\right)^{7/3}(ad+bc)}{7b^2d^2}+\frac{3\left(a+bx^3\right)^{10/3}}{10b^2d}-\frac{c^2(bc-ad)^4}{2}\right.$$

input
```
Int[(x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
((-3*c^2*(b*c - a*d)*(a + b*x^3)^(1/3))/d^4 + (3*c^2*(a + b*x^3)^(4/3))/(4
*d^3) - (3*(b*c + a*d)*(a + b*x^3)^(7/3))/(7*b^2*d^2) + (3*(a + b*x^3)^(10
/3))/(10*b^2*d) - (Sqrt[3]*c^2*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a
 + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/d^(13/3) - (c^2*(b*c - a*d)^
(4/3)*Log[c + d*x^3])/(2*d^(13/3)) + (3*c^2*(b*c - a*d)^(4/3)*Log[(b*c - a
*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(13/3)))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{3\left(d\left(\left(b\,x^{3}+a\right)^{2}\left(-\frac{7b\,x^{3}}{3}+a\right)d^{3}+\frac{10bc\left(b\,x^{3}+a\right)^{2}d^{2}}{3}-\frac{175c^{2}\left(\frac{b\,x^{3}}{5}+a\right)b^{2}d}{6}+\frac{70b^{3}c^{3}}{3}\right)\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\frac{35b^{2}c^{2}(ad-bc)}{}\right.}{70}$

input
```
int(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-3/70/((a*d-b*c)/d)^(2/3)*(d*((b*x^3+a)^2*(-7/3*b*x^3+a)*d^3+10/3*b*c*(b*x
^3+a)^2*d^2-175/6*c^2*(1/5*b*x^3+a)*b^2*d+70/3*b^3*c^3)*((a*d-b*c)/d)^(2/3
)*(b*x^3+a)^(1/3)+35/9*b^2*c^2*(a*d-b*c)^2*(2*arctan(1/3*3^(1/2)*(2*(b*x^3
+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*c)/d)^(1/3))*3^(1/2)+ln((b*x^3+a)^(
2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))-2*ln((b*x^3+
a)^(1/3)-((a*d-b*c)/d)^(1/3))))/d^5/b^2
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.47

$$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{140\sqrt{3}(b^3c^3-ab^2c^2d)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+70}{}$$

input
```
integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/420*(140*sqrt(3)*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*arctan(-
1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c -
 a*d))/(b*c - a*d)) + 70*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*lo
g((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c -
a*d)/d)^(2/3)) - 140*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*log((b
*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) + 3*(14*b^3*d^3*x^9 - 2*(10*b^3*
c*d^2 - 11*a*b^2*d^3)*x^6 - 140*b^3*c^3 + 175*a*b^2*c^2*d - 20*a^2*b*c*d^2
 - 6*a^3*d^3 + (35*b^3*c^2*d - 40*a*b^2*c*d^2 + 2*a^2*b*d^3)*x^3)*(b*x^3 +
 a)^(1/3))/(b^2*d^4)
```

Sympy [F]

$$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x^8(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**8*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**8*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.57

$$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx =$$
$$-\frac{(b^{24}c^4d^6-2\,ab^{23}c^3d^7+a^2b^{22}c^2d^8)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{23}cd^{10}-ab^{22}d^{11})}$$
$$+\frac{\sqrt{3}(bc^3-ac^2d)(-bcd^2+ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^5}$$
$$+\frac{(bc^3-ac^2d)(-bcd^2+ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^5}$$
$$-\frac{140\,(bx^3+a)^{\frac{1}{3}}b^{21}c^3d^6-35\,(bx^3+a)^{\frac{4}{3}}b^{20}c^2d^7-140\,(bx^3+a)^{\frac{1}{3}}ab^{20}c^2d^7+20\,(bx^3+a)^{\frac{7}{3}}b^{19}cd^8-14\,(bx^3}{140\,b^{20}d^{10}}$$

input
```
integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*(b^24*c^4*d^6 - 2*a*b^23*c^3*d^7 + a^2*b^22*c^2*d^8)*(-(b*c - a*d)/d)
^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^23*c*d^10 -
 a*b^22*d^11) + 1/3*sqrt(3)*(b*c^3 - a*c^2*d)*(-b*c*d^2 + a*d^3)^(1/3)*arc
tan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*
d)/d)^(1/3))/d^5 + 1/6*(b*c^3 - a*c^2*d)*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x
^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d
)^(2/3))/d^5 - 1/140*(140*(b*x^3 + a)^(1/3)*b^21*c^3*d^6 - 35*(b*x^3 + a)^
(4/3)*b^20*c^2*d^7 - 140*(b*x^3 + a)^(1/3)*a*b^20*c^2*d^7 + 20*(b*x^3 + a)
^(7/3)*b^19*c*d^8 - 14*(b*x^3 + a)^(10/3)*b^18*d^9 + 20*(b*x^3 + a)^(7/3)*
a*b^18*d^9)/(b^20*d^10)
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.90

$$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx = \left(\frac{a^2}{4\,b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b^3\,c-a\,b^2\,d\right)}{4\,b^2\,d}\right)\,\left(b\,x^3+a\right)^{4/3}$$
$$-\left(\frac{2\,a}{7\,b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{7\,b^4\,d^2}\right)\,\left(b\,x^3+a\right)^{7/3}+\frac{\left(b\,x^3+a\right)^{10/3}}{10\,b^2\,d}$$
$$+\frac{c^2\,\ln\left(\frac{3\,\left(b\,x^3+a\right)^{1/3}\,\left(a^2\,c^2\,d^2-2\,a\,b\,c^3\,d+b^2\,c^4\right)}{d^2}-\frac{c^2\,\left(a\,d-b\,c\right)^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{13/3}}\right)\,\left(a\,d-b\,c\right)^{4/3}}{3\,d^{13/3}}$$
$$-\frac{\left(\frac{a^2}{b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b^3\,c-a\,b^2\,d\right)}{b^2\,d}\right)\,\left(b\,x^3+a\right)^{1/3}\,\left(b^3\,c-a\,b^2\,d\right)}{b^2\,d}$$
$$-\frac{c^2\,\ln\left(\frac{3\,c^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{7/3}}{d^{7/3}}+\frac{3\,c^2\,\left(b\,x^3+a\right)^{1/3}\,\left(a\,d-b\,c\right)^2}{d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{4/3}}{3\,d^{13/3}}$$
$$+\frac{c^2\,\ln\left(\frac{3\,c^2\,\left(b\,x^3+a\right)^{1/3}\,\left(a\,d-b\,c\right)^2}{d^2}-\frac{9\,c^2\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,\left(a\,d-b\,c\right)^{7/3}}{d^{7/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,\left(a\,d-b\,c\right)^{4/3}}{d^{13/3}}$$

input
```
int((x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

output
```
(a^2/(4*b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a
*b^2*d))/(4*b^2*d))*(a + b*x^3)^(4/3) - ((2*a)/(7*b^2*d) + (b^3*c - a*b^2*
d)/(7*b^4*d^2))*(a + b*x^3)^(7/3) + (a + b*x^3)^(10/3)/(10*b^2*d) + (c^2*l
og((3*(a + b*x^3)^(1/3)*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d))/d^2 - (c^2*
(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(13/3)))*(a*d - b*c)^(4/3))/
(3*d^(13/3)) - ((a^2/(b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^
2))*(b^3*c - a*b^2*d))/(b^2*d))*(a + b*x^3)^(1/3)*(b^3*c - a*b^2*d))/(b^2*
d) - (c^2*log((3*c^2*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(7/3))/d^(7/3) + (
3*c^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^2)*((3^(1/2)*1i)/2 + 1/2)*(a*d -
b*c)^(4/3))/(3*d^(13/3)) + (c^2*log((3*c^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2
)/d^2 - (9*c^2*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(7/3))/d^(7/3))*((3^(1/2
)*1i)/6 - 1/6)*(a*d - b*c)^(4/3))/d^(13/3)
```

Reduce [F]

$$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{-6(b\,x^3+a)^{\frac{1}{3}}\,a^3d^2+120(b\,x^3+a)^{\frac{1}{3}}\,a^2bcd+2(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,d^2x^3-105(b\,x^3+a)^{\frac{1}{3}}}{}$$

input
```
int(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
( - 6*(a + b*x**3)**(1/3)*a**3*d**2 + 120*(a + b*x**3)**(1/3)*a**2*b*c*d +
 2*(a + b*x**3)**(1/3)*a**2*b*d**2*x**3 - 105*(a + b*x**3)**(1/3)*a*b**2*c
**2 - 40*(a + b*x**3)**(1/3)*a*b**2*c*d*x**3 + 22*(a + b*x**3)**(1/3)*a*b*
*2*d**2*x**6 + 35*(a + b*x**3)**(1/3)*b**3*c**2*x**3 - 20*(a + b*x**3)**(1
/3)*b**3*c*d*x**6 + 14*(a + b*x**3)**(1/3)*b**3*d**2*x**9 - 140*int(((a +
b*x**3)**(1/3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a**2*b**2*c
*d**2 + 280*int(((a + b*x**3)**(1/3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*
d*x**6),x)*a*b**3*c**2*d - 140*int(((a + b*x**3)**(1/3)*x**5)/(a*c + a*d*x
**3 + b*c*x**3 + b*d*x**6),x)*b**4*c**3)/(140*b**2*d**3)
```

3.712 $\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 211

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c(a+bx^3)^{4/3}}{4d^2} + \frac{(a+bx^3)^{7/3}}{7bd} + \frac{c(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}} + \frac{c(bc-ad)^{4/3}\log(c+dx^3)}{6d^{10/3}} - \frac{c(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}$$

output
```
c*(-a*d+b*c)*(b*x^3+a)^(1/3)/d^3-1/4*c*(b*x^3+a)^(4/3)/d^2+1/7*(b*x^3+a)^(
7/3)/b/d+1/3*c*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-
a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/d^(10/3)+1/6*c*(-a*d+b*c)^(4/3)*ln(d*x^3+
c)/d^(10/3)-1/2*c*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(
1/3))/d^(10/3)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.23

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}(4a^2d^2+abd(-35c+8dx^3)+b^2(28c^2-7cdx^3+4d^2x^6))}{b}+28\sqrt{3}c(bc-ad)^{4/3}\arctan$$

input
```
Integrate[(x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
((3*d^(1/3)*(a + b*x^3)^(1/3)*(4*a^2*d^2 + a*b*d*(-35*c + 8*d*x^3) + b^2*(
28*c^2 - 7*c*d*x^3 + 4*d^2*x^6)))/b + 28*Sqrt[3]*c*(b*c - a*d)^(4/3)*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 28*c*(b
*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + 14*c*
(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b
*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(84*d^(10/3))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {948, 90, 60, 60, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^3(bx^3+a)^{4/3}}{dx^3+c}dx^3$$

$$\Big\downarrow \text{ 90}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{7/3}}{7bd}-\frac{c\int\frac{\left(bx^3+a\right)^{4/3}}{dx^3+c}dx^3}{d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{7/3}}{7bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\int\frac{\sqrt[3]{bx^3+a}}{dx^3+c}dx^3}{d}\right)}{d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{7/3}}{7bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\int\frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{d}\right)}{d}\right)}{d}\right)$$

$$\Big\downarrow \text{70}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{7/3}}{7bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int\frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+}}{2d^{2/3}\sqrt[3]{bc-ad}}\right.}{}\right.}{d}\right.}{d}\right.$$

$\downarrow$ 16

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{7/3}}{7bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int\frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+}}{2d^{2/3}\sqrt[3]{bc-ad}}\right.}{d}\right.}{d}\right.}{d}\right.$$

$\downarrow$ 1082

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{7/3}}{7bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int\frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\,\mathrm{l}\cdots}{\cdots}\right.}{d}\right.}{d}\right.}{d}\right.$$

$\downarrow$ 217

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{7/3}}{7bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log \cdots}{\cdots}\right.}{d}\right.}{d}\right.}{d}\right.$$

input
```
Int[(x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
((3*(a + b*x^3)^(7/3))/(7*b*d) - (c*((3*(a + b*x^3)^(4/3))/(4*d) - ((b*c -
 a*d)*((3*(a + b*x^3)^(1/3))/d - ((b*c - a*d)*(-((Sqrt[3]*ArcTan[(1 - (2*d
^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(1/3)*(b*c - a*d
)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c - a*d)^(2/3)) + (3*Log[(b*c - a
*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1/3)*(b*c - a*d)^(2/3))))/d)
)/d))/d)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$\frac{d\left((b\,x^3+a)^2 d^2-\frac{35c\left(\frac{b\,x^3}{5}+a\right)bd}{4}+7b^2c^2\right)\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}(b\,x^3+a)^{\frac{1}{3}}+\frac{7bc(ad-bc)^2\left(2\arctan\left(\frac{\sqrt{3}\left(2(b\,x^3+a)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{}}{}}{7\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}d^4b}$

input
```
int(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/7/((a*d-b*c)/d)^(2/3)*(d*((b*x^3+a)^2*d^2-35/4*c*(1/5*b*x^3+a)*b*d+7*b^2
*c^2)*((a*d-b*c)/d)^(2/3)*(b*x^3+a)^(1/3)+7/6*b*c*(a*d-b*c)^2*(2*arctan(1/
3*3^(1/2)*(2*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*c)/d)^(1/3))*3^(
1/2)+ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^
(2/3))-2*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))))/d^4/b
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.41

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{28\sqrt{3}(b^2c^2-abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+14\,(b^2c^2-}{}$$

input
```
integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/84*(28*sqrt(3)*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*
sqrt(3)*(b*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(
b*c - a*d)) + 14*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)
^(2/3) - (b*x^3 + a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3))
- 28*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b
*c - a*d)/d)^(1/3)) + 3*(4*b^2*d^2*x^6 + 28*b^2*c^2 - 35*a*b*c*d + 4*a^2*d
^2 - (7*b^2*c*d - 8*a*b*d^2)*x^3)*(b*x^3 + a)^(1/3))/(b*d^3)
```

Sympy [F]

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x^5(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**5*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**5*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(171) = 342$.

Time = 0.14 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.65

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{(b^{10}c^3d^4 - 2\,ab^9c^2d^5 + a^2b^8cd^6)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^9cd^7 - ab^8d^8)} - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}(bc^2-acd)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^4} - \frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc^2-acd)\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^4} + \frac{28\,(bx^3+a)^{\frac{1}{3}}b^8c^2d^4 - 7\,(bx^3+a)^{\frac{4}{3}}b^7cd^5 - 28\,(bx^3+a)^{\frac{1}{3}}ab^7cd^5 + 4\,(bx^3+a)^{\frac{7}{3}}b^6d^6}{28\,b^7d^7}$$

input
```
integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*(b^10*c^3*d^4 - 2*a*b^9*c^2*d^5 + a^2*b^8*c*d^6)*(-(b*c - a*d)/d)^(1/3
)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^9*c*d^7 - a*b^8*
d^8) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c^2 - a*c*d)*arctan(1/3*sqr
t(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3)
)/d^4 - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c^2 - a*c*d)*log((b*x^3 + a)^(2/3)
 + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^4
+ 1/28*(28*(b*x^3 + a)^(1/3)*b^8*c^2*d^4 - 7*(b*x^3 + a)^(4/3)*b^7*c*d^5 -
 28*(b*x^3 + a)^(1/3)*a*b^7*c*d^5 + 4*(b*x^3 + a)^(7/3)*b^6*d^6)/(b^7*d^7)
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.65

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{\left(b\,x^3+a\right)^{7/3}}{7\,b\,d} - \left(b\,x^3+a\right)^{4/3}\left(\frac{a}{4\,b\,d}+\frac{b^2\,c-a\,b\,d}{4\,b^2\,d^2}\right) - \frac{c\,\ln\left(\frac{3\,\left(b\,x^3+a\right)^{1/3}\,\left(a^2\,c\,d^2-2\,a\,b\,c^2\,d+b^2\,c^3\right)}{d}-\frac{c\,\left(a\,d-b\,c\right)^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{10/3}}\right)\,(a\,d}{3\,d^{10/3}}$$

input
```
int((x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

output
```
(a + b*x^3)^(7/3)/(7*b*d) - (a + b*x^3)^(4/3)*(a/(4*b*d) + (b^2*c - a*b*d)
/(4*b^2*d^2)) - (c*log((3*(a + b*x^3)^(1/3)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c
^2*d))/d - (c*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(10/3)))*(a*d
- b*c)^(4/3))/(3*d^(10/3)) - (c*log((3*c*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/
d - (3*c*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(4/3))*((3^(1/2)*1i)/
2 - 1/2)*(a*d - b*c)^(4/3))/(3*d^(10/3)) + (c*log((3*c*(a + b*x^3)^(1/3)*(
a*d - b*c)^2)/d + (3*c*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(7/3))/d^(4/3))*
((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(4/3))/(3*d^(10/3)) + ((a + b*x^3)^(1/3
)*(b^2*c - a*b*d)*(a/(b*d) + (b^2*c - a*b*d)/(b^2*d^2)))/(b*d)
```

Reduce [F]

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{-24(b\,x^3+a)^{\frac{1}{3}}\,a^2d+21(b\,x^3+a)^{\frac{1}{3}}\,abc+8(b\,x^3+a)^{\frac{1}{3}}\,abd\,x^3-7(b\,x^3+a)^{\frac{1}{3}}\,b^2c\,x^3}{}$$

input
```
int(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
( - 24*(a + b*x**3)**(1/3)*a**2*d + 21*(a + b*x**3)**(1/3)*a*b*c + 8*(a +
b*x**3)**(1/3)*a*b*d*x**3 - 7*(a + b*x**3)**(1/3)*b**2*c*x**3 + 4*(a + b*x
**3)**(1/3)*b**2*d*x**6 + 28*int(((a + b*x**3)**(1/3)*x**5)/(a*c + a*d*x**
3 + b*c*x**3 + b*d*x**6),x)*a**2*b*d**2 - 56*int(((a + b*x**3)**(1/3)*x**5
)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b**2*c*d + 28*int(((a + b*x*
*3)**(1/3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**3*c**2)/(28*
b*d**2)
```

3.713 $\int \frac{x^2\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 187

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2}+\frac{(a+bx^3)^{4/3}}{4d} - \frac{(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}} - \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{7/3}} + \frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}$$

output

```
-(-a*d+b*c)*(b*x^3+a)^(1/3)/d^2+1/4*(b*x^3+a)^(4/3)/d-1/3*(-a*d+b*c)^(4/3)
*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2
)/d^(7/3)-1/6*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/d^(7/3)+1/2*(-a*d+b*c)^(4/3)*ln
((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(7/3)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.18

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}(-4bc+5ad+bdx^3)-4\sqrt{3}(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{}$$

input
```
Integrate[(x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
(3*d^(1/3)*(a + b*x^3)^(1/3)*(-4*b*c + 5*a*d + b*d*x^3) - 4*Sqrt[3]*(b*c -
 a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/S
qrt[3]] + 4*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^
(1/3)] - 2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(
1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(12*d^(7/3))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {946, 60, 60, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+bx^3\right)^{4/3}}{c+dx^3}dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{3}\int \frac{\left(bx^3+a\right)^{4/3}}{dx^3+c}dx^3$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\int \frac{\sqrt[3]{bx^3+a}}{dx^3+c}dx^3}{d}\right)$$

$$\downarrow 60$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{d}\right)}{d}\right)$$

$$\downarrow 70$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \cdots}{\cdots}\cdots\right)}{d}\cdots\right)}{d}\cdots\right)$$

$$\downarrow 16$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(c+d\cdots\right.}{2\sqrt[3]{d}(bc-\cdots}\cdots\right.}{d}\cdots\right.}{d}\cdots\right.$$

$\downarrow$ 1082

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-\cdots}\right.}{2\sqrt[3]{\cdots}}\cdots\right.}{d}\cdots\right.}{d}\cdots\right.$$

$\downarrow$ 217

$$\frac{1}{3}\left(\frac{3(a+bx^3)^{4/3}}{4d} - \frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d} - \frac{(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}} - \frac{\log(c+dx^3)}{2\sqrt[3]{d}(bc-ad)^{2/3}} + \frac{3\log\left(\sqrt[3]{bc-\right.}{2\sqrt[3]{}}\right.}{d}\right.}{d}\right.$$

input
```
Int[(x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
((3*(a + b*x^3)^(4/3))/(4*d) - ((b*c - a*d)*((3*(a + b*x^3)^(1/3))/d - ((b
*c - a*d)*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d
)^(1/3))/Sqrt[3]])/(d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3
)*(b*c - a*d)^(2/3)) + (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3
)])/(2*d^(1/3)*(b*c - a*d)^(2/3))))/d))/d)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{\frac{15\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}} d\left(\left(\frac{b\,x^{3}}{5}+a\right)d-\frac{4bc}{5}\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{2}+(ad-bc)^{2}\left(-2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}} d^{3}}$

input
```
int(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/d)^(2/3)*(15/2*((a*d-b*c)/d)^(2/3)*d*((1/5*b*x^3+a)*d-4/5*b
*c)*(b*x^3+a)^(1/3)+(a*d-b*c)^2*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+
((a*d-b*c)/d)^(1/3))/((a*d-b*c)/d)^(1/3))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-((a
*d-b*c)/d)^(1/3))-ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+(
(a*d-b*c)/d)^(2/3))))/d^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.32

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{4\sqrt{3}(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+2\,(bc-ad)}{}$$

input
```
integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/12*(4*sqrt(3)*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*
(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a
*d)) + 2*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3
 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 4*(b*c - a*
d)*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3))
 + 3*(b*d*x^3 - 4*b*c + 5*a*d)*(b*x^3 + a)^(1/3))/d^2
```

Sympy [F]

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x^2(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**2*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**2*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.59

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = -\frac{(b^2c^2d^2-2\,abcd^3+a^2d^4)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bcd^4-ad^5)} + \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^3} + \frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^3} - \frac{4\,(bx^3+a)^{\frac{1}{3}}bcd^2-(bx^3+a)^{\frac{4}{3}}d^3-4\,(bx^3+a)^{\frac{1}{3}}ad^3}{4\,d^4}$$

input
```
integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(-(b*c - a*d)/d)^(1/3)*log(abs(
(b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c*d^4 - a*d^5) + 1/3*sqrt(
3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^
(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^3 + 1/6*(-b*c*d^
2 + a*d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(
b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^3 - 1/4*(4*(b*x^3 + a)^(1/
3)*b*c*d^2 - (b*x^3 + a)^(4/3)*d^3 - 4*(b*x^3 + a)^(1/3)*a*d^3)/d^4
```

Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.63

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{\left(b\,x^3+a\right)^{4/3}}{4\,d} + \frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\left(3\,a^2\,d^2-6\,a\,b\,c\,d+3\,b^2\,c^2\right)-\frac{(a\,d-b\,c)^{4/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{3\,d^{7/3}}\right)\,(a\,d-b\,c)^{4/3}}{3\,d^{7/3}} + \frac{\left(b\,x^3+a\right)^{1/3}\,(a\,d-b\,c)}{d^2} - \frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\left(3\,a^2\,d^2-6\,a\,b\,c\,d+3\,b^2\,c^2\right)+\frac{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(a\,d-b\,c)^{4/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{3\,d^{7/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(a\,d-b\,c)^{4/}}{3\,d^{7/3}} + \frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\left(3\,a^2\,d^2-6\,a\,b\,c\,d+3\,b^2\,c^2\right)-\frac{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)(a\,d-b\,c)^{4/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{d^{7/3}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)(a\,d-b\,c}{d^{7/3}}$$

input
```
int((x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

output
```
(a + b*x^3)^(4/3)/(4*d) + (log((a + b*x^3)^(1/3)*(3*a^2*d^2 + 3*b^2*c^2 -
6*a*b*c*d) - ((a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*(a*d -
 b*c)^(4/3))/(3*d^(7/3)) + ((a + b*x^3)^(1/3)*(a*d - b*c))/d^2 - (log((a +
 b*x^3)^(1/3)*(3*a^2*d^2 + 3*b^2*c^2 - 6*a*b*c*d) + (((3^(1/2)*1i)/2 + 1/2
)*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*((3^(1/2)*1i)/2 +
1/2)*(a*d - b*c)^(4/3))/(3*d^(7/3)) + (log((a + b*x^3)^(1/3)*(3*a^2*d^2 +
3*b^2*c^2 - 6*a*b*c*d) - (((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(4/3)*(9*a*d^
3 - 9*b*c*d^2))/d^(7/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(4/3))/d^(7/3)
```

Reduce [F]

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{4(b\,x^3+a)^{\frac{1}{3}}\,a^2d-3(b\,x^3+a)^{\frac{1}{3}}\,abc+(b\,x^3+a)^{\frac{1}{3}}\,b^2c\,x^3-4\left(\int\frac{(b\,x^3+a)^{\frac{1}{3}}\,x^5}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right.}{4bcd}$$

input
```
int(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
(4*(a + b*x**3)**(1/3)*a**2*d - 3*(a + b*x**3)**(1/3)*a*b*c + (a + b*x**3)
**(1/3)*b**2*c*x**3 - 4*int(((a + b*x**3)**(1/3)*x**5)/(a*c + a*d*x**3 + b
*c*x**3 + b*d*x**6),x)*a**2*b*d**2 + 8*int(((a + b*x**3)**(1/3)*x**5)/(a*c
 + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b**2*c*d - 4*int(((a + b*x**3)**(1
/3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**3*c**2)/(4*b*c*d)
```

3.714 $\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 261

$$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx = \frac{b\sqrt[3]{a+bx^3}}{d} - \frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c} + \frac{(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}cd^{4/3}} - \frac{a^{4/3}\log(x)}{2c} + \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6cd^{4/3}} + \frac{a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} - \frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2cd^{4/3}}$$

output

```
b*(b*x^3+a)^(1/3)/d-1/3*a^(4/3)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(
1/2)/a^(1/3))*3^(1/2)/c+1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^
3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/c/d^(4/3)-1/2*a^(4/3)*ln(x)/
c+1/6*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c/d^(4/3)+1/2*a^(4/3)*ln(a^(1/3)-(b*x^3
+a)^(1/3))/c-1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1
/3))/c/d^(4/3)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx = \frac{6bc\sqrt[3]{d}\sqrt[3]{a+bx^3} - 2\sqrt{3}a^{4/3}d^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\sqrt{3}(bc-ad)^{4/3}\arctan\left(\right.}{}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x*(c + d*x^3)),x]
```

output
```
(6*b*c*d^(1/3)*(a + b*x^3)^(1/3) - 2*Sqrt[3]*a^(4/3)*d^(4/3)*ArcTan[(1 + (
2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 2*Sqrt[3]*(b*c - a*d)^(4/3)*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 2*a^(4/
3)*d^(4/3)*Log[-a^(1/3) + (a + b*x^3)^(1/3)] - 2*(b*c - a*d)^(4/3)*Log[(b*
c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - a^(4/3)*d^(4/3)*Log[a^(2/3)
+ a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + (b*c - a*d)^(4/3)*Log[(
b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(
a + b*x^3)^(2/3)])/(6*c*d^(4/3))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {948, 95, 174, 69, 16, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{4/3}}{x^3(dx^3+c)}dx^3$$

$$\Big\downarrow \ 95$$

$$\frac{1}{3}\left(\frac{\int \frac{a^2d-b(bc-2ad)x^3}{x^3(bx^3+a)^{2/3}(dx^3+c)}dx^3}{d}+\frac{3b\sqrt[3]{a+bx^3}}{d}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{3}\left(\frac{\frac{a^2d\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3}{c}-\frac{(bc-ad)^2\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{c}}{d}+\frac{3b\sqrt[3]{a+bx^3}}{d}\right)$$

$$\Big\downarrow \; 69$$

$$\frac{1}{3}\left(\frac{\frac{a^2d\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{(bc-ad)^2\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+\right.}}{c}}{d}\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(\frac{\frac{a^2d\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{(bc-ad)^2\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{c}}{d}+\right.$$

$$\Big\downarrow \; 70$$

$$\frac{1}{3}\left(\frac{\frac{a^2 d\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{(bc-ad)^2\left(\frac{3\int\frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-\cdots}}{\cdots}}\cdots}{2d^{2/3}\sqrt[3]{\cdots}}\cdots\right)}{\cdots}}{d}\cdots\right.$$

$\Big\downarrow$ 16

$$\frac{1}{3}\left(\frac{\frac{a^2 d\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{(bc-ad)^2\left(\frac{3\int\frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-\cdots}}{\cdots}}\cdots}{2d^{2/3}\sqrt[3]{\cdots}}\cdots\right)}{\cdots}}{d}\cdots\right.$$

$\Big\downarrow$ 1082

$$\frac{1}{3}\left(\frac{\frac{a^2 d\left(\frac{3\int\frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{(bc-ad)^2\left(\frac{3\int\frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log\cdots}{2\sqrt[3]{\cdots}}\cdots\right)}{\cdots}}{d}\cdots\right.$$

$\Big\downarrow$ 217

$$\frac{1}{3}\left(\frac{\frac{a^2d\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{(bc-ad)^2\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log}{2\sqrt[3]{d}}\cdots\right)}{c}}{d}\cdots\right.$$

input
```
Int[(a + b*x^3)^(4/3)/(x*(c + d*x^3)),x]
```

output
```
((3*b*(a + b*x^3)^(1/3))/d + ((a^2*d*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3
)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/
3) - (a + b*x^3)^(1/3)])/(2*a^(2/3))))/c - ((b*c - a*d)^2*(-((Sqrt[3]*ArcT
an[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(1/3
)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c - a*d)^(2/3)) + (3*
Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1/3)*(b*c - a*d)
^(2/3))))/c)/d)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{3\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}bcd+\frac{\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}d^{2}\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\right.}{2}}{}$

input
```
int((b*x^3+a)^(4/3)/x/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/d)^(2/3)*(3*((a*d-b*c)/d)^(2/3)*(b*x^3+a)^(1/3)*b*c*d+1/2*(
(a*d-b*c)/d)^(2/3)*d^2*(-2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/
a^(1/3))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-a^(1/3))-ln((b*x^3+a)^(2/3)+a^(1/3)*
(b*x^3+a)^(1/3)+a^(2/3)))*a^(4/3)+1/2*(a*d-b*c)^2*(2*arctan(1/3*3^(1/2)*(2
*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*c)/d)^(1/3))*3^(1/2)+ln((b*x
^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))-2*ln(
(b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))))/d^2/c
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx =$$

$$-\frac{2\sqrt{3}a^{\frac{4}{3}}d\arctan\left(\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+a^{\frac{4}{3}}d\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2a^{\frac{4}{3}}d\log\left((bx^3+\right.}{}$$

input
```
integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*a^(4/3)*d*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*a^(2/3)
+ sqrt(3)*a)/a) + a^(4/3)*d*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1
/3) + a^(2/3)) - 2*a^(4/3)*d*log((b*x^3 + a)^(1/3) - a^(1/3)) - 2*sqrt(3)*
(b*c - a*d)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)
*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 6*(b*x^3 +
a)^(1/3)*b*c - (b*c - a*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (
b*x^3 + a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) + 2*(b*c -
 a*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3))
)/(c*d)
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/x/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(x*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.37

$$\begin{aligned}\int \frac{(a+bx^3)^{4/3}}{x\,(c+dx^3)}\,dx = & -\frac{\sqrt{3}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,c} \\ & -\frac{a^{\frac{4}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,c}+\frac{a^{\frac{4}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,c} \\ & +\frac{(b^2c^2-2\,abcd+a^2d^2)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2d-acd^2)} \\ & +\frac{(bx^3+a)^{\frac{1}{3}}b}{d}-\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd^2} \\ & -\frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,cd^2}\end{aligned}$$

input
```
integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^
(1/3))/c - 1/6*a^(4/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) +
 a^(2/3))/c + 1/3*a^(4/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/c + 1/3*(b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(
1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^2*d - a*c*d^2) + (b*x^3 + a)^(1/3)*b/
d - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2
*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c*d^
2) - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x
^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c*d^2)
```

Mupad [B] (verification not implemented)

Time = 5.08 (sec) , antiderivative size = 796, normalized size of antiderivative = 3.05

$$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^3)^(4/3)/(x*(c + d*x^3)),x)
```

output
```
log(c*d*(-(a*d - b*c)^4/(c^3*d^4))^(1/3) + a*d*(a + b*x^3)^(1/3) - b*c*(a
+ b*x^3)^(1/3))*(-(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d -
 4*a^3*b*c*d^3)/(27*c^3*d^4))^(1/3) + log(c*(a^4/c^3)^(1/3) - a*(a + b*x^3
)^(1/3))*(a^4/(27*c^3))^(1/3) + (b*(a + b*x^3)^(1/3))/d - log(c*(a^4/c^3)^
(1/3) + 2*a*(a + b*x^3)^(1/3) + 3^(1/2)*c*(a^4/c^3)^(1/3)*1i)*((3^(1/2)*1i
)/2 + 1/2)*(a^4/(27*c^3))^(1/3) + log(c*(a^4/c^3)^(1/3)*1i + a*(a + b*x^3)
^(1/3)*2i + 3^(1/2)*c*(a^4/c^3)^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a^4/(27*c^3
))^(1/3) + log((3*a^2*b^4*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(2*a^4*d^4 + b^4
*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + 3*a^2*b^4*c
*((3^(1/2)*1i)/2 - 1/2)*(-(a*d - b*c)^4/(c^3*d^4))^(1/3)*(2*a^5*d^5 - b^5*
c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 6*a^4*b*c*
d^4))*((3^(1/2)*1i)/2 - 1/2)*(-(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*
a*b^3*c^3*d - 4*a^3*b*c*d^3)/(27*c^3*d^4))^(1/3) - log((3*a^2*b^4*(a + b*x
^3)^(1/3)*(a*d - b*c)^2*(2*a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3
*c^3*d - 4*a^3*b*c*d^3))/d - 3*a^2*b^4*c*((3^(1/2)*1i)/2 + 1/2)*(-(a*d - b
*c)^4/(c^3*d^4))^(1/3)*(2*a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*
b^2*c^2*d^3 + 5*a*b^4*c^4*d - 6*a^4*b*c*d^4))*((3^(1/2)*1i)/2 + 1/2)*(-(a^
4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(27*c
^3*d^4))^(1/3)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{4/3}}{x\,(c+dx^3)}\,dx = \frac{2(b\,x^3+a)^{\frac{1}{3}}\,a+\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{bd\,x^7+ad\,x^4+bc\,x^4+acx}dx\right)a^2c-2\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}x^5}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)abd+\left(\right.}{c}$$

input
```
int((b*x^3+a)^(4/3)/x/(d*x^3+c),x)
```

output
```
(2*(a + b*x**3)**(1/3)*a + int((a + b*x**3)**(1/3)/(a*c*x + a*d*x**4 + b*c
*x**4 + b*d*x**7),x)*a**2*c - 2*int(((a + b*x**3)**(1/3)*x**5)/(a*c + a*d*
x**3 + b*c*x**3 + b*d*x**6),x)*a*b*d + int(((a + b*x**3)**(1/3)*x**5)/(a*c
 + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**2*c)/c
```

3.715 $\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 340

$$\begin{aligned}\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx &= \frac{b\sqrt[3]{a+bx^3}}{3c}+\frac{b(a+bx^3)^{4/3}}{3ac}\\ &\quad-\frac{(a+bx^3)^{7/3}}{3acx^3}-\frac{\sqrt[3]{a}(4bc-3ad)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}c^2}\\ &\quad-\frac{(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2\sqrt[3]{d}}-\frac{\sqrt[3]{a}(4bc-3ad)\log(x)}{6c^2}\\ &\quad-\frac{(bc-ad)^{4/3}\log(c+dx^3)}{6c^2\sqrt[3]{d}}+\frac{\sqrt[3]{a}(4bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6c^2}\\ &\quad+\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{d}}\end{aligned}$$

output
```
1/3*b*(b*x^3+a)^(1/3)/c+1/3*b*(b*x^3+a)^(4/3)/a/c-1/3*(b*x^3+a)^(7/3)/a/c/
x^3-1/9*a^(1/3)*(-3*a*d+4*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1
/2)/a^(1/3))*3^(1/2)/c^2-1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x
^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/c^2/d^(1/3)-1/6*a^(1/3)*(-3
*a*d+4*b*c)*ln(x)/c^2-1/6*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^2/d^(1/3)+1/6*a^(
1/3)*(-3*a*d+4*b*c)*ln(a^(1/3)-(b*x^3+a)^(1/3))/c^2+1/2*(-a*d+b*c)^(4/3)*l
n((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c^2/d^(1/3)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx = \frac{-\frac{6ac\sqrt[3]{a+bx^3}}{x^3} + 2\sqrt{3}\sqrt[3]{a}(-4bc+3ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - \frac{6\sqrt{3}(bc-ad)^{4/3}\arctan\left(\cdots\right)}{\sqrt[3]{\cdots}}\cdots}{\cdots}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x]
```

output
```
((-6*a*c*(a + b*x^3)^(1/3))/x^3 + 2*Sqrt[3]*a^(1/3)*(-4*b*c + 3*a*d)*ArcTa
n[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - (6*Sqrt[3]*(b*c - a*d)^(4
/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/d^(1/3) - 2*a^(1/3)*(-4*b*c + 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)] +
(6*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/d
^(1/3) + a^(1/3)*(-4*b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3)
+ (a + b*x^3)^(2/3)] - (3*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3
)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/d^(1/3
))/(18*c^2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {948, 114, 27, 174, 60, 60, 69, 16, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{4/3}}{x^6(dx^3+c)}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int -\frac{(bx^3+a)^{4/3}(4bdx^3+4bc-3ad)}{3x^3(dx^3+c)}dx^3}{ac}-\frac{(a+bx^3)^{7/3}}{acx^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{(bx^3+a)^{4/3}(4bdx^3+4bc-3ad)}{x^3(dx^3+c)}dx^3}{3ac}-\frac{(a+bx^3)^{7/3}}{acx^3}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{\frac{3ad^2\int \frac{(bx^3+a)^{4/3}}{dx^3+c}dx^3}{c}+\frac{(4bc-3ad)\int \frac{(bx^3+a)^{4/3}}{x^3}dx^3}{c}}{3ac}-\frac{(a+bx^3)^{7/3}}{acx^3}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{\frac{3ad^2\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\int\frac{\sqrt[3]{bx^3+a}}{dx^3+c}dx^3}{d}\right)}{c}+\frac{(4bc-3ad)\left(a\int\frac{\sqrt[3]{bx^3+a}}{x^3}dx^3+\frac{3}{4}\left(a+bx^3\right)^{4/3}\right)}{c}}{3ac}-\frac{\left(a+bx^3\right)^{7/3}}{acx^3}\right)$$

$\downarrow$ 60

$$\frac{1}{3}\left(\frac{\frac{3ad^2\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\int\frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{d}\right)}{d}\right)}{c}+\frac{(4bc-3ad)\left(a\left(a\int\frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3+3\sqrt[3]{a+\cdots}\right)\right)}{c}}{3ac}\cdots\right)$$

$\downarrow$ 69

$$\frac{1}{3}\left(\frac{\frac{(4bc-3ad)\left(a\left(a\left(-\frac{3\int\frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+\cdots\right)}{c}}{3ac}\cdots\right)$$

$\downarrow$ 16

$$\frac{1}{3}\left(\frac{\frac{(4bc-3ad)\left(a\left(a\left(-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+\frac{3}{4}\left(a+bx^3\right)^4}{c}}{3ac}\right.$$

$$\Big\downarrow \; 70$$

$$\frac{1}{3}\left(\frac{\frac{(4bc-3ad)\left(a\left(a\left(-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+\frac{3}{4}\left(a+bx^3\right)^{4}}{c}\right.$$

↓ 16

$$\frac{1}{3}\left(\frac{(4bc-3ad)\left(a\left(a\left(-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+\frac{3}{4}\left(a+bx^3\right)^4}{c}\right.$$

$\downarrow$ 1082

$$\frac{1}{3}\left(\frac{\frac{(4bc-3ad)\left(a\left(a\left(\frac{3\int\frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+\frac{3}{4}\left(a+bx^3\right)^{4/3}\right)}{c}+\frac{3ad^2\left(\right.}{}}{}\right.$$

$\downarrow$ 217

$$\frac{1}{3}\left(\frac{(4bc-3ad)\left(a\left(a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+\frac{3}{4}\left(a+bx^3\right)^{4/3}\right)}{c}+\frac{3ad^2\left(\cdots\right.}{\cdots}\right.$$

input
```
Int[(a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x]
```

output
```
(-((a + b*x^3)^(7/3)/(a*c*x^3)) + (((4*b*c - 3*a*d)*((3*(a + b*x^3)^(4/3))
/4 + a*(3*(a + b*x^3)^(1/3) + a*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/
3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) -
(a + b*x^3)^(1/3)])/(2*a^(2/3))))))/c + (3*a*d^2*((3*(a + b*x^3)^(4/3))/(4
*d) - ((b*c - a*d)*((3*(a + b*x^3)^(1/3))/d - ((b*c - a*d)*(-((Sqrt[3]*Arc
Tan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(1/
3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c - a*d)^(2/3)) + (3
*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1/3)*(b*c - a*d
)^(2/3))))/d))/d))/c)/(3*a*c))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$$-\frac{\frac{x^3(ad-bc)^2\ln\left(\left(b\,x^3+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)}{2}-x^3\sqrt{3}\,(ad-bc)^2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^3+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)}{}$$

input
```
int((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/d)^(2/3)*(-1/2*x^3*(a*d-b*c)^2*ln((b*x^3+a)^(2/3)+((a*d-b*c
)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))-x^3*3^(1/2)*(a*d-b*c)^2*ar
ctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*c)/d)^(1/
3))+1/2*d*(-4/3*a^(1/3)*b*c+d*a^(4/3))*((a*d-b*c)/d)^(2/3)*x^3*ln((b*x^3+a
)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))+x^3*(a*d-b*c)^2*ln((b*x^3+a)^(1/3
)-((a*d-b*c)/d)^(1/3))-d*(-(-4/3*a^(1/3)*b*c+d*a^(4/3))*3^(1/2)*x^3*arctan
(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))+x^3*(-4/3*a^(1/3)*b*c+d*
a^(4/3))*ln((b*x^3+a)^(1/3)-a^(1/3))+(b*x^3+a)^(1/3)*a*c)*((a*d-b*c)/d)^(2
/3))/c^2/d/x^3
```

Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.13

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^4\left(c+dx^3\right)}\,dx=\frac{6\sqrt{3}(bc-ad)x^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+2\sqrt{3}(4\,bc-}{}$$

input
```
integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/18*(6*sqrt(3)*(b*c - a*d)*x^3*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt
(3)*(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c
 - a*d)) + 2*sqrt(3)*(4*b*c - 3*a*d)*(-a)^(1/3)*x^3*arctan(1/3*(2*sqrt(3)*
(b*x^3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + (4*b*c - 3*a*d)*(-a)^(1/3)*
x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 3
*(b*c - a*d)*x^3*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a
)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 2*(4*b*c - 3*a*
d)*(-a)^(1/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 6*(b*c - a*d)*x^3*
(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) - 6
*(b*x^3 + a)^(1/3)*a*c)/(c^2*x^3)
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x^4(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/x**4/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(x**4*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^4}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx = -\frac{(b^2c^2-2\,abcd+a^2d^2)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^3-ac^2d)}$$
$$-\frac{\sqrt{3}\left(4\,a^{\frac{1}{3}}bc-3\,a^{\frac{4}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,c^2}$$
$$-\frac{\left(4\,a^{\frac{1}{3}}bc-3\,a^{\frac{4}{3}}d\right)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,c^2}$$
$$+\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^2d}$$
$$+\frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^2d}$$
$$+\frac{(4\,abc-3\,a^2d)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{2}{3}}c^2}-\frac{(bx^3+a)^{\frac{1}{3}}a}{3\,cx^3}$$

input

```
integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

output

```
-1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3
 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^3 - a*c^2*d) - 1/9*sqrt(3)*(4*
a^(1/3)*b*c - 3*a^(4/3)*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/
3))/a^(1/3))/c^2 - 1/18*(4*a^(1/3)*b*c - 3*a^(4/3)*d)*log((b*x^3 + a)^(2/3
) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/c^2 + 1/3*sqrt(3)*(-b*c*d^2 + a*d
^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a
*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/
3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^
(1/3) + (-(b*c - a*d)/d)^(2/3))/(c^2*d) + 1/9*(4*a*b*c - 3*a^2*d)*log(abs(
(b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/3)*c^2) - 1/3*(b*x^3 + a)^(1/3)*a/(c*x
^3)
```

Mupad [B] (verification not implemented)

Time = 9.90 (sec) , antiderivative size = 2047, normalized size of antiderivative = 6.02

$$\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x)
```

output
```
log(c^2*(-(a*(3*a*d - 4*b*c)^3)/c^6)^(1/3) + 3*a*d*(a + b*x^3)^(1/3) - 4*b
*c*(a + b*x^3)^(1/3))*(-(27*a^4*d^3 - 64*a*b^3*c^3 + 144*a^2*b^2*c^2*d - 1
08*a^3*b*c*d^2)/(729*c^6))^(1/3) + log(((((81*a*b^4*c^4*d^3*(2*a^2*d^2 + b
^2*c^2 - 3*a*b*c*d)*((a*d - b*c)^4/(c^6*d))^(1/3) - 108*a*b^5*c^3*d^3*(a +
 b*x^3)^(1/3)*(a*d - b*c)^2)*((a*d - b*c)^4/(c^6*d))^(2/3))/9 + (a*b^5*d^2
*(27*a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 16
2*a*b^4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3
- (a*b^4*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 38
8*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^
4))/(9*c^4))*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a
^3*b*c*d^3)/(27*c^6*d))^(1/3) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i
)/2 + 1/2)*(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2 - 81*a*b^4*c
^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((a*d - b*
c)^4/(c^6*d))^(1/3))*((a*d - b*c)^4/(c^6*d))^(2/3))/9 + (a*b^5*d^2*(27*a^5
*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 162*a*b^4*
c^4*d - 153*a^4*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3 - (a*b^4
*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^
3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c
^4))*((3^(1/2)*1i)/2 - 1/2)*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*
b^3*c^3*d - 4*a^3*b*c*d^3)/(27*c^6*d))^(1/3) - log((a*b^4*d^2*(a + b*x^...
```

Reduce [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x)
```

output

```
( - 3*(a + b*x**3)**(1/3)*a**2*d - 2*(a + b*x**3)**(1/3)*a*b*c - 3*(a + b*
x**3)**(1/3)*a*b*d*x**3 + 6*(a + b*x**3)**(1/3)*b**2*c*x**3 - 27*int((a +
b*x**3)**(1/3)/(3*a**2*c*d*x + 3*a**2*d**2*x**4 + 2*a*b*c**2*x + 5*a*b*c*d
*x**4 + 3*a*b*d**2*x**7 + 2*b**2*c**2*x**4 + 2*b**2*c*d*x**7),x)*a**4*d**3
*x**3 + 36*int((a + b*x**3)**(1/3)/(3*a**2*c*d*x + 3*a**2*d**2*x**4 + 2*a*
b*c**2*x + 5*a*b*c*d*x**4 + 3*a*b*d**2*x**7 + 2*b**2*c**2*x**4 + 2*b**2*c*
d*x**7),x)*a**2*b**2*c**2*d*x**3 + 16*int((a + b*x**3)**(1/3)/(3*a**2*c*d*
x + 3*a**2*d**2*x**4 + 2*a*b*c**2*x + 5*a*b*c*d*x**4 + 3*a*b*d**2*x**7 + 2
*b**2*c**2*x**4 + 2*b**2*c*d*x**7),x)*a*b**3*c**3*x**3 + 9*int(((a + b*x**
3)**(1/3)*x**5)/(3*a**2*c*d + 3*a**2*d**2*x**3 + 2*a*b*c**2 + 5*a*b*c*d*x*
*3 + 3*a*b*d**2*x**6 + 2*b**2*c**2*x**3 + 2*b**2*c*d*x**6),x)*a**2*b**2*d*
*3*x**3 - 12*int(((a + b*x**3)**(1/3)*x**5)/(3*a**2*c*d + 3*a**2*d**2*x**3
 + 2*a*b*c**2 + 5*a*b*c*d*x**3 + 3*a*b*d**2*x**6 + 2*b**2*c**2*x**3 + 2*b*
*2*c*d*x**6),x)*a*b**3*c*d**2*x**3 - 12*int(((a + b*x**3)**(1/3)*x**5)/(3*
a**2*c*d + 3*a**2*d**2*x**3 + 2*a*b*c**2 + 5*a*b*c*d*x**3 + 3*a*b*d**2*x**
6 + 2*b**2*c**2*x**3 + 2*b**2*c*d*x**6),x)*b**4*c**2*d*x**3 - 18*int(((a +
 b*x**3)**(1/3)*x**2)/(3*a**2*c*d + 3*a**2*d**2*x**3 + 2*a*b*c**2 + 5*a*b*
c*d*x**3 + 3*a*b*d**2*x**6 + 2*b**2*c**2*x**3 + 2*b**2*c*d*x**6),x)*a**3*b
*d**3*x**3 + 12*int(((a + b*x**3)**(1/3)*x**2)/(3*a**2*c*d + 3*a**2*d**2*x
**3 + 2*a*b*c**2 + 5*a*b*c*d*x**3 + 3*a*b*d**2*x**6 + 2*b**2*c**2*x**3 ...
```

3.716 $\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 403

$$\begin{aligned}
\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx &= \frac{b(2bc-3ad)\sqrt[3]{a+bx^3}}{9ac^2} - \frac{(bc-6ad)(a+bx^3)^{4/3}}{18ac^2x^3} \\
&\quad - \frac{(a+bx^3)^{7/3}}{6acx^6} - \frac{(2b^2c^2-12abcd+9a^2d^2)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}c^3} \\
&\quad + \frac{d^{2/3}(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^3} \\
&\quad - \frac{(2b^2c^2-12abcd+9a^2d^2)\log(x)}{18a^{2/3}c^3} + \frac{d^{2/3}(bc-ad)^{4/3}\log(c+dx^3)}{6c^3} \\
&\quad + \frac{(2b^2c^2-12abcd+9a^2d^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{2/3}c^3} \\
&\quad - \frac{d^{2/3}(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3}
\end{aligned}$$

output
```
1/9*b*(-3*a*d+2*b*c)*(b*x^3+a)^(1/3)/a/c^2-1/18*(-6*a*d+b*c)*(b*x^3+a)^(4/
3)/a/c^2/x^3-1/6*(b*x^3+a)^(7/3)/a/c/x^6-1/27*(9*a^2*d^2-12*a*b*c*d+2*b^2*
c^2)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/
3)/c^3+1/3*d^(2/3)*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3
)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/c^3-1/18*(9*a^2*d^2-12*a*b*c*d+2*b^2*
c^2)*ln(x)/a^(2/3)/c^3+1/6*d^(2/3)*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^3+1/18*(
9*a^2*d^2-12*a*b*c*d+2*b^2*c^2)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(2/3)/c^3-1/
2*d^(2/3)*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c^
3
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx = \frac{\frac{3c\sqrt[3]{a+bx^3}(-3ac-7bcx^3+6adx^3)}{x^6} - \frac{2\sqrt{3}(2b^2c^2-12abcd+9a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + 18\sqrt{3}d^{2/}}{}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x]
```

output
```
((3*c*(a + b*x^3)^(1/3)*(-3*a*c - 7*b*c*x^3 + 6*a*d*x^3))/x^6 - (2*Sqrt[3]
*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^
(1/3))/Sqrt[3]])/a^(2/3) + 18*Sqrt[3]*d^(2/3)*(b*c - a*d)^(4/3)*ArcTan[(1
- (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + (2*(2*b^2*c^
2 - 12*a*b*c*d + 9*a^2*d^2)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(2/3) - 1
8*d^(2/3)*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1
/3)] + ((-2*b^2*c^2 + 12*a*b*c*d - 9*a^2*d^2)*Log[a^(2/3) + a^(1/3)*(a + b
*x^3)^(1/3) + (a + b*x^3)^(2/3)])/a^(2/3) + 9*d^(2/3)*(b*c - a*d)^(4/3)*Lo
g[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3
)*(a + b*x^3)^(2/3)])/(54*c^3)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {948, 114, 27, 166, 27, 174, 60, 69, 16, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^7\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{\left(bx^3+a\right)^{4/3}}{x^9\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \text{ 114}$$

$$\frac{1}{3}\left(-\frac{\int -\frac{\left(bx^3+a\right)^{4/3}\left(bdx^3+bc-6ad\right)}{3x^6\left(dx^3+c\right)}dx^3}{2ac}-\frac{\left(a+bx^3\right)^{7/3}}{2acx^6}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{\int \frac{\left(bx^3+a\right)^{4/3}\left(bdx^3+bc-6ad\right)}{x^6\left(dx^3+c\right)}dx^3}{6ac}-\frac{\left(a+bx^3\right)^{7/3}}{2acx^6}\right)$$

$$\Big\downarrow \text{ 166}$$

$$\frac{1}{3}\left(\frac{\frac{\int \frac{2\sqrt[3]{bx^3+a}\left(bd(2bc-3ad)x^3+2b^2c^2+9a^2d^2-12abcd\right)}{3x^3\left(dx^3+c\right)}dx^3}{c}-\frac{\left(a+bx^3\right)^{4/3}(bc-6ad)}{cx^3}}{6ac}-\frac{\left(a+bx^3\right)^{7/3}}{2acx^6}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{\frac{2\int \frac{\sqrt[3]{bx^3+a}\left(bd(2bc-3ad)x^3+2b^2c^2+9a^2d^2-12abcd\right)}{x^3\left(dx^3+c\right)}dx^3}{3c}-\frac{\left(a+bx^3\right)^{4/3}(bc-6ad)}{cx^3}}{6ac}-\frac{\left(a+bx^3\right)^{7/3}}{2acx^6}\right)$$

$\big\downarrow$ 174

$$\frac{1}{3}\left(\frac{\frac{2\left(\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\int \frac{\sqrt[3]{bx^3+a}}{x^3}dx^3}{c}+\frac{9ad^2(bc-ad)\int \frac{\sqrt[3]{bx^3+a}}{dx^3+c}dx^3}{c}\right)}{3c}-\frac{\left(a+bx^3\right)^{4/3}(bc-6ad)}{cx^3}}{6ac}-\frac{\left(a+bx^3\right)^{7/3}}{2acx^6}\right)$$

$\big\downarrow$ 60

$$\frac{1}{3}\left(\frac{\frac{2\left(\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\left(a\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{9ad^2(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{d}\right)}{c}\right)}{3c}-\cdots}{6ac}\cdots\right.$$

$\big\downarrow$ 69

$$\frac{1}{3}\left(\frac{\frac{2\left(\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\left(a\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+\cdots}\right.}{c}\cdots\right.}{3c}\cdots}{6ac}\cdots\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(\frac{\frac{2\left(\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\left(a\left(-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\cdots\right)}{3c}}{6ac}\right.$$

$$\Big\downarrow \; 70$$

$$\frac{1}{3}\left(\frac{2\left(\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\left(a\left(-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+ \cdots\right)}{\cdots}\right.$$

9a

$$\Big\downarrow 16$$

$$\frac{1}{3}\left(\frac{2\left(\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\left(a\left(-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+ - \cdots}{\cdots}\right.$$

9a

$$\Big\downarrow \; 1082$$

$$\frac{1}{3}\left(\frac{2\left(\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\left(a\left(\frac{3\int\frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{9ad^2(bc-ad)\left(\frac{3\sqrt[3]{\cdots}}{\cdots}\right.}{\cdots}\right.}{3c}\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}\left(\frac{\left(2\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\left(a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{9ad^2(bc-ad)\left(\frac{3\sqrt[3]{\cdots}}{\cdots}\right.}{\cdots}\right.}{3c}\right.$$

input
```
Int[(a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x]
```

output
```
(-1/2*(a + b*x^3)^(7/3)/(a*c*x^6) + (-(((b*c - 6*a*d)*(a + b*x^3)^(4/3))/(
c*x^3)) + (2*(((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(3*(a + b*x^3)^(1/3) +
 a*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3
)) - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3
)))))/c + (9*a*d^2*(b*c - a*d)*((3*(a + b*x^3)^(1/3))/d - ((b*c - a*d)*(-(
(Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt
[3]])/(d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c - a*d)
^(2/3)) + (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1/3
)*(b*c - a*d)^(2/3))))/d))/c))/(3*c))/(6*a*c))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$\frac{\frac{x^{6}\left(a^{\frac{8}{3}} d^{2}-2 a^{\frac{5}{3}} b c d+b^{2} c^{2} a^{\frac{2}{3}}\right) \ln\left(\left(b x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}} \left(b x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{a d-b c}{d}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\, x^{6} \left(a^{\frac{8}{3}} d^{2}-2 a^{\frac{5}{3}} b c d+b^{2} c^{2} a^{\frac{2}{3}}\right) \arctan\left(\right.}{}$

input
```
int((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/d)^(2/3)*(1/2*x^6*(a^(8/3)*d^2-2*a^(5/3)*b*c*d+b^2*c^2*a^(2
/3))*ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^
(2/3))+3^(1/2)*x^6*(a^(8/3)*d^2-2*a^(5/3)*b*c*d+b^2*c^2*a^(2/3))*arctan(2/
3*3^(1/2)/((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+1/3*3^(1/2))-1/2*((a*d-b*c)/
d)^(2/3)*x^6*(a^2*d^2-4/3*a*b*c*d+2/9*b^2*c^2)*ln((b*x^3+a)^(2/3)+a^(1/3)*
(b*x^3+a)^(1/3)+a^(2/3))-x^6*(a^(8/3)*d^2-2*a^(5/3)*b*c*d+b^2*c^2*a^(2/3))
*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))+(-3^(1/2)*x^6*(a^2*d^2-4/3*a*b*c*
d+2/9*b^2*c^2)*arctan(2/3*3^(1/2)/a^(1/3)*(b*x^3+a)^(1/3)+1/3*3^(1/2))+x^6
*(a^2*d^2-4/3*a*b*c*d+2/9*b^2*c^2)*ln((b*x^3+a)^(1/3)-a^(1/3))-7/6*c*(b*x^
3+a)^(1/3)*(3/7*(-2*d*x^3+c)*a^(5/3)+b*c*x^3*a^(2/3)))*((a*d-b*c)/d)^(2/3)
)/a^(2/3)/c^3/x^6
```

Fricas [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx = \frac{18\sqrt{3}(a^2bc-a^3d)(bcd^2-ad^3)^{\frac{1}{3}}x^6\arctan\left(-\frac{2\sqrt{3}(bcd^2-ad^3)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}-\sqrt{3}(bcd-ad^2)}{3(bcd-ad^2)}\right)-6}{}$$

input
```
integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/54*(18*sqrt(3)*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^(1/3)*x^6*arctan(-1/3
*(2*sqrt(3)*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a
*d^2))/(b*c*d - a*d^2)) - 6*sqrt(1/3)*(2*a*b^2*c^2 - 12*a^2*b*c*d + 9*a^3*
d^2)*(a^2)^(1/6)*x^6*arctan(sqrt(1/3)*(a^2)^(1/6)*((a^2)^(1/3)*a + 2*(b*x^
3 + a)^(1/3)*(a^2)^(2/3))/a^2) - (2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(a^2
)^(2/3)*x^6*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a
^2)^(2/3)) + 2*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(a^2)^(2/3)*x^6*log((b
*x^3 + a)^(1/3)*a - (a^2)^(2/3)) + 9*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^(
1/3)*x^6*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(
1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 18*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)
^(1/3)*x^6*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*(3*a^3*c
^2 + (7*a^2*b*c^2 - 6*a^3*c*d)*x^3)*(b*x^3 + a)^(1/3))/(a^2*c^3*x^6)
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x^7(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/x**7/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(x**7*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^7}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^7), x)
```

Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx = \frac{(b^2c^2d-2\,abcd^2+a^2d^3)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^4-ac^3d)} - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^3} - \frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^3} - \frac{\sqrt{3}(2\,b^2c^2-12\,abcd+9\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{2}{3}}c^3} - \frac{(2\,b^2c^2-12\,abcd+9\,a^2d^2)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{54\,a^{\frac{2}{3}}c^3} + \frac{(2\,b^2c^2-12\,abcd+9\,a^2d^2)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27\,a^{\frac{2}{3}}c^3} - \frac{7\,(bx^3+a)^{\frac{4}{3}}b^2c-4\,(bx^3+a)^{\frac{1}{3}}ab^2c-6\,(bx^3+a)^{\frac{4}{3}}abd+6\,(bx^3+a)^{\frac{1}{3}}a^2bd}{18\,b^2c^2x^6}$$

input
```
integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*
x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^4 - a*c^3*d) - 1/3*sqrt(3)*
(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/
3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/c^3 - 1/6*(-b*c*d^2 +
 a*d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c
 - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/c^3 - 1/27*sqrt(3)*(2*b^2*c^2 -
 12*a*b*c*d + 9*a^2*d^2)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3)
)/a^(1/3))/(a^(2/3)*c^3) - 1/54*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(2/3)*c^3) + 1/
27*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*log(abs((b*x^3 + a)^(1/3) - a^(1/3
)))/(a^(2/3)*c^3) - 1/18*(7*(b*x^3 + a)^(4/3)*b^2*c - 4*(b*x^3 + a)^(1/3)*
a*b^2*c - 6*(b*x^3 + a)^(4/3)*a*b*d + 6*(b*x^3 + a)^(1/3)*a^2*b*d)/(b^2*c^
2*x^6)
```

Mupad [B] (verification not implemented)

Time = 12.69 (sec) , antiderivative size = 2841, normalized size of antiderivative = 7.05

$$\int \frac{(a+bx^3)^{4/3}}{x^7 (c+dx^3)} \, dx = \text{Too large to display}$$

input
```
int((a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x)
```

output
```
log(((((18*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) + 9*a
*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((9*a^2*d^2 + 2*b^2*c^2 - 1
2*a*b*c*d)^3/(a^2*c^9))^(1/3))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^
2*c^9))^(2/3))/729 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*
d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645
*a^5*b*c*d^5))/(81*c^4))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9)
)^(1/3))/27 - (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170
*b^6*c^6 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^
2*d^4 - 1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((729*a^6*d^6 + 8
*b^6*c^6 + 972*a^2*b^4*c^4*d^2 - 3024*a^3*b^3*c^3*d^3 + 4374*a^4*b^2*c^2*d
^4 - 144*a*b^5*c^5*d - 2916*a^5*b*c*d^5)/(19683*a^2*c^9))^(1/3) + log(((((
18*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) + 81*a*b^4*c^
4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))*
(-(d^2*(a*d - b*c)^4)/c^9)^(2/3))/9 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 -
3258*a^2*b^4*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a
*b^5*c^5*d + 3645*a^5*b*c*d^5))/(81*c^4))*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3)
)/3 - (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6
 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 -
1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*(-(a^4*d^6 + b^4*c^4*d^2
- 4*a*b^3*c^3*d^3 + 6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5)/(27*c^9))^(1/3) ...
```

Reduce [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^7\,(c+dx^3)}\,dx = \frac{-(b\,x^3+a)^{\frac{1}{3}}\,a-36\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{6ab\,d^2x^{10}+5b^2cd\,x^{10}+6a^2d^2x^7+11abcd\,x^7+5b^2c^2x^7+6a^2cd\,x^4+5ab\,c^2x^4}dx\right)a^3c$$

input
```
int((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x)
```

output
```
( - (a + b*x**3)**(1/3)*a - 36*int((a + b*x**3)**(1/3)/(6*a**2*c*d*x**4 +
6*a**2*d**2*x**7 + 5*a*b*c**2*x**4 + 11*a*b*c*d*x**7 + 6*a*b*d**2*x**10 +
5*b**2*c**2*x**7 + 5*b**2*c*d*x**10),x)*a**3*d**2*x**6 + 12*int((a + b*x**
3)**(1/3)/(6*a**2*c*d*x**4 + 6*a**2*d**2*x**7 + 5*a*b*c**2*x**4 + 11*a*b*c
*d*x**7 + 6*a*b*d**2*x**10 + 5*b**2*c**2*x**7 + 5*b**2*c*d*x**10),x)*a**2*
b*c*d*x**6 + 35*int((a + b*x**3)**(1/3)/(6*a**2*c*d*x**4 + 6*a**2*d**2*x**
7 + 5*a*b*c**2*x**4 + 11*a*b*c*d*x**7 + 6*a*b*d**2*x**10 + 5*b**2*c**2*x**
7 + 5*b**2*c*d*x**10),x)*a*b**2*c**2*x**6 - 30*int((a + b*x**3)**(1/3)/(6*
a**2*c*d*x + 6*a**2*d**2*x**4 + 5*a*b*c**2*x + 11*a*b*c*d*x**4 + 6*a*b*d**
2*x**7 + 5*b**2*c**2*x**4 + 5*b**2*c*d*x**7),x)*a**2*b*d**2*x**6 + 11*int(
(a + b*x**3)**(1/3)/(6*a**2*c*d*x + 6*a**2*d**2*x**4 + 5*a*b*c**2*x + 11*a
*b*c*d*x**4 + 6*a*b*d**2*x**7 + 5*b**2*c**2*x**4 + 5*b**2*c*d*x**7),x)*a*b
**2*c*d*x**6 + 30*int((a + b*x**3)**(1/3)/(6*a**2*c*d*x + 6*a**2*d**2*x**4
 + 5*a*b*c**2*x + 11*a*b*c*d*x**4 + 6*a*b*d**2*x**7 + 5*b**2*c**2*x**4 + 5
*b**2*c*d*x**7),x)*b**3*c**2*x**6)/(6*c*x**6)
```

3.717 $\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 334

$$\begin{aligned}\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx &= -\frac{(6bc-7ad)x^2\sqrt[3]{a+bx^3}}{18d^2}+\frac{bx^5\sqrt[3]{a+bx^3}}{6d}\\ &\quad-\frac{(9b^2c^2-12abcd+2a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{2/3}d^3}\\ &\quad+\frac{c^{2/3}(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^3}-\frac{c^{2/3}(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^3}\\ &\quad-\frac{(9b^2c^2-12abcd+2a^2d^2)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{2/3}d^3}\\ &\quad+\frac{c^{2/3}(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^3}\end{aligned}$$

output
```
-1/18*(-7*a*d+6*b*c)*x^2*(b*x^3+a)^(1/3)/d^2+1/6*b*x^5*(b*x^3+a)^(1/3)/d-1
/27*(2*a^2*d^2-12*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(
1/3))*3^(1/2))*3^(1/2)/b^(2/3)/d^3+1/3*c^(2/3)*(-a*d+b*c)^(4/3)*arctan(1/3
*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d^3-1/6
*c^(2/3)*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/d^3-1/18*(2*a^2*d^2-12*a*b*c*d+9*b^2
*c^2)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)/d^3+1/2*c^(2/3)*(-a*d+b*c)^(4/
3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^3
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.81 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.57

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{6dx^2\sqrt[3]{a+bx^3}(-6bc+7ad+3bdx^3) - \frac{4\sqrt{3}(9b^2c^2-12abcd+2a^2d^2)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right.}{b^{2/3}}}{}$$

input
```
Integrate[(x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
(6*d*x^2*(a + b*x^3)^(1/3)*(-6*b*c + 7*a*d + 3*b*d*x^3) - (4*Sqrt[3]*(9*b^
2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*
(a + b*x^3)^(1/3))])/b^(2/3) - 18*Sqrt[-6 - (6*I)*Sqrt[3]]*c^(2/3)*(b*c -
a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (
3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] - (4*(9*b^2*c^2 - 12*a*b*c*d +
2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(2/3) + (18*I)*(I + Sq
rt[3])*c^(2/3)*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3
])*c^(1/3)*(a + b*x^3)^(1/3)] + (2*(9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*Lo
g[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(2/3)
+ 9*(1 - I*Sqrt[3])*c^(2/3)*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(2/3)*x^2
+ (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I +
Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(108*d^3)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {977, 25, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (a+bx^3)^{4/3}}{c+dx^3} \, dx$$

$$\Big\downarrow \text{977}$$

$$\frac{\int -\frac{x^4 (b(6bc-7ad)x^3+a(5bc-6ad))}{(bx^3+a)^{2/3}(dx^3+c)}dx}{6d}+\frac{bx^5\sqrt[3]{a+bx^3}}{6d}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx^5\sqrt[3]{a+bx^3}}{6d}-\frac{\int \frac{x^4 (b(6bc-7ad)x^3+a(5bc-6ad))}{(bx^3+a)^{2/3}(dx^3+c)}dx}{6d}$$

$$\Big\downarrow \text{1052}$$

$$\frac{bx^5\sqrt[3]{a+bx^3}}{6d}-\frac{\frac{x^2\sqrt[3]{a+bx^3}(6bc-7ad)}{3d}-\frac{\int \frac{2bx\left(\left(9b^2c^2-12abdc+2a^2d^2\right)x^3+ac(6bc-7ad)\right)}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{3bd}}{6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{bx^5\sqrt[3]{a+bx^3}}{6d}-\frac{\frac{x^2\sqrt[3]{a+bx^3}(6bc-7ad)}{3d}-\frac{2\int \frac{x\left(\left(9b^2c^2-12abdc+2a^2d^2\right)x^3+ac(6bc-7ad)\right)}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{3d}}{6d}$$

$$\Big\downarrow \text{1054}$$

$$\frac{bx^5\sqrt[3]{a+bx^3}}{6d}-\frac{\frac{x^2\sqrt[3]{a+bx^3}(6bc-7ad)}{3d}-\frac{2\int \left(\frac{\left(9b^2c^2-12abdc+2a^2d^2\right)x}{d\left(bx^3+a\right)^{2/3}}-\frac{9\left(b^2c^3-2abdc^2+a^2d^2c\right)x}{d\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}\right)dx}{3d}}{6d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bx^5\sqrt[3]{a+bx^3}}{6d} - \frac{\frac{x^2\sqrt[3]{a+bx^3}(6bc-7ad)}{3d} - \frac{2\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\left(2a^2d^2-12abcd+9b^2c^2\right)}{\sqrt{3}b^{2/3}d} - \frac{\left(2a^2d^2-12abcd+9b^2c^2\right)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d} + \frac{3\sqrt{3}c^{2/}}{}\right.}{}}{6d}$$

input
```
Int[(x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
(b*x^5*(a + b*x^3)^(1/3))/(6*d) - (((6*b*c - 7*a*d)*x^2*(a + b*x^3)^(1/3))
/(3*d) - (2*(-(((9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*ArcTan[(1 + (2*b^(1/3
)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)*d)) + (3*Sqrt[3]*c^(2/3
)*(b*c - a*d)^(4/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^
3)^(1/3)))/Sqrt[3]])/d - (3*c^(2/3)*(b*c - a*d)^(4/3)*Log[c + d*x^3])/(2*d
) - ((9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*Log[b^(1/3)*x - (a + b*x^3)^(1/3
)])/(2*b^(2/3)*d) + (9*c^(2/3)*(b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)
/c^(1/3) - (a + b*x^3)^(1/3)])/(2*d)))/(3*d))/(6*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 977
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c -
a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p,
q, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.37

method	result
pseudoelliptic	$\frac{\left(-\frac{b^{\frac{2}{3}}a^{2}d^{2}}{2}+b^{\frac{5}{3}}acd-\frac{b^{\frac{8}{3}}c^{2}}{2}\right)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)+\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}\left(a^{2}d^{2}-6abcd+\frac{9}{2}b^{2}c^{2}\right)\ln\left(\right.}{}}{}$

input
```
int(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(2/3)*((-1/2*b^(2/3)*a^2*d^2+b^(5/3)*a*c*d-1/2*b^(8/3)*c
^2)*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x
^3+a)^(2/3))/x^2)+1/9*((a*d-b*c)/c)^(2/3)*(a^2*d^2-6*a*b*c*d+9/2*b^2*c^2)*
ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-3^(1/2)*(-
2*b^(5/3)*a*c*d+b^(8/3)*c^2+b^(2/3)*a^2*d^2)*arctan(1/3*3^(1/2)*(-2/((a*d-
b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)+(-2*b^(5/3)*a*c*d+b^(8/3)*c^2+b^(2/3)*
a^2*d^2)*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-2/9*((a*d-b*c)/c)^(
2/3)*(-3^(1/2)*(a^2*d^2-6*a*b*c*d+9/2*b^2*c^2)*arctan(1/3*3^(1/2)*(2*(b*x^
3+a)^(1/3)/b^(1/3)+x)/x)+(a^2*d^2-6*a*b*c*d+9/2*b^2*c^2)*ln((-b^(1/3)*x+(b
*x^3+a)^(1/3))/x)-21/4*(3/7*(d*x^3-2*c)*b^(5/3)+a*d*b^(2/3))*d*(b*x^3+a)^(
1/3)*x^2))/b^(2/3)/d^3
```

Fricas [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.64

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{6\sqrt{\frac{1}{3}}(9\,b^3c^2-12\,ab^2cd+2\,a^2bd^2)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(-b^2\right)^{\frac{1}{3}}bx-2\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right.\right.}{b^2x}\right.}{}$$

input
```
integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/54*(6*sqrt(1/3)*(9*b^3*c^2 - 12*a*b^2*c*d + 2*a^2*b*d^2)*sqrt(-(-b^2)^(1
/3))*arctan(-sqrt(1/3)*((-b^2)^(1/3)*b*x - 2*(b*x^3 + a)^(1/3)*(-b^2)^(2/3
))*sqrt(-(-b^2)^(1/3))/(b^2*x)) - 18*sqrt(3)*(b^3*c - a*b^2*d)*(-b*c^3 + a
*c^2*d)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c^2 - a*c*d)*x + 2*sqrt(3)*(-b*c^3 +
 a*c^2*d)^(2/3)*(b*x^3 + a)^(1/3))/((b*c^2 - a*c*d)*x)) - 2*(9*b^2*c^2 - 1
2*a*b*c*d + 2*a^2*d^2)*(-b^2)^(2/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/
3)*b)/x) + (9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*(-b^2)^(2/3)*log(-((-b^2)^
(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2)
 - 18*(b^3*c - a*b^2*d)*(-b*c^3 + a*c^2*d)^(1/3)*log(((b*x^3 + a)^(1/3)*c
+ (-b*c^3 + a*c^2*d)^(1/3)*x)/x) + 9*(b^3*c - a*b^2*d)*(-b*c^3 + a*c^2*d)^
(1/3)*log(((b*x^3 + a)^(2/3)*c^2 - (-b*c^3 + a*c^2*d)^(1/3)*(b*x^3 + a)^(1
/3)*c*x + (-b*c^3 + a*c^2*d)^(2/3)*x^2)/x^2) + 3*(3*b^3*d^2*x^5 - (6*b^3*c
*d - 7*a*b^2*d^2)*x^2)*(b*x^3 + a)^(1/3))/(b^2*d^3)
```

Sympy [F]

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x^4(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**4*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**4*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}x^4}{dx^3+c}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)*x^4/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}x^4}{dx^3+c}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)*x^4/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{x^4\,(b\,x^3+a)^{4/3}}{d\,x^3+c}\,dx$$

input
```
int((x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

output
```
int((x^4*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx=\frac{7(b\,x^3+a)^{\frac{1}{3}}\,ad\,x^2-6(b\,x^3+a)^{\frac{1}{3}}\,bc\,x^2+3(b\,x^3+a)^{\frac{1}{3}}\,bd\,x^5+4\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}x^4}{bd\,x^6+ad\,x^3+bc\,x^3+a}\right.}{}$$

input
```
int(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
(7*(a + b*x**3)**(1/3)*a*d*x**2 - 6*(a + b*x**3)**(1/3)*b*c*x**2 + 3*(a +
b*x**3)**(1/3)*b*d*x**5 + 4*int(((a + b*x**3)**(1/3)*x**4)/(a*c + a*d*x**3
 + b*c*x**3 + b*d*x**6),x)*a**2*d**2 - 24*int(((a + b*x**3)**(1/3)*x**4)/(
a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b*c*d + 18*int(((a + b*x**3)**(
1/3)*x**4)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**2*c**2 - 14*int(((
a + b*x**3)**(1/3)*x)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a**2*c*d +
 12*int(((a + b*x**3)**(1/3)*x)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*
a*b*c**2)/(18*d**2)
```

3.718 $\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 277

$$\begin{aligned}\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx &= \frac{bx^2\sqrt[3]{a+bx^3}}{3d} + \frac{\sqrt[3]{b}(3bc-4ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}d^2} \\ &- \frac{(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}d^2} + \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6\sqrt[3]{c}d^2} \\ &+ \frac{\sqrt[3]{b}(3bc-4ad)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6d^2} \\ &- \frac{(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}d^2}\end{aligned}$$

output
```
1/3*b*x^2*(b*x^3+a)^(1/3)/d+1/9*b^(1/3)*(-4*a*d+3*b*c)*arctan(1/3*(1+2*b^(
1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d^2-1/3*(-a*d+b*c)^(4/3)*arctan(1
/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(1/
3)/d^2+1/6*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(1/3)/d^2+1/6*b^(1/3)*(-4*a*d+3*
b*c)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/d^2-1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^
(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(1/3)/d^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.20 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.69

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{12bdx^2\sqrt[3]{a+bx^3}+4\sqrt{3}\sqrt[3]{b}(3bc-4ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+\frac{6\sqrt{-6-6i\sqrt{3}}(bc-ad}{}}{}$$

input
```
Integrate[(x*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
(12*b*d*x^2*(a + b*x^3)^(1/3) + 4*Sqrt[3]*b^(1/3)*(3*b*c - 4*a*d)*ArcTan[(
Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + (6*Sqrt[-6 - (6*I)
*Sqrt[3]]*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
 a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/c^(1/3) + 4*b
^(1/3)*(3*b*c - 4*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + (6*(1 - I*S
qrt[3])*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1
/3)*(a + b*x^3)^(1/3)])/c^(1/3) - 2*b^(1/3)*(3*b*c - 4*a*d)*Log[b^(2/3)*x^
2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + ((3*I)*(I + Sqrt[3]
)*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)
*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^
3)^(2/3)])/c^(1/3))/(36*d^2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {977, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+bx^3\right)^{4/3}}{c+dx^3}dx$$

$$\Big\downarrow \text{ 977}$$

$$\frac{\int -\frac{x\left(b(3bc-4ad)x^3+a(2bc-3ad)\right)}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{3d}+\frac{bx^2\sqrt[3]{a+bx^3}}{3d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{bx^2\sqrt[3]{a+bx^3}}{3d}-\frac{\int \frac{x\left(b(3bc-4ad)x^3+a(2bc-3ad)\right)}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{3d}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{bx^2\sqrt[3]{a+bx^3}}{3d}-\frac{\int \left(\frac{b(3bc-4ad)x}{d\left(bx^3+a\right)^{2/3}}-\frac{3\left(b^2c^2-2abdc+a^2d^2\right)x}{d\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}\right)dx}{3d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{bx^2\sqrt[3]{a+bx^3}}{3d}-\frac{\frac{\sqrt{3}(bc-ad)^{4/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{c}d}-\frac{\sqrt[3]{b}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(3bc-4ad)}{\sqrt{3}d}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{2\sqrt[3]{c}d}+\frac{3(bc-ad)^{4/3}\log\Big(}{}}{3d}$$

input
```
Int[(x*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
(b*x^2*(a + b*x^3)^(1/3))/(3*d) - (-((b^(1/3)*(3*b*c - 4*a*d)*ArcTan[(1 +
(2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d)) + (Sqrt[3]*(b*c -
a*d)^(4/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3))
)/Sqrt[3]])/(c^(1/3)*d) - ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(2*c^(1/3)*d)
 - (b^(1/3)*(3*b*c - 4*a*d)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*d) + (3
*(b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])
/(2*c^(1/3)*d))/(3*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 977
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c -
a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p,
q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.39

method	result
pseudoelliptic	$-\frac{-\frac{(ad-bc)^2 \ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}} x+\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{2}-\sqrt{3}\,(ad-bc)^2 \arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x-2\left(b\,x^3+a\right)^{\frac{1}{3}}\right.}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x}\right.}{}$

input
```
int(x*(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/((a*d-b*c)/c)^(2/3)*(-1/2*(a*d-b*c)^2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a
*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-3^(1/2)*(a*d-b*c)
^2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)
/c)^(1/3)/x)-2/3*c*(b^(1/3)*a*d-3/4*b^(4/3)*c)*((a*d-b*c)/c)^(2/3)*ln((b^(
2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(a*d-b*c)^2*ln(((
(a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+4/3*c*(-(b^(1/3)*a*d-3/4*b^(4/3)*
c)*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+(b^
(1/3)*a*d-3/4*b^(4/3)*c)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-3/4*b*x^2*(b*x
^3+a)^(1/3)*d)*((a*d-b*c)/c)^(2/3))/c/d^2
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.43

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{6\,(bx^3+a)^{\frac{1}{3}} bdx^2 - 6\,\sqrt{3}(bc-ad)\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}} \arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\,\sqrt{3}(bx^3+a)^{\frac{1}{3}} c\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}}{3\,(bc-ad)x}\right.}{}$$

input
```
integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/18*(6*(b*x^3 + a)^(1/3)*b*d*x^2 - 6*sqrt(3)*(b*c - a*d)*((b*c - a*d)/c)^
(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*(
(b*c - a*d)/c)^(2/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(3*b*c - 4*a*d)*(-b)^(1
/3)*arctan(1/3*(sqrt(3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b)^(2/3))/(b*x
)) - 2*(3*b*c - 4*a*d)*(-b)^(1/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x
) - 6*(b*c - a*d)*((b*c - a*d)/c)^(1/3)*log(-(x*((b*c - a*d)/c)^(1/3) - (b
*x^3 + a)^(1/3))/x) + (3*b*c - 4*a*d)*(-b)^(1/3)*log(((-b)^(2/3)*x^2 - (b*
x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(b*c - a*d)*((b*
c - a*d)/c)^(1/3)*log((x^2*((b*c - a*d)/c)^(2/3) + (b*x^3 + a)^(1/3)*x*((b
*c - a*d)/c)^(1/3) + (b*x^3 + a)^(2/3))/x^2))/d^2
```

Sympy [F]

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}x}{dx^3+c}\,dx$$

input
```
integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)*x/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}x}{dx^3+c}\,dx$$

input
```
integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)*x/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{x\,(b\,x^3+a)^{4/3}}{d\,x^3+c}\,dx$$

input
```
int((x*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

output
```
int((x*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx=\frac{(b\,x^3+a)^{\frac{1}{3}}\,b\,x^2+4\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}x^4}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)abd-3\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}x^4}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)b^2c+}{3d}$$

input
```
int(x*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
((a + b*x**3)**(1/3)*b*x**2 + 4*int(((a + b*x**3)**(1/3)*x**4)/(a*c + a*d*
x**3 + b*c*x**3 + b*d*x**6),x)*a*b*d - 3*int(((a + b*x**3)**(1/3)*x**4)/(a
*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**2*c + 3*int(((a + b*x**3)**(1/3
)*x)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a**2*d - 2*int(((a + b*x**3
)**(1/3)*x)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b*c)/(3*d)
```

3.719 $\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 254

$$\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx = -\frac{a\sqrt[3]{a+bx^3}}{cx} - \frac{b^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d} + \frac{(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}d} - \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6c^{4/3}d} - \frac{b^{4/3}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d} + \frac{(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}d}$$

output
```
-a*(b*x^3+a)^(1/3)/c/x-1/3*b^(4/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/
3))*3^(1/2))*3^(1/2)/d+1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/
3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(4/3)/d-1/6*(-a*d+b*c)^(4
/3)*ln(d*x^3+c)/c^(4/3)/d-1/2*b^(4/3)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/d+1/2*
(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(4/3)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.67 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.80

$$\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx = \frac{-12a\sqrt[3]{c}d\sqrt[3]{a+bx^3}-4\sqrt{3}b^{4/3}c^{4/3}x\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-2\sqrt{-6-6i\sqrt{3}}(bc-$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x]
```

output
```
(-12*a*c^(1/3)*d*(a + b*x^3)^(1/3) - 4*Sqrt[3]*b^(4/3)*c^(4/3)*x*ArcTan[(S
qrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*Sqrt[-6 - (6*I)*S
qrt[3]]*(b*c - a*d)^(4/3)*x*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
 a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] - 4*b^(4/3)*c^
(4/3)*x*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + (2*I)*(I + Sqrt[3])*(b*c -
 a*d)^(4/3)*x*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x
^3)^(1/3)] + 2*b^(4/3)*c^(4/3)*x*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(
1/3) + (a + b*x^3)^(2/3)] + (1 - I*Sqrt[3])*(b*c - a*d)^(4/3)*x*Log[2*(b*c
 - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^
3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(4/3)*d*x)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {974, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx$$

$$\Big\downarrow \; 974$$

$$\frac{\int \frac{x(b^2cx^3+a(2bc-ad))}{(bx^3+a)^{2/3}(dx^3+c)}dx}{c} - \frac{a\sqrt[3]{a+bx^3}}{cx}$$

$$\Big\downarrow \text{1054}$$

$$\frac{\int \left(\frac{b^2cx}{d(bx^3+a)^{2/3}} - \frac{(b^2c^2-2abdc+a^2d^2)x}{d(bx^3+a)^{2/3}(dx^3+c)}\right)dx}{c} - \frac{a\sqrt[3]{a+bx^3}}{cx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^{4/3}c\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d} + \frac{(bc-ad)^{4/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}d} - \frac{b^{4/3}c\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d} - \frac{(bc-ad)^{4/3}\log(c+}{6\sqrt[3]{c}d}}{c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{cx}$$

input
```
Int[(a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x]
```

output
```
-((a*(a + b*x^3)^(1/3))/(c*x)) + (-((b^(4/3)*c*ArcTan[(1 + (2*b^(1/3)*x)/(
a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d)) + ((b*c - a*d)^(4/3)*ArcTan[(1 +
(2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^
(1/3)*d) - ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^(1/3)*d) - (b^(4/3)*c*L
og[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*d) + ((b*c - a*d)^(4/3)*Log[((b*c -
a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(1/3)*d))/c
```

Defintions of rubi rules used

rule 974
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.32

method	result
pseudoelliptic	$-\frac{3\left(b\,x^{3}+a\right)^{\frac{1}{3}} c a d \left(\frac{a d-b c}{c}\right)^{\frac{2}{3}}+\frac{\left(c^{2}\left(\frac{a d-b c}{c}\right)^{\frac{2}{3}}\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right)+2\ln\left(\frac{-b^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\ln\left(\frac{b^{\frac{2}{3}} x^{2}\cdots}{}\right)\cdots\right)\cdots\right)}{\cdots}}{\cdots}$

input
```
int((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*(b*x^3+a)^(1/3)*c*a*d*((a*d-b*c)/c)^(2/3)+1/2*(c^2*((a*d-b*c)/c)^(
2/3)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/
x)+2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^
(1/3)*x+(b*x^3+a)^(2/3))/x^2))*b^(4/3)+(a*d-b*c)^2*(2*3^(1/2)*arctan(1/3*3
^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)+ln
((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^
(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)))*x)/((a*d-b*c
)/c)^(2/3)/c^2/x/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x^2(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/x**2/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(x**2*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx=\int \frac{(b\,x^3+a)^{4/3}}{x^2\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(4/3)/(x^2*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx=\frac{-(b\,x^3+a)^{\frac{1}{3}}\,b+\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{bd\,x^8+ad\,x^5+bc\,x^5+ac\,x^2}dx\right)a^2dx-\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{bd\,x^8+ad\,x^5+bc\,x^5+ac\,x^2}dx\right)abcx}{dx}$$

input
```
int((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x)
```

output
```
( - (a + b*x**3)**(1/3)*b + int((a + b*x**3)**(1/3)/(a*c*x**2 + a*d*x**5 +
 b*c*x**5 + b*d*x**8),x)*a**2*d*x - int((a + b*x**3)**(1/3)/(a*c*x**2 + a*
d*x**5 + b*c*x**5 + b*d*x**8),x)*a*b*c*x + int(((a + b*x**3)**(1/3)*x)/(c
+ d*x**3),x)*b*d*x)/(d*x)
```

3.720 $\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 201

$$\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx = -\frac{a\sqrt[3]{a+bx^3}}{4cx^4} - \frac{(5bc-4ad)\sqrt[3]{a+bx^3}}{4c^2x} - \frac{(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}} + \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6c^{7/3}} - \frac{(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{7/3}}$$

output
```
-1/4*a*(b*x^3+a)^(1/3)/c/x^4-1/4*(-4*a*d+5*b*c)*(b*x^3+a)^(1/3)/c^2/x-1/3*
(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3
))*3^(1/2))*3^(1/2)/c^(7/3)+1/6*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(7/3)-1/2*(
-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(7/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.63

$$\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx = \frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}(-ac-5bcx^3+4adx^3)}{x^4}+2\sqrt{-6-6i\sqrt{3}}(bc-ad)^{4/3}\arctan\left(\frac{3\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{bc-ad}x-(3}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x]
```

output
```
((3*c^(1/3)*(a + b*x^3)^(1/3)*(-(a*c) - 5*b*c*x^3 + 4*a*d*x^3))/x^4 + 2*Sq
rt[-6 - (6*I)*Sqrt[3]]*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(S
qrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] +
 2*(1 - I*Sqrt[3])*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sq
rt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + I*(I + Sqrt[3])*(b*c - a*d)^(4/3)*Log[
2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a
+ b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(7/3))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {974, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx$$

$$\Big\downarrow \text{974}$$

$$\frac{\int \frac{b(4bc-3ad)x^3+a(5bc-4ad)}{x^2(bx^3+a)^{2/3}(dx^3+c)}dx}{4c} - \frac{a\sqrt[3]{a+bx^3}}{4cx^4}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int -\frac{4a(bc-ad)^2x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{ac}-\frac{\sqrt[3]{a+bx^3}(5bc-4ad)}{cx}}{4c}-\frac{a\sqrt[3]{a+bx^3}}{4cx^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{4(bc-ad)^2\int \frac{x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{c}-\frac{\sqrt[3]{a+bx^3}(5bc-4ad)}{cx}}{4c}-\frac{a\sqrt[3]{a+bx^3}}{4cx^4}$$

$$\Big\downarrow \; 992$$

$$\frac{\frac{4(bc-ad)^2\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{c}-\frac{\sqrt[3]{a+bx^3}(5bc-4ad)}{cx}}{4c}-\frac{a\sqrt[3]{a+bx^3}}{4cx^4}$$

input
```
Int[(a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x]
```

output
```
-1/4*(a*(a + b*x^3)^(1/3))/(c*x^4) + (-(((5*b*c - 4*a*d)*(a + b*x^3)^(1/3)
)/(c*x)) + (4*(b*c - a*d)^2*(-(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3
)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[
c + d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/
3) - (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a*d)^(2/3))))/c)/(4*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 974
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
+ b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$\frac{-x^4(ad-bc)^2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)-\frac{3c\left((-4ad+5bc)x^3+ac\right)\left(b\,x^3+a\right)^{\frac{1}{3}}\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}}{4}+(ad-bc)^2\left(\arctan\left(\frac{\sqrt{3}\left(-\frac{2(b\,\cdots}{(ad\cdots}\right.}{3\cdots}\right.\right.}{3\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}c^3x^4}$

input
```
int((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(2/3)*(-x^4*(a*d-b*c)^2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3
+a)^(1/3))/x)-3/4*c*((-4*a*d+5*b*c)*x^3+a*c)*(b*x^3+a)^(1/3)*((a*d-b*c)/c)
^(2/3)+(a*d-b*c)^2*(arctan(1/3*3^(1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(
1/3)+x)/x)*3^(1/2)+1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*
x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*x^4)/c^3/x^4
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x^5(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/x**5/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(x**5*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^5\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^5), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^5\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^5\,(c+dx^3)}\,dx = \int \frac{{\left(b\,x^3+a\right)}^{4/3}}{x^5\,\left(d\,x^3+c\right)}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(4/3)/(x^5*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^5\,(c+dx^3)}\,dx = \frac{-(b\,x^3+a)^{\frac{1}{3}}\,a^2d+3(b\,x^3+a)^{\frac{1}{3}}\,abd\,x^3-4(b\,x^3+a)^{\frac{1}{3}}\,b^2c\,x^3-4\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{bd\,x^8+ad\,x^5+bc\,x^5+ac}\right.}{}$$

input
```
int((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x)
```

output
```
( - (a + b*x**3)**(1/3)*a**2*d + 3*(a + b*x**3)**(1/3)*a*b*d*x**3 - 4*(a +
 b*x**3)**(1/3)*b**2*c*x**3 - 4*int((a + b*x**3)**(1/3)/(a*c*x**2 + a*d*x*
*5 + b*c*x**5 + b*d*x**8),x)*a**3*d**2*x**4 + 8*int((a + b*x**3)**(1/3)/(a
*c*x**2 + a*d*x**5 + b*c*x**5 + b*d*x**8),x)*a**2*b*c*d*x**4 - 4*int((a +
b*x**3)**(1/3)/(a*c*x**2 + a*d*x**5 + b*c*x**5 + b*d*x**8),x)*a*b**2*c**2*
x**4)/(4*a*c*d*x**4)
```

3.721 $\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 250

$$\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx = -\frac{a\sqrt[3]{a+bx^3}}{7cx^7} - \frac{(8bc-7ad)\sqrt[3]{a+bx^3}}{28c^2x^4} - \frac{(4b^2c^2-35abcd+28a^2d^2)\sqrt[3]{a+bx^3}}{28ac^3x} + \frac{d(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{10/3}} - \frac{d(bc-ad)^{4/3}\log(c+dx^3)}{6c^{10/3}} + \frac{d(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{10/3}}$$

output
```
-1/7*a*(b*x^3+a)^(1/3)/c/x^7-1/28*(-7*a*d+8*b*c)*(b*x^3+a)^(1/3)/c^2/x^4-1
/28*(28*a^2*d^2-35*a*b*c*d+4*b^2*c^2)*(b*x^3+a)^(1/3)/a/c^3/x+1/3*d*(-a*d+
b*c)^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(
1/2))*3^(1/2)/c^(10/3)-1/6*d*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(10/3)+1/2*d*(
-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(10/3)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.73 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx = \frac{-\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}(4b^2c^2x^6+abcx^3(8c-35dx^3)+a^2(4c^2-7cdx^3+28d^2x^6))}{ax^7} - 14\sqrt{-6-6i\sqrt{3}}d(bc-ad}{}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x]
```

output
```
((-3*c^(1/3)*(a + b*x^3)^(1/3)*(4*b^2*c^2*x^6 + a*b*c*x^3*(8*c - 35*d*x^3)
 + a^2*(4*c^2 - 7*c*d*x^3 + 28*d^2*x^6)))/(a*x^7) - 14*Sqrt[-6 - (6*I)*Sqr
t[3]]*d*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a
*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (14*I)*(I + Sq
rt[3])*d*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(
1/3)*(a + b*x^3)^(1/3)] + 7*(1 - I*Sqrt[3])*d*(b*c - a*d)^(4/3)*Log[2*(b*c
 - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^
3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(84*c^(10/3))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {974, 1053, 25, 27, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}dx$$

$$\Big\downarrow \text{974}$$

$$\frac{\int \frac{b(7bc-6ad)x^3+a(8bc-7ad)}{x^5(bx^3+a)^{2/3}(dx^3+c)}dx}{7c} - \frac{a\sqrt[3]{a+bx^3}}{7cx^7}$$

$$\Big\downarrow \text{1053}$$

$$-\frac{-\frac{\int -\frac{a\left(-3bd(8bc-7ad)x^3+4b^2c^2+28a^2d^2-35abcd\right)}{x^2\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{4ac}-\frac{\sqrt[3]{a+bx^3}(8bc-7ad)}{4cx^4}}{7c}-\frac{a\sqrt[3]{a+bx^3}}{7cx^7}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{a\left(-3bd(8bc-7ad)x^3+4b^2c^2+28a^2d^2-35abcd\right)}{x^2\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{4ac}-\frac{\sqrt[3]{a+bx^3}(8bc-7ad)}{4cx^4}}{7c}-\frac{a\sqrt[3]{a+bx^3}}{7cx^7}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{-3bd(8bc-7ad)x^3+4b^2c^2+28a^2d^2-35abcd}{x^2\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{4c}-\frac{\sqrt[3]{a+bx^3}(8bc-7ad)}{4cx^4}}{7c}-\frac{a\sqrt[3]{a+bx^3}}{7cx^7}$$

$\downarrow$ 1053

$$\frac{\frac{-\frac{\int \frac{28ad(bc-ad)^2x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{4b^2c}{a}+\frac{28ad^2}{c}-35bd\right)}{x}}{4c}-\frac{\sqrt[3]{a+bx^3}(8bc-7ad)}{4cx^4}}{7c}-\frac{a\sqrt[3]{a+bx^3}}{7cx^7}$$

$\downarrow$ 27

$$\frac{\frac{-\frac{28d(bc-ad)^2\int \frac{x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{c}-\frac{\sqrt[3]{a+bx^3}\left(\frac{4b^2c}{a}+\frac{28ad^2}{c}-35bd\right)}{x}}{4c}-\frac{\sqrt[3]{a+bx^3}(8bc-7ad)}{4cx^4}}{7c}-\frac{a\sqrt[3]{a+bx^3}}{7cx^7}$$

$\downarrow$ 992

$$\frac{\frac{-\frac{28d(bc-ad)^2\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{c}-\frac{\sqrt[3]{a+bx^3}\left(\frac{4b^2c}{a}+\frac{28ad^2}{c}-35bd\right)}{x}}{4c}-\frac{\sqrt[3]{a+bx^3}(8bc-7ad)}{4cx^4}}{7c}-\frac{a\sqrt[3]{a+bx^3}}{7cx^7}$$

input

```
Int[(a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x]
```

output
```
-1/7*(a*(a + b*x^3)^(1/3))/(c*x^7) + (-1/4*((8*b*c - 7*a*d)*(a + b*x^3)^(1
/3))/(c*x^4) + (-((((4*b^2*c)/a - 35*b*d + (28*a*d^2)/c)*(a + b*x^3)^(1/3)
)/x) - (28*d*(b*c - a*d)^2*(-(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)
*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[c
 + d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3
) - (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a*d)^(2/3))))/c)/(4*c))/(7*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 974
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
+ b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{c\left(\left(b x^3+a\right)^2 c^2-\frac{7 a d x^3\left(5 b x^3+a\right) c}{4}+7 a^2 d^2 x^6\right)\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}\left(b x^3+a\right)^{\frac{1}{3}}+\frac{7 a d x^7 (ad-bc)^2\left(2\sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x-2(\cdots}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\cdots}\right.\right.}{\cdots}}{7\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} x^7 c^4 \cdots}$

input
```
int((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/7/((a*d-b*c)/c)^(2/3)*(c*((b*x^3+a)^2*c^2-7/4*a*d*x^3*(5*b*x^3+a)*c+7*a
^2*d^2*x^6)*((a*d-b*c)/c)^(2/3)*(b*x^3+a)^(1/3)+7/6*a*d*x^7*(a*d-b*c)^2*(2
*3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*
d-b*c)/c)^(1/3)/x)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+
a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln(((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/
3))/x)))/x^7/c^4/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+b x^3\right)^{4/3}}{x^8\left(c+d x^3\right)}\, dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^8\,(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x^8\,(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/x**8/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(x**8*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^8\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^8}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^8), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^8\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^8}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx=\int \frac{(b\,x^3+a)^{4/3}}{x^8\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(4/3)/(x^8*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx=\frac{-4(b\,x^3+a)^{\frac{1}{3}}\,a^2c+7(b\,x^3+a)^{\frac{1}{3}}\,a^2d\,x^3-8(b\,x^3+a)^{\frac{1}{3}}\,abc\,x^3-21(b\,x^3+a)^{\frac{1}{3}}\,abd\,x^6-}{}$$

input
```
int((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x)
```

output
```
( - 4*(a + b*x**3)**(1/3)*a**2*c + 7*(a + b*x**3)**(1/3)*a**2*d*x**3 - 8*(
a + b*x**3)**(1/3)*a*b*c*x**3 - 21*(a + b*x**3)**(1/3)*a*b*d*x**6 + 24*(a
+ b*x**3)**(1/3)*b**2*c*x**6 + 28*int((a + b*x**3)**(1/3)/(a*c*x**2 + a*d*
x**5 + b*c*x**5 + b*d*x**8),x)*a**3*d**2*x**7 - 56*int((a + b*x**3)**(1/3)
/(a*c*x**2 + a*d*x**5 + b*c*x**5 + b*d*x**8),x)*a**2*b*c*d*x**7 + 28*int((
a + b*x**3)**(1/3)/(a*c*x**2 + a*d*x**5 + b*c*x**5 + b*d*x**8),x)*a*b**2*c
**2*x**7)/(28*a*c**2*x**7)
```

3.722 $\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 318

$$\begin{aligned}\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx &= -\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}-\frac{(11bc-10ad)\sqrt[3]{a+bx^3}}{70c^2x^7}\\ &-\frac{(2b^2c^2-40abcd+35a^2d^2)\sqrt[3]{a+bx^3}}{140ac^3x^4}\\ &+\frac{(6b^3c^3+20ab^2c^2d-175a^2bcd^2+140a^3d^3)\sqrt[3]{a+bx^3}}{140a^2c^4x}\\ &-\frac{d^2(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{13/3}}+\frac{d^2(bc-ad)^{4/3}\log(c+dx^3)}{6c^{13/3}}\\ &-\frac{d^2(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{13/3}}\end{aligned}$$

output
```
-1/10*a*(b*x^3+a)^(1/3)/c/x^10-1/70*(-10*a*d+11*b*c)*(b*x^3+a)^(1/3)/c^2/x
^7-1/140*(35*a^2*d^2-40*a*b*c*d+2*b^2*c^2)*(b*x^3+a)^(1/3)/a/c^3/x^4+1/140
*(140*a^3*d^3-175*a^2*b*c*d^2+20*a*b^2*c^2*d+6*b^3*c^3)*(b*x^3+a)^(1/3)/a^
2/c^4/x-1/3*d^2*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3
)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(13/3)+1/6*d^2*(-a*d+b*c)^(4/3)*ln(d
*x^3+c)/c^(13/3)-1/2*d^2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b
*x^3+a)^(1/3))/c^(13/3)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.54 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = \frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}(6b^3c^3x^9-2ab^2c^2x^6(c-10dx^3)+a^2bcx^3(-22c^2+40cdx^3-175d^2x^6)+a^3(-14c^3+20c^2dx^3-35cd^2}{a^2x^{10}}}{}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x]
```

output
```
((3*c^(1/3)*(a + b*x^3)^(1/3)*(6*b^3*c^3*x^9 - 2*a*b^2*c^2*x^6*(c - 10*d*x
^3) + a^2*b*c*x^3*(-22*c^2 + 40*c*d*x^3 - 175*d^2*x^6) + a^3*(-14*c^3 + 20
*c^2*d*x^3 - 35*c*d^2*x^6 + 140*d^3*x^9)))/(a^2*x^10) + 70*Sqrt[-6 - (6*I)
*Sqrt[3]]*d^2*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b
*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 70*(1 -
I*Sqrt[3])*d^2*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3
])*c^(1/3)*(a + b*x^3)^(1/3)] + (35*I)*(I + Sqrt[3])*d^2*(b*c - a*d)^(4/3)
*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*
x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(420*c^(
13/3))
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {974, 1053, 27, 1053, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}dx$$

$$\Big\downarrow \text{974}$$

$$\frac{\int \frac{b(10bc-9ad)x^3+a(11bc-10ad)}{x^8(bx^3+a)^{2/3}(dx^3+c)}dx}{10c}-\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int -\frac{2a\left(-3bd(11bc-10ad)x^3+2b^2c^2+35a^2d^2-40abcd\right)}{x^5\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{7ac}-\frac{\sqrt[3]{a+bx^3}(11bc-10ad)}{7cx^7}}{10c}-\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\int \frac{-3bd(11bc-10ad)x^3+2b^2c^2+35a^2d^2-40abcd}{x^5\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{7c}-\frac{\sqrt[3]{a+bx^3}(11bc-10ad)}{7cx^7}}{10c}-\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{\frac{2\left(-\frac{\int \frac{6b^3c^3+20ab^2dc^2-175a^2bd^2c+140a^3d^3+3bd\left(2b^2c^2-40abdc+35a^2d^2\right)x^3}{x^2\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{2b^2c}{a}+\frac{35ad^2}{c}-40bd\right)}{4x^4}\right)}{7c}-\frac{\sqrt[3]{a+bx^3}(11bc-10ad)}{7cx^7}}{10c}-$$
$$\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{2\left(-\frac{-\frac{\int \frac{140a^2d^2(bc-ad)^2x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{ac}-\frac{\sqrt[3]{a+bx^3}(140a^3d^3-175a^2bcd^2+20ab^2c^2d+6b^3c^3)}{acx}}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{2b^2c}{a}+\frac{35ad^2}{c}-40bd\right)}{4x^4}\right)}{7c}-\frac{\sqrt[3]{a+bx^3}(11}{7cx^7}}{10c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$\Big\downarrow$ 27

$$\frac{2\left(-\frac{-\frac{140ad^2(bc-ad)^2\int \frac{x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{c}-\frac{\sqrt[3]{a+bx^3}(140a^3d^3-175a^2bcd^2+20ab^2c^2d+6b^3c^3)}{acx}}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{2b^2c}{a}+\frac{35ad^2}{c}-40bd\right)}{4x^4}\right)}{7c}-}{10c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$\Big\downarrow$ 992

$$\frac{2\left(-\frac{-\frac{\sqrt[3]{a+bx^3}(140a^3d^3-175a^2bcd^2+20ab^2c^2d+6b^3c^3)}{acx}-\frac{140ad^2(bc-ad)^2\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log(c+dx^3)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-}}{\sqrt[3]{c}}\right.}{2\sqrt[3]{c}}\right.}{c}}{4ac}\right.}{7c}}{10c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}$$

input `Int[(a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x]`

output
```
-1/10*(a*(a + b*x^3)^(1/3))/(c*x^10) + (-1/7*((11*b*c - 10*a*d)*(a + b*x^3
)^(1/3))/(c*x^7) + (2*(-1/4*(((2*b^2*c)/a - 40*b*d + (35*a*d^2)/c)*(a + b*
x^3)^(1/3))/x^4 - (-(((6*b^3*c^3 + 20*a*b^2*c^2*d - 175*a^2*b*c*d^2 + 140*
a^3*d^3)*(a + b*x^3)^(1/3))/(a*c*x)) - (140*a*d^2*(b*c - a*d)^2*(-(ArcTan[
(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3
]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[c + d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3
)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*
c - a*d)^(2/3))))/c)/(4*a*c)))/(7*c))/(10*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 974
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
+ b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$-\frac{c\left(\left(b x^3+a\right)^2\left(-\frac{3 b x^3}{7}+a\right) c^3-\frac{10 a d x^3\left(b x^3+a\right)^2 c^2}{7}+\frac{5 a^2 d^2 x^6\left(5 b x^3+a\right) c}{2}-10 a^3 d^3 x^9\right)\left(\frac{a d-b c}{c}\right)^{\frac{2}{3}}\left(b x^3+a\right)^{\frac{1}{3}}-\frac{5 a^2 d^2 x^{10}(a}{}}{}$

input
```
int((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/10/((a*d-b*c)/c)^(2/3)*(c*((b*x^3+a)^2*(-3/7*b*x^3+a)*c^3-10/7*a*d*x^3*
(b*x^3+a)^2*c^2+5/2*a^2*d^2*x^6*(5*b*x^3+a)*c-10*a^3*d^3*x^9)*((a*d-b*c)/c
)^(2/3)*(b*x^3+a)^(1/3)-5/3*a^2*d^2*x^10*(a*d-b*c)^2*(2*3^(1/2)*arctan(1/3
*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)+
ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a
)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)))/x^10/c^5/a
^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^{11}\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(4/3)/x**11/(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^11), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^11), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{x^{11}\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(4/3)/(x^11*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = \frac{-14(b\,x^3+a)^{\frac{1}{3}}\,a^3c^2+20(b\,x^3+a)^{\frac{1}{3}}\,a^3cd\,x^3-35(b\,x^3+a)^{\frac{1}{3}}\,a^3d^2x^6-22(b\,x^3+a)^{\frac{1}{3}}\,a}{}$$

input
```
int((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x)
```

output
```
( - 14*(a + b*x**3)**(1/3)*a**3*c**2 + 20*(a + b*x**3)**(1/3)*a**3*c*d*x**
3 - 35*(a + b*x**3)**(1/3)*a**3*d**2*x**6 - 22*(a + b*x**3)**(1/3)*a**2*b*
c**2*x**3 + 40*(a + b*x**3)**(1/3)*a**2*b*c*d*x**6 + 105*(a + b*x**3)**(1/
3)*a**2*b*d**2*x**9 - 2*(a + b*x**3)**(1/3)*a*b**2*c**2*x**6 - 120*(a + b*
x**3)**(1/3)*a*b**2*c*d*x**9 + 6*(a + b*x**3)**(1/3)*b**3*c**2*x**9 - 140*
int((a + b*x**3)**(1/3)/(a*c*x**2 + a*d*x**5 + b*c*x**5 + b*d*x**8),x)*a**
4*d**3*x**10 + 280*int((a + b*x**3)**(1/3)/(a*c*x**2 + a*d*x**5 + b*c*x**5
 + b*d*x**8),x)*a**3*b*c*d**2*x**10 - 140*int((a + b*x**3)**(1/3)/(a*c*x**
2 + a*d*x**5 + b*c*x**5 + b*d*x**8),x)*a**2*b**2*c**2*d*x**10)/(140*a**2*c
**3*x**10)
```

3.723 $\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 392

$$\begin{aligned}
\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx = &-\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}} - \frac{(14bc-13ad)\sqrt[3]{a+bx^3}}{130c^2x^{10}} \\
&- \frac{(4b^2c^2-143abcd+130a^2d^2)\sqrt[3]{a+bx^3}}{910ac^3x^7} \\
&+ \frac{(12b^3c^3+26ab^2c^2d-520a^2bcd^2+455a^3d^3)\sqrt[3]{a+bx^3}}{1820a^2c^4x^4} \\
&- \frac{(36b^4c^4+78ab^3c^3d+260a^2b^2c^2d^2-2275a^3bcd^3+1820a^4d^4)\sqrt[3]{a+bx^3}}{1820a^3c^5x} \\
&+ \frac{d^3(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{16/3}} - \frac{d^3(bc-ad)^{4/3}\log(c+dx^3)}{6c^{16/3}} \\
&+ \frac{d^3(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{16/3}}
\end{aligned}$$

output
```
-1/13*a*(b*x^3+a)^(1/3)/c/x^13-1/130*(-13*a*d+14*b*c)*(b*x^3+a)^(1/3)/c^2/
x^10-1/910*(130*a^2*d^2-143*a*b*c*d+4*b^2*c^2)*(b*x^3+a)^(1/3)/a/c^3/x^7+1
/1820*(455*a^3*d^3-520*a^2*b*c*d^2+26*a*b^2*c^2*d+12*b^3*c^3)*(b*x^3+a)^(1
/3)/a^2/c^4/x^4-1/1820*(1820*a^4*d^4-2275*a^3*b*c*d^3+260*a^2*b^2*c^2*d^2+
78*a*b^3*c^3*d+36*b^4*c^4)*(b*x^3+a)^(1/3)/a^3/c^5/x+1/3*d^3*(-a*d+b*c)^(4
/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3
^(1/2)/c^(16/3)-1/6*d^3*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(16/3)+1/2*d^3*(-a*
d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(16/3)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.23 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx = \frac{-\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}\left(36b^4c^4x^{12}+6ab^3c^3x^9(-2c+13dx^3)+2a^2b^2c^2x^6(4c^2-13cdx^3+130d^2x^6)+a^3bcx^3(196c^3-286\cdots}{a^3x^{13}}}{}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x]
```

output
```
((-3*c^(1/3)*(a + b*x^3)^(1/3)*(36*b^4*c^4*x^12 + 6*a*b^3*c^3*x^9*(-2*c +
13*d*x^3) + 2*a^2*b^2*c^2*x^6*(4*c^2 - 13*c*d*x^3 + 130*d^2*x^6) + a^3*b*c
*x^3*(196*c^3 - 286*c^2*d*x^3 + 520*c*d^2*x^6 - 2275*d^3*x^9) + a^4*(140*c
^4 - 182*c^3*d*x^3 + 260*c^2*d^2*x^6 - 455*c*d^3*x^9 + 1820*d^4*x^12)))/(a
^3*x^13) - 910*Sqrt[-6 - (6*I)*Sqrt[3]]*d^3*(b*c - a*d)^(4/3)*ArcTan[(3*(b
*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*
(a + b*x^3)^(1/3))] + (910*I)*(I + Sqrt[3])*d^3*(b*c - a*d)^(4/3)*Log[2*(b
*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + 455*(1 -
I*Sqrt[3])*d^3*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqr
t[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2
/3)*(a + b*x^3)^(2/3)])/(5460*c^(16/3))
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {974, 1053, 25, 27, 1053, 27, 1053, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^{14}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{974}$$

$$\frac{\int \frac{b(13bc-12ad)x^3+a(14bc-13ad)}{x^{11}(bx^3+a)^{2/3}(dx^3+c)}dx}{13c}-\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int -\frac{a\left(-9bd(14bc-13ad)x^3+4b^2c^2+130a^2d^2-143abcd\right)}{x^8\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{10ac}-\frac{\sqrt[3]{a+bx^3}(14bc-13ad)}{10cx^{10}}}{13c}-\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{a\left(-9bd(14bc-13ad)x^3+4b^2c^2+130a^2d^2-143abcd\right)}{x^8\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{10ac}-\frac{\sqrt[3]{a+bx^3}(14bc-13ad)}{10cx^{10}}}{13c}-\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{-9bd(14bc-13ad)x^3+4b^2c^2+130a^2d^2-143abcd}{x^8\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{10c}-\frac{\sqrt[3]{a+bx^3}(14bc-13ad)}{10cx^{10}}}{13c}-\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{\frac{-\frac{\int \frac{2\left(12b^3c^3+26ab^2dc^2-520a^2bd^2c+455a^3d^3+3bd\left(4b^2c^2-143abdc+130a^2d^2\right)x^3\right)}{x^5\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{7ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{4b^2c}{a}+\frac{130ad^2}{c}-143bd\right)}{7x^7}}{10c}-\frac{\sqrt[3]{a+bx^3}(14bc-13}{10cx^{10}}}{13c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{-\frac{2\int \frac{12b^3c^3+26ab^2dc^2-520a^2bd^2c+455a^3d^3+3bd\left(4b^2c^2-143abdc+130a^2d^2\right)x^3}{x^5\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{7ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{4b^2c}{a}+\frac{130ad^2}{c}-143bd\right)}{7x^7}}{10c}-\frac{\sqrt[3]{a+bx^3}(14bc-13a\cdots}{10cx^{10}}}{13c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

$\downarrow$ 1053

$$\frac{-\frac{-\frac{2\left(-\frac{\int \frac{36b^4c^4+78ab^3dc^3+260a^2b^2d^2c^2-2275a^3bd^3c+1820a^4d^4+3bd\left(12b^3c^3+26ab^2dc^2-520a^2bd^2c+455a^3d^3\right)x^3}{x^2\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(455a^3d^3-520a^2bcd^2\cdots\right.}{4acx^4}\cdots}{7ac}\cdots}{10c}\cdots}{13c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

$\downarrow$ 1053

$$\frac{-\frac{-\frac{2\left(-\frac{-\frac{\int \frac{1820a^3d^3(bc-ad)^2x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{ac}-\frac{\sqrt[3]{a+bx^3}\left(1820a^4d^4-2275a^3bcd^3+260a^2b^2c^2d^2+78ab^3c^3d+36b^4c^4\right)}{acx}}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(455a^3d^3-520a^2bcd^2+26ab^2\cdots\right.}{4acx^4}\cdots}{7ac}\cdots}{10c}\cdots}{13c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

$\downarrow$ 27

$$\frac{-\frac{-\frac{2\left(-\frac{-\frac{1820a^2d^3(bc-ad)^2\int \frac{x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{c}-\frac{\sqrt[3]{a+bx^3}\left(1820a^4d^4-2275a^3bcd^3+260a^2b^2c^2d^2+78ab^3c^3d+36b^4c^4\right)}{acx}}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(455a^3d^3\cdots\right.}{\cdots}\cdots}{7ac}\cdots}{10c}\cdots}{13c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

$\downarrow$ 992

$$-\frac{2\left(-\frac{\sqrt[3]{a+bx^3}\left(455a^3d^3-520a^2bcd^2+26ab^2c^2d+12b^3c^3\right)}{4acx^4}-\frac{-\frac{1820a^2d^3(bc-ad)^2\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{b\cdots}}{\cdots}\right)}{2\cdots}\right)}{c}\cdots}{\cdots}\right)}{7ac}\cdots}{10c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

input
```
Int[(a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x]
```

output
```
-1/13*(a*(a + b*x^3)^(1/3))/(c*x^13) + (-1/10*((14*b*c - 13*a*d)*(a + b*x^
3)^(1/3))/(c*x^10) + (-1/7*(((4*b^2*c)/a - 143*b*d + (130*a*d^2)/c)*(a + b
*x^3)^(1/3))/x^7 - (2*(-1/4*((12*b^3*c^3 + 26*a*b^2*c^2*d - 520*a^2*b*c*d^
2 + 455*a^3*d^3)*(a + b*x^3)^(1/3))/(a*c*x^4) - (-(((36*b^4*c^4 + 78*a*b^3
*c^3*d + 260*a^2*b^2*c^2*d^2 - 2275*a^3*b*c*d^3 + 1820*a^4*d^4)*(a + b*x^3
)^(1/3))/(a*c*x)) - (1820*a^2*d^3*(b*c - a*d)^2*(-(ArcTan[(1 + (2*(b*c - a
*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(1/3)*(b*c -
 a*d)^(2/3))) + Log[c + d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) - Log[((b*c -
 a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a*d)^(2/3)))
)/c)/(4*a*c)))/(7*a*c))/(10*c))/(13*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 974
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
+ b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$-\frac{c\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}\left(\left(b\,x^{3}+a\right)^{2}\left(\frac{9}{35}b^{2}x^{6}-\frac{3}{5}ab\,x^{3}+a^{2}\right)c^{4}-\frac{13ad\left(b\,x^{3}+a\right)^{2}\left(-\frac{3b\,x^{3}}{7}+a\right)x^{3}c^{3}}{10}+\frac{13\left(b\,x^{3}+a\right)^{2}a^{2}c^{2}d^{2}x^{6}}{7}-\frac{13a^{3}d^{3}x^{9}\left(5\right.}{4}\right.}{}$

input
```
int((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/13/((a*d-b*c)/c)^(2/3)*(c*((a*d-b*c)/c)^(2/3)*((b*x^3+a)^2*(9/35*b^2*x^
6-3/5*a*b*x^3+a^2)*c^4-13/10*a*d*(b*x^3+a)^2*(-3/7*b*x^3+a)*x^3*c^3+13/7*(
b*x^3+a)^2*a^2*c^2*d^2*x^6-13/4*a^3*d^3*x^9*(5*b*x^3+a)*c+13*a^4*d^4*x^12)
*(b*x^3+a)^(1/3)+13/6*a^3*d^3*x^13*(a*d-b*c)^2*(2*3^(1/2)*arctan(1/3*3^(1/
2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)+ln((((
a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3
))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)))/x^13/c^6/a^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^{14}\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(4/3)/x**14/(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^{14}}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^14), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^{14}}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^14), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx=\int \frac{(b\,x^3+a)^{4/3}}{x^{14}\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(4/3)/(x^14*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx=\frac{-140(b\,x^3+a)^{\frac{1}{3}}\,a^4c^2+182(b\,x^3+a)^{\frac{1}{3}}\,a^4cd\,x^3-260(b\,x^3+a)^{\frac{1}{3}}\,a^4d^2x^6-196(b\,x^3+}{}$$

input
```
int((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x)
```

output
```
( - 140*(a + b*x**3)**(1/3)*a**4*c**2 + 182*(a + b*x**3)**(1/3)*a**4*c*d*x
**3 - 260*(a + b*x**3)**(1/3)*a**4*d**2*x**6 - 196*(a + b*x**3)**(1/3)*a**
3*b*c**2*x**3 + 286*(a + b*x**3)**(1/3)*a**3*b*c*d*x**6 + 390*(a + b*x**3)
**(1/3)*a**3*b*d**2*x**9 - 8*(a + b*x**3)**(1/3)*a**2*b**2*c**2*x**6 - 429
*(a + b*x**3)**(1/3)*a**2*b**2*c*d*x**9 - 1170*(a + b*x**3)**(1/3)*a**2*b*
*2*d**2*x**12 + 12*(a + b*x**3)**(1/3)*a*b**3*c**2*x**9 + 1287*(a + b*x**3
)**(1/3)*a*b**3*c*d*x**12 - 36*(a + b*x**3)**(1/3)*b**4*c**2*x**12 - 1820*
int((a + b*x**3)**(1/3)/(a*c*x**5 + a*d*x**8 + b*c*x**8 + b*d*x**11),x)*a*
*5*d**3*x**13 + 3640*int((a + b*x**3)**(1/3)/(a*c*x**5 + a*d*x**8 + b*c*x*
*8 + b*d*x**11),x)*a**4*b*c*d**2*x**13 - 1820*int((a + b*x**3)**(1/3)/(a*c
*x**5 + a*d*x**8 + b*c*x**8 + b*d*x**11),x)*a**3*b**2*c**2*d*x**13)/(1820*
a**3*c**3*x**13)
```

3.724 $\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{ax^7\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{7}{3},-\frac{4}{3},1,\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/7*a*x^7*(b*x^3+a)^(1/3)*AppellF1(7/3,-4/3,1,10/3,-b*x^3/a,-d*x^3/c)/c/(1
+b*x^3/a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 343 vs. 2(65) = 130.

Time = 8.46 (sec) , antiderivative size = 343, normalized size of antiderivative = 5.28

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{x\left(2(a+bx^3)\left(2a^2d^2+3abd(-8c+3dx^3)+b^2(20c^2-8cdx^3+5d^2x^6)\right)-\frac{(20b^3c^3-30\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(x^6*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
(x*(2*(a + b*x^3)*(2*a^2*d^2 + 3*a*b*d*(-8*c + 3*d*x^3) + b^2*(20*c^2 - 8*
c*d*x^3 + 5*d^2*x^6)) - ((20*b^3*c^3 - 30*a*b^2*c^2*d + 8*a^2*b*c*d^2 + a^
3*d^3)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
-((d*x^3)/c)])/c + (16*a^2*c^2*(10*b^2*c^2 - 12*a*b*c*d + a^2*d^2)*AppellF
1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*Appe
llF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4
/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(80*b*d^3*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 \left(a+b x^3\right)^{4/3}}{c+d x^3} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{a \sqrt[3]{a+b x^3} \int \frac{x^6 \left(\frac{b x^3}{a}+1\right)^{4/3}}{d x^3+c}dx}{\sqrt[3]{\frac{b x^3}{a}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{a x^7 \sqrt[3]{a+b x^3} \operatorname{AppellF1}\left(\frac{7}{3},-\frac{4}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right)}{7 c \sqrt[3]{\frac{b x^3}{a}+1}}$$

input
```
Int[(x^6*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
(a*x^7*(a + b*x^3)^(1/3)*AppellF1[7/3, -4/3, 1, 10/3, -((b*x^3)/a), -((d*x
^3)/c)])/(7*c*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{6} \left(b\,x^{3}+a\right)^{\frac{4}{3}}}{d\,x^{3}+c}dx$$

input
```
int(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6 (a+bx^3)^{4/3}}{c+dx^3}\,dx=\text{Timed out}$$

input
```
integrate(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x^6(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**6*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**6*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}x^6}{dx^3+c}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)*x^6/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}x^6}{dx^3+c}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)*x^6/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{x^6\,(b\,x^3+a)^{4/3}}{d\,x^3+c}\,dx$$

input
```
int((x^6*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

output
```
int((x^6*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx=\frac{2(b\,x^3+a)^{\frac{1}{3}}\,a^2d^2x-24(b\,x^3+a)^{\frac{1}{3}}\,abcdx+9(b\,x^3+a)^{\frac{1}{3}}\,ab\,d^2x^4+20(b\,x^3+a)^{\frac{1}{3}}\,b^2c}{}$$

input
```
int(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
(2*(a + b*x**3)**(1/3)*a**2*d**2*x - 24*(a + b*x**3)**(1/3)*a*b*c*d*x + 9*
(a + b*x**3)**(1/3)*a*b*d**2*x**4 + 20*(a + b*x**3)**(1/3)*b**2*c**2*x - 8
*(a + b*x**3)**(1/3)*b**2*c*d*x**4 + 5*(a + b*x**3)**(1/3)*b**2*d**2*x**7
- 2*int((a + b*x**3)**(1/3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a**3
*c*d**2 + 24*int((a + b*x**3)**(1/3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6
),x)*a**2*b*c**2*d - 20*int((a + b*x**3)**(1/3)/(a*c + a*d*x**3 + b*c*x**3
 + b*d*x**6),x)*a*b**2*c**3 - 2*int(((a + b*x**3)**(1/3)*x**3)/(a*c + a*d*
x**3 + b*c*x**3 + b*d*x**6),x)*a**3*d**3 - 16*int(((a + b*x**3)**(1/3)*x**
3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a**2*b*c*d**2 + 60*int(((a +
b*x**3)**(1/3)*x**3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b**2*c**2
*d - 40*int(((a + b*x**3)**(1/3)*x**3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x*
*6),x)*b**3*c**3)/(40*b*d**3)
```

3.725 $\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx=\frac{ax^4\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{4}{3},-\frac{4}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/4*a*x^4*(b*x^3+a)^(1/3)*AppellF1(4/3,-4/3,1,7/3,-b*x^3/a,-d*x^3/c)/c/(1+
b*x^3/a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 280 vs. 2(65) = 130.

Time = 8.14 (sec) , antiderivative size = 280, normalized size of antiderivative = 4.31

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx=\frac{x\left(4(a+bx^3)\left(-5bc+6ad+2bdx^3\right)+\frac{(10b^2c^2-15abcd+4a^2d^2)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3}\right.}{c}\right.}{}$$

input
```
Integrate[(x^3*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
(x*(4*(a + b*x^3)*(-5*b*c + 6*a*d + 2*b*d*x^3) + ((10*b^2*c^2 - 15*a*b*c*d
 + 4*a^2*d^2)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^
3)/a), -((d*x^3)/c)])/c + (16*a^2*c^2*(-5*b*c + 6*a*d)*AppellF1[1/3, 2/3,
1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/
3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2,
7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x
^3)/a), -((d*x^3)/c)])))))/(40*d^2*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (a+bx^3)^{4/3}}{c+dx^3} \, dx$$

$$\Big\downarrow \ 1013$$

$$\frac{a\sqrt[3]{a+bx^3}\int \frac{x^3\left(\frac{bx^3}{a}+1\right)^{4/3}}{dx^3+c}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \ 1012$$

$$\frac{ax^4\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{4}{3},-\frac{4}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(x^3*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
(a*x^4*(a + b*x^3)^(1/3)*AppellF1[4/3, -4/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(4*c*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{3} \left(b \,x^{3}+a \right)^{\frac{4}{3}}}{d \,x^{3}+c}d x$$

input
```
int(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx=\text{Timed out}$$

input
```
integrate(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x^3(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**3*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**3*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}x^3}{dx^3+c}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)*x^3/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}x^3}{dx^3+c}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)*x^3/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x^3\,(b\,x^3+a)^{4/3}}{d\,x^3+c}\,dx$$

input
```
int((x^3*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

output
```
int((x^3*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{6(b\,x^3+a)^{\frac{1}{3}}\,adx - 5(b\,x^3+a)^{\frac{1}{3}}\,bcx + 2(b\,x^3+a)^{\frac{1}{3}}\,bd\,x^4 - 6\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right.}{}$$

input
```
int(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
(6*(a + b*x**3)**(1/3)*a*d*x - 5*(a + b*x**3)**(1/3)*b*c*x + 2*(a + b*x**3
)**(1/3)*b*d*x**4 - 6*int((a + b*x**3)**(1/3)/(a*c + a*d*x**3 + b*c*x**3 +
 b*d*x**6),x)*a**2*c*d + 5*int((a + b*x**3)**(1/3)/(a*c + a*d*x**3 + b*c*x
**3 + b*d*x**6),x)*a*b*c**2 + 4*int(((a + b*x**3)**(1/3)*x**3)/(a*c + a*d*
x**3 + b*c*x**3 + b*d*x**6),x)*a**2*d**2 - 15*int(((a + b*x**3)**(1/3)*x**
3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b*c*d + 10*int(((a + b*x**3
)**(1/3)*x**3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**2*c**2)/(10*d*
*2)
```

3.726 $\int \frac{\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx=\frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/
a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 346 vs. 2(60) = 120.

Time = 0.21 (sec) , antiderivative size = 346, normalized size of antiderivative = 5.77

$$\int \frac{\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx=\frac{x\left(\frac{b(-2bc+3ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{4\left(-4ac\left(2a^2d+abdx^3+b^2x^3\left(c+dx^3\right)\right)\text{Appell}}{\left(c+dx^3\right)\left(-4ac\,\text{AppellF1}\right.}\right.}{8a}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

output
```
(x*((b*(-2*b*c + 3*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/
3, -((b*x^3)/a), -((d*x^3)/c)])/c + (4*(-4*a*c*(2*a^2*d + a*b*d*x^3 + b^2*
x^3*(c + d*x^3))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] +
b*x^3*(a + b*x^3)*(c + d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/
a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])))/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -(
(d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/
(8*d*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$$

$$\Big\downarrow \; 937$$

$$\frac{a\sqrt[3]{a+bx^3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{4/3}}{dx^3+c}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \; 936$$

$$\frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

output
```
(a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{4}{3}}}{d\,x^{3}+c}dx$$

input
```
int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(4/3)/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{(b\,x^3+a)^{\frac{1}{3}}\,bx + 2\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)a^2d - \left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)abc + 3\Big(}{2d}$$

input
```
int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
((a + b*x**3)**(1/3)*b*x + 2*int((a + b*x**3)**(1/3)/(a*c + a*d*x**3 + b*c
*x**3 + b*d*x**6),x)*a**2*d - int((a + b*x**3)**(1/3)/(a*c + a*d*x**3 + b*
c*x**3 + b*d*x**6),x)*a*b*c + 3*int(((a + b*x**3)**(1/3)*x**3)/(a*c + a*d*
x**3 + b*c*x**3 + b*d*x**6),x)*a*b*d - 2*int(((a + b*x**3)**(1/3)*x**3)/(a
*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**2*c)/(2*d)
```

3.727 $\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)} \, dx = -\frac{a\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{4}{3},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-1/2*a*(b*x^3+a)^(1/3)*AppellF1(-2/3,-4/3,1,1/3,-b*x^3/a,-d*x^3/c)/c/x^2/(
1+b*x^3/a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 341 vs. 2(65) = 130.

Time = 10.29 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.25

$$\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)} \, dx = -\frac{b(-2bc+ad)x^6\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4ac\left(-4ac(ac-2bcx^3+3adx^3+bdx^6)\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1\right.\right.}{(c+dx^3)\left(-4ac\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3}\right.\right.}}{8c^2x^2(a+bx^3)^{2/3}}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x^3*(c + d*x^3)),x]
```

output
```
-1/8*(b*(-2*b*c + a*d)*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3
, -((b*x^3)/a), -((d*x^3)/c)] + (4*a*c*(-4*a*c*(a*c - 2*b*c*x^3 + 3*a*d*x^
3 + b*d*x^6)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*
(a + b*x^3)*(c + d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -(
(d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
)))/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
+ 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(c^2*x^
2*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^3\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{a\sqrt[3]{a+bx^3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{4/3}}{x^3(dx^3+c)}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{a\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{4}{3},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(4/3)/(x^3*(c + d*x^3)),x]
```

output
```
-1/2*(a*(a + b*x^3)^(1/3)*AppellF1[-2/3, -4/3, 1, 1/3, -((b*x^3)/a), -((d*
x^3)/c)])/(c*x^2*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{x^{3}\left(d\,x^{3}+c\right)}d x$$

input
```
int((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^3\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x^3(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/x**3/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(x**3*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)}\,dx=\int \frac{(b\,x^3+a)^{4/3}}{x^3\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(x^3*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(4/3)/(x^3*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)}\,dx=\frac{-2(b\,x^3+a)^{\frac{1}{3}}\,ab+4\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{2ab\,d^2x^9+b^2cd\,x^9+2a^2d^2x^6+3abcd\,x^6+b^2c^2x^6+2a^2cd\,x^3+ab\,c^2x^3}dx\right)a^4d^2x^2$$

input
```
int((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x)
```

output
```
( - 2*(a + b*x**3)**(1/3)*a*b + 4*int((a + b*x**3)**(1/3)/(2*a**2*c*d*x**3
 + 2*a**2*d**2*x**6 + a*b*c**2*x**3 + 3*a*b*c*d*x**6 + 2*a*b*d**2*x**9 + b
**2*c**2*x**6 + b**2*c*d*x**9),x)*a**4*d**2*x**2 - 4*int((a + b*x**3)**(1/
3)/(2*a**2*c*d*x**3 + 2*a**2*d**2*x**6 + a*b*c**2*x**3 + 3*a*b*c*d*x**6 +
2*a*b*d**2*x**9 + b**2*c**2*x**6 + b**2*c*d*x**9),x)*a**3*b*c*d*x**2 - 3*i
nt((a + b*x**3)**(1/3)/(2*a**2*c*d*x**3 + 2*a**2*d**2*x**6 + a*b*c**2*x**3
 + 3*a*b*c*d*x**6 + 2*a*b*d**2*x**9 + b**2*c**2*x**6 + b**2*c*d*x**9),x)*a
**2*b**2*c**2*x**2 + 2*int(((a + b*x**3)**(1/3)*x**3)/(2*a**2*c*d + 2*a**2
*d**2*x**3 + a*b*c**2 + 3*a*b*c*d*x**3 + 2*a*b*d**2*x**6 + b**2*c**2*x**3
+ b**2*c*d*x**6),x)*a*b**3*c*d*x**2 + int(((a + b*x**3)**(1/3)*x**3)/(2*a*
*2*c*d + 2*a**2*d**2*x**3 + a*b*c**2 + 3*a*b*c*d*x**3 + 2*a*b*d**2*x**6 +
b**2*c**2*x**3 + b**2*c*d*x**6),x)*b**4*c**2*x**2)/(x**2*(2*a*d + b*c))
```

3.728 $\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)}\,dx=-\frac{a\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{5}{3},-\frac{4}{3},1,-\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-1/5*a*(b*x^3+a)^(1/3)*AppellF1(-5/3,-4/3,1,-2/3,-b*x^3/a,-d*x^3/c)/c/x^5/
(1+b*x^3/a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 286 vs. 2(65) = 130.

Time = 10.34 (sec) , antiderivative size = 286, normalized size of antiderivative = 4.40

$$\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)}\,dx=\frac{-\frac{4(a+bx^3)(2ac+6bcx^3-5adx^3)}{c^2x^5}+\frac{bd(-6bc+5ad)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}-\frac{\cdots}{c(c+dx^3)(\cdots}}{40(\cdots}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x^6*(c + d*x^3)),x]
```

output
```
((-4*(a + b*x^3)*(2*a*c + 6*b*c*x^3 - 5*a*d*x^3))/(c^2*x^5) + (b*d*(-6*b*c
 + 5*a*d)*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a
), -((d*x^3)/c)])/c^3 - (16*a*(4*b^2*c^2 - 15*a*b*c*d + 10*a^2*d^2)*x*Appe
llF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*(c + d*x^3)*(-4*a*c
*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*Appel
lF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/
3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(40*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^6\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{a\sqrt[3]{a+bx^3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{4/3}}{x^6\left(dx^3+c\right)}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{a\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{5}{3},-\frac{4}{3},1,-\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(4/3)/(x^6*(c + d*x^3)),x]
```

output
```
-1/5*(a*(a + b*x^3)^(1/3)*AppellF1[-5/3, -4/3, 1, -2/3, -((b*x^3)/a), -((d
*x^3)/c)])/(c*x^5*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{4}{3}}}{x^{6}\,(d\,x^{3}+c)}dx$$

input
```
int((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^6\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x^6(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/x**6/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(x**6*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^6), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{x^6\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(x^6*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(4/3)/(x^6*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)}\,dx = \frac{-(b\,x^3+a)^{\frac{1}{3}}\,a-25\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{5ab\,d^2x^9+4b^2cd\,x^9+5a^2d^2x^6+9abcd\,x^6+4b^2c^2x^6+5a^2cd\,x^3+4ab\,c^2x^3}dx\right)a^3d^2x$$

input
```
int((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x)
```

output
```
( - (a + b*x**3)**(1/3)*a - 25*int((a + b*x**3)**(1/3)/(5*a**2*c*d*x**3 +
5*a**2*d**2*x**6 + 4*a*b*c**2*x**3 + 9*a*b*c*d*x**6 + 5*a*b*d**2*x**9 + 4*
b**2*c**2*x**6 + 4*b**2*c*d*x**9),x)*a**3*d**2*x**5 + 10*int((a + b*x**3)*
*(1/3)/(5*a**2*c*d*x**3 + 5*a**2*d**2*x**6 + 4*a*b*c**2*x**3 + 9*a*b*c*d*x
**6 + 5*a*b*d**2*x**9 + 4*b**2*c**2*x**6 + 4*b**2*c*d*x**9),x)*a**2*b*c*d*
x**5 + 24*int((a + b*x**3)**(1/3)/(5*a**2*c*d*x**3 + 5*a**2*d**2*x**6 + 4*
a*b*c**2*x**3 + 9*a*b*c*d*x**6 + 5*a*b*d**2*x**9 + 4*b**2*c**2*x**6 + 4*b*
*2*c*d*x**9),x)*a*b**2*c**2*x**5 - 20*int((a + b*x**3)**(1/3)/(5*a**2*c*d
+ 5*a**2*d**2*x**3 + 4*a*b*c**2 + 9*a*b*c*d*x**3 + 5*a*b*d**2*x**6 + 4*b**
2*c**2*x**3 + 4*b**2*c*d*x**6),x)*a**2*b*d**2*x**5 + 9*int((a + b*x**3)**(
1/3)/(5*a**2*c*d + 5*a**2*d**2*x**3 + 4*a*b*c**2 + 9*a*b*c*d*x**3 + 5*a*b*
d**2*x**6 + 4*b**2*c**2*x**3 + 4*b**2*c*d*x**6),x)*a*b**2*c*d*x**5 + 20*in
t((a + b*x**3)**(1/3)/(5*a**2*c*d + 5*a**2*d**2*x**3 + 4*a*b*c**2 + 9*a*b*
c*d*x**3 + 5*a*b*d**2*x**6 + 4*b**2*c**2*x**3 + 4*b**2*c*d*x**6),x)*b**3*c
**2*x**5)/(5*c*x**5)
```

3.729 $\int \frac{x^{14}}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 290

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{(bc+ad)\,(b^2c^2+a^2d^2)\,(a+bx^3)^{2/3}}{2b^4d^4} + \frac{(b^2c^2+2abcd+3a^2d^2)\,(a+bx^3)^{5/3}}{5b^4d^3} - \frac{(bc+3ad)\,(a+bx^3)^{8/3}}{8b^4d^2} + \frac{(a+bx^3)^{11/3}}{11b^4d} - \frac{c^4\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{14/3}\sqrt[3]{bc-ad}} + \frac{c^4\log(c+dx^3)}{6d^{14/3}\sqrt[3]{bc-ad}} - \frac{c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{14/3}\sqrt[3]{bc-ad}}$$

output
```
-1/2*(a*d+b*c)*(a^2*d^2+b^2*c^2)*(b*x^3+a)^(2/3)/b^4/d^4+1/5*(3*a^2*d^2+2*
a*b*c*d+b^2*c^2)*(b*x^3+a)^(5/3)/b^4/d^3-1/8*(3*a*d+b*c)*(b*x^3+a)^(8/3)/b
^4/d^2+1/11*(b*x^3+a)^(11/3)/b^4/d-1/3*c^4*arctan(1/3*(1-2*d^(1/3)*(b*x^3+
a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/d^(14/3)/(-a*d+b*c)^(1/3)+1/6*
c^4*ln(d*x^3+c)/d^(14/3)/(-a*d+b*c)^(1/3)-1/2*c^4*ln((-a*d+b*c)^(1/3)+d^(1
/3)*(b*x^3+a)^(1/3))/d^(14/3)/(-a*d+b*c)^(1/3)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.06

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{-3d^{2/3}\sqrt[3]{bc-ad}(a+bx^3)^{2/3}\,(81a^3d^3+9a^2bd^2(11c-6dx^3)+3ab^2d(44c^2-22cdx^3+15d^2x^6)+b^3(220c}{}$$

input
```
Integrate[x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-3*d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3)*(81*a^3*d^3 + 9*a^2*b*d^2*
(11*c - 6*d*x^3) + 3*a*b^2*d*(44*c^2 - 22*c*d*x^3 + 15*d^2*x^6) + b^3*(220
*c^3 - 88*c^2*d*x^3 + 55*c*d^2*x^6 - 40*d^3*x^9)) - 440*Sqrt[3]*b^4*c^4*Ar
cTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 440*
b^4*c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + 220*b^4*c^4*L
og[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/
3)*(a + b*x^3)^(2/3)])/(1320*b^4*d^(14/3)*(b*c - a*d)^(1/3))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{x^{12}}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx^3$$

$$\Big\downarrow \; 99$$

$$\frac{1}{3}\int \left(\frac{c^4}{d^4\sqrt[3]{bx^3+a}\,(dx^3+c)}+\frac{(bx^3+a)^{8/3}}{b^3d}+\frac{(-bc-3ad)\,(bx^3+a)^{5/3}}{b^3d^2}+\frac{(b^2c^2+2abdc+3a^2d^2)\,(bx^3+a)^{2/3}}{b^3d^3}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(-\frac{3(a+bx^3)^{2/3}\,(ad+bc)\,(a^2d^2+b^2c^2)}{2b^4d^4}+\frac{3(a+bx^3)^{5/3}\,(3a^2d^2+2abcd+b^2c^2)}{5b^4d^3}-\frac{\sqrt{3}c^4\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{c}}{\sqrt[3]{bc}}}{\sqrt{3}}\right)}{d^{14/3}\sqrt[3]{bc-ad}}\right.$$

input
```
Int[x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
((-3*(b*c + a*d)*(b^2*c^2 + a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^4*d^4) + (3*(
b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*(a + b*x^3)^(5/3))/(5*b^4*d^3) - (3*(b*c
+ 3*a*d)*(a + b*x^3)^(8/3))/(8*b^4*d^2) + (3*(a + b*x^3)^(11/3))/(11*b^4*d
) - (Sqrt[3]*c^4*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/
3))/Sqrt[3]])/(d^(14/3)*(b*c - a*d)^(1/3)) + (c^4*Log[c + d*x^3])/(2*d^(14
/3)*(b*c - a*d)^(1/3)) - (3*c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3
)^(1/3)])/(2*d^(14/3)*(b*c - a*d)^(1/3)))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{\frac{243\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}} d\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(\frac{\left(-40d^{3}x^{9}+55c\,d^{2}x^{6}-88c^{2}d\,x^{3}+220c^{3}\right)b^{3}}{81}+\frac{44ad\left(\frac{15}{44}d^{2}x^{6}-\frac{1}{2}cd\,x^{3}+c^{2}\right)b^{2}}{27}+\frac{11a^{2}d^{2}\left(-\frac{6d\,x^{3}}{11}+c\right)b}{9}+c}{220}}{}$

input
```
int(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/d)^(1/3)*(-243/220*((a*d-b*c)/d)^(1/3)*d*(b*x^3+a)^(2/3)*(1
/81*(-40*d^3*x^9+55*c*d^2*x^6-88*c^2*d*x^3+220*c^3)*b^3+44/27*a*d*(15/44*d
^2*x^6-1/2*c*d*x^3+c^2)*b^2+11/9*a^2*d^2*(-6/11*d*x^3+c)*b+a^3*d^3)+b^4*c^
4*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*c)
/d)^(1/3))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))-ln((b*x^3+a)^
(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))))/d^5/b^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 1004, normalized size of antiderivative = 3.46

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/1320*(220*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(2/3)*d^2 +
(-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) -
 440*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2
+ a*d^3)^(1/3)) + 660*sqrt(1/3)*(b^5*c^5*d - a*b^4*c^4*d^2)*sqrt((-b*c*d^2
 + a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1
/3)*(2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c
*d - a*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3
)^(1/3)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^
3 + c)) - 3*(220*b^4*c^4*d^2 - 88*a*b^3*c^3*d^3 - 33*a^2*b^2*c^2*d^4 - 18*
a^3*b*c*d^5 - 81*a^4*d^6 - 40*(b^4*c*d^5 - a*b^3*d^6)*x^9 + 5*(11*b^4*c^2*
d^4 - 2*a*b^3*c*d^5 - 9*a^2*b^2*d^6)*x^6 - 2*(44*b^4*c^3*d^3 - 11*a*b^3*c^
2*d^4 - 6*a^2*b^2*c*d^5 - 27*a^3*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(b^5*c*d^6
 - a*b^4*d^7), 1/1320*(220*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a
)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a
*d^3)^(2/3)) - 440*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(1/3)*
d - (-b*c*d^2 + a*d^3)^(1/3)) + 1320*sqrt(1/3)*(b^5*c^5*d - a*b^4*c^4*d^2)
*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 +
a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c
 - a*d))/d) - 3*(220*b^4*c^4*d^2 - 88*a*b^3*c^3*d^3 - 33*a^2*b^2*c^2*d^4 -
 18*a^3*b*c*d^5 - 81*a^4*d^6 - 40*(b^4*c*d^5 - a*b^3*d^6)*x^9 + 5*(11*b...
```

Sympy [F]

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x**14/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**14/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.51

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{\frac{440\,b^4c^4d^7\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{bcd^{11}-ad^{12}}+\frac{1320\,(-bcd^2+ad^3)^{\frac{2}{3}}b^4c^4\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^6-\sqrt{3}ad^7}-\frac{220\,(\cdots}{\cdots}}{\cdots}$$

input
```
integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/1320*(440*b^4*c^4*d^7*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3)
- (-(b*c - a*d)/d)^(1/3)))/(b*c*d^11 - a*d^12) + 1320*(-b*c*d^2 + a*d^3)^(
2/3)*b^4*c^4*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1
/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^6 - sqrt(3)*a*d^7) - 220*(-b*c
*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b
*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^6 - a*d^7) + 3*(220*(b
*x^3 + a)^(2/3)*b^3*c^3*d^7 - 88*(b*x^3 + a)^(5/3)*b^2*c^2*d^8 + 220*(b*x^
3 + a)^(2/3)*a*b^2*c^2*d^8 + 55*(b*x^3 + a)^(8/3)*b*c*d^9 - 176*(b*x^3 + a
)^(5/3)*a*b*c*d^9 + 220*(b*x^3 + a)^(2/3)*a^2*b*c*d^9 - 40*(b*x^3 + a)^(11
/3)*d^10 + 165*(b*x^3 + a)^(8/3)*a*d^10 - 264*(b*x^3 + a)^(5/3)*a^2*d^10 +
 220*(b*x^3 + a)^(2/3)*a^3*d^10)/d^11)/b^4
```

Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.51

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \left(\frac{6\,a^2}{5\,b^4\,d}+\frac{\left(\frac{4\,a}{b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{b^8\,d^2}\right)\,\left(b^5\,c-a\,b^4\,d\right)}{5\,b^4\,d}\right)\,{\left(b\,x^3+a\right)}^{5/3}$$
$$-\left(\frac{a}{2\,b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{8\,b^8\,d^2}\right)\,{\left(b\,x^3+a\right)}^{8/3}$$
$$-{\left(b\,x^3+a\right)}^{2/3}\,\left(\frac{2\,a^3}{b^4\,d}+\frac{\left(\frac{6\,a^2}{b^4\,d}+\frac{\left(\frac{4\,a}{b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{b^8\,d^2}\right)\,\left(b^5\,c-a\,b^4\,d\right)}{b^4\,d}\right)\,\left(b^5\,c-a\,b^4\,d\right)}{2\,b^4\,d}\right)+\frac{{\left(b\,x^3+a\right)}^{11/3}}{11\,b^4\,d}+\frac{c^4\,\ln\left(\frac{c^8\,(b}{}\right.}{3\,d^1}$$

input
```
int(x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
((6*a^2)/(5*b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c
 - a*b^4*d))/(5*b^4*d))*(a + b*x^3)^(5/3) - (a/(2*b^4*d) + (b^5*c - a*b^4*
d)/(8*b^8*d^2))*(a + b*x^3)^(8/3) - (a + b*x^3)^(2/3)*((2*a^3)/(b^4*d) + (
((6*a^2)/(b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c -
 a*b^4*d))/(b^4*d))*(b^5*c - a*b^4*d))/(2*b^4*d)) + (a + b*x^3)^(11/3)/(11
*b^4*d) + (c^4*log((c^8*(a + b*x^3)^(1/3))/d^7 - (c^8*(a*d - b*c)^(1/3))/d
^(22/3)))/(3*d^(14/3)*(a*d - b*c)^(1/3)) - (log((c^8*(a + b*x^3)^(1/3))/d^
7 - (c^8*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d^(22/3)))*(3^(1/2)*c^4*
1i + c^4))/(6*d^(14/3)*(a*d - b*c)^(1/3)) + (c^4*log((c^8*(a + b*x^3)^(1/3
))/d^7 - (c^8*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(1/3))/(4*d^(22/3)))*((3^(1/2
)*1i)/6 - 1/6))/(d^(14/3)*(a*d - b*c)^(1/3))
```

Reduce [F]

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x^{14}}{(b\,x^3+a)^{\frac{1}{3}}\,c+(b\,x^3+a)^{\frac{1}{3}}\,d\,x^3}dx$$

input
```
int(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x**14/((a + b*x**3)**(1/3)*c + (a + b*x**3)**(1/3)*d*x**3),x)
```

3.730 $\int \frac{x^{11}}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 244

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\,(a+bx^3)^{5/3}}{5b^3d^2} + \frac{(a+bx^3)^{8/3}}{8b^3d} + \frac{c^3\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}\sqrt[3]{bc-ad}} - \frac{c^3\log(c+dx^3)}{6d^{11/3}\sqrt[3]{bc-ad}} + \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}\sqrt[3]{bc-ad}}$$

output

```
1/2*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(2/3)/b^3/d^3-1/5*(2*a*d+b*c)*(b*x
^3+a)^(5/3)/b^3/d^2+1/8*(b*x^3+a)^(8/3)/b^3/d+1/3*c^3*arctan(1/3*(1-2*d^(1
/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/d^(11/3)/(-a*d+b*c)
^(1/3)-1/6*c^3*ln(d*x^3+c)/d^(11/3)/(-a*d+b*c)^(1/3)+1/2*c^3*ln((-a*d+b*c)
^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(11/3)/(-a*d+b*c)^(1/3)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.08

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{3d^{2/3}\sqrt[3]{bc-ad}(a+bx^3)^{2/3}\,(9a^2d^2-6abd(-2c+dx^3)+b^2(20c^2-8cdx^3+5d^2x^6))+40\sqrt{3}b^3c^3\arctan\Big(}{}$$

input
```
Integrate[x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(3*d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3)*(9*a^2*d^2 - 6*a*b*d*(-2*c
+ d*x^3) + b^2*(20*c^2 - 8*c*d*x^3 + 5*d^2*x^6)) + 40*Sqrt[3]*b^3*c^3*ArcT
an[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 40*b^3
*c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - 20*b^3*c^3*Log[(
b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(
a + b*x^3)^(2/3)])/(120*b^3*d^(11/3)*(b*c - a*d)^(1/3))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^9}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(-\frac{c^3}{d^3\sqrt[3]{bx^3+a}\,(dx^3+c)}+\frac{\left(bx^3+a\right)^{5/3}}{b^2d}+\frac{(-bc-2ad)\left(bx^3+a\right)^{2/3}}{b^2d^2}+\frac{b^2c^2+abdc+a^2d^2}{b^2d^3\sqrt[3]{bx^3+a}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}\left(a^2d^2+abcd+b^2c^2\right)}{2b^3d^3}+\frac{\sqrt{3}c^3\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{11/3}\sqrt[3]{bc-ad}}-\frac{3\left(a+bx^3\right)^{5/3}(2ad+bc)}{5b^3d^2}+\frac{3(a\right.$$

input
```
Int[x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
((3*(b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^3*d^3) - (3*(b*c
 + 2*a*d)*(a + b*x^3)^(5/3))/(5*b^3*d^2) + (3*(a + b*x^3)^(8/3))/(8*b^3*d)
 + (Sqrt[3]*c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3
))/Sqrt[3]])/(d^(11/3)*(b*c - a*d)^(1/3)) - (c^3*Log[c + d*x^3])/(2*d^(11/
3)*(b*c - a*d)^(1/3)) + (3*c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)
^(1/3)])/(2*d^(11/3)*(b*c - a*d)^(1/3)))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{\frac{27\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}} d\left(\frac{\left(5d^2x^6-8cd\,x^3+20c^2\right)b^2}{9}+\frac{4a\left(-\frac{d\,x^3}{2}+c\right)db}{3}+a^2d^2\right)\left(b\,x^3+a\right)^{\frac{2}{3}}}{20}+b^3c^3\left(-2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^3+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\right)}{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}b^3d^4}$

input
```
int(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/d)^(1/3)*(27/20*((a*d-b*c)/d)^(1/3)*d*(1/9*(5*d^2*x^6-8*c*d
*x^3+20*c^2)*b^2+4/3*a*(-1/2*d*x^3+c)*d*b+a^2*d^2)*(b*x^3+a)^(2/3)+b^3*c^3
*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*c)
/d)^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+
((a*d-b*c)/d)^(2/3))-2*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))))/b^3/d^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 873, normalized size of antiderivative = 3.58

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/120*(20*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 + a)^(2/3)*d^2 - (b
*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 40*
(b*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3
)^(1/3)) - 60*sqrt(1/3)*(b^4*c^4*d - a*b^3*c^3*d^2)*sqrt(-(b*c*d^2 - a*d^3
)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(
b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^
2) - (b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b
*c - a*d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3
*(20*b^3*c^3*d^2 - 8*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4 - 9*a^3*d^5 + 5*(b^3*c*
d^4 - a*b^2*d^5)*x^6 - 2*(4*b^3*c^2*d^3 - a*b^2*c*d^4 - 3*a^2*b*d^5)*x^3)*
(b*x^3 + a)^(2/3))/(b^4*c*d^5 - a*b^3*d^6), -1/120*(20*(b*c*d^2 - a*d^3)^(
2/3)*b^3*c^3*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 +
a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 40*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^3
*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 120*sqrt(1/3)*(b^4*c
^4*d - a*b^3*c^3*d^2)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqr
t(1/3)*(2*(b*x^3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a
*d^3)^(1/3)/(b*c - a*d))/d) - 3*(20*b^3*c^3*d^2 - 8*a*b^2*c^2*d^3 - 3*a^2*
b*c*d^4 - 9*a^3*d^5 + 5*(b^3*c*d^4 - a*b^2*d^5)*x^6 - 2*(4*b^3*c^2*d^3 - a
*b^2*c*d^4 - 3*a^2*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(b^4*c*d^5 - a*b^3*d^6)]
```

Sympy [F]

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x**11/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**11/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.52

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{b^{27}c^3d^5\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{28}cd^8-ab^{27}d^9)} + \frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^5-\sqrt{3}ad^6} - \frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^5-ad^6)} + \frac{20\,(bx^3+a)^{\frac{2}{3}}b^{23}c^2d^5-8\,(bx^3+a)^{\frac{5}{3}}b^{22}cd^6+20\,(bx^3+a)^{\frac{2}{3}}ab^{22}cd^6+5\,(bx^3+a)^{\frac{8}{3}}b^{21}d^7-16\,(bx^3+a)^{\frac{5}{3}}}{40\,b^{24}d^8}$$

input
```
integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*b^27*c^3*d^5*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^28*c*d^8 - a*b^27*d^9) + (-b*c*d^2 + a*d^3)^(2/3)*c^
3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c
 - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^5 - sqrt(3)*a*d^6) - 1/6*(-b*c*d^2 + a*d^
3)^(2/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1
/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^5 - a*d^6) + 1/40*(20*(b*x^3 + a)^(2/
3)*b^23*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^22*c*d^6 + 20*(b*x^3 + a)^(2/3)*a*
b^22*c*d^6 + 5*(b*x^3 + a)^(8/3)*b^21*d^7 - 16*(b*x^3 + a)^(5/3)*a*b^21*d^
7 + 20*(b*x^3 + a)^(2/3)*a^2*b^21*d^7)/(b^24*d^8)
```

Mupad [B] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.39

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \left(\frac{3\,a^2}{2\,b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{2\,b^3\,d}\right)\,{\left(b\,x^3+a\right)}^{2/3} - \left(\frac{3\,a}{5\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{5\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{5/3} + \frac{{\left(b\,x^3+a\right)}^{8/3}}{8\,b^3\,d} - \frac{c^3\,\ln\left(\frac{c^6\,{\left(b\,x^3+a\right)}^{1/3}}{d^5}+\frac{b\,c^7-a\,c^6\,d}{d^{16/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)}{3\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{1/3}} + \frac{\ln\left(\frac{c^6\,{\left(b\,x^3+a\right)}^{1/3}}{d^5}-\frac{c^6\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(a\,d-b\,c\right)}^{1/3}}{4\,d^{16/3}}\right)\,\left(c^3+\sqrt{3}\,c^3\,1{}\mathrm{i}\right)}{6\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{1/3}} - \frac{c^3\,\ln\left(\frac{c^6\,{\left(b\,x^3+a\right)}^{1/3}}{d^5}+\frac{c^6\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{16/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{1/3}}$$

input
```
int(x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
((3*a^2)/(2*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
 - a*b^3*d))/(2*b^3*d))*(a + b*x^3)^(2/3) - ((3*a)/(5*b^3*d) + (b^4*c - a*
b^3*d)/(5*b^6*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^3*d) - (c^3
*log((c^6*(a + b*x^3)^(1/3))/d^5 + (b*c^7 - a*c^6*d)/(d^(16/3)*(a*d - b*c)
^(2/3))))/(3*d^(11/3)*(a*d - b*c)^(1/3)) + (log((c^6*(a + b*x^3)^(1/3))/d^
5 - (c^6*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d^(16/3)))*(3^(1/2)*c^3*
1i + c^3))/(6*d^(11/3)*(a*d - b*c)^(1/3)) - (c^3*log((c^6*(a + b*x^3)^(1/3
))/d^5 + (c^6*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3))/d^(16/3))*((3^(1/2
)*1i)/2 - 1/2))/(3*d^(11/3)*(a*d - b*c)^(1/3))
```

Reduce [F]

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^{11}}{(b\,x^3+a)^{\frac{1}{3}}\,c+(b\,x^3+a)^{\frac{1}{3}}\,d\,x^3}dx$$

input
```
int(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x**11/((a + b*x**3)**(1/3)*c + (a + b*x**3)**(1/3)*d*x**3),x)
```

3.731 $\int \frac{x^8}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 203

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2}+\frac{\left(a+bx^3\right)^{5/3}}{5b^2d} - \frac{c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}\sqrt[3]{bc-ad}}+\frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}} - \frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}\sqrt[3]{bc-ad}}$$

output
```
-1/2*(a*d+b*c)*(b*x^3+a)^(2/3)/b^2/d^2+1/5*(b*x^3+a)^(5/3)/b^2/d-1/3*c^2*a
rctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/
d^(8/3)/(-a*d+b*c)^(1/3)+1/6*c^2*ln(d*x^3+c)/d^(8/3)/(-a*d+b*c)^(1/3)-1/2*
c^2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(8/3)/(-a*d+b*c)^(1/3)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.14

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{-3d^{2/3}\sqrt[3]{bc-ad}(a+bx^3)^{2/3}\,(5bc+3ad-2bdx^3) - 10\sqrt{3}b^2c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) - 10b^2c^2\log}{30b^2d^{8/3}\sqrt[3]{b}}$$

input
```
Integrate[x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-3*d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3)*(5*b*c + 3*a*d - 2*b*d*x^3
) - 10*Sqrt[3]*b^2*c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*
d)^(1/3))/Sqrt[3]] - 10*b^2*c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3
)^(1/3)] + 5*b^2*c^2*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a
+ b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(30*b^2*d^(8/3)*(b*c - a*d)^(
1/3))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{c^2}{d^2\sqrt[3]{bx^3+a}\,(dx^3+c)}+\frac{(bx^3+a)^{2/3}}{bd}+\frac{-bc-ad}{bd^2\sqrt[3]{bx^3+a}}\right)dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{8/3}\sqrt[3]{bc-ad}}-\frac{3(a+bx^3)^{2/3}(ad+bc)}{2b^2d^2}+\frac{3(a+bx^3)^{5/3}}{5b^2d}+\frac{c^2\log(c+dx^3)}{2d^{8/3}\sqrt[3]{bc-ad}}-\frac{3c^2\log}{}\right.$$

input
```
Int[x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
((-3*(b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^2*d^2) + (3*(a + b*x^3)^(5/3))/(5
*b^2*d) - (Sqrt[3]*c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*
d)^(1/3))/Sqrt[3]])/(d^(8/3)*(b*c - a*d)^(1/3)) + (c^2*Log[c + d*x^3])/(2*
d^(8/3)*(b*c - a*d)^(1/3)) - (3*c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b
*x^3)^(1/3)])/(2*d^(8/3)*(b*c - a*d)^(1/3)))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-\frac{3\left(\frac{5\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)b^{2}c^{2}}{9}-\frac{10c^{2}\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\,\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)b^{2}}{9}-\frac{10\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}} \cdots\right.}{\cdots}\right.}{10\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}d^{3}b^{2}}$

input
```
int(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-3/10*(5/9*ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*
c)/d)^(2/3))*b^2*c^2-10/9*c^2*3^(1/2)*arctan(2/3*3^(1/2)/((a*d-b*c)/d)^(1/
3)*(b*x^3+a)^(1/3)+1/3*3^(1/2))*b^2-10/9*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^
(1/3))*b^2*c^2+((a*d-b*c)/d)^(1/3)*d*(1/3*(-2*d*x^3+5*c)*b+a*d)*(b*x^3+a)^
(2/3))/((a*d-b*c)/d)^(1/3)/d^3/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(164) = 328.

Time = 0.11 (sec) , antiderivative size = 768, normalized size of antiderivative = 3.78

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/30*(5*(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(2/3)*d^2 + (-b*
c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 10*
(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d
^3)^(1/3)) + 15*sqrt(1/3)*(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt((-b*c*d^2 + a*d
^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2
*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a
*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3
)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)
) - 3*(5*b^2*c^2*d^2 - 2*a*b*c*d^3 - 3*a^2*d^4 - 2*(b^2*c*d^3 - a*b*d^4)*x
^3)*(b*x^3 + a)^(2/3))/(b^3*c*d^4 - a*b^2*d^5), 1/30*(5*(-b*c*d^2 + a*d^3)
^(2/3)*b^2*c^2*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3
 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 10*(-b*c*d^2 + a*d^3)^(2/3)*b^
2*c^2*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) + 30*sqrt(1/3)*(
b^3*c^3*d - a*b^2*c^2*d^2)*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arc
tan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b
*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))/d) - 3*(5*b^2*c^2*d^2 - 2*a*b*c*d^3 - 3
*a^2*d^4 - 2*(b^2*c*d^3 - a*b*d^4)*x^3)*(b*x^3 + a)^(2/3))/(b^3*c*d^4 - a*
b^2*d^5)]
```

Sympy [F]

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x**8/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**8/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.52

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{\frac{10\,b^2c^2d^3\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{bcd^5-ad^6}+\frac{30\left(-bcd^2+ad^3\right)^{\frac{2}{3}}b^2c^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^4-\sqrt{3}ad^5}-\frac{5\,(-bcd^2}{}}{30\,b^2}$$

input
```
integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/30*(10*b^2*c^2*d^3*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (
-(b*c - a*d)/d)^(1/3)))/(b*c*d^5 - a*d^6) + 30*(-b*c*d^2 + a*d^3)^(2/3)*b^
2*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-
(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^4 - sqrt(3)*a*d^5) - 5*(-b*c*d^2 + a*
d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)
/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^4 - a*d^5) + 3*(5*(b*x^3 + a)^(
2/3)*b*c*d^3 - 2*(b*x^3 + a)^(5/3)*d^4 + 5*(b*x^3 + a)^(2/3)*a*d^4)/d^5)/b
^2
```

Mupad [B] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.32

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$= \frac{(b\,x^3+a)^{5/3}}{5\,b^2\,d} - \left(\frac{a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{2\,b^4\,d^2}\right)(b\,x^3+a)^{2/3} + \frac{c^2\,\ln\left(\frac{c^4\,(b\,x^3+a)^{1/3}}{d^3}+\frac{b\,c^5-a\,c^4\,d}{d^{10/3}\,(a\,d-b\,c)^{2/3}}\right)}{3\,d^{8/3}\,(a\,d-b\,c)^{1/3}} - \frac{\ln\left(\frac{c^4\,(b\,x^3+a)^{1/3}}{d^3}-\frac{c^4\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\,(a\,d-b\,c)^{1/3}}{4\,d^{10/3}}\right)\left(c^2+\sqrt{3}\,c^2\,1\mathrm{i}\right)}{6\,d^{8/3}\,(a\,d-b\,c)^{1/3}} + \frac{c^2\,\ln\left(\frac{c^4\,(b\,x^3+a)^{1/3}}{d^3}-\frac{c^4\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,(a\,d-b\,c)^{1/3}}{4\,d^{10/3}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{d^{8/3}\,(a\,d-b\,c)^{1/3}}$$

input
```
int(x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
(a + b*x^3)^(5/3)/(5*b^2*d) - (a/(b^2*d) + (b^3*c - a*b^2*d)/(2*b^4*d^2))*
(a + b*x^3)^(2/3) + (c^2*log((c^4*(a + b*x^3)^(1/3))/d^3 + (b*c^5 - a*c^4*
d)/(d^(10/3)*(a*d - b*c)^(2/3))))/(3*d^(8/3)*(a*d - b*c)^(1/3)) - (log((c^
4*(a + b*x^3)^(1/3))/d^3 - (c^4*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d
^(10/3)))*(3^(1/2)*c^2*1i + c^2))/(6*d^(8/3)*(a*d - b*c)^(1/3)) + (c^2*log
((c^4*(a + b*x^3)^(1/3))/d^3 - (c^4*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(1/3))/
(4*d^(10/3)))*((3^(1/2)*1i)/6 - 1/6))/(d^(8/3)*(a*d - b*c)^(1/3))
```

Reduce [F]

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^8}{(b\,x^3+a)^{\frac{1}{3}}\,c+(b\,x^3+a)^{\frac{1}{3}}\,d\,x^3}dx$$

input
```
int(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x**8/((a + b*x**3)**(1/3)*c + (a + b*x**3)**(1/3)*d*x**3),x)
```

3.732 $\int \frac{x^5}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{(a+bx^3)^{2/3}}{2bd} + \frac{c\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}\sqrt[3]{bc-ad}} - \frac{c\log(c+dx^3)}{6d^{5/3}\sqrt[3]{bc-ad}} + \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}}$$

output
```
1/2*(b*x^3+a)^(2/3)/b/d+1/3*c*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*
d+b*c)^(1/3))*3^(1/2))*3^(1/2)/d^(5/3)/(-a*d+b*c)^(1/3)-1/6*c*ln(d*x^3+c)/
d^(5/3)/(-a*d+b*c)^(1/3)+1/2*c*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3)
)/d^(5/3)/(-a*d+b*c)^(1/3)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.21

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{3d^{2/3}\sqrt[3]{bc-ad}(a+bx^3)^{2/3}+2\sqrt{3}bc\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)+2bc\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{6bd^{5/3}\sqrt[3]{bc-ad}}$$

input
```
Integrate[x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(3*d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3) + 2*Sqrt[3]*b*c*ArcTan[(1 -
 (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 2*b*c*Log[(b*
c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - b*c*Log[(b*c - a*d)^(2/3) -
d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/
(6*b*d^(5/3)*(b*c - a*d)^(1/3))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 90, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}}{2bd}-\frac{c\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{d}\right)$$

$\downarrow$ 68

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}}{2bd}-\frac{c\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+ \ldots}{2d} \ldots\right)}{d}\right.$$

$\downarrow$ 16

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}}{2bd}-\frac{c\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d} \ldots\right.}{2d^{2/3}\sqrt[3]{bc- \ldots}} \ldots\right)}{d}\right.$$

$\downarrow$ 1082

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}}{2bd}-\frac{c\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{d}\right)$$

$\downarrow$ 217

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}}{2bd}-\frac{c\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{d}\right)$$

input
```
Int[x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
((3*(a + b*x^3)^(2/3))/(2*b*d) - (c*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a
+ b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(2/3)*(b*c - a*d)^(1/3)))
+ Log[c + d*x^3]/(2*d^(2/3)*(b*c - a*d)^(1/3)) - (3*Log[(b*c - a*d)^(1/3)
+ d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c - a*d)^(1/3))))/d)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$\frac{3\left(b\,x^{3}+a\right)^{\frac{2}{3}} d \left(\frac{a d-b c}{d}\right)^{\frac{1}{3}}-2 \arctan\left(\frac{\sqrt{3}\,\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}\, b c-2 \ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}}\right) b c+\ln\left(\left(b\,x^{3}+\right.\right.}{6 b\, d^{2}\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}}}$

input
```
int(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(3*(b*x^3+a)^(2/3)*d*((a*d-b*c)/d)^(1/3)-2*arctan(1/3*3^(1/2)*(2*(b*x^
3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*c)/d)^(1/3))*3^(1/2)*b*c-2*ln((b*x
^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))*b*c+ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3
)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))*b*c)/b/d^2/((a*d-b*c)/d)^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(134) = 268$.

Time = 0.11 (sec) , antiderivative size = 667, normalized size of antiderivative = 3.97

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/6*((b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 -
a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 2*(b*c*d^2 -
 a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*s
qrt(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)
)*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2
/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d
^3)^(1/3)*(b*c - a*d))*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c
*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(b*c*d^2 - a*d^3)*
(b*x^3 + a)^(2/3))/(b^2*c*d^3 - a*b*d^4), -1/6*((b*c*d^2 - a*d^3)^(2/3)*b*
c*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d
+ (b*c*d^2 - a*d^3)^(2/3)) - 2*(b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*x^3 + a)
^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 6*sqrt(1/3)*(b^2*c^2*d - a*b*c*d^2)*
sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^
(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d
))/d) - 3*(b*c*d^2 - a*d^3)*(b*x^3 + a)^(2/3))/(b^2*c*d^3 - a*b*d^4)]
```

Sympy [F]

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x**5/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**5/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.53

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$=\frac{\frac{2\,bcd\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{bcd^2-ad^3}+\frac{6\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^3-\sqrt{3}ad^4}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\,\mathrm{lo}}{}}{6\,b}$$

input
```
integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/6*(2*b*c*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a
*d)/d)^(1/3)))/(b*c*d^2 - a*d^3) + 6*(-b*c*d^2 + a*d^3)^(2/3)*b*c*arctan(1
/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)
^(1/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) - (-b*c*d^2 + a*d^3)^(2/3)*b*c*l
og((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c -
 a*d)/d)^(2/3))/(b*c*d^3 - a*d^4) + 3*(b*x^3 + a)^(2/3)/d)/b
```

Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.30

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\left(b\,x^3+a\right)^{2/3}}{2\,b\,d} + \frac{\ln\left(\frac{c^2\,\left(b\,x^3+a\right)^{1/3}}{d}-\frac{c^2\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,\left(a\,d-b\,c\right)^{1/3}}{4\,d^{4/3}}\right)\,\left(c-\sqrt{3}\,c\,1\mathrm{i}\right)}{6\,d^{5/3}\,\left(a\,d-b\,c\right)^{1/3}} + \frac{\ln\left(\frac{c^2\,\left(b\,x^3+a\right)^{1/3}}{d}-\frac{c^2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\,\left(a\,d-b\,c\right)^{1/3}}{4\,d^{4/3}}\right)\,\left(c+\sqrt{3}\,c\,1\mathrm{i}\right)}{6\,d^{5/3}\,\left(a\,d-b\,c\right)^{1/3}} - \frac{c\,\ln\left(\frac{c^2\,\left(b\,x^3+a\right)^{1/3}}{d}+\frac{b\,c^3-a\,c^2\,d}{d^{4/3}\,\left(a\,d-b\,c\right)^{2/3}}\right)}{3\,d^{5/3}\,\left(a\,d-b\,c\right)^{1/3}}$$

input
```
int(x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
(a + b*x^3)^(2/3)/(2*b*d) + (log((c^2*(a + b*x^3)^(1/3))/d - (c^2*(3^(1/2)
*1i - 1)^2*(a*d - b*c)^(1/3))/(4*d^(4/3)))*(c - 3^(1/2)*c*1i))/(6*d^(5/3)*
(a*d - b*c)^(1/3)) + (log((c^2*(a + b*x^3)^(1/3))/d - (c^2*(3^(1/2)*1i + 1
)^2*(a*d - b*c)^(1/3))/(4*d^(4/3)))*(c + 3^(1/2)*c*1i))/(6*d^(5/3)*(a*d -
b*c)^(1/3)) - (c*log((c^2*(a + b*x^3)^(1/3))/d + (b*c^3 - a*c^2*d)/(d^(4/3
)*(a*d - b*c)^(2/3))))/(3*d^(5/3)*(a*d - b*c)^(1/3))
```

Reduce [F]

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x^5}{\left(b\,x^3+a\right)^{\frac{1}{3}} c+\left(b\,x^3+a\right)^{\frac{1}{3}} d\,x^3}dx$$

input
```
int(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x**5/((a + b*x**3)**(1/3)*c + (a + b*x**3)**(1/3)*d*x**3),x)
```

3.733 $\int \frac{x^2}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}$$

output
```
-1/3*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^
(1/2)/d^(2/3)/(-a*d+b*c)^(1/3)+1/6*ln(d*x^3+c)/d^(2/3)/(-a*d+b*c)^(1/3)-1/
2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(2/3)/(-a*d+b*c)^(1/3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{-2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-}\right.}{6d^{2/3}\sqrt[3]{bc-ad}}$$

input
```
Integrate[x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/S
qrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*c
- a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a +
b*x^3)^(2/3)])/(6*d^(2/3)*(b*c - a*d)^(1/3))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {946, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{1}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx^3$$

$$\Big\downarrow \text{68}$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-a}}\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)$$

input
```
Int[x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/S
qrt[3]])/(d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b*c - a
*d)^(1/3)) - (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(
2/3)*(b*c - a*d)^(1/3)))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)-\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\cdots\right.}{6d\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}$

input
```
int(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*
c)/d)^(1/3))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))-ln((b*x^3+a
)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3)))/d/((a*d-
b*c)/d)^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(114) = 228$.

Time = 0.12 (sec) , antiderivative size = 592, normalized size of antiderivative = 4.08

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*(b*c*d - a*d^2)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d
))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^
(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 +
a*d^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(
-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) + (-b*c*d^2 + a*d^
3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^
(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 2*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^
3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)))/(b*c*d^2 - a*d^3), 1/6*(6*sqrt
(1/3)*(b*c*d - a*d^2)*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(s
qrt(1/3)*(2*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^
2 + a*d^3)^(1/3)/(b*c - a*d))/d) + (-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a
)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a
*d^3)^(2/3)) - 2*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*
d^2 + a*d^3)^(1/3)))/(b*c*d^2 - a*d^3)]
```

Sympy [F]

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x**2/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**2/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.56

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$= -\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3}$$
$$+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^2-ad^3)}$$
$$-\frac{\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc-ad)}$$

input
```
integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c
 - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3
) + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)
*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^2 - a*d^3) - 1/3*
(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3))
)/(b*c - a*d)
```

Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.43

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\ln\left(d\,(b\,x^3+a)^{1/3}-\frac{9\,a\,d^3-9\,b\,c\,d^2}{9\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)}{3\,d^{2/3}\,(a\,d-b\,c)^{1/3}} + \frac{\ln\left(d\,(b\,x^3+a)^{1/3}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,(9\,a\,d^3-9\,b\,c\,d^2)}{36\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,d^{2/3}\,(a\,d-b\,c)^{1/3}} - \frac{\ln\left(d\,(b\,x^3+a)^{1/3}-\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\,(9\,a\,d^3-9\,b\,c\,d^2)}{36\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,d^{2/3}\,(a\,d-b\,c)^{1/3}}$$

input
```
int(x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
log(d*(a + b*x^3)^(1/3) - (9*a*d^3 - 9*b*c*d^2)/(9*d^(4/3)*(a*d - b*c)^(2/
3)))/(3*d^(2/3)*(a*d - b*c)^(1/3)) + (log(d*(a + b*x^3)^(1/3) - ((3^(1/2)*
1i - 1)^2*(9*a*d^3 - 9*b*c*d^2))/(36*d^(4/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*
1i - 1))/(6*d^(2/3)*(a*d - b*c)^(1/3)) - (log(d*(a + b*x^3)^(1/3) - ((3^(1
/2)*1i + 1)^2*(9*a*d^3 - 9*b*c*d^2))/(36*d^(4/3)*(a*d - b*c)^(2/3)))*(3^(1
/2)*1i + 1))/(6*d^(2/3)*(a*d - b*c)^(1/3))
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^2}{(b\,x^3+a)^{\frac{1}{3}}\,c+(b\,x^3+a)^{\frac{1}{3}}\,d\,x^3}dx$$

input
```
int(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x**2/((a + b*x**3)**(1/3)*c + (a + b*x**3)**(1/3)*d*x**3),x)
```

3.734 $\int \frac{1}{x\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 244

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}c} + \frac{\sqrt[3]{d}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{bc-ad}} - \frac{\log(x)}{2\sqrt[3]{a}c} - \frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}c} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{bc-ad}}$$

output

```
1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(1/3
)/c+1/3*d^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*
3^(1/2))*3^(1/2)/c/(-a*d+b*c)^(1/3)-1/2*ln(x)/a^(1/3)/c-1/6*d^(1/3)*ln(d*x
^3+c)/c/(-a*d+b*c)^(1/3)+1/2*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(1/3)/c+1/2*d^(
1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c/(-a*d+b*c)^(1/3)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.27

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{2\sqrt{3}\sqrt[3]{d}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{bc-ad}}+\frac{2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{a}}+\frac{2\sqrt[3]{d}\log\left(\sqrt[3]{bc-\right.}{\sqrt[3]{b}}$$

input
```
Integrate[1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
((2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) +
 (2*Sqrt[3]*d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^
(1/3))/Sqrt[3]])/(b*c - a*d)^(1/3) + (2*Log[-a^(1/3) + (a + b*x^3)^(1/3)])
/a^(1/3) + (2*d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/
(b*c - a*d)^(1/3) - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^
(2/3)]/a^(1/3) - (d^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3
)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(1/3))/(6*c)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {948, 97, 67, 16, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{1}{x^3\sqrt[3]{bx^3+a}\,(dx^3+c)}\,dx^3$$

$$\Big\downarrow 97$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3}{c}-\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{c}\right)$$

$$\Big\downarrow 67$$

$$\frac{1}{3}\left(\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}}{c}-\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{c}\right)$$

$$\Big\downarrow 16$$

$$\frac{1}{3}\left(\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}}{c}-\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{c}\right)$$

$$\Big\downarrow 68$$

$$\frac{1}{3}\left(\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}}{c}-\frac{d\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{\ldots}}{2d^{2/3}\sqrt[3]{bc-ad}}\ldots\right.}{\ldots}\right.$$

$$\Big\downarrow 16$$

$$\frac{1}{3}\left(\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}}{c}-\frac{d\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{b}}{\sqrt[3]{d}}}}{2d}\right.}{}\right.$$

$\downarrow$ 1082

$$\frac{1}{3}\left(\frac{-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}}{c}-\frac{d\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(}{2d^{2/3}\sqrt[3]{}}\right.}{}\right.$$

$\downarrow$ 217

$$\frac{1}{3}\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}}{c}-\frac{d\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+}{2d^{2/3}\sqrt[3]{b}}\right.}{c}\right.$$

input
```
Int[1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) -
Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3)))/c
 - (d*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1
/3))/Sqrt[3]])/(d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b
*c - a*d)^(1/3)) - (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/
(2*d^(2/3)*(b*c - a*d)^(1/3))))/c)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] && !IntegerQ[p]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{\frac{\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)a^{\frac{1}{3}}}{2}-\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)a^{\frac{1}{3}}-\frac{\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)\right.}{2}}{3\left(\frac{ad}{}\right.}$

input
```
int(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/d)^(1/3)*(1/2*ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3
+a)^(1/3)+((a*d-b*c)/d)^(2/3))*a^(1/3)-3^(1/2)*arctan(2/3*3^(1/2)/((a*d-b*
c)/d)^(1/3)*(b*x^3+a)^(1/3)+1/3*3^(1/2))*a^(1/3)-1/2*ln((b*x^3+a)^(2/3)+a^
(1/3)*(b*x^3+a)^(1/3)+a^(2/3))*((a*d-b*c)/d)^(1/3)-ln((b*x^3+a)^(1/3)-((a*
d-b*c)/d)^(1/3))*a^(1/3)+((a*d-b*c)/d)^(1/3)*(arctan(2/3*3^(1/2)/a^(1/3)*(
b*x^3+a)^(1/3)+1/3*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(1/3)-a^(1/3))))/a^(1/3)/
c
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 628, normalized size of antiderivative = 2.57

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3
+ a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(
b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) - 2*sqrt(3)*a*(d/(b*c - a*d))^(1/3)*a
rctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) - 1/3*sqrt(3)) -
 a*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*d)
)^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) + 2*a*(
d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(
1/3)*d) - a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2
/3)) + 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a*c), -1/6*(2*sqrt(3)*
a*(d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d
))^(1/3) - 1/3*sqrt(3)) + a*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(
b*c - a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b
*c - a*d))^(1/3)) - 2*a*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*
d))^(2/3) + (b*x^3 + a)^(1/3)*d) - 6*sqrt(1/3)*a^(2/3)*arctan(sqrt(1/3)*(2
*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) + (
b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^
(1/3)))/(a*c)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(1/x/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x}\,dx$$

input
```
integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.34

$$\begin{aligned}
&\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx \\
&= \frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} \\
&+ \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2d-\sqrt{3}acd^2} \\
&- \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^2d-acd^2)} \\
&+ \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}c} \\
&- \frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{1}{3}}c} + \frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{1}{3}}c}
\end{aligned}$$

input
```
integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^
(1/3)))/(b*c^2 - a*c*d) + (-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(
b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3
)*b*c^2*d - sqrt(3)*a*c*d^2) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a
)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3
))/(b*c^2*d - a*c*d^2) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/
3) + a^(1/3))/a^(1/3))/(a^(1/3)*c) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 +
a)^(1/3)*a^(1/3) + a^(2/3))/(a^(1/3)*c) + 1/3*log(abs((b*x^3 + a)^(1/3) -
a^(1/3)))/(a^(1/3)*c)
```

Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 702, normalized size of antiderivative = 2.88

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
log(b^5*d^4*(a + b*x^3)^(1/3) - (d*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a^
2*d^2 + b^2*c^2 - 2*a*b*c*d) - 243*a*b^4*c^4*d^3*(d/(27*b*c^4 - 27*a*c^3*d
))^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)))/(27*b*c^4 - 27*a*c^3*d))*(d/(
27*b*c^4 - 27*a*c^3*d))^(1/3) + log((a + b*x^3)^(1/3) - a*c^2*(1/(a*c^3))^
(2/3))*(1/(27*a*c^3))^(1/3) + (log(b^5*d^4*(a + b*x^3)^(1/3) - (d*(3^(1/2)
*1i - 1)^3*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a^2*d^2 + b^2*c^2 - 2*a*b*
c*d) - (243*a*b^4*c^4*d^3*(3^(1/2)*1i - 1)^2*(d/(27*b*c^4 - 27*a*c^3*d))^(
2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/4))/(8*(27*b*c^4 - 27*a*c^3*d)))*(
3^(1/2)*1i - 1)*(d/(27*b*c^4 - 27*a*c^3*d))^(1/3))/2 - (log(b^5*d^4*(a + b
*x^3)^(1/3) + (d*(3^(1/2)*1i + 1)^3*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a
^2*d^2 + b^2*c^2 - 2*a*b*c*d) - (243*a*b^4*c^4*d^3*(3^(1/2)*1i + 1)^2*(d/(
27*b*c^4 - 27*a*c^3*d))^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/4))/(8*(2
7*b*c^4 - 27*a*c^3*d)))*(3^(1/2)*1i + 1)*(d/(27*b*c^4 - 27*a*c^3*d))^(1/3)
)/2 - log((a + b*x^3)^(1/3)*2i + a*c^2*(1/(a*c^3))^(2/3)*1i + 3^(1/2)*a*c^
2*(1/(a*c^3))^(2/3))*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a*c^3))^(1/3) + log((a
+ b*x^3)^(1/3)*2i + a*c^2*(1/(a*c^3))^(2/3)*1i - 3^(1/2)*a*c^2*(1/(a*c^3))
^(2/3))*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a*c^3))^(1/3)
```

Reduce [F]

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,cx+(b\,x^3+a)^{\frac{1}{3}}\,d\,x^4}dx$$

input
```
int(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*c*x + (a + b*x**3)**(1/3)*d*x**4),x)
```

3.735 $\int \frac{1}{x^4 \sqrt[3]{a+bx^3}(c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 296

$$\begin{aligned}\int \frac{1}{x^4 \sqrt[3]{a+bx^3}\,(c+dx^3)} dx = & -\frac{(a+bx^3)^{2/3}}{3acx^3} - \frac{(bc+3ad)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}c^2} \\ & -\frac{d^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2\sqrt[3]{bc-ad}} + \frac{(bc+3ad)\log(x)}{6a^{4/3}c^2} \\ & + \frac{d^{4/3}\log(c+dx^3)}{6c^2\sqrt[3]{bc-ad}} - \frac{(bc+3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}c^2} \\ & - \frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{bc-ad}}\end{aligned}$$

output
```
-1/3*(b*x^3+a)^(2/3)/a/c/x^3-1/9*(3*a*d+b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+
a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(4/3)/c^2-1/3*d^(4/3)*arctan(1/3*(1-2
*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/c^2/(-a*d+b*c)
^(1/3)+1/6*(3*a*d+b*c)*ln(x)/a^(4/3)/c^2+1/6*d^(4/3)*ln(d*x^3+c)/c^2/(-a*d
+b*c)^(1/3)-1/6*(3*a*d+b*c)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(4/3)/c^2-1/2*d^
(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c^2/(-a*d+b*c)^(1/3)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{\frac{6c(a+bx^3)^{2/3}}{ax^3} + \frac{2\sqrt{3}(bc+3ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} + \frac{6\sqrt{3}d^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{bc-ad}} + \frac{2(bc+3ad)\log\left(-\sqrt[3]{a}+\right.}{a^{4/3}}}{}$$

input
```
Integrate[1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
-1/18*((6*c*(a + b*x^3)^(2/3))/(a*x^3) + (2*Sqrt[3]*(b*c + 3*a*d)*ArcTan[(
1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(4/3) + (6*Sqrt[3]*d^(4/3)*
ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(b*
c - a*d)^(1/3) + (2*(b*c + 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(4/
3) + (6*d^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c -
 a*d)^(1/3) - ((b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a
+ b*x^3)^(2/3)])/a^(4/3) - (3*d^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c
 - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^
(1/3))/c^2
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {948, 114, 27, 174, 67, 16, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{1}{x^6\sqrt[3]{bx^3+a}\,(dx^3+c)}dx^3$$

$\downarrow$ 114

$$\frac{1}{3}\left(-\frac{\int \frac{bdx^3+bc+3ad}{3x^3\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{ac}-\frac{\left(a+bx^3\right)^{2/3}}{acx^3}\right)$$

$\downarrow$ 27

$$\frac{1}{3}\left(-\frac{\int \frac{bdx^3+bc+3ad}{x^3\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{3ac}-\frac{\left(a+bx^3\right)^{2/3}}{acx^3}\right)$$

$\downarrow$ 174

$$\frac{1}{3}\left(-\frac{\frac{(3ad+bc)\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3}{c}-\frac{3ad^2\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx^3}{c}}{3ac}-\frac{\left(a+bx^3\right)^{2/3}}{acx^3}\right)$$

$\downarrow$ 67

$$\frac{1}{3}\left(-\frac{\frac{(3ad+bc)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{3ad^2\int \frac{1}{\sqrt[3]{bx^3+a}\left(d\right.}}{c}}{3ac}\right.$$

$\downarrow$ 16

$$\frac{1}{3}\left(-\frac{\frac{(3ad+bc)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{3ad^2\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx^3}{c}}{3ac}\right.$$

$\downarrow$ 68

$$\frac{1}{3}\left(-\frac{(3ad+bc)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{3ad^2\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx\cdots}}\cdots}{2d^{2/3}\sqrt[3]{bc\cdots}}\cdots\right.}{3ac}\cdots\right.$$

$\downarrow$ 16

$$\frac{1}{3}\left(-\frac{(3ad+bc)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{3ad^2\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc\cdots}}{\cdots}}\cdots}{\cdots}\cdots\right.}{3ac}\cdots\right.$$

$\downarrow$ 1082

$$\frac{1}{3}\left(-\frac{(3ad+bc)\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{3ad^2\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\cdots\right.}{3ac}\cdots\right.$$

$\downarrow$ 217

$$\frac{1}{3}\left(-\frac{\frac{(3ad+bc)\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{3ad^2\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\cdots}{2d^2\cdots}\right.}{\cdots}}{3ac}\right.$$

input
```
Int[1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-((a + b*x^3)^(2/3)/(a*c*x^3)) - (((b*c + 3*a*d)*((Sqrt[3]*ArcTan[(1 + (2
*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^3]/(2*a^(1/3)) + (3
*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3))))/c - (3*a*d^2*(-((Sqrt[3]*
ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^
(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b*c - a*d)^(1/3)) -
 (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c -
a*d)^(1/3))))/c)/(3*a*c))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\left(-a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{2}{3}}c+\frac{\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)\right)x}{2}\right.$

input
```
int(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/d)^(1/3)*((-a^(1/3)*(b*x^3+a)^(2/3)*c+1/2*(-2*arctan(1/3*(a
^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1
/3)*(b*x^3+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*x^3*(1/3*b*c+a
*d))*((a*d-b*c)/d)^(1/3)+d*(arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+((a*d-b*
c)/d)^(1/3))/((a*d-b*c)/d)^(1/3))*3^(1/2)+ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)
^(1/3))-1/2*ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b
*c)/d)^(2/3)))*a^(4/3)*x^3)/a^(4/3)/c^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 837, normalized size of antiderivative = 2.83

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/18*(6*sqrt(3)*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^
3 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) - 3*a^2*d*x^3*(-d/(b*c
- a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) +
(b*x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*a^2*d*x^3*(-
d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^
(1/3)*d) + 3*sqrt(1/3)*(a*b*c + 3*a^2*d)*x^3*sqrt((-a)^(1/3)/a)*log((2*b*x
^3 - 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (
-a)^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^
3) + (b*c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3
)*(-a)^(1/3) + (-a)^(2/3)) - 2*(b*c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a
)^(1/3) + (-a)^(1/3)) - 6*(b*x^3 + a)^(2/3)*a*c)/(a^2*c^2*x^3), 1/18*(6*sq
rt(3)*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3
)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) - 3*a^2*d*x^3*(-d/(b*c - a*d))^(1/
3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)
^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*a^2*d*x^3*(-d/(b*c - a*
d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d) -
6*sqrt(1/3)*(a*b*c + 3*a^2*d)*x^3*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*
(b*x^3 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + (b*c + 3*a*d)*(-a)^
(2/3)*x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3
)) - 2*(b*c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3))...
```

Sympy [F]

$$\int \frac{1}{x^4 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x^4 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(1/x**4/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**4*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx &= -\frac{d^2\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^3-ac^2d)} \\ &\quad - \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^3-\sqrt{3}ac^2d} \\ &\quad + \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^3-ac^2d)} \\ &\quad - \frac{\sqrt{3}(bc+3\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{4}{3}}c^2} \\ &\quad + \frac{(bc+3\,ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{4}{3}}c^2} \\ &\quad - \frac{\left(a^{\frac{1}{3}}bc+3\,a^{\frac{4}{3}}d\right)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{5}{3}}c^2} - \frac{(bx^3+a)^{\frac{2}{3}}}{3\,acx^3}\end{aligned}$$

input
```
integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*d^2*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/
d)^(1/3)))/(b*c^3 - a*c^2*d) - (-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)
*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(s
qrt(3)*b*c^3 - sqrt(3)*a*c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3
+ a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(
2/3))/(b*c^3 - a*c^2*d) - 1/9*sqrt(3)*(b*c + 3*a*d)*arctan(1/3*sqrt(3)*(2*
(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(4/3)*c^2) + 1/18*(b*c + 3*a*d)*l
og((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(4/3)*c^2)
- 1/9*(a^(1/3)*b*c + 3*a^(4/3)*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a
^(5/3)*c^2) - 1/3*(b*x^3 + a)^(2/3)/(a*c*x^3)
```

Mupad [B] (verification not implemented)

Time = 10.66 (sec) , antiderivative size = 1929, normalized size of antiderivative = 6.52

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
log(- (((((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c
^2*d^2 + 4*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - 3*a*b^4*c^4*d^3*(2*a^2*d^2
 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))*(-(3*a*d + b*c
)^3/(a^4*c^6))^(1/3))/9 + (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d +
 18*a^2*b*c*d^2))/(3*a^2*c))*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))/81 - (4*b
^5*d^7*(a + b*x^3)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*(-(27*a^3*d^3 + b^
3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2)/(729*a^4*c^6))^(1/3) + log(- (-d^4
/(27*b*c^7 - 27*a*c^6*d))^(2/3)*((-d^4/(27*b*c^7 - 27*a*c^6*d))^(1/3)*((3*
b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2 + 4*a*
b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - 243*a*b^4*c^4*d^3*(-d^4/(27*b*c^7 - 27*
a*c^6*d))^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)) + (b^5*d^4*(b^3*c^3 - 2
7*a^3*d^3 + 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^2*c)) - (4*b^5*d^7*(a +
b*x^3)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*(-d^4/(27*b*c^7 - 27*a*c^6*d))
^(1/3) - log((((((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^
2*b^2*c^2*d^2 + 4*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - 3*a*b^4*c^4*d^3*((3
^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + b*c)^3/(a
^4*c^6))^(2/3))*((3^(1/2)*1i)/2 + 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(1/3))
/9 - (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*
a^2*c))*((3^(1/2)*1i)/2 - 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))/81 - (4
*b^5*d^7*(a + b*x^3)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*((3^(1/2)*1i)...
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,c\,x^4+(b\,x^3+a)^{\frac{1}{3}}\,d\,x^7}dx$$

input
```
int(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*c*x**4 + (a + b*x**3)**(1/3)*d*x**7),x)
```

3.736 $\int \frac{x^6}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 273

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{x(a+bx^3)^{2/3}}{3bd} - \frac{(3bc+ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}d^2} + \frac{c^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2\sqrt[3]{bc-ad}} + \frac{c^{4/3}\log(c+dx^3)}{6d^2\sqrt[3]{bc-ad}} - \frac{c^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2d^2\sqrt[3]{bc-ad}} + \frac{(3bc+ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}d^2}$$

output
```
1/3*x*(b*x^3+a)^(2/3)/b/d-1/9*(a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3
+a)^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)/d^2+1/3*c^(4/3)*arctan(1/3*(1+2*(-a*d+
b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d^2/(-a*d+b*c)^(1/3
)+1/6*c^(4/3)*ln(d*x^3+c)/d^2/(-a*d+b*c)^(1/3)-1/2*c^(4/3)*ln((-a*d+b*c)^(
1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2/(-a*d+b*c)^(1/3)+1/6*(a*d+3*b*c)*ln(-b
^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)/d^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.87 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.71

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{\frac{12dx(a+bx^3)^{2/3}}{b} - \frac{4\sqrt{3}(3bc+ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{4/3}} - \frac{6\sqrt{-6+6i\sqrt{3}}c^{4/3}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+b}}\right.}{\sqrt[3]{bc-ad}}}{}$$

input
```
Integrate[x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
((12*d*x*(a + b*x^3)^(2/3))/b - (4*Sqrt[3]*(3*b*c + a*d)*ArcTan[(Sqrt[3]*b
^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/b^(4/3) - (6*Sqrt[-6 + (6*I)
*Sqrt[3]]*c^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3
)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(b*c - a*d)^(1/3) + (4*
(3*b*c + a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(4/3) + (6*(1 + I*S
qrt[3])*c^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b
*x^3)^(1/3)])/(b*c - a*d)^(1/3) - (2*(3*b*c + a*d)*Log[b^(2/3)*x^2 + b^(1/
3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(4/3) - ((3*I)*(-I + Sqrt[3
])*c^(4/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a
*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]
)/(b*c - a*d)^(1/3))/(36*d^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {979, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}dx$$

$$\Big\downarrow \text{ 979}$$

$$\frac{x\left(a+bx^3\right)^{2/3}}{3bd}-\frac{\int \frac{(3bc+ad)x^3+ac}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{3bd}$$

$$\Big\downarrow \text{ 1026}$$

$$\frac{x\left(a+bx^3\right)^{2/3}}{3bd}-\frac{\frac{(ad+3bc)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{3bc^2\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}}{3bd}$$

$$\Big\downarrow \text{ 769}$$

$$\frac{x\left(a+bx^3\right)^{2/3}}{3bd}-\frac{\frac{(ad+3bc)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{3bc^2\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}}{3bd}$$

$$\Big\downarrow \text{ 901}$$

$$\frac{x(a+bx^3)^{2/3}}{3bd} - \frac{\frac{(ad+3bc)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d} - \frac{3bc^2\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\frac{x\sqrt[3]{bc\cdots}}{\sqrt[3]{\cdots}}\cdots\right)}{2c^{\cdots}}\cdots\right)}{d}}{3bd}$$

input
```
Int[x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(x*(a + b*x^3)^(2/3))/(3*b*d) - ((-3*b*c^2*(ArcTan[(1 + (2*(b*c - a*d)^(1/
3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(
1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1
/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/d + ((
3*b*c + a*d)*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[
3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/d)/(3*b*
d)
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$-\frac{\left(-6dx\left(b\,x^3+a\right)^{\frac{2}{3}}b^{\frac{1}{3}}+(ad+3bc)\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\right)-\right.\right.}{}$

input
```
int(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/18/((a*d-b*c)/c)^(1/3)*((-6*d*x*(b*x^3+a)^(2/3)*b^(1/3)+(a*d+3*b*c)*(-2
*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((b
^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-b^(1/3)*
x+(b*x^3+a)^(1/3))/x)))*((a*d-b*c)/c)^(1/3)-6*c*b^(4/3)*(3^(1/2)*arctan(1/
3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)
+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((((a*d-b*c)/c)^(2/3)
*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))/b^(4/3)
/d^2
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 826, normalized size of antiderivative = 3.03

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/18*(6*sqrt(3)*b^2*c*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*
sqrt(3)*(b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 6*b^2*c*(-c/(b*c -
a*d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)
*c)/x) + 3*b^2*c*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x^2*(-c/(b*c - a
*d))^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x
^3 + a)^(2/3)*c)/x^2) - 6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c +
 a*b*d)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3
*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^
(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) - 2*(3*b*c + a*d)*b^(2/3)*log(-(b^(1/3)*x
 - (b*x^3 + a)^(1/3))/x) + (3*b*c + a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3
 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^2*d^2), -1/18*(6*sqrt(
3)*b^2*c*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(b*x^3
+ a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 6*b^2*c*(-c/(b*c - a*d))^(1/3)*log
(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) + 3*b^2*
c*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x^2*(-c/(b*c - a*d))^(1/3) - (b
*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(2/3)*c
)/x^2) - 6*(b*x^3 + a)^(2/3)*b*d*x - 2*(3*b*c + a*d)*b^(2/3)*log(-(b^(1/3)
*x - (b*x^3 + a)^(1/3))/x) + (3*b*c + a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x
^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 6*sqrt(1/3)*(3*b^2*c +
 a*b*d)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))...
```

Sympy [F]

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x**6/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**6/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^6/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^6/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^6}{{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(x^6/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}\,c+(b\,x^3+a)^{\frac{1}{3}}\,d\,x^3}dx$$

input
```
int(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x**6/((a + b*x**3)**(1/3)*c + (a + b*x**3)**(1/3)*d*x**3),x)
```

3.737 $\int \frac{x^3}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 233

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d}-\frac{\sqrt[3]{c}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d\sqrt[3]{bc-ad}}-\frac{\sqrt[3]{c}\log\left(c+dx^3\right)}{6d\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{c}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d\sqrt[3]{bc-ad}}-\frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}d}$$

output
```
1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(1/3)/d-
1/3*c^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^
(1/2))*3^(1/2)/d/(-a*d+b*c)^(1/3)-1/6*c^(1/3)*ln(d*x^3+c)/d/(-a*d+b*c)^(1/
3)+1/2*c^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d/(-a*d+b*c)
^(1/3)-1/2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.22 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.82

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$= \frac{\frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}} + \frac{2\sqrt{-6+6i\sqrt{3}}\sqrt[3]{c}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}} - \frac{4\log\left(-\sqrt[3]{b}x+\sqrt[3]{a \cdots}\right.}{\sqrt[3]{b}\cdots}}{\cdots}$$

input
```
Integrate[x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
((4*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])
/b^(1/3) + (2*Sqrt[-6 + (6*I)*Sqrt[3]]*c^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)
*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/
3))])/(b*c - a*d)^(1/3) - (4*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(1/3
) - ((2*I)*(-I + Sqrt[3])*c^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[
3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(1/3) + (2*Log[b^(2/3)*x^2 + b
^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(1/3) + ((1 + I*Sqrt[3]
)*c^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*
d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])
/(b*c - a*d)^(1/3))/(12*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {983, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \ 983$$

$$\frac{\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d} - \frac{c\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}$$

$$\Big\downarrow \ 769$$

$$\frac{\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}}{d} - \frac{c\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}$$

$$\Big\downarrow \ 901$$

$$\frac{\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}}{d} - \frac{c\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{d}$$

input
```
Int[x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
-((c*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqr
t[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c
 - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2
*c^(2/3)*(b*c - a*d)^(1/3))))/d) + (ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^
(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/
(2*b^(1/3)))/d
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.27

method	result
pseudoelliptic	$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}+x\right)}{3x}\right)b^{\frac{1}{3}}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{b^{\frac{1}{3}}}+x\right)}{3x}\right)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)\left(\frac{a}{}\right.}{}$

input
```
int(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/((a*d-b*c)/c)^(1/3)*(3^(1/2)*arctan(1/3*3^(1/2)*(-2/((a*d-b*c)/c)^(1/
3)*(b*x^3+a)^(1/3)+x)/x)*b^(1/3)+3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(
1/3)/b^(1/3)+x)/x)*((a*d-b*c)/c)^(1/3)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*
((a*d-b*c)/c)^(1/3)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a
)^(2/3))/x^2)*((a*d-b*c)/c)^(1/3)+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3
))/x)*b^(1/3)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a
)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*b^(1/3))/b^(1/3)/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 761, normalized size of antiderivative = 3.27

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(
-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 +
2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*sqrt(3)*b*
(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*
(c/(b*c - a*d))^(1/3))/x) + 2*b*(c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*
(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) - b*(c/(b*c - a*d))^(1/3)*
log(((b*c - a*d)*x^2*(c/(b*c - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)
*x*(c/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*c)/x^2) - 2*(-b)^(2/3)*log(((
-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*
x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b*d), -1/6*(6*sqrt
(1/3)*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a
)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) - 2*sqrt(3)*b*(c/(b*c - a*d))^(1/3)*arctan
(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) -
2*b*(c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x
^3 + a)^(1/3)*c)/x) + b*(c/(b*c - a*d))^(1/3)*log(((b*c - a*d)*x^2*(c/(b*c
 - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c - a*d))^(2/3) + (
b*x^3 + a)^(2/3)*c)/x^2) + 2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1
/3))/x) - (-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x
+ (b*x^3 + a)^(2/3))/x^2))/(b*d)]
```

Sympy [F]

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x**3/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**3/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^3}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^3}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^3/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x^3}{\left(b\,x^3+a\right)^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(x^3/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x^3}{\left(b\,x^3+a\right)^{\frac{1}{3}}c+\left(b\,x^3+a\right)^{\frac{1}{3}}d\,x^3}dx$$

input
```
int(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x**3/((a + b*x**3)**(1/3)*c + (a + b*x**3)**(1/3)*d*x**3),x)
```

3.738 $\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

output
```
1/3*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3
^(1/2)/c^(2/3)/(-a*d+b*c)^(1/3)+1/6*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(1/3)-1
/2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(1/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{-2\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+(1+i\sqrt{3})\left(2\log\left(2\sqrt[3]{bc-ad}x+(1+\right.\right.}{12}$$

input
```
Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-2*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c
- a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (1 + I*Sqrt
[3])*(2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1
/3)] - Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^
(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]))/(
12*c^(2/3)*(b*c - a*d)^(1/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \; 901$$

$$\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

input
```
Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/
(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d
)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/
3)*(b*c - a*d)^(1/3))
```

Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)+2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x^{2}}\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}c}$

input
```
int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3)
)/((a*d-b*c)/c)^(1/3)/x)+2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-l
n((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)
^(2/3))/x^2))/((a*d-b*c)/c)^(1/3)/c
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,c+(b\,x^3+a)^{\frac{1}{3}}\,d\,x^3}dx$$

input
```
int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*c + (a + b*x**3)**(1/3)*d*x**3),x)
```

3.739 $\int \frac{1}{x^3 \sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 176

$$\int \frac{1}{x^3 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{2acx^2} - \frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}} - \frac{d\log(c+dx^3)}{6c^{5/3}\sqrt[3]{bc-ad}} + \frac{d\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{5/3}\sqrt[3]{bc-ad}}$$

output
```
-1/2*(b*x^3+a)^(2/3)/a/c/x^2-1/3*d*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1
/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(5/3)/(-a*d+b*c)^(1/3)-1/6*d*ln(d*
x^3+c)/c^(5/3)/(-a*d+b*c)^(1/3)+1/2*d*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3
+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(1/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.82 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.78

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$= \frac{-6c^{2/3}\sqrt[3]{bc-ad}(a+bx^3)^{2/3}+2\sqrt{-6+6i\sqrt{3}}adx^2\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)-2i(-$$

input
```
Integrate[1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-6*c^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3) + 2*Sqrt[-6 + (6*I)*Sqrt[3
]]*a*d*x^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (
3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] - (2*I)*(-I + Sqrt[3])*a*d*x^2*
Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + a
*(d + I*Sqrt[3]*d)*x^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1
/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b
*x^3)^(2/3)])/(12*a*c^(5/3)*(b*c - a*d)^(1/3)*x^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {980, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$\Big\downarrow \; 980$$
$$\frac{\int -\frac{2ad}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{2ac}-\frac{(a+bx^3)^{2/3}}{2acx^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{c}-\frac{(a+bx^3)^{2/3}}{2acx^2}$$

$$\Big\downarrow \text{901}$$

$$-\frac{d\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{(a+bx^3)^{2/3}}{2acx^2}$$

input
```
Int[1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
-1/2*(a + b*x^3)^(2/3)/(a*c*x^2) - (d*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)
/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3))
 + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x
)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$-\frac{a\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)x^{2}d+\frac{3\left(b\,x^{3}+a\right)^{\frac{2}{3}}c\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}{2}+ad\,x^{2}\left(\arctan\left(\frac{\sqrt{3}\left(-\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}+x\right)}{3x}\right)\sqrt{3}-\frac{\ln\left(\left(\frac{ad-}{c}\right.\right.}{}\right.}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}a\,c^{2}x^{2}}$

input
```
int(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/((a*d-b*c)/c)^(1/3)*(a*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*
x^2*d+3/2*(b*x^3+a)^(2/3)*c*((a*d-b*c)/c)^(1/3)+a*d*x^2*(arctan(1/3*3^(1/2
)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*3^(1/2)-1/2*ln((((a*d-b*c)
/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))
)/a/c^2/x^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(1/x**3/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**3*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{x^3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)} \,d x$$

input
```
int(1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{1}{3}} c\,x^{3}+\left(b\,x^{3}+a\right)^{\frac{1}{3}} d\,x^{6}}d x$$

input
```
int(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*c*x**3 + (a + b*x**3)**(1/3)*d*x**6),x)
```

3.740 $\int \frac{1}{x^6 \sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 214

$$\begin{aligned}\int \frac{1}{x^6 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx &= -\frac{(a+bx^3)^{2/3}}{5acx^5}+\frac{(3bc+5ad)\,(a+bx^3)^{2/3}}{10a^2c^2x^2} \\ &\quad+\frac{d^2\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{8/3}\sqrt[3]{bc-ad}}+\frac{d^2\log(c+dx^3)}{6c^{8/3}\sqrt[3]{bc-ad}} \\ &\quad-\frac{d^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{8/3}\sqrt[3]{bc-ad}}\end{aligned}$$

output
```
-1/5*(b*x^3+a)^(2/3)/a/c/x^5+1/10*(5*a*d+3*b*c)*(b*x^3+a)^(2/3)/a^2/c^2/x^
2+1/3*d^2*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1
/2))*3^(1/2)/c^(8/3)/(-a*d+b*c)^(1/3)+1/6*d^2*ln(d*x^3+c)/c^(8/3)/(-a*d+b*
c)^(1/3)-1/2*d^2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-
a*d+b*c)^(1/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.59

$$\int \frac{1}{x^6 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\frac{6c^{2/3}(a+bx^3)^{2/3}(-2ac+3bcx^3+5adx^3)}{a^2x^5} - \frac{10\sqrt{-6+6i\sqrt{3}}\,d^2\arctan\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}} + \frac{10\left(1+i\sqrt{3}\right)d^2\log\left(\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
((6*c^(2/3)*(a + b*x^3)^(2/3)*(-2*a*c + 3*b*c*x^3 + 5*a*d*x^3))/(a^2*x^5)
- (10*Sqrt[-6 + (6*I)*Sqrt[3]]*d^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]
*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(b*c -
 a*d)^(1/3) + (10*(1 + I*Sqrt[3])*d^2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*S
qrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]])/(b*c - a*d)^(1/3) - ((5*I)*(-I + Sqrt[
3])*d^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)
^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(
b*c - a*d)^(1/3))/(60*c^(8/3))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {980, 25, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \text{ 980}$$

$$\frac{\int -\frac{3bdx^3+3bc+5ad}{x^3\sqrt[3]{bx^3+a}(dx^3+c)}dx}{5ac}-\frac{(a+bx^3)^{2/3}}{5acx^5}$$

$\Big\downarrow$ 25

$$-\frac{\int \frac{3bdx^3+3bc+5ad}{x^3\sqrt[3]{bx^3+a}(dx^3+c)}dx}{5ac}-\frac{(a+bx^3)^{2/3}}{5acx^5}$$

$\Big\downarrow$ 1053

$$-\frac{-\frac{\int \frac{10a^2d^2}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{2ac}-\frac{(a+bx^3)^{2/3}(5ad+3bc)}{2acx^2}}{5ac}-\frac{(a+bx^3)^{2/3}}{5acx^5}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{5ad^2\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{c}-\frac{(a+bx^3)^{2/3}(5ad+3bc)}{2acx^2}}{5ac}-\frac{(a+bx^3)^{2/3}}{5acx^5}$$

$\Big\downarrow$ 901

$$-\frac{-\frac{5ad^2\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{(a+bx^3)^{2/3}(5ad+3bc)}{2acx^2}}{5ac}-\frac{(a+bx^3)^{2/3}}{5acx^5}$$

input
```
Int[1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
-1/5*(a + b*x^3)^(2/3)/(a*c*x^5) - (-1/2*((3*b*c + 5*a*d)*(a + b*x^3)^(2/3
))/(a*c*x^2) - (5*a*d^2*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a +
 b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x
^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a
 + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/c)/(5*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$\frac{-\frac{d^{2}\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)a^{2}x^{5}}{2}+d^{2}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}+x\right)}{3x}\right)a^{2}x^{5}+d^{2}\ln\left(\left(\right.\right.}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}a^{2}c^{3}x^{5}}$

input
```
int(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-1/2*d^2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1
/3)*x+(b*x^3+a)^(2/3))/x^2)*a^2*x^5+d^2*3^(1/2)*arctan(1/3*3^(1/2)*(-2/((a
*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*a^2*x^5+d^2*ln((((a*d-b*c)/c)^(1/3)
*x+(b*x^3+a)^(1/3))/x)*a^2*x^5-3/5*((a*d-b*c)/c)^(1/3)*((-5/2*d*x^3+c)*a-3
/2*x^3*b*c)*c*(b*x^3+a)^(2/3))/((a*d-b*c)/c)^(1/3)/a^2/c^3/x^5
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(1/x**6/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**6*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x^6\,(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,c\,x^6+(b\,x^3+a)^{\frac{1}{3}}\,d\,x^9}dx$$

input
```
int(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*c*x**6 + (a + b*x**3)**(1/3)*d*x**9),x)
```

3.741 $\int \frac{1}{x^9 \sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 262

$$\int \frac{1}{x^9 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{8acx^8}+\frac{(3bc+4ad)\,(a+bx^3)^{2/3}}{20a^2c^2x^5} - \frac{(9b^2c^2+12abcd+20a^2d^2)\,(a+bx^3)^{2/3}}{40a^3c^3x^2} - \frac{d^3\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{11/3}\sqrt[3]{bc-ad}} - \frac{d^3\log(c+dx^3)}{6c^{11/3}\sqrt[3]{bc-ad}} + \frac{d^3\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{11/3}\sqrt[3]{bc-ad}}$$

output

```
-1/8*(b*x^3+a)^(2/3)/a/c/x^8+1/20*(4*a*d+3*b*c)*(b*x^3+a)^(2/3)/a^2/c^2/x^
5-1/40*(20*a^2*d^2+12*a*b*c*d+9*b^2*c^2)*(b*x^3+a)^(2/3)/a^3/c^3/x^2-1/3*d
^3*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^
(1/2)/c^(11/3)/(-a*d+b*c)^(1/3)-1/6*d^3*ln(d*x^3+c)/c^(11/3)/(-a*d+b*c)^(1
/3)+1/2*d^3*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(11/3)/(-a*d+
b*c)^(1/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.71 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^9 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{-\frac{3c^{2/3}(a+bx^3)^{2/3}(9b^2c^2x^6-6abcx^3(c-2dx^3)+a^2(5c^2-8cdx^3+20d^2x^6))}{a^3x^8} + \frac{20\sqrt{-6+6i\sqrt{3}}d^3\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a}}\right.}{\sqrt[3]{bc-ad}}}{}$$

input
```
Integrate[1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
((-3*c^(2/3)*(a + b*x^3)^(2/3)*(9*b^2*c^2*x^6 - 6*a*b*c*x^3*(c - 2*d*x^3)
+ a^2*(5*c^2 - 8*c*d*x^3 + 20*d^2*x^6)))/(a^3*x^8) + (20*Sqrt[-6 + (6*I)*S
qrt[3]]*d^3*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x -
(3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(b*c - a*d)^(1/3) - ((20*I)*(
-I + Sqrt[3])*d^3*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a +
 b*x^3)^(1/3)])/(b*c - a*d)^(1/3) + (10*(1 + I*Sqrt[3])*d^3*Log[2*(b*c - a
*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(
1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(1/3))/(120
*c^(11/3))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {980, 27, 1053, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \text{ 980}$$

$$\frac{\int -\frac{2(3bdx^3+3bc+4ad)}{x^6\sqrt[3]{bx^3+a}(dx^3+c)}dx}{8ac} - \frac{(a+bx^3)^{2/3}}{8acx^8}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{3bdx^3+3bc+4ad}{x^6\sqrt[3]{bx^3+a}(dx^3+c)}dx}{4ac} - \frac{(a+bx^3)^{2/3}}{8acx^8}$$

$$\Big\downarrow \; 1053$$

$$-\frac{-\frac{\int \frac{3bd(3bc+4ad)x^3+9b^2c^2+20a^2d^2+12abcd}{x^3\sqrt[3]{bx^3+a}(dx^3+c)}dx}{5ac} - \frac{(a+bx^3)^{2/3}(4ad+3bc)}{5acx^5}}{4ac} - \frac{(a+bx^3)^{2/3}}{8acx^8}$$

$$\Big\downarrow \; 1053$$

$$-\frac{-\frac{-\frac{\int \frac{40a^3d^3}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{2ac} - \frac{(a+bx^3)^{2/3}\left(\frac{9b^2c}{a}+\frac{20ad^2}{c}+12bd\right)}{2x^2}}{5ac} - \frac{(a+bx^3)^{2/3}(4ad+3bc)}{5acx^5}}{4ac} - \frac{(a+bx^3)^{2/3}}{8acx^8}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{-\frac{20a^2d^3\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{c} - \frac{(a+bx^3)^{2/3}\left(\frac{9b^2c}{a}+\frac{20ad^2}{c}+12bd\right)}{2x^2}}{5ac} - \frac{(a+bx^3)^{2/3}(4ad+3bc)}{5acx^5}}{4ac} - \frac{(a+bx^3)^{2/3}}{8acx^8}$$

$$\Big\downarrow \; 901$$

$$-\frac{-\frac{-\frac{20a^2d^3\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c} - \frac{(a+bx^3)^{2/3}\left(\frac{9b^2c}{a}+\frac{20ad^2}{c}+12bd\right)}{2x^2}}{5ac} - }{4ac} \\ \frac{(a+bx^3)^{2/3}}{8acx^8}$$

input
```
Int[1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
-1/8*(a + b*x^3)^(2/3)/(a*c*x^8) - (-1/5*((3*b*c + 4*a*d)*(a + b*x^3)^(2/3
))/(a*c*x^5) - (-1/2*(((9*b^2*c)/a + 12*b*d + (20*a*d^2)/c)*(a + b*x^3)^(2
/3))/x^2 - (20*a^2*d^3*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a +
b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^
3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a
+ b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/c)/(5*a*c))/(4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{3\left(\left(4a^2d^2+\frac{12}{5}abcd+\frac{9}{5}b^2c^2\right)x^6+\frac{2\left(-4a^2cd-3b\,c^2a\right)x^3}{5}+a^2c^2\right)c\left(b\,x^3+a\right)^{\frac{2}{3}}\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}+4a^3d^3x^8\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad}{}\right.\right.}{}\right.\right.}{24\left(\frac{ad-bc}{c}\right)}$

input
```
int(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/24/((a*d-b*c)/c)^(1/3)*(3*((4*a^2*d^2+12/5*a*b*c*d+9/5*b^2*c^2)*x^6+2/5
*(-4*a^2*c*d-3*a*b*c^2)*x^3+a^2*c^2)*c*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3)
+4*a^3*d^3*x^8*(2*3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x
^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)+2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^
(1/3))/x)-ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*
x+(b*x^3+a)^(2/3))/x^2)))/x^8/c^4/a^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^9 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x^9 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(1/x**9/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**9*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^9 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^9}\,dx$$

input
```
integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^9), x)
```

Giac [F]

$$\int \frac{1}{x^9 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^9}\,dx$$

input
```
integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^9 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x^9\,(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^9 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,c\,x^9+(b\,x^3+a)^{\frac{1}{3}}\,d\,x^{12}}dx$$

input
```
int(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*c*x**9 + (a + b*x**3)**(1/3)*d*x**12),x)
```

3.742 $\int \frac{x^7}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{8}{3},\frac{1}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{a+bx^3}}$$

output
```
1/8*x^8*(1+b*x^3/a)^(1/3)*AppellF1(8/3,1/3,1,11/3,-b*x^3/a,-d*x^3/c)/c/(b*
x^3+a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 144 vs. 2(64) = 128.

Time = 7.76 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.25

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$= \frac{5cx^2(a+bx^3) - 5acx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right) - 2(2bc+ad)x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}}{20bcd\sqrt[3]{a+bx^3}}$$

input
```
Integrate[x^7/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(5*c*x^2*(a + b*x^3) - 5*a*c*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3,
1, 5/3, -((b*x^3)/a), -((d*x^3)/c)] - 2*(2*b*c + a*d)*x^5*(1 + (b*x^3)/a)^
(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*b*c*d*(a
 + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x^7}{\sqrt[3]{\frac{bx^3}{a}+1}(dx^3+c)}dx}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{8}{3},\frac{1}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{a+bx^3}}$$

input
```
Int[x^7/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(x^8*(1 + (b*x^3)/a)^(1/3)*AppellF1[8/3, 1/3, 1, 11/3, -((b*x^3)/a), -((d*
x^3)/c)])/(8*c*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{7}}{\left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)}d x$$

input
```
int(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x**7/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**7/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^7/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^7/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^7}{\left(b\,x^3+a\right)^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(x^7/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(x^7/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^7}{\left(b\,x^3+a\right)^{\frac{1}{3}} c+\left(b\,x^3+a\right)^{\frac{1}{3}} d\,x^3}dx$$

input
```
int(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x**7/((a + b*x**3)**(1/3)*c + (a + b*x**3)**(1/3)*d*x**3),x)
```

3.743 $\int \frac{x^4}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

output
```
1/5*x^5*(1+b*x^3/a)^(1/3)*AppellF1(5/3,1/3,1,8/3,-b*x^3/a,-d*x^3/c)/c/(b*x
^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 7.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\frac{x^5\sqrt[3]{\frac{a+bx^3}{a}}\,\mathrm{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

input
```
Integrate[x^4/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(x^5*((a + b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x
^3)/c)])/(5*c*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x^4}{\sqrt[3]{\frac{bx^3}{a}+1}(dx^3+c)}dx}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

input
```
Int[x^4/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x
^3)/c)])/(5*c*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{4}}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(d\,x^{3}+c\right)}d x$$

input
```
int(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x**4/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**4/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^4}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

input
```
int(x^4/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(x^4/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^4}{(b\,x^3+a)^{\frac{1}{3}}\,c+(b\,x^3+a)^{\frac{1}{3}}\,d\,x^3}dx$$

input
```
int(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x**4/((a + b*x**3)**(1/3)*c + (a + b*x**3)**(1/3)*d*x**3),x)
```

3.744 $\int \frac{x}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

output
```
1/2*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,1/3,1,5/3,-b*x^3/a,-d*x^3/c)/c/(b*x
^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 9.75 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\frac{x^2\sqrt[3]{\frac{a+bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

input
```
Integrate[x/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(x^2*((a + b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x
^3)/c)])/(2*c*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x}{\sqrt[3]{\frac{bx^3}{a}+1}(dx^3+c)}dx}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

input
```
Int[x/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x
^3)/c)])/(2*c*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(d\,x^{3}+c\right)}d x$$

input
```
int(x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x}{{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(x/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(x/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}\,c+(b\,x^3+a)^{\frac{1}{3}}\,d\,x^3}dx$$

input
```
int(x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x/((a + b*x**3)**(1/3)*c + (a + b*x**3)**(1/3)*d*x**3),x)
```

3.745 $\int \frac{1}{x^2 \sqrt[3]{a+bx^3}(c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{3},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{a+bx^3}}$$

output
```
-(1+b*x^3/a)^(1/3)*AppellF1(-1/3,1/3,1,2/3,-b*x^3/a,-d*x^3/c)/c/x/(b*x^3+a
)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(62) = 124.

Time = 10.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{-10c(a+bx^3)+5(bc-ad)x^3\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bdx^6\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}}{10ac^2x\sqrt[3]{a+bx^3}}$$

input
```
Integrate[1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-10*c*(a + b*x^3) + 5*(b*c - a*d)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3,
 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*d*x^6*(1 + (b*x^3)/a)^(1/3
)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(10*a*c^2*x*(a +
 b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3} (c+dx^3)} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1} \int \frac{1}{x^2 \sqrt[3]{\frac{bx^3}{a}+1} (dx^3+c)} dx}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1} \operatorname{AppellF1}\left(-\frac{1}{3},\frac{1}{3},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx \sqrt[3]{a+bx^3}}$$

input
```
Int[1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
-(((1 + (b*x^3)/a)^(1/3)*AppellF1[-1/3, 1/3, 1, 2/3, -((b*x^3)/a), -((d*x^
3)/c)])/(c*x*(a + b*x^3)^(1/3)))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)}d x$$

input
```
int(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x^2\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(1/x**2/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**2*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x^2\,(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,c\,x^2+(b\,x^3+a)^{\frac{1}{3}}\,d\,x^5}dx$$

input
```
int(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*c*x**2 + (a + b*x**3)**(1/3)*d*x**5),x)
```

3.746 $\int \frac{1}{x^5 \sqrt[3]{a+bx^3}(c+dx^3)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3} (c+dx^3)} \, dx = -\frac{\sqrt[3]{1+\frac{bx^3}{a}} \operatorname{AppellF1}\left(-\frac{4}{3},\frac{1}{3},1,-\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4 \sqrt[3]{a+bx^3}}$$

output
```
-1/4*(1+b*x^3/a)^(1/3)*AppellF1(-4/3,1/3,1,-1/3,-b*x^3/a,-d*x^3/c)/c/x^4/(
b*x^3+a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 183 vs. 2(64) = 128.

Time = 10.20 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.86

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3} (c+dx^3)} \, dx$$
$$= \frac{5c(a+bx^3)(-ac+2bcx^3+4adx^3)+5(-b^2c^2-2abcd+2a^2d^2)x^6\sqrt[3]{1+\frac{bx^3}{a}} \operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},\right.}{20a^2c^3x^4\sqrt[3]{a+bx^3}}$$

input
```
Integrate[1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(5*c*(a + b*x^3)*(-(a*c) + 2*b*c*x^3 + 4*a*d*x^3) + 5*(-(b^2*c^2) - 2*a*b*
c*d + 2*a^2*d^2)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b
*x^3)/a), -((d*x^3)/c)] - 2*b*d*(b*c + 2*a*d)*x^9*(1 + (b*x^3)/a)^(1/3)*Ap
pellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*a^2*c^3*x^4*(a +
 b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3} \left(c+dx^3\right)} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1} \int \frac{1}{x^5 \sqrt[3]{\frac{bx^3}{a}+1}(dx^3+c)}dx}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1} \operatorname{AppellF1}\left(-\frac{4}{3},\frac{1}{3},1,-\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4 \sqrt[3]{a+bx^3}}$$

input
```
Int[1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
-1/4*((1 + (b*x^3)/a)^(1/3)*AppellF1[-4/3, 1/3, 1, -1/3, -((b*x^3)/a), -((
d*x^3)/c)])/(c*x^4*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{5} \left(b\,x^{3}+a \right)^{\frac{1}{3}} \left(d\,x^{3}+c \right)}d x$$

input
```
int(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3} \left(c+dx^3\right)} \, dx=\text{Timed out}$$

input
```
integrate(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x^5 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(1/x**5/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**5*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{x^5\,\left(b\,x^3+a\right)^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^5\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\,c\,x^5+\left(b\,x^3+a\right)^{\frac{1}{3}}\,d\,x^8}dx$$

input
```
int(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*c*x**5 + (a + b*x**3)**(1/3)*d*x**8),x)
```

3.747 $\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 241

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{(b^2c^2+abcd+a^2d^2)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{4/3}}{4b^3d^2} + \frac{(a+bx^3)^{7/3}}{7b^3d} + \frac{c^3\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}(bc-ad)^{2/3}} + \frac{c^3\log(c+dx^3)}{6d^{10/3}(bc-ad)^{2/3}} - \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}(bc-ad)^{2/3}}$$

output
```
(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(1/3)/b^3/d^3-1/4*(2*a*d+b*c)*(b*x^3+a
)^(4/3)/b^3/d^2+1/7*(b*x^3+a)^(7/3)/b^3/d+1/3*c^3*arctan(1/3*(1-2*d^(1/3)*
(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/d^(10/3)/(-a*d+b*c)^(2/
3)+1/6*c^3*ln(d*x^3+c)/d^(10/3)/(-a*d+b*c)^(2/3)-1/2*c^3*ln((-a*d+b*c)^(1/
3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(10/3)/(-a*d+b*c)^(2/3)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.10

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{3\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{a+bx^3}(18a^2d^2+3abd(7c-2dx^3)+b^2(28c^2-7cdx^3+4\cdots}{\cdots}$$

input
```
Integrate[x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(3*d^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3)*(18*a^2*d^2 + 3*a*b*d*(7*c
- 2*d*x^3) + b^2*(28*c^2 - 7*c*d*x^3 + 4*d^2*x^6)) + 28*Sqrt[3]*b^3*c^3*Ar
cTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 28*b
^3*c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + 14*b^3*c^3*Log
[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)
*(a + b*x^3)^(2/3)])/(84*b^3*d^(10/3)*(b*c - a*d)^(2/3))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^9}{(bx^3+a)^{2/3}(dx^3+c)}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(-\frac{c^3}{d^3\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}+\frac{\left(bx^3+a\right)^{4/3}}{b^2d}+\frac{(-bc-2ad)\sqrt[3]{bx^3+a}}{b^2d^2}+\frac{b^2c^2+abdc+a^2d^2}{b^2d^3\left(bx^3+a\right)^{2/3}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}(a^2d^2+abcd+b^2c^2)}{b^3d^3}+\frac{\sqrt{3}c^3\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{10/3}(bc-ad)^{2/3}}-\frac{3\left(a+bx^3\right)^{4/3}(2ad+bc)}{4b^3d^2}+\frac{3(a+b}{7b}\right.$$

input
```
Int[x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
((3*(b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(1/3))/(b^3*d^3) - (3*(b*c +
 2*a*d)*(a + b*x^3)^(4/3))/(4*b^3*d^2) + (3*(a + b*x^3)^(7/3))/(7*b^3*d) +
 (Sqrt[3]*c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))
/Sqrt[3]])/(d^(10/3)*(b*c - a*d)^(2/3)) + (c^3*Log[c + d*x^3])/(2*d^(10/3)
*(b*c - a*d)^(2/3)) - (3*c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(
1/3)])/(2*d^(10/3)*(b*c - a*d)^(2/3)))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{\frac{27\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\left(\frac{\left(2d^{2}x^{6}-\frac{7}{2}cd\,x^{3}+14c^{2}\right)b^{2}}{9}+\frac{7ad\left(-\frac{2d\,x^{3}}{7}+c\right)b}{6}+a^{2}d^{2}\right)d\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{7}+b^{3}c^{3}\left(2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right.\right.}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}b^{3}d^{4}}$

input
```
int(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/d)^(2/3)*(27/7*((a*d-b*c)/d)^(2/3)*(1/9*(2*d^2*x^6-7/2*c*d*
x^3+14*c^2)*b^2+7/6*a*d*(-2/7*d*x^3+c)*b+a^2*d^2)*d*(b*x^3+a)^(1/3)+b^3*c^
3*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*c)
/d)^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+
((a*d-b*c)/d)^(2/3))-2*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))))/b^3/d^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(201) = 402.

Time = 0.11 (sec) , antiderivative size = 1322, normalized size of antiderivative = 5.49

$$\int \frac{x^{11}}{\left(a+bx^{3}\right)^{2/3}\left(c+dx^{3}\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/84*(14*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 +
 a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*
c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) -
 28*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(1
/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 42*sqr
t(1/3)*(b^4*c^4*d - a*b^3*c^3*d^2)*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^
3)^(1/3)/d)*log((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x
^3 - 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*
b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3
)^(2/3)*(b*x^3 + a)^(1/3))*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)
/d) - 3*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c
- a*d))/(d*x^3 + c)) + 3*(28*b^4*c^4*d - 35*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*
d^3 - 15*a^3*b*c*d^4 + 18*a^4*d^5 + 4*(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b
^2*d^5)*x^6 - (7*b^4*c^3*d^2 - 8*a*b^3*c^2*d^3 - 5*a^2*b^2*c*d^4 + 6*a^3*b
*d^5)*x^3)*(b*x^3 + a)^(1/3))/(b^5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6),
 1/84*(14*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 +
 a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*
c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) -
 28*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(1
/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 84*...
```

Sympy [F]

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\int \frac{x^{11}}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**11/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**11/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.54

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{b^{24}c^3d^4\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{25}cd^7-ab^{24}d^8)} - \frac{(-bcd^2+ad^3)^{\frac{1}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^4-\sqrt{3}ad^5} - \frac{(-bcd^2+ad^3)^{\frac{1}{3}}c^3\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^4-ad^5)} + \frac{28\,(bx^3+a)^{\frac{1}{3}}b^{20}c^2d^4-7\,(bx^3+a)^{\frac{4}{3}}b^{19}cd^5+28\,(bx^3+a)^{\frac{1}{3}}ab^{19}cd^5+4\,(bx^3+a)^{\frac{7}{3}}b^{18}d^6-14\,(bx^3+a)^{\frac{4}{3}}ab^{1}}{28\,b^{21}d^7}$$

input
```
integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*b^24*c^3*d^4*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^25*c*d^7 - a*b^24*d^8) - (-b*c*d^2 + a*d^3)^(1/3)*c^
3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c
 - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^4 - sqrt(3)*a*d^5) - 1/6*(-b*c*d^2 + a*d^
3)^(1/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1
/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^4 - a*d^5) + 1/28*(28*(b*x^3 + a)^(1/
3)*b^20*c^2*d^4 - 7*(b*x^3 + a)^(4/3)*b^19*c*d^5 + 28*(b*x^3 + a)^(1/3)*a*
b^19*c*d^5 + 4*(b*x^3 + a)^(7/3)*b^18*d^6 - 14*(b*x^3 + a)^(4/3)*a*b^18*d^
6 + 28*(b*x^3 + a)^(1/3)*a^2*b^18*d^6)/(b^21*d^7)
```

Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.37

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \left(\frac{3\,a^2}{b^3\,d} + \frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{b^3\,d}\right)\,{\left(b\,x^3+a\right)}^{1/3} - \left(\frac{3\,a}{4\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{4\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{4/3} + \frac{{\left(b\,x^3+a\right)}^{7/3}}{7\,b^3\,d} + \frac{\ln\left(\frac{3\,c^3\,{\left(b\,x^3+a\right)}^{1/3}}{d}+\frac{3\,c^3\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{2\,d^{4/3}}\right)\,\left(c^3+\sqrt{3}\,c^3\,1{}\mathrm{i}\right)}{6\,d^{10/3}\,{\left(a\,d-b\,c\right)}^{2/3}} - \frac{c^3\,\ln\left(\frac{3\,c^3\,{\left(b\,x^3+a\right)}^{1/3}}{d}-\frac{3\,c^3\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{4/3}}\right)}{3\,d^{10/3}\,{\left(a\,d-b\,c\right)}^{2/3}} - \frac{c^3\,\ln\left(\frac{3\,c^3\,{\left(b\,x^3+a\right)}^{1/3}}{d}-\frac{3\,c^3\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{4/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,d^{10/3}\,{\left(a\,d-b\,c\right)}^{2/3}}$$

input
```
int(x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
((3*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c -
 a*b^3*d))/(b^3*d))*(a + b*x^3)^(1/3) - ((3*a)/(4*b^3*d) + (b^4*c - a*b^3*
d)/(4*b^6*d^2))*(a + b*x^3)^(4/3) + (a + b*x^3)^(7/3)/(7*b^3*d) + (log((3*
c^3*(a + b*x^3)^(1/3))/d + (3*c^3*(3^(1/2)*1i + 1)*(a*d - b*c)^(1/3))/(2*d
^(4/3)))*(3^(1/2)*c^3*1i + c^3))/(6*d^(10/3)*(a*d - b*c)^(2/3)) - (c^3*log
((3*c^3*(a + b*x^3)^(1/3))/d - (3*c^3*(a*d - b*c)^(1/3))/d^(4/3)))/(3*d^(1
0/3)*(a*d - b*c)^(2/3)) - (c^3*log((3*c^3*(a + b*x^3)^(1/3))/d - (3*c^3*((
3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(1/3))/d^(4/3))*((3^(1/2)*1i)/2 - 1/2))/(
3*d^(10/3)*(a*d - b*c)^(2/3))
```

Reduce [F]

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^{11}}{(b\,x^3+a)^{\frac{2}{3}}\,c+(b\,x^3+a)^{\frac{2}{3}}\,d\,x^3}dx$$

input
```
int(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x**11/((a + b*x**3)**(2/3)*c + (a + b*x**3)**(2/3)*d*x**3),x)
```

3.748 $\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 201

$$\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{(a+bx^3)^{4/3}}{4b^2d} - \frac{c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}(bc-ad)^{2/3}} - \frac{c^2\log(c+dx^3)}{6d^{7/3}(bc-ad)^{2/3}} + \frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}(bc-ad)^{2/3}}$$

output
```
-(a*d+b*c)*(b*x^3+a)^(1/3)/b^2/d^2+1/4*(b*x^3+a)^(4/3)/b^2/d-1/3*c^2*arcta
n(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/d^(7
/3)/(-a*d+b*c)^(2/3)-1/6*c^2*ln(d*x^3+c)/d^(7/3)/(-a*d+b*c)^(2/3)+1/2*c^2*
ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(7/3)/(-a*d+b*c)^(2/3)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.15

$$\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{-3\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{a+bx^3}(4bc+3ad-bdx^3)-4\sqrt{3}b^2c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\cdots}{\sqrt[3]{b\cdots}}}{\cdots}\right.\cdots}{\cdots}$$

input
```
Integrate[x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(-3*d^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3)*(4*b*c + 3*a*d - b*d*x^3)
- 4*Sqrt[3]*b^2*c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^
(1/3))/Sqrt[3]] + 4*b^2*c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1
/3)] - 2*b^2*c^2*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*
x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(12*b^2*d^(7/3)*(b*c - a*d)^(2/3)
)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{x^6}{(bx^3+a)^{2/3}(dx^3+c)}dx^3$$

$$\Big\downarrow \; 99$$

$$\frac{1}{3}\int \left(\frac{c^2}{d^2(bx^3+a)^{2/3}(dx^3+c)}+\frac{\sqrt[3]{bx^3+a}}{bd}+\frac{-bc-ad}{bd^2(bx^3+a)^{2/3}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{7/3}(bc-ad)^{2/3}}-\frac{3\sqrt[3]{a+bx^3}(ad+bc)}{b^2d^2}+\frac{3\left(a+bx^3\right)^{4/3}}{4b^2d}-\frac{c^2\log\left(c+dx^3\right)}{2d^{7/3}(bc-ad)^{2/3}}+\frac{3c^2\log}{}\right.$$

input
```
Int[x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
((-3*(b*c + a*d)*(a + b*x^3)^(1/3))/(b^2*d^2) + (3*(a + b*x^3)^(4/3))/(4*b
^2*d) - (Sqrt[3]*c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)
^(1/3))/Sqrt[3]])/(d^(7/3)*(b*c - a*d)^(2/3)) - (c^2*Log[c + d*x^3])/(2*d^
(7/3)*(b*c - a*d)^(2/3)) + (3*c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x
^3)^(1/3)])/(2*d^(7/3)*(b*c - a*d)^(2/3)))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{3\left(\frac{2\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)b^{2}c^{2}}{9}+\frac{4c^{2}\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)b^{2}}{9}-\frac{4\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\right.\right.}{9}}{4\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}d^{3}b^{2}}$

input
```
int(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-3/4/((a*d-b*c)/d)^(2/3)*(2/9*ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^
3+a)^(1/3)+((a*d-b*c)/d)^(2/3))*b^2*c^2+4/9*c^2*3^(1/2)*arctan(2/3*3^(1/2)
/((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+1/3*3^(1/2))*b^2-4/9*ln((b*x^3+a)^(1/
3)-((a*d-b*c)/d)^(1/3))*b^2*c^2+((a*d-b*c)/d)^(2/3)*d*(1/3*(-d*x^3+4*c)*b+
a*d)*(b*x^3+a)^(1/3))/d^3/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(164) = 328$.

Time = 0.13 (sec) , antiderivative size = 1156, normalized size of antiderivative = 5.75

$$\int \frac{x^8}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/12*(2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log(-(b*x^3 +
a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c
- a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 4*
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log(-(b*x^3 + a)^(1/3)*(
b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)) + 6*sqrt(1/3)*
(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3
)/d)*log((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 + 3*
sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2
+ a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(
b*x^3 + a)^(1/3))*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d) + 3*(
b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d
*x^3 + c)) + 3*(4*b^3*c^3*d - 5*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + 3*a^3*d^4
- (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(1/3))/(b^4*c
^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5), -1/12*(2*(b^2*c^2*d - 2*a*b*c*d^2 +
 a^2*d^3)^(2/3)*b^2*c^2*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 4*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^
(2/3)*b^2*c^2*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)^(2/3)) - 12*sqrt(1/3)*(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt((b
^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)*arctan(-sqrt(1/3)*((b^2*c^2*...
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^8}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**8/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**8/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.53

$$\int \frac{x^8}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx =$$

$$-\frac{\frac{4\,b^2c^2d^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{bcd^4-ad^5}-\frac{12\left(-bcd^2+ad^3\right)^{\frac{1}{3}}b^2c^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^3-\sqrt{3}ad^4}-\frac{2\left(-bcd^2+ad\right.}{}}{12\,b^2}$$

input
```
integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/12*(4*b^2*c^2*d^2*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-
(b*c - a*d)/d)^(1/3)))/(b*c*d^4 - a*d^5) - 12*(-b*c*d^2 + a*d^3)^(1/3)*b^2
*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(
b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) - 2*(-b*c*d^2 + a*d
^3)^(1/3)*b^2*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/
d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^3 - a*d^4) + 3*(4*(b*x^3 + a)^(1
/3)*b*c*d^2 - (b*x^3 + a)^(4/3)*d^3 + 4*(b*x^3 + a)^(1/3)*a*d^3)/d^4)/b^2
```

Mupad [B] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.45

$$\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{\left(b\,x^3+a\right)^{4/3}}{4\,b^2\,d} - \left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b\,x^3+a\right)^{1/3} - \frac{\ln\left(3\,c^2\,\left(b\,x^3+a\right)^{1/3}+\frac{\left(c^2+\sqrt{3}\,c^2\,1\mathrm{i}\right)\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{6\,d^{7/3}\,\left(a\,d-b\,c\right)^{2/3}}\right)\,\left(c^2+\sqrt{3}\,c^2\,1\mathrm{i}\right)}{6\,d^{7/3}\,\left(a\,d-b\,c\right)^{2/3}} + \frac{c^2\,\ln\left(3\,c^2\,\left(b\,x^3+a\right)^{1/3}-\frac{c^2\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}\,\left(a\,d-b\,c\right)^{2/3}}\right)}{3\,d^{7/3}\,\left(a\,d-b\,c\right)^{2/3}} + \frac{c^2\,\ln\left(3\,c^2\,\left(b\,x^3+a\right)^{1/3}-\frac{c^2\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{d^{7/3}\,\left(a\,d-b\,c\right)^{2/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{d^{7/3}\,\left(a\,d-b\,c\right)^{2/3}}$$

input
```
int(x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
(a + b*x^3)^(4/3)/(4*b^2*d) - ((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2)
)*(a + b*x^3)^(1/3) - (log(3*c^2*(a + b*x^3)^(1/3) + ((3^(1/2)*c^2*1i + c^
2)*(9*a*d^3 - 9*b*c*d^2))/(6*d^(7/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*c^2*1i +
 c^2))/(6*d^(7/3)*(a*d - b*c)^(2/3)) + (c^2*log(3*c^2*(a + b*x^3)^(1/3) -
(c^2*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)*(a*d - b*c)^(2/3))))/(3*d^(7/3)*(a*
d - b*c)^(2/3)) + (c^2*log(3*c^2*(a + b*x^3)^(1/3) - (c^2*((3^(1/2)*1i)/6
- 1/6)*(9*a*d^3 - 9*b*c*d^2))/(d^(7/3)*(a*d - b*c)^(2/3)))*((3^(1/2)*1i)/6
 - 1/6))/(d^(7/3)*(a*d - b*c)^(2/3))
```

Reduce [F]

$$\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^8}{\left(b\,x^3+a\right)^{\frac{2}{3}} c+\left(b\,x^3+a\right)^{\frac{2}{3}} d\,x^3}dx$$

input
```
int(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x**8/((a + b*x**3)**(2/3)*c + (a + b*x**3)**(2/3)*d*x**3),x)
```

3.749 $\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 165

$$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{4/3}(bc-ad)^{2/3}} + \frac{c\log(c+dx^3)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}}$$

output
```
(b*x^3+a)^(1/3)/b/d+1/3*c*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*
c)^(1/3))*3^(1/2))*3^(1/2)/d^(4/3)/(-a*d+b*c)^(2/3)+1/6*c*ln(d*x^3+c)/d^(4
/3)/(-a*d+b*c)^(2/3)-1/2*c*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^
(4/3)/(-a*d+b*c)^(2/3)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.22

$$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{6\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{a+bx^3}+2\sqrt{3}bc\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-2bc\log\left(\right.}{}$$

input
```
Integrate[x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(6*d^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3) + 2*Sqrt[3]*b*c*ArcTan[(1 -
 (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*b*c*Log[(b*
c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + b*c*Log[(b*c - a*d)^(2/3) -
d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/
(6*b*d^(4/3)*(b*c - a*d)^(2/3))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 90, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{x^3}{(bx^3+a)^{2/3}(dx^3+c)}dx^3$$

$$\Big\downarrow \ 90$$

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}}{bd}-\frac{c\int \frac{1}{(bx^3+a)^{2/3}(dx^3+c)}dx^3}{d}\right)$$

$$\Big\downarrow \text{70}$$

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}}{bd}-\frac{c\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{d}(bc-ad)^{2/3}}-\right.}{d}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}}{bd}-\frac{c\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log(c+dx^3)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a}\right.}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right.}{d}\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}}{bd}-\frac{c\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log(c+dx^3)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right)}{d}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}}{bd}-\frac{c\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log(c+dx^3)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right)}{d}\right)$$

input
```
Int[x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
((3*(a + b*x^3)^(1/3))/(b*d) - (c*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a +
b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(1/3)*(b*c - a*d)^(2/3))) -
Log[c + d*x^3]/(2*d^(1/3)*(b*c - a*d)^(2/3)) + (3*Log[(b*c - a*d)^(1/3) +
d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1/3)*(b*c - a*d)^(2/3))))/d)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{3(b x^3+a)^{\frac{1}{3}} d \left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}+\frac{bc \left(2 \arctan \left(\frac{2\sqrt{3}\left(b x^3+a\right)^{\frac{1}{3}}}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)\sqrt{3}-2\ln \left(\left(b x^3+a\right)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)+\ln \left(\left(b x^3+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}} (\right.\right.}{2}}{3\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}} b d^2}$

input
```
int(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/d)^(2/3)*(3*(b*x^3+a)^(1/3)*d*((a*d-b*c)/d)^(2/3)+1/2*b*c*(
2*arctan(2/3*3^(1/2)/((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+1/3*3^(1/2))*3^(1
/2)-2*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))+ln((b*x^3+a)^(2/3)+((a*d-b*c
)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))))/b/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(133) = 266$.

Time = 0.11 (sec) , antiderivative size = 1060, normalized size of antiderivative = 6.42

$$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/6*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(2/3
)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d)
 + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 2*(-b^2
*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(1/3)*(b*c*d -
a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 3*sqrt(1/3)*(b^2*c^
2*d - a*b*c*d^2)*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)*log((b
^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 - 3*sqrt(1/3)*(
2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)
^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a
)^(1/3))*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d) - 3*(-b^2*c^2*
d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*x^3 + c
)) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x^3 + a)^(1/3))/(b^3*c^2*d^2
 - 2*a*b^2*c*d^3 + a^2*b*d^4), 1/6*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(
2/3)*b*c*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^
2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3
)*(b*x^3 + a)^(1/3)) - 2*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*lo
g(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3
)^(2/3)) - 6*sqrt(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt(-(-b^2*c^2*d + 2*a*b*c
*d^2 - a^2*d^3)^(1/3)/d)*arctan(sqrt(1/3)*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2
*d^3)^(1/3)*(b*c - a*d) + 2*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*...
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^5}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**5/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**5/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.53

$$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx =$$

$$-\frac{\frac{6\left(-bcd^2+ad^3\right)^{\frac{1}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}bc\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{bcd^2-ad^3}}{6\,b}$$

input
```
integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/6*(6*(-b*c*d^2 + a*d^3)^(1/3)*b*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/
3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^2 - sq
rt(3)*a*d^3) + (-b*c*d^2 + a*d^3)^(1/3)*b*c*log((b*x^3 + a)^(2/3) + (b*x^3
 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^2 - a*
d^3) - 2*b*c*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a
*d)/d)^(1/3)))/(b*c*d - a*d^2) - 6*(b*x^3 + a)^(1/3)/d)/b
```

Mupad [B] (verification not implemented)

Time = 3.66 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.41

$$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{(b\,x^3+a)^{1/3}}{b\,d} - \frac{c\,\ln\left(3\,c\,d\,(b\,x^3+a)^{1/3}-\frac{c\,(9\,a\,d^3-9\,b\,c\,d^2)}{3\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)}{3\,d^{4/3}\,(a\,d-b\,c)^{2/3}} + \frac{\ln\left(3\,c\,d\,(b\,x^3+a)^{1/3}+\frac{(9\,a\,d^3-9\,b\,c\,d^2)\,\left(c-\sqrt{3}\,c\,1\mathrm{i}\right)}{6\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(c-\sqrt{3}\,c\,1\mathrm{i}\right)}{6\,d^{4/3}\,(a\,d-b\,c)^{2/3}} + \frac{\ln\left(3\,c\,d\,(b\,x^3+a)^{1/3}+\frac{(9\,a\,d^3-9\,b\,c\,d^2)\,\left(c+\sqrt{3}\,c\,1\mathrm{i}\right)}{6\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(c+\sqrt{3}\,c\,1\mathrm{i}\right)}{6\,d^{4/3}\,(a\,d-b\,c)^{2/3}}$$

input
```
int(x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
(a + b*x^3)^(1/3)/(b*d) - (c*log(3*c*d*(a + b*x^3)^(1/3) - (c*(9*a*d^3 - 9
*b*c*d^2))/(3*d^(4/3)*(a*d - b*c)^(2/3))))/(3*d^(4/3)*(a*d - b*c)^(2/3)) +
 (log(3*c*d*(a + b*x^3)^(1/3) + ((9*a*d^3 - 9*b*c*d^2)*(c - 3^(1/2)*c*1i))
/(6*d^(4/3)*(a*d - b*c)^(2/3)))*(c - 3^(1/2)*c*1i))/(6*d^(4/3)*(a*d - b*c)
^(2/3)) + (log(3*c*d*(a + b*x^3)^(1/3) + ((9*a*d^3 - 9*b*c*d^2)*(c + 3^(1/
2)*c*1i))/(6*d^(4/3)*(a*d - b*c)^(2/3)))*(c + 3^(1/2)*c*1i))/(6*d^(4/3)*(a
*d - b*c)^(2/3))
```

Reduce [F]

$$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^5}{(b\,x^3+a)^{\frac{2}{3}}\,c+(b\,x^3+a)^{\frac{2}{3}}\,d\,x^3}dx$$

input
```
int(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x**5/((a + b*x**3)**(2/3)*c + (a + b*x**3)**(2/3)*d*x**3),x)
```

3.750 $\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)^{2/3}} - \frac{\log(c+dx^3)}{6\sqrt[3]{d}(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}$$

output
```
-1/3*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^
(1/2)/d^(1/3)/(-a*d+b*c)^(2/3)-1/6*ln(d*x^3+c)/d^(1/3)/(-a*d+b*c)^(2/3)+1/
2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(1/3)/(-a*d+b*c)^(2/3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{6\sqrt[3]{d}(bc-ad)^{2/3}}$$

input
```
Integrate[x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3
))/Sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(
b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(
a + b*x^3)^(2/3)])/(d^(1/3)*(b*c - a*d)^(2/3))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {946, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}dx$$

$$\Big\downarrow \ 946$$

$$\frac{1}{3}\int \frac{1}{(bx^3+a)^{2/3}(dx^3+c)}dx^3$$

$$\Big\downarrow \ 70$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log(c+dx^3)}{2\sqrt[3]{d}(bc-ad)^{2/}}\right.$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log(c+dx^3)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right)$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log(c+dx^3)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log(c+dx^3)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right)$$

input
```
Int[x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/S
qrt[3]])/(d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c - a
*d)^(2/3)) + (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(
1/3)*(b*c - a*d)^(2/3)))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{-2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)-\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\right.}{6d\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}}$

input
```
int(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b
*c)/d)^(1/3))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))-ln((b*x^3+
a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3)))/d/((a*d
-b*c)/d)^(2/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(114) = 228$.

Time = 0.11 (sec) , antiderivative size = 927, normalized size of antiderivative = 6.39

$$\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/6*(3*sqrt(1/3)*(b*c*d - a*d^2)*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^
3)^(1/3)/d)*log((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x
^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b
*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^
(2/3)*(b*x^3 + a)^(1/3))*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d
) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a
*d))/(d*x^3 + c)) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3
+ a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*
c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) -
2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(1/3)*(b*c*d
- a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)))/(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3), 1/6*(6*sqrt(1/3)*(b*c*d - a*d^2)*sqrt((b^2*c^2*d - 2*a*b
*c*d^2 + a^2*d^3)^(1/3)/d)*arctan(-sqrt(1/3)*((b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)^(1/3)*(b*c - a*d) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b
*x^3 + a)^(1/3))*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)/(b^2*c^
2 - 2*a*b*c*d + a^2*d^2)) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(
-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(
1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(
1/3)) + 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(1/3)
*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)))/(b^2*c^2...
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx=\int \frac{x^2}{(a+bx^3)^{\frac{2}{3}}\,(c+dx^3)}\,dx$$

input
```
integrate(x**2/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**2/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.52

$$\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{(-bcd^2+ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd-\sqrt{3}ad^2} + \frac{(-bcd^2+ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd-ad^2)} - \frac{\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc-ad)}$$

input
```
integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
(-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c
- a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d - sqrt(3)*a*d^2) +
 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-
(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d - a*d^2) - 1/3*(-(b*
c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*
c - a*d)
```

Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.47

$$\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{\ln\left(3\,d^2\,(b\,x^3+a)^{1/3}-\frac{9\,a\,d^3-9\,b\,c\,d^2}{3\,d^{1/3}\,(a\,d-b\,c)^{2/3}}\right)}{3\,d^{1/3}\,(a\,d-b\,c)^{2/3}} + \frac{\ln\left(3\,d^2\,(b\,x^3+a)^{1/3}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,(9\,a\,d^3-9\,b\,c\,d^2)}{6\,d^{1/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,d^{1/3}\,(a\,d-b\,c)^{2/3}} - \frac{\ln\left(3\,d^2\,(b\,x^3+a)^{1/3}+\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)\,(9\,a\,d^3-9\,b\,c\,d^2)}{6\,d^{1/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,d^{1/3}\,(a\,d-b\,c)^{2/3}}$$

input
```
int(x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
log(3*d^2*(a + b*x^3)^(1/3) - (9*a*d^3 - 9*b*c*d^2)/(3*d^(1/3)*(a*d - b*c)
^(2/3)))/(3*d^(1/3)*(a*d - b*c)^(2/3)) + (log(3*d^2*(a + b*x^3)^(1/3) - ((
3^(1/2)*1i - 1)*(9*a*d^3 - 9*b*c*d^2))/(6*d^(1/3)*(a*d - b*c)^(2/3)))*(3^(
1/2)*1i - 1))/(6*d^(1/3)*(a*d - b*c)^(2/3)) - (log(3*d^2*(a + b*x^3)^(1/3)
 + ((3^(1/2)*1i + 1)*(9*a*d^3 - 9*b*c*d^2))/(6*d^(1/3)*(a*d - b*c)^(2/3)))
*(3^(1/2)*1i + 1))/(6*d^(1/3)*(a*d - b*c)^(2/3))
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^2}{(b\,x^3+a)^{\frac{2}{3}}\,c+(b\,x^3+a)^{\frac{2}{3}}\,d\,x^3}dx$$

input
```
int(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x**2/((a + b*x**3)**(2/3)*c + (a + b*x**3)**(2/3)*d*x**3),x)
```

3.751 $\int \frac{1}{x(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 245

$$\begin{aligned}\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx &= -\frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}c} \\ &+ \frac{d^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c(bc-ad)^{2/3}} - \frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}} \\ &+ \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}c} - \frac{d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{2/3}}\end{aligned}$$

output

```
-1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/
3)/c+1/3*d^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))
*3^(1/2))*3^(1/2)/c/(-a*d+b*c)^(2/3)-1/2*ln(x)/a^(2/3)/c+1/6*d^(2/3)*ln(d*
x^3+c)/c/(-a*d+b*c)^(2/3)+1/2*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(2/3)/c-1/2*d^
(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c/(-a*d+b*c)^(2/3)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.26

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{-\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{2\sqrt{3}d^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{(bc-ad)^{2/3}}+\frac{2\log\left(-\sqrt[3]{a}\right.}{}$$

input
```
Integrate[1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
((-2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)
+ (2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)
^(1/3))/Sqrt[3]])/(b*c - a*d)^(2/3) + (2*Log[-a^(1/3) + (a + b*x^3)^(1/3)]
)/a^(2/3) - (2*d^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(b*c - a*d)^(2/3) - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)
^(2/3)]/a^(2/3) + (d^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/
3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(2/3))/(6*c
)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 97, 69, 16, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \text{97}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{x^3(bx^3+a)^{2/3}}dx^3}{c}-\frac{d\int \frac{1}{(bx^3+a)^{2/3}(dx^3+c)}dx^3}{c}\right)$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{3}\left(\frac{-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2a^{2/3}}}{c}-\frac{d\int \frac{1}{(bx^3+a)^{2/3}(dx^3+c)}dx^3}{c}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}}{c}-\frac{d\int \frac{1}{(bx^3+a)^{2/3}(dx^3+c)}dx^3}{c}\right)$$

$$\Big\downarrow \text{70}$$

$$\frac{1}{3}\left(\frac{-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}}{c}-\frac{d\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{b}}{\sqrt[3]{d}}}}{2d^{2/3}\sqrt[3]{bc-a}}\right.}{}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}}{c}-\frac{d\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{b}}{\sqrt[3]{d}}}}{2d^{2/3}\sqrt[3]{bc-a}}\right.}{}\right.$$

$\downarrow$ 1082

$$\frac{1}{3}\left(\frac{\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}}{c}-\frac{d\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log(c+}{2\sqrt[3]{d}(bc-}\right.}{c}\right.$$

$\downarrow$ 217

$$\frac{1}{3}\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}}{c}-\frac{d\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log(c}{2\sqrt[3]{d}(b}\right.}{c}\right.$$

input
```
Int[1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
((-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3))
 - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3))
)/c - (d*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)
^(1/3))/Sqrt[3]])/(d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)
*(b*c - a*d)^(2/3)) + (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)
])/(2*d^(1/3)*(b*c - a*d)^(2/3))))/c)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$-\frac{-\frac{\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)a^{\frac{2}{3}}}{2}-\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\,\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)a^{\frac{2}{3}}+\frac{\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\ln\left(\left(b\,x^{3}+a\right.\right.}{}}{}$

input
```
int(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/((a*d-b*c)/d)^(2/3)*(-1/2*ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x
^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))*a^(2/3)-3^(1/2)*arctan(2/3*3^(1/2)/((a*d-
b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+1/3*3^(1/2))*a^(2/3)+1/2*((a*d-b*c)/d)^(2/3)
*ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))+ln((b*x^3+a)^(1/3)-((
a*d-b*c)/d)^(1/3))*a^(2/3)-((a*d-b*c)/d)^(2/3)*(-arctan(2/3*3^(1/2)/a^(1/3
)*(b*x^3+a)^(1/3)+1/3*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(1/3)-a^(1/3))))/a^(2/
3)/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(192) = 384$.

Time = 0.12 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.91

$$\int \frac{1}{x\,(a+bx^3)^{2/3}\,(c+dx^3)}\,dx =$$

$$-\frac{2\sqrt{3}a^2\left(-\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(bc-ad)\left(-\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3\,d}\right)+a^2\left(-\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)}{}$$

input
```
integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*a^2*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-1
/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d^2/(b^2*c^2 - 2*a*b*c*d + a
^2*d^2))^(2/3) - sqrt(3)*d)/d) + a^2*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)
)^(1/3)*log((b*x^3 + a)^(2/3)*d^2 + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2)*(-d^
2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)) - 2*a^2*(-d^2/(b^2*c^2 - 2*
a*b*c*d + a^2*d^2))^(1/3)*log((b*x^3 + a)^(1/3)*d - (b*c - a*d)*(-d^2/(b^2
*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)) + 6*sqrt(1/3)*(a^2)^(1/6)*a*arctan(sqr
t(1/3)*(a^2)^(1/6)*((a^2)^(1/3)*a + 2*(b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2)
+ (a^2)^(2/3)*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*
(a^2)^(2/3)) - 2*(a^2)^(2/3)*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)))/(a^2*
c)
```

Sympy [F]

$$\int \frac{1}{x\,(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \int \frac{1}{x\,(a+bx^3)^{\frac{2}{3}}\,(c+dx^3)}\,dx$$

input
```
integrate(1/x/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)x}\,dx$$

input
```
integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.31

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2-\sqrt{3}acd}-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^2-acd\right)}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{2}{3}}c}-\frac{\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}c}+\frac{\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3a^{\frac{2}{3}}c}$$

input
```
integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*d*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^
(1/3)))/(b*c^2 - a*c*d) - (-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(
b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3
)*b*c^2 - sqrt(3)*a*c*d) - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2
/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(
b*c^2 - a*c*d) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(
1/3))/a^(1/3))/(a^(2/3)*c) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)
*a^(1/3) + a^(2/3))/(a^(2/3)*c) + 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3))
)/(a^(2/3)*c)
```

Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 1413, normalized size of antiderivative = 5.77

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
log((((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - (243*a*b^6*
c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(1/(27*a^2*c^3))^(1/3
))*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2*d^4)*(1/(27*a^2*c^3))^(1/3) + 6*b^4*
d^5*(a + b*x^3)^(1/3))*(1/(27*a^2*c^3))^(1/3) + log(- (((81*b^6*c^5*d^3 -
162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - (-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2
- 54*a*b*c^4*d))^(1/3)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*
b^4*c^4*d^5))*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(2/3) -
9*b^5*c^2*d^4)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3) -
 6*b^4*d^5*(a + b*x^3)^(1/3))*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*
c^4*d))^(1/3) + log(((3^(1/2)*1i)/2 - 1/2)*(((3^(1/2)*1i)/2 - 1/2)^2*((81*
b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - ((3^(1/2)*1i)/2 - 1/2
)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(1/(27*a
^2*c^3))^(1/3))*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2*d^4)*(1/(27*a^2*c^3))^(
1/3) + 6*b^4*d^5*(a + b*x^3)^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a^2*c^3)
)^(1/3) - log(6*b^4*d^5*(a + b*x^3)^(1/3) - ((3^(1/2)*1i)/2 + 1/2)*(((3^(1
/2)*1i)/2 + 1/2)^2*((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3)
 + ((3^(1/2)*1i)/2 + 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a
^3*b^4*c^4*d^5)*(1/(27*a^2*c^3))^(1/3))*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2
*d^4)*(1/(27*a^2*c^3))^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a^2*c^3))^(1/3
) + (log(6*b^4*d^5*(a + b*x^3)^(1/3) + ((3^(1/2)*1i - 1)*(((3^(1/2)*1i ...
```

Reduce [F]

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}cx+\left(b\,x^3+a\right)^{\frac{2}{3}}d\,x^4}dx$$

input
```
int(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*c*x + (a + b*x**3)**(2/3)*d*x**4),x)
```

3.752 $\int \frac{1}{x^4 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 299

$$\int \frac{1}{x^4 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)} \, dx = -\frac{\sqrt[3]{a+bx^3}}{3acx^3} + \frac{(2bc+3ad)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}c^2} - \frac{d^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2(bc-ad)^{2/3}} + \frac{(2bc+3ad)\log(x)}{6a^{5/3}c^2} - \frac{d^{5/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{2/3}} - \frac{(2bc+3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{5/3}c^2} + \frac{d^{5/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)^{2/3}}$$

output

```
-1/3*(b*x^3+a)^(1/3)/a/c/x^3+1/9*(3*a*d+2*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^
3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/c^2-1/3*d^(5/3)*arctan(1/3*(1
-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/c^2/(-a*d+b*
c)^(2/3)+1/6*(3*a*d+2*b*c)*ln(x)/a^(5/3)/c^2-1/6*d^(5/3)*ln(d*x^3+c)/c^2/(
-a*d+b*c)^(2/3)-1/6*(3*a*d+2*b*c)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(5/3)/c^2+
1/2*d^(5/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c^2/(-a*d+b*c)^(2
/3)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^4 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \frac{-\frac{6c\sqrt[3]{a+bx^3}}{ax^3} + \frac{2\sqrt{3}(2bc+3ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} - \frac{6\sqrt{3}d^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{}}{\sqrt[3]{b}}}{}\right)}{(bc-ad)^{2/3}}}{}$$

input
```
Integrate[1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
((-6*c*(a + b*x^3)^(1/3))/(a*x^3) + (2*Sqrt[3]*(2*b*c + 3*a*d)*ArcTan[(1 +
 (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(5/3) - (6*Sqrt[3]*d^(5/3)*Arc
Tan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(b*c -
 a*d)^(2/3) - (2*(2*b*c + 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(5/3
) + (6*d^(5/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c -
a*d)^(2/3) + ((2*b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a
 + b*x^3)^(2/3)])/a^(5/3) - (3*d^(5/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*
c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)
^(2/3))/(18*c^2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {948, 114, 27, 174, 69, 16, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^3)^{2/3} (c+dx^3)} \, dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{1}{x^6 (bx^3+a)^{2/3} (dx^3+c)}dx^3$$

$$\Big\downarrow \; 114$$

$$\frac{1}{3}\left(-\frac{\int \frac{2bdx^3+2bc+3ad}{3x^3(bx^3+a)^{2/3}(dx^3+c)}dx^3}{ac}-\frac{\sqrt[3]{a+bx^3}}{acx^3}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(-\frac{\int \frac{2bdx^3+2bc+3ad}{x^3(bx^3+a)^{2/3}(dx^3+c)}dx^3}{3ac}-\frac{\sqrt[3]{a+bx^3}}{acx^3}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+2bc)\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3}{c}-\frac{3ad^2\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{c}}{3ac}-\frac{\sqrt[3]{a+bx^3}}{acx^3}\right)$$

$$\Big\downarrow \; 69$$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+2bc)\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{3ad^2\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(\ldots\right.}}{c}}{3ac}\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+2bc)\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{3ad^2\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{c}}{3ac}\right.$$

$$\Big\downarrow \; 70$$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+2bc)\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{3ad^2\left(\frac{3\int\frac{\cdots}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\sqrt[3]{b\cdots}}}{2d\cdots}\cdots\right.}{\cdots}}{3ac}\cdots\right.$$

↓ 16

$$\frac{1}{3}\left(-\frac{\frac{(3ad+2bc)\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{3ad^2\left(\frac{3\int\frac{\cdots}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\sqrt[3]{b\cdots}}}{2d\cdots}\cdots\right.}{\cdots}}{3ac}\cdots\right.$$

↓ 1082

$$\frac{1}{3}\left(-\frac{\frac{(3ad+2bc)\left(\frac{3\int\frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{3ad^2\left(\frac{3\int\frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\cdots\right.}{\cdots}}{3ac}\cdots\right.$$

↓ 217

$$\frac{1}{3}\left(-\frac{\frac{(3ad+2bc)\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{3ad^2\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\cdots\right.}{\cdots}}{3ac}\cdots\right.$$

input
```
Int[1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(-((a + b*x^3)^(1/3)/(a*c*x^3)) - (((2*b*c + 3*a*d)*(-((Sqrt[3]*ArcTan[(1
+ (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^3]/(2*a^(2/3))
 + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3))))/c - (3*a*d^2*(-((Sqr
t[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]
)/(d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c - a*d)^(2/
3)) + (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1/3)*(b
*c - a*d)^(2/3))))/c)/(3*a*c))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$\frac{-2\left(b\,x^{3}+a\right)^{\frac{1}{3}} c\,a^{\frac{2}{3}} \left(\frac{a d -b c}{d}\right)^{\frac{2}{3}}+\left(-d \left(2 \arctan\left(\frac{\sqrt{3}\,\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{a d -b c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a d -b c}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{a d -b c}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right.\right.\right.\right.}{}$

input
```
int(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/d)^(2/3)/a^(5/3)*(-2*(b*x^3+a)^(1/3)*c*a^(2/3)*((a*d-b*c)/d
)^(2/3)+(-d*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/
((a*d-b*c)/d)^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3
+a)^(1/3)+((a*d-b*c)/d)^(2/3))-2*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3)))*
a^(5/3)+1/3*((a*d-b*c)/d)^(2/3)*(3*a*d+2*b*c)*(2*arctan(1/3*(a^(1/3)+2*(b*
x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a
)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3))))*x^3)/c^2/x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(239) = 478$.

Time = 0.54 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.87

$$\int \frac{1}{x^4 (a+bx^3)^{2/3} (c+dx^3)}\,dx =$$

$$-\frac{6\sqrt{3}a^3 d\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}} x^3 \arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(bc-ad)\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3\,d}\right)+3\,a^3d\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right.}{}$$

input
```
integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/18*(6*sqrt(3)*a^3*d*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x^3*arc
tan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(d^2/(b^2*c^2 - 2*a*b*c*
d + a^2*d^2))^(2/3) - sqrt(3)*d)/d) + 3*a^3*d*(d^2/(b^2*c^2 - 2*a*b*c*d +
a^2*d^2))^(1/3)*x^3*log((b*x^3 + a)^(2/3)*d^2 - (b*x^3 + a)^(1/3)*(b*c*d -
 a*d^2)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3) + (b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)) - 6*a^3*d*(d^2/(b
^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x^3*log((b*x^3 + a)^(1/3)*d + (b*c -
a*d)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)) - 6*sqrt(1/3)*(2*a*b*c +
 3*a^2*d)*x^3*sqrt(-(-a^2)^(1/3))*arctan(-sqrt(1/3)*((-a^2)^(1/3)*a - 2*(b
*x^3 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2) - (-a^2)^(2/3)*(2*b
*c + 3*a*d)*x^3*log((b*x^3 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^3 + a)^(1/
3)*(-a^2)^(2/3)) + 2*(-a^2)^(2/3)*(2*b*c + 3*a*d)*x^3*log((b*x^3 + a)^(1/3
)*a - (-a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)*a^2*c)/(a^3*c^2*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4 (a+bx^3)^{2/3} (c+dx^3)}\,dx = \int \frac{1}{x^4 (a+bx^3)^{\frac{2}{3}} (c+dx^3)}\,dx$$

input
```
integrate(1/x**4/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**4*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^3)^{2/3} (c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^4 (a+bx^3)^{2/3} (c+dx^3)}\,dx = -\frac{d^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^3-ac^2d)} + \frac{(-bcd^2+ad^3)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^3-\sqrt{3}ac^2d} + \frac{(-bcd^2+ad^3)^{\frac{1}{3}}d\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^3-ac^2d)} + \frac{\sqrt{3}(2\,bc+3\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{5}{3}}c^2} + \frac{(2\,bc+3\,ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{5}{3}}c^2} - \frac{(2\,bc+3\,ad)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{5}{3}}c^2} - \frac{(bx^3+a)^{\frac{1}{3}}}{3\,acx^3}$$

input
```
integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*d^2*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/
d)^(1/3)))/(b*c^3 - a*c^2*d) + (-b*c*d^2 + a*d^3)^(1/3)*d*arctan(1/3*sqrt(
3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/
(sqrt(3)*b*c^3 - sqrt(3)*a*c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*d*log((b*
x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/
d)^(2/3))/(b*c^3 - a*c^2*d) + 1/9*sqrt(3)*(2*b*c + 3*a*d)*arctan(1/3*sqrt(
3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(5/3)*c^2) + 1/18*(2*b*c +
3*a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(5/
3)*c^2) - 1/9*(2*b*c + 3*a*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5/
3)*c^2) - 1/3*(b*x^3 + a)^(1/3)/(a*c*x^3)
```

Mupad [B] (verification not implemented)

Time = 10.98 (sec) , antiderivative size = 1959, normalized size of antiderivative = 6.55

$$\int \frac{1}{x^4 (a + bx^3)^{2/3} (c + dx^3)} dx = \text{Too large to display}$$

input
```
int(1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
log(- (((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c
*d))/a - 81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(d^5/(c^6*(a*d
 - b*c)^2))^(1/3))*(d^5/(c^6*(a*d - b*c)^2))^(2/3))/9 + (b^5*d^4*(8*b^3*c^
3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c))*(d^5/(c^6*(a
*d - b*c)^2))^(1/3))/3 - (2*b^4*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*
c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4))*(d^5/(27*b^2*c^8 + 27
*a^2*c^6*d^2 - 54*a*b*c^7*d))^(1/3) + log(- (((((27*b^5*c^3*d^3*(a + b*x^3
)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - 27*a*b^4*c^4*d^3*(2*a^2*d^2
 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))*(-(3*a*d + 2
*b*c)^3/(a^5*c^6))^(2/3))/81 + (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2
*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c))*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))
/9 - (2*b^4*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d
 + 36*a^2*b*c*d^2))/(9*a^3*c^4))*(-(27*a^3*d^3 + 8*b^3*c^3 + 36*a*b^2*c^2*
d + 54*a^2*b*c*d^2)/(729*a^5*c^6))^(1/3) + (log(((3^(1/2)*1i - 1)*((((27*b
^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - (81*a*
b^4*c^4*d^3*(3^(1/2)*1i - 1)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(d^5/(c^6*(
a*d - b*c)^2))^(1/3))/2)*(3^(1/2)*1i - 1)^2*(d^5/(c^6*(a*d - b*c)^2))^(2/3
))/36 + (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2
))/(3*a^3*c))*(d^5/(c^6*(a*d - b*c)^2))^(1/3))/6 + (2*b^4*d^6*(a + b*x^3)^
(1/3)*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^...
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^3)^{2/3} (c+dx^3)}\, dx = \int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,c\,x^4+(b\,x^3+a)^{\frac{2}{3}}\,d\,x^7}dx$$

input
```
int(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*c*x**4 + (a + b*x**3)**(2/3)*d*x**7),x)
```

3.753 $\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 279

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\frac{x^2\sqrt[3]{a+bx^3}}{3bd}+\frac{(3bc+2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}d^2}-\frac{c^{5/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2(bc-ad)^{2/3}}+\frac{c^{5/3}\log(c+dx^3)}{6d^2(bc-ad)^{2/3}}+\frac{(3bc+2ad)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{5/3}d^2}-\frac{c^{5/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2(bc-ad)^{2/3}}$$

output
```
1/3*x^2*(b*x^3+a)^(1/3)/b/d+1/9*(2*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b
*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(5/3)/d^2-1/3*c^(5/3)*arctan(1/3*(1+2*(-
a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d^2/(-a*d+b*c)^
(2/3)+1/6*c^(5/3)*ln(d*x^3+c)/d^2/(-a*d+b*c)^(2/3)+1/6*(2*a*d+3*b*c)*ln(b^
(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)/d^2-1/2*c^(5/3)*ln((-a*d+b*c)^(1/3)*x/c^(
1/3)-(b*x^3+a)^(1/3))/d^2/(-a*d+b*c)^(2/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.85 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.69

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{\frac{12dx^2\sqrt[3]{a+bx^3}}{b} + \frac{4\sqrt{3}(3bc+2ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{5/3}} + \frac{6\sqrt{-6-6i\sqrt{3}}c^{5/3}\arctan\left(\right.}{}}{}$$

input
```
Integrate[x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
((12*d*x^2*(a + b*x^3)^(1/3))/b + (4*Sqrt[3]*(3*b*c + 2*a*d)*ArcTan[(Sqrt[
3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]])/b^(5/3) + (6*Sqrt[-6 - (
6*I)*Sqrt[3]]*c^(5/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^
(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(b*c - a*d)^(2/3) +
 (4*(3*b*c + 2*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]])/b^(5/3) + (6*(1
 - I*Sqrt[3])*c^(5/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*
(a + b*x^3)^(1/3)]])/(b*c - a*d)^(2/3) - (2*(3*b*c + 2*a*d)*Log[b^(2/3)*x^2
 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]])/b^(5/3) + ((3*I)*(I +
 Sqrt[3])*c^(5/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(
b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)
^(2/3)]])/(b*c - a*d)^(2/3))/(36*d^2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {979, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$$

$$\Big\downarrow \; 979$$

$$\frac{x^2\sqrt[3]{a+bx^3}}{3bd}-\frac{\int \frac{x((3bc+2ad)x^3+2ac)}{(bx^3+a)^{2/3}(dx^3+c)}dx}{3bd}$$

$\big\downarrow$ 1054

$$\frac{x^2\sqrt[3]{a+bx^3}}{3bd}-\frac{\int \left(\frac{(3bc+2ad)x}{d(bx^3+a)^{2/3}}-\frac{3bc^2x}{d(bx^3+a)^{2/3}(dx^3+c)}\right)dx}{3bd}$$

$\big\downarrow$ 2009

$$\frac{x^2\sqrt[3]{a+bx^3}}{3bd}-\frac{-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(2ad+3bc)}{\sqrt{3}b^{2/3}d}+\frac{\sqrt{3}bc^{5/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{d(bc-ad)^{2/3}}-\frac{(2ad+3bc)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}-\frac{bc^{5/3}\log(c-}{2d(bc-ad}}{3bd}$$

input
```
Int[x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(x^2*(a + b*x^3)^(1/3))/(3*b*d) - (-(((3*b*c + 2*a*d)*ArcTan[(1 + (2*b^(1/
3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)*d)) + (Sqrt[3]*b*c^(5/
3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3
]])/(d*(b*c - a*d)^(2/3)) - (b*c^(5/3)*Log[c + d*x^3])/(2*d*(b*c - a*d)^(2
/3)) - ((3*b*c + 2*a*d)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*b^(2/3)*d)
+ (3*b*c^(5/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*
d*(b*c - a*d)^(2/3)))/(3*b*d)
```

Defintions of rubi rules used

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.18

method	result
pseudoelliptic	$\frac{2(b\,x^3+a)^{\frac{1}{3}} x^2 d\, b^{\frac{2}{3}} \left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}+c\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x-2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x}\right)+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^3+a\right)}{x^2}\right.\right.}{}$

input
```
int(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/c)^(2/3)*(2*(b*x^3+a)^(1/3)*x^2*d*b^(2/3)*((a*d-b*c)/c)^(2/
3)+c*(2*3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3
))/((a*d-b*c)/c)^(1/3)/x)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*
(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3
+a)^(1/3))/x))*b^(5/3)-1/3*((a*d-b*c)/c)^(2/3)*(2*a*d+3*b*c)*(2*3^(1/2)*ar
ctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((b^(2/3)*x^2+
b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-b^(1/3)*x+(b*x^3+a)
^(1/3))/x)))/b^(5/3)/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(225) = 450$.

Time = 0.53 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.99

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{6\sqrt{3}b^3c\left(-\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(bc-ad)\left(-\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}+}{3\,cx}\right.}{}$$

input
```
integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/18*(6*sqrt(3)*b^3*c*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(
-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(-c^2/(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2))^(2/3) + sqrt(3)*c*x)/(c*x)) + 6*(b*x^3 + a)^(1/3)*b^2*d*x^2 + 6
*b^3*c*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*log(((b*c - a*d)*(-c^2
/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x + (b*x^3 + a)^(1/3)*c)/x) - 3*b^
3*c*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*log(((b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)*x^2 + (b*x^3 + a)
^(2/3)*c^2 - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d)*(-c^2/(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2))^(1/3)*x)/x^2) - 6*sqrt(1/3)*(3*b^2*c + 2*a*b*d)*(b^2)^(1/6)*ar
ctan(sqrt(1/3)*((b^2)^(1/3)*b*x + 2*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(
1/6)/(b^2*x)) + 2*(b^2)^(2/3)*(3*b*c + 2*a*d)*log(-((b^2)^(2/3)*x - (b*x^3
 + a)^(1/3)*b)/x) - (b^2)^(2/3)*(3*b*c + 2*a*d)*log(((b^2)^(1/3)*b*x^2 + (
b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2))/(b^3*d^2)
```

Sympy [F]

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^7}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**7/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**7/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^7/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^7/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^7}{(b\,x^3+a)^{2/3}\,(d\,x^3+c)}\,dx$$

input
```
int(x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
int(x^7/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^7}{(b\,x^3+a)^{\frac{2}{3}}\,c+(b\,x^3+a)^{\frac{2}{3}}\,d\,x^3}dx$$

input
```
int(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x**7/((a + b*x**3)**(2/3)*c + (a + b*x**3)**(2/3)*d*x**3),x)
```

3.754 $\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 234

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d} + \frac{c^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d(bc-ad)^{2/3}} - \frac{c^{2/3}\log(c+dx^3)}{6d(bc-ad)^{2/3}} - \frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d} + \frac{c^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{2/3}}$$

output
```
-1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)/d
+1/3*c^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3
^(1/2))*3^(1/2)/d/(-a*d+b*c)^(2/3)-1/6*c^(2/3)*ln(d*x^3+c)/d/(-a*d+b*c)^(2
/3)-1/2*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)/d+1/2*c^(2/3)*ln((-a*d+b*c)^
(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d/(-a*d+b*c)^(2/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.52 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.81

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{-\frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{2/3}} - \frac{2\sqrt{-6-6i\sqrt{3}}c^{2/3}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}}\right)}{(bc-ad)^{2/3}}}{}$$

input
```
Integrate[x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
((-4*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]
)/b^(2/3) - (2*Sqrt[-6 - (6*I)*Sqrt[3]]*c^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3
)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1
/3))])/(b*c - a*d)^(2/3) - (4*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(2/
3) + ((2*I)*(I + Sqrt[3])*c^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[
3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(2/3) + (2*Log[b^(2/3)*x^2 + b
^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(2/3) + ((1 - I*Sqrt[3]
)*c^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*
d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])
/(b*c - a*d)^(2/3))/(12*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {983, 853, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$$

$$\Big\downarrow \ 983$$

$$\frac{\int \frac{x}{(bx^3+a)^{2/3}}dx}{d} - \frac{c\int \frac{x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{d}$$

$\Big\downarrow$ 853

$$\frac{-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} - \frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}}{d} - \frac{c\int \frac{x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{d}$$

$\Big\downarrow$ 992

$$\frac{-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} - \frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}}{d} - \frac{c\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\log(c+dx^3)}{6\sqrt[3]{c}(bc-ad)^{2/3}} - \frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{d}$$

input

```
Int[x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output

```
(-(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3))
) - Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b^(2/3)))/d - (c*(-(ArcTan[(1 +
(2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(
1/3)*(b*c - a*d)^(2/3))) + Log[c + d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) -
Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a
*d)^(2/3))))/d
```

Defintions of rubi rules used

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.28

method	result
pseudoelliptic	$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}+x\right)}{3x}\right)b^{\frac{2}{3}}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{b^{\frac{1}{3}}}+x\right)}{3x}\right)\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}-\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right.}{}$

input
```
int(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3^(1/2)*arctan(1/3*3^(1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x
)/x)*b^(2/3)-3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)/b^(1/3)+x)/x)*(
(a*d-b*c)/c)^(2/3)-ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*b^(2/3)+1
/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^
3+a)^(2/3))/x^2)*b^(2/3)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*((a*d-b*c)/c)^
(2/3)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*
((a*d-b*c)/c)^(2/3))/((a*d-b*c)/c)^(2/3)/b^(2/3)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(187) = 374$.

Time = 0.10 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.25

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{2\sqrt{3}b^2\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(bc-ad)\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}+\sqrt{3}cx}{3\,cx}\right.}{}$$

input
```
integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-1/3
*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*
d^2))^(2/3) + sqrt(3)*c*x)/(c*x)) + 2*b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*
d^2))^(1/3)*log(-((b*c - a*d)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*
x - (b*x^3 + a)^(1/3)*c)/x) - b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1
/3)*log(((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d
^2))^(2/3)*x^2 + (b*x^3 + a)^(2/3)*c^2 + (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d)
*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x)/x^2) + 6*sqrt(1/3)*b*sqrt(
-(-b^2)^(1/3))*arctan(-sqrt(1/3)*((-b^2)^(1/3)*b*x - 2*(b*x^3 + a)^(1/3)*(
-b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) - 2*(-b^2)^(2/3)*log(-((-b^2)^(2
/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(2/3)*log(-((-b^2)^(1/3)*b*x^2 -
(b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d)
```

Sympy [F]

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^4}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**4/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**4/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^4}{(b\,x^3+a)^{2/3}\,(d\,x^3+c)}\,dx$$

input
```
int(x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
int(x^4/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^4}{(b\,x^3+a)^{\frac{2}{3}}\,c+(b\,x^3+a)^{\frac{2}{3}}\,d\,x^3}dx$$

input
```
int(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x**4/((a + b*x**3)**(2/3)*c + (a + b*x**3)**(2/3)*d*x**3),x)
```

3.755 $\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\log(c+dx^3)}{6\sqrt[3]{c}(bc-ad)^{2/3}} - \frac{\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}$$

output
```
-1/3*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*
3^(1/2)/c^(1/3)/(-a*d+b*c)^(2/3)+1/6*ln(d*x^3+c)/c^(1/3)/(-a*d+b*c)^(2/3)-
1/2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(1/3)/(-a*d+b*c)^(2/3
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.71

$$\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{2\sqrt{-6-6i\sqrt{3}}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+\left(1-i\sqrt{3}\right)\left(\cdots\right.}{\cdots}$$

input
```
Integrate[x/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(2*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
 a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (1 - I*Sqrt[
3])*(2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/
3)] - Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(
1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]))/(1
2*c^(1/3)*(b*c - a*d)^(2/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$$

$$\Big\downarrow \; 992$$

$$-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log(c+dx^3)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}$$

input
```
Int[x/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
-(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]
]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[c + d*x^3]/(6*c^(1/3)*(b*c -
a*d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^
(1/3)*(b*c - a*d)^(2/3))
```

Defintions of rubi rules used

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)-2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\cdots}{\cdots}\right)}{6\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}c}$

input
```
int(x/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3)
)/((a*d-b*c)/c)^(1/3)/x)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(
b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln(((((a*d-b*c)/c)^(1/3)*x+(b*x^3+
a)^(1/3))/x))/((a*d-b*c)/c)^(2/3)/c
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x}{(b\,x^3+a)^{2/3}\,(d\,x^3+c)}\,dx$$

input
```
int(x/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
int(x/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x}{(b\,x^3+a)^{\frac{2}{3}}\,c+(b\,x^3+a)^{\frac{2}{3}}\,d\,x^3}dx$$

input
```
int(x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x/((a + b*x**3)**(2/3)*c + (a + b*x**3)**(2/3)*d*x**3),x)
```

3.756 $\int \frac{1}{x^2(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{1}{x^2(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{acx}+\frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}(bc-ad)^{2/3}} - \frac{d\log(c+dx^3)}{6c^{4/3}(bc-ad)^{2/3}}+\frac{d\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}(bc-ad)^{2/3}}$$

output
```
-(b*x^3+a)^(1/3)/a/c/x+1/3*d*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b
*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(4/3)/(-a*d+b*c)^(2/3)-1/6*d*ln(d*x^3+c)
/c^(4/3)/(-a*d+b*c)^(2/3)+1/2*d*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1
/3))/c^(4/3)/(-a*d+b*c)^(2/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.78

$$\int \frac{1}{x^2 (a+bx^3)^{2/3} (c+dx^3)} dx = \frac{-12\sqrt[3]{c}(bc-ad)^{2/3}\sqrt[3]{a+bx^3} - 2\sqrt{-6-6i\sqrt{3}}adx \arctan\left(\frac{3}{\sqrt{3}\sqrt[3]{bc-adx}}\right.}{}$$

input
```
Integrate[1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(-12*c^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3) - 2*Sqrt[-6 - (6*I)*Sqrt[
3]]*a*d*x*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3
*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (2*I)*(I + Sqrt[3])*a*d*x*Log[
2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + a*(d
- I*Sqrt[3]*d)*x*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b
*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^
(2/3)])/(12*a*c^(4/3)*(b*c - a*d)^(2/3)*x)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {980, 25, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^3)^{2/3} (c+dx^3)} dx$$

$$\Big\downarrow \text{980}$$

$$\frac{\int -\frac{adx}{(bx^3+a)^{2/3}(dx^3+c)}dx}{ac} - \frac{\sqrt[3]{a+bx^3}}{acx}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{adx}{(bx^3+a)^{2/3}(dx^3+c)}dx}{ac}-\frac{\sqrt[3]{a+bx^3}}{acx}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{d\int \frac{x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{c}-\frac{\sqrt[3]{a+bx^3}}{acx}$$

$$\Big\downarrow \textbf{992}$$

$$-\frac{d\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log(c+dx^3)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{c}-\frac{\sqrt[3]{a+bx^3}}{acx}$$

input
```
Int[1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
-((a + b*x^3)^(1/3)/(a*c*x)) - (d*(-(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(
c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3)))
+ Log[c + d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)
/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a*d)^(2/3))))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$-\frac{-a\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)xd+3\left(b\,x^{3}+a\right)^{\frac{1}{3}}c\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}+ad\left(\arctan\left(\frac{\sqrt{3}\left(-\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}+x\right)}{3x}\right)\sqrt{3}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right.}{} \cdots\right.\right.}{3\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}a\,c^{2}x}$

input
```
int(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/((a*d-b*c)/c)^(2/3)*(-a*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)
*x*d+3*(b*x^3+a)^(1/3)*c*((a*d-b*c)/c)^(2/3)+a*d*(arctan(1/3*3^(1/2)*(-2/(
(a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*3^(1/2)+1/2*ln((((a*d-b*c)/c)^(2/
3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*x)/a/c
^2/x
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \int \frac{1}{x^2 (a+bx^3)^{\frac{2}{3}} (c+dx^3)} \, dx$$

input
```
integrate(1/x**2/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**2*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}} (dx^3+c) x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}} (dx^3+c) x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \int \frac{1}{x^2 (b\,x^3+a)^{2/3} \,(d\,x^3+c)} \, dx$$

input
```
int(1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,c\,x^2 + (b\,x^3+a)^{\frac{2}{3}}\,d\,x^5} dx$$

input
```
int(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*c*x**2 + (a + b*x**3)**(2/3)*d*x**5),x)
```

3.757 $\int \frac{1}{x^5(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 215

$$\int \frac{1}{x^5(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{4acx^4} + \frac{(3bc+4ad)\sqrt[3]{a+bx^3}}{4a^2c^2x} - \frac{d^2\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}(bc-ad)^{2/3}} + \frac{d^2\log(c+dx^3)}{6c^{7/3}(bc-ad)^{2/3}} - \frac{d^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}(bc-ad)^{2/3}}$$

output

```
-1/4*(b*x^3+a)^(1/3)/a/c/x^4+1/4*(4*a*d+3*b*c)*(b*x^3+a)^(1/3)/a^2/c^2/x-1
/3*d^2*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2)
)*3^(1/2)/c^(7/3)/(-a*d+b*c)^(2/3)+1/6*d^2*ln(d*x^3+c)/c^(7/3)/(-a*d+b*c)^
(2/3)-1/2*d^2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(7/3)/(-a*d
+b*c)^(2/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.31 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^5 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}(-ac+3bcx^3+4adx^3)}{a^2x^4} + \frac{2\sqrt{-6-6i\sqrt{3}}d^2 \arctan\left(\frac{3\sqrt[3]{bc-ad}}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{}}\right.}{(bc-ad)^{2/3}}}{}$$

input
```
Integrate[1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
((3*c^(1/3)*(a + b*x^3)^(1/3)*(-(a*c) + 3*b*c*x^3 + 4*a*d*x^3))/(a^2*x^4)
+ (2*Sqrt[-6 - (6*I)*Sqrt[3]]*d^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*
(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(b*c -
a*d)^(2/3) + (2*(1 - I*Sqrt[3])*d^2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqr
t[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(2/3) + (I*(I + Sqrt[3])*d^2
*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*
x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a
*d)^(2/3))/(12*c^(7/3))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {980, 25, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^3)^{2/3} (c+dx^3)} \, dx$$

$$\Big\downarrow \; 980$$

$$\frac{\int -\frac{3bdx^3+3bc+4ad}{x^2(bx^3+a)^{2/3}(dx^3+c)}dx}{4ac} - \frac{\sqrt[3]{a+bx^3}}{4acx^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{3bdx^3+3bc+4ad}{x^2(bx^3+a)^{2/3}(dx^3+c)}dx}{4ac}-\frac{\sqrt[3]{a+bx^3}}{4acx^4}$$

$$\Big\downarrow \text{1053}$$

$$-\frac{-\frac{\int \frac{4a^2d^2x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{ac}-\frac{\sqrt[3]{a+bx^3}(4ad+3bc)}{acx}}{4ac}-\frac{\sqrt[3]{a+bx^3}}{4acx^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{4ad^2\int \frac{x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{c}-\frac{\sqrt[3]{a+bx^3}(4ad+3bc)}{acx}}{4ac}-\frac{\sqrt[3]{a+bx^3}}{4acx^4}$$

$$\Big\downarrow \text{992}$$

$$-\frac{-\frac{4ad^2\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log(c+dx^3)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{c}-\frac{\sqrt[3]{a+bx^3}(4ad+3bc)}{acx}}{4ac}-\frac{\sqrt[3]{a+bx^3}}{4acx^4}$$

input

```
Int[1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output

```
-1/4*(a + b*x^3)^(1/3)/(a*c*x^4) - (-(((3*b*c + 4*a*d)*(a + b*x^3)^(1/3))/
(a*c*x)) - (4*a*d^2*(-(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b
*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[c + d*x^
3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a
+ b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a*d)^(2/3))))/c)/(4*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$-\frac{-\frac{\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right) a^{2} d^{2} x^{4}}{2}-d^{2}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}+x\right)}{3x}\right) a^{2} x^{4}+\ln\left(\left(\cdots\right.\right.}{3\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} a^{2} c^{3} x^{4}}$

input
```
int(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/((a*d-b*c)/c)^(2/3)*(-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(
1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a^2*d^2*x^4-d^2*3^(1/2)*arcta
n(1/3*3^(1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*a^2*x^4+ln((((
a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^2*d^2*x^4+3/4*((a*d-b*c)/c)^(2/3
)*c*((-4*d*x^3+c)*a-3*x^3*b*c)*(b*x^3+a)^(1/3))/a^2/c^3/x^4
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^5 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \int \frac{1}{x^5 (a+bx^3)^{\frac{2}{3}} (c+dx^3)} \, dx$$

input
```
integrate(1/x**5/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**5*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}} (dx^3+c) x^5} \, dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}} (dx^3+c) x^5} \, dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^3)^{2/3} (c+dx^3)} dx = \int \frac{1}{x^5 (b x^3+a)^{2/3} (d x^3+c)} dx$$

input
```
int(1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^5 (a+bx^3)^{2/3} (c+dx^3)} dx = \int \frac{1}{(b x^3+a)^{\frac{2}{3}} c x^5+(b x^3+a)^{\frac{2}{3}} d x^8} dx$$

input
```
int(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*c*x**5 + (a + b*x**3)**(2/3)*d*x**8),x)
```

3.758 $\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\frac{x^7\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{7}{3},\frac{2}{3},1,\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\,(a+bx^3)^{2/3}}$$

output
```
1/7*x^7*(1+b*x^3/a)^(2/3)*AppellF1(7/3,2/3,1,10/3,-b*x^3/a,-d*x^3/c)/c/(b*
x^3+a)^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 249 vs. 2(64) = 128.

Time = 8.40 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.89

$$\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\frac{x\left(-\frac{(2bc+ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{bc}+4\left(\frac{a}{b}+x^3+\frac{}{b(c+dx^3)\left(-4ac\right.}\right.\right.}{}$$

input
```
Integrate[x^6/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(x*(-(((2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3,
-((b*x^3)/a), -((d*x^3)/c)])/(b*c)) + 4*(a/b + x^3 + (4*a^2*c^2*AppellF1[1
/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(b*(c + d*x^3)*(-4*a*c*Appel
lF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/
3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
7/3, -((b*x^3)/a), -((d*x^3)/c)]))))))/(8*d*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}dx$$
$$\Big\downarrow \text{1013}$$
$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{x^6}{\left(\frac{bx^3}{a}+1\right)^{2/3}(dx^3+c)}dx}{(a+bx^3)^{2/3}}$$
$$\Big\downarrow \text{1012}$$
$$\frac{x^7\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{7}{3},\frac{2}{3},1,\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c(a+bx^3)^{2/3}}$$

input
```
Int[x^6/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(x^7*(1 + (b*x^3)/a)^(2/3)*AppellF1[7/3, 2/3, 1, 10/3, -((b*x^3)/a), -((d*
x^3)/c)])/(7*c*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{6}}{\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(d\,x^{3}+c\right)}dx$$

input
```
int(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^6}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**6/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**6/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^6/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^6/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^6}{(b\,x^3+a)^{2/3}\,(d\,x^3+c)}\,dx$$

input
```
int(x^6/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
int(x^6/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^6}{(b\,x^3+a)^{\frac{2}{3}}\,c+(b\,x^3+a)^{\frac{2}{3}}\,d\,x^3}dx$$

input
```
int(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x**6/((a + b*x**3)**(2/3)*c + (a + b*x**3)**(2/3)*d*x**3),x)
```

3.759 $\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\frac{x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c(a+bx^3)^{2/3}}$$

output
```
1/4*x^4*(1+b*x^3/a)^(2/3)*AppellF1(4/3,2/3,1,7/3,-b*x^3/a,-d*x^3/c)/c/(b*x
^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 7.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\frac{x^4\left(\frac{a+bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c(a+bx^3)^{2/3}}$$

input
```
Integrate[x^3/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(x^4*((a + b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x
^3)/c)])/(4*c*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{x^3}{\left(\frac{bx^3}{a}+1\right)^{2/3}(dx^3+c)}dx}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c(a+bx^3)^{2/3}}$$

input
```
Int[x^3/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x
^3)/c)])/(4*c*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^3}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)}dx$$

input
```
int(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx=\int \frac{x^3}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

input
```
integrate(x**3/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**3/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \int \frac{x^3}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \int \frac{x^3}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^3/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \int \frac{x^3}{(b\,x^3+a)^{2/3}\,(d\,x^3+c)}\,dx$$

input
```
int(x^3/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
int(x^3/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}\,c+(b\,x^3+a)^{\frac{2}{3}}\,d\,x^3}dx$$

input
```
int(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x**3/((a + b*x**3)**(2/3)*c + (a + b*x**3)**(2/3)*d*x**3),x)
```

3.760 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\,(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(b*x^3+a)^
(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.

Time = 0.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx =$$
$$-\frac{4acx\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(a+bx^3)^{2/3}(c+dx^3)\left(-4ac\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3ad\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right.\right.}$$

input
```
Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(-4*a*c*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((a + b*
x^3)^(2/3)*(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -(
(d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}(dx^3+c)}dx}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c(a+bx^3)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(b\,x^{3}+a)^{\frac{2}{3}}\,(d\,x^{3}+c)}dx$$

input
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{2/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,c+(b\,x^3+a)^{\frac{2}{3}}\,d\,x^3}dx$$

input
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*c + (a + b*x**3)**(2/3)*d*x**3),x)
```

3.761 $\int \frac{1}{x^3(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^3\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=-\frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(-\frac{2}{3},\frac{2}{3},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\left(a+bx^3\right)^{2/3}}$$

output
```
-1/2*(1+b*x^3/a)^(2/3)*AppellF1(-2/3,2/3,1,1/3,-b*x^3/a,-d*x^3/c)/c/x^2/(b
*x^3+a)^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 338 vs. 2(64) = 128.

Time = 10.21 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.28

$$\int \frac{1}{x^3\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\frac{-bdx^6\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c\left(-4ac(ac+2bcx^3+3\cdots\right.}{(c+\cdots}}{\cdots}$$

input
```
Integrate[1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(-(b*d*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
-((d*x^3)/c)]) + (4*c*(-4*a*c*(a*c + 2*b*c*x^3 + 3*a*d*x^3 + b*d*x^6)*Appe
llF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(a + b*x^3)*(c +
d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b
*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*
(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d
*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4
/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(8*a*c^2*x^2*(a + b*x^3)^
(2/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)} \, dx$$
$$\Big\downarrow \text{ 1013}$$
$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3} \int \frac{1}{x^3 \left(\frac{bx^3}{a}+1\right)^{2/3} (dx^3+c)}dx}{\left(a+bx^3\right)^{2/3}}$$
$$\Big\downarrow \text{ 1012}$$
$$-\frac{\left(\frac{bx^3}{a}+1\right)^{2/3} \text{AppellF1}\left(-\frac{2}{3},\frac{2}{3},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2 \left(a+bx^3\right)^{2/3}}$$

input
```
Int[1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
-1/2*((1 + (b*x^3)/a)^(2/3)*AppellF1[-2/3, 2/3, 1, 1/3, -((b*x^3)/a), -((d
*x^3)/c)])/(c*x^2*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{3} \left(b\,x^{3}+a \right)^{\frac{2}{3}} \left(d\,x^{3}+c \right)}d x$$

input
```
int(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)} \, dx=\text{Timed out}$$

input
```
integrate(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^3\,(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \int \frac{1}{x^3\,(a+bx^3)^{\frac{2}{3}}\,(c+dx^3)}\,dx$$

input
```
integrate(1/x**3/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**3*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\,(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\,(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^3)^{2/3} (c + dx^3)} \, dx = \int \frac{1}{x^3 \left(b\,x^3 + a\right)^{2/3} \left(d\,x^3 + c\right)} \, dx$$

input
```
int(1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a + bx^3)^{2/3} (c + dx^3)} \, dx = \int \frac{1}{\left(b\,x^3 + a\right)^{\frac{2}{3}} c\,x^3 + \left(b\,x^3 + a\right)^{\frac{2}{3}} d\,x^6} dx$$

input
```
int(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*c*x**3 + (a + b*x**3)**(2/3)*d*x**6),x)
```

3.762 $\int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 275

$$\int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(b^2c^2+2abcd+3a^2d^2)(a+bx^3)^{2/3}}{2b^4d^3} - \frac{(bc+3ad)(a+bx^3)^{5/3}}{5b^4d^2} + \frac{(a+bx^3)^{8/3}}{8b^4d} + \frac{c^4\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}(bc-ad)^{4/3}} - \frac{c^4\log(c+dx^3)}{6d^{11/3}(bc-ad)^{4/3}} + \frac{c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}(bc-ad)^{4/3}}$$

output
```
-a^4/b^4/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/2*(3*a^2*d^2+2*a*b*c*d+b^2*c^2)*(b*x
^3+a)^(2/3)/b^4/d^3-1/5*(3*a*d+b*c)*(b*x^3+a)^(5/3)/b^4/d^2+1/8*(b*x^3+a)^
(8/3)/b^4/d+1/3*c^4*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/
3))*3^(1/2))*3^(1/2)/d^(11/3)/(-a*d+b*c)^(4/3)-1/6*c^4*ln(d*x^3+c)/d^(11/3
)/(-a*d+b*c)^(4/3)+1/2*c^4*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^
(11/3)/(-a*d+b*c)^(4/3)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.27

$$\int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\frac{3d^{2/3}\left(-81a^4d^3+9a^3bd^2(c-3dx^3)+3a^2b^2d(4c^2+cdx^3+3d^2x^6)+b^4cx^3(20c^2-8cdx^3+5d^2x^6)+ab^3(20c^3}{b^4(bc-ad)\sqrt[3]{a+bx^3}}}{}$$

input
```
Integrate[x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((3*d^(2/3)*(-81*a^4*d^3 + 9*a^3*b*d^2*(c - 3*d*x^3) + 3*a^2*b^2*d*(4*c^2
+ c*d*x^3 + 3*d^2*x^6) + b^4*c*x^3*(20*c^2 - 8*c*d*x^3 + 5*d^2*x^6) + a*b^
3*(20*c^3 + 4*c^2*d*x^3 - c*d^2*x^6 - 5*d^3*x^9)))/(b^4*(b*c - a*d)*(a + b
*x^3)^(1/3)) + (40*Sqrt[3]*c^4*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(
b*c - a*d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(4/3) + (40*c^4*Log[(b*c - a*d)^(1
/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) - (20*c^4*Log[(b*c - a
*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x
^3)^(2/3)])/(b*c - a*d)^(4/3))/(120*d^(11/3))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {948, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{x^{12}}{(bx^3+a)^{4/3}(dx^3+c)}dx^3$$

$\downarrow$ 98

$$\frac{1}{3}\int \left(\frac{x^6}{bd\sqrt[3]{bx^3+a}}-\frac{(bc+ad)x^3}{b^2d^2\sqrt[3]{bx^3+a}}+\frac{b^2c^2+abdc+a^2d^2}{b^3d^3\sqrt[3]{bx^3+a}}+\frac{c^4}{d^3(ad-bc)\sqrt[3]{bx^3+a}\,(dx^3+c)}+\frac{a^4}{b^3(bc-ad)\,(bx^3\right.$$

$\downarrow$ 2009

$$\frac{1}{3}\left(-\frac{3a^4}{b^4\sqrt[3]{a+bx^3}(bc-ad)}+\frac{3a^2\left(a+bx^3\right)^{2/3}}{2b^4d}+\frac{3\left(a+bx^3\right)^{2/3}\left(a^2d^2+abcd+b^2c^2\right)}{2b^4d^3}+\frac{\sqrt{3}c^4\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{}}{\sqrt[3]{bc}}}{\sqrt{}}\right.}{d^{11/3}(bc-ad)^4}\right.$$

input
```
Int[x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((-3*a^4)/(b^4*(b*c - a*d)*(a + b*x^3)^(1/3)) + (3*a^2*(a + b*x^3)^(2/3))/
(2*b^4*d) + (3*a*(b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^4*d^2) + (3*(b^2*c^2
+ a*b*c*d + a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^4*d^3) - (6*a*(a + b*x^3)^(5/
3))/(5*b^4*d) - (3*(b*c + a*d)*(a + b*x^3)^(5/3))/(5*b^4*d^2) + (3*(a + b*
x^3)^(8/3))/(8*b^4*d) + (Sqrt[3]*c^4*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1
/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(11/3)*(b*c - a*d)^(4/3)) - (c^4*Log[
c + d*x^3])/(2*d^(11/3)*(b*c - a*d)^(4/3)) + (3*c^4*Log[(b*c - a*d)^(1/3)
+ d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(11/3)*(b*c - a*d)^(4/3)))/3
```

Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$-\frac{-\frac{243d\left(a\left(\frac{5}{81}b^3x^9-\frac{1}{9}ab^2x^6+\frac{1}{3}a^2bx^3+a^3\right)d^3-\frac{\left(\frac{5}{9}b^2x^6-\frac{2}{3}abx^3+a^2\right)c\left(bx^3+a\right)bd^2}{9}-\frac{4c^2\left(bx^3+a\right)\left(-\frac{2bx^3}{3}+a\right)b^2d}{27}-\frac{20b^3c^3(b\ldots}{81}\ldots\right)}{20}\ldots}{\ldots}$

input
```
int(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/((a*d-b*c)/d)^(1/3)/(b*x^3+a)^(1/3)*(-243/20*d*(a*(5/81*b^3*x^9-1/9*a
*b^2*x^6+1/3*a^2*b*x^3+a^3)*d^3-1/9*(5/9*b^2*x^6-2/3*a*b*x^3+a^2)*c*(b*x^3
+a)*b*d^2-4/27*c^2*(b*x^3+a)*(-2/3*b*x^3+a)*b^2*d-20/81*b^3*c^3*(b*x^3+a))
*((a*d-b*c)/d)^(1/3)+b^4*c^4*(b*x^3+a)^(1/3)*(-2*arctan(1/3*3^(1/2)*(2*(b*
x^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*c)/d)^(1/3))*3^(1/2)+ln((b*x^3+a
)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))-2*ln((b*x
^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))))/d^4/(a*d-b*c)/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(231) = 462$.

Time = 0.15 (sec) , antiderivative size = 1300, normalized size of antiderivative = 4.73

$$\int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/120*(60*sqrt(1/3)*(a*b^5*c^5*d - a^2*b^4*c^4*d^2 + (b^6*c^5*d - a*b^5*
c^4*d^2)*x^3)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3
- b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2
/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c -
a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2
/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) + 20*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d
^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*
x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 40*(b^5*c^4*x^3 + a*b^4*c^4
)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1
/3)) - 3*(20*a*b^4*c^4*d^2 - 8*a^2*b^3*c^3*d^3 - 3*a^3*b^2*c^2*d^4 - 90*a^
4*b*c*d^5 + 81*a^5*d^6 + 5*(b^5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6)*x^9
 - (8*b^5*c^3*d^3 - 7*a*b^4*c^2*d^4 - 10*a^2*b^3*c*d^5 + 9*a^3*b^2*d^6)*x^
6 + (20*b^5*c^4*d^2 - 16*a*b^4*c^3*d^3 - a^2*b^3*c^2*d^4 - 30*a^3*b^2*c*d^
5 + 27*a^4*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(a*b^6*c^2*d^5 - 2*a^2*b^5*c*d^6
 + a^3*b^4*d^7 + (b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^3), -1/120*
(120*sqrt(1/3)*(a*b^5*c^5*d - a^2*b^4*c^4*d^2 + (b^6*c^5*d - a*b^5*c^4*d^2
)*x^3)*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*
x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3
)/(b*c - a*d))/d) + 20*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a*d^3)^(2/3)*
log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*...
```

Sympy [F]

$$\int \frac{x^{14}}{\left(a+b x^3\right)^{4/3} \left(c+d x^3\right)}\, dx = \int \frac{x^{14}}{\left(a+b x^{3}\right)^{\frac{4}{3}} \left(c+d x^{3}\right)}\, dx$$

input
```
integrate(x**14/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**14/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{14}}{\left(a+b x^3\right)^{4/3} \left(c+d x^3\right)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.55

$$\int \frac{x^{14}}{\left(a+b x^3\right)^{4/3} \left(c+d x^3\right)}\, dx = \frac{\frac{120 \left(-b c d^{2}+a d^{3}\right)^{\frac{2}{3}} b^{4} c^{4} \arctan\left(\frac{\sqrt{3} \left(2 \left(b x^{3}+a\right)^{\frac{1}{3}} + \left(-\frac{b c - a d}{d}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{b c - a d}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3} b^{2} c^{2} d^{5} - 2 \sqrt{3} a b c d^{6} + \sqrt{3} a^{2} d^{7}} - \frac{20 \left(-b c d^{2}+a d^{3}\right)^{\frac{2}{3}} b^{4} c^{4} \log\left(\left(b x^{3} \cdots\right.\right.}{b^{2} c^{\cdots}}}{\cdots}$$

input
```
integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/120*(120*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*arctan(1/3*sqrt(3)*(2*(b*x^3 +
 a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c
^2*d^5 - 2*sqrt(3)*a*b*c*d^6 + sqrt(3)*a^2*d^7) - 20*(-b*c*d^2 + a*d^3)^(2
/3)*b^4*c^4*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/
3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^2*d^5 - 2*a*b*c*d^6 + a^2*d^7) + 40*b^
4*c^4*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^
(1/3)))/(b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5) - 120*a^4/((b*x^3 + a)^(1/3)
*(b*c - a*d)) + 3*(20*(b*x^3 + a)^(2/3)*b^2*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*
b*c*d^6 + 40*(b*x^3 + a)^(2/3)*a*b*c*d^6 + 5*(b*x^3 + a)^(8/3)*d^7 - 24*(b
*x^3 + a)^(5/3)*a*d^7 + 60*(b*x^3 + a)^(2/3)*a^2*d^7)/d^8)/b^4
```

Mupad [B] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 564, normalized size of antiderivative = 2.05

$$\int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \left(\frac{3\,a^2}{b^4\,d}+\frac{\left(\frac{4\,a}{b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{b^8\,d^2}\right)\,\left(b^5\,c-a\,b^4\,d\right)}{2\,b^4\,d}\right)\,{\left(b\,x^3+a\right)}^{2/3} - \left(\frac{4\,a}{5\,b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{5\,b^8\,d^2}\right)\,{\left(b\,x^3+a\right)}^{5/3}+\frac{{\left(b\,x^3+a\right)}^{8/3}}{8\,b^4\,d}+\frac{a^4}{b^4\,{\left(b\,x^3+a\right)}^{1/3}\,\left(a\,d-b\,c\right)} + \frac{c^4\,\ln\left({\left(b\,x^3+a\right)}^{1/3}\,\left(a\,c^8\,d^5-b\,c^9\,d^4\right)-\frac{c^8\,\left(9\,a^4\,d^{15}-36\,a^3\,b\,c\,d^{14}+54\,a^2\,b^2\,c^2\,d^{13}-36\,a\,b^3\,c^3\,d^{12}+9\,b^4\,c^4\,d^{11}\right)}{9\,d^{22/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)}{3\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{4/3}} - \frac{\ln\left({\left(b\,x^3+a\right)}^{1/3}\,\left(a\,c^8\,d^5-b\,c^9\,d^4\right)-\frac{{\left(c^4+\sqrt{3}\,c^4\,1{}\mathrm{i}\right)}^2\,\left(9\,a^4\,d^{15}-36\,a^3\,b\,c\,d^{14}+54\,a^2\,b^2\,c^2\,d^{13}-36\,a\,b^3\,c^3\,d^{12}+9\,b^4\,c^4\,d^{11}\right)}{36\,d^{22/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)\,\left(c^4\right.}{6\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{4/3}} + \frac{c^4\,\ln\left({\left(b\,x^3+a\right)}^{1/3}\,\left(a\,c^8\,d^5-b\,c^9\,d^4\right)-\frac{c^8\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2\,\left(9\,a^4\,d^{15}-36\,a^3\,b\,c\,d^{14}+54\,a^2\,b^2\,c^2\,d^{13}-36\,a\,b^3\,c^3\,d^{12}+9\,b^4\,c^4\,d^{11}\right)}{d^{22/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)\,\left(\right.}{d^{11/3}\,{\left(a\,d-b\,c\right)}^{4/3}}$$

input
```
int(x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
((3*a^2)/(b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c -
 a*b^4*d))/(2*b^4*d))*(a + b*x^3)^(2/3) - ((4*a)/(5*b^4*d) + (b^5*c - a*b^
4*d)/(5*b^8*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^4*d) + a^4/(b
^4*(a + b*x^3)^(1/3)*(a*d - b*c)) + (c^4*log((a + b*x^3)^(1/3)*(a*c^8*d^5
- b*c^9*d^4) - (c^8*(9*a^4*d^15 + 9*b^4*c^4*d^11 - 36*a*b^3*c^3*d^12 + 54*
a^2*b^2*c^2*d^13 - 36*a^3*b*c*d^14))/(9*d^(22/3)*(a*d - b*c)^(8/3))))/(3*d
^(11/3)*(a*d - b*c)^(4/3)) - (log((a + b*x^3)^(1/3)*(a*c^8*d^5 - b*c^9*d^4
) - ((3^(1/2)*c^4*1i + c^4)^2*(9*a^4*d^15 + 9*b^4*c^4*d^11 - 36*a*b^3*c^3*
d^12 + 54*a^2*b^2*c^2*d^13 - 36*a^3*b*c*d^14))/(36*d^(22/3)*(a*d - b*c)^(8
/3)))*(3^(1/2)*c^4*1i + c^4))/(6*d^(11/3)*(a*d - b*c)^(4/3)) + (c^4*log((a
 + b*x^3)^(1/3)*(a*c^8*d^5 - b*c^9*d^4) - (c^8*((3^(1/2)*1i)/6 - 1/6)^2*(9
*a^4*d^15 + 9*b^4*c^4*d^11 - 36*a*b^3*c^3*d^12 + 54*a^2*b^2*c^2*d^13 - 36*
a^3*b*c*d^14))/(d^(22/3)*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/6 - 1/6))/(d^(1
1/3)*(a*d - b*c)^(4/3))
```

Reduce [F]

$$\int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^{14}}{(b\,x^3+a)^{\frac{1}{3}}\,ac+(b\,x^3+a)^{\frac{1}{3}}\,ad\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bc\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bd\,x^6}\,d$$

input
```
int(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x**14/((a + b*x**3)**(1/3)*a*c + (a + b*x**3)**(1/3)*a*d*x**3 + (a + b
*x**3)**(1/3)*b*c*x**3 + (a + b*x**3)**(1/3)*b*d*x**6),x)
```

3.763 $\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 232

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{(bc+2ad)(a+bx^3)^{2/3}}{2b^3d^2} + \frac{(a+bx^3)^{5/3}}{5b^3d} - \frac{c^3\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}(bc-ad)^{4/3}} + \frac{c^3\log(c+dx^3)}{6d^{8/3}(bc-ad)^{4/3}} - \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}(bc-ad)^{4/3}}$$

output
```
a^3/b^3/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/2*(2*a*d+b*c)*(b*x^3+a)^(2/3)/b^3/d^2
+1/5*(b*x^3+a)^(5/3)/b^3/d-1/3*c^3*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)
/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/d^(8/3)/(-a*d+b*c)^(4/3)+1/6*c^3*ln(d*
x^3+c)/d^(8/3)/(-a*d+b*c)^(4/3)-1/2*c^3*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3
+a)^(1/3))/d^(8/3)/(-a*d+b*c)^(4/3)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.28

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\frac{3d^{2/3}\left(18a^3d^2+b^3cx^3\left(-5c+2dx^3\right)+3a^2bd\left(-c+2dx^3\right)-ab^2\left(5c^2+cdx^3+2d^2x^6\right)\right)}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{10\sqrt{3}c^3\arctan\left(\frac{1-\cdots}{\cdots}\right)}{(bc-\cdots}\cdots}{\cdots}$$

input
```
Integrate[x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((3*d^(2/3)*(18*a^3*d^2 + b^3*c*x^3*(-5*c + 2*d*x^3) + 3*a^2*b*d*(-c + 2*d
*x^3) - a*b^2*(5*c^2 + c*d*x^3 + 2*d^2*x^6)))/(b^3*(b*c - a*d)*(a + b*x^3)
^(1/3)) - (10*Sqrt[3]*c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c -
 a*d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(4/3) - (10*c^3*Log[(b*c - a*d)^(1/3) +
 d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) + (5*c^3*Log[(b*c - a*d)^(2
/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2
/3)])/(b*c - a*d)^(4/3))/(30*d^(8/3))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{x^9}{(bx^3+a)^{4/3}(dx^3+c)}dx^3$$

$$\Big\downarrow \; 98$$

$$\frac{1}{3}\int \left(-\frac{a^3}{b^2(bc-ad)\left(bx^3+a\right)^{4/3}}+\frac{x^3}{bd\sqrt[3]{bx^3+a}}+\frac{-bc-ad}{b^2d^2\sqrt[3]{bx^3+a}}-\frac{c^3}{d^2(ad-bc)\sqrt[3]{bx^3+a}\left(dx^3+c\right)}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{3a^3}{b^3\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\sqrt{3}c^3\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{8/3}(bc-ad)^{4/3}}-\frac{3\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^3d^2}-\frac{3a\left(a+bx^3\right)^{2/3}}{2b^3d}+\cdots\right.$$

input
```
Int[x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((3*a^3)/(b^3*(b*c - a*d)*(a + b*x^3)^(1/3)) - (3*a*(a + b*x^3)^(2/3))/(2*
b^3*d) - (3*(b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^3*d^2) + (3*(a + b*x^3)^(5
/3))/(5*b^3*d) - (Sqrt[3]*c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b
*c - a*d)^(1/3))/Sqrt[3]])/(d^(8/3)*(b*c - a*d)^(4/3)) + (c^3*Log[c + d*x^
3])/(2*d^(8/3)*(b*c - a*d)^(4/3)) - (3*c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)
*(a + b*x^3)^(1/3)])/(2*d^(8/3)*(b*c - a*d)^(4/3)))/3
```

Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.25

method	result
pseudoelliptic	$-\frac{9\left(-\frac{5\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)b^{3}c^{3}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{54}+\frac{5\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)b^{3}c^{3}\left(b\,x^{3}+a\right)\cdots}{27}\cdots\right.}{5\left(\frac{ad-\cdots}{d}\right.\cdots}$

input
```
int(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-9/5/((a*d-b*c)/d)^(1/3)/(b*x^3+a)^(1/3)*(-5/54*ln((b*x^3+a)^(2/3)+((a*d-b
*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))*b^3*c^3*(b*x^3+a)^(1/3)+
5/27*3^(1/2)*arctan(2/3*3^(1/2)/((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+1/3*3^
(1/2))*b^3*c^3*(b*x^3+a)^(1/3)+5/27*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3)
)*b^3*c^3*(b*x^3+a)^(1/3)+(-5/18*c*(-2/5*d*x^3+c)*x^3*b^3-5/18*a*(2/5*d^2*
x^6+1/5*c*d*x^3+c^2)*b^2-1/6*a^2*d*(-2*d*x^3+c)*b+a^3*d^2)*d*((a*d-b*c)/d)
^(1/3))/d^3/(a*d-b*c)/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(191) = 382.

Time = 0.16 (sec) , antiderivative size = 1141, normalized size of antiderivative = 4.92

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/30*(15*sqrt(1/3)*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 + (b^5*c^4*d - a*b^4*c
^3*d^2)*x^3)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 -
 b*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3
) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d
))*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*
(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 5*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a
*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a
)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) + 10*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d
^2 - a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 3*(
5*a*b^3*c^3*d^2 - 2*a^2*b^2*c^2*d^3 - 21*a^3*b*c*d^4 + 18*a^4*d^5 - 2*(b^4
*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 + (5*b^4*c^3*d^2 - 4*a*b^3*c^2
*d^3 - 7*a^2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(a*b^5*c^2*d
^4 - 2*a^2*b^4*c*d^5 + a^3*b^3*d^6 + (b^6*c^2*d^4 - 2*a*b^5*c*d^5 + a^2*b^
4*d^6)*x^3), 1/30*(30*sqrt(1/3)*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 + (b^5*c^4*
d - a*b^4*c^3*d^2)*x^3)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(s
qrt(1/3)*(2*(b*x^3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 -
 a*d^3)^(1/3)/(b*c - a*d))/d) + 5*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d
^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^
(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 10*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2
 - a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*...
```

Sympy [F]

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^{11}}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**11/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**11/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.60

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = -\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d^4-2\sqrt{3}abcd^5+\sqrt{3}a^2d^6} + \frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(b^2c^2d^4-2\,abcd^5+a^2d^6)} - \frac{c^3\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^2c^2d^2-2\,abcd^3+a^2d^4)} + \frac{a^3}{(b^4c-ab^3d)(bx^3+a)^{\frac{1}{3}}} - \frac{5\,(bx^3+a)^{\frac{2}{3}}b^{13}cd^3-2\,(bx^3+a)^{\frac{5}{3}}b^{12}d^4+10\,(bx^3+a)^{\frac{2}{3}}ab^{12}d^4}{10\,b^{15}d^5}$$

input
```
integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-(-b*c*d^2 + a*d^3)^(2/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-
(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^4 - 2*sqr
t(3)*a*b*c*d^5 + sqrt(3)*a^2*d^6) + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^3*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d
)/d)^(2/3))/(b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6) - 1/3*c^3*(-(b*c - a*d)/
d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d^2
 - 2*a*b*c*d^3 + a^2*d^4) + a^3/((b^4*c - a*b^3*d)*(b*x^3 + a)^(1/3)) - 1/
10*(5*(b*x^3 + a)^(2/3)*b^13*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^12*d^4 + 10*(b*
x^3 + a)^(2/3)*a*b^12*d^4)/(b^15*d^5)
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.12

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{(b\,x^3+a)^{5/3}}{5\,b^3\,d} - \left(\frac{3\,a}{2\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{2\,b^6\,d^2}\right)\,(b\,x^3+a)^{2/3} - \frac{a^3}{b^3\,(b\,x^3+a)^{1/3}\,(a\,d-b\,c)} - \frac{c^3\,\ln\left((b\,x^3+a)^{1/3}\,(a\,c^6\,d^4-b\,c^7\,d^3)-\frac{c^6\,(9\,a^4\,d^{12}-36\,a^3\,b\,c\,d^{11}+54\,a^2\,b^2\,c^2\,d^{10}-36\,a\,b^3\,c^3\,d^9+9\,b^4\,c^4\,d^8)}{9\,d^{16/3}\,(a\,d-b\,c)^{8/3}}\right)}{3\,d^{8/3}\,(a\,d-b\,c)^{4/3}} + \frac{\ln\left((b\,x^3+a)^{1/3}\,(a\,c^6\,d^4-b\,c^7\,d^3)-\frac{\left(c^3+\sqrt{3}\,c^3\,1\mathrm{i}\right)^2\,(9\,a^4\,d^{12}-36\,a^3\,b\,c\,d^{11}+54\,a^2\,b^2\,c^2\,d^{10}-36\,a\,b^3\,c^3\,d^9+9\,b^4\,c^4\,d^8)}{36\,d^{16/3}\,(a\,d-b\,c)^{8/3}}\right)\,(c^3+}{6\,d^{8/3}\,(a\,d-b\,c)^{4/3}} - \frac{c^3\,\ln\left((b\,x^3+a)^{1/3}\,(a\,c^6\,d^4-b\,c^7\,d^3)-\frac{c^6\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\,(9\,a^4\,d^{12}-36\,a^3\,b\,c\,d^{11}+54\,a^2\,b^2\,c^2\,d^{10}-36\,a\,b^3\,c^3\,d^9+9\,b^4\,c^4\,d^8)}{9\,d^{16/3}\,(a\,d-b\,c)^{8/3}}\right)\,(}{3\,d^{8/3}\,(a\,d-b\,c)^{4/3}}$$

input
```
int(x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
(a + b*x^3)^(5/3)/(5*b^3*d) - ((3*a)/(2*b^3*d) + (b^4*c - a*b^3*d)/(2*b^6*
d^2))*(a + b*x^3)^(2/3) - a^3/(b^3*(a + b*x^3)^(1/3)*(a*d - b*c)) - (c^3*l
og((a + b*x^3)^(1/3)*(a*c^6*d^4 - b*c^7*d^3) - (c^6*(9*a^4*d^12 + 9*b^4*c^
4*d^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*c^2*d^10 - 36*a^3*b*c*d^11))/(9*d^(1
6/3)*(a*d - b*c)^(8/3))))/(3*d^(8/3)*(a*d - b*c)^(4/3)) + (log((a + b*x^3)
^(1/3)*(a*c^6*d^4 - b*c^7*d^3) - ((3^(1/2)*c^3*1i + c^3)^2*(9*a^4*d^12 + 9
*b^4*c^4*d^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*c^2*d^10 - 36*a^3*b*c*d^11))/
(36*d^(16/3)*(a*d - b*c)^(8/3)))*(3^(1/2)*c^3*1i + c^3))/(6*d^(8/3)*(a*d -
 b*c)^(4/3)) - (c^3*log((a + b*x^3)^(1/3)*(a*c^6*d^4 - b*c^7*d^3) - (c^6*(
(3^(1/2)*1i)/2 - 1/2)^2*(9*a^4*d^12 + 9*b^4*c^4*d^8 - 36*a*b^3*c^3*d^9 + 5
4*a^2*b^2*c^2*d^10 - 36*a^3*b*c*d^11))/(9*d^(16/3)*(a*d - b*c)^(8/3)))*((3
^(1/2)*1i)/2 - 1/2))/(3*d^(8/3)*(a*d - b*c)^(4/3))
```

Reduce [F]

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx=\int \frac{x^{11}}{(b\,x^3+a)^{\frac{1}{3}}\,ac+(b\,x^3+a)^{\frac{1}{3}}\,ad\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bc\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bd\,x^6}dx$$

input
```
int(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x**11/((a + b*x**3)**(1/3)*a*c + (a + b*x**3)**(1/3)*a*d*x**3 + (a + b
*x**3)**(1/3)*b*c*x**3 + (a + b*x**3)**(1/3)*b*d*x**6),x)
```

3.764 $\int \frac{x^8}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 203

$$\int \frac{x^8}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\left(a+bx^3\right)^{2/3}}{2b^2d} + \frac{c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}(bc-ad)^{4/3}} - \frac{c^2\log\left(c+dx^3\right)}{6d^{5/3}(bc-ad)^{4/3}} + \frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}(bc-ad)^{4/3}}$$

output
```
-a^2/b^2/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/2*(b*x^3+a)^(2/3)/b^2/d+1/3*c^2*arct
an(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/d^(
5/3)/(-a*d+b*c)^(4/3)-1/6*c^2*ln(d*x^3+c)/d^(5/3)/(-a*d+b*c)^(4/3)+1/2*c^2
*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(5/3)/(-a*d+b*c)^(4/3)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.25

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\frac{3d^{2/3}(-3a^2d+b^2cx^3+ab(c-dx^3))}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{2\sqrt{3}c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{(bc-ad)^{4/3}} + \frac{2c^2\log\left(\sqrt[3]{bc-\right.}{(bc}}{6d^5}$$

input
```
Integrate[x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((3*d^(2/3)*(-3*a^2*d + b^2*c*x^3 + a*b*(c - d*x^3)))/(b^2*(b*c - a*d)*(a
+ b*x^3)^(1/3)) + (2*Sqrt[3]*c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))
/(b*c - a*d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(4/3) + (2*c^2*Log[(b*c - a*d)^(
1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) - (c^2*Log[(b*c - a*d
)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3
)^(2/3)])/(b*c - a*d)^(4/3))/(6*d^(5/3))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^6}{(bx^3+a)^{4/3}(dx^3+c)}dx^3$$

$$\Big\downarrow \text{ 98}$$

$$\frac{1}{3}\int \left(\frac{a^2}{b(bc-ad)\left(bx^3+a\right)^{4/3}}+\frac{1}{bd\sqrt[3]{bx^3+a}}+\frac{c^2}{d(ad-bc)\sqrt[3]{bx^3+a}\left(dx^3+c\right)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{3a^2}{b^2\sqrt[3]{a+bx^3}(bc-ad)}+\frac{\sqrt{3}c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{5/3}(bc-ad)^{4/3}}+\frac{3\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{c^2\log\left(c+dx^3\right)}{2d^{5/3}(bc-ad)^{4/3}}+\frac{3c^2\log\ldots}{\ldots}\right.$$

input
```
Int[x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((-3*a^2)/(b^2*(b*c - a*d)*(a + b*x^3)^(1/3)) + (3*(a + b*x^3)^(2/3))/(2*b
^2*d) + (Sqrt[3]*c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)
^(1/3))/Sqrt[3]])/(d^(5/3)*(b*c - a*d)^(4/3)) - (c^2*Log[c + d*x^3])/(2*d^
(5/3)*(b*c - a*d)^(4/3)) + (3*c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x
^3)^(1/3)])/(2*d^(5/3)*(b*c - a*d)^(4/3)))/3
```

Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.09

method	result
pseudoelliptic	$-\frac{-9d\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(-\frac{b^{2}c\,x^{3}}{3}-\frac{a\left(-d\,x^{3}+c\right)b}{3}+d\,a^{2}\right)+b^{2}c^{2}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(-2\arctan\left(\frac{2\sqrt{3}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}d^{2}(ad-bc)b^{2}}$

input
```
int(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/((a*d-b*c)/d)^(1/3)*(-9*d*((a*d-b*c)/d)^(1/3)*(-1/3*b^2*c*x^3-1/3*a*(
-d*x^3+c)*b+d*a^2)+b^2*c^2*(b*x^3+a)^(1/3)*(-2*arctan(2/3*3^(1/2)/((a*d-b*
c)/d)^(1/3)*(b*x^3+a)^(1/3)+1/3*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+((a*d-
b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))-2*ln((b*x^3+a)^(1/3)-((
a*d-b*c)/d)^(1/3))))/(b*x^3+a)^(1/3)/d^2/(a*d-b*c)/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(167) = 334$.

Time = 0.12 (sec) , antiderivative size = 1004, normalized size of antiderivative = 4.95

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/6*(3*sqrt(1/3)*(a*b^3*c^3*d - a^2*b^2*c^2*d^2 + (b^4*c^3*d - a*b^3*c^2
*d^2)*x^3)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b
*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3)
 + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d
))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)
*(b*x^3 + a)^(1/3))/(d*x^3 + c)) + (b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a
*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 +
a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 2*(b^3*c^2*x^3 + a*b^2*c^2)*(-b*c
*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) -
3*(a*b^2*c^2*d^2 - 4*a^2*b*c*d^3 + 3*a^3*d^4 + (b^3*c^2*d^2 - 2*a*b^2*c*d^
3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(2/3))/(a*b^4*c^2*d^3 - 2*a^2*b^3*c*d^4 +
a^3*b^2*d^5 + (b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5)*x^3), -1/6*(6*sq
rt(1/3)*(a*b^3*c^3*d - a^2*b^2*c^2*d^2 + (b^4*c^3*d - a*b^3*c^2*d^2)*x^3)*
sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a
)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c
- a*d))/d) + (b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3
 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2
 + a*d^3)^(2/3)) - 2*(b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^(2/3)*lo
g((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*(a*b^2*c^2*d^2 - 4*a
^2*b*c*d^3 + 3*a^3*d^4 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)...
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\int \frac{x^8}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**8/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**8/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.64

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\frac{6\,(-bcd^2+ad^3)^{\frac{2}{3}}b^2c^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d^3-2\,\sqrt{3}abcd^4+\sqrt{3}a^2d^5} - \frac{(-bcd^2+ad^3)^{\frac{2}{3}}b^2c^2\log\left((bx^3+a)^{\frac{2}{3}}\right.}{b^2c^2d^3\cdots}}{\cdots}$$

input
```
integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/6*(6*(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^
(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d
^3 - 2*sqrt(3)*a*b*c*d^4 + sqrt(3)*a^2*d^5) - (-b*c*d^2 + a*d^3)^(2/3)*b^2
*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-
(b*c - a*d)/d)^(2/3))/(b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5) + 2*b^2*c^2*(-
(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) - 6*a^2/((b*x^3 + a)^(1/3)*(b*c - a*d)
) + 3*(b*x^3 + a)^(2/3)/d)/b^2
```

Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.21

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{(b\,x^3+a)^{2/3}}{2\,b^2\,d} + \frac{a^2}{b^2\,(b\,x^3+a)^{1/3}\,(a\,d-b\,c)} + \frac{c^2\,\ln\left((b\,x^3+a)^{1/3}\,(a\,c^4\,d^3-b\,c^5\,d^2)-\frac{c^4\,(9\,a^4\,d^9-36\,a^3\,b\,c\,d^8+54\,a^2\,b^2\,c^2\,d^7-36\,a\,b^3\,c^3\,d^6+9\,b^4\,c^4\,d^5)}{9\,d^{10/3}\,(a\,d-b\,c)^{8/3}}\right)}{3\,d^{5/3}\,(a\,d-b\,c)^{4/3}} - \frac{\ln\left((b\,x^3+a)^{1/3}\,(a\,c^4\,d^3-b\,c^5\,d^2)-\frac{\left(c^2+\sqrt{3}\,c^2\,1\mathrm{i}\right)^2\,(9\,a^4\,d^9-36\,a^3\,b\,c\,d^8+54\,a^2\,b^2\,c^2\,d^7-36\,a\,b^3\,c^3\,d^6+9\,b^4\,c^4\,d^5)}{36\,d^{10/3}\,(a\,d-b\,c)^{8/3}}\right)\,\left(c^2+\sqrt{\right.}{6\,d^{5/3}\,(a\,d-b\,c)^{4/3}} + \frac{c^2\,\ln\left((b\,x^3+a)^{1/3}\,(a\,c^4\,d^3-b\,c^5\,d^2)-\frac{c^4\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\,(9\,a^4\,d^9-36\,a^3\,b\,c\,d^8+54\,a^2\,b^2\,c^2\,d^7-36\,a\,b^3\,c^3\,d^6+9\,b^4\,c^4\,d^5)}{d^{10/3}\,(a\,d-b\,c)^{8/3}}\right)\,\left(-\frac{1}{6}\right.}{d^{5/3}\,(a\,d-b\,c)^{4/3}}$$

input
```
int(x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
(a + b*x^3)^(2/3)/(2*b^2*d) + a^2/(b^2*(a + b*x^3)^(1/3)*(a*d - b*c)) + (c
^2*log((a + b*x^3)^(1/3)*(a*c^4*d^3 - b*c^5*d^2) - (c^4*(9*a^4*d^9 + 9*b^4
*c^4*d^5 - 36*a*b^3*c^3*d^6 + 54*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8))/(9*d^(
10/3)*(a*d - b*c)^(8/3))))/(3*d^(5/3)*(a*d - b*c)^(4/3)) - (log((a + b*x^3
)^(1/3)*(a*c^4*d^3 - b*c^5*d^2) - ((3^(1/2)*c^2*1i + c^2)^2*(9*a^4*d^9 + 9
*b^4*c^4*d^5 - 36*a*b^3*c^3*d^6 + 54*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8))/(3
6*d^(10/3)*(a*d - b*c)^(8/3)))*(3^(1/2)*c^2*1i + c^2))/(6*d^(5/3)*(a*d - b
*c)^(4/3)) + (c^2*log((a + b*x^3)^(1/3)*(a*c^4*d^3 - b*c^5*d^2) - (c^4*((3
^(1/2)*1i)/6 - 1/6)^2*(9*a^4*d^9 + 9*b^4*c^4*d^5 - 36*a*b^3*c^3*d^6 + 54*a
^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8))/(d^(10/3)*(a*d - b*c)^(8/3)))*((3^(1/2)*
1i)/6 - 1/6))/(d^(5/3)*(a*d - b*c)^(4/3))
```

Reduce [F]

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^8}{(bx^3+a)^{\frac{1}{3}}ac+(bx^3+a)^{\frac{1}{3}}adx^3+(bx^3+a)^{\frac{1}{3}}bcx^3+(bx^3+a)^{\frac{1}{3}}bdx^6}d$$

input
```
int(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x**8/((a + b*x**3)**(1/3)*a*c + (a + b*x**3)**(1/3)*a*d*x**3 + (a + b*
x**3)**(1/3)*b*c*x**3 + (a + b*x**3)**(1/3)*b*d*x**6),x)
```

3.765 $\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 174

$$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} - \frac{c\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{2/3}(bc-ad)^{4/3}} + \frac{c\log(c+dx^3)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}}$$

output
```
a/b/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/3*c*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/
3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/d^(2/3)/(-a*d+b*c)^(4/3)+1/6*c*ln(d*
x^3+c)/d^(2/3)/(-a*d+b*c)^(4/3)-1/2*c*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a
)^(1/3))/d^(2/3)/(-a*d+b*c)^(4/3)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.30

$$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{6}\left(\frac{6a}{(b^2c-abd)\sqrt[3]{a+bx^3}} - \frac{2\sqrt{3}c\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}(bc-ad)^{4/3}} - \frac{2c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{d^{2/3}(bc-ad)^{4/3}} + \frac{c\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{d^{2/3}(bc-ad)^{4/3}}\right)$$

input
```
Integrate[x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((6*a)/((b^2*c - a*b*d)*(a + b*x^3)^(1/3)) - (2*Sqrt[3]*c*ArcTan[(1 - (2*d
^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(2/3)*(b*c - a*d
)^(4/3)) - (2*c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(d^(2/
3)*(b*c - a*d)^(4/3)) + (c*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/
3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(d^(2/3)*(b*c - a*d)^(4
/3)))/6
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 87, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^3}{(bx^3+a)^{4/3}(dx^3+c)}dx^3$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{3}\left(\frac{c\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{bc-ad}+\frac{3a}{b\sqrt[3]{a+bx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{ 68}$$

$$\frac{1}{3}\left(\frac{c\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-a}}\right.}{bc-ad}\right.$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{3}\left(\frac{c\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}+\right.$$

$\downarrow$ 1082

$$\frac{1}{3}\left(\frac{c\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}+\frac{3a}{b\sqrt[3]{a+bx^3}(bc-a\right.$$

$\downarrow$ 217

$$\frac{1}{3}\left(\frac{c\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}+\frac{3a}{b\sqrt[3]{a+bx^3}(bc-\right.$$

input
```
Int[x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((3*a)/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) + (c*(-((Sqrt[3]*ArcTan[(1 - (2*d
^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(2/3)*(b*c - a*d
)^(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b*c - a*d)^(1/3)) - (3*Log[(b*c - a
*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c - a*d)^(1/3))))/(b
*c - a*d))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$-\frac{\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)bc\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{3}+\frac{\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)bc\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{3}-\frac{\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad}{}\right.\right.}{}}{\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}(ad-bc)db}$

input
```
int(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/((a*d-b*c)/d)^(1/3)*(1/3*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+
((a*d-b*c)/d)^(1/3))/((a*d-b*c)/d)^(1/3))*b*c*(b*x^3+a)^(1/3)+1/3*ln((b*x^
3+a)^(1/3)-((a*d-b*c)/d)^(1/3))*b*c*(b*x^3+a)^(1/3)-1/6*ln((b*x^3+a)^(2/3)
+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3))*b*c*(b*x^3+a)^(1
/3)+((a*d-b*c)/d)^(1/3)*a*d)/(b*x^3+a)^(1/3)/(a*d-b*c)/d/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(141) = 282$.

Time = 0.13 (sec) , antiderivative size = 872, normalized size of antiderivative = 5.01

$$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/6*(3*sqrt(1/3)*(a*b^2*c^2*d - a^2*b*c*d^2 + (b^3*c^2*d - a*b^2*c*d^2)*
x^3)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d +
 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x
^3 + a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*sqrt(
-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 +
 a)^(1/3))/(d*x^3 + c)) - (b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2/3)*log(
(b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c
*d^2 - a*d^3)^(2/3)) + 2*(b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2/3)*log((
b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 6*(a*b*c*d^2 - a^2*d^3)*(b
*x^3 + a)^(2/3))/(a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + a^3*b*d^4 + (b^4*c^2*d
^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^3), 1/6*(6*sqrt(1/3)*(a*b^2*c^2*d - a^
2*b*c*d^2 + (b^3*c^2*d - a*b^2*c*d^2)*x^3)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b
*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/
3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))/d) + (b^2*c*x^3 + a*b*c)*(b*
c*d^2 - a*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(
b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 2*(b^2*c*x^3 + a*b*c)*(b*c
*d^2 - a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 6
*(a*b*c*d^2 - a^2*d^3)*(b*x^3 + a)^(2/3))/(a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3
 + a^3*b*d^4 + (b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^3)]
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^5}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**5/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**5/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(141) = 282$.

Time = 0.15 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.73

$$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{\frac{6\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d^2-2\sqrt{3}abcd^3+\sqrt{3}a^2d^4}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{b^2c^2d^2-2abcd^3+a^2d^4}}{6b}$$

input
```
integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/6*(6*(-b*c*d^2 + a*d^3)^(2/3)*b*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/
3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^2
- 2*sqrt(3)*a*b*c*d^3 + sqrt(3)*a^2*d^4) - (-b*c*d^2 + a*d^3)^(2/3)*b*c*lo
g((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c -
a*d)/d)^(2/3))/(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4) + 2*b*c*(-(b*c - a*d)
/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2 -
2*a*b*c*d + a^2*d^2) - 6*a/((b*x^3 + a)^(1/3)*(b*c - a*d)))/b
```

Mupad [B] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.37

$$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{a}{b\,(b\,x^3+a)^{1/3}\,(a\,d-b\,c)} - \frac{c\,\ln\left((b\,x^3+a)^{1/3}\,(a\,c^2\,d^2-b\,c^3\,d)-\frac{c^2\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{9\,d^{4/3}\,(a\,d-b\,c)^{8/3}}\right)}{3\,d^{2/3}\,(a\,d-b\,c)^{4/3}} + \frac{\ln\left((b\,x^3+a)^{1/3}\,(a\,c^2\,d^2-b\,c^3\,d)-\frac{(c-\sqrt{3}\,c\,1i)^2\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{36\,d^{4/3}\,(a\,d-b\,c)^{8/3}}\right)\,(c-\sqrt{3}\,c\,1}{6\,d^{2/3}\,(a\,d-b\,c)^{4/3}} + \frac{\ln\left((b\,x^3+a)^{1/3}\,(a\,c^2\,d^2-b\,c^3\,d)-\frac{(c+\sqrt{3}\,c\,1i)^2\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{36\,d^{4/3}\,(a\,d-b\,c)^{8/3}}\right)\,(c+\sqrt{3}\,c\,1}{6\,d^{2/3}\,(a\,d-b\,c)^{4/3}}$$

input
```
int(x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
(log((a + b*x^3)^(1/3)*(a*c^2*d^2 - b*c^3*d) - ((c - 3^(1/2)*c*1i)^2*(9*a^
4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b*c
*d^5))/(36*d^(4/3)*(a*d - b*c)^(8/3)))*(c - 3^(1/2)*c*1i))/(6*d^(2/3)*(a*d
 - b*c)^(4/3)) - (c*log((a + b*x^3)^(1/3)*(a*c^2*d^2 - b*c^3*d) - (c^2*(9*
a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b
*c*d^5))/(9*d^(4/3)*(a*d - b*c)^(8/3))))/(3*d^(2/3)*(a*d - b*c)^(4/3)) - a
/(b*(a + b*x^3)^(1/3)*(a*d - b*c)) + (log((a + b*x^3)^(1/3)*(a*c^2*d^2 - b
*c^3*d) - ((c + 3^(1/2)*c*1i)^2*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*
d^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(36*d^(4/3)*(a*d - b*c)^(8/3))
)*(c + 3^(1/2)*c*1i))/(6*d^(2/3)*(a*d - b*c)^(4/3))
```

Reduce [F]

$$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^5}{(b\,x^3+a)^{\frac{1}{3}}\,ac+(b\,x^3+a)^{\frac{1}{3}}\,ad\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bc\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bd\,x^6}d$$

input
```
int(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x**5/((a + b*x**3)**(1/3)*a*c + (a + b*x**3)**(1/3)*a*d*x**3 + (a + b*
x**3)**(1/3)*b*c*x**3 + (a + b*x**3)**(1/3)*b*d*x**6),x)
```

3.766 $\int \frac{x^2}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 167

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{d}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}} - \frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}$$

output
```
-1/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/3*d^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a
)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/(-a*d+b*c)^(4/3)-1/6*d^(1/3)*ln
(d*x^3+c)/(-a*d+b*c)^(4/3)+1/2*d^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+
a)^(1/3))/(-a*d+b*c)^(4/3)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.32

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{d}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3(bc-ad)^{4/3}} - \frac{\sqrt[3]{d}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{6(bc-ad)^{4/3}}$$

input
```
Integrate[x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
-(1/((b*c - a*d)*(a + b*x^3)^(1/3))) + (d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a
+ b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*(b*c - a*d)^(4/3)) +
 (d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*(b*c - a*
d)^(4/3)) - (d^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a
+ b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*(b*c - a*d)^(4/3))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {946, 61, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{3}\int \frac{1}{(bx^3+a)^{4/3}(dx^3+c)}dx^3$$

$$\Big\downarrow \; 61$$

$$\frac{1}{3}\left(-\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{bc-ad}-\frac{3}{\sqrt[3]{a+bx^3}(bc-ad)}\right)$$

$$\Big\downarrow \; 68$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}+\frac{\log(c+dx^3}{2d^{2/3}\sqrt[3]{bc-}}\right.}{bc-ad}\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}\right.$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}-\frac{3}{\sqrt[3]{a+bx^3}(bc-}\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}-\frac{3}{\sqrt[3]{a+bx^3}(bc-\right.$$

input
```
Int[x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(-3/((b*c - a*d)*(a + b*x^3)^(1/3)) - (d*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3
)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(2/3)*(b*c - a*d)^(1/
3))) + Log[c + d*x^3]/(2*d^(2/3)*(b*c - a*d)^(1/3)) - (3*Log[(b*c - a*d)^(
1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c - a*d)^(1/3))))/(b*c -
a*d))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.18

method	result
pseudoelliptic	$\frac{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}+\left(2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)-\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)\right.\right.}{\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(6ad-6bc\right)}$

input
```
int(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
2/((a*d-b*c)/d)^(1/3)*(3*((a*d-b*c)/d)^(1/3)+1/2*(2*arctan(1/3*3^(1/2)*(2*
(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*c)/d)^(1/3))*3^(1/2)+2*ln((b*
x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))-ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(
b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3)))*(b*x^3+a)^(1/3))/(b*x^3+a)^(1/3)/(6*a
*d-6*b*c)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.57

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx =$$
$$-\frac{2\sqrt{3}(bx^3+a)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-(bx^3+a)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\log\left(-(bx^3\right.$$

input
```
integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(b*x^3 + a)*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x
^3 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) - (b*x^3 + a)*(-d/(b*c
 - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) +
 (b*x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 2*(b*x^3 + a)
*(-d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 +
a)^(1/3)*d) + 6*(b*x^3 + a)^(2/3))/((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^2}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**2/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**2/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(135) = 270.

Time = 0.14 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.71

$$\begin{aligned}\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx &= \frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^2c^2-2\,abcd+a^2d^2)} \\ &+ \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d-2\,\sqrt{3}abcd^2+\sqrt{3}a^2d^3} \\ &- \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(b^2c^2d-2\,abcd^2+a^2d^3)} \\ &- \frac{1}{(bx^3+a)^{\frac{1}{3}}(bc-ad)}\end{aligned}$$

input
```
integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^
(1/3)))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + (-b*c*d^2 + a*d^3)^(2/3)*arctan(
1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d
)^(1/3))/(sqrt(3)*b^2*c^2*d - 2*sqrt(3)*a*b*c*d^2 + sqrt(3)*a^2*d^3) - 1/6
*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c
 - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3) - 1/((b*x^3 + a)^(1/3)*(b*c - a*d))
```

Mupad [B] (verification not implemented)

Time = 3.65 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.33

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{(b\,x^3+a)^{1/3}\,(a\,d-b\,c)}$$
$$+\frac{d^{1/3}\,\ln\left((b\,x^3+a)^{1/3}\,(a\,d^4-b\,c\,d^3)-\frac{d^{2/3}\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{9\,(a\,d-b\,c)^{8/3}}\right)}{3\,(a\,d-b\,c)^{4/3}}$$
$$-\frac{d^{1/3}\,\ln\left((b\,x^3+a)^{1/3}\,(a\,d^4-b\,c\,d^3)-\frac{d^{2/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)^2\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{9\,(a\,d-b\,c)^{8/3}}\right)\,\left(\frac{1}{2}+\right.}{3\,(a\,d-b\,c)^{4/3}}$$
$$+\frac{d^{1/3}\,\ln\left((b\,x^3+a)^{1/3}\,(a\,d^4-b\,c\,d^3)-\frac{d^{2/3}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^2\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{(a\,d-b\,c)^{8/3}}\right)\,\left(-\frac{1}{6}\right.}{(a\,d-b\,c)^{4/3}}$$

input
```
int(x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
1/((a + b*x^3)^(1/3)*(a*d - b*c)) + (d^(1/3)*log((a + b*x^3)^(1/3)*(a*d^4
- b*c*d^3) - (d^(2/3)*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a
^2*b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(9*(a*d - b*c)^(8/3))))/(3*(a*d - b*c)^(
4/3)) - (d^(1/3)*log((a + b*x^3)^(1/3)*(a*d^4 - b*c*d^3) - (d^(2/3)*((3^(1
/2)*1i)/2 + 1/2)^2*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2*
b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(9*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/2 + 1/
2))/(3*(a*d - b*c)^(4/3)) + (d^(1/3)*log((a + b*x^3)^(1/3)*(a*d^4 - b*c*d^
3) - (d^(2/3)*((3^(1/2)*1i)/6 - 1/6)^2*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b
^3*c^3*d^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(a*d - b*c)^(8/3))*((3^
(1/2)*1i)/6 - 1/6))/(a*d - b*c)^(4/3)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^2}{(b\,x^3+a)^{\frac{1}{3}}\,ac+(b\,x^3+a)^{\frac{1}{3}}\,ad\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bc\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bd\,x^6}d$$

input
```
int(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x**2/((a + b*x**3)**(1/3)*a*c + (a + b*x**3)**(1/3)*a*d*x**3 + (a + b*
x**3)**(1/3)*b*c*x**3 + (a + b*x**3)**(1/3)*b*d*x**6),x)
```

3.767 $\int \frac{1}{x(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 271

$$\int \frac{1}{x(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}c} - \frac{d^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c(bc-ad)^{4/3}} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3}\log(c+dx^3)}{6c(bc-ad)^{4/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{4/3}c} - \frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{4/3}}$$

output
```
b/a/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*
3^(1/2)/a^(1/3))*3^(1/2)/a^(4/3)/c-1/3*d^(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*
x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/c/(-a*d+b*c)^(4/3)-1/2*ln(
x)/a^(4/3)/c+1/6*d^(4/3)*ln(d*x^3+c)/c/(-a*d+b*c)^(4/3)+1/2*ln(a^(1/3)-(b*
x^3+a)^(1/3))/a^(4/3)/c-1/2*d^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^
(1/3))/c/(-a*d+b*c)^(4/3)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{6}\left(\frac{6b}{\left(abc-a^2d\right)\sqrt[3]{a+bx^3}} + \frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}c} - \frac{2\sqrt{3}d^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{c(bc-ad)^{4/3}} + \frac{2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{a^{4/3}c} - \frac{2d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{c(bc-ad)^{4/3}} - \frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{a^{4/3}c} + \frac{d^{4/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{c(bc-ad)^{4/3}}\right)$$

input
```
Integrate[1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((6*b)/((a*b*c - a^2*d)*(a + b*x^3)^(1/3)) + (2*Sqrt[3]*ArcTan[(1 + (2*(a
+ b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/(a^(4/3)*c) - (2*Sqrt[3]*d^(4/3)*ArcTan
[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(c*(b*c -
 a*d)^(4/3)) + (2*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/(a^(4/3)*c) - (2*d^(4
/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(c*(b*c - a*d)^(4/
3)) - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(a^(4/3
)*c) + (d^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x
^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(c*(b*c - a*d)^(4/3)))/6
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {948, 96, 25, 174, 67, 16, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{1}{x^3\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \ 96$$

$$\frac{1}{3}\left(\frac{3b}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\int -\frac{bdx^3+bc-ad}{x^3\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{a(bc-ad)}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{3}\left(\frac{\int \frac{bdx^3+bc-ad}{x^3\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{a(bc-ad)}+\frac{3b}{a\sqrt[3]{a+bx^3}(bc-ad)}\right)$$

$$\Big\downarrow \ 174$$

$$\frac{1}{3}\left(\frac{\frac{ad^2\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{c}+\frac{(bc-ad)\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3}{c}}{a(bc-ad)}+\frac{3b}{a\sqrt[3]{a+bx^3}(bc-ad)}\right)$$

$$\Big\downarrow \ 67$$

$$\frac{1}{3}\left(\frac{\frac{(bc-ad)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}+\frac{ad^2\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right.}}{c}}{a(bc-ad)}\right.$$

$\Big\downarrow$ 16

$$\frac{1}{3}\left(\frac{\frac{(bc-ad)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}+\frac{ad^2\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx^3}{c}}{a(bc-ad)}+\right.$$

$\Big\downarrow$ 68

$$\frac{1}{3}\left(\frac{\frac{(bc-ad)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}+\frac{ad^2\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+}}}{2d^{2/3}\sqrt[3]{bc-a}}\right.}{}}{a(bc-ad)}\right.$$

$\Big\downarrow$ 16

$$\frac{1}{3}\left(\frac{\frac{(bc-ad)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}+\frac{ad^2\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-a\cdots}}{\cdots}}\cdots}{2d\cdots}\cdots\right.}{\cdots}}{a(bc-ad)}\cdots\right.$$

$\big\downarrow$ 1082

$$\frac{1}{3}\left(\frac{\frac{ad^2\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{c}+\frac{(bc-ad)\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{\cdots}}{\cdots}\right)}{\sqrt[3]{\cdots}}\cdots\right.}{\cdots}}{a(bc-ad)}\cdots\right.$$

$\big\downarrow$ 217

$$\frac{1}{3}\left(\frac{\frac{ad^2\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{c}+\frac{(bc-ad)\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a}\cdots}{\sqrt[3]{\cdots}}}{\cdots}\right)}{\sqrt[3]{a}\cdots}\cdots\right.}{\cdots}}{a(bc-ad)}\cdots\right.$$

input
```
Int[1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((3*b)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) + (((b*c - a*d)*((Sqrt[3]*ArcTan[
(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^3]/(2*a^(1/3
)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3))))/c + (a*d^2*(-((Sqr
t[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]
)/(d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b*c - a*d)^(1/
3)) - (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b
*c - a*d)^(1/3))))/c)/(a*(b*c - a*d)))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{\left(-(ad-bc)\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)\right)(b}{}$

input
```
int(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/d)^(1/3)/(b*x^3+a)^(1/3)/a^(4/3)*((-(a*d-b*c)*(-2*arctan(1/
3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+
a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*(b*x^3+a)^
(1/3)-6*a^(1/3)*b*c)*((a*d-b*c)/d)^(1/3)+d*(b*x^3+a)^(1/3)*(-2*arctan(1/3*
3^(1/2)*(2*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*c)/d)^(1/3))*3^(1/
2)+ln((b*x^3+a)^(2/3)+((a*d-b*c)/d)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2
/3))-2*ln((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3)))*a^(4/3))/(a*d-b*c)/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(216) = 432$.

Time = 0.13 (sec) , antiderivative size = 975, normalized size of antiderivative = 3.60

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/6*(6*(b*x^3 + a)^(2/3)*a*b*c + 3*sqrt(1/3)*(a^2*b*c - a^3*d + (a*b^2*c
- a^2*b*d)*x^3)*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)
^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^
3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) + 2*sqrt(3)*(a^2*b*d*x^3 + a^3*d)*(d/(b*c
 - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3)
- 1/3*sqrt(3)) - ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3
+ a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*((b^2*c - a*b*d)*x^3
 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + (a^2*b*d*x^3
+ a^3*d)*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c
- a*d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) -
 2*(a^2*b*d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a
*d))^(2/3) + (b*x^3 + a)^(1/3)*d))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a
^3*b*c*d)*x^3), 1/6*(6*(b*x^3 + a)^(2/3)*a*b*c + 2*sqrt(3)*(a^2*b*d*x^3 +
a^3*d)*(d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c
- a*d))^(1/3) - 1/3*sqrt(3)) - ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/
3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*((b^2*
c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) +
 (a^2*b*d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c -
 a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c -
a*d))^(1/3)) - 2*(a^2*b*d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log((b*c -...
```

Sympy [F]

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{x\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

input
```
integrate(1/x/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x}\,dx$$

input
```
integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.44

$$\begin{aligned}\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= -\frac{d^2\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^3-2\,abc^2d+a^2cd^2\right)} \\
&\quad-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^3-2\sqrt{3}abc^2d+\sqrt{3}a^2cd^2} \\
&\quad+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^3-2\,abc^2d+a^2cd^2\right)} \\
&\quad+\frac{b}{(bx^3+a)^{\frac{1}{3}}(abc-a^2d)}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{4}{3}}c} \\
&\quad-\frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{4}{3}}c}+\frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{4}{3}}c}\end{aligned}$$

input
```
integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*d^2*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/
d)^(1/3)))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2) - (-b*c*d^2 + a*d^3)^(2/3)*
arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
 a*d)/d)^(1/3))/(sqrt(3)*b^2*c^3 - 2*sqrt(3)*a*b*c^2*d + sqrt(3)*a^2*c*d^2
) + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)
*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^3 - 2*a*b*c^2*d +
 a^2*c*d^2) + b/((b*x^3 + a)^(1/3)*(a*b*c - a^2*d)) + 1/3*sqrt(3)*arctan(1
/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(4/3)*c) - 1/6*log(
(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(4/3)*c) + 1/3
*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(4/3)*c)
```

Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 3804, normalized size of antiderivative = 14.04

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
log(9*a^7*b^14*c^11*d^4 - ((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a
^8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937
*a^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 98
55*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 4
86*a^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14) - (-d^4/(27*b^4*c^7 + 27*a^4*
c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(2/3
)*(243*a^10*b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13
*d^5 - 51030*a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*
b^10*c^10*d^8 + 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255
*a^18*b^7*c^7*d^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 48
6*a^21*b^4*c^4*d^14))*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d
^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3) - 90*a^8*b^13*c^10*d^5
+ 405*a^9*b^12*c^9*d^6 - 1071*a^10*b^11*c^8*d^7 + 1827*a^11*b^10*c^7*d^8 -
 2079*a^12*b^9*c^6*d^9 + 1575*a^13*b^8*c^5*d^10 - 765*a^14*b^7*c^4*d^11 +
216*a^15*b^6*c^3*d^12 - 27*a^16*b^5*c^2*d^13)*(-d^4/(27*b^4*c^7 + 27*a^4*c
^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3)
 + log(9*a^7*b^14*c^11*d^4 - ((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 29
7*a^8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8
937*a^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 -
 9855*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^...
```

Reduce [F]

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}acx+\left(b\,x^3+a\right)^{\frac{1}{3}}ad\,x^4+\left(b\,x^3+a\right)^{\frac{1}{3}}bc\,x^4+\left(b\,x^3+a\right)^{\frac{1}{3}}bd\,x^7}\,dx$$

input
```
int(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a*c*x + (a + b*x**3)**(1/3)*a*d*x**4 + (a + b*x
**3)**(1/3)*b*c*x**4 + (a + b*x**3)**(1/3)*b*d*x**7),x)
```

3.768 $\int \frac{1}{x^4(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 339

$$
\begin{aligned}
\int \frac{1}{x^4(a+bx^3)^{4/3}(c+dx^3)}\,dx &= -\frac{b(4bc-ad)}{3a^2c(bc-ad)\sqrt[3]{a+bx^3}} \\
&\quad -\frac{1}{3acx^3\sqrt[3]{a+bx^3}}-\frac{(4bc+3ad)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}c^2} \\
&\quad +\frac{d^{7/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2(bc-ad)^{4/3}}+\frac{(4bc+3ad)\log(x)}{6a^{7/3}c^2}-\frac{d^{7/3}\log(c+dx^3)}{6c^2(bc-ad)^{4/3}} \\
&\quad -\frac{(4bc+3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{7/3}c^2}+\frac{d^{7/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)^{4/3}}
\end{aligned}
$$

output
```
-1/3*b*(-a*d+4*b*c)/a^2/c/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/3/a/c/x^3/(b*x^3+a)
^(1/3)-1/9*(3*a*d+4*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^
(1/3))*3^(1/2)/a^(7/3)/c^2+1/3*d^(7/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(
1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/c^2/(-a*d+b*c)^(4/3)+1/6*(3*a*d+4*
b*c)*ln(x)/a^(7/3)/c^2-1/6*d^(7/3)*ln(d*x^3+c)/c^2/(-a*d+b*c)^(4/3)-1/6*(3
*a*d+4*b*c)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(7/3)/c^2+1/2*d^(7/3)*ln((-a*d+b
*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c^2/(-a*d+b*c)^(4/3)
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^4 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \frac{\frac{6c(-a^2d+4b^2cx^3+ab(c-dx^3))}{a^2(-bc+ad)x^3\sqrt[3]{a+bx^3}} - \frac{2\sqrt{3}(4bc+3ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}} + \frac{6\sqrt{3}d^{7/3}\arctan}{}}{}$$

input
```
Integrate[1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((6*c*(-(a^2*d) + 4*b^2*c*x^3 + a*b*(c - d*x^3)))/(a^2*(-(b*c) + a*d)*x^3*
(a + b*x^3)^(1/3)) - (2*Sqrt[3]*(4*b*c + 3*a*d)*ArcTan[(1 + (2*(a + b*x^3)
^(1/3))/a^(1/3))/Sqrt[3]])/a^(7/3) + (6*Sqrt[3]*d^(7/3)*ArcTan[(1 - (2*d^(
1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(4/3) - (
2*(4*b*c + 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(7/3) + (6*d^(7/3)*
Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) + ((
4*b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)
])/a^(7/3) - (3*d^(7/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*
(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))/(18*c^2
)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {948, 114, 27, 174, 61, 67, 16, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^3)^{4/3} (c+dx^3)}\,dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{1}{x^6\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \; 114$$

$$\frac{1}{3}\left(-\frac{\int \frac{4bdx^3+4bc+3ad}{3x^3\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)}dx^3}{ac}-\frac{1}{acx^3\sqrt[3]{a+bx^3}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(-\frac{\int \frac{4bdx^3+4bc+3ad}{x^3\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)}dx^3}{3ac}-\frac{1}{acx^3\sqrt[3]{a+bx^3}}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+4bc)\int \frac{1}{x^3\left(bx^3+a\right)^{4/3}}dx^3}{c}-\frac{3ad^2\int \frac{1}{\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)}dx^3}{c}}{3ac}-\frac{1}{acx^3\sqrt[3]{a+bx^3}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+4bc)\left(\frac{\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{c}-\frac{3ad^2\left(-\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx^3}{bc-ad}-\frac{3}{\sqrt[3]{a+bx^3}(bc-ad)}\right)}{c}}{3ac}-\frac{1}{acx^3\sqrt[3]{a\ldots}}\right.$$

$$\Big\downarrow \; 67$$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+4bc)\left(\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{c}-\frac{3ad^2\ldots}{\ldots}}{3ac}\right.$$

$\downarrow$ 16

$$\frac{1}{3}\left(-\frac{\frac{(3ad+4bc)\left(\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{c}-\frac{3ad^2\left(-\frac{d\int \frac{}{\sqrt[3]{}}}{}\right.}{}}{3ac}\right.$$

$\downarrow$ 68

$$\frac{1}{3}\left(-\frac{\frac{(3ad+4bc)\left(\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{c}-\frac{3ad^2\left(-\frac{d\left(-\frac{3\int}{}\right.}{}\right.}{}}{}\right.$$

$\downarrow$ 16

$$\frac{1}{3}\left(-\frac{(3ad+4bc)\left(\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{c}-\frac{3ad^2\left(-\frac{d\left(\frac{3\int -}{x}\right)}{}\right)}{}\right.$$

$\downarrow$ 1082

$$\frac{1}{3}\left(-\frac{(3ad+4bc)\left(\frac{-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{c}-\frac{3ad^2\left(-\frac{d\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\right.}{d^{2/3}\sqrt[3]{}}\right)}{}\right)}{3ac}\right.$$

$\Big\downarrow$ 217

$$\frac{1}{3}\left(-\frac{(3ad+4bc)\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{c}-\frac{3ad^2\left(-\frac{d\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\cdots}}{\cdots}}{\cdots}\right)}{d^{2/3}\sqrt[3]{b\cdots}}\cdots\right)}{\cdots}\cdots\right)}{3ac}\cdots\right.$$

input
```
Int[1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(-(1/(a*c*x^3*(a + b*x^3)^(1/3))) - (((4*b*c + 3*a*d)*(3/(a*(a + b*x^3)^(1
/3)) + ((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1
/3) - Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/
3)))/a))/c - (3*a*d^2*(-3/((b*c - a*d)*(a + b*x^3)^(1/3)) - (d*(-((Sqrt[3]
*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d
^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b*c - a*d)^(1/3))
- (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c -
 a*d)^(1/3))))/(b*c - a*d)))/c)/(3*a*c))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{\left((ad-bc)\left(ad+\frac{4bc}{3}\right)\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\right.\right.\right.}{}$

input
```
int(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/(b*x^3+a)^(1/3)/a^(7/3)/((a*d-b*c)/d)^(1/3)*(((a*d-b*c)*(a*d+4/3*b*c)*
(-2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)+ln((b*
x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)
))*x^3*(b*x^3+a)^(1/3)+2*c*(-a^(7/3)*d+((-d*x^3+c)*a^(4/3)+4*b*c*x^3*a^(1/
3))*b))*((a*d-b*c)/d)^(1/3)+2*(b*x^3+a)^(1/3)*a^(7/3)*d^2*(arctan(1/3*3^(1
/2)*(2*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(1/3))/((a*d-b*c)/d)^(1/3))*3^(1/2)+l
n((b*x^3+a)^(1/3)-((a*d-b*c)/d)^(1/3))-1/2*ln((b*x^3+a)^(2/3)+((a*d-b*c)/d
)^(1/3)*(b*x^3+a)^(1/3)+((a*d-b*c)/d)^(2/3)))*x^3)/c^2/x^3/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. 2(276) = 552.

Time = 0.78 (sec) , antiderivative size = 1386, normalized size of antiderivative = 4.09

$$\int \frac{1}{x^4 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(1/3)*((4*a*b^3*c^2 - a^2*b^2*c*d - 3*a^3*b*d^2)*x^6 + (4*a^2
*b^2*c^2 - a^3*b*c*d - 3*a^4*d^2)*x^3)*sqrt((-a)^(1/3)/a)*log((2*b*x^3 - 3
*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)^(1
/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) - 6
*sqrt(3)*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*arctan(2/3*s
qrt(3)*(b*x^3 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) + ((4*b^3*c
^2 - a*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*
x^3)*(-a)^(2/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a
)^(2/3)) - 2*((4*b^3*c^2 - a*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - a
^2*b*c*d - 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*x^3 + a)^(1/3) + (-a)^(1/3))
 + 3*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^
(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d - (b*c - a*
d)*(-d/(b*c - a*d))^(1/3)) - 6*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*
d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d) -
6*(a^2*b*c^2 - a^3*c*d + (4*a*b^2*c^2 - a^2*b*c*d)*x^3)*(b*x^3 + a)^(2/3))
/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/18*(6
*sqrt(1/3)*((4*a*b^3*c^2 - a^2*b^2*c*d - 3*a^3*b*d^2)*x^6 + (4*a^2*b^2*c^2
 - a^3*b*c*d - 3*a^4*d^2)*x^3)*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*
x^3 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 6*sqrt(3)*(a^3*b*d^2*x
^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)...
```

Sympy [F]

$$\int \frac{1}{x^4 \left(a+b x^3\right)^{4/3} \left(c+d x^3\right)}\, dx = \int \frac{1}{x^4 \left(a+b x^3\right)^{\frac{4}{3}} \left(c+d x^3\right)}\, dx$$

input
```
integrate(1/x**4/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**4*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a + bx^3)^{4/3} (c + dx^3)} \, dx = \int \frac{1}{(bx^3 + a)^{\frac{4}{3}} (dx^3 + c)x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.42

$$\begin{aligned}\int \frac{1}{x^4 (a + bx^3)^{4/3} (c + dx^3)} \, dx &= \frac{d^3 \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}} \log \left(\left|(bx^3 + a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2)} \\ &+ \frac{(-bcd^2 + ad^3)^{\frac{2}{3}} d \arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^4 - 2\,\sqrt{3}abc^3d + \sqrt{3}a^2c^2d^2} \\ &- \frac{(-bcd^2 + ad^3)^{\frac{2}{3}} d \log\left((bx^3 + a)^{\frac{2}{3}} + (bx^3 + a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2)} \\ &- \frac{4\,(bx^3 + a)b^2c - 3\,ab^2c - (bx^3 + a)abd}{3\,(a^2bc^2 - a^3cd)\left((bx^3 + a)^{\frac{4}{3}} - (bx^3 + a)^{\frac{1}{3}}a\right)} \\ &- \frac{\sqrt{3}(4\,bc + 3\,ad) \arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{7}{3}}c^2} \\ &+ \frac{(4\,bc + 3\,ad) \log\left((bx^3 + a)^{\frac{2}{3}} + (bx^3 + a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{18\,a^{\frac{7}{3}}c^2} \\ &- \frac{\left(4\,a^{\frac{1}{3}}bc + 3\,a^{\frac{4}{3}}d\right) \log\left(\left|(bx^3 + a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{8}{3}}c^2}\end{aligned}$$

input
```
integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*d^3*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) + (-b*c*d^2 + a*d^3)^(2/3)
*d*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*
c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^4 - 2*sqrt(3)*a*b*c^3*d + sqrt(3)*a^2*c^
2*d^2) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*d*log((b*x^3 + a)^(2/3) + (b*x^3 + a
)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^4 - 2*a*b*
c^3*d + a^2*c^2*d^2) - 1/3*(4*(b*x^3 + a)*b^2*c - 3*a*b^2*c - (b*x^3 + a)*
a*b*d)/((a^2*b*c^2 - a^3*c*d)*((b*x^3 + a)^(4/3) - (b*x^3 + a)^(1/3)*a)) -
 1/9*sqrt(3)*(4*b*c + 3*a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(
1/3))/a^(1/3))/(a^(7/3)*c^2) + 1/18*(4*b*c + 3*a*d)*log((b*x^3 + a)^(2/3)
+ (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(7/3)*c^2) - 1/9*(4*a^(1/3)*b*c
+ 3*a^(4/3)*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(8/3)*c^2)
```

Mupad [B] (verification not implemented)

Time = 5.12 (sec) , antiderivative size = 5875, normalized size of antiderivative = 17.33

$$\int \frac{1}{x^4 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
log((d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c
^8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(419904*a^13*b^17*c^20*d^4 - ((a + b*x^3)
^(1/3)*(8975448*a^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*
a^16*b^15*c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^1
8*d^7 + 56509893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 937107
63*a^21*b^10*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c
^13*d^12 - 38736144*a^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 814
8762*a^26*b^5*c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + (d^7/(27*b^4*c^10 +
 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*
d))^(2/3)*(4782969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 3108
92985*a^21*b^13*c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23
*b^11*c^20*d^7 - 3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*
d^9 - 2582803260*a^26*b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 4782
96900*a^28*b^6*c^15*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4
*c^13*d^14))*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*
a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(1/3) - 3254256*a^14*b^16*c^19*d^5 + 1
0156428*a^15*b^15*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^
13*c^16*d^8 + 15529887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 +
 5412825*a^20*b^10*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22
*b^8*c^11*d^13 + 7801029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15...
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,ac\,x^4+(b\,x^3+a)^{\frac{1}{3}}\,ad\,x^7+(b\,x^3+a)^{\frac{1}{3}}\,bc\,x^7+(b\,x^3+a)^{\frac{1}{3}}\,b$$

input
```
int(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a*c*x**4 + (a + b*x**3)**(1/3)*a*d*x**7 + (a +
b*x**3)**(1/3)*b*c*x**7 + (a + b*x**3)**(1/3)*b*d*x**10),x)
```

3.769 $\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 322

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{ax^4}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(bc-4ad)x(a+bx^3)^{2/3}}{3b^2d(bc-ad)} - \frac{(3bc+4ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{7/3}d^2} + \frac{c^{7/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2(bc-ad)^{4/3}} + \frac{c^{7/3}\log(c+dx^3)}{6d^2(bc-ad)^{4/3}} - \frac{c^{7/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2(bc-ad)^{4/3}} + \frac{(3bc+4ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{7/3}d^2}$$

output
```
a*x^4/b/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/3*(-4*a*d+b*c)*x*(b*x^3+a)^(2/3)/b^2/
d/(-a*d+b*c)-1/9*(4*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*
3^(1/2))*3^(1/2)/b^(7/3)/d^2+1/3*c^(7/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*
x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d^2/(-a*d+b*c)^(4/3)+1/6*c^(7/
3)*ln(d*x^3+c)/d^2/(-a*d+b*c)^(4/3)-1/2*c^(7/3)*ln((-a*d+b*c)^(1/3)*x/c^(1
/3)-(b*x^3+a)^(1/3))/d^2/(-a*d+b*c)^(4/3)+1/6*(4*a*d+3*b*c)*ln(-b^(1/3)*x+
(b*x^3+a)^(1/3))/b^(7/3)/d^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.09 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.57

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\frac{12d(-4a^2dx+b^2cx^4+abx(c-dx^3))}{b^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4\sqrt{3}(3bc+4ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{7/3}} - \frac{6\sqrt{-6+6i}}{}}{}$$

input
```
Integrate[x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((12*d*(-4*a^2*d*x + b^2*c*x^4 + a*b*x*(c - d*x^3)))/(b^2*(b*c - a*d)*(a +
 b*x^3)^(1/3)) - (4*Sqrt[3]*(3*b*c + 4*a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^
(1/3)*x + 2*(a + b*x^3)^(1/3))])/b^(7/3) - (6*Sqrt[-6 + (6*I)*Sqrt[3]]*c^(
7/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I +
Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(b*c - a*d)^(4/3) + (4*(3*b*c + 4*a*
d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(7/3) + (6*(1 + I*Sqrt[3])*c^(
7/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)
])/(b*c - a*d)^(4/3) - (2*(3*b*c + 4*a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a +
 b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(7/3) - ((3*I)*(-I + Sqrt[3])*c^(7/3
)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)
*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c -
a*d)^(4/3))/(36*d^2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {970, 1052, 25, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{970}$$

$$\frac{ax^4}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\int \frac{x^3(4ac-(bc-4ad)x^3)}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{b(bc-ad)}$$

$$\Big\downarrow \text{1052}$$

$$\frac{ax^4}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{-\frac{\int -\frac{(bc-ad)(3bc+4ad)x^3+ac(bc-4ad)}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{3bd}-\frac{x(a+bx^3)^{2/3}(bc-4ad)}{3bd}}{b(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{ax^4}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\frac{\int \frac{(bc-ad)(3bc+4ad)x^3+ac(bc-4ad)}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{3bd}-\frac{x(a+bx^3)^{2/3}(bc-4ad)}{3bd}}{b(bc-ad)}$$

$$\Big\downarrow \text{1026}$$

$$\frac{ax^4}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\frac{\frac{(bc-ad)(4ad+3bc)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{3b^2c^3\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}}{3bd}-\frac{x(a+bx^3)^{2/3}(bc-4ad)}{3bd}}{b(bc-ad)}$$

$$\Big\downarrow \text{769}$$

$$\frac{\frac{ax^4}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\frac{(bc-ad)(4ad+3bc)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{3b^2c^3\int\frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}}{3bd}-\frac{x(a+bx^3)^{2/3}(bc-4ad)}{3bd}}{b(bc-ad)}$$

$\big\downarrow$ 901

$$\frac{\frac{ax^4}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\frac{(bc-ad)(4ad+3bc)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{3b^2c^3\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{\ldots}}{\ldots}\right.}{\ldots}\right.}{d}}{3bd}}{b(bc-ad)}$$

input
```
Int[x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(a*x^4)/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) - (-1/3*((b*c - 4*a*d)*x*(a + b*
x^3)^(2/3))/(b*d) + ((-3*b^2*c^3*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(
1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Lo
g[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(
1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/d + ((b*c - a*d)
*(3*b*c + 4*a*d)*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(S
qrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/d)/(
3*b*d))/(b*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.52

method	result
pseudoelliptic	$-\frac{-\frac{\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right) c^{2} b^{\frac{13}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}}}{2}+\frac{2(ad-bc)\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(ad+\frac{3bc}{4}\right)b^{2}\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} \ln\left(\cdots\right.}{3}\cdots}{\cdots}$

input
```
int(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)
*x+(b*x^3+a)^(2/3))/x^2)*c^2*b^(13/3)*(b*x^3+a)^(1/3)+2/3*(a*d-b*c)*(b*x^3
+a)^(1/3)*(a*d+3/4*b*c)*b^2*((a*d-b*c)/c)^(1/3)*ln((b^(2/3)*x^2+b^(1/3)*(b
*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^
(1/3))/x)*c^2*b^(13/3)*(b*x^3+a)^(1/3)-4/3*(a*d-b*c)*(b*x^3+a)^(1/3)*(a*d+
3/4*b*c)*b^2*3^(1/2)*((a*d-b*c)/c)^(1/3)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(
1/3)/b^(1/3)+x)/x)-4/3*(a*d-b*c)*(b*x^3+a)^(1/3)*(a*d+3/4*b*c)*b^2*((a*d-b
*c)/c)^(1/3)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)+b^(7/3)*(-4*(-1/4*b^2*c*x^
3-1/4*a*(-d*x^3+c)*b+d*a^2)*d*x*((a*d-b*c)/c)^(1/3)+b^2*arctan(1/3*3^(1/2)
*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*(b*x^3+a)^(1/3)*c^2*3^(1/2)
))/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3)/d^2/b^(13/3)/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. 2(267) = 534.

Time = 0.68 (sec) , antiderivative size = 1329, normalized size of antiderivative = 4.13

$$\int \frac{x^9}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(1/3)*(3*a*b^3*c^2 + a^2*b^2*c*d - 4*a^3*b*d^2 + (3*b^4*c^2 +
 a*b^3*c*d - 4*a^2*b^2*d^2)*x^3)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 +
 a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2
 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) - 6*sqrt(3)*(b^4
*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt
(3)*(b*x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) + 2*(3*a*b^2*c^2 + a^2*b*c
*d - 4*a^3*d^2 + (3*b^3*c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log(-(
b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 +
 (3*b^3*c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (b*
x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 6*(b^4*c^2*x^3 + a*b^
3*c^2)*(c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (
b*x^3 + a)^(1/3)*c)/x) + 3*(b^4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)
*log(((b*c - a*d)*x^2*(c/(b*c - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d
)*x*(c/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*c)/x^2) + 6*((b^3*c*d - a*b^
2*d^2)*x^4 + (a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*c*d^2
- a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^3), -1/18*(6*sqrt(3)*(b^4*c^2*x^
3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*
x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) - 2*(3*a*b^2*c^2 + a^2*b*c*d - 4*
a^3*d^2 + (3*b^3*c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)
*x - (b*x^3 + a)^(1/3))/x) + (3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*...
```

Sympy [F]

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^9}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**9/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**9/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^9}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^9/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^9}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^9/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^9}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

input
```
int(x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(x^9/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^9}{(b\,x^3+a)^{\frac{1}{3}}\,ac+(b\,x^3+a)^{\frac{1}{3}}\,ad\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bc\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bd\,x^6}\,d$$

input
```
int(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x**9/((a + b*x**3)**(1/3)*a*c + (a + b*x**3)**(1/3)*a*d*x**3 + (a + b*
x**3)**(1/3)*b*c*x**3 + (a + b*x**3)**(1/3)*b*d*x**6),x)
```

3.770 $\int \frac{x^6}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 260

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{ax}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}d} - \frac{c^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d(bc-ad)^{4/3}} - \frac{c^{4/3}\log\left(c+dx^3\right)}{6d(bc-ad)^{4/3}} + \frac{c^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{4/3}} - \frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2b^{4/3}d}$$

output
```
a*x/b/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(
1/3))*3^(1/2))*3^(1/2)/b^(4/3)/d-1/3*c^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1
/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d/(-a*d+b*c)^(4/3)-1/6*c^(
4/3)*ln(d*x^3+c)/d/(-a*d+b*c)^(4/3)+1/2*c^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1
/3)-(b*x^3+a)^(1/3))/d/(-a*d+b*c)^(4/3)-1/2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))
/b^(4/3)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.56 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.79

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{12}\left(\frac{12ax}{(b^2c-abd)\sqrt[3]{a+bx^3}} + \frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{4/3}d} + \frac{2\sqrt{-6+6i\sqrt{3}}c^{4/3}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{d(bc-ad)^{4/3}} - \frac{4\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{b^{4/3}d} - \frac{2i\left(-i+\sqrt{3}\right)c^{4/3}\log\left(2\sqrt[3]{bc-ad}x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{d(bc-ad)^{4/3}} + \frac{2\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{b^{4/3}d} + \frac{\left(1+i\sqrt{3}\right)c^{4/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{d(bc-ad)^{4/3}}\right)$$

input
```
Integrate[x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((12*a*x)/((b^2*c - a*b*d)*(a + b*x^3)^(1/3)) + (4*Sqrt[3]*ArcTan[(Sqrt[3]
*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(b^(4/3)*d) + (2*Sqrt[-6 +
 (6*I)*Sqrt[3]]*c^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d
)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(d*(b*c - a*d)^(4
/3)) - (4*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(b^(4/3)*d) - ((2*I)*(-I
+ Sqrt[3])*c^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a
+ b*x^3)^(1/3)])/(d*(b*c - a*d)^(4/3)) + (2*Log[b^(2/3)*x^2 + b^(1/3)*x*(a
 + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(b^(4/3)*d) + ((1 + I*Sqrt[3])*c^(4/
3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3
)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(d*(b*
c - a*d)^(4/3)))/12
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {970, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{970}$$

$$\frac{ax}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\int \frac{ac-(bc-ad)x^3}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{b(bc-ad)}$$

$$\Big\downarrow \text{1026}$$

$$\frac{ax}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\frac{bc^2\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}}{b(bc-ad)}$$

$$\Big\downarrow \text{769}$$

$$\frac{ax}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\frac{bc^2\int \frac{1}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{d}-\frac{(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}}{b(bc-ad)}$$

$\downarrow$ 901

$$\frac{ax}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\frac{bc^2\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{d}-\frac{(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\log\left(\right.\right.}{d}}{b(bc-ad)}$$

input
```
Int[x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(a*x)/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) - ((b*c^2*(ArcTan[(1 + (2*(b*c - a
*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c -
 a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c -
a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))
/d - ((b*c - a*d)*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(
Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/d)/
(b*(b*c - a*d))
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.31

method	result
pseudoelliptic	$\frac{\left(-(ad-bc)\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}\right)\right.}{}$

input
```
int(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3)*((-(a*d-b*c)*(3^(1/2)*arctan(1/3*3
^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((-b^(1/3)*x+(b*x^3+a)^(
1/3))/x)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^
2))*(b*x^3+a)^(1/3)-3*b^(1/3)*a*d*x)*((a*d-b*c)/c)^(1/3)+c*(b*x^3+a)^(1/3)
*(3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a
*d-b*c)/c)^(1/3)/x)+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((
((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2
/3))/x^2))*b^(4/3))/b^(4/3)/(a*d-b*c)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(211) = 422$.

Time = 0.12 (sec) , antiderivative size = 1127, normalized size of antiderivative = 4.33

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/6*(6*(b*x^3 + a)^(2/3)*a*b*d*x + 3*sqrt(1/3)*(a*b^2*c - a^2*b*d + (b^3*
c - a*b^2*d)*x^3)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b
)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*
(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*sqrt(3)*(b^3
*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3
)*(b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 2*((b^2*c - a*b*d)*x^3 +
a*b*c - a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + ((b^
2*c - a*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3
+ a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 2*(b^3*c*x^3 + a*b^2*c
)*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x
^3 + a)^(1/3)*c)/x) + (b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*log(-((
b*c - a*d)*x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-
c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2))/(a*b^3*c*d - a^2*b^2*d^2
 + (b^4*c*d - a*b^3*d^2)*x^3), 1/6*(6*(b*x^3 + a)^(2/3)*a*b*d*x - 6*sqrt(1
/3)*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-(-b)^(1/3)/b)*arctan
(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) +
2*sqrt(3)*(b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3
)*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 2*((b^2*c -
 a*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1
/3))/x) + ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((-b)^(2...
```

Sympy [F]

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\int \frac{x^6}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**6/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**6/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^6/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^6/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^6}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

input
```
int(x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(x^6/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}\,ac+(b\,x^3+a)^{\frac{1}{3}}\,ad\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bc\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bd\,x^6}d$$

input
```
int(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x**6/((a + b*x**3)**(1/3)*a*c + (a + b*x**3)**(1/3)*a*d*x**3 + (a + b*
x**3)**(1/3)*b*c*x**3 + (a + b*x**3)**(1/3)*b*d*x**6),x)
```

3.771 $\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 172

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)} dx = -\frac{x}{(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{c}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}} + \frac{\sqrt[3]{c}\log(c+dx^3)}{6(bc-ad)^{4/3}} - \frac{\sqrt[3]{c}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}$$

output
```
-x/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/3*c^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)
*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/(-a*d+b*c)^(4/3)+1/6*c^(1/3)*
ln(d*x^3+c)/(-a*d+b*c)^(4/3)-1/2*c^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*
x^3+a)^(1/3))/(-a*d+b*c)^(4/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.26 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.87

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{12}\left(-\frac{12x}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{2\sqrt{-6+6i\sqrt{3}}\sqrt[3]{c}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{(bc-ad)^{4/3}} + \frac{2(1+i\sqrt{3})\sqrt[3]{c}\log\left(2\sqrt[3]{bc-ad}x+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{4/3}} - \frac{i(-i+\sqrt{3})\sqrt[3]{c}\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{(bc-ad)^{4/3}}\right)$$

input

```
Integrate[x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output

```
((-12*x)/((b*c - a*d)*(a + b*x^3)^(1/3)) - (2*Sqrt[-6 + (6*I)*Sqrt[3]]*c^(
1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I +
Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(b*c - a*d)^(4/3) + (2*(1 + I*Sqrt[3
])*c^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)
^(1/3)]])/(b*c - a*d)^(4/3) - (I*(-I + Sqrt[3])*c^(1/3)*Log[2*(b*c - a*d)^(
2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3)
+ I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))/12
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {971, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{971}$$

$$\frac{\int \frac{c}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{bc-ad}-\frac{x}{\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{bc-ad}-\frac{x}{\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \text{901}$$

$$\frac{c\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}-\frac{x}{\sqrt[3]{a+bx^3}(bc-ad)}$$

input
```
Int[x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
-(x/((b*c - a*d)*(a + b*x^3)^(1/3))) + (c*(ArcTan[(1 + (2*(b*c - a*d)^(1/3
)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1
/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/
3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/(b*c -
a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.26

method	result
pseudoelliptic	$-\frac{2\left(-3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)+2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-}{}\right.}{2}\right.}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(6ad-6bc\right)}$

input
```
int(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-2/((a*d-b*c)/c)^(1/3)*(-3*((a*d-b*c)/c)^(1/3)*x+1/2*(2*3^(1/2)*arctan(1/3
*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)+
2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-ln((((a*d-b*c)/c)^(2/3)*x^
2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(b*x^3+a)^(
1/3))/(b*x^3+a)^(1/3)/(6*a*d-6*b*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^3}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**3/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**3/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^3}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^3}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^3/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^3}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

input
```
int(x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(x^3/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,ac+(b\,x^3+a)^{\frac{1}{3}}\,ad\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bc\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bd\,x^6}\,d$$

input
```
int(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x**3/((a + b*x**3)**(1/3)*a*c + (a + b*x**3)**(1/3)*a*d*x**3 + (a + b*
x**3)**(1/3)*b*c*x**3 + (a + b*x**3)**(1/3)*b*d*x**6),x)
```

3.772 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} - \frac{d\log(c+dx^3)}{6c^{2/3}(bc-ad)^{4/3}} + \frac{d\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}$$

output
```
b*x/a/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/3*d*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/
c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(2/3)/(-a*d+b*c)^(4/3)-1/6*d*l
n(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(4/3)+1/2*d*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b
*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(4/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.83

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{12}\left(\frac{12bx}{(abc-a^2d)\sqrt[3]{a+bx^3}} + \frac{2\sqrt{-6+6i\sqrt{3}}d\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{4/3}} - \frac{2i(-i+\sqrt{3})\,d\log\left(2\sqrt[3]{bc-ad}x+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}(bc-ad)^{4/3}} + \frac{(d+i\sqrt{3}d)\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}(bc-ad)^{4/3}}\right)$$

input
```
Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((12*b*x)/((a*b*c - a^2*d)*(a + b*x^3)^(1/3)) + (2*Sqrt[-6 + (6*I)*Sqrt[3]
]*d*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + S
qrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(c^(2/3)*(b*c - a*d)^(4/3)) - ((2*I)*
(-I + Sqrt[3])*d*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a +
b*x^3)^(1/3)])/(c^(2/3)*(b*c - a*d)^(4/3)) + ((d + I*Sqrt[3]*d)*Log[2*(b*c
 - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^
3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(2/3)*(b*c - a*d
)^(4/3)))/12
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {907, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{907}$$

$$\frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{bc-ad}$$

$$\Big\downarrow \text{901}$$

$$\frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{d\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}$$

input
```
Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(b*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (d*(ArcTan[(1 + (2*(b*c - a*d)^(
1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)
^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^
(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/(b*c
 - a*d)
```

Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)ad\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)ad\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\frac{\ln\left(\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}\cdots\right.}{\cdots}}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(ad-bc\right)ca}$

input
```
int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(1/3)*(3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x
-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*a*d*(b*x^3+a)^(1/3)+ln((((a*d-b
*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*a*d*(b*x^3+a)^(1/3)-1/2*ln((((a*d-b*c)/
c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a
*d*(b*x^3+a)^(1/3)-3*b*x*c*((a*d-b*c)/c)^(1/3))/(b*x^3+a)^(1/3)/(a*d-b*c)/
c/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,ac+(b\,x^3+a)^{\frac{1}{3}}\,ad\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bc\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bd\,x^6}\,d$$

input
```
int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a*c + (a + b*x**3)**(1/3)*a*d*x**3 + (a + b*x**
3)**(1/3)*b*c*x**3 + (a + b*x**3)**(1/3)*b*d*x**6),x)
```

3.773 $\int \frac{1}{x^3(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 229

$$\int \frac{1}{x^3(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{b}{a(bc-ad)x^2\sqrt[3]{a+bx^3}} - \frac{(3bc-ad)(a+bx^3)^{2/3}}{2a^2c(bc-ad)x^2} + \frac{d^2\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}(bc-ad)^{4/3}} + \frac{d^2\log(c+dx^3)}{6c^{5/3}(bc-ad)^{4/3}} - \frac{d^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{5/3}(bc-ad)^{4/3}}$$

output
```
b/a/(-a*d+b*c)/x^2/(b*x^3+a)^(1/3)-1/2*(-a*d+3*b*c)*(b*x^3+a)^(2/3)/a^2/c/
(-a*d+b*c)/x^2+1/3*d^2*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a
)^(1/3))*3^(1/2))*3^(1/2)/c^(5/3)/(-a*d+b*c)^(4/3)+1/6*d^2*ln(d*x^3+c)/c^(
5/3)/(-a*d+b*c)^(4/3)-1/2*d^2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3
))/c^(5/3)/(-a*d+b*c)^(4/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.65 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.56

$$\int \frac{1}{x^3 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \frac{\frac{6c^{2/3}(-a^2d+3b^2cx^3+ab(c-dx^3))}{a^2(-bc+ad)x^2\sqrt[3]{a+bx^3}} - \frac{2\sqrt{-6+6i\sqrt{3}}d^2 \arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{\cdots}}\right.}{(bc-ad)^{4/3}}}{\cdots}$$

input
```
Integrate[1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((6*c^(2/3)*(-(a^2*d) + 3*b^2*c*x^3 + a*b*(c - d*x^3)))/(a^2*(-(b*c) + a*d
)*x^2*(a + b*x^3)^(1/3)) - (2*Sqrt[-6 + (6*I)*Sqrt[3]]*d^2*ArcTan[(3*(b*c
- a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a
+ b*x^3)^(1/3))])/(b*c - a*d)^(4/3) + (2*(1 + I*Sqrt[3])*d^2*Log[2*(b*c -
a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/
3) - (I*(-I + Sqrt[3])*d^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*
c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a
 + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))/(12*c^(5/3))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {972, 25, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^3)^{4/3} (c+dx^3)} \, dx$$

$$\Big\downarrow \; 972$$

$$\frac{b}{ax^2\sqrt[3]{a+bx^3}(bc-ad)} - \frac{\int -\frac{3bdx^3+3bc-ad}{x^3\sqrt[3]{bx^3+a}(dx^3+c)}dx}{a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3bdx^3+3bc-ad}{x^3\sqrt[3]{bx^3+a}(dx^3+c)}dx}{a(bc-ad)}+\frac{b}{ax^2\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \; 1053$$

$$\frac{-\frac{\int -\frac{2a^2d^2}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{2ac}-\frac{(a+bx^3)^{2/3}(3bc-ad)}{2acx^2}}{a(bc-ad)}+\frac{b}{ax^2\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{ad^2\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{c}-\frac{(a+bx^3)^{2/3}(3bc-ad)}{2acx^2}}{a(bc-ad)}+\frac{b}{ax^2\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \; 901$$

$$\frac{\frac{ad^2\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{(a+bx^3)^{2/3}(3bc-ad)}{2acx^2}}{a(bc-ad)}+\frac{b}{ax^2\sqrt[3]{a+bx^3}(bc-ad)}$$

input
```
Int[1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
b/(a*(b*c - a*d)*x^2*(a + b*x^3)^(1/3)) + (-1/2*((3*b*c - a*d)*(a + b*x^3)
^(2/3))/(a*c*x^2) + (a*d^2*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(
a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c +
d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
 (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/c)/(a*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$-\frac{\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a^{2}d^{2}x^{2}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\frac{3c\left(\left(abd-3b^{2}c\right)x^{3}+d\,a^{2}-abc\right)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}{2}+a^{2}d^{2}\left(\arctan\left(\frac{\sqrt{3}\left(-\frac{2\left(b\,x^{3}\right.}{\left(\frac{ad-}{c}\right.}\right.}{3x}\right.\right.}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}c^{2}x^{2}(ad-bc}$

input
```
int(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/((a*d-b*c)/c)^(1/3)*(ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^
2*d^2*x^2*(b*x^3+a)^(1/3)+3/2*c*((a*b*d-3*b^2*c)*x^3+d*a^2-a*b*c)*((a*d-b*
c)/c)^(1/3)+a^2*d^2*(arctan(1/3*3^(1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^
(1/3)+x)/x)*3^(1/2)-1/2*ln(((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b
*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(b*x^3+a)^(1/3)*x^2)/(b*x^3+a)^(1/3
)/c^2/x^2/(a*d-b*c)/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{x^3 (a+bx^3)^{\frac{4}{3}} (c+dx^3)} \, dx$$

input
```
integrate(1/x**3/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**3*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}} (dx^3+c)x^3} \, dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}} (dx^3+c)x^3} \, dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{x^3 \left(b x^3+a\right)^{4/3} \left(d x^3+c\right)} \, dx$$

input
```
int(1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{\left(b x^3+a\right)^{\frac{1}{3}} a c x^3+\left(b x^3+a\right)^{\frac{1}{3}} a d x^6+\left(b x^3+a\right)^{\frac{1}{3}} b c x^6+\left(b x^3+a\right)^{\frac{1}{3}} b}$$

input
```
int(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a*c*x**3 + (a + b*x**3)**(1/3)*a*d*x**6 + (a +
b*x**3)**(1/3)*b*c*x**6 + (a + b*x**3)**(1/3)*b*d*x**9),x)
```

3.774 $\int \frac{1}{x^6 (a+bx^3)^{4/3} (c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 287

$$\int \frac{1}{x^6 (a+bx^3)^{4/3} (c+dx^3)} dx = \frac{b}{a(bc-ad)x^5\sqrt[3]{a+bx^3}} - \frac{(6bc-ad)(a+bx^3)^{2/3}}{5a^2c(bc-ad)x^5} + \frac{(18b^2c^2-3abcd-5a^2d^2)(a+bx^3)^{2/3}}{10a^3c^2(bc-ad)x^2} - \frac{d^3\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{8/3}(bc-ad)^{4/3}} - \frac{d^3\log(c+dx^3)}{6c^{8/3}(bc-ad)^{4/3}} + \frac{d^3\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{8/3}(bc-ad)^{4/3}}$$

output
```
b/a/(-a*d+b*c)/x^5/(b*x^3+a)^(1/3)-1/5*(-a*d+6*b*c)*(b*x^3+a)^(2/3)/a^2/c/
(-a*d+b*c)/x^5+1/10*(-5*a^2*d^2-3*a*b*c*d+18*b^2*c^2)*(b*x^3+a)^(2/3)/a^3/
c^2/(-a*d+b*c)/x^2-1/3*d^3*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x
^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(8/3)/(-a*d+b*c)^(4/3)-1/6*d^3*ln(d*x^3+c)
/c^(8/3)/(-a*d+b*c)^(4/3)+1/2*d^3*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^
(1/3))/c^(8/3)/(-a*d+b*c)^(4/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.38 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^6 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \frac{\frac{6c^{2/3}(-18b^3c^2x^6+3ab^2cx^3(-2c+dx^3)+a^3d(-2c+5dx^3)+a^2b(2c^2+cdx^3+5d^2x^6))}{a^3(-bc+ad)x^5\sqrt[3]{a+bx^3}} + \frac{10\sqrt{-6+6i}}{}}{}$$

input
```
Integrate[1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((6*c^(2/3)*(-18*b^3*c^2*x^6 + 3*a*b^2*c*x^3*(-2*c + d*x^3) + a^3*d*(-2*c
+ 5*d*x^3) + a^2*b*(2*c^2 + c*d*x^3 + 5*d^2*x^6)))/(a^3*(-(b*c) + a*d)*x^5
*(a + b*x^3)^(1/3)) + (10*Sqrt[-6 + (6*I)*Sqrt[3]]*d^3*ArcTan[(3*(b*c - a*
d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*
x^3)^(1/3))]])/(b*c - a*d)^(4/3) - ((10*I)*(-I + Sqrt[3])*d^3*Log[2*(b*c -
a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]])/(b*c - a*d)^(4/
3) + (5*(1 + I*Sqrt[3])*d^3*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])
*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(
a + b*x^3)^(2/3)]])/(b*c - a*d)^(4/3))/(60*c^(8/3))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {972, 25, 1053, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^3)^{4/3} (c+dx^3)} \, dx$$

$$\Big\downarrow \text{ 972}$$

$$\frac{b}{ax^5\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\int -\frac{6bdx^3+6bc-ad}{x^6\sqrt[3]{bx^3+a}(dx^3+c)}dx}{a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{6bdx^3+6bc-ad}{x^6\sqrt[3]{bx^3+a}(dx^3+c)}dx}{a(bc-ad)}+\frac{b}{ax^5\sqrt[3]{a+bx^3}(bc-ad)}$$

$\Big\downarrow$ 1053

$$\frac{-\frac{\int \frac{3bd(6bc-ad)x^3+18b^2c^2-5a^2d^2-3abcd}{x^3\sqrt[3]{bx^3+a}(dx^3+c)}dx}{5ac}-\frac{(a+bx^3)^{2/3}(6bc-ad)}{5acx^5}}{a(bc-ad)}+\frac{b}{ax^5\sqrt[3]{a+bx^3}(bc-ad)}$$

$\Big\downarrow$ 1053

$$\frac{-\frac{-\frac{\int -\frac{10a^3d^3}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{2ac}-\frac{(a+bx^3)^{2/3}\left(\frac{18b^2c}{a}-\frac{5ad^2}{c}-3bd\right)}{2x^2}}{5ac}-\frac{(a+bx^3)^{2/3}(6bc-ad)}{5acx^5}}{a(bc-ad)}+\frac{b}{ax^5\sqrt[3]{a+bx^3}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\frac{5a^2d^3\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{c}-\frac{(a+bx^3)^{2/3}\left(\frac{18b^2c}{a}-\frac{5ad^2}{c}-3bd\right)}{2x^2}}{5ac}-\frac{(a+bx^3)^{2/3}(6bc-ad)}{5acx^5}}{a(bc-ad)}+\frac{b}{ax^5\sqrt[3]{a+bx^3}(bc-ad)}$$

$\Big\downarrow$ 901

$$\frac{-\frac{\frac{5a^2d^3\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{(a+bx^3)^{2/3}\left(\frac{18b^2c}{a}-\frac{5ad^2}{c}-3bd\right)}{2x^2}}{5ac}-\frac{(a+bx^3)^{2/3}(6bc-ad)}{5acx^5}}{a(bc-ad)}+\frac{b}{ax^5\sqrt[3]{a+bx^3}(bc-ad)}$$

input
```
Int[1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
b/(a*(b*c - a*d)*x^5*(a + b*x^3)^(1/3)) + (-1/5*((6*b*c - a*d)*(a + b*x^3)
^(2/3))/(a*c*x^5) - (-1/2*(((18*b^2*c)/a - 3*b*d - (5*a*d^2)/c)*(a + b*x^3
)^(2/3))/x^2 + (5*a^2*d^3*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a
 + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d
*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
(a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/c)/(5*a*c))/(a*(b*c - a
*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$\frac{-\frac{\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)a^{3}d^{3}x^{5}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{2}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a^{3}d^{3}x^{5}\left(b\,x^{3}+a\right.}{3\left(\frac{ac}{}\right.}$

input
```
int(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(1/3)*(-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1
/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a^3*d^3*x^5*(b*x^3+a)^(1/3)+ln
((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^3*d^3*x^5*(b*x^3+a)^(1/3)-3/
5*c*((-5/2*a^2*b*d^2-3/2*a*b^2*c*d+9*c^2*b^3)*x^6+(-5/2*a^3*d^2-1/2*a^2*b*
c*d+3*a*b^2*c^2)*x^3+(a*d-b*c)*a^2*c)*((a*d-b*c)/c)^(1/3)+3^(1/2)*arctan(1
/3*3^(1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*a^3*d^3*x^5*(b*x^
3+a)^(1/3))/(b*x^3+a)^(1/3)/c^3/x^5/(a*d-b*c)/a^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a + bx^3)^{4/3} (c + dx^3)} dx = \text{Timed out}$$

input
```
integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^6 (a + bx^3)^{4/3} (c + dx^3)} dx = \int \frac{1}{x^6 (a + bx^3)^{\frac{4}{3}} (c + dx^3)} dx$$

input
```
integrate(1/x**6/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**6*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^6 (a + bx^3)^{4/3} (c + dx^3)} dx = \int \frac{1}{(bx^3 + a)^{\frac{4}{3}} (dx^3 + c) x^6} dx$$

input
```
integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 \left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx=\int \frac{1}{{\left(b x^{3} + a\right)}^{\frac{4}{3}} {\left(d x^{3} + c\right)} x^{6}} \,d x$$

input
```
integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx=\int \frac{1}{x^6\,{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)} \,d x$$

input
```
int(1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 \left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx=\int \frac{1}{\left(b \,x^{3}+a \right)^{\frac{1}{3}} a c \,x^{6}+\left(b \,x^{3}+a \right)^{\frac{1}{3}} a d \,x^{9}+\left(b \,x^{3}+a \right)^{\frac{1}{3}} b c \,x^{9}+\left(b \,x^{3}+a \right)^{\frac{1}{3}} b$$

input
```
int(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a*c*x**6 + (a + b*x**3)**(1/3)*a*d*x**9 + (a +
b*x**3)**(1/3)*b*c*x**9 + (a + b*x**3)**(1/3)*b*d*x**12),x)
```

3.775 $\int \frac{1}{x^9(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 351

$$\int \frac{1}{x^9\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{b}{a(bc-ad)x^8\sqrt[3]{a+bx^3}} - \frac{(9bc-ad)\left(a+bx^3\right)^{2/3}}{8a^2c(bc-ad)x^8} + \frac{(9bc-4ad)(3bc+ad)\left(a+bx^3\right)^{2/3}}{20a^3c^2(bc-ad)x^5} - \frac{\left(81b^3c^3-9ab^2c^2d-12a^2bcd^2-20a^3d^3\right)\left(a+bx^3\right)^{2/3}}{40a^4c^3(bc-ad)x^2} + \frac{d^4\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{11/3}(bc-ad)^{4/3}} + \frac{d^4\log\left(c+dx^3\right)}{6c^{11/3}(bc-ad)^{4/3}} - \frac{d^4\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{11/3}(bc-ad)^{4/3}}$$

output
```
b/a/(-a*d+b*c)/x^8/(b*x^3+a)^(1/3)-1/8*(-a*d+9*b*c)*(b*x^3+a)^(2/3)/a^2/c/
(-a*d+b*c)/x^8+1/20*(-4*a*d+9*b*c)*(a*d+3*b*c)*(b*x^3+a)^(2/3)/a^3/c^2/(-a
*d+b*c)/x^5-1/40*(-20*a^3*d^3-12*a^2*b*c*d^2-9*a*b^2*c^2*d+81*b^3*c^3)*(b*
x^3+a)^(2/3)/a^4/c^3/(-a*d+b*c)/x^2+1/3*d^4*arctan(1/3*(1+2*(-a*d+b*c)^(1/
3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(11/3)/(-a*d+b*c)^(4/3)+1
/6*d^4*ln(d*x^3+c)/c^(11/3)/(-a*d+b*c)^(4/3)-1/2*d^4*ln((-a*d+b*c)^(1/3)*x
/c^(1/3)-(b*x^3+a)^(1/3))/c^(11/3)/(-a*d+b*c)^(4/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.34 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^9 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \frac{-\frac{3c^{2/3}(-81b^4c^3x^9+9ab^3c^2x^6(-3c+dx^3)+3a^2b^2cx^3(3c^2+cdx^3+4d^2x^6)+a^4d(5c^2-8cdx^3+20d^2x^6 \cdots}{a^4(-bc+ad)x^8\sqrt[3]{a+bx^3}} \cdots}{\cdots}$$

input
```
Integrate[1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((-3*c^(2/3)*(-81*b^4*c^3*x^9 + 9*a*b^3*c^2*x^6*(-3*c + d*x^3) + 3*a^2*b^2
*c*x^3*(3*c^2 + c*d*x^3 + 4*d^2*x^6) + a^4*d*(5*c^2 - 8*c*d*x^3 + 20*d^2*x
^6) + a^3*b*(-5*c^3 - c^2*d*x^3 + 4*c*d^2*x^6 + 20*d^3*x^9)))/(a^4*(-(b*c)
 + a*d)*x^8*(a + b*x^3)^(1/3)) - (20*Sqrt[-6 + (6*I)*Sqrt[3]]*d^4*ArcTan[(
3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1
/3)*(a + b*x^3)^(1/3))])/(b*c - a*d)^(4/3) + (20*(1 + I*Sqrt[3])*d^4*Log[2
*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c -
a*d)^(4/3) - ((10*I)*(-I + Sqrt[3])*d^4*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1
- I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3
])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))/(120*c^(11/3))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {972, 25, 1053, 27, 1053, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 (a+bx^3)^{4/3} (c+dx^3)} \, dx$$

$$\Big\downarrow \text{972}$$

$$\frac{b}{ax^8 \sqrt[3]{a+bx^3}(bc-ad)} - \frac{\int -\frac{9bdx^3+9bc-ad}{x^9 \sqrt[3]{bx^3+a}(dx^3+c)}dx}{a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{9bdx^3+9bc-ad}{x^9 \sqrt[3]{bx^3+a}(dx^3+c)}dx}{a(bc-ad)} + \frac{b}{ax^8 \sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int \frac{2\left(3bd(9bc-ad)x^3+(9bc-4ad)(3bc+ad)\right)}{x^6 \sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{8ac} - \frac{(a+bx^3)^{2/3}(9bc-ad)}{8acx^8}}{a(bc-ad)} + \frac{b}{ax^8 \sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{3bd(9bc-ad)x^3+(9bc-4ad)(3bc+ad)}{x^6 \sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{4ac} - \frac{(a+bx^3)^{2/3}(9bc-ad)}{8acx^8}}{a(bc-ad)} + \frac{b}{ax^8 \sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{-\frac{\int \frac{81b^3c^3-9ab^2dc^2-12a^2bd^2c-20a^3d^3+3bd(9bc-4ad)(3bc+ad)x^3}{x^3 \sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{5ac} - \frac{\left(a+bx^3\right)^{2/3}(9bc-4ad)(ad+3bc)}{5acx^5}}{4ac} - \frac{(a+bx^3)^{2/3}(9bc-ad)}{8acx^8}}{a(bc-ad)} + \frac{b}{ax^8 \sqrt[3]{a+bx^3}(bc-ad)}$$

$\big\downarrow$ 1053

$$-\frac{-\frac{-\frac{-\frac{\int -\frac{40a^4d^4}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{2ac}-\frac{(a+bx^3)^{2/3}(-20a^3d^3-12a^2bcd^2-9ab^2c^2d+81b^3c^3)}{2acx^2}}{5ac}-\frac{(a+bx^3)^{2/3}(9bc-4ad)(ad+3bc)}{5acx^5}}{4ac}-\frac{(a+bx^3)^{2/3}(9bc-ad)}{8acx^8}}{a(bc-ad)} \cdots$$

$$\frac{b}{ax^8\sqrt[3]{a+bx^3}(bc-ad)}$$

$\big\downarrow$ 27

$$-\frac{-\frac{-\frac{\frac{20a^3d^4\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{c}-\frac{(a+bx^3)^{2/3}(-20a^3d^3-12a^2bcd^2-9ab^2c^2d+81b^3c^3)}{2acx^2}}{5ac}-\frac{(a+bx^3)^{2/3}(9bc-4ad)(ad+3bc)}{5acx^5}}{4ac}-\frac{(a+bx^3)^{2/3}(9bc-a}{8acx^8}}{a(bc-ad)} \cdots$$

$$\frac{b}{ax^8\sqrt[3]{a+bx^3}(bc-ad)}$$

$\big\downarrow$ 901

$$-\frac{-\frac{\frac{20a^3d^4\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{(a+bx^3)^{2/3}(-20a^3d^3-12a^2bcd^2-9ab^2c^2 \cdots}{2acx^2}}{5ac} \cdots}{4ac} \cdots}{a(bc-ad)}$$

$$\frac{b}{ax^8\sqrt[3]{a+bx^3}(bc-ad)}$$

input
```
Int[1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
b/(a*(b*c - a*d)*x^8*(a + b*x^3)^(1/3)) + (-1/8*((9*b*c - a*d)*(a + b*x^3)
^(2/3))/(a*c*x^8) - (-1/5*((9*b*c - 4*a*d)*(3*b*c + a*d)*(a + b*x^3)^(2/3)
)/(a*c*x^5) - (-1/2*((81*b^3*c^3 - 9*a*b^2*c^2*d - 12*a^2*b*c*d^2 - 20*a^3
*d^3)*(a + b*x^3)^(2/3))/(a*c*x^2) + (20*a^3*d^4*(ArcTan[(1 + (2*(b*c - a*
d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c -
a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a
*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/
c)/(5*a*c))/(4*a*c))/(a*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{3c\left(d\left(4d^2x^6-\frac{8}{5}cdx^3+c^2\right)a^4-\left(-4d^3x^9-\frac{4}{5}cd^2x^6+\frac{1}{5}c^2dx^3+c^3\right)ba^3+\frac{9c\left(\frac{4}{3}d^2x^6+\frac{1}{3}cdx^3+c^2\right)b^2x^3a^2}{5}-\frac{27c^2\left(-\frac{dx^3}{3}+c\right)b^3x^6a}{5}-}{4}$

input
```
int(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/c)^(1/3)*(-3/4*c*(d*(4*d^2*x^6-8/5*c*d*x^3+c^2)*a^4-(-4*d^3
*x^9-4/5*c*d^2*x^6+1/5*c^2*d*x^3+c^3)*b*a^3+9/5*c*(4/3*d^2*x^6+1/3*c*d*x^3
+c^2)*b^2*x^3*a^2-27/5*c^2*(-1/3*d*x^3+c)*b^3*x^6*a-81/5*b^4*c^3*x^9)*((a*
d-b*c)/c)^(1/3)+a^4*d^4*x^8*(b*x^3+a)^(1/3)*(-2*3^(1/2)*arctan(1/3*3^(1/2)
*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)+ln((((a*
d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))
/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)))/(b*x^3+a)^(1/3)/x^
8/c^4/(a*d-b*c)/a^4
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^9 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^9 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{1}{x^9 (a+bx^3)^{\frac{4}{3}} (c+dx^3)}\,dx$$

input
```
integrate(1/x**9/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**9*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^9 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}} (dx^3+c)x^9}\,dx$$

input
```
integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^9), x)
```

Giac [F]

$$\int \frac{1}{x^9 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}} (dx^3+c) x^9} \, dx$$

input
```
integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^9 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{x^9 (b\,x^3+a)^{4/3} (d\,x^3+c)} \, dx$$

input
```
int(1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^9 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}} ac\,x^9 + (b\,x^3+a)^{\frac{1}{3}} ad\,x^{12} + (b\,x^3+a)^{\frac{1}{3}} bc\,x^{12} + (b\,x^3+a)^{\frac{1}{3}}}$$

input
```
int(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a*c*x**9 + (a + b*x**3)**(1/3)*a*d*x**12 + (a +
 b*x**3)**(1/3)*b*c*x**12 + (a + b*x**3)**(1/3)*b*d*x**15),x)
```

3.776 $\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{x^{11}\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{11}{3},\frac{4}{3},1,\frac{14}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{11ac\sqrt[3]{a+bx^3}}$$

output
```
1/11*x^11*(1+b*x^3/a)^(1/3)*AppellF1(11/3,4/3,1,14/3,-b*x^3/a,-d*x^3/c)/a/
c/(b*x^3+a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 194 vs. 2(67) = 134.

Time = 10.17 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.90

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{x^2\left(5c(-5a^2d+b^2cx^3+ab(c-dx^3))+5ac(-bc+5ad)\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF}\right.}{}$$

input
```
Integrate[x^10/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(x^2*(5*c*(-5*a^2*d + b^2*c*x^3 + a*b*(c - d*x^3)) + 5*a*c*(-(b*c) + 5*a*d
)*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)
/c)] + 2*(-2*b^2*c^2 - a*b*c*d + 5*a^2*d^2)*x^3*(1 + (b*x^3)/a)^(1/3)*Appe
llF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]))/(20*b^2*c*d*(b*c - a*
d)*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x^{10}}{\left(\frac{bx^3}{a}+1\right)^{4/3}(dx^3+c)}dx}{a\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x^{11}\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{11}{3},\frac{4}{3},1,\frac{14}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{11ac\sqrt[3]{a+bx^3}}$$

input
```
Int[x^10/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(x^11*(1 + (b*x^3)/a)^(1/3)*AppellF1[11/3, 4/3, 1, 14/3, -((b*x^3)/a), -((
d*x^3)/c)])/(11*a*c*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{10}}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}d x$$

input
```
int(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^{10}}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**10/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**10/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^{10}}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^10/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^{10}}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^10/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx=\int \frac{x^{10}}{{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)} \,d x$$

input
```
int(x^10/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(x^10/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx=\int \frac{x^{10}}{\left(b\,x^{3}+a\right)^{\frac{1}{3}} a c +\left(b\,x^{3}+a\right)^{\frac{1}{3}} a d\,x^{3}+\left(b\,x^{3}+a\right)^{\frac{1}{3}} b c\,x^{3}+\left(b\,x^{3}+a\right)^{\frac{1}{3}} b d\,x^{6}}d$$

input
```
int(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x**10/((a + b*x**3)**(1/3)*a*c + (a + b*x**3)**(1/3)*a*d*x**3 + (a + b
*x**3)**(1/3)*b*c*x**3 + (a + b*x**3)**(1/3)*b*d*x**6),x)
```

3.777 $\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8ac\sqrt[3]{a+bx^3}}$$

output
```
1/8*x^8*(1+b*x^3/a)^(1/3)*AppellF1(8/3,4/3,1,11/3,-b*x^3/a,-d*x^3/c)/a/c/(
b*x^3+a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 144 vs. 2(67) = 134.

Time = 10.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.15

$$\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\frac{5acx^2-5acx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+(bc-2ad)x^5\sqrt[3]{\cdots}}{5bc(bc-ad)\sqrt[3]{a+bx^3}}$$

input
```
Integrate[x^7/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(5*a*c*x^2 - 5*a*c*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -(
(b*x^3)/a), -((d*x^3)/c)] + (b*c - 2*a*d)*x^5*(1 + (b*x^3)/a)^(1/3)*Appell
F1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(5*b*c*(b*c - a*d)*(a +
b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x^7}{\left(\frac{bx^3}{a}+1\right)^{4/3}(dx^3+c)}dx}{a\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8ac\sqrt[3]{a+bx^3}}$$

input
```
Int[x^7/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(x^8*(1 + (b*x^3)/a)^(1/3)*AppellF1[8/3, 4/3, 1, 11/3, -((b*x^3)/a), -((d*
x^3)/c)])/(8*a*c*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{7}}{\left(b \,x^{3}+a \right)^{\frac{4}{3}} \left(d \,x^{3}+c \right)}d x$$

input
```
int(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^7}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**7/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**7/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^7/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^7/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx=\int \frac{x^7}{{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)} \,d x$$

input
```
int(x^7/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(x^7/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^7}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx=\int \frac{x^7}{\left(b\,x^3+a\right)^{\frac{1}{3}} a c+\left(b\,x^3+a\right)^{\frac{1}{3}} a d\,x^3+\left(b\,x^3+a\right)^{\frac{1}{3}} b c\,x^3+\left(b\,x^3+a\right)^{\frac{1}{3}} b d\,x^6}\,d$$

input
```
int(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x**7/((a + b*x**3)**(1/3)*a*c + (a + b*x**3)**(1/3)*a*d*x**3 + (a + b*
x**3)**(1/3)*b*c*x**3 + (a + b*x**3)**(1/3)*b*d*x**6),x)
```

3.778 $\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5ac\sqrt[3]{a+bx^3}}$$

output
```
1/5*x^5*(1+b*x^3/a)^(1/3)*AppellF1(5/3,4/3,1,8/3,-b*x^3/a,-d*x^3/c)/a/c/(b
*x^3+a)^(1/3)
```

Mathematica [A] (warning: unable to verify)

Time = 9.37 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.93

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{x^2\left(-5c+5c\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+dx^3\sqrt[3]{1+\frac{bx^3}{a}}\,A\right.}{5c(bc-ad)\sqrt[3]{a+bx^3}}$$

input
```
Integrate[x^4/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(x^2*(-5*c + 5*c*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3
)/a), -((d*x^3)/c)] + d*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/
3, -((b*x^3)/a), -((d*x^3)/c)]))/(5*c*(b*c - a*d)*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x^4}{\left(\frac{bx^3}{a}+1\right)^{4/3}(dx^3+c)}dx}{a\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}\operatorname{AppellF1}\left(\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5ac\sqrt[3]{a+bx^3}}$$

input
```
Int[x^4/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 4/3, 1, 8/3, -((b*x^3)/a), -((d*x
^3)/c)])/(5*a*c*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{4}}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}d x$$

input
```
int(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^4}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**4/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**4/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^4}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

input
```
int(x^4/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(x^4/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^4}{(b\,x^3+a)^{\frac{1}{3}}\,ac+(b\,x^3+a)^{\frac{1}{3}}\,ad\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bc\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bd\,x^6}\,d$$

input
```
int(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x**4/((a + b*x**3)**(1/3)*a*c + (a + b*x**3)**(1/3)*a*d*x**3 + (a + b*
x**3)**(1/3)*b*c*x**3 + (a + b*x**3)**(1/3)*b*d*x**6),x)
```

3.779 $\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{4}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt[3]{a+bx^3}}$$

output
```
1/2*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,4/3,1,5/3,-b*x^3/a,-d*x^3/c)/a/c/(b
*x^3+a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(67) = 134.

Time = 10.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.10

$$\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\frac{x^2\left(-10bc+5(bc+ad)\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bdx\right.}{10ac(-bc+ad)\sqrt[3]{a+bx^3}}$$

input
```
Integrate[x/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(x^2*(-10*b*c + 5*(b*c + a*d)*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1,
5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*d*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF
1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]))/(10*a*c*(-(b*c) + a*d)*(
a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x}{\left(\frac{bx^3}{a}+1\right)^{4/3}(dx^3+c)}dx}{a\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\operatorname{AppellF1}\left(\frac{2}{3},\frac{4}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt[3]{a+bx^3}}$$

input
```
Int[x/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 4/3, 1, 5/3, -((b*x^3)/a), -((d*x
^3)/c)])/(2*a*c*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}d x$$

input
```
int(x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{x}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

input
```
integrate(x/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{x}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

input
```
integrate(x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{x}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

input
```
integrate(x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\int \frac{x}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

input
```
int(x/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(x/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}\,ac+(b\,x^3+a)^{\frac{1}{3}}\,ad\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bc\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bd\,x^6}\,dx$$

input
```
int(x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x/((a + b*x**3)**(1/3)*a*c + (a + b*x**3)**(1/3)*a*d*x**3 + (a + b*x**
3)**(1/3)*b*c*x**3 + (a + b*x**3)**(1/3)*b*d*x**6),x)
```

3.780 $\int \frac{1}{x^2(a+bx^3)^{4/3}(c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{1}{x^2 (a+bx^3)^{4/3} (c+dx^3)} dx = -\frac{\sqrt[3]{1+\frac{bx^3}{a}} \text{AppellF1}\left(-\frac{1}{3},\frac{4}{3},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt[3]{a+bx^3}}$$

output
```
-(1+b*x^3/a)^(1/3)*AppellF1(-1/3,4/3,1,2/3,-b*x^3/a,-d*x^3/c)/a/c/x/(b*x^3
+a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 193 vs. 2(65) = 130.

Time = 10.16 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.97

$$\int \frac{1}{x^2 (a+bx^3)^{4/3} (c+dx^3)} dx = \frac{10c(-a^2d+2b^2cx^3+ab(c-dx^3)) - 5(2b^2c^2-abcd+a^2d^2)\, x^3 \sqrt[3]{1+\frac{bx^3}{a}} \cdots}{1 \cdots}$$

input
```
Integrate[1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(10*c*(-(a^2*d) + 2*b^2*c*x^3 + a*b*(c - d*x^3)) - 5*(2*b^2*c^2 - a*b*c*d
+ a^2*d^2)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/
a), -((d*x^3)/c)] + 2*b*d*(-2*b*c + a*d)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF
1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(10*a^2*c^2*(-(b*c) + a*d
)*x*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^3)^{4/3} (c+dx^3)} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1} \int \frac{1}{x^2 \left(\frac{bx^3}{a}+1\right)^{4/3} (dx^3+c)}dx}{a\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1} \operatorname{AppellF1}\left(-\frac{1}{3},\frac{4}{3},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt[3]{a+bx^3}}$$

input
```
Int[1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
-(((1 + (b*x^3)/a)^(1/3)*AppellF1[-1/3, 4/3, 1, 2/3, -((b*x^3)/a), -((d*x^
3)/c)])/(a*c*x*(a + b*x^3)^(1/3)))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}dx$$

input
```
int(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{1}{x^2 (a+bx^3)^{\frac{4}{3}} (c+dx^3)}\,dx$$

input
```
integrate(1/x**2/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**2*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{x^2 \left(b\,x^3+a\right)^{4/3} \left(d\,x^3+c\right)} \, dx$$

input
```
int(1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,ac\,x^2 + (b\,x^3+a)^{\frac{1}{3}}\,ad\,x^5 + (b\,x^3+a)^{\frac{1}{3}}\,bc\,x^5 + (b\,x^3+a)^{\frac{1}{3}}\,b$$

input
```
int(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a*c*x**2 + (a + b*x**3)**(1/3)*a*d*x**5 + (a +
b*x**3)**(1/3)*b*c*x**5 + (a + b*x**3)**(1/3)*b*d*x**8),x)
```

3.781 $\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 220

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d}-\frac{a^2(a+bx^3)^{4/3}}{4b^4d}+\frac{a(a+bx^3)^{7/3}}{7b^4d}-\frac{(a+bx^3)^{10/3}}{10b^4d}+\frac{\sqrt[3]{2}a^{10/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^4d}+\frac{a^{10/3}\log(a-bx^3)}{3\,2^{2/3}b^4d}-\frac{a^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^4d}$$

output
```
-a^3*(b*x^3+a)^(1/3)/b^4/d-1/4*a^2*(b*x^3+a)^(4/3)/b^4/d+1/7*a*(b*x^3+a)^(
7/3)/b^4/d-1/10*(b*x^3+a)^(10/3)/b^4/d+1/3*2^(1/3)*a^(10/3)*arctan(1/3*(a^
(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/b^4/d+1/6*a^(10/3)
*ln(-b*x^3+a)*2^(1/3)/b^4/d-1/2*a^(10/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3
))*2^(1/3)/b^4/d
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.92

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{-3\sqrt[3]{a+bx^3}(169a^3+37a^2bx^3+22ab^2x^6+14b^3x^9)+140\sqrt[3]{2}\sqrt{3}a^{10/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-140\cdots}{420b^4\cdots}$$

input
```
Integrate[(x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
(-3*(a + b*x^3)^(1/3)*(169*a^3 + 37*a^2*b*x^3 + 22*a*b^2*x^6 + 14*b^3*x^9)
 + 140*2^(1/3)*Sqrt[3]*a^(10/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^
(1/3))/Sqrt[3]] - 140*2^(1/3)*a^(10/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3
)^(1/3)] + 70*2^(1/3)*a^(10/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)
^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(420*b^4*d)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {948, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^9\sqrt[3]{bx^3+a}}{d(a-bx^3)}dx^3$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{x^9 \sqrt[3]{bx^3+a}}{a-bx^3}dx^3}{3d}$$

$\big\downarrow$ 99

$$\frac{\int \left(\frac{\sqrt[3]{bx^3+a}a^3}{b^3(a-bx^3)} - \frac{\sqrt[3]{bx^3+a}a^2}{b^3} + \frac{(bx^3+a)^{4/3}a}{b^3} - \frac{(bx^3+a)^{7/3}}{b^3}\right)dx^3}{3d}$$

$\big\downarrow$ 2009

$$\frac{\frac{\sqrt[3]{2}\sqrt{3}a^{10/3}\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^4} + \frac{a^{10/3}\log(a-bx^3)}{2^{2/3}b^4} - \frac{3a^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^4} - \frac{3a^3\sqrt[3]{a+bx^3}}{b^4} - \frac{3a^2(a \ldots}{\ldots}}{3d}$$

input
```
Int[(x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
((-3*a^3*(a + b*x^3)^(1/3))/b^4 - (3*a^2*(a + b*x^3)^(4/3))/(4*b^4) + (3*a
*(a + b*x^3)^(7/3))/(7*b^4) - (3*(a + b*x^3)^(10/3))/(10*b^4) + (2^(1/3)*S
qrt[3]*a^(10/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/b^4 + (a^(10/3)*Log[a - b*x^3])/(2^(2/3)*b^4) - (3*a^(10/3)*Log[2^(
1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^4))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 5.78 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$\frac{\left(-42b^3x^9-66a\,b^2x^6-111a^2b\,x^3-507a^3\right)\left(b\,x^3+a\right)^{\frac{1}{3}}+70a^{\frac{10}{3}}\left(2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^3+a\right)^{\frac{2}{3}}+2\right.\right.}{420b^4d}$

input
```
int(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/420*((-42*b^3*x^9-66*a*b^2*x^6-111*a^2*b*x^3-507*a^3)*(b*x^3+a)^(1/3)+70
*a^(10/3)*(2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))
*3^(1/2)+ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3
))-2*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3)))*2^(1/3))/b^4/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{140\sqrt{3}2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^3\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+70\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^3\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}(\cdots\right.}{\cdots}$$

input
```
integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/420*(140*sqrt(3)*2^(1/3)*(-a)^(1/3)*a^3*arctan(1/3*(sqrt(3)*2^(2/3)*(b*
x^3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 70*2^(1/3)*(-a)^(1/3)*a^3*log(
2^(2/3)*(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2
/3)) - 140*2^(1/3)*(-a)^(1/3)*a^3*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/
3)) + 3*(14*b^3*x^9 + 22*a*b^2*x^6 + 37*a^2*b*x^3 + 169*a^3)*(b*x^3 + a)^(
1/3))/(b^4*d)
```

Sympy [F]

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^{11}\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**11*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**11*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.83

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\frac{140\,\sqrt{3}2^{\frac{1}{3}}a^{\frac{10}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d} + \frac{70\cdot 2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d} - \frac{140\cdot 2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\right.}{d}}{420\,b^4}$$

input
```
integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
1/420*(140*sqrt(3)*2^(1/3)*a^(10/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^
(1/3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 70*2^(1/3)*a^(10/3)*log(2^(2/3)*
a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 140*2
^(1/3)*a^(10/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 3*(14*(b*x^3
 + a)^(10/3) - 20*(b*x^3 + a)^(7/3)*a + 35*(b*x^3 + a)^(4/3)*a^2 + 140*(b*
x^3 + a)^(1/3)*a^3)/d)/b^4
```

Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt{3}2^{\frac{1}{3}}a^{\frac{10}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,b^4d} + \frac{2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,b^4d} - \frac{2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,b^4d} - \frac{14\,(bx^3+a)^{\frac{10}{3}}b^{36}d^9-20\,(bx^3+a)^{\frac{7}{3}}ab^{36}d^9+35\,(bx^3+a)^{\frac{4}{3}}a^2b^{36}d^9+140\,(bx^3+a)^{\frac{1}{3}}a^3b^{36}d^9}{140\,b^{40}d^{10}}$$

input
```
integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*2^(1/3)*a^(10/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
 2*(b*x^3 + a)^(1/3))/a^(1/3))/(b^4*d) + 1/6*2^(1/3)*a^(10/3)*log(2^(2/3)*
a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b^4*d) -
 1/3*2^(1/3)*a^(10/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b^4*
d) - 1/140*(14*(b*x^3 + a)^(10/3)*b^36*d^9 - 20*(b*x^3 + a)^(7/3)*a*b^36*d
^9 + 35*(b*x^3 + a)^(4/3)*a^2*b^36*d^9 + 140*(b*x^3 + a)^(1/3)*a^3*b^36*d^
9)/(b^40*d^10)
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.09

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{a\,{\left(b\,x^3+a\right)}^{7/3}}{7\,b^4\,d}-\frac{a^3\,{\left(b\,x^3+a\right)}^{1/3}}{b^4\,d}-\frac{a^2\,{\left(b\,x^3+a\right)}^{4/3}}{4\,b^4\,d}-\frac{{\left(b\,x^3+a\right)}^{10/3}}{10\,b^4\,d}-\frac{2^{1/3}\,a^{10/3}\,\ln\left({\left(b\,x^3+a\right)}^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b^4\,d}-\frac{2^{1/3}\,a^{10/3}\,\ln\left(\frac{6\,a^4\,{\left(b\,x^3+a\right)}^{1/3}}{b^4\,d}-\frac{6\,2^{1/3}\,a^{13/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^4\,d}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^4\,d}+\frac{2^{1/3}\,a^{10/3}\,\ln\left(\frac{6\,a^4\,{\left(b\,x^3+a\right)}^{1/3}}{b^4\,d}+\frac{18\,2^{1/3}\,a^{13/3}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^4\,d}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^4\,d}$$

input
```
int((x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

output
```
(a*(a + b*x^3)^(7/3))/(7*b^4*d) - (a^3*(a + b*x^3)^(1/3))/(b^4*d) - (a^2*(
a + b*x^3)^(4/3))/(4*b^4*d) - (a + b*x^3)^(10/3)/(10*b^4*d) - (2^(1/3)*a^(
10/3)*log((a + b*x^3)^(1/3) - 2^(1/3)*a^(1/3)))/(3*b^4*d) - (2^(1/3)*a^(10
/3)*log((6*a^4*(a + b*x^3)^(1/3))/(b^4*d) - (6*2^(1/3)*a^(13/3)*((3^(1/2)*
1i)/2 - 1/2))/(b^4*d))*((3^(1/2)*1i)/2 - 1/2))/(3*b^4*d) + (2^(1/3)*a^(10/
3)*log((6*a^4*(a + b*x^3)^(1/3))/(b^4*d) + (18*2^(1/3)*a^(13/3)*((3^(1/2)*
1i)/6 + 1/6))/(b^4*d))*((3^(1/2)*1i)/6 + 1/6))/(b^4*d)
```

Reduce [F]

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{111(b\,x^3+a)^{\frac{1}{3}}\,a^3-37(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3-22(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6-14(b\,x^3+a)^{\frac{1}{3}}\,b^3x^9+280\left(\int\frac{(b\,x^3+a)^{\frac{1}{3}}x^5}{-b^2x^6+a^2}\right.}{140b^4d}$$

input
```
int(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

output
```
(111*(a + b*x**3)**(1/3)*a**3 - 37*(a + b*x**3)**(1/3)*a**2*b*x**3 - 22*(a
 + b*x**3)**(1/3)*a*b**2*x**6 - 14*(a + b*x**3)**(1/3)*b**3*x**9 + 280*int
(((a + b*x**3)**(1/3)*x**5)/(a**2 - b**2*x**6),x)*a**3*b**2)/(140*b**4*d)
```

3.782 $\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 174

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx = -\frac{a^2 \sqrt[3]{a+bx^3}}{b^3 d} - \frac{(a+bx^3)^{7/3}}{7b^3 d} + \frac{\sqrt[3]{2} a^{7/3} \arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3} b^3 d} + \frac{a^{7/3} \log(a-bx^3)}{3\ 2^{2/3} b^3 d} - \frac{a^{7/3} \log\left(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2^{2/3} b^3 d}$$

output
```
-a^2*(b*x^3+a)^(1/3)/b^3/d-1/7*(b*x^3+a)^(7/3)/b^3/d+1/3*2^(1/3)*a^(7/3)*a
rctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/b^3/d
+1/6*a^(7/3)*ln(-b*x^3+a)*2^(1/3)/b^3/d-1/2*a^(7/3)*ln(2^(1/3)*a^(1/3)-(b*
x^3+a)^(1/3))*2^(1/3)/b^3/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.21

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx =$$

$$-\frac{48a^2\sqrt[3]{a+bx^3}+12abx^3\sqrt[3]{a+bx^3}+6b^2x^6\sqrt[3]{a+bx^3}-14\sqrt[3]{2}\sqrt{3}a^{7/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+14\sqrt[3]{\ldots}}{42b^3d\ldots}$$

input
```
Integrate[(x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
-1/42*(48*a^2*(a + b*x^3)^(1/3) + 12*a*b*x^3*(a + b*x^3)^(1/3) + 6*b^2*x^6
*(a + b*x^3)^(1/3) - 14*2^(1/3)*Sqrt[3]*a^(7/3)*ArcTan[(1 + (2^(2/3)*(a +
b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 14*2^(1/3)*a^(7/3)*Log[-2*a^(1/3) + 2^(2
/3)*(a + b*x^3)^(1/3)] - 7*2^(1/3)*a^(7/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)
*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^3*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {948, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6 \sqrt[3]{bx^3+a}}{d\left(a-bx^3\right)}dx^3$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^6\sqrt[3]{bx^3+a}}{a-bx^3}dx^3}{3d}$$

$$\Big\downarrow \; 99$$

$$\frac{\int \left(\frac{a^2\sqrt[3]{bx^3+a}}{b^2(a-bx^3)}-\frac{(bx^3+a)^{4/3}}{b^2}\right)dx^3}{3d}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\sqrt[3]{2}\sqrt{3}a^{7/3}\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^3}+\frac{a^{7/3}\log(a-bx^3)}{2^{2/3}b^3}-\frac{3a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^3}-\frac{3a^2\sqrt[3]{a+bx^3}}{b^3}-\frac{3(a+bx^3)}{7b^3}}{3d}$$

input
```
Int[(x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
((-3*a^2*(a + b*x^3)^(1/3))/b^3 - (3*(a + b*x^3)^(7/3))/(7*b^3) + (2^(1/3)
*Sqrt[3]*a^(7/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(
1/3))])/b^3 + (a^(7/3)*Log[a - b*x^3])/(2^(2/3)*b^3) - (3*a^(7/3)*Log[2^(1
/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^3))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\left(-6b^2x^6-12ab\,x^3-48a^2\right)\left(b\,x^3+a\right)^{\frac{1}{3}}+7a^{\frac{7}{3}}\left(2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^3+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}+\right.\right.}{42b^3d}$

input
```
int(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/42*((-6*b^2*x^6-12*a*b*x^3-48*a^2)*(b*x^3+a)^(1/3)+7*a^(7/3)*(2*arctan(1
/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)+ln((b*x^3+a)
^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))-2*ln((b*x^3+a)^(1/
3)-2^(1/3)*a^(1/3)))*2^(1/3))/b^3/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{14\sqrt{3}2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\,a^2\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}a}{3\,a}\right)+7\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\,a^2\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}(-a\right.}{42}$$

input
```
integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/42*(14*sqrt(3)*2^(1/3)*(-a)^(1/3)*a^2*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^
3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 7*2^(1/3)*(-a)^(1/3)*a^2*log(2^(
2/3)*(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)
) - 14*2^(1/3)*(-a)^(1/3)*a^2*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3))
+ 6*(b^2*x^6 + 2*a*b*x^3 + 8*a^2)*(b*x^3 + a)^(1/3))/(b^3*d)
```

Sympy [F]

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^8 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**8*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**8*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.89

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\frac{14\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{7}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d} + \frac{7\cdot 2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d} - \frac{14\cdot 2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+\right.}{d}}{42\,b^3}$$

input
```
integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
1/42*(14*sqrt(3)*2^(1/3)*a^(7/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/
3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 7*2^(1/3)*a^(7/3)*log(2^(2/3)*a^(2/
3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 14*2^(1/3)
*a^(7/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 6*((b*x^3 + a)^(7/3
) + 7*(b*x^3 + a)^(1/3)*a^2)/d)/b^3
```

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.01

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx = \frac{\sqrt{3}2^{\frac{1}{3}} a^{\frac{7}{3}} \arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}} a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,b^3 d} + \frac{2^{\frac{1}{3}} a^{\frac{7}{3}} \log\left(2^{\frac{2}{3}} a^{\frac{2}{3}} + 2^{\frac{1}{3}} (bx^3+a)^{\frac{1}{3}} a^{\frac{1}{3}} + (bx^3+a)^{\frac{2}{3}}\right)}{6\,b^3 d} - \frac{2^{\frac{1}{3}} a^{\frac{7}{3}} \log\left(\left|-2^{\frac{1}{3}} a^{\frac{1}{3}} + (bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,b^3 d} - \frac{(bx^3+a)^{\frac{7}{3}} b^{18} d^6 + 7\,(bx^3+a)^{\frac{1}{3}} a^2 b^{18} d^6}{7\,b^{21} d^7}$$

input
```
integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*2^(1/3)*a^(7/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
2*(b*x^3 + a)^(1/3))/a^(1/3))/(b^3*d) + 1/6*2^(1/3)*a^(7/3)*log(2^(2/3)*a^
(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b^3*d) - 1
/3*2^(1/3)*a^(7/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b^3*d)
- 1/7*((b*x^3 + a)^(7/3)*b^18*d^6 + 7*(b*x^3 + a)^(1/3)*a^2*b^18*d^6)/(b^2
1*d^7)
```

Mupad [B] (verification not implemented)

Time = 3.45 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.26

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx = \frac{2^{1/3}\,{(-a)}^{7/3}\,\ln\left(6\,a^3\,{\left(b\,x^3+a\right)}^{1/3}-6\,2^{1/3}\,{(-a)}^{10/3}\right)}{3\,b^3\,d} - \frac{a^2\,{\left(b\,x^3+a\right)}^{1/3}}{b^3\,d} - \frac{{\left(b\,x^3+a\right)}^{7/3}}{7\,b^3\,d} - \frac{2^{1/3}\,{(-a)}^{7/3}\,\ln\left(\frac{6\,a^3\,{\left(b\,x^3+a\right)}^{1/3}}{b^3\,d}+\frac{6\,2^{1/3}\,{(-a)}^{10/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^3\,d}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^3\,d} + \frac{2^{1/3}\,{(-a)}^{7/3}\,\ln\left(\frac{6\,a^3\,{\left(b\,x^3+a\right)}^{1/3}}{b^3\,d}-\frac{18\,2^{1/3}\,{(-a)}^{10/3}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^3\,d}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^3\,d}$$

input
```
int((x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

output
```
(2^(1/3)*(-a)^(7/3)*log(6*a^3*(a + b*x^3)^(1/3) - 6*2^(1/3)*(-a)^(10/3)))/
(3*b^3*d) - (a^2*(a + b*x^3)^(1/3))/(b^3*d) - (a + b*x^3)^(7/3)/(7*b^3*d)
- (2^(1/3)*(-a)^(7/3)*log((6*a^3*(a + b*x^3)^(1/3))/(b^3*d) + (6*2^(1/3)*(
-a)^(10/3)*((3^(1/2)*1i)/2 + 1/2))/(b^3*d))*((3^(1/2)*1i)/2 + 1/2))/(3*b^3
*d) + (2^(1/3)*(-a)^(7/3)*log((6*a^3*(a + b*x^3)^(1/3))/(b^3*d) - (18*2^(1
/3)*(-a)^(10/3)*((3^(1/2)*1i)/6 - 1/6))/(b^3*d))*((3^(1/2)*1i)/6 - 1/6))/(
b^3*d)
```

Reduce [F]

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{6(b\,x^3+a)^{\frac{1}{3}}\,a^2-2(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3-(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6+14\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}x^5}{-b^2x^6+a^2}dx\right)a^2b^2}{7b^3d}$$

input
```
int(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

output
```
(6*(a + b*x**3)**(1/3)*a**2 - 2*(a + b*x**3)**(1/3)*a*b*x**3 - (a + b*x**3
)**(1/3)*b**2*x**6 + 14*int(((a + b*x**3)**(1/3)*x**5)/(a**2 - b**2*x**6),
x)*a**2*b**2)/(7*b**3*d)
```

3.783 $\int \frac{x^5 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 172

$$\int \frac{x^5 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{a\sqrt[3]{a+bx^3}}{b^2d}-\frac{(a+bx^3)^{4/3}}{4b^2d}+\frac{\sqrt[3]{2}a^{4/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^2d}+\frac{a^{4/3}\log(a-bx^3)}{3\ 2^{2/3}b^2d}-\frac{a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d}$$

output

```
-a*(b*x^3+a)^(1/3)/b^2/d-1/4*(b*x^3+a)^(4/3)/b^2/d+1/3*2^(1/3)*a^(4/3)*arc
tan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/b^2/d+1
/6*a^(4/3)*ln(-b*x^3+a)*2^(1/3)/b^2/d-1/2*a^(4/3)*ln(2^(1/3)*a^(1/3)-(b*x^
3+a)^(1/3))*2^(1/3)/b^2/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.09

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx =$$

$$-\frac{15a\sqrt[3]{a+bx^3}+3bx^3\sqrt[3]{a+bx^3}-4\sqrt[3]{2}\sqrt{3}a^{4/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{2}a^{4/3}\log\left(-2\sqrt[3]{a}+2^{2/3}\right.}{12b^2d}$$

input
```
Integrate[(x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
-1/12*(15*a*(a + b*x^3)^(1/3) + 3*b*x^3*(a + b*x^3)^(1/3) - 4*2^(1/3)*Sqrt
[3]*a^(4/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 4*
2^(1/3)*a^(4/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] - 2*2^(1/3)*a^
(4/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x
^3)^(2/3)])/(b^2*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {948, 27, 90, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3\sqrt[3]{bx^3+a}}{d\left(a-bx^3\right)}dx^3$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^3 \sqrt[3]{bx^3+a}}{a-bx^3}dx^3}{3d}$$

$$\Big\downarrow \ 90$$

$$\frac{\frac{a\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx^3}{b}-\frac{3\left(a+bx^3\right)^{4/3}}{4b^2}}{3d}$$

$$\Big\downarrow \ 60$$

$$\frac{\frac{a\left(2a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx^3-\frac{3\sqrt[3]{a+bx^3}}{b}\right)}{b}-\frac{3\left(a+bx^3\right)^{4/3}}{4b^2}}{3d}$$

$$\Big\downarrow \ 69$$

$$\frac{\frac{a\left(2a\left(\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\ 2^{2/3}a^{2/3}b}+\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log\left(a-bx^3\right)}{2\ 2^{2/3}a^{2/3}b}\right)-\frac{3\sqrt[3]{a+bx^3}}{b}\right)}{b}-}{3d}$$

$$\Big\downarrow \ 16$$

$$\frac{\frac{a\left(2a\left(\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log\left(a-bx^3\right)}{2\ 2^{2/3}a^{2/3}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}a^{2/3}b}\right)-\frac{3\sqrt[3]{a+bx^3}}{b}\right)}{b}-\frac{3(a+bx^3}{4b^2}}{3d}$$

$$\Big\downarrow \ 1082$$

$$\frac{\frac{a\left(2a\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{2^{2/3}a^{2/3}b}+\frac{\log\left(a-bx^3\right)}{2\ 2^{2/3}a^{2/3}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}a^{2/3}b}\right)-\frac{3\sqrt[3]{a+bx^3}}{b}\right)}{b}-\frac{3\left(a+bx^3\right)^{4/3}}{4b^2}}{3d}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{a\left(2a\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{2^{2/3}a^{2/3}b}+\frac{\log\left(a-bx^3\right)}{2\ 2^{2/3}a^{2/3}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}a^{2/3}b}\right)-\frac{3\sqrt[3]{a+bx^3}}{b}\right)}{b}-\frac{3\left(a+bx^3\right)^{4/3}}{4b^2}}{3d}$$

input
```
Int[(x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
((-3*(a + b*x^3)^(4/3))/(4*b^2) + (a*((-3*(a + b*x^3)^(1/3))/b + 2*a*((Sqr
t[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/(2^(2/3)*a
^(2/3)*b) + Log[a - b*x^3]/(2*2^(2/3)*a^(2/3)*b) - (3*Log[2^(1/3)*a^(1/3)
- (a + b*x^3)^(1/3)])/(2*2^(2/3)*a^(2/3)*b))))/b)/(3*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{\left(-3 b x^3-15 a\right)\left(b x^3+a\right)^{\frac{1}{3}}+2 a^{\frac{4}{3}}\left(2 \arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b x^3+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3 a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b x^3+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}} a^{\frac{1}{3}}\left(b x^3+a\right)^{\frac{1}{3}}+2^{\frac{2}{3}} a^{\frac{2}{3}}\right)-2}{12 b^2 d}$

input
```
int(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/12*((-3*b*x^3-15*a)*(b*x^3+a)^(1/3)+2*a^(4/3)*(2*arctan(1/3*(a^(1/3)+2^(
2/3)*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+2^(1/3)*
a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))-2*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1
/3)))*2^(1/3))/b^2/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.91

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{4\sqrt{3}2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+2\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}\right.}{12b^2d}$$

input
```
integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/12*(4*sqrt(3)*2^(1/3)*(-a)^(1/3)*a*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3 +
 a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 2*2^(1/3)*(-a)^(1/3)*a*log(2^(2/3)*
(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) - 4
*2^(1/3)*(-a)^(1/3)*a*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 3*(b*x
^3 + 5*a)*(b*x^3 + a)^(1/3))/(b^2*d)
```

Sympy [F]

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^5\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**5*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**5*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$
$$=\frac{\frac{4\sqrt{3}2^{\frac{1}{3}}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}+\frac{2\cdot 2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}-\frac{4\cdot 2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)\right.}{d}}{12\,b^2}$$

input
```
integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
1/12*(4*sqrt(3)*2^(1/3)*a^(4/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3
) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 2*2^(1/3)*a^(4/3)*log(2^(2/3)*a^(2/3
) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 4*2^(1/3)*a
^(4/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 3*((b*x^3 + a)^(4/3)
+ 4*(b*x^3 + a)^(1/3)*a)/d)/b^2
```

Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.01

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\frac{\sqrt{3}2^{\frac{1}{3}}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,b^2d}+\frac{2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,b^2d}-\frac{2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,b^2d}-\frac{(bx^3+a)^{\frac{4}{3}}b^6d^3+4\,(bx^3+a)^{\frac{1}{3}}ab^6d^3}{4\,b^8d^4}$$

input
```
integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*2^(1/3)*a^(4/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
2*(b*x^3 + a)^(1/3))/a^(1/3))/(b^2*d) + 1/6*2^(1/3)*a^(4/3)*log(2^(2/3)*a^
(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b^2*d) - 1
/3*2^(1/3)*a^(4/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b^2*d)
- 1/4*((b*x^3 + a)^(4/3)*b^6*d^3 + 4*(b*x^3 + a)^(1/3)*a*b^6*d^3)/(b^8*d^4
)
```

Mupad [B] (verification not implemented)

Time = 3.49 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.16

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\left(b\,x^3+a\right)^{4/3}}{4\,b^2\,d}-\frac{a\,\left(b\,x^3+a\right)^{1/3}}{b^2\,d} -\frac{2^{1/3}\,a^{4/3}\,\ln\left(\left(b\,x^3+a\right)^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b^2\,d} -\frac{2^{1/3}\,a^{4/3}\,\ln\left(\frac{6\,a^2\,\left(b\,x^3+a\right)^{1/3}}{b^2\,d}-\frac{6\,2^{1/3}\,a^{7/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^2\,d}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^2\,d} +\frac{2^{1/3}\,a^{4/3}\,\ln\left(\frac{6\,a^2\,\left(b\,x^3+a\right)^{1/3}}{b^2\,d}+\frac{18\,2^{1/3}\,a^{7/3}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^2\,d}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^2\,d}$$

input
```
int((x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

output
```
(2^(1/3)*a^(4/3)*log((6*a^2*(a + b*x^3)^(1/3))/(b^2*d) + (18*2^(1/3)*a^(7/
3)*((3^(1/2)*1i)/6 + 1/6))/(b^2*d))*((3^(1/2)*1i)/6 + 1/6))/(b^2*d) - (a*(
a + b*x^3)^(1/3))/(b^2*d) - (2^(1/3)*a^(4/3)*log((a + b*x^3)^(1/3) - 2^(1/
3)*a^(1/3)))/(3*b^2*d) - (2^(1/3)*a^(4/3)*log((6*a^2*(a + b*x^3)^(1/3))/(b
^2*d) - (6*2^(1/3)*a^(7/3)*((3^(1/2)*1i)/2 - 1/2))/(b^2*d))*((3^(1/2)*1i)/
2 - 1/2))/(3*b^2*d) - (a + b*x^3)^(4/3)/(4*b^2*d)
```

Reduce [F]

$$\int \frac{x^5 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{3(b\,x^3+a)^{\frac{1}{3}}\,a-(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3+8\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}x^5}{-b^2x^6+a^2}dx\right)a\,b^2}{4b^2d}$$

input
```
int(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

output
```
(3*(a + b*x**3)**(1/3)*a - (a + b*x**3)**(1/3)*b*x**3 + 8*int(((a + b*x**3
)**(1/3)*x**5)/(a**2 - b**2*x**6),x)*a*b**2)/(4*b**2*d)
```

3.784 $\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 150

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{\sqrt[3]{2}\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}bd} + \frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\ 2^{2/3}bd} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd}$$

output
```
-(b*x^3+a)^(1/3)/b/d+1/3*2^(1/3)*a^(1/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^
3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/b/d+1/6*a^(1/3)*ln(-b*x^3+a)*2^(1/3)/
b/d-1/2*a^(1/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/b/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.13

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$
$$= \frac{-6\sqrt[3]{a+bx^3} + 2\sqrt[3]{2}\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\sqrt[3]{2}\sqrt[3]{a}\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right) + \sqrt[3]{2}\sqrt[3]{a}\cdots}{6bd}$$

input
```
Integrate[(x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
(-6*(a + b*x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*a^(1/3)*ArcTan[(1 + (2^(2/3)*(a
+ b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(1/3)*a^(1/3)*Log[-2*a^(1/3) + 2^(
2/3)*(a + b*x^3)^(1/3)] + 2^(1/3)*a^(1/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*
(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(6*b*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {946, 27, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3} \int \frac{\sqrt[3]{bx^3+a}}{d(a-bx^3)}dx^3$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx^3}{3d}$$

$$\Big\downarrow \text{60}$$

$$\frac{2a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx^3 - \frac{3\sqrt[3]{a+bx^3}}{b}}{3d}$$

$$\Big\downarrow \text{69}$$

$$\frac{2a\left(\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\ 2^{2/3}a^{2/3}b} + \frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{2}\sqrt[3]{a}b} + \frac{\log(a-bx^3)}{2\ 2^{2/3}a^{2/3}b}\right) - \frac{3\sqrt[3]{a+bx^3}}{b}}{3d}$$

$$\Big\downarrow \ 16$$

$$\frac{2a\left(\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log(a-bx^3)}{2\ 2^{2/3}a^{2/3}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}a^{2/3}b}\right)-\frac{3\sqrt[3]{a+bx^3}}{b}}{3d}$$

$$\Big\downarrow \ 1082$$

$$\frac{2a\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{2^{2/3}a^{2/3}b}+\frac{\log(a-bx^3)}{2\ 2^{2/3}a^{2/3}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}a^{2/3}b}\right)-\frac{3\sqrt[3]{a+bx^3}}{b}}{3d}$$

$$\Big\downarrow \ 217$$

$$\frac{2a\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{2^{2/3}a^{2/3}b}+\frac{\log(a-bx^3)}{2\ 2^{2/3}a^{2/3}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}a^{2/3}b}\right)-\frac{3\sqrt[3]{a+bx^3}}{b}}{3d}$$

input
```
Int[(x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
((-3*(a + b*x^3)^(1/3))/b + 2*a*((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)
^(1/3))/a^(1/3))/Sqrt[3]])/(2^(2/3)*a^(2/3)*b) + Log[a - b*x^3]/(2*2^(2/3)
*a^(2/3)*b) - (3*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2*2^(2/3)*a^(2
/3)*b)))/(3*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{2\,2^{\frac{1}{3}} a^{\frac{1}{3}} \arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}-2a^{\frac{1}{3}} 2^{\frac{1}{3}} \ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}} a^{\frac{1}{3}}\right)+a^{\frac{1}{3}} 2^{\frac{1}{3}} \ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}} a^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}}\right.}{6bd}$

input
```
int(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*2^(1/3)*a^(1/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2
)/a^(1/3))*3^(1/2)-2*a^(1/3)*2^(1/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))+a
^(1/3)*2^(1/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*
a^(2/3))-6*(b*x^3+a)^(1/3))/b/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.96

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{2\,\sqrt{3}2^{\frac{1}{3}}(-a)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}a}{3\,a}\right)+2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(bx^3\right.}{6\,bd}$$

input
```
integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*2^(1/3)*(-a)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3 + a)
^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 2^(1/3)*(-a)^(1/3)*log(2^(2/3)*(-a)^(2
/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) - 2*2^(1/3
)*(-a)^(1/3)*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 6*(b*x^3 + a)^(
1/3))/(b*d)
```

Sympy [F]

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^2 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**2*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**2*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$
$$= \frac{\frac{2\sqrt{3}2^{\frac{1}{3}}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d} + \frac{2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d} - \frac{2\cdot 2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}}{d}}{6b}$$

input
```
integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
1/6*(2*sqrt(3)*2^(1/3)*a^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
 + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 2^(1/3)*a^(1/3)*log(2^(2/3)*a^(2/3) +
 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 2*2^(1/3)*a^(1
/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 6*(b*x^3 + a)^(1/3)/d)/b
```

Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt{3}2^{\frac{1}{3}}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{3bd} + \frac{2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6bd} - \frac{2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3bd} - \frac{(bx^3+a)^{\frac{1}{3}}}{bd}$$

input
```
integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*2^(1/3)*a^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
2*(b*x^3 + a)^(1/3))/a^(1/3))/(b*d) + 1/6*2^(1/3)*a^(1/3)*log(2^(2/3)*a^(2
/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b*d) - 1/3*2
^(1/3)*a^(1/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b*d) - (b*x
^3 + a)^(1/3)/(b*d)
```

Mupad [B] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.29

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{2^{1/3}(-a)^{1/3}\ln\left(6a\left(bx^3+a\right)^{1/3}-6\,2^{1/3}(-a)^{4/3}\right)}{3bd} - \frac{\left(bx^3+a\right)^{1/3}}{bd} + \frac{2^{1/3}(-a)^{1/3}\ln\left(\frac{6a\left(bx^3+a\right)^{1/3}}{bd}-\frac{6\,2^{1/3}(-a)^{4/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{bd}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3bd} - \frac{2^{1/3}(-a)^{1/3}\ln\left(\frac{6a\left(bx^3+a\right)^{1/3}}{bd}+\frac{6\,2^{1/3}(-a)^{4/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{bd}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3bd}$$

input
```
int((x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

output
```
(2^(1/3)*(-a)^(1/3)*log(6*a*(a + b*x^3)^(1/3) - 6*2^(1/3)*(-a)^(4/3)))/(3*
b*d) - (a + b*x^3)^(1/3)/(b*d) + (2^(1/3)*(-a)^(1/3)*log((6*a*(a + b*x^3)^
(1/3))/(b*d) - (6*2^(1/3)*(-a)^(4/3)*((3^(1/2)*1i)/2 - 1/2))/(b*d))*((3^(1
/2)*1i)/2 - 1/2))/(3*b*d) - (2^(1/3)*(-a)^(1/3)*log((6*a*(a + b*x^3)^(1/3)
)/(b*d) + (6*2^(1/3)*(-a)^(4/3)*((3^(1/2)*1i)/2 + 1/2))/(b*d))*((3^(1/2)*1
i)/2 + 1/2))/(3*b*d)
```

Reduce [F]

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{(b\,x^3+a)^{\frac{1}{3}}+2\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}x^5}{-b^2x^6+a^2}dx\right)b^2}{bd}$$

input
```
int(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

output
```
((a + b*x**3)**(1/3) + 2*int(((a + b*x**3)**(1/3)*x**5)/(a**2 - b**2*x**6)
,x)*b**2)/(b*d)
```

3.785 $\int \frac{\sqrt[3]{a+bx^3}}{x(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 214

$$\int \frac{\sqrt[3]{a+bx^3}}{x(ad-bdx^3)}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}d} + \frac{\sqrt[3]{2}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}d} - \frac{\log(x)}{2a^{2/3}d} + \frac{\log(a-bx^3)}{3\ 2^{2/3}a^{2/3}d} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d}$$

output
```
-1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/
3)/d+1/3*2^(1/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2)/a^(1
/3))*3^(1/2)/a^(2/3)/d-1/2*ln(x)/a^(2/3)/d+1/6*ln(-b*x^3+a)*2^(1/3)/a^(2/3
)/d+1/2*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(2/3)/d-1/2*ln(2^(1/3)*a^(1/3)-(b*x^
3+a)^(1/3))*2^(1/3)/a^(2/3)/d
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt[3]{a+bx^3}}{x(ad-bdx^3)}\,dx =$$

$$-\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)+2\cdots}{\cdots}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^
(1/3)*Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] -
2*Log[-a^(1/3) + (a + b*x^3)^(1/3)] + 2*2^(1/3)*Log[-2*a^(1/3) + 2^(2/3)*(
a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^
(2/3)] - 2^(1/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/
3)*(a + b*x^3)^(2/3)])/(a^(2/3)*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 27, 94, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x(ad-bdx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt[3]{bx^3+a}}{dx^3(a-bx^3)}dx^3$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt[3]{bx^3+a}}{x^3(a-bx^3)}dx^3}{3d}$$

$\downarrow$ 94

$$\frac{\int \frac{1}{x^3(bx^3+a)^{2/3}}dx^3+2b\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx^3}{3d}$$

$\downarrow$ 69

$$\frac{-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+2b\left(\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\ 2^{2/3}a^{2/3}b}\right.}{3d}$$

$\downarrow$ 16

$$\frac{-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+2b\left(\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log(a-bx^3)}{2\ 2^{2/3}a^{2/3}b}-\frac{3\log\left(\sqrt[3]{2}\right.}{}\right.}{3d}$$

$\downarrow$ 1082

$$\frac{\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+2b\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{2^{2/3}a^{2/3}b}+\frac{\log(a-bx^3)}{2\ 2^{2/3}a^{2/3}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}a^{2/3}b}\right)+}{3d}$$

$\downarrow$ 217

$$\frac{-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+2b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{2^{2/3}a^{2/3}b}+\frac{\log(a-bx^3)}{2\ 2^{2/3}a^{2/3}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}a^{2/3}b}\right)+\frac{3\log}{}}{3d}$$

input
```
Int[(a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x]
```

output
```
(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3))
- Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3))
+ 2*b*((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])
/(2^(2/3)*a^(2/3)*b) + Log[a - b*x^3]/(2*2^(2/3)*a^(2/3)*b) - (3*Log[2^(1/
3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2*2^(2/3)*a^(2/3)*b)))/(3*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$\frac{2\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}-2\,2^{\frac{1}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)+2^{\frac{1}{3}}\ln}{6d\,a^{\frac{2}{3}}}$

input
```
int((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*2^(1/3)*3^(1/2)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2
)/a^(1/3))-2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/
2)-2*2^(1/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))+2^(1/3)*ln((b*x^3+a)^(2/3
)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))+2*ln((b*x^3+a)^(1/3)-a^
(1/3))-ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3)))/d/a^(2/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(161) = 322$.

Time = 0.10 (sec) , antiderivative size = 634, normalized size of antiderivative = 2.96

$$\int \frac{\sqrt[3]{a+bx^3}}{x(ad-bdx^3)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/6*(1/2)^(1/3)*(sqrt(-3) + 1)*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d
^3))^(1/3)*log(-(1/2)^(1/3)*(sqrt(-3)*a^3*d^4 + a^3*d^4)*(-(3*a^2*d^3*sqrt
(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 2*(b*x^3 + a)^(1/3
)) + 1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a
^2*d^3))^(1/3)*log((1/2)^(1/3)*(sqrt(-3)*a^3*d^4 - a^3*d^4)*(-(3*a^2*d^3*s
qrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 2*(b*x^3 + a)^(
1/3)) - 1/6*(1/2)^(1/3)*(sqrt(-3) + 1)*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/
(a^2*d^3))^(1/3)*log((1/2)^(1/3)*(sqrt(-3)*a^3*d^4 + a^3*d^4)*((3*a^2*d^3*
sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 2*(b*x^3 + a)^
(1/3)) + 1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)
/(a^2*d^3))^(1/3)*log(-(1/2)^(1/3)*(sqrt(-3)*a^3*d^4 - a^3*d^4)*((3*a^2*d^
3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 2*(b*x^3 + a
)^(1/3)) + 1/3*(1/2)^(1/3)*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^
(1/3)*log((1/2)^(1/3)*a^3*d^4*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3
))^(1/3)*sqrt(1/(a^4*d^6)) + (b*x^3 + a)^(1/3)) + 1/3*(1/2)^(1/3)*((3*a^2*
d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*log(-(1/2)^(1/3)*a^3*d^4*((3*a
^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + (b*x^3
+ a)^(1/3))
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x(ad-bdx^3)}\,dx = -\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax+bx^4}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/x/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a*x + b*x**4), x)/d
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(ad-bdx^3)}\,dx = \frac{\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}d} - \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}d} + \frac{2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,a^{\frac{2}{3}}d} - \frac{2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{2}{3}}d} - \frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{2}{3}}d} + \frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{2}{3}}d}$$

input
```
integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b*x^3
 + a)^(1/3))/a^(1/3))/(a^(2/3)*d) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x
^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(2/3)*d) + 1/6*2^(1/3)*log(2^(2/3)*a^
(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(a^(2/3)*d)
 - 1/3*2^(1/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(a^(2/3)*d)
- 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(2/3
)*d) + 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/3)*d)
```

Mupad [B] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(ad-bdx^3)}\,dx = \ln\left(\left(b\,x^3+a\right)^{1/3}-a\,d\left(\frac{1}{a^2\,d^3}\right)^{1/3}\right)\left(\frac{1}{27\,a^2\,d^3}\right)^{1/3} +\ln\left(\left(b\,x^3+a\right)^{1/3}+2^{1/3}\,a\,d\left(-\frac{1}{a^2\,d^3}\right)^{1/3}\right)\left(-\frac{2}{27\,a^2\,d^3}\right)^{1/3}-\ln\left(2^{1/3}\,a\,d\left(-\frac{1}{a^2\,d^3}\right)^{1/3}-2\left(b\,x^3+a\right)^{1/}\right.$$

input
```
int((a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x)
```

output
```
log((a + b*x^3)^(1/3) - a*d*(1/(a^2*d^3))^(1/3))*(1/(27*a^2*d^3))^(1/3) +
log((a + b*x^3)^(1/3) + 2^(1/3)*a*d*(-1/(a^2*d^3))^(1/3))*(-2/(27*a^2*d^3)
)^(1/3) - log(2^(1/3)*a*d*(-1/(a^2*d^3))^(1/3) - 2*(a + b*x^3)^(1/3) + 2^(
1/3)*3^(1/2)*a*d*(-1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(-2/(27*a
^2*d^3))^(1/3) + log(2*(a + b*x^3)^(1/3) - 2^(1/3)*a*d*(-1/(a^2*d^3))^(1/3
) + 2^(1/3)*3^(1/2)*a*d*(-1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(-
2/(27*a^2*d^3))^(1/3) + log(2*(a + b*x^3)^(1/3) + a*d*(1/(a^2*d^3))^(1/3)
- 3^(1/2)*a*d*(1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a^2*d^
3))^(1/3) - log(2*(a + b*x^3)^(1/3) + a*d*(1/(a^2*d^3))^(1/3) + 3^(1/2)*a*
d*(1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a^2*d^3))^(1/3)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(ad-bdx^3\right)}\,dx=\frac{\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{-b\,x^4+ax}dx}{d}$$

input

```
int((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x)
```

output

```
int((a + b*x**3)**(1/3)/(a*x - b*x**4),x)/d
```

3.786 $\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 268

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx = {} & \frac{b\sqrt[3]{a+bx^3}}{3a^2d} - \frac{(a+bx^3)^{4/3}}{3a^2dx^3} - \frac{4b\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}d} \\ & + \frac{\sqrt[3]{2}b\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}d} - \frac{2b\log(x)}{3a^{5/3}d} + \frac{b\log(a-bx^3)}{3\ 2^{2/3}a^{5/3}d} \\ & + \frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}d} - \frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{5/3}d}\end{aligned}$$

output

```
1/3*b*(b*x^3+a)^(1/3)/a^2/d-1/3*(b*x^3+a)^(4/3)/a^2/d/x^3-4/9*b*arctan(1/3
*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/d+1/3*2^(1/3
)*b*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/
a^(5/3)/d-2/3*b*ln(x)/a^(5/3)/d+1/6*b*ln(-b*x^3+a)*2^(1/3)/a^(5/3)/d+2/3*b
*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(5/3)/d-1/2*b*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^
(1/3))*2^(1/3)/a^(5/3)/d
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx =$$

$$-\frac{6a^{2/3}\sqrt[3]{a+bx^3}+8\sqrt{3}bx^3\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-6\sqrt[3]{2}\sqrt{3}bx^3\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-8bx^3\,\mathrm{lo}}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x]
```

output
```
-1/18*(6*a^(2/3)*(a + b*x^3)^(1/3) + 8*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b
*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 6*2^(1/3)*Sqrt[3]*b*x^3*ArcTan[(1 + (2^(2
/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 8*b*x^3*Log[-a^(1/3) + (a + b*x
^3)^(1/3)] + 6*2^(1/3)*b*x^3*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] +
 4*b*x^3*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 3*
2^(1/3)*b*x^3*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*
(a + b*x^3)^(2/3)])/(a^(5/3)*d*x^3)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {948, 27, 114, 27, 174, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{\sqrt[3]{bx^3+a}}{dx^6(a-bx^3)}dx^3$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\sqrt[3]{bx^3+a}}{x^6(a-bx^3)}dx^3}{3d}$$

$\Big\downarrow$ 114

$$\frac{-\frac{\int -\frac{b\left(4a-bx^3\right)\sqrt[3]{bx^3+a}}{3x^3\left(a-bx^3\right)}dx^3}{a^2}-\frac{\left(a+bx^3\right)^{4/3}}{a^2x^3}}{3d}$$

$\Big\downarrow$ 27

$$\frac{\frac{b\int \frac{\left(4a-bx^3\right)\sqrt[3]{bx^3+a}}{x^3\left(a-bx^3\right)}dx^3}{3a^2}-\frac{\left(a+bx^3\right)^{4/3}}{a^2x^3}}{3d}$$

$\Big\downarrow$ 174

$$\frac{\frac{b\left(4\int \frac{\sqrt[3]{bx^3+a}}{x^3}dx^3+3b\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx^3\right)}{3a^2}-\frac{\left(a+bx^3\right)^{4/3}}{a^2x^3}}{3d}$$

$\Big\downarrow$ 60

$$\frac{\frac{b\left(4\left(a\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3+3\sqrt[3]{a+bx^3}\right)+3b\left(2a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx^3-\frac{3\sqrt[3]{a+bx^3}}{b}\right)\right)}{3a^2}-\frac{\left(a+bx^3\right)^{4/3}}{a^2x^3}}{3d}$$

$\Big\downarrow$ 69

$$b\left(4\left(a\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+3b\left(2a\left(\frac{3\int \cdots\right.\right.$$

$\Big\downarrow$ 16

$$b\left(4\left(a\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+3b\left(2a\left(\frac{3\int \frac{\cdots}{x^6+2^{2/3}a^{2/3}\cdots}\right.\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{b\left(4\left(a\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+3b\left(2a\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}\cdots\right)}{2^{2/3}a^{2/3}b}\cdots\right)\cdots\right)\right)}{3a^2}\cdots}{3d}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{b\left(4\left(a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+3b\left(2a\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\cdots\right)}{2^{2/3}a^{2/3}b}\cdots\right)\cdots\right)\right)}{3a^2}\cdots}{3d}$$

input

```
Int[(a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x]
```

output

```
(-((a + b*x^3)^(4/3)/(a^2*x^3)) + (b*(4*(3*(a + b*x^3)^(1/3) + a*(-((Sqrt[
3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^3
]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3)))) + 3*b*(
(-3*(a + b*x^3)^(1/3))/b + 2*a*((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^
(1/3))/a^(1/3))/Sqrt[3]])/(2^(2/3)*a^(2/3)*b) + Log[a - b*x^3]/(2*2^(2/3)*
a^(2/3)*b) - (3*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2*2^(2/3)*a^(2/
3)*b)))))/(3*a^2))/(3*d)
```

Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{-2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)b\,x^{3}+2^{\frac{1}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)b\,x^{3}+\frac{4\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{3}}{}$

input
```
int((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^(5/3)*(-2^(1/3)*3^(1/2)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3)
)*3^(1/2)/a^(1/3))*b*x^3+2^(1/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))*b*x^3
+4/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)*b*x^3
-1/2*2^(1/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^
(2/3))*b*x^3-4/3*ln((b*x^3+a)^(1/3)-a^(1/3))*b*x^3+2/3*ln((b*x^3+a)^(2/3)+
a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))*b*x^3+(b*x^3+a)^(1/3)*a^(2/3))/x^3/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx =$$

$$-\frac{6\sqrt{3}2^{\frac{1}{3}}a^2bx^3\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}a\left(-\frac{1}{a^2}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)+3\cdot 2^{\frac{1}{3}}a^2bx^3\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^2\left(-\frac{1}{a^2}\right.\right.}{}$$

input
```
integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/18*(6*sqrt(3)*2^(1/3)*a^2*b*x^3*(-1/a^2)^(1/3)*arctan(1/3*sqrt(3)*2^(2/
3)*(b*x^3 + a)^(1/3)*a*(-1/a^2)^(2/3) + 1/3*sqrt(3)) + 3*2^(1/3)*a^2*b*x^3
*(-1/a^2)^(1/3)*log(2^(2/3)*a^2*(-1/a^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)
*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(2/3)) - 6*2^(1/3)*a^2*b*x^3*(-1/a^2)^(1/3
)*log(2^(1/3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(1/3)) + 24*sqrt(1/3)*(a^2)^(
1/6)*a*b*x^3*arctan(sqrt(1/3)*(a^2)^(1/6)*((a^2)^(1/3)*a + 2*(b*x^3 + a)^(
1/3)*(a^2)^(2/3))/a^2) + 4*(a^2)^(2/3)*b*x^3*log((b*x^3 + a)^(2/3)*a + (a^
2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 8*(a^2)^(2/3)*b*x^3*log((b*x
^3 + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)*a^2)/(a^3*d*x^3)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx = -\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^4+bx^7}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/x**4/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a*x**4 + b*x**7), x)/d
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^4}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx = \frac{\sqrt{3}2^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2(bx^3+a)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{3a^{\frac{5}{3}}d} - \frac{4\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{5}{3}}d} + \frac{2^{\frac{1}{3}}b\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6a^{\frac{5}{3}}d} - \frac{2^{\frac{1}{3}}b\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3a^{\frac{5}{3}}d} - \frac{2b\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{5}{3}}d} + \frac{4b\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9a^{\frac{5}{3}}d} - \frac{(bx^3+a)^{\frac{1}{3}}}{3adx^3}$$

input
```
integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*2^(1/3)*b*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b*x
^3 + a)^(1/3))/a^(1/3))/(a^(5/3)*d) - 4/9*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*
(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(5/3)*d) + 1/6*2^(1/3)*b*log(2^(2
/3)*a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(a^(5
/3)*d) - 1/3*2^(1/3)*b*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(a^(
5/3)*d) - 2/9*b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3
))/(a^(5/3)*d) + 4/9*b*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5/3)*d) -
 1/3*(b*x^3 + a)^(1/3)/(a*d*x^3)
```

Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\,(ad-bdx^3)}\,dx = \frac{4\,\ln\left(b\,\left(b\,x^3+a\right)^{1/3}-a^2\,d\,\left(\frac{b^3}{a^5\,d^3}\right)^{1/3}\right)\,\left(\frac{b^3}{a^5\,d^3}\right)^{1/3}}{9} +\ln\left(b\,\left(b\,x^3+a\right)^{1/3}+2^{1/3}\,a^2\,d\,\left(-\frac{b^3}{a^5\,d^3}\right)^{1/3}\right)\,\left(-\frac{2\,b^3}{27\,a^5\,d^3}\right)^{1/3}+\ln\left(2\,b\,\left(b\,x^3+a\right)^{1/3}+a^2\,d\,\left(\frac{b^3}{a^5\,d^3}\right)^{1/3}\right.$$

input
```
int((a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x)
```

output
```
(4*log(b*(a + b*x^3)^(1/3) - a^2*d*(b^3/(a^5*d^3))^(1/3))*(b^3/(a^5*d^3))^
(1/3))/9 + log(b*(a + b*x^3)^(1/3) + 2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(1/3))
*(-(2*b^3)/(27*a^5*d^3))^(1/3) + log(2*b*(a + b*x^3)^(1/3) + a^2*d*(b^3/(a
^5*d^3))^(1/3) - 3^(1/2)*a^2*d*(b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 -
 1/2)*((64*b^3)/(729*a^5*d^3))^(1/3) - log(2*b*(a + b*x^3)^(1/3) + a^2*d*(
b^3/(a^5*d^3))^(1/3) + 3^(1/2)*a^2*d*(b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1
i)/2 + 1/2)*((64*b^3)/(729*a^5*d^3))^(1/3) - log(2^(1/3)*a^2*d*(-b^3/(a^5*
d^3))^(1/3) - 2*b*(a + b*x^3)^(1/3) + 2^(1/3)*3^(1/2)*a^2*d*(-b^3/(a^5*d^3
))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(-(2*b^3)/(27*a^5*d^3))^(1/3) + log(2*
b*(a + b*x^3)^(1/3) - 2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(1/3) + 2^(1/3)*3^(1/
2)*a^2*d*(-b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(-(2*b^3)/(27*a
^5*d^3))^(1/3) - (b*(a + b*x^3)^(1/3))/(3*a*(d*(a + b*x^3) - a*d))
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(ad-bdx^3\right)}\,dx=\frac{\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{-b\,x^7+a\,x^4}dx}{d}$$

input
```
int((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x)
```

output
```
int((a + b*x**3)**(1/3)/(a*x**4 - b*x**7),x)/d
```

3.787 $\int \frac{\sqrt[3]{a+bx^3}}{x^7(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 283

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7(ad-bdx^3)}\,dx=-\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3}-\frac{(a+bx^3)^{4/3}}{6a^2dx^6}-\frac{11b^2\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}d}+\frac{\sqrt[3]{2}b^2\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}d}-\frac{11b^2\log(x)}{18a^{8/3}d}+\frac{b^2\log(a-bx^3)}{3\,2^{2/3}a^{8/3}d}+\frac{11b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{8/3}d}-\frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{8/3}d}$$

output
```
-2/9*b*(b*x^3+a)^(1/3)/a^2/d/x^3-1/6*(b*x^3+a)^(4/3)/a^2/d/x^6-11/27*b^2*a
rctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3)/d+1
/3*2^(1/3)*b^2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3
))*3^(1/2)/a^(8/3)/d-11/18*b^2*ln(x)/a^(8/3)/d+1/6*b^2*ln(-b*x^3+a)*2^(1/3
)/a^(8/3)/d+11/18*b^2*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(8/3)/d-1/2*b^2*ln(2^(
1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/a^(8/3)/d
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(ad-bdx^3\right)}\,dx = -\frac{9a^{5/3}\sqrt[3]{a+bx^3}+21a^{2/3}bx^3\sqrt[3]{a+bx^3}+22\sqrt{3}b^2x^6\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-18\sqrt[3]{2}\sqrt{3}b^2x^6\arctan\left(\right.}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x]
```

output
```
-1/54*(9*a^(5/3)*(a + b*x^3)^(1/3) + 21*a^(2/3)*b*x^3*(a + b*x^3)^(1/3) +
22*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 1
8*2^(1/3)*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))
/Sqrt[3]] - 22*b^2*x^6*Log[-a^(1/3) + (a + b*x^3)^(1/3)] + 18*2^(1/3)*b^2*
x^6*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] + 11*b^2*x^6*Log[a^(2/3) +
 a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 9*2^(1/3)*b^2*x^6*Log[2*
a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/
(a^(8/3)*d*x^6)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {948, 27, 114, 27, 166, 27, 174, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(ad-bdx^3\right)}\,dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{\sqrt[3]{bx^3+a}}{dx^9\left(a-bx^3\right)}dx^3$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\sqrt[3]{bx^3+a}}{x^9(a-bx^3)}dx^3}{3d}$$

$\Big\downarrow$ 114

$$\frac{-\frac{\int -\frac{2b\sqrt[3]{bx^3+a}(bx^3+2a)}{3x^6(a-bx^3)}dx^3}{2a^2}-\frac{(a+bx^3)^{4/3}}{2a^2x^6}}{3d}$$

$\Big\downarrow$ 27

$$\frac{\frac{b\int \frac{\sqrt[3]{bx^3+a}(bx^3+2a)}{x^6(a-bx^3)}dx^3}{3a^2}-\frac{(a+bx^3)^{4/3}}{2a^2x^6}}{3d}$$

$\Big\downarrow$ 166

$$\frac{\frac{b\left(\frac{\int \frac{ab(7bx^3+11a)}{3x^3(a-bx^3)(bx^3+a)^{2/3}}dx^3}{a}-\frac{2\sqrt[3]{a+bx^3}}{x^3}\right)}{3a^2}-\frac{(a+bx^3)^{4/3}}{2a^2x^6}}{3d}$$

$\Big\downarrow$ 27

$$\frac{\frac{b\left(\frac{1}{3}b\int \frac{7bx^3+11a}{x^3(a-bx^3)(bx^3+a)^{2/3}}dx^3-\frac{2\sqrt[3]{a+bx^3}}{x^3}\right)}{3a^2}-\frac{(a+bx^3)^{4/3}}{2a^2x^6}}{3d}$$

$\Big\downarrow$ 174

$$\frac{\frac{b\left(\frac{1}{3}b\left(11\int \frac{1}{x^3(bx^3+a)^{2/3}}dx^3+18b\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx^3\right)-\frac{2\sqrt[3]{a+bx^3}}{x^3}\right)}{3a^2}-\frac{(a+bx^3)^{4/3}}{2a^2x^6}}{3d}$$

$\Big\downarrow$ 69

$$\frac{\frac{b\left(\frac{1}{3}b\left(11\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2a^{2/3}}\right)+18b\left(\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{bx^3+}}}{2\ 2^{2/3}a^{2/}}\right.\right.\right.}{3a^2}}{3d}$$

$\Big\downarrow$ 16

$$\frac{b\left(\frac{1}{3}b\left(11\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+18b\left(\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3}}}{2\sqrt[3]{2}\sqrt[3]{a}b}\right.\right.}{3a^2}}{3d}$$

↓ 1082

$$\frac{b\left(\frac{1}{3}b\left(11\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+18b\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{2^{2/3}a^{2/3}b}+\frac{\log\left(a-bx^3\right)}{2\ 2^{2/3}a^{2/3}b}\right.\right.}{3a^2}}{3d}$$

↓ 217

$$\frac{b\left(\frac{1}{3}b\left(11\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+18b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{2^{2/3}a^{2/3}b}+\frac{\log\left(a-bx^3\right)}{2\ 2^{2/3}a^{2/3}b}-\right.\right.}{3a^2}}{3d}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x]
```

output
```
(-1/2*(a + b*x^3)^(4/3)/(a^2*x^6) + (b*((-2*(a + b*x^3)^(1/3))/x^3 + (b*(1
1*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)
) - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3)
)) + 18*b*((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[
3]])/(2^(2/3)*a^(2/3)*b) + Log[a - b*x^3]/(2*2^(2/3)*a^(2/3)*b) - (3*Log[2
^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2*2^(2/3)*a^(2/3)*b))))/3))/(3*a^2))
/(3*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{18\, 2^{\frac{1}{3}} \sqrt{3}\, \arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}}\right) \sqrt{3}}{3 a^{\frac{1}{3}}}\right) b^{2} x^{6}-22 \sqrt{3}\, \arctan\left(\frac{\left(a^{\frac{1}{3}}+2 \left(b\,x^{3}+a\right)^{\frac{1}{3}}\right) \sqrt{3}}{3 a^{\frac{1}{3}}}\right) b^{2} x^{6}-18\, 2^{\frac{1}{3}} \ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}} \cdots\right.}{\cdots}$

input
```
int((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/54*(18*2^(1/3)*3^(1/2)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1
/2)/a^(1/3))*b^2*x^6-22*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(
1/2)/a^(1/3))*b^2*x^6-18*2^(1/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))*b^2*x
^6+9*2^(1/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^
(2/3))*b^2*x^6+22*ln((b*x^3+a)^(1/3)-a^(1/3))*b^2*x^6-11*ln((b*x^3+a)^(2/3
)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))*b^2*x^6-21*b*x^3*(b*x^3+a)^(1/3)*a^(2/3
)-9*(b*x^3+a)^(1/3)*a^(5/3))/a^(8/3)/x^6/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(ad-bdx^3\right)}\,dx =$$

$$-\frac{18\,\sqrt{3}2^{\frac{1}{3}}a^2b^2x^6\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\,\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}a\left(-\frac{1}{a^2}\right)^{\frac{2}{3}}+\frac{1}{3}\,\sqrt{3}\right)+9\cdot2^{\frac{1}{3}}a^2b^2x^6\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^2\left(\right.\right.}{}$$

input
```
integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/54*(18*sqrt(3)*2^(1/3)*a^2*b^2*x^6*(-1/a^2)^(1/3)*arctan(1/3*sqrt(3)*2^
(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a^2)^(2/3) + 1/3*sqrt(3)) + 9*2^(1/3)*a^2*b^
2*x^6*(-1/a^2)^(1/3)*log(2^(2/3)*a^2*(-1/a^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^
(1/3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(2/3)) - 18*2^(1/3)*a^2*b^2*x^6*(-1/a
^2)^(1/3)*log(2^(1/3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(1/3)) + 66*sqrt(1/3)
*(a^2)^(1/6)*a*b^2*x^6*arctan(sqrt(1/3)*(a^2)^(1/6)*((a^2)^(1/3)*a + 2*(b*
x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) + 11*(a^2)^(2/3)*b^2*x^6*log((b*x^3 + a)^
(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 22*(a^2)^(2/3)*
b^2*x^6*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)) + 3*(7*a^2*b*x^3 + 3*a^3)*(
b*x^3 + a)^(1/3))/(a^4*d*x^6)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(ad-bdx^3\right)}\,dx = -\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^7+bx^{10}}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/x**7/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a*x**7 + b*x**10), x)/d
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^7}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^7), x)
```

Giac [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7(ad-bdx^3)}\,dx = \frac{\sqrt{3}2^{\frac{1}{3}}b^2\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{8}{3}}d} - \frac{11\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{8}{3}}d} + \frac{2^{\frac{1}{3}}b^2\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,a^{\frac{8}{3}}d} - \frac{2^{\frac{1}{3}}b^2\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{8}{3}}d} - \frac{11\,b^2\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{54\,a^{\frac{8}{3}}d} + \frac{11\,b^2\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27\,a^{\frac{8}{3}}d} - \frac{7\,(bx^3+a)^{\frac{4}{3}}b^2-4\,(bx^3+a)^{\frac{1}{3}}ab^2}{18\,a^2b^2dx^6}$$

input
```
integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*2^(1/3)*b^2*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b
*x^3 + a)^(1/3))/a^(1/3))/(a^(8/3)*d) - 11/27*sqrt(3)*b^2*arctan(1/3*sqrt(
3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(8/3)*d) + 1/6*2^(1/3)*b^2*
log(2^(2/3)*a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3
))/(a^(8/3)*d) - 1/3*2^(1/3)*b^2*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1
/3)))/(a^(8/3)*d) - 11/54*b^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^
(1/3) + a^(2/3))/(a^(8/3)*d) + 11/27*b^2*log(abs((b*x^3 + a)^(1/3) - a^(1/
3)))/(a^(8/3)*d) - 1/18*(7*(b*x^3 + a)^(4/3)*b^2 - 4*(b*x^3 + a)^(1/3)*a*b
^2)/(a^2*b^2*d*x^6)
```

Mupad [B] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7 (ad-bdx^3)} \, dx = \frac{\frac{2\,b^2\,\left(b\,x^3+a\right)^{1/3}}{9\,a}-\frac{7\,b^2\,\left(b\,x^3+a\right)^{4/3}}{18\,a^2}}{d\,\left(b\,x^3+a\right)^2+a^2\,d-2\,a\,d\,\left(b\,x^3+a\right)} + \frac{11\,\ln\left(b^2\,\left(b\,x^3+a\right)^{1/3}-a^3\,d\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}\right)\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}}{27} + \ln\left(b^2\,\left(b\,x^3+a\right)^{1/3}+2^{1/3}\,a^3\,d\,{\left(-\frac{b^6}{a^8\,d^3}\right)}^{1/3}\right)\,{\left(-\frac{2\,b^6}{27\,a^8\,d^3}\right)}^{1/3}-\ln\left(2^{1/3}\,a^3\,d\,{\left(-\frac{b^6}{a^8\,d^3}\right)}^{1/3}-2\,b^2\,\left(b\,x^3\right.\right.$$

input
```
int((a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x)
```

output
```
((2*b^2*(a + b*x^3)^(1/3))/(9*a) - (7*b^2*(a + b*x^3)^(4/3))/(18*a^2))/(d*
(a + b*x^3)^2 + a^2*d - 2*a*d*(a + b*x^3)) + (11*log(b^2*(a + b*x^3)^(1/3)
 - a^3*d*(b^6/(a^8*d^3))^(1/3))*(b^6/(a^8*d^3))^(1/3))/27 + log(b^2*(a + b
*x^3)^(1/3) + 2^(1/3)*a^3*d*(-b^6/(a^8*d^3))^(1/3))*(-(2*b^6)/(27*a^8*d^3)
)^(1/3) - log(2^(1/3)*a^3*d*(-b^6/(a^8*d^3))^(1/3) - 2*b^2*(a + b*x^3)^(1/
3) + 2^(1/3)*3^(1/2)*a^3*d*(-b^6/(a^8*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/
2)*(-(2*b^6)/(27*a^8*d^3))^(1/3) + log(2*b^2*(a + b*x^3)^(1/3) - 2^(1/3)*a
^3*d*(-b^6/(a^8*d^3))^(1/3) + 2^(1/3)*3^(1/2)*a^3*d*(-b^6/(a^8*d^3))^(1/3)
*1i)*((3^(1/2)*1i)/2 - 1/2)*(-(2*b^6)/(27*a^8*d^3))^(1/3) + (11*log(2*b^2*
(a + b*x^3)^(1/3) + a^3*d*(b^6/(a^8*d^3))^(1/3) - 3^(1/2)*a^3*d*(b^6/(a^8*
d^3))^(1/3)*1i)*(3^(1/2)*1i - 1)*(b^6/(a^8*d^3))^(1/3))/54 - (11*log(2*b^2
*(a + b*x^3)^(1/3) + a^3*d*(b^6/(a^8*d^3))^(1/3) + 3^(1/2)*a^3*d*(b^6/(a^8
*d^3))^(1/3)*1i)*(3^(1/2)*1i + 1)*(b^6/(a^8*d^3))^(1/3))/54
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\,(ad-bdx^3)}\,dx=\frac{\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{-b\,x^{10}+a\,x^7}dx}{d}$$

input
```
int((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x)
```

output
```
int((a + b*x**3)**(1/3)/(a*x**7 - b*x**10),x)/d
```

3.788 $\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 268

$$\begin{aligned}\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx =& -\frac{7ax^2\sqrt[3]{a+bx^3}}{18b^2d}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}+\frac{11a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{8/3}d}\\&-\frac{\sqrt[3]{2}a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{8/3}d}+\frac{a^2\log\left(ad-bdx^3\right)}{3\ 2^{2/3}b^{8/3}d}\\&+\frac{11a^2\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{8/3}d}-\frac{a^2\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{8/3}d}\end{aligned}$$

output
```
-7/18*a*x^2*(b*x^3+a)^(1/3)/b^2/d-1/6*x^5*(b*x^3+a)^(1/3)/b/d+11/27*a^2*ar
ctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(8/3)/d-1/3*2^
(1/3)*a^2*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1
/2)/b^(8/3)/d+1/6*a^2*ln(-b*d*x^3+a*d)*2^(1/3)/b^(8/3)/d+11/18*a^2*ln(b^(1
/3)*x-(b*x^3+a)^(1/3))/b^(8/3)/d-1/2*a^2*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1
/3))*2^(1/3)/b^(8/3)/d
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.22

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx =$$
$$-\frac{21ab^{2/3}x^2\sqrt[3]{a+bx^3}+9b^{5/3}x^5\sqrt[3]{a+bx^3}-22\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+18\sqrt[3]{2}\sqrt{3}a^2\arctan\left(\right.}{}$$

input
```
Integrate[(x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
-1/54*(21*a*b^(2/3)*x^2*(a + b*x^3)^(1/3) + 9*b^(5/3)*x^5*(a + b*x^3)^(1/3
) - 22*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(
1/3))] + 18*2^(1/3)*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^
(2/3)*(a + b*x^3)^(1/3))] - 22*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] +
 18*2^(1/3)*a^2*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] + 11*a^2*Log
[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 9*2^(1/3
)*a^2*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a
 + b*x^3)^(2/3)])/(b^(8/3)*d)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {978, 27, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$

$$\Big\downarrow \ 978$$

$$\frac{\int \frac{ax^4(7bx^3+5a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{6bd}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}$$

$$\Big\downarrow \ 27$$

$$\frac{a\int \frac{x^4(7bx^3+5a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{6bd}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}$$

$\Big\downarrow$ 1052

$$\frac{a\left(\frac{\int \frac{2abx(11bx^3+7a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{3b^2}-\frac{7x^2\sqrt[3]{a+bx^3}}{3b}\right)}{6bd}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}$$

$\Big\downarrow$ 27

$$\frac{a\left(\frac{2a\int \frac{x(11bx^3+7a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{3b}-\frac{7x^2\sqrt[3]{a+bx^3}}{3b}\right)}{6bd}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}$$

$\Big\downarrow$ 1054

$$\frac{a\left(\frac{2a\int \left(\frac{18ax}{(a-bx^3)(bx^3+a)^{2/3}}-\frac{11x}{(bx^3+a)^{2/3}}\right)dx}{3b}-\frac{7x^2\sqrt[3]{a+bx^3}}{3b}\right)}{6bd}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}$$

$\Big\downarrow$ 2009

$$\frac{a\left(\frac{2a\left(\frac{11\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{3\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{b^{2/3}}+\frac{3\log(a-bx^3)}{2^{2/3}b^{2/3}}+\frac{11\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}-\frac{9\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a\right.}}{2^{2/3}b^{2/3}}\right.}{3b}\right.}{6bd}$$

$$\frac{x^5\sqrt[3]{a+bx^3}}{6bd}$$

input
```
Int[(x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
-1/6*(x^5*(a + b*x^3)^(1/3))/(b*d) + (a*((-7*x^2*(a + b*x^3)^(1/3))/(3*b)
+ (2*a*((11*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3
]*b^(2/3)) - (3*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x
^3)^(1/3))/Sqrt[3]])/b^(2/3) + (3*Log[a - b*x^3])/(2^(2/3)*b^(2/3)) + (11*
Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*b^(2/3)) - (9*Log[2^(1/3)*b^(1/3)*x
 - (a + b*x^3)^(1/3)])/(2^(2/3)*b^(2/3))))/(3*b)))/(6*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 7.09 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{-3(3b x^3+7a)(b x^3+a)^{\frac{1}{3}} x^2 b^{\frac{8}{3}}+a^2 b^2\left(9\left(\left(2\arctan\left(\frac{\sqrt{3}\left(\frac{2^{\frac{2}{3}}(b x^3+a)^{\frac{1}{3}}}{b^{\frac{1}{3}}}+x\right)}{3x}\right)\sqrt{3}+\ln\left(\frac{b^{\frac{2}{3}}2^{\frac{2}{3}}x^2+(b x^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}2^{\frac{1}{3}}x+(b x^3+\cdots}{x^2}\right.\right.\right.\right.}{}$

input
```
int(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/54*(-3*(3*b*x^3+7*a)*(b*x^3+a)^(1/3)*x^2*b^(8/3)+a^2*b^2*(9*(2*arctan(1/
3*3^(1/2)*(2^(2/3)/b^(1/3)*(b*x^3+a)^(1/3)+x)/x)*3^(1/2)+ln((b^(2/3)*2^(2/
3)*x^2+(b*x^3+a)^(1/3)*b^(1/3)*2^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-2^(1
/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x))*2^(1/3)-22*arctan(1/3*3^(1/2)*(2*(b*x^3
+a)^(1/3)/b^(1/3)+x)/x)*3^(1/2)-11*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)
*x+(b*x^3+a)^(2/3))/x^2)+22*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)))/b^(14/3)/
d
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.34

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx =$$

$$-\frac{18\sqrt{3}2^{\frac{1}{3}}a^2b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}b\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}x}{3x}\right)-18\cdot 2^{\frac{1}{3}}a^2b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+(bx^3+a)\cdots}{x}\right.}{}$$

input
```
integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/54*(18*sqrt(3)*2^(1/3)*a^2*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/
3)*(b*x^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 18*2^(1/3)*a^2*b^2
*(-1/b^2)^(1/3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) +
9*2^(1/3)*a^2*b^2*(-1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(
1/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 66*s
qrt(1/3)*a^2*(b^2)^(1/6)*b*arctan(sqrt(1/3)*((b^2)^(1/3)*b*x + 2*(b*x^3 +
a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 22*a^2*(b^2)^(2/3)*log(-((b^2
)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 11*a^2*(b^2)^(2/3)*log(((b^2)^(1/3)*
b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2) + 3*(3
*b^3*x^5 + 7*a*b^2*x^2)*(b*x^3 + a)^(1/3))/(b^4*d)
```

Sympy [F]

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^7 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**7*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**7*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x^7}{bdx^3-ad}\,dx$$

input
```
integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)*x^7/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x^7}{bdx^3-ad}\,dx$$

input
```
integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)*x^7/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int \frac{x^7\,(b\,x^3+a)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

output
```
int((x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

Reduce [F]

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{-7(b\,x^3+a)^{\frac{1}{3}}\,a\,x^2-3(b\,x^3+a)^{\frac{1}{3}}\,b\,x^5+22\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}\,x^4}{-b^2x^6+a^2}dx\right)a^2b+14\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}\,x}{-b^2x^6+a^2}dx\right)a^3}{18b^2d}$$

input
```
int(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

output
```
( - 7*(a + b*x**3)**(1/3)*a*x**2 - 3*(a + b*x**3)**(1/3)*b*x**5 + 22*int((
(a + b*x**3)**(1/3)*x**4)/(a**2 - b**2*x**6),x)*a**2*b + 14*int(((a + b*x*
*3)**(1/3)*x)/(a**2 - b**2*x**6),x)*a**3)/(18*b**2*d)
```

3.789 $\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 233

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{x^2\sqrt[3]{a+bx^3}}{3bd} + \frac{4a\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}d} - \frac{\sqrt[3]{2}a\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{5/3}d} + \frac{a\log\left(ad-bdx^3\right)}{3\ 2^{2/3}b^{5/3}d} + \frac{2a\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3}d} - \frac{a\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{5/3}d}$$

output
```
-1/3*x^2*(b*x^3+a)^(1/3)/b/d+4/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/
3))*3^(1/2))*3^(1/2)/b^(5/3)/d-1/3*2^(1/3)*a*arctan(1/3*(1+2*2^(1/3)*b^(1/
3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(5/3)/d+1/6*a*ln(-b*d*x^3+a*d)*2^
(1/3)/b^(5/3)/d+2/3*a*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)/d-1/2*a*ln(2^(
1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/b^(5/3)/d
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.26

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx =$$
$$-\frac{6b^{2/3}x^2\sqrt[3]{a+bx^3}-8\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+6\sqrt[3]{2}\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)-8a\,\mathrm{l}}{}$$

input
```
Integrate[(x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
-1/18*(6*b^(2/3)*x^2*(a + b*x^3)^(1/3) - 8*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^(1/
3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 6*2^(1/3)*Sqrt[3]*a*ArcTan[(Sqr
t[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] - 8*a*Log[-(b^(1/
3)*x) + (a + b*x^3)^(1/3)] + 6*2^(1/3)*a*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b
*x^3)^(1/3)] + 4*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*
x^3)^(2/3)] - 3*2^(1/3)*a*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3
)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^(5/3)*d)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {978, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3}dx$$
$$\Big\downarrow \text{978}$$
$$\frac{\int \frac{2ax(2bx^3+a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{3bd}-\frac{x^2\sqrt[3]{a+bx^3}}{3bd}$$
$$\Big\downarrow \text{27}$$

$$\frac{2a\int \frac{x(2bx^3+a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{3bd}-\frac{x^2\sqrt[3]{a+bx^3}}{3bd}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{2a\int \left(\frac{3ax}{(a-bx^3)(bx^3+a)^{2/3}}-\frac{2x}{(bx^3+a)^{2/3}}\right)dx}{3bd}-\frac{x^2\sqrt[3]{a+bx^3}}{3bd}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a\left(\frac{2\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}b^{2/3}}+\frac{\log(a-bx^3)}{2\ 2^{2/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{\ }\right.}{2\ 2^{2/3}b^{2/3}}\right.}{3bd}$$

$$\frac{x^2\sqrt[3]{a+bx^3}}{3bd}$$

input
```
Int[(x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
-1/3*(x^2*(a + b*x^3)^(1/3))/(b*d) + (2*a*((2*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) - (Sqrt[3]*ArcTan[(1 + (2*2^(
1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(2/3)*b^(2/3)) + Log[a - b
*x^3]/(2*2^(2/3)*b^(2/3)) + Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/b^(2/3) - (
3*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*2^(2/3)*b^(2/3))))/(3*b*d
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{-6\left(b\,x^{3}+a\right)^{\frac{1}{3}} x^{2} b^{\frac{2}{3}}+6\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}} x\right)}{3 b^{\frac{1}{3}} x}\right) a-8\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right) a-6\,2^{\frac{1}{3}}\ln\left(\right.}{}$

input
```
int(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/18*(-6*(b*x^3+a)^(1/3)*x^2*b^(2/3)+6*2^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*
(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)*x)/b^(1/3)/x)*a-8*3^(1/2)*arctan(1/3*3^(1
/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a-6*2^(1/3)*ln((-2^(1/3)*b^(1
/3)*x+(b*x^3+a)^(1/3))/x)*a+3*2^(1/3)*ln((b^(2/3)*2^(2/3)*x^2+(b*x^3+a)^(1
/3)*b^(1/3)*2^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a+8*ln((-b^(1/3)*x+(b*x^3+a)^(
1/3))/x)*a-4*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^
2)*a)/b^(5/3)/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.43

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{6\sqrt{3}2^{\frac{1}{3}}ab^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}b\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}x}{3x}\right)-6\cdot 2^{\frac{1}{3}}ab^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}}{x}\right)}{}$$

input
```
integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/18*(6*sqrt(3)*2^(1/3)*a*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*
(b*x^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 6*2^(1/3)*a*b^2*(-1/b
^2)^(1/3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 3*2^(1
/3)*a*b^2*(-1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)*(b*
x^3 + a)^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 6*(b*x^3 + a
)^(1/3)*b^2*x^2 + 24*sqrt(1/3)*a*(b^2)^(1/6)*b*arctan(sqrt(1/3)*((b^2)^(1/
3)*b*x + 2*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 8*a*(b^2)
^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 4*a*(b^2)^(2/3)*log
(((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*
b)/x^2))/(b^3*d)
```

Sympy [F]

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^4 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**4*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**4*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x^4}{bdx^3-ad}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)*x^4/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x^4}{bdx^3-ad}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)*x^4/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx = \int \frac{x^4 \left(b\,x^3+a\right)^{1/3}}{a\,d-b\,d\,x^3} \, dx$$

input
```
int((x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

output
```
int((x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

Reduce [F]

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx = \frac{-(b\,x^3+a)^{\frac{1}{3}} x^2 + 4\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}} x^4}{-b^2x^6+a^2}dx\right) ab + 2\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}} x}{-b^2x^6+a^2}dx\right) a^2}{3bd}$$

input
```
int(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

output
```
( - (a + b*x**3)**(1/3)*x**2 + 4*int(((a + b*x**3)**(1/3)*x**4)/(a**2 - b*
*2*x**6),x)*a*b + 2*int(((a + b*x**3)**(1/3)*x)/(a**2 - b**2*x**6),x)*a**2
)/(3*b*d)
```

3.790 $\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 201

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d}-\frac{\sqrt[3]{2}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d}+\frac{\log\left(ad-bdx^3\right)}{3\ 2^{2/3}b^{2/3}d}+\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{2/3}d}$$

output
```
1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)/d-
1/3*2^(1/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^
(1/2)/b^(2/3)/d+1/6*ln(-b*d*x^3+a*d)*2^(1/3)/b^(2/3)/d+1/2*ln(b^(1/3)*x-(b
*x^3+a)^(1/3))/b^(2/3)/d-1/2*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)
/b^(2/3)/d
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.32

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$
$$= \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)+2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)-}{}$$

input
```
Integrate[(x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] -
 2*2^(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*
x^3)^(1/3))] + 2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - 2*2^(1/3)*Log[-2*
b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] - Log[b^(2/3)*x^2 + b^(1/3)*x*(a +
b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + 2^(1/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^
(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(6*b^(2/3)*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {984, 27, 853, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$
$$\Big\downarrow \text{984}$$
$$2a\int \frac{x}{d(a-bx^3)(bx^3+a)^{2/3}}dx-\frac{\int \frac{x}{(bx^3+a)^{2/3}}dx}{d}$$
$$\Big\downarrow \text{27}$$

$$\frac{2a\int \frac{x}{(a-bx^3)(bx^3+a)^{2/3}}dx}{d}-\frac{\int \frac{x}{(bx^3+a)^{2/3}}dx}{d}$$

$\big\downarrow$ 853

$$\frac{2a\int \frac{x}{(a-bx^3)(bx^3+a)^{2/3}}dx}{d}-\frac{-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}}{d}$$

$\big\downarrow$ 992

$$\frac{2a\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}ab^{2/3}}+\frac{\log(a-bx^3)}{6\ 2^{2/3}ab^{2/3}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}ab^{2/3}}\right)}{d}-\frac{-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}}{d}$$

input
```
Int[(x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
-((-(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3
))) - Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b^(2/3)))/d) + (2*a*(-(ArcTan[
(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]*a*
b^(2/3))) + Log[a - b*x^3]/(6*2^(2/3)*a*b^(2/3)) - Log[2^(1/3)*b^(1/3)*x -
 (a + b*x^3)^(1/3)]/(2*2^(2/3)*a*b^(2/3))))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.09

method	result
pseudoelliptic	$\frac{2\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)-2\,2^{\frac{1}{3}}\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)+}{6d\,b^{\frac{2}{3}}}$

input
```
int(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*2^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)
*x)/b^(1/3)/x)-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/
b^(1/3)/x)-2*2^(1/3)*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)+2^(1/3)*ln
((b^(2/3)*2^(2/3)*x^2+(b*x^3+a)^(1/3)*b^(1/3)*2^(1/3)*x+(b*x^3+a)^(2/3))/x
^2)+2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)
^(1/3)*x+(b*x^3+a)^(2/3))/x^2))/d/b^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.54

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{2\sqrt{3}2^{\frac{1}{3}}b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}b\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}x}{3x}\right)-2\cdot 2^{\frac{1}{3}}b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}}{x}\right)+2}{}$$

input
```
integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*2^(1/3)*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(b*
x^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 2*2^(1/3)*b^2*(-1/b^2)^(
1/3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 2^(1/3)*b^2
*(-1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^
(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(b^2)^(1/
6)*b*arctan(sqrt(1/3)*((b^2)^(1/3)*b*x + 2*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*
(b^2)^(1/6)/(b^2*x)) - 2*(b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/
3)*b)/x) + (b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2
/3)*x + (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d)
```

Sympy [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=-\frac{\int \frac{x\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}x}{bdx^3-ad}\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)*x/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}x}{bdx^3-ad}\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)*x/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\int \frac{x\,{\left(b\,x^3+a\right)}^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

output
```
int((x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

Reduce [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\frac{\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}x}{-b\,x^{3}+a}dx}{d}$$

input
```
int(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

output
```
int(((a + b*x**3)**(1/3)*x)/(a - b*x**3),x)/d
```

3.791 $\int \frac{\sqrt[3]{a+bx^3}}{x^2(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 156

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2(ad-bdx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{adx} - \frac{\sqrt[3]{2}\sqrt[3]{b}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}ad} + \frac{\sqrt[3]{b}\log(ad-bdx^3)}{3\ 2^{2/3}ad} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}ad}$$

output
```
-(b*x^3+a)^(1/3)/a/d/x-1/3*2^(1/3)*b^(1/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)
*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/a/d+1/6*b^(1/3)*ln(-b*d*x^3+a*d)*2^(1
/3)/a/d-1/2*b^(1/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/a/d
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2(ad-bdx^3)}\,dx =$$
$$-\frac{6\sqrt[3]{a+bx^3}+2\sqrt[3]{2}\sqrt{3}\sqrt[3]{b}x\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)+2\sqrt[3]{2}\sqrt[3]{b}x\log\left(-2\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}\right)-}{6adx}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x]
```

output
```
-1/6*(6*(a + b*x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*b^(1/3)*x*ArcTan[(Sqrt[3]*b^
(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] + 2*2^(1/3)*b^(1/3)*x*Lo
g[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] - 2^(1/3)*b^(1/3)*x*Log[2*b^(2
/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)]
)/(a*d*x)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {975, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2(ad-bdx^3)}\,dx$$
$$\Big\downarrow \; 975$$
$$\frac{\int \frac{2abx}{(a-bx^3)(bx^3+a)^{2/3}}dx}{ad}-\frac{\sqrt[3]{a+bx^3}}{adx}$$
$$\Big\downarrow \; 27$$
$$\frac{2b\int \frac{x}{(a-bx^3)(bx^3+a)^{2/3}}dx}{d}-\frac{\sqrt[3]{a+bx^3}}{adx}$$

$\downarrow$ 992

$$\frac{2b\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}ab^{2/3}}+\frac{\log(a-bx^3)}{6\ 2^{2/3}ab^{2/3}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}ab^{2/3}}\right)}{d}-\frac{\sqrt[3]{a+bx^3}}{adx}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x]
```

output
```
-((a + b*x^3)^(1/3)/(a*d*x)) + (2*b*(-(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(
a + b*x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]*a*b^(2/3))) + Log[a - b*x^3]/(
6*2^(2/3)*a*b^(2/3)) - Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(2/
3)*a*b^(2/3))))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{2b^{\frac{1}{3}}2^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)x-2b^{\frac{1}{3}}2^{\frac{1}{3}}\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)x+b^{\frac{1}{3}}2^{\frac{1}{3}}\ln\left(\frac{b^{\frac{2}{3}}2^{\frac{2}{3}}x^{2}+\left(b\,x^{3}+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}2 \cdots}{x^{2}}\right.}{6adx}$

input
```
int((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*b^(1/3)*2^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)
+b^(1/3)*x)/b^(1/3)/x)*x-2*b^(1/3)*2^(1/3)*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a
)^(1/3))/x)*x+b^(1/3)*2^(1/3)*ln((b^(2/3)*2^(2/3)*x^2+(b*x^3+a)^(1/3)*b^(1
/3)*2^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*x-6*(b*x^3+a)^(1/3))/a/d/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(125) = 250.

Time = 88.97 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.53

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx = -\frac{2\sqrt{3}2^{\frac{1}{3}}(-b)^{\frac{1}{3}}\,x\arctan\left(\frac{6\sqrt{3}2^{\frac{2}{3}}\left(19\,b^2x^8+16\,abx^5+a^2x^2\right)\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}+6\sqrt{3}2^{\frac{1}{3}}\left(5\,b^2x^7-4\,abx^4-a^2x\right)\left(bx^3+a\right)^{\frac{2}{3}}(-b)^{\frac{1}{3}}+\sqrt{3}(\cdots}{3\left(109\,b^3x^9+105\,ab^2x^6+3\,a^2bx^3-a^3\right)}\right.\cdots}{\cdots}$$

input
```
integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/18*(2*sqrt(3)*2^(1/3)*(-b)^(1/3)*x*arctan(1/3*(6*sqrt(3)*2^(2/3)*(19*b^
2*x^8 + 16*a*b*x^5 + a^2*x^2)*(b*x^3 + a)^(1/3)*(-b)^(2/3) + 6*sqrt(3)*2^(
1/3)*(5*b^2*x^7 - 4*a*b*x^4 - a^2*x)*(b*x^3 + a)^(2/3)*(-b)^(1/3) + sqrt(3
)*(71*b^3*x^9 + 111*a*b^2*x^6 + 33*a^2*b*x^3 + a^3))/(109*b^3*x^9 + 105*a*
b^2*x^6 + 3*a^2*b*x^3 - a^3)) - 2*2^(1/3)*(-b)^(1/3)*x*log(-(6*2^(1/3)*(b*
x^3 + a)^(1/3)*(-b)^(1/3)*b*x^2 + 6*(b*x^3 + a)^(2/3)*b*x + 2^(2/3)*(b*x^3
 - a)*(-b)^(2/3))/(b*x^3 - a)) + 2^(1/3)*(-b)^(1/3)*x*log((3*2^(2/3)*(5*b*
x^4 + a*x)*(b*x^3 + a)^(2/3)*(-b)^(2/3) - 2^(1/3)*(19*b^2*x^6 + 16*a*b*x^3
 + a^2)*(-b)^(1/3) + 12*(2*b^2*x^5 + a*b*x^2)*(b*x^3 + a)^(1/3))/(b^2*x^6
- 2*a*b*x^3 + a^2)) + 18*(b*x^3 + a)^(1/3))/(a*d*x)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx=-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^2+bx^5}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/x**2/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a*x**2 + b*x**5), x)/d
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx=\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^2\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx=\frac{(b\,x^3+a)^{\frac{1}{3}}+2\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{-b^2x^8+a^2x^2}dx\right)a^2x}{adx}$$

input
```
int((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x)
```

output
```
((a + b*x**3)**(1/3) + 2*int((a + b*x**3)**(1/3)/(a**2*x**2 - b**2*x**8),x
)*a**2*x)/(a*d*x)
```

3.792 $\int \frac{\sqrt[3]{a+bx^3}}{x^5(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 183

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5(ad-bdx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{4adx^4}-\frac{5b\sqrt[3]{a+bx^3}}{4a^2dx}-\frac{\sqrt[3]{2}b^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^2d}+\frac{b^{4/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^2d}-\frac{b^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^2d}$$

output

```
-1/4*(b*x^3+a)^(1/3)/a/d/x^4-5/4*b*(b*x^3+a)^(1/3)/a^2/d/x-1/3*2^(1/3)*b^(
4/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/a
^2/d+1/6*b^(4/3)*ln(-b*d*x^3+a*d)*2^(1/3)/a^2/d-1/2*b^(4/3)*ln(2^(1/3)*b^(
1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/a^2/d
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5(ad-bdx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}(a+5bx^3)}{4a^2dx^4} - \frac{\sqrt[3]{2}b^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}a^2d} - \frac{\sqrt[3]{2}b^{4/3}\log\left(-2\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}\right)}{3a^2d} + \frac{b^{4/3}\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\sqrt[3]{2}(a+bx^3)^{2/3}\right)}{3\ 2^{2/3}a^2d}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x]
```

output
```
-1/4*((a + b*x^3)^(1/3)*(a + 5*b*x^3))/(a^2*d*x^4) - (2^(1/3)*b^(4/3)*ArcT
an[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))])/(Sqrt[3]*
a^2*d) - (2^(1/3)*b^(4/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(
3*a^2*d) + (b^(4/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3
) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(2/3)*a^2*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {975, 27, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5(ad-bdx^3)}\,dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{b(3bx^3+5a)}{x^2(a-bx^3)(bx^3+a)^{2/3}}dx}{4ad} - \frac{\sqrt[3]{a+bx^3}}{4adx^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{3bx^3+5a}{x^2(a-bx^3)(bx^3+a)^{2/3}}dx}{4ad}-\frac{\sqrt[3]{a+bx^3}}{4adx^4}$$

$\Big\downarrow$ 1053

$$\frac{b\left(-\frac{\int -\frac{8a^2bx}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{a^2}-\frac{5\sqrt[3]{a+bx^3}}{ax}\right)}{4ad}-\frac{\sqrt[3]{a+bx^3}}{4adx^4}$$

$\Big\downarrow$ 27

$$\frac{b\left(8b\int \frac{x}{(a-bx^3)(bx^3+a)^{2/3}}dx-\frac{5\sqrt[3]{a+bx^3}}{ax}\right)}{4ad}-\frac{\sqrt[3]{a+bx^3}}{4adx^4}$$

$\Big\downarrow$ 992

$$\frac{b\left(8b\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}ab^{2/3}}+\frac{\log\left(a-bx^3\right)}{6\ 2^{2/3}ab^{2/3}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}ab^{2/3}}\right)-\frac{5\sqrt[3]{a+bx^3}}{ax}\right)}{4ad}-\frac{\sqrt[3]{a+bx^3}}{4adx^4}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x]
```

output
```
-1/4*(a + b*x^3)^(1/3)/(a*d*x^4) + (b*((-5*(a + b*x^3)^(1/3))/(a*x) + 8*b*
(-(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*
Sqrt[3]*a*b^(2/3))) + Log[a - b*x^3]/(6*2^(2/3)*a*b^(2/3)) - Log[2^(1/3)*b
^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(2/3)*a*b^(2/3)))))/(4*a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$\frac{\left(-15 b\,x^{3}-3 a\right) \left(b\,x^{3}+a\right)^{\frac{1}{3}}+2 x^{4} b^{\frac{4}{3}} \left(2 \arctan\left(\frac{\sqrt{3}\,\left(2^{\frac{2}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}} x\right)}{3 b^{\frac{1}{3}} x}\right) \sqrt{3}+\ln\left(\frac{b^{\frac{2}{3}} 2^{\frac{2}{3}} x^{2}+\left(b\,x^{3}+a\right)^{\frac{1}{3}} b^{\frac{1}{3}} 2^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right.\right.}{12 a^{2} d\,x^{4}}$

input
```
int((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/12*((-15*b*x^3-3*a)*(b*x^3+a)^(1/3)+2*x^4*b^(4/3)*(2*arctan(1/3*3^(1/2)*
(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)*x)/b^(1/3)/x)*3^(1/2)+ln((b^(2/3)*2^(2/3)
*x^2+(b*x^3+a)^(1/3)*b^(1/3)*2^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-2^(1/3
)*b^(1/3)*x+(b*x^3+a)^(1/3))/x))*2^(1/3))/a^2/d/x^4
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx = -\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^5+bx^8}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/x**5/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a*x**5 + b*x**8), x)/d
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx=\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^5), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx=\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx=\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^5\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx = \frac{-(b\,x^3+a)^{\frac{1}{3}}\,a+3(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3+8\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{-b^2x^8+a^2x^2}dx\right)a^2b\,x^4}{4a^2d\,x^4}$$

input
```
int((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x)
```

output
```
( - (a + b*x**3)**(1/3)*a + 3*(a + b*x**3)**(1/3)*b*x**3 + 8*int((a + b*x*
*3)**(1/3)/(a**2*x**2 - b**2*x**8),x)*a**2*b*x**4)/(4*a**2*d*x**4)
```

3.793 $\int \frac{\sqrt[3]{a+bx^3}}{x^8(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 210

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx=-\frac{\sqrt[3]{a+bx^3}}{7adx^7}-\frac{2b\sqrt[3]{a+bx^3}}{7a^2dx^4}-\frac{8b^2\sqrt[3]{a+bx^3}}{7a^3dx}-\frac{\sqrt[3]{2}b^{7/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^3d}+\frac{b^{7/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^3d}-\frac{b^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^3d}$$

output
```
-1/7*(b*x^3+a)^(1/3)/a/d/x^7-2/7*b*(b*x^3+a)^(1/3)/a^2/d/x^4-8/7*b^2*(b*x^
3+a)^(1/3)/a^3/d/x-1/3*2^(1/3)*b^(7/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(
b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/a^3/d+1/6*b^(7/3)*ln(-b*d*x^3+a*d)*2^(1/3
)/a^3/d-1/2*b^(7/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/a^3/d
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8 (ad-bdx^3)}\,dx =$$
$$-\frac{\frac{6\sqrt[3]{a+bx^3}(a^2+2abx^3+8b^2x^6)}{x^7}+14\sqrt[3]{2}\sqrt{3}b^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)+14\sqrt[3]{2}b^{7/3}\log\left(-2\sqrt[3]{b}x+2^{2/}\right.}{42a^3d}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x]
```

output
```
-1/42*((6*(a + b*x^3)^(1/3)*(a^2 + 2*a*b*x^3 + 8*b^2*x^6))/x^7 + 14*2^(1/3
)*Sqrt[3]*b^(7/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x
^3)^(1/3))] + 14*2^(1/3)*b^(7/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1
/3)] - 7*2^(1/3)*b^(7/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)
^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(a^3*d)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {975, 27, 1053, 27, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8 (ad-bdx^3)}\,dx$$
$$\Big\downarrow \ 975$$
$$\frac{\int \frac{2b(3bx^3+4a)}{x^5(a-bx^3)(bx^3+a)^{2/3}}dx}{7ad}-\frac{\sqrt[3]{a+bx^3}}{7adx^7}$$
$$\Big\downarrow \ 27$$
$$\frac{2b\int \frac{3bx^3+4a}{x^5(a-bx^3)(bx^3+a)^{2/3}}dx}{7ad}-\frac{\sqrt[3]{a+bx^3}}{7adx^7}$$

$$\Big\downarrow \text{1053}$$

$$\frac{2b\left(-\frac{\int -\frac{4ab(3bx^3+4a)}{x^2(a-bx^3)(bx^3+a)^{2/3}}dx}{4a^2}-\frac{\sqrt[3]{a+bx^3}}{ax^4}\right)}{7ad}-\frac{\sqrt[3]{a+bx^3}}{7adx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\left(\frac{b\int \frac{3bx^3+4a}{x^2(a-bx^3)(bx^3+a)^{2/3}}dx}{a}-\frac{\sqrt[3]{a+bx^3}}{ax^4}\right)}{7ad}-\frac{\sqrt[3]{a+bx^3}}{7adx^7}$$

$$\Big\downarrow \text{1053}$$

$$\frac{2b\left(\frac{b\left(-\frac{\int -\frac{7a^2bx}{(a-bx^3)(bx^3+a)^{2/3}}dx}{a^2}-\frac{4\sqrt[3]{a+bx^3}}{ax}\right)}{a}-\frac{\sqrt[3]{a+bx^3}}{ax^4}\right)}{7ad}-\frac{\sqrt[3]{a+bx^3}}{7adx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\left(\frac{b\left(7b\int \frac{x}{(a-bx^3)(bx^3+a)^{2/3}}dx-\frac{4\sqrt[3]{a+bx^3}}{ax}\right)}{a}-\frac{\sqrt[3]{a+bx^3}}{ax^4}\right)}{7ad}-\frac{\sqrt[3]{a+bx^3}}{7adx^7}$$

$$\Big\downarrow \text{992}$$

$$\frac{2b\left(\frac{b\left(7b\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}ab^{2/3}}+\frac{\log\left(a-bx^3\right)}{6\ 2^{2/3}ab^{2/3}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}ab^{2/3}}\right)-\frac{4\sqrt[3]{a+bx^3}}{ax}\right)}{a}-\frac{\sqrt[3]{a+bx^3}}{ax^4}\right)}{7ad}-\frac{\sqrt[3]{a+bx^3}}{7adx^7}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x]
```

output
```
-1/7*(a + b*x^3)^(1/3)/(a*d*x^7) + (2*b*(-((a + b*x^3)^(1/3)/(a*x^4)) + (b
*((-4*(a + b*x^3)^(1/3))/(a*x) + 7*b*(-(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/
(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]*a*b^(2/3))) + Log[a - b*x^3]/
(6*2^(2/3)*a*b^(2/3)) - Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(2
/3)*a*b^(2/3)))))/a))/(7*a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{\left(-48 b^{2} x^{6}-12 a b\, x^{3}-6 a^{2}\right) \left(b\, x^{3}+a \right)^{\frac{1}{3}}+7 x^{7} \left(2 \arctan \left(\frac{\sqrt{3}\, \left(2^{\frac{2}{3}} \left(b\, x^{3}+a \right)^{\frac{1}{3}}+b^{\frac{1}{3}} x \right)}{3 b^{\frac{1}{3}} x}\right) \sqrt{3}+\ln \left(\frac{b^{\frac{2}{3}} 2^{\frac{2}{3}} x^{2}+\left(b\, x^{3}+a \right)^{\frac{1}{3}} b^{\frac{1}{3}} 2^{\frac{1}{3}} x+\left(b\right.}{x^{2}}\right.\right.}{42 x^{7} a^{3} d}$

input
```
int((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/42*((-48*b^2*x^6-12*a*b*x^3-6*a^2)*(b*x^3+a)^(1/3)+7*x^7*(2*arctan(1/3*3
^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)*x)/b^(1/3)/x)*3^(1/2)+ln((b^(2/3)*
2^(2/3)*x^2+(b*x^3+a)^(1/3)*b^(1/3)*2^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((
-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x))*b^(7/3)*2^(1/3))/x^7/a^3/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx = -\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^8+bx^{11}}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/x**8/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a*x**8 + b*x**11), x)/d
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\,(ad-bdx^3)}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^8}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^8), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\,(ad-bdx^3)}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^8}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\,(ad-bdx^3)}\,dx=\int \frac{(b\,x^3+a)^{1/3}}{x^8\,(a\,d-b\,d\,x^3)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8(ad-bdx^3)}\,dx$$
$$= \frac{-(b\,x^3+a)^{\frac{1}{3}}\,a^2-2(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3+6(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6+14\left(\int\frac{(b\,x^3+a)^{\frac{1}{3}}}{-b^2x^8+a^2x^2}dx\right)a^2b^2x^7}{7a^3d\,x^7}$$

input
```
int((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x)
```

output
```
( - (a + b*x**3)**(1/3)*a**2 - 2*(a + b*x**3)**(1/3)*a*b*x**3 + 6*(a + b*x
**3)**(1/3)*b**2*x**6 + 14*int((a + b*x**3)**(1/3)/(a**2*x**2 - b**2*x**8)
,x)*a**2*b**2*x**7)/(7*a**3*d*x**7)
```

3.794 $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 237

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}-\frac{11b\sqrt[3]{a+bx^3}}{70a^2dx^7}-\frac{37b^2\sqrt[3]{a+bx^3}}{140a^3dx^4}-\frac{169b^3\sqrt[3]{a+bx^3}}{140a^4dx}-\frac{\sqrt[3]{2}b^{10/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^4d}+\frac{b^{10/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^4d}-\frac{b^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^4d}$$

output
```
-1/10*(b*x^3+a)^(1/3)/a/d/x^10-11/70*b*(b*x^3+a)^(1/3)/a^2/d/x^7-37/140*b^
2*(b*x^3+a)^(1/3)/a^3/d/x^4-169/140*b^3*(b*x^3+a)^(1/3)/a^4/d/x-1/3*2^(1/3
)*b^(10/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(
1/2)/a^4/d+1/6*b^(10/3)*ln(-b*d*x^3+a*d)*2^(1/3)/a^4/d-1/2*b^(10/3)*ln(2^(
1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/a^4/d
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(ad-bdx^3)}\,dx = -\frac{\frac{3\sqrt[3]{a+bx^3}(14a^3+22a^2bx^3+37ab^2x^6+169b^3x^9)}{x^{10}} + 140\sqrt[3]{2}\sqrt{3}b^{10/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right) + 140\sqrt[3]{2}b^{10/3}\,\mathrm{lo}}{420a^4d}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x]
```

output
```
-1/420*((3*(a + b*x^3)^(1/3)*(14*a^3 + 22*a^2*b*x^3 + 37*a*b^2*x^6 + 169*b
^3*x^9))/x^10 + 140*2^(1/3)*Sqrt[3]*b^(10/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b
^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] + 140*2^(1/3)*b^(10/3)*Log[-2*b^(1/
3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] - 70*2^(1/3)*b^(10/3)*Log[2*b^(2/3)*x^2
+ 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(a^4*d
)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {975, 27, 1053, 27, 1053, 25, 27, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(ad-bdx^3)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{b(9bx^3+11a)}{x^8(a-bx^3)(bx^3+a)^{2/3}}dx}{10ad} - \frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{9bx^3+11a}{x^8(a-bx^3)(bx^3+a)^{2/3}}dx}{10ad}-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

$\Big\downarrow$ 1053

$$\frac{b\left(-\frac{\int -\frac{2ab\left(33bx^3+37a\right)}{x^5\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{7a^2}-\frac{11\sqrt[3]{a+bx^3}}{7ax^7}\right)}{10ad}-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

$\Big\downarrow$ 27

$$\frac{b\left(\frac{2b\int \frac{33bx^3+37a}{x^5\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{7a}-\frac{11\sqrt[3]{a+bx^3}}{7ax^7}\right)}{10ad}-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

$\Big\downarrow$ 1053

$$\frac{b\left(\frac{2b\left(-\frac{\int -\frac{ab\left(111bx^3+169a\right)}{x^2\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{4a^2}-\frac{37\sqrt[3]{a+bx^3}}{4ax^4}\right)}{7a}-\frac{11\sqrt[3]{a+bx^3}}{7ax^7}\right)}{10ad}-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{2b\left(\frac{\int \frac{ab\left(111bx^3+169a\right)}{x^2\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{4a^2}-\frac{37\sqrt[3]{a+bx^3}}{4ax^4}\right)}{7a}-\frac{11\sqrt[3]{a+bx^3}}{7ax^7}\right)}{10ad}-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

$\Big\downarrow$ 27

$$\frac{b\left(\frac{2b\left(\frac{b\int \frac{111bx^3+169a}{x^2\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{4a}-\frac{37\sqrt[3]{a+bx^3}}{4ax^4}\right)}{7a}-\frac{11\sqrt[3]{a+bx^3}}{7ax^7}\right)}{10ad}-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

$\downarrow$ 1053

$$\frac{b\left(\frac{2b\left(\frac{b\left(-\frac{\int -\frac{280a^2bx}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{a^2}-\frac{169\sqrt[3]{a+bx^3}}{ax}\right)}{4a}-\frac{37\sqrt[3]{a+bx^3}}{4ax^4}\right)}{7a}-\frac{11\sqrt[3]{a+bx^3}}{7ax^7}\right)}{10ad}-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

$\downarrow$ 27

$$\frac{b\left(\frac{2b\left(\frac{b\left(280b\int \frac{x}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-\frac{169\sqrt[3]{a+bx^3}}{ax}\right)}{4a}-\frac{37\sqrt[3]{a+bx^3}}{4ax^4}\right)}{7a}-\frac{11\sqrt[3]{a+bx^3}}{7ax^7}\right)}{10ad}-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

$\downarrow$ 992

$$\frac{b\left(\frac{2b\left(\frac{b\left(280b\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}ab^{2/3}}+\frac{\log\left(a-bx^3\right)}{6\ 2^{2/3}ab^{2/3}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}ab^{2/3}}\right)-\frac{169\sqrt[3]{a+bx^3}}{ax}\right)}{4a}-\frac{37\sqrt[3]{a+bx^3}}{4ax^4}\right)}{7a}-\frac{11\sqrt[3]{(}}{7}\right.}{10ad}$$

$$\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x]
```

output
```
-1/10*(a + b*x^3)^(1/3)/(a*d*x^10) + (b*((-11*(a + b*x^3)^(1/3))/(7*a*x^7)
 + (2*b*((-37*(a + b*x^3)^(1/3))/(4*a*x^4) + (b*((-169*(a + b*x^3)^(1/3))/
(a*x) + 280*b*(-(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt
[3]]/(2^(2/3)*Sqrt[3]*a*b^(2/3))) + Log[a - b*x^3]/(6*2^(2/3)*a*b^(2/3)) -
 Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(2/3)*a*b^(2/3)))))/(4*a)
))/(7*a)))/(10*a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{\left(-507 b^{3} x^{9}-111 a \,b^{2} x^{6}-66 a^{2} b \,x^{3}-42 a^{3}\right) \left(b \,x^{3}+a \right)^{\frac{1}{3}}+70 x^{10} \left(2 \arctan \left(\frac{\sqrt{3}\, \left(2^{\frac{2}{3}} \left(b \,x^{3}+a \right)^{\frac{1}{3}}+b^{\frac{1}{3}} x \right)}{3 b^{\frac{1}{3}} x}\right) \sqrt{3}+\ln \left(\frac{b^{\frac{2}{3}} 2^{\frac{2}{3}} x^{2}+\left(b x\right.}{}\right.\right.}{420 x^{10} a^{4} d}$

input
```
int((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/420*((-507*b^3*x^9-111*a*b^2*x^6-66*a^2*b*x^3-42*a^3)*(b*x^3+a)^(1/3)+70
*x^10*(2*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)*x)/b^(1/3)/x)
*3^(1/2)+ln((b^(2/3)*2^(2/3)*x^2+(b*x^3+a)^(1/3)*b^(1/3)*2^(1/3)*x+(b*x^3+
a)^(2/3))/x^2)-2*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x))*b^(10/3)*2^(1
/3))/x^10/a^4/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(ad-bdx^3)}\,dx = -\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^{11}+bx^{14}}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/x**11/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a*x**11 + b*x**14), x)/d
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^11), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^11), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx=\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^{11}\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx$$
$$=\frac{-14(b\,x^3+a)^{\frac{1}{3}}\,a^3-22(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3-37(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6+111(b\,x^3+a)^{\frac{1}{3}}\,b^3x^9+280\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{-b^2x^8+a^2x^2}\right.}{140a^4d\,x^{10}}$$

input
```
int((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x)
```

output
```
( - 14*(a + b*x**3)**(1/3)*a**3 - 22*(a + b*x**3)**(1/3)*a**2*b*x**3 - 37*
(a + b*x**3)**(1/3)*a*b**2*x**6 + 111*(a + b*x**3)**(1/3)*b**3*x**9 + 280*
int((a + b*x**3)**(1/3)/(a**2*x**2 - b**2*x**8),x)*a**2*b**3*x**10)/(140*a
**4*d*x**10)
```

3.795 $\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 521

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{3ax\sqrt[3]{a+bx^3}}{5b^2d}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd} - \frac{\sqrt[3]{2}a^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}d} - \frac{a^{5/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}b^{7/3}d} - \frac{2a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b^2d\left(a+bx^3\right)^{2/3}} - \frac{a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{7/3}d} + \frac{a^{5/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{7/3}d} - \frac{\sqrt[3]{2}a^{5/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{7/3}d} + \frac{a^{5/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}b^{7/3}d}$$

output
```
-3/5*a*x*(b*x^3+a)^(1/3)/b^2/d-1/5*x^4*(b*x^3+a)^(1/3)/b/d-1/3*2^(1/3)*a^(
5/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))
*3^(1/2)/b^(7/3)/d-1/6*a^(5/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(
b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/b^(7/3)/d-2/5*a^2*x*(1+b*x^3/a)^(
2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/b^2/d/(b*x^3+a)^(2/3)-1/6*a^(5/3
)*ln(2^(2/3)-(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(7/3)/d+1/6*a^
(5/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(7/3)/d-1/3*2^(1/3)*a^(5/3)*ln(1+2^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(7/3)/d+1/12*a^(5/3)*ln(2*2^(1
/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x
^3+a)^(1/3))*2^(1/3)/b^(7/3)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 6.74 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.45

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{-4(a+bx^3)(3ax+bx^4)+7abx^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+\frac{}{(a-bx^3)\left(4a\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3}\right.\right.}}{20b^2d\,(a+bx^3)^{2/3}}$$

input
```
Integrate[(x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
(-4*(a + b*x^3)*(3*a*x + b*x^4) + 7*a*b*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1
[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a] + (48*a^4*x*AppellF1[1/3, 2/3,
 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1,
 4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*
x^3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a
]))))/(20*b^2*d*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.11, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {978, 27, 1052, 27, 1026, 779, 778, 928, 779, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$$

$$\Big\downarrow \text{978}$$

$$\frac{\int \frac{2ax^3(3bx^3+2a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{5bd}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a\int \frac{x^3(3bx^3+2a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{5bd}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow \text{1052}$$

$$\frac{2a\left(\frac{\int \frac{ab(7bx^3+3a)}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{2b^2}-\frac{3x\sqrt[3]{a+bx^3}}{2b}\right)}{5bd}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a\left(\frac{a\int \frac{7bx^3+3a}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{2b}-\frac{3x\sqrt[3]{a+bx^3}}{2b}\right)}{5bd}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow \text{1026}$$

$$\frac{2a\left(\frac{a\left(10a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-7\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx\right)}{2b}-\frac{3x\sqrt[3]{a+bx^3}}{2b}\right)}{5bd}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow \text{779}$$

$$\frac{2a\left(\frac{a\left(10a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx-\frac{7\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}\right)}{2b}-\frac{3x\sqrt[3]{a+bx^3}}{2b}\right)}{5bd}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$\downarrow$ 778

$$\frac{2a\left(\frac{a\left(10a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx-\frac{7x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2b}-\frac{3x\sqrt[3]{a+bx^3}}{2b}\right)}{5bd}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$\downarrow$ 928

$$\frac{2a\left(\frac{a\left(10a\left(\frac{\int \frac{1}{(bx^3+a)^{2/3}}dx}{2a}+\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}\right)-\frac{7x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2b}-\frac{3x\sqrt[3]{a+bx^3}}{2b}\right)}{5bd}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$\downarrow$ 779

$$\frac{2a\left(\frac{a\left(10a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a(a+bx^3)^{2/3}}\right)-\frac{7x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2b}-\frac{3x\sqrt[3]{a+bx^3}}{2b}\right)}{5bd}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow \; 778$$

$$\frac{2a\left(\frac{a\left(10a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}\right)-\frac{7x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)}{2b}-\frac{3x}{}\right.}{5bd}$$

$$\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow \; 927$$

$$\frac{2a\left(\frac{a\left(10a\left(\frac{9\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\right.}{2a\left(a+bx^3\right)^{2/3}}\right.\right.}{2b}\right.}{5bd}$$

$$\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow \; 982$$

$$2a\left(\frac{a\left(\frac{10a\left(\frac{9\left(\frac{1}{9}\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3-}}\right.}{2a^{2/3}\sqrt[3]{b}}\right.}{2b}\right.}{5bd}\right.$$

$$\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$\downarrow$ 821

$$2a\left(\frac{a\left(10a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)+\frac{1}{9}\left(\int \cdots\right.\right.}{2a^{2/3}\sqrt[3]{b}\cdots}\right.\right.}{\cdots}\right.$$

$$\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$\downarrow$ 16

$$2a\left(\frac{a\left(10a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2/}\cdots}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\cdots}{\left(bx^3+a\right)^{2/3}}\cdots}\cdots}{\cdots}\right.\cdots}{2a^{2/3}\sqrt[3]{b}}\cdots}{\cdots}\right.\right.\cdots$$

$$\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$\downarrow$ 1142

$\int -$

$$2a\left(a\left(10a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+ -}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.}{}\right.\right.\right.$$

$\Big\downarrow$ 25

$$2a\left(a\left(10a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\cdots}{3\sqrt[3]{2}\sqrt[3]{a}\cdots}\cdots\right.\right.}{\cdots}\cdots\right.\right.\right.}{\cdots}$$

↓ 27

$$2a\left(\frac{a\left(10a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int\cdots}{3\sqrt[3]{2}\sqrt[3]{a}\cdots}\cdots\right.\right.}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}$$

$$\frac{x^4\sqrt[3]{bx^3+a}}{5bd}$$

↓ 1082

$$2a\left(a\left(10a\left(9\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.\right.\right.\right.$$

↓ 217

$$2a \Bigg(a \Bigg(10a \Bigg(9 \Bigg(\frac{2}{9} \Bigg(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}\, d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}} - \frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{\ldots}\right.}{\sqrt[3]{a}\ldots}\right.}{3\ 2\ldots} \ldots$$

↓ 1103

$$2a\left(a\left(10a\left(9\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right.\right.\right.\right.\right.$$

$$\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

input
```
Int[(x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
-1/5*(x^4*(a + b*x^3)^(1/3))/(b*d) + (2*a*((-3*x*(a + b*x^3)^(1/3))/(2*b)
+ (a*((-7*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^
3)/a)])/(a + b*x^3)^(2/3) + 10*a*((x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2
F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*(a + b*x^3)^(2/3)) + (9*((2*((-((Sqr
t[3]*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt
[3]])/(2^(1/3)*a^(1/3))) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a +
b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*2^(1/
3)*a^(1/3))))/(3*2^(1/3)*a^(1/3)) - Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))
/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(2/3))))/9 + (-1/3*Log[2^(2/3) - (a^(1/3)
 + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3)) - ((Sqrt[3]*ArcTan[(1 +
 (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/a^(1/3) - Lo
g[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3
) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*a^(1/3)))/(3*2^(2/3)*a^(1/3)))/9))/(
2*a^(2/3)*b^(1/3)))))/(2*b)))/(5*b*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [F]

$$\int \frac{x^6 (b\,x^3+a)^{\frac{1}{3}}}{-bd\,x^3+ad}dx$$

input
```
int(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

output
```
int(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \text{Timed out}$$

input
```
integrate(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^6 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**6*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**6*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x^6}{bdx^3-ad}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)*x^6/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x^6}{bdx^3-ad}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)*x^6/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int \frac{x^6\,(b\,x^3+a)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

output
```
int((x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

Reduce [F]

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$
$$= \frac{-3(b\,x^3+a)^{\frac{1}{3}}\,ax-(b\,x^3+a)^{\frac{1}{3}}\,b\,x^4+3\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{-b^2x^6+a^2}dx\right)a^3+7\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}x^3}{-b^2x^6+a^2}dx\right)a^2b}{5b^2d}$$

input
```
int(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

output
```
( - 3*(a + b*x**3)**(1/3)*a*x - (a + b*x**3)**(1/3)*b*x**4 + 3*int((a + b*
x**3)**(1/3)/(a**2 - b**2*x**6),x)*a**3 + 7*int(((a + b*x**3)**(1/3)*x**3)
/(a**2 - b**2*x**6),x)*a**2*b)/(5*b**2*d)
```

3.796 $\int \frac{x^3 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 494

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{x\sqrt[3]{a+bx^3}}{2bd}-\frac{\sqrt[3]{2}a^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}d}-\frac{a^{2/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}b^{4/3}d}-\frac{ax\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2bd\left(a+bx^3\right)^{2/3}}-\frac{a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{4/3}d}+\frac{a^{2/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{4/3}d}-\frac{\sqrt[3]{2}a^{2/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{4/3}d}+\frac{a^{2/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}b^{4/3}d}$$

output
```
-1/2*x*(b*x^3+a)^(1/3)/b/d-1/3*2^(1/3)*a^(2/3)*arctan(1/3*(1-2*2^(1/3)*(a^
(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)/d-1/6*a^(2/3)*a
rctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)
*3^(1/2)/b^(4/3)/d-1/2*a*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b
*x^3/a)/b/d/(b*x^3+a)^(2/3)-1/6*a^(2/3)*ln(2^(2/3)-(a^(1/3)+b^(1/3)*x)/(b*
x^3+a)^(1/3))*2^(1/3)/b^(4/3)/d+1/6*a^(2/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*
x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/
b^(4/3)/d-1/3*2^(1/3)*a^(2/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(
1/3))/b^(4/3)/d+1/12*a^(2/3)*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^
(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(4/3)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 5.94 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.46

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$
$$= \frac{x\left(3x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+\frac{4\left(-a-bx^3+\frac{4a^3\,\text{AppellF1}\left(\cdots\right.}{\left(a-bx^3\right)\left(4a\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\left(3\,\text{Appe}\cdots\right.\right.}\right.}{b}\right.}{8d\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
(x*(3*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (
b*x^3)/a] + (4*(-a - b*x^3 + (4*a^3*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a
), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a),
(b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a]
- 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))))/b))/(8*d*(a +
 b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.11, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {978, 27, 1026, 779, 778, 928, 779, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$

$$\Big\downarrow \text{978}$$

$$\frac{\int \frac{a(3bx^3+a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{2bd}-\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{3bx^3+a}{(a-bx^3)(bx^3+a)^{2/3}}dx}{2bd}-\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow \text{1026}$$

$$\frac{a\left(4a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx-3\int \frac{1}{(bx^3+a)^{2/3}}dx\right)}{2bd}-\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow \text{779}$$

$$\frac{a\left(4a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx-\frac{3\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}\right)}{2bd}-\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow \text{778}$$

$$\frac{a\left(4a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx-\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2bd}-\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow \text{928}$$

$$\frac{a\left(4a\left(\frac{\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx}{2a}+\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}\right)-\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2bd}-\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$\Big\downarrow$ 779

$$\frac{a\left(4a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a(a+bx^3)^{2/3}}\right)-\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2bd}-\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$\Big\downarrow$ 778

$$\frac{a\left(4a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(a+bx^3)^{2/3}}\right)-\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2bd}$$

$$\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$\Big\downarrow$ 927

$$\frac{a\left(4a\left(\frac{9\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3}\right.}{2a(a+bx^3)^{2/3}}\right.\right.}{2bd}$$

$$\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$\Big\downarrow$ 982

$$a\left(4a\left(\frac{9\left(\frac{1}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}} \cdots\right.}{2a^{2/3}\sqrt[3]{b}} \cdots\right.\right. \Big/ \; 2bd$$

$$\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$\downarrow$ 821

$$a\left(4a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)+\frac{1}{9}\left(\int \frac{\cdots}{2^{2/3}\cdots} \cdots\right.\right.}{2a^{2/3}\sqrt[3]{b}} \cdots\right.\right. \Big/ \cdots$$

$$\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$\downarrow$ 16

$$a\left(4a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2/3}\ \cdots}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}+\cdots}\cdots}{\cdots}\right.\right.\cdots}{2a^{2/3}\sqrt[3]{b}\ \cdots}\right.\right.\cdots}{\cdots}$$

$$\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\downarrow\ 1142$$

$$a\left(4a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(bx^3+a)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+-\cdots \quad \int}{3\sqrt[3]{2}\sqrt[3]{\cdots}}\right.\right.}{}\right.\right.$$

$$\frac{x\sqrt[3]{bx^3+a}}{2bd}$$

$\downarrow$ 25

$$a\left(4a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(bx^3+a)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\cdots \int}{3\sqrt[3]{2}\sqrt[3]{\cdots}}\right.\right.}{\cdots}\right.\right.$$

$$\frac{x\sqrt[3]{bx^3+a}}{2bd}$$

$\downarrow$ 27

$$a\left(4a\left(9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}\ldots}{\sqrt[3]{a}\sqrt[3]{\ldots}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.\right.\right.$$

$$\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$\downarrow$ 1082

$$a\left(4a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\cdots$$

lo

$$\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$\downarrow$ 217

$$a\left(4a\left(9\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\ldots\right.}{\sqrt[3]{a+\ldots}}\right.}{3\ 2^{2/3}a\ldots}\right.\right.\right.\right.\right.$$

$$\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$\downarrow$ 1103

$$a\left(4a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\right.}{2a^{2/3}\sqrt[3]{b}}$$

$$\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

input
```
Int[(x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
-1/2*(x*(a + b*x^3)^(1/3))/(b*d) + (a*((-3*x*(1 + (b*x^3)/a)^(2/3)*Hyperge
ometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) + 4*a*((x*(1 +
(b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*(a +
 b*x^3)^(2/3)) + (9*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(
1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*a^(1/3))) + Log[1 + (2^(2/3
)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)
*x))/(a + b*x^3)^(1/3)]/(2*2^(1/3)*a^(1/3))))/(3*2^(1/3)*a^(1/3)) - Log[1 +
 (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(2/3))))/
9 + (-1/3*Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(2^(2/3)*
a^(2/3)) - ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^
3)^(1/3))/Sqrt[3]])/a^(1/3) - Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a +
 b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*a^(1
/3)))/(3*2^(2/3)*a^(1/3)))/9))/(2*a^(2/3)*b^(1/3)))))/(2*b*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [F]

$$\int \frac{x^3 (b\,x^3+a)^{\frac{1}{3}}}{-bd\,x^3+ad}dx$$

input
```
int(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

output
```
int(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^3 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**3*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**3*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x^3}{bdx^3-ad}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)*x^3/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}x^3}{bdx^3-ad}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)*x^3/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\int \frac{x^3\,(b\,x^3+a)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

output
```
int((x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

Reduce [F]

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\frac{-(b\,x^3+a)^{\frac{1}{3}}\,x+\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{-b^2x^6+a^2}dx\right)a^2+3\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}x^3}{-b^2x^6+a^2}dx\right)ab}{2bd}$$

input
```
int(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

output
```
( - (a + b*x**3)**(1/3)*x + int((a + b*x**3)**(1/3)/(a**2 - b**2*x**6),x)*
a**2 + 3*int(((a + b*x**3)**(1/3)*x**3)/(a**2 - b**2*x**6),x)*a*b)/(2*b*d)
```

3.797 $\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 416

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}d} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}d} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d} + \frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d} - \frac{\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d} + \frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}$$

output
```
-1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*
3^(1/2))*3^(1/2)/a^(1/3)/b^(1/3)/d-1/6*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/a^(1/3)/b^(1/3)/d-1/6*ln(
2^(2/3)-(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/3)/d+1/6
*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/
3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/3)/d-1/3*2^(1/3)*ln(1+2^(1/3)*
(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(1/3)/b^(1/3)/d+1/12*ln(2*2^(1/3)+(
a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)
^(1/3))*2^(1/3)/a^(1/3)/b^(1/3)/d
```

Mathematica [A] (verified)

Time = 2.66 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3} dx$$
$$= \frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{-2\sqrt[3]{2}\sqrt[3]{a}-2\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{\sqrt[3]{2}\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}}\right)-4\log\left(\sqrt[3]{2}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(a*d - b*d*x^3),x]
```

output
```
(4*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x^3)^(1/3))/(-2*2^(1/3)*a^(1/3) - 2*2^(1
/3)*b^(1/3)*x + (a + b*x^3)^(1/3))] + 2*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x^3
)^(1/3))/(2^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3))] - 4*Lo
g[2^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3)] - 2*Log[-(2^(1/
3)*a^(1/3)) - 2^(1/3)*b^(1/3)*x + 2*(a + b*x^3)^(1/3)] + Log[2^(2/3)*a^(2/
3) + 2^(2/3)*b^(2/3)*x^2 + 2*2^(1/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 4*(a +
b*x^3)^(2/3) + 2*2^(1/3)*a^(1/3)*(2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3))]
 + 2*Log[2^(2/3)*a^(2/3) + 2^(2/3)*b^(2/3)*x^2 - 2^(1/3)*b^(1/3)*x*(a + b*x
^3)^(1/3) + (a + b*x^3)^(2/3) + a^(1/3)*(2*2^(2/3)*b^(1/3)*x - 2^(1/3)*(a
 + b*x^3)^(1/3))])/(6*2^(2/3)*a^(1/3)*b^(1/3)*d)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3} dx$$
$$\Big\downarrow \text{ 927}$$

$$\frac{9\sqrt[3]{a}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{\sqrt[3]{b}d}$$

$\downarrow$ 982

$$\frac{9\sqrt[3]{a}\left(\frac{1}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+}}$$

$$\sqrt[3]{b}d$$

$\downarrow$ 821

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)+\frac{1}{9}\left(\int -\right.\right.$$

$$\sqrt[3]{b}d$$

$\downarrow$ 16

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2}}{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}}{\left(bx^3+a\right)^{2/3}}\right.\right.$$

$$\sqrt[3]{b}d$$

$\downarrow$ 1142

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{\int -\frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+}{\sqrt[3]{a}\sqrt[3]{bx}}}{2\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.$$

$\downarrow$ 25

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{}}{\sqrt[3]{a}\sqrt[3]{bx^3}}}{2\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.$$

$\downarrow$ 27

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+ \cdots}{\sqrt[3]{a}\sqrt[3]{b \cdots}}}{3\sqrt[3]{2}\sqrt[3]{a}} \cdots\right.\right.$$

$$\downarrow \; 1082$$

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log \cdots}{\cdots}\right.\right.$$

$$\downarrow \; 217$$

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\ldots\right.}{\sqrt[3]{a+\ldots}}\right.}{3\ 2^{2/3}a^{\ldots}}\right.\right.$$

$\downarrow$ 1103

$$\frac{9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\ldots\right)}{\sqrt[3]{b}d}$$

input
```
Int[(a + b*x^3)^(1/3)/(a*d - b*d*x^3),x]
```

output
```
(9*a^(1/3)*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/
(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*a^(1/3))) + Log[1 + (2^(2/3)*(a^(1/3
) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a +
 b*x^3)^(1/3)]/(2*2^(1/3)*a^(1/3)))/(3*2^(1/3)*a^(1/3)) - Log[1 + (2^(1/3)
*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(2/3))))/9 + (-1/3
*Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3))
- ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))
/Sqrt[3]])/a^(1/3) - Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(
2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*a^(1/3)))/(3*
2^(2/3)*a^(1/3)))/9))/(b^(1/3)*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{-bd\,x^3+ad}dx$$

input
```
int((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

output
```
int((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{bdx^3-ad}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{bdx^3-ad}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(a*d - b*d*x^3),x)
```

output
```
int((a + b*x^3)^(1/3)/(a*d - b*d*x^3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{-b\,x^3+a}dx}{d}$$

input
```
int((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

output
```
int((a + b*x**3)**(1/3)/(a - b*x**3),x)/d
```

3.798 $\int \frac{\sqrt[3]{a+bx^3}}{x^3(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 496

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3(ad-bdx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{2adx^2} - \frac{\sqrt[3]{2}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}d} - \frac{b^{2/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a^{4/3}d} + \frac{bx\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2ad(a+bx^3)^{2/3}} - \frac{b^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{4/3}d} + \frac{b^{2/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{4/3}d} - \frac{\sqrt[3]{2}b^{2/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{4/3}d} + \frac{b^{2/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{4/3}d}$$

output
```
-1/2*(b*x^3+a)^(1/3)/a/d/x^2-1/3*2^(1/3)*b^(2/3)*arctan(1/3*(1-2*2^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/a^(4/3)/d-1/6*b^(2/3)
*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/
3)*3^(1/2)/a^(4/3)/d+1/2*b*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],
-b*x^3/a)/a/d/(b*x^3+a)^(2/3)-1/6*b^(2/3)*ln(2^(2/3)-(a^(1/3)+b^(1/3)*x)/(
b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/d+1/6*b^(2/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3
)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3
)/a^(4/3)/d-1/3*2^(1/3)*b^(2/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)
^(1/3))/a^(4/3)/d+1/12*b^(2/3)*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a
)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx$$
$$= \frac{-4a(a+bx^3)+b^2x^6\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+\frac{48a^3bx^3 \cdots}{(a-bx^3)\left(4a\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3 \cdots\right.}}{8a^2dx^2\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)),x]
```

output
```
(-4*a*(a + b*x^3) + b^2*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/
3, -((b*x^3)/a), (b*x^3)/a] + (48*a^3*b*x^3*AppellF1[1/3, 2/3, 1, 4/3, -((
b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x
^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*
x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]))))/(8*a^2
*d*x^2*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.11, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {975, 27, 1026, 779, 778, 928, 779, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{b(bx^3+3a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{2ad}-\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{bx^3+3a}{(a-bx^3)(bx^3+a)^{2/3}}dx}{2ad}-\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{1026}$$

$$\frac{b\left(4a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx-\int \frac{1}{(bx^3+a)^{2/3}}dx\right)}{2ad}-\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{779}$$

$$\frac{b\left(4a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx-\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}\right)}{2ad}-\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{778}$$

$$\frac{b\left(4a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx-\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2ad}-\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{928}$$

$$\frac{b\left(4a\left(\frac{\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx}{2a}+\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}\right)-\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2ad}-\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{ 779}$$

$$\frac{b\left(4a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a(a+bx^3)^{2/3}}\right)-\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2ad}-\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{b\left(4a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(a+bx^3)^{2/3}}\right)-\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2ad}-\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{ 927}$$

$$\frac{b\left(4a\left(\frac{9\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3}\right.}{2a(a+bx^3)^{2/3}}\right.\right.}{2ad}$$

$$\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{ 982}$$

$$b\left(4a\left(\frac{9\left(\frac{1}{9}\int \frac{\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}}{}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}\right.}{2a^{2/3}\sqrt[3]{b}}\right.\right.}{2ad}$$

$$\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$\big\downarrow$ 821

$$b\left(4a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)+\frac{1}{9}\left(\int \frac{}{2^{2/3}\cdots}\right.\right.}{2a^{2/3}\sqrt[3]{b}}\right.\right.$$

$$\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$\big\downarrow$ 16

$$b\left(4a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2/3}\ldots}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}+\ldots}\ldots}{\ldots}\right.\right.}{2a^{2/3}\sqrt[3]{b}}\ldots\right.\right.}{\ldots}$$

$$\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$\downarrow$ 1142

$$b\left(4a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(bx^3+a)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{(bx^3+a)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\cdots}{3\sqrt[3]{2}\sqrt[3]{\cdots}}\right.\right.}{\cdots}\right.\right.$$

$$\frac{\sqrt[3]{bx^3+a}}{2adx^2}$$

$\downarrow$ 25

$$\frac{b\left(4a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(bx^3+a)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{(bx^3+a)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\cdots\int}{3\sqrt[3]{2}\sqrt[3]{\cdots}}\cdots\right)\cdots\right)}{\cdots}\cdots\right)\cdots\right)}{\cdots}$$

$$\frac{\sqrt[3]{bx^3+a}}{2adx^2}$$

$\downarrow$ 27

$$b\left(4a\left(9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}\ldots}{\sqrt[3]{a}\sqrt[3]{\ldots}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.\right.\right.$$

$$\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$\downarrow$ 1082

$$b\left(4a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}} - \cdots\right.\right.}{\cdots}\right.\right.$$

$$\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \; 217$$

$$b\left(4a\left(9\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\cdots\right.}{\sqrt[3]{a+\cdots}}\right.}{3\cdot 2^{2/3}a\cdots}\right.\right.\right.\right.\right.$$

$$\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$\downarrow$ 1103

$$b\left(4a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\cdots}{2a^{2/3}\sqrt[3]{b}\cdots}\right.\right.\right.$$

$$\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)),x]
```

output
```
-1/2*(a + b*x^3)^(1/3)/(a*d*x^2) + (b*(-((x*(1 + (b*x^3)/a)^(2/3)*Hypergeo
metric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3)) + 4*a*((x*(1 +
(b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*(a +
 b*x^3)^(2/3)) + (9*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(
1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*a^(1/3))) + Log[1 + (2^(2/3
)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)
*x))/(a + b*x^3)^(1/3)]/(2*2^(1/3)*a^(1/3))))/(3*2^(1/3)*a^(1/3)) - Log[1 +
 (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(2/3))))/
9 + (-1/3*Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(2^(2/3)*
a^(2/3)) - ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^
3)^(1/3))/Sqrt[3]])/a^(1/3) - Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a +
 b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*a^(1
/3)))/(3*2^(2/3)*a^(1/3)))/9))/(2*a^(2/3)*b^(1/3)))))/(2*a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}}{x^{3}\left(-bd\,x^{3}+ad\right)}dx$$

input
```
int((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x)
```

output
```
int((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3(ad-bdx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3(ad-bdx^3)}\,dx = -\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^3+bx^6}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/x**3/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a*x**3 + b*x**6), x)/d
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx = \int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx = \int \frac{\left(b\,x^3+a\right)^{1/3}}{x^3\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx = \frac{\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{-b\,x^6+a\,x^3}dx}{d}$$

input
```
int((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x)
```

output
```
int((a + b*x**3)**(1/3)/(a*x**3 - b*x**6),x)/d
```

3.799 $\int \frac{\sqrt[3]{a+bx^3}}{x^6(ad-bdx^3)} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 523

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6 (ad-bdx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{5adx^5} - \frac{3b\sqrt[3]{a+bx^3}}{5a^2dx^2} - \frac{\sqrt[3]{2}b^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{7/3}d} - \frac{b^{5/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a^{7/3}d} + \frac{2b^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5a^2d(a+bx^3)^{2/3}} - \frac{b^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{7/3}d} + \frac{b^{5/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{7/3}d} - \frac{\sqrt[3]{2}b^{5/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{7/3}d} + \frac{b^{5/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{7/3}d}$$

output
```
-1/5*(b*x^3+a)^(1/3)/a/d/x^5-3/5*b*(b*x^3+a)^(1/3)/a^2/d/x^2-1/3*2^(1/3)*b
^(5/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2
))*3^(1/2)/a^(7/3)/d-1/6*b^(5/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)
/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/a^(7/3)/d+2/5*b^2*x*(1+b*x^3/a)
^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/a^2/d/(b*x^3+a)^(2/3)-1/6*b^(5
/3)*ln(2^(2/3)-(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/d+1/6*
b^(5/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3
)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/d-1/3*2^(1/3)*b^(5/3)*ln(1+2
^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(7/3)/d+1/12*b^(5/3)*ln(2*2^
(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b
*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.15 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx$$

$$= \frac{-\frac{4\left(a^2+4abx^3+3b^2x^6\right)}{a^2x^5}+\frac{3b^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{a^3}+\frac{112b^2x\,\text{AppellF1}\left(\ldots\right.}{\left(a-bx^3\right)\left(4a\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\left(3\,\text{Appell}\ldots\right.\right.}}{20d\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)),x]
```

output
```
((-4*(a^2 + 4*a*b*x^3 + 3*b^2*x^6))/(a^2*x^5) + (3*b^3*x^4*(1 + (b*x^3)/a)
^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])/a^3 + (112*b^2
*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*
AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/
3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -(
(b*x^3)/a), (b*x^3)/a]))))/(20*d*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.11, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {975, 27, 1053, 25, 27, 1026, 779, 778, 928, 779, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6 \left(ad-bdx^3\right)} \, dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{2b(2bx^3+3a)}{x^3(a-bx^3)(bx^3+a)^{2/3}}dx}{5ad}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\int \frac{2bx^3+3a}{x^3(a-bx^3)(bx^3+a)^{2/3}}dx}{5ad}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \text{1053}$$

$$\frac{2b\left(-\frac{\int -\frac{ab\left(3bx^3+7a\right)}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{2a^2}-\frac{3\sqrt[3]{a+bx^3}}{2ax^2}\right)}{5ad}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b\left(\frac{\int \frac{ab\left(3bx^3+7a\right)}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{2a^2}-\frac{3\sqrt[3]{a+bx^3}}{2ax^2}\right)}{5ad}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\left(\frac{b\int \frac{3bx^3+7a}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{2a}-\frac{3\sqrt[3]{a+bx^3}}{2ax^2}\right)}{5ad}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \text{1026}$$

$$\frac{2b\left(\frac{b\left(10a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx-3\int \frac{1}{(bx^3+a)^{2/3}}dx\right)}{2a}-\frac{3\sqrt[3]{a+bx^3}}{2ax^2}\right)}{5ad}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$\downarrow$ 779

$$\frac{2b\left(\frac{b\left(10a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx-\frac{3\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}\right)}{2a}-\frac{3\sqrt[3]{a+bx^3}}{2ax^2}\right)}{5ad}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$\downarrow$ 778

$$\frac{2b\left(\frac{b\left(10a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx-\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2a}-\frac{3\sqrt[3]{a+bx^3}}{2ax^2}\right)}{5ad}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$\downarrow$ 928

$$\frac{2b\left(\frac{b\left(10a\left(\frac{\int \frac{1}{(bx^3+a)^{2/3}}dx}{2a}+\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}\right)-\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2a}-\frac{3\sqrt[3]{a+bx^3}}{2ax^2}\right)}{5ad}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$\downarrow$ 779

$$\frac{2b\left(\frac{b\left(10a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a\left(a+bx^3\right)^{2/3}}\right)-\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)}{2a}-\frac{3\sqrt[3]{a+bx^3}}{2ax^2}\right)}{5ad}-$$

$$\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$\downarrow$ 778

$$\frac{2b\left(\frac{b\left(10a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}\right)-\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)}{2a}-\frac{3\sqrt[3]{}}{}\right.}{5ad}$$

$$\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$\downarrow$ 927

$$\frac{2b\left(\frac{b\left(10a\left(\frac{9\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\right.}{2a\left(a+bx^3\right)^{2/3}}\right.\right.}{2a}\right.}{5ad}$$

$$\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$\downarrow$ 982

$$2b\left(\frac{b\left(10a\left(\frac{9\left(\frac{1}{9}\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+}}\right.}{2a^{2/3}\sqrt[3]{b}}\right.}{2a}\right.}{5ad}\right.$$

$$\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$\downarrow$ 821

$$2b\left(\frac{b\left(10a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)+\frac{1}{9}\left(\int -\frac{\cdots}{2^{\cdots}\cdots}\cdots\right.\right.}{2a^{2/3}\sqrt[3]{b}}\cdots}{\cdots}\cdots}{\cdots}\cdots$$

$$\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$\downarrow$ 16

$$
2b\left(\frac{b\left(10a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2/\cdots}}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^{2\cdots}}{\left(bx^3+a\right)^{2/3}}\cdots}}{\cdots}\cdots\right.\right)}{2a^{2/3}\sqrt[3]{b}}\cdots\right.\right)}{\cdots}\cdots\right.}{\cdots}
$$

$$
\frac{\sqrt[3]{a+bx^3}}{5adx^5}
$$

$$\big\downarrow \; 1142$$

$$2b\left(b\left(10a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\cdots\int-\cdots}{3\sqrt[3]{2}\sqrt[3]{a}\cdots}\cdots\right.\right.}{\cdots}\right.\right.\right.$$

$\Big\downarrow$ 25

$$2b\left(\frac{b\left(10a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\cdots}{3\sqrt[3]{2}\sqrt[3]{a}\cdots}\cdots\right.\right.}{\cdots}\right.\right.}{\cdots}\cdots\right.$$

$\Big\downarrow$ 27

$$2b\left(b\left(10a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.}{}\right.\right.\right.$$

$$\frac{\sqrt[3]{bx^3+a}}{5adx^5}$$

↓ 1082

$$2b\left(b\left(10a\left(9\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}\,d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}\,d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.\right.\right.\right.$$

↓ 217

$$2b\left(b\left(10a\left(9\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{\cdots}\right.}{\sqrt[3]{a}\cdots}\right.}{3\ 2^{2\cdots}}\cdots\right.\right.\right.\right.\right.$$

↓ 1103

$$2b\left(\frac{b\left(\frac{10a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\cdots}{2a^{2/3}\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)),x]
```

output
```
-1/5*(a + b*x^3)^(1/3)/(a*d*x^5) + (2*b*((-3*(a + b*x^3)^(1/3))/(2*a*x^2)
+ (b*((-3*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^
3)/a)])/(a + b*x^3)^(2/3) + 10*a*((x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2
F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*(a + b*x^3)^(2/3)) + (9*((2*((-((Sqr
t[3]*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt
[3]])/(2^(1/3)*a^(1/3))) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a +
b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*2^(1/
3)*a^(1/3))))/(3*2^(1/3)*a^(1/3)) - Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))
/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(2/3))))/9 + (-1/3*Log[2^(2/3) - (a^(1/3)
 + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3)) - ((Sqrt[3]*ArcTan[(1 +
 (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/a^(1/3) - Lo
g[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3
) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*a^(1/3)))/(3*2^(2/3)*a^(1/3)))/9))/(
2*a^(2/3)*b^(1/3))))))/(2*a)))/(5*a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{x^6\,(-bd\,x^3+ad)}dx$$

input
```
int((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x)
```

output
```
int((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\,(ad-bdx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\,(ad-bdx^3)}\,dx = -\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^6+bx^9}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/x**6/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a*x**6 + b*x**9), x)/d
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\,(ad-bdx^3)}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\,(ad-bdx^3)}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\,(ad-bdx^3)}\,dx=\int \frac{(b\,x^3+a)^{1/3}}{x^6\,(a\,d-b\,d\,x^3)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx = \frac{\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{-b\,x^9+a\,x^6}dx}{d}$$

input
```
int((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x)
```

output
```
int((a + b*x**3)**(1/3)/(a*x**6 - b*x**9),x)/d
```

3.800 $\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 223

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{a^3(a+bx^3)^{2/3}}{2b^4d}-\frac{a^2(a+bx^3)^{5/3}}{5b^4d}+\frac{a(a+bx^3)^{8/3}}{8b^4d} -\frac{(a+bx^3)^{11/3}}{11b^4d}-\frac{2^{2/3}a^{11/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^4d} +\frac{a^{11/3}\log(a-bx^3)}{3\sqrt[3]{2}b^4d}-\frac{a^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^4d}$$

output
```
-1/2*a^3*(b*x^3+a)^(2/3)/b^4/d-1/5*a^2*(b*x^3+a)^(5/3)/b^4/d+1/8*a*(b*x^3+
a)^(8/3)/b^4/d-1/11*(b*x^3+a)^(11/3)/b^4/d-1/3*2^(2/3)*a^(11/3)*arctan(1/3
*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/b^4/d+1/6*a^(1
1/3)*ln(-b*x^3+a)*2^(2/3)/b^4/d-1/2*a^(11/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^
(1/3))*2^(2/3)/b^4/d
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{3(a+bx^3)^{2/3}(293a^3+98a^2bx^3+65ab^2x^6+40b^3x^9)+440\,2^{2/3}\sqrt{3}a^{11/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+44}{132}$$

input
```
Integrate[(x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/1320*(3*(a + b*x^3)^(2/3)*(293*a^3 + 98*a^2*b*x^3 + 65*a*b^2*x^6 + 40*b
^3*x^9) + 440*2^(2/3)*Sqrt[3]*a^(11/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1
/3))/a^(1/3))/Sqrt[3]] + 440*2^(2/3)*a^(11/3)*Log[-2*a^(1/3) + 2^(2/3)*(a
+ b*x^3)^(1/3)] - 220*2^(2/3)*a^(11/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a
+ b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^4*d)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {948, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^9(bx^3+a)^{2/3}}{d(a-bx^3)}dx^3$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^9\left(bx^3+a\right)^{2/3}}{a-bx^3}dx^3}{3d}$$

$$\Big\downarrow \; 99$$

$$\frac{\int \left(\frac{\left(bx^3+a\right)^{2/3}a^3}{b^3\left(a-bx^3\right)}-\frac{\left(bx^3+a\right)^{2/3}a^2}{b^3}+\frac{\left(bx^3+a\right)^{5/3}a}{b^3}-\frac{\left(bx^3+a\right)^{8/3}}{b^3}\right)dx^3}{3d}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{2^{2/3}\sqrt{3}a^{11/3}\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^4}+\frac{a^{11/3}\log\left(a-bx^3\right)}{\sqrt[3]{2}b^4}-\frac{3a^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^4}-\frac{3a^3\left(a+bx^3\right)^{2/3}}{2b^4}-\frac{3a^2(\cdots}{\cdots}}{3d}$$

input
```
Int[(x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
((-3*a^3*(a + b*x^3)^(2/3))/(2*b^4) - (3*a^2*(a + b*x^3)^(5/3))/(5*b^4) +
(3*a*(a + b*x^3)^(8/3))/(8*b^4) - (3*(a + b*x^3)^(11/3))/(11*b^4) - (2^(2/
3)*Sqrt[3]*a^(11/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*
a^(1/3))])/b^4 + (a^(11/3)*Log[a - b*x^3])/(2^(1/3)*b^4) - (3*a^(11/3)*Log
[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^4))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$\frac{-220\, 2^{\frac{2}{3}} \left(2 \arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}} \left(b\, x^{3}+a\right)^{\frac{1}{3}}\right) \sqrt{3}}{3 a^{\frac{1}{3}}}\right) \sqrt{3}+2 \ln\left(\left(b\, x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}} a^{\frac{1}{3}}\right)-\ln\left(\left(b\, x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}} a^{\frac{1}{3}} \left(b\, x^{3}+a\right)^{\frac{1}{3}}+2^{\frac{2}{3}} a^{\frac{2}{3}}\right.\right.}{1320 b^{4} d}$

input
```
int(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/1320*(-220*2^(2/3)*(2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/
2)/a^(1/3))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))-ln((b*x^3+a)^(2/
3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3)))*a^(11/3)-3*(b*x^3+a)^
(2/3)*(40*b^3*x^9+65*a*b^2*x^6+98*a^2*b*x^3+293*a^3))/b^4/d
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.94

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{440 \cdot 4^{\frac{1}{3}} \sqrt{3} (-a^2)^{\frac{1}{3}} a^3 \arctan\left(\frac{4^{\frac{1}{3}} \sqrt{3} (bx^3+a)^{\frac{1}{3}} (-a^2)^{\frac{1}{3}} - \sqrt{3} a}{3\, a}\right) + 220 \cdot 4^{\frac{1}{3}} (-a^2)^{\frac{1}{3}} a^3 \log\left(4^{\frac{2}{3}} (bx^3+a)^{\frac{1}{3}} (-a^2)^{\frac{2}{3}} + \right.}{}$$

input
```
integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/1320*(440*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a^3*arctan(1/3*(4^(1/3)*sqrt(3)*
(b*x^3 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 220*4^(1/3)*(-a^2)^(1/3)*
a^3*log(4^(2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2
*4^(1/3)*(-a^2)^(1/3)*a) - 440*4^(1/3)*(-a^2)^(1/3)*a^3*log(-4^(2/3)*(-a^2
)^(2/3) + 2*(b*x^3 + a)^(1/3)*a) + 3*(40*b^3*x^9 + 65*a*b^2*x^6 + 98*a^2*b
*x^3 + 293*a^3)*(b*x^3 + a)^(2/3))/(b^4*d)
```

Sympy [F]

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^{11}(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**11*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**11*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.82

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\frac{440\sqrt{3}2^{\frac{2}{3}}a^{\frac{11}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d}-\frac{220\cdot 2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}+\frac{440\cdot 2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\right.}{d}}{1320\,b^4}$$

input
```
integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-1/1320*(440*sqrt(3)*2^(2/3)*a^(11/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*
a^(1/3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 220*2^(2/3)*a^(11/3)*log(2^(2/
3)*a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 44
0*2^(2/3)*a^(11/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*(40*(b*
x^3 + a)^(11/3) - 55*(b*x^3 + a)^(8/3)*a + 88*(b*x^3 + a)^(5/3)*a^2 + 220*
(b*x^3 + a)^(2/3)*a^3)/d)/b^4
```

Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.96

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\sqrt{3}2^{\frac{2}{3}}a^{\frac{11}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,b^4d} + \frac{2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,b^4d} - \frac{2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,b^4d} - \frac{40\,(bx^3+a)^{\frac{11}{3}}b^{40}d^{10}-55\,(bx^3+a)^{\frac{8}{3}}ab^{40}d^{10}+88\,(bx^3+a)^{\frac{5}{3}}a^2b^{40}d^{10}+220\,(bx^3+a)^{\frac{2}{3}}a^3b^{40}d^{10}}{440\,b^{44}d^{11}}$$

input
```
integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*2^(2/3)*a^(11/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
+ 2*(b*x^3 + a)^(1/3))/a^(1/3))/(b^4*d) + 1/6*2^(2/3)*a^(11/3)*log(2^(2/3)
*a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b^4*d)
- 1/3*2^(2/3)*a^(11/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b^4
*d) - 1/440*(40*(b*x^3 + a)^(11/3)*b^40*d^10 - 55*(b*x^3 + a)^(8/3)*a*b^40
*d^10 + 88*(b*x^3 + a)^(5/3)*a^2*b^40*d^10 + 220*(b*x^3 + a)^(2/3)*a^3*b^4
0*d^10)/(b^44*d^11)
```

Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{a\,(b\,x^3+a)^{8/3}}{8\,b^4\,d} - \frac{a^3\,(b\,x^3+a)^{2/3}}{2\,b^4\,d} - \frac{a^2\,(b\,x^3+a)^{5/3}}{5\,b^4\,d} - \frac{(b\,x^3+a)^{11/3}}{11\,b^4\,d} + \frac{4^{1/3}\,(-a)^{11/3}\,\ln\left(4\,a^8\,(b\,x^3+a)^{1/3}+4\,2^{1/3}\,(-a)^{25/3}\right)}{3\,b^4\,d} - \frac{4^{1/3}\,(-a)^{11/3}\,\ln\left(\frac{4\,a^8\,(b\,x^3+a)^{1/3}}{b^8\,d^2}+\frac{2\,4^{2/3}\,(-a)^{25/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^8\,d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^4\,d} + \frac{4^{1/3}\,(-a)^{11/3}\,\ln\left(\frac{4\,a^8\,(b\,x^3+a)^{1/3}}{b^8\,d^2}+\frac{18\,4^{2/3}\,(-a)^{25/3}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2}{b^8\,d^2}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^4\,d}$$

input
```
int((x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

output
```
(a*(a + b*x^3)^(8/3))/(8*b^4*d) - (a^3*(a + b*x^3)^(2/3))/(2*b^4*d) - (a^2
*(a + b*x^3)^(5/3))/(5*b^4*d) - (a + b*x^3)^(11/3)/(11*b^4*d) + (4^(1/3)*(
-a)^(11/3)*log(4*a^8*(a + b*x^3)^(1/3) + 4*2^(1/3)*(-a)^(25/3)))/(3*b^4*d)
 - (4^(1/3)*(-a)^(11/3)*log((4*a^8*(a + b*x^3)^(1/3))/(b^8*d^2) + (2*4^(2/
3)*(-a)^(25/3)*((3^(1/2)*1i)/2 + 1/2)^2)/(b^8*d^2))*((3^(1/2)*1i)/2 + 1/2)
)/(3*b^4*d) + (4^(1/3)*(-a)^(11/3)*log((4*a^8*(a + b*x^3)^(1/3))/(b^8*d^2)
 + (18*4^(2/3)*(-a)^(25/3)*((3^(1/2)*1i)/6 - 1/6)^2)/(b^8*d^2))*((3^(1/2)*
1i)/6 - 1/6))/(b^4*d)
```

Reduce [F]

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{147(b\,x^3+a)^{\frac{2}{3}}\,a^3 - 98(b\,x^3+a)^{\frac{2}{3}}\,a^2b\,x^3 - 65(b\,x^3+a)^{\frac{2}{3}}\,a\,b^2x^6 - 40(b\,x^3+a)^{\frac{2}{3}}\,b^3x}{440b^4d}$$

input
```
int(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

output
```
(147*(a + b*x**3)**(2/3)*a**3 - 98*(a + b*x**3)**(2/3)*a**2*b*x**3 - 65*(a
 + b*x**3)**(2/3)*a*b**2*x**6 - 40*(a + b*x**3)**(2/3)*b**3*x**9 + 880*int
(((a + b*x**3)**(2/3)*x**5)/(a**2 - b**2*x**6),x)*a**3*b**2)/(440*b**4*d)
```

3.801 $\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 177

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{a^2(a+bx^3)^{2/3}}{2b^3d}-\frac{(a+bx^3)^{8/3}}{8b^3d}-\frac{2^{2/3}a^{8/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^3d}+\frac{a^{8/3}\log(a-bx^3)}{3\sqrt[3]{2}b^3d}-\frac{a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^3d}$$

output
```
-1/2*a^2*(b*x^3+a)^(2/3)/b^3/d-1/8*(b*x^3+a)^(8/3)/b^3/d-1/3*2^(2/3)*a^(8/
3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/b
^3/d+1/6*a^(8/3)*ln(-b*x^3+a)*2^(2/3)/b^3/d-1/2*a^(8/3)*ln(2^(1/3)*a^(1/3)
-(b*x^3+a)^(1/3))*2^(2/3)/b^3/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.19

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{15a^2(a+bx^3)^{2/3}+6abx^3(a+bx^3)^{2/3}+3b^2x^6(a+bx^3)^{2/3}+8\,2^{2/3}\sqrt{3}a^{8/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\cdots}{24b^3\cdots}$$

input
```
Integrate[(x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/24*(15*a^2*(a + b*x^3)^(2/3) + 6*a*b*x^3*(a + b*x^3)^(2/3) + 3*b^2*x^6*
(a + b*x^3)^(2/3) + 8*2^(2/3)*Sqrt[3]*a^(8/3)*ArcTan[(1 + (2^(2/3)*(a + b*
x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 8*2^(2/3)*a^(8/3)*Log[-2*a^(1/3) + 2^(2/3)
*(a + b*x^3)^(1/3)] - 4*2^(2/3)*a^(8/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a
 + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^3*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {948, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6(bx^3+a)^{2/3}}{d(a-bx^3)}dx^3$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^6\left(bx^3+a\right)^{2/3}}{a-bx^3}dx^3}{3d}$$

$\Big\downarrow$ 99

$$\frac{\int \left(\frac{a^2\left(bx^3+a\right)^{2/3}}{b^2\left(a-bx^3\right)}-\frac{\left(bx^3+a\right)^{5/3}}{b^2}\right)dx^3}{3d}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{2^{2/3}\sqrt{3}a^{8/3}\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^3}+\frac{a^{8/3}\log\left(a-bx^3\right)}{\sqrt[3]{2}b^3}-\frac{3a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^3}-\frac{3a^2\left(a+bx^3\right)^{2/3}}{2b^3}-\frac{3(a+bx^3}{8b^3}}{3d}$$

input
```
Int[(x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
((-3*a^2*(a + b*x^3)^(2/3))/(2*b^3) - (3*(a + b*x^3)^(8/3))/(8*b^3) - (2^(
2/3)*Sqrt[3]*a^(8/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]
*a^(1/3))])/b^3 + (a^(8/3)*Log[a - b*x^3])/(2^(1/3)*b^3) - (3*a^(8/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^3))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{-4\,2^{\frac{2}{3}}\left(2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)-\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{24b^{3}d}$

input
```
int(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-4*2^(2/3)*(2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2)/a
^(1/3))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))-ln((b*x^3+a)^(2/3)+2
^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3)))*a^(8/3)-3*(b*x^3+a)^(2/3)
*(b^2*x^6+2*a*b*x^3+5*a^2))/b^3/d
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.11

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$
$$-\frac{8\cdot 4^{\frac{1}{3}}\sqrt{3}(-a^2)^{\frac{1}{3}}\,a^2\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)+4\cdot 4^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}\,a^2\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{2}{3}}+2\,(bx\right.}{}$$

input
```
integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/24*(8*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a^2*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x
^3 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 4*4^(1/3)*(-a^2)^(1/3)*a^2*lo
g(4^(2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/
3)*(-a^2)^(1/3)*a) - 8*4^(1/3)*(-a^2)^(1/3)*a^2*log(-4^(2/3)*(-a^2)^(2/3)
+ 2*(b*x^3 + a)^(1/3)*a) + 3*(b^2*x^6 + 2*a*b*x^3 + 5*a^2)*(b*x^3 + a)^(2/
3))/(b^3*d)
```

Sympy [F]

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^8(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**8*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**8*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.88

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\frac{8\sqrt{3}2^{\frac{2}{3}}a^{\frac{8}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d}-\frac{4\cdot 2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}+\frac{8\cdot 2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}}{24\,b^3}$$

input
```
integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-1/24*(8*sqrt(3)*2^(2/3)*a^(8/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/
3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 4*2^(2/3)*a^(8/3)*log(2^(2/3)*a^(2/
3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 8*2^(2/3)*
a^(8/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*((b*x^3 + a)^(8/3)
 + 4*(b*x^3 + a)^(2/3)*a^2)/d)/b^3
```

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.99

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\sqrt{3}2^{\frac{2}{3}}a^{\frac{8}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,b^3d} + \frac{2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,b^3d} - \frac{2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,b^3d} - \frac{(bx^3+a)^{\frac{8}{3}}b^{21}d^7+4\,(bx^3+a)^{\frac{2}{3}}a^2b^{21}d^7}{8\,b^{24}d^8}$$

input
```
integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*2^(2/3)*a^(8/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
 2*(b*x^3 + a)^(1/3))/a^(1/3))/(b^3*d) + 1/6*2^(2/3)*a^(8/3)*log(2^(2/3)*a
^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b^3*d) -
1/3*2^(2/3)*a^(8/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b^3*d)
 - 1/8*((b*x^3 + a)^(8/3)*b^21*d^7 + 4*(b*x^3 + a)^(2/3)*a^2*b^21*d^7)/(b^
24*d^8)
```

Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.16

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{(b\,x^3+a)^{8/3}}{8\,b^3\,d} - \frac{a^2\,(b\,x^3+a)^{2/3}}{2\,b^3\,d} - \frac{4^{1/3}\,a^{8/3}\,\ln\left((b\,x^3+a)^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b^3\,d} - \frac{4^{1/3}\,a^{8/3}\,\ln\left(\frac{4\,a^6\,(b\,x^3+a)^{1/3}}{b^6\,d^2}-\frac{2\,4^{2/3}\,a^{19/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)^2}{b^6\,d^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^3\,d} + \frac{4^{1/3}\,a^{8/3}\,\ln\left(\frac{4\,a^6\,(b\,x^3+a)^{1/3}}{b^6\,d^2}-\frac{18\,4^{2/3}\,a^{19/3}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)^2}{b^6\,d^2}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^3\,d}$$

input
```
int((x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

output
```
(4^(1/3)*a^(8/3)*log((4*a^6*(a + b*x^3)^(1/3))/(b^6*d^2) - (18*4^(2/3)*a^(
19/3)*((3^(1/2)*1i)/6 + 1/6)^2)/(b^6*d^2))*((3^(1/2)*1i)/6 + 1/6))/(b^3*d)
 - (a^2*(a + b*x^3)^(2/3))/(2*b^3*d) - (4^(1/3)*a^(8/3)*log((a + b*x^3)^(1
/3) - 2^(1/3)*a^(1/3)))/(3*b^3*d) - (4^(1/3)*a^(8/3)*log((4*a^6*(a + b*x^3
)^(1/3))/(b^6*d^2) - (2*4^(2/3)*a^(19/3)*((3^(1/2)*1i)/2 - 1/2)^2)/(b^6*d^
2))*((3^(1/2)*1i)/2 - 1/2))/(3*b^3*d) - (a + b*x^3)^(8/3)/(8*b^3*d)
```

Reduce [F]

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{3(b\,x^3+a)^{\frac{2}{3}}\,a^2-2(b\,x^3+a)^{\frac{2}{3}}\,ab\,x^3-(b\,x^3+a)^{\frac{2}{3}}\,b^2x^6+16\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}x^5}{-b^2x^6+a^2}dx\right)a^2b^2}{8b^3d}$$

input
```
int(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

output
```
(3*(a + b*x**3)**(2/3)*a**2 - 2*(a + b*x**3)**(2/3)*a*b*x**3 - (a + b*x**3
)**(2/3)*b**2*x**6 + 16*int(((a + b*x**3)**(2/3)*x**5)/(a**2 - b**2*x**6),
x)*a**2*b**2)/(8*b**3*d)
```

3.802 $\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 175

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{a(a+bx^3)^{2/3}}{2b^2d}-\frac{(a+bx^3)^{5/3}}{5b^2d} - \frac{2^{2/3}a^{5/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^2d} + \frac{a^{5/3}\log(a-bx^3)}{3\sqrt[3]{2}b^2d}-\frac{a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^2d}$$

output

```
-1/2*a*(b*x^3+a)^(2/3)/b^2/d-1/5*(b*x^3+a)^(5/3)/b^2/d-1/3*2^(2/3)*a^(5/3)
*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/b^2
/d+1/6*a^(5/3)*ln(-b*x^3+a)*2^(2/3)/b^2/d-1/2*a^(5/3)*ln(2^(1/3)*a^(1/3)-(
b*x^3+a)^(1/3))*2^(2/3)/b^2/d
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.07

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{21a(a+bx^3)^{2/3}+6bx^3(a+bx^3)^{2/3}+10\ 2^{2/3}\sqrt{3}a^{5/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+10\ 2^{2/3}a^{5/3}\log\left(-2\sqrt[3]{\cdots}\right.}{30b^2d}$$

input
```
Integrate[(x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/30*(21*a*(a + b*x^3)^(2/3) + 6*b*x^3*(a + b*x^3)^(2/3) + 10*2^(2/3)*Sqr
t[3]*a^(5/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 1
0*2^(2/3)*a^(5/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] - 5*2^(2/3)*
a^(5/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b
*x^3)^(2/3)])/(b^2*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {948, 27, 90, 60, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+bx^3\right)^{2/3}}{ad-bdx^3}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^3\left(bx^3+a\right)^{2/3}}{d\left(a-bx^3\right)}dx^3$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{x^3(bx^3+a)^{2/3}}{a-bx^3}dx^3}{3d}$$

$$\Big\downarrow \; 90$$

$$\frac{\frac{a\int \frac{(bx^3+a)^{2/3}}{a-bx^3}dx^3}{b}-\frac{3(a+bx^3)^{5/3}}{5b^2}}{3d}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{a\left(2a\int \frac{1}{(a-bx^3)\sqrt[3]{bx^3+a}}dx^3-\frac{3(a+bx^3)^{2/3}}{2b}\right)}{b}-\frac{3(a+bx^3)^{5/3}}{5b^2}}{3d}$$

$$\Big\downarrow \; 67$$

$$\frac{\frac{a\left(2a\left(-\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2b}+\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log(a-bx^3)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)-\frac{3(a+bx^3)^{2/3}}{2b}\right)}{b}-\frac{3(a+bx^3)^{5/3}}{5b^2}}{3d}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{a\left(2a\left(-\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2b}+\frac{\log(a-bx^3)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)-\frac{3(a+bx^3)^{2/3}}{2b}\right)}{b}-\frac{3(a+bx^3)^{5/3}}{5b^2}}{3d}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{a\left(2a\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log(a-bx^3)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)-\frac{3(a+bx^3)^{2/3}}{2b}\right)}{b}-\frac{3(a+bx^3)^{5/3}}{5b^2}}{3d}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{a\left(2a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log\left(a-bx^3\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)-\frac{3\left(a+bx^3\right)^{2/3}}{2b}\right)}{b}-\frac{3\left(a+bx^3\right)^{5/3}}{5b^2}}{3d}$$

input
```
Int[(x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
((-3*(a + b*x^3)^(5/3))/(5*b^2) + (a*((-3*(a + b*x^3)^(2/3))/(2*b) + 2*a*(
-((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/(2^(
1/3)*a^(1/3)*b)) + Log[a - b*x^3]/(2*2^(1/3)*a^(1/3)*b) - (3*Log[2^(1/3)*a
^(1/3) - (a + b*x^3)^(1/3)])/(2*2^(1/3)*a^(1/3)*b))))/b)/(3*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{5\,2^{\frac{2}{3}}\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{30b^{2}d}$

input
```
int(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/30*(5*2^(2/3)*(-2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2)/a
^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)
*a^(2/3))-2*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3)))*a^(5/3)-3*(b*x^3+a)^(2/3)
*(2*b*x^3+7*a))/b^2/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.03

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$
$$-\frac{10\cdot 4^{\frac{1}{3}}\sqrt{3}(-a^2)^{\frac{1}{3}}\,a\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)+5\cdot 4^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}\,a\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{2}{3}}+2\,(bx^3}{3}$$

input
```
integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/30*(10*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^
3 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 5*4^(1/3)*(-a^2)^(1/3)*a*log(4
^(2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*
(-a^2)^(1/3)*a) - 10*4^(1/3)*(-a^2)^(1/3)*a*log(-4^(2/3)*(-a^2)^(2/3) + 2*
(b*x^3 + a)^(1/3)*a) + 3*(2*b*x^3 + 7*a)*(b*x^3 + a)^(2/3))/(b^2*d)
```

Sympy [F]

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^5(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**5*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**5*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.89

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\frac{10\sqrt{3}2^{\frac{2}{3}}a^{\frac{5}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d} - \frac{5\cdot 2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d} + \frac{10\cdot 2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a\right.}{d}}{30\,b^2}$$

input
```
integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-1/30*(10*sqrt(3)*2^(2/3)*a^(5/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1
/3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 5*2^(2/3)*a^(5/3)*log(2^(2/3)*a^(2
/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 10*2^(2/3
)*a^(5/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*(2*(b*x^3 + a)^(
5/3) + 5*(b*x^3 + a)^(2/3)*a)/d)/b^2
```

Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\sqrt{3}2^{\frac{2}{3}}a^{\frac{5}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{3\,b^2d} + \frac{2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{6\,b^2d} - \frac{2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right|\right)}{3\,b^2d} - \frac{2\left(bx^3+a\right)^{\frac{5}{3}}b^8d^4+5\left(bx^3+a\right)^{\frac{2}{3}}ab^8d^4}{10\,b^{10}d^5}$$

input
```
integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*2^(2/3)*a^(5/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
 2*(b*x^3 + a)^(1/3))/a^(1/3))/(b^2*d) + 1/6*2^(2/3)*a^(5/3)*log(2^(2/3)*a
^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b^2*d) -
1/3*2^(2/3)*a^(5/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b^2*d)
 - 1/10*(2*(b*x^3 + a)^(5/3)*b^8*d^4 + 5*(b*x^3 + a)^(2/3)*a*b^8*d^4)/(b^1
0*d^5)
```

Mupad [B] (verification not implemented)

Time = 3.83 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.26

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{4^{1/3}\,(-a)^{5/3}\,\ln\left(4\,a^4\,\left(b\,x^3+a\right)^{1/3}+4\,2^{1/3}\,(-a)^{13/3}\right)}{3\,b^2\,d} - \frac{a\,\left(b\,x^3+a\right)^{2/3}}{2\,b^2\,d} - \frac{\left(b\,x^3+a\right)^{5/3}}{5\,b^2\,d} - \frac{4^{1/3}\,(-a)^{5/3}\,\ln\left(\frac{4\,a^4\,\left(b\,x^3+a\right)^{1/3}}{b^4\,d^2}+\frac{2\,4^{2/3}\,(-a)^{13/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^4\,d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^2\,d} + \frac{4^{1/3}\,(-a)^{5/3}\,\ln\left(\frac{4\,a^4\,\left(b\,x^3+a\right)^{1/3}}{b^4\,d^2}+\frac{18\,4^{2/3}\,(-a)^{13/3}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2}{b^4\,d^2}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^2\,d}$$

input
```
int((x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

output
```
(4^(1/3)*(-a)^(5/3)*log(4*a^4*(a + b*x^3)^(1/3) + 4*2^(1/3)*(-a)^(13/3)))/
(3*b^2*d) - (a*(a + b*x^3)^(2/3))/(2*b^2*d) - (a + b*x^3)^(5/3)/(5*b^2*d)
- (4^(1/3)*(-a)^(5/3)*log((4*a^4*(a + b*x^3)^(1/3))/(b^4*d^2) + (2*4^(2/3)
*(-a)^(13/3)*((3^(1/2)*1i)/2 + 1/2)^2)/(b^4*d^2))*((3^(1/2)*1i)/2 + 1/2))/
(3*b^2*d) + (4^(1/3)*(-a)^(5/3)*log((4*a^4*(a + b*x^3)^(1/3))/(b^4*d^2) +
(18*4^(2/3)*(-a)^(13/3)*((3^(1/2)*1i)/6 - 1/6)^2)/(b^4*d^2))*((3^(1/2)*1i)
/6 - 1/6))/(b^2*d)
```

Reduce [F]

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{3(b\,x^3+a)^{\frac{2}{3}}\,a-2(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3+20\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}x^5}{-b^2x^6+a^2}dx\right)a\,b^2}{10b^2d}$$

input
```
int(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

output
```
(3*(a + b*x**3)**(2/3)*a - 2*(a + b*x**3)**(2/3)*b*x**3 + 20*int(((a + b*x
**3)**(2/3)*x**5)/(a**2 - b**2*x**6),x)*a*b**2)/(10*b**2*d)
```

3.803 $\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 153

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{(a+bx^3)^{2/3}}{2bd} - \frac{2^{2/3}a^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}bd} + \frac{a^{2/3}\log(a-bx^3)}{3\sqrt[3]{2}bd} - \frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}bd}$$

output
```
-1/2*(b*x^3+a)^(2/3)/b/d-1/3*2^(2/3)*a^(2/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(
b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/b/d+1/6*a^(2/3)*ln(-b*x^3+a)*2^(2
/3)/b/d-1/2*a^(2/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/b/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.11

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{3(a+bx^3)^{2/3} + 2\,2^{2/3}\sqrt{3}a^{2/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\,2^{2/3}a^{2/3}\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right) - 2^{2/}}{6bd}$$

input
```
Integrate[(x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/6*(3*(a + b*x^3)^(2/3) + 2*2^(2/3)*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2^(2/3)
*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 2*2^(2/3)*a^(2/3)*Log[-2*a^(1/3) +
 2^(2/3)*(a + b*x^3)^(1/3)] - 2^(2/3)*a^(2/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1
/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b*d)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {946, 27, 60, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(a+bx^3\right)^{2/3}}{ad-bdx^3} \, dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3} \int \frac{\left(bx^3+a\right)^{2/3}}{d\left(a-bx^3\right)}dx^3$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(bx^3+a\right)^{2/3}}{a-bx^3}dx^3}{3d}$$

$$\Big\downarrow \text{60}$$

$$\frac{2a \int \frac{1}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx^3-\frac{3\left(a+bx^3\right)^{2/3}}{2b}}{3d}$$

$$\Big\downarrow \text{67}$$

$$\frac{2a \left(-\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2b}+\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log\left(a-bx^3\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)-\frac{3\left(a+bx^3\right)^{2/3}}{2b}}{3d}$$

$$\Big\downarrow \textbf{16}$$

$$\frac{2a\left(-\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2b}+\frac{\log(a-bx^3)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)-\frac{3(a+bx^3)^{2/3}}{2b}}{3d}$$

$$\Big\downarrow \textbf{1082}$$

$$\frac{2a\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log(a-bx^3)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)-\frac{3(a+bx^3)^{2/3}}{2b}}{3d}$$

$$\Big\downarrow \textbf{217}$$

$$\frac{2a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log(a-bx^3)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)-\frac{3(a+bx^3)^{2/3}}{2b}}{3d}$$

input
```
Int[(x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
((-3*(a + b*x^3)^(2/3))/(2*b) + 2*a*(-((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a +
b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/(2^(1/3)*a^(1/3)*b)) + Log[a - b*x^3]/(2*
2^(1/3)*a^(1/3)*b) - (3*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2*2^(1/
3)*a^(1/3)*b)))/(3*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{-2\,2^{\frac{2}{3}} a^{\frac{2}{3}} \arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3 a^{\frac{1}{3}}}\right)\sqrt{3}-2 a^{\frac{2}{3}} 2^{\frac{2}{3}} \ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}} a^{\frac{1}{3}}\right)+a^{\frac{2}{3}} 2^{\frac{2}{3}} \ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}} a^{\frac{1}{3}} \left(b\,x^{3}+a\right)\right.}{6bd}$

input
```
int(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-2*2^(2/3)*a^(2/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/
2)/a^(1/3))*3^(1/2)-2*a^(2/3)*2^(2/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))+
a^(2/3)*2^(2/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)
*a^(2/3))-3*(b*x^3+a)^(2/3))/b/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}(-a^2)^{\frac{1}{3}}\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)+4^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{2}{3}}+2\,(bx^3+a)^{\frac{2}{3}}a\right.}{6\,bd}$$

input
```
integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/6*(2*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^3 +
a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 4^(1/3)*(-a^2)^(1/3)*log(4^(2/3)*(
b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*(-a^2)^(
1/3)*a) - 2*4^(1/3)*(-a^2)^(1/3)*log(-4^(2/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)
^(1/3)*a) + 3*(b*x^3 + a)^(2/3))/(b*d)
```

Sympy [F]

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^2(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**2*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**2*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\frac{2\sqrt{3}2^{\frac{2}{3}}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d} - \frac{2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d} + \frac{2\cdot 2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}}{6b}$$

input
```
integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*2^(2/3)*a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3
) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 2^(2/3)*a^(2/3)*log(2^(2/3)*a^(2/3)
+ 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 2*2^(2/3)*a^(
2/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*(b*x^3 + a)^(2/3)/d)/
b
```

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=-\frac{\sqrt{3}2^{\frac{2}{3}}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{3bd}+\frac{2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6bd}-\frac{2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3bd}-\frac{(bx^3+a)^{\frac{2}{3}}}{2bd}$$

input
```
integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*2^(2/3)*a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
 2*(b*x^3 + a)^(1/3))/a^(1/3))/(b*d) + 1/6*2^(2/3)*a^(2/3)*log(2^(2/3)*a^(
2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b*d) - 1/3*
2^(2/3)*a^(2/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b*d) - 1/2
*(b*x^3 + a)^(2/3)/(b*d)
```

Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.22

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=-\frac{(b\,x^3+a)^{2/3}}{2\,b\,d}-\frac{4^{1/3}\,a^{2/3}\,\ln\left(\left(b\,x^3+a\right)^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b\,d}-\frac{4^{1/3}\,a^{2/3}\,\ln\left(\frac{4\,a^2\,\left(b\,x^3+a\right)^{1/3}}{b^2\,d^2}-\frac{2\,4^{2/3}\,a^{7/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^2\,d^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b\,d}+\frac{4^{1/3}\,a^{2/3}\,\ln\left(\frac{4\,a^2\,\left(b\,x^3+a\right)^{1/3}}{b^2\,d^2}-\frac{18\,4^{2/3}\,a^{7/3}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2}{b^2\,d^2}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b\,d}$$

input
```
int((x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

output
```
(4^(1/3)*a^(2/3)*log((4*a^2*(a + b*x^3)^(1/3))/(b^2*d^2) - (18*4^(2/3)*a^(
7/3)*((3^(1/2)*1i)/6 + 1/6)^2)/(b^2*d^2))*((3^(1/2)*1i)/6 + 1/6))/(b*d) -
(4^(1/3)*a^(2/3)*log((a + b*x^3)^(1/3) - 2^(1/3)*a^(1/3)))/(3*b*d) - (4^(1
/3)*a^(2/3)*log((4*a^2*(a + b*x^3)^(1/3))/(b^2*d^2) - (2*4^(2/3)*a^(7/3)*(
(3^(1/2)*1i)/2 - 1/2)^2)/(b^2*d^2))*((3^(1/2)*1i)/2 - 1/2))/(3*b*d) - (a +
 b*x^3)^(2/3)/(2*b*d)
```

Reduce [F]

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{(b\,x^3+a)^{\frac{2}{3}}+4\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}x^5}{-b^2x^6+a^2}dx\right)b^2}{2bd}$$

input
```
int(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

output
```
((a + b*x**3)**(2/3) + 4*int(((a + b*x**3)**(2/3)*x**5)/(a**2 - b**2*x**6)
,x)*b**2)/(2*b*d)
```

3.804 $\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 214

$$\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx = \frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}d} - \frac{2^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}d} - \frac{\log(x)}{2\sqrt[3]{a}d} + \frac{\log(a-bx^3)}{3\sqrt[3]{2}\sqrt[3]{a}d} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}d} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{a}d}$$

output
```
1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(1/3
)/d-1/3*2^(2/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/
3))*3^(1/2)/a^(1/3)/d-1/2*ln(x)/a^(1/3)/d+1/6*ln(-b*x^3+a)*2^(2/3)/a^(1/3)
/d+1/2*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(1/3)/d-1/2*ln(2^(1/3)*a^(1/3)-(b*x^3
+a)^(1/3))*2^(2/3)/a^(1/3)/d
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\log\left(-\sqrt[3]{a}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(2/3)
*Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 2*Log
[-a^(1/3) + (a + b*x^3)^(1/3)] - 2*2^(2/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b
*x^3)^(1/3)] - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)
] + 2^(2/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a
 + b*x^3)^(2/3)])/(6*a^(1/3)*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 27, 94, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{2/3}}{dx^3(a-bx^3)}dx^3$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(bx^3+a)^{2/3}}{x^3(a-bx^3)}dx^3}{3d}$$

$\downarrow$ 94

$$\frac{\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3+2b\int \frac{1}{(a-bx^3)\sqrt[3]{bx^3+a}}dx^3}{3d}$$

$\downarrow$ 67

$$\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+2b\left(-\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2b}+\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{bx^3+}}}{2\sqrt[3]{2}\sqrt[3]{a}}\right.}{3d}$$

$\downarrow$ 16

$$\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+2b\left(-\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2b}+\frac{\log(a-bx^3)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\sqrt[3]{}\right.}{}\right.}{3d}$$

$\downarrow$ 1082

$$\frac{-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+2b\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log(a-bx^3)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)+-\frac{3}{}}{3d}$$

$\downarrow$ 217

$$\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+2b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log(a-bx^3)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)+\frac{3\log}{}}{3d}$$

input
```
Int[(a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x]
```

output
```
((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - L
og[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3)) + 2
*b*(-((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/
(2^(1/3)*a^(1/3)*b)) + Log[a - b*x^3]/(2*2^(1/3)*a^(1/3)*b) - (3*Log[2^(1/
3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2*2^(1/3)*a^(1/3)*b)))/(3*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$\frac{-2\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)-2\,2^{\frac{2}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)+2^{\frac{2}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right.}{6d\,a^{\frac{1}{3}}}$

input
```
int((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-2*3^(1/2)*2^(2/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/
2)/a^(1/3))-2*2^(2/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))+2^(2/3)*ln((b*x^
3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))+2*arctan(1/3*(
a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-a
^(1/3))-ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3)))/d/a^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.48

$$\int \frac{(a+bx^3)^{2/3}}{x\left(ad-bdx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
[-1/6*(2*4^(1/3)*sqrt(3)*a*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(b*x^3
+ a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*lo
g((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*
a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) +
4^(1/3)*a*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^
(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 2*4^(1/3)*a*(-1/a)^(1/3)*log
(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) + a^(2/3)*log((b*x^3 + a)^
(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(
1/3) - a^(1/3)))/(a*d), -1/6*(2*4^(1/3)*sqrt(3)*a*(-1/a)^(1/3)*arctan(1/3*
4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) + 4^(1/3)*a*
(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-
1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 2*4^(1/3)*a*(-1/a)^(1/3)*log(-4^(2/3)*
a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) - 6*sqrt(1/3)*a^(2/3)*arctan(sqrt(1/
3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3
) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3)
 - a^(1/3)))/(a*d)]
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x\,(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax+bx^4}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/x/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a*x + b*x**4), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x\,(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx = -\frac{\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}d} + \frac{2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,a^{\frac{1}{3}}d} - \frac{2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{1}{3}}d} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}d} - \frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{1}{3}}d} + \frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{1}{3}}d}$$

input
```
integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b*x^
3 + a)^(1/3))/a^(1/3))/(a^(1/3)*d) + 1/6*2^(2/3)*log(2^(2/3)*a^(2/3) + 2^(
1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(a^(1/3)*d) - 1/3*2^(2
/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(a^(1/3)*d) + 1/3*sqrt(
3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(1/3)*d)
 - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(1/
3)*d) + 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(1/3)*d)
```

Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.72

$$\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx = \ln\left(2\left(b\,x^3+a\right)^{1/3} -2\,2^{1/3}\,a\,d^2\left(-\frac{1}{a\,d^3}\right)^{2/3}\right)\left(-\frac{4}{27\,a\,d^3}\right)^{1/3}+\ln\left(\left(b\,x^3+a\right)^{1/3}-a\,d^2\left(\frac{1}{a\,d^3}\right)^{2/3}\right)\left(\frac{1}{27\,a\,d^3}\right)^{1/3}-\ln\left(4\left(b\,x\right.\right.$$

input
```
int((a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x)
```

output
```
log(2*(a + b*x^3)^(1/3) - 2*2^(1/3)*a*d^2*(-1/(a*d^3))^(2/3))*(-4/(27*a*d^
3))^(1/3) + log((a + b*x^3)^(1/3) - a*d^2*(1/(a*d^3))^(2/3))*(1/(27*a*d^3)
)^(1/3) - log(4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a*d^2*(-1/(a*d^3))^(2/3) - 2
^(1/3)*3^(1/2)*a*d^2*(-1/(a*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 + 1/2)*(-4/(27
*a*d^3))^(1/3) + log(4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a*d^2*(-1/(a*d^3))^(2
/3) + 2^(1/3)*3^(1/2)*a*d^2*(-1/(a*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)*
(-4/(27*a*d^3))^(1/3) - log(2*(a + b*x^3)^(1/3) + a*d^2*(1/(a*d^3))^(2/3)
- 3^(1/2)*a*d^2*(1/(a*d^3))^(2/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a*d^3)
)^(1/3) + log(2*(a + b*x^3)^(1/3) + a*d^2*(1/(a*d^3))^(2/3) + 3^(1/2)*a*d^
2*(1/(a*d^3))^(2/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a*d^3))^(1/3)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx = \frac{\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{-b\,x^4+ax}dx}{d}$$

input
```
int((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x)
```

output
```
int((a + b*x**3)**(2/3)/(a*x - b*x**4),x)/d
```

3.805 $\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 269

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx = \frac{b(a+bx^3)^{2/3}}{3a^2d} - \frac{(a+bx^3)^{5/3}}{3a^2dx^3} + \frac{5b\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}d} - \frac{2^{2/3}b\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}d} - \frac{5b\log(x)}{6a^{4/3}d} + \frac{b\log(a-bx^3)}{3\sqrt[3]{2}a^{4/3}d} + \frac{5b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}d} - \frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{4/3}d}$$

output

```
1/3*b*(b*x^3+a)^(2/3)/a^2/d-1/3*(b*x^3+a)^(5/3)/a^2/d/x^3+5/9*b*arctan(1/3
*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(4/3)/d-1/3*2^(2/3
)*b*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/
a^(4/3)/d-5/6*b*ln(x)/a^(4/3)/d+1/6*b*ln(-b*x^3+a)*2^(2/3)/a^(4/3)/d+5/6*b
*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(4/3)/d-1/2*b*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^
(1/3))*2^(2/3)/a^(4/3)/d
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx = \frac{-6\sqrt[3]{a}(a+bx^3)^{2/3}+10\sqrt{3}bx^3\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-6\,2^{2/3}\sqrt{3}bx^3\arctan\left(\frac{1+}{}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x]
```

output
```
(-6*a^(1/3)*(a + b*x^3)^(2/3) + 10*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b*x^3
)^(1/3))/a^(1/3))/Sqrt[3]] - 6*2^(2/3)*Sqrt[3]*b*x^3*ArcTan[(1 + (2^(2/3)*
(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 10*b*x^3*Log[-a^(1/3) + (a + b*x^3)
^(1/3)] - 6*2^(2/3)*b*x^3*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] - 5*
b*x^3*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + 3*2^(
2/3)*b*x^3*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a
+ b*x^3)^(2/3)])/(18*a^(4/3)*d*x^3)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {948, 27, 114, 27, 174, 60, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{2/3}}{dx^6(a-bx^3)}dx^3$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(bx^3+a)^{2/3}}{x^6(a-bx^3)}dx^3}{3d}$$

$$\Big\downarrow \ 114$$

$$\frac{-\frac{\int -\frac{b(5a-2bx^3)(bx^3+a)^{2/3}}{3x^3(a-bx^3)}dx^3}{a^2}-\frac{(a+bx^3)^{5/3}}{a^2x^3}}{3d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{b\int \frac{(5a-2bx^3)(bx^3+a)^{2/3}}{x^3(a-bx^3)}dx^3}{3a^2}-\frac{(a+bx^3)^{5/3}}{a^2x^3}}{3d}$$

$$\Big\downarrow \ 174$$

$$\frac{\frac{b\left(5\int \frac{(bx^3+a)^{2/3}}{x^3}dx^3+3b\int \frac{(bx^3+a)^{2/3}}{a-bx^3}dx^3\right)}{3a^2}-\frac{(a+bx^3)^{5/3}}{a^2x^3}}{3d}$$

$$\Big\downarrow \ 60$$

$$\frac{\frac{b\left(5\left(a\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3+\frac{3}{2}(a+bx^3)^{2/3}\right)+3b\left(2a\int \frac{1}{(a-bx^3)\sqrt[3]{bx^3+a}}dx^3-\frac{3(a+bx^3)^{2/3}}{2b}\right)\right)}{3a^2}-\frac{(a+bx^3)^{5/3}}{a^2x^3}}{3d}$$

$$\Big\downarrow \ 67$$

$$b\left(5\left(a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)+\frac{3}{2}(a+bx^3)^{2/3}\right)+3b\left(2a\left(-\frac{3\int}{x^6 \ldots}\right.\right.\right. \ldots$$

$$\Big\downarrow \ 16$$

$$b\left(5\left(a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)+\frac{3}{2}(a+bx^3)^{2/3}\right)+3b\left(2a\left(-\frac{3\int}{x^6+2^{2/3}a^{2/3}+\ldots}\right.\right.\right. \ldots$$

$$\Big\downarrow \ 1082$$

$$\frac{\frac{b\left(5\left(a\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)+3b\left(2a\left(\frac{3\int\frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}\right.}{\sqrt[3]{2}\sqrt[3]{a}b}\right.\right.\right.}{3a^2}}{3d}$$

$\big\downarrow$ 217

$$\frac{\frac{b\left(5\left(a\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)+3b\left(2a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right.}{\sqrt[3]{2}\sqrt[3]{a}b}\right.\right.\right.}{3a^2}}{3d}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x]
```

output
```
(-((a + b*x^3)^(5/3)/(a^2*x^3)) + (b*(5*((3*(a + b*x^3)^(2/3))/2 + a*((Sqr
t[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^
3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3)))) + 3*b*
((-3*(a + b*x^3)^(2/3))/(2*b) + 2*a*(-((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a +
b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/(2^(1/3)*a^(1/3)*b)) + Log[a - b*x^3]/(2*
2^(1/3)*a^(1/3)*b) - (3*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2*2^(1/
3)*a^(1/3)*b)))))/(3*a^2))/(3*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 67

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$-\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)b\,x^{3}+2^{\frac{2}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)b\,x^{3}-\frac{5\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}\,b\,x}{3}}{}$

input
```
int((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3^(1/2)*2^(2/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2)
/a^(1/3))*b*x^3+2^(2/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))*b*x^3-5/3*arct
an(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)*b*x^3-1/2*2^(2
/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))*b*
x^3-5/3*ln((b*x^3+a)^(1/3)-a^(1/3))*b*x^3+5/6*ln((b*x^3+a)^(2/3)+a^(1/3)*(
b*x^3+a)^(1/3)+a^(2/3))*b*x^3+(b*x^3+a)^(2/3)*a^(1/3))/a^(4/3)/x^3/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 612, normalized size of antiderivative = 2.28

$$\int \frac{(a+bx^3)^{2/3}}{x^4\,(ad-bdx^3)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
[-1/18*(6*4^(1/3)*sqrt(3)*a*b*x^3*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*
(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 15*sqrt(1/3)*a*b*x^3*sqrt(
-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x
^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3)
+ 3*a)/x^3) + 3*4^(1/3)*a*b*x^3*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)
*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 6*4^(1
/3)*a*b*x^3*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)
) + 5*a^(2/3)*b*x^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^
(2/3)) - 10*a^(2/3)*b*x^3*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*(b*x^3 + a)
^(2/3)*a)/(a^2*d*x^3), -1/18*(6*4^(1/3)*sqrt(3)*a*b*x^3*(-1/a)^(1/3)*arcta
n(1/3*4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) + 3*4^
(1/3)*a*b*x^3*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) -
2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 6*4^(1/3)*a*b*x^3*(-1/a)
^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) - 30*sqrt(1/3)*a
^(2/3)*b*x^3*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)) + 5
*a^(2/3)*b*x^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)
) - 10*a^(2/3)*b*x^3*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*(b*x^3 + a)^(2/3
)*a)/(a^2*d*x^3)]
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^4+bx^7}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/x**4/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a*x**4 + b*x**7), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^4}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx = -\frac{\sqrt{3}2^{\frac{2}{3}}b\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{4}{3}}d} + \frac{2^{\frac{2}{3}}b\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,a^{\frac{4}{3}}d} - \frac{2^{\frac{2}{3}}b\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{4}{3}}d} + \frac{5\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{4}{3}}d} - \frac{5\,b\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{4}{3}}d} + \frac{5\,b\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{4}{3}}d} - \frac{(bx^3+a)^{\frac{2}{3}}}{3\,adx^3}$$

input
```
integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*2^(2/3)*b*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b*
x^3 + a)^(1/3))/a^(1/3))/(a^(4/3)*d) + 1/6*2^(2/3)*b*log(2^(2/3)*a^(2/3) +
 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(a^(4/3)*d) - 1/3*
2^(2/3)*b*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(a^(4/3)*d) + 5/9
*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^
(4/3)*d) - 5/18*b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2
/3))/(a^(4/3)*d) + 5/9*b*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(4/3)*d)
 - 1/3*(b*x^3 + a)^(2/3)/(a*d*x^3)
```

Mupad [B] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx = \ln\left(2\,b^2\,\left(b\,x^3+a\right)^{1/3} -2\,2^{1/3}\,a^3\,d^2\,\left(-\frac{b^3}{a^4\,d^3}\right)^{2/3}\right)\,\left(-\frac{4\,b^3}{27\,a^4\,d^3}\right)^{1/3} + \frac{5\,\ln\left(b^2\,\left(b\,x^3+a\right)^{1/3}-a^3\,d^2\,\left(\frac{b^3}{a^4\,d^3}\right)^{2/3}\right)\,\left(\frac{b^3}{a^4\,d^3}\right)^{1/3}}{9} - \ln\Bigg(4$$

input
```
int((a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x)
```

output
```
log(2*b^2*(a + b*x^3)^(1/3) - 2*2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3))*(-
(4*b^3)/(27*a^4*d^3))^(1/3) + (5*log(b^2*(a + b*x^3)^(1/3) - a^3*d^2*(b^3/
(a^4*d^3))^(2/3))*(b^3/(a^4*d^3))^(1/3))/9 - log(4*b^2*(a + b*x^3)^(1/3) +
 2*2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3) - 2^(1/3)*3^(1/2)*a^3*d^2*(-b^3/
(a^4*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 + 1/2)*(-(4*b^3)/(27*a^4*d^3))^(1/3)
+ log(4*b^2*(a + b*x^3)^(1/3) + 2*2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3) +
 2^(1/3)*3^(1/2)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)
*(-(4*b^3)/(27*a^4*d^3))^(1/3) - log(2*b^2*(a + b*x^3)^(1/3) + a^3*d^2*(b^
3/(a^4*d^3))^(2/3) - 3^(1/2)*a^3*d^2*(b^3/(a^4*d^3))^(2/3)*1i)*((3^(1/2)*1
i)/2 + 1/2)*((125*b^3)/(729*a^4*d^3))^(1/3) + log(2*b^2*(a + b*x^3)^(1/3)
+ a^3*d^2*(b^3/(a^4*d^3))^(2/3) + 3^(1/2)*a^3*d^2*(b^3/(a^4*d^3))^(2/3)*1i
)*((3^(1/2)*1i)/2 - 1/2)*((125*b^3)/(729*a^4*d^3))^(1/3) - (b*(a + b*x^3)^
(2/3))/(3*a*(d*(a + b*x^3) - a*d))
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx = \frac{\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{-b\,x^7+a\,x^4}dx}{d}$$

input
```
int((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x)
```

output
```
int((a + b*x**3)**(2/3)/(a*x**4 - b*x**7),x)/d
```

3.806 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(ad-bdx^3\right)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 284

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(ad-bdx^3\right)}\,dx=-\frac{5b\left(a+bx^3\right)^{2/3}}{18a^2dx^3}-\frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6}+\frac{14b^2\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}d}-\frac{2^{2/3}b^2\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}d}-\frac{7b^2\log(x)}{9a^{7/3}d}+\frac{b^2\log\left(a-bx^3\right)}{3\sqrt[3]{2}a^{7/3}d}+\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{9a^{7/3}d}-\frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{7/3}d}$$

output
```
-5/18*b*(b*x^3+a)^(2/3)/a^2/d/x^3-1/6*(b*x^3+a)^(5/3)/a^2/d/x^6+14/27*b^2*
arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(7/3)/d-
1/3*2^(2/3)*b^2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/
3))*3^(1/2)/a^(7/3)/d-7/9*b^2*ln(x)/a^(7/3)/d+1/6*b^2*ln(-b*x^3+a)*2^(2/3)
/a^(7/3)/d+7/9*b^2*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(7/3)/d-1/2*b^2*ln(2^(1/3
)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/a^(7/3)/d
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx = \frac{-9a^{4/3}(a+bx^3)^{2/3}-24\sqrt[3]{a}bx^3(a+bx^3)^{2/3}+28\sqrt{3}b^2x^6\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x]
```

output
```
(-9*a^(4/3)*(a + b*x^3)^(2/3) - 24*a^(1/3)*b*x^3*(a + b*x^3)^(2/3) + 28*Sq
rt[3]*b^2*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 18*2^(
2/3)*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt
[3]] + 28*b^2*x^6*Log[-a^(1/3) + (a + b*x^3)^(1/3)] - 18*2^(2/3)*b^2*x^6*L
og[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] - 14*b^2*x^6*Log[a^(2/3) + a^(1
/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + 9*2^(2/3)*b^2*x^6*Log[2*a^(2/
3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(54*a
^(7/3)*d*x^6)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {948, 27, 114, 27, 166, 27, 174, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{2/3}}{dx^9(a-bx^3)}dx^3$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(bx^3+a)^{2/3}}{x^9(a-bx^3)}dx^3}{3d}$$

$\Big\downarrow$ 114

$$\frac{-\frac{\int -\frac{b(bx^3+a)^{2/3}(bx^3+5a)}{3x^6(a-bx^3)}dx^3}{2a^2}-\frac{(a+bx^3)^{5/3}}{2a^2x^6}}{3d}$$

$\Big\downarrow$ 27

$$\frac{\frac{b\int \frac{(bx^3+a)^{2/3}(bx^3+5a)}{x^6(a-bx^3)}dx^3}{6a^2}-\frac{(a+bx^3)^{5/3}}{2a^2x^6}}{3d}$$

$\Big\downarrow$ 166

$$\frac{\frac{b\left(\frac{\int \frac{4ab(2bx^3+7a)}{3x^3(a-bx^3)\sqrt[3]{bx^3+a}}dx^3}{a}-\frac{5(a+bx^3)^{2/3}}{x^3}\right)}{6a^2}-\frac{(a+bx^3)^{5/3}}{2a^2x^6}}{3d}$$

$\Big\downarrow$ 27

$$\frac{\frac{b\left(\frac{4}{3}b\int \frac{2bx^3+7a}{x^3(a-bx^3)\sqrt[3]{bx^3+a}}dx^3-\frac{5(a+bx^3)^{2/3}}{x^3}\right)}{6a^2}-\frac{(a+bx^3)^{5/3}}{2a^2x^6}}{3d}$$

$\Big\downarrow$ 174

$$\frac{\frac{b\left(\frac{4}{3}b\left(7\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3+9b\int \frac{1}{(a-bx^3)\sqrt[3]{bx^3+a}}dx^3\right)-\frac{5(a+bx^3)^{2/3}}{x^3}\right)}{6a^2}-\frac{(a+bx^3)^{5/3}}{2a^2x^6}}{3d}$$

$\Big\downarrow$ 67

$$b\left(\frac{4}{3}b\left(7\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)+9b\left(-\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\ldots}\ldots}{2\ldots}\right.\right.\right. \quad \ldots \quad 6a^2 \quad \ldots \quad 3d$$

$\Big\downarrow$ 16

$$\frac{b\left(\frac{4}{3}b\left(7\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+9b\left(-\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+}}}{2b}\right.\right.\right.}{\frac{6a^2}{3d}}$$

$\Big\downarrow$ 1082

$$\frac{b\left(\frac{4}{3}b\left(7\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+9b\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log\left(a-bx^3\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\right.\right.\right.}{\frac{6a^2}{3d}}$$

$\Big\downarrow$ 217

$$\frac{b\left(\frac{4}{3}b\left(7\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+9b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log\left(a-bx^3\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log}{}\right.\right.\right.}{\frac{6a^2}{3d}}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x]
```

output
```
(-1/2*(a + b*x^3)^(5/3)/(a^2*x^6) + (b*((-5*(a + b*x^3)^(2/3))/x^3 + (4*b*
(7*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3)
- Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3)))
 + 9*b*(-((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3
]])/(2^(1/3)*a^(1/3)*b)) + Log[a - b*x^3]/(2*2^(1/3)*a^(1/3)*b) - (3*Log[2
^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2*2^(1/3)*a^(1/3)*b))))/3))/(6*a^2))
/(3*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-18\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)b^{2}x^{6}-18\,2^{\frac{2}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)b^{2}x^{6}+9\,2^{\frac{2}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}\right.\right.$

input
```
int((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/54*(-18*3^(1/2)*2^(2/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))*3^(
1/2)/a^(1/3))*b^2*x^6-18*2^(2/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))*b^2*x
^6+9*2^(2/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^
(2/3))*b^2*x^6+28*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a
^(1/3))*b^2*x^6+28*ln((b*x^3+a)^(1/3)-a^(1/3))*b^2*x^6-14*ln((b*x^3+a)^(2/
3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))*b^2*x^6-24*b*x^3*(b*x^3+a)^(2/3)*a^(1/
3)-9*(b*x^3+a)^(2/3)*a^(4/3))/a^(7/3)/x^6/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 660, normalized size of antiderivative = 2.32

$$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
[-1/54*(18*4^(1/3)*sqrt(3)*a*b^2*x^6*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(
3)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 42*sqrt(1/3)*a*b^2*x^6*
sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) -
 (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(
2/3) + 3*a)/x^3) + 9*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a
)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) -
 18*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3
+ a)^(1/3)) + 14*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)
*a^(1/3) + a^(2/3)) - 28*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(1/3) - a^(1/3))
+ 3*(8*a*b*x^3 + 3*a^2)*(b*x^3 + a)^(2/3))/(a^3*d*x^6), -1/54*(18*4^(1/3)*
sqrt(3)*a*b^2*x^6*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3
)*(-1/a)^(1/3) - 1/3*sqrt(3)) + 9*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(4^(2/
3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3
+ a)^(2/3)) - 18*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3
) + 2*(b*x^3 + a)^(1/3)) - 84*sqrt(1/3)*a^(2/3)*b^2*x^6*arctan(sqrt(1/3)*(
2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)) + 14*a^(2/3)*b^2*x^6*log((b*x^3 +
a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 28*a^(2/3)*b^2*x^6*log((
b*x^3 + a)^(1/3) - a^(1/3)) + 3*(8*a*b*x^3 + 3*a^2)*(b*x^3 + a)^(2/3))/(a^
3*d*x^6)]
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^7+bx^{10}}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/x**7/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a*x**7 + b*x**10), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^7}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^7), x)
```

Giac [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx &= -\frac{\sqrt{3}2^{\frac{2}{3}}b^2\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{7}{3}}d} \\ &\quad + \frac{2^{\frac{2}{3}}b^2\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,a^{\frac{7}{3}}d} \\ &\quad - \frac{2^{\frac{2}{3}}b^2\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{7}{3}}d} + \frac{14\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{7}{3}}d} \\ &\quad - \frac{7\,b^2\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{27\,a^{\frac{7}{3}}d} \\ &\quad + \frac{14\,b^2\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27\,a^{\frac{7}{3}}d} - \frac{8\,(bx^3+a)^{\frac{5}{3}}b^2-5\,(bx^3+a)^{\frac{2}{3}}ab^2}{18\,a^2b^2dx^6}\end{aligned}$$

input
```
integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*2^(2/3)*b^2*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(
b*x^3 + a)^(1/3))/a^(1/3))/(a^(7/3)*d) + 1/6*2^(2/3)*b^2*log(2^(2/3)*a^(2/
3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(a^(7/3)*d) -
1/3*2^(2/3)*b^2*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(a^(7/3)*d)
 + 14/27*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^
(1/3))/(a^(7/3)*d) - 7/27*b^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^
(1/3) + a^(2/3))/(a^(7/3)*d) + 14/27*b^2*log(abs((b*x^3 + a)^(1/3) - a^(1/
3)))/(a^(7/3)*d) - 1/18*(8*(b*x^3 + a)^(5/3)*b^2 - 5*(b*x^3 + a)^(2/3)*a*b
^2)/(a^2*b^2*d*x^6)
```

Mupad [B] (verification not implemented)

Time = 4.62 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx = \frac{\frac{5\,b^2\,\left(b\,x^3+a\right)^{2/3}}{18\,a}-\frac{4\,b^2\,\left(b\,x^3+a\right)^{5/3}}{9\,a^2}}{d\,\left(b\,x^3+a\right)^2+a^2\,d-2\,a\,d\,\left(b\,x^3+a\right)}$$

$$+\ln\left(2\,b^4\,\left(b\,x^3+a\right)^{1/3}-2\,2^{1/3}\,a^5\,d^2\,\left(-\frac{b^6}{a^7\,d^3}\right)^{2/3}\right)\,\left(-\frac{4\,b^6}{27\,a^7\,d^3}\right)^{1/3}+\frac{14\,\ln\left(b^4\,\left(b\,x^3+a\right)^{1/3}-a^5\,d^2\,\left(\frac{b^6}{a^7\,d^3}\right.\right.}{27}$$

input
```
int((a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x)
```

output
```
((5*b^2*(a + b*x^3)^(2/3))/(18*a) - (4*b^2*(a + b*x^3)^(5/3))/(9*a^2))/(d*
(a + b*x^3)^2 + a^2*d - 2*a*d*(a + b*x^3)) + log(2*b^4*(a + b*x^3)^(1/3) -
 2*2^(1/3)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3))*(-(4*b^6)/(27*a^7*d^3))^(1/3) +
 (14*log(b^4*(a + b*x^3)^(1/3) - a^5*d^2*(b^6/(a^7*d^3))^(2/3))*(b^6/(a^7*
d^3))^(1/3))/27 - log(4*b^4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a^5*d^2*(-b^6/(a
^7*d^3))^(2/3) - 2^(1/3)*3^(1/2)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3)*2i)*((3^(1
/2)*1i)/2 + 1/2)*(-(4*b^6)/(27*a^7*d^3))^(1/3) + log(4*b^4*(a + b*x^3)^(1/
3) + 2*2^(1/3)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3) + 2^(1/3)*3^(1/2)*a^5*d^2*(-
b^6/(a^7*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)*(-(4*b^6)/(27*a^7*d^3))^(1
/3) - (7*log(2*b^4*(a + b*x^3)^(1/3) + a^5*d^2*(b^6/(a^7*d^3))^(2/3) - 3^(
1/2)*a^5*d^2*(b^6/(a^7*d^3))^(2/3)*1i)*(3^(1/2)*1i + 1)*(b^6/(a^7*d^3))^(1
/3))/27 + (7*log(2*b^4*(a + b*x^3)^(1/3) + a^5*d^2*(b^6/(a^7*d^3))^(2/3) +
 3^(1/2)*a^5*d^2*(b^6/(a^7*d^3))^(2/3)*1i)*(3^(1/2)*1i - 1)*(b^6/(a^7*d^3)
)^(1/3))/27
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx = \frac{\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{-b\,x^{10}+a\,x^7}dx}{d}$$

input
```
int((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x)
```

output
```
int((a + b*x**3)**(2/3)/(a*x**7 - b*x**10),x)/d
```

3.807 $\int \frac{x^6\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 264

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{4ax(a+bx^3)^{2/3}}{9b^2d}-\frac{x^4(a+bx^3)^{2/3}}{6bd} - \frac{14a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{7/3}d} + \frac{2^{2/3}a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}d} + \frac{a^2\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}b^{7/3}d} - \frac{a^2\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^{7/3}d} + \frac{7a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{9b^{7/3}d}$$

output
```
-4/9*a*x*(b*x^3+a)^(2/3)/b^2/d-1/6*x^4*(b*x^3+a)^(2/3)/b/d-14/27*a^2*arcta
n(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(7/3)/d+1/3*2^(2/
3)*a^2*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)
/b^(7/3)/d+1/6*a^2*ln(-b*d*x^3+a*d)*2^(2/3)/b^(7/3)/d-1/2*a^2*ln(2^(1/3)*b
^(1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/b^(7/3)/d+7/9*a^2*ln(-b^(1/3)*x+(b*x^3+a
)^(1/3))/b^(7/3)/d
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.23

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{24a\sqrt[3]{b}x(a+bx^3)^{2/3}+9b^{4/3}x^4(a+bx^3)^{2/3}+28\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-18\,2^{2/3}\sqrt{3}a^2\arctan\Big($$

input
```
Integrate[(x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/54*(24*a*b^(1/3)*x*(a + b*x^3)^(2/3) + 9*b^(4/3)*x^4*(a + b*x^3)^(2/3)
+ 28*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/
3))] - 18*2^(2/3)*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2
/3)*(a + b*x^3)^(1/3))] - 28*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 1
8*2^(2/3)*a^2*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] + 14*a^2*Log[b
^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 9*2^(2/3)*
a^2*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a +
 b*x^3)^(2/3)])/(b^(7/3)*d)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {978, 27, 1052, 27, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$$

$$\Big\downarrow \ 978$$

$$\frac{\int \frac{4ax^3\left(2bx^3+a\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{6bd}-\frac{x^4\left(a+bx^3\right)^{2/3}}{6bd}$$

$$\Big\downarrow \ 27$$

$$\frac{2a\int \frac{x^3(2bx^3+a)}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{3bd}-\frac{x^4(a+bx^3)^{2/3}}{6bd}$$

$$\Big\downarrow \ 1052$$

$$\frac{2a\left(\frac{\int \frac{ab(7bx^3+2a)}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{3b^2}-\frac{2x(a+bx^3)^{2/3}}{3b}\right)}{3bd}-\frac{x^4(a+bx^3)^{2/3}}{6bd}$$

$$\Big\downarrow \ 27$$

$$\frac{2a\left(\frac{a\int \frac{7bx^3+2a}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{3b}-\frac{2x(a+bx^3)^{2/3}}{3b}\right)}{3bd}-\frac{x^4(a+bx^3)^{2/3}}{6bd}$$

$$\Big\downarrow \ 1026$$

$$\frac{2a\left(\frac{a\left(9a\int \frac{1}{(a-bx^3)\sqrt[3]{bx^3+a}}dx-7\int \frac{1}{\sqrt[3]{bx^3+a}}dx\right)}{3b}-\frac{2x(a+bx^3)^{2/3}}{3b}\right)}{3bd}-\frac{x^4(a+bx^3)^{2/3}}{6bd}$$

$$\Big\downarrow \ 769$$

$$\frac{2a\left(\frac{a\left(9a\int\frac{1}{(a-bx^3)\sqrt[3]{bx^3+a}}dx-7\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)\right)}{3b}-\frac{2x(a+bx^3)^{2/3}}{3b}\right)}{3bd}-\frac{x^4(a+bx^3)^{2/3}}{6bd}$$

$\big\downarrow$ 901

$$\frac{2a\left(\frac{a\left(9a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\sqrt[3]{b}}+\frac{\log(a-bx^3)}{6\sqrt[3]{2}a\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}a\sqrt[3]{b}}\right)-7\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{\ldots}\right.}{2\sqrt[3]{b}}\right.\right.}{3b}\ldots}{3bd}$$

$$\frac{x^4(a+bx^3)^{2/3}}{6bd}$$

input
```
Int[(x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/6*(x^4*(a + b*x^3)^(2/3))/(b*d) + (2*a*((-2*x*(a + b*x^3)^(2/3))/(3*b)
+ (a*(9*a*(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(
2^(1/3)*Sqrt[3]*a*b^(1/3)) + Log[a - b*x^3]/(6*2^(1/3)*a*b^(1/3)) - Log[2^
(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(1/3)*a*b^(1/3))) - 7*(ArcTan[(1
 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(
1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3)))))/(3*b)))/(3*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{-9\left(b\,x^{3}+a\right)^{\frac{2}{3}} b^{\frac{4}{3}} x^{4}-18\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}} x\right)}{3 b^{\frac{1}{3}} x}\right) a^{2}-18\,2^{\frac{2}{3}} \ln\left(\frac{-2^{\frac{1}{3}} b^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right) a^{2}+9\,2^{\frac{2}{3}} \ln\left(\frac{b^{\frac{2}{3}} 2 \cdots}{\cdots}\right.}{\cdots}$

input
```
int(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/54*(-9*(b*x^3+a)^(2/3)*b^(4/3)*x^4-18*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)
*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)*x)/b^(1/3)/x)*a^2-18*2^(2/3)*ln((-2^(1/3
)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^2+9*2^(2/3)*ln((b^(2/3)*2^(2/3)*x^2+(b*x
^3+a)^(1/3)*b^(1/3)*2^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a^2-24*a*x*(b*x^3+a)^(
2/3)*b^(1/3)+28*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b
^(1/3)/x)*a^2+28*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^2-14*ln((b^(2/3)*x^2
+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a^2)/d/b^(7/3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 701, normalized size of antiderivative = 2.66

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
[-1/54*(18*4^(1/3)*sqrt(3)*a^2*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(
1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 42*sqrt(1/3)*a^2*b*sqrt(
-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b
^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt
(-1/b^(2/3)) + 2*a) - 18*4^(1/3)*a^2*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/
b)^(2/3) - 2*(b*x^3 + a)^(1/3))/x) + 9*4^(1/3)*a^2*b*(-1/b)^(1/3)*log(-(2*
4^(1/3)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) -
2*(b*x^3 + a)^(2/3))/x^2) - 28*a^2*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(
1/3))/x) + 14*a^2*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x +
 (b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*x^4 + 8*a*b*x)*(b*x^3 + a)^(2/3))/(b^3
*d), -1/54*(18*4^(1/3)*sqrt(3)*a^2*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x -
 4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 18*4^(1/3)*a^2*b*(-1
/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(1/3))/x) + 9*4^(
1/3)*a^2*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^
3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^2) - 84*sqrt(1/3)*a
^2*b^(2/3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))
 - 28*a^2*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + 14*a^2*b^(2/3)
*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2)
+ 3*(3*b^2*x^4 + 8*a*b*x)*(b*x^3 + a)^(2/3))/(b^3*d)]
```

Sympy [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^6(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**6*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**6*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x^6}{bdx^3-ad}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)*x^6/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x^6}{bdx^3-ad}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)*x^6/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int \frac{x^6\,(b\,x^3+a)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

output
```
int((x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

Reduce [F]

$$\int \frac{x^6 (a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{-8(b\,x^3+a)^{\frac{2}{3}}\,ax-3(b\,x^3+a)^{\frac{2}{3}}\,b\,x^4+8\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{-b^2x^6+a^2}dx\right)a^3+28\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}x^3}{-b^2x^6+a^2}dx\right)}{18b^2d}$$

input
```
int(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

output
```
( - 8*(a + b*x**3)**(2/3)*a*x - 3*(a + b*x**3)**(2/3)*b*x**4 + 8*int((a +
b*x**3)**(2/3)/(a**2 - b**2*x**6),x)*a**3 + 28*int(((a + b*x**3)**(2/3)*x*
*3)/(a**2 - b**2*x**6),x)*a**2*b)/(18*b**2*d)
```

3.808 $\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 229

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3} \, dx = -\frac{x(a+bx^3)^{2/3}}{3bd} - \frac{5a\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}d} + \frac{2^{2/3}a\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}d} + \frac{a\log(ad-bdx^3)}{3\sqrt[3]{2}b^{4/3}d} - \frac{a\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^{4/3}d} + \frac{5a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}d}$$

output
```
-1/3*x*(b*x^3+a)^(2/3)/b/d-5/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3)
)*3^(1/2))*3^(1/2)/b^(4/3)/d+1/3*2^(2/3)*a*arctan(1/3*(1+2*2^(1/3)*b^(1/3)
*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)/d+1/6*a*ln(-b*d*x^3+a*d)*2^(2
/3)/b^(4/3)/d-1/2*a*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/b^(4/3)/
d+5/6*a*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)/d
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.27

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$
$$-\frac{6\sqrt[3]{b}x(a+bx^3)^{2/3}+10\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-6\,2^{2/3}\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)-10a}{}$$

input
```
Integrate[(x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/18*(6*b^(1/3)*x*(a + b*x^3)^(2/3) + 10*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^(1/3
)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 6*2^(2/3)*Sqrt[3]*a*ArcTan[(Sqrt
[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] - 10*a*Log[-(b^(1/
3)*x) + (a + b*x^3)^(1/3)] + 6*2^(2/3)*a*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b
*x^3)^(1/3)] + 5*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*
x^3)^(2/3)] - 3*2^(2/3)*a*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3
)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^(4/3)*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {978, 27, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+bx^3\right)^{2/3}}{ad-bdx^3}dx$$
$$\Big\downarrow \; 978$$
$$\frac{\int \frac{a\left(5bx^3+a\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{3bd}-\frac{x\left(a+bx^3\right)^{2/3}}{3bd}$$
$$\Big\downarrow \; 27$$

$$\frac{a\int \frac{5bx^3+a}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{3bd}-\frac{x(a+bx^3)^{2/3}}{3bd}$$

$\Big\downarrow$ 1026

$$\frac{a\left(6a\int \frac{1}{(a-bx^3)\sqrt[3]{bx^3+a}}dx-5\int \frac{1}{\sqrt[3]{bx^3+a}}dx\right)}{3bd}-\frac{x(a+bx^3)^{2/3}}{3bd}$$

$\Big\downarrow$ 769

$$\frac{a\left(6a\int \frac{1}{(a-bx^3)\sqrt[3]{bx^3+a}}dx-5\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)\right)}{3bd}-\frac{x(a+bx^3)^{2/3}}{3bd}$$

$\Big\downarrow$ 901

$$\frac{a\left(6a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\sqrt[3]{b}}+\frac{\log(a-bx^3)}{6\sqrt[3]{2}a\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}a\sqrt[3]{b}}\right)-5\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a-\right.}{\right.}\right.}{3bd}$$

$$\frac{x(a+bx^3)^{2/3}}{3bd}$$

input
```
Int[(x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/3*(x*(a + b*x^3)^(2/3))/(b*d) + (a*(6*a*(ArcTan[(1 + (2*2^(1/3)*b^(1/3)
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]*a*b^(1/3)) + Log[a - b*x^
3]/(6*2^(1/3)*a*b^(1/3)) - Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2
^(1/3)*a*b^(1/3))) - 5*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[
3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3)))
))/(3*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{-6\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)a-6\,2^{\frac{2}{3}}\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a+3\,2^{\frac{2}{3}}\ln\left(\frac{b^{\frac{2}{3}}2^{\frac{2}{3}}x^{2}+\left(b\,x^{3}+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}2^{\frac{1}{3}}x+}{x^{2}}\right.}{}$

input
```
int(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/18*(-6*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/
3)*x)/b^(1/3)/x)*a-6*2^(2/3)*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a+
3*2^(2/3)*ln((b^(2/3)*2^(2/3)*x^2+(b*x^3+a)^(1/3)*b^(1/3)*2^(1/3)*x+(b*x^3
+a)^(2/3))/x^2)*a-6*(b*x^3+a)^(2/3)*x*b^(1/3)+10*3^(1/2)*arctan(1/3*3^(1/2
)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a+10*ln((-b^(1/3)*x+(b*x^3+a)^(
1/3))/x)*a-5*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^
2)*a)/d/b^(4/3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 653, normalized size of antiderivative = 2.85

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
[-1/18*(6*4^(1/3)*sqrt(3)*a*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3
)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 15*sqrt(1/3)*a*b*sqrt(-1/b^
(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3
)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b
^(2/3)) + 2*a) - 6*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3)
 - 2*(b*x^3 + a)^(1/3))/x) + 3*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*
x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 +
 a)^(2/3))/x^2) + 6*(b*x^3 + a)^(2/3)*b*x - 10*a*b^(2/3)*log(-(b^(1/3)*x -
 (b*x^3 + a)^(1/3))/x) + 5*a*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*
b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^2*d), -1/18*(6*4^(1/3)*sqrt(3)*a*b
*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(
-1/b)^(1/3))/x) - 6*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3
) - 2*(b*x^3 + a)^(1/3))/x) + 3*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b
*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3
+ a)^(2/3))/x^2) - 30*sqrt(1/3)*a*b^(2/3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*
(b*x^3 + a)^(1/3))/(b^(1/3)*x)) + 6*(b*x^3 + a)^(2/3)*b*x - 10*a*b^(2/3)*l
og(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + 5*a*b^(2/3)*log((b^(2/3)*x^2 + (b
*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^2*d)]
```

Sympy [F]

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^3(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**3*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**3*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{2}{3}}x^3}{bdx^3-ad}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)*x^3/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{2}{3}}x^3}{bdx^3-ad}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)*x^3/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\int \frac{x^3\,{\left(b\,x^3+a\right)}^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

output
```
int((x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

Reduce [F]

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{-(b\,x^3+a)^{\frac{2}{3}}\,x+\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{-b^2x^6+a^2}dx\right)a^2+5\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}x^3}{-b^2x^6+a^2}dx\right)ab}{3bd}$$

input
```
int(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

output
```
( - (a + b*x**3)**(2/3)*x + int((a + b*x**3)**(2/3)/(a**2 - b**2*x**6),x)*
a**2 + 5*int(((a + b*x**3)**(2/3)*x**3)/(a**2 - b**2*x**6),x)*a*b)/(3*b*d)
```

3.809 $\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 200

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d} + \frac{2^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d} + \frac{\log(ad-bdx^3)}{3\sqrt[3]{2}\sqrt[3]{b}d} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{b}d} + \frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}d}$$

output
```
-1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(1/3)/d
+1/3*2^(2/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3
^(1/2)/b^(1/3)/d+1/6*ln(-b*d*x^3+a*d)*2^(2/3)/b^(1/3)/d-1/2*ln(2^(1/3)*b^(
1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/b^(1/3)/d+1/2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3
))/b^(1/3)/d
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$
$$-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)-2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(a*d - b*d*x^3),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3
))] - 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a
 + b*x^3)^(1/3))] - 2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 2*2^(2/3)*Lo
g[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] + Log[b^(2/3)*x^2 + b^(1/3)*x*
(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 2^(2/3)*Log[2*b^(2/3)*x^2 + 2^(2/
3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^(1/3)*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {916, 27, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}dx$$
$$\Big\downarrow \text{ 916}$$
$$2a\int \frac{1}{d(a-bx^3)\sqrt[3]{bx^3+a}}dx-\frac{\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}$$
$$\Big\downarrow \text{ 27}$$

$$\frac{2a\int \frac{1}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{d}-\frac{\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}$$

$$\Big\downarrow \text{769}$$

$$\frac{2a\int \frac{1}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{d}-\frac{\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}}{d}$$

$$\Big\downarrow \text{901}$$

$$\frac{2a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\sqrt[3]{b}}+\frac{\log(a-bx^3)}{6\sqrt[3]{2}a\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}a\sqrt[3]{b}}\right)}{d}-\frac{\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}}{d}$$

input
```
Int[(a + b*x^3)^(2/3)/(a*d - b*d*x^3),x]
```

output
```
(2*a*(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(1/
3)*Sqrt[3]*a*b^(1/3)) + Log[a - b*x^3]/(6*2^(1/3)*a*b^(1/3)) - Log[2^(1/3)
*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(1/3)*a*b^(1/3))))/d - (ArcTan[(1 + (
2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)
*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3)))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 916
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[b/d   Int[(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*
x^n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c -
a*d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{-2\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}} x\right)}{3 b^{\frac{1}{3}} x}\right)-2\,2^{\frac{2}{3}} \ln\left(\frac{-2^{\frac{1}{3}} b^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)+2^{\frac{2}{3}} \ln\left(\frac{b^{\frac{2}{3}} 2^{\frac{2}{3}} x^{2}+\left(b\,x^{3}+a\right)^{\frac{1}{3}} b^{\frac{1}{3}} 2^{\frac{1}{3}} x+\left(b\,x\right.}{x^{2}}\right.}{6 d\,b}$

input
```
int((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-2*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3
)*x)/b^(1/3)/x)-2*2^(2/3)*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)+2^(2/
3)*ln((b^(2/3)*2^(2/3)*x^2+(b*x^3+a)^(1/3)*b^(1/3)*2^(1/3)*x+(b*x^3+a)^(2/
3))/x^2)+2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3
)/x)+2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a
)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))/d/b^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 611, normalized size of antiderivative = 3.06

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
[-1/6*(2*4^(1/3)*sqrt(3)*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*s
qrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 3*sqrt(1/3)*b*sqrt(-1/b^(2/3))
*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3
+ (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)
) + 2*a) - 2*4^(1/3)*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*
x^3 + a)^(1/3))/x) + 4^(1/3)*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(
1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x
^2) - 2*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + b^(2/3)*log((b^(
2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b*d), -
1/6*(2*4^(1/3)*sqrt(3)*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqr
t(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 2*4^(1/3)*b*(-1/b)^(1/3)*log(-(4
^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(1/3))/x) + 4^(1/3)*b*(-1/b)^(1/3)
*log(-(2*4^(1/3)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)
^(2/3) - 2*(b*x^3 + a)^(2/3))/x^2) - 6*sqrt(1/3)*b^(2/3)*arctan(sqrt(1/3)*
(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x)) - 2*b^(2/3)*log(-(b^(1/3)*x
 - (b*x^3 + a)^(1/3))/x) + b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^
(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b*d)]
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{bdx^3-ad}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{bdx^3-ad}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(a*d - b*d*x^3),x)
```

output
```
int((a + b*x^3)^(2/3)/(a*d - b*d*x^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\frac{\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{-b\,x^{3}+a}d x}{d}$$

input
```
int((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

output
```
int((a + b*x**3)**(2/3)/(a - b*x**3),x)/d
```

3.810 $\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 157

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{2adx^2}+\frac{2^{2/3}b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}ad}+\frac{b^{2/3}\log(ad-bdx^3)}{3\sqrt[3]{2}ad}-\frac{b^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}ad}$$

output
```
-1/2*(b*x^3+a)^(2/3)/a/d/x^2+1/3*2^(2/3)*b^(2/3)*arctan(1/3*(1+2*2^(1/3)*b
^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/a/d+1/6*b^(2/3)*ln(-b*d*x^3+a*d
)*2^(2/3)/a/d-1/2*b^(2/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/a/
d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx = \frac{-3(a+bx^3)^{2/3}+2\,2^{2/3}\sqrt{3}b^{2/3}x^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)-2\,2^{2/3}b^{2/3}x^2\log\left(\right.}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x]
```

output
```
(-3*(a + b*x^3)^(2/3) + 2*2^(2/3)*Sqrt[3]*b^(2/3)*x^2*ArcTan[(Sqrt[3]*b^(1
/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] - 2*2^(2/3)*b^(2/3)*x^2*Lo
g[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] + 2^(2/3)*b^(2/3)*x^2*Log[2*b^
(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3
)])/(6*a*d*x^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {975, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{4ab}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{2ad}-\frac{(a+bx^3)^{2/3}}{2adx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\int \frac{1}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{d}-\frac{(a+bx^3)^{2/3}}{2adx^2}$$

$$\Big\downarrow \text{901}$$

$$\frac{2b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\sqrt[3]{b}}+\frac{\log(a-bx^3)}{6\sqrt[3]{2}a\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}a\sqrt[3]{b}}\right)}{d}-\frac{(a+bx^3)^{2/3}}{2adx^2}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x]
```

output
```
-1/2*(a + b*x^3)^(2/3)/(a*d*x^2) + (2*b*(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)
/(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]*a*b^(1/3)) + Log[a - b*x^3]/
(6*2^(1/3)*a*b^(1/3)) - Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(1
/3)*a*b^(1/3))))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$\frac{-2b^{\frac{2}{3}}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)x^2-2b^{\frac{2}{3}}2^{\frac{2}{3}}\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)x^2+b^{\frac{2}{3}}2^{\frac{2}{3}}\ln\left(\frac{b^{\frac{2}{3}}2^{\frac{2}{3}}x^2+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right.}{6ad\,x^2}$

input
```
int((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-2*b^(2/3)*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3
)+b^(1/3)*x)/b^(1/3)/x)*x^2-2*b^(2/3)*2^(2/3)*ln((-2^(1/3)*b^(1/3)*x+(b*x^
3+a)^(1/3))/x)*x^2+b^(2/3)*2^(2/3)*ln((b^(2/3)*2^(2/3)*x^2+(b*x^3+a)^(1/3)
*b^(1/3)*2^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*x^2-3*(b*x^3+a)^(2/3))/a/d/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(125) = 250$.

Time = 76.82 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.76

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^3\left(ad-bdx^3\right)}\,dx =$$

$$-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}(-b^2)^{\frac{1}{3}}\,x^2\arctan\left(\frac{3\cdot 4^{\frac{2}{3}}\sqrt{3}(5\,b^2x^7-4\,abx^4-a^2x)\,(bx^3+a)^{\frac{2}{3}}\,(-b^2)^{\frac{2}{3}}+6\cdot 4^{\frac{1}{3}}\sqrt{3}(19\,b^3x^8+16\,ab^2x^5+a^2bx^2)\,(bx^3+a)^{\frac{1}{3}}\,(-b^2)^{\frac{1}{3}}}{3\,(109\,b^4x^9+105\,ab^3x^6+3\,a^2b^2x^3-a^3b)}\right.}{}$$

input
```
integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/18*(2*4^(1/3)*sqrt(3)*(-b^2)^(1/3)*x^2*arctan(1/3*(3*4^(2/3)*sqrt(3)*(5
*b^2*x^7 - 4*a*b*x^4 - a^2*x)*(b*x^3 + a)^(2/3)*(-b^2)^(2/3) + 6*4^(1/3)*s
qrt(3)*(19*b^3*x^8 + 16*a*b^2*x^5 + a^2*b*x^2)*(b*x^3 + a)^(1/3)*(-b^2)^(1
/3) - sqrt(3)*(71*b^4*x^9 + 111*a*b^3*x^6 + 33*a^2*b^2*x^3 + a^3*b))/(109*
b^4*x^9 + 105*a*b^3*x^6 + 3*a^2*b^2*x^3 - a^3*b)) - 2*4^(1/3)*(-b^2)^(1/3)
*x^2*log((3*4^(2/3)*(b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x^2 - 6*(b*x^3 + a)^(2/
3)*b*x + 4^(1/3)*(b*x^3 - a)*(-b^2)^(1/3))/(b*x^3 - a)) + 4^(1/3)*(-b^2)^(
1/3)*x^2*log(-(6*4^(1/3)*(5*b^2*x^4 + a*b*x)*(b*x^3 + a)^(2/3)*(-b^2)^(1/3
) - 4^(2/3)*(19*b^2*x^6 + 16*a*b*x^3 + a^2)*(-b^2)^(2/3) - 24*(2*b^3*x^5 +
 a*b^2*x^2)*(b*x^3 + a)^(1/3))/(b^2*x^6 - 2*a*b*x^3 + a^2)) + 9*(b*x^3 + a
)^(2/3))/(a*d*x^2)
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^3+bx^6}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/x**3/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a*x**3 + b*x**6), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^3), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3\,(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^3\,(ad-bdx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^3\,(a\,d-b\,d\,x^3)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3\,(ad-bdx^3)}\,dx = \frac{(b\,x^3+a)^{\frac{2}{3}}+4\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{-b^2x^9+a^2x^3}dx\right)a^2x^2}{2ad\,x^2}$$

input
```
int((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x)
```

output
```
((a + b*x**3)**(2/3) + 4*int((a + b*x**3)**(2/3)/(a**2*x**3 - b**2*x**9),x
)*a**2*x**2)/(2*a*d*x**2)
```

3.811 $\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 182

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{5adx^5} - \frac{7b(a+bx^3)^{2/3}}{10a^2dx^2} + \frac{2^{2/3}b^{5/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^2d} + \frac{b^{5/3}\log(ad-bdx^3)}{3\sqrt[3]{2}a^2d} - \frac{b^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^2d}$$

output
```
-1/5*(b*x^3+a)^(2/3)/a/d/x^5-7/10*b*(b*x^3+a)^(2/3)/a^2/d/x^2+1/3*2^(2/3)*
b^(5/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2
)/a^2/d+1/6*b^(5/3)*ln(-b*d*x^3+a*d)*2^(2/3)/a^2/d-1/2*b^(5/3)*ln(2^(1/3)*
b^(1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/a^2/d
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx = -\frac{(a+bx^3)^{2/3}(2a+7bx^3)}{10a^2dx^5} + \frac{2^{2/3}b^{5/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}a^2d} - \frac{2^{2/3}b^{5/3}\log\left(-2\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}\right)}{3a^2d} + \frac{b^{5/3}\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\sqrt[3]{2}(a+bx^3)^{2/3}\right)}{3\sqrt[3]{2}a^2d}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x]
```

output
```
-1/10*((a + b*x^3)^(2/3)*(2*a + 7*b*x^3))/(a^2*d*x^5) + (2^(2/3)*b^(5/3)*A
rcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))])/(Sqrt[
3]*a^2*d) - (2^(2/3)*b^(5/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)]
)/(3*a^2*d) + (b^(5/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(
1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(1/3)*a^2*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {975, 27, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx$$

$$\Big\downarrow \; 975$$

$$\frac{\int \frac{b(3bx^3+7a)}{x^3(a-bx^3)\sqrt[3]{bx^3+a}}dx}{5ad} - \frac{(a+bx^3)^{2/3}}{5adx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{3bx^3+7a}{x^3(a-bx^3)\sqrt[3]{bx^3+a}}dx}{5ad}-\frac{\left(a+bx^3\right)^{2/3}}{5adx^5}$$

$$\Big\downarrow \text{1053}$$

$$\frac{b\left(-\frac{\int -\frac{20a^2b}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{2a^2}-\frac{7\left(a+bx^3\right)^{2/3}}{2ax^2}\right)}{5ad}-\frac{\left(a+bx^3\right)^{2/3}}{5adx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(10b\int \frac{1}{(a-bx^3)\sqrt[3]{bx^3+a}}dx-\frac{7\left(a+bx^3\right)^{2/3}}{2ax^2}\right)}{5ad}-\frac{\left(a+bx^3\right)^{2/3}}{5adx^5}$$

$$\Big\downarrow \text{901}$$

$$\frac{b\left(10b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\sqrt[3]{b}}+\frac{\log\left(a-bx^3\right)}{6\sqrt[3]{2}a\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}a\sqrt[3]{b}}\right)-\frac{7\left(a+bx^3\right)^{2/3}}{2ax^2}\right)}{5ad}-\frac{\left(a+bx^3\right)^{2/3}}{5adx^5}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x]
```

output
```
-1/5*(a + b*x^3)^(2/3)/(a*d*x^5) + (b*((-7*(a + b*x^3)^(2/3))/(2*a*x^2) +
10*b*(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(1/
3)*Sqrt[3]*a*b^(1/3)) + Log[a - b*x^3]/(6*2^(1/3)*a*b^(1/3)) - Log[2^(1/3)
*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(1/3)*a*b^(1/3)))))/(5*a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{5x^{5}2^{\frac{2}{3}}\left(-2\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)\sqrt{3}+\ln\left(\frac{b^{\frac{2}{3}}2^{\frac{2}{3}}x^{2}+\left(b\,x^{3}+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}2^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)-2\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(b\,x^{3}\right.}{x}\right.\right.}{30x^{5}d\,a^{2}}$

input
```
int((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/30*(5*x^5*2^(2/3)*(-2*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3
)*x)/b^(1/3)/x)*3^(1/2)+ln((b^(2/3)*2^(2/3)*x^2+(b*x^3+a)^(1/3)*b^(1/3)*2^
(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)
)*b^(5/3)-3*(b*x^3+a)^(2/3)*(7*b*x^3+2*a))/x^5/d/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^6+bx^9}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/x**6/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a*x**6 + b*x**9), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^6), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^6\,(a\,d-b\,d\,x^3)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx = \frac{-2(b\,x^3+a)^{\frac{2}{3}}\,a+3(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3+20\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{-b^2x^9+a^2x^3}dx\right)a^2b\,x^5}{10a^2d\,x^5}$$

input
```
int((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x)
```

output
```
( - 2*(a + b*x**3)**(2/3)*a + 3*(a + b*x**3)**(2/3)*b*x**3 + 20*int((a + b
*x**3)**(2/3)/(a**2*x**3 - b**2*x**9),x)*a**2*b*x**5)/(10*a**2*d*x**5)
```

3.812 $\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 209

$$\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{8adx^8}-\frac{b(a+bx^3)^{2/3}}{4a^2dx^5}-\frac{5b^2(a+bx^3)^{2/3}}{8a^3dx^2}+\frac{2^{2/3}b^{8/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^3d}+\frac{b^{8/3}\log(ad-bdx^3)}{3\sqrt[3]{2}a^3d}-\frac{b^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^3d}$$

output
```
-1/8*(b*x^3+a)^(2/3)/a/d/x^8-1/4*b*(b*x^3+a)^(2/3)/a^2/d/x^5-5/8*b^2*(b*x^
3+a)^(2/3)/a^3/d/x^2+1/3*2^(2/3)*b^(8/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x
/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/a^3/d+1/6*b^(8/3)*ln(-b*d*x^3+a*d)*2^(2
/3)/a^3/d-1/2*b^(8/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/a^3/d
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx = \frac{-\frac{3(a+bx^3)^{2/3}(a^2+2abx^3+5b^2x^6)}{x^8} + 8\,2^{2/3}\sqrt{3}b^{8/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right) - 8\,2^{2/3}b^{8}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x]
```

output
```
((-3*(a + b*x^3)^(2/3)*(a^2 + 2*a*b*x^3 + 5*b^2*x^6))/x^8 + 8*2^(2/3)*Sqrt
[3]*b^(8/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1
/3))] - 8*2^(2/3)*b^(8/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] +
4*2^(2/3)*b^(8/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3)
+ 2^(1/3)*(a + b*x^3)^(2/3)])/(24*a^3*d)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {975, 27, 1053, 27, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{2b(3bx^3+5a)}{x^6(a-bx^3)\sqrt[3]{bx^3+a}}dx}{8ad} - \frac{(a+bx^3)^{2/3}}{8adx^8}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{3bx^3+5a}{x^6(a-bx^3)\sqrt[3]{bx^3+a}}dx}{4ad} - \frac{(a+bx^3)^{2/3}}{8adx^8}$$

$$\Big\downarrow \text{1053}$$

$$\frac{b\left(-\frac{\int -\frac{5ab(3bx^3+5a)}{x^3(a-bx^3)\sqrt[3]{bx^3+a}}dx}{5a^2}-\frac{(a+bx^3)^{2/3}}{ax^5}\right)}{4ad}-\frac{(a+bx^3)^{2/3}}{8adx^8}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\left(\frac{b\int \frac{3bx^3+5a}{x^3(a-bx^3)\sqrt[3]{bx^3+a}}dx}{a}-\frac{(a+bx^3)^{2/3}}{ax^5}\right)}{4ad}-\frac{(a+bx^3)^{2/3}}{8adx^8}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{b\left(\frac{b\left(-\frac{\int -\frac{16a^2b}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{2a^2}-\frac{5(a+bx^3)^{2/3}}{2ax^2}\right)}{a}-\frac{(a+bx^3)^{2/3}}{ax^5}\right)}{4ad}-\frac{(a+bx^3)^{2/3}}{8adx^8}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\left(\frac{b\left(8b\int \frac{1}{(a-bx^3)\sqrt[3]{bx^3+a}}dx-\frac{5(a+bx^3)^{2/3}}{2ax^2}\right)}{a}-\frac{(a+bx^3)^{2/3}}{ax^5}\right)}{4ad}-\frac{(a+bx^3)^{2/3}}{8adx^8}$$

$$\Big\downarrow \text{ 901}$$

$$\frac{b\left(\frac{b\left(8b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\sqrt[3]{b}}+\frac{\log\left(a-bx^3\right)}{6\sqrt[3]{2}a\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}a\sqrt[3]{b}}\right)-\frac{5\left(a+bx^3\right)^{2/3}}{2ax^2}\right)}{a}-\frac{\left(a+bx^3\right)^{2/3}}{ax^5}\right)}{4ad}-\frac{\left(a+bx^3\right)^{2/3}}{8adx^8}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x]
```

output
```
-1/8*(a + b*x^3)^(2/3)/(a*d*x^8) + (b*(-((a + b*x^3)^(2/3)/(a*x^5)) + (b*(
(-5*(a + b*x^3)^(2/3))/(2*a*x^2) + 8*b*(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/
(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]*a*b^(1/3)) + Log[a - b*x^3]/(
6*2^(1/3)*a*b^(1/3)) - Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(1/
3)*a*b^(1/3)))))/a))/(4*a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{4x^{8}2^{\frac{2}{3}}\left(-2\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)\sqrt{3}+\ln\left(\frac{b^{\frac{2}{3}}2^{\frac{2}{3}}x^{2}+\left(b\,x^{3}+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}2^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)-2\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(b\,x^{3}\right.}{x}\right.}{24x^{8}a^{3}d}$

input
```
int((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/24*(4*x^8*2^(2/3)*(-2*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3
)*x)/b^(1/3)/x)*3^(1/2)+ln((b^(2/3)*2^(2/3)*x^2+(b*x^3+a)^(1/3)*b^(1/3)*2^
(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)
)*b^(8/3)-3*(b*x^3+a)^(2/3)*(5*b^2*x^6+2*a*b*x^3+a^2))/x^8/a^3/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^9+bx^{12}}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/x**9/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a*x**9 + b*x**12), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^9}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^9), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9 (ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^9}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^9 (ad-bdx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^9\,(a\,d-b\,d\,x^3)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9 (ad-bdx^3)}\,dx = \frac{-(b\,x^3+a)^{\frac{2}{3}}\,a^2-2(b\,x^3+a)^{\frac{2}{3}}\,ab\,x^3+3(b\,x^3+a)^{\frac{2}{3}}\,b^2x^6+16\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{-b^2x^9+a^2x^3}dx\right)a^2b}{8a^3d\,x^8}$$

input
```
int((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x)
```

output
```
( - (a + b*x**3)**(2/3)*a**2 - 2*(a + b*x**3)**(2/3)*a*b*x**3 + 3*(a + b*x
**3)**(2/3)*b**2*x**6 + 16*int((a + b*x**3)**(2/3)/(a**2*x**3 - b**2*x**9)
,x)*a**2*b**2*x**8)/(8*a**3*d*x**8)
```

3.813 $\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 236

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{11adx^{11}} - \frac{13b(a+bx^3)^{2/3}}{88a^2dx^8} - \frac{49b^2(a+bx^3)^{2/3}}{220a^3dx^5} - \frac{293b^3(a+bx^3)^{2/3}}{440a^4dx^2} + \frac{2^{2/3}b^{11/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^4d} + \frac{b^{11/3}\log(ad-bdx^3)}{3\sqrt[3]{2}a^4d} - \frac{b^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^4d}$$

output
```
-1/11*(b*x^3+a)^(2/3)/a/d/x^11-13/88*b*(b*x^3+a)^(2/3)/a^2/d/x^8-49/220*b^
2*(b*x^3+a)^(2/3)/a^3/d/x^5-293/440*b^3*(b*x^3+a)^(2/3)/a^4/d/x^2+1/3*2^(2
/3)*b^(11/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3
^(1/2)/a^4/d+1/6*b^(11/3)*ln(-b*d*x^3+a*d)*2^(2/3)/a^4/d-1/2*b^(11/3)*ln(2
^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/a^4/d
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx = \frac{-\frac{3(a+bx^3)^{2/3}(40a^3+65a^2bx^3+98ab^2x^6+293b^3x^9)}{x^{11}} + 440\,2^{2/3}\sqrt{3}b^{11/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+}}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x]
```

output
```
((-3*(a + b*x^3)^(2/3)*(40*a^3 + 65*a^2*b*x^3 + 98*a*b^2*x^6 + 293*b^3*x^9
))/x^11 + 440*2^(2/3)*Sqrt[3]*b^(11/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)
*x + 2^(2/3)*(a + b*x^3)^(1/3))] - 440*2^(2/3)*b^(11/3)*Log[-2*b^(1/3)*x +
 2^(2/3)*(a + b*x^3)^(1/3)] + 220*2^(2/3)*b^(11/3)*Log[2*b^(2/3)*x^2 + 2^(
2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(1320*a^4*d
)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {975, 27, 1053, 27, 1053, 25, 27, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{b(9bx^3+13a)}{x^9(a-bx^3)\sqrt[3]{bx^3+a}}dx}{11ad} - \frac{(a+bx^3)^{2/3}}{11adx^{11}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{9bx^3+13a}{x^9(a-bx^3)\sqrt[3]{bx^3+a}}dx}{11ad} - \frac{(a+bx^3)^{2/3}}{11adx^{11}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{b\left(-\frac{\int -\frac{2ab(39bx^3+49a)}{x^6(a-bx^3)\sqrt[3]{bx^3+a}}dx}{8a^2}-\frac{13(a+bx^3)^{2/3}}{8ax^8}\right)}{11ad}-\frac{(a+bx^3)^{2/3}}{11adx^{11}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{b\int \frac{39bx^3+49a}{x^6(a-bx^3)\sqrt[3]{bx^3+a}}dx}{4a}-\frac{13(a+bx^3)^{2/3}}{8ax^8}\right)}{11ad}-\frac{(a+bx^3)^{2/3}}{11adx^{11}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{b\left(\frac{b\left(-\frac{\int -\frac{ab(147bx^3+293a)}{x^3(a-bx^3)\sqrt[3]{bx^3+a}}dx}{5a^2}-\frac{49(a+bx^3)^{2/3}}{5ax^5}\right)}{4a}-\frac{13(a+bx^3)^{2/3}}{8ax^8}\right)}{11ad}-\frac{(a+bx^3)^{2/3}}{11adx^{11}}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{b\left(\frac{\int \frac{ab(147bx^3+293a)}{x^3(a-bx^3)\sqrt[3]{bx^3+a}}dx}{5a^2}-\frac{49(a+bx^3)^{2/3}}{5ax^5}\right)}{4a}-\frac{13(a+bx^3)^{2/3}}{8ax^8}\right)}{11ad}-\frac{(a+bx^3)^{2/3}}{11adx^{11}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{b\left(\frac{b\int \frac{147bx^3+293a}{x^3\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{5a}-\frac{49\left(a+bx^3\right)^{2/3}}{5ax^5}\right)}{4a}-\frac{13\left(a+bx^3\right)^{2/3}}{8ax^8}\right)}{11ad}-\frac{\left(a+bx^3\right)^{2/3}}{11adx^{11}}$$

$\Big\downarrow$ 1053

$$\frac{b\left(\frac{b\left(\frac{b\left(-\frac{\int -\frac{880a^2b}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{2a^2}-\frac{293\left(a+bx^3\right)^{2/3}}{2ax^2}\right)}{5a}-\frac{49\left(a+bx^3\right)^{2/3}}{5ax^5}\right)}{4a}-\frac{13\left(a+bx^3\right)^{2/3}}{8ax^8}\right)}{11ad}-\frac{\left(a+bx^3\right)^{2/3}}{11adx^{11}}$$

$\Big\downarrow$ 27

$$\frac{b\left(\frac{b\left(\frac{b\left(440b\int \frac{1}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx-\frac{293\left(a+bx^3\right)^{2/3}}{2ax^2}\right)}{5a}-\frac{49\left(a+bx^3\right)^{2/3}}{5ax^5}\right)}{4a}-\frac{13\left(a+bx^3\right)^{2/3}}{8ax^8}\right)}{11ad}-\frac{\left(a+bx^3\right)^{2/3}}{11adx^{11}}$$

$\Big\downarrow$ 901

$$\frac{b\left(\frac{b\left(\frac{b\left(440b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\sqrt[3]{b}}+\frac{\log\left(a-bx^3\right)}{6\sqrt[3]{2}a\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}a\sqrt[3]{b}}\right)-\frac{293\left(a+bx^3\right)^{2/3}}{2ax^2}\right)}{5a}-\frac{49\left(a+bx^3\right)^{2/3}}{5ax^5}\right)}{4a}-\frac{13\left(a+bx^3\right)^{2/}}{8ax^8}}{11ad}$$

$$\frac{\left(a+bx^3\right)^{2/3}}{11adx^{11}}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x]
```

output
```
-1/11*(a + b*x^3)^(2/3)/(a*d*x^11) + (b*((-13*(a + b*x^3)^(2/3))/(8*a*x^8)
 + (b*((-49*(a + b*x^3)^(2/3))/(5*a*x^5) + (b*((-293*(a + b*x^3)^(2/3))/(2
*a*x^2) + 440*b*(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt
[3]]/(2^(1/3)*Sqrt[3]*a*b^(1/3)) + Log[a - b*x^3]/(6*2^(1/3)*a*b^(1/3)) -
Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(1/3)*a*b^(1/3)))))/(5*a))
)/(4*a)))/(11*a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{220x^{11}2^{\frac{2}{3}}\left(-2\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)\sqrt{3}+\ln\left(\frac{b^{\frac{2}{3}}2^{\frac{2}{3}}x^{2}+\left(bx^{3}+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}2^{\frac{1}{3}}x+\left(bx^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)-2\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+(b}{x}\right.}{1320x^{11}a^{4}d}$

input
```
int((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/1320*(220*x^11*2^(2/3)*(-2*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b
^(1/3)*x)/b^(1/3)/x)*3^(1/2)+ln((b^(2/3)*2^(2/3)*x^2+(b*x^3+a)^(1/3)*b^(1/
3)*2^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3
))/x))*b^(11/3)-3*(b*x^3+a)^(2/3)*(293*b^3*x^9+98*a*b^2*x^6+65*a^2*b*x^3+4
0*a^3))/x^11/a^4/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(ad-bdx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x**3+a)**(2/3)/x**12/(-b*d*x**3+a*d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(ad-bdx^3\right)}\,dx=\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^{12}}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^12), x)
```

Giac [F]

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(ad-bdx^3\right)}\,dx=\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^{12}}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}\left(ad-bdx^3\right)}\,dx=\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^{12}\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}\left(ad-bdx^3\right)}\,dx=\frac{-40(b\,x^3+a)^{\frac{2}{3}}\,a^3-65(b\,x^3+a)^{\frac{2}{3}}\,a^2b\,x^3-98(b\,x^3+a)^{\frac{2}{3}}\,a\,b^2x^6+147(b\,x^3+a)^{\frac{2}{3}}\,b}{440a^4d\,x^{11}}$$

input
```
int((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x)
```

output
```
( - 40*(a + b*x**3)**(2/3)*a**3 - 65*(a + b*x**3)**(2/3)*a**2*b*x**3 - 98*
(a + b*x**3)**(2/3)*a*b**2*x**6 + 147*(a + b*x**3)**(2/3)*b**3*x**9 + 880*
int((a + b*x**3)**(2/3)/(a**2*x**3 - b**2*x**9),x)*a**2*b**3*x**11)/(440*a
**4*d*x**11)
```

3.814 $\int \frac{x^7 (a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 512

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{9ax^2(a+bx^3)^{2/3}}{28b^2d} - \frac{x^5(a+bx^3)^{2/3}}{7bd} + \frac{2^{2/3}a^{7/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{8/3}d} + \frac{a^{7/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{8/3}d} - \frac{19a^2x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{28b^2d\sqrt[3]{a+bx^3}} + \frac{a^{7/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{8/3}d} + \frac{a^{7/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}b^{8/3}d} - \frac{2^{2/3}a^{7/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{8/3}d} - \frac{a^{7/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{8/3}d}$$

output
```
-9/28*a*x^2*(b*x^3+a)^(2/3)/b^2/d-1/7*x^5*(b*x^3+a)^(2/3)/b/d+1/3*2^(2/3)*
a^(7/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/
2))*3^(1/2)/b^(8/3)/d+1/6*a^(7/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x
)/(b*x^3+a)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)/b^(8/3)/d-19/28*a^2*x^2*(1+b*x
^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/b^2/d/(b*x^3+a)^(1/3)+1/1
2*a^(7/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/3)/b^(8/3)/
d+1/6*a^(7/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/3)/b^(8/3)/d-1/3*2^(2/3)*a^(7/3)*
ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(8/3)/d-1/4*a^(7/3)*ln
(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/a^(1/
3))*2^(2/3)/b^(8/3)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 7.44 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.29

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{-5(9a^2x^2+13abx^5+4b^2x^8)+45a^2x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+}{140b^2d\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
(-5*(9*a^2*x^2 + 13*a*b*x^5 + 4*b^2*x^8) + 45*a^2*x^2*(1 + (b*x^3)/a)^(1/3
)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a] + 38*a*b*x^5*(1 + (b
*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a])/(140*b
^2*d*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {978, 27, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \left(a+bx^3\right)^{2/3}}{ad-bdx^3}dx$$

$\Big\downarrow$ 978

$$\frac{\int \frac{ax^4\left(9bx^3+5a\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{7bd}-\frac{x^5\left(a+bx^3\right)^{2/3}}{7bd}$$

$\Big\downarrow$ 27

$$\frac{a\int \frac{x^4\left(9bx^3+5a\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{7bd}-\frac{x^5\left(a+bx^3\right)^{2/3}}{7bd}$$

$\Big\downarrow$ 1052

$$\frac{a\left(\frac{\int \frac{2abx\left(19bx^3+9a\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{4b^2}-\frac{9x^2\left(a+bx^3\right)^{2/3}}{4b}\right)}{7bd}-\frac{x^5\left(a+bx^3\right)^{2/3}}{7bd}$$

$\Big\downarrow$ 27

$$\frac{a\left(\frac{a\int \frac{x\left(19bx^3+9a\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{2b}-\frac{9x^2\left(a+bx^3\right)^{2/3}}{4b}\right)}{7bd}-\frac{x^5\left(a+bx^3\right)^{2/3}}{7bd}$$

$\Big\downarrow$ 1054

$$\frac{a\left(\frac{a\int \left(\frac{28ax}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}-\frac{19x}{\sqrt[3]{bx^3+a}}\right)dx}{2b}-\frac{9x^2\left(a+bx^3\right)^{2/3}}{4b}\right)}{7bd}-\frac{x^5\left(a+bx^3\right)^{2/3}}{7bd}$$

$\Big\downarrow$ 2009

$$a\left(a\left(\frac{14\ 2^{2/3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{7\ 2^{2/3}\sqrt[3]{a}\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{7\ 2^{2/3}\sqrt[3]{a}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}}{\sqrt[3]{\ }}\right.}{3b^{2/3}}\right.\right.$$

$$\frac{x^5\left(a+bx^3\right)^{2/3}}{7bd}$$

input
```
Int[(x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/7*(x^5*(a + b*x^3)^(2/3))/(b*d) + (a*((-9*x^2*(a + b*x^3)^(2/3))/(4*b)
+ (a*((14*2^(2/3)*a^(1/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) + (7*2^(2/3)*a^(1/3)*ArcTan[(
1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*
b^(2/3)) - (19*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3,
-((b*x^3)/a)])/(2*(a + b*x^3)^(1/3)) + (7*a^(1/3)*Log[((a^(1/3) - b^(1/3)*
x)^2*(a^(1/3) + b^(1/3)*x))/a])/(3*2^(1/3)*b^(2/3)) + (7*2^(2/3)*a^(1/3)*L
og[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(
1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]])/(3*b^(2/3)) - (14*2^(2/3)*a^(1/3)*L
og[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]])/(3*b^(2/3)) - (
7*a^(1/3)*Log[(b^(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(2/3)*b^(1/3)*(
a + b*x^3)^(1/3))/a^(1/3)]])/(2^(1/3)*b^(2/3))))/(2*b)))/(7*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^7 (b\,x^3+a)^{\frac{2}{3}}}{-bd\,x^3+ad}dx$$

input
```
int(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

output
```
int(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^7 (a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \text{Timed out}$$

input
```
integrate(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^7 (a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^7 (a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**7*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**7*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x^7}{bdx^3-ad}\,dx$$

input
```
integrate(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)*x^7/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x^7}{bdx^3-ad}\,dx$$

input
```
integrate(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)*x^7/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int \frac{x^7\,(b\,x^3+a)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

output
```
int((x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

Reduce [F]

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{-9(b\,x^3+a)^{\frac{2}{3}}\,a\,x^2-4(b\,x^3+a)^{\frac{2}{3}}\,b\,x^5+38\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}\,x^4}{-b^2x^6+a^2}dx\right)a^2b+18\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}\,x}{-b^2x^6+a^2}\right.}{28b^2d}$$

input
```
int(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

output
```
( - 9*(a + b*x**3)**(2/3)*a*x**2 - 4*(a + b*x**3)**(2/3)*b*x**5 + 38*int((
(a + b*x**3)**(2/3)*x**4)/(a**2 - b**2*x**6),x)*a**2*b + 18*int(((a + b*x*
*3)**(2/3)*x)/(a**2 - b**2*x**6),x)*a**3)/(28*b**2*d)
```

3.815 $\int \frac{x^4 (a+bx^3)^{2/3}}{ad-bdx^3} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 485

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{x^2(a+bx^3)^{2/3}}{4bd} + \frac{2^{2/3}a^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{5/3}d} + \frac{a^{4/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{5/3}d} - \frac{3ax^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{4bd\sqrt[3]{a+bx^3}} + \frac{a^{4/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{5/3}d} + \frac{a^{4/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}b^{5/3}d} - \frac{2^{2/3}a^{4/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{5/3}d} - \frac{a^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{5/3}d}$$

output
```
-1/4*x^2*(b*x^3+a)^(2/3)/b/d+1/3*2^(2/3)*a^(4/3)*arctan(1/3*(1-2*2^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(5/3)/d+1/6*a^(4/3)
*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(2/
3)*3^(1/2)/b^(5/3)/d-3/4*a*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3
],-b*x^3/a)/b/d/(b*x^3+a)^(1/3)+1/12*a^(4/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(
1/3)+b^(1/3)*x)/a)*2^(2/3)/b^(5/3)/d+1/6*a^(4/3)*ln(1+2^(2/3)*(a^(1/3)+b^(
1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(
2/3)/b^(5/3)/d-1/3*2^(2/3)*a^(4/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3
+a)^(1/3))/b^(5/3)/d-1/4*a^(4/3)*ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^
(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/a^(1/3))*2^(2/3)/b^(5/3)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 6.95 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.26

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{x^2\left(-5(a+bx^3)+5a\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+6bx^3\sqrt[3]{1+\frac{bx^3}{a}}\,\text{A}\right.}{20bd\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
(x^2*(-5*(a + b*x^3) + 5*a*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3
, -((b*x^3)/a), (b*x^3)/a] + 6*b*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1
/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a]))/(20*b*d*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {978, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(a+b x^3\right)^{2/3}}{a d-b d x^3} \, dx$$

$\Big\downarrow$ 978

$$\frac{\int \frac{2 a x \left(3 b x^3+a\right)}{\left(a-b x^3\right) \sqrt[3]{b x^3+a}}dx}{4 b d}-\frac{x^2 \left(a+b x^3\right)^{2/3}}{4 b d}$$

$\Big\downarrow$ 27

$$\frac{a \int \frac{x \left(3 b x^3+a\right)}{\left(a-b x^3\right) \sqrt[3]{b x^3+a}}dx}{2 b d}-\frac{x^2 \left(a+b x^3\right)^{2/3}}{4 b d}$$

$\Big\downarrow$ 1054

$$\frac{a \int \left(\frac{4 a x}{\left(a-b x^3\right) \sqrt[3]{b x^3+a}}-\frac{3 x}{\sqrt[3]{b x^3+a}}\right)dx}{2 b d}-\frac{x^2 \left(a+b x^3\right)^{2/3}}{4 b d}$$

$\Big\downarrow$ 2009

$$\frac{a \left(\frac{2\ 2^{2/3} \sqrt[3]{a} \arctan\left(\frac{1-\frac{2 \sqrt[3]{2} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\sqrt[3]{a+b x^3}}}{\sqrt{3}}\right)}{\sqrt{3} b^{2/3}}+\frac{2^{2/3} \sqrt[3]{a} \arctan\left(\frac{\frac{\sqrt[3]{2} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\sqrt[3]{a+b x^3}}+1}{\sqrt{3}}\right)}{\sqrt{3} b^{2/3}}+\frac{2^{2/3} \sqrt[3]{a} \log\left(\frac{2^{2/3} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{\sqrt[3]{2} \cdots}{\cdots}\right.}{3 b^{2/3}} \cdots\right.}{\cdots}-\frac{x^2 \left(a+b x^3\right)^{2/3}}{4 b d}$$

input
```
Int[(x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/4*(x^2*(a + b*x^3)^(2/3))/(b*d) + (a*((2*2^(2/3)*a^(1/3)*ArcTan[(1 - (2
*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2
/3)) + (2^(2/3)*a^(1/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b
*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) - (3*x^2*(1 + (b*x^3)/a)^(1/3)*Hy
pergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(1/3)) + (a^(
1/3)*Log[((a^(1/3) - b^(1/3)*x)^2*(a^(1/3) + b^(1/3)*x))/a])/(3*2^(1/3)*b^
(2/3)) + (2^(2/3)*a^(1/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b
*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*b^(2/
3)) - (2*2^(2/3)*a^(1/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^
3)^(1/3)])/(3*b^(2/3)) - (a^(1/3)*Log[(b^(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1
/3) - (2^(2/3)*b^(1/3)*(a + b*x^3)^(1/3))/a^(1/3)])/(2^(1/3)*b^(2/3))))/(2
*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^4 (b\,x^3+a)^{\frac{2}{3}}}{-bd\,x^3+ad}dx$$

input
```
int(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

output
```
int(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4 (a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4 (a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^4 (a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**4*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**4*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{2}{3}}x^4}{bdx^3-ad}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)*x^4/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{2}{3}}x^4}{bdx^3-ad}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)*x^4/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\int \frac{x^4\,(b\,x^3+a)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

output
```
int((x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

Reduce [F]

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{-(b\,x^3+a)^{\frac{2}{3}}\,x^2+6\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}x^4}{-b^2x^6+a^2}dx\right)ab+2\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}x}{-b^2x^6+a^2}dx\right)a^2}{4bd}$$

input

```
int(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

output

```
( - (a + b*x**3)**(2/3)*x**2 + 6*int(((a + b*x**3)**(2/3)*x**4)/(a**2 - b*
*2*x**6),x)*a*b + 2*int(((a + b*x**3)**(2/3)*x)/(a**2 - b**2*x**6),x)*a**2
)/(4*b*d)
```

3.816 $\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 457

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{2^{2/3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d} + \frac{\sqrt[3]{a}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{2/3}d} - \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{a}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{2/3}d} + \frac{\sqrt[3]{a}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}b^{2/3}d} - \frac{2^{2/3}\sqrt[3]{a}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d} - \frac{\sqrt[3]{a}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{2/3}d}$$

output
```
1/3*2^(2/3)*a^(1/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^
(1/3))*3^(1/2))*3^(1/2)/b^(2/3)/d+1/6*a^(1/3)*arctan(1/3*(1+2^(1/3)*(a^(1/
3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)/b^(2/3)/d-1/2*x^2*
(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/d/(b*x^3+a)^(1/3)+1
/12*a^(1/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/3)/b^(2/3
)/d+1/6*a^(1/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)
*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/3)/b^(2/3)/d-1/3*2^(2/3)*a^(1/3
)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(2/3)/d-1/4*a^(1/3)*
ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/a^(
1/3))*2^(2/3)/b^(2/3)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 9.70 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.14

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
(x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), (b*x^
3)/a])/(2*d*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.01, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {984, 27, 889, 888, 991, 27, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103, 2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$$

$\Big\downarrow$ 984

$$2a\int \frac{x}{d\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx-\frac{\int \frac{x}{\sqrt[3]{bx^3+a}}dx}{d}$$

$\Big\downarrow$ 27

$$\frac{2a\int \frac{x}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{d}-\frac{\int \frac{x}{\sqrt[3]{bx^3+a}}dx}{d}$$

$\Big\downarrow$ 889

$$\frac{2a\int \frac{x}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{d}-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x}{\sqrt[3]{\frac{bx^3}{a}+1}}dx}{d\sqrt[3]{a+bx^3}}$$

$\Big\downarrow$ 888

$$\frac{2a\int \frac{x}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{d}-\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$\Big\downarrow$ 991

$$\frac{2a\left(\frac{\int \frac{\sqrt[3]{a}}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3a^{2/3}\sqrt[3]{b}}-\frac{\int \frac{1}{\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{\sqrt[3]{a}b^{2/3}}\right)}{d}-\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$\Big\downarrow$ 27

$$\frac{2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{\sqrt[3]{a}b^{2/3}}\right)}{d}-\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$\Big\downarrow$ 750

$$\frac{2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{1}{3}\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\cdots}{\sqrt[3]{a}b^{2/3}}\cdots\right)}{d}\cdots$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$\Big\downarrow$ 16

$$\frac{2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}\sqrt[3]{a}}}{\sqrt[3]{a}b^{2/3}}\cdots\right)}{d}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$\Big\downarrow$ 27

$$\frac{2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\sqrt[3]{2}\int \frac{2^{2/3}-\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+\right.}{3\sqrt[3]{2}\sqrt[3]{a}}}{\sqrt[3]{a}b^{2/3}}\right.}{d}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$\Big\downarrow$ 1142

$$2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2\sqrt[3]{2}}-\frac{\int -\frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\cdots\right.}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\cdots}{\left(bx^3+a\right)^{2/3}}\cdots}\cdots}{\cdots}\right.}{\sqrt[3]{a}b^{2/3}\cdots}\right.$$

$$\frac{\cdots}{d}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$\downarrow$ 25

$$\frac{2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2\sqrt[3]{2}}+\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\frac{2\cdots}{\cdots}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}\cdots}\cdots}{\cdots}\cdots\right)}{\sqrt[3]{a}b^{2/3}\cdots}\cdots\right)}{d\cdots}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$\downarrow$ 27

$$\frac{2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2\sqrt[3]{2}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}\left(\cdots\right.}{\sqrt[3]{\cdots}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}\cdots}\cdots}{\cdots}\cdots\right.}{\sqrt[3]{a}b^{2/3}}\cdots\right.}{d}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$\Big\downarrow$ 1082

$$2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{2^{2/3}\sqrt[3]{a}} + \frac{\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\right.}{\sqrt[3]{bx^3+}}}}{2\sqrt[3]{2}}}{\sqrt[3]{a}b^{2/3}}\right.$$

$$\overline{\qquad d \qquad}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$\downarrow$ 217

$$\frac{2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\sqrt[3]{2}\left(\frac{\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{\cdots}\right.}{\sqrt[3]{a-\cdots}}}{\sqrt{3}\cdots}\right.}{2^{2/3}\sqrt[3]{a}\cdots}\right.}{\sqrt[3]{a}b^{2/3}\cdots}\right.}{d}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$\downarrow$ 1103

$$\frac{2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt[3]{a}}-\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\ 2^{2/3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}\right)}{d}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$\downarrow$ 2574

$$\frac{2a\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}}-\frac{3\log\left(\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}\right)}{4\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}}+\frac{\log\left(\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\right)}{4\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\sqrt[3]{2}\ldots}{}\right.}{d}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

input
```
Int[(x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/2*(x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)
/a)])/(d*(a + b*x^3)^(1/3)) + (2*a*(-(((2^(1/3)*(-((Sqrt[3]*ArcTan[(1 - (2
*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(2/3)*a^(1
/3))) - Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(
1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*2^(2/3)*a^(1/3))))/3 + L
og[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(1/3)*a^(1/
3)))/(a^(1/3)*b^(2/3))) + ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3
)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3)*a^(1/3)*b^(1/3)) + Log[(a^(1
/3) - b^(1/3)*x)^2*(a^(1/3) + b^(1/3)*x)]/(4*2^(1/3)*a^(1/3)*b^(1/3)) - (3
*Log[b^(1/3)*(a^(1/3) + b^(1/3)*x) - 2^(2/3)*b^(1/3)*(a + b*x^3)^(1/3)])/(
4*2^(1/3)*a^(1/3)*b^(1/3))))/(3*a^(1/3)*b^(1/3))))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 991
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Simp[-q^2/(3*d)   Int[1/((1 - q*x)*(a + b*x^3)^(1/3)
), x], x] + Simp[q/d   Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x
^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c +
 a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Maple [F]

$$\int \frac{x(b\,x^3+a)^{\frac{2}{3}}}{-bd\,x^3+ad}dx$$

input
```
int(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

output
```
int(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x}{bdx^3-ad}\,dx$$

input
```
integrate(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)*x/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x}{bdx^3-ad}\,dx$$

input
```
integrate(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)*x/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int \frac{x\,(b\,x^3+a)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

output
```
int((x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

Reduce [F]

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{\int \frac{(b\,x^3+a)^{\frac{2}{3}}x}{-b\,x^3+a}dx}{d}$$

input
```
int(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

output
```
int(((a + b*x**3)**(2/3)*x)/(a - b*x**3),x)/d
```

3.817 $\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 483

$$\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{adx} + \frac{2^{2/3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}d} + \frac{\sqrt[3]{b}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a^{2/3}d} + \frac{bx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2ad\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{b}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}a^{2/3}d} + \frac{\sqrt[3]{b}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}a^{2/3}d} - \frac{2^{2/3}\sqrt[3]{b}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{2/3}d} - \frac{\sqrt[3]{b}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}a^{2/3}d}$$

output
```
-(b*x^3+a)^(2/3)/a/d/x+1/3*2^(2/3)*b^(1/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3
)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/a^(2/3)/d+1/6*b^(1/3)*arcta
n(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(2/3)*3^(
1/2)/a^(2/3)/d+1/2*b*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x
^3/a)/a/d/(b*x^3+a)^(1/3)+1/12*b^(1/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(1/3)+b
^(1/3)*x)/a)*2^(2/3)/a^(2/3)/d+1/6*b^(1/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x
)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/3)/a
^(2/3)/d-1/3*2^(2/3)*b^(1/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1
/3))/a^(2/3)/d-1/4*b^(1/3)*ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3)*
b^(1/3)*(b*x^3+a)^(1/3)/a^(1/3))*2^(2/3)/a^(2/3)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.28

$$\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx = \frac{15abx^3\sqrt[3]{1+\frac{bx^3}{a}}\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2\left(5a(a+bx^3)+b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\right.}{10a^2dx\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)),x]
```

output
```
(15*a*b*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a),
 (b*x^3)/a] - 2*(5*a*(a + b*x^3) + b^2*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[
5/3, 1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a]))/(10*a^2*d*x*(a + b*x^3)^(1/3)
)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 467, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {975, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^2\left(ad-bdx^3\right)}dx$$

$$\Big\downarrow \; 975$$

$$\frac{\int \frac{bx\left(3a-bx^3\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{ad}-\frac{\left(a+bx^3\right)^{2/3}}{adx}$$

$$\Big\downarrow \; 27$$

$$\frac{b\int \frac{x\left(3a-bx^3\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{ad}-\frac{\left(a+bx^3\right)^{2/3}}{adx}$$

$$\Big\downarrow \; 1054$$

$$\frac{b\int \left(\frac{2ax}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}+\frac{x}{\sqrt[3]{bx^3+a}}\right)dx}{ad}-\frac{\left(a+bx^3\right)^{2/3}}{adx}$$

$$\Big\downarrow \; 2009$$

$$\frac{b\left(\frac{2^{2/3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{\sqrt[3]{a}\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{2/3}}+\frac{\sqrt[3]{a}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}{\sqrt[3]{a+bx^3}}\cdots\right.}{3\sqrt[3]{2}b^{2/3}}\cdots\right.}{\cdots}$$

$$\frac{\left(a+bx^3\right)^{2/3}}{adx}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)),x]
```

output
```
-((a + b*x^3)^(2/3)/(a*d*x)) + (b*((2^(2/3)*a^(1/3)*ArcTan[(1 - (2*2^(1/3)
*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) + (
a^(1/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqr
t[3]])/(2^(1/3)*Sqrt[3]*b^(2/3)) + (x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometr
ic2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(1/3)) + (a^(1/3)*Log[(
(a^(1/3) - b^(1/3)*x)^2*(a^(1/3) + b^(1/3)*x))/a])/(6*2^(1/3)*b^(2/3)) + (
a^(1/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(
1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*2^(1/3)*b^(2/3)) - (2^(
2/3)*a^(1/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(
3*b^(2/3)) - (a^(1/3)*Log[(b^(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(2/
3)*b^(1/3)*(a + b*x^3)^(1/3))/a^(1/3)])/(2*2^(1/3)*b^(2/3))))/(a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{2}{3}}}{x^{2}\,(-bd\,x^{3}+ad)}dx$$

input
```
int((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x)
```

output
```
int((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^2\,(ad-bdx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2\,(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^2+bx^5}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/x**2/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a*x**2 + b*x**5), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^2), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^2\,(a\,d-b\,d\,x^3)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx = \frac{\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{-b\,x^5+a\,x^2}dx}{d}$$

input
```
int((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x)
```

output
```
int((a + b*x**3)**(2/3)/(a*x**2 - b*x**5),x)/d
```

3.818 $\int \frac{(a+bx^3)^{2/3}}{x^5(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 512

$$\int \frac{(a+bx^3)^{2/3}}{x^5(ad-bdx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{4adx^4} - \frac{3b(a+bx^3)^{2/3}}{2a^2dx} + \frac{2^{2/3}b^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{5/3}d} + \frac{b^{4/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a^{5/3}d} + \frac{3b^2x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{4a^2d\sqrt[3]{a+bx^3}} + \frac{b^{4/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}a^{5/3}d} + \frac{b^{4/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}a^{5/3}d} - \frac{2^{2/3}b^{4/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{5/3}d} - \frac{b^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}a^{5/3}d}$$

output
```
-1/4*(b*x^3+a)^(2/3)/a/d/x^4-3/2*b*(b*x^3+a)^(2/3)/a^2/d/x+1/3*2^(2/3)*b^(
4/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))
*3^(1/2)/a^(5/3)/d+1/6*b^(4/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(
b*x^3+a)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)/a^(5/3)/d+3/4*b^2*x^2*(1+b*x^3/a)
^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/a^2/d/(b*x^3+a)^(1/3)+1/12*b^(
4/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/3)/a^(5/3)/d+1/6
*b^(4/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/
3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/3)/a^(5/3)/d-1/3*2^(2/3)*b^(4/3)*ln(1+
2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(5/3)/d-1/4*b^(4/3)*ln(b^(1
/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/a^(1/3))*2
^(2/3)/a^(5/3)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.29

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(ad-bdx^3)}\,dx = \frac{-5a(a^2+7abx^3+6b^2x^6)+35ab^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)-6b}{20a^3dx^4\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)),x]
```

output
```
(-5*a*(a^2 + 7*a*b*x^3 + 6*b^2*x^6) + 35*a*b^2*x^6*(1 + (b*x^3)/a)^(1/3)*A
ppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a] - 6*b^3*x^9*(1 + (b*x^3
)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a])/(20*a^3*d*
x^4*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 496, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {975, 27, 1053, 25, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^5(ad-bdx^3)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{2b(bx^3+3a)}{x^2(a-bx^3)\sqrt[3]{bx^3+a}}dx}{4ad}-\frac{(a+bx^3)^{2/3}}{4adx^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{bx^3+3a}{x^2(a-bx^3)\sqrt[3]{bx^3+a}}dx}{2ad}-\frac{(a+bx^3)^{2/3}}{4adx^4}$$

$$\Big\downarrow \text{1053}$$

$$\frac{b\left(-\frac{\int -\frac{abx(7a-3bx^3)}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{a^2}-\frac{3(a+bx^3)^{2/3}}{ax}\right)}{2ad}-\frac{(a+bx^3)^{2/3}}{4adx^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\int \frac{abx(7a-3bx^3)}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{a^2}-\frac{3(a+bx^3)^{2/3}}{ax}\right)}{2ad}-\frac{(a+bx^3)^{2/3}}{4adx^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{b\int \frac{x(7a-3bx^3)}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{a}-\frac{3(a+bx^3)^{2/3}}{ax}\right)}{2ad}-\frac{(a+bx^3)^{2/3}}{4adx^4}$$

$$\Big\downarrow \text{1054}$$

$$\frac{b\left(\frac{b\int \left(\frac{4ax}{(a-bx^3)\sqrt[3]{bx^3+a}}+\frac{3x}{\sqrt[3]{bx^3+a}}\right)dx}{a}-\frac{3(a+bx^3)^{2/3}}{ax}\right)}{2ad}-\frac{(a+bx^3)^{2/3}}{4adx^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{b\left(\frac{2\,2^{2/3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{2^{2/3}\sqrt[3]{a}\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{2^{2/3}\sqrt[3]{a}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}\cdots\right.}{\sqrt[3]{a+\cdots}}\right.}{3b^{2/3}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{\left(a+bx^3\right)^{2/3}}{4adx^4}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)),x]
```

output
```
-1/4*(a + b*x^3)^(2/3)/(a*d*x^4) + (b*((-3*(a + b*x^3)^(2/3))/(a*x) + (b*(
(2*2^(2/3)*a^(1/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^
3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) + (2^(2/3)*a^(1/3)*ArcTan[(1 + (2^(1
/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3))
 + (3*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/
a)])/(2*(a + b*x^3)^(1/3)) + (a^(1/3)*Log[((a^(1/3) - b^(1/3)*x)^2*(a^(1/3
) + b^(1/3)*x))/a])/(3*2^(1/3)*b^(2/3)) + (2^(2/3)*a^(1/3)*Log[1 + (2^(2/3
)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)
*x))/(a + b*x^3)^(1/3)])/(3*b^(2/3)) - (2*2^(2/3)*a^(1/3)*Log[1 + (2^(1/3)
*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*b^(2/3)) - (a^(1/3)*Log[(b^
(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(2/3)*b^(1/3)*(a + b*x^3)^(1/3))
/a^(1/3)])/(2^(1/3)*b^(2/3))))/a))/(2*a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{2}{3}}}{x^{5}\left(-bd\,x^{3}+ad\right)}dx$$

input
```
int((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x)
```

output
```
int((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(ad-bdx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^5+bx^8}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/x**5/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a*x**5 + b*x**8), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^5), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^5(ad-bdx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^5\,(a\,d-b\,d\,x^3)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5(ad-bdx^3)}\,dx = \frac{\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{-b\,x^8+a\,x^5}dx}{d}$$

input

```
int((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x)
```

output

```
int((a + b*x**3)**(2/3)/(a*x**5 - b*x**8),x)/d
```

3.819 $\int \frac{x^{14}}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{2}{5}\left(1-x^3\right)^{5/3} - \frac{1}{4}\left(1-x^3\right)^{8/3} + \frac{1}{11}\left(1-x^3\right)^{11/3} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
2/5*(-x^3+1)^(5/3)-1/4*(-x^3+1)^(8/3)+1/11*(-x^3+1)^(11/3)+1/6*arctan(1/3*
(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/12*ln(x^3+1)*2^(2/3)
+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.14

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{220}\left(1-x^3\right)^{2/3}\left(53-38x^3+5x^6-20x^9\right) + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}} - \frac{\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x^3)^{2/3}\right)}{6\sqrt[3]{2}}$$

input
```
Integrate[x^14/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((1 - x^3)^(2/3)*(53 - 38*x^3 + 5*x^6 - 20*x^9))/220 + ArcTan[(1 + 2^(2/3)
*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[-2 + 2^(2/3)*(1 - x^3)^
(1/3)]/(3*2^(1/3)) - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(
2/3)]/(6*2^(1/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(x^3+1)}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{x^{12}}{\sqrt[3]{1-x^3}\,(x^3+1)}dx^3$$

$$\Big\downarrow \; 99$$

$$\frac{1}{3}\int \left(-\left(1-x^3\right)^{8/3}+2\left(1-x^3\right)^{5/3}-2\left(1-x^3\right)^{2/3}+\frac{1}{\left(x^3+1\right)\sqrt[3]{1-x^3}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{3}{11}\left(1-x^3\right)^{11/3}-\frac{3}{4}\left(1-x^3\right)^{8/3}+\frac{6}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}\right.}{2}\right.$$

input
```
Int[x^14/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((6*(1 - x^3)^(5/3))/5 - (3*(1 - x^3)^(8/3))/4 + (3*(1 - x^3)^(11/3))/11 +
 (Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3) - Log[1 +
 x^3]/(2*2^(1/3)) + (3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*2^(1/3)))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 12.56 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{\left(2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}-\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)+2\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\right)2^{\frac{2}{3}}}{12}-\frac{\left(-x^{3}+\right.}{}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int(x^14/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*(2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-ln((-x^3+1)
^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))+2*ln((-x^3+1)^(1/3)-2^(1/3)))*2^(2/
3)-1/220*(-x^3+1)^(2/3)*(20*x^9-5*x^6+38*x^3-53)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = &-\frac{1}{220}\left(20\,x^9-5\,x^6+38\,x^3-53\right)\left(-x^3+1\right)^{\frac{2}{3}}\\ &+2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\arctan\left(2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)\\ &-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)\\ &+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)\end{aligned}$$

input
```
integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/220*(20*x^9 - 5*x^6 + 38*x^3 - 53)*(-x^3 + 1)^(2/3) + 2^(1/6)*sqrt(1/6)
*arctan(2^(1/6)*sqrt(1/6)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) - 1/12*2^(2/3)*l
og(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*lo
g(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Sympy [F]

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^{14}}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(x**14/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x**14/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{1}{11}\left(-x^3+1\right)^{\frac{11}{3}} - \frac{1}{4}\left(-x^3+1\right)^{\frac{8}{3}} \\ &\quad + \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) \\ &\quad + \frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}} - \frac{1}{12} \\ &\quad \cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) \\ &\quad + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)\end{aligned}$$

input
```
integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
1/11*(-x^3 + 1)^(11/3) - 1/4*(-x^3 + 1)^(8/3) + 1/6*sqrt(3)*2^(2/3)*arctan
(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 2/5*(-x^3 + 1)^(5/3
) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)
) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{11}\left(x^3-1\right)^3\left(-x^3+1\right)^{\frac{2}{3}} - \frac{1}{4}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{2}{3}} + \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}} - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

input
```
integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/11*(x^3 - 1)^3*(-x^3 + 1)^(2/3) - 1/4*(x^3 - 1)^2*(-x^3 + 1)^(2/3) + 1/
6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3)
)) + 2/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^
(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3
)))
```

Mupad [B] (verification not implemented)

Time = 3.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6} + \frac{2\left(1-x^3\right)^{5/3}}{5} - \frac{\left(1-x^3\right)^{8/3}}{4} + \frac{\left(1-x^3\right)^{11/3}}{11} + \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} - \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

input
```
int(x^14/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
(2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2*(1 - x^3)^(5/3))/5 - (1 -
x^3)^(8/3)/4 + (1 - x^3)^(11/3)/11 + (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/
3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x^3)^(1
/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

Reduce [F]

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^{14}}{(-x^3+1)^{\frac{1}{3}}\,x^3+(-x^3+1)^{\frac{1}{3}}}dx$$

input
```
int(x^14/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(x**14/(( - x**3 + 1)**(1/3)*x**3 + ( - x**3 + 1)**(1/3)),x)
```

3.820 $\int \frac{x^{11}}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 128

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{5}\left(1-x^3\right)^{5/3} - \frac{1}{8}\left(1-x^3\right)^{8/3} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output

```
-1/2*(-x^3+1)^(2/3)+1/5*(-x^3+1)^(5/3)-1/8*(-x^3+1)^(8/3)-1/6*arctan(1/3*(
1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*ln(x^3+1)*2^(2/3)-
1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.09

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{120}\left(-3\left(1-x^3\right)^{2/3}\left(17-2x^3+5x^6\right) -20\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-20\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+10\,2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}\right.\right.$$

input
```
Integrate[x^11/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(-3*(1 - x^3)^(2/3)*(17 - 2*x^3 + 5*x^6) - 20*2^(2/3)*Sqrt[3]*ArcTan[(1 +
2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 20*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^
(1/3)] + 10*2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2
/3)])/120
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{x^9}{\sqrt[3]{1-x^3}\,(x^3+1)}dx^3$$

$$\Big\downarrow \; 99$$

$$\frac{1}{3}\int \left(\left(1-x^3\right)^{5/3}-\left(1-x^3\right)^{2/3}-\frac{1}{(x^3+1)\sqrt[3]{1-x^3}}+\frac{1}{\sqrt[3]{1-x^3}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{3}{8}\left(1-x^3\right)^{8/3}+\frac{3}{5}\left(1-x^3\right)^{5/3}-\frac{3}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}-\frac{3\log\left(\sqrt[3]{2}\right.}{2}\right.$$

input
```
Int[x^11/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((-3*(1 - x^3)^(2/3))/2 + (3*(1 - x^3)^(5/3))/5 - (3*(1 - x^3)^(8/3))/8 -
(Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3) + Log[1 +
x^3]/(2*2^(1/3)) - (3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*2^(1/3)))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 13.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{\left(-2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}-2\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)+\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)\right)2^{\frac{2}{3}}}{12}-\frac{\left(-x^{3}\right.}{}$
trager	$\left(-\frac{1}{8}x^{6}+\frac{1}{20}x^{3}-\frac{17}{40}\right)\left(-x^{3}+1\right)^{\frac{2}{3}}+\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^{3}+4\right)^{2}+6\_Z\,\mathrm{RootOf}\left(\_Z^{3}+\right.\right.$
risch	Expression too large to display

input
```
int(x^11/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*(-2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-2*ln((-x^3
+1)^(1/3)-2^(1/3))+ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3)))*2^(2
/3)-1/40*(-x^3+1)^(2/3)*(5*x^6-2*x^3+17)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = & -\frac{1}{40}\left(5\,x^{6}-2\,x^{3}+17\right)\left(-x^{3}+1\right)^{\frac{2}{3}} \\ & -2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\arctan\left(2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right) \\ & +\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right) \\ & -\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\end{aligned}$$

input
```
integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/40*(5*x^6 - 2*x^3 + 17)*(-x^3 + 1)^(2/3) - 2^(1/6)*sqrt(1/6)*arctan(2^(
1/6)*sqrt(1/6)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/12*2^(2/3)*log(2^(2/3)
+ 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(2/3)*log(-2^(1/3)
+ (-x^3 + 1)^(1/3))
```

Sympy [F]

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^{11}}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(x**11/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = &-\frac{1}{8}\left(-x^3+1\right)^{\frac{8}{3}} \\ &-\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) \\ &+\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}}+\frac{1}{12} \\ &\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6} \\ &\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)-\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}\end{aligned}$$

input
```
integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
-1/8*(-x^3 + 1)^(8/3) - 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^
(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/5*(-x^3 + 1)^(5/3) + 1/12*2^(2/3)*log(2^(
2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(2/3)*log(-2^(
1/3) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 + 1)^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.99

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{8}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{2}{3}} - \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}} + \frac{1}{12} \cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{6} \cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) - \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}$$

input
```
integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/8*(x^3 - 1)^2*(-x^3 + 1)^(2/3) - 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)
*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/5*(-x^3 + 1)^(5/3) + 1/12*2^(
2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(2
/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/2*(-x^3 + 1)^(2/3)
```

Mupad [B] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.04

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{(1-x^3)^{5/3}}{5} - \frac{(1-x^3)^{2/3}}{2} - \frac{2^{2/3}\ln\left((1-x^3)^{1/3}-2^{1/3}\right)}{6} - \frac{(1-x^3)^{8/3}}{8} - \frac{2^{2/3}\ln\left((1-x^3)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} + \frac{2^{2/3}\ln\left((1-x^3)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

input
```
int(x^11/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
(1 - x^3)^(5/3)/5 - (1 - x^3)^(2/3)/2 - (2^(2/3)*log((1 - x^3)^(1/3) - 2^(
1/3)))/6 - (1 - x^3)^(8/3)/8 - (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^
(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 + (2^(2/3)*log((1 - x^3)^(1/3) -
(2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

Reduce [F]

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^{11}}{(-x^3+1)^{\frac{1}{3}}x^3+(-x^3+1)^{\frac{1}{3}}}dx$$

input
```
int(x^11/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(x**11/(( - x**3 + 1)**(1/3)*x**3 + ( - x**3 + 1)**(1/3)),x)
```

3.821 $\int \frac{x^8}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{x^8}{\sqrt[3]{1-x^3}(1+x^3)}\,dx = \frac{1}{5}(1-x^3)^{5/3} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
1/5*(-x^3+1)^(5/3)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2
/3)*3^(1/2)-1/12*ln(x^3+1)*2^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.31

$$\int \frac{x^8}{\sqrt[3]{1-x^3}(1+x^3)}\,dx = \frac{1}{60}\left(12(1-x^3)^{5/3} + 10\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 10\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - 5\,2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\right.\right.$$

input
```
Integrate[x^8/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(12*(1 - x^3)^(5/3) + 10*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/
3))/Sqrt[3]] + 10*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 5*2^(2/3)*Lo
g[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/60
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{x^6}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx^3$$

$$\Big\downarrow \ 99$$

$$\frac{1}{3}\int \left(\frac{1}{\sqrt[3]{1-x^3}\left(x^3+1\right)}-\left(1-x^3\right)^{2/3}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{3}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)$$

input
```
Int[x^8/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((3*(1 - x^3)^(5/3))/5 + (Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqr
t[3]])/2^(1/3) - Log[1 + x^3]/(2*2^(1/3)) + (3*Log[2^(1/3) - (1 - x^3)^(1/
3)])/(2*2^(1/3)))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 8.55 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$-\frac{x^{3}\left(-x^{3}+1\right)^{\frac{2}{3}}}{5}+\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{5}+\frac{2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)}{6}-\frac{2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{12}+\frac{\arctan\Big(}{}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int(x^8/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/5*x^3*(-x^3+1)^(2/3)+1/5*(-x^3+1)^(2/3)+1/6*2^(2/3)*ln((-x^3+1)^(1/3)-2
^(1/3))-1/12*2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))+1/6
*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{5}\,(x^3-1)\,(-x^3+1)^{\frac{2}{3}} + 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\arctan\left(2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(2^{\frac{1}{3}}+2\,(-x^3+1)^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\,(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)$$

input
```
integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/5*(x^3 - 1)*(-x^3 + 1)^(2/3) + 2^(1/6)*sqrt(1/6)*arctan(2^(1/6)*sqrt(1/
6)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-
x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)
^(1/3))
```

Sympy [F]

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^8}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(x**8/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}} - \frac{1}{12} \cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

input
```
integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1
)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}} - \frac{1}{12} \cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

input
```
integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1
)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1
/3)))
```

Mupad [B] (verification not implemented)

Time = 3.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.14

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}+\frac{\left(1-x^3\right)^{5/3}}{5} + \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} - \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

input
```
int(x^8/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
(2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (1 - x^3)^(5/3)/5 + (2^(2/3)*
log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12
 - (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)
*1i + 1))/12
```

Reduce [F]

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^8}{\left(-x^3+1\right)^{\frac{1}{3}}x^3+\left(-x^3+1\right)^{\frac{1}{3}}}dx$$

input
```
int(x^8/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(x**8/(( - x**3 + 1)**(1/3)*x**3 + ( - x**3 + 1)**(1/3)),x)
```

3.822 $\int \frac{x^5}{\sqrt[3]{1-x^3}(1+x^3)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)} \, dx = -\frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
-1/2*(-x^3+1)^(2/3)-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(
2/3)*3^(1/2)+1/12*ln(x^3+1)*2^(2/3)-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.29

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)} \, dx = \frac{1}{12}\Bigg(-6\left(1-x^3\right)^{2/3} - 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) + 2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\right.$$

input
```
Integrate[x^5/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(-6*(1 - x^3)^(2/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3
))/Sqrt[3]] - 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(2/3)*Log[2
+ 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 90, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{\sqrt[3]{1-x^3}\,(x^3+1)}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(-\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx^3-\frac{3}{2}\left(1-x^3\right)^{2/3}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{3}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{3}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}-\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{\sqrt[3]{2}}-\frac{3}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}-\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{3}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}-\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)$$

input

```
Int[x^5/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output

```
((-3*(1 - x^3)^(2/3))/2 - (Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sq
rt[3]])/2^(1/3) + Log[1 + x^3]/(2*2^(1/3)) - (3*Log[2^(1/3) - (1 - x^3)^(1
/3)])/(2*2^(1/3)))/3
```

Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 90

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 8.72 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$-\frac{(-x^3+1)^{\frac{2}{3}}}{2}-\frac{2^{\frac{2}{3}}\ln\left((-x^3+1)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)}{6}+\frac{2^{\frac{2}{3}}\ln\left((-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{12}-\frac{\arctan\left(\frac{\left(1+2^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}\right.}{3}\right.}{6}$
trager	$-\frac{(-x^3+1)^{\frac{2}{3}}}{2}+\frac{\operatorname{RootOf}(\_Z^3+4)\ln\left(\frac{-45\operatorname{RootOf}\left(\operatorname{RootOf}(\_Z^3+4)^2+6\_Z\operatorname{RootOf}(\_Z^3+4)+36\_Z^2\right)^2\operatorname{RootOf}(\_\right.}{}}{}$
risch	Expression too large to display

input
```
int(x^5/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-x^3+1)^(2/3)-1/6*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))+1/12*2^(2/3)*ln
((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))-1/6*arctan(1/3*(1+2^(2/3)*
(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\arctan\left(2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) - \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}$$

input
```
integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-2^(1/6)*sqrt(1/6)*arctan(2^(1/6)*sqrt(1/6)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))
) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)
) - 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 + 1)^(2/3)
```

Sympy [F]

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^5}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(x**5/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) - \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}$$

input
```
integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) - 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 + 1)^(2/3
)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) - \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}$$

input
```
integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) - 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/2*(-x^3 + 1)
^(2/3)
```

Mupad [B] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}-\frac{\left(1-x^3\right)^{2/3}}{2} - \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} + \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

input
```
int(x^5/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
(2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i
 + 1))/12 - (1 - x^3)^(2/3)/2 - (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3
^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x^3)^(1/3) -
 2^(1/3)))/6
```

Reduce [F]

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^5}{\left(-x^3+1\right)^{\frac{1}{3}}x^3+\left(-x^3+1\right)^{\frac{1}{3}}}dx$$

input
```
int(x^5/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(x**5/(( - x**3 + 1)**(1/3)*x**3 + ( - x**3 + 1)**(1/3)),x)
```

3.823 $\int \frac{x^2}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(1+x^3)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/12*ln
(x^3+1)*2^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$
$$=\frac{2\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+2\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x^3)^{2/3}\right)}{6\sqrt[3]{2}}$$

input
```
Integrate[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 2*Log[-2 + 2^(2
/3)*(1 - x^3)^(1/3)] - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)
^(2/3)])/(6*2^(1/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {946, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{1}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx^3$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}+\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)$$

input
```
Int[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3) - Log[1 +
 x^3]/(2*2^(1/3)) + (3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*2^(1/3)))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 6.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$\frac{\left(2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}-\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)+2\ln\left(\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\right)2^{\frac{2}{3}}}{12}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^3-4\right)\ln\left(\frac{-6\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_Z^3-4\right)^2+6\_Z\operatorname{RootOf}\left(\_Z^3-4\right)+36\_Z^2\right)\operatorname{RootOf}\left(\_Z^3-4\right)^3x^3-45\operatorname{RootO}}{}\right.}{}$

input
```
int(x^2/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*(2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-ln((-x^3+1)
^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))+2*ln((-x^3+1)^(1/3)-2^(1/3)))*2^(2/
3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\arctan\left(2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

input
```
integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
2^(1/6)*sqrt(1/6)*arctan(2^(1/6)*sqrt(1/6)*(2^(1/3) + 2*(-x^3 + 1)^(1/3)))
 - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3))
 + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Sympy [F]

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^2}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)\\&\quad-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)\\&\quad+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)\end{aligned}$$

input
```
integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}} \arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

input
```
integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6} + \frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} - \frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

input
```
int(x^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
(2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2^(2/3)*log((1 - x^3)^(1/3)
- (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 -
 x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^2}{(-x^3+1)^{\frac{1}{3}}\,x^3+(-x^3+1)^{\frac{1}{3}}}dx$$

input
```
int(x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(x**2/(( - x**3 + 1)**(1/3)*x**3 + ( - x**3 + 1)**(1/3)),x)
```

3.824 $\int \frac{1}{x\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 137

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{2} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(1+2^(
2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/2*ln(x)+1/12*ln(x^3+1)*2^(
2/3)+1/2*ln(1-(-x^3+1)^(1/3))-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.35

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{12}\left(4\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 4\log\left(-1+\sqrt[3]{1-x^3}\right) - 2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - 2\log\left(\right.\right.$$

input
```
Integrate[1/(x*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] - 2*2^(2/3)*Sqrt[3]*Arc
Tan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 4*Log[-1 + (1 - x^3)^(1/3)] -
 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 2*Log[1 + (1 - x^3)^(1/3) +
 (1 - x^3)^(2/3)] + 2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 -
 x^3)^(2/3)])/12
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {948, 97, 67, 16, 1082, 217, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^3\sqrt[3]{1-x^3}\,(x^3+1)}dx^3$$

$$\Big\downarrow \text{97}$$

$$\frac{1}{3}\left(\int \frac{1}{x^3\sqrt[3]{1-x^3}}dx^3-\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx^3\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}+\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}+\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}-\frac{3}{2}\int \frac{}{x^6\ldots}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{1}{2}\log\left(x^3\right)+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3}{2}\right.$$

$\Big\downarrow$ 1082

$$\frac{1}{3}\left(\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{\sqrt[3]{2}}+\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}-\frac{1}{2}\log\left(x^3\right)+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3}{2}\log\left(1-\right.\right.$$

$\Big\downarrow$ 217

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3\right)}{2}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3}{2}\log\left(1-\sqrt[3]{}\right.\right.$$

$\Big\downarrow$ 1083

$$\frac{1}{3}\left(-3\int \frac{1}{-x^6-3}d\left(2\sqrt[3]{1-x^3}+1\right)-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3\right)}{2}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3}{2}\log\left(1-\sqrt[3]{}\right.\right.$$

$\Big\downarrow$ 217

$$\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3\right)}{2}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3}{2}\log\left(1-\sqrt[3]{1-}\right.\right.$$

input
```
Int[1/(x*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] - (Sqrt[3]*ArcTan[(1 + 2^
(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3) - Log[x^3]/2 + Log[1 + x^3]/(2*2^
(1/3)) + (3*Log[1 - (1 - x^3)^(1/3)])/2 - (3*Log[2^(1/3) - (1 - x^3)^(1/3)
])/(2*2^(1/3)))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 3.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$-\frac{2^{\frac{2}{3}} \ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)}{6}+\frac{2^{\frac{2}{3}} \ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{12}-\frac{\arctan\left(\frac{\left(1+2^{\frac{2}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}\right) \sqrt{3}}{3}\right) 2^{\frac{2}{3}} \sqrt{3}}{6}+$

input
```
int(1/x/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))+1/12*2^(2/3)*ln((-x^3+1)^(2/3)+2^(
1/3)*(-x^3+1)^(1/3)+2^(2/3))-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(
1/2))*2^(2/3)*3^(1/2)+1/3*ln(-1+(-x^3+1)^(1/3))-1/6*ln((-x^3+1)^(2/3)+(-x^
3+1)^(1/3)+1)+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.99

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/12*2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))*log(1/8*(I*sqrt(3)*(-1)^(
1/3) - (-1)^(1/3))^3 - 3/4*2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2 +
 3*(-x^3 + 1)^(1/3) + 1) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3
)) - 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2))*log
(3/8*2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2
)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 3/8*2^(1/3)*(I*sqrt(3)*(-1)^(1/3)
- (-1)^(1/3))^2 + 3*(-x^3 + 1)^(1/3)) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3
) - (-1)^(1/3)) + 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(
1/3))^2))*log(-3/8*2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) -
 (-1)^(1/3))^2)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 3/8*2^(1/3)*(I*sqrt(
3)*(-1)^(1/3) - (-1)^(1/3))^2 + 3*(-x^3 + 1)^(1/3)) + 1/3*sqrt(3)*arctan(2
/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 1/3*log(-1/24*(I*sqrt(3)*(-1)
^(1/3) - (-1)^(1/3))^3 + (-x^3 + 1)^(1/3) - 4/3) - 1/6*log((-x^3 + 1)^(2/3
) + (-x^3 + 1)^(1/3) + 1)
```

Sympy [F]

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(1/x/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x}\,dx$$

input
```
integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) + \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{1}{6}\,\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right) + \frac{1}{3}\,\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) - 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/3*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 + 1)^(2/3) + (-
x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.87

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{\ln\left(6-6\left(1-x^3\right)^{1/3}\right)}{3}$$

$$+\ln\left(\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^3\left(1458\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2-135\left(1-x^3\right)^{1/3}\right)-\left(1-x^3\right)^{1/3}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln$$

input
```
int(1/(x*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
log(6 - 6*(1 - x^3)^(1/3))/3 + log(((3^(1/2)*1i)/6 - 1/6)^3*(1458*((3^(1/2
)*1i)/6 - 1/6)^2 - 135*(1 - x^3)^(1/3)) - (1 - x^3)^(1/3))*((3^(1/2)*1i)/6
 - 1/6) - log(- ((3^(1/2)*1i)/6 + 1/6)^3*(1458*((3^(1/2)*1i)/6 + 1/6)^2 -
135*(1 - x^3)^(1/3)) - (1 - x^3)^(1/3))*((3^(1/2)*1i)/6 + 1/6) - (2^(2/3)*
log((3*(1 - x^3)^(1/3))/2 - (3*2^(1/3))/2))/6 + ((-1)^(1/3)*2^(2/3)*log((3
*(1 - x^3)^(1/3))/2 - (3*(-1)^(2/3)*2^(1/3))/2))/6 - ((-1)^(1/3)*2^(2/3)*l
og(- ((3^(1/2)*1i + 1)^3*(135*(1 - x^3)^(1/3) - (81*(-1)^(2/3)*2^(1/3)*(3^
(1/2)*1i + 1)^2)/4))/432 - (1 - x^3)^(1/3))*(3^(1/2)*1i + 1))/12
```

Reduce [F]

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(-x^3+1)^{\frac{1}{3}}\,x^4+(-x^3+1)^{\frac{1}{3}}\,x}dx$$

input
```
int(1/x/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(1/(( - x**3 + 1)**(1/3)*x**4 + ( - x**3 + 1)**(1/3)*x),x)
```

3.825 $\int \frac{1}{x^4 \sqrt[3]{1-x^3}(1+x^3)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{1}{x^4 \sqrt[3]{1-x^3} (1+x^3)} \, dx = -\frac{(1-x^3)^{2/3}}{3x^3} - \frac{2\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(x)}{3} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
-1/3*(-x^3+1)^(2/3)/x^3-2/9*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/
2)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/3*
ln(x)-1/12*ln(x^3+1)*2^(2/3)-1/3*ln(1-(-x^3+1)^(1/3))+1/4*ln(2^(1/3)-(-x^3
+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{36}\left(-\frac{12(1-x^3)^{2/3}}{x^3} - 8\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)\right.$$
$$+6\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-8\log\left(-1+\sqrt[3]{1-x^3}\right)+6\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+4\log\left(\right.$$

input
```
Integrate[1/(x^4*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((-12*(1 - x^3)^(2/3))/x^3 - 8*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt
[3]] + 6*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 8
*Log[-1 + (1 - x^3)^(1/3)] + 6*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] +
 4*Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 3*2^(2/3)*Log[2 + 2^(2/3)*
(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/36
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {948, 114, 27, 174, 67, 16, 1082, 217, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(x^3+1)}dx^3$$

$$\Big\downarrow \; 114$$

$$\frac{1}{3}\left(-\int \frac{2-x^3}{3x^3\sqrt[3]{1-x^3}\,(x^3+1)}dx^3-\frac{(1-x^3)^{2/3}}{x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{1}{3}\int \frac{2-x^3}{x^3\sqrt[3]{1-x^3}\left(x^3+1\right)}dx^3-\frac{\left(1-x^3\right)^{2/3}}{x^3}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(3\int \frac{1}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx^3-2\int \frac{1}{x^3\sqrt[3]{1-x^3}}dx^3\right)-\frac{\left(1-x^3\right)^{2/3}}{x^3}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\Bigg(\frac{1}{3}\Bigg(3\left(-\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}+\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}\right)-2\Bigg(-\frac{3}{2}\int$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\Bigg(\frac{1}{3}\Bigg(3\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)-2\Bigg(\frac{3}{2}\int \frac{}{x^6+\sqrt[3]{}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\Bigg(\frac{1}{3}\Bigg(3\left(-\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)-2\Bigg(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\Bigg(\frac{1}{3}\Bigg(3\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)-2\Bigg(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\Bigg(\frac{1}{3}\Bigg(3\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)-2\Bigg(-3\int \frac{1}{-x^6-3}d\Big(2\sqrt[3]{1}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(3\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)-2\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^3}}{\sqrt{3}}\right.\right.\right.\right.$$

input
```
Int[1/(x^4*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(-((1 - x^3)^(2/3)/x^3) + (-2*(Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt
[3]] - Log[x^3]/2 + (3*Log[1 - (1 - x^3)^(1/3)])/2) + 3*((Sqrt[3]*ArcTan[(
1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3) - Log[1 + x^3]/(2*2^(1/3))
+ (3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*2^(1/3))))/3)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 5.13 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.34

method	result
pseudoelliptic	$\frac{-6\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right) x^{3}-6\,2^{\frac{2}{3}} \ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right) x^{3}+3\,2^{\frac{2}{3}} \ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right) \cdots}{36\left(-1+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\left(\left(\cdots\right.\right.}$

input
```
int(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/36*(-6*3^(1/2)*2^(2/3)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*x^
3-6*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*x^3+3*2^(2/3)*ln((-x^3+1)^(2/3)+2^(
1/3)*(-x^3+1)^(1/3)+2^(2/3))*x^3+8*3^(1/2)*arctan(1/3*(1+2*(-x^3+1)^(1/3))
*3^(1/2))*x^3+8*ln(-1+(-x^3+1)^(1/3))*x^3-4*ln((-x^3+1)^(2/3)+(-x^3+1)^(1/
3)+1)*x^3+12*(-x^3+1)^(2/3))/(-1+(-x^3+1)^(1/3))/((-x^3+1)^(2/3)+(-x^3+1)^
(1/3)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{36\cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}x^3\arctan\left(2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-3\cdot 2^{\frac{2}{3}}x^3\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+}{}$$

input
```
integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/36*(36*2^(1/6)*sqrt(1/6)*x^3*arctan(2^(1/6)*sqrt(1/6)*(2^(1/3) + 2*(-x^3
 + 1)^(1/3))) - 3*2^(2/3)*x^3*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x
^3 + 1)^(2/3)) + 6*2^(2/3)*x^3*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - 8*sqrt(3
)*x^3*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 4*x^3*log((-x^3
 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 8*x^3*log((-x^3 + 1)^(1/3) - 1) - 12
*(-x^3 + 1)^(2/3))/x^3
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^4\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(1/x**4/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^4}\,dx$$

input
```
integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = {} & \frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\,(-x^3+1)^{\frac{1}{3}}\right)\right) \\ & - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right) \\ & + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right|\right) \\ & - \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,(-x^3+1)^{\frac{1}{3}}+1\right)\right) - \frac{(-x^3+1)^{\frac{2}{3}}}{3\,x^3} \\ & + \frac{1}{9}\,\log\left((-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}+1\right) \\ & - \frac{2}{9}\,\log\left(\left|(-x^3+1)^{\frac{1}{3}}-1\right|\right)\end{aligned}$$

input
```
integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 2/9*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/3*(-x^3 + 1)^(2/3)/x^3 + 1/9
*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 2/9*log(abs((-x^3 + 1)^(1/
3) - 1))
```

Mupad [B] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.43

$$\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{2^{2/3}\ln\left(\frac{2^{1/3}\left(\frac{2^{2/3}\left(81\,2^{1/3}-75\,(1-x^3)^{1/3}\right)}{6}-\frac{38}{3}\right)}{18}+\frac{16\,(1-x^3)^{1/3}}{27}\right)}{6} - \frac{(1-x^3)^{2/3}}{3\,x^3} - \frac{2\ln\left(\frac{344\,(1-x^3)^{1/3}}{243}-\frac{344}{243}\right)}{9}$$

$$+\ln\left(\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2\left(\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)\left(1458\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2-75\,(1-x^3)^{1/3}\right)-\frac{38}{3}\right)+\frac{16\,(1-x^3)^{1/3}}{27}\right)$$

input
```
int(1/(x^4*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
(2^(2/3)*log((2^(1/3)*((2^(2/3)*(81*2^(1/3) - 75*(1 - x^3)^(1/3)))/6 - 38/
3))/18 + (16*(1 - x^3)^(1/3))/27))/6 - (1 - x^3)^(2/3)/(3*x^3) - (2*log((3
44*(1 - x^3)^(1/3))/243 - 344/243))/9 + log(((3^(1/2)*1i)/9 + 1/9)^2*(((3^
(1/2)*1i)/9 + 1/9)*(1458*((3^(1/2)*1i)/9 + 1/9)^2 - 75*(1 - x^3)^(1/3)) -
38/3) + (16*(1 - x^3)^(1/3))/27)*((3^(1/2)*1i)/9 + 1/9) - log((16*(1 - x^3
)^(1/3))/27 - ((3^(1/2)*1i)/9 - 1/9)^2*(((3^(1/2)*1i)/9 - 1/9)*(1458*((3^(
1/2)*1i)/9 - 1/9)^2 - 75*(1 - x^3)^(1/3)) + 38/3))*((3^(1/2)*1i)/9 - 1/9)
+ (2^(2/3)*log((16*(1 - x^3)^(1/3))/27 + (2^(1/3)*(3^(1/2)*1i - 1)^2*((2^(
2/3)*(3^(1/2)*1i - 1)*((81*2^(1/3)*(3^(1/2)*1i - 1)^2)/4 - 75*(1 - x^3)^(1
/3)))/12 - 38/3))/72)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((16*(1 - x^3)^(1
/3))/27 - (2^(1/3)*(3^(1/2)*1i + 1)^2*((2^(2/3)*(3^(1/2)*1i + 1)*((81*2^(1
/3)*(3^(1/2)*1i + 1)^2)/4 - 75*(1 - x^3)^(1/3)))/12 + 38/3))/72)*(3^(1/2)*
1i + 1))/12
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(-x^3+1)^{\frac{1}{3}}\,x^7+(-x^3+1)^{\frac{1}{3}}\,x^4}dx$$

input
```
int(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(1/(( - x**3 + 1)**(1/3)*x**7 + ( - x**3 + 1)**(1/3)*x**4),x)
```

3.826 $\int \frac{x^6}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 154

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{3}x\left(1-x^3\right)^{2/3} + \frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{1}{3}\log\left(x+\sqrt[3]{1-x^3}\right)$$

output
```
-1/3*x*(-x^3+1)^(2/3)+2/9*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/
2)-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-
1/12*ln(x^3+1)*2^(2/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/3*ln(x+
(-x^3+1)^(1/3))
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.43

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{36}\left(-12x(1-x^3)^{2/3} + 8\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)\right.$$
$$-6\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-8\log\left(x+\sqrt[3]{1-x^3}\right)+6\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+4\log\left(x^2\right.$$

input
```
Integrate[x^6/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(-12*x*(1 - x^3)^(2/3) + 8*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/
3))] - 6*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))]
 - 8*Log[x + (1 - x^3)^(1/3)] + 6*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3
)] + 4*Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 3*2^(2/3)*Log[-2*x
^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/36
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {979, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(x^3+1)}\,dx$$
$$\Big\downarrow \text{979}$$
$$\frac{1}{3}\int \frac{1-2x^3}{\sqrt[3]{1-x^3}\,(x^3+1)}dx - \frac{1}{3}x(1-x^3)^{2/3}$$
$$\Big\downarrow \text{1026}$$
$$\frac{1}{3}\left(3\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx - 2\int \frac{1}{\sqrt[3]{1-x^3}}dx\right) - \frac{1}{3}x(1-x^3)^{2/3}$$

$$\Big\downarrow \text{769}$$

$$\frac{1}{3}\left(3\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-2\left(\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)-\frac{1}{3}x\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \text{901}$$

$$\frac{1}{3}\left(3\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}\right)-2\left(\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\text{arc}\right.\right.$$

$$\frac{1}{3}x\left(1-x^3\right)^{2/3}$$

input
```
Int[x^6/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-1/3*(x*(1 - x^3)^(2/3)) + (3*(-(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3)
)/Sqrt[3]]/(2^(1/3)*Sqrt[3])) - Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x
) - (1 - x^3)^(1/3)]/(2*2^(1/3))) - 2*(-(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3)
)/Sqrt[3]]/Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/2))/3
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

Maple [A] (verified)

Time = 6.04 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.51

method	result
pseudoelliptic	$\frac{6\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)+6\,2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)-3\,2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)-}{36\left(\left(-x^{3}+1\right)^{\frac{2}{3}}-\left(-x^{3}+1\right)^{\frac{1}{3}}\right.}$

input
```
int(x^6/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/36*(6*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)+
6*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)-3*2^(2/3)*ln((2^(2/3)*x^2-2^(1/
3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)-12*x*(-x^3+1)^(2/3)-8*3^(1/2)*arc
tan(1/3*(-2*(-x^3+1)^(1/3)+x)*3^(1/2)/x)+4*ln(((-x^3+1)^(2/3)-(-x^3+1)^(1/
3)*x+x^2)/x^2)-8*ln((x+(-x^3+1)^(1/3))/x))/((-x^3+1)^(2/3)-(-x^3+1)^(1/3)*
x+x^2)/(x+(-x^3+1)^(1/3))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.29

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{3}\left(-x^3+1\right)^{\frac{2}{3}}x - 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\arctan\left(-\frac{2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(2^{\frac{1}{3}}x-2\left(-x^3+1\right)^{\frac{1}{3}}\right)}{x}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right) + \frac{2}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3\,x}\right) - \frac{2}{9}\log\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right) + \frac{1}{9}\log\left(\frac{x^2-\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

input
```
integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/3*(-x^3 + 1)^(2/3)*x - 2^(1/6)*sqrt(1/6)*arctan(-2^(1/6)*sqrt(1/6)*(2^(
1/3)*x - 2*(-x^3 + 1)^(1/3))/x) + 1/6*2^(2/3)*log((2^(1/3)*x + (-x^3 + 1)^
(1/3))/x) - 1/12*2^(2/3)*log((2^(2/3)*x^2 - 2^(1/3)*(-x^3 + 1)^(1/3)*x + (
-x^3 + 1)^(2/3))/x^2) + 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x
^3 + 1)^(1/3))/x) - 2/9*log((x + (-x^3 + 1)^(1/3))/x) + 1/9*log((x^2 - (-x
^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [F]

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^6}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(x**6/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [F]

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\int \frac{x^6}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(x^6/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(x^6/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\int \frac{x^6}{(-x^3+1)^{\frac{1}{3}}\,x^3+(-x^3+1)^{\frac{1}{3}}}dx$$

input
```
int(x^6/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(x**6/(( - x**3 + 1)**(1/3)*x**3 + ( - x**3 + 1)**(1/3)),x)
```

3.827 $\int \frac{x^3}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

output
```
-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1-
2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*ln(x^3+1)*2^(2/3
)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+1/2*ln(x+(-x^3+1)^(1/3))
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.52

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{12}\left(-4\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)\right.$$
$$+2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)+4\log\left(x+\sqrt[3]{1-x^3}\right)-2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)-2\log\left(x^2\right.$$

input
```
Integrate[x^3/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(-4*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/3))] + 2*2^(2/3)*Sqrt[3
]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] + 4*Log[x + (1 - x^3)^
(1/3)] - 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] - 2*Log[x^2 - x*(1 -
 x^3)^(1/3) + (1 - x^3)^(2/3)] + 2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^
(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {983, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(x^3+1)}\,dx$$
$$\Big\downarrow \text{983}$$
$$\int \frac{1}{\sqrt[3]{1-x^3}}dx-\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$
$$\Big\downarrow \text{769}$$

$$-\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx - \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)$$

$$\Big\downarrow \text{901}$$

$$-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}} + \frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)$$

input
```
Int[x^3/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 - (2*2^
(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(
1/3)) - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3)) + Log[x + (1 - x^3
)^(1/3)]/2
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.33

method	result
pseudoelliptic	$-\frac{\ln\left(\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}-\left(-x^{3}+1\right)^{\frac{1}{3}} x+x^{2}}{x^{2}}\right)}{6}+\frac{\sqrt{3}\, \arctan\left(\frac{\left(-2\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)\sqrt{3}}{3 x}\right)}{3}-\frac{2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)}{6}+\frac{2^{\frac{2}{3}} \ln\left(\cdots\right.}{}$ …

input
```
int(x^3/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(((-x^3+1)^(2/3)-(-x^3+1)^(1/3)*x+x^2)/x^2)+1/3*3^(1/2)*arctan(1/3*
(-2*(-x^3+1)^(1/3)+x)*3^(1/2)/x)-1/6*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))
/x)+1/12*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/
x^2)-1/6*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)
+1/3*ln((x+(-x^3+1)^(1/3))/x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 452, normalized size of antiderivative = 3.35

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/12*2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))*log(-1/8*(x*(I*sqrt(3)*(-
1)^(1/3) - (-1)^(1/3))^3 - 6*2^(1/3)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))
^2 + 8*x - 24*(-x^3 + 1)^(1/3))/x) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) -
 (-1)^(1/3)) - 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3
))^2))*log(-3/8*(2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (
-1)^(1/3))^2)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 2^(1/3)*x*(I*sqrt(3)
*(-1)^(1/3) - (-1)^(1/3))^2 - 8*(-x^3 + 1)^(1/3))/x) - 1/24*(2^(2/3)*(I*sq
rt(3)*(-1)^(1/3) - (-1)^(1/3)) + 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)
^(1/3) - (-1)^(1/3))^2))*log(3/8*(2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(
3)*(-1)^(1/3) - (-1)^(1/3))^2)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) - 2^(
1/3)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2 + 8*(-x^3 + 1)^(1/3))/x) - 1/
3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/3*lo
g(1/24*(x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^3 + 32*x + 24*(-x^3 + 1)^(1/
3))/x) - 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^3}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(x**3/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^3}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(x^3/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(x^3/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^3}{(-x^3+1)^{\frac{1}{3}}\,x^3+(-x^3+1)^{\frac{1}{3}}}dx$$

input
```
int(x^3/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(x**3/(( - x**3 + 1)**(1/3)*x**3 + ( - x**3 + 1)**(1/3)),x)
```

3.828 $\int \frac{1}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 88

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output

```
-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/
12*ln(x^3+1)*2^(2/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 2\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + \log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}(1-x\right.}{6\sqrt[3]{2}}$$

input
```
Integrate[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*Log[
2*x + 2^(2/3)*(1 - x^3)^(1/3)] + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) -
2^(1/3)*(1 - x^3)^(2/3)])/2^(1/3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{901}$$

$$-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}$$

input
```
Int[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) -
 Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3)
)
```

Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

method	result	size
pseudoelliptic	$\frac{2^{\frac{2}{3}}\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)+\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{2^{\frac{2}{3}}x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}\right)}{6}$	95
trager	Expression too large to display	616

input
```
int(1/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(2/3)*(3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)+ln(
(2^(1/3)*x+(-x^3+1)^(1/3))/x)-1/2*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x
+(-x^3+1)^(2/3))/x^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(67) = 134$.

Time = 1.58 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.80

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{3}$$
$$\cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(6\cdot 2^{\frac{2}{3}}(5\,x^7+4\,x^4-x)(-x^3+1)^{\frac{2}{3}}-2^{\frac{1}{3}}(71\,x^9-111\,x^6+33\,x^3-1)+12\,(19\,x^8}{109\,x^9-105\,x^6+3\,x^3+1}\right.$$
$$+\frac{1}{18}\cdot 2^{\frac{2}{3}}\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}x^2+2^{\frac{2}{3}}(x^3+1)+6\,(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\right)-\frac{1}{36}$$
$$\cdot 2^{\frac{2}{3}}\log\left(\frac{3\cdot 2^{\frac{2}{3}}(5\,x^4-x)(-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(19\,x^6-16\,x^3+1)-12\,(2\,x^5-x^2)(-x^3+1)^{\frac{1}{3}}}{x^6+2\,x^3+1}\right)$$

input
```
integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/3*2^(1/6)*sqrt(1/6)*arctan(2^(1/6)*sqrt(1/6)*(6*2^(2/3)*(5*x^7 + 4*x^4
- x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1) + 12*(19*x
^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) + 1/
18*2^(2/3)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2/3)*(x^3 + 1) + 6*(-x
^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/36*2^(2/3)*log((3*2^(2/3)*(5*x^4 - x)*(-x^
3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)*(-x^3 + 1)
^(1/3))/(x^6 + 2*x^3 + 1))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(1/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(1/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(1/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(-x^3+1)^{\frac{1}{3}}\,x^3 + (-x^3+1)^{\frac{1}{3}}}dx$$

input
```
int(1/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(1/(( - x**3 + 1)**(1/3)*x**3 + ( - x**3 + 1)**(1/3)),x)
```

3.829 $\int \frac{1}{x^3\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{1}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{(1-x^3)^{2/3}}{2x^2} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
-1/2*(-x^3+1)^(2/3)/x^2+1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1
/2))*2^(2/3)*3^(1/2)+1/12*ln(x^3+1)*2^(2/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/
3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{12}\left(-\frac{6(1-x^3)^{2/3}}{x^2} + 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + 2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}\cdots\right.\right.$$

input
```
Integrate[1/(x^3*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((-6*(1 - x^3)^(2/3))/x^2 + 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2
/3)*(1 - x^3)^(1/3))] - 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(
2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {980, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{ 980}$$

$$\frac{1}{2}\int -\frac{2}{\sqrt[3]{1-x^3}\,(x^3+1)}dx - \frac{(1-x^3)^{2/3}}{2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx - \frac{(1-x^3)^{2/3}}{2x^2}$$

$$\Big\downarrow \text{ 901}$$

$$\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}-\frac{\left(1-x^3\right)^{2/3}}{2x^2}$$

input
```
Int[1/(x^3*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-1/2*(1 - x^3)^(2/3)/x^2 + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt
[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)) - Log[-(2^(1/3)*x) - (1
- x^3)^(1/3)]/(2*2^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

Maple [A] (verified)

Time = 27.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.20

method	result
pseudoelliptic	$\frac{-2\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)x^{2}-2\,2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)x^{2}+2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}}x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)}{x^{2}}\right.}{12x^{2}}$
risch	Expression too large to display
trager	Expression too large to display

input
```
int(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*(-2*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)
*x^2-2*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*x^2+2^(2/3)*ln((2^(2/3)*x^
2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)*x^2-6*(-x^3+1)^(2/3))/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(81) = 162$.

Time = 1.83 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.57

$$\int \frac{1}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{12\cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}x^2\arctan\left(\frac{2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(6\cdot 2^{\frac{2}{3}}\left(5x^{7}+4x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(71x^{9}-111x^{6}+33x^{3}-1\right)+12\left(19x^{8}-16x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}\right)}{109x^{9}-105x^{6}+3x^{3}+1}\right) -}{}$$

input
```
integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/36*(12*2^(1/6)*sqrt(1/6)*x^2*arctan(2^(1/6)*sqrt(1/6)*(6*2^(2/3)*(5*x^7
+ 4*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1) +
12*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 + 3*x^3 +
1)) - 2*2^(2/3)*x^2*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2/3)*(x^3 + 1
) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*x^2*log((3*2^(2/3)*(5*x^4 -
 x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)*(-
x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 18*(-x^3 + 1)^(2/3))/x^2
```

Sympy [F]

$$\int \frac{1}{x^3 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^3 \sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(1/x**3/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

Maxima [F]

$$\int \frac{1}{x^3 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^3}\,dx$$

input
```
integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^3}\,dx$$

input
```
integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^3\,(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(1/(x^3*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^3*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(-x^3+1)^{\frac{1}{3}}\,x^6+(-x^3+1)^{\frac{1}{3}}\,x^3}dx$$

input
```
int(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(1/(( - x**3 + 1)**(1/3)*x**6 + ( - x**3 + 1)**(1/3)*x**3),x)
```

3.830 $\int \frac{1}{x^6 \sqrt[3]{1-x^3}(1+x^3)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 124

$$\int \frac{1}{x^6 \sqrt[3]{1-x^3}\,(1+x^3)} \, dx = -\frac{(1-x^3)^{2/3}}{5x^5} + \frac{(1-x^3)^{2/3}}{5x^2} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output

```
-1/5*(-x^3+1)^(2/3)/x^5+1/5*(-x^3+1)^(2/3)/x^2-1/6*arctan(1/3*(1-2*2^(1/3)
*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/12*ln(x^3+1)*2^(2/3)+1/4*ln(
-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{60}\left(-\frac{12(1-x^3)^{5/3}}{x^5}\right.$$
$$-10\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)+10\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)-5\,2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-\right.$$

input
```
Integrate[1/(x^6*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((-12*(1 - x^3)^(5/3))/x^5 - 10*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^
(2/3)*(1 - x^3)^(1/3))] + 10*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] -
5*2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)
])/60
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {980, 25, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(x^3+1)}\,dx$$
$$\Big\downarrow \text{980}$$
$$\frac{1}{5}\int -\frac{2-3x^3}{x^3\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{(1-x^3)^{2/3}}{5x^5}$$
$$\Big\downarrow \text{25}$$
$$-\frac{1}{5}\int \frac{2-3x^3}{x^3\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{(1-x^3)^{2/3}}{5x^5}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{5}\left(\frac{1}{2}\int \frac{10}{\sqrt[3]{1-x^3}\,(x^3+1)}dx+\frac{(1-x^3)^{2/3}}{x^2}\right)-\frac{(1-x^3)^{2/3}}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(5\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx+\frac{(1-x^3)^{2/3}}{x^2}\right)-\frac{(1-x^3)^{2/3}}{5x^5}$$

$$\Big\downarrow \text{901}$$

$$\frac{1}{5}\left(5\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(x^3+1)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}\right)+\frac{(1-x^3)^{2/3}}{x^2}\right)-\frac{(1-x^3)^{2/3}}{5x^5}$$

input
```
Int[1/(x^6*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-1/5*(1 - x^3)^(2/3)/x^5 + ((1 - x^3)^(2/3)/x^2 + 5*(-(ArcTan[(1 - (2*2^(1
/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) - Log[1 + x^3]/(6*2^(1
/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 26.22 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{10\, 2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right) x^{5}+12\left(x^{3}-1\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-5\, 2^{\frac{2}{3}} x^{5}\left(-2\sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)+\ln\left(\frac{2^{\frac{2}{3}} x^{2}-2 \cdots}{\cdots}\right.\right.}{60x^{5}}$
risch	Expression too large to display
trager	Expression too large to display

input
```
int(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/60*(10*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*x^5+12*(x^3-1)*(-x^3+1)^
(2/3)-5*2^(2/3)*x^5*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3
)+x)/x)+ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)))/x^
5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(95) = 190$.

Time = 1.80 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.23

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{60\cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}x^5\arctan\left(\frac{2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(6\cdot 2^{\frac{2}{3}}\left(5\,x^7+4\,x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(71\,x^9-111\,x^6+33\,x^3-1\right)+12\left(19\,x^8-16\,x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}\right)}{109\,x^9-105\,x^6+3\,x^3+1}\right)$$

input
```
integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/180*(60*2^(1/6)*sqrt(1/6)*x^5*arctan(2^(1/6)*sqrt(1/6)*(6*2^(2/3)*(5*x^
7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1)
+ 12*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 + 3*x^3
+ 1)) - 10*2^(2/3)*x^5*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2/3)*(x^3
+ 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(2/3)*x^5*log((3*2^(2/3)*(5*
x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^
2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 36*(x^3 - 1)*(-x^3 + 1)^(2/3))/x
^5
```

Sympy [F]

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^6\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(1/x**6/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(1/(x**6*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

Maxima [F]

$$\int \frac{1}{x^6 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^6}\,dx$$

input
```
integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^6}\,dx$$

input
```
integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^6\,(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(1/(x^6*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^6*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(-x^3+1)^{\frac{1}{3}}\,x^9+(-x^3+1)^{\frac{1}{3}}\,x^6}dx$$

input
```
int(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(1/(( - x**3 + 1)**(1/3)*x**9 + ( - x**3 + 1)**(1/3)*x**6),x)
```

3.831 $\int \frac{1}{x^9 \sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 141

$$\int \frac{1}{x^9 \sqrt[3]{1-x^3}(1+x^3)}\,dx = -\frac{(1-x^3)^{2/3}}{8x^8}+\frac{(1-x^3)^{2/3}}{20x^5} -\frac{17(1-x^3)^{2/3}}{40x^2}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} +\frac{\log(1+x^3)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
-1/8*(-x^3+1)^(2/3)/x^8+1/20*(-x^3+1)^(2/3)/x^5-17/40*(-x^3+1)^(2/3)/x^2+1
/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12
*ln(x^3+1)*2^(2/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^9\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{120}\left(-\frac{3(1-x^3)^{2/3}\,(5-2x^3+17x^6)}{x^8}\right.$$
$$+20\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-20\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+10\,2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1\right.$$

input
```
Integrate[1/(x^9*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((-3*(1 - x^3)^(2/3)*(5 - 2*x^3 + 17*x^6))/x^8 + 20*2^(2/3)*Sqrt[3]*ArcTan
[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 20*2^(2/3)*Log[2*x + 2^(2/3)
*(1 - x^3)^(1/3)] + 10*2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^
(1/3)*(1 - x^3)^(2/3)])/120
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {980, 27, 1053, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9\sqrt[3]{1-x^3}\,(x^3+1)}dx$$
$$\Big\downarrow \text{980}$$
$$\frac{1}{8}\int -\frac{2(1-3x^3)}{x^6\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{(1-x^3)^{2/3}}{8x^8}$$
$$\Big\downarrow \text{27}$$
$$-\frac{1}{4}\int \frac{1-3x^3}{x^6\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{(1-x^3)^{2/3}}{8x^8}$$

$$\Big\downarrow \; 1053$$

$$\frac{1}{4}\left(\frac{1}{5}\int \frac{17-3x^3}{x^3\sqrt[3]{1-x^3}\,(x^3+1)}dx+\frac{\left(1-x^3\right)^{2/3}}{5x^5}\right)-\frac{\left(1-x^3\right)^{2/3}}{8x^8}$$

$$\Big\downarrow \; 1053$$

$$\frac{1}{4}\left(\frac{1}{5}\left(-\frac{1}{2}\int \frac{40}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{17\left(1-x^3\right)^{2/3}}{2x^2}\right)+\frac{\left(1-x^3\right)^{2/3}}{5x^5}\right)-\frac{\left(1-x^3\right)^{2/3}}{8x^8}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\left(\frac{1}{5}\left(-20\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{17\left(1-x^3\right)^{2/3}}{2x^2}\right)+\frac{\left(1-x^3\right)^{2/3}}{5x^5}\right)-\frac{\left(1-x^3\right)^{2/3}}{8x^8}$$

$$\Big\downarrow \; 901$$

$$\frac{1}{4}\left(\frac{1}{5}\left(-20\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}\right)-\frac{17\left(1-x^3\right)^{2/3}}{2x^2}\right)+\frac{\left(1-x^3\right)^{2/3}}{5x^5}\right)-\frac{\left(1-x^3\right)^{2/3}}{8x^8}$$

input
```
Int[1/(x^9*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-1/8*(1 - x^3)^(2/3)/x^8 + ((1 - x^3)^(2/3)/(5*x^5) + ((-17*(1 - x^3)^(2/3
))/(2*x^2) - 20*(-(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^
(1/3)*Sqrt[3])) - Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^
(1/3)]/(2*2^(1/3))))/5)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 26.48 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{-20\,2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)x^{8}+\left(-51x^{6}+6x^{3}-15\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-10\,2^{\frac{2}{3}}x^{8}\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)-\ln\right.}{120x^{8}}$
risch	Expression too large to display
trager	Expression too large to display

input
```
int(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/120*(-20*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*x^8+(-51*x^6+6*x^3-15)
*(-x^3+1)^(2/3)-10*2^(2/3)*x^8*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x
^3+1)^(1/3)+x)/x)-ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))
/x^2)))/x^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(109) = 218.

Time = 1.75 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.01

$$\int \frac{1}{x^9\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{120\cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}x^8\arctan\left(\frac{2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(6\cdot 2^{\frac{2}{3}}\left(5\,x^7+4\,x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(71\,x^9-111\,x^6+33\,x^3-1\right)+12\left(19\,x^8-16\,x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}\right)}{109\,x^9-105\,x^6+3\,x^3+1}\right)}{}$$

input
```
integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/360*(120*2^(1/6)*sqrt(1/6)*x^8*arctan(2^(1/6)*sqrt(1/6)*(6*2^(2/3)*(5*x^
7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1)
+ 12*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 + 3*x^3
+ 1)) - 20*2^(2/3)*x^8*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2/3)*(x^3
+ 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 10*2^(2/3)*x^8*log((3*2^(2/3)*(5
*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5 - x
^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 9*(17*x^6 - 2*x^3 + 5)*(-x^3 +
1)^(2/3))/x^8
```

Sympy [F]

$$\int \frac{1}{x^9 \sqrt[3]{1-x^3} (1+x^3)} \, dx = \int \frac{1}{x^9 \sqrt[3]{-(x-1)(x^2+x+1)} (x+1)(x^2-x+1)} \, dx$$

input
```
integrate(1/x**9/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(1/(x**9*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

Maxima [F]

$$\int \frac{1}{x^9 \sqrt[3]{1-x^3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}} x^9} \, dx$$

input
```
integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^9), x)
```

Giac [F]

$$\int \frac{1}{x^9 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^9}\,dx$$

input
```
integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^9 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^9\,(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(1/(x^9*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^9*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^9 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(-x^3+1)^{\frac{1}{3}}\,x^{12}+(-x^3+1)^{\frac{1}{3}}\,x^9}dx$$

input
```
int(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(1/(( - x**3 + 1)**(1/3)*x**12 + ( - x**3 + 1)**(1/3)*x**9),x)
```

3.832 $\int \frac{x^7}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 271

$$\begin{aligned}\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = &-\frac{1}{4}x^2\left(1-x^3\right)^{2/3}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} \\ &+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} \\ &-\frac{1}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) \\ &+\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} \\ &-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\end{aligned}$$

output
```
-1/4*x^2*(-x^3+1)^(2/3)+1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*
3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*
3^(1/2))*2^(2/3)*3^(1/2)-1/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/24*ln((
1-x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-
x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-
1/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.15

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{4}x^2\left(-\left(1-x^3\right)^{2/3} + \operatorname{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},x^3,-x^3\right)\right)$$

input
```
Integrate[x^7/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(x^2*(-(1 - x^3)^(2/3) + AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3]))/4
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {979, 27, 984, 888, 991, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103, 2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(x^3+1)}\,dx$$

$$\Big\downarrow \text{979}$$

$$\frac{1}{4}\int \frac{2x\left(1-x^3\right)^{2/3}}{x^3+1}dx-\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\int \frac{x\left(1-x^3\right)^{2/3}}{x^3+1}dx-\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \; 984$$

$$\frac{1}{2}\left(2\int \frac{x}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx-\int \frac{x}{\sqrt[3]{1-x^3}}dx\right)-\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \; 888$$

$$\frac{1}{2}\left(2\int \frac{x}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\right)-\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \; 991$$

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\int \frac{1}{\frac{2(1-x)^3}{1-x^3}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}\right)-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\right)- \frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \; 750$$

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{3}\int \frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}\right)\right. \frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \; 16$$

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}\right)-\frac{1}{2}x^2\,\text{H}\right. \frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\int \frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}\right)-\frac{1}{2}\right.$$

$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$\downarrow$ 1142

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\int -\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\right.}{2\ 2^{2/3}}\right.\right.$$

$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$\downarrow$ 25

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int \frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{}{\sqrt[3]{}}\right.}{2\ 2^{2/3}}\right.\right.$$

$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$\downarrow$ 27

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}\right.\right.\right.$$

$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$\big\downarrow$ 1082

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{-\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}\right.\right.\right.$$

$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$\big\downarrow$ 217

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\sqrt[3]{2}\left(\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}\right)-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx\right.\right.$$

$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$\big\downarrow$ 1103

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\int\frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)-\right.\right.$$

$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$\downarrow$ 2574

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)+\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right.}{2\sqrt[3]{2}}\right.\right.\right.$$

$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

input
```
Int[x^7/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-1/4*(x^2*(1 - x^3)^(2/3)) + (-1/2*(x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x
^3]) + 2*(-1/3*(2^(1/3)*(-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x
^3)^(1/3))/Sqrt[3]])/2^(2/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3
) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(2*2^(2/3)))) - Log[1 + (2^(1/3)*(1
 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) + ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 -
x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3)) + Log[(1 - x)*(1 + x)^2]/(4*2^(
1/3)) - (3*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)))/3))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 991
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Simp[-q^2/(3*d)   Int[1/((1 - q*x)*(a + b*x^3)^(1/3)
), x], x] + Simp[q/d   Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x
^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c +
 a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Maple [F]

$$\int \frac{x^7}{(-x^3+1)^{\frac{1}{3}}(x^3+1)}dx$$

input
```
int(x^7/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(x^7/(-x^3+1)^(1/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^7/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral(-(-x^3 + 1)^(2/3)*x^7/(x^6 - 1), x)
```

Sympy [F]

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^7}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(x**7/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [F]

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^7/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^7/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^7}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(x^7/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(x^7/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^7}{(-x^3+1)^{\frac{1}{3}}\,x^3+(-x^3+1)^{\frac{1}{3}}}dx$$

input
```
int(x^7/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(x**7/(( - x**3 + 1)**(1/3)*x**3 + ( - x**3 + 1)**(1/3)),x)
```

3.833 $\int \frac{x^4}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 254

$$\int \frac{x^4}{\sqrt[3]{1-x^3}(1+x^3)}\,dx=-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)-\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}-\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}+\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
-1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2
)-1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2
)+1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/24*ln((1-x)*(1+x)^2)*2^(2/3)-1
/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2
/3)+1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/8*ln(-1+x+2^(2/3)*(-x
^3+1)^(1/3))*2^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.10

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{5}x^5 \operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)$$

input
```
Integrate[x^4/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/5
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {983, 888, 991, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103, 2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(x^3+1)}\,dx$$

$$\Big\downarrow \text{983}$$

$$\int \frac{x}{\sqrt[3]{1-x^3}}dx - \int \frac{x}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{2}x^2 \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \int \frac{x}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{991}$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx + \int \frac{1}{\frac{2(1-x)^3}{1-x^3}+1}d\frac{1-x}{\sqrt[3]{1-x^3}} + \frac{1}{2}x^2 \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{1}{3}\int \frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{3}\sqrt[3]{2}\int \frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\int -\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\ 2^{2/3}}\right)+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int \frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\ 2^{2/3}}\right)+\frac{1}{2}x^2 \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}\right)+\frac{1}{2}x^2 \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \ 1082$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{-\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}\right)+\frac{1}{2}x^2 \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\sqrt[3]{2}\left(\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}\right)+$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+$$

$$\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)+$$

$$\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{2574}$$

$$\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\,2^{2/3}}\right)+$$
$$\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}+\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}\right)+$$
$$\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

input
```
Int[x^4/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 + (2^(1/3)*(-((Sqrt[3]*ArcTa
n[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(2/3)) - Log[1 + (
2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(2
*2^(2/3))))/3 + Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) + (
-1/2*(Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(
1/3) - Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + (3*Log[-1 + x + 2^(2/3)*(1 - x
^3)^(1/3)])/(4*2^(1/3)))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 991
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Simp[-q^2/(3*d)   Int[1/((1 - q*x)*(a + b*x^3)^(1/3)
), x], x] + Simp[q/d   Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x
^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c +
 a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Maple [F]

$$\int \frac{x^4}{(-x^3+1)^{\frac{1}{3}}(x^3+1)}dx$$

input
```
int(x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral(-(-x^3 + 1)^(2/3)*x^4/(x^6 - 1), x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^4}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(x**4/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^4}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(x^4/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(x^4/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^4}{(-x^3+1)^{\frac{1}{3}}\,x^3+(-x^3+1)^{\frac{1}{3}}}dx$$

input
```
int(x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(x**4/(( - x**3 + 1)**(1/3)*x**3 + ( - x**3 + 1)**(1/3)),x)
```

3.834 $\int \frac{x}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 233

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output

```
1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
+1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
+1/24*ln((1-x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2
^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3)
)*2^(2/3)-1/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.21

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\right.}{}$$

input
```
Integrate[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1 - x
^3)^(1/3))] - 4*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2*2
^(1/3)*x + (1 - x^3)^(1/3))] - 4*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/3
)] - 2*Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)] + 2*Log[2^(2/3) - 2*2
^(2/3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)] + L
og[2^(2/3) - 2*2^(2/3)*x + 2^(2/3)*x^2 - 2*(-1 + x)*(2 - 2*x^3)^(1/3) + 4*
(1 - x^3)^(2/3)])/(12*2^(1/3))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {991, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103, 2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(x^3+1)}\,dx$$

$$\Big\downarrow \text{991}$$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\int \frac{1}{\frac{2(1-x)^3}{1-x^3}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{750}$$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{3}\int \frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}$$

$\Big\downarrow$ 16

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$\Big\downarrow$ 27

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\int \frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$\Big\downarrow$ 1142

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\int -\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\ 2^{2/3}}\right)-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$\Big\downarrow$ 25

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int \frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\ 2^{2/3}}\right)-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}\right)-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \ 1082$$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{-\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}\right)-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{3}\sqrt[3]{2}\left(\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}\right)-$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-$$

$$\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)-$$

$$\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{2574}$$

$$-\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)+\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}\right)-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

input
```
Int[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-1/3*(2^(1/3)*(-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]])/2^(2/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/
3)*(1 - x))/(1 - x^3)^(1/3)]/(2*2^(2/3)))) - Log[1 + (2^(1/3)*(1 - x))/(1
- x^3)^(1/3)]/(3*2^(1/3)) + ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x
^3)^(1/3))/Sqrt[3]])/(2*2^(1/3)) + Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) - (3
*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 991
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Simp[-q^2/(3*d)   Int[1/((1 - q*x)*(a + b*x^3)^(1/3)
), x], x] + Simp[q/d   Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x
^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c +
 a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Maple [F]

$$\int \frac{x}{(-x^3+1)^{\frac{1}{3}}(x^3+1)}dx$$

input
```
int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

Fricas [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.44

$$\int \frac{x}{\sqrt[3]{1-x^3}(1+x^3)}\,dx = \frac{1}{6} \cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(24\cdot 2^{\frac{2}{3}}(x^{14}-2x^{11}-6x^8-2x^5+x^2)(-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(x^{18}+42x^{15}-417x^{12}\right.}{x^{18}-102x^{15}+447x^{12}}\right) - \frac{1}{36} \cdot 2^{\frac{2}{3}}\log\left(-\frac{12(-x^3+1)^{\frac{2}{3}}x^2+2^{\frac{2}{3}}(x^6+2x^3+1)-6\cdot 2^{\frac{1}{3}}(x^4-x)(-x^3+1)^{\frac{1}{3}}}{x^6+2x^3+1}\right) + \frac{1}{72} \cdot 2^{\frac{2}{3}}\log\left(\frac{12\cdot 2^{\frac{2}{3}}(x^8-4x^5+x^2)(-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(x^{12}-32x^9+78x^6-32x^3+1)+6(x^{10}-11x^7+11}{x^{12}+4x^9+6x^6+4x^3+1}\right)$$

input
```
integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/6*2^(1/6)*sqrt(1/6)*arctan(2^(1/6)*sqrt(1/6)*(24*2^(2/3)*(x^14 - 2*x^11
- 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) + 2^(1/3)*(x^18 + 42*x^15 - 417*x^
12 + 812*x^9 - 417*x^6 + 42*x^3 + 1) - 12*(x^16 - 33*x^13 + 110*x^10 - 110
*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9
 + 447*x^6 - 102*x^3 + 1)) - 1/36*2^(2/3)*log(-(12*(-x^3 + 1)^(2/3)*x^2 +
2^(2/3)*(x^6 + 2*x^3 + 1) - 6*2^(1/3)*(x^4 - x)*(-x^3 + 1)^(1/3))/(x^6 + 2
*x^3 + 1)) + 1/72*2^(2/3)*log((12*2^(2/3)*(x^8 - 4*x^5 + x^2)*(-x^3 + 1)^(
2/3) + 2^(1/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) + 6*(x^10 - 11*x^7 +
11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1))
```

Sympy [F]

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(x/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(x/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(x/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{(-x^3+1)^{\frac{1}{3}}\,x^3+(-x^3+1)^{\frac{1}{3}}}dx$$

input
```
int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(x/(( - x**3 + 1)**(1/3)*x**3 + ( - x**3 + 1)**(1/3)),x)
```

3.835 $\int \frac{1}{x^2\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 270

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{(1-x^3)^{2/3}}{x}-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)-\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}-\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}+\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
-(-x^3+1)^(2/3)/x-1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2
))*2^(2/3)*3^(1/2)-1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2
))*2^(2/3)*3^(1/2)-1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/24*ln((1-x)*(
1+x)^2)*2^(2/3)-1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x
^3+1)^(1/3))*2^(2/3)+1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/8*ln
(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{(1-x^3)^{2/3}}{x} - x^2\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right) - \frac{1}{5}x^5\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)$$

input
```
Integrate[1/(x^2*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-((1 - x^3)^(2/3)/x) - x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] - (x^5*Ap
pellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/5
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {980, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{980}$$

$$\int -\frac{x(x^3+2)}{\sqrt[3]{1-x^3}\,(x^3+1)}dx - \frac{(1-x^3)^{2/3}}{x}$$

$$\Big\downarrow \; 25$$

$$-\int \frac{x(x^3+2)}{\sqrt[3]{1-x^3}\,(x^3+1)}dx - \frac{(1-x^3)^{2/3}}{x}$$

$$\Big\downarrow \; 1054$$

$$-\int \left(\frac{x}{\sqrt[3]{1-x^3}} + \frac{x}{\sqrt[3]{1-x^3}\,(x^3+1)}\right)dx - \frac{(1-x^3)^{2/3}}{x}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{(1-x^3)^{2/3}}{x} - \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}} + \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}} + \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}} - \frac{\log\left((1-x)(x+1)^2\right)}{12\sqrt[3]{2}}$$

input
```
Int[1/(x^2*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-((1 - x^3)^(2/3)/x) - ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sq
rt[3]]/(2^(1/3)*Sqrt[3]) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/
Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])
/2 - Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1
- x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[1 + (2
^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) + Log[-1 + x + 2^(2/3)*(1 - x
^3)^(1/3)]/(4*2^(1/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(-x^{3}+1\right)^{\frac{1}{3}} \left(x^{3}+1\right)}dx$$

input
```
int(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral(-(-x^3 + 1)^(2/3)/(x^8 - x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^2\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(1/x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^2\,(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(1/(x^2*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^2*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(-x^3+1)^{\frac{1}{3}}\,x^5+(-x^3+1)^{\frac{1}{3}}\,x^2}dx$$

input
```
int(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(1/(( - x**3 + 1)**(1/3)*x**5 + ( - x**3 + 1)**(1/3)*x**2),x)
```

3.836 $\int \frac{1}{x^5 \sqrt[3]{1-x^3}(1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 289

$$\begin{aligned}\int \frac{1}{x^5 \sqrt[3]{1-x^3}(1+x^3)} dx = & -\frac{(1-x^3)^{2/3}}{4x^4} + \frac{(1-x^3)^{2/3}}{2x} \\ & + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} \\ & + \frac{1}{4}x^2 \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) \\ & + \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} \\ & - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\end{aligned}$$

output
```
-1/4*(-x^3+1)^(2/3)/x^4+1/2*(-x^3+1)^(2/3)/x+1/6*arctan(1/3*(1-2*2^(1/3)*(
1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/3*(1+2^(1/3)*(
1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/4*x^2*hypergeom([1/3, 2/3]
,[5/3],x^3)+1/24*ln((1-x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3
+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x
^3+1)^(1/3))*2^(2/3)-1/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^5\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$
$$= \frac{5(1-x^3)^{2/3}\,(-1+2x^3)+15x^6\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right)+2x^9\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)}{20x^4}$$

input
```
Integrate[1/(x^5*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(5*(1 - x^3)^(2/3)*(-1 + 2*x^3) + 15*x^6*AppellF1[2/3, 1/3, 1, 5/3, x^3, -
x^3] + 2*x^9*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/(20*x^4)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {980, 27, 975, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt[3]{1-x^3}\,(x^3+1)}\,dx$$
$$\Big\downarrow \; 980$$

$$\frac{1}{4}\int -\frac{2(1-x^3)^{2/3}}{x^2(x^3+1)}dx - \frac{(1-x^3)^{2/3}}{4x^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{2}\int \frac{(1-x^3)^{2/3}}{x^2(x^3+1)}dx - \frac{(1-x^3)^{2/3}}{4x^4}$$

$$\Big\downarrow \; 975$$

$$\frac{1}{2}\left(\frac{(1-x^3)^{2/3}}{x} - \int -\frac{x(x^3+3)}{\sqrt[3]{1-x^3}(x^3+1)}dx\right) - \frac{(1-x^3)^{2/3}}{4x^4}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\int \frac{x(x^3+3)}{\sqrt[3]{1-x^3}(x^3+1)}dx + \frac{(1-x^3)^{2/3}}{x}\right) - \frac{(1-x^3)^{2/3}}{4x^4}$$

$$\Big\downarrow \; 1054$$

$$\frac{1}{2}\left(\int \left(\frac{x}{\sqrt[3]{1-x^3}} + \frac{2x}{\sqrt[3]{1-x^3}(x^3+1)}\right)dx + \frac{(1-x^3)^{2/3}}{x}\right) - \frac{(1-x^3)^{2/3}}{4x^4}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{(1-x^3)}{x}\right.$$

$$\frac{(1-x^3)^{2/3}}{4x^4}$$

input
```
Int[1/(x^5*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-1/4*(1 - x^3)^(2/3)/x^4 + ((1 - x^3)^(2/3)/x + (2^(2/3)*ArcTan[(1 - (2*2^
(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + ArcTan[(1 + (2^(1/3)*(
1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + (x^2*Hypergeometric2
F1[1/3, 2/3, 5/3, x^3])/2 + Log[(1 - x)*(1 + x)^2]/(6*2^(1/3)) + Log[1 + (
2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3
*2^(1/3)) - (2^(2/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 - Log[-
1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(2*2^(1/3)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{1}{x^5 (-x^3+1)^{\frac{1}{3}} (x^3+1)} dx$$

input
```
int(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{1}{x^5 \sqrt[3]{1-x^3} (1+x^3)} dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}} x^5} dx$$

input
```
integrate(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral(-(-x^3 + 1)^(2/3)/(x^11 - x^5), x)
```

Sympy [F]

$$\int \frac{1}{x^5\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^5\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(1/x**5/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

Maxima [F]

$$\int \frac{1}{x^5\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^5}\,dx$$

input
```
integrate(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^5}\,dx$$

input
```
integrate(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^5\,(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(1/(x^5*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^5*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^5 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(-x^3+1)^{\frac{1}{3}}\,x^8+(-x^3+1)^{\frac{1}{3}}\,x^5}dx$$

input
```
int(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(1/(( - x**3 + 1)**(1/3)*x**8 + ( - x**3 + 1)**(1/3)*x**5),x)
```

3.837 $\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\sqrt[3]{1-x^3}+\frac{1}{4}(1-x^3)^{4/3} - \frac{1}{7}(1-x^3)^{7/3}+\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log(1+x^3)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
-(-x^3+1)^(1/3)+1/4*(-x^3+1)^(4/3)-1/7*(-x^3+1)^(7/3)+1/6*arctan(1/3*(1+2^
(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)+1/12*ln(x^3+1)*2^(1/3)-1/4*
ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{84}\left(3\sqrt[3]{1-x^3}(-25+x^3-4x^6) +14\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-14\sqrt[3]{2}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+7\sqrt[3]{2}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-\right.\right.$$

input
```
Integrate[x^11/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(3*(1 - x^3)^(1/3)*(-25 + x^3 - 4*x^6) + 14*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^
(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 14*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1
/3)] + 7*2^(1/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)
])/84
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^9}{(1-x^3)^{2/3}(x^3+1)}dx^3$$

$$\Big\downarrow \text{ 99}$$

$$\frac{1}{3}\int \left((1-x^3)^{4/3}-\sqrt[3]{1-x^3}-\frac{1}{(x^3+1)(1-x^3)^{2/3}}+\frac{1}{(1-x^3)^{2/3}}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{3}{7}(1-x^3)^{7/3}+\frac{3}{4}(1-x^3)^{4/3}-3\sqrt[3]{1-x^3}+\frac{\log(x^3+1)}{2\ 2^{2/3}}-\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{\right.}}{2\ 2^{2/3}}\right.$$

input
```
Int[x^11/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-3*(1 - x^3)^(1/3) + (3*(1 - x^3)^(4/3))/4 - (3*(1 - x^3)^(7/3))/7 + (Sqr
t[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(2/3) + Log[1 + x^3]
/(2*2^(2/3)) - (3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*2^(2/3)))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 12.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{\left(-4x^{6}+x^{3}-25\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}{28}+\frac{2^{\frac{1}{3}}\left(2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}+\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)-2\ln\left(\right.}{12}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int(x^11/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/28*(-4*x^6+x^3-25)*(-x^3+1)^(1/3)+1/12*2^(1/3)*(2*arctan(1/3*(1+2^(2/3)*
(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+
2^(2/3))-2*ln((-x^3+1)^(1/3)-2^(1/3)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94

$$\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{2}\cdot 4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\arctan\left(\frac{1}{2}\cdot 4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\left(4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\right)\right) + \frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right) - \frac{1}{12} \cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{2}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right) - \frac{1}{28}\left(4\,x^6-x^3+25\right)\left(-x^3+1\right)^{\frac{1}{3}}$$

input
```
integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/2*4^(1/6)*sqrt(1/3)*arctan(1/2*4^(1/6)*sqrt(1/3)*(4^(2/3)*(-x^3 + 1)^(1/
3) + 4^(1/3))) + 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3 + 1)^
(2/3) + 2*4^(1/3)) - 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3)) - 1/2
8*(4*x^6 - x^3 + 25)*(-x^3 + 1)^(1/3)
```

Sympy [F]

$$\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^{11}}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**11/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.95

$$\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{7}\left(-x^3+1\right)^{\frac{7}{3}} + \frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}} + \frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) - \left(-x^3+1\right)^{\frac{1}{3}}$$

input

```
integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output

```
-1/7*(-x^3 + 1)^(7/3) + 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^
(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/4*(-x^3 + 1)^(4/3) + 1/12*2^(1/3)*log(2^(
2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1/3)*log(-2^(
1/3) + (-x^3 + 1)^(1/3)) - (-x^3 + 1)^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{7}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{1}{3}} + \frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}} + \frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) - \left(-x^3+1\right)^{\frac{1}{3}}$$

input

```
integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/7*(x^3 - 1)^2*(-x^3 + 1)^(1/3) + 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)
*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/4*(-x^3 + 1)^(4/3) + 1/12*2^(
1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1
/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - (-x^3 + 1)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 3.67 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08

$$\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{(1-x^3)^{4/3}}{4} - (1-x^3)^{1/3} - \frac{2^{1/3}\ln\left(3\,2^{1/3}-3\,(1-x^3)^{1/3}\right)}{6} - \frac{(1-x^3)^{7/3}}{7} - \frac{2^{1/3}\ln\left(3\,(1-x^3)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} + \frac{2^{1/3}\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3\,(1-x^3)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

input
```
int(x^11/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
(1 - x^3)^(4/3)/4 - (1 - x^3)^(1/3) - (2^(1/3)*log(3*2^(1/3) - 3*(1 - x^3)
^(1/3)))/6 - (1 - x^3)^(7/3)/7 - (2^(1/3)*log(3*(1 - x^3)^(1/3) - (3*2^(1/
3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 + (2^(1/3)*log((3*2^(1/3)*(3^
(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^(1/2)*1i + 1))/12
```

Reduce [F]

$$\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^{11}}{(-x^3+1)^{\frac{2}{3}}x^3+(-x^3+1)^{\frac{2}{3}}}dx$$

input
```
int(x^11/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(x**11/(( - x**3 + 1)**(2/3)*x**3 + ( - x**3 + 1)**(2/3)),x)
```

3.838 $\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
1/4*(-x^3+1)^(4/3)-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1
/3)*3^(1/2)-1/12*ln(x^3+1)*2^(1/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.30

$$\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{12}\Bigg(3\left(1-x^3\right)^{4/3} - 2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 2\sqrt[3]{2}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - \sqrt[3]{2}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x^3\right.$$

input
```
Integrate[x^8/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(3*(1 - x^3)^(4/3) - 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3)
)/Sqrt[3]] + 2*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 2^(1/3)*Log[2 +
 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{(1-x^3)^{2/3}(x^3+1)}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{1}{(1-x^3)^{2/3}(x^3+1)}-\sqrt[3]{1-x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}+\frac{3}{4}(1-x^3)^{4/3}-\frac{\log(x^3+1)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\right)$$

input
```
Int[x^8/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
((3*(1 - x^3)^(4/3))/4 - (Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqr
t[3]])/2^(2/3) - Log[1 + x^3]/(2*2^(2/3)) + (3*Log[2^(1/3) - (1 - x^3)^(1/
3)])/(2*2^(2/3)))/3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 11.57 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$-\frac{x^{3}\left(-x^{3}+1\right)^{\frac{1}{3}}}{4}+\frac{\left(-x^{3}+1\right)^{\frac{1}{3}}}{4}+\frac{2^{\frac{1}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)}{6}-\frac{2^{\frac{1}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{12}-\frac{\arctan\left(\right.}{}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int(x^8/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^3*(-x^3+1)^(1/3)+1/4*(-x^3+1)^(1/3)+1/6*2^(1/3)*ln((-x^3+1)^(1/3)-2
^(1/3))-1/12*2^(1/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))-1/6
*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.12

$$\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{2} \cdot 4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\arctan\left(\frac{1}{2}\cdot 4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\left(4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}+4^{\frac{1}{3}}\right)\right) - \frac{1}{24} \cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}+2(-x^3+1)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right) + \frac{1}{12} \cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{2}{3}}+2(-x^3+1)^{\frac{1}{3}}\right) - \frac{1}{4}(x^3-1)(-x^3+1)^{\frac{1}{3}}$$

input
```
integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/2*4^(1/6)*sqrt(1/3)*arctan(1/2*4^(1/6)*sqrt(1/3)*(4^(2/3)*(-x^3 + 1)^(1
/3) + 4^(1/3))) - 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3 + 1)
^(2/3) + 2*4^(1/3)) + 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3)) - 1/
4*(x^3 - 1)*(-x^3 + 1)^(1/3)
```

Sympy [F]

$$\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^8}{(-(x-1)(x^2+x+1))^{\frac{2}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**8/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}}-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

input
```
integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/4*(-x^3 + 1)^(4/3) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 +
1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)
)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}}-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

input
```
integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/4*(-x^3 + 1)^(4/3) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 +
1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(
1/3)))
```

Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15

$$\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{2^{1/3}\ln\left(\frac{(1-x^3)^{1/3}}{2}-\frac{2^{1/3}}{2}\right)}{6}+\frac{(1-x^3)^{4/3}}{4} + \frac{2^{1/3}\ln\left(3(1-x^3)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} - \frac{2^{1/3}\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3(1-x^3)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

input
```
int(x^8/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
(2^(1/3)*log((1 - x^3)^(1/3)/2 - 2^(1/3)/2))/6 + (1 - x^3)^(4/3)/4 + (2^(1
/3)*log(3*(1 - x^3)^(1/3) - (3*2^(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i -
1))/12 - (2^(1/3)*log((3*2^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*
(3^(1/2)*1i + 1))/12
```

Reduce [F]

$$\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^8}{(-x^3+1)^{\frac{2}{3}}x^3+(-x^3+1)^{\frac{2}{3}}}dx$$

input
```
int(x^8/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(x**8/(( - x**3 + 1)**(2/3)*x**3 + ( - x**3 + 1)**(2/3)),x)
```

3.839 $\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\sqrt[3]{1-x^3}+\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(1+x^3)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
-(-x^3+1)^(1/3)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)
*3^(1/2)+1/12*ln(x^3+1)*2^(1/3)-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.33

$$\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{12}\left(-12\sqrt[3]{1-x^3} +2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-2\sqrt[3]{2}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+\sqrt[3]{2}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x^3\right.\right.$$

input
```
Integrate[x^5/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-12*(1 - x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/
3))/Sqrt[3]] - 2*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(1/3)*Log[2
 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 90, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{(1-x^3)^{2/3}(x^3+1)}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(-\int \frac{1}{(1-x^3)^{2/3}(x^3+1)}dx^3-3\sqrt[3]{1-x^3}\right)$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\ 2^{2/3}}+\frac{3\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}-3\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}-3\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}-\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{2^{2/3}}-3\sqrt[3]{1-x^3}+\frac{\log \left(x^3+1\right)}{2\ 2^{2/3}}-\frac{3\log \left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan \left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}-3\sqrt[3]{1-x^3}+\frac{\log \left(x^3+1\right)}{2\ 2^{2/3}}-\frac{3\log \left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\right)$$

input
```
Int[x^5/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-3*(1 - x^3)^(1/3) + (Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3
]])/2^(2/3) + Log[1 + x^3]/(2*2^(2/3)) - (3*Log[2^(1/3) - (1 - x^3)^(1/3)]
)/(2*2^(2/3)))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 9.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-(-x^3+1)^{\frac{1}{3}}-\frac{2^{\frac{1}{3}}\ln\left((-x^3+1)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)}{6}+\frac{2^{\frac{1}{3}}\ln\left((-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{12}+\frac{\arctan\left(\frac{\left(1+2^{\frac{2}{3}}(-x^3+1\right.}{3}\right.}{6}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int(x^5/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-(-x^3+1)^(1/3)-1/6*2^(1/3)*ln((-x^3+1)^(1/3)-2^(1/3))+1/12*2^(1/3)*ln((-x
^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))+1/6*arctan(1/3*(1+2^(2/3)*(-x^
3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11

$$\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{2}\cdot 4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\arctan\left(\frac{1}{2}\cdot 4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\left(4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\right)\right) + \frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right) - \frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{2}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right) - \left(-x^3+1\right)^{\frac{1}{3}}$$

input
```
integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/2*4^(1/6)*sqrt(1/3)*arctan(1/2*4^(1/6)*sqrt(1/3)*(4^(2/3)*(-x^3 + 1)^(1/
3) + 4^(1/3))) + 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3 + 1)^
(2/3) + 2*4^(1/3)) - 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3)) - (-x
^3 + 1)^(1/3)
```

Sympy [F]

$$\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^5}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**5/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02

$$\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) - \left(-x^3+1\right)^{\frac{1}{3}}$$

input
```
integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) - 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - (-x^3 + 1)^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.03

$$\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) - \left(-x^3+1\right)^{\frac{1}{3}}$$

input
```
integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) - 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - (-x^3 + 1)^(1/3
)
```

Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

$$\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{2^{1/3}\ln\left(\frac{(1-x^3)^{1/3}}{2}-\frac{2^{1/3}}{2}\right)}{6}-\left(1-x^3\right)^{1/3} - \frac{2^{1/3}\ln\left(3\left(1-x^3\right)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} + \frac{2^{1/3}\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3\left(1-x^3\right)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

input
```
int(x^5/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
(2^(1/3)*log((3*2^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^(1/2)*
1i + 1))/12 - (1 - x^3)^(1/3) - (2^(1/3)*log(3*(1 - x^3)^(1/3) - (3*2^(1/3
)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 - (2^(1/3)*log((1 - x^3)^(1/3)
/2 - 2^(1/3)/2))/6
```

Reduce [F]

$$\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^5}{(-x^3+1)^{\frac{2}{3}}x^3+(-x^3+1)^{\frac{2}{3}}}dx$$

input
```
int(x^5/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(x**5/(( - x**3 + 1)**(2/3)*x**3 + ( - x**3 + 1)**(2/3)),x)
```

3.840 $\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(1+x^3)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)-1/12*l
n(x^3+1)*2^(1/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 2\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) + \log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x^3)^{2/3}\right)}{6\ 2^{2/3}}$$

input
```
Integrate[x^2/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 2*Log[-2 +
 2^(2/3)*(1 - x^3)^(1/3)] + Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 -
 x^3)^(2/3)])/2^(2/3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {946, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{1}{(1-x^3)^{2/3}(x^3+1)}dx^3$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\ 2^{2/3}}-\frac{3\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}-\frac{\log(x^3+1)}{2\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}-\frac{\log(x^3+1)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{2^{2/3}}-\frac{\log(x^3+1)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\right)$$

input
```
Int[x^2/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-((Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(2/3)) - Log[
1 + x^3]/(2*2^(2/3)) + (3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*2^(2/3)))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

method	result	size
pseudoelliptic	$-\frac{2^{\frac{1}{3}}\left(2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}+\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)-2\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\right)}{12}$	78
trager	Expression too large to display	734

input
```
int(x^2/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/12*2^(1/3)*(2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+ln
((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))-2*ln((-x^3+1)^(1/3)-2^(1/3
)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{2} \cdot 4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\arctan\left(\frac{1}{2}\cdot 4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\left(4^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\right)\right)-\frac{1}{24} \cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{2}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)$$

input
```
integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/2*4^(1/6)*sqrt(1/3)*arctan(1/2*4^(1/6)*sqrt(1/3)*(4^(2/3)*(-x^3 + 1)^(1
/3) + 4^(1/3))) - 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3 + 1)
^(2/3) + 2*4^(1/3)) + 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3))
```

Sympy [F]

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^2}{(-(x-1)(x^2+x+1))^{\frac{2}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**2/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

input
```
integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) + 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

input
```
integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) + 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{2^{1/3}\ln\left(3\,2^{1/3}-3\,(1-x^3)^{1/3}\right)}{6}+\frac{2^{1/3}\ln\left(3\,(1-x^3)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{1/3}\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3\,(1-x^3)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

input
```
int(x^2/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
(2^(1/3)*log(3*2^(1/3) - 3*(1 - x^3)^(1/3)))/6 + (2^(1/3)*log(3*(1 - x^3)^
(1/3) - (3*2^(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 - (2^(1/3)*lo
g((3*2^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^(1/2)*1i + 1))/12
```

Reduce [F]

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^2}{(-x^3+1)^{\frac{2}{3}}x^3+(-x^3+1)^{\frac{2}{3}}}dx$$

input
```
int(x^2/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(x**2/(( - x**3 + 1)**(2/3)*x**3 + ( - x**3 + 1)**(2/3)),x)
```

3.841 $\int \frac{1}{x(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 137

$$\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=-\frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
-1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2^
(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)-1/2*ln(x)+1/12*ln(x^3+1)*2^
(1/3)+1/2*ln(1-(-x^3+1)^(1/3))-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.35

$$\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\frac{1}{12}\left(-4\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+4\log\left(-1+\sqrt[3]{1-x^3}\right)-2\sqrt[3]{2}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-2\log\left(1+\sqrt[3]{1-x^3}\right.\right.$$

input
```
Integrate[1/(x*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] + 2*2^(1/3)*Sqrt[3]*Ar
cTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 4*Log[-1 + (1 - x^3)^(1/3)]
- 2*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 2*Log[1 + (1 - x^3)^(1/3)
+ (1 - x^3)^(2/3)] + 2^(1/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1
- x^3)^(2/3)])/12
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {948, 97, 69, 16, 1082, 217, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^3(1-x^3)^{2/3}(x^3+1)}dx^3$$

$$\Big\downarrow \text{97}$$

$$\frac{1}{3}\left(\int \frac{1}{x^3(1-x^3)^{2/3}}dx^3-\int \frac{1}{(1-x^3)^{2/3}(x^3+1)}dx^3\right)$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}+\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\ 2^{2/3}}-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+\frac{3\int \frac{}{x^6+}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+\frac{3\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}-\frac{1}{2}\log\left(x^3\right)+\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3}{2}\mathrm{lo}\right.$$

$$\Big\downarrow 1082$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{2^{2/3}}-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}-\frac{1}{2}\log\left(x^3\right)+\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3}{2}\log\left(1\right.\right.$$

$$\Big\downarrow 217$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^3\right)}{2}+\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3}{2}\log\left(1-\right.\right.$$

$$\Big\downarrow 1083$$

$$\frac{1}{3}\left(3\int \frac{1}{-x^6-3}d\left(2\sqrt[3]{1-x^3}+1\right)+\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^3\right)}{2}+\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3}{2}\log\left(1-\sqrt[3]{1}\right.\right.$$

$$\Big\downarrow 217$$

$$\frac{1}{3}\left(-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)+\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^3\right)}{2}+\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3}{2}\log\left(1-\sqrt[3]{1}\right.\right.$$

input
```
Int[1/(x*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-(Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]) + (Sqrt[3]*ArcTan[(1 +
 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(2/3) - Log[x^3]/2 + Log[1 + x^3]/(2
*2^(2/3)) + (3*Log[1 - (1 - x^3)^(1/3)])/2 - (3*Log[2^(1/3) - (1 - x^3)^(1
/3)])/(2*2^(2/3)))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$-\frac{2^{\frac{1}{3}} \ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)}{6}+\frac{2^{\frac{1}{3}} \ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{12}+\frac{\arctan\left(\frac{\left(1+2^{\frac{2}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}\right) \sqrt{3}}{3}\right) 2^{\frac{1}{3}} \sqrt{3}}{6}+$

input
```
int(1/x/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/3)*ln((-x^3+1)^(1/3)-2^(1/3))+1/12*2^(1/3)*ln((-x^3+1)^(2/3)+2^(
1/3)*(-x^3+1)^(1/3)+2^(2/3))+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(
1/2))*2^(1/3)*3^(1/2)+1/3*ln(-1+(-x^3+1)^(1/3))-1/6*ln((-x^3+1)^(2/3)+(-x^
3+1)^(1/3)+1)-1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{1}{2} \cdot 4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\arctan\left(\frac{1}{2}\cdot 4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\left(4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\right)\right)+\frac{1}{24} \cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right)-\frac{1}{12} \cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{2}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

input
```
integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/2*4^(1/6)*sqrt(1/3)*arctan(1/2*4^(1/6)*sqrt(1/3)*(4^(2/3)*(-x^3 + 1)^(1/
3) + 4^(1/3))) + 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3 + 1)^
(2/3) + 2*4^(1/3)) - 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3)) - 1/3
*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) - 1/6*log((-x^
3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

Sympy [F]

$$\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{1}{x\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(1/x/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}x}\,dx$$

input
```
integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right) + \frac{1}{12} \cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) - \frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right) + \frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/12*2^(
1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1
/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/6*log((-x^3 + 1)^(2/3) + (-x
^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.51

$$\int \frac{1}{x(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{\ln\left(5-5\left(1-x^3\right)^{1/3}\right)}{3} - \frac{2^{1/3}\ln\left(6\left(1-x^3\right)^{1/3} - \frac{2^{1/3}\left(\frac{2^{2/3}\left(243\,2^{1/3}+243\left(1-x^3\right)^{1/3}\right)}{36}+9\right)}{6}\right)}{6}$$

$$+\ln\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\left(243\left(1-x^3\right)^{1/3}+243-\sqrt{3}\,243\mathrm{i}\right)+9\right)+6\left(1-x^3\right)^{1/3}\right)\left(-\frac{1}{6}+\right.$$

input
```
int(1/(x*(1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
log(5 - 5*(1 - x^3)^(1/3))/3 - (2^(1/3)*log(6*(1 - x^3)^(1/3) - (2^(1/3)*(
(2^(2/3)*(243*2^(1/3) + 243*(1 - x^3)^(1/3)))/36 + 9))/6))/6 + log(((3^(1/
2)*1i)/6 - 1/6)*(((3^(1/2)*1i)/6 - 1/6)^2*(243*(1 - x^3)^(1/3) - 3^(1/2)*2
43i + 243) + 9) + 6*(1 - x^3)^(1/3))*((3^(1/2)*1i)/6 - 1/6) - log(6*(1 - x
^3)^(1/3) - ((3^(1/2)*1i)/6 + 1/6)*(((3^(1/2)*1i)/6 + 1/6)^2*(3^(1/2)*243i
 + 243*(1 - x^3)^(1/3) + 243) + 9))*((3^(1/2)*1i)/6 + 1/6) + ((-1)^(1/3)*2
^(1/3)*log(6*(1 - x^3)^(1/3) - ((-1)^(1/3)*2^(1/3)*(((-1)^(2/3)*2^(2/3)*(2
43*(-1)^(1/3)*2^(1/3) - 243*(1 - x^3)^(1/3)))/36 - 9))/6))/6 - ((-1)^(1/3)
*2^(1/3)*log(6*(1 - x^3)^(1/3) - ((-1)^(1/3)*2^(1/3)*(3^(1/2)*1i + 1)*(((-
1)^(2/3)*2^(2/3)*(3^(1/2)*1i + 1)^2*(243*(1 - x^3)^(1/3) + (243*(-1)^(1/3)
*2^(1/3)*(3^(1/2)*1i + 1))/2))/144 + 9))/12)*(3^(1/2)*1i + 1))/12
```

Reduce [F]

$$\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{1}{\left(-x^3+1\right)^{\frac{2}{3}}x^4+\left(-x^3+1\right)^{\frac{2}{3}}x}dx$$

input
```
int(1/x/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(1/(( - x**3 + 1)**(2/3)*x**4 + ( - x**3 + 1)**(2/3)*x),x)
```

3.842 $\int \frac{1}{x^4(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 158

$$\int \frac{1}{x^4(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\sqrt[3]{1-x^3}}{3x^3}+\frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log(x)}{6}-\frac{\log(1+x^3)}{6\ 2^{2/3}} - \frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
-1/3*(-x^3+1)^(1/3)/x^3+1/9*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/
2)-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)+1/6*
ln(x)-1/12*ln(x^3+1)*2^(1/3)-1/6*ln(1-(-x^3+1)^(1/3))+1/4*ln(2^(1/3)-(-x^3
+1)^(1/3))*2^(1/3)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^4 (1-x^3)^{2/3} (1+x^3)} dx = \frac{1}{36} \left(-\frac{12\sqrt[3]{1-x^3}}{x^3} + 4\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right) \right.$$
$$-6\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 4\log\left(-1+\sqrt[3]{1-x^3}\right) + 6\sqrt[3]{2}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) + 2\log\left(1+\sqrt[3]{\right.$$

input
```
Integrate[1/(x^4*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
((-12*(1 - x^3)^(1/3))/x^3 + 4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt
[3]] - 6*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 4
*Log[-1 + (1 - x^3)^(1/3)] + 6*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] +
 2*Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 3*2^(1/3)*Log[2 + 2^(2/3)*
(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/36
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {948, 114, 27, 174, 69, 16, 1082, 217, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (1-x^3)^{2/3} (x^3+1)} dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{1}{x^6 (1-x^3)^{2/3} (x^3+1)} dx^3$$

$$\Big\downarrow \ 114$$

$$\frac{1}{3}\left(-\int \frac{1-2x^3}{3x^3 (1-x^3)^{2/3} (x^3+1)} dx^3 - \frac{\sqrt[3]{1-x^3}}{x^3}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(-\frac{1}{3}\int \frac{1-2x^3}{x^3\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx^3-\frac{\sqrt[3]{1-x^3}}{x^3}\right)$$

$$\Big\downarrow \ 174$$

$$\frac{1}{3}\left(\frac{1}{3}\left(3\int \frac{1}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx^3-\int \frac{1}{x^3\left(1-x^3\right)^{2/3}}dx^3\right)-\frac{\sqrt[3]{1-x^3}}{x^3}\right)$$

$$\Big\downarrow \ 69$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{1-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}+\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+3\left(-\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\ 2^{2/3}}-\right.\right.\right.$$

$$\Big\downarrow \ 16$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+3\left(-\frac{3\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}\right.}{2}\right.\right.\right.$$

$$\Big\downarrow \ 1082$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+3\left(\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{2^{2/3}}-\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{\right.}}{2\ 2^{2/3}}\right.\right.\right.$$

$$\Big\downarrow \ 217$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+3\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{\right.}}{2\ 2^{2/3}}\right.\right.\right.$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{3}\left(\frac{1}{3}\left(-3\int \frac{1}{-x^6-3}d\left(2\sqrt[3]{1-x^3}+1\right)+3\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{\right.}}{2\ 2^{2/3}}\right.\right.\right.$$

$$\Big\downarrow \ 217$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)+3\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-\right.}}{2\ 2^{2/3}}\right.\right.\right.$$

input
```
Int[1/(x^4*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-((1 - x^3)^(1/3)/x^3) + (Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]
 + Log[x^3]/2 - (3*Log[1 - (1 - x^3)^(1/3)])/2 + 3*(-((Sqrt[3]*ArcTan[(1 +
 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(2/3)) - Log[1 + x^3]/(2*2^(2/3)) +
(3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*2^(2/3))))/3)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.34

method	result
pseudoelliptic	$\frac{6\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)x^{3}-4\sqrt{3}\arctan\left(\frac{\left(1+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)x^{3}-6\,2^{\frac{1}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)x^{3}+3\,2^{\frac{1}{3}}}{36\left(-1+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\left(\left(-\right.\right.}$

input
```
int(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/36*(6*3^(1/2)*2^(1/3)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*x^3
-4*3^(1/2)*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*x^3-6*2^(1/3)*ln((-x^3
+1)^(1/3)-2^(1/3))*x^3+3*2^(1/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+
2^(2/3))*x^3+4*ln(-1+(-x^3+1)^(1/3))*x^3-2*ln((-x^3+1)^(2/3)+(-x^3+1)^(1/3
)+1)*x^3+12*(-x^3+1)^(1/3))/(-1+(-x^3+1)^(1/3))/((-x^3+1)^(2/3)+(-x^3+1)^(
1/3)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^4 (1-x^3)^{2/3} (1+x^3)} dx = -\frac{36 \cdot 4^{\frac{1}{6}} \sqrt{\frac{1}{3}} x^3 \arctan\left(\frac{1}{2} \cdot 4^{\frac{1}{6}} \sqrt{\frac{1}{3}} \left(4^{\frac{2}{3}} (-x^3+1)^{\frac{1}{3}} + 4^{\frac{1}{3}}\right)\right) + 3 \cdot 4^{\frac{2}{3}} x^3 \log\left(4^{\frac{2}{3}} (-x^3+1)^{\frac{1}{3}} + 2 (-x^3+1)^{\frac{2}{3}} + 2\right.}{}$$

input
```
integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/72*(36*4^(1/6)*sqrt(1/3)*x^3*arctan(1/2*4^(1/6)*sqrt(1/3)*(4^(2/3)*(-x^
3 + 1)^(1/3) + 4^(1/3))) + 3*4^(2/3)*x^3*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*
(-x^3 + 1)^(2/3) + 2*4^(1/3)) - 6*4^(2/3)*x^3*log(-4^(2/3) + 2*(-x^3 + 1)^
(1/3)) - 8*sqrt(3)*x^3*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3))
- 4*x^3*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 8*x^3*log((-x^3 + 1
)^(1/3) - 1) + 24*(-x^3 + 1)^(1/3))/x^3
```

Sympy [F]

$$\int \frac{1}{x^4 (1-x^3)^{2/3} (1+x^3)} dx = \int \frac{1}{x^4 (-(x-1)(x^2+x+1))^{\frac{2}{3}} (x+1)(x^2-x+1)} dx$$

input
```
integrate(1/x**4/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

Maxima [F]

$$\int \frac{1}{x^4 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}} x^4} \, dx$$

input
```
integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \frac{1}{x^4 (1-x^3)^{2/3} (1+x^3)} \, dx = & -\frac{1}{6} \sqrt{3} 2^{\frac{1}{3}} \arctan\left(\frac{1}{6} \sqrt{3} 2^{\frac{2}{3}} \left(2^{\frac{1}{3}} + 2 \left(-x^3+1\right)^{\frac{1}{3}}\right)\right) \\ & + \frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2 \left(-x^3+1\right)^{\frac{1}{3}} + 1\right)\right) - \frac{1}{12} \\ & \cdot 2^{\frac{1}{3}} \log\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}} \left(-x^3+1\right)^{\frac{1}{3}} + \left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6} \\ & \cdot 2^{\frac{1}{3}} \log\left(\left|-2^{\frac{1}{3}} + \left(-x^3+1\right)^{\frac{1}{3}}\right|\right) - \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{3 x^3} \\ & + \frac{1}{18} \log\left(\left(-x^3+1\right)^{\frac{2}{3}} + \left(-x^3+1\right)^{\frac{1}{3}} + 1\right) - \frac{1}{9} \log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}} - 1\right|\right)\end{aligned}$$

input
```
integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/12*2^
(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(
1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/3*(-x^3 + 1)^(1/3)/x^3 + 1/
18*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 1/9*log(abs((-x^3 + 1)^(
1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.33

$$\int \frac{1}{x^4 (1-x^3)^{2/3} (1+x^3)}\,dx = \frac{2^{1/3} \ln \left(\frac{10 (1-x^3)^{1/3}}{9} - \frac{2^{1/3} \left(\frac{2^{2/3} \left(243\, 2^{1/3}+27 (1-x^3)^{1/3}\right)}{36} - \frac{25}{3}\right)}{6}\right)}{6} - \frac{(1-x^3)^{1/3}}{3\,x^3} - \frac{\ln \left(\frac{31 (1-x^3)^{1/3}}{243} - \frac{31}{243}\right)}{9}$$

$$-\ln\left(\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)\left(\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)^2\left(27\left(1-x^3\right)^{1/3}+81-\sqrt{3}\,81\mathrm{i}\right)-\frac{25}{3}\right)+\frac{10\left(1-x^3\right)^{1/3}}{9}\right)\left(-\frac{1}{1\ldots}\right.$$

input
```
int(1/(x^4*(1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
(2^(1/3)*log((10*(1 - x^3)^(1/3))/9 - (2^(1/3)*((2^(2/3)*(243*2^(1/3) + 27
*(1 - x^3)^(1/3)))/36 - 25/3))/6))/6 - (1 - x^3)^(1/3)/(3*x^3) - log((31*(
1 - x^3)^(1/3))/243 - 31/243)/9 - log(((3^(1/2)*1i)/18 - 1/18)*(((3^(1/2)*
1i)/18 - 1/18)^2*(27*(1 - x^3)^(1/3) - 3^(1/2)*81i + 81) - 25/3) + (10*(1
- x^3)^(1/3))/9)*((3^(1/2)*1i)/18 - 1/18) + log((10*(1 - x^3)^(1/3))/9 - (
(3^(1/2)*1i)/18 + 1/18)*(((3^(1/2)*1i)/18 + 1/18)^2*(3^(1/2)*81i + 27*(1 -
 x^3)^(1/3) + 81) - 25/3))*((3^(1/2)*1i)/18 + 1/18) + (2^(1/3)*log((10*(1
- x^3)^(1/3))/9 - (2^(1/3)*(3^(1/2)*1i - 1)*((2^(2/3)*(3^(1/2)*1i - 1)^2*(
(243*2^(1/3)*(3^(1/2)*1i - 1))/2 + 27*(1 - x^3)^(1/3)))/144 - 25/3))/12)*(
3^(1/2)*1i - 1))/12 - (2^(1/3)*log((10*(1 - x^3)^(1/3))/9 - (2^(1/3)*(3^(1
/2)*1i + 1)*((2^(2/3)*(3^(1/2)*1i + 1)^2*((243*2^(1/3)*(3^(1/2)*1i + 1))/2
 - 27*(1 - x^3)^(1/3)))/144 + 25/3))/12)*(3^(1/2)*1i + 1))/12
```

Reduce [F]

$$\int \frac{1}{x^4 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{(-x^3+1)^{\frac{2}{3}} x^7 + (-x^3+1)^{\frac{2}{3}} x^4} dx$$

input
```
int(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(1/(( - x**3 + 1)**(2/3)*x**7 + ( - x**3 + 1)**(2/3)*x**4),x)
```

3.843 $\int \frac{x^7}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(1+x^3)}{6\ 2^{2/3}} + \frac{1}{6}\log\left(-x-\sqrt[3]{1-x^3}\right) - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
-1/3*x^2*(-x^3+1)^(1/3)+1/9*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(
1/2)-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2
)+1/12*ln(x^3+1)*2^(1/3)+1/6*ln(-x-(-x^3+1)^(1/3))-1/4*ln(-2^(1/3)*x-(-x^3
+1)^(1/3))*2^(1/3)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.39

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{36}\left(-12x^2\sqrt[3]{1-x^3}+4\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)\right.$$
$$-6\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)+4\log\left(x+\sqrt[3]{1-x^3}\right)-6\sqrt[3]{2}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)-2\log\left(x^2-x\sqrt[3]{\ldots}\right.$$

input
```
Integrate[x^7/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-12*x^2*(1 - x^3)^(1/3) + 4*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(
1/3))] - 6*2^(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3)
)] + 4*Log[x + (1 - x^3)^(1/3)] - 6*2^(1/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1
/3)] - 2*Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)] + 3*2^(1/3)*Log[-2
*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/36
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {979, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(1-x^3)^{2/3}(x^3+1)}dx$$
$$\Big\downarrow \text{979}$$
$$\frac{1}{3}\int \frac{x(2-x^3)}{(1-x^3)^{2/3}(x^3+1)}dx-\frac{1}{3}x^2\sqrt[3]{1-x^3}$$
$$\Big\downarrow \text{1054}$$
$$\frac{1}{3}\int \left(\frac{3x}{(1-x^3)^{2/3}(x^3+1)}-\frac{x}{(1-x^3)^{2/3}}\right)dx-\frac{1}{3}x^2\sqrt[3]{1-x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)-\frac{3\log\left(-\sqrt[3]{\right.}}{}\right.$$

$$\frac{1}{3}x^2\sqrt[3]{1-x^3}$$

input
```
Int[x^7/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
-1/3*(x^2*(1 - x^3)^(1/3)) + (ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/
Sqrt[3] - (Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/2^
(2/3) + Log[1 + x^3]/(2*2^(2/3)) + Log[-x - (1 - x^3)^(1/3)]/2 - (3*Log[-(
2^(1/3)*x) - (1 - x^3)^(1/3)])/(2*2^(2/3)))/3
```

Defintions of rubi rules used

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.46

method	result
pseudoelliptic	$\frac{3\,2^{\frac{1}{3}}\ln\left(\frac{2^{\frac{2}{3}}x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)-2\ln\left(\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}-\left(-x^{3}+1\right)^{\frac{1}{3}}x+x^{2}}{x^{2}}\right)-6\,2^{\frac{1}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)+4\ln\left(\right.}{36\left(x+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\left(\left(-x^{3}+\right.\right.}$

input
```
int(x^7/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/36*(3*2^(1/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x
^2)-2*ln(((-x^3+1)^(2/3)-(-x^3+1)^(1/3)*x+x^2)/x^2)-6*2^(1/3)*ln((2^(1/3)*
x+(-x^3+1)^(1/3))/x)+4*ln((x+(-x^3+1)^(1/3))/x)-12*x^2*(-x^3+1)^(1/3)+(6*a
rctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)*2^(1/3)-4*arctan(1/3*(-2*
(-x^3+1)^(1/3)+x)*3^(1/2)/x))*3^(1/2))/(x+(-x^3+1)^(1/3))/((-x^3+1)^(2/3)+
x*(x-(-x^3+1)^(1/3)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.30

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{3}\left(-x^3+1\right)^{\frac{1}{3}}x^2 - \frac{1}{2} \cdot 4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\arctan\left(-\frac{4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\left(4^{\frac{1}{3}}x - 4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)}{2x}\right) - \frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}x + 2\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right) + \frac{1}{24} \cdot 4^{\frac{2}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}x^2 - 4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x + 2\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right) + \frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x - 2\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3x}\right) + \frac{1}{9}\log\left(\frac{x + \left(-x^3+1\right)^{\frac{1}{3}}}{x}\right) - \frac{1}{18}\log\left(\frac{x^2 - \left(-x^3+1\right)^{\frac{1}{3}}x + \left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

input
```
integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/3*(-x^3 + 1)^(1/3)*x^2 - 1/2*4^(1/6)*sqrt(1/3)*arctan(-1/2*4^(1/6)*sqrt
(1/3)*(4^(1/3)*x - 4^(2/3)*(-x^3 + 1)^(1/3))/x) - 1/12*4^(2/3)*log((4^(2/3
)*x + 2*(-x^3 + 1)^(1/3))/x) + 1/24*4^(2/3)*log((2*4^(1/3)*x^2 - 4^(2/3)*(
-x^3 + 1)^(1/3)*x + 2*(-x^3 + 1)^(2/3))/x^2) + 1/9*sqrt(3)*arctan(-1/3*(sq
rt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/9*log((x + (-x^3 + 1)^(1/3))/
x) - 1/18*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [F]

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^7}{\left(-(x-1)(x^2+x+1)\right)^{\frac{2}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**7/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [F]

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Giac [F]

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^7}{(1-x^3)^{2/3}(x^3+1)}\,dx$$

input
```
int(x^7/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
int(x^7/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^7}{(-x^3+1)^{\frac{2}{3}}x^3+(-x^3+1)^{\frac{2}{3}}}dx$$

input
```
int(x^7/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(x**7/(( - x**3 + 1)**(2/3)*x**3 + ( - x**3 + 1)**(2/3)),x)
```

3.844 $\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 139

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(1+x^3)}{6\ 2^{2/3}} - \frac{1}{2}\log\left(-x-\sqrt[3]{1-x^3}\right) + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1-
2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)-1/12*ln(x^3+1)*2^(1/3
)-1/2*ln(-x-(-x^3+1)^(1/3))+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(1/3)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.48

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{12}\left(-4\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)\right.$$
$$+2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-4\log\left(x+\sqrt[3]{1-x^3}\right)+2\sqrt[3]{2}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+2\log\left(x^2-x\sqrt[3]{\right.$$

input
```
Integrate[x^4/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-4*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/3))] + 2*2^(1/3)*Sqrt[3
]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 4*Log[x + (1 - x^3)^
(1/3)] + 2*2^(1/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + 2*Log[x^2 - x*(1 -
 x^3)^(1/3) + (1 - x^3)^(2/3)] - 2^(1/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^
(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {983, 853, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(1-x^3)^{2/3}(x^3+1)}\,dx$$
$$\Big\downarrow \text{983}$$
$$\int \frac{x}{(1-x^3)^{2/3}}dx-\int \frac{x}{(1-x^3)^{2/3}(x^3+1)}dx$$
$$\Big\downarrow \text{853}$$

$$-\int \frac{x}{(1-x^3)^{2/3}(x^3+1)}dx-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)$$

$$\Big\downarrow \text{992}$$

$$-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x^3+1)}{6\ 2^{2/3}}-\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}$$

input
```
Int[x^4/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
-(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 - (2*2^
(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^(
2/3)) - Log[-x - (1 - x^3)^(1/3)]/2 + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/
(2*2^(2/3))
```

Defintions of rubi rules used

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$-\frac{2^{\frac{1}{3}} \ln\left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)}{12}+\frac{\ln\left(\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}-\left(-x^{3}+1\right)^{\frac{1}{3}} x+x^{2}}{x^{2}}\right)}{6}+\frac{2^{\frac{1}{3}} \ln\left(\frac{2^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)}{6}-$

input
```
int(x^4/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/12*2^(1/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2
)+1/6*ln(((-x^3+1)^(2/3)-(-x^3+1)^(1/3)*x+x^2)/x^2)+1/6*2^(1/3)*ln((2^(1/3
)*x+(-x^3+1)^(1/3))/x)-1/3*ln((x+(-x^3+1)^(1/3))/x)+1/6*(-arctan(1/3*3^(1/
2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)*2^(1/3)+2*arctan(1/3*(-2*(-x^3+1)^(1/3)+
x)*3^(1/2)/x))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.40

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{2} \cdot 4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\arctan\left(-\frac{4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\left(4^{\frac{1}{3}}x-4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}\right)}{2\,x}\right) + \frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}x+2\,(-x^3+1)^{\frac{1}{3}}}{x}\right) - \frac{1}{24} \cdot 4^{\frac{2}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}x^2-4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}x+2\,(-x^3+1)^{\frac{2}{3}}}{x^2}\right) - \frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3\,x}\right) - \frac{1}{3}\log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right) + \frac{1}{6}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)$$

input
```
integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/2*4^(1/6)*sqrt(1/3)*arctan(-1/2*4^(1/6)*sqrt(1/3)*(4^(1/3)*x - 4^(2/3)*(
-x^3 + 1)^(1/3))/x) + 1/12*4^(2/3)*log((4^(2/3)*x + 2*(-x^3 + 1)^(1/3))/x)
 - 1/24*4^(2/3)*log((2*4^(1/3)*x^2 - 4^(2/3)*(-x^3 + 1)^(1/3)*x + 2*(-x^3
+ 1)^(2/3))/x^2) - 1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 +
1)^(1/3))/x) - 1/3*log((x + (-x^3 + 1)^(1/3))/x) + 1/6*log((x^2 - (-x^3 +
1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [F]

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^4}{\left(-(x-1)(x^2+x+1)\right)^{\frac{2}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**4/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [F]

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Giac [F]

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^4}{(1-x^3)^{2/3}(x^3+1)}\,dx$$

input
```
int(x^4/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
int(x^4/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^4}{(-x^3+1)^{\frac{2}{3}}x^3+(-x^3+1)^{\frac{2}{3}}}dx$$

input
```
int(x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(x**4/(( - x**3 + 1)**(2/3)*x**3 + ( - x**3 + 1)**(2/3)),x)
```

3.845 $\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(1+x^3)}{6\ 2^{2/3}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)+1/
12*ln(x^3+1)*2^(1/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(1/3)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

$$\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{-2\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 2\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + \log\left(-2x^2+2\right.}{6\ 2^{2/3}}$$

input
```
Integrate[x/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*Log[2*x
+ 2^(2/3)*(1 - x^3)^(1/3)] + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1
/3)*(1 - x^3)^(2/3)])/(6*2^(2/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow 992$$

$$-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log(x^3+1)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}$$

input
```
Int[x/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
-(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) +
 Log[1 + x^3]/(6*2^(2/3)) - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(2/3)
)
```

Defintions of rubi rules used

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 3.84 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09

method	result	si
pseudoelliptic	$\frac{2^{\frac{1}{3}}\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)-2\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)+\ln\left(\frac{2^{\frac{2}{3}}x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)\right)}{12}$	9
trager	Expression too large to display	9

input
```
int(x/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*2^(1/3)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)-
2*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)+ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*
x+(-x^3+1)^(2/3))/x^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(67) = 134$.

Time = 1.29 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.81

$$\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{6}$$

$$\cdot 4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\left(6\cdot 4^{\frac{2}{3}}(19\,x^8-16\,x^5+x^2)(-x^3+1)^{\frac{1}{3}}+12\,(5\,x^7+4\,x^4-x)(-x^3+1)^{\frac{2}{3}}-4^{\frac{1}{3}}(71\,x\right.}{2\,(109\,x^9-105\,x^6+3\,x^3+1)}\right.$$

$$-\frac{1}{36}\cdot 4^{\frac{2}{3}}\log\left(\frac{3\cdot 4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}x^2+6\,(-x^3+1)^{\frac{2}{3}}x+4^{\frac{1}{3}}(x^3+1)}{x^3+1}\right)+\frac{1}{72}$$

$$\cdot 4^{\frac{2}{3}}\log\left(\frac{6\cdot 4^{\frac{1}{3}}(5\,x^4-x)(-x^3+1)^{\frac{2}{3}}+4^{\frac{2}{3}}(19\,x^6-16\,x^3+1)-24\,(2\,x^5-x^2)(-x^3+1)^{\frac{1}{3}}}{x^6+2\,x^3+1}\right)$$

input
```
integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/6*4^(1/6)*sqrt(1/3)*arctan(1/2*4^(1/6)*sqrt(1/3)*(6*4^(2/3)*(19*x^8 - 1
6*x^5 + x^2)*(-x^3 + 1)^(1/3) + 12*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) -
4^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1))
 - 1/36*4^(2/3)*log((3*4^(2/3)*(-x^3 + 1)^(1/3)*x^2 + 6*(-x^3 + 1)^(2/3)*x
 + 4^(1/3)*(x^3 + 1))/(x^3 + 1)) + 1/72*4^(2/3)*log((6*4^(1/3)*(5*x^4 - x)*
(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 - 16*x^3 + 1) - 24*(2*x^5 - x^2)*(-x^3
 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

Sympy [F]

$$\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x}{(-(x-1)(x^2+x+1))^{\frac{2}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Giac [F]

$$\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x}{(1-x^3)^{2/3}(x^3+1)}\,dx$$

input
```
int(x/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
int(x/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x}{(-x^3+1)^{\frac{2}{3}}x^3+(-x^3+1)^{\frac{2}{3}}}dx$$

input
```
int(x/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(x/(( - x**3 + 1)**(2/3)*x**3 + ( - x**3 + 1)**(2/3)),x)
```

3.846 $\int \frac{1}{x^2(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{1}{x^2(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\sqrt[3]{1-x^3}}{x}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(1+x^3)}{6\ 2^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
-(-x^3+1)^(1/3)/x+1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2
^(1/3)*3^(1/2)-1/12*ln(x^3+1)*2^(1/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^
(1/3)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^2 (1-x^3)^{2/3} (1+x^3)} dx = -\frac{\sqrt[3]{1-x^3}}{x} - \frac{\arctan\left(\frac{\sqrt{3}x}{-x+2^{2/3}\sqrt[3]{1-x^3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)}{3\ 2^{2/3}} - \frac{\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}(1-x^3)^{2/3}\right)}{6\ 2^{2/3}}$$

input
```
Integrate[1/(x^2*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
-((1 - x^3)^(1/3)/x) - ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/
(2^(2/3)*Sqrt[3]) + Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[-
2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {980, 25, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (1-x^3)^{2/3} (x^3+1)} dx$$

$$\Big\downarrow \text{980}$$

$$\int -\frac{x}{(1-x^3)^{2/3} (x^3+1)} dx - \frac{\sqrt[3]{1-x^3}}{x}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{x}{(1-x^3)^{2/3} (x^3+1)} dx - \frac{\sqrt[3]{1-x^3}}{x}$$

$$\Big\downarrow \text{992}$$

$$\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\sqrt[3]{1-x^3}}{x}-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}$$

input
```
Int[1/(x^2*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
-((1 - x^3)^(1/3)/x) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]
/(2^(2/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^(2/3)) + Log[-(2^(1/3)*x) - (1 - x^
3)^(1/3)]/(2*2^(2/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 26.87 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.17

method	result
pseudoelliptic	$\frac{-2\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)x+2\,2^{\frac{1}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)x-2^{\frac{1}{3}}\ln\left(\frac{2^{\frac{2}{3}}x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right.}{12x}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*(-2*2^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)
*x+2*2^(1/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*x-2^(1/3)*ln((2^(2/3)*x^2-2^
(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)*x-12*(-x^3+1)^(1/3))/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(81) = 162$.

Time = 1.13 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.58

$$\int \frac{1}{x^2(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{12\cdot 4^{\frac{1}{6}}\sqrt{\frac{1}{3}}x\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\left(6\cdot 4^{\frac{2}{3}}\left(19\,x^{8}-16\,x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}+12\left(5\,x^{7}+4\,x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}\right.}{2\left(109\,x^{9}-105\,x^{6}+3\,x^{3}+1\right)}\right.}{}$$

input
```
integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/72*(12*4^(1/6)*sqrt(1/3)*x*arctan(1/2*4^(1/6)*sqrt(1/3)*(6*4^(2/3)*(19*x
^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) + 12*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2
/3) - 4^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3
+ 1)) + 2*4^(2/3)*x*log((3*4^(2/3)*(-x^3 + 1)^(1/3)*x^2 + 6*(-x^3 + 1)^(2/
3)*x + 4^(1/3)*(x^3 + 1))/(x^3 + 1)) - 4^(2/3)*x*log((6*4^(1/3)*(5*x^4 - x
)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 - 16*x^3 + 1) - 24*(2*x^5 - x^2)*(-x^
3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 72*(-x^3 + 1)^(1/3))/x
```

Sympy [F]

$$\int \frac{1}{x^2 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{x^2 \left(-(x-1)(x^2+x+1)\right)^{\frac{2}{3}} (x+1)(x^2-x+1)} \, dx$$

input
```
integrate(1/x**2/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

Maxima [F]

$$\int \frac{1}{x^2 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}} x^2} \, dx$$

input
```
integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}} x^2} \, dx$$

input
```
integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{x^2 (1-x^3)^{2/3} (x^3+1)} \, dx$$

input
```
int(1/(x^2*(1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^2*(1 - x^3)^(2/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{(-x^3+1)^{\frac{2}{3}} x^5 + (-x^3+1)^{\frac{2}{3}} x^2} dx$$

input
```
int(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(1/(( - x**3 + 1)**(2/3)*x**5 + ( - x**3 + 1)**(2/3)*x**2),x)
```

3.847 $\int \frac{1}{x^5(1-x^3)^{2/3}(1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 124

$$\int \frac{1}{x^5\left(1-x^3\right)^{2/3}\left(1+x^3\right)} dx = -\frac{\sqrt[3]{1-x^3}}{4x^4}+\frac{\sqrt[3]{1-x^3}}{4x} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output

```
-1/4*(-x^3+1)^(1/3)/x^4+1/4*(-x^3+1)^(1/3)/x-1/6*arctan(1/3*(1-2*2^(1/3)*x
/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)+1/12*ln(x^3+1)*2^(1/3)-1/4*ln(-2
^(1/3)*x-(-x^3+1)^(1/3))*2^(1/3)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^5 (1-x^3)^{2/3} (1+x^3)} \, dx = \frac{1}{12}\left(-\frac{3(1-x^3)^{4/3}}{x^4}\right.$$
$$-2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-2\sqrt[3]{2}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+\sqrt[3]{2}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}(\right.$$

input
```
Integrate[1/(x^5*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
((-3*(1 - x^3)^(4/3))/x^4 - 2*2^(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2
/3)*(1 - x^3)^(1/3))] - 2*2^(1/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(
1/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {980, 25, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (1-x^3)^{2/3} (x^3+1)} \, dx$$
$$\Big\downarrow \text{ 980}$$
$$\frac{1}{4}\int -\frac{1-3x^3}{x^2 (1-x^3)^{2/3} (x^3+1)}dx-\frac{\sqrt[3]{1-x^3}}{4x^4}$$
$$\Big\downarrow \text{ 25}$$
$$-\frac{1}{4}\int \frac{1-3x^3}{x^2 (1-x^3)^{2/3} (x^3+1)}dx-\frac{\sqrt[3]{1-x^3}}{4x^4}$$
$$\Big\downarrow \text{ 1053}$$

$$\frac{1}{4}\left(\int \frac{4x}{(1-x^3)^{2/3}(x^3+1)}dx+\frac{\sqrt[3]{1-x^3}}{x}\right)-\frac{\sqrt[3]{1-x^3}}{4x^4}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{4}\left(4\int \frac{x}{(1-x^3)^{2/3}(x^3+1)}dx+\frac{\sqrt[3]{1-x^3}}{x}\right)-\frac{\sqrt[3]{1-x^3}}{4x^4}$$

$$\Big\downarrow \textbf{992}$$

$$\frac{1}{4}\left(4\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log(x^3+1)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}\right)+\frac{\sqrt[3]{1-x^3}}{x}\right)-\frac{\sqrt[3]{1-x^3}}{4x^4}$$

input
```
Int[1/(x^5*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
-1/4*(1 - x^3)^(1/3)/x^4 + ((1 - x^3)^(1/3)/x + 4*(-(ArcTan[(1 - (2*2^(1/3
)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) + Log[1 + x^3]/(6*2^(2/3
)) - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(2/3))))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 26.91 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{-2\,2^{\frac{1}{3}} \ln\left(\frac{2^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right) x^{4}+\left(3 x^{3}-3\right) \left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{1}{3}} x^{4} \left(2 \sqrt{3}\, \arctan\left(\frac{\sqrt{3}\, \left(-2^{\frac{2}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3 x}\right)+\ln\left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}} (\cdots}{\cdots}\right.\right.}{12 x^{4}}$
risch	Expression too large to display
trager	Expression too large to display

input
```
int(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*(-2*2^(1/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*x^4+(3*x^3-3)*(-x^3+1)^(
1/3)+2^(1/3)*x^4*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)
/x)+ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)))/x^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(95) = 190$.

Time = 1.12 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.23

$$\int \frac{1}{x^5 (1-x^3)^{2/3} (1+x^3)} dx = -\frac{12 \cdot 4^{\frac{1}{6}} \sqrt{\frac{1}{3}} x^4 \arctan\left(\frac{4^{\frac{1}{6}} \sqrt{\frac{1}{3}} \left(6 \cdot 4^{\frac{2}{3}} (19 x^8 - 16 x^5 + x^2)(-x^3+1)^{\frac{1}{3}} + 12 (5 x^7 + 4 x^4 - x)(-x^3+1)^{\frac{2}{3}} - 4^{\frac{1}{3}} (71 x^9 - 111 x^6 + 33 x^3 - 1)\right)}{2 (109 x^9 - 105 x^6 + 3 x^3 + 1)}\right) + \cdots}{}$$

input
```
integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/72*(12*4^(1/6)*sqrt(1/3)*x^4*arctan(1/2*4^(1/6)*sqrt(1/3)*(6*4^(2/3)*(1
9*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) + 12*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)
^(2/3) - 4^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x
^3 + 1)) + 2*4^(2/3)*x^4*log((3*4^(2/3)*(-x^3 + 1)^(1/3)*x^2 + 6*(-x^3 + 1
)^(2/3)*x + 4^(1/3)*(x^3 + 1))/(x^3 + 1)) - 4^(2/3)*x^4*log((6*4^(1/3)*(5*
x^4 - x)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 - 16*x^3 + 1) - 24*(2*x^5 - x^
2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 18*(x^3 - 1)*(-x^3 + 1)^(1/3))/x
^4
```

Sympy [F]

$$\int \frac{1}{x^5 (1-x^3)^{2/3} (1+x^3)} dx = \int \frac{1}{x^5 (-(x-1)(x^2+x+1))^{\frac{2}{3}} (x+1)(x^2-x+1)} dx$$

input
```
integrate(1/x**5/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

Maxima [F]

$$\int \frac{1}{x^5 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}} x^5} \, dx$$

input
```
integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}} x^5} \, dx$$

input
```
integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{x^5 (1-x^3)^{2/3} (x^3+1)} \, dx$$

input
```
int(1/(x^5*(1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^5*(1 - x^3)^(2/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^5 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{(-x^3+1)^{\frac{2}{3}} x^8 + (-x^3+1)^{\frac{2}{3}} x^5} dx$$

input
```
int(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(1/(( - x**3 + 1)**(2/3)*x**8 + ( - x**3 + 1)**(2/3)*x**5),x)
```

3.848 $\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 291

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{2}x\sqrt[3]{1-x^3} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\,2^{2/3}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\,2^{2/3}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\,2^{2/3}} - \frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\,2^{2/3}}$$

output

```
-1/2*x*(-x^3+1)^(1/3)+1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^
(1/2))*2^(1/3)*3^(1/2)+1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^
(1/2))*2^(1/3)*3^(1/2)+1/12*ln(2^(2/3)-(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/12*
ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+
1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/24*ln(2*2^(1/3)+(1-x)^2/(
-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.40

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{2}x\sqrt[3]{1-x^3}\Bigg(-1$$

$$-\frac{4\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right)}{(1+x^3)\left(-4\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right)+x^3\left(3\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},x^3,-x^3\right)+\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\right.\right.\right.}$$

input
```
Integrate[x^6/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(x*(1 - x^3)^(1/3)*(-1 - (4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1 +
x^3)*(-4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -1/
3, 2, 7/3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))))/2
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {979, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow \text{979}$$

$$\frac{1}{2}\int \frac{\sqrt[3]{1-x^3}}{x^3+1}dx-\frac{1}{2}x\sqrt[3]{1-x^3}$$

$$\Big\downarrow \text{927}$$

$$-\frac{9}{2}\int \frac{1-x}{\sqrt[3]{1-x^3}\left(4-\frac{(1-x)^3}{1-x^3}\right)\left(\frac{2(1-x)^3}{1-x^3}+1\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}\sqrt[3]{1-x^3}x$$

$$\Big\downarrow \ 982$$

$$-\frac{9}{2}\left(\frac{1}{9}\int \frac{1-x}{\sqrt[3]{1-x^3}\left(4-\frac{(1-x)^3}{1-x^3}\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{2}{9}\int \frac{1-x}{\sqrt[3]{1-x^3}\left(\frac{2(1-x)^3}{1-x^3}+1\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}\right)-\frac{1}{2}\sqrt[3]{1-x^3}x$$

$$\Big\downarrow \ 821$$

$$-\frac{9}{2}\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}\right)+\frac{1}{9}\left(\frac{\int \frac{1}{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\ 2^{2/3}} - \right.\right.$$

$$\frac{1}{2}\sqrt[3]{1-x^3}x$$

$$\Big\downarrow \ 16$$

$$-\frac{9}{2}\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}}d\frac{}{\sqrt[3]{}}}{3\ 2^{2/3}}\right.\right.$$

$$\frac{1}{2}\sqrt[3]{1-x^3}x$$

$$\Big\downarrow \ 1142$$

$$-\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{\int -\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)\right.$$

$$\frac{1}{2}\sqrt[3]{1-x^3}x$$

$\downarrow$ 25

$$-\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\int \frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)-\right.$$

$$\frac{1}{2}\sqrt[3]{1-x^3}x$$

$\downarrow$ 27

$$-\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right.\right.$$

$$\frac{1}{2}\sqrt[3]{1-x^3}x$$

$$\Big\downarrow \ 1082$$

$$-\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\right.$$

$$\frac{1}{2}\sqrt[3]{1-x^3}x$$

$$\Big\downarrow \ 217$$

$$-\frac{9}{2}\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\sqrt{\right.\right.$$

$$\frac{1}{2}\sqrt[3]{1-x^3}x$$

$$\downarrow \text{1103}$$

$$-\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\sqrt{3}\arctan\left(\right.}{}\right.\right.$$

$$\frac{1}{2}\sqrt[3]{1-x^3}x$$

input
```
Int[x^6/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
-1/2*(x*(1 - x^3)^(1/3)) - (9*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(1 -
 x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1
- x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(2*2^(1/3))))/(3*2^(1/3))
 - Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(2/3))))/9 + (-1/3*Log[
2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/2^(2/3) - (Sqrt[3]*ArcTan[(1 + (2^(1/3)
*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]] - Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^
(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/2)/(3*2^(2/3)))/9))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 22.78 (sec) , antiderivative size = 696, normalized size of antiderivative = 2.39

method	result	size
risch	Expression too large to display	696
trager	Expression too large to display	1163

input
```
int(x^6/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x^3-1)/(-x^3+1)^(2/3)+(1/4*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3
-2)+9*_Z^2)*ln(-(18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*
RootOf(_Z^3-2)^2*x^3+12*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2
)*RootOf(_Z^3-2)^3*x^3+3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^
2)*x^6+2*RootOf(_Z^3-2)*x^6-9*(x^6-2*x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+
3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^2*x-18*(x^6-2*x^3+1)^(1/3)*Root
Of(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)*x^2-6*(x^6-
2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x^2-6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(
_Z^3-2)+9*_Z^2)*x^3-4*RootOf(_Z^3-2)*x^3+3*RootOf(RootOf(_Z^3-2)^2+3*_Z*Ro
otOf(_Z^3-2)+9*_Z^2)+2*RootOf(_Z^3-2))/(1+x)^2/(x^2-x+1)^2)+1/12*RootOf(_Z
^3-2)*ln((36*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(
_Z^3-2)^2*x^3+6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf
(_Z^3-2)^3*x^3-6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^6-R
ootOf(_Z^3-2)*x^6-9*(x^6-2*x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootO
f(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^2*x-6*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^
2*x^2+36*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^3+6*RootOf(
_Z^3-2)*x^3-6*(x^6-2*x^3+1)^(2/3)*x-6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(
_Z^3-2)+9*_Z^2)-RootOf(_Z^3-2))/(1+x)^2/(x^2-x+1)^2))/(-x^3+1)^(2/3)*((x^3
-1)^2)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.20

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{12}$$

$$\cdot 4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\arctan\left(-\frac{4^{\frac{1}{6}}\sqrt{\frac{1}{3}}\left(6\cdot 4^{\frac{2}{3}}(x^{16}-33\,x^{13}+110\,x^{10}-110\,x^{7}+33\,x^{4}-x)(-x^{3}+1)^{\frac{1}{3}}-48\,(x^{14}-2\,x^{11}\right.}{2\,(x^{18}-102\,x^{15}+447\,x^{12}}\right.$$

$$+\frac{1}{72}\cdot 4^{\frac{2}{3}}\log\left(-\frac{12\,(-x^{3}+1)^{\frac{2}{3}}x^{2}-3\cdot 4^{\frac{2}{3}}(x^{4}-x)(-x^{3}+1)^{\frac{1}{3}}+4^{\frac{1}{3}}(x^{6}+2\,x^{3}+1)}{x^{6}+2\,x^{3}+1}\right)$$

$$-\frac{1}{144}$$

$$\cdot 4^{\frac{2}{3}}\log\left(\frac{24\cdot 4^{\frac{1}{3}}(x^{8}-4\,x^{5}+x^{2})(-x^{3}+1)^{\frac{2}{3}}+4^{\frac{2}{3}}(x^{12}-32\,x^{9}+78\,x^{6}-32\,x^{3}+1)+12\,(x^{10}-11\,x^{7}+11\,x\right.}{x^{12}+4\,x^{9}+6\,x^{6}+4\,x^{3}+1}$$

$$-\frac{1}{2}\left(-x^{3}+1\right)^{\frac{1}{3}}x$$

input
```
integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/12*4^(1/6)*sqrt(1/3)*arctan(-1/2*4^(1/6)*sqrt(1/3)*(6*4^(2/3)*(x^16 - 33
*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3) - 48*(x^14 - 2*x
^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) - 4^(1/3)*(x^18 + 42*x^15 - 41
7*x^12 + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 62
8*x^9 + 447*x^6 - 102*x^3 + 1)) + 1/72*4^(2/3)*log(-(12*(-x^3 + 1)^(2/3)*x
^2 - 3*4^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) + 4^(1/3)*(x^6 + 2*x^3 + 1))/(x^
6 + 2*x^3 + 1)) - 1/144*4^(2/3)*log((24*4^(1/3)*(x^8 - 4*x^5 + x^2)*(-x^3
+ 1)^(2/3) + 4^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) + 12*(x^10 - 11
*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1)) -
 1/2*(-x^3 + 1)^(1/3)*x
```

Sympy [F]

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx=\int \frac{x^6}{\left(-(x-1)(x^2+x+1)\right)^{\frac{2}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**6/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [F]

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx=\int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Giac [F]

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^6}{(1-x^3)^{2/3}(x^3+1)}\,dx$$

input
```
int(x^6/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
int(x^6/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^6}{(-x^3+1)^{\frac{2}{3}}x^3+(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
int(x^6/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(x**6/(( - x**3 + 1)**(2/3)*x**3 + ( - x**3 + 1)**(2/3)),x)
```

3.849 $\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 294

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right) - \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \text{log}$$

output
```
-1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2
)-1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2
)+1/2*x*hypergeom([1/3, 2/3],[4/3],x^3)-1/12*ln(2^(2/3)-(1-x)/(-x^3+1)^(1/
3))*2^(1/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1
)^(1/3))*2^(1/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/24*ln(2*
2^(1/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.09

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{4}x^4 \operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},x^3,-x^3\right)$$

input
```
Integrate[x^3/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(x^4*AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])/4
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {983, 778, 928, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow \text{983}$$

$$\int \frac{1}{(1-x^3)^{2/3}}dx-\int \frac{1}{(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow \text{778}$$

$$x \operatorname{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-\int \frac{1}{(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow \text{928}$$

$$-\frac{1}{2}\int \frac{1}{(1-x^3)^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{1-x^3}}{x^3+1}dx+x \operatorname{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$$\Big\downarrow \text{778}$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-\frac{1}{2}\int\frac{\sqrt[3]{1-x^3}}{x^3+1}dx$$

$\downarrow$ 927

$$\frac{9}{2}\int\frac{1-x}{\sqrt[3]{1-x^3}\left(4-\frac{(1-x)^3}{1-x^3}\right)\left(\frac{2(1-x)^3}{1-x^3}+1\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$\downarrow$ 982

$$\frac{9}{2}\left(\frac{1}{9}\int\frac{1-x}{\sqrt[3]{1-x^3}\left(4-\frac{(1-x)^3}{1-x^3}\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{2}{9}\int\frac{1-x}{\sqrt[3]{1-x^3}\left(\frac{2(1-x)^3}{1-x^3}+1\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}\right)+\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$\downarrow$ 821

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\int\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\int\frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}\right)+\frac{1}{9}\left(\frac{\int\frac{1}{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\ 2^{2/3}}-\cdots\right.\right.$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$\downarrow$ 16

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\int\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int\frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}}d\cdots}{3\ 2^{2/3}}\cdots\right.\right.$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$\downarrow$ 1142

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{\int -\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\right.$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$\downarrow$ 25

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\int \frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\right.$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$\downarrow$ 27

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)\right.$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{(1-x)^2}{(1-x^3)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(\right.\right.$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\sqrt{3}\ldots}{}\right.\right.$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\sqrt{3}\arctan\left(\cdots\right.}{\cdots}\right.\right.$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

input
```
Int[x^3/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(x*Hypergeometric2F1[1/3, 2/3, 4/3, x^3])/2 + (9*((2*((-((Sqrt[3]*ArcTan[(
1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3)) + Log[1 + (2^(
2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(2*2^
(1/3)))/(3*2^(1/3)) - Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(2/3
))))/9 + (-1/3*Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/2^(2/3) - (Sqrt[3]*A
rcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]] - Log[2*2^(1/3) + (
1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/2)/(3*2^(2/3
)))/9))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [F]

$$\int \frac{x^3}{(-x^3+1)^{\frac{2}{3}}(x^3+1)}dx$$

input
```
int(x^3/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(x^3/(-x^3+1)^(2/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral(-(-x^3 + 1)^(1/3)*x^3/(x^6 - 1), x)
```

Sympy [F]

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^3}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**3/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [F]

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Giac [F]

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^3}{(1-x^3)^{2/3}(x^3+1)}\,dx$$

input
```
int(x^3/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
int(x^3/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^3}{(-x^3+1)^{\frac{2}{3}}x^3+(-x^3+1)^{\frac{2}{3}}}dx$$

input
```
int(x^3/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(x**3/(( - x**3 + 1)**(2/3)*x**3 + ( - x**3 + 1)**(2/3)),x)
```

3.850 $\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 293

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\,2^{2/3}}-\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\,2^{2/3}}+\frac{\log}{}$$

output
```
1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
+1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
+1/2*x*hypergeom([1/3, 2/3],[4/3],x^3)+1/12*ln(2^(2/3)-(1-x)/(-x^3+1)^(1/3
))*2^(1/3)-1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)
^(1/3))*2^(1/3)+1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/24*ln(2*2
^(1/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.38

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{4x\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},x^3,-x^3\right)}{(1-x^3)^{2/3}(1+x^3)\left(-4\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},x^3,-x^3\right)+x^3\left(3\,\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},x^3,-x^3\right)-2\,\text{AppellF}\right.\right.}$$

input
```
Integrate[1/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-4*x*AppellF1[1/3, 2/3, 1, 4/3, x^3, -x^3])/((1 - x^3)^(2/3)*(1 + x^3)*(-
4*AppellF1[1/3, 2/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, 2/3, 2, 7/3
, x^3, -x^3] - 2*AppellF1[4/3, 5/3, 1, 7/3, x^3, -x^3])))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {928, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow \text{ 928}$$

$$\frac{1}{2}\int \frac{1}{(1-x^3)^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{1-x^3}}{x^3+1}dx$$

$$\Big\downarrow \text{ 778}$$

$$\frac{1}{2}\int \frac{\sqrt[3]{1-x^3}}{x^3+1}dx+\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$$\Big\downarrow \text{ 927}$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-\frac{9}{2}\int\frac{1-x}{\sqrt[3]{1-x^3}\left(4-\frac{(1-x)^3}{1-x^3}\right)\left(\frac{2(1-x)^3}{1-x^3}+1\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{982}$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-$$
$$\frac{9}{2}\left(\frac{1}{9}\int\frac{1-x}{\sqrt[3]{1-x^3}\left(4-\frac{(1-x)^3}{1-x^3}\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{2}{9}\int\frac{1-x}{\sqrt[3]{1-x^3}\left(\frac{2(1-x)^3}{1-x^3}+1\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-$$
$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\int\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\int\frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}\right)+\frac{1}{9}\left(\frac{\int\frac{1}{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\ 2^{2/3}}-\right.\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-$$
$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\int\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int\frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}}d\frac{1-}{\sqrt[3]{1}}}{3\ 2^{2/3}}\right.\right.$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-$$
$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{\int-\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\right.$$

$\Big\downarrow$ 25

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-$$
$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\int\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\right.$$

$\Big\downarrow$ 27

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-$$
$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)\right.$$

$$\Big\downarrow 1082$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(\right.\right.$$

$$\Big\downarrow 217$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\sqrt{3}\ldots}{\ldots}\right.\right.$$

$$\Big\downarrow 1103$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\sqrt{3}\arctan\left(\ldots\right)}{\ldots}\right.\right.$$

input
```
Int[1/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(x*Hypergeometric2F1[1/3, 2/3, 4/3, x^3])/2 - (9*((2*((-((Sqrt[3]*ArcTan[(
1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3)) + Log[1 + (2^(
2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(2*2^
(1/3))))/(3*2^(1/3)) - Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(2/3
))))/9 + (-1/3*Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/2^(2/3) - (Sqrt[3]*A
rcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]] - Log[2*2^(1/3) + (
1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/2)/(3*2^(2/3
)))/9))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [F]

$$\int \frac{1}{(-x^3+1)^{\frac{2}{3}}(x^3+1)}dx$$

input
```
int(1/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(1/(-x^3+1)^(2/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral(-(-x^3 + 1)^(1/3)/(x^6 - 1), x)
```

Sympy [F]

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{1}{(-(x-1)(x^2+x+1))^{\frac{2}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(1/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(1/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Giac [F]

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{1}{(1-x^3)^{2/3}(x^3+1)}\,dx$$

input
```
int(1/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
int(1/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{1}{(-x^3+1)^{\frac{2}{3}}x^3+(-x^3+1)^{\frac{2}{3}}}dx$$

input
```
int(1/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(1/(( - x**3 + 1)**(2/3)*x**3 + ( - x**3 + 1)**(2/3)),x)
```

3.851 $\int \frac{1}{x^3(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 294

$$\int \frac{1}{x^3(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\sqrt[3]{1-x^3}}{2x^2} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\,2^{2/3}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\,2^{2/3}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\,2^{2/3}} + \frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\,2^{2/3}}$$

output
```
-1/2*(-x^3+1)^(1/3)/x^2-1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*
3^(1/2))*2^(1/3)*3^(1/2)-1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*
3^(1/2))*2^(1/3)*3^(1/2)-1/12*ln(2^(2/3)-(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/1
2*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3
)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/24*ln(2*2^(1/3)+(1-x)^2
/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 11.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)} \, dx = \frac{\sqrt[3]{1-x^3} \left(-1+\frac{4x^3 \text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right)}{(1+x^3)\left(-4 \text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right)+x^3\left(3 \text{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},x^3,-x^3\right)+\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},x^3,-x^3\right)\right)\right)}\right)}{2x^2}$$

input
```
Integrate[1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
((1 - x^3)^(1/3)*(-1 + (4*x^3*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1
+ x^3)*(-4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -
1/3, 2, 7/3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))))/(2*x^
2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {980, 25, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (x^3+1)} \, dx$$

$$\Big\downarrow \ 980$$

$$\frac{1}{2}\int -\frac{\sqrt[3]{1-x^3}}{x^3+1}dx-\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{1}{2}\int \frac{\sqrt[3]{1-x^3}}{x^3+1}dx-\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$$\Big\downarrow \ 927$$

$$\frac{9}{2}\int \frac{1-x}{\sqrt[3]{1-x^3}\left(4-\frac{(1-x)^3}{1-x^3}\right)\left(\frac{2(1-x)^3}{1-x^3}+1\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$\downarrow$ 982

$$\frac{9}{2}\left(\frac{1}{9}\int \frac{1-x}{\sqrt[3]{1-x^3}\left(4-\frac{(1-x)^3}{1-x^3}\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{2}{9}\int \frac{1-x}{\sqrt[3]{1-x^3}\left(\frac{2(1-x)^3}{1-x^3}+1\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}\right)-\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$\downarrow$ 821

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}\right)+\frac{1}{9}\left(\frac{\int \frac{1}{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\ 2^{2/3}}-\right.\right.$$

$$\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$\downarrow$ 16

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}}d\frac{1-}{\sqrt[3]{1}}}{3\ 2^{2/3}}\right.\right.$$

$$\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$\downarrow$ 1142

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{\int -\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\right.$$

$$\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$$\Big\downarrow \ 25$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\int \frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\right.$$

$$\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$$\Big\downarrow \ 27$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)\right)$$

$$\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$\downarrow$ 1082

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(\right.\right.$$

$$\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$\downarrow$ 217

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\sqrt{3}\ldots}{\ldots}\right.\right.$$

$$\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$\downarrow$ 1103

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\sqrt{3}\arctan\left(\frac{\ldots}{\ldots}\right.}{\ldots}\right.\right.$$

$$\frac{\sqrt[3]{1-x^3}}{2x^2}$$

input
```
Int[1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
-1/2*(1 - x^3)^(1/3)/x^2 + (9*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(1 -
 x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1
- x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(2*2^(1/3)))/(3*2^(1/3))
 - Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(2/3))))/9 + (-1/3*Log[
2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/2^(2/3) - (Sqrt[3]*ArcTan[(1 + (2^(1/3)
*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]] - Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^
(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/2)/(3*2^(2/3)))/9))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 73.19 (sec) , antiderivative size = 695, normalized size of antiderivative = 2.36

method	result	size
risch	Expression too large to display	695
trager	Expression too large to display	1150

input
```
int(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^3-1)/x^2/(-x^3+1)^(2/3)+(1/12*RootOf(_Z^3+2)*ln((6*RootOf(RootOf(_Z
^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)^3*x^3+36*RootOf(RootOf(
_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)^2*RootOf(_Z^3+2)^2*x^3+RootOf(_Z^3+2
)*x^6+6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^6+9*(x^6-2*x
^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9
*_Z^2)*x-6*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3+2)^2*x^2-6*RootOf(_Z^3+2)*x^3-3
6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^3-6*(x^6-2*x^3+1)^
(2/3)*x+RootOf(_Z^3+2)+6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^
2))/(1+x)^2/(x^2-x+1)^2)+1/4*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9
*_Z^2)*ln(-(12*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(
_Z^3+2)^3*x^3+18*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)^2*Roo
tOf(_Z^3+2)^2*x^3-2*RootOf(_Z^3+2)*x^6-3*RootOf(RootOf(_Z^3+2)^2+3*_Z*Root
Of(_Z^3+2)+9*_Z^2)*x^6+9*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootO
f(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x-6*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^
3+2)^2*x^2-18*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3
+2)+9*_Z^2)*RootOf(_Z^3+2)*x^2+4*RootOf(_Z^3+2)*x^3+6*RootOf(RootOf(_Z^3+2
)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^3-2*RootOf(_Z^3+2)-3*RootOf(RootOf(_Z^3+
2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2))/(1+x)^2/(x^2-x+1)^2))/(-x^3+1)^(2/3)*((x
^3-1)^2)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)} \, dx =$$

$$-\frac{12 \cdot 4^{\frac{1}{6}} \sqrt{\frac{1}{3}} x^2 \arctan\left(-\frac{4^{\frac{1}{6}} \sqrt{\frac{1}{3}} \left(6 \cdot 4^{\frac{2}{3}} (x^{16} - 33 x^{13} + 110 x^{10} - 110 x^7 + 33 x^4 - x) (-x^3+1)^{\frac{1}{3}} - 48 (x^{14} - 2 x^{11} - 6 x^8 - 2 x^5 + x^2) (-x^3+1)^{\frac{2}{3}}}{2 (x^{18} - 102 x^{15} + 447 x^{12} - 628 x^9 + 447 x^6 - 102 x^3 + 1)}}{}$$

input
```
integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/144*(12*4^(1/6)*sqrt(1/3)*x^2*arctan(-1/2*4^(1/6)*sqrt(1/3)*(6*4^(2/3)*
(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3) - 48*(
x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) - 4^(1/3)*(x^18 + 42
*x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447
*x^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) + 2*4^(2/3)*x^2*log(-(12*(-x^3 +
 1)^(2/3)*x^2 - 3*4^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) + 4^(1/3)*(x^6 + 2*x^
3 + 1))/(x^6 + 2*x^3 + 1)) - 4^(2/3)*x^2*log((24*4^(1/3)*(x^8 - 4*x^5 + x^
2)*(-x^3 + 1)^(2/3) + 4^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) + 12*(
x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^
3 + 1)) + 72*(-x^3 + 1)^(1/3))/x^2
```

Sympy [F]

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{x^3 \left(-(x-1)(x^2+x+1)\right)^{\frac{2}{3}} (x+1)(x^2-x+1)} \, dx$$

input
```
integrate(1/x**3/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

Maxima [F]

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}} x^3} \, dx$$

input
```
integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}} x^3} \, dx$$

input
```
integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{x^3 (1-x^3)^{2/3} (x^3+1)} \, dx$$

input
```
int(1/(x^3*(1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^3*(1 - x^3)^(2/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{(-x^3+1)^{\frac{2}{3}} x^6 + (-x^3+1)^{\frac{2}{3}} x^3} dx$$

input
```
int(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(1/(( - x**3 + 1)**(2/3)*x**6 + ( - x**3 + 1)**(2/3)*x**3),x)
```

3.852 $\int \frac{x^{14}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)} dx$

Optimal result

Integrand size = 22, antiderivative size = 141

$$\int \frac{x^{14}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{1}{2\sqrt[3]{1-x^3}}+\left(1-x^3\right)^{2/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2/(-x^3+1)^(1/3)+(-x^3+1)^(2/3)-2/5*(-x^3+1)^(5/3)+1/8*(-x^3+1)^(8/3)+1/
12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/24*ln(
x^3+1)*2^(2/3)+1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\int \frac{x^{14}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{1}{120}\left(-\frac{3(-49+23x^3+x^6+5x^9)}{\sqrt[3]{1-x^3}}+10\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+10\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-5\,2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\right.\right.$$

input
```
Integrate[x^14/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((-3*(-49 + 23*x^3 + x^6 + 5*x^9))/(1 - x^3)^(1/3) + 10*2^(2/3)*Sqrt[3]*Ar
cTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 10*2^(2/3)*Log[-2 + 2^(2/3)*
(1 - x^3)^(1/3)] - 5*2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1
- x^3)^(2/3)])/120
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(x^3+1)}dx$$
$$\Big\downarrow \text{948}$$
$$\frac{1}{3}\int \frac{x^{12}}{(1-x^3)^{4/3}(x^3+1)}dx^3$$
$$\Big\downarrow \text{98}$$
$$\frac{1}{3}\int \left(-\frac{x^6}{\sqrt[3]{1-x^3}}-\frac{1}{\sqrt[3]{1-x^3}}+\frac{1}{\sqrt[3]{1-x^3}(1-x^6)}\right)dx^3$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}+\frac{3}{8}(1-x^3)^{8/3}-\frac{6}{5}(1-x^3)^{5/3}+3(1-x^3)^{2/3}+\frac{3}{2\sqrt[3]{1-x^3}}-\frac{\log(x^3+1)}{4\sqrt[3]{2}}+\right.$$

input
```
Int[x^14/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(3/(2*(1 - x^3)^(1/3)) + 3*(1 - x^3)^(2/3) - (6*(1 - x^3)^(5/3))/5 + (3*(1
 - x^3)^(8/3))/8 + (Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])
/(2*2^(1/3)) - Log[1 + x^3]/(4*2^(1/3)) + (3*Log[2^(1/3) - (1 - x^3)^(1/3)
])/(4*2^(1/3)))/3
```

Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 8.71 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$\frac{-15x^9-3x^6+10\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+10\,2^{\frac{2}{3}}\ln\left(\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\left(-x^3+1\right)^{\frac{1}{3}}-5\,2^{\frac{2}{3}}\ln\left(\left(-\right.\right.}{120\left(-x^3+1\right)^{\frac{1}{3}}}$
risch	$-\frac{5x^9+x^6+23x^3-49}{40\left(-x^3+1\right)^{\frac{1}{3}}}+\frac{\mathrm{RootOf}\left(\_Z^3-4\right)\ln\left(-\frac{6\,\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^3-4\right)^2+6\_Z\,\mathrm{RootOf}\left(\_Z^3-4\right)+36\_Z^2\right)\mathrm{Root}}{}\right.}{}$
trager	Expression too large to display

input
```
int(x^14/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/120*(-15*x^9-3*x^6+10*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(
2/3)*3^(1/2)*(-x^3+1)^(1/3)+10*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*(-x^3+1)
^(1/3)-5*2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))*(-x^3+1
)^(1/3)-69*x^3+147)/(-x^3+1)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{60\cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}(x^3-1)\arctan\left(2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(2^{\frac{1}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)\right)-5\cdot 2^{\frac{2}{3}}(x^3-1)\log\left(\right.}{}$$

input
```
integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/120*(60*2^(1/6)*sqrt(1/6)*(x^3 - 1)*arctan(2^(1/6)*sqrt(1/6)*(2^(1/3) +
2*(-x^3 + 1)^(1/3))) - 5*2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1
)^(1/3) + (-x^3 + 1)^(2/3)) + 10*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3 +
1)^(1/3)) + 3*(5*x^9 + x^6 + 23*x^3 - 49)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{14}}{\left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**14/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**14/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{8}\left(-x^3+1\right)^{\frac{8}{3}} + \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}} - \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) + \left(-x^3+1\right)^{\frac{2}{3}} + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

input
```
integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
1/8*(-x^3 + 1)^(8/3) + 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^
(1/3) + 2*(-x^3 + 1)^(1/3))) - 2/5*(-x^3 + 1)^(5/3) - 1/24*2^(2/3)*log(2^(
2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/12*2^(2/3)*log(-2^
(1/3) + (-x^3 + 1)^(1/3)) + (-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{8}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{2}{3}} + \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}} - \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) + \left(-x^3+1\right)^{\frac{2}{3}} + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

input
```
integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/8*(x^3 - 1)^2*(-x^3 + 1)^(2/3) + 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)
*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) - 2/5*(-x^3 + 1)^(5/3) - 1/24*2^(
2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/12*2^(
2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + (-x^3 + 1)^(2/3) + 1/2/(-x^3
+ 1)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.05

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12} + \frac{1}{2\,(1-x^3)^{1/3}} + (1-x^3)^{2/3} - \frac{2\,(1-x^3)^{5/3}}{5} + \frac{(1-x^3)^{8/3}}{8} + \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24} - \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

input
```
int(x^14/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
(2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) +
(1 - x^3)^(2/3) - (2*(1 - x^3)^(5/3))/5 + (1 - x^3)^(8/3)/8 + (2^(2/3)*log
((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/24
 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/16)*(3^(1
/2)*1i + 1))/24
```

Reduce [F]

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\left(\int \frac{x^{14}}{(-x^3+1)^{\frac{1}{3}}x^6-(-x^3+1)^{\frac{1}{3}}}dx\right)$$

input
```
int(x^14/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
- int(x**14/(( - x**3 + 1)**(1/3)*x**6 - ( - x**3 + 1)**(1/3)),x)
```

3.853 $\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 130

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3} -\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2/(-x^3+1)^(1/3)+1/2*(-x^3+1)^(2/3)-1/5*(-x^3+1)^(5/3)-1/12*arctan(1/3*(
1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/24*ln(x^3+1)*2^(2/3)-
1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{120}\left(-\frac{12(-8+x^3+2x^6)}{\sqrt[3]{1-x^3}} -10\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-10\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+5\,2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\right.\right.$$

input
```
Integrate[x^11/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((-12*(-8 + x^3 + 2*x^6))/(1 - x^3)^(1/3) - 10*2^(2/3)*Sqrt[3]*ArcTan[(1 +
 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 10*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)
^(1/3)] + 5*2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2
/3)])/120
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}dx^3$$

$$\Big\downarrow \text{98}$$

$$\frac{1}{3}\int \left(-\frac{x^3}{\sqrt[3]{1-x^3}}-\frac{x^3}{\sqrt[3]{1-x^3}\left(x^6-1\right)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{3}{5}\left(1-x^3\right)^{5/3}+\frac{3}{2}\left(1-x^3\right)^{2/3}+\frac{3}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{4\sqrt[3]{2}}-\frac{3\log\left(\sqrt[3]{2}-\right.}{4\sqrt[3]{2}}\right.$$

input
```
Int[x^11/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(3/(2*(1 - x^3)^(1/3)) + (3*(1 - x^3)^(2/3))/2 - (3*(1 - x^3)^(5/3))/5 - (
Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3)) + Log[1
 + x^3]/(4*2^(1/3)) - (3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(4*2^(1/3)))/3
```

Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 10.87 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{-24x^{6}-10\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}-10\,2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}+5\,2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right.\right.}{120\left(-x^{3}+1\right)^{\frac{1}{3}}}$
risch	$-\frac{2x^{6}+x^{3}-8}{10\left(-x^{3}+1\right)^{\frac{1}{3}}}+\frac{\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_Z^{3}+4\right)^{2}+6\_Z\operatorname{RootOf}\left(\_Z^{3}+4\right)+36\_Z^{2}\right)\ln\left(\frac{72\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_Z^{3}+4\right.\right.}{}\right.}{}$
trager	Expression too large to display

input
```
int(x^11/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/120*(-24*x^6-10*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3
^(1/2)*(-x^3+1)^(1/3)-10*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*(-x^3+1)^(1/3)
+5*2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))*(-x^3+1)^(1/3
)-12*x^3+96)/(-x^3+1)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{60\cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}(x^3-1)\arctan\left(2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(2^{\frac{1}{3}}+2\,(-x^3+1)^{\frac{1}{3}}\right)\right)-5\cdot 2^{\frac{2}{3}}(x^3-1)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x\right.}{120\,(x^3-1)}$$

input
```
integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/120*(60*2^(1/6)*sqrt(1/6)*(x^3 - 1)*arctan(2^(1/6)*sqrt(1/6)*(2^(1/3) +
 2*(-x^3 + 1)^(1/3))) - 5*2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3 +
1)^(1/3) + (-x^3 + 1)^(2/3)) + 10*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3 +
 1)^(1/3)) - 12*(2*x^6 + x^3 - 8)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{11}}{\left(-(x-1)\,(x^2+x+1)\right)^{\frac{4}{3}}(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(x**11/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}} + \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) + \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}} + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

input
```
integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(
1/3))) - 1/5*(-x^3 + 1)^(5/3) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 +
 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/
3)) + 1/2*(-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.92

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}} + \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) + \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}} + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

input
```
integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(
1/3))) - 1/5*(-x^3 + 1)^(5/3) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 +
 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)
^(1/3))) + 1/2*(-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 3.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.07

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2(1-x^3)^{1/3}} - \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4} - \frac{2^{1/3}}{4}\right)}{12} + \frac{(1-x^3)^{2/3}}{2} - \frac{(1-x^3)^{5/3}}{5} - \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4} - \frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24} + \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4} - \frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

input
```
int(x^11/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
1/(2*(1 - x^3)^(1/3)) - (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 +
(1 - x^3)^(2/3)/2 - (1 - x^3)^(5/3)/5 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (
2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/24 + (2^(2/3)*log((1 - x
^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*1i + 1))/24
```

Reduce [F]

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\left(\int \frac{x^{11}}{(-x^3+1)^{\frac{1}{3}}x^6 - (-x^3+1)^{\frac{1}{3}}}dx\right)$$

input
```
int(x^11/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
- int(x**11/(( - x**3 + 1)**(1/3)*x**6 - ( - x**3 + 1)**(1/3)),x)
```

3.854 $\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}(1-x^3)^{2/3} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2/(-x^3+1)^(1/3)+1/2*(-x^3+1)^(2/3)+1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^
(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(2^(1/3)-(-x^
3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.15

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{24}\left(-\frac{12(-2+x^3)}{\sqrt[3]{1-x^3}} + 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - 2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-\right.\right.$$

input
```
Integrate[x^8/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((-12*(-2 + x^3))/(1 - x^3)^(1/3) + 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*
(1 - x^3)^(1/3))/Sqrt[3]] + 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] -
2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(1-x^3)^{4/3}(x^3+1)}dx$$
$$\Big\downarrow \text{948}$$
$$\frac{1}{3}\int \frac{x^6}{(1-x^3)^{4/3}(x^3+1)}dx^3$$
$$\Big\downarrow \text{98}$$
$$\frac{1}{3}\int \left(\frac{1}{\sqrt[3]{1-x^3}(1-x^6)}-\frac{1}{\sqrt[3]{1-x^3}}\right)dx^3$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}+\frac{3}{2}(1-x^3)^{2/3}+\frac{3}{2\sqrt[3]{1-x^3}}-\frac{\log(x^3+1)}{4\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\right)$$

input
```
Int[x^8/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(3/(2*(1 - x^3)^(1/3)) + (3*(1 - x^3)^(2/3))/2 + (Sqrt[3]*ArcTan[(1 + 2^(2
/3)*(1 - x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3)) - Log[1 + x^3]/(4*2^(1/3)) + (3
*Log[2^(1/3) - (1 - x^3)^(1/3)])/(4*2^(1/3)))/3
```

Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 8.65 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$-\frac{-2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}-2\,2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\right.}{24\left(-x^{3}+1\right)^{\frac{1}{3}}}$
risch	$-\frac{x^{3}-2}{2\left(-x^{3}+1\right)^{\frac{1}{3}}}+\frac{\operatorname{RootOf}\left(\_Z^{3}-4\right)\ln\left(-\frac{6\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_Z^{3}-4\right)^{2}+6\_Z\operatorname{RootOf}\left(\_Z^{3}-4\right)+36\_Z^{2}\right)\operatorname{RootOf}\left(\_\right.}{}\right.}{}$
trager	Expression too large to display

input
```
int(x^8/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)*(
-x^3+1)^(1/3)-2*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*(-x^3+1)^(1/3)+2^(2/3)*
ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))*(-x^3+1)^(1/3)+12*x^3-24
)/(-x^3+1)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{12\cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}(x^3-1)\arctan\left(2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-2^{\frac{2}{3}}(x^3-1)\log\left(2^{\frac{2}{3}}\right.}{}$$

input
```
integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/24*(12*2^(1/6)*sqrt(1/6)*(x^3 - 1)*arctan(2^(1/6)*sqrt(1/6)*(2^(1/3) + 2
*(-x^3 + 1)^(1/3))) - 2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(
1/3) + (-x^3 + 1)^(2/3)) + 2*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3 + 1)^(
1/3)) + 12*(x^3 - 2)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^8}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**8/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) + \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}} + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

input
```
integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) + 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2*(-x^3 + 1)^(2/
3) + 1/2/(-x^3 + 1)^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) + \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}} + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

input
```
integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) + 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/2*(-x^3 + 1
)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{1}{2(1-x^3)^{1/3}} + \frac{(1-x^3)^{2/3}}{2}+\frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24} - \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

input
```
int(x^8/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
(2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) +
(1 - x^3)^(2/3)/2 + (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i
- 1)^2)/16)*(3^(1/2)*1i - 1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/
3)*(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*1i + 1))/24
```

Reduce [F]

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\left(\int \frac{x^8}{(-x^3+1)^{\frac{1}{3}}x^6-(-x^3+1)^{\frac{1}{3}}}dx\right)$$

input
```
int(x^8/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
- int(x**8/(( - x**3 + 1)**(1/3)*x**6 - ( - x**3 + 1)**(1/3)),x)
```

3.855 $\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2/(-x^3+1)^(1/3)-1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(
2/3)*3^(1/2)+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.26

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{24}\left(\frac{12}{\sqrt[3]{1-x^3}} - 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) + 2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-\right.\right.$$

input
```
Integrate[x^5/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(12/(1 - x^3)^(1/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3
))/Sqrt[3]] - 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(2/3)*Log[2
+ 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(1-x^3)^{4/3}(x^3+1)}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{x^3}{(1-x^3)^{4/3}(x^3+1)}dx^3$$

$$\Big\downarrow \ 87$$

$$\frac{1}{3}\left(\frac{3}{2\sqrt[3]{1-x^3}}-\frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}(x^3+1)}dx^3\right)$$

$$\Big\downarrow \ 67$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}+\frac{\log(x^3+1)}{2\sqrt[3]{2}}\right)+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \ 16$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}+\frac{\log(x^3+1)}{2\sqrt[3]{2}}-\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \ 1082$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{\sqrt[3]{2}}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}-\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \ 217$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}-\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

input
```
Int[x^5/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(3/(2*(1 - x^3)^(1/3)) + (-((Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/
Sqrt[3]])/2^(1/3)) + Log[1 + x^3]/(2*2^(1/3)) - (3*Log[2^(1/3) - (1 - x^3)
^(1/3)])/(2*2^(1/3)))/2)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)    Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)    Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 8.77 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$\frac{-2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}-2\,2^{\frac{2}{3}}\ln\left(\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\left(-x^3+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-\right.\right.}{24\left(-x^3+1\right)^{\frac{1}{3}}}$
trager	$-\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{2\left(x^3-1\right)}+\frac{\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_Z^3+4\right)^2+6\_Z\operatorname{RootOf}\left(\_Z^3+4\right)+36\_Z^2\right)\ln\left(\frac{72\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_Z^3+4\right)^2\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int(x^5/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)*(-
x^3+1)^(1/3)-2*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*(-x^3+1)^(1/3)+2^(2/3)*l
n((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))*(-x^3+1)^(1/3)+12)/(-x^3+
1)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{12\cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}(x^3-1)\arctan\left(2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-2^{\frac{2}{3}}(x^3-1)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3\right.}{24\,(x^3-1)}$$

input
```
integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/24*(12*2^(1/6)*sqrt(1/6)*(x^3 - 1)*arctan(2^(1/6)*sqrt(1/6)*(2^(1/3) +
2*(-x^3 + 1)^(1/3))) - 2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^
(1/3) + (-x^3 + 1)^(2/3)) + 2*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3 + 1)^
(1/3)) + 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^5}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**5/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

input
```
integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(
1/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^
(2/3)) - 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2/(-x^3 + 1)^(1
/3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

input
```
integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(
1/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^
(2/3)) - 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/2/(-x^3 +
1)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2(1-x^3)^{1/3}} - \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12} - \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24} + \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

input
```
int(x^5/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
1/(2*(1 - x^3)^(1/3)) - (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 -
(2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)
*1i - 1))/24 + (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^
2)/16)*(3^(1/2)*1i + 1))/24
```

Reduce [F]

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\left(\int \frac{x^5}{(-x^3+1)^{\frac{1}{3}}x^6-(-x^3+1)^{\frac{1}{3}}}dx\right)$$

input
```
int(x^5/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
- int(x**5/(( - x**3 + 1)**(1/3)*x**6 - ( - x**3 + 1)**(1/3)),x)
```

3.856 $\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2/(-x^3+1)^(1/3)+1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(
2/3)*3^(1/2)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{24}\left(\frac{12}{\sqrt[3]{1-x^3}} + 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - 2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-\right.\right.$$

input
```
Integrate[x^2/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(12/(1 - x^3)^(1/3) + 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3
))/Sqrt[3]] + 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 2^(2/3)*Log[2
+ 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {946, 61, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-x^3)^{4/3}(x^3+1)}dx$$

$$\Big\downarrow \ 946$$

$$\frac{1}{3}\int \frac{1}{(1-x^3)^{4/3}(x^3+1)}dx^3$$

$$\Big\downarrow \ 61$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}(x^3+1)}dx^3+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \ 67$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}+\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{\log(x^3+1)}{2\sqrt[3]{2}}\right)+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \ 16$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{\log(x^3+1)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

input
```
Int[x^2/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(3/(2*(1 - x^3)^(1/3)) + ((Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sq
rt[3]])/2^(1/3) - Log[1 + x^3]/(2*2^(1/3)) + (3*Log[2^(1/3) - (1 - x^3)^(1
/3)])/(2*2^(1/3)))/2)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 6.99 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$-\frac{-2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}-2\,2^{\frac{2}{3}}\ln\left(\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\left(-x^3+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\right.}{24\left(-x^3+1\right)^{\frac{1}{3}}}$
risch	$\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}+\frac{\mathrm{RootOf}\left(\_Z^3-4\right)\ln\left(-\frac{6\,\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^3-4\right)^2+6\_Z\,\mathrm{RootOf}\left(\_Z^3-4\right)+36\_Z^2\right)\mathrm{RootOf}\left(\_Z^3\right.}{}\right.}{}$
trager	$-\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{2\left(x^3-1\right)}+\frac{\mathrm{RootOf}\left(\_Z^3-4\right)\ln\left(-\frac{6\,\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^3-4\right)^2+6\_Z\,\mathrm{RootOf}\left(\_Z^3-4\right)+36\_Z^2\right)\mathrm{RootOf}\left(\_Z\right.}{}\right.}{}$

input
```
int(x^2/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)*(
-x^3+1)^(1/3)-2*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*(-x^3+1)^(1/3)+2^(2/3)*
ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))*(-x^3+1)^(1/3)-12)/(-x^3
+1)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{12\cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(x^3-1\right)\arctan\left(2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-2^{\frac{2}{3}}\left(x^3-1\right)\log\left(2^{\frac{2}{3}}\right.}{}$$

input
```
integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/24*(12*2^(1/6)*sqrt(1/6)*(x^3 - 1)*arctan(2^(1/6)*sqrt(1/6)*(2^(1/3) + 2
*(-x^3 + 1)^(1/3))) - 2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(
1/3) + (-x^3 + 1)^(2/3)) + 2*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3 + 1)^(
1/3)) - 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]

$$\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^2}{\left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**2/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx &= \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)\\ &\quad-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)\\ &\quad+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}\end{aligned}$$

input
```
integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) + 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2/(-x^3 + 1)^(1/
3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

input
```
integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) + 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/2/(-x^3 + 1
)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12} + \frac{1}{2\,(1-x^3)^{1/3}} + \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24} - \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

input
```
int(x^2/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
(2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) +
(2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)
*1i - 1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^
2)/16)*(3^(1/2)*1i + 1))/24
```

Reduce [F]

$$\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\left(\int \frac{x^2}{(-x^3+1)^{\frac{1}{3}}x^6-(-x^3+1)^{\frac{1}{3}}}dx\right)$$

input
```
int(x^2/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
- int(x**2/(( - x**3 + 1)**(1/3)*x**6 - ( - x**3 + 1)**(1/3)),x)
```

3.857 $\int \frac{1}{x(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 154

$$\int \frac{1}{x(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2\sqrt[3]{1-x^3}}+\frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{2}+\frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2/(-x^3+1)^(1/3)+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/
12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/2*ln(x
)+1/24*ln(x^3+1)*2^(2/3)+1/2*ln(1-(-x^3+1)^(1/3))-1/8*ln(2^(1/3)-(-x^3+1)^
(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.29

$$\int \frac{1}{x(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{24}\left(\frac{12}{\sqrt[3]{1-x^3}}+8\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)\right.$$
$$-2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+8\log\left(-1+\sqrt[3]{1-x^3}\right)-2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-4\log\left(1+\right.$$

input
```
Integrate[1/(x*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(12/(1 - x^3)^(1/3) + 8*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] -
2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 8*Log[-1
 + (1 - x^3)^(1/3)] - 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 4*Log[
1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)] + 2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)
^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {948, 96, 174, 67, 16, 1082, 217, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(1-x^3)^{4/3}(x^3+1)}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{1}{x^3(1-x^3)^{4/3}(x^3+1)}dx^3$$

$$\Big\downarrow \; 96$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{x^3+2}{x^3\sqrt[3]{1-x^3}(x^3+1)}dx^3+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(2\int \frac{1}{x^3\sqrt[3]{1-x^3}}dx^3-\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx^3\right)+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}+2\left(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{1-x^3}}d\sqrt[3]{1-\right.\right.\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}+2\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}-\frac{1}{2}\log\left(x^3\right)+\frac{3}{2}\log\left(1\right.\right.\right.\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{\sqrt[3]{2}}+2\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}-\frac{1}{2}\log\left(x^3\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-\right.\right.\right.\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(2\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}-\frac{1}{2}\log\left(x^3\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^3}\right)\right)-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+}{\sqrt{3}}\right.}{\sqrt[3]{2}}\right.\right.$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(2\left(-3\int \frac{1}{-x^6-3}d\left(2\sqrt[3]{1-x^3}+1\right)-\frac{1}{2}\log\left(x^3\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^3}\right)\right)-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+}{\sqrt{3}}\right.}{\sqrt[3]{2}}\right.\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}+2\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)-\frac{\log\left(x^3\right)}{2}+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^3}\right)\right)\right.\right.$$

input
```
Int[1/(x*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(3/(2*(1 - x^3)^(1/3)) + (-((Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/
Sqrt[3]])/2^(1/3)) + Log[1 + x^3]/(2*2^(1/3)) + 2*(Sqrt[3]*ArcTan[(1 + 2*(
1 - x^3)^(1/3))/Sqrt[3]] - Log[x^3]/2 + (3*Log[1 - (1 - x^3)^(1/3)])/2) -
(3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*2^(1/3)))/2)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 5.30 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$\frac{-2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}-2\,2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-\right.\right.}{}$

input
```
int(1/x/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)*(-
x^3+1)^(1/3)-2*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*(-x^3+1)^(1/3)+2^(2/3)*l
n((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))*(-x^3+1)^(1/3)+8*arctan(1
/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*(-x^3+1)^(1/3)+8*ln(-1+(-x^3+1)^(
1/3))*(-x^3+1)^(1/3)-4*ln((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)*(-x^3+1)^(1/3)+
12)/(-x^3+1)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx =$$
$$-\frac{12\cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}(x^3-1)\arctan\left(2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-2^{\frac{2}{3}}(x^3-1)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3\right.}{}$$

input
```
integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/24*(12*2^(1/6)*sqrt(1/6)*(x^3 - 1)*arctan(2^(1/6)*sqrt(1/6)*(2^(1/3) +
2*(-x^3 + 1)^(1/3))) - 2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^
(1/3) + (-x^3 + 1)^(2/3)) + 2*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3 + 1)^
(1/3)) - 8*sqrt(3)*(x^3 - 1)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqr
t(3)) + 4*(x^3 - 1)*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 8*(x^3
- 1)*log((-x^3 + 1)^(1/3) - 1) + 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]

$$\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \int \frac{1}{x\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(1/x/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x}\,dx$$

input
```
integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = -\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(
1/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^
(2/3)) - 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/3*sqrt(3)*
arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2/(-x^3 + 1)^(1/3) - 1/6*
log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3
) - 1))
```

Mupad [B] (verification not implemented)

Time = 3.47 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.64

$$\int \frac{1}{x(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{\ln\left(\frac{17}{4}-\frac{17(1-x^3)^{1/3}}{4}\right)}{3}$$

$$+\ln\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\left(1458\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2-\frac{459(1-x^3)^{1/3}}{4}\right)-\frac{63}{4}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right.\right.$$

input
```
int(1/(x*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
log(17/4 - (17*(1 - x^3)^(1/3))/4)/3 + log(((3^(1/2)*1i)/6 - 1/6)*(1458*((
3^(1/2)*1i)/6 - 1/6)^2 - (459*(1 - x^3)^(1/3))/4) - 63/4)*((3^(1/2)*1i)/6
- 1/6) - log(((3^(1/2)*1i)/6 + 1/6)*(1458*((3^(1/2)*1i)/6 + 1/6)^2 - (459*
(1 - x^3)^(1/3))/4) + 63/4)*((3^(1/2)*1i)/6 + 1/6) - (2^(2/3)*log((2^(2/3)
*((81*2^(1/3))/4 - (459*(1 - x^3)^(1/3))/4))/12 + 63/4))/12 + 1/(2*(1 - x^
3)^(1/3)) + ((-1)^(1/3)*2^(2/3)*log(((-1)^(1/3)*2^(2/3)*((81*(-1)^(2/3)*2^
(1/3))/4 - (459*(1 - x^3)^(1/3))/4))/12 - 63/4))/12 - ((-1)^(1/3)*2^(2/3)*
log(((-1)^(1/3)*2^(2/3)*(3^(1/2)*1i + 1)*((459*(1 - x^3)^(1/3))/4 - (81*(-
1)^(2/3)*2^(1/3)*(3^(1/2)*1i + 1)^2)/16))/24 - 63/4)*(3^(1/2)*1i + 1))/24
```

Reduce [F]

$$\int \frac{1}{x(1-x^3)^{4/3}(1+x^3)}\,dx = -\left(\int \frac{1}{(-x^3+1)^{\frac{1}{3}}x^7-(-x^3+1)^{\frac{1}{3}}x}dx\right)$$

input
```
int(1/x/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
- int(1/(( - x**3 + 1)**(1/3)*x**7 - ( - x**3 + 1)**(1/3)*x),x)
```

3.858 $\int \frac{1}{x^4(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 175

$$\int \frac{1}{x^4(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{6} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
5/6/(-x^3+1)^(1/3)-1/3/x^3/(-x^3+1)^(1/3)+1/9*arctan(1/3*(1+2*(-x^3+1)^(1/
3))*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2
^(2/3)*3^(1/2)-1/6*ln(x)-1/24*ln(x^3+1)*2^(2/3)+1/6*ln(1-(-x^3+1)^(1/3))+1
/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^4(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{72}\left(\frac{12(-2+5x^3)}{x^3\sqrt[3]{1-x^3}}+8\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)\right.$$
$$+6\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+8\log\left(-1+\sqrt[3]{1-x^3}\right)+6\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-4\log\left(1+\right.$$

input
```
Integrate[1/(x^4*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((12*(-2 + 5*x^3))/(x^3*(1 - x^3)^(1/3)) + 8*Sqrt[3]*ArcTan[(1 + 2*(1 - x^
3)^(1/3))/Sqrt[3]] + 6*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3)
)/Sqrt[3]] + 8*Log[-1 + (1 - x^3)^(1/3)] + 6*2^(2/3)*Log[-2 + 2^(2/3)*(1 -
 x^3)^(1/3)] - 4*Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 3*2^(2/3)*Lo
g[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/72
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {948, 114, 27, 174, 61, 67, 16, 1082, 217, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(1-x^3)^{4/3}(x^3+1)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{1}{x^6(1-x^3)^{4/3}(x^3+1)}dx^3$$

$$\Big\downarrow \text{ 114}$$

$$\frac{1}{3}\left(-\int -\frac{4x^3+1}{3x^3(1-x^3)^{4/3}(x^3+1)}dx^3-\frac{1}{x^3\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{1}{3}\int \frac{4x^3+1}{x^3\left(1-x^3\right)^{4/3}\left(x^3+1\right)}dx^3-\frac{1}{x^3\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\int \frac{1}{x^3\left(1-x^3\right)^{4/3}}dx^3+3\int \frac{1}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}dx^3\right)-\frac{1}{x^3\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\int \frac{1}{x^3\sqrt[3]{1-x^3}}dx^3+3\left(\frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx^3+\frac{3}{2\sqrt[3]{1-x^3}}\right)+\frac{3}{\sqrt[3]{1-x^3}}\right)-\frac{1}{x^3\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \; 67$$

$$\frac{1}{3}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}+\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+3\left(\frac{1}{2}\left(-\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-\ldots}}{2\sqrt[3]{2}}\ldots\right.\right.\right.\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+3\left(\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\ldots}{\ldots}\right.\right.\right.\right.$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+3\left(\frac{1}{2}\left(-\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}\ldots\right.}{\ldots}\right.\right.\right.\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+3\left(\frac{1}{2}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\ldots\right.}{2\sqrt[3]{\ldots}}\right.\right.\right.\right.$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{3}\left(\frac{1}{3}\left(-3\int \frac{1}{-x^6-3}d\left(2\sqrt[3]{1-x^3}+1\right)+3\left(\frac{1}{2}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\right.}{2\sqrt[3]{2}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)+3\left(\frac{1}{2}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-\right.}{2\sqrt[3]{2}}\right.\right.\right.\right.$$

input
```
Int[1/(x^4*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(-(1/(x^3*(1 - x^3)^(1/3))) + (3/(1 - x^3)^(1/3) + Sqrt[3]*ArcTan[(1 + 2*(
1 - x^3)^(1/3))/Sqrt[3]] - Log[x^3]/2 + (3*Log[1 - (1 - x^3)^(1/3)])/2 + 3
*(3/(2*(1 - x^3)^(1/3)) + ((Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/S
qrt[3]])/2^(1/3) - Log[1 + x^3]/(2*2^(1/3)) + (3*Log[2^(1/3) - (1 - x^3)^(
1/3)])/(2*2^(1/3)))/2))/3)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(131) = 262$.

Time = 4.66 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.53

method	result
pseudoelliptic	$-\frac{\frac{3\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)x^{3}\left(-x^{3}+1\right)^{\frac{1}{3}}}{4}+\sqrt{3}\arctan\left(\frac{\left(1+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)x^{3}\left(-x^{3}+1\right)^{\frac{1}{3}}-\frac{3\,2^{\frac{2}{3}}\ln\left(\left(-x^{3}\right.\right.}{}}{9(-}$

input
```
int(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(3/4*3^(1/2)*2^(2/3)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*x
^3*(-x^3+1)^(1/3)+3^(1/2)*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*x^3*(-x
^3+1)^(1/3)-3/8*2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))*
x^3*(-x^3+1)^(1/3)+3/4*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*x^3*(-x^3+1)^(1/
3)-1/2*ln((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)*x^3*(-x^3+1)^(1/3)+ln(-1+(-x^3+
1)^(1/3))*x^3*(-x^3+1)^(1/3)+15/2*x^3-3)/(-x^3+1)^(1/3)/(-1+(-x^3+1)^(1/3)
)/((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^4 (1-x^3)^{4/3} (1+x^3)}\,dx = \frac{36\cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}(x^6-x^3)\arctan\left(2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-3\cdot 2^{\frac{2}{3}}(x^6-x^3}{}$$

input
```
integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/72*(36*2^(1/6)*sqrt(1/6)*(x^6 - x^3)*arctan(2^(1/6)*sqrt(1/6)*(2^(1/3) +
 2*(-x^3 + 1)^(1/3))) - 3*2^(2/3)*(x^6 - x^3)*log(2^(2/3) + 2^(1/3)*(-x^3
+ 1)^(1/3) + (-x^3 + 1)^(2/3)) + 6*2^(2/3)*(x^6 - x^3)*log(-2^(1/3) + (-x^
3 + 1)^(1/3)) + 8*sqrt(3)*(x^6 - x^3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3)
+ 1/3*sqrt(3)) - 4*(x^6 - x^3)*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1
) + 8*(x^6 - x^3)*log((-x^3 + 1)^(1/3) - 1) - 12*(5*x^3 - 2)*(-x^3 + 1)^(2
/3))/(x^6 - x^3)
```

Sympy [F]

$$\int \frac{1}{x^4 (1-x^3)^{4/3} (1+x^3)}\,dx = \int \frac{1}{x^4 \left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}} \left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(1/x**4/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

Maxima [F]

$$\int \frac{1}{x^4 (1-x^3)^{4/3} (1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^4}\,dx$$

input
```
integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4 (1-x^3)^{4/3} (1+x^3)} \, dx = \frac{1}{12} \sqrt{3} 2^{\frac{2}{3}} \arctan\left(\frac{1}{6} \sqrt{3} 2^{\frac{2}{3}} \left(2^{\frac{1}{3}} + 2 \left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{24} \cdot 2^{\frac{2}{3}} \log\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}} \left(-x^3+1\right)^{\frac{1}{3}} + \left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{12} \cdot 2^{\frac{2}{3}} \log\left(\left|-2^{\frac{1}{3}} + \left(-x^3+1\right)^{\frac{1}{3}}\right|\right) + \frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2 \left(-x^3+1\right)^{\frac{1}{3}} + 1\right)\right) - \frac{5 x^3 - 2}{6 \left(\left(-x^3+1\right)^{\frac{4}{3}} - \left(-x^3+1\right)^{\frac{1}{3}}\right)} - \frac{1}{18} \log\left(\left(-x^3+1\right)^{\frac{2}{3}} + \left(-x^3+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{9} \log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}} - 1\right|\right)$$

input
```
integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) + 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/9*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*(5*x^3 - 2)/((-x^3 + 1)^
(4/3) - (-x^3 + 1)^(1/3)) - 1/18*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) +
 1) + 1/9*log(abs((-x^3 + 1)^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.28

$$\int \frac{1}{x^4 (1-x^3)^{4/3} (1+x^3)} \, dx = \frac{\ln \left(\frac{11 (1-x^3)^{1/3}}{972} - \frac{11}{972}\right)}{9} + \frac{2^{2/3} \ln \left(\frac{2^{1/3} \left(\frac{2^{2/3} \left(\frac{81\, 2^{1/3}}{4} - \frac{75 (1-x^3)^{1/3}}{4}\right)}{12} - \frac{35}{12}\right)}{72} + \frac{(1-x^3)^{1/3}}{27}\right)}{12}$$

$$+ \ln \left(\left(-\frac{1}{18} + \frac{\sqrt{3}\, 1i}{18}\right)^2 \left(\left(-\frac{1}{18} + \frac{\sqrt{3}\, 1i}{18}\right) \left(1458 \left(-\frac{1}{18} + \frac{\sqrt{3}\, 1i}{18}\right)^2 - \frac{75 (1-x^3)^{1/3}}{4}\right) - \frac{35}{12}\right) + \frac{(1-x^3)^{1/3}}{27}\right.$$

input
```
int(1/(x^4*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
log((11*(1 - x^3)^(1/3))/972 - 11/972)/9 + (2^(2/3)*log((2^(1/3)*((2^(2/3)
*((81*2^(1/3))/4 - (75*(1 - x^3)^(1/3))/4))/12 - 35/12))/72 + (1 - x^3)^(1
/3)/27))/12 + log(((3^(1/2)*1i)/18 - 1/18)^2*(((3^(1/2)*1i)/18 - 1/18)*(14
58*((3^(1/2)*1i)/18 - 1/18)^2 - (75*(1 - x^3)^(1/3))/4) - 35/12) + (1 - x^
3)^(1/3)/27)*((3^(1/2)*1i)/18 - 1/18) - log((1 - x^3)^(1/3)/27 - ((3^(1/2)
*1i)/18 + 1/18)^2*(((3^(1/2)*1i)/18 + 1/18)*(1458*((3^(1/2)*1i)/18 + 1/18)
^2 - (75*(1 - x^3)^(1/3))/4) + 35/12))*((3^(1/2)*1i)/18 + 1/18) + ((5*x^3)
/6 - 1/3)/((1 - x^3)^(1/3) - (1 - x^3)^(4/3)) + (2^(2/3)*log((1 - x^3)^(1/
3)/27 + (2^(1/3)*(3^(1/2)*1i - 1)^2*((2^(2/3)*(3^(1/2)*1i - 1)*((81*2^(1/3
)*(3^(1/2)*1i - 1)^2)/16 - (75*(1 - x^3)^(1/3))/4))/24 - 35/12))/288)*(3^(
1/2)*1i - 1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/27 - (2^(1/3)*(3^(1/2)*1i
+ 1)^2*((2^(2/3)*(3^(1/2)*1i + 1)*((81*2^(1/3)*(3^(1/2)*1i + 1)^2)/16 - (7
5*(1 - x^3)^(1/3))/4))/24 + 35/12))/288)*(3^(1/2)*1i + 1))/24
```

Reduce [F]

$$\int \frac{1}{x^4 (1-x^3)^{4/3} (1+x^3)} \, dx = -\left(\int \frac{1}{(-x^3+1)^{\frac{1}{3}} x^{10} - (-x^3+1)^{\frac{1}{3}} x^4} dx \right)$$

input
```
int(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
 - int(1/(( - x**3 + 1)**(1/3)*x**10 - ( - x**3 + 1)**(1/3)*x**4),x)
```

3.859 $\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 174

$$\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)} dx = \frac{x^4}{2\sqrt[3]{1-x^3}} + \frac{5}{6}x(1-x^3)^{2/3} + \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{1}{6}\log\left(x+\sqrt[3]{1-x^3}\right)$$

output
```
1/2*x^4/(-x^3+1)^(1/3)+5/6*x*(-x^3+1)^(2/3)+1/9*arctan(1/3*(1-2*x/(-x^3+1)
^(1/3))*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^
(1/2))*2^(2/3)*3^(1/2)+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-2^(1/3)*x-(-x^3+1)^(
1/3))*2^(2/3)-1/6*ln(x+(-x^3+1)^(1/3))
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.30

$$\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{72}\left(-\frac{12x(-5+2x^3)}{\sqrt[3]{1-x^3}} + 8\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)\right.$$
$$+6\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 8\log\left(x+\sqrt[3]{1-x^3}\right) - 6\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + 4\log\left(x^2-x\right.$$

input
```
Integrate[x^9/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((-12*x*(-5 + 2*x^3))/(1 - x^3)^(1/3) + 8*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x -
2*(1 - x^3)^(1/3))] + 6*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1
 - x^3)^(1/3))] - 8*Log[x + (1 - x^3)^(1/3)] - 6*2^(2/3)*Log[2*x + 2^(2/3)
*(1 - x^3)^(1/3)] + 4*Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)] + 3*2
^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/
72
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {970, 1052, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(1-x^3)^{4/3}(x^3+1)}\,dx$$

$$\Big\downarrow \text{ 970}$$

$$\frac{x^4}{2\sqrt[3]{1-x^3}} - \frac{1}{2}\int \frac{x^3(5x^3+4)}{\sqrt[3]{1-x^3}(x^3+1)}dx$$

$$\Big\downarrow \text{ 1052}$$

$$\frac{1}{2}\left(\frac{5}{3}x\left(1-x^3\right)^{2/3}-\frac{1}{3}\int \frac{2x^3+5}{\sqrt[3]{1-x^3}\,(x^3+1)}dx\right)+\frac{x^4}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \; 1026$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-2\int \frac{1}{\sqrt[3]{1-x^3}}dx-3\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx\right)+\frac{5}{3}\left(1-x^3\right)^{2/3}x\right)+\frac{x^4}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \; 769$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-3\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-2\left(\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+\frac{5}{3}\left(1-x^3\right)^{2/3}x\right)$$

$$\frac{x^4}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \; 901$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-3\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}\right)-2\left(\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)\right.\right.\right.$$

$$\frac{x^4}{2\sqrt[3]{1-x^3}}$$

input
```
Int[x^9/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
x^4/(2*(1 - x^3)^(1/3)) + ((5*x*(1 - x^3)^(2/3))/3 + (-3*(-(ArcTan[(1 - (2
*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) - Log[1 + x^3]/(6
*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))) - 2*(-(ArcTan
[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/
2))/3)/2
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(132) = 264.

Time = 3.92 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.68

method	result
pseudoelliptic	$\frac{-6\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-6\,2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}+3\,2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{2}{3}}x^{2}-2^{\frac{1}{3}}\left(-\right.}{}\right.}{7}$

input
```
int(x^9/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/72*(-6*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)
*(-x^3+1)^(1/3)-6*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*(-x^3+1)^(1/3)+
3*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)*(-
x^3+1)^(1/3)-24*x^4-8*3^(1/2)*arctan(1/3*(-2*(-x^3+1)^(1/3)+x)*3^(1/2)/x)*
(-x^3+1)^(1/3)+4*ln(((-x^3+1)^(2/3)-(-x^3+1)^(1/3)*x+x^2)/x^2)*(-x^3+1)^(1
/3)-8*ln((x+(-x^3+1)^(1/3))/x)*(-x^3+1)^(1/3)+60*x)/((-x^3+1)^(2/3)-(-x^3+
1)^(1/3)*x+x^2)/(x+(-x^3+1)^(1/3))/(-x^3+1)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.41

$$\int \frac{x^9}{(1-x^3)^{4/3}\,(1+x^3)}\,dx = \frac{36\cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}(x^3-1)\arctan\left(-\frac{2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(2^{\frac{1}{3}}x-2\left(-x^3+1\right)^{\frac{1}{3}}\right)}{x}\right)-6\cdot 2^{\frac{2}{3}}(x^3-1)\log\left(\frac{2^{\frac{1}{3}}}{}\right.}{}$$

input
```
integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/72*(36*2^(1/6)*sqrt(1/6)*(x^3 - 1)*arctan(-2^(1/6)*sqrt(1/6)*(2^(1/3)*x
- 2*(-x^3 + 1)^(1/3))/x) - 6*2^(2/3)*(x^3 - 1)*log((2^(1/3)*x + (-x^3 + 1)
^(1/3))/x) + 3*2^(2/3)*(x^3 - 1)*log((2^(2/3)*x^2 - 2^(1/3)*(-x^3 + 1)^(1/
3)*x + (-x^3 + 1)^(2/3))/x^2) + 8*sqrt(3)*(x^3 - 1)*arctan(-1/3*(sqrt(3)*x
 - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) - 8*(x^3 - 1)*log((x + (-x^3 + 1)^(1/3))
/x) + 4*(x^3 - 1)*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2) +
 12*(2*x^4 - 5*x)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]

$$\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^9}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**9/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**9/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [F]

$$\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^9}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^9/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Giac [F]

$$\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^9}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^9/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^9}{(1-x^3)^{4/3}(x^3+1)}\,dx$$

input
```
int(x^9/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(x^9/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\left(\int \frac{x^9}{(-x^3+1)^{\frac{1}{3}}x^6-(-x^3+1)^{\frac{1}{3}}}dx\right)$$

input
```
int(x^9/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
- int(x**9/(( - x**3 + 1)**(1/3)*x**6 - ( - x**3 + 1)**(1/3)),x)
```

3.860 $\int \frac{x^6}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 153

$$\int \frac{x^6}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{x}{2\sqrt[3]{1-x^3}}+\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

output

```
1/2*x/(-x^3+1)^(1/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2
)-1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-
1/24*ln(x^3+1)*2^(2/3)+1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(x+
(-x^3+1)^(1/3))
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.44

$$\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{24}\left(\frac{12x}{\sqrt[3]{1-x^3}} + 8\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right) -2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) -8\log\left(x+\sqrt[3]{1-x^3}\right) +2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) +4\log\left(x^2-x\right.\right.$$

input
```
Integrate[x^6/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((12*x)/(1 - x^3)^(1/3) + 8*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1
/3))] - 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))
] - 8*Log[x + (1 - x^3)^(1/3)] + 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/
3)] + 4*Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 2^(2/3)*Log[-2*x^
2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {970, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(1-x^3)^{4/3}(x^3+1)}\,dx$$

$$\Big\downarrow \text{ 970}$$

$$\frac{x}{2\sqrt[3]{1-x^3}} - \frac{1}{2}\int \frac{2x^3+1}{\sqrt[3]{1-x^3}(x^3+1)}dx$$

$$\Big\downarrow \text{ 1026}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt[3]{1-x^3}(x^3+1)}dx - 2\int \frac{1}{\sqrt[3]{1-x^3}}dx\right) + \frac{x}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \ 769$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-2\left(\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+\frac{x}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \ 901$$

$$\frac{1}{2}\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-2\left(\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{\log(x^3+1)}{6\sqrt[3]{2}}+\frac{\log\left(-\right.}{}\right.$$

$$\frac{x}{2\sqrt[3]{1-x^3}}$$

input
```
Int[x^6/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
x/(2*(1 - x^3)^(1/3)) + (-(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt
[3]]/(2^(1/3)*Sqrt[3])) - Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1
 - x^3)^(1/3)]/(2*2^(1/3)) - 2*(-(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[
3]]/Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/2))/2
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.43

method	result
pseudoelliptic	$-\frac{-2\,2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right) \left(-x^{3}+1\right)^{\frac{1}{3}}+8 \ln\left(\frac{x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right) \left(-x^{3}+1\right)^{\frac{1}{3}}+\left(\left(-2 \sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\cdots\right.}{3x}\cdots\right.\right.\right.}{\cdots}$

input
```
int(x^6/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-2*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*(-x^3+1)^(1/3)+8*ln((x+
(-x^3+1)^(1/3))/x)*(-x^3+1)^(1/3)+((-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3
)*(-x^3+1)^(1/3)+x)/x)+ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(
2/3))/x^2))*2^(2/3)+8*3^(1/2)*arctan(1/3*(-2*(-x^3+1)^(1/3)+x)*3^(1/2)/x)-
4*ln(((-x^3+1)^(2/3)-(-x^3+1)^(1/3)*x+x^2)/x^2))*(-x^3+1)^(1/3)-12*x)/(-x^
3+1)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.54

$$\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{12\cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}(x^3-1)\arctan\left(-\frac{2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(2^{\frac{1}{3}}x-2\left(-x^3+1\right)^{\frac{1}{3}}\right)}{x}\right)-2\cdot 2^{\frac{2}{3}}(x^3-1)\log\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)+2^{\frac{2}{3}}(x^3-1)}{}$$

input
```
integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/24*(12*2^(1/6)*sqrt(1/6)*(x^3 - 1)*arctan(-2^(1/6)*sqrt(1/6)*(2^(1/3)*x
 - 2*(-x^3 + 1)^(1/3))/x) - 2*2^(2/3)*(x^3 - 1)*log((2^(1/3)*x + (-x^3 + 1
)^(1/3))/x) + 2^(2/3)*(x^3 - 1)*log((2^(2/3)*x^2 - 2^(1/3)*(-x^3 + 1)^(1/3
)*x + (-x^3 + 1)^(2/3))/x^2) - 8*sqrt(3)*(x^3 - 1)*arctan(-1/3*(sqrt(3)*x
- 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 8*(x^3 - 1)*log((x + (-x^3 + 1)^(1/3))/
x) - 4*(x^3 - 1)*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2) +
12*(-x^3 + 1)^(2/3)*x)/(x^3 - 1)
```

Sympy [F]

$$\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^6}{\left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**6/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [F]

$$\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Giac [F]

$$\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^6}{(1-x^3)^{4/3}(x^3+1)}\,dx$$

input
```
int(x^6/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(x^6/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\left(\int \frac{x^6}{(-x^3+1)^{\frac{1}{3}}x^6-(-x^3+1)^{\frac{1}{3}}}dx\right)$$

input

```
int(x^6/(-x^3+1)^(4/3)/(x^3+1),x)
```

output

```
- int(x**6/(( - x**3 + 1)**(1/3)*x**6 - ( - x**3 + 1)**(1/3)),x)
```

3.861 $\int \frac{x^3}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \frac{x^3}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{x}{2\sqrt[3]{1-x^3}}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2*x/(-x^3+1)^(1/3)+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2
))*2^(2/3)*3^(1/2)+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3)
)*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.31

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{24}\left(\frac{12x}{\sqrt[3]{1-x^3}} + 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + 2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{\right.\right.$$

input
```
Integrate[x^3/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((12*x)/(1 - x^3)^(1/3) + 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3
)*(1 - x^3)^(1/3))] - 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(2/
3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {971, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(1-x^3)^{4/3}(x^3+1)}dx$$

$$\Big\downarrow \text{ 971}$$

$$\frac{x}{2\sqrt[3]{1-x^3}} - \frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}(x^3+1)}dx$$

$$\Big\downarrow \text{ 901}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}\right)+\frac{x}{2\sqrt[3]{1-x^3}}$$

input
```
Int[x^3/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
x/(2*(1 - x^3)^(1/3)) + (ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3
]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)) - Log[-(2^(1/3)*x) - (1 -
x^3)^(1/3)]/(2*2^(1/3)))/2
```

Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Maple [A] (verified)

Time = 7.83 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.33

method	result
pseudoelliptic	$-\frac{\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-\frac{2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}{2}+2^{\frac{2}{3}} \ln\left(\right.}{12\left(-x^{3}+1\right)^{\frac{1}{3}}}$
risch	Expression too large to display
trager	Expression too large to display

input
```
int(x^3/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)*(
-x^3+1)^(1/3)-1/2*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1
)^(2/3))/x^2)*(-x^3+1)^(1/3)+2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*(-x^
3+1)^(1/3)-6*x)/(-x^3+1)^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(79) = 158.

Time = 1.61 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.65

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{12\cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}(x^3-1)\arctan\left(\frac{2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(6\cdot 2^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-\right.\right.}{109x^9-105x^6+3x^3+1}\right.}{}$$

input
```
integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/72*(12*2^(1/6)*sqrt(1/6)*(x^3 - 1)*arctan(2^(1/6)*sqrt(1/6)*(6*2^(2/3)*(
5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 -
 1) + 12*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 + 3*
x^3 + 1)) - 2*2^(2/3)*(x^3 - 1)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2
/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(x^3 - 1)*log((
3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 1
2*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 36*(-x^3 + 1)^(2/3)
*x)/(x^3 - 1)
```

Sympy [F]

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^3}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**3/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [F]

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Giac [F]

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^3}{(1-x^3)^{4/3}(x^3+1)}\,dx$$

input
```
int(x^3/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(x^3/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\left(\int \frac{x^3}{(-x^3+1)^{\frac{1}{3}}x^6-(-x^3+1)^{\frac{1}{3}}}dx\right)$$

input
```
int(x^3/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
- int(x**3/(( - x**3 + 1)**(1/3)*x**6 - ( - x**3 + 1)**(1/3)),x)
```

3.862 $\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 106

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{x}{2\sqrt[3]{1-x^3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2*x/(-x^3+1)^(1/3)-1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2
))*2^(2/3)*3^(1/2)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3)
)*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.32

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{24}\left(\frac{12x}{\sqrt[3]{1-x^3}}\right.$$
$$-2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)+2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)-2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{\ldots}\right.$$

input
```
Integrate[1/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((12*x)/(1 - x^3)^(1/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3
)*(1 - x^3)^(1/3))] + 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] - 2^(2/
3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {907, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x^3)^{4/3}(x^3+1)}\,dx$$
$$\Big\downarrow \text{907}$$
$$\frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}(x^3+1)}dx+\frac{x}{2\sqrt[3]{1-x^3}}$$
$$\Big\downarrow \text{901}$$

$$\frac{1}{2}\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}\right)+\frac{x}{2\sqrt[3]{1-x^3}}$$

input
```
Int[1/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
x/(2*(1 - x^3)^(1/3)) + (-(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt
[3]]/(2^(1/3)*Sqrt[3])) - Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1
 - x^3)^(1/3)]/(2*2^(1/3)))/2
```

Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

Maple [A] (verified)

Time = 6.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.33

method	result
pseudoelliptic	$\frac{\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-\frac{2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}{2}+2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{1}{3}}}{}\right.}{12\left(-x^{3}+1\right)^{\frac{1}{3}}}$
risch	Expression too large to display
trager	Expression too large to display

input
```
int(1/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*(3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)*(-
x^3+1)^(1/3)-1/2*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)
^(2/3))/x^2)*(-x^3+1)^(1/3)+2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*(-x^3
+1)^(1/3)+6*x)/(-x^3+1)^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(79) = 158$.

Time = 1.46 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.65

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx =$$

$$-\frac{12\cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}(x^3-1)\arctan\left(\frac{2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(6\cdot 2^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-1\right)+12\left(19x^8-16x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}\right.}{109x^9-105x^6+3x^3+1}\right.}{}$$

input
```
integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/72*(12*2^(1/6)*sqrt(1/6)*(x^3 - 1)*arctan(2^(1/6)*sqrt(1/6)*(6*2^(2/3)*
(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3
- 1) + 12*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 + 3
*x^3 + 1)) - 2*2^(2/3)*(x^3 - 1)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(
2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(x^3 - 1)*log(
(3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) -
12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) + 36*(-x^3 + 1)^(2/3
)*x)/(x^3 - 1)
```

Sympy [F]

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{1}{(-(x-1)(x^2+x+1))^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(1/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(1/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Giac [F]

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{1}{(1-x^3)^{4/3}(x^3+1)}\,dx$$

input
```
int(1/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(1/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\left(\int \frac{1}{(-x^3+1)^{\frac{1}{3}}x^6-(-x^3+1)^{\frac{1}{3}}}dx\right)$$

input
```
int(1/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
- int(1/(( - x**3 + 1)**(1/3)*x**6 - ( - x**3 + 1)**(1/3)),x)
```

3.863 $\int \frac{1}{x^3(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 124

$$\int \frac{1}{x^3(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2x^2\sqrt[3]{1-x^3}} - \frac{(1-x^3)^{2/3}}{x^2} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2/x^2/(-x^3+1)^(1/3)-(-x^3+1)^(2/3)/x^2+1/12*arctan(1/3*(1-2*2^(1/3)*x/(
-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-2^(
1/3)*x-(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^3(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{24}\left(\frac{12(-1+2x^3)}{x^2\sqrt[3]{1-x^3}}\right.$$
$$+2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{\ }\right.$$

input
```
Integrate[1/(x^3*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((12*(-1 + 2*x^3))/(x^2*(1 - x^3)^(1/3)) + 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[
3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^
3)^(1/3)] + 2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 -
x^3)^(2/3)])/24
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {972, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(1-x^3)^{4/3}(x^3+1)}dx$$
$$\Big\downarrow \text{ 972}$$
$$\frac{1}{2}\int \frac{3x^3+4}{x^3\sqrt[3]{1-x^3}(x^3+1)}dx+\frac{1}{2x^2\sqrt[3]{1-x^3}}$$
$$\Big\downarrow \text{ 1053}$$
$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{2}{\sqrt[3]{1-x^3}(x^3+1)}dx-\frac{2(1-x^3)^{2/3}}{x^2}\right)+\frac{1}{2x^2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{2(1-x^3)^{2/3}}{x^2}\right)+\frac{1}{2x^2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{901}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}-\frac{2(1-x^3)^{2/3}}{x^2}\right)+\frac{1}{2x^2\sqrt[3]{1-x^3}}$$

input
```
Int[1/(x^3*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
1/(2*x^2*(1 - x^3)^(1/3)) + ((-2*(1 - x^3)^(2/3))/x^2 + ArcTan[(1 - (2*2^(
1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1
/3)) - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 26.74 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.26

method	result
pseudoelliptic	$-\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)x^2\left(-x^3+1\right)^{\frac{1}{3}}-\frac{2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)x^2\left(-x^3+1\right)^{\frac{1}{3}}}{2}+2^{\frac{2}{3}}}{12\left(-x^3+1\right)^{\frac{1}{3}}x^2}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)*x
^2*(-x^3+1)^(1/3)-1/2*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x
^3+1)^(2/3))/x^2)*x^2*(-x^3+1)^(1/3)+2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))
/x)*x^2*(-x^3+1)^(1/3)-12*x^3+6)/(-x^3+1)^(1/3)/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(95) = 190$.

Time = 1.47 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^3 (1-x^3)^{4/3} (1+x^3)} \, dx = \frac{12 \cdot 2^{\frac{1}{6}} \sqrt{\frac{1}{6}} (x^5 - x^2) \arctan \left(\frac{2^{\frac{1}{6}} \sqrt{\frac{1}{6}} \left(6 \cdot 2^{\frac{2}{3}} (5x^7 + 4x^4 - x)(-x^3+1)^{\frac{2}{3}} - 2^{\frac{1}{3}} (71x^9 - 111x^6 + 3}{109x^9 - 105x^6 + 3x^3}\right.}{}$$

input
```
integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/72*(12*2^(1/6)*sqrt(1/6)*(x^5 - x^2)*arctan(2^(1/6)*sqrt(1/6)*(6*2^(2/3)
*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3
 - 1) + 12*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 +
3*x^3 + 1)) - 2*2^(2/3)*(x^5 - x^2)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 +
2^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(x^5 - x^2)
*log((3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 +
1) - 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 36*(2*x^3 - 1
)*(-x^3 + 1)^(2/3))/(x^5 - x^2)
```

Sympy [F]

$$\int \frac{1}{x^3 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{x^3 \left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}} (x+1)(x^2-x+1)} \, dx$$

input
```
integrate(1/x**3/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

Maxima [F]

$$\int \frac{1}{x^3 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}} x^3} \, dx$$

input
```
integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}} x^3} \, dx$$

input
```
integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{x^3 (1-x^3)^{4/3} (x^3+1)} \, dx$$

input
```
int(1/(x^3*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^3*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (1-x^3)^{4/3} (1+x^3)} \, dx = -\left(\int \frac{1}{(-x^3+1)^{\frac{1}{3}} x^9 - (-x^3+1)^{\frac{1}{3}} x^3} dx\right)$$

input
```
int(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
- int(1/(( - x**3 + 1)**(1/3)*x**9 - ( - x**3 + 1)**(1/3)*x**3),x)
```

3.864 $\int \frac{1}{x^6(1-x^3)^{4/3}(1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 144

$$\int \frac{1}{x^6 (1-x^3)^{4/3} (1+x^3)} dx = \frac{1}{2x^5\sqrt[3]{1-x^3}} - \frac{7(1-x^3)^{2/3}}{10x^5} - \frac{4(1-x^3)^{2/3}}{5x^2} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x - \sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2/x^5/(-x^3+1)^(1/3)-7/10*(-x^3+1)^(2/3)/x^5-4/5*(-x^3+1)^(2/3)/x^2-1/12
*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/24*l
n(x^3+1)*2^(2/3)+1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^6 (1-x^3)^{4/3} (1+x^3)} dx = \frac{1}{120} \left(-\frac{12(2+x^3-8x^6)}{x^5 \sqrt[3]{1-x^3}} \right.$$
$$-10\, 2^{2/3} \sqrt{3} \arctan \left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}} \right) + 10\, 2^{2/3} \log \left(2x+2^{2/3}\sqrt[3]{1-x^3} \right) - 5\, 2^{2/3} \log \left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3} \right.$$

input
```
Integrate[1/(x^6*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((-12*(2 + x^3 - 8*x^6))/(x^5*(1 - x^3)^(1/3)) - 10*2^(2/3)*Sqrt[3]*ArcTan
[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] + 10*2^(2/3)*Log[2*x + 2^(2/3)
*(1 - x^3)^(1/3)] - 5*2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(
1/3)*(1 - x^3)^(2/3)])/120
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {972, 1053, 25, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (1-x^3)^{4/3} (x^3+1)} dx$$
$$\Big\downarrow \text{972}$$
$$\frac{1}{2} \int \frac{6x^3+7}{x^6 \sqrt[3]{1-x^3} (x^3+1)} dx + \frac{1}{2x^5 \sqrt[3]{1-x^3}}$$
$$\Big\downarrow \text{1053}$$
$$\frac{1}{2} \left(-\frac{1}{5} \int -\frac{21x^3+16}{x^3 \sqrt[3]{1-x^3} (x^3+1)} dx - \frac{7(1-x^3)^{2/3}}{5x^5} \right) + \frac{1}{2x^5 \sqrt[3]{1-x^3}}$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(\frac{1}{5}\int\frac{21x^3+16}{x^3\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{7(1-x^3)^{2/3}}{5x^5}\right)+\frac{1}{2x^5\sqrt[3]{1-x^3}}$$

$\Big\downarrow$ 1053

$$\frac{1}{2}\left(\frac{1}{5}\left(-\frac{1}{2}\int-\frac{10}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{8(1-x^3)^{2/3}}{x^2}\right)-\frac{7(1-x^3)^{2/3}}{5x^5}\right)+\frac{1}{2x^5\sqrt[3]{1-x^3}}$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(\frac{1}{5}\left(5\int\frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{8(1-x^3)^{2/3}}{x^2}\right)-\frac{7(1-x^3)^{2/3}}{5x^5}\right)+\frac{1}{2x^5\sqrt[3]{1-x^3}}$$

$\Big\downarrow$ 901

$$\frac{1}{2}\left(\frac{1}{5}\left(5\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}\right)-\frac{8(1-x^3)^{2/3}}{x^2}\right)-\frac{7(1-x^3)^{2/3}}{5x^5}\right)+\frac{1}{2x^5\sqrt[3]{1-x^3}}$$

input
```
Int[1/(x^6*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
1/(2*x^5*(1 - x^3)^(1/3)) + ((-7*(1 - x^3)^(2/3))/(5*x^5) + ((-8*(1 - x^3)
^(2/3))/x^2 + 5*(-(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^
(1/3)*Sqrt[3])) - Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^
(1/3)]/(2*2^(1/3))))/5)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 26.46 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right) x^{5}\left(-x^{3}+1\right)^{\frac{1}{3}}-\frac{2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right) x^{5}\left(-x^{3}+1\right)^{\frac{1}{3}}}{2}+2^{\frac{2}{3}} \ln}{12\left(-x^{3}+1\right)^{\frac{1}{3}} x^{5}}$
risch	Expression too large to display
trager	Expression too large to display

input
```
int(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*(3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)*x^
5*(-x^3+1)^(1/3)-1/2*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^
3+1)^(2/3))/x^2)*x^5*(-x^3+1)^(1/3)+2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/
x)*x^5*(-x^3+1)^(1/3)+48/5*x^6-6/5*x^3-12/5)/(-x^3+1)^(1/3)/x^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(109) = 218$.

Time = 1.75 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.15

$$\int \frac{1}{x^{6}\left(1-x^{3}\right)^{4/3}\left(1+x^{3}\right)}\,dx =$$

$$-\frac{60 \cdot 2^{\frac{1}{6}} \sqrt{\frac{1}{6}}\left(x^{8}-x^{5}\right) \arctan\left(\frac{2^{\frac{1}{6}} \sqrt{\frac{1}{6}}\left(6 \cdot 2^{\frac{2}{3}}\left(5 x^{7}+4 x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(71 x^{9}-111 x^{6}+33 x^{3}-1\right)+12\left(19 x^{8}-16 x^{5}+x^{2}\right)\left(-x^{3}+1\right)}{109 x^{9}-105 x^{6}+3 x^{3}+1}\right.}{}$$

input
```
integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/360*(60*2^(1/6)*sqrt(1/6)*(x^8 - x^5)*arctan(2^(1/6)*sqrt(1/6)*(6*2^(2/
3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(71*x^9 - 111*x^6 + 33*x
^3 - 1) + 12*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6
+ 3*x^3 + 1)) - 10*2^(2/3)*(x^8 - x^5)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2
 + 2^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(2/3)*(x^8 -
 x^5)*log((3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x
^3 + 1) - 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) + 36*(8*x^
6 - x^3 - 2)*(-x^3 + 1)^(2/3))/(x^8 - x^5)
```

Sympy [F]

$$\int \frac{1}{x^6 \left(1-x^3\right)^{4/3} \left(1+x^3\right)}\,dx = \int \frac{1}{x^6 \left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}} \left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(1/x**6/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(1/(x**6*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

Maxima [F]

$$\int \frac{1}{x^6 \left(1-x^3\right)^{4/3} \left(1+x^3\right)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^6}\,dx$$

input
```
integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}} x^6} \, dx$$

input
```
integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{x^6 (1-x^3)^{4/3} (x^3+1)} \, dx$$

input
```
int(1/(x^6*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^6*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (1-x^3)^{4/3} (1+x^3)} \, dx = -\left(\int \frac{1}{(-x^3+1)^{\frac{1}{3}} x^{12} - (-x^3+1)^{\frac{1}{3}} x^6} dx \right)$$

input
```
int(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
- int(1/(( - x**3 + 1)**(1/3)*x***12 - ( - x**3 + 1)**(1/3)*x***6),x)
```

3.865 $\int \frac{1}{x^9 (1-x^3)^{4/3} (1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 162

$$\int \frac{1}{x^9 (1-x^3)^{4/3} (1+x^3)} dx = \frac{1}{2x^8 \sqrt[3]{1-x^3}} - \frac{5(1-x^3)^{2/3}}{8x^8} - \frac{13(1-x^3)^{2/3}}{20x^5} - \frac{49(1-x^3)^{2/3}}{40x^2} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{2}x - \sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2/x^8/(-x^3+1)^(1/3)-5/8*(-x^3+1)^(2/3)/x^8-13/20*(-x^3+1)^(2/3)/x^5-49/
40*(-x^3+1)^(2/3)/x^2+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/
2))*2^(2/3)*3^(1/2)+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3
))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^9 (1-x^3)^{4/3} (1+x^3)} \, dx = \frac{1}{120}\left(-\frac{3(5+x^3+23x^6-49x^9)}{x^8\sqrt[3]{1-x^3}} \right.$$
$$+10\, 2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 10\, 2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + 5\, 2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}\right.$$

input
```
Integrate[1/(x^9*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((-3*(5 + x^3 + 23*x^6 - 49*x^9))/(x^8*(1 - x^3)^(1/3)) + 10*2^(2/3)*Sqrt[
3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 10*2^(2/3)*Log[2*x
+ 2^(2/3)*(1 - x^3)^(1/3)] + 5*2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1
/3) - 2^(1/3)*(1 - x^3)^(2/3)])/120
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {972, 1053, 27, 1053, 25, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 (1-x^3)^{4/3} (x^3+1)} \, dx$$

$$\Big\downarrow \text{ 972}$$

$$\frac{1}{2}\int \frac{9x^3+10}{x^9\sqrt[3]{1-x^3}\,(x^3+1)}dx + \frac{1}{2x^8\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{1}{2}\left(-\frac{1}{8}\int -\frac{4(15x^3+13)}{x^6\sqrt[3]{1-x^3}\,(x^3+1)}dx - \frac{5(1-x^3)^{2/3}}{4x^8}\right) + \frac{1}{2x^8\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{15x^3+13}{x^6\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{5(1-x^3)^{2/3}}{4x^8}\right)+\frac{1}{2x^8\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{5}\int -\frac{39x^3+49}{x^3\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{13(1-x^3)^{2/3}}{5x^5}\right)-\frac{5(1-x^3)^{2/3}}{4x^8}\right)+\frac{1}{2x^8\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{5}\int \frac{39x^3+49}{x^3\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{13(1-x^3)^{2/3}}{5x^5}\right)-\frac{5(1-x^3)^{2/3}}{4x^8}\right)+\frac{1}{2x^8\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{5}\left(-\frac{1}{2}\int \frac{20}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{49(1-x^3)^{2/3}}{2x^2}\right)-\frac{13(1-x^3)^{2/3}}{5x^5}\right)-\frac{5(1-x^3)^{2/3}}{4x^8}\right)+\frac{1}{2x^8\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{5}\left(-10\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{49(1-x^3)^{2/3}}{2x^2}\right)-\frac{13(1-x^3)^{2/3}}{5x^5}\right)-\frac{5(1-x^3)^{2/3}}{4x^8}\right)+\frac{1}{2x^8\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{901}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{5}\left(-10\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(x^3+1)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}\right)-\frac{49(1-x^3)^{2/3}}{2x^2}\right)-\frac{1}{}\right.\right.$$

$$\frac{1}{2x^8\sqrt[3]{1-x^3}}$$

input
```
Int[1/(x^9*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
1/(2*x^8*(1 - x^3)^(1/3)) + ((-5*(1 - x^3)^(2/3))/(4*x^8) + ((-13*(1 - x^3
)^(2/3))/(5*x^5) + ((-49*(1 - x^3)^(2/3))/(2*x^2) - 10*(-(ArcTan[(1 - (2*2
^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) - Log[1 + x^3]/(6*2
^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))))/5)/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 27.64 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{-10\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+x\right)}{3x}\right) x^8\left(-x^3+1\right)^{\frac{1}{3}} - 10\,2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right) x^8\left(-x^3+1\right)^{\frac{1}{3}} + 5\,2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{2}{3}}x^2}{\cdots}\right.}{120x^8\left(-x^3+1\right)^{\frac{1}{3}}}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/120*(-10*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/
x)*x^8*(-x^3+1)^(1/3)-10*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*x^8*(-x^
3+1)^(1/3)+5*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/
3))/x^2)*x^8*(-x^3+1)^(1/3)+147*x^9-69*x^6-3*x^3-15)/x^8/(-x^3+1)^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(123) = 246$.

Time = 1.45 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.94

$$\int \frac{1}{x^9\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{60\cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(x^{11}-x^8\right)\arctan\left(\frac{2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(6\cdot 2^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(71x^9-111x^6+\cdots\right.\right.}{109x^9-105x^6+3x\cdots}\right.}{\cdots}$$

input
```
integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/360*(60*2^(1/6)*sqrt(1/6)*(x^11 - x^8)*arctan(2^(1/6)*sqrt(1/6)*(6*2^(2/
3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(71*x^9 - 111*x^6 + 33*x
^3 - 1) + 12*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6
+ 3*x^3 + 1)) - 10*2^(2/3)*(x^11 - x^8)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^
2 + 2^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(2/3)*(x^11
 - x^8)*log((3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16
*x^3 + 1) - 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 9*(49*
x^9 - 23*x^6 - x^3 - 5)*(-x^3 + 1)^(2/3))/(x^11 - x^8)
```

Sympy [F]

$$\int \frac{1}{x^9 (1-x^3)^{4/3} (1+x^3)} dx = \int \frac{1}{x^9 \left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}} (x+1)(x^2-x+1)} dx$$

input
```
integrate(1/x**9/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(1/(x**9*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

Maxima [F]

$$\int \frac{1}{x^9 (1-x^3)^{4/3} (1+x^3)} dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}} x^9} dx$$

input
```
integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^9), x)
```

Giac [F]

$$\int \frac{1}{x^9 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}} x^9} \, dx$$

input
```
integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^9 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{x^9 (1-x^3)^{4/3} (x^3+1)} \, dx$$

input
```
int(1/(x^9*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^9*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^9 (1-x^3)^{4/3} (1+x^3)} \, dx = -\left(\int \frac{1}{(-x^3+1)^{\frac{1}{3}} x^{15} - (-x^3+1)^{\frac{1}{3}} x^9} dx \right)$$

input
```
int(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
- int(1/(( - x**3 + 1)**(1/3)*x**15 - ( - x**3 + 1)**(1/3)*x**9),x)
```

3.866 $\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 292

$$\begin{aligned}\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx &= \frac{x^5}{2\sqrt[3]{1-x^3}}\\ &+\frac{3}{4}x^2\left(1-x^3\right)^{2/3}-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}\\ &-\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\\ &-\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}-\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}\\ &+\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}\end{aligned}$$

output
```
1/2*x^5/(-x^3+1)^(1/3)+3/4*x^2*(-x^3+1)^(2/3)-1/12*arctan(1/3*(1-2*2^(1/3)
*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/24*arctan(1/3*(1+2^(1/3)
*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/2*x^2*hypergeom([1/3, 2/
3],[5/3],x^3)-1/48*ln((1-x)*(1+x)^2)*2^(2/3)-1/24*ln(1+2^(2/3)*(1-x)^2/(-x
^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/12*ln(1+2^(1/3)*(1-x)/
(-x^3+1)^(1/3))*2^(2/3)+1/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.24

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{20}x^2\left(-\frac{5(-3+x^3)}{\sqrt[3]{1-x^3}} - 15\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right) - 4x^3\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)\right)$$

input
```
Integrate[x^10/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(x^2*((-5*(-3 + x^3))/(1 - x^3)^(1/3) - 15*AppellF1[2/3, 1/3, 1, 5/3, x^3,
 -x^3] - 4*x^3*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3]))/20
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {970, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(x^3+1)}dx$$

$$\Big\downarrow \text{ 970}$$

$$\frac{x^5}{2\sqrt[3]{1-x^3}}-\frac{1}{2}\int \frac{x^4(6x^3+5)}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{1052}$$

$$\frac{1}{2}\left(\frac{3}{2}x^2(1-x^3)^{2/3}-\frac{1}{4}\int \frac{4x(2x^3+3)}{\sqrt[3]{1-x^3}\,(x^3+1)}dx\right)+\frac{x^5}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{3}{2}x^2(1-x^3)^{2/3}-\int \frac{x(2x^3+3)}{\sqrt[3]{1-x^3}\,(x^3+1)}dx\right)+\frac{x^5}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{1054}$$

$$\frac{1}{2}\left(\frac{3}{2}x^2(1-x^3)^{2/3}-\int \left(\frac{2x}{\sqrt[3]{1-x^3}}+\frac{x}{\sqrt[3]{1-x^3}\,(x^3+1)}\right)dx\right)+\frac{x^5}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+x^2\left(-\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\right)-\frac{\log\left(\frac{2^{2}}{(}\right.}{}\right.$$

$$\frac{x^5}{2\sqrt[3]{1-x^3}}$$

input
```
Int[x^10/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
x^5/(2*(1 - x^3)^(1/3)) + ((3*x^2*(1 - x^3)^(2/3))/2 - ArcTan[(1 - (2*2^(1
/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - ArcTan[(1 + (2^
(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - x^2*Hyperge
ometric2F1[1/3, 2/3, 5/3, x^3] - Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) - Log
[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/
3)]/(6*2^(1/3)) + Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) +
 Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(4*2^(1/3)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^{10}}{(-x^3+1)^{\frac{4}{3}}(x^3+1)}dx$$

input
```
int(x^10/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
int(x^10/(-x^3+1)^(4/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{10}}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^10/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)*x^10/(x^9 - x^6 - x^3 + 1), x)
```

Sympy [F]

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{10}}{(-(x-1)(x^2+x+1))^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**10/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**10/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [F]

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{10}}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^10/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^10/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Giac [F]

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{10}}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^10/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^10/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{10}}{(1-x^3)^{4/3}(x^3+1)}\,dx$$

input
```
int(x^10/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(x^10/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\left(\int \frac{x^{10}}{(-x^3+1)^{\frac{1}{3}}x^6-(-x^3+1)^{\frac{1}{3}}}dx\right)$$

input
```
int(x^10/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
- int(x**10/(( - x**3 + 1)**(1/3)*x**6 - ( - x**3 + 1)**(1/3)),x)
```

3.867 $\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 274

$$\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{x^2}{2\sqrt[3]{1-x^3}}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{3}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}$$

output
```
1/2*x^2/(-x^3+1)^(1/3)+1/12*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*
3^(1/2))*2^(2/3)*3^(1/2)+1/24*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*
3^(1/2))*2^(2/3)*3^(1/2)-3/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/48*ln((
1-x)*(1+x)^2)*2^(2/3)+1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-
x)/(-x^3+1)^(1/3))*2^(2/3)-1/12*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)
-1/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.24

$$\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{10}x^2\left(\frac{5}{\sqrt[3]{1-x^3}} - 5\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right) - 3x^3\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)\right)$$

input
```
Integrate[x^7/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(x^2*(5/(1 - x^3)^(1/3) - 5*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] - 3*x^3*
AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3]))/10
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {970, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(1-x^3)^{4/3}(x^3+1)}dx$$

$$\Big\downarrow \text{ 970}$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{1}{2}\int \frac{x(3x^3+2)}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$\Big\downarrow$ 1054

$$\frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{1}{2}\int \left(\frac{3x}{\sqrt[3]{1-x^3}}-\frac{x}{\sqrt[3]{1-x^3}\,(x^3+1)}\right)dx$$

$\Big\downarrow$ 2009

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{3}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{2^{2/3}(1-\right.}{(1-x^3)^2}}{}\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

input
```
Int[x^7/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
x^2/(2*(1 - x^3)^(1/3)) + (ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3)
)/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/
3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - (3*x^2*Hypergeometric2F1[1/3, 2/3, 5/3,
 x^3])/2 + Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^
2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[
1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-1 + x + 2^(2/3)*
(1 - x^3)^(1/3)]/(4*2^(1/3)))/2
```

Defintions of rubi rules used

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^{7}}{\left(-x^{3}+1\right)^{\frac{4}{3}} \left(x^{3}+1\right)}dx$$

input
```
int(x^7/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
int(x^7/(-x^3+1)^(4/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^7/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)*x^7/(x^9 - x^6 - x^3 + 1), x)
```

Sympy [F]

$$\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^7}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**7/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [F]

$$\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^7/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Giac [F]

$$\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^7/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^7}{(1-x^3)^{4/3}(x^3+1)}\,dx$$

input
```
int(x^7/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(x^7/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\left(\int \frac{x^7}{(-x^3+1)^{\frac{1}{3}}x^6-(-x^3+1)^{\frac{1}{3}}}dx\right)$$

input
```
int(x^7/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
- int(x**7/(( - x**3 + 1)**(1/3)*x**6 - ( - x**3 + 1)**(1/3)),x)
```

3.868 $\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 274

$$\begin{aligned}\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx &= \frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}\\&-\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\\&-\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}-\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}\\&+\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}\end{aligned}$$

output
```
1/2*x^2/(-x^3+1)^(1/3)-1/12*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*
3^(1/2))*2^(2/3)*3^(1/2)-1/24*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*
3^(1/2))*2^(2/3)*3^(1/2)-1/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/48*ln((
1-x)*(1+x)^2)*2^(2/3)-1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-
x)/(-x^3+1)^(1/3))*2^(2/3)+1/12*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)
+1/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.24

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{10}x^2\left(\frac{5}{\sqrt[3]{1-x^3}} - 5\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right) - x^3\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)\right)$$

input
```
Integrate[x^4/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(x^2*(5/(1 - x^3)^(1/3) - 5*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] - x^3*Ap
pellF1[5/3, 1/3, 1, 8/3, x^3, -x^3]))/10
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {971, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(1-x^3)^{4/3}(x^3+1)}dx$$

$$\Big\downarrow \text{971}$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{1}{2}\int \frac{x\left(x^3+2\right)}{\sqrt[3]{1-x^3}\,\left(x^3+1\right)}dx$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{1}{2}\int \left(\frac{x}{\sqrt[3]{1-x^3}}+\frac{x}{\sqrt[3]{1-x^3}\,\left(x^3+1\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)-\frac{\log\left(\frac{2^{2/3}(1}{(1-x^3}\right.}{}\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

input

```
Int[x^4/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output

```
x^2/(2*(1 - x^3)^(1/3)) + (-(ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/
3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^
(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3
, x^3])/2 - Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) - Log[1 + (2^(2/3)*(1 - x)
^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log
[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) + Log[-1 + x + 2^(2/3)
*(1 - x^3)^(1/3)]/(4*2^(1/3)))/2
```

Defintions of rubi rules used

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^{4}}{\left(-x^{3}+1\right)^{\frac{4}{3}} \left(x^{3}+1\right)}dx$$

input
```
int(x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
int(x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{x^4}{(1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{x^{4}}{{\left(x^{3} + 1\right)} {\left(-x^{3} + 1\right)}^{\frac{4}{3}}} \,d x$$

input
```
integrate(x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)*x^4/(x^9 - x^6 - x^3 + 1), x)
```

Sympy [F]

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^4}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**4/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [F]

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Giac [F]

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx=\int \frac{x^4}{(1-x^3)^{4/3}(x^3+1)}\,dx$$

input
```
int(x^4/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(x^4/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx=-\left(\int \frac{x^4}{(-x^3+1)^{\frac{1}{3}}x^6-(-x^3+1)^{\frac{1}{3}}}dx\right)$$

input
```
int(x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
- int(x**4/(( - x**3 + 1)**(1/3)*x**6 - ( - x**3 + 1)**(1/3)),x)
```

3.869 $\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 274

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{x^2}{2\sqrt[3]{1-x^3}}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}$$

output
```
1/2*x^2/(-x^3+1)^(1/3)+1/12*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*
3^(1/2))*2^(2/3)*3^(1/2)+1/24*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*
3^(1/2))*2^(2/3)*3^(1/2)-1/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/48*ln((
1-x)*(1+x)^2)*2^(2/3)+1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-
x)/(-x^3+1)^(1/3))*2^(2/3)-1/12*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)
-1/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.16

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{x^2}{2\sqrt[3]{1-x^3}} - \frac{1}{10}x^5\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)$$

input
```
Integrate[x/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
x^2/(2*(1 - x^3)^(1/3)) - (x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/10
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {972, 25, 983, 888, 991, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103, 2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-x^3)^{4/3}(x^3+1)}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{1}{2}\int -\frac{x^4}{\sqrt[3]{1-x^3}\,(x^3+1)}dx + \frac{x^2}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{1}{2}\int \frac{x^4}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$\Big\downarrow$ 983

$$\frac{1}{2}\left(\int \frac{x}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\int \frac{x}{\sqrt[3]{1-x^3}}dx\right)+\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$\Big\downarrow$ 888

$$\frac{1}{2}\left(\int \frac{x}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\right)+\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$\Big\downarrow$ 991

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\int \frac{1}{\frac{2(1-x)^3}{1-x^3}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\right)+$$
$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$\Big\downarrow$ 750

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{3}\int \frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}\right.$$
$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$\Big\downarrow$ 16

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},\right.\right.$$
$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\int \frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}x^2 \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3}\right.\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$\downarrow$ 1142

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\int -\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1}{\sqrt[3]{1}}}{2\ 2^{2/3}}\right.\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$\downarrow$ 25

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int \frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-}}}{2\ 2^{2/3}}\right.\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$\downarrow$ 27

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}\right.\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$\downarrow$ 1082

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{-\frac{(1-x)^2}{(1-x^3)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}\right.\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$\downarrow$ 217

$$\frac{1}{2}\left(-\frac{1}{3}\sqrt[3]{2}\left(\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}\right)-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{2}\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$\downarrow$ 1103

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)-\frac{1}{2}\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{2574}$$

$$\frac{1}{2}\left(-\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)+\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right.}{2\sqrt[3]{2}}\right.\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

input
```
Int[x/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
x^2/(2*(1 - x^3)^(1/3)) + (-1/2*(x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3]
) - (2^(1/3)*(-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/
Sqrt[3]])/2^(2/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3
)*(1 - x))/(1 - x^3)^(1/3)]/(2*2^(2/3))))/3 - Log[1 + (2^(1/3)*(1 - x))/(1
 - x^3)^(1/3)]/(3*2^(1/3)) + ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 -
x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3)) + Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) - (
3*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)))/3)/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 991
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Simp[-q^2/(3*d)   Int[1/((1 - q*x)*(a + b*x^3)^(1/3)
), x], x] + Simp[q/d   Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x
^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c +
 a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Maple [F]

$$\int \frac{x}{(-x^3+1)^{\frac{4}{3}}(x^3+1)}dx$$

input
```
int(x/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
int(x/(-x^3+1)^(4/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)*x/(x^9 - x^6 - x^3 + 1), x)
```

Sympy [F]

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x}{(-(x-1)(x^2+x+1))^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Giac [F]

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x}{(1-x^3)^{4/3}(x^3+1)}\,dx$$

input
```
int(x/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(x/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\left(\int \frac{x}{(-x^3+1)^{\frac{1}{3}}x^6 - (-x^3+1)^{\frac{1}{3}}}dx\right)$$

input
```
int(x/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
 - int(x/(( - x**3 + 1)**(1/3)*x**6 - ( - x**3 + 1)**(1/3)),x)
```

3.870 $\int \frac{1}{x^2(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 292

$$\begin{aligned}\int \frac{1}{x^2(1-x^3)^{4/3}(1+x^3)}\,dx &= \frac{1}{2x\sqrt[3]{1-x^3}}-\frac{3(1-x^3)^{2/3}}{2x} \\ &\quad -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} \\ &\quad -\frac{3}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) \\ &\quad -\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}-\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}} \\ &\quad +\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}\end{aligned}$$

output
```
1/2/x/(-x^3+1)^(1/3)-3/2*(-x^3+1)^(2/3)/x-1/12*arctan(1/3*(1-2*2^(1/3)*(1-
x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/24*arctan(1/3*(1+2^(1/3)*(1-
x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-3/4*x^2*hypergeom([1/3, 2/3],[
5/3],x^3)-1/48*ln((1-x)*(1+x)^2)*2^(2/3)-1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1
)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/12*ln(1+2^(1/3)*(1-x)/(-x^
3+1)^(1/3))*2^(2/3)+1/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^2(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{-2+3x^3}{2x\sqrt[3]{1-x^3}} - x^2\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right) - \frac{3}{10}x^5\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)$$

input
```
Integrate[1/(x^2*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(-2 + 3*x^3)/(2*x*(1 - x^3)^(1/3)) - x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -
x^3] - (3*x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/10
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {972, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(1-x^3)^{4/3}(x^3+1)}\,dx$$

$$\Big\downarrow \text{972}$$

$$\frac{1}{2}\int \frac{2x^3+3}{x^2\sqrt[3]{1-x^3}\,(x^3+1)}dx+\frac{1}{2x\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{2}\left(-\int \frac{x(3x^3+4)}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{3(1-x^3)^{2/3}}{x}\right)+\frac{1}{2x\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{1054}$$

$$\frac{1}{2}\left(-\int \left(\frac{3x}{\sqrt[3]{1-x^3}}+\frac{x}{\sqrt[3]{1-x^3}\,(x^3+1)}\right)dx-\frac{3(1-x^3)^{2/3}}{x}\right)+\frac{1}{2x\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{3}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)-\frac{3(1-x^3)^{2}}{x}\right.$$

$$\frac{1}{2x\sqrt[3]{1-x^3}}$$

input
```
Int[1/(x^2*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
1/(2*x*(1 - x^3)^(1/3)) + ((-3*(1 - x^3)^(2/3))/x - ArcTan[(1 - (2*2^(1/3)
*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - ArcTan[(1 + (2^(1/
3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - (3*x^2*Hyperge
ometric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) -
Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^
(1/3)]/(6*2^(1/3)) + Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)
) + Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(4*2^(1/3)))/2
```

Defintions of rubi rules used

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(-x^{3}+1\right)^{\frac{4}{3}} \left(x^{3}+1\right)}d x$$

input
```
int(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
int(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}} x^2} \, dx$$

input
```
integrate(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)/(x^11 - x^8 - x^5 + x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{x^2 \left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}} (x+1)(x^2-x+1)} \, dx$$

input
```
integrate(1/x**2/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

Maxima [F]

$$\int \frac{1}{x^2 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}} x^2} \, dx$$

input
```
integrate(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\int \frac{1}{x^2\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\,dx$$

input
```
int(1/(x^2*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^2*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=-\left(\int \frac{1}{(-x^3+1)^{\frac{1}{3}}x^8-(-x^3+1)^{\frac{1}{3}}x^2}dx\right)$$

input
```
int(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
- int(1/(( - x**3 + 1)**(1/3)*x**8 - ( - x**3 + 1)**(1/3)*x**2),x)
```

3.871 $\int \frac{1}{x^5(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 308

$$\int \frac{1}{x^5(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2x^4\sqrt[3]{1-x^3}} - \frac{3(1-x^3)^{2/3}}{4x^4} - \frac{(1-x^3)^{2/3}}{x} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{\log((1-x)(1+x)^2)}{24\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}} - \frac{\log\left(\right.}{}$$

output
```
1/2/x^4/(-x^3+1)^(1/3)-3/4*(-x^3+1)^(2/3)/x^4-(-x^3+1)^(2/3)/x+1/12*arctan
(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/24*arct
an(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/2*x^2*h
ypergeom([1/3, 2/3],[5/3],x^3)+1/48*ln((1-x)*(1+x)^2)*2^(2/3)+1/24*ln(1+2^
(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/12*ln
(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/
3))*2^(2/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^5 (1-x^3)^{4/3} (1+x^3)} \, dx = -\frac{\frac{5(1+x^3-4x^6)}{\sqrt[3]{1-x^3}} + 5x^6 \operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right) + 4x^9 \operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)}{20x^4}$$

input
```
Integrate[1/(x^5*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
-1/20*((5*(1 + x^3 - 4*x^6))/(1 - x^3)^(1/3) + 5*x^6*AppellF1[2/3, 1/3, 1,
 5/3, x^3, -x^3] + 4*x^9*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/x^4
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {972, 1053, 27, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (1-x^3)^{4/3} (x^3+1)} \, dx$$

$$\Big\downarrow \text{ 972}$$

$$\frac{1}{2} \int \frac{5x^3+6}{x^5 \sqrt[3]{1-x^3} (x^3+1)} dx + \frac{1}{2x^4 \sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{1}{2} \left(-\frac{1}{4} \int -\frac{4(3x^3+2)}{x^2 \sqrt[3]{1-x^3} (x^3+1)} dx - \frac{3(1-x^3)^{2/3}}{2x^4} \right) + \frac{1}{2x^4 \sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\int \frac{3x^3+2}{x^2\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{3\left(1-x^3\right)^{2/3}}{2x^4}\right)+\frac{1}{2x^4\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{1}{2}\left(-\int \frac{x\left(2x^3+1\right)}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{2\left(1-x^3\right)^{2/3}}{x}-\frac{3\left(1-x^3\right)^{2/3}}{2x^4}\right)+\frac{1}{2x^4\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{1}{2}\left(-\int \left(\frac{2x}{\sqrt[3]{1-x^3}}-\frac{x}{\sqrt[3]{1-x^3}\,(x^3+1)}\right)dx-\frac{2\left(1-x^3\right)^{2/3}}{x}-\frac{3\left(1-x^3\right)^{2/3}}{2x^4}\right)+\frac{1}{2x^4\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+x^2\left(-\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\right)-\frac{2\left(1-x^3\right.}{x}\right.$$

$$\frac{1}{2x^4\sqrt[3]{1-x^3}}$$

input
```
Int[1/(x^5*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
1/(2*x^4*(1 - x^3)^(1/3)) + ((-3*(1 - x^3)^(2/3))/(2*x^4) - (2*(1 - x^3)^(
2/3))/x + ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/
3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2
^(1/3)*Sqrt[3]) - x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3] + Log[(1 - x)*
(1 + x)^2]/(12*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2
^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (2^(1/3)*(1 - x))/(
1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(4*2^(
1/3)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{1}{x^5(-x^3+1)^{\frac{4}{3}}(x^3+1)}dx$$

input
```
int(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
int(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{1}{x^5(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^5}\,dx$$

input
```
integrate(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)/(x^14 - x^11 - x^8 + x^5), x)
```

Sympy [F]

$$\int \frac{1}{x^5(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{1}{x^5(-(x-1)(x^2+x+1))^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(1/x**5/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

Maxima [F]

$$\int \frac{1}{x^5 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^5} \, dx$$

input
```
integrate(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^5} \, dx$$

input
```
integrate(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{x^5 (1-x^3)^{4/3} (x^3+1)} \, dx$$

input
```
int(1/(x^5*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^5*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^5 (1-x^3)^{4/3} (1+x^3)}\, dx = -\left(\int \frac{1}{(-x^3+1)^{\frac{1}{3}} x^{11} - (-x^3+1)^{\frac{1}{3}} x^5} dx\right)$$

input
```
int(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
 - int(1/(( - x**3 + 1)**(1/3)*x**11 - ( - x**3 + 1)**(1/3)*x**5),x)
```

3.872 $\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}$$

output
```
1/3*(b*x^3+a)^(1/2)*(d*x^3+c)^(1/2)/b/d-1/3*(a*d+b*c)*arctanh(d^(1/2)*(b*x
^3+a)^(1/2)/b^(1/2)/(d*x^3+c)^(1/2))/b^(3/2)/d^(3/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{d}\sqrt{a+bx^3}}\right)}{3b^{3/2}d^{3/2}}$$

input
```
Integrate[x^5/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(3*b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[b]*
Sqrt[c + d*x^3])/(Sqrt[d]*Sqrt[a + b*x^3])])/(3*b^(3/2)*d^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {948, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{x^3}{\sqrt{bx^3+a}\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \ 90$$

$$\frac{1}{3}\left(\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{bd}-\frac{(ad+bc)\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}}dx^3}{2bd}\right)$$

$$\Big\downarrow \ 66$$

$$\frac{1}{3}\left(\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{bd}-\frac{(ad+bc)\int \frac{1}{b-dx^6}d\frac{\sqrt{bx^3+a}}{\sqrt{dx^3+c}}}{bd}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{3}\left(\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{bd}-\frac{(ad+bc)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{b^{3/2}d^{3/2}}\right)$$

input
```
Int[x^5/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
((Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^3])/(Sqrt[b]*Sqrt[c + d*x^3])])/(b^(3/2)*d^(3/2)))/3
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^5}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}dx$$

input
```
int(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.91

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$
$$= \left[\frac{4\sqrt{bx^3+a}\sqrt{dx^3+c}bd + (bc+ad)\sqrt{bd}\log\left(8\,b^2d^2x^6 + b^2c^2 + 6\,abcd + a^2d^2 + 8\,(b^2cd+abd^2)x^3 - 4\,(2\right.}{12\,b^2d^2}\right.$$

input
```
integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(4*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*b*d + (b*c + a*d)*sqrt(b*d)*log(8
*b^2*d^2*x^6 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^3 -
 4*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(b*d)))/(b^
2*d^2), 1/6*(2*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*b*d + (b*c + a*d)*sqrt(-b*d
)*arctan(1/2*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(
-b*d)/(b^2*d^2*x^6 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^3)))/(b^2*d^2)]
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**5/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**5/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\frac{(bc+ad)\log\left(\left|-\sqrt{bx^3+a}\sqrt{bd}+\sqrt{b^2c+(bx^3+a)bd-abd}\right|\right)}{\sqrt{bd}d} + \frac{\sqrt{bx^3+a}\sqrt{b^2c+(bx^3+a)bd-abd}}{bd}}{3|b|}$$

input
```
integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
1/3*((b*c + a*d)*log(abs(-sqrt(b*x^3 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^3
+ a)*b*d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b*x^3 + a)*sqrt(b^2*c + (b*x^3 +
a)*b*d - a*b*d)/(b*d))/abs(b)
```

Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.22

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\frac{2\,a\,d}{3}+\frac{2\,b\,c}{3}\right)}{d^3\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}+\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\frac{2\,a\,d}{3}+\frac{2\,b\,c}{3}\right)}{b\,d^2\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3}-\frac{8\,\sqrt{a}\,\sqrt{c}\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^2}{3\,d^2\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^4}{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^4}+\frac{b^2}{d^2}-\frac{2\,b\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^2}{d\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^2}} - \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{\sqrt{b}\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}\right)(a\,d+b\,c)}{3\,b^{3/2}\,d^{3/2}}$$

input
```
int(x^5/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output
```
((((a + b*x^3)^(1/2) - a^(1/2))*((2*a*d)/3 + (2*b*c)/3))/(d^3*((c + d*x^3)
^(1/2) - c^(1/2))) + (((a + b*x^3)^(1/2) - a^(1/2))^3*((2*a*d)/3 + (2*b*c)
/3))/(b*d^2*((c + d*x^3)^(1/2) - c^(1/2))^3) - (8*a^(1/2)*c^(1/2)*((a + b*
x^3)^(1/2) - a^(1/2))^2)/(3*d^2*((c + d*x^3)^(1/2) - c^(1/2))^2))/(((a + b
*x^3)^(1/2) - a^(1/2))^4/((c + d*x^3)^(1/2) - c^(1/2))^4 + b^2/d^2 - (2*b*
((a + b*x^3)^(1/2) - a^(1/2))^2)/(d*((c + d*x^3)^(1/2) - c^(1/2))^2)) - (2
*atanh((d^(1/2)*((a + b*x^3)^(1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^3)^(1/2)
 - c^(1/2))))*(a*d + b*c))/(3*b^(3/2)*d^(3/2))
```

Reduce [F]

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}\,x^5}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx$$

input
```
int(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*sqrt(a + b*x**3)*x**5)/(a*c + a*d*x**3 + b*c*x**3 +
b*d*x**6),x)
```

3.873 $\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 48

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3\sqrt{b}\sqrt{d}}$$

output
```
2/3*arctanh(d^(1/2)*(b*x^3+a)^(1/2)/b^(1/2)/(d*x^3+c)^(1/2))/b^(1/2)/d^(1/
2)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{d}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\sqrt{d}}$$

input
```
Integrate[x^2/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/(Sqrt[d]*Sqrt[a + b*x^3])])/(3*Sqrt[b
]*Sqrt[d])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {946, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}dx$$
$$\Big\downarrow \text{946}$$
$$\frac{1}{3}\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}}dx^3$$
$$\Big\downarrow \text{66}$$
$$\frac{2}{3}\int \frac{1}{b-dx^6}d\frac{\sqrt{bx^3+a}}{\sqrt{dx^3+c}}$$
$$\Big\downarrow \text{221}$$
$$\frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3\sqrt{b}\sqrt{d}}$$

input
```
Int[x^2/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^3])/(Sqrt[b]*Sqrt[c + d*x^3])])/(3*Sqrt[b
]*Sqrt[d])
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}dx$$

input
```
int(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(34) = 68$.

Time = 0.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 4.04

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^6+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^3+4\,(2\,bdx^3+bc+ad)\sqrt{bx^3+a}\sqrt{dx^3+c}\right.}{6\,bd} - \frac{\sqrt{-bd}\arctan\left(\frac{(2\,bdx^3+bc+ad)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{-bd}}{2\,(b^2d^2x^6+abcd+(b^2cd+abd^2)x^3)}\right)}{3\,bd}\right]$$

input
```
integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*sqrt(b*d)*log(8*b^2*d^2*x^6 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*
c*d + a*b*d^2)*x^3 + 4*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3
+ c)*sqrt(b*d))/(b*d), -1/3*sqrt(-b*d)*arctan(1/2*(2*b*d*x^3 + b*c + a*d)*
sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-b*d)/(b^2*d^2*x^6 + a*b*c*d + (b^2*c
*d + a*b*d^2)*x^3))/(b*d)]
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**2/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{2\,b\log\left(\left|-\sqrt{bx^3+a}\sqrt{bd}+\sqrt{b^2c+(bx^3+a)bd-abd}\right|\right)}{3\,\sqrt{bd}|b|}$$

input
```
integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-2/3*b*log(abs(-sqrt(b*x^3 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^3 + a)*b*d -
 a*b*d)))/(sqrt(b*d)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 3.76 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}{\sqrt{-b\,d}\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}\right)}{3\,\sqrt{-b\,d}}$$

input
```
int(x^2/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output
```
-(4*atan((b*((c + d*x^3)^(1/2) - c^(1/2)))/((-b*d)^(1/2)*((a + b*x^3)^(1/2
) - a^(1/2)))))/(3*(-b*d)^(1/2))
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}\,x^2}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx$$

input
```
int(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*sqrt(a + b*x**3)*x**2)/(a*c + a*d*x**3 + b*c*x**3 +
b*d*x**6),x)
```

3.874 $\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 48

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

output
```
-2/3*arctanh(c^(1/2)*(b*x^3+a)^(1/2)/a^(1/2)/(d*x^3+c)^(1/2))/a^(1/2)/c^(1
/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c+dx^3}}{\sqrt{c}\sqrt{a+bx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

input
```
Integrate[1/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^3])/(Sqrt[c]*Sqrt[a + b*x^3])])/(3*Sqrt[
a]*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {948, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{1}{x^3\sqrt{bx^3+a}\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \; 104$$

$$\frac{2}{3}\int \frac{1}{cx^6-a}d\frac{\sqrt{bx^3+a}}{\sqrt{dx^3+c}}$$

$$\Big\downarrow \; 221$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

input
```
Int[1/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*Sqrt[
a]*Sqrt[c])
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{1}{x\sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}}dx$$

input
```
int(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(34) = 68.

Time = 0.11 (sec) , antiderivative size = 204, normalized size of antiderivative = 4.25

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \left[\frac{\sqrt{ac}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^6+8\,a^2c^2+8\,(abc^2+a^2cd)x^3-4\,((bc+ad)x^3+2\,ac)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{ac}}{x^6}\right)}{6\,ac}, \frac{\sqrt{-ac}\arctan\left(\frac{((bc+ad}{2\,(ab}\right.}{}\right.$$

input
```
integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*sqrt(a*c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 + 8*a^2*c^2 + 8*(a
*b*c^2 + a^2*c*d)*x^3 - 4*((b*c + a*d)*x^3 + 2*a*c)*sqrt(b*x^3 + a)*sqrt(d
*x^3 + c)*sqrt(a*c))/x^6)/(a*c), 1/3*sqrt(-a*c)*arctan(1/2*((b*c + a*d)*x^
3 + 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-a*c)/(a*b*c*d*x^6 + a^2*c
^2 + (a*b*c^2 + a^2*c*d)*x^3))/(a*c)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(34) = 68$.

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.85

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{2\sqrt{bd}b\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{3\sqrt{-abcd}|b|}$$

input
```
integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(b*d)*b*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^3 + a)*sqrt(b*d) -
 sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*
c*d)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 6.58 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.83

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{\ln\left(\frac{\sqrt{b\,x^3+a}-\sqrt{a}}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)-\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^3+a}-\sqrt{a}\,\sqrt{d\,x^3+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\,\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)}{3\,\sqrt{a}\,\sqrt{c}}$$

input
```
int(1/(x*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output
```
-(log(((a + b*x^3)^(1/2) - a^(1/2))/((c + d*x^3)^(1/2) - c^(1/2))) - log((
(c^(1/2)*(a + b*x^3)^(1/2) - a^(1/2)*(c + d*x^3)^(1/2))*(b*c^(1/2) - (a^(1
/2)*d*((a + b*x^3)^(1/2) - a^(1/2)))/((c + d*x^3)^(1/2) - c^(1/2))))/((c +
 d*x^3)^(1/2) - c^(1/2))))/(3*a^(1/2)*c^(1/2))
```

Reduce [F]

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}}{bd\,x^7+ad\,x^4+bc\,x^4+acx}dx$$

input
```
int(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*sqrt(a + b*x**3))/(a*c*x + a*d*x**4 + b*c*x**4 + b*d
*x**7),x)
```

3.875 $\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3}+\frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}$$

output
```
-1/3*(b*x^3+a)^(1/2)*(d*x^3+c)^(1/2)/a/c/x^3+1/3*(a*d+b*c)*arctanh(c^(1/2)
*(b*x^3+a)^(1/2)/a^(1/2)/(d*x^3+c)^(1/2))/a^(3/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3}+\frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
-1/3*(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(a*c*x^3) + ((b*c + a*d)*ArcTanh[(S
qrt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*a^(3/2)*c^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {948, 107, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^6\sqrt{bx^3+a}\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{107}$$

$$\frac{1}{3}\left(-\frac{(ad+bc)\int \frac{1}{x^3\sqrt{bx^3+a}\sqrt{dx^3+c}}dx^3}{2ac}-\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{acx^3}\right)$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{3}\left(-\frac{(ad+bc)\int \frac{1}{cx^6-a}d\frac{\sqrt{bx^3+a}}{\sqrt{dx^3+c}}}{ac}-\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{acx^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{(ad+bc)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{a^{3/2}c^{3/2}}-\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{acx^3}\right)$$

input
```
Int[1/(x^4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(-((Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(a*c*x^3)) + ((b*c + a*d)*ArcTanh[(Sq
rt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(a^(3/2)*c^(3/2)))/3
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{1}{x^{4}\sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}}dx$$

input
```
int(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.05

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \left[\frac{\sqrt{ac}(bc+ad)x^3\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^6+8\,a^2c^2+8\,(abc^2+a^2cd)x^3+4\,((bc+ad)x^3+2\,ac)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{ac}}{x^6}\right)-4\sqrt{bx^3}}{12\,a^2c^2x^3}\right.$$

$$\left.-\frac{\sqrt{-ac}(bc+ad)x^3\arctan\left(\frac{((bc+ad)x^3+2\,ac)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{-ac}}{2\,(abcdx^6+a^2c^2+(abc^2+a^2cd)x^3)}\right)+2\sqrt{bx^3+a}\sqrt{dx^3+c}ac}{6\,a^2c^2x^3}\right]$$

input
```
integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(sqrt(a*c)*(b*c + a*d)*x^3*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6
+ 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^3 + 4*((b*c + a*d)*x^3 + 2*a*c)*sqrt
(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(a*c))/x^6) - 4*sqrt(b*x^3 + a)*sqrt(d*x^3
 + c)*a*c)/(a^2*c^2*x^3), -1/6*(sqrt(-a*c)*(b*c + a*d)*x^3*arctan(1/2*((b*
c + a*d)*x^3 + 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-a*c)/(a*b*c*d*
x^6 + a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^3)) + 2*sqrt(b*x^3 + a)*sqrt(d*x^3 +
 c)*a*c)/(a^2*c^2*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**4/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(71) = 142$.

Time = 0.14 (sec) , antiderivative size = 413, normalized size of antiderivative = 4.54

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\sqrt{bd}b^4d\left(\frac{(bc+ad)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^3+a\right)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}ab^3cd} - \frac{2\left(b^3c^2-2ab^2cd+a^2bd^2-\left(\cdots\right.\right.}{\left(b^4c^2-2ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3\cdots}\right.\right.}\cdots\right.}{3|b|\cdots}$$

input
```
integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(b*d)*b^4*d*((b*c + a*d)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^3
+ a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)
*b))/(sqrt(-a*b*c*d)*a*b^3*c*d) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2 - (
sqrt(b*x^3 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*b*c -
 (sqrt(b*x^3 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*a*d
)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^3 + a)*sqrt(b*d) - s
qrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^3 + a)*sqrt(b*
d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^3 + a)*sqr
t(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^4)*a*b^2*c*d))/abs(b)
```

Mupad [B] (verification not implemented)

Time = 10.00 (sec) , antiderivative size = 481, normalized size of antiderivative = 5.29

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\,\left(\frac{c\,b^2}{12}+\frac{a\,d\,b}{12}\right)}{a^{3/2}\,c^{3/2}\,d\,\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}-\frac{b^2}{12\,a\,c\,d}+\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^2\,\left(\frac{a^2\,d^2}{12}-\frac{a\,b\,c\,d}{4}+\frac{b^2\,c^2}{12}\right)}{a^2\,c^2\,d\,{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}^2}}{\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3}{{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}^3}+\frac{b\,\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{d\,\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}-\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^2\,\left(a\,d+b\,c\right)}{\sqrt{a}\,\sqrt{c}\,d\,{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}^2}} + \frac{\ln\left(\frac{\sqrt{b\,x^3+a}-\sqrt{a}}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)\,\left(\sqrt{a}\,b\,c^{3/2}+a^{3/2}\,\sqrt{c}\,d\right)}{6\,a^2\,c^2} - \frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^3+a}-\sqrt{a}\,\sqrt{d\,x^3+c}\right)\,\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\,\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)\,\left(\sqrt{a}\,b\,c^{3/2}+a^{3/2}\,\sqrt{c}\,d\right)}{6\,a^2\,c^2} - \frac{d\,\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{12\,a\,c\,\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}$$

input
```
int(1/(x^4*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output
```
((((a + b*x^3)^(1/2) - a^(1/2))*((b^2*c)/12 + (a*b*d)/12))/(a^(3/2)*c^(3/2
)*d*((c + d*x^3)^(1/2) - c^(1/2))) - b^2/(12*a*c*d) + (((a + b*x^3)^(1/2)
- a^(1/2))^2*((a^2*d^2)/12 + (b^2*c^2)/12 - (a*b*c*d)/4))/(a^2*c^2*d*((c +
 d*x^3)^(1/2) - c^(1/2))^2))/(((a + b*x^3)^(1/2) - a^(1/2))^3/((c + d*x^3)
^(1/2) - c^(1/2))^3 + (b*((a + b*x^3)^(1/2) - a^(1/2)))/(d*((c + d*x^3)^(1
/2) - c^(1/2))) - (((a + b*x^3)^(1/2) - a^(1/2))^2*(a*d + b*c))/(a^(1/2)*c
^(1/2)*d*((c + d*x^3)^(1/2) - c^(1/2))^2)) + (log(((a + b*x^3)^(1/2) - a^(
1/2))/((c + d*x^3)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2)*c^(1/2)*
d))/(6*a^2*c^2) - (log(((c^(1/2)*(a + b*x^3)^(1/2) - a^(1/2)*(c + d*x^3)^(
1/2))*(b*c^(1/2) - (a^(1/2)*d*((a + b*x^3)^(1/2) - a^(1/2)))/((c + d*x^3)^
(1/2) - c^(1/2))))/((c + d*x^3)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(
3/2)*c^(1/2)*d))/(6*a^2*c^2) - (d*((a + b*x^3)^(1/2) - a^(1/2)))/(12*a*c*(
(c + d*x^3)^(1/2) - c^(1/2)))
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{-2\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}-3\left(\int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}}{bd\,x^7+ad\,x^4+bc\,x^4+acx}dx\right)ad\,x^3-3\left(\int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}}{bd\,x^7+ad\,x^4+bc\,x^4+acx}dx\right)bc\,x^3}{6ac\,x^3}$$

input
```
int(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**3)*sqrt(a + b*x**3) - 3*int((sqrt(c + d*x**3)*sqrt(a +
 b*x**3))/(a*c*x + a*d*x**4 + b*c*x**4 + b*d*x**7),x)*a*d*x**3 - 3*int((sq
rt(c + d*x**3)*sqrt(a + b*x**3))/(a*c*x + a*d*x**4 + b*c*x**4 + b*d*x**7),
x)*b*c*x**3)/(6*a*c*x**3)
```

3.876 $\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\frac{x^5\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},\frac{1}{2},\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

output
```
1/5*x^5*(1+b*x^3/a)^(1/2)*(1+d*x^3/c)^(1/2)*AppellF1(5/3,1/2,1/2,8/3,-b*x^
3/a,-d*x^3/c)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [A] (verified)

Time = 2.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\frac{x^5\sqrt{\frac{a+bx^3}{a}}\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},\frac{1}{2},\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Integrate[x^4/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(x^5*Sqrt[(a + b*x^3)/a]*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1/2, 8/3,
-((b*x^3)/a), -((d*x^3)/c)])/(5*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \ 1013$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{x^4}{\sqrt{\frac{bx^3}{a}+1}\sqrt{dx^3+c}}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 1013$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\int \frac{x^4}{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$\Big\downarrow \ 1012$$

$$\frac{x^5\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},\frac{1}{2},\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Int[x^4/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(x^5*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1/2, 8/3,
-((b*x^3)/a), -((d*x^3)/c)])/(5*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}dx$$

input
```
int(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^4/(b*d*x^6 + (b*c + a*d)*x^3 +
a*c), x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**4/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**4/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(x^4/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output
```
int(x^4/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}\,x^4}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx$$

input
```
int(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*sqrt(a + b*x**3)*x**4)/(a*c + a*d*x**3 + b*c*x**3 +
b*d*x**6),x)
```

3.877 $\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

output
```
1/4*x^4*(1+b*x^3/a)^(1/2)*(1+d*x^3/c)^(1/2)*AppellF1(4/3,1/2,1/2,7/3,-b*x^
3/a,-d*x^3/c)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{\frac{a+bx^3}{a}}\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Integrate[x^3/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(x^4*Sqrt[(a + b*x^3)/a]*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/3,
-((b*x^3)/a), -((d*x^3)/c)])/(4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{x^3}{\sqrt{\frac{bx^3}{a}+1}\sqrt{dx^3+c}}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\int \frac{x^3}{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Int[x^3/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(x^4*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/3,
-((b*x^3)/a), -((d*x^3)/c)])/(4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^3}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}dx$$

input
```
int(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{x^3}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^3/(b*d*x^6 + (b*c + a*d)*x^3 +
a*c), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**3/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(x^3/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output
```
int(x^3/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}\,x^3}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx$$

input
```
int(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*sqrt(a + b*x**3)*x**3)/(a*c + a*d*x**3 + b*c*x**3 +
b*d*x**6),x)
```

3.878 $\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

output
```
1/2*x^2*(1+b*x^3/a)^(1/2)*(1+d*x^3/c)^(1/2)*AppellF1(2/3,1/2,1/2,5/3,-b*x^
3/a,-d*x^3/c)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{\frac{a+bx^3}{a}}\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Integrate[x/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(x^2*Sqrt[(a + b*x^3)/a]*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3,
-((b*x^3)/a), -((d*x^3)/c)])/(2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{x}{\sqrt{\frac{bx^3}{a}+1}\sqrt{dx^3+c}}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\int \frac{x}{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Int[x/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(x^2*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3,
-((b*x^3)/a), -((d*x^3)/c)])/(2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x}{\sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}}dx$$

input
```
int(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{x}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x/(b*d*x^6 + (b*c + a*d)*x^3 + a*
c), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(x/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output
```
int(x/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}\,x}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx$$

input
```
int(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*sqrt(a + b*x**3)*x)/(a*c + a*d*x**3 + b*c*x**3 + b*d
*x**6),x)
```

3.879 $\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\frac{x\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

output
```
x*(1+b*x^3/a)^(1/2)*(1+d*x^3/c)^(1/2)*AppellF1(1/3,1/2,1/2,4/3,-b*x^3/a,-d
*x^3/c)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 170 vs. 2(83) = 166.

Time = 0.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx =$$

$$-\frac{8acx\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(ad\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right.\right.\right.}$$

input
```
Integrate[1/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(-8*a*c*x*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(Sqrt[
a + b*x^3]*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/
a), -((d*x^3)/c)] + 3*x^3*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a),
-((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)])))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{1}{\sqrt{\frac{bx^3}{a}+1}\sqrt{dx^3+c}}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Int[1/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1/2, 1/2, 4/3, -(
(b*x^3)/a), -((d*x^3)/c)])/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}}dx$$

input
```
int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^6 + (b*c + a*d)*x^3 + a*c)
, x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(1/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output
```
int(1/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx$$

input
```
int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*sqrt(a + b*x**3))/(a*c + a*d*x**3 + b*c*x**3 + b*d*x
**6),x)
```

3.880 $\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(-\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

output
```
-(1+b*x^3/a)^(1/2)*(1+d*x^3/c)^(1/2)*AppellF1(-1/3,1/2,1/2,2/3,-b*x^3/a,-d
*x^3/c)/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 189 vs. 2(86) = 172.

Time = 2.44 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$
$$=\frac{-20(a+bx^3)\left(c+dx^3\right)+5(bc+ad)x^3\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+8bdx^6\sqrt{1}}{20acx\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(-20*(a + b*x^3)*(c + d*x^3) + 5*(b*c + a*d)*x^3*Sqrt[1 + (b*x^3)/a]*Sqrt[
1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c)] +
8*b*d*x^6*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1/2,
8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*a*c*x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3
])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{1}{x^2\sqrt{\frac{bx^3}{a}+1}\sqrt{dx^3+c}}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^2\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Int[1/(x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
-((Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1/2, 1/2, 2/3, -
((b*x^3)/a), -((d*x^3)/c)])/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2}\sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}}dx$$

input
```
int(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^8 + (b*c + a*d)*x^5 + a*c*
x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**2/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{x^2\,\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(1/(x^2*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}}{bd\,x^8+ad\,x^5+bc\,x^5+ac\,x^2}dx$$

input
```
int(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*sqrt(a + b*x**3))/(a*c*x**2 + a*d*x**5 + b*c*x**5 +
b*d*x**8),x)
```

3.881 $\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

output
```
-1/2*(1+b*x^3/a)^(1/2)*(1+d*x^3/c)^(1/2)*AppellF1(-2/3,1/2,1/2,1/3,-b*x^3/
a,-d*x^3/c)/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 365 vs. 2(88) = 176.

Time = 2.55 (sec) , antiderivative size = 365, normalized size of antiderivative = 4.15

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$
$$= \frac{bdx^6\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4\left(-4ac(2ac+3bcx^3+3adx^3+2bdx^6)\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\right.\right.}{8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\right.}}{8acx^2\sqrt{a+bx^3}\sqrt{c+d}}$$

input
```
Integrate[1/(x^3*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(b*d*x^6*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7
/3, -((b*x^3)/a), -((d*x^3)/c)] + (4*(-4*a*c*(2*a*c + 3*b*c*x^3 + 3*a*d*x^
3 + 2*b*d*x^6)*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)] +
3*x^3*(a + b*x^3)*(c + d*x^3)*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/
a), -((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])))/(8*a*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)]
- 3*x^3*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*
c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/(8*a*c*x^2*S
qrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{1}{x^3\sqrt{\frac{bx^3}{a}+1}\sqrt{dx^3+c}}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^3\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\text{AppellF1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Int[1/(x^3*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
-1/2*(Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1/2, 1/2, 1/3
, -((b*x^3)/a), -((d*x^3)/c)])/(x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{3}\sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}}dx$$

input
```
int(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^9 + (b*c + a*d)*x^6 + a*c*
x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**3/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{x^3\,\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(1/(x^3*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}}{bd\,x^9+ad\,x^6+bc\,x^6+ac\,x^3}dx$$

input
```
int(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*sqrt(a + b*x**3))/(a*c*x**3 + a*d*x**6 + b*c*x**6 +
b*d*x**9),x)
```

3.882 $\int \frac{x^3}{\sqrt{a-bx^3}\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 551

$$\int \frac{x^3}{\sqrt{a-bx^3}\sqrt{a+bx^3}}\,dx = \frac{a^2-b^2x^6}{b^{4/3}\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x^2\right)\sqrt{a-bx^3}\sqrt{a+bx^3}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{2/3}\left(a^{2/3}-b^{2/3}x^2\right)\sqrt{\frac{a^{4/3}+a^{2/3}b^{2/3}x^2+b^{4/3}x^4}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x^2\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x^2}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x^2}\right)|-7-4\sqrt{3}\right)}{2b^{4/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x^2\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x^2\right)^2}}\sqrt{a-bx^3}\sqrt{a+bx^3}} + \frac{\sqrt{2}a^{2/3}\left(a^{2/3}-b^{2/3}x^2\right)\sqrt{\frac{a^{4/3}+a^{2/3}b^{2/3}x^2+b^{4/3}x^4}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x^2\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x^2}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x^2}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{4/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x^2\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x^2\right)^2}}\sqrt{a-bx^3}\sqrt{a+bx^3}}$$

output
```
(-b^2*x^6+a^2)/b^(4/3)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x^2)/(-b*x^3+a)^(1/2)/
(b*x^3+a)^(1/2)-1/2*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(2/3)*(a^(2/3)-b^(
2/3)*x^2)*((a^(4/3)+a^(2/3)*b^(2/3)*x^2+b^(4/3)*x^4)/((1+3^(1/2))*a^(2/3)-
b^(2/3)*x^2)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(2/3)-b^(2/3)*x^2)/((1+3^(1
/2))*a^(2/3)-b^(2/3)*x^2),I*3^(1/2)+2*I)/b^(4/3)/(a^(2/3)*(a^(2/3)-b^(2/3)
*x^2)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x^2)^2)^(1/2)/(-b*x^3+a)^(1/2)/(b*x^3+a
)^(1/2)+1/3*2^(1/2)*a^(2/3)*(a^(2/3)-b^(2/3)*x^2)*((a^(4/3)+a^(2/3)*b^(2/3
)*x^2+b^(4/3)*x^4)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x^2)^2)^(1/2)*EllipticF(((
1-3^(1/2))*a^(2/3)-b^(2/3)*x^2)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x^2),I*3^(1/2
)+2*I)*3^(3/4)/b^(4/3)/(a^(2/3)*(a^(2/3)-b^(2/3)*x^2)/((1+3^(1/2))*a^(2/3)
-b^(2/3)*x^2)^2)^(1/2)/(-b*x^3+a)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.13

$$\int \frac{x^3}{\sqrt{a-bx^3}\sqrt{a+bx^3}}\,dx = \frac{x^4\sqrt{1-\frac{b^2x^6}{a^2}}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{b^2x^6}{a^2}\right)}{4\sqrt{a-bx^3}\sqrt{a+bx^3}}$$

input
```
Integrate[x^3/(Sqrt[a - b*x^3]*Sqrt[a + b*x^3]),x]
```

output
```
(x^4*Sqrt[1 - (b^2*x^6)/a^2]*HypergeometricPFQ[{1/2, 2/3}, {5/3}, (b^2*x^6
)/a^2])/(4*Sqrt[a - b*x^3]*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.01 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {808, 890, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a-bx^3}\sqrt{a+bx^3}}\,dx$$

$\big\downarrow$ 808

$$\frac{1}{2}\int \frac{x^2}{\sqrt{a-b\left(x^2\right)^{3/2}}\sqrt{b\left(x^2\right)^{3/2}+a}}dx^2$$

$\big\downarrow$ 890

$$\frac{\sqrt{a^2-b^2x^6}\int \frac{x^2}{\sqrt{a^2-b^2x^6}}dx^2}{2\sqrt{a-b\left(x^2\right)^{3/2}}\sqrt{a+b\left(x^2\right)^{3/2}}}$$

$\big\downarrow$ 832

$$\frac{\sqrt{a^2-b^2x^6}\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{a^2-b^2x^6}}dx^2}{b^{2/3}}-\frac{\int \frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x^2}{\sqrt{a^2-b^2x^6}}dx^2}{b^{2/3}}\right)}{2\sqrt{a-b\left(x^2\right)^{3/2}}\sqrt{a+b\left(x^2\right)^{3/2}}}$$

$\big\downarrow$ 759

$$\frac{\sqrt{a^2-b^2x^6}\left(-\frac{\int \frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x^2}{\sqrt{a^2-b^2x^6}}dx^2}{b^{2/3}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}a^{2/3}\left(a^{2/3}-b^{2/3}x^2\right)\sqrt{\frac{a^{2/3}b^{2/3}x^2+a^{4/3}+b^{4/3}x^4}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x^2\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\cdots}{\left(1+\sqrt{3}\right)\cdots}\right.\right.}{\sqrt[4]{3}b^{4/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x^2\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x^2\right)^2}}\sqrt{a^2-b^2x^6}}\cdots\right.}{2\sqrt{a-b\left(x^2\right)^{3/2}}\sqrt{a+b\left(x^2\right)^{3/2}}}$$

$\big\downarrow$ 2416

$$\frac{\sqrt{a^2-b^2x^6}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}a^{2/3}\left(a^{2/3}-b^{2/3}x^2\right)\sqrt{\frac{a^{2/3}b^{2/3}x^2+a^{4/3}+b^{4/3}x^4}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x^2\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x^2}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x^2}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{4/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x^2\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x^2\right)^2}}\sqrt{a^2-b^2x^6}}-\cdots\right.}{2\sqrt{a-b\left(x^2\right)^{3/2}}\sqrt{a\cdots}}$$

input
```
Int[x^3/(Sqrt[a - b*x^3]*Sqrt[a + b*x^3]),x]
```

output
```
(Sqrt[a^2 - b^2*x^6]*(-(((-2*Sqrt[a^2 - b^2*x^6])/(b^(2/3)*((1 + Sqrt[3])*
a^(2/3) - b^(2/3)*x^2)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(2/3)*(a^(2/3) - b^
(2/3)*x^2)*Sqrt[(a^(4/3) + a^(2/3)*b^(2/3)*x^2 + b^(4/3)*x^4)/((1 + Sqrt[3
])*a^(2/3) - b^(2/3)*x^2)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(2/3) - b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x^2)], -7 - 4*Sqrt[3]])/(b^(2/3
)*Sqrt[(a^(2/3)*(a^(2/3) - b^(2/3)*x^2))/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*
x^2)^2]*Sqrt[a^2 - b^2*x^6]))/b^(2/3)) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]
]*a^(2/3)*(a^(2/3) - b^(2/3)*x^2)*Sqrt[(a^(4/3) + a^(2/3)*b^(2/3)*x^2 + b^
(4/3)*x^4)/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x^2)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*a^(2/3) - b^(2/3)*x^2)/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x^2)], -
7 - 4*Sqrt[3]])/(3^(1/4)*b^(4/3)*Sqrt[(a^(2/3)*(a^(2/3) - b^(2/3)*x^2))/((
1 + Sqrt[3])*a^(2/3) - b^(2/3)*x^2)^2]*Sqrt[a^2 - b^2*x^6])))/(2*Sqrt[a -
b*(x^2)^(3/2)]*Sqrt[a + b*(x^2)^(3/2)])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 808
```
Int[(x_)^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(
p_), x_Symbol] :> With[{k = GCD[m + 1, 2*n]}, Simp[1/k   Subst[Int[x^((m +
1)/k - 1)*(a1 + b1*x^(n/k))^p*(a2 + b2*x^(n/k))^p, x], x, x^k], x] /; k !=
1] /; FreeQ[{a1, b1, a2, b2, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0
] && IntegerQ[m]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 890

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^Fra
cPart[p]/(a1*a2 + b1*b2*x^(2*n))^FracPart[p])   Int[(c*x)^m*(a1*a2 + b1*b2*
x^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
 a1*b2, 0] &&  !IntegerQ[p]
```

rule 2416

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x^3}{\sqrt{-b\,x^3+a}\,\sqrt{b\,x^3+a}}dx$$

input

```
int(x^3/(-b*x^3+a)^(1/2)/(b*x^3+a)^(1/2),x)
```

output

```
int(x^3/(-b*x^3+a)^(1/2)/(b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.06

$$\int \frac{x^3}{\sqrt{a-bx^3}\sqrt{a+bx^3}}\,dx = \frac{\sqrt{-b^2}\text{weierstrassZeta}\left(0,\frac{4\,a^2}{b^2},\text{weierstrassPInverse}\left(0,\frac{4\,a^2}{b^2},x^2\right)\right)}{b^2}$$

input

```
integrate(x^3/(-b*x^3+a)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(-b^2)*weierstrassZeta(0, 4*a^2/b^2, weierstrassPInverse(0, 4*a^2/b^2,
 x^2))/b^2
```

Sympy [A] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.22

$$\int \frac{x^3}{\sqrt{a-bx^3}\sqrt{a+bx^3}}\,dx = \frac{i\sqrt[3]{a}G_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{12}, \frac{7}{12}, 1 & \frac{1}{3}, \frac{1}{3}, \frac{5}{6} \\ -\frac{1}{6}, \frac{1}{12}, \frac{1}{3}, \frac{7}{12}, \frac{5}{6} & 0 \end{matrix}\middle|\frac{a^2}{b^2x^6}\right)}{12\pi^{\frac{3}{2}}b^{\frac{4}{3}}} + \frac{\sqrt[3]{a}G_{6,6}^{2,6}\left(\begin{matrix} -\frac{2}{3}, -\frac{5}{12}, -\frac{1}{6}, \frac{1}{12}, \frac{1}{3}, 1 & \\ -\frac{5}{12}, \frac{1}{12} & -\frac{2}{3}, -\frac{1}{6}, -\frac{1}{6}, 0 \end{matrix}\middle|\frac{a^2e^{-2i\pi}}{b^2x^6}\right)e^{-\frac{i\pi}{3}}}{12\pi^{\frac{3}{2}}b^{\frac{4}{3}}}$$

input
```
integrate(x**3/(-b*x**3+a)**(1/2)/(b*x**3+a)**(1/2),x)
```

output
```
I*a**(1/3)*meijerg(((1/12, 7/12, 1), (1/3, 1/3, 5/6)), ((-1/6, 1/12, 1/3,
7/12, 5/6), (0,)), a**2/(b**2*x**6))/(12*pi**(3/2)*b**(4/3)) + a**(1/3)*me
ijerg(((-2/3, -5/12, -1/6, 1/12, 1/3, 1), ()), ((-5/12, 1/12), (-2/3, -1/6
, -1/6, 0)), a**2*exp_polar(-2*I*pi)/(b**2*x**6))*exp(-I*pi/3)/(12*pi**(3/
2)*b**(4/3))
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a-bx^3}\sqrt{a+bx^3}}\,dx = \int \frac{x^3}{\sqrt{bx^3+a}\sqrt{-bx^3+a}}\,dx$$

input
```
integrate(x^3/(-b*x^3+a)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(b*x^3 + a)*sqrt(-b*x^3 + a)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{a-bx^3}\sqrt{a+bx^3}}\,dx=\int \frac{x^3}{\sqrt{bx^3+a}\sqrt{-bx^3+a}}\,dx$$

input
```
integrate(x^3/(-b*x^3+a)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/(sqrt(b*x^3 + a)*sqrt(-b*x^3 + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a-bx^3}\sqrt{a+bx^3}}\,dx=\int \frac{x^3}{\sqrt{b\,x^3+a}\,\sqrt{a-b\,x^3}}\,dx$$

input
```
int(x^3/((a + b*x^3)^(1/2)*(a - b*x^3)^(1/2)),x)
```

output
```
int(x^3/((a + b*x^3)^(1/2)*(a - b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{a-bx^3}\sqrt{a+bx^3}}\,dx=\int \frac{\sqrt{b\,x^3+a}\,\sqrt{-b\,x^3+a}\,x^3}{-b^2x^6+a^2}dx$$

input
```
int(x^3/(-b*x^3+a)^(1/2)/(b*x^3+a)^(1/2),x)
```

output
```
int((sqrt(a + b*x**3)*sqrt(a - b*x**3)*x**3)/(a**2 - b**2*x**6),x)
```

3.883 $\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx = \frac{b(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a(bc-ad)e(1+m)} - \frac{d(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{dx^3}{c}\right)}{c(bc-ad)e(1+m)}$$

output
```
b*(e*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a/(-a*d+b*c)/
e/(1+m)-d*(e*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-d*x^3/c)/c/(-a
*d+b*c)/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx = \frac{x(ex)^m\left(-bc\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+ad\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{dx^3}{c}\right)\right)}{ac(-bc+ad)(1+m)}$$

input
```
Integrate[(e*x)^m/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(x*(e*x)^m*(-(b*c*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)]
) + a*d*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((d*x^3)/c)]))/(a*c*(-
(b*c) + a*d)*(1 + m))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {982, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}dx$$

$$\Big\downarrow \text{ 982}$$

$$\frac{b\int \frac{(ex)^m}{bx^3+a}dx}{bc-ad}-\frac{d\int \frac{(ex)^m}{dx^3+c}dx}{bc-ad}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{b(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{ae(m+1)(bc-ad)}-\frac{d(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{dx^3}{c}\right)}{ce(m+1)(bc-ad)}$$

input
```
Int[(e*x)^m/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(b*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])
/(a*(b*c - a*d)*e*(1 + m)) - (d*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)
/3, (4 + m)/3, -((d*x^3)/c)])/(c*(b*c - a*d)*e*(1 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{(ex)^m}{(b\,x^3+a)\,(d\,x^3+c)}dx$$

input
```
int((e*x)^m/(b*x^3+a)/(d*x^3+c),x)
```

output
```
int((e*x)^m/(b*x^3+a)/(d*x^3+c),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^3)\,(c+dx^3)}\,dx=\int \frac{(ex)^m}{(bx^3+a)(dx^3+c)}\,dx$$

input
```
integrate((e*x)^m/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((e*x)^m/(b*d*x^6 + (b*c + a*d)*x^3 + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx = \int \frac{(ex)^m}{(bx^3+a)(dx^3+c)}\,dx$$

input
```
integrate((e*x)^m/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/((b*x^3 + a)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx = \int \frac{(ex)^m}{(bx^3+a)(dx^3+c)}\,dx$$

input
```
integrate((e*x)^m/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((e*x)^m/((b*x^3 + a)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx=\int \frac{(e\,x)^m}{(b\,x^3+a)\,(d\,x^3+c)}\,dx$$

input
```
int((e*x)^m/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
int((e*x)^m/((a + b*x^3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx=e^m\left(\int \frac{x^m}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)$$

input
```
int((e*x)^m/(b*x^3+a)/(d*x^3+c),x)
```

output
```
e**m*int(x**m/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)
```

3.884 $\int x^{2-3p}(a+bx^3)^p (c+dx^3)^2 \, dx$

Optimal result

Integrand size = 26, antiderivative size = 176

$$\int x^{2-3p}(a+bx^3)^p (c+dx^3)^2 \, dx = \frac{c^2x^{3-3p}(a+bx^3)^{1+p}}{3a(1-p)} + \frac{d^2x^{6-3p}(a+bx^3)^{1+p}}{9b} - \frac{(6b^2c^2-6abcd(1-p)+a^2d^2(2-3p+p^2))\, x^{6-3p}(a+bx^3)^p \left(1+\frac{bx^3}{a}\right)^{-p} \text{Hypergeometric2F1}\left(2-p, \ldots\right.}{9ab(1-p)(2-p)}$$

output
```
1/3*c^2*x^(3-3*p)*(b*x^3+a)^(p+1)/a/(1-p)+1/9*d^2*x^(6-3*p)*(b*x^3+a)^(p+1
)/b-1/9*(6*b^2*c^2-6*a*b*c*d*(1-p)+a^2*d^2*(p^2-3*p+2))*x^(6-3*p)*(b*x^3+a
)^p*hypergeom([-p, 2-p],[3-p],-b*x^3/a)/a/b/(1-p)/(2-p)/((1+b*x^3/a)^p)
```

Mathematica [A] (verified)

Time = 5.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.88

$$\int x^{2-3p}(a+bx^3)^p (c+dx^3)^2 \, dx = -\frac{x^{3-3p}(a+bx^3)^p \left(1+\frac{bx^3}{a}\right)^{-p} \left(c^2(6-5p+p^2)\, \text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^3}{a}\right) + d(-1+\ldots\right.}{3\ldots}$$

input
```
Integrate[x^(2 - 3*p)*(a + b*x^3)^p*(c + d*x^3)^2,x]
```

output
```
-1/3*(x^(3 - 3*p)*(a + b*x^3)^p*(c^2*(6 - 5*p + p^2)*Hypergeometric2F1[1 -
 p, -p, 2 - p, -((b*x^3)/a)] + d*(-1 + p)*x^3*(2*c*(-3 + p)*Hypergeometric
2F1[2 - p, -p, 3 - p, -((b*x^3)/a)] + d*(-2 + p)*x^3*Hypergeometric2F1[3 -
 p, -p, 4 - p, -((b*x^3)/a)])))/((-3 + p)*(-2 + p)*(-1 + p)*(1 + (b*x^3)/a
)^p)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {964, 27, 959, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2-3p} \left(c+dx^3\right)^2 \left(a+bx^3\right)^p \, dx$$

$$\Big\downarrow \text{964}$$

$$\frac{\int 3x^{2-3p} \left(bx^3+a\right)^p \left(d(6bc-ad(2-p))x^3+3bc^2\right) dx}{9b}+\frac{d^2 x^{6-3p} \left(a+bx^3\right)^{p+1}}{9b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int x^{2-3p} \left(bx^3+a\right)^p \left(d(6bc-ad(2-p))x^3+3bc^2\right) dx}{3b}+\frac{d^2 x^{6-3p} \left(a+bx^3\right)^{p+1}}{9b}$$

$$\Big\downarrow \text{959}$$

$$\frac{\frac{\left(a^2d^2\left(p^2-3p+2\right)-6abcd(1-p)+6b^2c^2\right)\int x^{2-3p}\left(bx^3+a\right)^p dx}{2b}+\frac{dx^{3-3p}\left(a+bx^3\right)^{p+1}(6bc-ad(2-p))}{6b}}{3b}+\frac{d^2 x^{6-3p} \left(a+bx^3\right)^{p+1}}{9b}$$

$$\Big\downarrow \text{882}$$

$$\frac{\frac{ax^{-3p}\left(\frac{x^3}{a+bx^3}\right)^p\left(a+bx^3\right)^p\left(a^2d^2\left(p^2-3p+2\right)-6abcd(1-p)+6b^2c^2\right)\int \frac{\left(\frac{x^3}{bx^3+a}\right)^{-p}}{\left(1-\frac{bx^3}{bx^3+a}\right)^2}d\frac{x^3}{bx^3+a}}{6b}+\frac{dx^{3-3p}\left(a+bx^3\right)^{p+1}(6bc-ad(2-p))}{6b}}{3b}+\frac{d^2 x^{6-3p} \left(a+bx^3\right)^{p+1}}{9b}$$

$$\downarrow 74$$

$$\frac{\frac{ax^{3-3p}(a+bx^3)^{p-1}(a^2d^2(p^2-3p+2)-6abcd(1-p)+6b^2c^2)\text{ Hypergeometric2F1}\left(2,1-p,2-p,\frac{bx^3}{bx^3+a}\right)}{6b(1-p)}+\frac{dx^{3-3p}(a+bx^3)^{p+1}(6bc-ad(2-p))}{6b}}{3b}+\frac{d^2x^{6-3p}(a+bx^3)^{p+1}}{9b}$$

input
```
Int[x^(2 - 3*p)*(a + b*x^3)^p*(c + d*x^3)^2,x]
```

output
```
(d^2*x^(6 - 3*p)*(a + b*x^3)^(1 + p))/(9*b) + ((d*(6*b*c - a*d*(2 - p))*x^
(3 - 3*p)*(a + b*x^3)^(1 + p))/(6*b) + (a*(6*b^2*c^2 - 6*a*b*c*d*(1 - p) +
 a^2*d^2*(2 - 3*p + p^2))*x^(3 - 3*p)*(a + b*x^3)^(-1 + p)*Hypergeometric2
F1[2, 1 - p, 2 - p, (b*x^3)/(a + b*x^3)])/(6*b*(1 - p)))/(3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 964
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Simp[1/(b*(m + n*(p + 2) + 1))   Int[(e*x)
^m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) - d*(a*d*(m + n + 1) - 2*b*
c*(m + n*(p + 2) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Maple [F]

$$\int x^{2-3p} \left(b\,x^{3}+a\right)^{p} \left(d\,x^{3}+c\right)^{2}d x$$

input
```
int(x^(2-3*p)*(b*x^3+a)^p*(d*x^3+c)^2,x)
```

output
```
int(x^(2-3*p)*(b*x^3+a)^p*(d*x^3+c)^2,x)
```

Fricas [F]

$$\int x^{2-3p}\left(a+bx^3\right)^p\left(c+dx^3\right)^2\,dx=\int \left(dx^{3}+c\right)^{2}\left(bx^{3}+a\right)^{p}x^{-3\,p+2}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^p*x^(-3*p + 2), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{2-3p}\left(a+bx^3\right)^p\left(c+dx^3\right)^2\,dx=\text{Timed out}$$

input
```
integrate(x**(2-3*p)*(b*x**3+a)**p*(d*x**3+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{2-3p}\left(a+bx^3\right)^p\left(c+dx^3\right)^2\,dx=\int\left(dx^3+c\right)^2\left(bx^3+a\right)^p x^{-3\,p+2}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^2*(b*x^3 + a)^p*x^(-3*p + 2), x)
```

Giac [F]

$$\int x^{2-3p}\left(a+bx^3\right)^p\left(c+dx^3\right)^2\,dx=\int\left(dx^3+c\right)^2\left(bx^3+a\right)^p x^{-3\,p+2}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2*(b*x^3 + a)^p*x^(-3*p + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{2-3p}(a+bx^3)^p(c+dx^3)^2\,dx=\int x^{2-3\,p}\,(b\,x^3+a)^p\,(d\,x^3+c)^2\,dx$$

input
```
int(x^(2 - 3*p)*(a + b*x^3)^p*(c + d*x^3)^2,x)
```

output
```
int(x^(2 - 3*p)*(a + b*x^3)^p*(c + d*x^3)^2, x)
```

Reduce [F]

$$\int x^{2-3p}(a+bx^3)^p(c+dx^3)^2\,dx$$
$$=\frac{(b\,x^3+a)^p\,a^2d^2p^2x^3-2(b\,x^3+a)^p\,a^2d^2p\,x^3+6(b\,x^3+a)^p\,abcdp\,x^3+(b\,x^3+a)^p\,ab\,d^2p\,x^6+6(b\,x^3+a)^p}{}$$

input
```
int(x^(2-3*p)*(b*x^3+a)^p*(d*x^3+c)^2,x)
```

output
```
((a + b*x**3)**p*a**2*d**2*p**2*x**3 - 2*(a + b*x**3)**p*a**2*d**2*p*x**3
+ 6*(a + b*x**3)**p*a*b*c*d*p*x**3 + (a + b*x**3)**p*a*b*d**2*p*x**6 + 6*(
a + b*x**3)**p*b**2*c**2*x**3 + 6*(a + b*x**3)**p*b**2*c*d*x**6 + 2*(a + b
*x**3)**p*b**2*d**2*x**9 + 3*x**(3*p)*int(((a + b*x**3)**p*x**2)/(x**(3*p)
*a + x**(3*p)*b*x**3),x)*a**3*d**2*p**3 - 9*x**(3*p)*int(((a + b*x**3)**p*
x**2)/(x**(3*p)*a + x**(3*p)*b*x**3),x)*a**3*d**2*p**2 + 6*x**(3*p)*int(((
a + b*x**3)**p*x**2)/(x**(3*p)*a + x**(3*p)*b*x**3),x)*a**3*d**2*p + 18*x*
*(3*p)*int(((a + b*x**3)**p*x**2)/(x**(3*p)*a + x**(3*p)*b*x**3),x)*a**2*b
*c*d*p**2 - 18*x**(3*p)*int(((a + b*x**3)**p*x**2)/(x**(3*p)*a + x**(3*p)*
b*x**3),x)*a**2*b*c*d*p + 18*x**(3*p)*int(((a + b*x**3)**p*x**2)/(x**(3*p)
*a + x**(3*p)*b*x**3),x)*a*b**2*c**2*p)/(18*x**(3*p)*b**2)
```

3.885 $\int x^{2-3p}(a+bx^3)^p\left(c+dx^3\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 101

$$\int x^{2-3p}\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx=\frac{dx^{3-3p}\left(a+bx^3\right)^{1+p}}{6b}-\frac{1}{6}\left(\frac{ad}{b}-\frac{2c}{1-p}\right)x^{3-3p}\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^3}{a}\right)$$

output
```
1/6*d*x^(3-3*p)*(b*x^3+a)^(p+1)/b-1/6*(a*d/b-2*c/(1-p))*x^(3-3*p)*(b*x^3+a
)^p*hypergeom([-p, 1-p],[2-p],-b*x^3/a)/((1+b*x^3/a)^p)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int x^{2-3p}\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx=-\frac{x^{3-3p}(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(c(-2+p)\,\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^3}{a}\right)+d(-1+p)x^3\right.}{3(-2+p)(-1+p)}$$

input
```
Integrate[x^(2 - 3*p)*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
-1/3*(x^(3 - 3*p)*(a + b*x^3)^p*(c*(-2 + p)*Hypergeometric2F1[1 - p, -p, 2
 - p, -((b*x^3)/a)] + d*(-1 + p)*x^3*Hypergeometric2F1[2 - p, -p, 3 - p, -
((b*x^3)/a)]))/((-2 + p)*(-1 + p)*(1 + (b*x^3)/a)^p)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {959, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2-3p}\left(c+dx^3\right)\left(a+bx^3\right)^p\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(2bc-ad(1-p))\int x^{2-3p}\left(bx^3+a\right)^p dx}{2b}+\frac{dx^{3-3p}\left(a+bx^3\right)^{p+1}}{6b}$$

$$\Big\downarrow \; 882$$

$$\frac{ax^{-3p}\left(\frac{x^3}{a+bx^3}\right)^p\left(a+bx^3\right)^p(2bc-ad(1-p))\int \frac{\left(\frac{x^3}{bx^3+a}\right)^{-p}}{\left(1-\frac{bx^3}{bx^3+a}\right)^2}d\frac{x^3}{bx^3+a}}{6b}+\frac{dx^{3-3p}\left(a+bx^3\right)^{p+1}}{6b}$$

$$\Big\downarrow \; 74$$

$$\frac{ax^{3-3p}\left(a+bx^3\right)^{p-1}(2bc-ad(1-p))\text{Hypergeometric2F1}\left(2,1-p,2-p,\frac{bx^3}{bx^3+a}\right)}{6b(1-p)}+\frac{dx^{3-3p}\left(a+bx^3\right)^{p+1}}{6b}$$

input
```
Int[x^(2 - 3*p)*(a + b*x^3)^p*(c + d*x^3),x]
```

output
```
(d*x^(3 - 3*p)*(a + b*x^3)^(1 + p))/(6*b) + (a*(2*b*c - a*d*(1 - p))*x^(3
- 3*p)*(a + b*x^3)^(-1 + p)*Hypergeometric2F1[2, 1 - p, 2 - p, (b*x^3)/(a
+ b*x^3)])/(6*b*(1 - p))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^{2-3p} \left(b\,x^{3}+a\right)^{p} \left(d\,x^{3}+c\right)dx$$

input
```
int(x^(2-3*p)*(b*x^3+a)^p*(d*x^3+c),x)
```

output
```
int(x^(2-3*p)*(b*x^3+a)^p*(d*x^3+c),x)
```

Fricas [F]

$$\int x^{2-3p}(a+bx^3)^p\left(c+dx^3\right)\,dx=\int \left(dx^3+c\right)\left(bx^3+a\right)^p x^{-3\,p+2}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p*(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*(b*x^3 + a)^p*x^(-3*p + 2), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{2-3p}(a+bx^3)^p\left(c+dx^3\right)\,dx=\text{Timed out}$$

input
```
integrate(x**(2-3*p)*(b*x**3+a)**p*(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{2-3p}(a+bx^3)^p\left(c+dx^3\right)\,dx=\int \left(dx^3+c\right)\left(bx^3+a\right)^p x^{-3\,p+2}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p*(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p*x^(-3*p + 2), x)
```

Giac [F]

$$\int x^{2-3p}\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx=\int \left(dx^3+c\right)\left(bx^3+a\right)^p x^{-3\,p+2}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^p*x^(-3*p + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{2-3p}\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx=\int x^{2-3\,p}\,\left(b\,x^3+a\right)^p\,\left(d\,x^3+c\right)\,dx$$

input
```
int(x^(2 - 3*p)*(a + b*x^3)^p*(c + d*x^3),x)
```

output
```
int(x^(2 - 3*p)*(a + b*x^3)^p*(c + d*x^3), x)
```

Reduce [F]

$$\int x^{2-3p}\left(a+bx^3\right)^p\left(c+dx^3\right)\,dx$$
$$=\frac{(b\,x^3+a)^p\,adp\,x^3+2(b\,x^3+a)^p\,bc\,x^3+(b\,x^3+a)^p\,bd\,x^6+3x^{3p}\left(\int\frac{(b\,x^3+a)^p x^2}{x^{3p}a+x^{3p}b\,x^3}dx\right)a^2d\,p^2-3x^{3p}\left(\int\frac{(b\,x^3+\cdots}{x^{3p}a+\cdots}\right.}{6x^{3p}b}$$

input
```
int(x^(2-3*p)*(b*x^3+a)^p*(d*x^3+c),x)
```

output

```
((a + b*x**3)**p*a*d*p*x**3 + 2*(a + b*x**3)**p*b*c*x**3 + (a + b*x**3)**p
*b*d*x**6 + 3*x**(3*p)*int(((a + b*x**3)**p*x**2)/(x**(3*p)*a + x**(3*p)*b
*x**3),x)*a**2*d*p**2 - 3*x**(3*p)*int(((a + b*x**3)**p*x**2)/(x**(3*p)*a
+ x**(3*p)*b*x**3),x)*a**2*d*p + 6*x**(3*p)*int(((a + b*x**3)**p*x**2)/(x*
*(3*p)*a + x**(3*p)*b*x**3),x)*a*b*c*p)/(6*x**(3*p)*b)
```

3.886 $\int x^{2-3p}(a+bx^3)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 64

$$\int x^{2-3p}\left(a+bx^3\right)^p\,dx = \frac{x^{3-3p}(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^3}{a}\right)}{3(1-p)}$$

output
```
1/3*x^(3-3*p)*(b*x^3+a)^p*hypergeom([-p, 1-p],[2-p],-b*x^3/a)/(1-p)/((1+b*
x^3/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int x^{2-3p}\left(a+bx^3\right)^p\,dx = \frac{x^{3-3p}(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^3}{a}\right)}{3-3p}$$

input
```
Integrate[x^(2 - 3*p)*(a + b*x^3)^p,x]
```

output
```
(x^(3 - 3*p)*(a + b*x^3)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x^3)/a
)])/((3 - 3*p)*(1 + (b*x^3)/a)^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2-3p}\left(a+bx^3\right)^p\,dx$$

$$\Big\downarrow \text{ 882}$$

$$\frac{1}{3}ax^{-3p}\left(\frac{x^3}{a+bx^3}\right)^p\left(a+bx^3\right)^p\int \frac{\left(\frac{x^3}{bx^3+a}\right)^{-p}}{\left(1-\frac{bx^3}{bx^3+a}\right)^2}d\frac{x^3}{bx^3+a}$$

$$\Big\downarrow \text{ 74}$$

$$\frac{ax^{3-3p}\left(a+bx^3\right)^{p-1}\text{Hypergeometric2F1}\left(2,1-p,2-p,\frac{bx^3}{bx^3+a}\right)}{3(1-p)}$$

input
```
Int[x^(2 - 3*p)*(a + b*x^3)^p,x]
```

output
```
(a*x^(3 - 3*p)*(a + b*x^3)^(-1 + p)*Hypergeometric2F1[2, 1 - p, 2 - p, (b*
x^3)/(a + b*x^3)])/(3*(1 - p))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

Maple [F]

$$\int x^{2-3p} \left(b\,x^{3}+a\right)^{p} dx$$

input
```
int(x^(2-3*p)*(b*x^3+a)^p,x)
```

output
```
int(x^(2-3*p)*(b*x^3+a)^p,x)
```

Fricas [F]

$$\int x^{2-3p} \left(a+bx^3\right)^p \, dx = \int \left(bx^{3}+a\right)^{p} x^{-3\,p+2} \, dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^p*x^(-3*p + 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 98.81 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.61

$$\int x^{2-3p}(a+bx^3)^p\,dx = \frac{a^p x^{3-3p}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix} -p, 1-p \\ 2-p \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma(2-p)}$$

input
```
integrate(x**(2-3*p)*(b*x**3+a)**p,x)
```

output
```
a**p*x**(3 - 3*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(2 - p))
```

Maxima [F]

$$\int x^{2-3p}(a+bx^3)^p\,dx = \int (bx^3+a)^p x^{-3p+2}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^p*x^(-3*p + 2), x)
```

Giac [F]

$$\int x^{2-3p}(a+bx^3)^p\,dx = \int (bx^3+a)^p x^{-3p+2}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^p*x^(-3*p + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{2-3p}\left(a+bx^3\right)^p\,dx=\int x^{2-3\,p}\,{\left(b\,x^3+a\right)}^p\,dx$$

input
```
int(x^(2 - 3*p)*(a + b*x^3)^p,x)
```

output
```
int(x^(2 - 3*p)*(a + b*x^3)^p, x)
```

Reduce [F]

$$\int x^{2-3p}\left(a+bx^3\right)^p\,dx=\frac{\left(b\,x^3+a\right)^p x^3+3x^{3p}\left(\int \frac{\left(b\,x^3+a\right)^p x^2}{x^{3p}a+x^{3p}b\,x^3}dx\right)ap}{3x^{3p}}$$

input
```
int(x^(2-3*p)*(b*x^3+a)^p,x)
```

output
```
((a + b*x**3)**p*x**3 + 3*x**(3*p)*int(((a + b*x**3)**p*x**2)/(x**(3*p)*a
+ x**(3*p)*b*x**3),x)*a*p)/(3*x**(3*p))
```

3.887 $\int \frac{x^{2-3p}(a+bx^3)^p}{c+dx^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 118

$$\int \frac{x^{2-3p}(a+bx^3)^p}{c+dx^3} dx = \frac{x^{-3p}(a+bx^3)^p \text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-ad)x^3}{c(a+bx^3)}\right)}{3dp} - \frac{x^{-3p}(a+bx^3)^p \left(1+\frac{bx^3}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^3}{a}\right)}{3dp}$$

output
```
1/3*(b*x^3+a)^p*hypergeom([1, -p],[1-p],(-a*d+b*c)*x^3/c/(b*x^3+a))/d/p/(x
^(3*p))-1/3*(b*x^3+a)^p*hypergeom([-p, -p],[1-p],-b*x^3/a)/d/p/(x^(3*p))/(
(1+b*x^3/a)^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

$$\int \frac{x^{2-3p}(a+bx^3)^p}{c+dx^3}\,dx = \frac{x^{3-3p}(a+bx^3)^p\left(\frac{a+bx^3}{a}\right)^{-p}\text{AppellF1}\left(1-p,-p,1,2-p,-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c(3-3p)}$$

input
```
Integrate[(x^(2 - 3*p)*(a + b*x^3)^p)/(c + d*x^3),x]
```

output
```
(x^(3 - 3*p)*(a + b*x^3)^p*AppellF1[1 - p, -p, 1, 2 - p, -((b*x^3)/a), -((
d*x^3)/c)])/(c*(3 - 3*p)*((a + b*x^3)/a)^p)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.65, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2-3p}(a+bx^3)^p}{c+dx^3}\,dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}\int \frac{x^{2-3p}\left(\frac{bx^3}{a}+1\right)^p}{dx^3+c}dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^{3-3p}(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}\text{AppellF1}\left(1-p,-p,1,2-p,-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{3c(1-p)}$$

input
```
Int[(x^(2 - 3*p)*(a + b*x^3)^p)/(c + d*x^3),x]
```

output
```
(x^(3 - 3*p)*(a + b*x^3)^p*AppellF1[1 - p, -p, 1, 2 - p, -((b*x^3)/a), -((
d*x^3)/c)])/(3*c*(1 - p)*(1 + (b*x^3)/a)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{2-3p}(b\,x^{3}+a)^{p}}{d\,x^{3}+c}dx$$

input
```
int(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c),x)
```

output
```
int(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c),x)
```

Fricas [F]

$$\int \frac{x^{2-3p}(a+bx^3)^p}{c+dx^3}\,dx = \int \frac{(bx^3+a)^p x^{-3\,p+2}}{dx^3+c}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^p*x^(-3*p + 2)/(d*x^3 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{2-3p}(a+bx^3)^p}{c+dx^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(2-3*p)*(b*x**3+a)**p/(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{2-3p}(a+bx^3)^p}{c+dx^3}\,dx = \int \frac{(bx^3+a)^p x^{-3\,p+2}}{dx^3+c}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^p*x^(-3*p + 2)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^{2-3p}(a+bx^3)^p}{c+dx^3}\,dx = \int \frac{(bx^3+a)^p x^{-3\,p+2}}{dx^3+c}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^p*x^(-3*p + 2)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{2-3p}(a+bx^3)^p}{c+dx^3}\,dx = \int \frac{x^{2-3\,p}\,(b\,x^3+a)^p}{d\,x^3+c}\,dx$$

input
```
int((x^(2 - 3*p)*(a + b*x^3)^p)/(c + d*x^3),x)
```

output
```
int((x^(2 - 3*p)*(a + b*x^3)^p)/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{x^{2-3p}(a+bx^3)^p}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^p\,x^2}{x^{3p}c+x^{3p}d\,x^3}dx$$

input
```
int(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c),x)
```

output
```
int(((a + b*x**3)**p*x**2)/(x**(3*p)*c + x**(3*p)*d*x**3),x)
```

3.888 $\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^2}\,dx = \frac{ax^{3-3p}(a+bx^3)^{-1+p}\,\text{Hypergeometric2F1}\left(2,1-p,2-p,\frac{(bc-ad)x^3}{c(a+bx^3)}\right)}{3c^2(1-p)}$$

output
```
1/3*a*x^(3-3*p)*(b*x^3+a)^(-1+p)*hypergeom([2, 1-p],[2-p],(-a*d+b*c)*x^3/c
/(b*x^3+a))/c^2/(1-p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.46

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^2}\,dx = -\frac{x^{3-3p}(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(1+\frac{dx^3}{c}\right)^p\text{Hypergeometric2F1}\left(1-p,-p,2-p,\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{3c(-1+p)(c+dx^3)}$$

input
```
Integrate[(x^(2 - 3*p)*(a + b*x^3)^p)/(c + d*x^3)^2,x]
```

output
```
-1/3*(x^(3 - 3*p)*(a + b*x^3)^p*(1 + (d*x^3)/c)^p*Hypergeometric2F1[1 - p,
 -p, 2 - p, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c*(-1 + p)*(1 + (b*x^3
)/a)^p*(c + d*x^3))
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.48, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^2} \, dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^3)^p \left(\frac{bx^3}{a}+1\right)^{-p} \int \frac{x^{2-3p}\left(\frac{bx^3}{a}+1\right)^p}{(dx^3+c)^2}dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^{3-3p}(a+bx^3)^p \left(\frac{bx^3}{a}+1\right)^{-p} \left(\frac{dx^3}{c}+1\right)^{p-1} \text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{dx^3+c}\right)}{3c^2(1-p)}$$

input
```
Int[(x^(2 - 3*p)*(a + b*x^3)^p)/(c + d*x^3)^2,x]
```

output
```
(x^(3 - 3*p)*(a + b*x^3)^p*(1 + (d*x^3)/c)^(-1 + p)*Hypergeometric2F1[1 -
p, -p, 2 - p, -((c*((b*x^3)/a - (d*x^3)/c))/(c + d*x^3))])/(3*c^2*(1 - p)*
(1 + (b*x^3)/a)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{2-3p} (b\,x^{3}+a)^{p}}{(d\,x^{3}+c)^{2}}dx$$

input
```
int(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c)^2,x)
```

output
```
int(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c)^2,x)
```

Fricas [F]

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^p x^{-3\,p+2}}{(dx^3+c)^2}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^p*x^(-3*p + 2)/(d^2*x^6 + 2*c*d*x^3 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(2-3*p)*(b*x**3+a)**p/(d*x**3+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^p x^{-3p+2}}{(dx^3+c)^2}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^p*x^(-3*p + 2)/(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^p x^{-3p+2}}{(dx^3+c)^2}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^p*x^(-3*p + 2)/(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^2}\,dx = \int \frac{x^{2-3\,p}\,(b\,x^3+a)^p}{(d\,x^3+c)^2}\,dx$$

input
```
int((x^(2 - 3*p)*(a + b*x^3)^p)/(c + d*x^3)^2,x)
```

output
```
int((x^(2 - 3*p)*(a + b*x^3)^p)/(c + d*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^2}\,dx = \text{too large to display}$$

input
```
int(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c)^2,x)
```

output
```
( - (a + b*x**3)**p*b*x**3 + 3*x**(3*p)*int(((a + b*x**3)**p*x**5)/(x**(3*
p)*a**2*c**2*d*p + x**(3*p)*a**2*c**2*d + 2*x**(3*p)*a**2*c*d**2*p*x**3 +
2*x**(3*p)*a**2*c*d**2*x**3 + x**(3*p)*a**2*d**3*p*x**6 + x**(3*p)*a**2*d*
*3*x**6 - 2*x**(3*p)*a*b*c**3 + x**(3*p)*a*b*c**2*d*p*x**3 - 3*x**(3*p)*a*
b*c**2*d*x**3 + 2*x**(3*p)*a*b*c*d**2*p*x**6 + x**(3*p)*a*b*d**3*p*x**9 +
x**(3*p)*a*b*d**3*x**9 - 2*x**(3*p)*b**2*c**3*x**3 - 4*x**(3*p)*b**2*c**2*
d*x**6 - 2*x**(3*p)*b**2*c*d**2*x**9),x)*a**2*b*c*d**2*p + 3*x**(3*p)*int(
((a + b*x**3)**p*x**5)/(x**(3*p)*a**2*c**2*d*p + x**(3*p)*a**2*c**2*d + 2*
x**(3*p)*a**2*c*d**2*p*x**3 + 2*x**(3*p)*a**2*c*d**2*x**3 + x**(3*p)*a**2*
d**3*p*x**6 + x**(3*p)*a**2*d**3*x**6 - 2*x**(3*p)*a*b*c**3 + x**(3*p)*a*b
*c**2*d*p*x**3 - 3*x**(3*p)*a*b*c**2*d*x**3 + 2*x**(3*p)*a*b*c*d**2*p*x**6
 + x**(3*p)*a*b*d**3*p*x**9 + x**(3*p)*a*b*d**3*x**9 - 2*x**(3*p)*b**2*c**
3*x**3 - 4*x**(3*p)*b**2*c**2*d*x**6 - 2*x**(3*p)*b**2*c*d**2*x**9),x)*a**
2*b*c*d**2 + 3*x**(3*p)*int(((a + b*x**3)**p*x**5)/(x**(3*p)*a**2*c**2*d*p
 + x**(3*p)*a**2*c**2*d + 2*x**(3*p)*a**2*c*d**2*p*x**3 + 2*x**(3*p)*a**2*
c*d**2*x**3 + x**(3*p)*a**2*d**3*p*x**6 + x**(3*p)*a**2*d**3*x**6 - 2*x**(
3*p)*a*b*c**3 + x**(3*p)*a*b*c**2*d*p*x**3 - 3*x**(3*p)*a*b*c**2*d*x**3 +
2*x**(3*p)*a*b*c*d**2*p*x**6 + x**(3*p)*a*b*d**3*p*x**9 + x**(3*p)*a*b*d**
3*x**9 - 2*x**(3*p)*b**2*c**3*x**3 - 4*x**(3*p)*b**2*c**2*d*x**6 - 2*x**(3
*p)*b**2*c*d**2*x**9),x)*a**2*b*d**3*p*x**3 + 3*x**(3*p)*int(((a + b*x*...
```

3.889 $\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^3}\,dx = \frac{x^{3-3p}(a+bx^3)^{1+p}}{3ac(1-p)\,(c+dx^3)^2} - \frac{a(2bc-ad(1+p))x^{6-3p}(a+bx^3)^{-2+p}\,\text{Hypergeometric2F1}\left(3,2-p,3-p,\frac{(bc-ad)x^3}{c(a+bx^3)}\right)}{3c^4(1-p)(2-p)}$$

output
```
1/3*x^(3-3*p)*(b*x^3+a)^(p+1)/a/c/(1-p)/(d*x^3+c)^2-1/3*a*(2*b*c-a*d*(p+1)
)*x^(6-3*p)*(b*x^3+a)^(-2+p)*hypergeom([3, 2-p],[3-p],(-a*d+b*c)*x^3/c/(b*
x^3+a))/c^4/(1-p)/(2-p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.17 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.19

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^3}\,dx$$

$$= \frac{px^{3-3p}(a+bx^3)^p\left(2(c+dpx^3)\,\Phi\left(\frac{(bc-ad)x^3}{c(a+bx^3)},1,1-p\right) - d(-1+p)x^3\Phi\left(\frac{(bc-ad)x^3}{c(a+bx^3)},1,2-p\right) - (2c+d(1+\right.}{6c^2\,(c+dx^3)^2}$$

input
```
Integrate[(x^(2 - 3*p)*(a + b*x^3)^p)/(c + d*x^3)^3,x]
```

output
```
(p*x^(3 - 3*p)*(a + b*x^3)^p*(2*(c + d*p*x^3)*HurwitzLerchPhi[((b*c - a*d)
*x^3)/(c*(a + b*x^3)), 1, 1 - p] - d*(-1 + p)*x^3*HurwitzLerchPhi[((b*c -
a*d)*x^3)/(c*(a + b*x^3)), 1, 2 - p] - (2*c + d*(1 + p)*x^3)*HurwitzLerchP
hi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1, -p]))/(6*c^2*(c + d*x^3)^2)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.66 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2-3p}\left(a+bx^3\right)^p}{\left(c+dx^3\right)^3}dx$$

$$\Big\downarrow \text{ 1013}$$

$$\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\int \frac{x^{2-3p}\left(\frac{bx^3}{a}+1\right)^p}{\left(dx^3+c\right)^3}dx$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{px^{3-3p}\left(a+bx^3\right)^p\left(d(1-p)x^3\Phi\left(\frac{(bc-ad)x^3}{c(bx^3+a)},1,2-p\right)+2\left(c+dpx^3\right)\Phi\left(\frac{(bc-ad)x^3}{c(bx^3+a)},1,1-p\right)-\left(2c+d(p+1)x^3\right)\Phi}{6c^2\left(c+dx^3\right)^2}$$

input
```
Int[(x^(2 - 3*p)*(a + b*x^3)^p)/(c + d*x^3)^3,x]
```

output
```
(p*x^(3 - 3*p)*(a + b*x^3)^p*(2*(c + d*p*x^3)*HurwitzLerchPhi[((b*c - a*d)
*x^3)/(c*(a + b*x^3)), 1, 1 - p] + d*(1 - p)*x^3*HurwitzLerchPhi[((b*c - a
*d)*x^3)/(c*(a + b*x^3)), 1, 2 - p] - (2*c + d*(1 + p)*x^3)*HurwitzLerchPh
i[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1, -p]))/(6*c^2*(c + d*x^3)^2)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{2-3p}(b\,x^3+a)^p}{(d\,x^3+c)^3}dx$$

input
```
int(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c)^3,x)
```

output
```
int(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c)^3,x)
```

Fricas [F]

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^p x^{-3\,p+2}}{(dx^3+c)^3}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^p*x^(-3*p + 2)/(d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 +
 c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(2-3*p)*(b*x**3+a)**p/(d*x**3+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^p x^{-3\,p+2}}{(dx^3+c)^3}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^p*x^(-3*p + 2)/(d*x^3 + c)^3, x)
```

Giac [F]

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^p x^{-3\,p+2}}{(dx^3+c)^3}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^p*x^(-3*p + 2)/(d*x^3 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^3}\,dx = \int \frac{x^{2-3\,p}\,(b\,x^3+a)^p}{(d\,x^3+c)^3}\,dx$$

input
```
int((x^(2 - 3*p)*(a + b*x^3)^p)/(c + d*x^3)^3,x)
```

output
```
int((x^(2 - 3*p)*(a + b*x^3)^p)/(c + d*x^3)^3, x)
```

Reduce [F]

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^3}\,dx = \text{too large to display}$$

input
```
int(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c)^3,x)
```

output

```
( - (a + b*x**3)**p*a**3*b*d**3*p**4*x**3 - 5*(a + b*x**3)**p*a**3*b*d**3*
p**3*x**3 - 8*(a + b*x**3)**p*a**3*b*d**3*p**2*x**3 - 4*(a + b*x**3)**p*a*
*3*b*d**3*p*x**3 + 10*(a + b*x**3)**p*a**2*b**2*c*d**2*p**3*x**3 + 26*(a +
 b*x**3)**p*a**2*b**2*c*d**2*p**2*x**3 + 28*(a + b*x**3)**p*a**2*b**2*c*d*
*2*p*x**3 + 8*(a + b*x**3)**p*a**2*b**2*c*d**2*x**3 + (a + b*x**3)**p*a**2
*b**2*d**3*p**3*x**6 + 5*(a + b*x**3)**p*a**2*b**2*d**3*p**2*x**6 + 8*(a +
 b*x**3)**p*a**2*b**2*d**3*p*x**6 + 4*(a + b*x**3)**p*a**2*b**2*d**3*x**6
- 26*(a + b*x**3)**p*a*b**3*c**2*d*p**2*x**3 - 40*(a + b*x**3)**p*a*b**3*c
**2*d*p*x**3 - 24*(a + b*x**3)**p*a*b**3*c**2*d*x**3 - 8*(a + b*x**3)**p*a
*b**3*c*d**2*p**2*x**6 - 16*(a + b*x**3)**p*a*b**3*c*d**2*p*x**6 - 12*(a +
 b*x**3)**p*a*b**3*c*d**2*x**6 + 20*(a + b*x**3)**p*b**4*c**3*p*x**3 + 16*
(a + b*x**3)**p*b**4*c**3*x**3 + 10*(a + b*x**3)**p*b**4*c**2*d*p*x**6 + 8
*(a + b*x**3)**p*b**4*c**2*d*x**6 - 12*x**(3*p)*int(((a + b*x**3)**p*x**5)
/(x**(3*p)*a**5*c**3*d**4*p**4 + 6*x**(3*p)*a**5*c**3*d**4*p**3 + 13*x**(3
*p)*a**5*c**3*d**4*p**2 + 12*x**(3*p)*a**5*c**3*d**4*p + 4*x**(3*p)*a**5*c
**3*d**4 + 3*x**(3*p)*a**5*c**2*d**5*p**4*x**3 + 18*x**(3*p)*a**5*c**2*d**
5*p**3*x**3 + 39*x**(3*p)*a**5*c**2*d**5*p**2*x**3 + 36*x**(3*p)*a**5*c**2
*d**5*p*x**3 + 12*x**(3*p)*a**5*c**2*d**5*x**3 + 3*x**(3*p)*a**5*c*d**6*p*
*4*x**6 + 18*x**(3*p)*a**5*c*d**6*p**3*x**6 + 39*x**(3*p)*a**5*c*d**6*p**2
*x**6 + 36*x**(3*p)*a**5*c*d**6*p*x**6 + 12*x**(3*p)*a**5*c*d**6*x**6 +...
```

3.890 $\int \frac{x^{2-3p}\left(a+bx^3\right)^p}{\left(c+dx^3\right)^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 231

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^4}\,dx = \frac{x^{3-3p}(a+bx^3)^{1+p}}{3ac(1-p)\left(c+dx^3\right)^3} + \frac{d(3bc-ad(2+p))x^{6-3p}(a+bx^3)^{1+p}}{9ac^2(bc-ad)(1-p)\left(c+dx^3\right)^3} - \frac{a(6b^2c^2-6abcd(1+p)+a^2d^2(2+3p+p^2))\,x^{6-3p}(a+bx^3)^{-2+p}\,\text{Hypergeometric2F1}\left(3,2-p,3-p,\right.}{9c^5(bc-ad)(1-p)(2-p)}$$

output
```
1/3*x^(3-3*p)*(b*x^3+a)^(p+1)/a/c/(1-p)/(d*x^3+c)^3+1/9*d*(3*b*c-a*d*(2+p)
)*x^(6-3*p)*(b*x^3+a)^(p+1)/a/c^2/(-a*d+b*c)/(1-p)/(d*x^3+c)^3-1/9*a*(6*b^
2*c^2-6*a*b*c*d*(p+1)+a^2*d^2*(p^2+3*p+2))*x^(6-3*p)*(b*x^3+a)^(-2+p)*hype
rgeom([3, 2-p],[3-p],(-a*d+b*c)*x^3/c/(b*x^3+a))/c^5/(-a*d+b*c)/(1-p)/(2-p
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.35 (sec) , antiderivative size = 484, normalized size of antiderivative = 2.10

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^4}\,dx$$
$$= \frac{px^{3-3p}(a+bx^3)^p\left(3(2c^2+4cdpx^3+d^2p(1+p)x^6)\,\Phi\left(\frac{(bc-ad)x^3}{c(a+bx^3)},1,1-p\right)-3d(-1+p)x^3(2c+dpx^3)\,\Phi\left(\right.\right.}{}$$

input
```
Integrate[(x^(2 - 3*p)*(a + b*x^3)^p)/(c + d*x^3)^4,x]
```

output
```
(p*x^(3 - 3*p)*(a + b*x^3)^p*(3*(2*c^2 + 4*c*d*p*x^3 + d^2*p*(1 + p)*x^6)*
HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1, 1 - p] - 3*d*(-1 + p
)*x^3*(2*c + d*p*x^3)*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1
, 2 - p] + 2*d^2*x^6*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1,
 3 - p] - 3*d^2*p*x^6*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1
, 3 - p] + d^2*p^2*x^6*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)),
1, 3 - p] - 6*c^2*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1, -p
] - 6*c*d*x^3*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1, -p] -
6*c*d*p*x^3*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1, -p] - 2*
d^2*x^6*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1, -p] - 3*d^2*
p*x^6*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1, -p] - d^2*p^2*
x^6*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1, -p]))/(18*c^3*(c
 + d*x^3)^3)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.26 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2-3p}\left(a+bx^3\right)^p}{\left(c+dx^3\right)^4}dx$$

$$\Big\downarrow \text{1013}$$

$$\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\int \frac{x^{2-3p}\left(\frac{bx^3}{a}+1\right)^p}{\left(dx^3+c\right)^4}dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{px^{3-3p}\left(a+bx^3\right)^p\left(3\left(2c^2+4cdpx^3+d^2p(p+1)x^6\right)\Phi\left(\frac{(bc-ad)x^3}{c(bx^3+a)},1,1-p\right)-6c^2\Phi\left(\frac{(bc-ad)x^3}{c(bx^3+a)},1,-p\right)+d^2p^2x^6\Phi\left(\right.\right.}{}$$

input
```
Int[(x^(2 - 3*p)*(a + b*x^3)^p)/(c + d*x^3)^4,x]
```

output
```
(p*x^(3 - 3*p)*(a + b*x^3)^p*(3*(2*c^2 + 4*c*d*p*x^3 + d^2*p*(1 + p)*x^6)*
HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1, 1 - p] + 3*d*(1 - p)
*x^3*(2*c + d*p*x^3)*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1,
 2 - p] + 2*d^2*x^6*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1,
3 - p] - 3*d^2*p*x^6*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1,
 3 - p] + d^2*p^2*x^6*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1
, 3 - p] - 6*c^2*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1, -p]
 - 6*c*d*x^3*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1, -p] - 6
*c*d*p*x^3*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1, -p] - 2*d
^2*x^6*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1, -p] - 3*d^2*p
*x^6*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1, -p] - d^2*p^2*x
^6*HurwitzLerchPhi[((b*c - a*d)*x^3)/(c*(a + b*x^3)), 1, -p]))/(18*c^3*(c
+ d*x^3)^3)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{2-3p}(b\,x^{3}+a)^{p}}{(d\,x^{3}+c)^{4}}dx$$

input
```
int(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c)^4,x)
```

output
```
int(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c)^4,x)
```

Fricas [F]

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^4}\,dx=\int \frac{(bx^3+a)^p x^{-3\,p+2}}{(dx^3+c)^4}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c)^4,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^p*x^(-3*p + 2)/(d^4*x^12 + 4*c*d^3*x^9 + 6*c^2*d^2*x^
6 + 4*c^3*d*x^3 + c^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^4}\,dx = \text{Timed out}$$

input
```
integrate(x**(2-3*p)*(b*x**3+a)**p/(d*x**3+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^4}\,dx = \int \frac{(bx^3+a)^p x^{-3\,p+2}}{(dx^3+c)^4}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c)^4,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^p*x^(-3*p + 2)/(d*x^3 + c)^4, x)
```

Giac [F]

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^4}\,dx = \int \frac{(bx^3+a)^p x^{-3\,p+2}}{(dx^3+c)^4}\,dx$$

input
```
integrate(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c)^4,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^p*x^(-3*p + 2)/(d*x^3 + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^4}\,dx = \int \frac{x^{2-3\,p}\,\left(b\,x^3+a\right)^p}{\left(d\,x^3+c\right)^4}\,dx$$

input
```
int((x^(2 - 3*p)*(a + b*x^3)^p)/(c + d*x^3)^4,x)
```

output
```
int((x^(2 - 3*p)*(a + b*x^3)^p)/(c + d*x^3)^4, x)
```

Reduce [F]

$$\int \frac{x^{2-3p}(a+bx^3)^p}{(c+dx^3)^4}\,dx = \text{too large to display}$$

input
```
int(x^(2-3*p)*(b*x^3+a)^p/(d*x^3+c)^4,x)
```

output

```
( - (a + b*x**3)**p*a**2*b*d**2*p**2*x**3 - (a + b*x**3)**p*a**2*b*d**2*p*
x**3 + 6*(a + b*x**3)**p*a*b**2*c*d*p*x**3 + 2*(a + b*x**3)**p*a*b**2*d**2
*p*x**6 - 6*(a + b*x**3)**p*b**3*c**2*x**3 - 6*(a + b*x**3)**p*b**3*c*d*x*
*6 - 2*(a + b*x**3)**p*b**3*d**2*x**9 + 9*x**(3*p)*int(((a + b*x**3)**p*x*
*5)/(x**(3*p)*a**7*c**4*d**6*p**6 + 12*x**(3*p)*a**7*c**4*d**6*p**5 + 58*x
**(3*p)*a**7*c**4*d**6*p**4 + 144*x**(3*p)*a**7*c**4*d**6*p**3 + 193*x**(3
*p)*a**7*c**4*d**6*p**2 + 132*x**(3*p)*a**7*c**4*d**6*p + 36*x**(3*p)*a**7
*c**4*d**6 + 4*x**(3*p)*a**7*c**3*d**7*p**6*x**3 + 48*x**(3*p)*a**7*c**3*d
**7*p**5*x**3 + 232*x**(3*p)*a**7*c**3*d**7*p**4*x**3 + 576*x**(3*p)*a**7*
c**3*d**7*p**3*x**3 + 772*x**(3*p)*a**7*c**3*d**7*p**2*x**3 + 528*x**(3*p)
*a**7*c**3*d**7*p*x**3 + 144*x**(3*p)*a**7*c**3*d**7*x**3 + 6*x**(3*p)*a**
7*c**2*d**8*p**6*x**6 + 72*x**(3*p)*a**7*c**2*d**8*p**5*x**6 + 348*x**(3*p
)*a**7*c**2*d**8*p**4*x**6 + 864*x**(3*p)*a**7*c**2*d**8*p**3*x**6 + 1158*
x**(3*p)*a**7*c**2*d**8*p**2*x**6 + 792*x**(3*p)*a**7*c**2*d**8*p*x**6 + 2
16*x**(3*p)*a**7*c**2*d**8*x**6 + 4*x**(3*p)*a**7*c*d**9*p**6*x**9 + 48*x*
*(3*p)*a**7*c*d**9*p**5*x**9 + 232*x**(3*p)*a**7*c*d**9*p**4*x**9 + 576*x*
*(3*p)*a**7*c*d**9*p**3*x**9 + 772*x**(3*p)*a**7*c*d**9*p**2*x**9 + 528*x*
*(3*p)*a**7*c*d**9*p*x**9 + 144*x**(3*p)*a**7*c*d**9*x**9 + x**(3*p)*a**7*
d**10*p**6*x**12 + 12*x**(3*p)*a**7*d**10*p**5*x**12 + 58*x**(3*p)*a**7*d*
*10*p**4*x**12 + 144*x**(3*p)*a**7*d**10*p**3*x**12 + 193*x**(3*p)*a**7...
```

3.891 $\int (ex)^m \left(a+bx^3\right)^p \left(c+dx^3\right)^q \, dx$

Optimal result

Integrand size = 24, antiderivative size = 101

$$\int (ex)^m \left(a+bx^3\right)^p \left(c+dx^3\right)^q \, dx = \frac{(ex)^{1+m} \left(a+bx^3\right)^p \left(1+\frac{bx^3}{a}\right)^{-p} \left(c+dx^3\right)^q \left(1+\frac{dx^3}{c}\right)^{-q} \text{AppellF1}\left(\frac{1+m}{3},-p,-q,\frac{4+m}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{e(1+m)}$$

output
```
(e*x)^(1+m)*(b*x^3+a)^p*(d*x^3+c)^q*AppellF1(1/3+1/3*m,-p,-q,4/3+1/3*m,-b*
x^3/a,-d*x^3/c)/e/(1+m)/((1+b*x^3/a)^p)/((1+d*x^3/c)^q)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int (ex)^m \left(a+bx^3\right)^p \left(c+dx^3\right)^q \, dx = \frac{x(ex)^m \left(a+bx^3\right)^p \left(1+\frac{bx^3}{a}\right)^{-p} \left(c+dx^3\right)^q \left(1+\frac{dx^3}{c}\right)^{-q} \text{AppellF1}\left(\frac{1+m}{3},-p,-q,\frac{4+m}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{1+m}$$

input
```
Integrate[(e*x)^m*(a + b*x^3)^p*(c + d*x^3)^q,x]
```

output
```
(x*(e*x)^m*(a + b*x^3)^p*(c + d*x^3)^q*AppellF1[(1 + m)/3, -p, -q, (4 + m)
/3, -((b*x^3)/a), -((d*x^3)/c)])/((1 + m)*(1 + (b*x^3)/a)^p*(1 + (d*x^3)/c
)^q)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+bx^3\right)^p \left(c+dx^3\right)^q \, dx$$

$$\Big\downarrow \text{1013}$$

$$\left(a+bx^3\right)^p \left(\frac{bx^3}{a}+1\right)^{-p} \int (ex)^m \left(\frac{bx^3}{a}+1\right)^p \left(dx^3+c\right)^q dx$$

$$\Big\downarrow \text{1013}$$

$$\left(a+bx^3\right)^p \left(\frac{bx^3}{a}+1\right)^{-p} \left(c+dx^3\right)^q \left(\frac{dx^3}{c}+1\right)^{-q} \int (ex)^m \left(\frac{bx^3}{a}+1\right)^p \left(\frac{dx^3}{c}+1\right)^q dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{(ex)^{m+1} \left(a+bx^3\right)^p \left(\frac{bx^3}{a}+1\right)^{-p} \left(c+dx^3\right)^q \left(\frac{dx^3}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{m+1}{3},-p,-q,\frac{m+4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{e(m+1)}$$

input
```
Int[(e*x)^m*(a + b*x^3)^p*(c + d*x^3)^q,x]
```

output
```
((e*x)^(1 + m)*(a + b*x^3)^p*(c + d*x^3)^q*AppellF1[(1 + m)/3, -p, -q, (4
+ m)/3, -((b*x^3)/a), -((d*x^3)/c)])/(e*(1 + m)*(1 + (b*x^3)/a)^p*(1 + (d*
x^3)/c)^q)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int (ex)^m \left(b\,x^3+a\right)^p \left(d\,x^3+c\right)^q dx$$

input
```
int((e*x)^m*(b*x^3+a)^p*(d*x^3+c)^q,x)
```

output
```
int((e*x)^m*(b*x^3+a)^p*(d*x^3+c)^q,x)
```

Fricas [F]

$$\int (ex)^m \left(a+bx^3\right)^p \left(c+dx^3\right)^q \, dx = \int \left(bx^3+a\right)^p \left(dx^3+c\right)^q (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^p*(d*x^3+c)^q,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^p*(d*x^3 + c)^q*(e*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^3\right)^p \left(c+dx^3\right)^q \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(b*x**3+a)**p*(d*x**3+c)**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m \left(a+bx^3\right)^p \left(c+dx^3\right)^q \, dx = \int \left(bx^3+a\right)^p \left(dx^3+c\right)^q (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^p*(d*x^3+c)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^p*(d*x^3 + c)^q*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m \left(a+bx^3\right)^p \left(c+dx^3\right)^q \, dx = \int \left(bx^3+a\right)^p \left(dx^3+c\right)^q (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^p*(d*x^3+c)^q,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^p*(d*x^3 + c)^q*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^3\right)^p \left(c+dx^3\right)^q \, dx = \int (e\,x)^m \,\left(b\,x^3+a\right)^p \,\left(d\,x^3+c\right)^q \,dx$$

input
```
int((e*x)^m*(a + b*x^3)^p*(c + d*x^3)^q,x)
```

output
```
int((e*x)^m*(a + b*x^3)^p*(c + d*x^3)^q, x)
```

Reduce [F]

$$\int (ex)^m \left(a+bx^3\right)^p \left(c+dx^3\right)^q \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^3+a)^p*(d*x^3+c)^q,x)
```

output

```
(e**m*(x**m*(c + d*x**3)**q*(a + b*x**3)**p*x + 3*int((x**m*(c + d*x**3)**
q*(a + b*x**3)**p*x**3)/(a*c*m + 3*a*c*p + 3*a*c*q + a*c + a*d*m*x**3 + 3*
a*d*p*x**3 + 3*a*d*q*x**3 + a*d*x**3 + b*c*m*x**3 + 3*b*c*p*x**3 + 3*b*c*q
*x**3 + b*c*x**3 + b*d*m*x**6 + 3*b*d*p*x**6 + 3*b*d*q*x**6 + b*d*x**6),x)
*a*d*m*p + 9*int((x**m*(c + d*x**3)**q*(a + b*x**3)**p*x**3)/(a*c*m + 3*a*
c*p + 3*a*c*q + a*c + a*d*m*x**3 + 3*a*d*p*x**3 + 3*a*d*q*x**3 + a*d*x**3
+ b*c*m*x**3 + 3*b*c*p*x**3 + 3*b*c*q*x**3 + b*c*x**3 + b*d*m*x**6 + 3*b*d
*p*x**6 + 3*b*d*q*x**6 + b*d*x**6),x)*a*d*p**2 + 9*int((x**m*(c + d*x**3)*
*q*(a + b*x**3)**p*x**3)/(a*c*m + 3*a*c*p + 3*a*c*q + a*c + a*d*m*x**3 + 3
*a*d*p*x**3 + 3*a*d*q*x**3 + a*d*x**3 + b*c*m*x**3 + 3*b*c*p*x**3 + 3*b*c*
q*x**3 + b*c*x**3 + b*d*m*x**6 + 3*b*d*p*x**6 + 3*b*d*q*x**6 + b*d*x**6),x
)*a*d*p*q + 3*int((x**m*(c + d*x**3)**q*(a + b*x**3)**p*x**3)/(a*c*m + 3*a
*c*p + 3*a*c*q + a*c + a*d*m*x**3 + 3*a*d*p*x**3 + 3*a*d*q*x**3 + a*d*x**3
 + b*c*m*x**3 + 3*b*c*p*x**3 + 3*b*c*q*x**3 + b*c*x**3 + b*d*m*x**6 + 3*b*
d*p*x**6 + 3*b*d*q*x**6 + b*d*x**6),x)*a*d*p + 3*int((x**m*(c + d*x**3)**q
*(a + b*x**3)**p*x**3)/(a*c*m + 3*a*c*p + 3*a*c*q + a*c + a*d*m*x**3 + 3*a
*d*p*x**3 + 3*a*d*q*x**3 + a*d*x**3 + b*c*m*x**3 + 3*b*c*p*x**3 + 3*b*c*q*
x**3 + b*c*x**3 + b*d*m*x**6 + 3*b*d*p*x**6 + 3*b*d*q*x**6 + b*d*x**6),x)*
b*c*m*q + 9*int((x**m*(c + d*x**3)**q*(a + b*x**3)**p*x**3)/(a*c*m + 3*a*c
*p + 3*a*c*q + a*c + a*d*m*x**3 + 3*a*d*p*x**3 + 3*a*d*q*x**3 + a*d*x**...
```

3.892 $\int x^{-1-3(3+2p)}(a+bx^3)^p (c+dx^3)^p\,dx$

Optimal result

Integrand size = 30, antiderivative size = 252

$$\int x^{-1-3(3+2p)}(a+bx^3)^p (c+dx^3)^p\,dx = \frac{(bc+ad)(2+p)x^{-6(1+p)}(a+bx^3)^{1+p}(c+dx^3)^{1+p}}{6a^2c^2(1+p)(3+2p)} - \frac{x^{-3(3+2p)}(a+bx^3)^{1+p}(c+dx^3)^{1+p}}{3ac(3+2p)} - \frac{(2abcd(1+p)+b^2c^2(2+p)+a^2d^2(2+p))\,x^{-3(1+2p)}(a+bx^3)^{1+p}(c+dx^3)^p\left(\frac{a(c+dx^3)}{c(a+bx^3)}\right)^{-p}\text{Hypergeome}}{6a^3c^2(3+8p+4p^2)}$$

output
```
1/6*(a*d+b*c)*(2+p)*(b*x^3+a)^(p+1)*(d*x^3+c)^(p+1)/a^2/c^2/(p+1)/(3+2*p)/
(x^(6*p+6))-1/3*(b*x^3+a)^(p+1)*(d*x^3+c)^(p+1)/a/c/(3+2*p)/(x^(9+6*p))-1/
6*(2*a*b*c*d*(p+1)+b^2*c^2*(2+p)+a^2*d^2*(2+p))*(b*x^3+a)^(p+1)*(d*x^3+c)^
p*hypergeom([-p, -1-2*p],[-2*p],(-a*d+b*c)*x^3/c/(b*x^3+a))/a^3/c^2/(4*p^2
+8*p+3)/(x^(3+6*p))/((a*(d*x^3+c)/c/(b*x^3+a))^p)
```

Mathematica [F]

$$\int x^{-1-3(3+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\int x^{-1-3(3+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx$$

input
```
Integrate[x^(-1 - 3*(3 + 2*p))*(a + b*x^3)^p*(c + d*x^3)^p,x]
```

output
```
Integrate[x^(-1 - 3*(3 + 2*p))*(a + b*x^3)^p*(c + d*x^3)^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3(2p+3)-1}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx$$

$$\Big\downarrow \text{1013}$$

$$\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\int x^{-2(3p+5)}\left(\frac{bx^3}{a}+1\right)^p\left(dx^3+c\right)^p\,dx$$

$$\Big\downarrow \text{1013}$$

$$\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(c+dx^3\right)^p\left(\frac{dx^3}{c}+1\right)^{-p}\int x^{-2(3p+5)}\left(\frac{bx^3}{a}+1\right)^p\left(\frac{dx^3}{c}+1\right)^p\,dx$$

$$\Big\downarrow \text{7299}$$

$$\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(c+dx^3\right)^p\left(\frac{dx^3}{c}+1\right)^{-p}\int x^{-2(3p+5)}\left(\frac{bx^3}{a}+1\right)^p\left(\frac{dx^3}{c}+1\right)^p\,dx$$

input
```
Int[x^(-1 - 3*(3 + 2*p))*(a + b*x^3)^p*(c + d*x^3)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int x^{-10-6p}\left(b\,x^{3}+a\right)^{p}\left(d\,x^{3}+c\right)^{p}dx$$

input
```
int(x^(-10-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x)
```

output
```
int(x^(-10-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x)
```

Fricas [F]

$$\int x^{-1-3(3+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\int\left(bx^3+a\right)^p\left(dx^3+c\right)^p x^{-6\,p-10}\,dx$$

input
```
integrate(x^(-10-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^p*(d*x^3 + c)^p*x^(-6*p - 10), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-3(3+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**(-10-6*p)*(b*x**3+a)**p*(d*x**3+c)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-3(3+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\int\left(bx^3+a\right)^p\left(dx^3+c\right)^px^{-6\,p-10}\,dx$$

input
```
integrate(x^(-10-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^p*(d*x^3 + c)^p*x^(-6*p - 10), x)
```

Giac [F]

$$\int x^{-1-3(3+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\int\left(bx^3+a\right)^p\left(dx^3+c\right)^px^{-6\,p-10}\,dx$$

input
```
integrate(x^(-10-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^p*(d*x^3 + c)^p*x^(-6*p - 10), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-3(3+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\int\frac{\left(b\,x^3+a\right)^p\left(d\,x^3+c\right)^p}{x^{6\,p+10}}\,dx$$

input
```
int(((a + b*x^3)^p*(c + d*x^3)^p)/x^(6*p + 10),x)
```

output
```
int(((a + b*x^3)^p*(c + d*x^3)^p)/x^(6*p + 10), x)
```

Reduce [F]

$$\int x^{-1-3(3+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\int x^{-10-6p}\left(b\,x^3+a\right)^p\left(d\,x^3+c\right)^p dx$$

input
```
int(x^(-10-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x)
```

output
```
int(x^(-10-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x)
```

3.893 $\int x^{-1-3(2+2p)}(a+bx^3)^p(c+dx^3)^p\,dx$

Optimal result

Integrand size = 30, antiderivative size = 159

$$\int x^{-1-3(2+2p)}(a+bx^3)^p(c+dx^3)^p\,dx=-\frac{x^{-6(1+p)}(a+bx^3)^{1+p}(c+dx^3)^{1+p}}{6ac(1+p)}$$
$$+\frac{(bc+ad)x^{-3(1+2p)}(a+bx^3)^{1+p}(c+dx^3)^p\left(\frac{a(c+dx^3)}{c(a+bx^3)}\right)^{-p}\text{Hypergeometric2F1}\left(-1-2p,-p,-2p,\frac{(bc-ad}{c(a+bx}\right.}{6a^2c(1+2p)}$$

output
```
-1/6*(b*x^3+a)^(p+1)*(d*x^3+c)^(p+1)/a/c/(p+1)/(x^(6*p+6))+1/6*(a*d+b*c)*(
b*x^3+a)^(p+1)*(d*x^3+c)^p*hypergeom([-p, -1-2*p],[-2*p],(-a*d+b*c)*x^3/c/
(b*x^3+a))/a^2/c/(1+2*p)/(x^(3+6*p))/((a*(d*x^3+c)/c/(b*x^3+a))^p)
```

Mathematica [F]

$$\int x^{-1-3(2+2p)}(a+bx^3)^p(c+dx^3)^p\,dx=\int x^{-1-3(2+2p)}(a+bx^3)^p(c+dx^3)^p\,dx$$

input
```
Integrate[x^(-1 - 3*(2 + 2*p))*(a + b*x^3)^p*(c + d*x^3)^p,x]
```

output
```
Integrate[x^(-1 - 3*(2 + 2*p))*(a + b*x^3)^p*(c + d*x^3)^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3(2p+2)-1}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx$$

$$\Big\downarrow \text{ 1013}$$

$$\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\int x^{-6p-7}\left(\frac{bx^3}{a}+1\right)^p\left(dx^3+c\right)^p\,dx$$

$$\Big\downarrow \text{ 1013}$$

$$\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(c+dx^3\right)^p\left(\frac{dx^3}{c}+1\right)^{-p}\int x^{-6p-7}\left(\frac{bx^3}{a}+1\right)^p\left(\frac{dx^3}{c}+1\right)^p\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(c+dx^3\right)^p\left(\frac{dx^3}{c}+1\right)^{-p}\int x^{-6p-7}\left(\frac{bx^3}{a}+1\right)^p\left(\frac{dx^3}{c}+1\right)^p\,dx$$

input
```
Int[x^(-1 - 3*(2 + 2*p))*(a + b*x^3)^p*(c + d*x^3)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int x^{-7-6p}(b\,x^3+a)^p\,(d\,x^3+c)^p\,dx$$

input
```
int(x^(-7-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x)
```

output
```
int(x^(-7-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x)
```

Fricas [F]

$$\int x^{-1-3(2+2p)}(a+bx^3)^p\,(c+dx^3)^p\,dx = \int (bx^3+a)^p(dx^3+c)^p x^{-6\,p-7}\,dx$$

input
```
integrate(x^(-7-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^p*(d*x^3 + c)^p*x^(-6*p - 7), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-3(2+2p)}(a+bx^3)^p\,(c+dx^3)^p\,dx = \text{Timed out}$$

input
```
integrate(x**(-7-6*p)*(b*x**3+a)**p*(d*x**3+c)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-3(2+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\int \left(bx^3+a\right)^p\left(dx^3+c\right)^p x^{-6\,p-7}\,dx$$

input
```
integrate(x^(-7-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^p*(d*x^3 + c)^p*x^(-6*p - 7), x)
```

Giac [F]

$$\int x^{-1-3(2+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\int \left(bx^3+a\right)^p\left(dx^3+c\right)^p x^{-6\,p-7}\,dx$$

input
```
integrate(x^(-7-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^p*(d*x^3 + c)^p*x^(-6*p - 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-3(2+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\int \frac{\left(b\,x^3+a\right)^p\left(d\,x^3+c\right)^p}{x^{6\,p+7}}\,dx$$

input
```
int(((a + b*x^3)^p*(c + d*x^3)^p)/x^(6*p + 7),x)
```

output
```
int(((a + b*x^3)^p*(c + d*x^3)^p)/x^(6*p + 7), x)
```

Reduce [F]

$$\int x^{-1-3(2+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\text{too large to display}$$

input
```
int(x^(-7-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x)
```

output
```
( - 2*(c + d*x**3)**p*(a + b*x**3)**p*a*c*p - (c + d*x**3)**p*(a + b*x**3)
**p*a*c - (c + d*x**3)**p*(a + b*x**3)**p*a*d*p*x**3 - (c + d*x**3)**p*(a
+ b*x**3)**p*b*c*p*x**3 + (c + d*x**3)**p*(a + b*x**3)**p*b*d*x**6 - 6*x**
(6*p)*int(((c + d*x**3)**p*(a + b*x**3)**p)/(2*x**(6*p)*a**2*c*d*p*x + x**
(6*p)*a**2*c*d*x + 2*x**(6*p)*a**2*d**2*p*x**4 + x**(6*p)*a**2*d**2*x**4 +
 2*x**(6*p)*a*b*c**2*p*x + x**(6*p)*a*b*c**2*x + 4*x**(6*p)*a*b*c*d*p*x**4
 + 2*x**(6*p)*a*b*c*d*x**4 + 2*x**(6*p)*a*b*d**2*p*x**7 + x**(6*p)*a*b*d**
2*x**7 + 2*x**(6*p)*b**2*c**2*p*x**4 + x**(6*p)*b**2*c**2*x**4 + 2*x**(6*p
)*b**2*c*d*p*x**7 + x**(6*p)*b**2*c*d*x**7),x)*a**3*d**3*p**3*x**6 - 9*x**
(6*p)*int(((c + d*x**3)**p*(a + b*x**3)**p)/(2*x**(6*p)*a**2*c*d*p*x + x**
(6*p)*a**2*c*d*x + 2*x**(6*p)*a**2*d**2*p*x**4 + x**(6*p)*a**2*d**2*x**4 +
 2*x**(6*p)*a*b*c**2*p*x + x**(6*p)*a*b*c**2*x + 4*x**(6*p)*a*b*c*d*p*x**4
 + 2*x**(6*p)*a*b*c*d*x**4 + 2*x**(6*p)*a*b*d**2*p*x**7 + x**(6*p)*a*b*d**
2*x**7 + 2*x**(6*p)*b**2*c**2*p*x**4 + x**(6*p)*b**2*c**2*x**4 + 2*x**(6*p
)*b**2*c*d*p*x**7 + x**(6*p)*b**2*c*d*x**7),x)*a**3*d**3*p**2*x**6 - 3*x**
(6*p)*int(((c + d*x**3)**p*(a + b*x**3)**p)/(2*x**(6*p)*a**2*c*d*p*x + x**
(6*p)*a**2*c*d*x + 2*x**(6*p)*a**2*d**2*p*x**4 + x**(6*p)*a**2*d**2*x**4 +
 2*x**(6*p)*a*b*c**2*p*x + x**(6*p)*a*b*c**2*x + 4*x**(6*p)*a*b*c*d*p*x**4
 + 2*x**(6*p)*a*b*c*d*x**4 + 2*x**(6*p)*a*b*d**2*p*x**7 + x**(6*p)*a*b*d**
2*x**7 + 2*x**(6*p)*b**2*c**2*p*x**4 + x**(6*p)*b**2*c**2*x**4 + 2*x**(...
```

3.894 $\int x^{-1-3(1+2p)}(a+bx^3)^p\,(c+dx^3)^p\;dx$

Optimal result

Integrand size = 30, antiderivative size = 104

$$\int x^{-1-3(1+2p)}(a+bx^3)^p\,(c+dx^3)^p\;dx = -\frac{x^{-3(1+2p)}(a+bx^3)^{1+p}\,(c+dx^3)^p\left(\frac{a(c+dx^3)}{c(a+bx^3)}\right)^{-p}\text{Hypergeometric2F1}\left(-1-2p,-p,-2p,\frac{(bc-ad)x^3}{c(a+bx^3)}\right)}{3a(1+2p)}$$

output
```
-1/3*(b*x^3+a)^(p+1)*(d*x^3+c)^p*hypergeom([-p, -1-2*p],[-2*p],(-a*d+b*c)*
x^3/c/(b*x^3+a))/a/(1+2*p)/(x^(3+6*p))/((a*(d*x^3+c)/c/(b*x^3+a))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int x^{-1-3(1+2p)}(a+bx^3)^p\,(c+dx^3)^p\;dx = -\frac{x^{-3-6p}(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}(c+dx^3)^{1+p}\left(1+\frac{dx^3}{c}\right)^p\text{Hypergeometric2F1}\left(-1-2p,-p,-2p,\frac{(-bc+a\cdots}{a(c+d\cdots}\right.}{3(c+2cp)}$$

input
```
Integrate[x^(-1 - 3*(1 + 2*p))*(a + b*x^3)^p*(c + d*x^3)^p,x]
```

output
```
-1/3*(x^(-3 - 6*p)*(a + b*x^3)^p*(c + d*x^3)^(1 + p)*(1 + (d*x^3)/c)^p*Hyp
ergeometric2F1[-1 - 2*p, -p, -2*p, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/
((c + 2*c*p)*(1 + (b*x^3)/a)^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3(2p+1)-1}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx$$

$$\Big\downarrow \text{1013}$$

$$\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\int x^{-2(3p+2)}\left(\frac{bx^3}{a}+1\right)^p\left(dx^3+c\right)^p dx$$

$$\Big\downarrow \text{1013}$$

$$\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(c+dx^3\right)^p\left(\frac{dx^3}{c}+1\right)^{-p}\int x^{-2(3p+2)}\left(\frac{bx^3}{a}+1\right)^p\left(\frac{dx^3}{c}+1\right)^p dx$$

$$\Big\downarrow \text{1012}$$

$$-\frac{x^{-3(2p+1)}\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(c+dx^3\right)^p\left(\frac{dx^3}{c}+1\right)^{p+1}\text{Hypergeometric2F1}\left(-2p-1,-p,-2p,-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{dx^3+c}\right.}{3(2p+1)}$$

input
```
Int[x^(-1 - 3*(1 + 2*p))*(a + b*x^3)^p*(c + d*x^3)^p,x]
```

output
```
-1/3*((a + b*x^3)^p*(c + d*x^3)^p*(1 + (d*x^3)/c)^(1 + p)*Hypergeometric2F
1[-1 - 2*p, -p, -2*p, -((c*((b*x^3)/a - (d*x^3)/c))/(c + d*x^3))])/((1 + 2
*p)*x^(3*(1 + 2*p))*(1 + (b*x^3)/a)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int x^{-4-6p} \left(b\,x^{3}+a\right)^{p} \left(d\,x^{3}+c\right)^{p} dx$$

input
```
int(x^(-4-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x)
```

output
```
int(x^(-4-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x)
```

Fricas [F]

$$\int x^{-1-3(1+2p)} \left(a+bx^3\right)^p \left(c+dx^3\right)^p \,dx = \int \left(bx^3+a\right)^p \left(dx^3+c\right)^p x^{-6\,p-4}\,dx$$

input
```
integrate(x^(-4-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^p*(d*x^3 + c)^p*x^(-6*p - 4), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-3(1+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**(-4-6*p)*(b*x**3+a)**p*(d*x**3+c)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-3(1+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\int\left(bx^3+a\right)^p\left(dx^3+c\right)^px^{-6p-4}\,dx$$

input
```
integrate(x^(-4-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^p*(d*x^3 + c)^p*x^(-6*p - 4), x)
```

Giac [F]

$$\int x^{-1-3(1+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\int\left(bx^3+a\right)^p\left(dx^3+c\right)^px^{-6p-4}\,dx$$

input
```
integrate(x^(-4-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^p*(d*x^3 + c)^p*x^(-6*p - 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-3(1+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\int\frac{\left(b\,x^3+a\right)^p\,\left(d\,x^3+c\right)^p}{x^{6\,p+4}}\,dx$$

input
```
int(((a + b*x^3)^p*(c + d*x^3)^p)/x^(6*p + 4),x)
```

output
```
int(((a + b*x^3)^p*(c + d*x^3)^p)/x^(6*p + 4), x)
```

Reduce [F]

$$\int x^{-1-3(1+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\text{too large to display}$$

input
```
int(x^(-4-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x)
```

output
```
( - (c + d*x**3)**p*(a + b*x**3)**p*a*d - (c + d*x**3)**p*(a + b*x**3)**p*
b*c - 2*(c + d*x**3)**p*(a + b*x**3)**p*b*d*x**3 + 6*x**(6*p)*int(((c + d*
x**3)**p*(a + b*x**3)**p)/(2*x**(6*p)*a**2*c*d*p*x + x**(6*p)*a**2*c*d*x +
 2*x**(6*p)*a**2*d**2*p*x**4 + x**(6*p)*a**2*d**2*x**4 + 2*x**(6*p)*a*b*c*
*2*p*x + x**(6*p)*a*b*c**2*x + 4*x**(6*p)*a*b*c*d*p*x**4 + 2*x**(6*p)*a*b*
c*d*x**4 + 2*x**(6*p)*a*b*d**2*p*x**7 + x**(6*p)*a*b*d**2*x**7 + 2*x**(6*p
)*b**2*c**2*p*x**4 + x**(6*p)*b**2*c**2*x**4 + 2*x**(6*p)*b**2*c*d*p*x**7
+ x**(6*p)*b**2*c*d*x**7),x)*a**3*d**3*p**2*x**3 + 3*x**(6*p)*int(((c + d*
x**3)**p*(a + b*x**3)**p)/(2*x**(6*p)*a**2*c*d*p*x + x**(6*p)*a**2*c*d*x +
 2*x**(6*p)*a**2*d**2*p*x**4 + x**(6*p)*a**2*d**2*x**4 + 2*x**(6*p)*a*b*c*
*2*p*x + x**(6*p)*a*b*c**2*x + 4*x**(6*p)*a*b*c*d*p*x**4 + 2*x**(6*p)*a*b*
c*d*x**4 + 2*x**(6*p)*a*b*d**2*p*x**7 + x**(6*p)*a*b*d**2*x**7 + 2*x**(6*p
)*b**2*c**2*p*x**4 + x**(6*p)*b**2*c**2*x**4 + 2*x**(6*p)*b**2*c*d*p*x**7
+ x**(6*p)*b**2*c*d*x**7),x)*a**3*d**3*p*x**3 - 6*x**(6*p)*int(((c + d*x**
3)**p*(a + b*x**3)**p)/(2*x**(6*p)*a**2*c*d*p*x + x**(6*p)*a**2*c*d*x + 2*
x**(6*p)*a**2*d**2*p*x**4 + x**(6*p)*a**2*d**2*x**4 + 2*x**(6*p)*a*b*c**2*
p*x + x**(6*p)*a*b*c**2*x + 4*x**(6*p)*a*b*c*d*p*x**4 + 2*x**(6*p)*a*b*c*d
*x**4 + 2*x**(6*p)*a*b*d**2*p*x**7 + x**(6*p)*a*b*d**2*x**7 + 2*x**(6*p)*b
**2*c**2*p*x**4 + x**(6*p)*b**2*c**2*x**4 + 2*x**(6*p)*b**2*c*d*p*x**7 + x
**(6*p)*b**2*c*d*x**7),x)*a**2*b*c*d**2*p**2*x**3 - 3*x**(6*p)*int(((c ...
```

3.895 $\int x^{-1-6p}(a+bx^3)^p (c+dx^3)^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 91

$$\int x^{-1-6p}(a+bx^3)^p (c+dx^3)^p \, dx = -\frac{x^{-6p}(a+bx^3)^p \left(1+\frac{bx^3}{a}\right)^{-p} (c+dx^3)^p \left(1+\frac{dx^3}{c}\right)^{-p} \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{6p}$$

output
```
-1/6*(b*x^3+a)^p*(d*x^3+c)^p*AppellF1(-2*p,-p,-p,1-2*p,-b*x^3/a,-d*x^3/c)/
p/(x^(6*p))/((1+b*x^3/a)^p)/((1+d*x^3/c)^p)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02

$$\int x^{-1-6p}(a+bx^3)^p (c+dx^3)^p \, dx = -\frac{x^{-6p}(a+bx^3)^p \left(\frac{a+bx^3}{a}\right)^{-p} (c+dx^3)^p \left(\frac{c+dx^3}{c}\right)^{-p} \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{6p}$$

input
```
Integrate[x^(-1 - 6*p)*(a + b*x^3)^p*(c + d*x^3)^p,x]
```

output
```
-1/6*((a + b*x^3)^p*(c + d*x^3)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -((b*x^3
)/a), -((d*x^3)/c)])/(p*x^(6*p)*((a + b*x^3)/a)^p*((c + d*x^3)/c)^p)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-6p-1} \left(a+bx^3\right)^p \left(c+dx^3\right)^p \, dx$$

$$\Big\downarrow \text{ 1013}$$

$$\left(a+bx^3\right)^p \left(\frac{bx^3}{a}+1\right)^{-p} \int x^{-6p-1} \left(\frac{bx^3}{a}+1\right)^p \left(dx^3+c\right)^p dx$$

$$\Big\downarrow \text{ 1013}$$

$$\left(a+bx^3\right)^p \left(\frac{bx^3}{a}+1\right)^{-p} \left(c+dx^3\right)^p \left(\frac{dx^3}{c}+1\right)^{-p} \int x^{-6p-1} \left(\frac{bx^3}{a}+1\right)^p \left(\frac{dx^3}{c}+1\right)^p dx$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{x^{-6p} \left(a+bx^3\right)^p \left(\frac{bx^3}{a}+1\right)^{-p} \left(c+dx^3\right)^p \left(\frac{dx^3}{c}+1\right)^{-p} \text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{6p}$$

input
```
Int[x^(-1 - 6*p)*(a + b*x^3)^p*(c + d*x^3)^p,x]
```

output
```
-1/6*((a + b*x^3)^p*(c + d*x^3)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -((b*x^3
)/a), -((d*x^3)/c)])/(p*x^(6*p)*(1 + (b*x^3)/a)^p*(1 + (d*x^3)/c)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int x^{-1-6p} \left(b\,x^{3}+a\right)^{p} \left(d\,x^{3}+c\right)^{p} dx$$

input
```
int(x^(-1-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x)
```

output
```
int(x^(-1-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x)
```

Fricas [F]

$$\int x^{-1-6p} \left(a+bx^3\right)^p \left(c+dx^3\right)^p \, dx = \int \left(bx^3+a\right)^p \left(dx^3+c\right)^p x^{-6\,p-1}\, dx$$

input
```
integrate(x^(-1-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^p*(d*x^3 + c)^p*x^(-6*p - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-6p}(a+bx^3)^p\,(c+dx^3)^p\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-6*p)*(b*x**3+a)**p*(d*x**3+c)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-6p}(a+bx^3)^p\,(c+dx^3)^p\,dx = \int (bx^3+a)^p(dx^3+c)^p x^{-6\,p-1}\,dx$$

input
```
integrate(x^(-1-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^p*(d*x^3 + c)^p*x^(-6*p - 1), x)
```

Giac [F]

$$\int x^{-1-6p}(a+bx^3)^p\,(c+dx^3)^p\,dx = \int (bx^3+a)^p(dx^3+c)^p x^{-6\,p-1}\,dx$$

input
```
integrate(x^(-1-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^p*(d*x^3 + c)^p*x^(-6*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-6p}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\int\frac{\left(b\,x^3+a\right)^p\,\left(d\,x^3+c\right)^p}{x^{6\,p+1}}\,dx$$

input
```
int(((a + b*x^3)^p*(c + d*x^3)^p)/x^(6*p + 1),x)
```

output
```
int(((a + b*x^3)^p*(c + d*x^3)^p)/x^(6*p + 1), x)
```

Reduce [F]

$$\int x^{-1-6p}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\int\frac{\left(d\,x^3+c\right)^p\left(b\,x^3+a\right)^p}{x^{6p}x}dx$$

input
```
int(x^(-1-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x)
```

output
```
int(((c + d*x**3)**p*(a + b*x**3)**p)/(x**(6*p)*x),x)
```

3.896 $\int x^{-1-3(-1+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx$

Optimal result

Integrand size = 30, antiderivative size = 103

$$\int x^{-1-3(-1+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx = \frac{x^{3(1-2p)}\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(c+dx^3\right)^p\left(1+\frac{dx^3}{c}\right)^{-p}\operatorname{AppellF1}\left(1-2p,-p,-p,2(1-p),-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{3(1-2p)}$$

output
```
1/3*x^(3-6*p)*(b*x^3+a)^p*(d*x^3+c)^p*AppellF1(1-2*p,-p,-p,2-2*p,-b*x^3/a,
-d*x^3/c)/(1-2*p)/((1+b*x^3/a)^p)/((1+d*x^3/c)^p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95

$$\int x^{-1-3(-1+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx = \frac{x^{3-6p}\left(a+bx^3\right)^p\left(\frac{a+bx^3}{a}\right)^{-p}\left(c+dx^3\right)^p\left(\frac{c+dx^3}{c}\right)^{-p}\operatorname{AppellF1}\left(1-2p,-p,-p,2-2p,-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{3-6p}$$

input
```
Integrate[x^(-1 - 3*(-1 + 2*p))*(a + b*x^3)^p*(c + d*x^3)^p,x]
```

output
```
(x^(3 - 6*p)*(a + b*x^3)^p*(c + d*x^3)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p
, -((b*x^3)/a), -((d*x^3)/c)])/((3 - 6*p)*((a + b*x^3)/a)^p*((c + d*x^3)/c
)^p)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3(2p-1)-1}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx$$

$$\Big\downarrow \text{1013}$$

$$\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\int x^{2(1-3p)}\left(\frac{bx^3}{a}+1\right)^p\left(dx^3+c\right)^p\,dx$$

$$\Big\downarrow \text{1013}$$

$$\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(c+dx^3\right)^p\left(\frac{dx^3}{c}+1\right)^{-p}\int x^{2(1-3p)}\left(\frac{bx^3}{a}+1\right)^p\left(\frac{dx^3}{c}+1\right)^p\,dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^{3(1-2p)}\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(c+dx^3\right)^p\left(\frac{dx^3}{c}+1\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2(1-p),-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{3(1-2p)}$$

input
```
Int[x^(-1 - 3*(-1 + 2*p))*(a + b*x^3)^p*(c + d*x^3)^p,x]
```

output
```
(x^(3*(1 - 2*p))*(a + b*x^3)^p*(c + d*x^3)^p*AppellF1[1 - 2*p, -p, -p, 2*(
1 - p), -((b*x^3)/a), -((d*x^3)/c)])/(3*(1 - 2*p)*(1 + (b*x^3)/a)^p*(1 + (
d*x^3)/c)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int x^{2-6p} (b\,x^3+a)^p \,(d\,x^3+c)^p\,dx$$

input
```
int(x^(2-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x)
```

output
```
int(x^(2-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x)
```

Fricas [F]

$$\int x^{-1-3(-1+2p)}(a+bx^3)^p\,(c+dx^3)^p\,dx=\int (bx^3+a)^p(dx^3+c)^p x^{-6\,p+2}\,dx$$

input
```
integrate(x^(2-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^p*(d*x^3 + c)^p*x^(-6*p + 2), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-3(-1+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**(2-6*p)*(b*x**3+a)**p*(d*x**3+c)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-3(-1+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\int\left(bx^3+a\right)^p\left(dx^3+c\right)^px^{-6\,p+2}\,dx$$

input
```
integrate(x^(2-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^p*(d*x^3 + c)^p*x^(-6*p + 2), x)
```

Giac [F]

$$\int x^{-1-3(-1+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\int\left(bx^3+a\right)^p\left(dx^3+c\right)^px^{-6\,p+2}\,dx$$

input
```
integrate(x^(2-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^p*(d*x^3 + c)^p*x^(-6*p + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-3(-1+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\int x^{2-6\,p}\,\left(b\,x^3+a\right)^p\,\left(d\,x^3+c\right)^p\,dx$$

input
```
int(x^(2 - 6*p)*(a + b*x^3)^p*(c + d*x^3)^p,x)
```

output
```
int(x^(2 - 6*p)*(a + b*x^3)^p*(c + d*x^3)^p, x)
```

Reduce [F]

$$\int x^{-1-3(-1+2p)}\left(a+bx^3\right)^p\left(c+dx^3\right)^p\,dx=\text{Too large to display}$$

input
```
int(x^(2-6*p)*(b*x^3+a)^p*(d*x^3+c)^p,x)
```

output
```
( - 2*(c + d*x**3)**p*(a + b*x**3)**p*a*c + (c + d*x**3)**p*(a + b*x**3)**
p*a*d*x**3 + (c + d*x**3)**p*(a + b*x**3)**p*b*c*x**3 + 3*x**(6*p)*int(((c
 + d*x**3)**p*(a + b*x**3)**p*x**5)/(x**(6*p)*a**2*c*d + x**(6*p)*a**2*d**
2*x**3 + x**(6*p)*a*b*c**2 + 2*x**(6*p)*a*b*c*d*x**3 + x**(6*p)*a*b*d**2*x
**6 + x**(6*p)*b**2*c**2*x**3 + x**(6*p)*b**2*c*d*x**6),x)*a**3*d**3*p + 9
*x**(6*p)*int(((c + d*x**3)**p*(a + b*x**3)**p*x**5)/(x**(6*p)*a**2*c*d +
x**(6*p)*a**2*d**2*x**3 + x**(6*p)*a*b*c**2 + 2*x**(6*p)*a*b*c*d*x**3 + x*
*(6*p)*a*b*d**2*x**6 + x**(6*p)*b**2*c**2*x**3 + x**(6*p)*b**2*c*d*x**6),x
)*a**2*b*c*d**2*p + 9*x**(6*p)*int(((c + d*x**3)**p*(a + b*x**3)**p*x**5)/
(x**(6*p)*a**2*c*d + x**(6*p)*a**2*d**2*x**3 + x**(6*p)*a*b*c**2 + 2*x**(6
*p)*a*b*c*d*x**3 + x**(6*p)*a*b*d**2*x**6 + x**(6*p)*b**2*c**2*x**3 + x**(
6*p)*b**2*c*d*x**6),x)*a*b**2*c**2*d*p + 3*x**(6*p)*int(((c + d*x**3)**p*(
a + b*x**3)**p*x**5)/(x**(6*p)*a**2*c*d + x**(6*p)*a**2*d**2*x**3 + x**(6*
p)*a*b*c**2 + 2*x**(6*p)*a*b*c*d*x**3 + x**(6*p)*a*b*d**2*x**6 + x**(6*p)*
b**2*c**2*x**3 + x**(6*p)*b**2*c*d*x**6),x)*b**3*c**3*p - 12*x**(6*p)*int(
((c + d*x**3)**p*(a + b*x**3)**p)/(x**(6*p)*a**2*c*d*x + x**(6*p)*a**2*d**
2*x**4 + x**(6*p)*a*b*c**2*x + 2*x**(6*p)*a*b*c*d*x**4 + x**(6*p)*a*b*d**2
*x**7 + x**(6*p)*b**2*c**2*x**4 + x**(6*p)*b**2*c*d*x**7),x)*a**3*c**2*d*p
 - 12*x**(6*p)*int(((c + d*x**3)**p*(a + b*x**3)**p)/(x**(6*p)*a**2*c*d*x
+ x**(6*p)*a**2*d**2*x**4 + x**(6*p)*a*b*c**2*x + 2*x**(6*p)*a*b*c*d*x*...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:    #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file